Exam_2 Q1

Contents

Pavel 'Pasha' Koprov

Q1

```
clear
XY = uscity('XY', mand({'Amarillo' 'Anderson'}, uscity('Name'),...
    {'TX' 'SC'}, uscity('ST')));
d = dists(XY(1,:),XY(2,:),'mi')*1.2;
cu = [6 3];         % unit cubic volume ft3
wt = [12 96];       % unit weight lb

T = 13;
D = ([64 64 56 31 166 62 53 96 126 143 39 45 55;
    125 101 40 26 89 104 87 22 57 81 297 163 113].*wt'/2000)'; % demand in tons

ppiTL = 136.3; % Jan 2020 (P)
r = 2*(ppiTL/102.7);
tr = struct('r',r,'Kwt',25,'Kcu',2750);
s = wt./cu; % lb/cft
v = [120 80].*(2000./wt); % $/ton
hobs = [0.2 0.1]/13;
h = (hobs + 0.11)/(365.2/7);

cTL=tr.r*d;         % transportation cost per truck per ton
qmax = maxpayld(s,tr)
Q = [sum(D);qmax];
Cp=[v;0 0]      % cost to fabricate a ton
Ct=[0 0;cTL cTL]
yinit = [0 0;[77 100].*wt/2000];   % initial storage (ton)
yfinal = yinit;     % final storage
ymax=[sum(D);qmax]

Ci = cumsum(Cp+Ct./Q).*h  % inventory cost
M = 2;              % number of stages
G = 2;              % number of products produced = 2
```

```
qmax =

    2.7500   25.0000


Cp =

   1.0e+04 *

    2.0000    0.1667
         0         0


Ct =

   1.0e+03 *
```

```
         0         0
    3.4590    3.4590


ymax =

    6.0000   62.6400
    2.7500   25.0000


Ci =

   48.0664    3.7598
   51.0894    4.0719
```

Create MILP model

```
Cp = reshape(repmat(Cp,[T 1 1]),M,T,G)     % create M x T x G array (3-D)
Ci = reshape(repmat(Ci,[T+1 1 1]),M,T+1,G) % create M x (T+1) x G array
Ci(:,1,:) = 0   % intital inventory cost already accounted for last period
Ct=reshape(repmat(Ct,[T 1 1]),M,T,G)
clear mp
mp = Milp('PPlan');
mp.addobj('min',Cp,Ci,Ct)  % Objective
for g = 1:G
    for t = 1:T  % Flow balance constraints
        for m = 1:M-1
            mp.addcstr({[1 -1],{[m m+1],t,g}},{[1 -1],{m,[t t+1],g}},0,'=',0)
        end
        mp.addcstr({M,t,g},{[1 -1],{M,[t t+1],g}},0,'=',D(t,g))
   for m = 1:M
       mp.addcstr({m,t,g},0,'<=',{Q(m,g),{m,t,g}})
   end
    end
end
mp.addlb(0,horzcat(reshape(yinit,M,1,G),zeros(M,T-1,G),reshape(yfinal,M,1,G)),0)  % Lower bounds
mp.addub(Inf,horzcat(reshape(yinit,M,1,G),repmat(reshape(ymax,M,1,G),1,T-1),reshape(yfinal,M,1,G)), Inf)
mp.addctype('C','C','I');
```

```
Cp(:,:,1) =

  Columns 1 through 6

       20000       20000       20000       20000       20000       20000
           0           0           0           0           0           0

  Columns 7 through 12

       20000       20000       20000       20000       20000       20000
           0           0           0           0           0           0

  Column 13

       20000
           0


Cp(:,:,2) =

   1.0e+03 *

  Columns 1 through 7

    1.6667    1.6667    1.6667    1.6667    1.6667    1.6667    1.6667
         0         0         0         0         0         0         0

  Columns 8 through 13

    1.6667    1.6667    1.6667    1.6667    1.6667    1.6667
         0         0         0         0         0         0


Ci(:,:,1) =

  Columns 1 through 7

   48.0664   48.0664   48.0664   48.0664   48.0664   48.0664   48.0664
   51.0894   51.0894   51.0894   51.0894   51.0894   51.0894   51.0894

  Columns 8 through 14

   48.0664   48.0664   48.0664   48.0664   48.0664   48.0664   48.0664
   51.0894   51.0894   51.0894   51.0894   51.0894   51.0894   51.0894


Ci(:,:,2) =
  Columns 1 through 7

    3.7598    3.7598    3.7598    3.7598    3.7598    3.7598    3.7598
    4.0719    4.0719    4.0719    4.0719    4.0719    4.0719    4.0719

  Columns 8 through 14

    3.7598    3.7598    3.7598    3.7598    3.7598    3.7598    3.7598
    4.0719    4.0719    4.0719    4.0719    4.0719    4.0719    4.0719


Ci(:,:,1) =

  Columns 1 through 7

         0   48.0664   48.0664   48.0664   48.0664   48.0664   48.0664
         0   51.0894   51.0894   51.0894   51.0894   51.0894   51.0894

  Columns 8 through 14

   48.0664   48.0664   48.0664   48.0664   48.0664   48.0664   48.0664
   51.0894   51.0894   51.0894   51.0894   51.0894   51.0894   51.0894


Ci(:,:,2) =

  Columns 1 through 7

         0    3.7598    3.7598    3.7598    3.7598    3.7598    3.7598
         0    4.0719    4.0719    4.0719    4.0719    4.0719    4.0719

  Columns 8 through 14

    3.7598    3.7598    3.7598    3.7598    3.7598    3.7598    3.7598
    4.0719    4.0719    4.0719    4.0719    4.0719    4.0719    4.0719


Ct(:,:,1) =

   1.0e+03 *

  Columns 1 through 7

         0         0         0         0         0         0         0
    3.4590    3.4590    3.4590    3.4590    3.4590    3.4590    3.4590

  Columns 8 through 13

         0         0         0         0         0         0
    3.4590    3.4590    3.4590    3.4590    3.4590    3.4590


Ct(:,:,2) =

   1.0e+03 *

  Columns 1 through 7

         0         0         0         0         0         0         0
    3.4590    3.4590    3.4590    3.4590    3.4590    3.4590    3.4590

  Columns 8 through 13

         0         0         0         0         0         0
    3.4590    3.4590    3.4590    3.4590    3.4590    3.4590
```

Solve using Gurobi

```
clear params
model = mp.milp2gb
params.outputflag = 1;
result = gurobi(model, params);
x = mp.namesolution(result.x)
TC = result.objval
```

```
model = 

  struct with fields:

        name: 'PPlan'
   modelsense: 'minimize'
         obj: [160×1 double]
          lb: [160×1 double]
          ub: [160×1 double]
       vtype: 'CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII
           A: [104×160 double]
       sense: [104×1 char]
```

```
           rhs: [104×1 double]


--------------------------------------------
Warning: your license will expire in 14 days
--------------------------------------------

Academic license - for non-commercial use only
Gurobi Optimizer version 9.0.3 build v9.0.3rc0 (win64)
Optimize a model with 104 rows, 160 columns and 286 nonzeros
Model fingerprint: 0x830f3373
Variable types: 108 continuous, 52 integer (0 binary)
Coefficient statistics:
  Matrix range     [1e+00, 6e+01]
  Objective range  [1e-08, 2e+04]
  Bounds range     [5e-01, 6e+01]
  RHS range        [2e-01, 1e+01]
Found heuristic solution: objective 270019.51497
Presolve removed 56 rows and 88 columns
Presolve time: 0.04s
Presolved: 48 rows, 72 columns, 132 nonzeros
Found heuristic solution: objective 259731.18652
Variable types: 46 continuous, 26 integer (5 binary)

Root relaxation: objective 2.444819e+05, 44 iterations, 0.00 seconds

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

     0     0 244481.897    0    6 259731.187 244481.897  5.87%     -    0s
H    0     0                    253110.12756 244481.897  3.41%     -    0s
H    0     0                    246646.37495 244481.897  0.88%     -    0s
H    0     0                    246644.33489 244481.897  0.88%     -    0s
     0     0 244553.550    0    6 246644.335 244553.550  0.85%     -    0s
     0     0 244657.869    0    2 246644.335 244657.869  0.81%     -    0s
H    0     0                    246368.01852 244657.869  0.69%     -    0s
     0     0 244702.135    0    8 246368.019 244702.135  0.68%     -    0s
H    0     0                    246252.01763 244702.135  0.63%     -    0s
     0     0 244859.429    0   10 246252.018 244859.429  0.57%     -    0s
     0     0 245441.440    0    8 246252.018 245441.440  0.33%     -    0s
     0     0 245494.512    0   10 246252.018 245494.512  0.31%     -    0s
     0     0 245516.982    0   10 246252.018 245516.982  0.30%     -    0s
     0     0 245519.228    0   12 246252.018 245519.228  0.30%     -    0s
     0     0 245525.940    0   12 246252.018 245525.940  0.29%     -    0s
     0     0 245527.949    0   14 246252.018 245527.949  0.29%     -    0s
     0     0 245530.611    0   12 246252.018 245530.611  0.29%     -    0s
     0     0 245530.611    0   12 246252.018 245530.611  0.29%     -    0s
     0     2 245530.611    0   12 246252.018 245530.611  0.29%     -    0s
H    3     3                    246225.40356 245661.133  0.23%   4.7    0s
H   10     2                    246208.52952 245944.776  0.11%   4.9    0s
*   15     0               5    246187.38810 246187.388  0.00%   4.9    0s

Cutting planes:
  Gomory: 2
  Implied bound: 1
  MIR: 12
  Flow cover: 1
  Relax-and-lift: 1

Explored 16 nodes (171 simplex iterations) in 0.11 seconds
Thread count was 4 (of 4 available processors)

Solution count 9: 246187 246209 246225 ... 270020

Optimal solution found (tolerance 1.00e-04)
Best objective 2.461873881011e+05, best bound 2.461873881011e+05, gap 0.0000%

x = 

  struct with fields:

    Cp: [2×13×2 double]
    Ci: [2×14×2 double]
    Ct: [2×13×2 double]


TC =

   2.4619e+05
```

Report results

```
Fp = x.Cp;
Fi = x.Ci;
Ft = x.Ct;
for g = 1:G
   mdisp(D(:,g)',[],[],['D' num2str(g)])
   mdisp(Fp(:,:,g),[],[],['Fp' num2str(g)])
   mdisp(Fi(:,:,g),[],[],['Fi' num2str(g)])
   mdisp(Ft(:,:,g),[],[],['Ft' num2str(g)])
end
TCp = sum(sum(sum(Cp.*Fp)));
TCi = sum(sum(sum(Ci.*Fi)));
TCt = sum(sum(sum(Ct.*Ft)));
vdisp('TCp,TCi,TCt,TC')
```

```
D1:     1      2      3      4      5      6      7      8      9     10     11    12    13
--:-------------------------------------------------------------------------------------------
 1:  0.384  0.384  0.336  0.186  0.996  0.372  0.318  0.576  0.756  0.858  0.234  0.27  0.33

Fp1:   1      2     3   4     5    6   7   8    9   10  11  12  13
---:----------------------------------------------------------------
  1:   0  0.8280   0   0  2.42   0   0   0  2.75   0   0   0   0
  2:   0  0.8280   0   0  2.42   0   0   0  2.75   0   0   0   0

Fi1:     1      2       3       4      5    6     7      8       9     10    11    12     13     14
---:--------------------------------------------------------------------------------------------------
  1:  0.000  0.0000  0.0000  0.0000   0  0.00  0.00  0.0000  0.0000  0.00  0.00  0.00  0.000  0.000
  2:  0.462  0.0780  0.5220  0.1860   0  1.43  1.05  0.7360  0.1600  2.15  1.30  1.06  0.792  0.462

Ft1:   1   2   3   4   5   6   7   8   9  10  11  12  13
---:--------------------------------------------------------
  1:   0   1   0   0   1   0   0   0   1   0   0   0   0
  2:   0   1   0   0   1   0   0   0   1   0   0   0   0

D2:   1    2     3     4     5     6     7     8     9    10     11    12    13
--:--------------------------------------------------------------------------------
 1:   6  4.85  1.92  1.25  4.27  4.99  4.18  1.06  2.74  3.89  14.26  7.82  5.42

Fp2:     1    2   3   4   5     6    7   8   9  10  11  12  13
---:--------------------------------------------------------------
  1:  13.49   0   0   0   0  24.15   0   0   0   0  25   0   0
  2:  13.49   0   0   0   0  24.15   0   0   0   0  25   0   0

Fi2:    1      2     3     4     5    6     7      8      9     10    11     12     13    14
---:-------------------------------------------------------------------------------------------
  1:  0.00   0.00  0.00  0.00  0.00   0   0.00   0.00   0.00   0.00  0.00   0.00   0.00  0.00
  2:  4.80  12.29  7.44  5.52  4.27   0  19.16  14.98  13.93  11.19  7.30  18.05  10.22  4.80

Ft2:   1   2   3   4   5   6   7   8   9  10  11  12  13
---:--------------------------------------------------------
  1:   1  -0   0  -0   0   1  -0  -0  -0  -0   1   0  -0
  2:   1  -0  -0  -0  -0   1  -0  -0  -0  -0   1   0  -0

 :    TCp       TCi       TCt         TC
-:---------------------------------------------
1:  224,400  1,033.14  20,754.24  246,187.39
```

Q2

```
clear, close all
df = table2struct(readtable('Exam2DataF20.xlsx'));

XY = [[df.Longitude]' [df.Latitude]'];
q = [df(2:end).Weight]'/2000;
s = [df(2:end).Density]';
tL = 20/60;
tU = 5/60;

sh = vec2struct('b',1,'e',[df(2:end).Customer]', 'q', q, 's', s);
tr = struct('b',1,'e',1,'tbmin',7,'temax',17,'Kwt',25,'Kcu',2750,...
    'maxTC', 10);

i = find([sh.q]'*2000./[sh.s]'/tr.Kcu > 1) % which shipment is above truck cubic capacity
srpls = sh(i).q*2000/sh(i).s/tr.Kcu - 1 % fraction of the surpluss

sh(end+1) = sh(i); % add additional shipment
sh(end).q = srpls*sh(i).q;

sh(i).q = sh(i).q - sh(end).q; % subtract additional shipment from overcubic shipment

sh = vec2struct(sh,'tU',[sh.q]'*tU, 'tbmin',7,'temax',17);
sdisp(sh)
```

```
i =

    28


srpls =

    0.0160


sh:  b   e    q     s      tU    tbmin  temax
```

```
--:-----------------------------------------
 1:  1   2  1.53   6.39  0.1276    7      17
 2:  1   3  1.22  18.06  0.1018    7      17
 3:  1   4  1.47  12.64  0.1225    7      17
 4:  1   5  2.07  18.53  0.1722    7      17
 5:  1   6  1.26   7.77  0.1053    7      17
 6:  1   7  1.26   2.65  0.1047    7      17
 7:  1   8  1.98  13.32  0.1652    7      17
 8:  1   9  1.22   2.70  0.1020    7      17
 9:  1  10  1.61   6.68  0.1340    7      17
10:  1  11  1.24   5.59  0.1034    7      17
11:  1  12  1.75   4.68  0.1458    7      17
12:  1  13  1.26   3.48  0.1050    7      17
13:  1  14  2.16   3.41  0.1798    7      17
14:  1  15  2.09   8.37  0.1743    7      17
15:  1  16  1.38  12.83  0.1152    7      17
16:  1  17  1.40   7.18  0.1169    7      17
17:  1  18  1.75   3.88  0.1459    7      17
18:  1  19  1.86   2.18  0.1548    7      17
19:  1  20  1.12   5.70  0.0935    7      17
20:  1  21  2.02  20.14  0.1681    7      17
21:  1  22  1.79  11.94  0.1489    7      17
22:  1  23  1.22  14.27  0.1019    7      17
23:  1  24  1.37   1.60  0.1144    7      17
24:  1  25  1.33   9.47  0.1111    7      17
25:  1  26  1.51  17.86  0.1260    7      17
26:  1  27  2.70   5.93  0.2253    7      17
27:  1  28  1.37   9.37  0.1139    7      17
28:  1  29  1.61   1.17  0.1340    7      17
29:  1  30  1.45   4.23  0.1210    7      17
30:  1  31  2.22  10.49  0.1852    7      17
31:  1  32  1.32   8.69  0.1097    7      17
32:  1  33  1.25  10.32  0.1042    7      17
33:  1  34  1.26   2.37  0.1050    7      17
34:  1  35  1.49   5.35  0.1240    7      17
35:  1  36  1.87   5.54  0.1558    7      17
36:  1  37  1.47   7.37  0.1229    7      17
37:  1  38  1.63  10.77  0.1357    7      17
38:  1  39  1.25  10.91  0.1043    7      17
39:  1  40  1.62   4.33  0.1347    7      17
40:  1  41  1.33  10.06  0.1107    7      17
41:  1  42  1.34   7.35  0.1118    7      17
42:  1  43  1.73   8.23  0.1441    7      17
43:  1  44  1.63  24.33  0.1358    7      17
44:  1  45  1.39   8.40  0.1158    7      17
45:  1  46  2.38  21.38  0.1981    7      17
46:  1  29  0.03   1.17  0.0022    7      17
```

Get road network

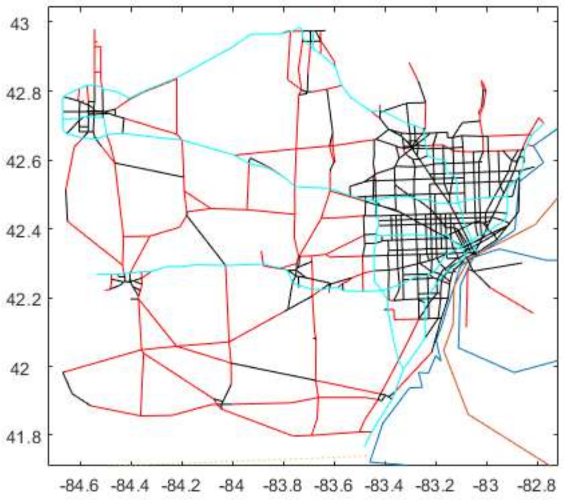
```
expansionAroundXY = 0.1;
[XY2,IJD,isXY,isIJD] = subgraph(usrdnode('XY'),...
   isinrect(usrdnode('XY'),boundrect(XY,expansionAroundXY)),...
   usrdlink('IJD'));
```

Label type of road

```
s = usrdlink(isIJD);
isI = s.Type == 'I';         % Interstate highways
isIR = isI & s.Urban == ' '; % Rural Interstate highways
isIU = isI & ~isIR;          % Urban Interstate highways
isR = s.Urban == ' ' & ~isI; % Rural non-Interstate roads
isU = ~isI & ~isR;           % Urban non-Interstate roads
```

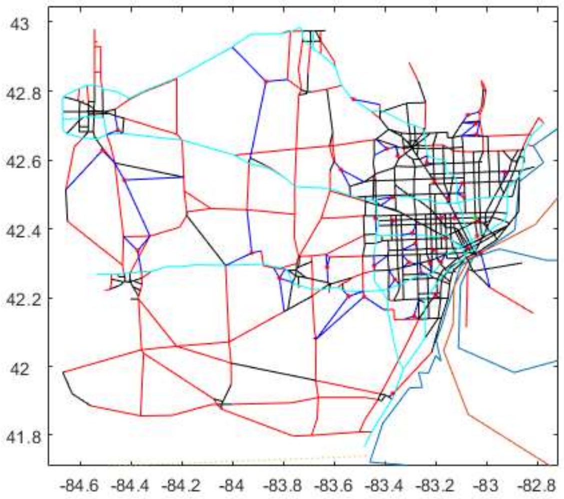
Plot roads

```
makemap(XY2,0.03)  % 3% expansion
h = [];  % Keep handle to each plot for legend
h = [h pplot(IJD(isR,:),XY2,'r-','DisplayName','Rural Roads')];
h = [h pplot(IJD(isU,:),XY2,'k-','DisplayName','Urban Roads')];
h = [h pplot(IJD(isI,:),XY2,'c-','DisplayName','Interstate Roads')];
```



Add connector roads from customers to road network

```
[IJD11,IJD12,IJD22] = addconnector(XY,XY2,IJD);
h = [h pplot(IJD12,[XY; XY2],'b-','DisplayName','Connector Roads')];
h = [h pplot(XY(2:end,:),'r.','DisplayName','Customers')];
h = [h pplot(XY(1,:),'g.','DisplayName','DC')];
```



Convert road distances to travel times (needs to be after ADDCONNECTOR)

```
v.IR = 75;  % Rural Interstate highways average speed (mph)
v.IU = 65;  % Urban Interstate highways average speed (mph)
v.R = 50;   % Rural non-Interstate roads average speed (mph)
v.U = 25;   % Urban non-Interstate roads average speed (mph)
v.C = 20;   % Facility to road connector average speed (mph)

IJT = IJD;
IJT(isIR,3) = IJD(isIR,3)/v.IR;
IJT(isIU,3) = IJD(isIU,3)/v.IU;
IJT(isR,3) = IJD(isR,3)/v.R;
IJT(isU,3) = IJD(isU,3)/v.U;

IJT22 = IJD22;                  % road to road
IJT22(:,3) = IJT(:,3);
IJT12 = IJD12;                  % facility to road
IJT12(:,3) = IJD12(:,3)/v.C;  % (IJD11 facility to facility arcs ignored)
```

Shortest time routes

```
n = size(XY,1);
[T,P] = dijk(list2adj([IJT12; IJT22]),1:n,1:n);
T = T+5/60;
```

Construct & improve routes:

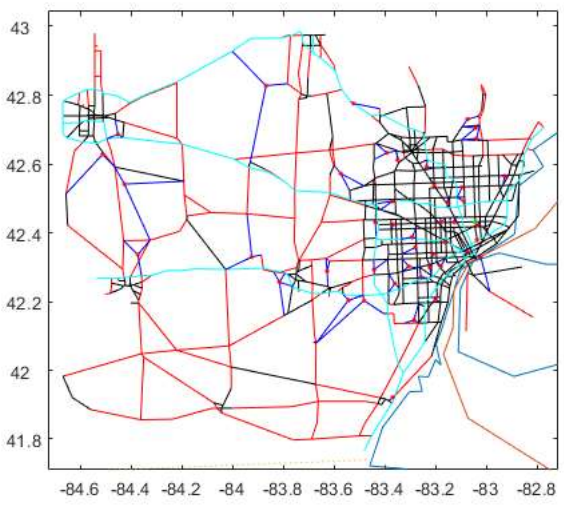
```
rTCh = @(rte) rteTC(rte,sh,T,tr);
tic
IJS = pairwisesavings(rTCh,sh); toc
tic
r = twoopt(savings(rTCh,sh,IJS),rTCh); toc
```

```
Elapsed time is 8.692237 seconds.
Elapsed time is 7.420696 seconds.
```

add any single-shipment routes

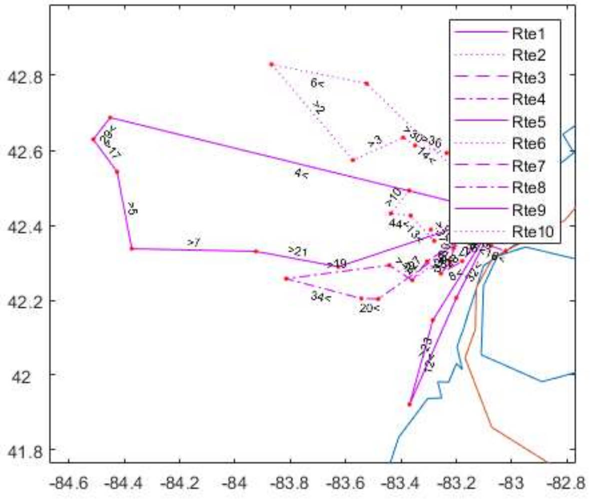
```
[r,~,Time] = sh2rte(sh,r,rTCh);
```

```
ADD SINGLE-SHIPMENT ROUTES:
34.487040: Added shipments 18 28
```



Plot routes

```
plotshmt(sh,XY,r,tr)
```



Display route output structure

```
[TC,Xflg,out] = rTCh(r);
for i = 1:length(out), sdisp(out(i),false,i), end
```

```
 1:  Rte  Loc  Cost  Arrive  Wait  TWmin  Start    LU    Depart  TWmax  Total
--:--------------------------------------------------------------------------
 1:    0    1  0.00    0.00    0       7   7.00  0.0000    7.00   Inf    0.00
 2:   21    1  0.00    7.00    0       7   7.00  0.0000    7.00   Inf    0.00
 3:    7    1  0.00    7.00    0       7   7.00  0.0000    7.00   Inf    0.00
 4:    5    1  0.00    7.00    0       7   7.00  0.0000    7.00   Inf    0.00
 5:    4    1  0.00    7.00    0       7   7.00  0.0000    7.00   Inf    0.00
 6:   29    1  0.00    7.00    0       7   7.00  0.0000    7.00   Inf    0.00
 7:   17    1  0.00    7.00    0       7   7.00  0.0000    7.00   Inf    0.00
 8:    4    5  1.33    8.33    0    -Inf   8.33  0.1722    8.50    17    1.50
 9:   29   30  0.42    8.92    0    -Inf   8.92  0.1210    9.04    17    0.54
10:   17   18  0.52    9.56    0    -Inf   9.56  0.1459    9.71    17    0.67
11:    5    6  0.98   10.69    0    -Inf  10.69  0.1053   10.80    17    1.09
12:    7    8  0.94   11.73    0    -Inf  11.73  0.1652   11.90    17    1.10
13:   21   22  0.72   12.62    0    -Inf  12.62  0.1489   12.77    17    0.87
14:    0    1  1.03   13.80    0    -Inf  13.80  0.0000   13.80    17    1.03
```

```
 2:  Rte  Loc  Cost  Arrive  Wait  TWmin  Start    LU    Depart  TWmax  Total
--:--------------------------------------------------------------------------
 1:    0    1  0.00    0.00    0       7   7.00  0.0000    7.00   Inf    0.00
 2:   36    1  0.00    7.00    0       7   7.00  0.0000    7.00   Inf    0.00
 3:    3    1  0.00    7.00    0       7   7.00  0.0000    7.00   Inf    0.00
 4:   30    1  0.00    7.00    0       7   7.00  0.0000    7.00   Inf    0.00
 5:    2    1  0.00    7.00    0       7   7.00  0.0000    7.00   Inf    0.00
 6:   14    1  0.00    7.00    0       7   7.00  0.0000    7.00   Inf    0.00
 7:    6    1  0.00    7.00    0       7   7.00  0.0000    7.00   Inf    0.00
 8:   14   15  0.82    7.82    0    -Inf   7.82  0.1743    8.00    17    1.00
 9:    6    7  0.80    8.80    0    -Inf   8.80  0.1047    8.90    17    0.91
10:    2    3  1.09   10.00    0    -Inf  10.00  0.1018   10.10    17    1.19
11:    3    4  0.70   10.80    0    -Inf  10.80  0.1225   10.92    17    0.82
12:   30   31  0.36   11.28    0    -Inf  11.28  0.1852   11.47    17    0.55
13:   36   37  0.45   11.91    0    -Inf  11.91  0.1229   12.04    17    0.57
14:    0    1  0.57   12.60    0    -Inf  12.60  0.0000   12.60    17    0.57
```

```
 3:  Rte  Loc    Cost   Arrive  Wait  TWmin  Start    LU    Depart  TWmax   Total
--:------------------------------------------------------------------------------
 1:    0    1  0.0000    0.00    0       7   7.00  0.0000    7.00   Inf   0.0000
 2:    1    1  0.0000    7.00    0       7   7.00  0.0000    7.00   Inf   0.0000
 3:   25    1  0.0000    7.00    0       7   7.00  0.0000    7.00   Inf   0.0000
 4:   45    1  0.0000    7.00    0       7   7.00  0.0000    7.00   Inf   0.0000
 5:   40    1  0.0000    7.00    0       7   7.00  0.0000    7.00   Inf   0.0000
 6:   41    1  0.0000    7.00    0       7   7.00  0.0000    7.00   Inf   0.0000
 7:   26    1  0.0000    7.00    0       7   7.00  0.0000    7.00   Inf   0.0000
 8:   22    1  0.0000    7.00    0       7   7.00  0.0000    7.00   Inf   0.0000
 9:   43    1  0.0000    7.00    0       7   7.00  0.0000    7.00   Inf   0.0000
10:   41   42  0.5888    7.59    0    -Inf   7.59  0.1118    7.70    17   0.7005
11:   26   27  0.5516    8.25    0    -Inf   8.25  0.2253    8.48    17   0.7769
12:   22   23  0.3114    8.79    0    -Inf   8.79  0.1019    8.89    17   0.4133
13:   43   44  0.3586    9.25    0    -Inf   9.25  0.1358    9.39    17   0.4945
14:   45   46  0.7890   10.17    0    -Inf  10.17  0.1981   10.37    17   0.9871
15:    1    2  0.2578   10.63    0    -Inf  10.63  0.1276   10.76    17   0.3854
16:   40   41  0.4116   11.17    0    -Inf  11.17  0.1107   11.28    17   0.5224
17:   25   26  0.3225   11.60    0    -Inf  11.60  0.1260   11.73    17   0.4485
18:    0    1  0.3572   12.09    0    -Inf  12.09  0.0000   12.09    17   0.3572
```

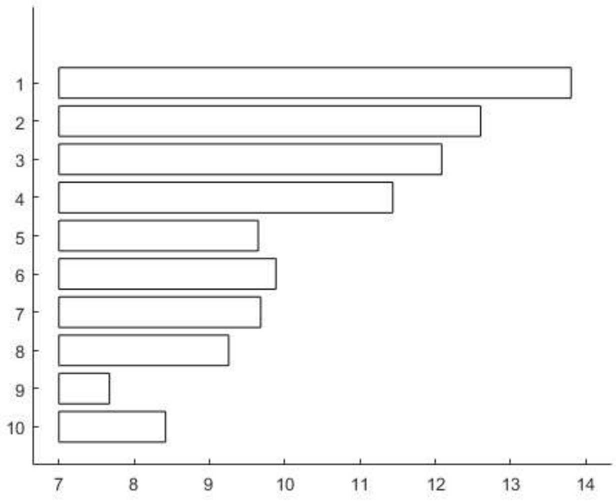
```
 4:  Rte  Loc    Cost   Arrive  Wait  TWmin  Start    LU    Depart  TWmax   Total
--:------------------------------------------------------------------------------
 1:    0    1  0.0000    0.00    0       7   7.00  0.0000    7.00   Inf   0.0000
 2:   27    1  0.0000    7.00    0       7   7.00  0.0000    7.00   Inf   0.0000
 3:   46    1  0.0000    7.00    0       7   7.00  0.0000    7.00   Inf   0.0000
 4:   19    1  0.0000    7.00    0       7   7.00  0.0000    7.00   Inf   0.0000
 5:   33    1  0.0000    7.00    0       7   7.00  0.0000    7.00   Inf   0.0000
 6:   34    1  0.0000    7.00    0       7   7.00  0.0000    7.00   Inf   0.0000
 7:   20    1  0.0000    7.00    0       7   7.00  0.0000    7.00   Inf   0.0000
 8:   33   34  0.7256    7.73    0    -Inf   7.73  0.1050    7.83    17   0.8305
 9:   20   21  0.3326    8.16    0    -Inf   8.16  0.1681    8.33    17   0.5007
10:   34   35  0.5569    8.89    0    -Inf   8.89  0.1240    9.01    17   0.6809
11:   19   20  0.6639    9.68    0    -Inf   9.68  0.0935    9.77    17   0.7574
12:   46   29  0.3973   10.17    0    -Inf  10.17  0.0022   10.17    17   0.3995
13:   27   28  0.4514   10.62    0    -Inf  10.62  0.1139   10.73    17   0.5653
14:    0    1  0.7024   11.44    0    -Inf  11.44  0.0000   11.44    17   0.7024
```

```
5:  Rte  Loc    Cost   Arrive  Wait  TWmin  Start    LU    Depart  TWmax   Total
-:------------------------------------------------------------------------------
1:    0    1  0.0000   0.00     0       7   7.00  0.0000   7.00    Inf   0.0000
2:   23    1  0.0000   7.00     0       7   7.00  0.0000   7.00    Inf   0.0000
3:   32    1  0.0000   7.00     0       7   7.00  0.0000   7.00    Inf   0.0000
4:   12    1  0.0000   7.00     0       7   7.00  0.0000   7.00    Inf   0.0000
5:   32   33  0.7153   7.72     0    -Inf   7.72  0.1042   7.82     17   0.8195
6:   12   13  0.4951   8.31     0    -Inf   8.31  0.1050   8.42     17   0.6001
7:   23   24  0.5409   8.96     0    -Inf   8.96  0.1144   9.07     17   0.6553
8:    0    1  0.5744   9.65     0    -Inf   9.65  0.0000   9.65     17   0.5744
```

```
 6:  Rte  Loc    Cost   Arrive  Wait  TWmin  Start    LU    Depart  TWmax   Total
--:------------------------------------------------------------------------------
 1:    0    1  0.0000   0.00     0       7   7.00  0.0000   7.00    Inf   0.0000
 2:   10    1  0.0000   7.00     0       7   7.00  0.0000   7.00    Inf   0.0000
 3:   15    1  0.0000   7.00     0       7   7.00  0.0000   7.00    Inf   0.0000
 4:   37    1  0.0000   7.00     0       7   7.00  0.0000   7.00    Inf   0.0000
 5:   44    1  0.0000   7.00     0       7   7.00  0.0000   7.00    Inf   0.0000
 6:   13    1  0.0000   7.00     0       7   7.00  0.0000   7.00    Inf   0.0000
 7:   15   16  0.5184   7.52     0    -Inf   7.52  0.1152   7.63     17   0.6336
 8:   37   38  0.2515   7.89     0    -Inf   7.89  0.1357   8.02     17   0.3873
 9:   13   14  0.4055   8.43     0    -Inf   8.43  0.1798   8.61     17   0.5853
10:   44   45  0.2804   8.89     0    -Inf   8.89  0.1158   9.00     17   0.3962
11:   10   11  0.2803   9.28     0    -Inf   9.28  0.1034   9.39     17   0.3837
12:    0    1  0.4965   9.88     0    -Inf   9.88  0.0000   9.88     17   0.4965

 7:  Rte  Loc   Cost   Arrive  Wait  TWmin  Start    LU    Depart  TWmax   Total
--:---------------------------------------------------------------------------
 1:    0    1  0.0000   0.00     0       7   7.00  0.0000   7.00    Inf   0.0000
 2:   24    1  0.0000   7.00     0       7   7.00  0.0000   7.00    Inf   0.0000
 3:    8    1  0.0000   7.00     0       7   7.00  0.0000   7.00    Inf   0.0000
 4:    9    1  0.0000   7.00     0       7   7.00  0.0000   7.00    Inf   0.0000
 5:   39    1  0.0000   7.00     0       7   7.00  0.0000   7.00    Inf   0.0000
 6:   38    1  0.0000   7.00     0       7   7.00  0.0000   7.00    Inf   0.0000
 7:    9   10  0.4134   7.41     0    -Inf   7.41  0.1340   7.55     17   0.5474
 8:    8    9  0.2759   7.82     0    -Inf   7.82  0.1020   7.93     17   0.3780
 9:   38   39  0.3531   8.28     0    -Inf   8.28  0.1043   8.38     17   0.4573
10:   39   40  0.2575   8.64     0    -Inf   8.64  0.1347   8.77     17   0.3921
11:   24   25  0.4093   9.18     0    -Inf   9.18  0.1111   9.30     17   0.5204
12:    0    1  0.3856   9.68     0    -Inf   9.68  0.0000   9.68     17   0.3856

 8:  Rte  Loc   Cost   Arrive  Wait  TWmin  Start    LU    Depart  TWmax   Total
--:---------------------------------------------------------------------------
 1:    0    1  0.0000   0.00     0       7   7.00  0.0000   7.00    Inf   0.0000
 2:   11    1  0.0000   7.00     0       7   7.00  0.0000   7.00    Inf   0.0000
 3:   42    1  0.0000   7.00     0       7   7.00  0.0000   7.00    Inf   0.0000
 4:   31    1  0.0000   7.00     0       7   7.00  0.0000   7.00    Inf   0.0000
 5:   35    1  0.0000   7.00     0       7   7.00  0.0000   7.00    Inf   0.0000
 6:   16    1  0.0000   7.00     0       7   7.00  0.0000   7.00    Inf   0.0000
 7:   11   12  0.1463   7.15     0    -Inf   7.15  0.1458   7.29     17   0.2921
 8:   31   32  0.3417   7.63     0    -Inf   7.63  0.1097   7.74     17   0.4513
 9:   16   17  0.2346   7.98     0    -Inf   7.98  0.1169   8.09     17   0.3516
10:   35   36  0.2426   8.34     0    -Inf   8.34  0.1558   8.49     17   0.3984
11:   42   43  0.3426   8.84     0    -Inf   8.84  0.1441   8.98     17   0.4868
12:    0    1  0.2747   9.25     0    -Inf   9.25  0.0000   9.25     17   0.2747

9:  Rte  Loc   Cost   Arrive  Wait  TWmin  Start    LU    Depart  TWmax   Total
-:---------------------------------------------------------------------------
1:    0    1  0.0000   0.00     0       7   7.00  0.0000   7.00    Inf   0.0000
2:   18    1  0.0000   7.00     0       7   7.00  0.0000   7.00    Inf   0.0000
3:   18   19  0.2587   7.26     0    -Inf   7.26  0.1548   7.41     17   0.4135
4:    0    1  0.2587   7.67     0    -Inf   7.67  0.0000   7.67     17   0.2587

10:  Rte  Loc   Cost   Arrive  Wait  TWmin  Start    LU    Depart  TWmax   Total
--:---------------------------------------------------------------------------
 1:    0    1  0.0000   0.00     0       7   7.00  0.0000   7.00    Inf   0.0000
 2:   28    1  0.0000   7.00     0       7   7.00  0.0000   7.00    Inf   0.0000
 3:   28   29  0.6417   7.64     0    -Inf   7.64  0.1340   7.78     17   0.7757
 4:    0    1  0.6417   8.42     0    -Inf   8.42  0.0000   8.42     17   0.6417
```

Display Gantt chort of route spans

```
b= arrayfun(@(x) (x.Start(1)),out); b = b(:);
e= arrayfun(@(x) (x.Depart(end)),out); e = e(:);
figure
gantt([b e])
```



Route time and delivery cubic ft

```
for i = 1:length(r)
    idx = r{i}(isorigin(r{i}));
    Maxload(i) = sum([sh(idx).q]'*2000./[sh(idx).s]');
end
vdisp('Time, Maxload')
```

```
  :  Time   Maxload
--:----------------
 1:  6.80  2,734.53
 2:  5.60  2,639.77
 3:  5.09  2,717.57
 4:  4.44  2,549.38
 5:  2.65  2,682.73
 6:  2.88  2,558.49
 7:  2.68  2,645.69
 8:  2.25  2,536.57
 9:  0.67  1,704.13
10:  1.42  2,749.30
```

number of trucks

```
m = length(Time);
M = 1:m;
V = 10;
v = Time;
mp = Milp('# of Trucks');
mp.addobj('min',ones(1,m),zeros(m));
for i = M
   mp.addcstr({V,{i}},'>=',{v+tL,{i,':'}})   % V*yi >= sum_j(vj*xij)
end
for j = M
   mp.addcstr(0,{':',j},'=',1)            % sum_i(xij) = 1
end
mp.addctype('B','B')
```

Use INTLINPROG to solve

```
ilp = mp.milp2ilp;
x = intlinprog(ilp{:});
x = mp.namesolution(x);
B = arrayfun(@(i) find(x.arg2(i,:)),find(x.arg1),'UniformOutput',false);
B{:}
fprintf('Number of required trucks is %d.\n', length(B))
```

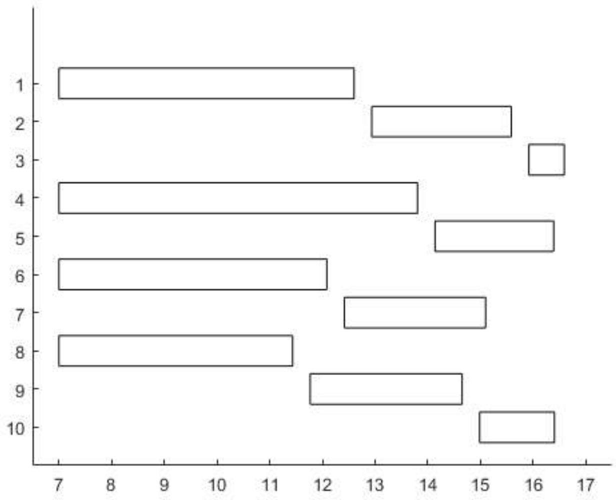
```
LP:                Optimal objective value is 3.782037.

Cut Generation:    Applied 1 clique cut, 2 cover cuts,
                   and 2 mir cuts.
                   Lower bound is 4.000000.

Heuristics:        Found 1 solution using ZI round.
                   Upper bound is 5.000000.
                   Relative gap is 16.67%.

Cut Generation:    Applied 4 clique cuts.
                   Lower bound is 4.000000.
                   Relative gap is 16.67%.

Branch and Bound:

   nodes     total   num int        integer       relative
explored  time (s)  solution           fval        gap (%)
      34      0.04         2   4.000000e+00   2.980232e-06

Optimal solution found.

Intlinprog stopped because the objective value is within a gap tolerance of the
optimal value, options.AbsoluteGapTolerance = 0 (the default value). The intcon
variables are integer within tolerance, options.IntegerTolerance = 1e-05 (the
default value).


ans =

     2     5     9


ans =

     1     8


ans =
     3     7


ans =

     4     6    10

Number of required trucks is 4.
```

Chart for Trucks;

```
b=[]
for i=1:length(B)
    b = [b; 7 7+Time(B{i}(1))]
    for j=2:length(B{i})
        b = [b; b(end)+tL b(end)+tL+Time(B{i}(j))]
    end
end

figure
gantt([b])
```

```
b =

     []


b =

    7.0000   12.6032


b =

    7.0000   12.6032
   12.9366   15.5859


b =

    7.0000   12.6032
   12.9366   15.5859
   15.9192   16.5914


b =

    7.0000   12.6032
   12.9366   15.5859
   15.9192   16.5914
    7.0000   13.8044


b =

    7.0000   12.6032
   12.9366   15.5859
   15.9192   16.5914
    7.0000   13.8044
   14.1377   16.3926


b =

    7.0000   12.6032
   12.9366   15.5859
   15.9192   16.5914
    7.0000   13.8044
   14.1377   16.3926
    7.0000   12.0857


b =

    7.0000   12.6032
   12.9366   15.5859
   15.9192   16.5914
    7.0000   13.8044
   14.1377   16.3926
    7.0000   12.0857
   12.4191   15.0998


b =

    7.0000   12.6032
   12.9366   15.5859
   15.9192   16.5914
```

```
    7.0000   13.8044
   14.1377   16.3926
    7.0000   12.0857
   12.4191   15.0998
    7.0000   11.4367


b =

    7.0000   12.6032
   12.9366   15.5859
   15.9192   16.5914
    7.0000   13.8044
   14.1377   16.3926
    7.0000   12.0857
   12.4191   15.0998
    7.0000   11.4367
   11.7700   14.6526


b =

    7.0000   12.6032
   12.9366   15.5859
   15.9192   16.5914
    7.0000   13.8044
   14.1377   16.3926
    7.0000   12.0857
   12.4191   15.0998
    7.0000   11.4367
   11.7700   14.6526
   14.9859   16.4033
```



Published with MATLAB® R2020a