Sets

Stuff from Chapter 1

Intro to sets: This chapter is about sets. A set is nothing more than a collection of items or objects. We study sets in computer science because so many CS problems involve manipulating groups of objects. In this class, we’ll mostly work with sets of numbers, but most of the stuff we’ll talk about here apply to sets containing any type of object.

Terminology:

* Set: a collection of objects.
  + The order of the objects does not matter.
  + In CS, this is what distinguishes a SET from a LIST.
* Defining sets explicitly with braces.
  + Order of elements does not matter.
* Furthermore, sets only ever contain one copy of an item. Adding an extra copy of an item to a set doesn’t change the set.
  + Ex: {1, 1, 2, 2} = {1, 2}.
  + In CS terms, there is a different data structure called a bag or a multiset that allows for multiple copies of the same item.
* Defining infinite sets explicitly with braces.
* Sets can contain anything – numbers, words, even other sets.
* Set builder notation
  + {x in S | P(x) } means all the members of S that make P true.
  + {x in Z | Even(x) }
    - also with existential quantifier { x in Z | exists k x = 3k+1 }
  + { a in R | a > 0 and a < 5 }
  + { p in Z | Prime(p) and p > 4 and 2 | p }
* Notation for the empty set = zeroslash = { }

* Subset predicate
  + A subseteq B <-> for all x in U, x in A -> x in B
  + Introduce symbol
  + All sets are implicitly subsets of the universal set, which contains everything we would ever care about.
  + Not subset symbol
* Examples distinguishing in from subseteq
  + 2 in {1, 2, 3} {2} subsetof {1,2,3}
  + {2} is not an element of {1, 2, 3} and 2 is not a subset of {1, 2, 3}
* Ordered pairs.
  + Uses parentheses. Order does matter.
  + Ordered pairs result from something called the Cartesian product of two sets.
  + A x B = {(x, y) | x in A and y in B}
  + This is where we get points in the Cartesian plane.

6.1

* Def’n of subset: for all x in U, x in A –> x in B
* Def’n of not a subset
* How to prove subset
  + Comes from the def’n.
  + Suppose x is arbitrary member of U. Assume x in A. Prove x in B.
  + Typically shorten first part to be suppose x is arbitrary element of A. Prove x in B.
* Let B = Z^even
* Let A = { n in Z | exists k n = 4k+2 }
* Prove A subseteq B.
* Visualize with venn diagram.
* **Disprove B subseteq A**.
* Set equality
* A=B means A subset B and B subset A
* Prove odds can be written as 2k+1 or 2k+3.

Put up definitions:

* Set builder notation {x in S | P(x) } = the set of everything in S that makes P true.
* Subset definition: A subseteq B <-> for all x in U, x in A -> x in B
* Set equality: A subseteq B and B subseteq A.
* Cartesian product: A x B = {(x, y) | x in A and y in B}

New things:

* Definitions.
  + Union of A, B is {x in U | x in A or x in B }

Ex: A = {1,2,3,4} B = {2,4,6,8}

* + Intersection
  + Difference A-B is {x in U | x in A and x not in B }
  + Complement A^c = {x in U |x not in A }
  + Show venn diagrams
    - A-B = A intersect B^c

Practice with these:

P = primes

E = evens

O = odds

M = multiples of 4 = {...-12, -8, -4, 0, 4, 8, 12, ...}

What is P intersect A? = {2,3}

What is P intersect B? = {2}

What is P intersect E? = {2}

What is E union O = Z

What is E intersect O? = empty set

What is E – M? = {2, 6, 10, …}

What is M – E? = empty set

* + Two sets are disjoint iff A intersect B = \emptyset.
  + A collection of sets are called pairwise disjoint or mututally disjoint is every possible pair of sets is disjoint (none of the sets h ave any elements in common).
  + Partition of a set A is a collection of subsets of A A1 A2 A3.. such that A is the union of all the sets ,and the subsets are all pairwise disjoint.
    - **Def** in two parts: {A1…An} is a partition of A iff
    - A = A1 union A2 union …. An
    - A1, A2…an are all mutually disjoint.
  + Powerset of A is the set of all subsets of A.
    - **Def P(A) = { X | X subseteq A }**
    - Example powerset of {1,2,3}

6.2

Proofs with sets: most common proofs are subset and set equality.

* Procedural def’ns of union, intersection, difference, complement, powerset, and Cartesian product
  + Union: x in A u B = x in A or x in B
  + Intersection: x in A cap B = x in A and x in B
  + Complement: x in A^c = x not in A = ~(x in A)
  + Difference x in A – B = x in A and x not in B
  + Powerset: X in P(A) = X subseteq A
  + Cartesian: (x,y) in AxB = x in A and y in B
* Proof: A cap B subseteq A. (Element argument)
  + Draw venn diagram.
* Show set identities. (handout)
* Prove Demorgan’s law for sets.
  + Prove (A u B)^c = A^c cap B^c
* Sample proof with empty set.
  + Do by contradiction.
  + For all sets *A*, *B*, and *C*, if *A* ⊆ *B* and *B* ⊆ *Cc*, then *A* ∩ *C* = ∅.
* Sample proof with Cartesian product
  + A subset B -> A x B subset B x B
* Sample disproof
  + Prove A-(B-A) not eq A-B

6.3

* Cardinality of a set
* Prove powerset of A has 2^n elements.
  + Use weak induction
* Algebraic proofs
  + (A u B) – C = (A - C) u (B - C)
* Students try
  + A-(A cap B) = A-B

Types of Set proofs

Element method proof (subset or equality)

A subseteq B and B subseteq C 🡪 A subseteq C

A subset B 🡪 (A cap C) subset (B cap C)

A subset B 🡪 (A cup C) subset (B cup C)

A subset B 🡪 B^c subset A^c [prove with contrapositive]

(A subset B) and (A subset C) 🡪 A subset (B cap C)

(A subset C) and (B subset C) 🡪 (A cup B) subset C [need cases]

(A cap B) cup (A cap B^c) = A [cases]

Proof with cart prod

Proof with power set

Proof proving equality to empty set

(do with element method if subsets, algebraic if no subsets)

Algebraic

(A – B) cup (C – B) = (A cup C) – B [easy]

(A – B) cap (C – B) = (A cap C) – B [easy]

(A cup (B-A)) = A cup B

A – (A cup B) = emptyset [also do with element method]

A cap (A-B) = A-B

A cup (A-B) = A

Disproof (with and without subsets on left)