Back Tracking

n - Queen Problem.

The problem is to find arrangement of n - queens on a chessboard such that no queen can attack any other queen on the board.
The chess queen can attack in any direction: horizontal, vertical and diagonal way

4x4 queen problem.

We have 4 queens to be placed on 4x4 chess-board satisfying the constraint that no 2 queens should in the same row, same column or same diagonal.

$S_1 = \{1, 2, 3, 4\}$ $\quad\quad 1 \le l \le 4$
Space

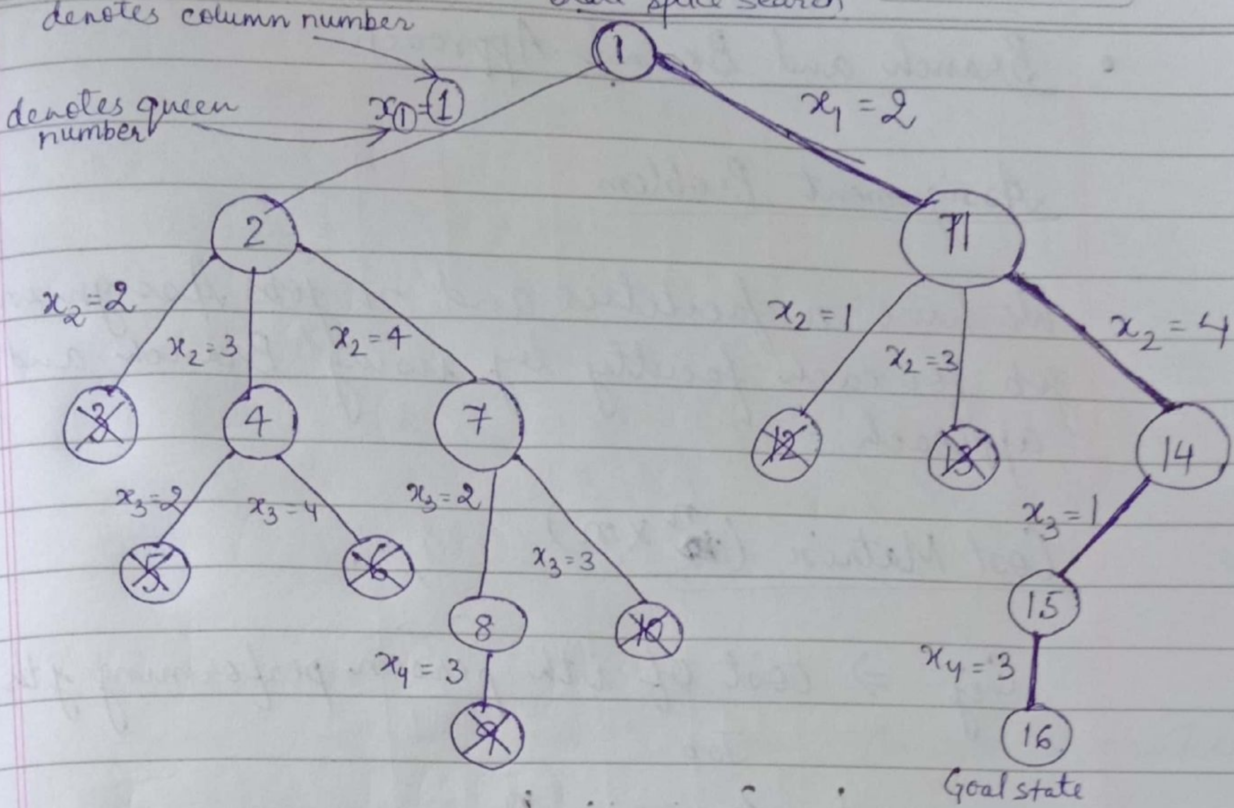
Solution ~~is placed~~ according to external constraint consists of $4^4 = 256$

As according to internal constraint we have $4!$ possible solution $= \underline{\underline{24}}$

Constraint

→ Queen 1 to be placed in row 1, Queen 2 in row 2 and so on Queen n in row n.
→ No 2 queen should be in same column.
→ No 2 queen in same diagonal.

state space search



Column	2	4	1	3
Queen	1	2	3	4

	1	2	3	4
1	●	Q_1		
2				Q_2
3	Q_3			
4			Q_4	

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- ## Branch and Bound Approach

Assignment Problem

We have n facilities and n jobs. Assign in each job for each faculty by using Branch and Bound approach.

Cost Matrix ($n \times n$)

$C_{ij} \Rightarrow$ cost of ith person performing jth job

$$\begin{array}{cc|c|c|c|c|}
 & & \multicolumn{4}{c}{\text{Job}} \\
 & & 1 & 2 & \cdots & n \\
\hline
 & 1 & C_{11} & C_{12} & \cdots & C_{1n} \\
\hline
 & 2 & C_{21} & C_{22} & \cdots & C_{2n} \\
\hline
\text{Person} & \vdots & & & & \\
\hline
 & \vdots & & & & \\
\hline
 & n & C_{n1} & C_{n2} & \cdots & C_{nn} \\
\hline
\end{array}$$

Cost: Minimize the total cost

$$\sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij} X_{ij}$$

$$x_{ij} = \begin{cases} 1 & \text{if ith person assigned jth job} \\ 0 & \text{else} \end{cases}$$

$\sum_{j=1}^{n} x_{ij} = 1$ 1 job done by ith person

$\sum_{i=1}^{n} x_{ij} = 1$ 1 person done jth job

job (j→)

Person (i)	1	2	3	4
1	33	40	43	32
2	45	28	31	23
3	42	29	36	29
4	27	42	44	38

(i) No. of persons = No. of jobs.
Else add dummy row or column to make symmetric matrix.

(ii) Make 0 in each row and column

	1	2	3	4	min
1	1	8	11	0	32
2	22	5	8	0	23
3	13	0	7	0	29
4	0	15	17	11	27

	1	2	3	4
1	1	8	4	0
2	22	5	1	0
3	13	0	0	0
4	0	15	10	11
min :	0	0	7	0

(iii) Select the row containing exactly 1 uncovered '0' and draw a vertical line through the column containing these 0.
and similarly for column

1	8	4	$\boxed{0}$
22	5	1	0
13	$\boxed{0}$	0	0
$\boxed{0}$	15	10	11

∴ 4 ≠ 3.

minimum uncrossed value.

- Substract it from all uncrossed value
- Add it to intersection point

(iv) If Total lines equal in size and solution is optimal we don't repeat the steps.

	1	2	3	4
1	1	7	3	$\boxed{0}$
2	22	4	$\boxed{0}$	0
3	14	$\boxed{0}$	0	1
4	$\boxed{0}$	14	9	11

$1 \rightarrow 4$
$2 \rightarrow 3$
$3 \rightarrow 2$
$4 \rightarrow 1$

Total cost $= 32 + 31 + 29 + 27$
$= 119$

Q.

	1	2	3	4
A	90	12	50	51
B	70	10	58	80
C	16	85	8	70
D	11	37	80	2

(1) Row reduction

	1	2	3	4
A	78	0	38	39
B	60	0	48	70
C	8	77	0	62
D	9	35	78	0

(2) Column reduction :

A	70	$\boxed{0}$	38	39
B	52	0	48	70
C	$\boxed{0}$	77	0	62
D	1	35	78	$\boxed{0}$

	1	2	3	4
A	79	$\boxed{0}$	38	39
B	59	0	48	70
C	0	77	$\boxed{0}$	62
D	$\boxed{0}$	35	78	0

$4 \neq 3$

Min value = 1

	1	2	3	4
A	69	$\boxed{0}$	37	39
B	51	0	47	70
C	0	78	$\boxed{0}$	63
D	0	35	77	$\boxed{0}$

$3 \neq 4$

Min value = 37

	1	2	3	4
A	32	0	$\boxed{0}$	2
B	14	$\boxed{0}$	10	33
C	$\boxed{0}$	115	0	63
D	0	72	77	$\boxed{0}$

$\boxed{4 = 4}$

Optimal solution

	cost
$A \rightarrow 3$	50
$B \rightarrow 2$	10
$C \rightarrow 1$	16
$D \rightarrow 4$	2

Total cost : 78