

Contents

- Define Ai
- Calculate A
- calculate partial derivatives
- linear and angular velocities using the O and Z vector from the A Matrix
- all the links are 1 unit and Theta1 and Theta2 is at zero degrees, determine the numerical Jacobian Matrix
- Determine the inverse Jacobian for the Matrix found in Question 4.
- Write a MATLAB script to solve for the inverse kinematics, (q1, q2, q3) to position the robot at 0.25x + 0.25y + 1.354z

Define Ai

```
syms q1 q2 q3 L1
```

```
A0 = eye(4)
```

```
A1 = [cos(q1) 0 sin(q1) 0; ...  
      sin(q1) 0 -cos(q1) 0; ...  
      0 1 0 L1; ...  
      0 0 0 1]
```

```
A2 = [cos(q2+pi/2) 0 sin(q2+pi/2) 0; ...  
      sin(q2+pi/2) 0 -cos(q2+pi/2) 0; ...  
      0 1 0 0; ...  
      0 0 0 1]
```

```
A3 = [1 0 0 0; ...  
      0 1 0 0; ...  
      0 0 1 q3; ...  
      0 0 0 1]
```

A0 =

1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1

A1 =

cos(q1)	0	sin(q1)	0
sin(q1)	0	-cos(q1)	0
0	1	0	L1
0	0	0	1

A2 =

cos(q2 + pi/2)	0	sin(q2 + pi/2)	0
sin(q2 + pi/2)	0	-cos(q2 + pi/2)	0
0	1	0	0
0	0	0	1

A3 =

```
[1, 0, 0, 0]
[0, 1, 0, 0]
[0, 0, 1, q3]
[0, 0, 0, 1]
```

Calculate A

```
A01 = A0*A1
A012 = A01*A2
A0123 = A012*A3
A = A0123;
```

A01 =

```
[cos(q1), 0, sin(q1), 0]
[sin(q1), 0, -cos(q1), 0]
[      0, 1,      0, L1]
[      0, 0,      0, 1]
```

A012 =

```
[cos(q1)*cos(q2 + pi/2), sin(q1), cos(q1)*sin(q2 + pi/2), 0]
[cos(q2 + pi/2)*sin(q1), -cos(q1), sin(q1)*sin(q2 + pi/2), 0]
[      sin(q2 + pi/2),      0,      -cos(q2 + pi/2), L1]
[      0,      0,      0, 1]
```

A0123 =

```
[cos(q1)*cos(q2 + pi/2), sin(q1), cos(q1)*sin(q2 + pi/2), q3*cos(q1)*sin(q2 + pi/2)]
[cos(q2 + pi/2)*sin(q1), -cos(q1), sin(q1)*sin(q2 + pi/2), q3*sin(q1)*sin(q2 + pi/2)]
[      sin(q2 + pi/2),      0,      -cos(q2 + pi/2), L1 - q3*cos(q2 + pi/2)]
[      0,      0,      0, 1]
```

calculate partial derivatives

```
dx = diff(A(1,4), q1)+diff(A(1,4), q2)+diff(A(1,4), q3)
dy = diff(A(2,4), q1)+diff(A(2,4), q2)+diff(A(2,4), q3)
dz = diff(A(3,4), q1)+diff(A(3,4), q2)+diff(A(3,4), q3)
```

dx =

$\cos(q_1) \sin(q_2 + \pi/2) + q_3 \cos(q_1) \cos(q_2 + \pi/2) - q_3 \sin(q_1) \sin(q_2 + \pi/2)$

dy =

$\sin(q_1) \sin(q_2 + \pi/2) + q_3 \cos(q_1) \sin(q_2 + \pi/2) + q_3 \cos(q_2 + \pi/2) \sin(q_1)$

dz =

$$q_3 \sin(q_2 + \pi/2) - \cos(q_2 + \pi/2)$$

linear and angular velocities using the O and Z vector from the A Matrix

```
J1 = [cross(A0(1:3,3),(A(1:3,4)-A0(1:3,4))); A0(1:3,3)]
J2 = [cross(A01(1:3,3),(A(1:3,4)-A01(1:3,4))); A01(1:3,3)]
J3 = [A012(1:3,3); [0 0 0]']
J = [J1 J2 J3]
```

J1 =

```
-q3*sin(q1)*sin(q2 + pi/2)
q3*cos(q1)*sin(q2 + pi/2)
0
0
0
1
```

J2 =

```
q3*cos(q1)*cos(q2 + pi/2)
q3*cos(q2 + pi/2)*sin(q1)
q3*sin(q1)^2*sin(q2 + pi/2) + q3*cos(q1)^2*sin(q2 + pi/2)
sin(q1)
-cos(q1)
0
```

J3 =

```
cos(q1)*sin(q2 + pi/2)
sin(q1)*sin(q2 + pi/2)
-cos(q2 + pi/2)
0
0
0
```

J =

```
[-q3*sin(q1)*sin(q2 + pi/2), q3*cos(q1)*cos(q2 + pi/2), cos(q1)*sin(q2 + pi/2)]
[ q3*cos(q1)*sin(q2 + pi/2), q3*cos(q2 + pi/2)*sin(q1), sin(q1)*sin(q2 + pi/2)]
[ 0, q3*sin(q1)^2*sin(q2 + pi/2) + q3*cos(q1)^2*sin(q2 + pi/2), -cos(q2 + pi/2)]
[ 0, sin(q1), 0]
[ 0, -cos(q1), 0]
[ 1, 0, 0]
```

all the links are 1 unit and Theta1 and Theta2 is at zero degrees, determine the numerical Jacobian Matrix

```
q1 = 0;
q2 = 0;
q3 = 0;
J = @(q1,q2,q3)([-q3*sin(q1)*sin(q2 + pi/2), q3*cos(q1)*cos(q2 + pi/2), cos(q1)*sin(q2 + pi/2);
q3*cos(q1)*sin(q2 + pi/2), q3*cos(q2 + pi/2)*sin(q1), sin(q1)*sin(q2 + pi/2);
```

```

0, q3*sin(q1)^2*sin(q2 + pi/2) + q3*cos(q1)^2*sin(q2 + pi/2), -cos(q2 + pi/2);
0, sin(q1), 0;
0, -cos(q1), 0;
1, 0, 0;]);
J(q1,q2,q3)

```

ans =

```

0      0      1.0000
0      0      0
0      0     -0.0000
0      0      0
0     -1.0000      0
1.0000      0      0

```

Determine the inverse Jacobian for the Matrix found in Question 4.

```
pinv(J(q1,q2,q3))
```

ans =

```

0      0      0      0      0      1
0      0      0      0     -1      0
1      0      0      0      0      0

```

Write a MATLAB script to solve for the inverse kinematics, (q1, q2, q3) to position the robot at 0.25x + 0.25y + 1.354z

```

L1 = 1;
L(1) = Link('alpha', pi/2, 'a', 0, 'd', L1);
L(2) = Link('alpha', pi/2, 'a', 0, 'd', 0, 'offset', pi/2);
L(3) = Link('alpha', 0, 'a', 0, 'theta', 0);
L(3).qlim = [0 1];
R1 = SerialLink(L, 'name', "Koprov HW5");
R1.plotopt = {'workspace' [-1,1,-1,1,0,1.5]};
R1.teach

Xg = 0.25;
Yg = 0.25;
Zg = 1.354;

qk1 = 0;
qk2 = 0;
qk3 = 0;

doLoop = true;
while doLoop
    FK = R1.fkine([qk1 qk2 qk3]);
    xk = FK.t(1);
    yk = FK.t(2);
    zk = FK.t(3);

    J = R1.jacob0([qk1 qk2 qk3]);
    J_inv = pinv(J);
    X = Xg - xk;

```

```

Y = Yg - yk;
Z = Zg - zk;
Xv = [X Y Z 0 0 0]';
Q = J_inv*Xv;

qk1 = qk1+Q(1);
qk2 = qk2+Q(2);
qk3 = qk3+Q(3);

R1.plot([qk1 qk2 qk3])

pos_error_x = xk -Xg;
pos_error_y = yk -Yg;
pos_error_z = zk -Zg;
p_sse = sqrt(pos_error_x^2+pos_error_y^2+pos_error_z^2);

if p_sse <= 0.001
    doLoop = false;
end
end
disp([qk1*180/pi, qk2*180/pi, qk3])

```

44.8644 45.0362 0.5003

