CSE4102: Programming Languages **Spring 2017**

Homework 1: ML Basics

Out Date: 01/19/2017
Due Date: 01/26/2017

Objectives

The purpose of this homework is to get you up to speed with ML. By now, you should be already familiar with the SML environment. Feel free to use the course web page to find additional information and tutorials if needed. Handin a single text file (`.sml` extension) with your answers. Do respect the name of the functions to ease grading. If you need helper (auxiliary) functions, feel free to add them. Do comment your code.

1 Question 1

1. Write an ML function `sum` which, given an integer n computes the sum of the first n naturals. Namely, it computes:

$$\sum_{i=0}^{n} i$$

---

fun sum n = ...

---

2. Extent your `sum` function into a `sumsq` function. Given a natural n, `sumsq` computes the sum of the squares of the first n naturals:

$$\sum_{i=0}^{n} i^2$$

---

fun sumsq n = ...

---

3. Write an ML function `sumOdd` which, given a natural n computes the sum of the first n odd naturals. For instance, `sumOdd 4` $= 1 + 3 + 5 + 7$ (i.e., we have four terms).

---

fun sumOdd n = ...

---

4. Write an ML function `fib` that computes, from an integer input n, the n^{th} Fibonacci number (as always, $fib(i) = fib(i-1) + fib(i-2), fib(0) = 0, fib(1) = 1$).

---

fun fib n = ...

---

5. Write an ML function `fibFast` that computes, in linear time, from an integer input n, the n^{th} Fibonacci's number.

---

fun fibFast n = ...

---

2 Question 2

Consider the Taylor expansion of the `sin` function

$$\sin(x) \approx x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

As you can see, it is *also* a sum of terms with a couple of twists. First, the signs alternate, second and the terms for even numbers are *missing*. The formula in sigma notation is

$$\sin(x) \approx \sum_{i=0}^{n} \frac{(-1)^i}{(2 \cdot i + 1)!} . x^{2 \cdot i + 1}$$

where the order$-n$ summation contains the first $n + 1$ terms of the expansion. Write a linear time ML function `sinappx` that, given a value n, computes in linear time in n, the Taylor expansion up to order n. As you surely realized, computing (for instance) factorials from scratch for each term would not deliver the desired complexity.

---

fun sinappx n x = ...

---

3 Question 3

Your task is to write a function that compute a definite integral, namely,

$$\int_a^b f(x)dx$$

where f is an arbitrary continuous unary function. Naturally, you should provide a numerical solution using a simple method such as the trapezoidal method over a collection of boxes that sub-divide the integration interval $[a, b]$. Formally, given f, a, b and n, subdivide the range $[a, b]$ into n sub-intervals (of width $\frac{b-a}{n}$) and compute the integral as the sum of the surfaces of the trapezoid embedded in each sub-interval.

---

fun integrate f a b n = ...

---

4 Question 4

The variance of a list of reals $[a_1, a_2, \cdots, a_n]$ is the average of the squares minus the square of the average, i.e.,

$$\frac{\sum_{i=1}^{n} a_i^2}{n} - \left(\frac{\sum_{i=1}^{n} a_i}{n} \right)^2$$

Write the most elegant ML function `variance` you can that computes the variance. (Short and clear is ideal). The function takes as input a list of integers. Remember that ML types all the expressions it encounters and does not mix integers and reals. If you have an integer that you wish to convert to a real for arithmetic purposes you can use the function `fn real : int -> real`. Similarly, do recall that the types of the two literals 0 and 0.0 are `0 : int` while `0.0 : real`.

Have fun!