HW 7 Solution - ISE 754 Fall 2020

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Question 1

(a) Unconstrained DC capacity

```
DCXY = uszip5('XY',mand([20548 26149 36317],uszip5('Code5')));
CXY = uszip5('XY',mand([30669 38339 30732 23830 23154],uszip5('Code5')));
dem = [40 55 35 70 25];
D = dists(DCXY,CXY,'mi');
F = full(sparse(argmin(D),1:size(D,2),dem));
mdisp(F)
TD = sum(sum(F.*D))
```

```
F:   1   2   3   4   5
-:--------------------
1:   0   0   0  70  25
2:   0   0   0   0   0
3:  40  55  35   0   0
TD =
   4.3046e+04
```

(b) Constrained DC capacity

```
sup = [60 90 80];
F = trans(D,sup,dem);
mdisp(F)
TD = sum(sum(F.*D))
```

```
F:   1   2   3   4   5
-:--------------------
1:   0   0   0  35  25
2:   0  15  35  35   0
3:  40  40   0   0   0
TD =
   5.7539e+04
```

(c) Constrained lane capacity

```
IJC = lev2list(D);
IJCU = [IJC repmat(30,size(IJC,1),1)]
s = [sup -dem]
lp = mcnf2lp(IJCU,s);
[x,TC,XFlg,out] = lplog(lp{:}); TC,XFlg,out
f = lp2mcnf(x,IJCU,s);
F = reshape(f,size(D));
mdisp(F)
TD = sum(sum(F.*D))
```

```
IJCU =
    1.0000    4.0000  494.3776   30.0000
    2.0000    4.0000  420.8226   30.0000
    3.0000    4.0000  192.9579   30.0000
    1.0000    5.0000  685.9578   30.0000
    2.0000    5.0000  521.2202   30.0000
    3.0000    5.0000  295.9040   30.0000
    1.0000    6.0000  519.2360   30.0000
    2.0000    6.0000  401.5780   30.0000
    3.0000    6.0000  205.3318   30.0000
    1.0000    7.0000  130.8135   30.0000
    2.0000    7.0000  252.0927   30.0000
    3.0000    7.0000  587.9471   30.0000
    1.0000    8.0000  108.3849   30.0000
    2.0000    8.0000  277.6360   30.0000
    3.0000    8.0000  645.5673   30.0000
s =
    60    90    80   -40   -55   -35   -70   -25
TC =
   6.4074e+04
XFlg =
     1
out =
  struct with fields:

    idxB: [6 20 13 16 23 30 25 8 27 26 3 12 31 32 33 10 11 17 2 5 9 19 22]
       r: [10×1 double]
       w: [23×1 double]
    iter: 23

F:   1   2   3   4   5
-:--------------------
1:   0   0   0  30  25
2:  10  25  25  30   0
3:  30  30  10  10   0
TD =
   6.4074e+04
```

(d) Constrained supplier capacity and unconstrained DC and lane capacities

```
SXY = uszip5('XY',mand([28124 27325 37421 27513],uszip5('Code5')));
DS2DC = dists(SXY,DCXY,'mi');
DDC2C = D;
IJC = lev2list(DS2DC,DDC2C)
s = [repmat(60,1,size(SXY,1)) zeros(1,size(DCXY,1)) -dem];
lp = mcnf2lp(IJC,s);
[x,TC,XFlg,out] = lplog(lp{:}); TC,XFlg,out
f = lp2mcnf(x,IJC,s);
IJF = [IJC(:,[1 2]) f(:)];
[FS2DC,FDC2C] = adj2lev(list2adj(IJF),[size(SXY,1) size(DCXY,1) size(CXY,1)]);
mdisp(FS2DC)
mdisp(FDC2C)
```

```
IJC =
    1.0000    5.0000  304.2874
    2.0000    5.0000  274.8856
    3.0000    5.0000  520.6931
    4.0000    5.0000  234.3006
    1.0000    6.0000  282.8247
    2.0000    6.0000  285.7097
    3.0000    6.0000  388.2559
    4.0000    6.0000  277.1106
    1.0000    7.0000  387.1087
    2.0000    7.0000  426.3990
    3.0000    7.0000  234.3984
    4.0000    7.0000  475.3527
    5.0000    8.0000  494.3776
    6.0000    8.0000  420.8226
    7.0000    8.0000  192.9579
    5.0000    9.0000  685.9578
    6.0000    9.0000  521.2202
    7.0000    9.0000  295.9040
    5.0000   10.0000  519.2360
    6.0000   10.0000  401.5780
    7.0000   10.0000  205.3318
    5.0000   11.0000  130.8135
```

```
    6.0000   11.0000  252.0927
    7.0000   11.0000  587.9471
    5.0000   12.0000  108.3849
    6.0000   12.0000  277.6360
    7.0000   12.0000  645.5673
TC =
   1.0828e+05
XFlg =
     1
out =
  struct with fields:

    idxB: [9 10 22 29 11 20 15 21 2 18 4 25]
       r: [19×1 double]
       w: [12×1 double]
    iter: 21

FS2DC:   1  2   3
-----:-----------
    1:   0  0  60
    2:  35  0  10
    3:   0  0  60
    4:  60  0   0

FDC2C:   1   2   3   4   5
-----:--------------------
    1:   0   0   0  70  25
    2:   0   0   0   0   0
    3:  40  55  35   0   0
```

Question 2(b)

Create Network

```
IJD = [1    -2    14;
1    -6    17;
1    -8    13;
1    -9     1;
1   -10    16;
2    -3     2;
2    -4     6;
2    -7    16;
2    -9    10;
2   -10     7;
3    -4     5;
3    -8     8;
3    -9     9;
4    -5    14;
4    -7    14;
5    -6     7;
5    -7     1;
5   -10    16;
6    -8    19;
6   -10    10;
7   -10     1;
8    -9    10];
```

Solve

```
[d,p] = dijkdemo(list2adj(IJD),3,6)
```

```
Node:    1    2    3    4    5    6    7    8    9   10

   S:    0    0    1*   0    0    0    0    0    0    0
   d:  Inf  Inf    0  Inf  Inf  Inf  Inf  Inf  Inf  Inf
pred:    0    0    0    0    0    0    0    0    0    0

   S:    0    1*   1    0    0    0    0    0    0    0
   d:  Inf    2    0    5  Inf  Inf  Inf    8    9  Inf
pred:    0    3    0    3    0    0    0    3    3    0

   S:    0    1    1    1*   0    0    0    0    0    0
   d:   16    2    0    5  Inf  Inf   18    8    9    9
pred:    2    3    0    3    0    0    2    3    3    2

   S:    0    1    1    1    0    0    0    1*   0    0
   d:   16    2    0    5   19  Inf   18    8    9    9
pred:    2    3    0    3    4    0    2    3    3    2

   S:    0    1    1    1    0    0    0    1    1*   0
   d:   16    2    0    5   19   27   18    8    9    9
pred:    2    3    0    3    4    8    2    3    3    2

   S:    0    1    1    1    0    0    0    1    1    1*
   d:   10    2    0    5   19   27   18    8    9    9
pred:    9    3    0    3    4    8    2    3    3    2
```

```
   S:    1*   1    1    1    0    0    0    1    1    1
   d:   10    2    0    5   19   19   10    8    9    9
pred:    9    3    0    3    4   10   10    3    3    2

   S:    1    1    1    1    0    0    1*   1    1    1
   d:   10    2    0    5   19   19   10    8    9    9
pred:    9    3    0    3    4   10   10    3    3    2

   S:    1    1    1    1    1*   0    1    1    1    1
   d:   10    2    0    5   11   19   10    8    9    9
pred:    9    3    0    3    7   10   10    3    3    2

   S:    1    1    1    1    1    1*   1    1    1    1
   d:   10    2    0    5   11   18   10    8    9    9
pred:    9    3    0    3    7    5   10    3    3    2

d =
    18
p =
     3     2    10     7     5     6
```

Question 3

Convert deg to dec

```
deg2dec = @(d,m,s) d + m/60 + s/60^2;
XY1 = [-deg2dec(78,42,5) deg2dec(35,46,21);  % Raleigh
   -deg2dec(84,23,24) deg2dec(33,46,19)]     % Ga Tech
```

```
XY1 =
  -78.7014   35.7725
  -84.3900   33.7719
```

Get road network

```
expansionAroundXY = 0.1;
[XY2,IJD,isXY,isIJD] = subgraph(usrdnode('XY'),...
   isinrect(usrdnode('XY'),boundrect(XY1,expansionAroundXY)),...
   usrdlink('IJD'));
```

Label type of road

```
s = usrdlink(isIJD);
isI = s.Type == 'I';         % Interstate highways
isIR = isI & s.Urban == ' '; % Rural Interstate highways
isIU = isI & ~isIR;          % Urban Interstate highways
isR = s.Urban == ' ' & ~isI; % Rural non-Interstate roads
isU = ~isI & ~isR;           % Urban non-Interstate roads
```

Add connector roads from cities to road network

```
[IJD11,IJD12,IJD22] = addconnector(XY1,XY2,IJD);
```

Convert road distances to travel times (needs to be after ADDCONNECTOR)

```
v.IR = 75;  % Rural Interstate highways average speed (mph)
v.IU = 65;  % Urban Interstate highways average speed (mph)
v.R = 50;   % Rural non-Interstate roads average speed (mph)
v.U = 25;   % Urban non-Interstate roads average speed (mph)
v.C = 20;   % Facility to road connector average speed (mph)

IJT = IJD;
IJT(isIR,3) = IJD(isIR,3)/v.IR;
IJT(isIU,3) = IJD(isIU,3)/v.IU;
IJT(isR,3) = IJD(isR,3)/v.R;
IJT(isU,3) = IJD(isU,3)/v.U;

IJT22 = IJD22;
IJT22(:,3) = IJT(:,3);
IJT12 = IJD12;
IJT12(:,3) = IJD12(:,3)/v.C;  % (IJD11 arcs ignored)
```

Shortest time routes

```
n = size(XY1,1);
[T,P] = dijk(list2adj([IJT12; IJT22]),1:n,1:n);
mdisp(T)
```

```
T:    1     2
```

```
-:------------
1:  0.00  5.54
2:  5.54  0.00
```

Distance of shortest time route

```
A = list2adj([IJD12; IJD22]);
D = zeros(n);
for i = 1:n
   for j = 1:n
      D(i,j) = locTC(pred2path(P,i,j),A);
   end
end
mdisp(D)
```

```
D:     1       2
-:----------------
1:    0.00  406.54
2:  406.54    0.00
```

Question 4

Input data

```
K = [60 50;
     55 45;
     50 35];
T = 26;
rng(1964)
D = round([gamrnd(6,4,T,1) gamrnd(4,3,T,1)])
Cp = [12  20;
      75 130;
      35  60];
h = 0.4/(365.25/7)   % If you know the year, can use 365/7 or 366/7
Ci = cumsum(Cp,1)*h
Cs = [400 600;
       90 110;
       50  70];
yinit = [0  0;
         0  0;
         0 sum(D(1:2,2))]
yfinal = zeros(3,2);
k0 = [1 0;
      1 0;
      1 0];
M = size(K,1);
T = size(D,1);
G = size(K,2);
```

```
D =
    27     6
    15     5
    22     8
    45     9
    28     7
    11     4
    24    12
    42    14
    45    21
    20     8
    18     6
    18    13
    47    25
    27     7
    15     5
    18    14
    27    10
    23     9
    29    25
    12     4
    24    13
    26    12
    33    12
    25    19
    27    19
    16    17
h =
    0.0077
Ci =
    0.0920    0.1533
    0.6669    1.1499
    0.9352    1.6099
yinit =
     0     0
```

```
     0     0
     0    11
```

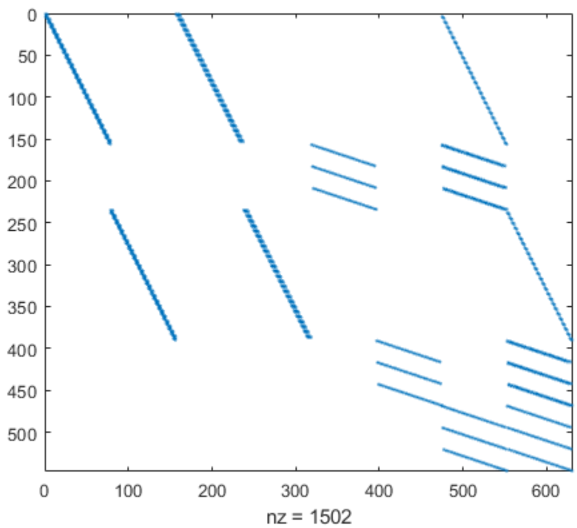
Create MILP model

```
Cp = reshape(repmat(Cp,[T 1 1]),M,T,G);
Ci = reshape(repmat(Ci,[T+1 1 1]),M,T+1,G);
Ci(:,1,:) = 0
Cs = reshape(repmat(Cs,[T 1 1]),M,T,G);
mp = Milp('PPlan');
mp.addobj('min',Cp,Ci,Cs,zeros(M,T,G))
for g = 1:G
   for t = 1:T
      for m = 1:M-1
         mp.addcstr({[1 -1],{[m m+1],t,g}},{[1 -1],{m,[t t+1],g}},0,0,'=',0)
      end
      mp.addcstr({M,t,g},{[1 -1],{M,[t t+1],g}},0,0,'=',D(t,g))
      for m = 1:M
         mp.addcstr({m,t,g},0,0,'<=',{K(m,g),{m,t,g}})
      end
   end
   for m = 1:M
      mp.addcstr(0,0,{-1,{m,1,g}},{m,1,g},'<=',k0(m,g))
      for t = 2:T
         mp.addcstr(0,0,{-1,{m,t,g}},{[1 -1],{m,[t t-1],g}},'<=',0)
      end
   end
end
for m = 1:M, for t = 1:T, mp.addcstr(0,0,0,{m,t,':'},'=',1), end, end
mp.addlb(0,horzcat(reshape(yinit,M,1,G),zeros(M,T-1,G),reshape(yfinal,M,1,G)),0,0)
mp.addub(Inf,horzcat(reshape(yinit,M,1,G),inf(M,T-1,G),reshape(yfinal,M,1,G)),1,1)
mp.addctype('C','C','B','B')
```

```
Ci(:,:,1) =
  Columns 1 through 7
         0    0.0920    0.0920    0.0920    0.0920    0.0920    0.0920
         0    0.6669    0.6669    0.6669    0.6669    0.6669    0.6669
         0    0.9352    0.9352    0.9352    0.9352    0.9352    0.9352
  Columns 8 through 14
    0.0920    0.0920    0.0920    0.0920    0.0920    0.0920    0.0920
    0.6669    0.6669    0.6669    0.6669    0.6669    0.6669    0.6669
    0.9352    0.9352    0.9352    0.9352    0.9352    0.9352    0.9352
  Columns 15 through 21
    0.0920    0.0920    0.0920    0.0920    0.0920    0.0920    0.0920
    0.6669    0.6669    0.6669    0.6669    0.6669    0.6669    0.6669
    0.9352    0.9352    0.9352    0.9352    0.9352    0.9352    0.9352
  Columns 22 through 27
    0.0920    0.0920    0.0920    0.0920    0.0920    0.0920
    0.6669    0.6669    0.6669    0.6669    0.6669    0.6669
    0.9352    0.9352    0.9352    0.9352    0.9352    0.9352
Ci(:,:,2) =
  Columns 1 through 7
         0    0.1533    0.1533    0.1533    0.1533    0.1533    0.1533
         0    1.1499    1.1499    1.1499    1.1499    1.1499    1.1499
         0    1.6099    1.6099    1.6099    1.6099    1.6099    1.6099
  Columns 8 through 14
    0.1533    0.1533    0.1533    0.1533    0.1533    0.1533    0.1533
    1.1499    1.1499    1.1499    1.1499    1.1499    1.1499    1.1499
    1.6099    1.6099    1.6099    1.6099    1.6099    1.6099    1.6099
  Columns 15 through 21
    0.1533    0.1533    0.1533    0.1533    0.1533    0.1533    0.1533
    1.1499    1.1499    1.1499    1.1499    1.1499    1.1499    1.1499
    1.6099    1.6099    1.6099    1.6099    1.6099    1.6099    1.6099
  Columns 22 through 27
    0.1533    0.1533    0.1533    0.1533    0.1533    0.1533
    1.1499    1.1499    1.1499    1.1499    1.1499    1.1499
    1.6099    1.6099    1.6099    1.6099    1.6099    1.6099
```

Display contraint matrix

```
spy(mp.Model.A)
```



Solve using Gurobi

```
clear params
model = mp.milp2gb
params.outputflag = 1;
result = gurobi(model, params);
x = mp.namesolution(result.x)
TC = result.objval
```

```
model =
  struct with fields:

         name: 'PPlan'
    modelsense: 'minimize'
          obj: [630×1 double]
           lb: [630×1 double]
           ub: [630×1 double]
        vtype: 'CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
            A: [546×630 double]
        sense: [546×1 char]
          rhs: [546×1 double]
Gurobi Optimizer version 9.0.0 build v9.0.0rc2 (win64)
Optimize a model with 546 rows, 630 columns and 1502 nonzeros
Model fingerprint: 0x02ea0060
Variable types: 318 continuous, 312 integer (312 binary)
Coefficient statistics:
  Matrix range     [1e+00, 6e+01]
  Objective range  [1e-08, 6e+02]
  Bounds range     [1e+00, 1e+01]
  RHS range        [1e+00, 5e+01]
Presolve removed 121 rows and 135 columns
Presolve time: 0.03s
Presolved: 425 rows, 495 columns, 1232 nonzeros
Variable types: 286 continuous, 209 integer (209 binary)

Root relaxation: objective 1.436247e+05, 491 iterations, 0.00 seconds

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

     0     0 143624.666    0   78          - 143624.666      -     -    0s
     0     0 144283.858    0   80          - 144283.858      -     -    0s
     0     0 144287.967    0   80          - 144287.967      -     -    0s
     0     0 144540.181    0   84          - 144540.181      -     -    0s
     0     0 144598.507    0   86          - 144598.507      -     -    0s
     0     0 144598.507    0   87          - 144598.507      -     -    0s
     0     0 144598.507    0   87          - 144598.507      -     -    0s
     0     0 144598.507    0   87          - 144598.507      -     -    0s
     0     0 144598.507    0   86          - 144598.507      -     -    0s
     0     0 144639.669    0   86          - 144639.669      -     -    0s
     0     0 144696.345    0   87          - 144696.345      -     -    0s
     0     0 144957.358    0   89          - 144957.358      -     -    0s
     0     0 144957.358    0   89          - 144957.358      -     -    0s
     0     0 144957.358    0   88          - 144957.358      -     -    0s
     0     0 144957.358    0   89          - 144957.358      -     -    0s
     0     0 144957.358    0   89          - 144957.358      -     -    0s
     0     0 144957.358    0   89          - 144957.358      -     -    0s
H    0     0                    177231.65147 144957.358  18.2%     -    0s
     0     0 144957.358    0   87 177231.651 144957.358  18.2%     -    0s
     0     2 144957.358    0   87 177231.651 144957.358  18.2%     -    0s
H  255   231                    149008.95907 145078.352  2.64%  23.1    0s
*  263   231              54    148837.16441 145078.352  2.53%  22.6    0s
```

```
*  291   257              53    148460.59959 145078.352  2.28%  22.1    0s
*  837   542              47    148131.63231 145591.485  1.71%  22.6    0s
H  926   586                    148038.93525 145598.075  1.65%  22.4    0s
*  964   582              44    147971.33032 145598.075  1.60%  22.2    0s
H 1494   780                    147958.79644 145808.379  1.45%  28.3    1s
* 1527   764              46    147830.99110 145808.488  1.37%  28.3    1s
H 1579   735                    147776.04463 145808.488  1.33%  29.1    2s
H 1825   731                    147740.01451 145826.861  1.29%  31.9    2s
H 3756  1162                    147735.41492 146250.151  1.01%  41.7    3s
H 3835  1064                    147461.66270 146273.437  0.81%  41.9    3s
  6045  1616 146556.468   22   67 147461.663 146510.277  0.65%  44.4    5s
* 6231  1603              42    147444.27625 146517.015  0.63%  44.2    5s
* 7383  1792              39    147414.04162 146579.441  0.57%  43.8    5s
H 7792  1819                    147404.15250 146599.282  0.55%  43.8    6s
 16539  2621 infeasible   33      147404.152 146894.682  0.35%  44.7   10s
*22888  2460              44    147386.99932 147027.097  0.24%  44.4   13s
 27158  1900     cutoff   33      147386.999 147120.223  0.18%  43.9   15s
*33731     0              39    147385.69610 147299.100  0.06%  42.4   16s

Cutting planes:
  Gomory: 52
  Cover: 5
  Implied bound: 5
  Clique: 2
  MIR: 92
  StrongCG: 1
  Flow cover: 157
  Flow path: 3
  Inf proof: 29
  Zero half: 3
  Relax-and-lift: 2

Explored 33805 nodes (1432495 simplex iterations) in 16.72 seconds
Thread count was 8 (of 8 available processors)

Solution count 10: 147386 147387 147404 ... 147831

Optimal solution found (tolerance 1.00e-04)
Best objective 1.473856960999e+05, best bound 1.473856960999e+05, gap 0.0000%
x =
  struct with fields:

     Cp: [3×26×2 double]
     Ci: [3×27×2 double]
     Cs: [3×26×2 double]
   arg4: [3×26×2 double]
TC =
   1.4739e+05
```

Report results

```
Fp = x.Cp;
Fi = x.Ci;
Fs = x.Cs;
Fk = x.arg4;
for g = 1:G
   mdisp(D(:,g)',[],[],['D' num2str(g)])
   mdisp(Fp(:,:,g),[],[],['Fp' num2str(g)])
   mdisp(Fi(:,:,g),[],[],['Fi' num2str(g)])
   mdisp(Fs(:,:,g),[],[],['Fs' num2str(g)])
   mdisp(Fk(:,:,g),[],[],['Fk' num2str(g)])
end
```

```
D1:   1   2   3   4   5   6   7   8   9  10  11  12  13  14  15  16  17  18  19  20  21  22  23  24  25  26
--:-------------------------------------------------------------------------------------------------------
 1:  27  15  22  45  28  11  24  42  45  20  18  18  47  27  15  18  27  23  29  12  24  26  33  25  27  16

Fp1:   1   2   3   4   5   6   7   8   9  10  11  12  13  14  15  16  17  18  19  20  21  22  23  24  25  26
---:-------------------------------------------------------------------------------------------------------
  1:  54  60   0   0  50  50  60  60  60  60  60   0   0   0   0   0   0   0   0  50  50  50   0   0   0
  2:  27  37   0  50  50  50   0   0  50  50  50   0   0  50  50  50   0   0   0   0  50  50  50   0   0   0
  3:  27  37   0  50  50  50   0   0  50  50  50   0   0  50  50  50   0   0   0   0  50  50  50   0   0   0

Fi1:   1   2   3   4   5   6   7   8   9   10   11   12   13   14   15  16  17  18  19  20  21  22  23  24  25  26  27
---:-------------------------------------------------------------------------------------------------------------------
  1:   0  27  50  50   0   0   0  60  120  130  140  150  150  150  100  50   0   0   0   0   0   0   0   0   0   0   0
  2:   0   0   0   0   0   0   0   0    0    0    0    0    0    0    0   0   0   0   0   0   0   0   0   0   0   0   0
  3:   0   0  22   0   5  27  66  42    0    5   35   67   49    2   25  60  92  65  42  13   1  27  51  68  43  16   0

Fs1:   1   2   3   4   5   6   7   8   9  10  11  12  13  14  15  16  17  18  19  20  21  22  23  24  25  26
---:-------------------------------------------------------------------------------------------------------
  1:   0   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1   0   0   0   0   0
  2:   0   0   0   1   0   0   0   0   1   0   0   0   0   1   0   0   0   0   0   0   1   0   0   0   0   0
  3:   0   0   0   1   0   0   0   0   1   0   0   0   0   1   0   0   0   0   0   0   1   0   0   0   0   0

Fk1:   1   2   3   4   5   6   7   8   9  10  11  12  13  14  15  16  17  18  19  20  21  22  23  24  25  26
---:-------------------------------------------------------------------------------------------------------
```

```
  1:   1   1   0   0   1   1   1   1   1   1   1   0   0   0   0   0   0   0   0   0   1   1   1   1   1   1
  2:   1   1   0   1   1   1   0   0   1   1   1   0   0   1   1   1   0   0   0   0   1   1   1   0   0   0
  3:   1   1   0   1   1   1   0   0   1   1   1   0   0   1   1   1   0   0   0   0   1   1   1   0   0   0

D2:   1   2   3   4   5   6   7   8   9  10  11  12  13  14  15  16  17  18  19  20  21  22  23  24  25  26
--:-------------------------------------------------------------------------------------------------------
 1:   6   5   8   9   7   4  12  14  21   8   6  13  25   7   5  14  10   9  25   4  13  12  12  19  19  17

Fp2:   1   2   3   4   5   6   7   8   9  10  11  12  13  14  15  16  17  18  19  20  21  22  23  24  25  26
---:-------------------------------------------------------------------------------------------------------
  1:   0   0  39  50   0   0   0   0   0   0   0  29  35   0   0   0  10  30  50  50   0   0   0   0   0   0
  2:   0   0  28   0   0   0  26  35   0   0   0  29  35   0   0   0  10   9  31  35   0   0   0  19  19  17
  3:   0   0  28   0   0   0  26  35   0   0   0  29  35   0   0   0  10   9  31  35   0   0   0  19  19  17

Fi2:   1   2   3   4   5   6   7   8   9  10  11  12  13  14  15  16  17  18  19  20  21  22  23  24  25  26  27
---:-----------------------------------------------------------------------------------------------------------
  1:   0   0   0  11  61  61  61  35   0   0   0   0   0   0   0   0   0   0  21  40  55  55  55  55  36  17   0
  2:   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
  3:  11   5   0  20  11   4   0  14  35  14   6   0  16  26  19  14   0   0   0   6  37  24  12   0   0   0   0

Fs2:   1   2   3   4   5   6   7   8   9  10  11  12  13  14  15  16  17  18  19  20  21  22  23  24  25  26
---:-------------------------------------------------------------------------------------------------------
  1:   0   0   1   0   0   0   0   0   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0
  2:   0   0   1   0   0   0   1   0   0   0   0   1   0   0   0   0   1   0   0   0   0   0   0   1   0   0
  3:   0   0   1   0   0   0   1   0   0   0   0   1   0   0   0   0   1   0   0   0   0   0   0   1   0   0

Fk2:   1   2   3   4   5   6   7   8   9  10  11  12  13  14  15  16  17  18  19  20  21  22  23  24  25  26
---:-------------------------------------------------------------------------------------------------------
  1:   0   0   1   1   0   0   0   0   0   0   0   1   1   1   1   1   1   1   1   0   0   0   0   0   0   0
  2:   0   0   1   0   0   0   1   1   0   0   0   1   1   0   0   0   1   1   1   1   0   0   0   1   1   1
  3:   0   0   1   0   0   0   1   1   0   0   0   1   1   0   0   0   1   1   1   1   0   0   0   1   1   1
```

Question 5

Input data

```
Q = [20; 30];
D = [25 15 10 50 25 15]';
Cp = [200; 800];
Cf = [3000; 9000];
h_ob = 0.08/(3/12)
h = (0.05 + 0.06 + h_ob)/12
Ci = cumsum(Cp + Cf./Q)*h     % Allocated fixed cost on per-ton basis
yinit = [5; 0];
yfinal = [7; 4];
ymax = 30;
M = size(Q,1);
T = size(D,1);
```

```
h_ob =
    0.3200
h =
    0.0358
Ci =
   12.5417
   51.9583
```

Create MILP model

```
Cp = repmat(Cp,1,T)
Ci = repmat(Ci,1,T+1);
Ci(:,1,:) = 0              % Initial inv cost not included
Cf = repmat(Cf,1,T)
clear mp
mp = Milp('PPlan');
mp.addobj('min',Cp,Ci,Cf)  % Objective
for t = 1:T  % Flow balance constraints
   for m = 1:M-1
      mp.addcstr({[1 -1],{[m m+1],t}},{[1 -1],{m,[t t+1]}},0,'=',0)
   end
   mp.addcstr({M,t},{[1 -1],{M,[t t+1]}},0,'=',D(t))
   for m = 1:M
      mp.addcstr({m,t},0,'<=',{Q(m),{m,t}})
   end
end
mp.addlb(0,[yinit zeros(M,T-1) yfinal],0)  % Lower bounds
mp.addub(Inf,[yinit repmat(ymax,M,T-1) yfinal],2)    % Upper bounds
mp.addctype('C','C','I')
```

```
Cp =
   200   200   200   200   200   200
   800   800   800   800   800   800
Ci =
         0   12.5417   12.5417   12.5417   12.5417   12.5417   12.5417
         0   51.9583   51.9583   51.9583   51.9583   51.9583   51.9583
Cf =
```

```
        3000        3000        3000        3000        3000        3000
        9000        9000        9000        9000        9000        9000
```

Display model

```
mp.dispmodel
```

```
PPlan:  lhs  C   C   C   C   C   C   C   C   C   C   C   C    C    C    C     C     C     C     C     C     C     C     C     C     C     I     I     I     I
-----:-------------------------------------------------------------------------------------------------------------------------------------------------------------
  Min:      200 800 200 800 200 800 200 800 200 800 200 800 0.00 0.00 12.54 51.96 12.54 51.96 12.54 51.96 12.54 51.96 12.54 51.96 12.54 51.96 3,000 9,000 3,000 9,00
    1:    0   1  -1   0   0   0   0   0   0   0   0   0   0 1.00 0.00 -1.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00     0     0     0
    2:   25   0   1   0   0   0   0   0   0   0   0   0   0 0.00 1.00  0.00 -1.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00     0     0     0
    3: -Inf   1   0   0   0   0   0   0   0   0   0   0   0 0.00 0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00   -20     0     0
    4: -Inf   0   1   0   0   0   0   0   0   0   0   0   0 0.00 0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00     0   -30     0
    5:    0   0   0   1  -1   0   0   0   0   0   0   0   0 0.00 0.00  1.00  0.00 -1.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00     0     0     0
    6:   15   0   0   0   1   0   0   0   0   0   0   0   0 0.00 0.00  0.00  1.00  0.00 -1.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00     0     0     0
    7: -Inf   0   0   1   0   0   0   0   0   0   0   0   0 0.00 0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00     0     0   -20
    8: -Inf   0   0   0   1   0   0   0   0   0   0   0   0 0.00 0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00     0     0     0   -3
    9:    0   0   0   0   0   1  -1   0   0   0   0   0   0 0.00 0.00  0.00  0.00  1.00  0.00 -1.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00     0     0     0
   10:   10   0   0   0   0   0   1   0   0   0   0   0   0 0.00 0.00  0.00  0.00  0.00  1.00  0.00 -1.00  0.00  0.00  0.00  0.00  0.00  0.00     0     0     0
   11: -Inf   0   0   0   0   1   0   0   0   0   0   0   0 0.00 0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00     0     0     0
   12: -Inf   0   0   0   0   0   1   0   0   0   0   0   0 0.00 0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00     0     0     0
   13:    0   0   0   0   0   0   0   1  -1   0   0   0   0 0.00 0.00  0.00  0.00  0.00  0.00  1.00  0.00 -1.00  0.00  0.00  0.00  0.00  0.00     0     0     0
   14:   50   0   0   0   0   0   0   0   1   0   0   0   0 0.00 0.00  0.00  0.00  0.00  0.00  0.00  1.00  0.00 -1.00  0.00  0.00  0.00  0.00     0     0     0
   15: -Inf   0   0   0   0   0   0   1   0   0   0   0   0 0.00 0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00     0     0     0
   16: -Inf   0   0   0   0   0   0   0   1   0   0   0   0 0.00 0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00     0     0     0
   17:    0   0   0   0   0   0   0   0   0   1  -1   0   0 0.00 0.00  0.00  0.00  0.00  0.00  0.00  0.00  1.00  0.00 -1.00  0.00  0.00  0.00     0     0     0
   18:   25   0   0   0   0   0   0   0   0   0   1   0   0 0.00 0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  1.00  0.00 -1.00  0.00  0.00     0     0     0
   19: -Inf   0   0   0   0   0   0   0   0   1   0   0   0 0.00 0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00     0     0     0
   20: -Inf   0   0   0   0   0   0   0   0   0   1   0   0 0.00 0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00     0     0     0
   21:    0   0   0   0   0   0   0   0   0   0   0   1  -1 0.00 0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  1.00  0.00 -1.00  0.00     0     0     0
   22:   15   0   0   0   0   0   0   0   0   0   0   0   1 0.00 0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  1.00  0.00 -1.00     0     0     0
   23: -Inf   0   0   0   0   0   0   0   0   0   0   1   0 0.00 0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00     0     0     0
   24: -Inf   0   0   0   0   0   0   0   0   0   0   0   1 0.00 0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00     0     0     0
   lb:        0   0   0   0   0   0   0   0   0   0   0   0 5.00 0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  7.00  4.00     0     0     0
   ub:      Inf Inf Inf Inf Inf Inf Inf Inf Inf Inf Inf Inf 5.00 0.00 30.00 30.00 30.00 30.00 30.00 30.00 30.00 30.00 30.00 30.00  7.00  4.00     2     2     2
```

Solve using INTLINPROG

```
ilp = mp.milp2ilp;
[x,TC,exitflag,output] = intlinprog(ilp{:});
TC,output
x = mp.namesolution(x);
```

```
LP:                Optimal objective value is 210180.833333.

Heuristics:        Found 2 solutions using ZI round.
                   Upper bound is 231823.875000.
                   Relative gap is 7.15%.

Heuristics:        Found 2 solutions using ZI round.
                   Upper bound is 226187.583333.
                   Relative gap is 4.83%.

Heuristics:        Found 1 solution using ZI round.
                   Upper bound is 220047.833333.
                   Relative gap is 2.18%.

Heuristics:        Found 2 solutions using ZI round.
                   Upper bound is 216834.625000.
                   Relative gap is 0.73%.

Cut Generation:    Applied 8 Gomory cuts, and 18 mir cuts.
                   Lower bound is 215253.400741.
                   Relative gap is 0.73%.

Heuristics:        Found 2 solutions using rss.
                   Upper bound is 216684.125000.
                   Relative gap is 0.66%.

Cut Generation:    Applied 3 Gomory cuts.
                   Lower bound is 215272.594557.
                   Relative gap is 0.65%.

Branch and Bound:

   nodes     total   num int        integer       relative
explored  time (s)  solution           fval        gap (%)
      35      0.05         9   2.166841e+05   0.000000e+00

Optimal solution found.

Intlinprog stopped because the objective value is within a gap tolerance of the
optimal value, options.AbsoluteGapTolerance = 0 (the default value). The intcon
variables are integer within tolerance, options.IntegerTolerance = 1e-05 (the
```

```
default value).

TC =
   2.1668e+05
output =
  struct with fields:

        relativegap: 0
        absolutegap: 0
      numfeaspoints: 9
           numnodes: 35
    constrviolation: 2.8422e-14
            message: 'Optimal solution found.↵↵Intlinprog stopped because the objective value is within a gap tolerance of the optimal value, options.AbsoluteGapTol
```

Report results

```
Fp = x.Cp; mdisp(Fp)
Fi = x.Ci; mdisp(Fi)
Ff = x.Cf; mdisp(Ff)
TCp = sum(sum(Cp.*Fp));
TCi = sum(sum(Ci.*Fi));
TCf = sum(sum(Cf.*Ff));
vdisp('TCp,TCi,TCf,TC')
```

```
Fp:   1   2   3   4   5  6
--:----------------------
 1:  20  29  20  40  37  0
 2:  25  29   0  60  30  0

Fi:  1   2   3   4   5   6  7
--:--------------------------
 1:  5   0   0  20   0   7  7
 2:  0  -0  14   4  14  19  4

Ff:  1  2   3  4  5   6
--:-------------------
 1:  1  2   1  2  2  -0
 2:  1  1  -0  2  1  -0

 :    TCp       TCi      TCf       TC
-:-------------------------------------
1:  144,400  3,284.12  69,000  216,684.13
```

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