Labolatorium 5

Autorzy:

Patryk Klatka
Wojciech Łoboda

Generacja macierzy o strukturze opisującej topologię trójwymiarowej siatki zbudowanej z elementów sześciennych

```
import numpy as np

def generate(k):
    n = 2 ** (3 * k)
    M = np.zeros((n, n))
    jumps = [0, -1, 1, -2**(k), 2**(k), -2**(k*2), 2**(k*2)]
    for v in range(n):
        for j in jumps:
            if v + j >= 0 and v + j < n:
                M[v][v + j] = 1
    return M * np.random.random((n, n))
```

Rekurencyjna procedura kompresji macierzy poszerzona o metody mnożenia macierzy skompresowanej przez wektor i przez inną macierz

Pseudokod algorytmu mnożenia skompresowanej macierzy przez wektor

```
def matrix_vector_mult(v, X):
    if v.sons == Ø:
        if v.rank == 0:
            return zeros(size(A).rows)
        return v.U * (v.V * X)
    rows = size(X).rows
    X1 = X[1:rows/2, *]
    X2 = X[rows/2 + 1 : rows, *]
    Y11 = matrix_vector_mult(v.sons(1), X1) # Istotny fragment
    Y12 = matrix_vector_mult(v.sons(2), X2) # Istotny fragment
    Y21 = matrix_vector_mult(v.sons(3), X1) # Istotny fragment
    Y22 = matrix_vector_mult(v.sons(4), X2) # Istotny fragment
    return [Y11 + Y12, Y21 + Y22]
```

Pseudokod algorytmu mnożenia skompresowanej macierzy przez

macierz

```
def mult(A,B):
    if v.sons == 0 and w.sons == 0:
        if v.rank == 0 and w.rank == 0:
            return Zero matrix of proper dimentions # Istotny fragment
        else if v.rank != 0 and w.rank != 0:
            return v.U(v.V ∗ w.U) ∗ w.V  # Istotny fragment
    if v.sons > and w.sons>0:
        A = [[A1, A2], [A3, A4]]
        B = [[B1, B2], [B3, B4]]
        return [[mult(A1, B1) + mult(A2, B3), mult(A1, B2) + mult(A2,
B4)], [mult(A3, B1) + mult(A4, B3), mult(A3, B2) + mult(A4, B4)]] #
Istotny fragment
    if v.sons == 0 and w.sons > 0:
        A = U1V1
        U1 = U1’U1’’
        V1 = V1’V1’’
        A =  [[U1’  ∗ V1’, U1’  ∗ V1’’ ], [ U1’’  ∗ V1’, U1’’  ∗ V1’’ ]]
        return [[mult(A1, B1) + mult(A2, B3), mult(A1, B2) + mult(A2,
B4)], [mult(A3, B1) + mult(A4, B3), mult(A3, B2) + mult(A4, B4)]] #
Istotny fragment
    if v.sons>0 and w = 0:
        # Analogicznie dla macierzy B
```

```python
from sklearn.utils.extmath import randomized_svd
import numpy as np

class MatrixTree:
    def __init__(self, matrix, row_min, row_max, col_min, col_max):
        self.matrix = matrix
        self.row_min = row_min
        self.row_max = row_max
        self.col_min = col_min
        self.col_max = col_max
        self.leaf = False
        self.children = [None, None, None, None]


    def compress(self, r, eps):
        U, Sigma, V = randomized_svd(self.matrix[self.row_min:self.row_max, self.cc
        if self.row_min + r == self.row_max or Sigma[r] <= eps:
            self.leaf = True
            self.rank = len(Sigma)
            self.u = U
            self.s = Sigma
            self.v = V
        else:
            self.children = []
            row_newmax = (self.row_min + self.row_max)//2
            col_newmax = (self.col_min + self.col_max)//2
            self.children.append(MatrixTree(self.matrix, self.row_min, row_newmax,
            self.children.append(MatrixTree(self.matrix, self.row_min, row_newmax,
            self.children.append(MatrixTree(self.matrix, row_newmax, self.row_max,
            self.children.append(MatrixTree(self.matrix, row_newmax, self.row_max,
```

```python
        for child in self.children:
            child.compress(r, eps)

    def decompress(self, output_matrix):
        if self.leaf:
            if self.rank != 0:
                sigma = np.zeros((self.rank,self.rank))
                np.fill_diagonal(sigma, self.s)
                output_matrix[self.row_min:self.row_max, self.col_min: self.col_max
            else:
                output_matrix[self.row_min:self.row_max, self.col_min: self.col_max
        else:
            for child in self.children:
                child.decompress(output_matrix)

    @staticmethod
    def multiply_vector(node, vector):
        if node.leaf:
            if node.rank != 0:
                a = node.v @ vector
                b = node.u @ a
                return (node.u @ (node.v @ vector)) * node.s[0]
            else:
                return node.matrix[node.row_min:node.row_max, node.col_min: node.cc
        else:
            n = len(vector)
            upper, lower = vector[:n//2], vector[n//2:]
            out_upper = MatrixTree.multiply_vector(node.children[0], upper) + Matri
            out_lower = MatrixTree.multiply_vector(node.children[2], upper) + Matri
            return np.append(out_upper, out_lower, axis=0)

    @staticmethod
    def add_matrices(node1, node2):
        result_node = MatrixTree(None, None, None, None, None)

        if node1.leaf and node2.leaf:
            r = node1.u.shape[1]
            u1 = node1.u  @ np.diag(node1.s)
            v1 = node1.v
            u2 = node2.u @ np.diag(node2.s)
            v2 = node2.v

            A = np.append(u1, u2, axis = 1)
            B = np.append(v1, v2, axis = 0)

            U, S, V = randomized_svd(A @ B, n_components=u1.shape[1])

            # Make leaf
            result_node.leaf = True
            result_node.rank = len(S)
            result_node.u = U
            result_node.s = S
            result_node.v = V

            return result_node
```

```
        elif node1.leaf or node2.leaf:
            node = node1 if node1.leaf else node2

            n, r = node.u.shape

            u_part = [node.u[:n//2], node.u[n//2:]]
            v_part = [node.v[:,:n//2], node.v[:,n//2:]]

            result_node.children = [MatrixTree(None, None, None, None, None) for _

            for i in range(2):
                for j in range(2):
                    fake_child = MatrixTree(None, None, None, None, None)
                    fake_child.leaf = True
                    fake_child.rank = r
                    fake_child.u = u_part[i]
                    fake_child.v = v_part[j]
                    fake_child.s = np.ones(r)

                    if node1.leaf:
                        result_node.children[i*2+j] = MatrixTree.add_matrices(fake_
                    else:
                        result_node.children[i*2+j] = MatrixTree.add_matrices(node1

            return result_node

        else:
            result_node.children = [MatrixTree(None, None, None, None, None) for _

            for i in range(4):
                result_node.children[i] = MatrixTree.add_matrices(node1.children[i]

            return result_node


    @staticmethod
    def multiply_matrices(node1, node2):
        result_node = MatrixTree(None, None, None, None, None)

        if node1.leaf and node2.leaf:
            r = node1.u.shape[1]
            u1 = node1.u  @ np.diag(node1.s)
            v1 = node1.v
            u2 = node2.u @ np.diag(node2.s)
            v2 = node2.v

            temp = v1 @ u2
            U = u1 @ temp

            U, V = U.reshape(u1.shape), v2

            # Make leaf
            result_node.leaf = True
            result_node.rank = r
            result_node.u = U
            result_node.s = np.ones((r,))
            result_node.v = V
```

```
            return result_node

        elif node1.leaf or node2.leaf:
            node = node1 if node1.leaf else node2

            n, r = node.u.shape

            u_part = [node.u[:n//2], node.u[n//2:]]
            v_part = [node.v[:,:n//2], node.v[:,n//2:]]

            fake_children = [MatrixTree(None, None, None, None, None) for _ in rang

            for i in range(4):
                fake_children[i].leaf = True
                fake_children[i].rank = r
                fake_children[i].u = u_part[i//2]
                fake_children[i].v = v_part[i%2]
                fake_children[i].s = np.ones(r)

            result_node.children = [MatrixTree(None, None, None, None, None) for _

            for i in range(4):
                if node1.leaf:
                    result_node.children[i] = MatrixTree.multiply_matrices(fake_chi
                else:
                    result_node.children[i] = MatrixTree.multiply_matrices(node1.ch

            return result_node
        else:
            result_node.children = [MatrixTree(None, None, None, None, None) for _

            for i in range(4):
                result_node.children[i] = MatrixTree.multiply_matrices(node1.childr


            return result_node

    @staticmethod
    def update_matrices_for_nodes(root,n):
        root.row_min = 0
        root.row_max = n
        root.col_min = 0
        root.col_max = n
        M = np.zeros((n,n))
        Q = []
        Q.append(root)
        while Q:
            node = Q.pop(0)
            node.matrix = M
            rows = [node.row_min, (node.row_min + node.row_max)//2, node.row_max]
            cols = [node.col_min, (node.col_min + node.col_max)//2, node.col_max]
            for i in range(2):
                for j in range(2):
                    if not node.leaf:
                        node.children[i*2+j].row_min = rows[i]
                        node.children[i*2+j].row_max = rows[i+1]
```

```
                    node.children[i*2+j].col_min = cols[j]
                    node.children[i*2+j].col_max = cols[j+1]
                    Q.append(node.children[i*2+j])
        root.decompress(M)
```

Rysowacz

```
import numpy as np
import matplotlib.pyplot as plt
from collections import deque

def draw_tree(root, title=''):
    image = np.ones(root.matrix.shape)*255
    Q = deque()
    Q.append(root)
    while Q:
        v = Q.pop()
        if v.leaf:
            image[v.row_min:v.row_max, v.col_min:v.col_min+v.rank] = np.zeros((v.ro
            image[v.row_min:v.row_min+v.rank, v.col_min:v.col_max] = np.zeros((v.ra
            image[v.row_min, v.col_min:v.col_max] = np.zeros((1,v.col_max - v.col_m
            image[v.row_max-1, v.col_min:v.col_max] = np.zeros((1,v.col_max - v.col
            image[v.row_min:v.row_max,v.col_min] = np.zeros(v.row_max-v.row_min)
            image[v.row_min:v.row_max,v.col_max-1] = np.zeros(v.row_max-v.row_min)
        else:
            for child in v.children:
                Q.append(child)

    plt.imshow(image, cmap="gist_gray", vmin=0, vmax=255)
    plt.title(title)
    plt.show()
```

Generacja macierzy

```
k = [2, 3, 4]
test_matrices = [generate(i) for i in k]
```

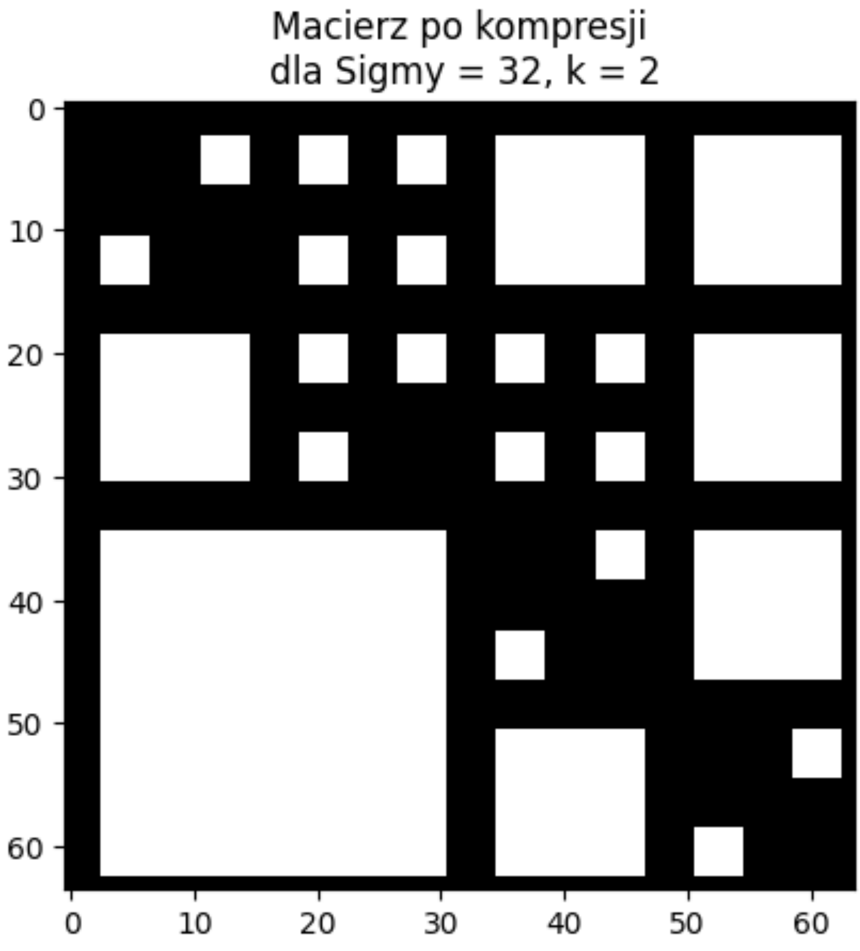
```
for x in test_matrices:
    print(x.shape)
```

```
(64, 64)
(512, 512)
(4096, 4096)
```

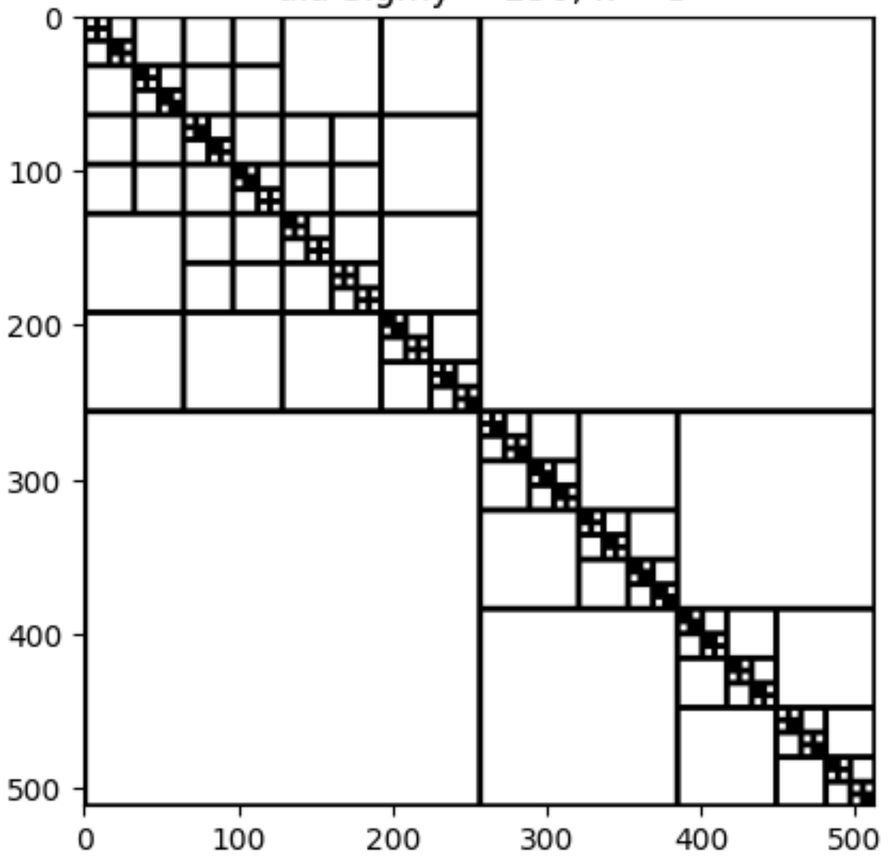
Pomiary mnożenia macierzy przez wektor

```
from time import perf_counter
from numpy.linalg import svd
import pandas as pd

results = []
for k in [2,3,4]:
```

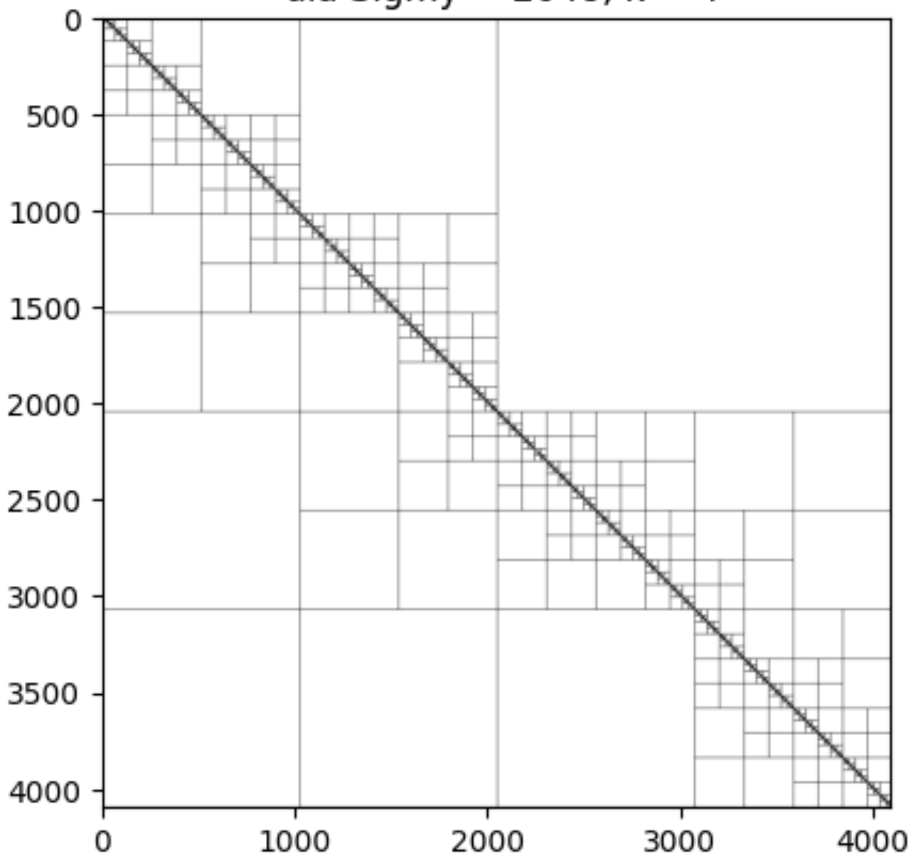
```
    X = generate(k)
    U, Sigma, V = svd(X)
    s = len(Sigma)//2
    root = MatrixTree(X, 0, len(X), 0, len(X[0]))
    root.compress(2, Sigma[s])
    draw_tree(root, f'Macierz po kompresji\n dla Sigmy = {s}, k = {k}')

    st = perf_counter()
    vector = np.random.uniform(low=10**(-8), high=1, size = (2**(3*k), 1))
    result = MatrixTree.multiply_vector(root,vector)
    end = perf_counter()

    N = np.zeros_like(X)
    result_true = X@vector
    err = np.sum(np.square(result - result_true))
    results.append([k, s, end-st, err])

df = pd.DataFrame(results, columns=["k", "s", "time", "error"])
df
```



Macierz po kompresji
dla Sigmy = 256, k = 3



Macierz po kompresji
dla Sigmy = 2048, k = 4



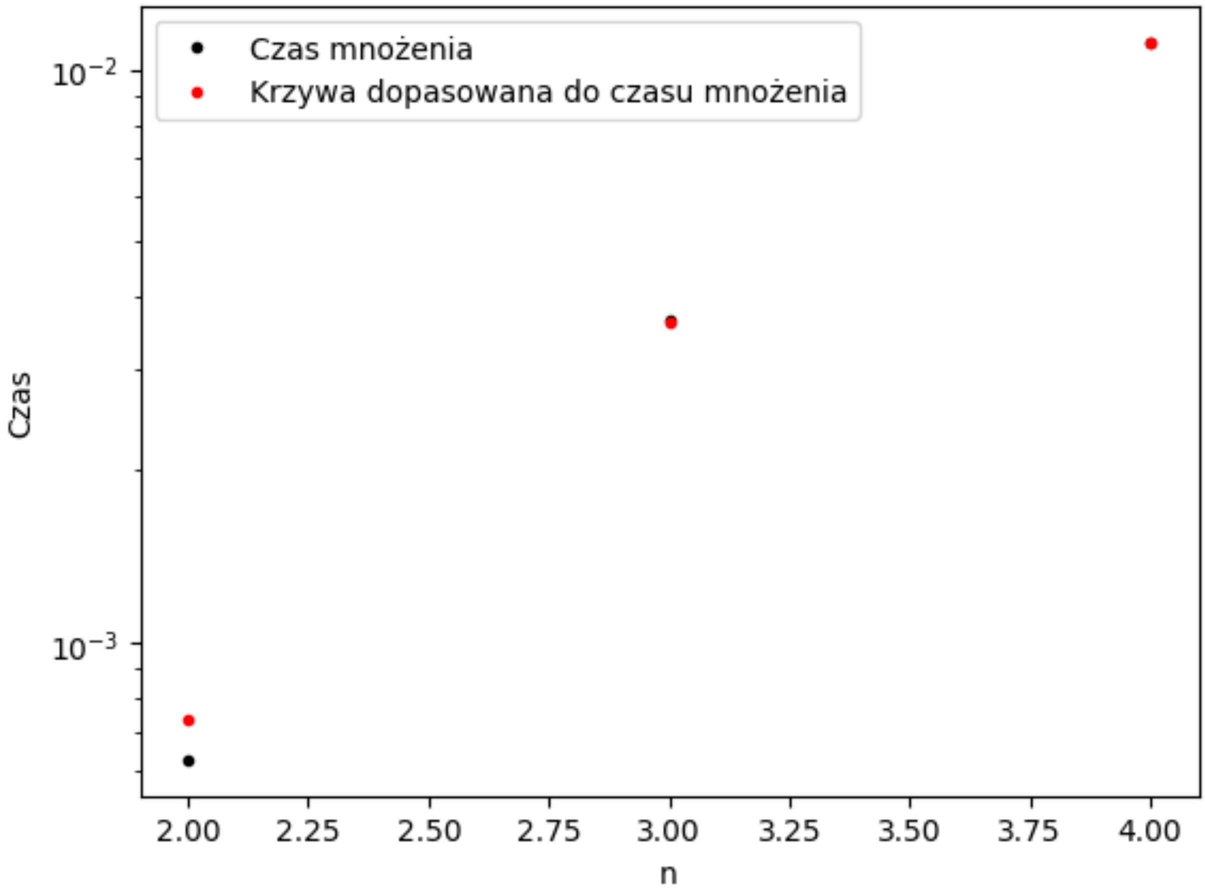
Out[ ]:

	k	s	time	error
0	2	32	0.000625	15.535239
1	3	256	0.003666	236.063876
2	4	2048	0.011131	2923.376622

Dopasowanie wykresu αN^{β} do wyników pomiarów

```
from scipy.optimize import curve_fit

def fit_curve(field1, field2, dataframe):
    x_data = dataframe[field1]
    y_data = dataframe[field2]
    params, covariance = curve_fit(lambda x, a, k: a*x**k, x_data, y_data, p0=[1.0,
    a, k = params
    print(f"Exponent beta is approximately: {k}")
    print(f"Constant alpha is approximately: {a}")
    return a, k

def plot_with_fit(a, k, data_frame, name1, name2, name):
    powers = data_frame[name1]
    time = data_frame[name2]

    time_spprox = [a * i ** k for i in powers]
    plt.plot(powers, time, ".", label = f"Czas mnożenia", color="black", )
    plt.plot(powers, time_spprox, ".", label = f"Krzywa dopasowana do czasu mnożeni
    plt.xlabel("n")
    plt.ylabel("Czas")
    plt.legend()
    plt.semilogy()
    plt.show()

a, k = fit_curve("k", "time", df)
plot_with_fit(a, k , df, "k", "time", "Bineta")
```

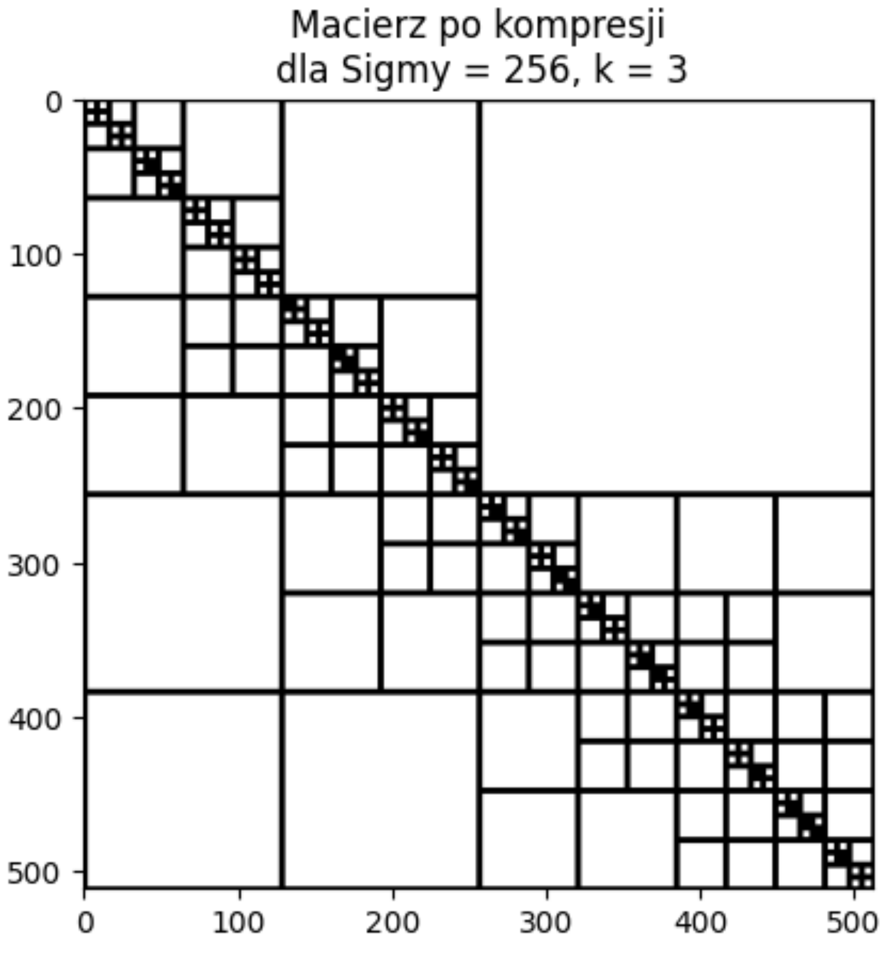
```
Exponent beta is approximately: 3.916838028964846
Constant alpha is approximately: 4.884099117976246 6e-05
```

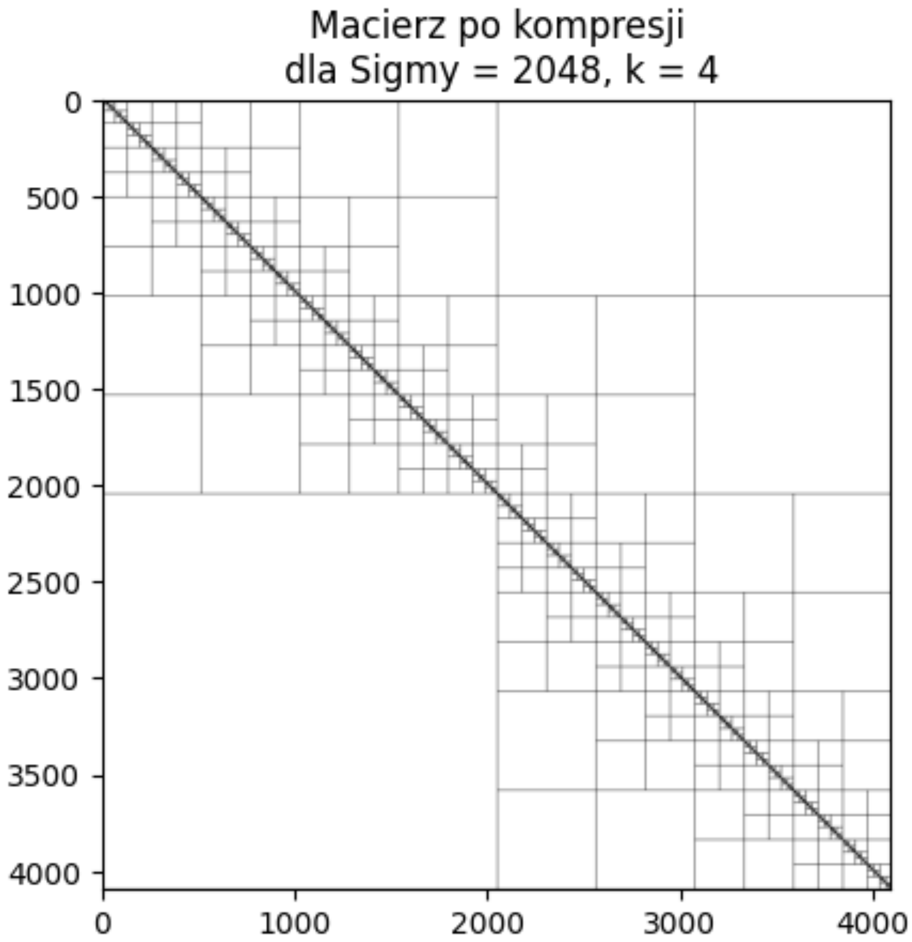


Pomiary mnożenia macierzy przez siebie samą

```
results = []
for k in [2,3,4]:
    X = generate(k)
    U, Sigma, V = svd(X)
    s = len(Sigma)//2
    root = MatrixTree(X, 0, len(X), 0, len(X[0]))
    root.compress(2, Sigma[s])
    draw_tree(root, f'Macierz po kompresji\n dla Sigmy = {s}, k = {k}')

    st = perf_counter()
    result = MatrixTree.multiply_matrices(root,root)
    end = perf_counter()

    # error
    N = np.zeros_like(X)
    result_true = X@X
    MatrixTree.update_matrices_for_nodes(result, 2**(3*k))
    err = np.sum(np.square(result.matrix - result_true))
    results.append([k, s, end-st, err])

df = pd.DataFrame(results, columns=["k", "s", "time", "error"])
df
```


Macierz po kompresji
dla Sigmy = 32, k = 2
0
10
20
30
40
50
60
0
10
20
30
40
50
60


Macierz po kompresji
dla Sigmy = 256, k = 3
0
100
200
300
400
500
0
100
200
300
400
500

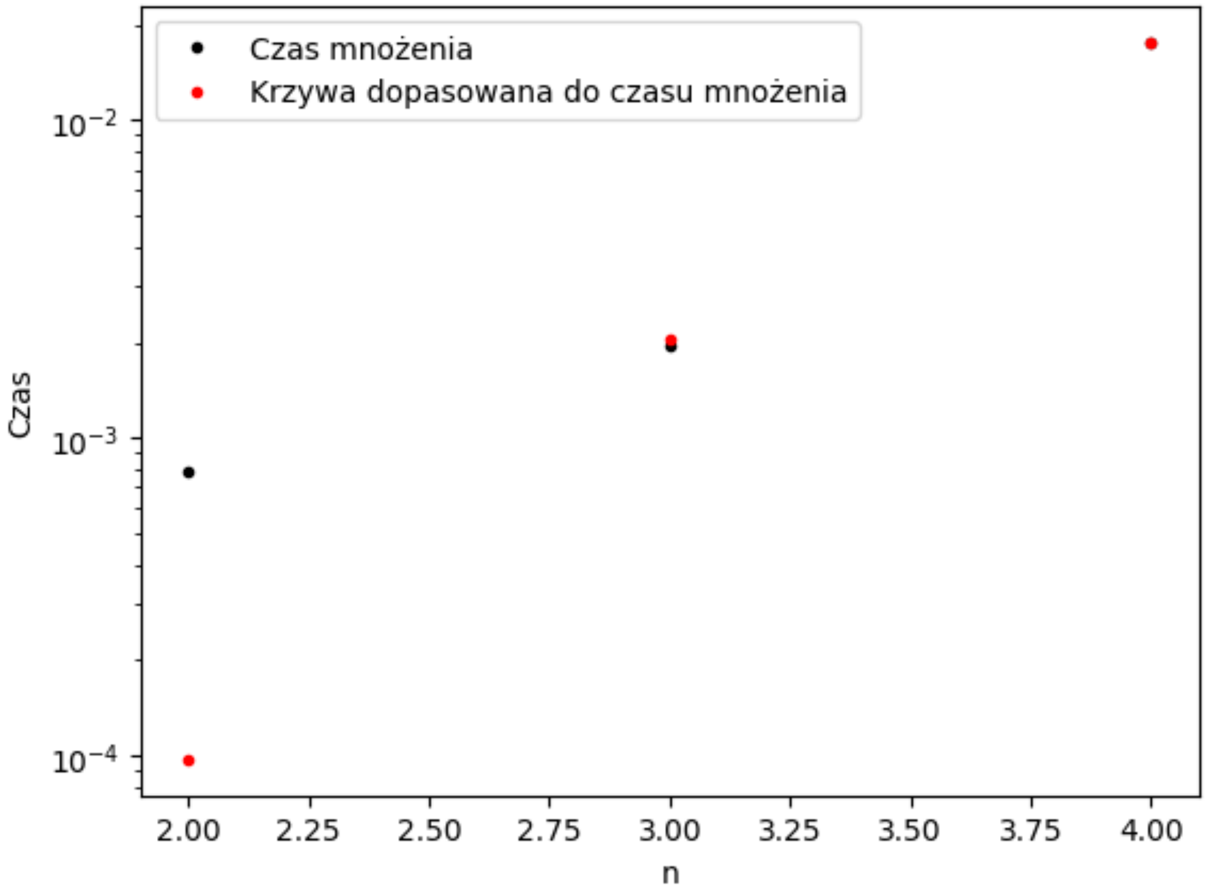


Out[ ]:

	k	s	time	error
0	2	32	0.000781	420.557835
1	3	256	0.001957	3652.719769
2	4	2048	0.017587	30468.092839

```
a, k = fit_curve("k", "time", df)
plot_with_fit(a, k , df, "k", "time", "Bineta")
```

```
Exponent beta is approximately: 7.494484167017299
Constant alpha is approximately: 5.406654366685517e-07
```


Czas mnożenia
Krzywa dopasowana do czasu mnożenia
Czas
10^{-2}
10^{-3}
10^{-4}
2.00
2.25
2.50
2.75
3.00
3.25
3.50
3.75
4.00
n