Chapter 9 – Counting (and Probability)

Counting (from a purely mathematical point of view) is a set of rules that we can use to figure out how big a set is. For example, how many even numbers are there between 1 and 100? Now we’re not restricted to counting numbers. For example, we can ask if I am playing a card game, like poker or blackjack, how many different cards are left in the deck that can help me win the game?

From a CS perspective, we use these counting in a number of different ways:

* In developing data structures, to know how much data a structure can hold. For instance, if I have a binary tree of a certain height and a certain depth, how many pieces of information can it hold?
* We use it to analyze how fast algorithms run, or how much memory they might use. For example if you know you need to visit five different professors offices on campus, you may want to develop an algorithm to figure out the order of the offices to visit in order to minimize time spent walking between them. In order to develop an efficient algorithm, you will probably want to know that total number of possible orders in which you can visit the offices.
* Or think about algorithms for playing games, like tic-tac-toe, or checkers or chess. What makes a game like tic-tac-toe “easier” than a game like checkers or chess? [size of board, number of possible positions, length of game, number of possible moves at each step]. All of these things can be counted.

Related to counting is the idea of probability, which is really nothing more of the ratio of the sizes of two different sets that we’ve counted. For instance, if I’m playing poker and I need to draw a king to obtain a certain hand, the probability that I get a king is just the ratio of the number of kings in the deck to the total number of cards, both of which I figure out using counting.

So much of CS is based around probability these days, especially in artificial intelligence. AI algorithms often use probability to make decisions, for instance, to learn how to play games. If I make this move, what is the probability of a win?

* Random process or experiment – something where the outcome is unknown before we take some action.
* Sample space: set of all possible outcomes of an experiment
* Event: A set that is a subset of the sample space.
* Ex: experiment – tossing a coin.
  + Sample space = {h, t}.
  + Event = any subset of {h, t}.
* Ex: experiment – tossing two coins
  + Sample space = {hh, ht, th, tt}
    - Sample space can be defined in multiple ways. Sample space = {2 heads, 1 each, 2 tails}
  + Event = any subset of sample space.
* Equally likely probabilities formula:
  + If S is a finite sample space **where all events are equally likely**, and If E subseteq S, then P(E) = |E|/|S|.
* Ex: 52 card deck.
  + Sample space = {…write it out
  + What is the the event of choosing a red face card: 6 outcomes
  + Prob of this event = 6/52
* Ex: Dice
  + Roll two dice.
  + What is the sample space? (set of 36 items)
  + What is the probability our sum of the two dice is 10, 11, 12, 13?
* Number of things between n and m is m-n+1
* How many 3-digit integers are divisible by 5?
  + Write out 100, 101, 102…999
  + 100=5\*20
  + 105 = 5\* 21
  + 110 = 5\*22
* Prob that if we randomly choose a 3 digit int it is div by 5?
* Pretend a friend of mine decides to have ice cream for dessert 3 days in a row. they have a carton of chocolate ice cream and a carton of vanilla ice cream in the freezer. Each day they reach in and grab one of the two cartons at random, so there’s an equal chance at grabbing each one.
  + If they do this 3 days in a row, what is sample space? Set of 6 things.
    - {CCC, CCV, CVC, CVV, VCC, VCV, VVC, VVV}
  + What is the probability that they ate vanilla on exactly one day? 3/6 = ½.
  + At least 2 days ate vanilla? = 4/6 = 2/3.
  + None of the three ate vanilla? = 1/6
* Monty hall problem. (you should always switch, which seems strange. Most people say it doesn’t matter if you stay).
  + Explanation: Three doors. 1/3 chance of car, 2/3 chance of goat. You pick a door. Monty always shows you a goat.
  + If you picked a goat originally, then switching wins. If you picked car, switching loses. So 2/3 chance to win If you switch.

9.2 – possibilities trees and mult rule

* A way to keep track of an experiment that has multiple steps that proceed in some order is called a possibility tree.
  + Say you must take 142, 172, 241, 231. But 172 and 142 are prereqs for 241.
  + Draw tree
* Can impose order on simultaneous events, like toss two dice.
* Multiplication rule corresponds to a certain type of tree, where the number of outcomes at a single step is always the same, in all branches. In other words, what you did in an earlier step does not change the number of possibilities for a later step.
* Total number of ways = multiply number at each step.
* Ex: how many Rhodes College student email addresses are there?
  + 3 of last name, first name, middle initial, digit, digit
  + 26 \* 26 \* 26 \* 26 \* 26 \* 10 \* 10.
* Ex: how many ways are there to pick a red face card?
  + = 2 \* 3 = 6.
* Ex: Number of elements in Cartesian product, relate to dice.
* Ex: Initial password of 6 letters. What is prob that password does not have any repeated letters?
  + passwords without regards to repeats = 26^6.
  + passwords without repeats = 26\*25\*24\*23\*22\*21.
* Ex: something where mult rule doesn’t work. pre-requisites. (tree is unbalanced).
* Ex: a best of 5 tournament between 2 sports teams. The first team to win three games wins the tournament. How many different sequences of Wins and Losses are there, from the perspective of my team?
  + It’s not

Takeaway = many counting problems can be reduced to a situation where you have **n different objects**, and you need to **choose k of them**. (My favorite example is doughnuts).

* The two most common criteria we care about in this situation are CAN WE REPEAT OBJECTS?/PICK SAME OBJECT MORE THAN ONCE, and DOES ORDER MATTER?
* Repeats allowed, order matters = n \* n \*…\* n (k times). = n^k. [like email addresses, passwords]
* PERMUTATIONS
  + Now we will examine the situation where repeats are NOT allowed.
  + So I went to the University of Maryland, where the mascot is the terrapin, which is a kind of turtle. We were known as the TERPS. At the football games, in coordination with the marching band, these people would run around the perimeter of the field carrying flags with the letters T-E-R-P-S on them to get the crowd excited.
  + Imagine I want to do the same thing at Rhodes, so I hire six students to carry flags with the letters RHODES on them. Unfortunately, the students don’t look at which flags they get before they run out onto the field, so the letters could be in any order. How many possible orders are there?
  + 6!
* A permutation of a set is a specific **ordering** of the elements within that set.
* Number of permutations of a set with n objects is n! = n\*(n-1)(n-2)..1.
* Go to a football game and I give 6 people flags with letters RHODES on it.
* Ex: how many ways can the flag holders run out onto the field?
  + 6! = 120\*6 = 720.
  + What is the prob of getting order 100% correct? 1/6!
* Ex: Suppose I make sure that the person holding the R flag runs out first, and the person holding the S flag runs out last. But the folks in the middle don’t bother to arrange themselves ahead of time. How many ways might they run out now? [R first, S last, 4 middle letters any order]
  + 1 \* 4 \* 3 \* 2 \* 1 \* 1 = 4!
  + What is probability that this will happen from all orders? = 4!/6!
* Ex: how many ways can I rearrange so that the ordering is CCVCVC?
  + 4 consonants, 2 vowels.
  + 4 \* 3 \* 2 \* 2 \* 1 \* 1.
  + note we can rearrange to get. 4!\*2!.
  + What is probability this will happen from all orders? = 4! \* 2! / 6!
* Ex: what if the people with the flags run in a circle continuously, so there is no end and no beginning? How many orders now?
  + 6!/6 = 5!
* PERMUTATIONS OF SELECTED OBJECTS
  + What if I have a set of n objects but I want a permutation of only a few of them, say r?
  + Called an r-permutation.
  + Formula = P(n,r) = n\*n-1\*n-2…(n-r+1).
  + = n!/(n-r)!
* Ex: An animal shelter has 14 dogs.
  + Suppose six people walk in and each person wants to adopt exactly one dog. So I need to pick 6 dogs from the 14, but the order matters that I pick them because giving dog A to person 1 is not the same as giving dog A to person 2.
    - P(14, 6)
* What is a different shelter has 14 dogs. 8 are large dogs, and 6 are small dogs.
  + Suppose 6 people walk in and want to adopt dogs. 3 want large dogs and 3 want small dogs. How many different ways can I adopt out the dogs?  
    Answer: Do this in 2 steps.
    - Do large dogs first. Pick 3 large dogs from 8 = P(8, 3)
    - Now pick small dogs: P(6, 3).
    - Multiply together:
* Ex: passwords with non-repeating letters. (did earlier)
  + 8-letter passwords?
  + Prob of no repeated digits?
* Ex: how many ints from 1000 to 9999 are there? (9000) or (9 \* 10 \* 10 \* 10) (also 9999-1000+1=9000)
  + How many are odd? = 9\*10\*10\*5 = 4500 (half!)
  + How many have distinct digits? (not necessarily odd) = tempting to say 9\*8\*7\*6, but
    - It’s actually 9\*9\*8\*7=4536
  + How many odd have distinct digits?
    - Go right to left instead of left to right.
    - 5 for rightmost digit.
    - Leftmost digit has 8 possibilities (1-9, minus whatever digit rightmost was)
    - Next digit has 8 possibilities (0-9, minus other two digits)
    - Next digit has 7 possibilities.
  + What is the prob that a randomly chosen int btw 1000 and 9999 has distinct digits? Is odd and has distinct digits?

9.3 counting elements of disjoint sets

* Remember an event is a set.
* We are interested in the size of that set, book denotes this by N(A). Sometimes I use |A|.
* Two sets A and B are disjoint if A cap B = emptyset.
* Addition rule:
  + If a finite set A = the union of k distinct mutually disjoint subsets A1..An.
  + Then N(A) = N(A1) + … + N(An).
  + In picture = draw partition.
  + Clue when to use addition rule = the word OR.
* Ex: How many passwords have 3 or fewer letters? (26 letters, add up 1 digit pws, 2 dig pws, 3 dig pws) = 26 + 26^2 + 26^3.
* Ex: How many 3 digit ints 100-999 are divis by 5?
  + Old way and new way
  + Old way = 100…999 divis by 5 = 100…995 = (divide by 5) 20…199 = numbers in this range is 199-20+1=180
  + New way = three dig ints that end in zero + three dig ints that end in 5.
    - = 9\*10\*1 + 9\*10\*1
* The difference rule:
  + If A is a finite set and B is a subset of A, then
  + N(A-B) = N(A) – N(B).
* Example: Select a 4-digit bank PIN.
  + How many pins CONTAIN AT LEAST ONE repeated digit?
    - (opposite of problem we’ve looked at before, which is no repeats)
  + Let A = total pins.
  + Let B = pins with repeated digits.
  + A – B = total pins MINUS pins with repeats.
  + A = 10^4 B = 10\*9\*8\*7 = permutation of 10, 4 = 10!/6!
  + Answer = 10^4 - 10!/6!
  + Prob of choosing a PIN at random that happens to not have any repeats =(10!/6!)/( 10^4 - 10!/6!)
* Formula for prob of complementary event.
  + P(A^c) = 1 – P(A)
  + Derivable from P(A^c) = |A^c|/|Sample| = (Sample-A)/sample   
    = sample/sample – a/sample = 1-A/Sample.
* Formulas:
  + N objects, Pick k of them.
  + Order matters, repeats allowed-> n^k
  + Order matters, no repeats allowed -> Permutation P(n,k) = n!/(n-k)! = n\*(n-1)\*\*\*(n-k+1).
* Mult rule:
  + If you have a two-step process, and step 1 can be done in n ways, and step 2 can be done in m ways, then the total number of ways to do the entire process is n\*m.
* Add rule:
  + If a finite set A consists of two disjoint subsets B and C (meaning A = B u C and   
    B intersect C = emptyset), then |A| = |B| + |C|.
* Difference rule:
  + If A is a finite set and B \subseteq A, then |A-B| = |A| - |B|.
* Ex: Birthday paradox. (50% chance with 23 people)
  + What is the probability that 2 ppl in this room share a birthday?
  + Number each day of the year from 1 to 365.
  + How many ways can we assign these numbers to people in the class so there is at least ONE repeated number? (hard question) (doughnut metaphor – there are 365 flavors of doughnuts, and everyone in the class picks a doughnut. How many ways can we do this so at least two people share a flavor?)
  + Easier: how many ways can we assign these numbers/flavors to people to there are no repeats?
    - = P(365, # in class)
  + What is total number of ways to assign birthdays = 365^(# in class)
  + Prob no matching birthdays = P(365, n)/365^n
  + Prob at least one matching birthday = 1 - P(365, n)/365^n
  + 50% as 23 ppl,

|  |  |
| --- | --- |
| 30 | 70.6% |
| 40 | 89.1% |
| 50 | 97.0% |
| 60 | 99.4% |
| 70 | 99.9% |

* Inclusion/exclusion rule:
  + Draw venn diagram
  + N(A cup B) = N(A) + N(B) – N(A cap B)
  + Ex: How many #s btw 1 and 100 are divis by 6 or 7?
    - Divis by 6 = 6…96 (16)
    - Divis by 7 = 7…98 (14)
    - Answer is not 30.
    - Divis by 6 and 7 = divis by 42 = {42, 84}
    - Answer = 16+14-2 = 28.
  + Extends to 3 sets.
  + N(A cup B cup C) = N(A) + N(B) + N(C) – N(AB) – N(AC) – N(BC) + N(ABC)
* Ex:

A professor in surveys a class of 50 students to find out what kind of ice cream flavors they like. The survey comes back with the following information:

30 liked vanilla; 18 liked chocolate; 26 liked Strawberry; 9 liked both vanilla and chocolate;

* 16 liked both vanilla and Strawberry; 8 liked both chocolate and Strawberry; 47 liked at least one of the three flavors.
* How many students do not like any flavors?
* How many students like all three flavors?
* Total = 50  
  |V| = 30
* |C| = 18
* |S| = 26
* V intersect C = 9
* V intersect S = 16
* C intersect S = 8
* 47 liked at least one = |V union C union S|

In Venn diagram: middle = 6

V and C minus middle = 3

V and S minus middle = 10

C and S minus middle = 2

V all alone = 30-(10+6+3) = 30-19=11

C all alone = 18-(3+6+2) = 18-11=7

S all alone = 26-(10+6+2)=26-18=8

Outside any region=50-47=3.

**9.4 Pigeonhole**

Pigeonhole principle:

If n pigeons fly into m pigeonholes and n > m, then at least one hole must at least 2 pigeons.

Math version:

If f:A->B and N(A) > N(B), then f cannot be 1-1 – there must be two elements of the domain that map to the same element in the co-domain.

Ex: In a group of 13 people (use # in class), must there be two people who were born in the same month?

Ex: If you have a drawer with brown socks, black socks, and white socks, how many socks must you take out such that you’re guaranteed to have a matching pair?

Key – don’t know what the color of the pair is, you just know they’re matching.

Are there two people in NYC with the same number of hairs on their heads?

People in NYC = 8.5 million

Hairs on a head = 300,000.

Make two sets, one = all people in NYC, two = all hairs, from 0 to 300,000.

Since 8.5 mill > 300K, there must be 2 people in NYC with the same # of hairs on their heads.

Ex: A = ints from 1-8

If I select 5 ints, is it true that among the 5 ints there is a pair that adds to 9?

Yes. Map from a1,a2,a3,a4,a5 to {1,8}, {2,7}…

Generalized pigeon hole

If n pigeons fly into m pigeonholes and n > km, then at least one hole must contain at least k+1 pigeons.

If f:A->B and n > km, then there must be k+1 elements in the domain that map to the same element in the co-domain.

Ex: Suppose I gather together 14 people. I do a survey with ice cream flavors (V, C, S). Can I be sure that at least 4 people will like the same flavor? Yes.

F: people -> flavors

14 -> 3

14 > 3\*4

Play a tournament that requires 3 wins to win the whole thing. How many games do I have to play?

Map games to w/l

Size of games = n

W/L = 2 = m

Solve n > k\*m. k+1 = 3, so k=2 n > 2\*2. So n=5.

Contrapositive form:

If f:A->B |A|=n, |B|=m, and each element of the domain is mapped to at most k elements, then n <= km.

Ex: Suppose in this class I ask everyone what month they were born in and we find out that no four people share a birth month. What is the largest possible class size in which this could happen?

N leq 3\*12

N leq 36

N can be 36, but no more.

Ex: There are 42 students who go eat lunch at 12 tables. Each student sits at a table. Each table only has six chairs, so no more than 6 students can sit at a table. Show that at least 5 tables have 3 or more students.

Wrong approach: Left set is (n=42) students. Right set is (m=12) tables. Largest k that works is k=3, because 42 > 3\*12. So there must be at least one table with 4 students. But we need a stronger statement.

So we approach by contradiction.

Suppose not. Suppose 4 or fewer tables have 3 or more students. So these four tables can hold at most 24 students (because there can be up to six students at a table). So there are 42-24 = 18 students left over to sit at 12-4=8 tables. If we try to seat 18 students at 8 tables, there must be a table with 3 students, bc 18 > 2\*6. This is a contradiction because now there is another table with 4 (3 or more) students.

9.5 combinations

* Suppose we have a bowl with n colored marbles in it. How many orderings can we pick r marbles from the bowl?
  + R-permutations: P(n, r) = n!/(n-r)! = n(n-1)…(n-r+1)
* Suppose after each pick, we return the marble to the bowl so it can be picked again.
  + N^r.
* Same problem, but the distinction is with replacement/without replacement. Note that in both problems, order matters.
* Today we’re going to look at selecting some items from a group where order does not matter.
* This is called an r-combination.
* Ex: I have a club with 4 people (A, B, C, D) and I need to choose a president and a vp. Ways to do this = 4\*3=12.
* Now choose co-presidents: ways to do this = 6.
* (n choose r) = n!/(r!(n-r)!)
* Ex: [**Sometimes the concept of “order not mattering” can be replaced with the concept of “picking identical things**”]. I need to choose 6 of a class of 14 ppl to run around with flags at the football game, but now all the flags are identical. How many ways can I do this?
* 14!/(6!8!)
* Suppose you have a group of ten students and I need to pick four to help clean erasers after class.
  + # = (10 choose 4)
* Suppose 2 people insist on doing it together or not at all.
  + Wrong way = combine two ppl into one, so we have 9 units…but now if we choose from the groups, (9 choose 4) will make some groups of 4 and some groups of 5.
  + Right way = split into two separate sets.
    - Set A = groups with the 2 ppl = \_ \_ \_ \_ = **1** \* (8 choose 2)
    - Set B = groups without = (8 choose 4)
    - Add together
* Suppose the 2 ppl refuse to be picked together.
  + Call the 2 ppl person X and person Y.
  + # = group with only X + grp with only Y + grp with neither
  + = 1 \* (8 choose 3) + 1 \* (8 choose 3) + (8 choose 4)
  + OR
  + Group of anyone – group with both
  + = (10 choose 4) – (8 choose 2)
* Problems with at most or at least
* Group of 14 dogs at shelter, 6 large dogs, 8 small dogs. Want to make a group of 5 to take to an adoption event.
  + How many groups contain 3 large and 2 small?

6 choose 3 \* 8 choose 2.

* + 2 large and 3 small?  
    6 choose 2 \* 8 choose 3.
  + How many groups have 2/3 or 3/2? Two previous answers added together.
  + How many groups contain at least one large dog?  
    **Wrong way**: Pick a large dog = 6 choose 1. 13 dogs left, so pick a group of 4 = 13 choose 4. Add together.  
      
    This double counts. Imagine large dogs are ABCDEF. Small dogs are GHIJKLMN.   
     Pick large dog A. Then pick remaining group = BCDEFG. Total = ABCDEFG.  
    (OR) Pick large dog B. Then pick remaining group = ACDEFG. Total = A through G.  
      
    Right way: Split into groups, and use addition rule.  
     1L/4S + 2L/3S + 3L/2S + 4L/1S + 5L.
  + OR, use subtraction rule.
  + # of groups with at least one large dog = (all possible groups) – (groups with zero large dogs)
  + = (14 choose 5) – (8 ch 5)
  + How many teams contain at most one man?
* How many ways can I rearrange the letters of TENNESSEE?
  + Total letters = 9. T – 1, E – 4, N – 2, S – 2
  + Answer = (9 choose 1)\*(8 choose 4)\*(4 choose 2)\*(2 choose 2)
* Why does this make sense?
  + Contrast with ways to rearrange the letters of ABC? = 3 \* 2 \* 1 =
  + (3 choose 1)\*(2 choose 1)\*(1 choose 1)
* Probability problems where you can’t tell if order matters/order doesn’t matter?
  + Suppose you have a bowl with 10 red marbles and 5 blue marbles.
  + I reach in and pull out 2.
  + What is the probability they are the same color?
  + Order matters
    - Ways to pick 2 marbles = 15 \* 14
    - Ways to pick a red then a blue = 10 \* 5
    - Ways to pick a blue then a red = 5 \* 10
    - Final answer = (10\* 5 + 5 \* 10)/(15 \* 14)
  + Order doesn’t matter
    - Ways to pick two marbles = (15 choose 2)
    - Ways to pick two marbles of diff color = (10 choose 1)(5 choose 1)
    - Final answer = (10 choose 1)(5 choose 1) / (15 choose 2)
  + **AS long as you are consistent in numerator and denominator, it won’t matter.**
* 10 snickers, 10 milky ways, 10 reeses.
  + Prob of choosing 2 candies, having them be the same?
  + Method 1 = order matters = 10

**REVIEW**

n objects, pick r of them:  
  
Order matters, repeats OK: n^r

Order matters, repeats not OK: P(n, r) = n!/(n-r)!

Order doesn’t matter, repeats not OK: combination, n choose r = n!/(n-r!)r!

[not covered] Order doesn’t matter, repeats OK: n+r-1 choose r.

Patterns with combinations:

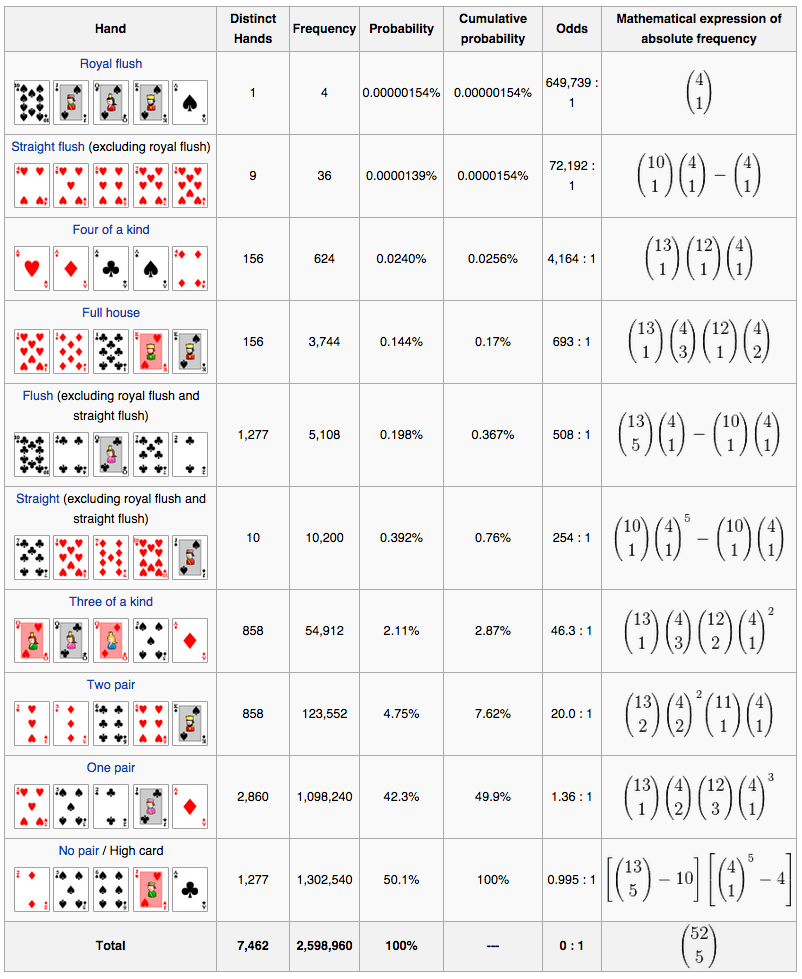
(n choose 1) = 1

(n choose n) = 1

(n choose k) = (n choose (n-k))

Best order to cover poker hands

* “Matching ranks” = 4 of kind, full house, three of a kind, two pair, one pair
* “flushes” = flush
* “straights” = straight, straight flush, royal [straight flush]



**Derivation of frequencies of 5-card poker hands**

The following computations show how the above frequencies for 5-card poker hands were determined. To understand these derivations, the reader should be familiar with the basic properties of the [binomial coefficients](http://en.wikipedia.org/wiki/Binomial_coefficient) and their interpretation as the number of ways of choosing elements from a given set. See also: [sample space](http://en.wikipedia.org/wiki/Sample_space) and [event (probability theory)](http://en.wikipedia.org/wiki/Event_(probability_theory)).

* Total number of hands = (52 choose 5) = 52!/(47!5!) = 2,598,960
* Royal flush. There are exactly four of these, one for each suit.
* *Straight flush* — Each straight flush is uniquely determined by its highest-ranking card. These ranks go from **5** (**A-2-3-4-5**) up to **A** (**10-J-Q-K-A**) in each of the 4 suits. Thus, the total number of straight flushes is:

10 \choose 1}{4 \choose 1} = 40

* + *Royal straight flush* — A royal straight flush is a subset of all straight flushes in which the ace is the highest card (i.e. **10-J-Q-K-A** in any of the four suits). Thus, the total number of royal straight flushes is

5 \choose 5}{8 \choose 0}{4 \choose 1} = 4

or simply 4 \choose 1} = 4. *Note*: this means that the total number of non-Royal straight flushes is 36.

* *Four of a kind* — Pick a rank = 13 ways. Pick all four suits within that rank = 1 way or (4 choose 4). Now we need a fifth card that doesn’t match in rank. So pick a different rank = 13-1=12 ways. Pick a suit = 4 ways.  
  + Any one of the thirteen ranks can form the four of a kind by selecting all four of the suits in that rank. The final card can have any one of the twelve remaining ranks, and any suit. Thus, the total number of four-of-a-kinds is:

13 \choose 1}{4 \choose 4}{12 \choose 1}{4 \choose 1} = 624

* *Full house* — The full house comprises a triple (three of a kind) and a pair. The triple can be any one of the thirteen ranks, and consists of three of the four suits. The pair can be any one of the remaining twelve ranks, and consists of two of the four suits. Thus, the total number of full houses is:

13 \choose 1}{4 \choose 3}{12 \choose 1}{4 \choose 2} = 3,744

* *Flush* — The flush contains any five of the thirteen ranks, all of which belong to one of the four suits, minus the 40 straight flushes. Thus, the total number of flushes is:

13 \choose 5}{4 \choose 1} - 40 = 5,108

* *Straight* — The straight consists of any one of the ten possible sequences of five consecutive cards, from **5-4-3-2-A** to **A-K-Q-J-10**. Each of these five cards can have any one of the four suits. Finally, as with the flush, the 40 straight flushes must be excluded, giving:

10 \choose 1}{4 \choose 1}^5 - 40 = 10,200

* *Three of a kind* — Any of the thirteen ranks can form the three of a kind, which can contain any three of the four suits. The remaining two cards can have any two of the remaining twelve ranks, and each can have any of the four suits. Thus, the total number of three-of-a-kinds is:

13 \choose 1}{4 \choose 3}{12 \choose 2}{4 \choose 1}^2 = 54,912

* *Two pair* — The pairs can have any two of the thirteen ranks, and each pair can have two of the four suits. The final card can have any one of the eleven remaining ranks, and any suit. Thus, the total number of two-pairs is:

13 \choose 2}{4 \choose 2}^2{11 \choose 1}{4 \choose 1} = 123,552

* *Pair* — The pair can have any one of the thirteen ranks, and any two of the four suits. The remaining three cards can have any three of the remaining twelve ranks, and each can have any of the four suits. Thus, the total number of pair hands is:

13 \choose 1}{4 \choose 2}{12 \choose 3}{4 \choose 1}^3 = 1,098,240

* *No pair* — A no-pair hand contains five of the thirteen ranks, discounting the ten possible straights, and each card can have any of the four suits, discounting the four possible flushes. Alternatively, a no-pair hand is any hand that does not fall into one of the above categories; that is, the [complement](http://en.wikipedia.org/wiki/Complement_(set_theory)) of the [union](http://en.wikipedia.org/wiki/Union_(set_theory)) of all the above hands, where the [universe](http://en.wikipedia.org/wiki/Universe_(mathematics)) is any way to choose five out of 52 cards. Thus, the total number of no-pair hands is:

left[{13 \choose 5} - 10\right]\left[{4 \choose 1}^5 - 4\right] = {52 \choose 5} - 1,296,420 = 1,302,540

* *Any five card poker hand* — The total number of five card hands that can be drawn from a deck of cards is found using a [combination](http://en.wikipedia.org/wiki/Combinatorics) selecting five cards, in any order where n refers to the number of items that can be selected and r to the sample size; the "!" is the [factorial](http://en.wikipedia.org/wiki/Factorial) operator:

n\choose r} = {{n!} \over {r!(n - r)!}} = {52 \choose 5} = {{52!} \over {5!(52 - 5)!}} = 2,598,960