



Closures & Continuations



Overview

- Controlling the Control
 - Closures
 - Continuation

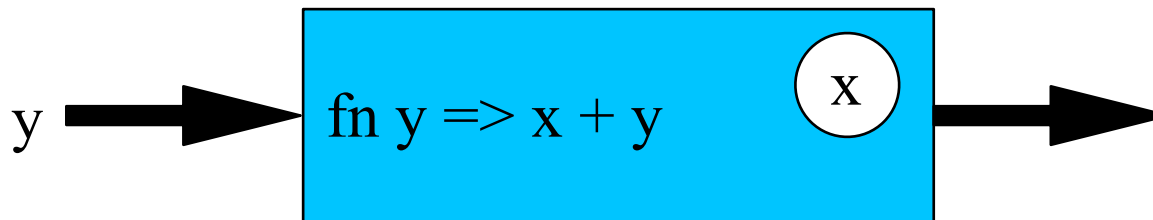


Bindings Revisited

- Consider the following fragment

```
fun foo f = f 10;  
  
val special = let val x = 100  
               in fn y => x + y  
               end;  
  
foo special;
```

- What is going on here ?





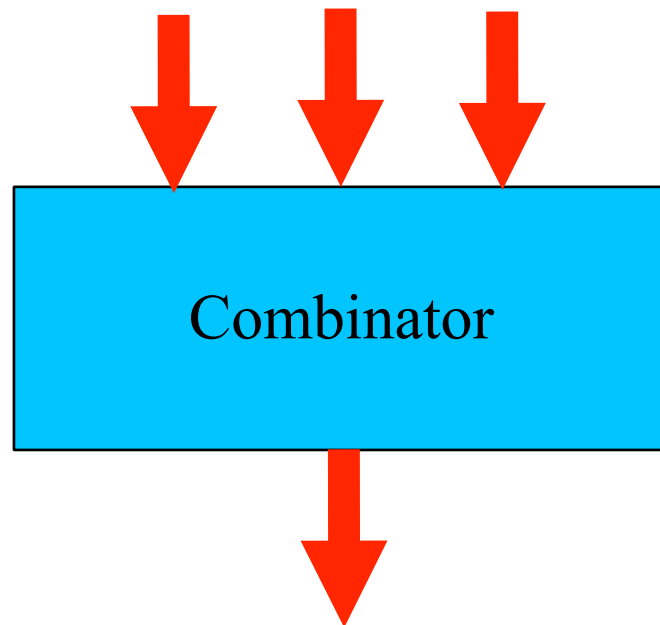
Environment Semantics

- The SML environment is....
 - A “Stack” of bindings
- Each time you state “val x = ...”
 - You push a new binding on the stack
- Each time you state “let val x = ... in ... end”
 - You *temporarily* push a binding on the stack
 - That is popped when reaching the “end”
- Each time you refer to a binding by name
 - You hold onto the matching environment
 - The environment sticks around as long as something refers to it!



Combinators

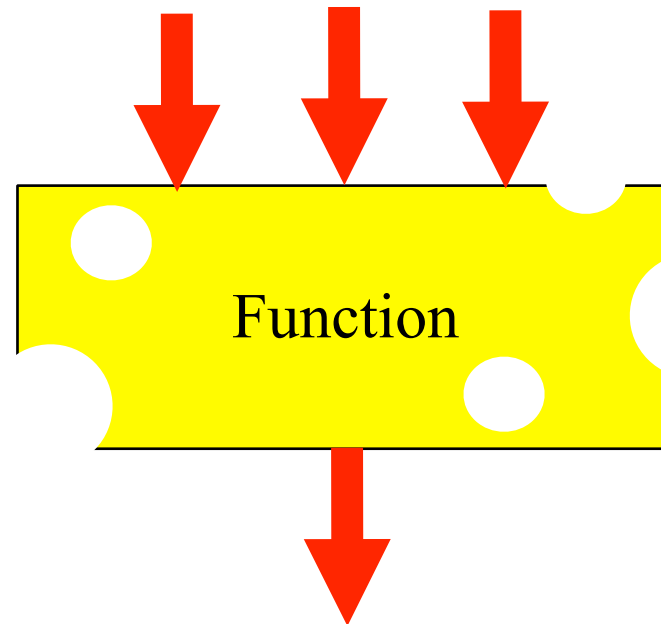
- What are combinators ?





Plain Old Functions

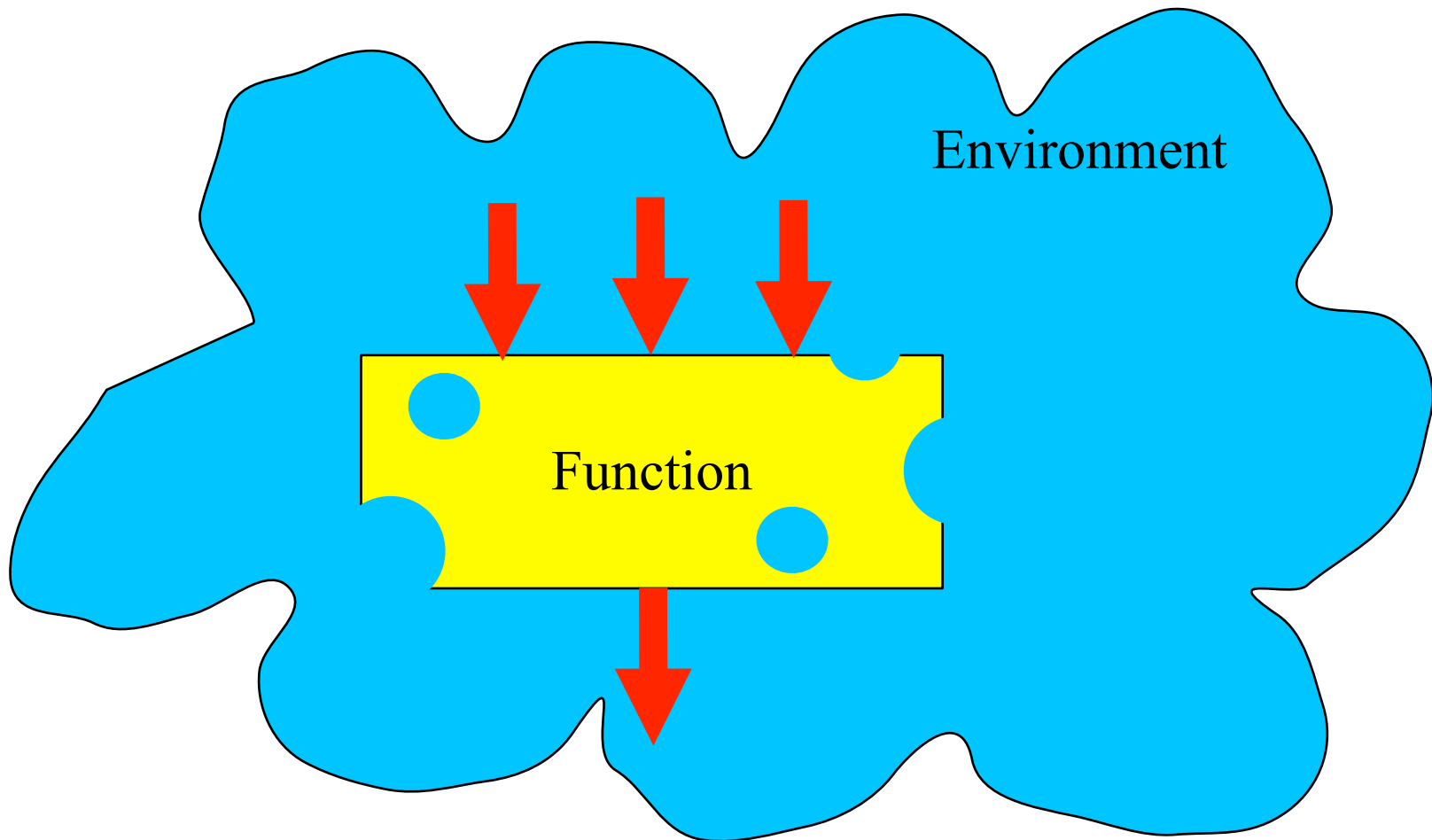
- Any Function
 - What is the difference ?





Environment

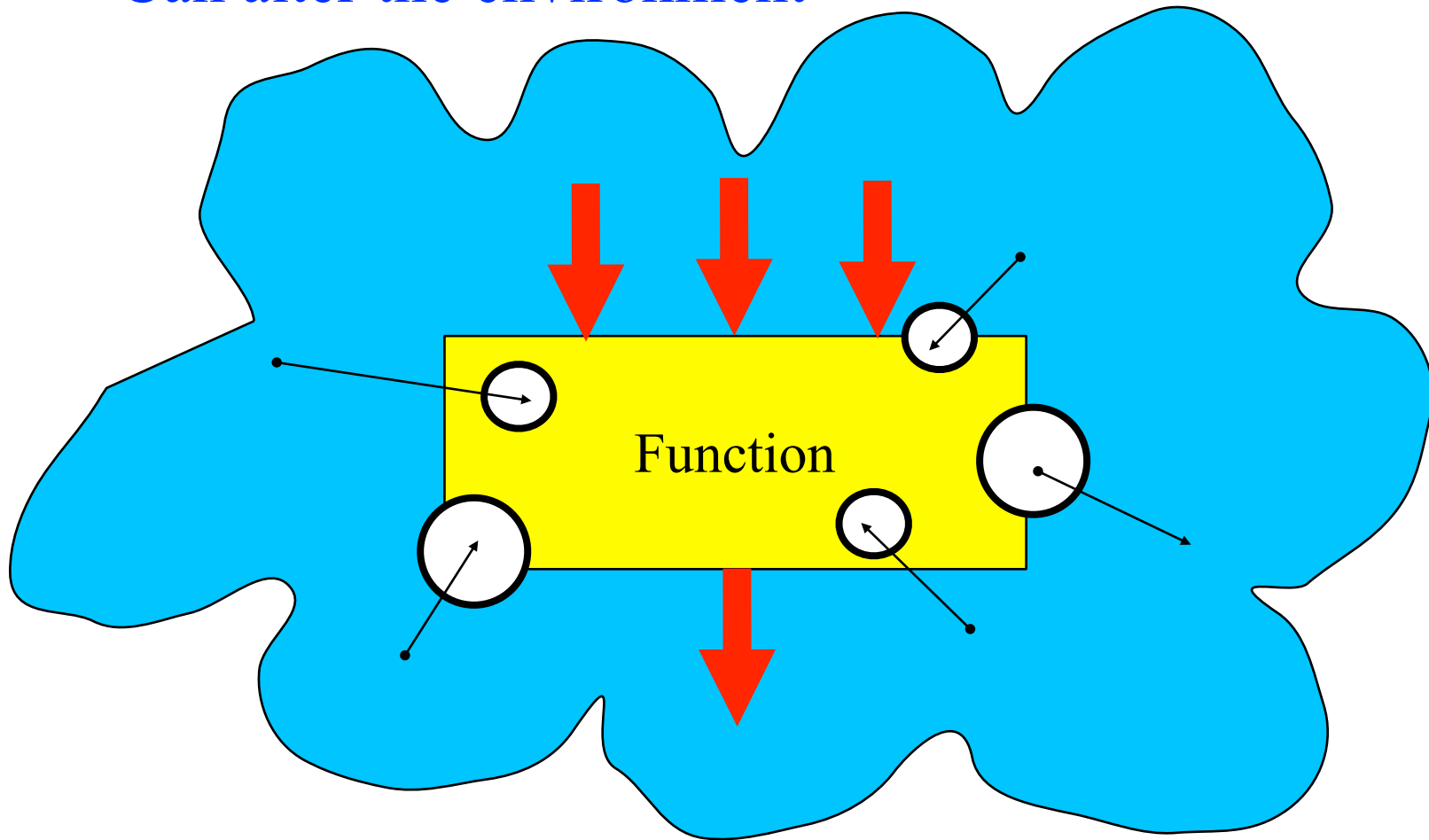
- The environment Provides
 - The “filling”





Alternative View

- **Functions**
 - Depend on the environment
 - Can alter the environment





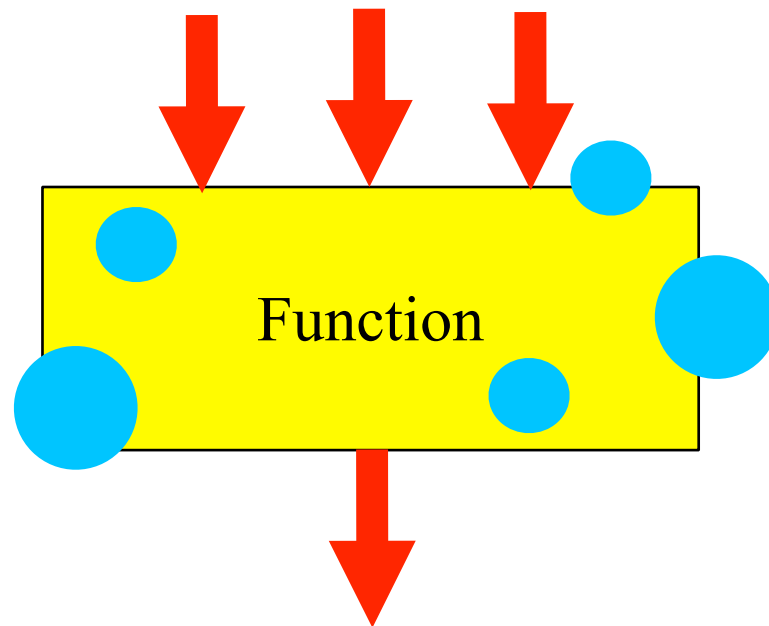
Combinators

- Can be composed
 - Easily
 - Just feed the output of one
 - Into the input of others!
- Can be dealt with
 - Independently
 - Without any reference to any context
- What about “plain functions” ?



Closures

- **Function in context**
 - A way to deal with a function
 - Package
 - The function
 - Its context
 - Deal with “the package” as a whole





What About ML...

- What is ML really doing ?
 - ML **never** let you use functions with “holes”
 - ML **always** provides closures
- Corollary
 - In ML, **every** “function” is actually a “combinator”



Example

- A direct example
 - What is the context ?

```
val x = 10
fun add n = x + n

val y1 = add 1
val z1 = add 2
val x = 20
val y2 = add 1
val z2 = add 2
```

Output ?



Example

- A more subtle example
 - What is the context ?
 - What is so subtle about it ?

```
val f = let val x = 20
          fun add n = x + n
        in add
        end

val y = f 10
```

Output ?



Closures & Context

- Which context should be used ?
 - The context current at closure creation time
- What constitutes the context ?
 - Must take into account
 - Model
 - Flat
 - Nested
 - Scoping
 - Static
 - Dynamic

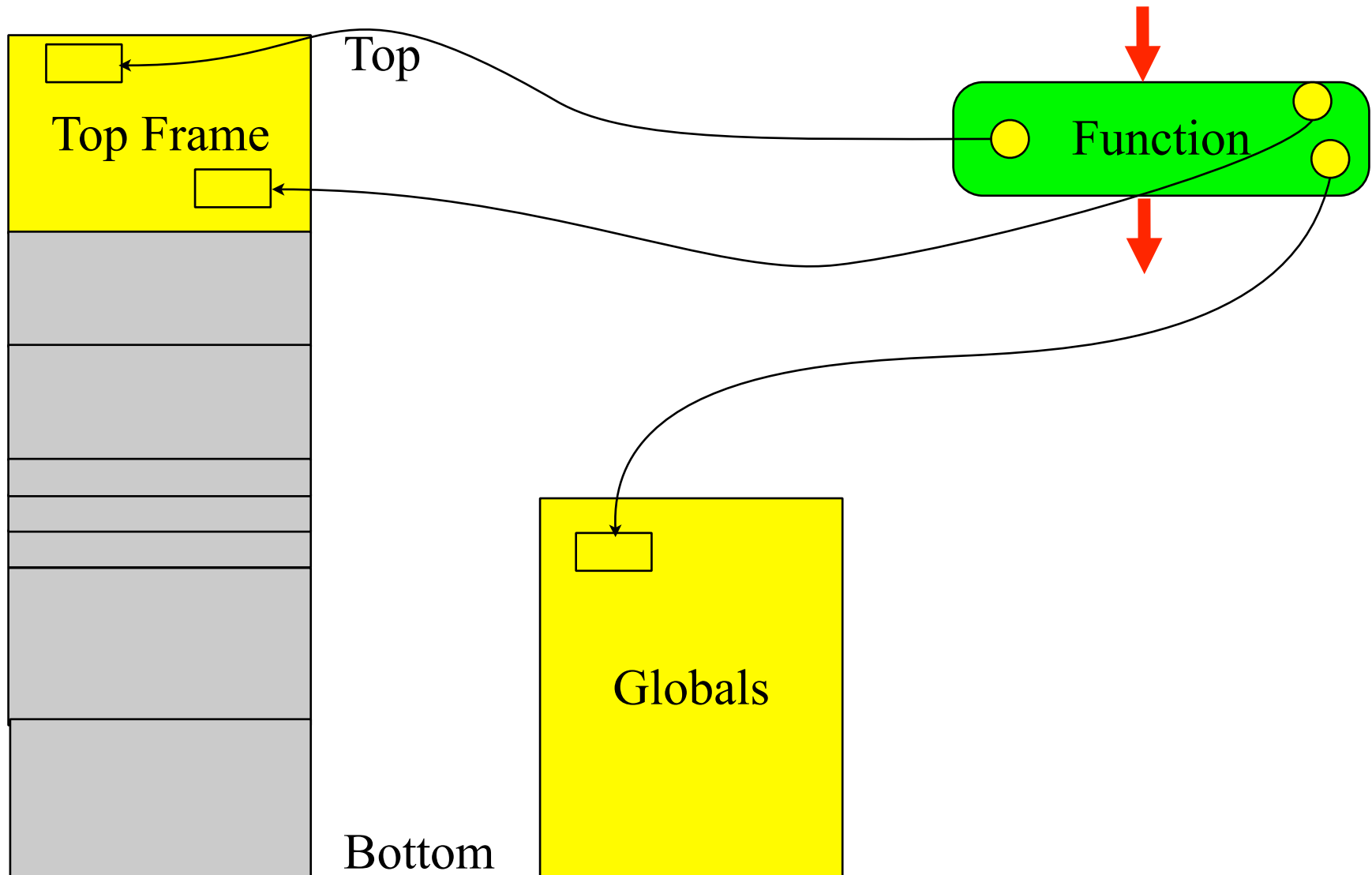


Flat Model / Static Scoping

- Issue
 - What is visible inside the function ?
- What must be saved...
 - In a stack-based implementation ?
 - In a functional implementation ?

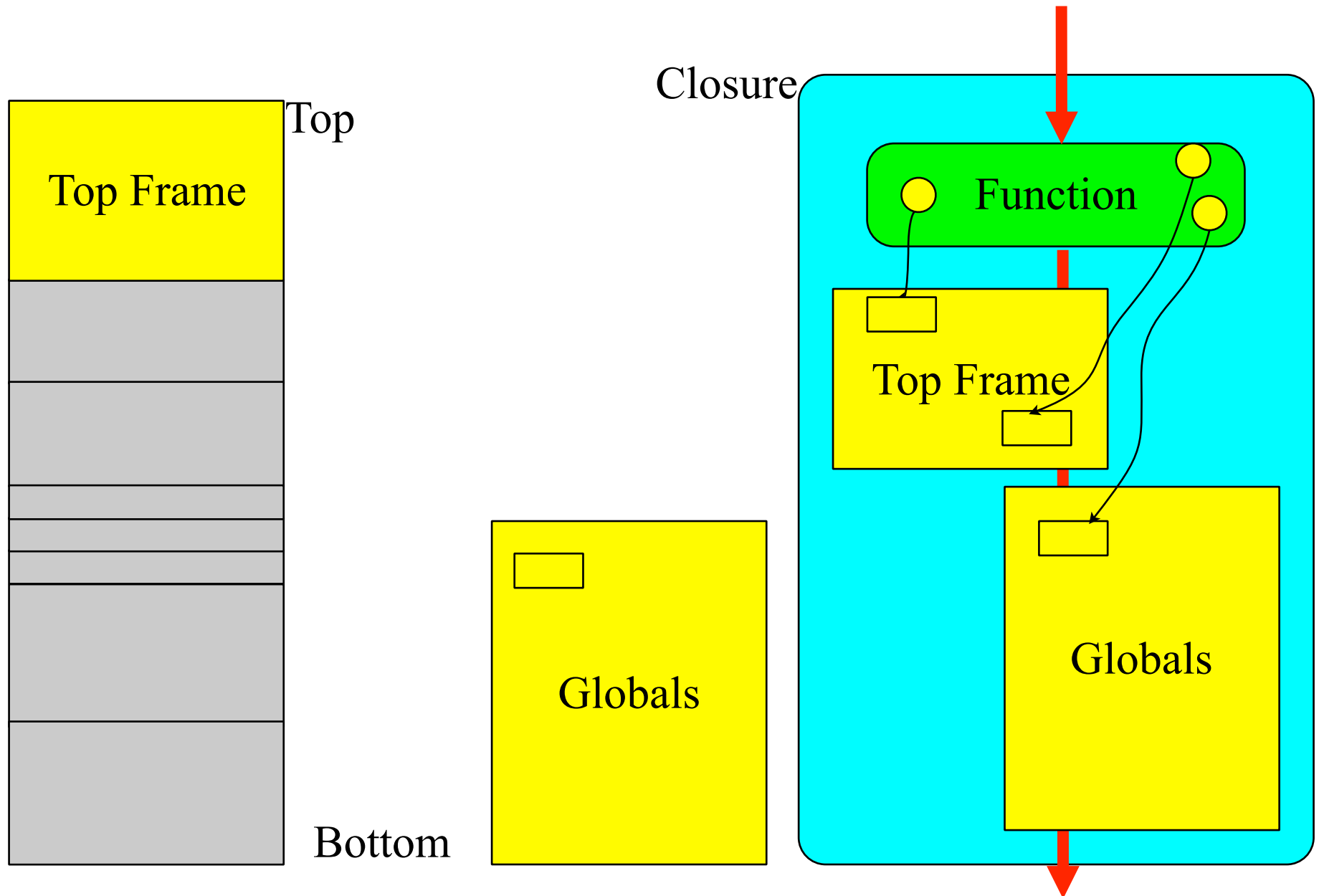


Stack-Based





Stack-Based





Implication

- We must be able to
 - Manipulate the stack
 - Copy frames from the stack
 - Easy to see...
 - Copy frames back to the stack
 - Why ?
 - What about the globals ?
- Questions
 - How expensive is a closure creation ?
 - Can we improve ?



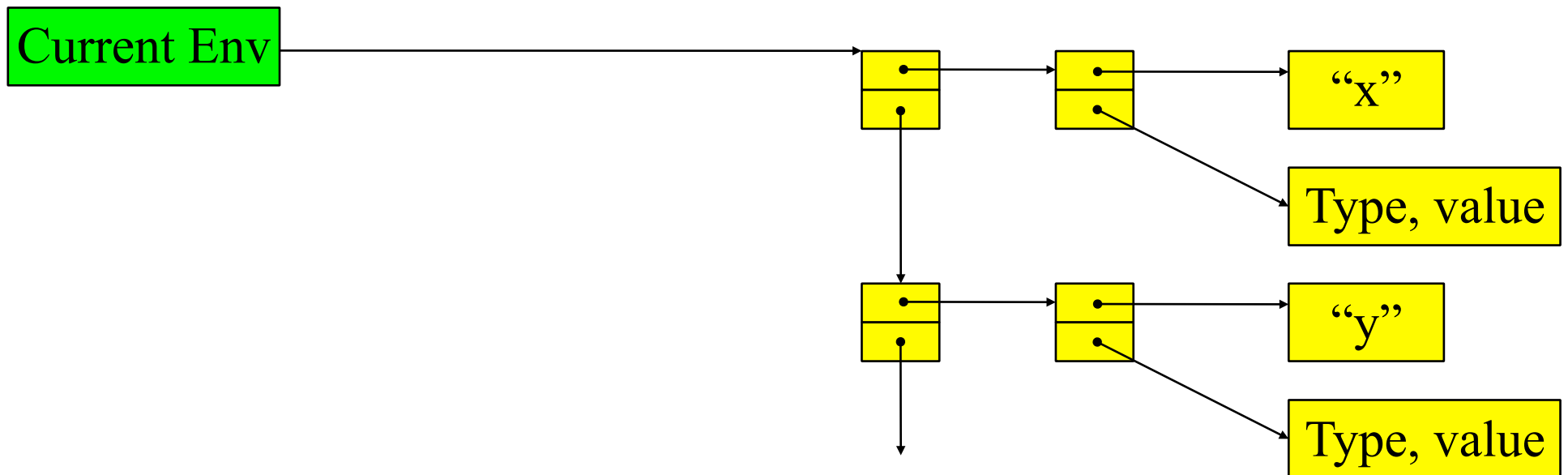
Functional Context

- Do we really need a stack ?
 - Of course not!
- Stacks are good for
 - Languages with destructive operations
 - e.g., assignments
 - Efficient update
 - variable lifetime = variable scope
- With functional languages
 - No destructive operation
 - Variable lifetime can exceed scope
- We can take advantage of that to do away with the stack!



Environments

- Implementation with either
 - Association list
 - Central reference table
- Association list

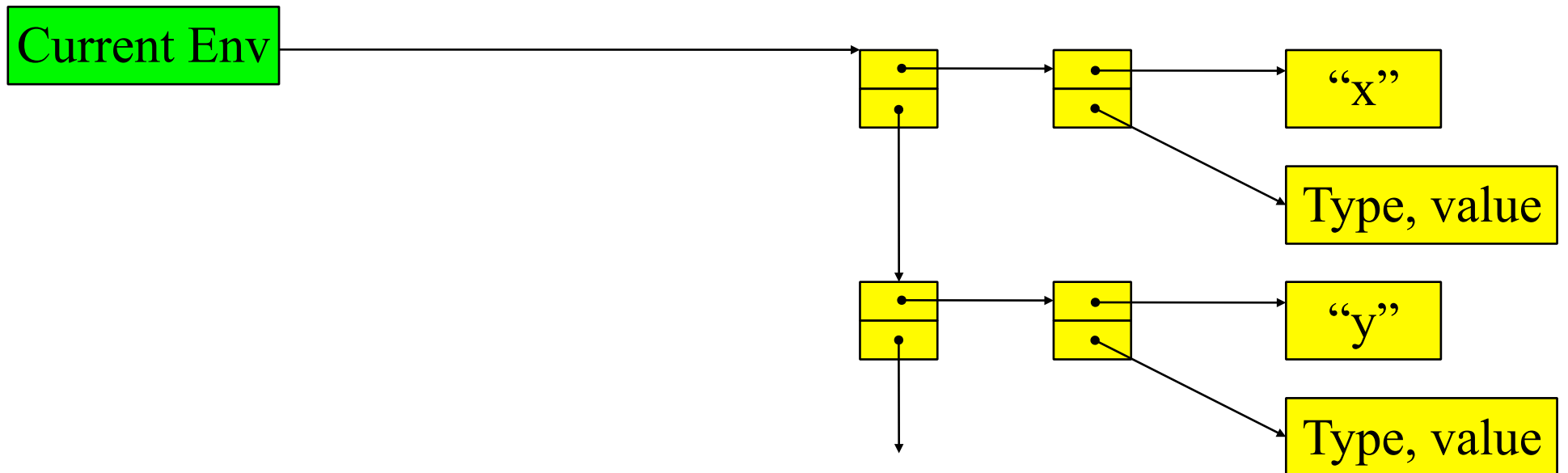




Binding a Name

- What It does

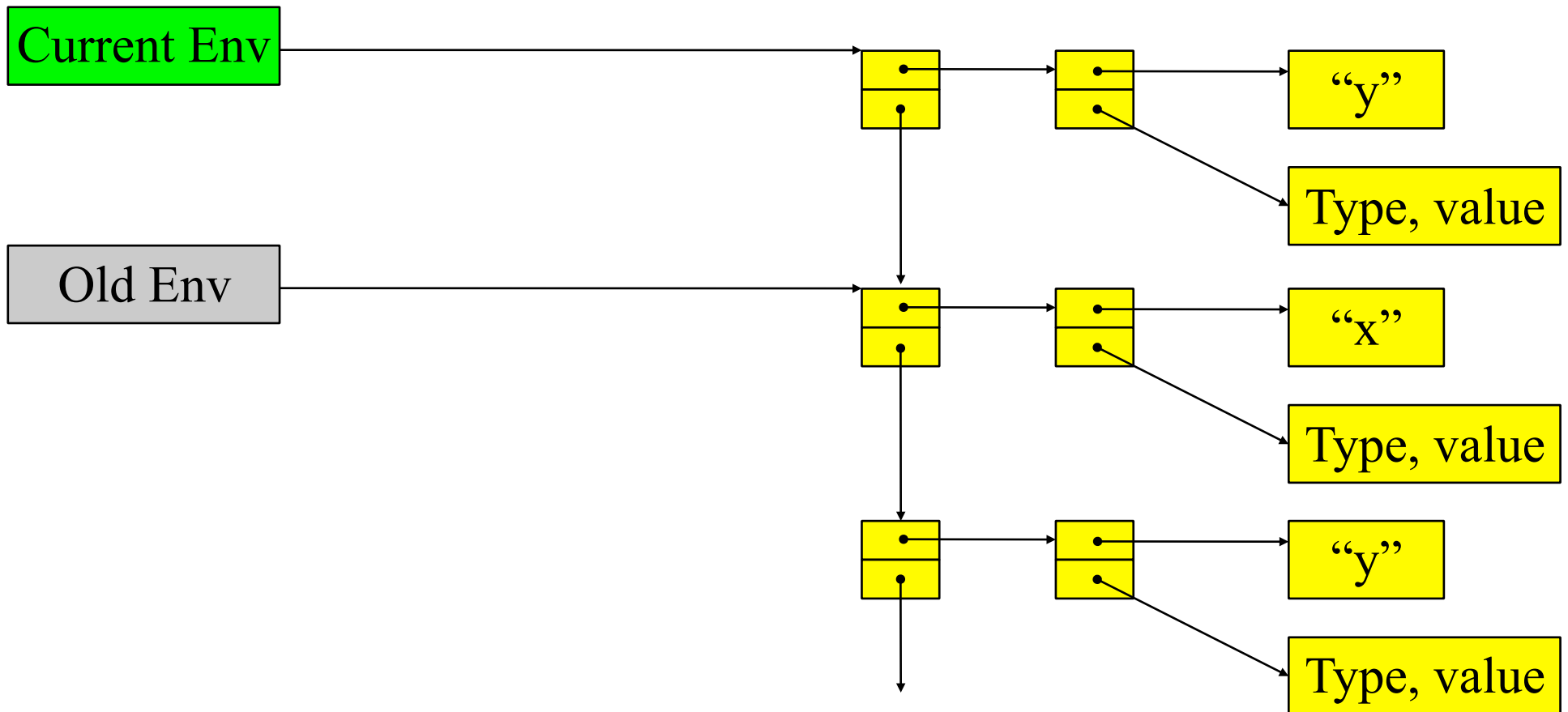
```
...  
let y = <some expr>  
in  <some other expr>  
end  
...
```





Binding a Name

- **What It does**
 - When we leave the let statement: restore the oldEnv
 - When we need to lookup a value, scan the env.



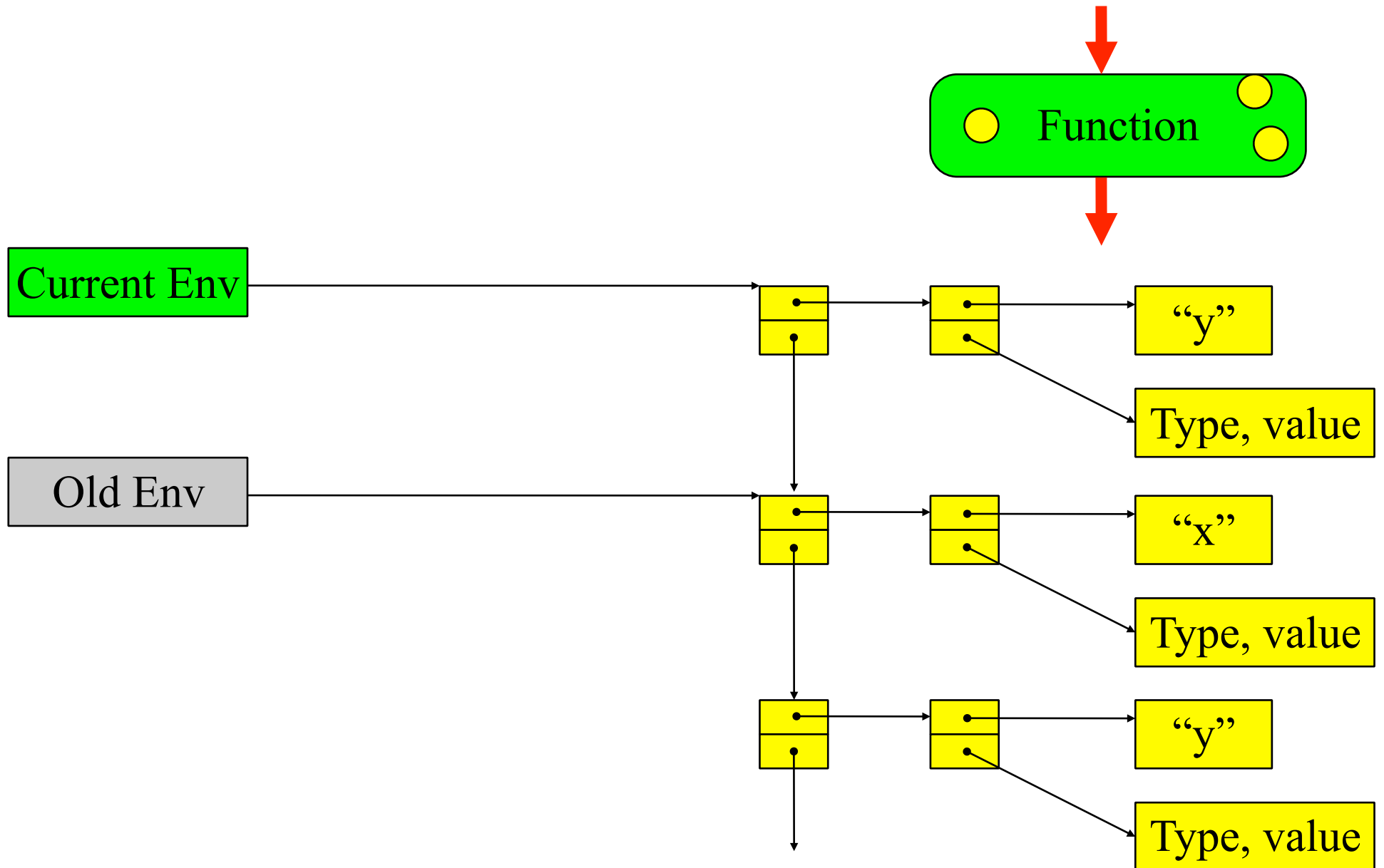


Creating a Closure

- What must be done exactly ?

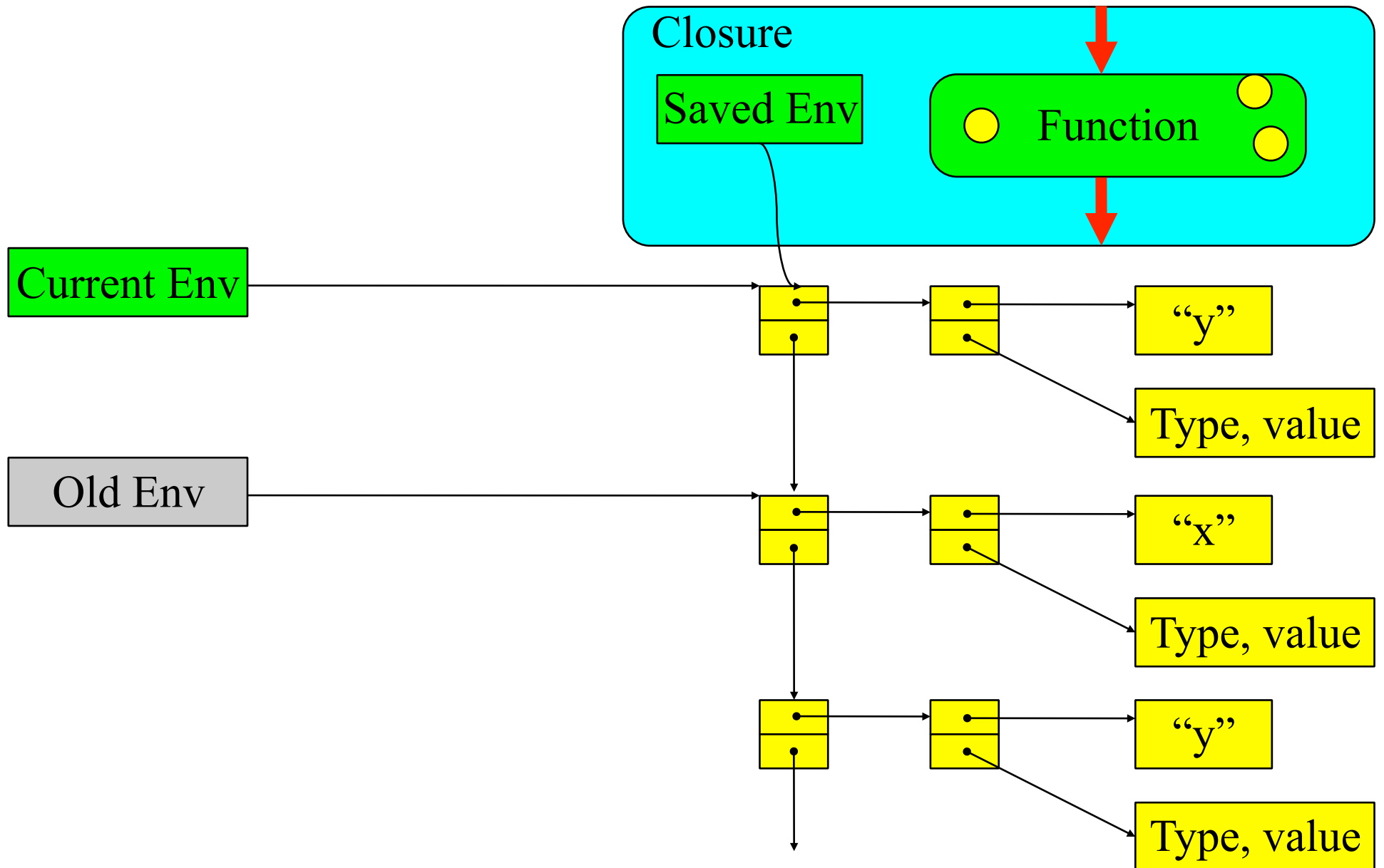


The Function to Close



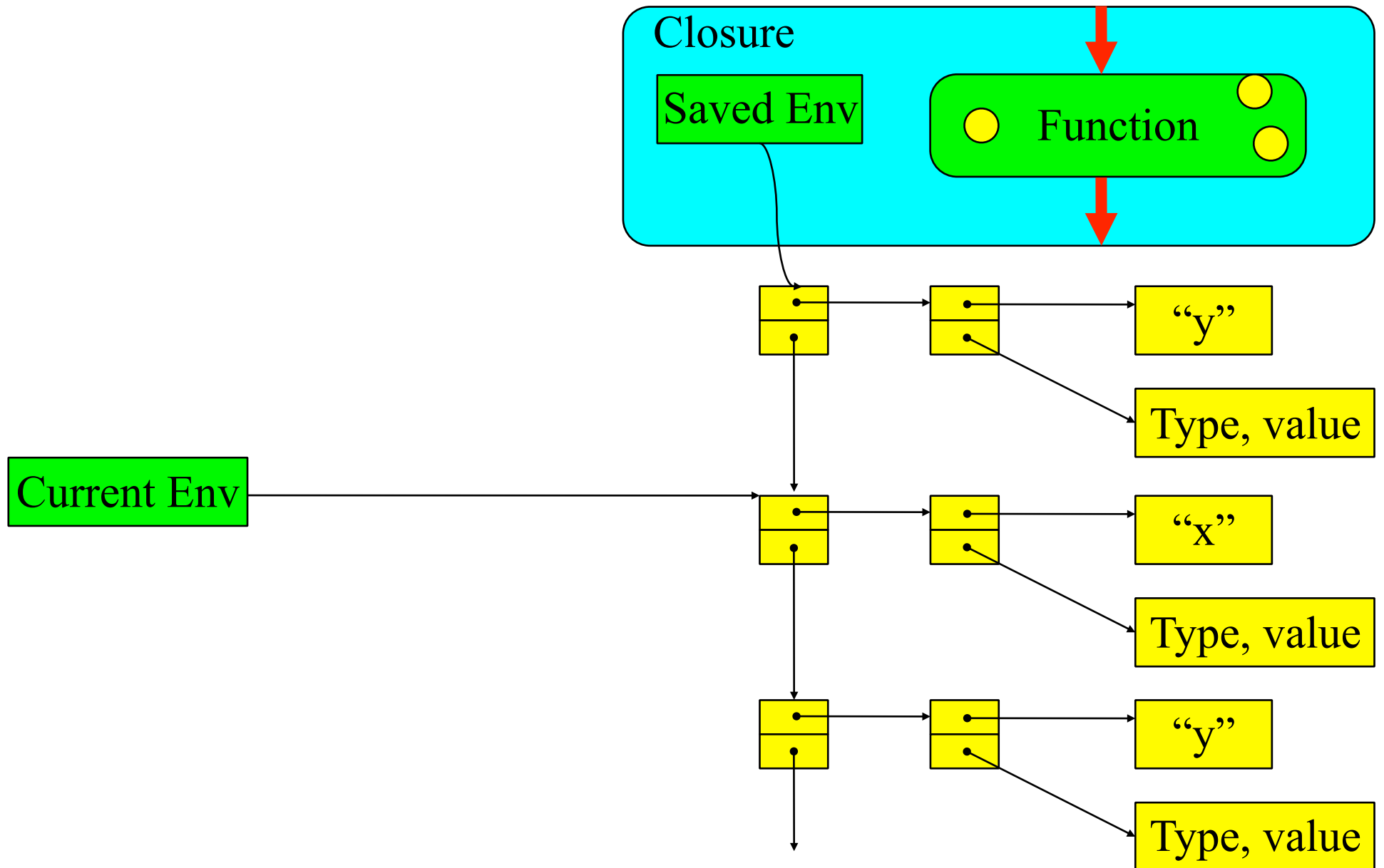


Closing





Leaving the Let





Association List

- Any downside ?



Applications

- Are closures really *that* useful ?
 - Their value is in the eye of the beholder
 - With functional languages
 - You use closures without even noticing!
- Applications
 - Reactive Programming, event handlers, GUI
 - Flexible control flow
 - Delayed evaluation of statements/blocks
 - Execution in context
 - Lazy computation
 - Parallel computation (body of loops! — GCD —)



Closures in Mainstream

- Closures (or close approximation to closures)
 - In Java
 - Event Handlers. Anonymous classes. Native in JDK1.8
 - In C#
 - Anonymous blocks
 - Syntactic sugar. Rewritten as Java anonymous classes
 - In Python
 - Lambdas: Python
- SML
- In C++
 - Lambdas in standard! (C++0x11)

```
lambda x : x + 1
```

```
fn x => x + 1
```

```
[=] (int x) -> int { return x + 1; }
```



Summary

- **Closures**
 - Allow lazy/delayed evaluation
 - Modulate the control flow
 - Extend classic functions naturally
 - Doable in functional, imperative, O.O. Languages
 - Language must provide the support though!
 - Moving (increasingly fast) in the mainstream
 - Java, C++, C#, Python, Rust, Nim, Go, Ruby,
 - Limitation
 - Deal only with a single function
 - No major alteration to control-flow



Overview

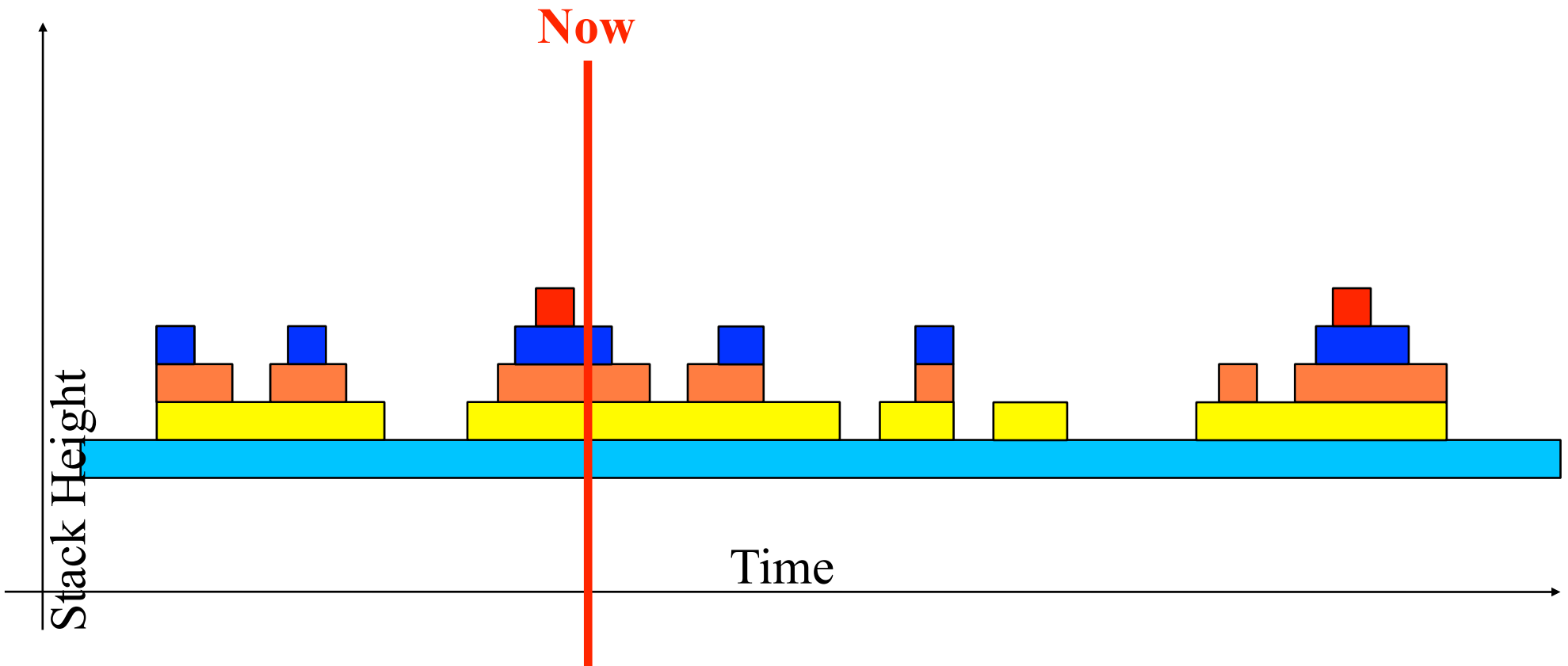
Continuations

- Definition
- Application
 - Compilation
 - First Class Objects
- Examples



Definition

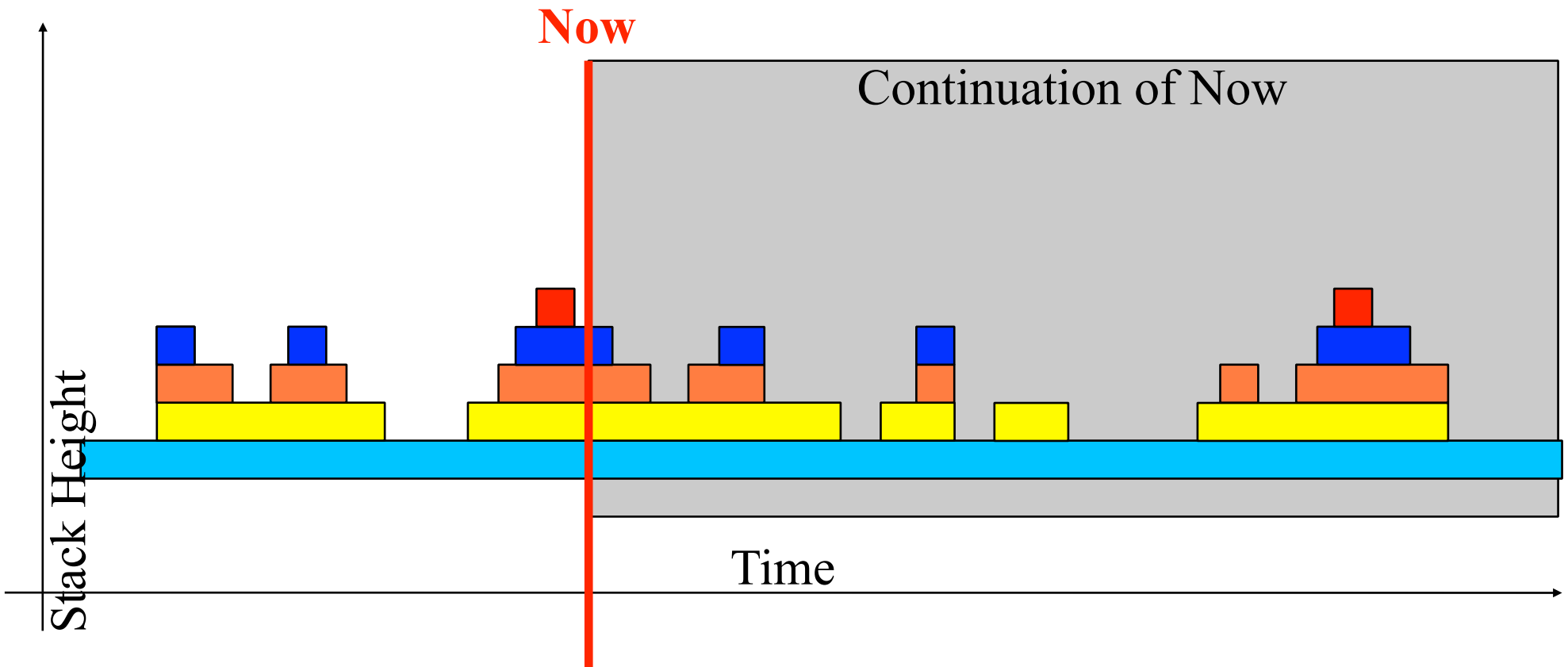
- What is a continuation ?
 - Informally
 - What is left to do from *now* to complete the computation.





Definition

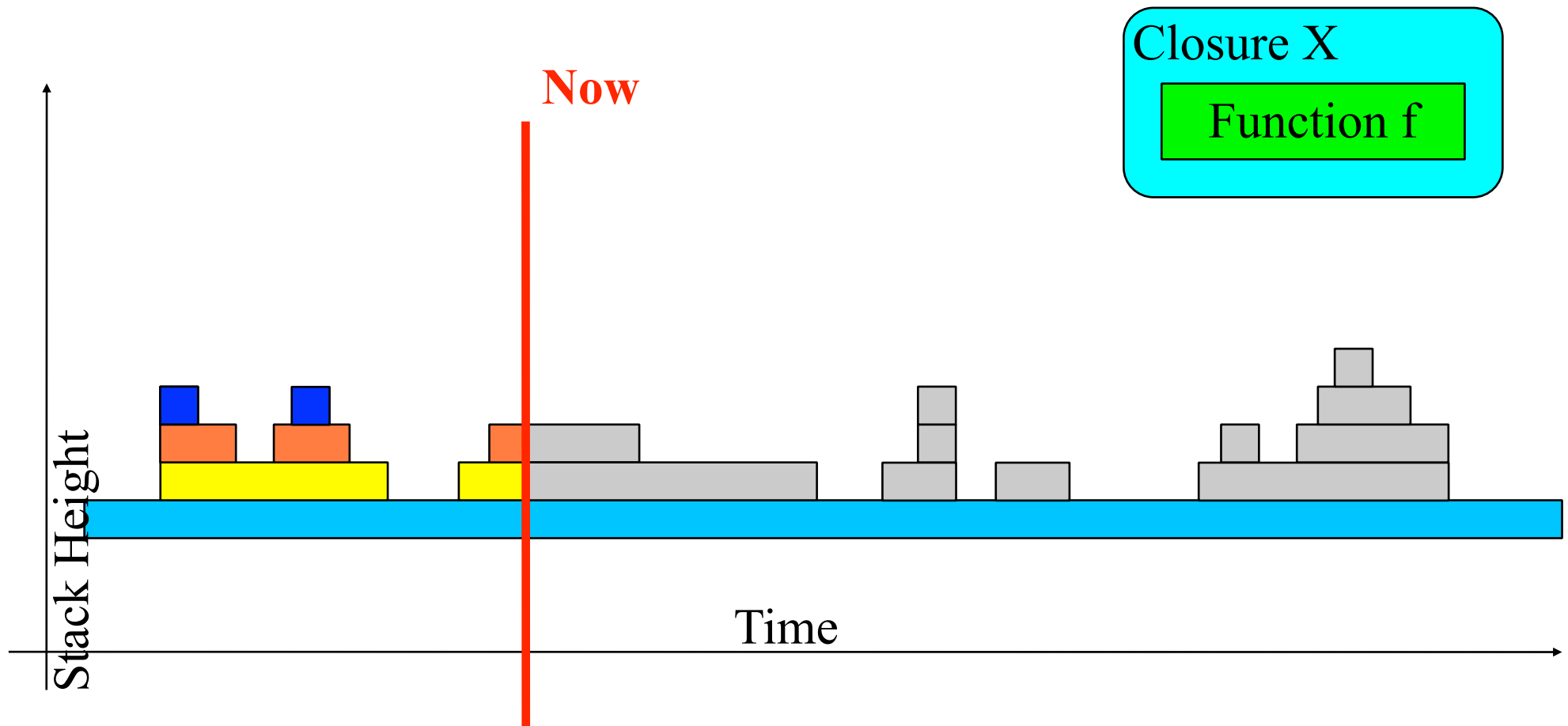
- What is a continuation ?
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 - What is left to do from *now* to complete the computation.





Continuation vs. Closure

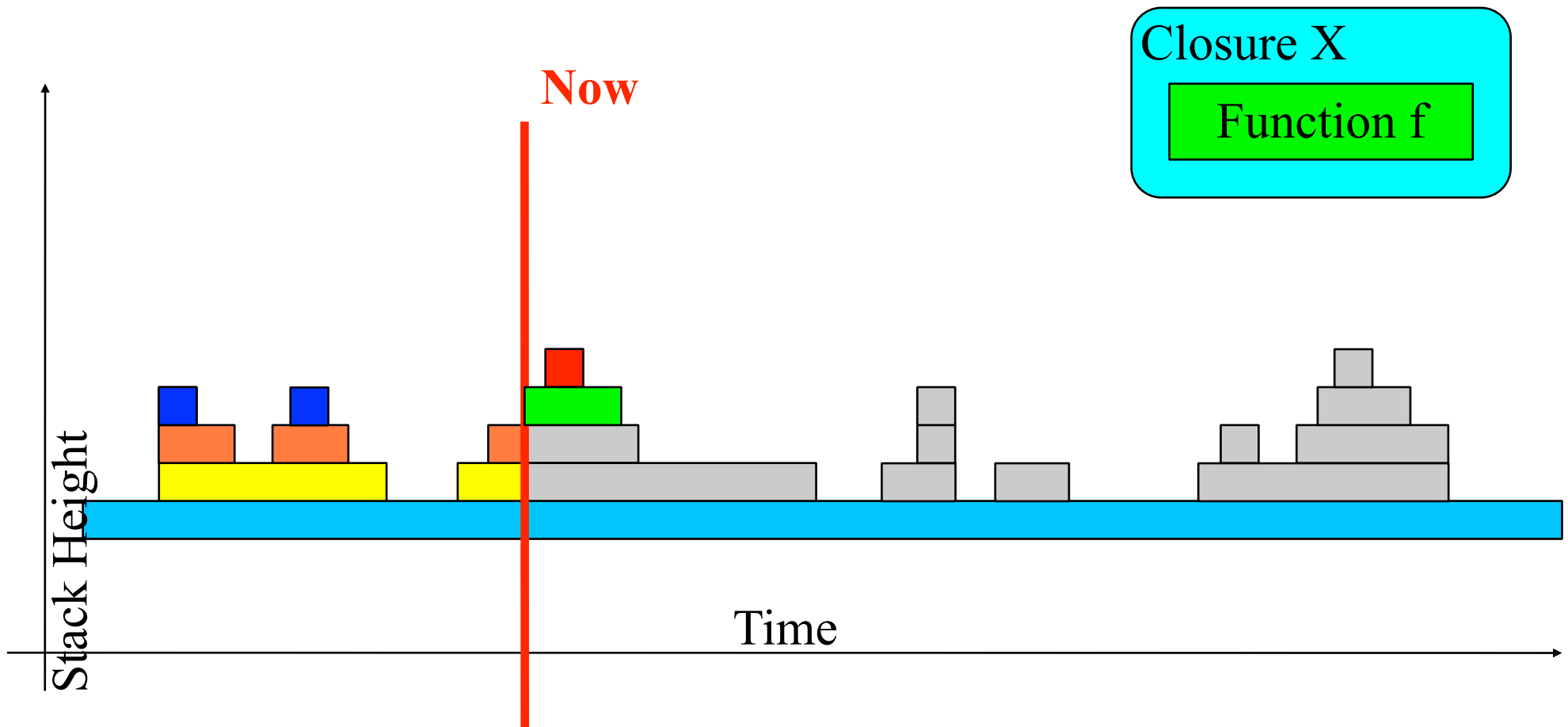
- Continuation is
 - The future of the computation.
- What is the difference with a closure ?





Continuation vs. Closure

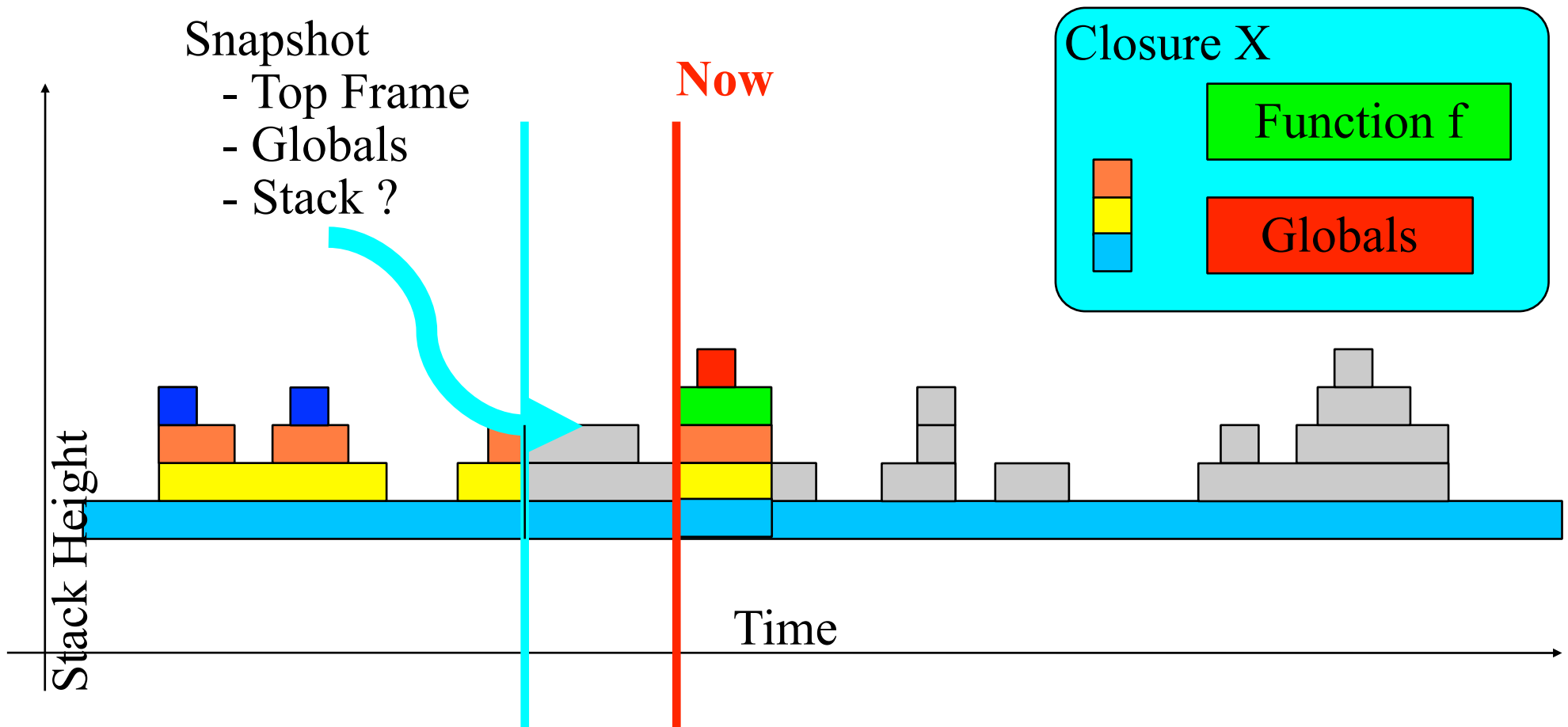
- **Picture of**
 - Calling function *f* right now
 - Function *f* executes in the current context





Continuation vs. Closure

- **Picture of**
 - **Creating closure X right now**
 - Closure X capture the current environment : Snapshot!





Difference

- A Continuation captures
 - The whole future
- A closure captures
 - A snapshot of the context
 - Allow us to execute a function in context
 - It does not manipulate the future.



Content of a Continuation

- What must be saved exactly ?
- Answer
 - Everything necessary to resume execution
 - Stack
 - Processor state
 - Stack pointers
 - Registers
 - PI-counter
 - Runtime state
 - Additional info such as
 - Pending events



Application

- What can we do with continuations ?
- Two Application areas
 - Compilation technique
 - Many control structure can be described in terms of continuations
 - A method to rewrite programs and eliminate the stack altogether
 - First class objects to
 - Manipulate your future
 - Control the control-flow



Control Primitives

- Consider a few primitives
 - Non-local exit [aborts]
 - Goto
 - setjmp/longjmp
 - Exceptions



Non-local Exit

- Purpose
 - Abort a computation when an error is detected
- The continuation view
 - The program has two possible future
 - Carrying on with whatever it is doing
 - Stopping
 - The abort primitive
 - Replaces the default continuation [carry on] by
 - The continuation that stops.
- Bottom line
 - We must be able to override the current continuation



Goto

- **Purpose**
 - Transfer control from the current point to a labeled instruction already seen
- **The continuation view**
 - **When seeing a label**
 - Save the current continuation into a table indexed by Lbl.
 - **When seeing a jump**
 - Lookup the continuation table at lbl and transfer control to that continuation
 - Jump could be non local (from a nested function call)
- **Any catch ?**

```
Lbl : x = x + 1
      if B
      then
          <body>
          jump Lbl
      endif
      <goon>
```



setjmp/longjmp

- **Purpose**
 - C functions that achieve non-local jumps
 - Setjmp creates a “label”
 - Longjmp jumps to the specified “label”
- **Continuation view**
 - Same as goto really.
 - Again
 - Only save the control
 - Only save the stack pointer
- **Catch?**
 - The stack can only shrink.



Exceptions

- **Purpose**
 - Transfer control non-locally on error detection
 - Try – Catch construction to specify
 - The guarded block, the exception handler
 - Throw construction to trigger an exception
- **Required data structure**
 - A table indexed by exception types
 - As many entries as types of exceptions
 - Content of an entry
 - Closure
 - Continuation



Exceptions

- Continuation view
 - Entering a try-catch
 - Capture a closure for the exception handler
 - Capture the current continuation
 - Save the content of exception table at the exception offset
 - Save (Closure, Continuation) in the table (at correct offset)
 - When throwing
 - Execute the closure handler
 - Transfer control to saved continuation
 - When leaving a try-catch block (normally)
 - Restore the table entry for the exception type to its old value
 - Old Handler
 - Old Continuation



Program Rewrites

- Continuation can be helpful to rewrite programs
- Objective of rewrite
 - Eliminate the stack
 - Make programs tail-recursive
 - Allow optimizations
- Technique name
 - Continuation Passing Style or CPS
- More on this a little later....



Overview

- Definition
- Application
 - Compilation
 - First Class Objects
- Examples



First Class Object

- Continuation are useful inside the compiler
- Continuation are also useful for end-user
 - It gives control over the control-flow of the program
- Primitives
 - Two key abstraction
 - call/cc
 - Captures the current continuation
 - throw
 - Transfer control to a specified continuation
- From now on
 - Study continuation in functional language (ML)
 - Continuations are functions
 - Continuation take one argument



Basic Example

- Motivation
 - A starting example

```
fun bar x = print ("bar:" ^ (Int.toString x) ^ "\n")
fun zoo x = print ("zoo:" ^ (Int.toString x) ^ "\n")

fun test a b = a orelse b
fun foo a b x = if test a b
                  then bar x
                  else zoo x

val z = foo true false 5
```

Output => “bar: 5”



Basic Example

- **Motivation**
 - Get familiar with CPS

```
fun test a b = a orelse b
fun foo a b x = if test a b
                  then bar x
                  else zoo x
val z = foo true false 5
```

```
fun test' a b k = k (a orelse b)
fun foo' a b x = let fun f b = if b
                        then bar x
                        else zoo x
                  in test' a b f
                  end
val z = foo' true false 5
```

Output => “bar: 5”



Basic Example With call/cc

- **Motivation**
 - Get familiar with call/cc

```
fun test a b = a orelse b
fun foo a b x = if test a b
                  then bar x
                  else zoo x
val z = foo true false 5
```

```
val callcc = SMLofNJ.Cont.callcc
val throw  = SMLofNJ.Cont.throw
type 'a cont = 'a SMLofNJ.Cont.cont
```

```
fun test'' a b = (callcc (fn k => throw k (a orelse b)))
fun foo'' a b x = if test'' a b
                    then bar x
                    else zoo x
val z = foo'' true false 5
```



What we did...

- It doesn't look like much....
- But
 - We captured the continuation and bound it to k
 - Done with callcc
 - We can now manipulate k
 - Save it
 - Wrap it in a closure
 - Alter its argument
 - Call it
 - We finally called k with the original argument
 - Done with throw

λ Saving & Using Continuations

- How can we save a continuation...
 - Can we bind it to a local variable ?



References

- ML offers destructive variables
 - So we finally see assignments
- ML destructive variables are references
 - A reference is a pointer to the real object
 - Ref keyword to create a reference
 - := operator to modify the content of a reference
 - ! to lookup the content of a reference.

```
val int x = 10;  
  
val int ref x = ref 10;  
  
x := 20;  
  
val z = 2 * !x
```



Reference and Continuation

- If we want to save a continuation....
 - Store it in a reference!
- A catch
 - A reference must always be initialized
 - What should we store at declaration time ?
- Solution
 - Wrap the continuation in a data type with 2 alternatives
 - Empty
 - A real continuation



Reference and Continuations

```
val callcc = SMLofNJ.Cont.callcc
val throw  = SMLofNJ.Cont.throw
type 'a cont = 'a SMLofNJ.Cont.cont
exception Bad of string
datatype 'a SCont = Null
                  | Cont of 'a cont

fun Sthrow (Null)      v = raise Bad("No continuation")
  | Sthrow (Cont a)    v = throw a v

val myc : int SCont ref = ref Null
```

The variable myc holds a reference to a datastructure that could be null, or a continuation. The continuation stored in myc takes an integer as input.



Second Example

- Factorial... revisited

```
exception Bad of string
datatype 'a SCont = Null
                    | Cont of 'a cont
fun Sthrow (Null)      v = raise Bad("No continuation")
  | Sthrow (Cont a)    v = throw a v
val myc : int SCont ref = ref Null

fun fact 0 = (callcc (fn k => (myc := Cont k;1)))
  | fact n = let val _ = print("Going down (n= " ^
                                (Int.toString n) ^ ")\n")
              in n * fact(n-1)
              end;

let val y = fact 5
in
  print("y = " ^ (Int.toString y) ^ "\n");
  Sthrow (!myc) 1
end
```