

# **Reactive Trajectory Planning in Structured Dynamic Urban Scenarios with Static and Dynamic Obstacles**

**Pradeep Korivi**

From the Faculty IV - Electrical Engineering and Computer Science

Technische Universität Berlin

Department of Telecommunication Systems -Communication and  
Operating Systems

**Master of Science**



Guides : Prof. Dr.-Ing. Reinhardt Karnapke  
Prof. Dr. habil. Odej Kao



# Abstract

The goal of any motion planner is to achieve a driving trajectory that is collision free, smooth and responsive (reactive), at the same time being far-sighted to maintain consistency, deliberative and allow smooth transitions. Prior approaches solved this problem through different methods having various complexity levels. This thesis proposes an on road motion planner with combination of path-velocity combination along with a low computational overhead collision avoidance system. The reactive nature tunability of the proposed thesis is suitable for urban driving scenarios.



# Abstrakt

To be translated



**Eidesstattliche Erklärung**

Ich bestätige, dass diese Masterarbeit meine eigene Arbeit ist und ich alle verwendeten Quellen und Materialien dokumentiert habe. Diese Arbeit wurde zuvor keinem anderen Prüfungsausschuss vorgelegt und ist nicht veröffentlicht worden.

**Statutory Declaration**

I confirm that this Master's thesis is my own work and I have documented all sources and material used. This thesis was not previously presented to another examination board and has not been published.

.....

Pradeep Korivi

Berlin, xxxx yyyyyyyyyy 2018





# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
1.1	Motivation . . . . .	1
1.2	Problem Statement . . . . .	1
1.3	Thesis Statement . . . . .	1
1.4	Thesis Contributions . . . . .	1
1.5	Thesis Structure . . . . .	1
<b>2</b>	<b>Related Work</b>	<b>3</b>
2.1	Planning Approaches . . . . .	3
2.1.1	Incremental Search Approaches . . . . .	4
2.1.2	Lattice Planners . . . . .	4
2.1.3	Local Search . . . . .	5
2.2	Trajectory Evaluation . . . . .	5
<b>3</b>	<b>Vehicle Setup</b>	<b>7</b>
3.1	Vehicle Base . . . . .	7
3.2	Sensors . . . . .	7
3.3	Computational Power and Software Architecture . . . . .	7
3.4	Vehicle Control . . . . .	7
3.5	Localisation . . . . .	7
<b>4</b>	<b>Planning Algorithm</b>	<b>9</b>
4.1	Introduction . . . . .	9
4.2	Localization . . . . .	9
4.3	Prediction . . . . .	10
4.4	Route Planner . . . . .	11
4.4.1	RNDF . . . . .	11
4.4.2	Path Representation and calculation . . . . .	13
4.4.3	Behavioral Layer . . . . .	14
4.5	Motion Planner . . . . .	14
4.5.1	Temporal Horizon . . . . .	15
4.5.2	Path Modelling . . . . .	16
4.5.3	Trajectory Creation . . . . .	17
4.5.4	Checking for Static Obstacles . . . . .	21
4.5.5	Checking for Dynamic Obstacles . . . . .	22
4.5.6	Cost Functions and Trajectory Selection . . . . .	23
4.5.7	Velocity Planning . . . . .	25
4.6	Trajectory Follower . . . . .	25

---

<b>5</b>	<b>Implementation</b>	<b>27</b>
<b>6</b>	<b>Evaluation</b>	<b>29</b>
6.1	Experiments . . . . .	29
6.1.1	Lane blocked . . . . .	29
6.1.2	Slow Moving Traffic . . . . .	30
6.1.3	Merging into traffic . . . . .	30
6.1.4	Merging into next lane with opposite traffic . . . . .	31
6.1.5	Road Blocked or Pedestrian Ahead . . . . .	32
6.1.6	Dynamic Obstacles - other vehicles . . . . .	32
6.2	Criteria Based Evaluation . . . . .	33
6.2.1	Optimality . . . . .	34
6.2.2	Feasibility . . . . .	36
6.2.3	Completeness . . . . .	36
6.2.4	Runtime . . . . .	37
6.2.5	Deliberative Approach . . . . .	38
6.3	Comparisons to other Planners . . . . .	38
<b>7</b>	<b>Conclusions and Future Work</b>	<b>39</b>
7.1	Conclusions . . . . .	39
7.2	Future Work . . . . .	39
	<b>Bibliography</b>	<b>41</b>

CHAPTER 1

# Introduction

---

1.1 Motivation

1.2 Problem Statement

1.3 Thesis Statement

1.4 Thesis Contributions

1.5 Thesis Structure



## CHAPTER 2

# Related Work

---

Motion planning is a widely researched topic with roots in control, artificial intelligence, computational geometry and Robotics. The literature presented in [18], [19] review significant portion of standard planning techniques. Due to the complexity of the autonomous cars and the environments, general techniques from robotics cannot be applied. Planning for autonomous cars in general is sub divided into two categories namely planning in unstructured environments such as parking areas etc and structured environments such as Road Networks where the traffic rules have to be followed. This chapter mainly focuses on the planning techniques in structured urban environments.

## 2.1 Planning Approaches

Autonomous vehicles have been in research communities from 80's with projects such as PROMETHEUS [1], the research was accelerated by the DARPA Grand Challenge in 2006 and DARPA Urban Challenge(DUC) in 2007 [4]. In DUC, six autonomous vehicles from different universities have completed the challenge and have used various techniques to accomplish the task. These techniques have formed the basis for modern day autonomous cars which perform much sophisticated compared to the DUC participants.

Approach followed by different participants of DUC are described in [4]. Planning module of Boss, autonomous vehicle from Carnegie Melon University which won the DUC is described in general here. Its planning framework is mainly subdivided into 3 sub modules

- Mission control is the higher level module which creates a global path, assigns lane to the vehicle to reach the check points, detects blockades.
- Behavioral module is responsible for decisions such as precedence at intersections, lane change decision, speed planning, look ahead point assignment etc.
- Motion planning module is responsible for generating local trajectories and converting them to steering and acceleration values. It initially creates a forward looking path and velocity planning is performed on top of that.

After DUC, there has been a huge research into this accelerated by the industry because of benefits from autonomous driving. Planning for autonomous driving in

general can be classified as shown in Figure 2.1. The following sub sections discuss research in each of the described sub-categories.

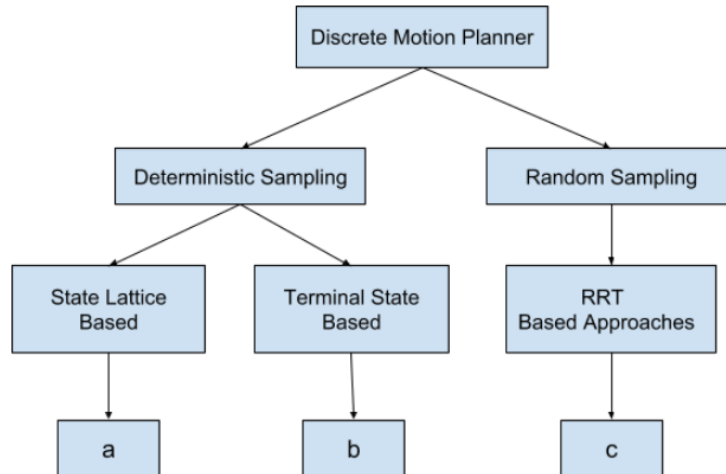


Figure 2.1: General Overview of On-road planners. a - [23] [27] [29] [13] [3] , b - [16] [20] [4], c - [10] [17] [9]

### 2.1.1 Incremental Search Approaches

different RRT approaches - start from MIT planner

RRT\*

RRT with Replanning

advantages and disadvantages of RRT

Check Davids thesis

### 2.1.2 Lattice Planners

The algorithm uses a discrete representation of the planning area with multiple states, often a multi dimensional one with dimensions such as position, acceleration, velocity, time etc. All of these states are connected together and the problem then reduces to finding a path from the initial state to the final state in the lattice.

write different improvements in different planners proposed

howard initial lattice

werling et al - next

mcnought - cmu

cmu traj optimization

diss shui planner

Advantages and disadvantages of Lattice

**2.1.3 Local Search**

Local Search or forward projecting trajectories Different types of curves, forward samples,

- Velocity planning - different approaches - check if needed

- sampling Action space - something similar to this thesis, State Space - final states

- Combining Path and Velocity Werling, Lattices, Unit A\* approach

- Other approaches like Voroni Diagrams

- Driving Corridors

**2.2 Trajectory Evaluation**

collison checking - different approaches

- cost functions - for different parameters





## CHAPTER 3

# Vehicle Setup

---

Check if needed, and how much is needed

Describe about different components, their function, limitations etc

### **3.1 Vehicle Base**

### **3.2 Sensors**

### **3.3 Architecture & Computational Power**

### **3.4 Vehicle Control**

### **3.5 Localisation**



## CHAPTER 4

# Planning Algorithm

---

### 4.1 Introduction

The goal of the path planner is to navigate the robot from the start configuration to destination by blending in the traffic. Path planning module is dependent on various modules to receive the data regarding the perceived environment and invoke a set of modules to move the robot safely. It has to drive the robot ahead considering the traffic rules, obstacles, kinematic and dynamic constraints of the robot and not compromising on the safety and comfort of the passengers inside. The main aim of this chapter is to derive path planning techniques to drive the robot safely to destination.

This chapter is organised to provide an overview of different modules needed for path planning and methods used in each module to achieve the goal. Path planning starts from the initial understanding of where the ego vehicle is and where to go, subsection 4.2 describes regarding localization of ego vehicle to provide this data, subsection 4.4 provides the information on how a global path to the destination is calculated. An autonomous vehicle should have the understanding of surroundings in terms of where other vehicles are, where pedestrians are, traffic signal information etc., Prediction module described in 4.3 details further on how the ego vehicle perceives environment. The next subsection 4.5 describes further in-depth details about the short term planning algorithm or the trajectory planner. The final module in the discussion is control unit, described in subsection 4.6. It is responsible for translating the path in space-time into steering and acceleration values to drive the robot.

Figure 4.1 represents the general architecture of the planning module. It details on the flow of information, dependencies, relative execution frequency. All the modules are implemented as independent nodes in Robot Operating System(ROS) and communicate with each other using ROS messages.

### 4.2 Localization

Localization module is responsible for providing the current state of the vehicle in terms of position, orientation, speed(linear and angular) and acceleration. The localization module implemented on the modelcar has two sub components Vehicle Odometry and Global Position estimation using Visual GPS. Odometry is calculated

Mention the ego vehicle, model car, robot are used interchangeably in the document

Add reference as udacity course material for path planning overview picture add different color to highlight the main modules I am working on

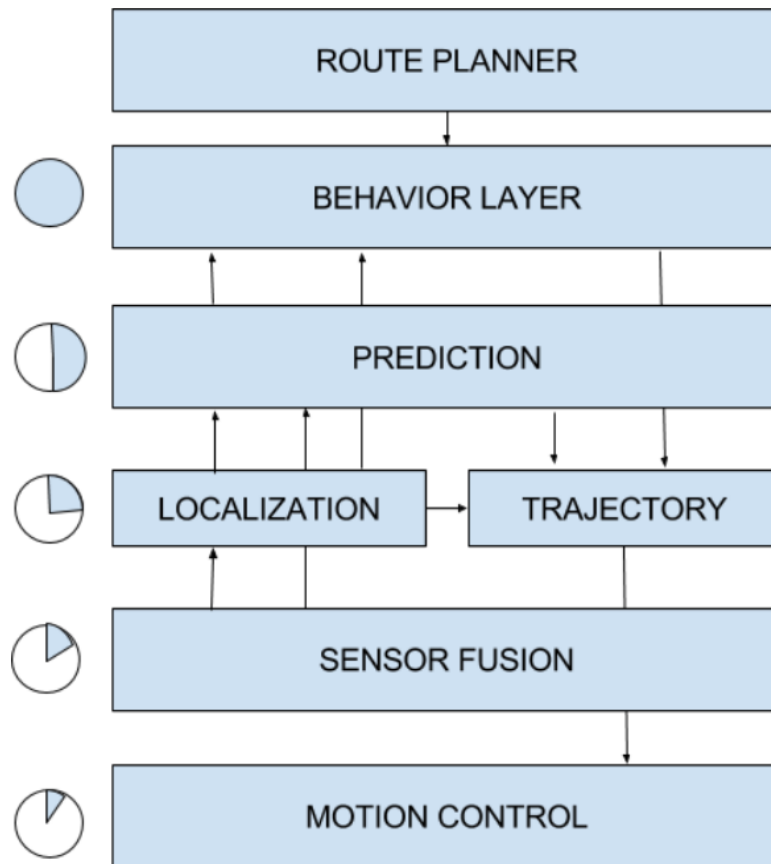


Figure 4.1: Path Planning Module

with dead reckoning [6] with speed information from motor and yaw information from Inertial Measurement Unit(IMU). The localization module combines odometry with the information received from a visual GPS node(tracks markers on roof) to correctly estimate the state of the ego vehicle.

### 4.3 Prediction

Prediction and Sensor fusion modules receive the data from various sensors such as Cameras, LIDAR etc and fuse them together to create an environment model, classify objects into different categories and predict the state of obstacles in the surroundings. Due to time constraints this thesis simulates a prediction module to provide motion planner with obstacle information in different traffic scenarios.

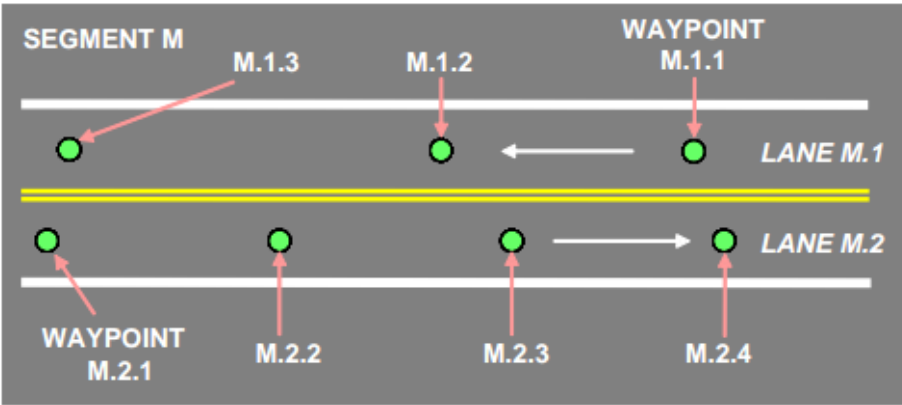


Figure 4.2: Segment representation in RNDF - Segement M has two lanes M1, M2 and each lane has way points 1-N

4.4 Route Planner

Route Planner is responsible for finding a global route between the vehicle current state and the goal state based on the static characteristics of the environment/map such as lane information, speed limits etc. Route planner obtains this information generally from the maps or other formats to represent the road network. In this thesis a simple model called " Road Navigation Definition File(RNDF)" [8] [7] is used to represent the route network. Next subsections details further about RNDF and how global reference route is calculated.

4.4.1 RNDF

This chapter details about the RNDF file [8] that defines the road network(set of roads/ areas connected together) over which the vehicle can traverse. This representation of road is developed by DARPA for its Autonomous Vehicles Urban Grand Challenge. RNDF representation first divides the traversable areas into two parts, segments and free zones and mentions regarding connections across these areas. Free zones represent areas such as parking lots and road segments are drivable lanes. Each segment has multiple lanes, each lane has way points along the driving direction. More significant information about way points such as whether it is a stop sign etc can be added. Each segment/zone is connected to one another using exits, they represent the connections between one segment way points at start/end to another. Figures 4.2 4.3 4.4 [8] represent various portions of the route representation and Figure 4.5 details regarding one of the route network of map used in Lab experiments.

foot note -  
source of im-  
ages

Add Appen-  
for how to c-  
ate the RND-  
file for mod-

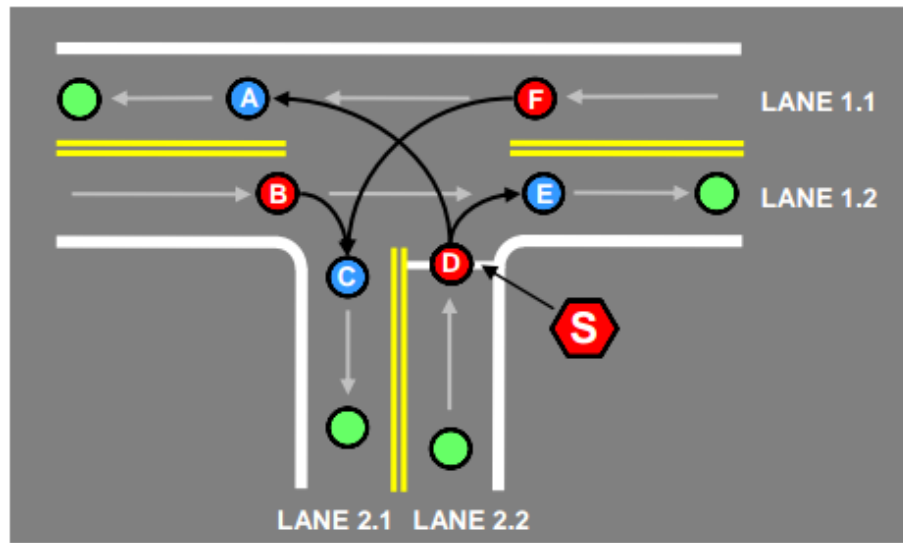


Figure 4.3: Exit representation in RNDF - Connections between two segments in a T-Junction

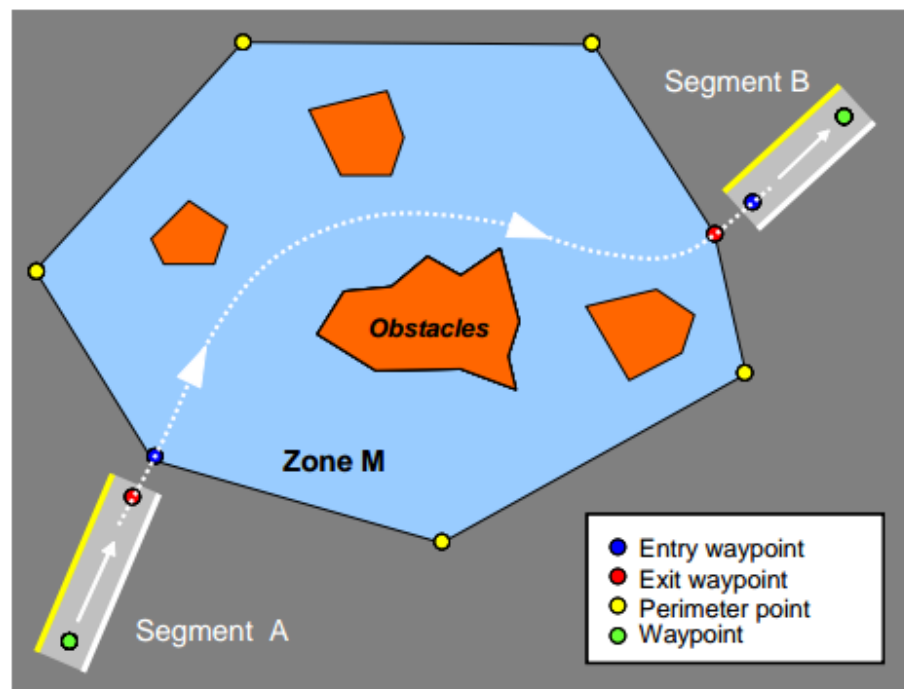


Figure 4.4: Zone Representation in RNDF - Connection between Segments and Zone



Figure 4.5: Road network for the map used to test model Car

#### 4.4.2 Path Representation and calculation

The data in RNDF is represented in the form of a tree with connections across way-points which in turn are related to their parent objects lanes and segments. The global path from source to destination is the shortest path between the closest way-point to ego vehicles current position and closest way-point to the destination in graph. Then the shortest path found is sub divided into sub-paths based on which segment the way points lie. A sub-path represents a set of way points in one segment, once the ego vehicle is at the end of one sub-path it receives a notification from the trajectory planner that a goal has been reached, then the route planner transmits the next sub path to the trajectory planner, this process is repeated till destination is reached. Figure 4.6 details further about the division of shortest path across different segments. This method also reduces the memory needed in modeling the road in trajectory planning stage.

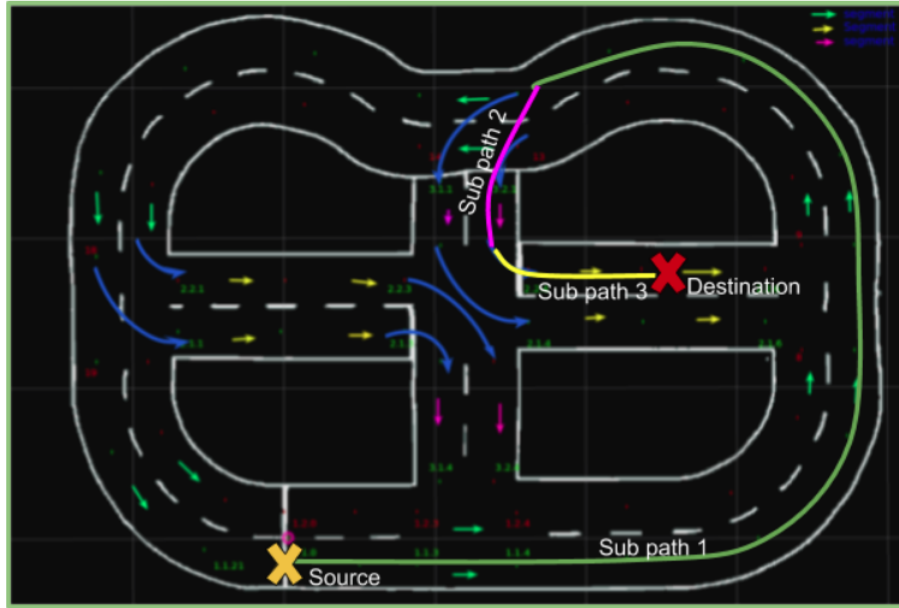


Figure 4.6: Division of shortest path in road network into Sub paths across different segments

### 4.4.3 Behavioral Layer

Behaviour layer plays an important role in making path planning, it is responsible for understanding the scenario and makes decisions according to various traffic rules, constraints and choices that will make driving more efficient for the vehicle. One such decision example is which lane the vehicle should drive, the decision is made by understanding whether the lane is empty for a significant time, if the ego vehicle is about to take an exit then a right lane is preferred, lane based on driving speed etc. Behavioural layer is a huge research topic in itself and not in the scope of this thesis, currently, a simple simulated approach is implemented where the user can provide inputs to decide behaviour using mouse clicks etc.

## 4.5 Motion Planner

This section discusses the planning algorithm to create short-term trajectories in accordance with the global path to reach destination. The subsection 4.5.1 provides an overview of the timing Horizon and constraints in dynamic environments for different modules in planning. The next subsection 4.5.2 details regarding path modeling and how this will improve the efficiency of planning along with constraints. It also discusses on approximation method used to convert coordinates from Cartesian to Frenet frame. The core of this chapter is creation of trajectories which is discussed in detail in subsection 4.5.3. The next sections 4.5.4 and 4.5.5 explain further on how



the created trajectories are evaluated for collision with static and dynamic obstacles. Next subsection 4.5.6 details on how a final trajectory is selected from the set of evaluated trajectories for the trajectory follower to follow.

### 4.5.1 Temporal Horizon

Time is an important aspect for planning in dynamic environments and there are several timing variables associated with planning. This section is mainly adopted from the doctoral thesis [22]. These timing parameters define how far into the future different sub modules of planning will be valid.

The initial timing variable in motion planning is timing horizon  $T_m$ . It is measure of how far into the future trajectory of the ego vehicle is planned. Second is the prediction horizon  $T_p$ , it is measure of how far into the future the motion of dynamic obstacles around can be predicted. The fundamental requirement of planning to be valid is that  $T_m \leq T_p$  such that planning is done only so far into the future as the environment is predictable.

Thirdly  $T_d$  indicates the computation time of the motion plan. Assuming planning is done in cycles, the plan created in previous cycle is executed in current cycle, thus  $T_d \leq T_m$ . If this condition fails then the planner will run out of path for the next cycle. In general  $T_d \ll T_m$ .  $T_s$  is the perception update cycle time, i.e., perception module updates the state of surrounding dynamic obstacles every  $T_s$  seconds. In general world the predicted trajectories for duration  $T_p$  will not hold true as the behavior of these vehicles is not controlled by the ego vehicle. Thus the constraint  $T_s \leq T_p$  should be valid. This creates an uncertainty in modeling of the environment, thus the execution duration of current plan  $T_e$  beyond  $T_s$  is not sensible. This is due to fact that obstacle trajectories may have changed in  $T_s$  and executing the old trajectory may lead to collisions invalidating the trajectory created for  $T_m$ .

The next timing constraint in consideration is  $T_e$ , control execution time of the current plan.  $T_e$  should not exceed the perception update time  $T_s$ . This restriction also imposes additional constraint on  $T_d$  (motion plan computation time),  $T_d \leq T_e$ .

In summary, timing constraints described above identify the relation between different modules such as motion planning, motion prediction and execution. It is also important to predict farther into future than  $T_s$  or  $T_e$  for completeness of motion planner with respect to goal objective, uncertainty also increases with time. In general a farsighted uncertain motion plan potentially directing the vehicle towards goal is better, but this plan needs to be re-evaluated and re-executed in short intervals for correctness.

In general behaviour of other vehicles can be predicted for up-to 5s probabilistically, thus the temporal  $T_m$  & prediction  $T_p$  horizon are chosen to be 5s. The planner has an execution time  $T_d$  far less than 100ms on a low power computational hardware which allows a high update rate allowing lower values for  $T_e$ . As obstacle detection

check if this name is needed and add footnote to link "Autonomous vehicle navigation in dynamic urban environment for increased traffic safety" of Macek kristijan doctoral work

check if it is needed to write on time discretization

check if this should be moved to implementation

is simulated a pessimistic value of 250ms for  $T_s$  is chosen and even higher update frequency can be chosen as the planner is fast enough to react.

### 4.5.2 Path Modelling

Planned global path is in Cartesian coordinate system, one of the problems with the Cartesian coordinate system is that the due to variation in curvature local planning becomes complex. To address this issue planning in curvilinear system or Frenet Frame or lane adoptive (SL) coordinate system has been adopted by researchers, [29] [30] [27] [20] [5] [15] are some of the research works in which Frenet frame is adopted. In this method center of the lane/road or preplanned global path is used as reference longitudinal coordinate(S) and perpendicular distance with respect to the lane center is considered as lateral coordinate(L/D) as represented in Figure 4.7 [27]. Thus once converted, (S,L) coordinate system essentially is a straight road.

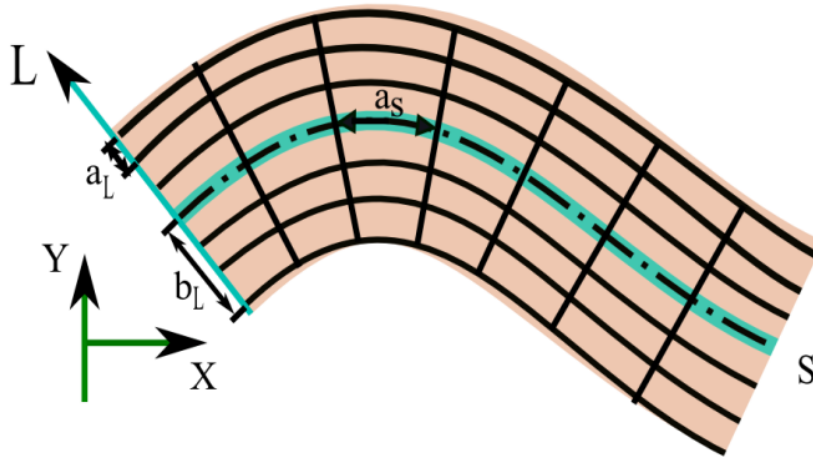


Figure 4.7: SL coordinate system laid over XY coordinate system

Conversion from Frenet frame to Cartesian and vice versa is a widely researched topic and many techniques exist which offer different level of complexity and accuracy. In this thesis an approximation method is used to convert between these two coordinate systems similar to [5]. This method is computationally inexpensive and provides required level of accuracy for modelcar. To convert an xy coordinate to SL coordinate, (x,y) is projected onto the current path represented by different way points as in figure 4.8, cumulative distance till this point gives the S coordinate and the perpendicular distance between the projected point and the current point provides the L coordinate. A similar process is used to convert S,L coordinate to x,y coordinate. S is used to find a point on a segment represented by way points, a

point at a perpendicular distance  $L$  gives the  $x, y$  coordinate. We assume that the path between two way points is linear which reduces the computational complexity in approximation. This approximation however approaches zero error when the spacing between two way points approaches zero. Adding dense way points in the curves significantly reduces the approximation error. There are different methods discussed in [26] [14] which provide better accuracy in calculating the paths.

As observed in Figure 4.7, in SL coordinate system the size of the unit distance is not constant, it stretches in the convex side of road and gets compressed in concave side of the reference line. This is especially an issue in curves with lower radius of curvature, this will affect the velocity planning thus causing discomfort in some cases. There is a wide research in topic of velocity and path smoothening which counter these affects.

In summary, curvilinear coordinate system makes planning easier but needs extra computation in conversion from one format to other. It also introduces errors and inefficiencies in planning if the complete planning is done in SL coordinate system.

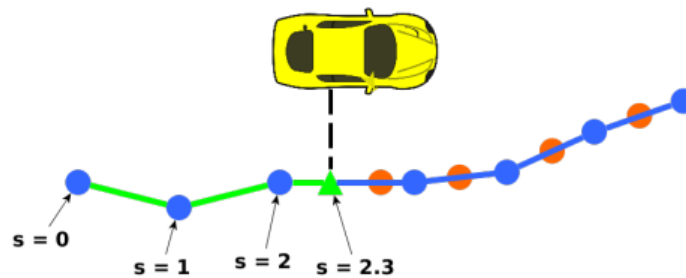


Figure 4.8: Showing projection of the current position of the car shown by green triangle to  $s$  coordinate system. Each circle represents a node, and the lines between them are links.

### 4.5.3 Trajectory Creation

The core of this thesis is the trajectory planner that drives the robot from source to destination. Understanding from the behavior of human drivers in structured environments (road networks) it is necessary for the planner to create trajectories that avoid collisions, align with the road network, smooth, continuous and comfortable. Chapter 2 discusses a great amount of literature in motion planning techniques.

The approach proposed in this thesis is inspired by how human drive, i.e., the driver tries to maintain an optimal speed, next shift laterally based on obstacles

add VOLVO  
reference re  
ference in fo  
note and als  
remove the  
ange dots

motion plan  
ning tech-  
niques be  
presented  
properly in  
related work

ahead and brake if a collision is predicted with current driving state or perform an evasive maneuver. To reach a speed vehicle need to accelerate/decelerate, this can be achieved with various levels of values based on current state as, each acceleration/deceleration level chosen will result in different final states. The initial step is to sample this set of acceleration values, Figure 4.9 shows how a vehicle which approached acceleration A3 at time  $t_0$  can continue to different levels of accelerations from A1 to A8. Generally, A1 represents an acceleration of around  $2m^{-1}$  and A8 of up to  $-8m^{-1}$  which are on the higher end of decelerations which not most of the cars are capable of performing. Generally deceleration values are up to  $-4.5m^{-1}$  [12] [24] [2]. Applying each of this acceleration profile to current ego vehicle state for planning horizon  $T_m$  leads to different final states of ego vehicle. The final sates will have different final velocity as shown in Figure 4.10, different distances traversed as in Figure .

Change of acceleration is defined as jerk and to create smooth trajectories it is important that the trajectories generated by the motion planner must have least jerk. There are various techniques to create these Jerk free trajectories as discussed in chapter 2. The selection of smoothness depends also on the capabilities of the ego vehicle and controller to track these fine trajectories.

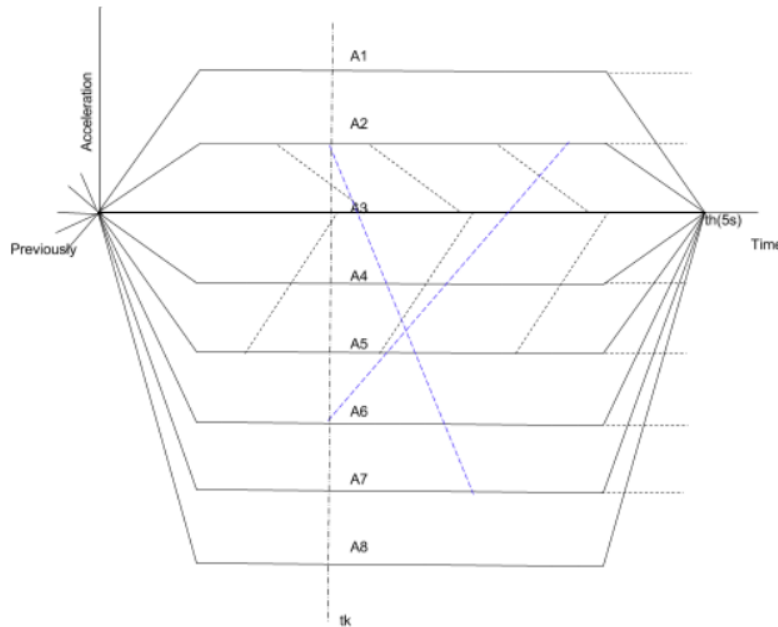


Figure 4.9: Different acceleration profiles a car can follow from current state.

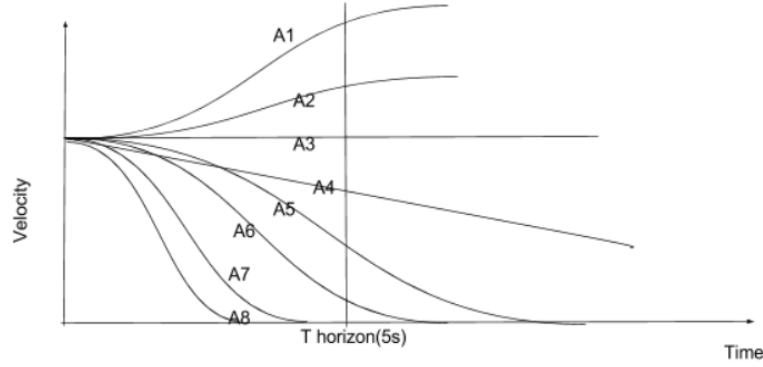


Figure 4.10: Different acceleration profiles a car can follow from current state.

Acceleration profiles discussed above solve the problem of longitudinal planning but to avoid obstacles the ego vehicle should also plan lateral(sideways) shifts in its trajectory. Similar to different accelerations, lateral shifts are sampled and combined along with acceleration samples to create final states. Lateral shifts can be mapped either as a function of time or distance traversed by ego vehicle. The research of Werling et al. [28] suggests that at lower speeds it is advantageous to map lateral shift as a function of distance and at higher speed as a function of time. As this thesis is intended towards urban environments with limited speeds, lateral shift is mapped as a function of distance traversed. Lateral shift planning in this thesis is adopted from [20], which uses cubic splines and models lateral shift as a parameter of longitudinal distance as shown in equation 4.1.

Redraw these profiles neatly with proper labels - acceleration and velocity

$$l(s) = c_0 + c_1s + c_2s^2 + c_3s^3 \quad (4.1)$$

The first and second derivative of the equation 4.1 are equations for lateral velocity 4.2 and acceleration 4.3.

$$\frac{dl}{ds} = c_1 + 2c_2s + 3c_3s^2 \quad (4.2)$$

$$\frac{d^2l}{ds^2} = 2c_2 + 6c_3s. \quad (4.3)$$

Form the boundary conditions(0- initial state, f - final state), we have

$$l(s_0) = l_0, l(s_f) = l_f \quad (4.4)$$

The angle between the road frame and the vehicle is defines as  $\theta(s)$ , it can be derived from the first derivative of the lateral shift with respect to s.

$$\theta(s) = \arctan\left(\frac{dl}{ds}\right) \quad (4.5)$$

To ensure the generated path follows current curvature and orientation of car and the final orientation is parallel to the road segment, following conditions should be satisfied.

$$\theta(s_0) = \theta_0, \theta(s_f) = 0 \quad (4.6)$$

The figure 4.11 indicates how the initial orientation will affect the shape of the trajectory.

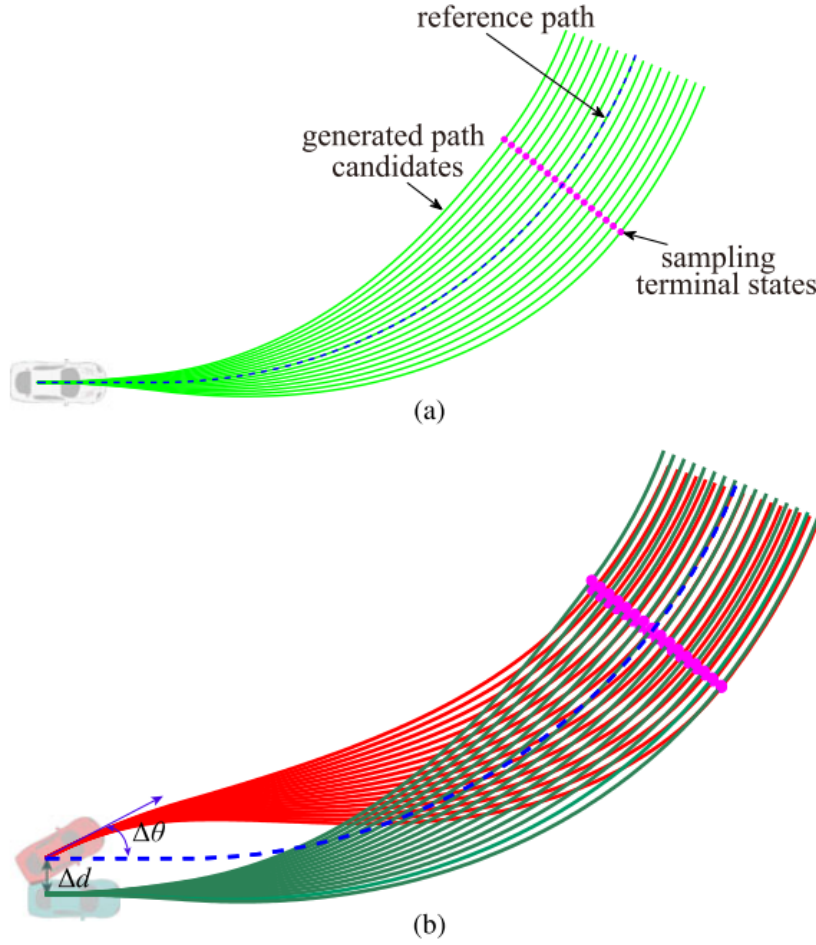


Figure 4.11: Path candidates generation results. (a)  $l_0 = 0$  and  $\theta_0 = 0$  (b)  $l_0 = \Delta_d$  and  $\theta_0 = \Delta\theta$ .

ate own fig-

The constants  $c_0, c_1, c_2, c_3$  in equation 4.1 can be obtained by solving equations 4.2 to 4.6.

In summary, combining samples in acceleration and lateral shifts, multiple trajectories with different final states are created over the time horizon. In the next

sections how these trajectories are tested for collision with respect to other dynamic obstacles is discussed.

add a picture of how this will look in end - various trajectories planning

#### 4.5.4 Checking for Static Obstacles

The main objective of the motion planner is to derive a path that is collision free. The generated trajectories must be evaluated for collision or driving close to obstacles. There are various techniques for collision detection as discussed in the background study. This thesis developed a simple two step collision checking technique for static obstacles. A road parallel model in Frenet frame is employed for collision checking ignoring the orientation of obstacles with respect to road as described in [23]. Using a road parallel method and dilating the obstacle for safety reduces computational complexity.

In the first step of collision checking, obstacle coordinates are transformed into Frenet frame and represented by a length and width parallel to road. In second step, the trajectory in consideration is checked if it has a collision in longitudinal dimension(s coordinate) for length of the obstacle. As shown in Figure 4.12 trajectories T0,T1 intersect in S for obstacle O1 and not for obstacle O2. The next step is to find this intersection region, I1 to I2 in Figure 4.12, the values are dilated for safety. In final it is checked whether from I1 to I2 there is a collision in lateral dimension(d) for trajectory and obstacle. This is computed by checking if at any point between I1 and I2, the lateral distance between trajectory and obstacle must always be greater than safety value as described in 4.7.

check if all things described in road parallel model for collision checking should be discussed here

$$|d_{\text{ego}} - d_{\text{obst}}| > \text{car\_width}/2 + \text{obstacle\_width}/2 + \text{safety\_margin} \quad (4.7)$$

It is clearly representative from figure 4.12 that the trajectory T1 has collision and trajectory T0 has no collision. Different costs can be added based on how close the ego vehicle is with respect to the static obstacle. As per trajectory creation lateral shift changes in only one direction thus it is sufficient to check for the collision at start, end and middle of the intersected path.

represent the parameters in terms of  $d_e, c_w, o_w$  etc

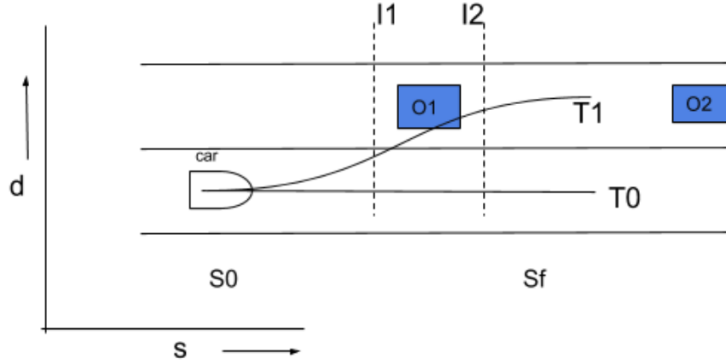


Figure 4.12: Collision check for static obstacles

#### 4.5.5 Checking for Dynamic Obstacles

In dynamic environments such as cities it is key to plan trajectories predicting the position of other obstacles and not colliding into them. Predicting the future behavior of obstacles is a challenging part in urban driving, some of the existing approaches assume dynamic obstacles as quasi-static, some assume obstacles to continue in their current detected heading, in contrast this thesis models dynamic obstacles as squares continuing with their current speed in their detected lane for planning duration similar to ?? or moving in opposite direction in lane or moving across the lane based on angle between the obstacle and road. This assumption can be justified by the fact that the trajectories are re-evaluated at high frequencies and any changes in obstacles lateral distance will be evaluated in next cycle thus keeping the vehicle safe from collision.

The collision check for dynamic obstacles has one extra check in time over checking for static obstacles, it is inspired from forbidden regions calculation as discussed in [11]. In step one the intersection in  $S$  coordinate for obstacle and ego vehicle is found, here the length of obstacle is dilated over the distance traveled by obstacle as represented by dotted line ahead of obstacle in figure 4.13. Then if the collision in  $S$  dimension exists, then the collision between ego vehicle and obstacle in lateral dimension( $d$ ) in the intersection region  $I1$  to  $I2$  is tested in similar way to static obstacle collision check. If the collision in  $d$  dimension exists then the  $s$  dimension where there is collision in lateral dimension( $d$ ) is found, represented with  $J1$ - $J2$  in figure 4.13 (generally this will be shorter than  $I1 - I2$ ). For the range  $J1$ - $J2$  it is checked if they collide in time also i.e., if they reach the same location in same time, buffer of time is added to be safe. As per instructions for safe driving it is required for the car to maintain a minimum time gap  $2s$  with the vehicle ahead. There are more formal methods [25] on safety distances for self driving cars. This thesis implements



simple 2s rule to safety and this is a tunable parameter which can be used to increase driving aggression. Sub Figure d of 4.14 indicates the collision in time dimension for scenario presented in Figure 4.13.

To check collision in time dimension, time difference between the ego vehicle and the obstacle at S dimension intersection borders is checked as shown in figure 4.14. If the gap is greater than 2 seconds all times and doesn't change sign then there is no collision, Figure 4.14 a) presents a situation where the obstacles get close but does not collide. Extra costs are added if the ego vehicle gets too close to obstacle. A collision occurs when the sign of time gap between the ego vehicle and the obstacle changes as shown in sub figure b and c of 4.14. Figure e. of 4.14 shows the collision when obstacle is moving in opposite direction.

If a dynamic obstacle is found moving laterally across the road then it is represented as a static obstacle occupying the length of the road. Thus if there is a pedestrian crossing the road, trajectory is considered to be in collision if there is a collision in s dimension and there will anyways by collision in d dimension due to dilation of the pedestrian width to width of the road and no time dimension is checked.

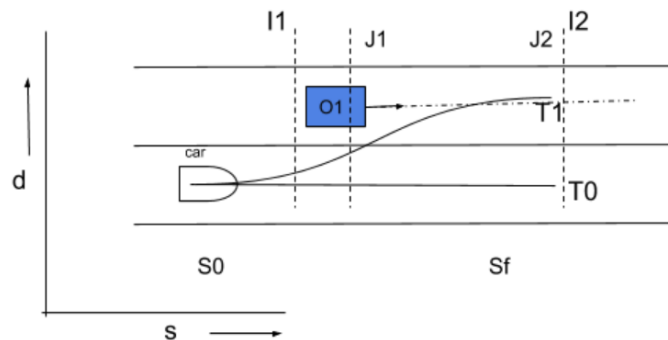


Figure 4.13: Collision check for Dynamic obstacles

#### 4.5.6 Cost Functions and Trajectory Selection

Selection of final trajectory is based on the different costs associated with it, costs can be static and dynamic. Static costs are known prior to trajectory creation and dynamic costs are known after the trajectory is created and evaluated. There is a wide research on different costs involved in trajectory selection as discussed in section

write why  
simulating  
over time and  
checking for  
collision is not  
a good idea  
- how com-  
putational  
complexity  
increases

Remove the  
pictures and  
add further  
pictures with  
evaluation for  
obstacles in  
same lane and  
opposite lane

add refer-  
ence to Dan  
Thesis for for-  
bidden regions  
etc

add link to  
sub chapter  
costs in rela-  
works

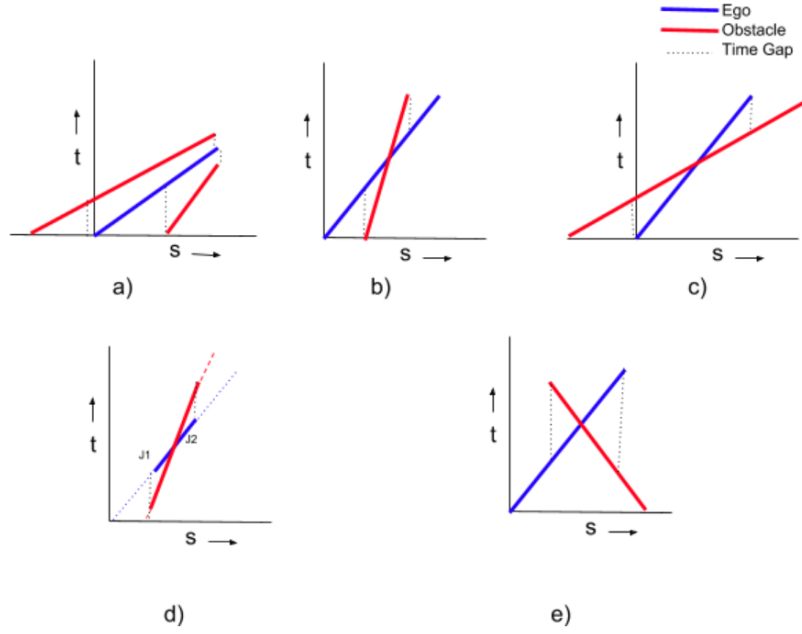


Figure 4.14: Collision check in space Time a) indicates obstacles getting close to the ego vehicle but not colliding. b) Ego vehicle hits the slow moving obstacle ahead. c) Ego vehicle is hit by fast moving obstacle behind - Situation during unchecked lane change by ego vehicle

This thesis implements a simple cost function as represented in 4.8, explicit smoothness costs are not considered as the target validation platform is a model car with no humans inside and limitations of model car to track a fine trajectory.

$$cost = |V_a - V_t| + |a_t| + |(d_t - d_e) * k1| + |(d_p - d_e) * k2| \quad (4.8)$$

$V_a$  - velocity achieved by trajectory.  $V_t$  - Target Velocity.  $a_t$  - Target Acceleration.  $d_t$  - Target lateral distance.  $d_e$  - Trajectory lateral distance.  $d_p$  - Previous target lateral for trajectory.  $k_1, k_2$  - Factors to adjust weights, currently used at 0.8 and 0.2

The Velocity and acceleration terms in cost function 4.8 promote higher accelerations if the target and current velocity difference is higher and lower accelerations if the difference is low. The next lateral terms promote the trajectories that are closer to the target lateral distance and also closer to the previous selection thus limiting the shifts in path selection and maintaining continuity.

Initially all the sampled trajectories are assigned the costs based on the cost function 4.8 and sorted, then the trajectory with the lowest cost is evaluated for collisions first. The list is sorted again and evaluated till the top of the list has lowest cost and the trajectory is evaluated.

### 4.5.7 Velocity Planning

Velocity planning is an important aspect of the planer, in general behavioral layer defines the target velocity based on speed regulation, other traffic participants, required behavior, road condition etc. In this thesis a simple approach of velocity limiting is used based on road curvature to limit lateral accelerations. Max velocity  $V_{\max}$  is calculated based on the equation 4.9.

$$V_{\max} = \min(\sqrt{Acc_{\text{MaxLat}}/|k(s)|}, V_{\text{limit}}) \quad (4.9)$$

$V_{\text{limit}}$  - Velocity limit mentioned in road network,  $Acc_{\text{MaxLat}}$  - Maximum Lateral acceleration(based on comfort and vehicle dynamics),  $k(s)$  - Road curvature.

## 4.6 Trajectory Follower

Check whether to refer to MIG trajectory controller or write in short about the controller.



## CHAPTER 5

# Implementation

---

Check if this chapter is needed,

- Write a flow chart describing various functions implemented

- How the collision check is implemented

- Further details on how these trajectories are converted to x,y coordinates used by the trajectory follower and the constraints in implementation for model car will be discussed in

- How costs are added

- Which messages are used for what, who sends what and who receives what - flow diagrams

- Speak about splines, how the state machine works, write a flowchart etc

- Inform when which order splines are good to implement

- Need for adding extra costs if this has to be ported for full scale cars

- Issues -



## CHAPTER 6

# Evaluation

---

In previous chapters detailed working of the planning algorithm has been discussed, this chapter discusses the evaluation criteria and results in detail. Various concepts discussed previously will be examined here through a series of experiments reflecting real life driving scenarios. This chapter is organized as follows: Section 6.1 discusses systematic evaluation of the planner by exposing it to various scenarios equivalent to on-road driving conditions. The next section 6.2 discusses a criteria based evaluation for the planner similar to any algorithm in the form of feasibility, optimality, completeness, run-time etc.

### 6.1 Experiments

Due to time and resource constraint most of the experiments to evaluate the planner are performed on simulator. The test cases involve finding a collision free path with obstruction in driving lane, avoiding slow moving traffic, merging into ongoing traffic, lane changes etc. The following subsections detail further on each experiment.

#### 6.1.1 Lane blocked

In driving scenario 6.1, driving lane is blocked by a static obstacle and a slow moving obstacle is in the next lane, here ego vehicle drives slowly till it finds enough room in the next lane, once obstacle is avoided the robot continues to shift into intended lane and drives with increasing speed.

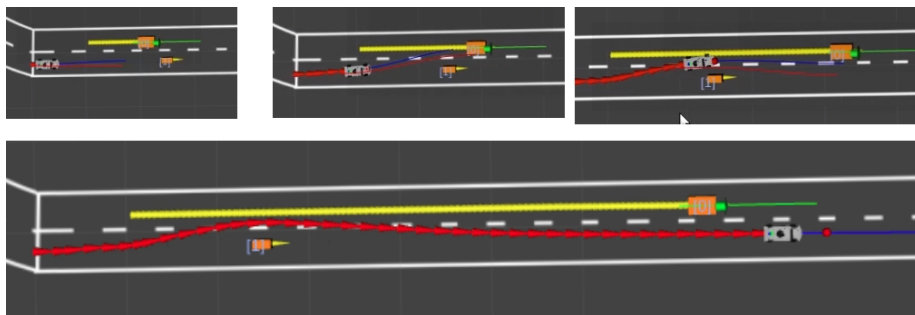


Figure 6.1: Driving Lane Blocked

### 6.1.2 Slow Moving Traffic

In situation 1 presented in Figure 6.2 ego vehicle starts changing into left lane once a slow moving obstacle is encountered, once the obstacle is passed it starts driving into right lane again. In situation 2 presented in Figure 6.3 there are two slow moving obstacles ahead, once the ego vehicle overtakes the initial obstacle it shifts to original lane as intended but it encounters the second slow moving obstacle and shifts to left lane again. This behavior is caused because of locally optimal cost functions driving the ego vehicle into intended lane without knowledge of long term planning information. A behavioral layer with longer scenario analysis horizon will result in better path selection.



Figure 6.2: Slow Moving Traffic Situation 1

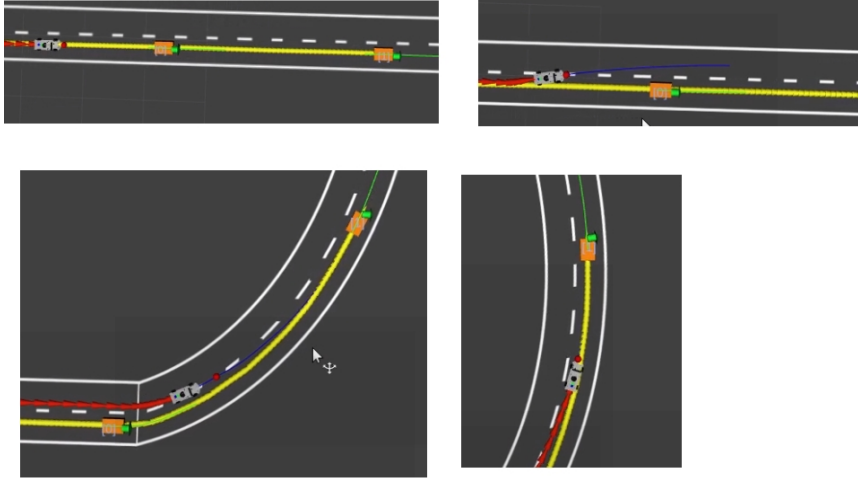


Figure 6.3: Slow Moving Traffic Situation 2

### 6.1.3 Merging into traffic

In scenario presented in Figure 6.4, lane changing is requested to merge into the traffic in left lane. Here ego vehicle speed is  $1ms^{(}-1)$  and obstacle speed is  $0.6m^{(}-1)$ . Initially lane change does not occur as cost functions are tuned to maintain speed over maintaining required lane. As the vehicle enters the curve, target driving speed



is reduced and the vehicle merges into the traffic in left lane. Depending on which portion of the lane the ego vehicle is in i.e, near intersections or exits target lane should have higher priority over maintaining speed and during rest of the regions target speed should be of higher priority to reach destination quickly. Cost functions implemented in this thesis provide flexibility in tuning behavior of the ego vehicle.

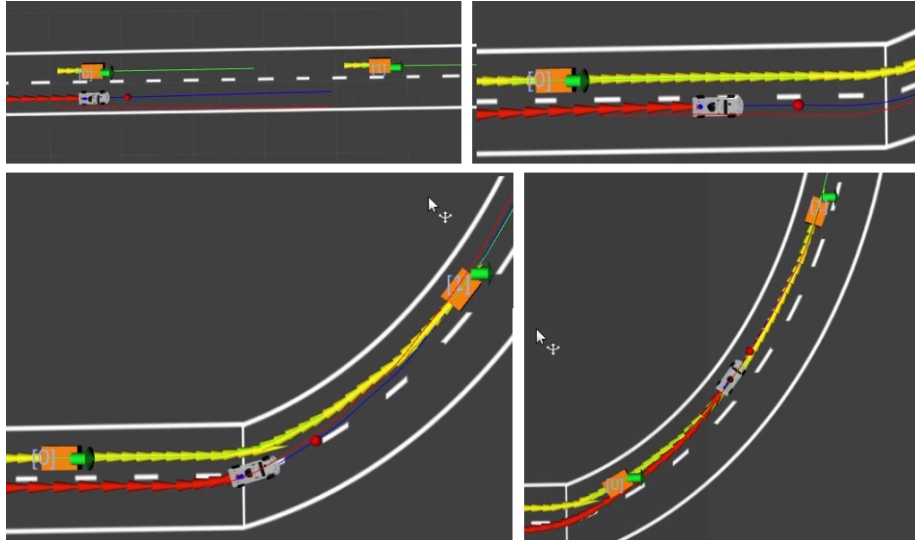


Figure 6.4: Merging into Traffic

#### 6.1.4 Merging into next lane with opposite traffic

In scenario presented in Figure 6.5 driving lane is blocked by a series of obstacles and the left lane is occupied by a moving obstacle. Ego vehicle starts slow in the driving lane and waits till the obstacle is passed in the left lane and starts driving forward.

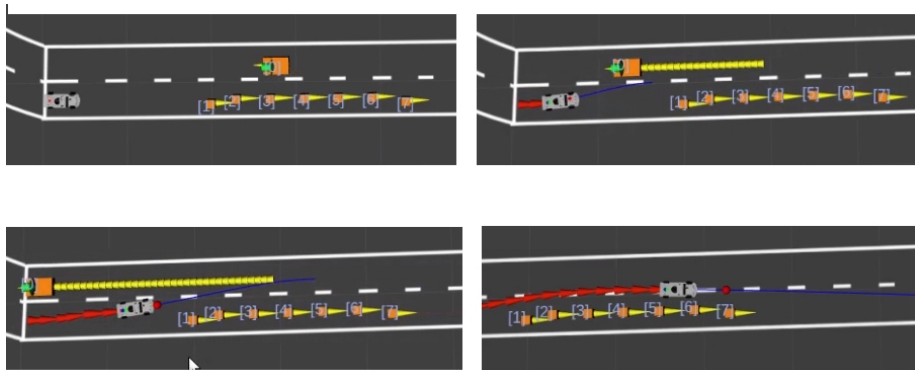


Figure 6.5: Lane blocked by series of static obstacles and vehicle in next lane driving opposite

This situation may lead to ego vehicle getting stuck in middle of the road. If driving speed of ego vehicle is low, temporal horizon limits ego vehicles lookahead distance into future. If a fast moving obstacle in left lane not visible in 7 seconds(5 planning and 2s of safety) of temporal horizon then the ego vehicle starts lane change and if there is time to abort it will abort and if not the ego vehicle will stop in middle of the lane due to no path ahead, if the obstacle proceeds without stopping for ego vehicle. This can be avoided by a behavioral layer with longer spatial scenario analysis horizon. As the planner discussed in this thesis is not created for controlling the vehicle to drive backwards, a different planner equivalent to off road planner must be used.

### 6.1.5 Road Blocked or Pedestrian Ahead

In scenario presented in Figure 6.6, road is blocked by a series of static obstacles, the vehicle enters empty left lane, slows down and finally stops when it cannot find route ahead.

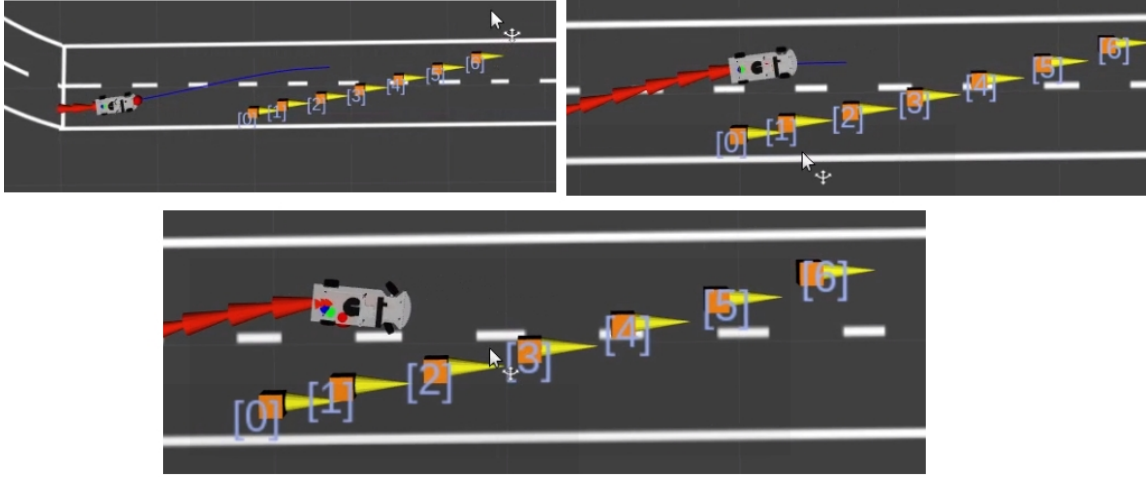


Figure 6.6: Road Blocked by series of obstacles

A pedestrian on road is considered similar to a road blocking, in this case as shown in Figure 6.7 the ego vehicle initially drives at full speed, then the vehicle slows down(shorter blue line representing a slower speed) and the robot finally comes to halt few meters ahead of the pedestrian. This is a tunable parameter and currently at maximum value for safety.

### 6.1.6 Dynamic Obstacles - other vehicles

The main objective of the planner is to adjust to the sudden changes in the environment caused by the dynamic obstacles in surroundings, here two sub scenarios are described where the ego vehicle has to react to sudden breaking of vehicles ahead.

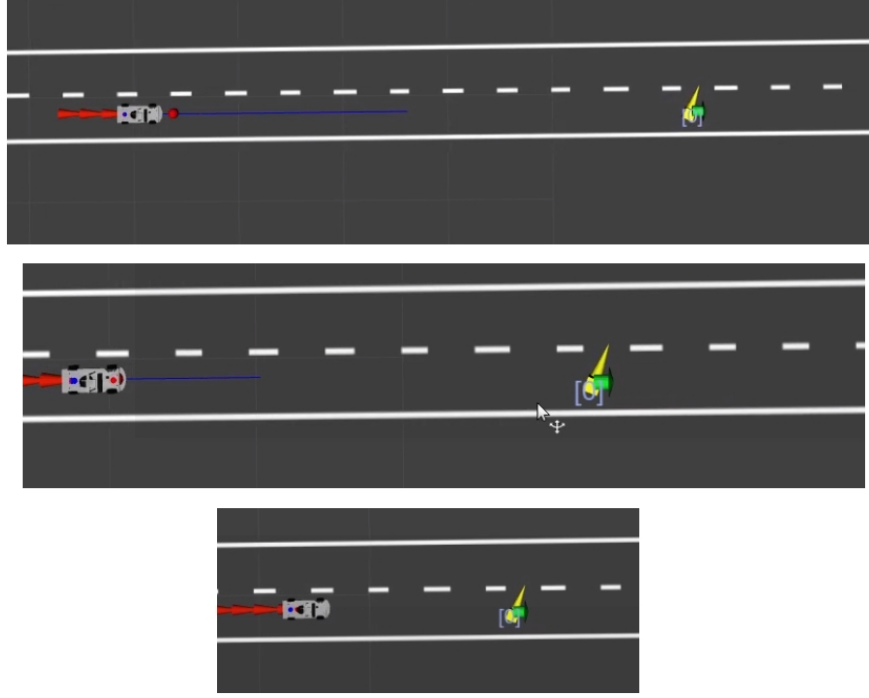


Figure 6.7: Pedestrian Ahead on Road

In scenario presented in Figure 6.8, there are two situations. In situation 1 the car aborts a lane change when the slow moving dynamic obstacle in left lane is detected, then once the dynamic obstacle is passed the vehicle shifts to left lane to avoid the stopped dynamic obstacle in driving lane. This situation is similar when a vehicle ahead stops to drop off a passenger or waiting for parking spot. In situation 2, the car doesn't choose lane change initially and slows down till it finds enough room in left lane to drive ahead of the stopped dynamic obstacle.

In scenario presented in Figure 6.9 there are two situations with different thresholds for safety, in situation 1 a safe  $2s+$  distance to obstacles ahead is chosen, here the ego vehicle stays far away from the vehicles ahead and when it stops it stops relatively farther from the vehicles ahead. In situation 2, the threshold has been adjusted to  $0.5s$  leading to a aggressive behavior of ego vehicle. The ego vehicle drives closer to the obstacles ahead and when the dynamic obstacles ahead stop suddenly, distance between the ego vehicle and the obstacles ahead is very narrow.

## 6.2 Criteria Based Evaluation

In this section, proposed planner is validated against the common criteria of evaluating any algorithm, i.e. optimality, feasibility, completeness, runtime and approach.



Figure 6.8: Dynamic Obstacle Ahead stops in middle of road

### 6.2.1 Optimality

In this thesis we discuss about the optimality of the time horizon, subsection 4.5.1 already defines regarding various timing constraints chosen in this planner. A larger planning horizon will enable planner to create a longer and better path but due to the unpredictability of the environment, plan created will not be valid after certain duration, a larger horizon will also increase the run-time of algorithm. The planner proposed in this thesis is only a local planner and always needs inputs from a behavioral layer or a global planner to choose target lane, velocity etc thus a short planning horizon is suitable for this proposed planner.

An example of how horizon will affect optimal planning for current planner is shown in figure 6.10. Here  $T_0, T_1$  are the trajectories with horizon "T" and  $T_2, T_3$  are trajectories with horizon "T'". In this condition if a lane change has been requested then trajectory  $T_1$  is chosen but with increased horizon trajectory  $T_3$  will be chosen, depending on situation one is efficient sometimes and other in others. These situations can be improved by lane selection algorithm in behavioral layer which looks for occupancy of different lanes and suggest the one best suitable lane. Similarly if an exit has to be taken on road, a long horizon would choose a plan with reduced speed compared to high speed path with short horizon. This can also be solved by having a velocity planner in the behavioral layer.

Another horizon generally in discussion for a planner is spatial horizon which discusses how long is the path generated, as per this planning criteria at low velocities the spatial horizon considered is very small thus the planner may not make right

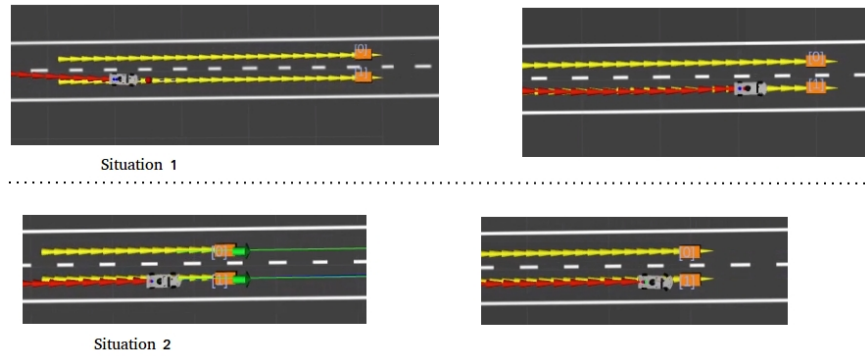


Figure 6.9: Two Dynamic Obstacles ahead stop suddenly

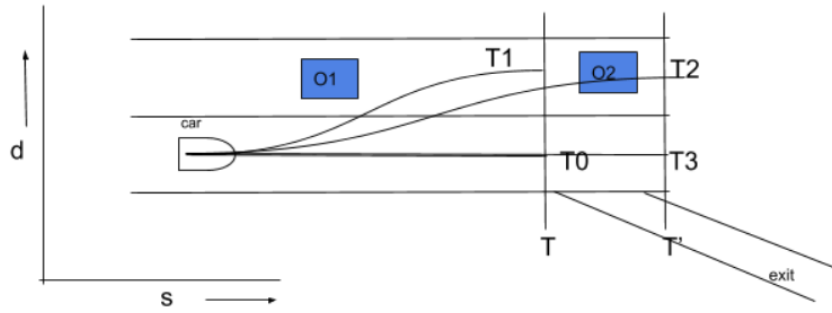


Figure 6.10: Horizon Optimality reference

decisions because of conditions like missing the obstacles ahead etc. This shortcoming can be improved by creating a spatial path with longer horizon in behavioral layer at lower speeds and allowing the local planner to follow new spatial path than following the global reference path. This is an efficient method as the path planning is generally less expensive than trajectory planning. As shown in figure 6.11, following the original reference path(solid line) will lead to trajectories that turn a lot causing discomfort due to obstacles on the side of road that enter the road, thus by using an optimized reference path(dotted line), ego vehicle can plan efficiently even using short horizons.

The resolution of the sampling in acceleration selection and lateral distance selection will also affect the optimality of planning, a chosen plan can only be optimal of the trajectories created by sampling, higher the number of samples, larger are the possibilities and a best selection is possible.

From the above discussion it can be stated that a planner that has a longer spatial horizon for path planning and short time horizons for trajectory planning will lead to an efficient planner.

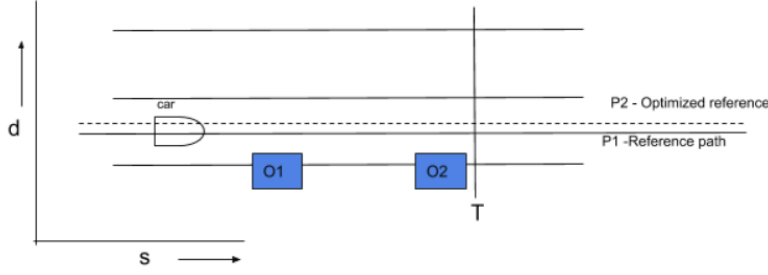


Figure 6.11: Optimized Reference Path

### 6.2.2 Feasibility

Ability of the vehicle to traverse the created trajectory determines feasibility, generally curvature of the path, smoothness and accelerations determine whether a trajectory is feasible or not. The proposed planner creates feasible trajectories at lower speeds because of the third order splines used, higher speeds require fifth or higher order splines to maintain continuity in path and speed. Discussions in [23] [21] throw light on how to achieve higher degrees of smoothness, which approach is better in which driving conditions. As the intended application for this thesis is a modelcar, constant acceleration profiles are used due to limitation in ability of the car to track small changes in velocity and inaccuracies in measurement. These can be easily replaced by a smoother higher order polynomials with a better hardware platform. Consistency in paths evaluated with respect to previous plan is another factor in feasibility, the current planner penalizes trajectories deviating from previous plan and also takes into account current orientation of the vehicle in choosing a path. Thus creating smoother transitions from one state to another by respecting current driving orientation.

### 6.2.3 Completeness

An algorithm is said to be complete if it can result in a solution every time. A motion planner can be called complete if it returns path if it exists in the space searched. Like many other sampling based approaches the planner proposed in this thesis only probabilistically complete. That is, probability of finding a solution approaches to one as the number of samples increases. If there are higher number of samples in the configuration space then higher are the chances of finding a solution. If a planner cannot find a solution within sampled region it forces the car to go into emergency manoeuvre.

In Figure 6.12, there are only two sampled end states and there is no solution found by the vehicle, by increasing the number of lateral samples a solution can be easily found. In general condition of completeness can be improved in two ways, first

is to sample as many points as possible and as closely as possible in the solution space. Second method is to keep on sampling till an end solution is found or timeout has been reached. The former method will reduce the computational performance while the later can be complex and expensive also. The planner proposed in this thesis implements a combination of both methods, as many samples as possible but evaluates only till a feasible solution is found. It is recommended to achieve completeness for safety purposes in autonomous driving.

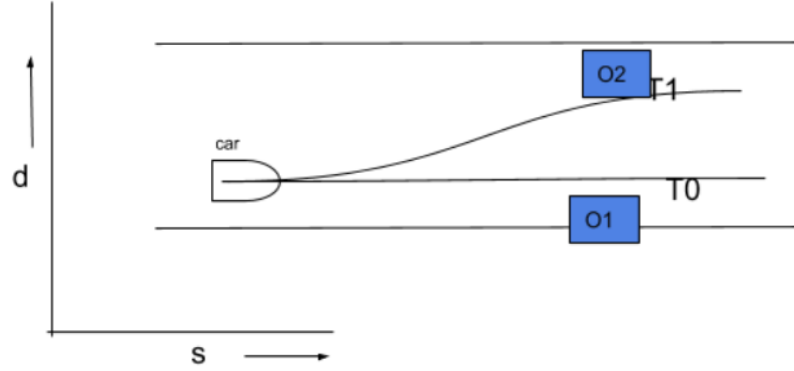


Figure 6.12: Probabilistic Completeness

#### 6.2.4 Runtime

Though computation power is available cheaply, it is important to create solutions which are cheaper and can be employed in large scale. In this case sampling based approaches generally fare well and run on a low computational hardware. In contrast lattice based approaches such as [23] [27] [28] computationally expensive and require a GPU to run. Low computational costs mean larger chances to be adopted to a greater number of platforms.

To compute the complexity of planner, let's consider  $n_a$  denote the number of acceleration/deceleration profiles,  $n_s$  denote the stopping deceleration profiles and  $n_l$  denote the number of lateral distance samples. Then the maximum number of samples created is  $(n_a + n_s) * n_l$ , generally stopping profiles are less as at higher deceleration lower number of lateral samples are chosen due to limitations in vehicle dynamics. This thesis employs a hybrid combination of two methods mentioned in 6.2.3 to achieve completeness. Therefore in best case only one trajectory is evaluated and complexity is  $O(1)$  and the worst case complexity is  $O((n_a + n_s) * n_l)$ .

Trajectory evaluation with respect to dynamic obstacles is an expensive process in evaluation of trajectories, generally simulation based methods as discussed in [16] are generally expensive generally in terms of  $O(n_o * n_n)$  where  $n_o$  is the number of obstacles and  $n_n$  is the number of simulation steps. This thesis employs a simple

collision checking algorithm with constant time for evaluating one obstacle thus reducing the complexity to  $O(n_o)$ , number of obstacles. This process is not effective in intersections, currently a conservative approach to wait for other obstacles to pass is used, a simple approach as discussed in [10] which has a performance better than the simulation based algorithms can also be employed in future.

### 6.2.5 Deliberative Approach

The planner proposed in this thesis maintains a mix of deliberative and reactive approach. Deliberative by evaluating a trajectory completely before committing to it, this is important to create trajectories adhering to traffic, safe and comfortable. In general all the planners evaluate all the sampled trajectories then choose the best based on different costs. The planner proposed in this thesis does not follow this convention and once it finds a best trajectory it stops evaluating the other trajectories as presented in subsection 4.5.6. This does not limit the real-time response of the trajectory as the sampling is chosen such that the worst case response time is within the hard real-time response required by the planner.

## 6.3 Comparisons to other Planners



# Conclusions and Future Work

---

## 7.1 Conclusions

## 7.2 Future Work

Replace by smoother polynomials over splines, especially in curves and when not following centre lane, they tend to be very bad.

// Diss shui thesis - read though page 80 and understand further on benefits and demerits of polynomials vs splines. Add some in evaluation and some in future work

Prediction of state from where the planner should start planning instead of current position. Due to inaccuracies in current planners measurement of speed and acceleration it is tough to estimate where the vehicle will be when the planner is under execution. Currently based on assumption that the vehicle will follow the current path for next few ms, it is made offset in control node. This can be improved to have better synchronizaton between planner and controller.

Create two functions to map lateral shift as a function of time and distance based on speed over current function only mapping based on distance.

Improve the lateral shift function to have an option to include where the ego vehicle is in current state with respect to lane change. If lets say the ego vehicle creates a trajectory to go from  $d_s$  to  $d_t$  in  $5s$ , then when the trajectory is evaluated next time the ego vehicle is at  $d_1$  and it will choose  $5s$  to travel to  $d_t$  from there thus delaying the lane shift and this process repeats indefinitely and the robot will never exactly reach the target  $d$ . Thus the  $d\_final$  for lane shift from previous trajectory should be associated with the  $s\_final$  previous thus making the trajectory to be executed in the required distance ahead. This is similar to classical Proportional controller issue.

Use quintic polynomials at high speeds and cubic at low speeds.

Improve collision checking with respect to pedestrians following the huge research published in this specific domain of pedestrian tracking in autonomous cars.



# Bibliography

- [1] R. Behringer and N. Muller. Autonomous road vehicle guidance from autobahnen to narrow curves. *IEEE Transactions on Robotics and Automation*, 14(5):810–815, Oct 1998. [3](#)
- [2] P.S. Bokare and A.K. Maurya. Acceleration-deceleration behaviour of various vehicle types. *Transportation Research Procedia*, 25:4733 – 4749, 2017. World Conference on Transport Research - WCTR 2016 Shanghai. 10-15 July 2016. [18](#)
- [3] Z. Boroujeni, D. Goehring, F. Ulbrich, D. Neumann, and R. Rojas. Flexible unit a-star trajectory planning for autonomous vehicles on structured road maps. In *2017 IEEE International Conference on Vehicular Electronics and Safety (ICVES)*, pages 7–12, June 2017. [4](#)
- [4] Martin Buehler, Karl Iagnemma, and Sanjiv Singh. *The DARPA Urban Challenge: Autonomous Vehicles in City Traffic*. Springer Publishing Company, Incorporated, 1st edition, 2009. [3](#), [4](#)
- [5] R. G. Cofield and R. Gupta. Reactive trajectory planning and tracking for pedestrian-aware autonomous driving in urban environments. In *2016 IEEE Intelligent Vehicles Symposium (IV)*, pages 747–754, June 2016. [16](#)
- [6] Gerald Cook. *Robot Navigation*, pages 324–. Wiley-IEEE Press, 2011. [10](#)
- [7] P. Czerwionka, M. Wang, and F. Wiesel. Optimized route network graph as map reference for autonomous cars operating on german autobahn. In *The 5th International Conference on Automation, Robotics and Applications*, pages 78–83, Dec 2011. [11](#)
- [8] DARPA. Route network definition file (rndf) and mission data file (mdf) formats. [11](#)
- [9] Gaston A. Fiore and David L. Darmofal. A robust motion planning approach for autonomous driving in urban areas. 2008. [4](#)
- [10] L. Garrote, C. Premebida, M. Silva, and U. Nunes. An rrt-based navigation approach for mobile robots and automated vehicles. In *2014 12th IEEE International Conference on Industrial Informatics (INDIN)*, pages 326–331, July 2014. [4](#), [38](#)
- [11] Daniel Graff. *Programming and Managing Swarms of Mobile Robots: A Systemic Approach*. PhD thesis, Technischen Universitat Berlin, 2016. [22](#)

- [12] Poul Greibe. Braking distance, friction and behavior. 18
- [13] Tianyu Gu and John M. Dolan. On-road motion planning for autonomous vehicles. In *Proceedings of the 5th International Conference on Intelligent Robotics and Applications (ICIRA 2012)*, pages 588–597, October 2012. 4
- [14] Joseph Kearney, Hongling Wang, and Kendall Atkinson. Robust and efficient computation of the closest point on a spline curve. In *In in Proc. 5th International Conference on Curves and Surfaces*, pages 397–406, 2002. 17
- [15] J. Kim, K. Jo, W. Lim, M. Lee, and M. Sunwoo. Curvilinear-coordinate-based object and situation assessment for highly automated vehicles. *IEEE Transactions on Intelligent Transportation Systems*, 16(3):1559–1575, June 2015. 16
- [16] Sascha Kolski. *Autonomous Driving in Dynamic Environments*. PhD thesis, ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE, 2008. 4, 37
- [17] Y. Kuwata, G. A. Fiore, J. Teo, E. Frazzoli, and J.P. How. Motion planning for urban driving using rrt. In *2008 IEEE/RSJ International Conference on Intelligent Robots and Systems*, pages 1681–1686, Sept 2008. 4
- [18] Jean-Claude Latombe. *Robot Motion Planning*. Kluwer Academic Publishers, Norwell, MA, USA, 1991. 3
- [19] Steven M. LaValle. *Planning Algorithms*. Cambridge University Press, New York, NY, USA, 2006. 3
- [20] X. Li, Z. Sun, D. Cao, Z. He, and Q. Zhu. Real-time trajectory planning for autonomous urban driving: Framework, algorithms, and verifications. *IEEE/ASME Transactions on Mechatronics*, 21(2):740–753, April 2016. 4, 16, 19
- [21] D. Madàs, M. Nosratinia, M. Keshavarz, P. Sundström, R. Philippsen, A. Eidehall, and K. M. Dahlén. On path planning methods for automotive collision avoidance. In *2013 IEEE Intelligent Vehicles Symposium (IV)*, pages 931–937, June 2013. 36
- [22] Kristijan Maček. *Autonomous vehicle navigation in dynamic urban environments for increased traffic safety*. PhD thesis, ETH Zurich, 2010. 15
- [23] Matthew McNaughton. *Parallel Algorithms for Real-time Motion Planning*. PhD thesis, Carnegie Mellon University, 2011. 4, 21, 36, 37
- [24] Arpan Mehar, Satish Chandra, and Senathipathi Velmurugan. Speed and acceleration characteristics of different types of vehicles on multi-lane highways. In *European Transport, 2013 - istiee.org*, 2013. 18

- [25] Shai Shalev-Shwartz, Shaked Shammah, and Amnon Shashua. On a formal model of safe and scalable self-driving cars. *CoRR*, abs/1708.06374, 2017. 22
- [26] Hongling Wang, Joseph Kearney, and Kendall Atkinson. Arc-length parameterized spline curves for real-time simulation. In *In in Proc. 5th International Conference on Curves and Surfaces*, pages 387–396, 2002. 17
- [27] Shuiying. Wang. *State Lattice-based Motion Planning for Autonomous On-Road Driving*. PhD thesis, Freie Universität Berlin, 2015. 4, 16, 37
- [28] M. Werling, J. Ziegler, S. Kammel, and S. Thrun. Optimal trajectory generation for dynamic street scenarios in a frenet frame. In *2010 IEEE International Conference on Robotics and Automation*, pages 987–993, May 2010. 19, 37
- [29] Wenda Xu, Junqing Wei, J. M. Dolan, Huijing Zhao, and Hongbin Zha. A real-time motion planner with trajectory optimization for autonomous vehicles. In *2012 IEEE International Conference on Robotics and Automation*, pages 2061–2067, May 2012. 4, 16
- [30] Julius Ziegler and Christoph Stiller. Spatiotemporal state lattices for fast trajectory planning in dynamic on-road driving scenarios. In *Proceedings of the 2009 IEEE/RSJ International Conference on Intelligent Robots and Systems, IROS'09*, pages 1879–1884, Piscataway, NJ, USA, 2009. IEEE Press. 16