Discrete Structures, Fall 2017, Homework 2 Solutions

You must write the solutions to these problems legibly on your own paper, with the problems in sequential order, and with all sheets stapled together.

1. Construct a complete truth table to help you determine if the following argument is valid or not. State whether it is valid or not, indicate the critical rows in the truth table, and explain why those rows support your answer.

   Premise: $\sim q \vee (p \wedge \sim r)$
   Premise: $\sim r \to p$
   Conclusion: $q \to r$

   Solution:

p	q	r	$\sim q$	$\sim r$	$p \wedge \sim r$	Premise $\sim q \vee (p \wedge \sim r)$	Premise $\sim r \to p$	Conclusion $q \to r$	
T	T	T	F	F	F	F	T	T	
T	T	F	F	T	T	T	T	F	$\leftarrow$ critical row
T	F	T	T	F	F	T	T	T	$\leftarrow$ critical row
T	F	F	T	T	T	T	T	T	$\leftarrow$ critical row
F	T	T	F	F	F	F	T	T	
F	T	F	F	T	F	F	F	F	
F	F	T	T	F	F	T	T	T	$\leftarrow$ critical row
F	F	F	T	T	F	T	F	T	

   The critical rows are the rows of the truth table where both premises are true. Because the conclusion is not true in *all* of the critical rows, the argument is invalid. (The conclusion is false in the first labeled critical row.)

2. For each of the following, if a *single*, valid rule of inference can lead from the given premises to the given conclusion, state what rule of inference would be used. If no valid rule could be used, write "no rule."

   (a) Premise: $(m \to p) \vee (n \to p)$
   Premise: n
   Conclusion: p

   Solution: No rule. (Not a valid argument anyway.)

   (b) Premise: $(a \wedge b) \to (z \vee y)$
   Premise: $\sim(z \vee y)$
   Conclusion: $\sim(a \wedge b)$

   Solution: Modus tollens.

(c) Premise: $r \wedge (q \vee p)$
Premise: $\sim q$
Conclusion: p

Solution: No rule. (It's a valid argument, but you would need to use conjunctive simplification first, then disjunctive syllogism.)

(d) Premise: $k \wedge m$
Conclusion: $(k \wedge m) \vee (k \rightarrow (n \rightarrow m))$

Solution: Disjunctive addition.

(e) Premise: $r \wedge (q \vee p)$
Premise: $\sim q$
Conclusion: $r \wedge p$

Solution: No rule. (It's a valid argument, but just like part (c), you'd need to use conjunctive simplification, then disjunctive syllogism, then conjunctive addition.)

(f) Premise: $g \rightarrow h$
Premise: $(e \wedge k) \vee g$
Premise: $(e \wedge k) \rightarrow h$
Conclusion: h

Solution: Dilemma (or proof by division into cases).

3. Complete the following proofs using the framework discussed in class. Each line of your proof must be justified with a rule of inference or logical equivalence and appropriate line numbers.

(a) P1 $\sim p$
P2 $(q \wedge \sim p) \rightarrow m$
P3 $\sim r \vee q$
P4 r

Prove: m

Solution:

Line	Statement	Rule	Lines Used
1	q	Disjunctive syllogism	P3, P4
2	$q \wedge \sim p$	Conjunctive addition	1, P1
3	m	Modus ponens	P2, 2

(b)

P1	$\sim k \wedge g$
P2	$m \to \sim g$
P3	$(\sim f \to k) \vee m$
Prove:	f

Solution:

Line	Statement	Rule	Lines Used
1	$\sim k$	Conjunctive simplification	P1
2	g	Conjunctive simplification	P1
3	$\sim m$	Modus tollens	2, P2
4	$\sim f \to k$	Disjunctive syllogism	3, P3
5	f	Modus tollens	1, 4