1. easy-med

P1	$p \vee q$
P2	$\sim(\sim r \wedge q)$
P3	$\sim p \vee r$
Prove:	r

Solution A:

Line	Statement	Rule	Lines Used
1	$r \vee \sim q$	DeMorgan's law	P2
2	$q \rightarrow r$	Definition of implication	1
3	$p \rightarrow r$	Definition of implication	P3
4	r	Dilemma	P1, 2, 3

Solution B:

Line	Statement	Rule	Lines Used
1	$\sim r$	Assume	—
2	$\sim p$	Disjunctive syllogism	P3, 1
3	q	Disjunctive syllogism	P1, 2
4	$r \vee \sim q$	DeMorgan's law	P2
5	r	Disjunctive syllogism	3, 4
6	$r \wedge \sim r$	Conjunctive addition	4, 5
7	r	Closing cond world with contra	1–6

2. medium

P1	$\sim b \to \sim d$
P2	$\sim b \to e$
P3	$(\sim b \to (\sim d \wedge e)) \to d$
Prove:	b

Solution:

Line	Statement	Rule	Lines Used
1	$\sim b$	Assume	—
2	$\sim d$	Modus ponens	P1, 1
3	e	Modus ponens	P2, 1
4	$\sim(\sim b \to (\sim d \wedge e))$	Modus tollens	P3, 2
5	$\sim(b \vee (\sim d \wedge e))$	Definition of implication	4
6	$\sim b \wedge \sim(\sim d \wedge e))$	DeMorgan's law	5
7	$\sim(\sim d \wedge e))$	Conjunctive simplification	6
8	$d \vee \sim e$	DeMorgan's law	7
9	$\sim e$	Disjunctive syllogism	2, 8
10	$e \wedge \sim e$	Conjunctive addition	3, 9
11	b	Closing cond world with contra	1–10

3. medium

P1	$(p \to q) \land (r \to s)$
P2	v
P3	$(s \land q) \to \sim v$
Prove:	$\sim p \lor \sim r$

Solution A:

Line	Statement	Rule	Lines Used
1	$p \to q$	Conjunctive simplification	P1
2	$r \to s$	Conjunctive simplification	P2
3	$\sim(s \land q)$	Modus tollens	P2, P3
4	$\sim s \lor \sim q$	DeMorgan's law	3
5	$q \to \sim s$	Definition of implication	4
6	$p \to \sim s$	Hypothetical syllogism	1, 5
7	$\sim s \to \sim r$	Contrapositive	2
8	$p \to \sim r$	Hypothetical syllogism	6, 7
9	$\sim p \lor \sim r$	Definition of implication	8

Solution B:

Line	Statement	Rule	Lines Used
1	$p \to q$	Conjunctive simplification	P1
2	$r \to s$	Conjunctive simplification	P2
3	$\sim(s \land q)$	Modus tollens	P2, P3
4	$\quad p \land r$	Assume	—
5	$\quad p$	Conjunctive simplification	4
6	$\quad r$	Conjunctive simplification	4
7	$\quad q$	Modus ponens	1, 5
8	$\quad s$	Modus ponens	2, 6
9	$\quad s \land q$	Conjunctive addition	7, 8
10	$\quad (s \land q) \land \sim(s \land q)$	Conjunctive addition	9, 3
11	$\sim(p \land r)$	Closing cond world with contra	4–10
12	$\sim p \lor \sim r$	DeMorgan's law	11

Solution C:

Line	Statement	Rule	Lines Used
1	$p \to q$	Conjunctive simplification	P1
2	$r \to s$	Conjunctive simplification	P2
3	$\sim(s \land q)$	Modus tollens	P2, P3
4	$\quad p$	Assume	—
5	$\quad q$	Modus ponens	1, 4
6	$\quad \sim s \lor \sim q$	DeMorgan's law	3
7	$\quad \sim s$	Disjunctive syllogism	5, 6
8	$\quad \sim r$	Modus tollens	2, 7
9	$p \to \sim r$	Closing cond world without contra	4–9
10	$\sim p \lor \sim r$	Definition of implication	9

4. medium+

P1	$p \to q$
P2	$\sim q \vee r$
P3	$s \vee (v \wedge \sim r)$
Prove:	$\sim s \to \sim(p \vee \sim v)$

Solution:

Line	Statement	Rule	Lines Used
1	$\sim s$	Assume	—
2	$v \wedge \sim r$	Disjunctive syllogism	P3, 1
3	v	Conjunctive simplification	2
4	$\sim r$	Conjunctive simplification	2
5	$\sim q$	Disjunctive syllogism	P2, 4
6	$\sim p$	Modus tollens	P1, 5
7	$\sim p \wedge v$	Conjunctive addition	6, 3
8	$\sim(p \vee \sim v)$	DeMorgan's law	7
9	$\sim s \to \sim(p \vee \sim v)$	Closing cond world without contra	1–8

5. medium

P1	$a \wedge \sim d$
P2	$b \rightarrow (e \rightarrow d)$
Prove:	$(a \rightarrow b) \rightarrow \sim e$

Solution A:

Line	Statement	Rule	Lines Used
1	$\quad a \rightarrow b$	Assume	—
2	$\quad a$	Conjunctive simplification	P1
3	$\quad \sim d$	Conjunctive simplification	P1
4	$\quad b$	Modus ponens	1, 2
5	$\quad e \rightarrow d$	Modus ponens	P2, 4
6	$\quad \sim e$	Modus tollens	3, 5
7	$(a \rightarrow b) \rightarrow \sim e$	Closing cond world without contra	1–5

Solution B:

Line	Statement	Rule	Lines Used
2	a	Conjunctive simplification	P1
3	$\sim d$	Conjunctive simplification	P1
4	$\sim b \vee (e \rightarrow d)$	DeMorgan's law	P2
5	$\sim b \vee (\sim e \vee d)$	DeMorgan's law	4
6	$(\sim b \vee \sim e) \vee d$	Associative law	5
7	$\sim b \vee \sim e$	Disjunctive syllogism	3, 6
8	$a \vee \sim e$	Disjunctive addition	2
9	$(a \vee \sim e) \wedge (\sim b \vee \sim e)$	Conjunctive addition	7, 8
10	$(a \wedge \sim b) \vee \sim e$	Distributive law	9
11	$\sim(a \wedge \sim b) \rightarrow \sim e$	Definition of implication	10
12	$(\sim a \vee b) \rightarrow \sim e$	DeMorgan's law	11
13	$(a \rightarrow b) \rightarrow \sim e$	Definition of implication	12