Discrete Structures, Fall 2017, Homework 3 Solutions

You must write the solutions to these problems legibly on your own paper, with the problems in sequential order, and with all sheets stapled together.

1. Complete the following proofs using the framework discussed in class. Each line of your proof must be justified with a rule of inference or logical equivalence and appropriate line numbers.

(a)

P1	$s \wedge e$
P2	$e \to b$
P3	$(b \wedge {\sim}m) \to {\sim}s$
Prove:	m

Solution:

Line	Statement	Rule	Lines Used
1	s	Conjunctive simplification	P1
2	e	Conjunctive simplification	P1
3	b	Modus ponens	2, P2
4	${\sim}(b \wedge {\sim}m)$	Modus tollens	1, P3
5	${\sim}b \vee m$	DeMorgan's law	4
6	m	Disjunctive syllogism	3, 5

(b)

P1	$(p \to q) \wedge (r \to s)$
P2	y
P3	$(s \wedge q) \to {\sim}y$
Prove:	${\sim}p \vee {\sim}r$

Solution A:

Line	Statement	Rule	Lines Used
1	$p \to q$	Conjunctive simplification	P1
2	$r \to s$	Conjunctive simplification	P2
3	${\sim}(s \wedge q)$	Modus tollens	P2, P3
4	${\sim}s \vee {\sim}q$	DeMorgan's law	3
5	$q \to {\sim}s$	Definition of implication	4
6	$p \to {\sim}s$	Hypothetical syllogism	1, 5
7	${\sim}s \to {\sim}r$	Contrapositive	2
8	$p \to {\sim}r$	Hypothetical syllogism	6, 7
9	${\sim}p \vee {\sim}r$	Definition of implication	8

Solution B:

Line	Statement	Rule	Lines Used
1	$p \to q$	Conjunctive simplification	P1
2	$r \to s$	Conjunctive simplification	P2
3	$\sim(s \wedge q)$	Modus tollens	P2, P3
4	$\quad p \wedge r$	Assume	—
5	$\quad p$	Conjunctive simplification	4
6	$\quad r$	Conjunctive simplification	4
7	$\quad q$	Modus ponens	1, 5
8	$\quad s$	Modus ponens	2, 6
9	$\quad s \wedge q$	Conjunctive addition	7, 8
10	$\quad (s \wedge q) \wedge \sim(s \wedge q)$	Conjunctive addition	9, 3
11	$\sim(p \wedge r)$	Closing cond world with contra	4–10
12	$\sim p \vee \sim r$	DeMorgan's law	11

Solution C:

Line	Statement	Rule	Lines Used
1	$p \to q$	Conjunctive simplification	P1
2	$r \to s$	Conjunctive simplification	P2
3	$\sim(s \wedge q)$	Modus tollens	P2, P3
4	$\quad p$	Assume	—
5	$\quad q$	Modus ponens	1, 4
6	$\quad \sim s \vee \sim q$	DeMorgan's law	3
7	$\quad \sim s$	Disjunctive syllogism	5, 6
8	$\quad \sim r$	Modus tollens	2, 7
9	$p \to \sim r$	Closing cond world without contra	4–9
10	$\sim p \vee \sim r$	Definition of implication	9

(c) P1 $p \to q$
P2 $\sim q \vee r$
P3 $s \vee (y \wedge \sim r)$

Prove: $\sim s \to \sim(p \vee \sim y)$

Solution:

Line	Statement	Rule	Lines Used
1	$\quad \sim s$	Assume	—
2	$\quad y \wedge \sim r$	Disjunctive syllogism	P3, 1
3	$\quad y$	Conjunctive simplification	2
4	$\quad \sim r$	Conjunctive simplification	2
5	$\quad \sim q$	Disjunctive syllogism	P2, 4
6	$\quad \sim p$	Modus tollens	P1, 5
7	$\quad \sim p \wedge y$	Conjunctive addition	6, 3
8	$\quad \sim(p \vee \sim y)$	DeMorgan's law	7
9	$\sim s \to \sim(p \vee \sim y)$	Closing cond world without contra	1–8

(d)

P1	$a \wedge \sim d$
P2	$b \to (e \to d)$
Prove:	$(a \to b) \to \sim e$

Solution A:

Line	Statement	Rule	Lines Used
1	$a \to b$	Assume	—
2	a	Conjunctive simplification	P1
3	$\sim d$	Conjunctive simplification	P1
4	b	Modus ponens	1, 2
5	$e \to d$	Modus ponens	P2, 4
6	$\sim e$	Modus tollens	3, 5
7	$(a \to b) \to \sim e$	Closing cond world without contra	1–5

Solution B:

Line	Statement	Rule	Lines Used
2	a	Conjunctive simplification	P1
3	$\sim d$	Conjunctive simplification	P1
4	$\sim b \vee (e \to d)$	DeMorgan's law	P2
5	$\sim b \vee (\sim e \vee d)$	DeMorgan's law	4
6	$(\sim b \vee \sim e) \vee d$	Associative law	5
7	$\sim b \vee \sim e$	Disjunctive syllogism	3, 6
8	$a \vee \sim e$	Disjunctive addition	2
9	$(a \vee \sim e) \wedge (\sim b \vee \sim e)$	Conjunctive addition	7, 8
10	$(a \wedge \sim b) \vee \sim e$	Distributive law	9
11	$\sim(a \wedge \sim b) \to \sim e$	Definition of implication	10
12	$(\sim a \vee b) \to \sim e$	DeMorgan's law	11
13	$(a \to b) \to \sim e$	Definition of implication	12

2. Translate each of the following English sentences into formal language – that is, using the symbols $\forall, \exists, \in$, etc. Use the following predicates:

   $B(s)$ means "s is an business major,"
   $C(s)$ means "s is a computer science major," and
   $M(s)$ means "s is a math major."

   Use the domain S = the set of all students at Rhodes College.

   (a) There is an business major who is also a math major.
   Solution: $\exists x \in S\ B(x) \wedge M(x)$

   (b) Every computer science major is also an business major.
   Solution: $\forall x \in S\ C(x) \to B(x)$

   (c) No computer science majors also major in business.
   Solution: $\forall x \in S\ C(x) \to \sim B(x) \equiv \forall x \in S\ \sim C(x) \vee \sim B(x)$

(d) Some people majoring in CS are also majoring in math.

Solution: $\exists x \in S\ C(x) \wedge M(x)$

(e) Some computer science majors are business majors as well, but some are not. (Think carefully; this is tricky.)

Solution: $\exists x \in S\ \exists y \in S\ C(x) \wedge C(y) \wedge B(x) \wedge \sim B(y)$