

For instructions on how to checkout the template code for the assignment and submit solutions using git, see <http://dna.engr.uconn.edu/moodle/mod/page/view.php?id=129>

Exercise 1. (50 points) Consider the sequence of integers defined by a positive integer x_0 using the recurrence

$$x_k = \begin{cases} x_{k-1}/2 & \text{if } x_{k-1} \text{ is even} \\ 3x_{k-1} + 1 & \text{if } x_{k-1} \text{ is odd} \end{cases}$$

For example, starting with $x_0 = 13$, this produces the following sequence:

$$\begin{aligned} x_0 &= 13 \\ x_1 &= 40 \\ x_2 &= 20 \\ x_3 &= 10 \\ x_4 &= 5 \\ x_5 &= 16 \\ x_6 &= 8 \\ x_7 &= 4 \\ x_8 &= 2 \\ x_9 &= 1 \\ &\dots \end{aligned}$$

If a value of 1 is reached, then the sequence becomes repetitive (1, 4, 2, 1, ...). It has been conjectured by Collatz that the sequence reaches 1 for any positive integer x_0 .¹

Write a C program that reads from the standard input two positive integers a and b , and then prints to the standard output the integer x between a and b that takes the largest number of steps to reach 1. In case x is not unique, print the smallest one.

Exercise 2. (50 points) Recall that each term in the Fibonacci sequence is generated by adding the previous two terms. When starting with 1 and 2, the first terms of the sequence are:

$$1, 2, 3, 5, 8, 13, 21, 34, 55, 89, \dots$$

Write a program to compute and print to the standard output

- a) the sum of all even Fibonacci numbers smaller than 1 billion, and
- b) the sum of all prime Fibonacci numbers smaller than 1 billion.

¹For more on Collatz' conjecture see https://en.wikipedia.org/wiki/Collatz_conjecture.