**Multivalued dependencies and 4th Normal Form**

* Multivalued dependencies
  + A ***MVD*** is a constraint that two sets of attributes are ***independent*** of each other.
  + A MVD A1…An ->-> B1…Bm holds in R if in every instance of R:
    - for every pair of tuples t and u that agree on all the As, we can find a tuple v in R that agrees
      * with both t and u on the As
      * with t on the Bs
      * with u on all those attributes of R that are not As or Bs
  + In other words, the information in A1..An determines the values of the set of tuples for B1..Bm ***and*** those tuples are independent of any other attributes in the relation.
* Details
  + Consider a MVD A1…An ->-> B1…Bm.
  + Call attributes not in A’s or B’s the Cs.
  + This MVD holds in R if:
    - whenever we have two tuples of R that agree on the A’s but differ on the B’s and C’s we should be able to find (or create) two new tuples with the same A’s but swapped B’s and C’s.
  + Equivalently:
    - If knowing A1…An determines a unique set of tuples for B1..Bm that is independent of the C’s.
* A FD A -> B says "Each A determines a unique B"
  + or, "Each A determines 0 or 1 Bs."
* A MVD A ->-> B says "Each A determines a set of Bs ***where the Bs are independent of anything in the relation that is not an A or a B.***"
* Rules
  + **FD promotion:** Every FD A🡪B is an MD A🡪🡪B
  + **Trivial MDs:** 
    - If A🡪🡪B, then A🡪🡪AB
    - If A1, A2…, An and B1, B2, …, Bm make up *all* the attributes of a relation, then   
      A1, A2, …An 🡪🡪 B1, B2, …Bm holds in the relation
  + **Transitive rule:** Given A🡪🡪B and B🡪🡪C, we can infer A🡪🡪C.
  + **Complementation rule:** if we know A🡪🡪B, then we know A🡪🡪C, where all the Cs are attributes not among the As or Bs.
* 4th normal form
  + "Stronger" than BCNF.
  + A relation R is in 4NF iff:
  + for all MVDs A1…An ->-> B1…Bm,   
     {A1, …, An} is a superkey of R.
* 4NF decomposition
  + Consider relation R with set of attributes X
  + A1 A2 … An 🡪🡪 B1 B2 … Bm violates 4NF
  + Decompose R into two relations whose attributes are:
    - The As and Bs together, i.e., {A1 A2 … An, B1, B2, …, Bm}
    - All the attributes of R which are not Bs, i.e. X – {B1, B2 …, Bm}
    - Recursively check if the new relations are in 4NF and repeat