**More notes on BCNF/3NF/4NF/MVDs**

**Boyce-Codd Normal Form (BCNF)**

* A relation R is in BCNF iff whenever there is a nontrivial FD A1…An->B1…Bm for R, {A1, …, An} is a superkey for R.
* Informally, the left side of every nontrivial FD must be a superkey.

**Checking for BCNF violations**

* List all nontrivial FDs in R.
* Ensure left side of each nontrivial FD is a superkey.
* (First have to find all the keys!)  
    
  Note: a relation with two attributes is always in BCNF.

**BCNF Decomposition**

Given relation R and set of FDs F:

* Check if R is in BCNF, if not, do:
* If there are FDs that violate BCNF, call one   
  X -> Y. Compute X+. Let R1 = X+ and R2 = X and all other attributes not in X+.
* Compute FDs for R1 and R2 (projection algorithm for FDs).
* Check if R1 and R2 are in BCNF, and repeat if needed.

**3rd Normal Form (3NF)**

* A relation R is in 3NF iff for every nontrivial FD A1…An -> B for R, one of the following is true:
  + A1…An is a superkey for R (BCNF test)
  + Each B is a ***prime*** attribute (an attribute in *some* key for R)

**3NF Decomposition**

Given a relation R and set F of functional dependencies:

1. Find a minimal basis, G, for F.
2. For each FD X -> A in G, use XA as the schema of one of the relations in the decomposition.
3. If none of the sets of schemas from Step 2 is a superkey for R, add another relation whose schema is a key for R.

**Multivalued dependencies**

* A ***MVD*** is a constraint that two sets of attributes are ***independent*** of each other.
* A MVD A1…An ->-> B1…Bm holds in R if in every instance of R:
  + for every pair of tuples t and u that agree on all the As, we can find a tuple v in R that agrees
    - with both t and u on the As
    - with t on the Bs
    - with u on all those attributes of R that are not As or Bs
* In other words, the information in A1..An determines the values of the set of tuples for B1..Bm ***and*** those tuples are independent of any other attributes in the relation.

**Rules for MVDs**

* **FD promotion:** Every FD A🡪B is an MVD A🡪🡪B
* **Trivial MVDs:**

1. If A🡪🡪B, then A🡪🡪AB
2. If A1, A2…, An and B1, B2, …, Bm make up *all* the attributes of a relation, then   
   A1, A2, …An 🡪🡪 B1, B2, …Bm holds in the relation

* **Transitive rule:** Given A🡪🡪B and B🡪🡪C, we can infer A🡪🡪C.
* **Complementation rule:** if we know A🡪🡪B, then we know A🡪🡪C, where all the Cs are attributes not among the As or Bs.
* Note that the **splitting rule does not hold!** If A🡪🡪BC, then it is not true that A🡪🡪B and A🡪🡪C.

**4th Normal Form (4NF)**

* "Stronger" than BCNF.
* A relation R is in 4NF iff:
  + for all MVDs A1…An ->-> B1…Bm,   
     {A1, …, An} is a superkey of R.

**4NF Decomposition**

* Consider relation R with set of attributes X
* A1 A2 … An 🡪🡪 B1 B2 … Bm violates 4NF
* Decompose R into two relations whose attributes are:

1. The As and Bs together, i.e., {A1 A2 … An, B1, B2, …, Bm}
2. All the attributes of R which are not Bs, i.e. X – {B1, B2 …, Bm}
3. Recursively check if the new relations are in 4NF and repeat