Discrete Structures, Fall 2017, Homework 1 Solutions

You must write the solutions to these problems legibly on your own paper, with the problems in sequential order, and with all sheets stapled together.

1. Convert the following sentences to logical statements using symbols assuming that "p", "b" and "m" represent the propositions below.

   $p =$ "Morgan is taking a physics class."
   $b =$ "Morgan is taking a biology class."
   $m =$ "Morgan is taking a math class."

   (a) Morgan is taking a biology class and a math class, but not a physics class.
   Answer: $b \wedge m \wedge \sim p$.

   (b) Morgan is taking a physics class, and either a biology or math class (but not both bio and math).
   Answer: $p \wedge (b \vee m) \wedge (\sim b \vee \sim m)$
   OR
   Answer: $p \wedge [(b \wedge \sim m) \vee (\sim b \wedge m)]$

   (c) Morgan is taking a physics class, and either a biology or math class (perhaps both bio and math).
   Answer: $p \wedge (b \vee m)$.

   (d) If Morgan is taking a biology class, then they are also taking a math class, but if Morgan is not taking a biology class, then they are taking a physics class.
   Answer: $(b \to m) \wedge (\sim b \to p)$

2. For each of the sentences below, determine if the sentence is a statement. If the sentence is a statement, tell whether it is true or false.

   (a) If a tree falls in the forest and no one is around to hear it, does it make a sound?
   Answer: Not a statement; it's a question.

   (b) If $2 + 2 = 5$, then I am the very model of a modern major-general.
   Answer: True statement (left side of this implication is clearly false, which means the whole statement is true).

   (c) This statement refers to itself.
   Answer: True statement.

   (d) This statement is false.
   Answer: Not a statement (it's a paradox).

3. Express the negations of the following statements in normal English sentences.

   (a) Sally is a computer science major and Sally's brother is a math major.
   Answer: Sally is not a CS major or Sally's brother is not a math major.

(b) Either the professor is late or my watch is fast.
Answer: The professor is not late and my watch is not fast.
Answer: OR The professor is on time (or early) and my watch is accurate (or slow).

4. For each of the following statements, give the contrapositive, converse, and inverse statements (label them) in normal English, using the syntax "If ..., then ..." You may change verb tenses to improve the grammar.

(a) "If you conquer yourself, then you conquer the world."[1]

(b) I will be able to retire if I save enough money.

(c) You can go to the party only if you get good grades.

Solution:

(a) Original statement: If you conquer yourself, then you conquer the world.
Contrapositive: If you don't conquer the world, then you didn't conquer yourself.
Converse: If you conquer the world, then you conquer yourself.
Inverse: If you don't conquer yourself, you won't conquer the world.

(b) Original statement: I will be able to retire if I save enough money..
Equivalent original statement, rephrased: If I save enough money, then I will be able to retire.
Contrapositive: If I'm not able to retire, then I didn't save enough money.
Converse: If I'm able to retire, then I saved enough money.
Inverse: If I don't save enough money, then I won't be able to retire.

(c) Original statement: You can go to the party only if you get good grades..
Equivalent original statement, rephrased: If you don't get good grades, then you can't go to the party.
Contrapositive: If you go to the party, then you must have gotten good grades.
Converse: If you don't go to the party, then you didn't get good grades.
Inverse: If you get good grades, then you can go to the party.

Note: The statement "p only if q" means "if not q then not p," as well as "if p then q" (see Epp page 44); clearly $\sim q \to \sim p \equiv p \to q$. Therefore, it's OK if you switched the original statement and the contrapositive above (because those are logically equivalent to each other) as well as the converse and inverse (the converse and inverse are also always logically equivalent to each other).

5. Let x, y and z be statements. Construct a complete truth table for the statement $(x \to \sim y) \wedge (\sim x \vee z)$.

[1] From *Aleph*, by Paulo Coelho.

x	y	z	$\sim y$	$x \to \sim y$	$\sim x$	$\sim x \vee z$	$(x \to \sim y) \wedge (\sim x \vee z)$
T	T	T	F	F	F	T	F
T	T	F	F	F	F	F	F
T	F	T	T	T	F	T	T
T	F	F	T	T	F	F	F
F	T	T	F	T	T	T	T
F	T	F	F	T	T	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

6. Are the statements $\sim(p \vee q)$ and $\sim p \vee \sim q$ logically equivalent? Use a complete truth table to justify your answer, and explain (in English) why the truth table supports your answer.

   Answer: No, the statements are not equivalent.

p	q	$p \vee q$	$\sim(p \vee q)$	$\sim p$	$\sim q$	$\sim p \vee \sim q$
T	T	T	F	F	F	F
T	F	T	F	F	T	T
F	T	T	F	T	F	T
F	F	F	T	T	T	T

The truth table supports the fact that the two statements are not equivalent because the columns for $\sim(p \vee q)$ and $\sim p \vee \sim q$ are different (they disagree on true/false for the two middle rows).