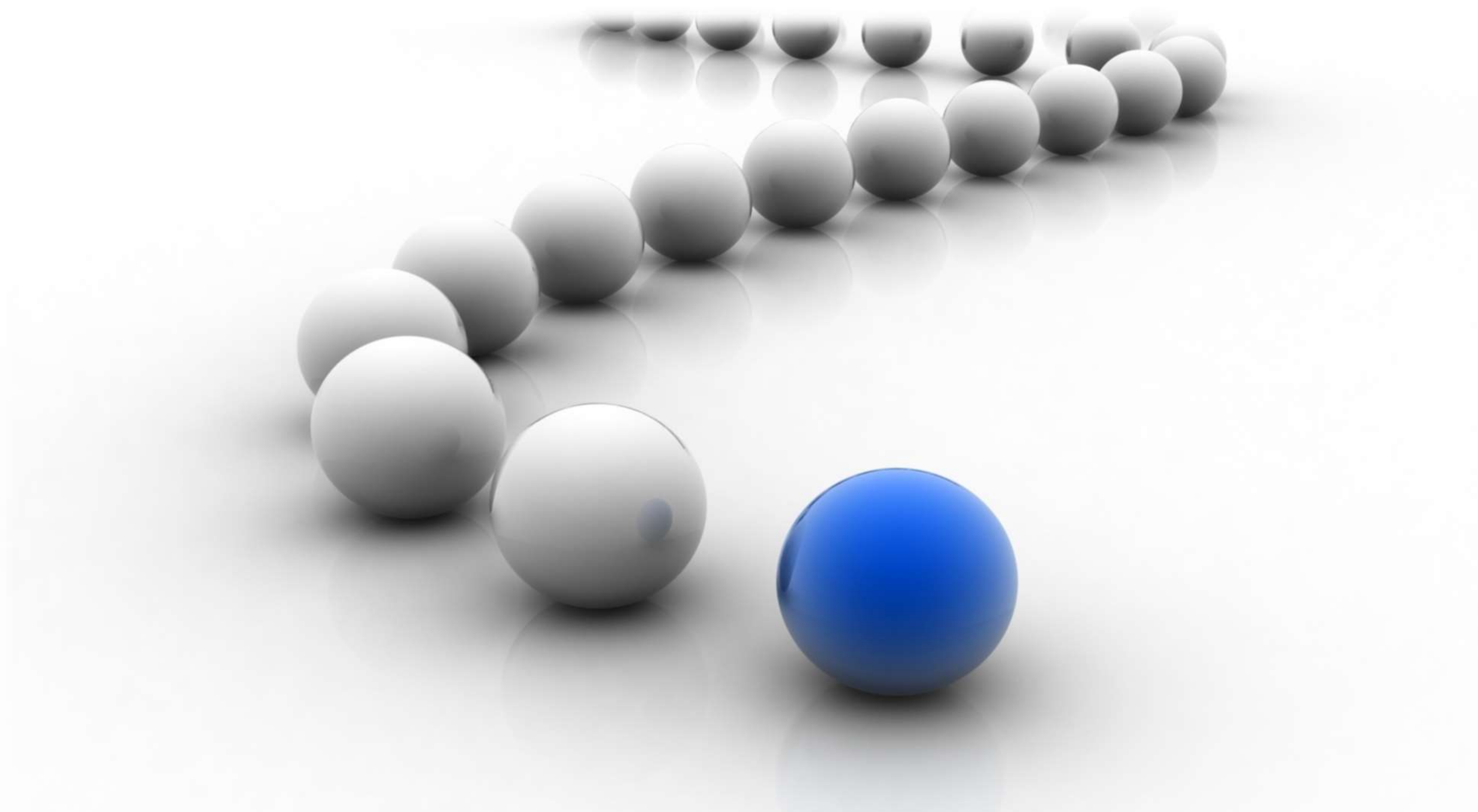


Certificate of Quantitative Finance

xVA Theory

Jon Gregory, 14th June 2016

In This Lecture

i) Historical background and xVA overview

ii) Credit and debt value adjustments (CVA and DVA)

iii) Funding value adjustment (FVA)

iv) Capital value adjustments (KVA)

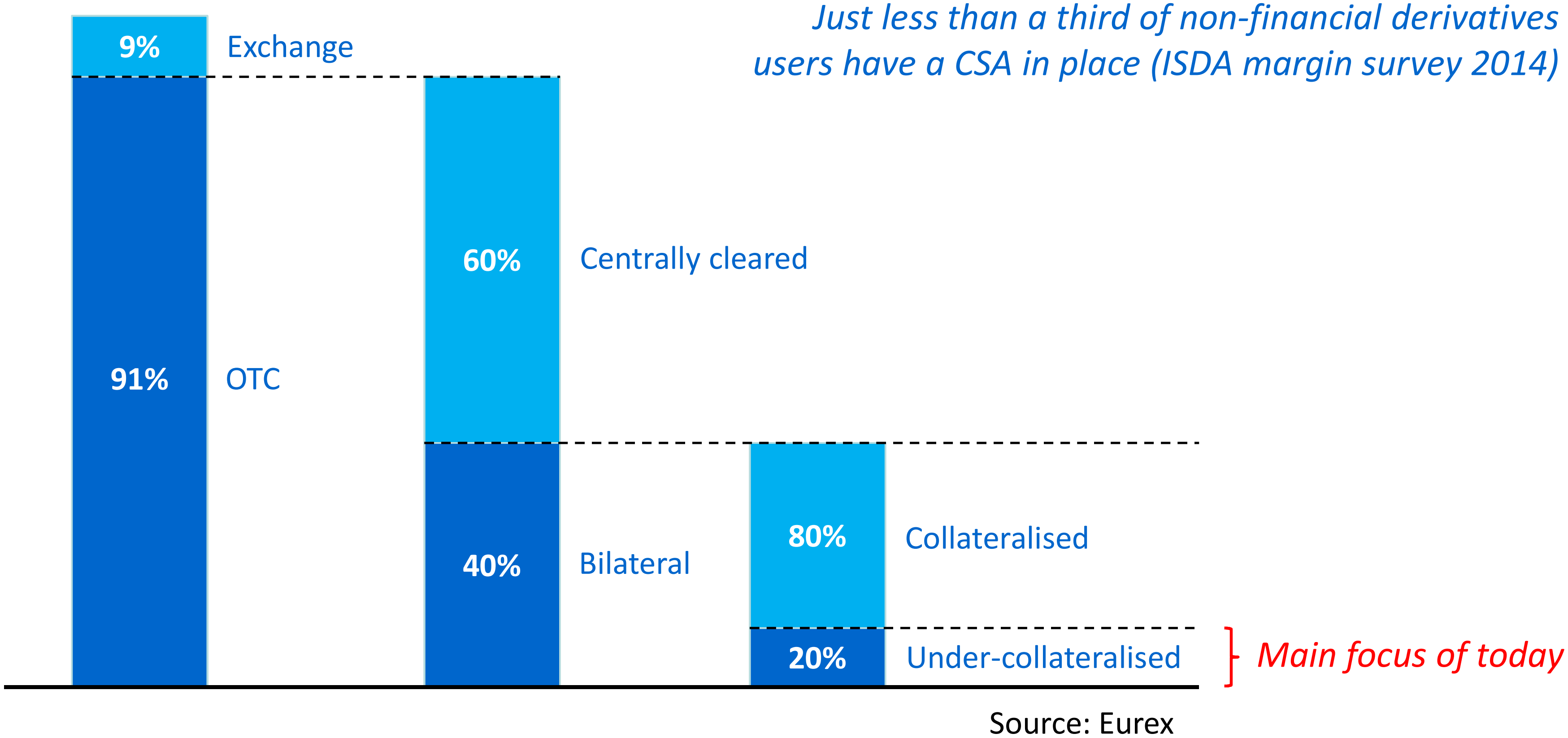
v) Current market practice and the future

History of xVA

Late 1990s	2000s	2007 onwards
• Asian crisis and LTCM failure • Some banks large price CVA into trades • Only passive CVA management (historical data, no hedging)	• Large banks refine approach to CVA • Lack of interest otherwise • Accounting rules (IAS39 and FABS157) imply CVA should be considered	• Major financial crisis • LIBOR no longer considered risk-free • IFRS 13 accounting standards (CVA/DVA) • Funding costs important (FVA) • Basel III and CVA capital charge (KVA) • Central clearing mandate and bilateral margin rules (MVA)

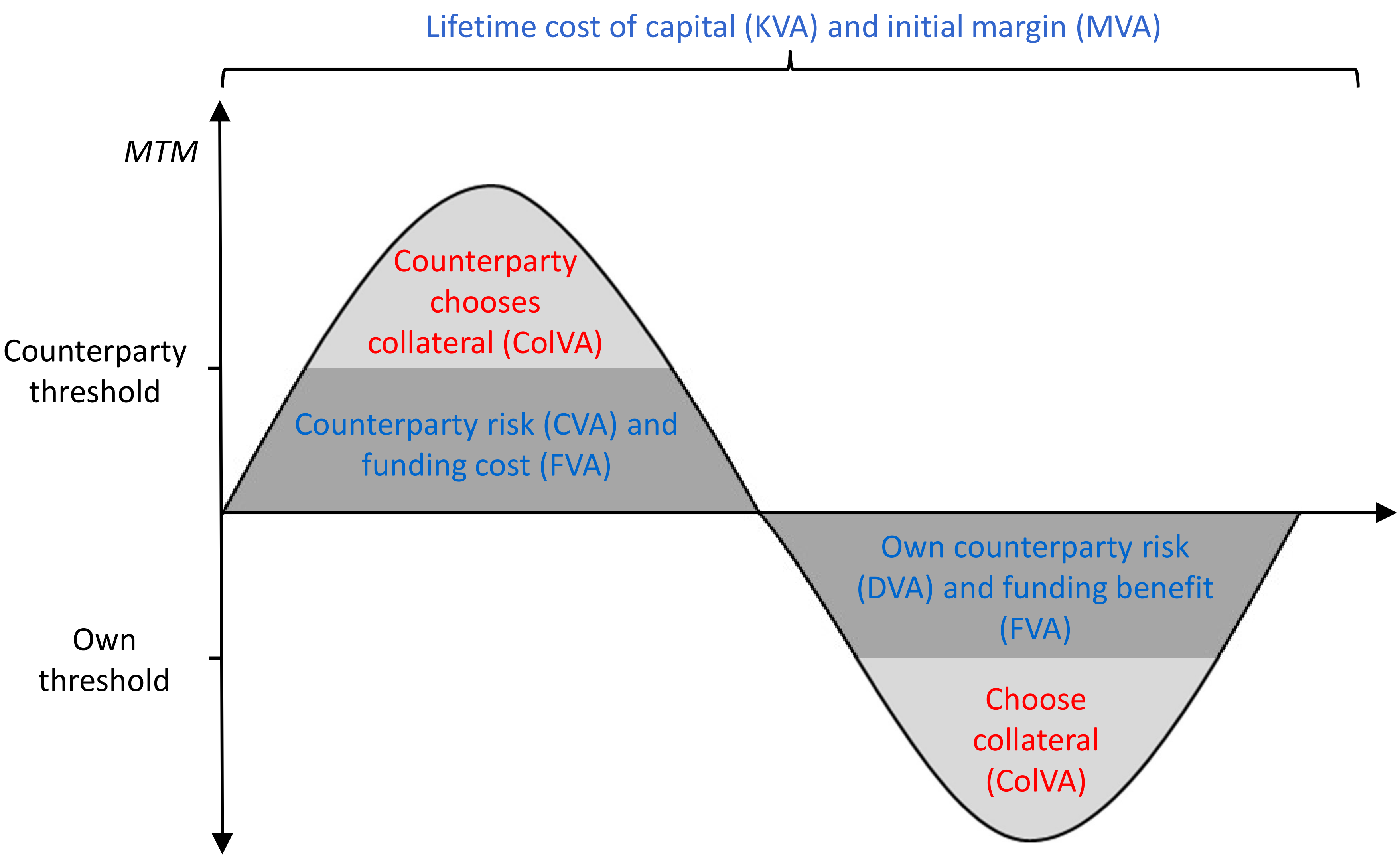
The Derivative Market

Just less than a third of non-financial derivatives users have a CSA in place (ISDA margin survey 2014)

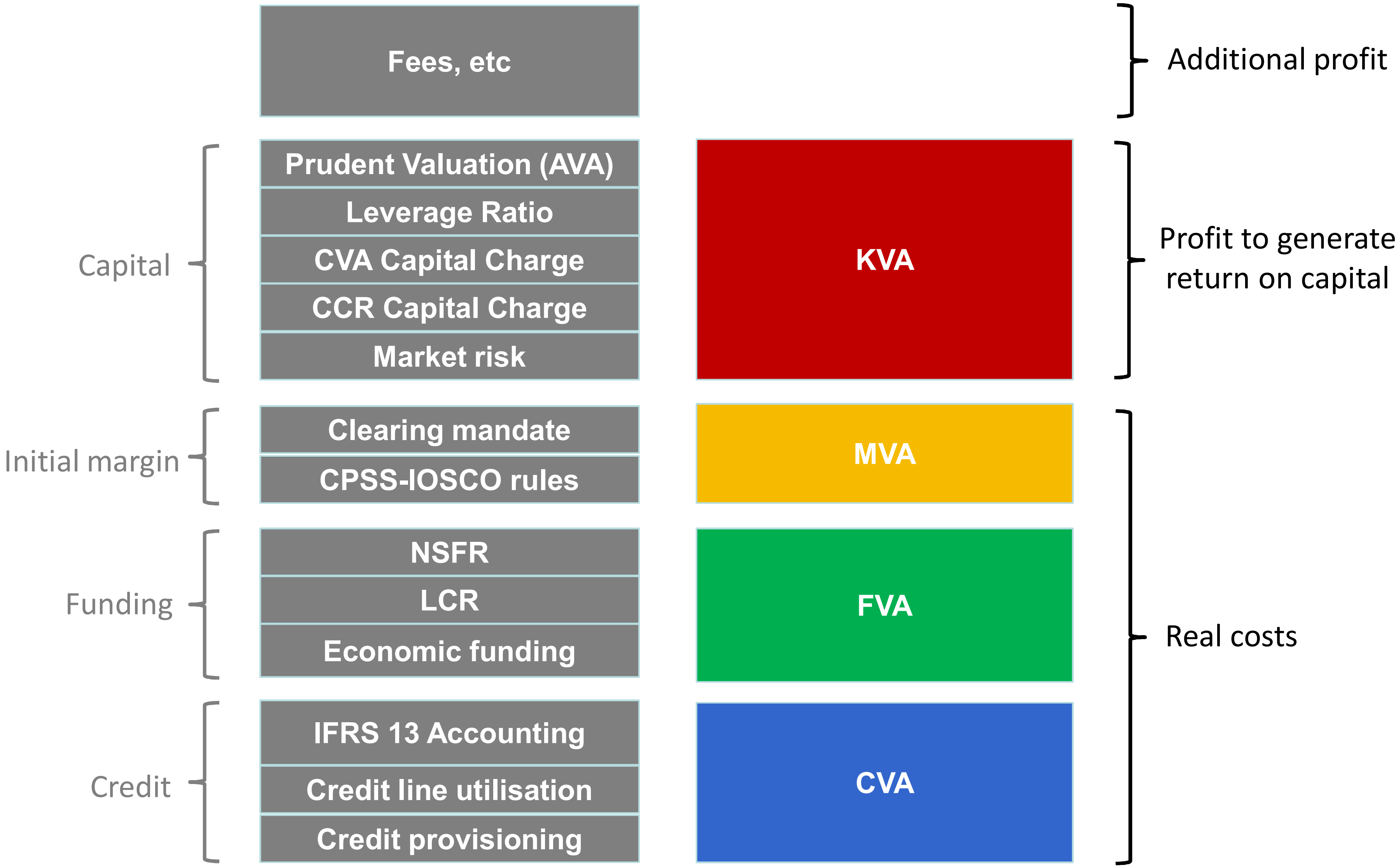


- **Note**
  - Under-collateralised OTC derivatives have a lot more risk than the others!

Economic Costs of Holding a Derivative Transaction

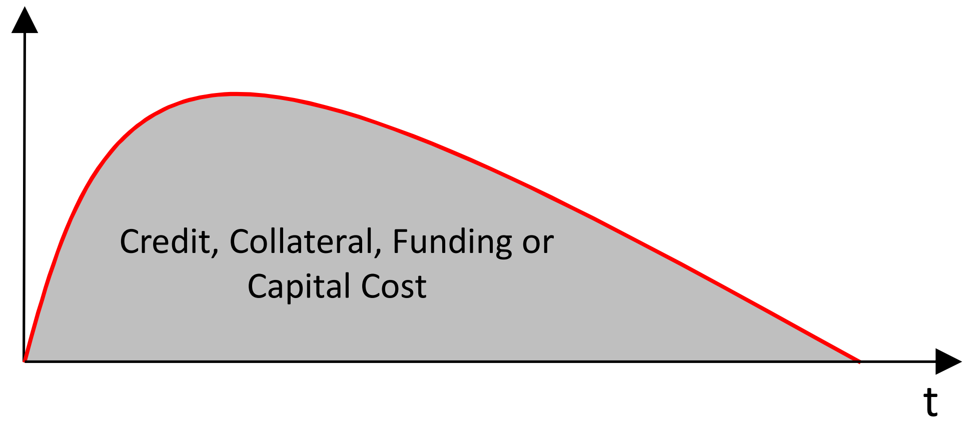


Pricing a Transaction with xVA



The xVA Calculation – General Comments

$$xVA = \int_0^\infty C(t) e^{-\int_0^t \beta(u)du} E_t[X(t)]dt$$



- **xVA computation involves**
  - Determination of curves, $C(t)$
  - Calculation of underlying profile, $X(t)$
- **The first is more qualitative, the second is very quantitative (option pricing)**
  - Numerical aspects are a big challenge (GPU, AAD)
- **In some special case we are only really pricing forward contracts**
  - xVA can be implemented by the correct choice of discount factor
- **Recursive aspects, non-linear behaviour and overlaps are all important**
  - Close-out assumptions, discounting assumptions, $\beta(u)$
  - Eg: DVA/FBA, can capital be used for funding, how much capital relief do xVA hedges provide?

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OIS Discounting

- **OIS discounting is the correct way to value a "perfectly collateralised" transaction (and is the closest thing to the risk-free rate)**
- **xVA can be seen as an adjustment for non-perfect collateralisation**
- **Credit**
  - Counterparty risk after any CSA is applied (CVA and DVA)
- **Funding**
  - Funding of uncollateralised exposure (FVA)
- **Capital**
  - The cost of holding (regulatory) capital against the transaction (KVA)
- **Also (but given less emphasis today)**
  - Optionality around collateral terms (ColVA)
  - Funding of initial margin (MVA)

Tradition Credit Pricing Formula

- **We can price the expected loss on an instrument such as a loan via**

$$EL = \underbrace{LGD}_{\text{Loss given default}} \times \underbrace{Exposure}_{\text{Notional of the loan}} \times \underbrace{PD}_{\text{Default probability}}$$

- **This is easy because:**
  - The notional of the loan can only be positive
  - The notional of the loan is deterministic (more or less)
  - It can be implemented (partially) by changing the discount factor

Deriving the CVA Formula (I)

- **Time of default is denoted by $\boldsymbol{\tau}$**
- **Base value of derivatives portfolio at time $\boldsymbol{t}$ with final maturity $\boldsymbol{T}$:** $V(\tau, T)$
- **Default payoff is:**

$$\delta V(\tau,T)_+ + V(\tau,T)_-$$

Recovery rate (%)

- **Risky value:**

$$\tilde{V}(t,T) = E_t\begin{bmatrix} \overbrace{I(\tau > T)}^{\text{No default before } T} V(t,T) + \\ I(\tau \leq T) V(t,\tau) + \\ \underbrace{I(\tau \leq T)(\delta V(\tau,T)_+ + V(\tau,T)_-)}_{\text{Default payoff}} \end{bmatrix}$$

Full value if no default

Cashflows before default

Exposure (don’t worry we’ll get back to the CVA formula)

- **What happens when a counterparty defaults?**
  - We close-out and replace the contracts
  - What determines the value of the contracts at this time?
  - It is normally the net value but depends on the contractual terms (and a legal interpretation of them)

<table>
<tr><th colspan="2">1992 ISDA</th><th colspan="2">2002 ISDA</th></tr>
<tr><td>Method</td><td>Description</td><td>Method</td><td>Description</td></tr>
<tr><td>Market Quotation</td><td>Obtain at least 3 firm quotes for the portfolio in question and use average</td><td rowspan="2">Close out amount</td><td rowspan="2">Indicative quotations, public sources of price information, models</td></tr>
<tr><td>Loss Method</td><td>Assess own losses as a result of the default in good faith</td></tr>
<tr><td colspan="2">Cure period 3 days</td><td colspan="2">Cure period 1 day</td></tr>
</table>

Exposure Definition

- **Economic loss incurred on all outstanding transactions if a counterparty defaults**

Counterparty default → Close-out and net transactions (V_{MTM})

Positive value
Claim on asset

Negative value
Pay liability

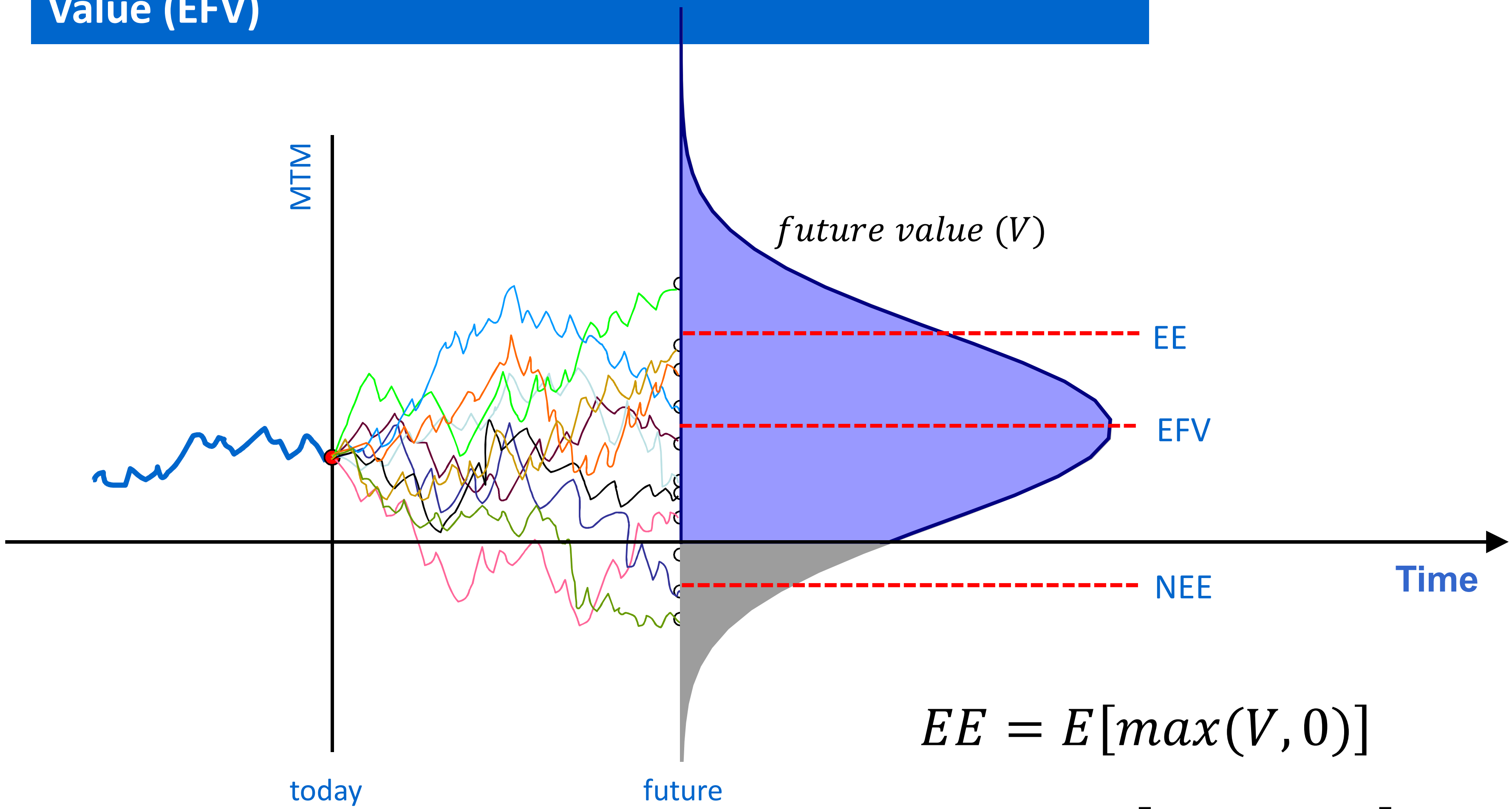
$$\boxed{E = max(V_{MTM}, 0)}$$

- Note that we do not lose precisely the mark-to-market value of the contracts and the actual amount depends on aspects such as the documentation

$$V_{MTM} \neq V_{close\ out}$$

- But we cannot model the true close out value!
- Should also take collateral into account where relevant

Expected Exposure (EE), Negative EE and Expected Future Value (EFV)



More details in Alonso Pena's lecture on simulating exposure

$$EE = E[max(V,0)]$$

$$NEE = E[min(V,0)]$$

$$EFV = E[V] = EE + NEE$$

Deriving the CVA Formula (I)

- **Time of default is denoted by τ**
- **Base value of derivatives portfolio at time t with final maturity T:** $V(\tau,T)$
- **Default payoff is:**

$$\delta V(\tau,T)_+ + V(\tau,T)_-$$

Recovery rate (%)

- **Risky value:**

No default before T

$$\tilde{V}(t,T) = E_t\begin{bmatrix} I(\tau > T)V(t,T) + \\ I(\tau \le T)V(t,\tau) + \\ I(\tau \le T)(\delta V(\tau,T)_+ + V(\tau,T)_-) \end{bmatrix}$$

Full value if no default

Cashflows before default

Default payoff

Deriving the CVA Formula (II)

- **Extract the risk-free value**

$$\tilde{V}(t,T) = E_t\begin{bmatrix} I(\tau > T)V(t,T) + \\ I(\tau \le T)V(t,\tau) + \\ I(\tau \le T)(\delta V(\tau,T)_+ + V(\tau,T)_-) \end{bmatrix}$$

$$\tilde{V}(t,T) = E_t\begin{bmatrix} I(\tau > T)V(t,T) + \\ I(\tau \le T)V(t,\tau) + \\ I(\tau \le T)(\delta V(\tau,T)_+ + V(\tau,T) - V(\tau,T)_+) \end{bmatrix}$$

$$\tilde{V}(t,T) = V(t,T) + \underbrace{E_t[I(\tau \le T)(1-\delta)V(\tau,T)_+]}_{\text{CVA}}$$

CVA Formula

$$CVA(t,T) = E_t[\underbrace{I(\tau \le T)}_{\text{Default}}\underbrace{(1-\delta)}_{\text{Loss given default (LGD)}}\underbrace{V(t,T)}_{\text{Value at default}}]$$

- **If we assume independence (no wrong-way risk)**

$$CVA = LGD \int_t^T \underbrace{EE(u)}_{\text{Discounted expected exposure}}\underbrace{dPD_C(u)}_{\text{Default probability}}$$

- **To compute CVA we require**
  - Expected exposure
  - Default probability of counterparty
  - LGD (second order effect)

$\left.\begin{array}{l}\text{Default probability of counterparty}\\ \text{LGD (second order effect)}\end{array}\right\}$ *Credit spread*

CVA and Accounting Rules

- **IFRS 13 (1st January 2013)**
- **CVA**
  - "The entity shall include the effect of the entity's ***net exposure to the credit risk of that counterparty*** or the counterparty's net exposure to the credit risk of the entity in the fair value measurement when market participants would take into account any existing arrangements that mitigate credit risk exposure in the event of default"

Traders shocked by $712m CVA loss at StanChart

Feature - Tue 15th Mar

Bank's new methodology has been used by some rivals for more than a decade

- **DVA**
  - Non-performance risk includes, but may not be limited to, an entity's ***own credit risk***"
  - Explicit that own credit must be incorporated into the fair value measurement based on the concept of ***"exit price"***
  - Exit price also implies the use of ***market implied parameters***

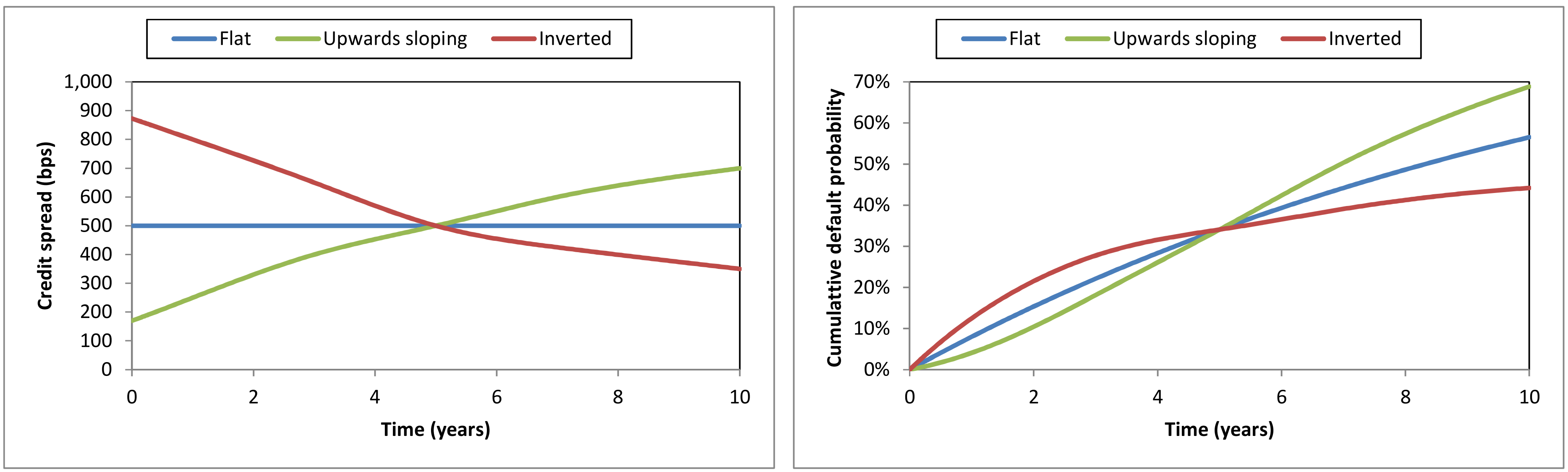
Risk-Neutral Default Probabilities

- **Approximate formula for market implied default probability**

$$\Delta PD(t_{i-1}, t_i) = PD(0, t_i) - PD(0, t_{i-1}) \approx \exp\left(-\frac{s_{i-1}}{LGD} t_{i-1}\right) - \exp\left(-\frac{s_i}{LGD} t_i\right)$$

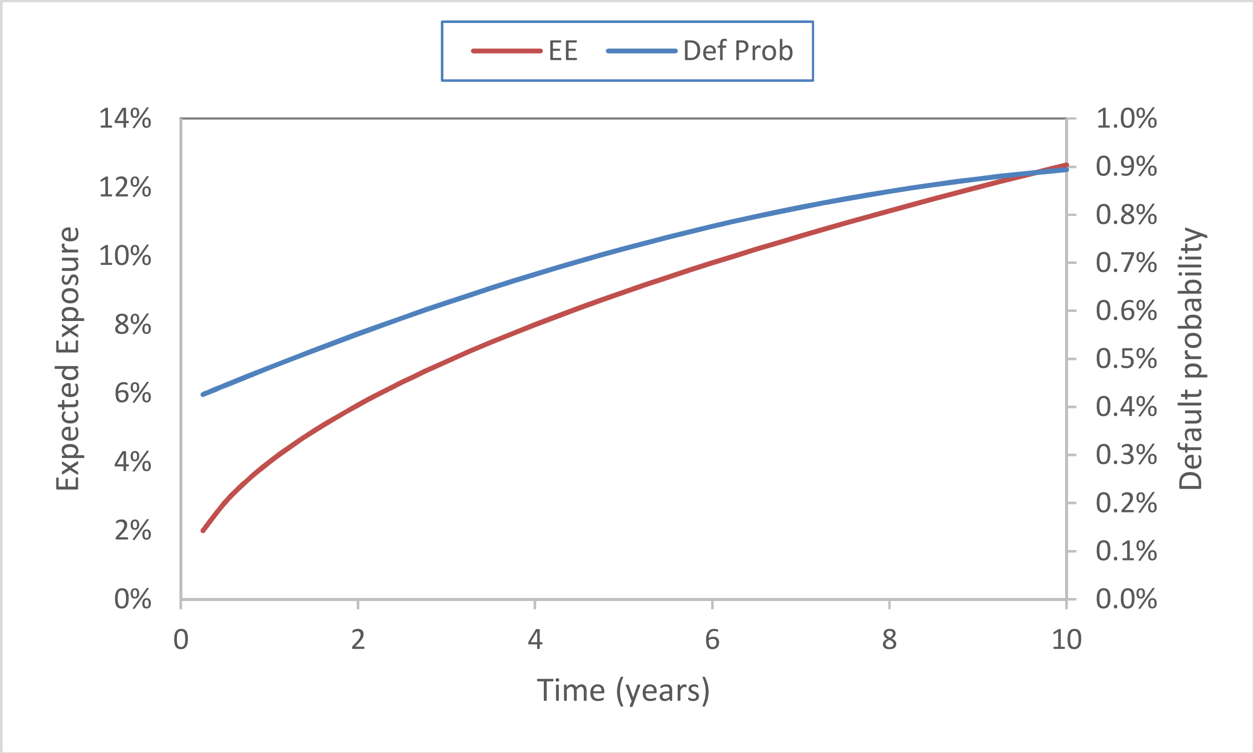
Spread at time *i-1* Spread at time *i*

- Accurate calculation has to be done numerically (more detail from Alonso Pena)



CVA Example (see spreadsheet 14.1)

- 10-year cross currency swap, 200 bps (upwards sloping) counterparty spread



Tenor	Spread (bps)
1Y	110
3Y	130
5Y	150
7Y	170
10Y	200

- CVA is -1.557% of notional up-front
- Or alternatively 17.6 bps (basis points per annum)

Bilateral CVA Formula

- **Considering an institutions (I) own default**

$$BCVA = CVA + DVA =$$

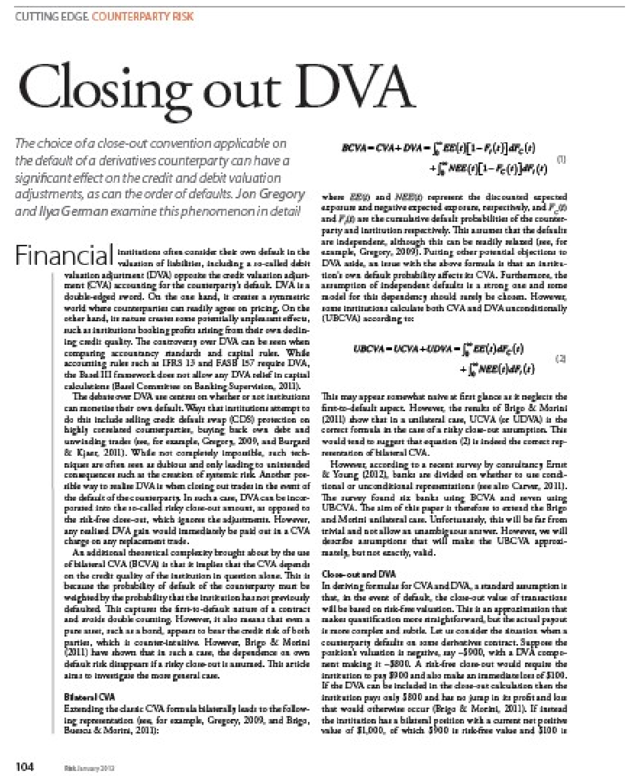
Do we include survival probabilities (grey terms)?

CVA

$$-LGD_C \int_t^T \underbrace{EE(u)}_{\text{Expected exposure}} S_I(u) \underbrace{dPD_C(u)}_{\text{Probability counterparty defaults}}$$

DVA

$$-LGD_I \int_t^T \underbrace{NEE(u)}_{\text{Negative expected exposure}} S_C(u) \underbrace{dPD_I(u)}_{\text{Probability we default}}$$

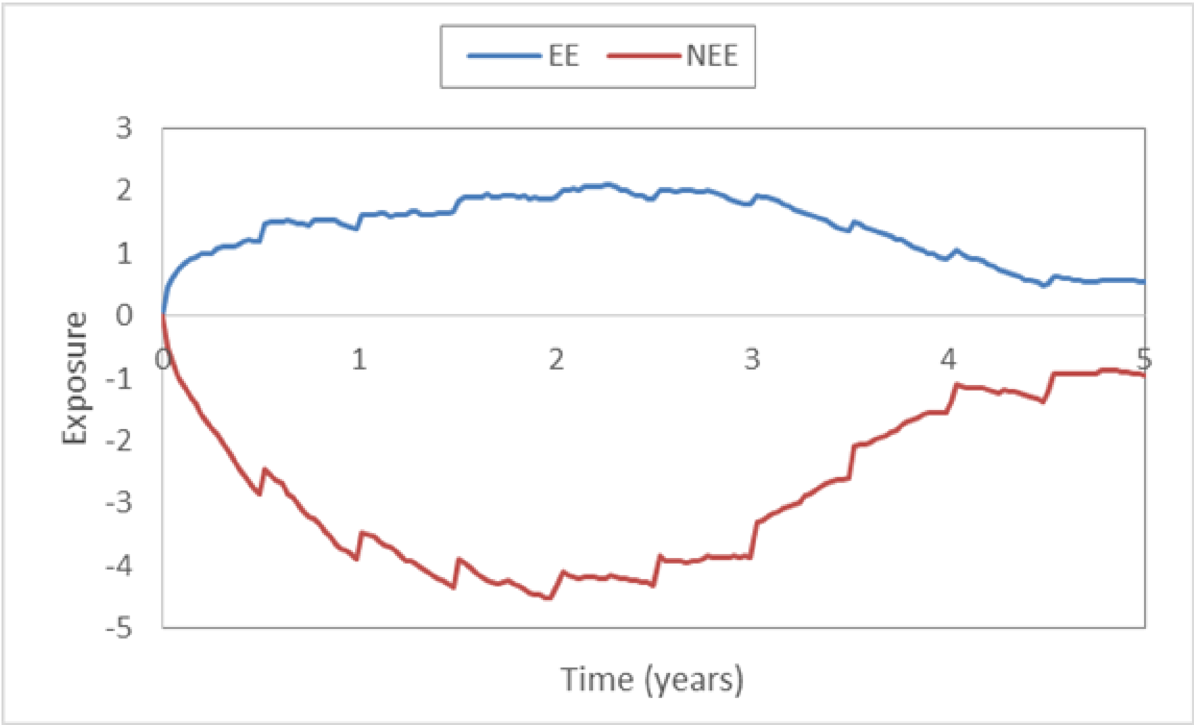

CUTTING EDGE. COUNTERPARTY RISK

Closing out DVA

The choice of a close-out convention applicable on the default of a derivatives counterparty can have a significant effect on the credit and debit valuation adjustments, as can the order of defaults. Jon Gregory and Ilya German examine this phenomenon in detail

CVA and DVA Example

- **Swap portfolio, counterparty spread = 200 bps, own spread = 100 bps**



CVA = -0.1309

DVA = +0.1357

BCVA = +0.0048

- **Is the BCVA a real profit?**

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Funding Costs – No CSA

recent paper [Fleming et al., 2012] in which the authors obtained access to interest rate derivative market transactions over the period between June 1st and August 31st 2010 found that the execution of a large swap trade by one bank was typically followed within 30 minutes by the execution of offsetting transactions with other counterparties.

- **Assumptions**
  - Trade with non-CSA client
  - Hedge trade with CSA counterparty (e.g. inter-dealer)
  - Assume trade moves against client

- **Implication**
  - Funding cost
  - Funding benefit when the trade moves in the favour of the client

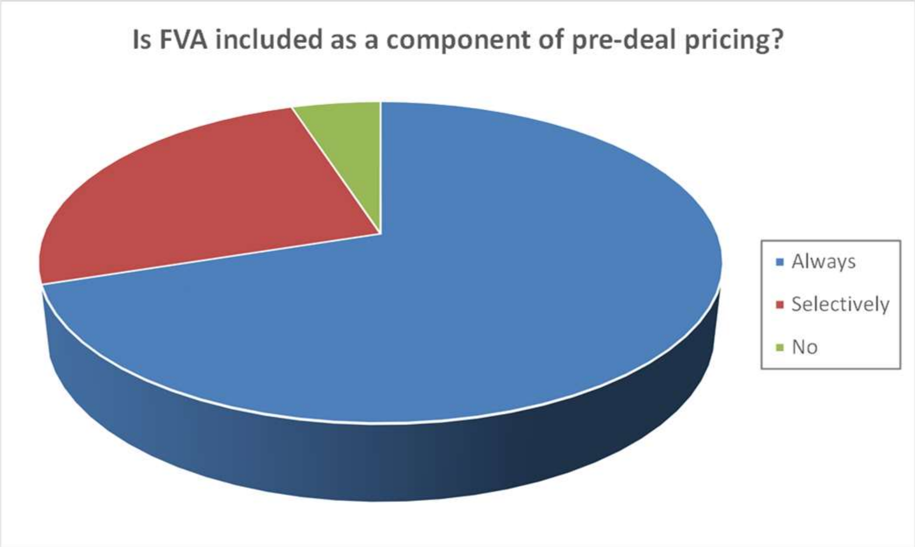
The Use of FVA

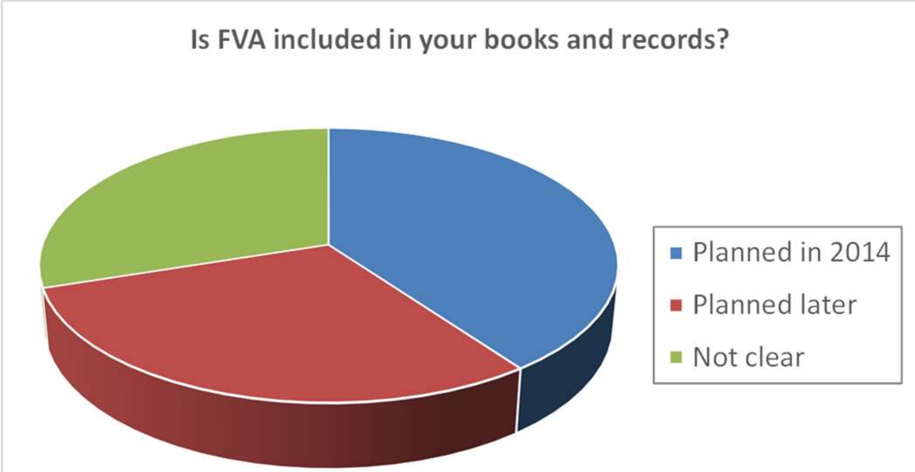
"During 2012, a fair-value adjustment was applied to account for the impact of incorporating the cost of funding into the valuation of uncollateralised derivatives"

"Valuation adjustments are integral to determining the fair value of derivatives …… [including] credit valuation adjustments and funding valuation adjustments."

"The group has recognised a funding valuation adjustment [of £143 million] to adjust for the net cost of funding certain uncollateralised derivative positions where the group considers that this cost is included in market pricing.

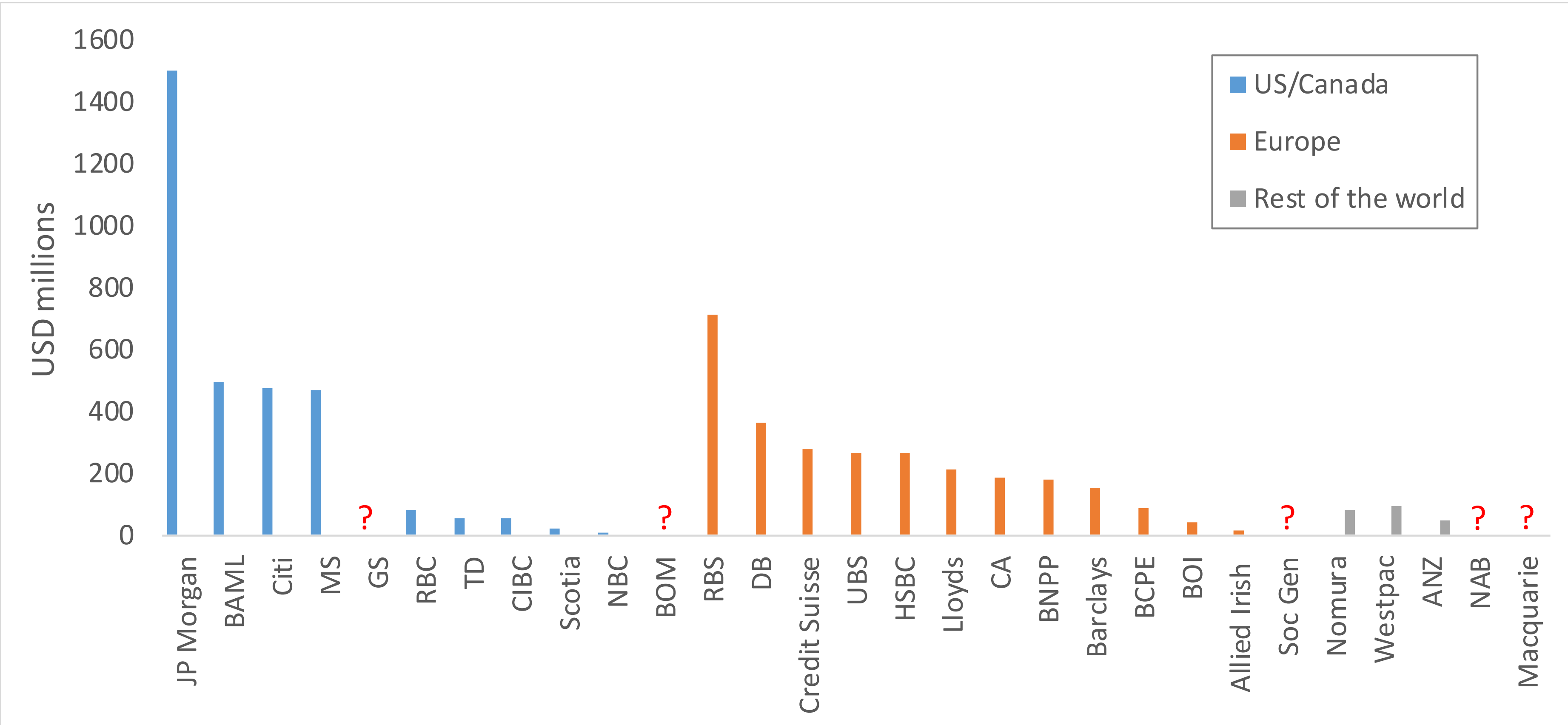
In general, FVA reflects a market funding risk premium inherent in the uncollateralized portion of derivative portfolios, and in collateralized derivatives where the terms of the agreement do not permit the reuse of the collateral received.





Source : Solum FVA Survey 2014

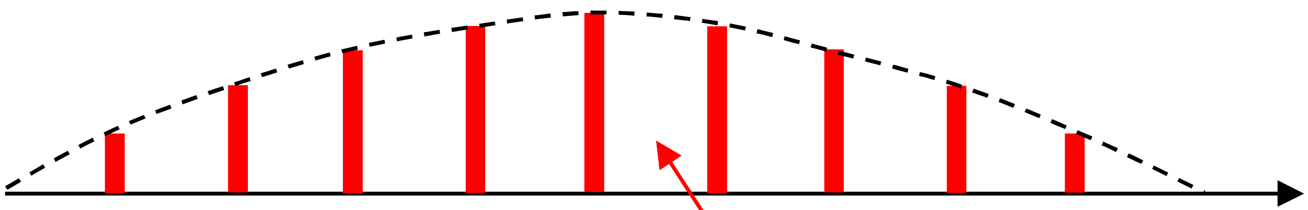
FVA and Market Practice



Source: Risk Magazine, April 2015
(converted to approx. USD equivalent)

FVA Formula

- **FVA obtained by integrating over "funding profile" (EFV) wrt funding cost**



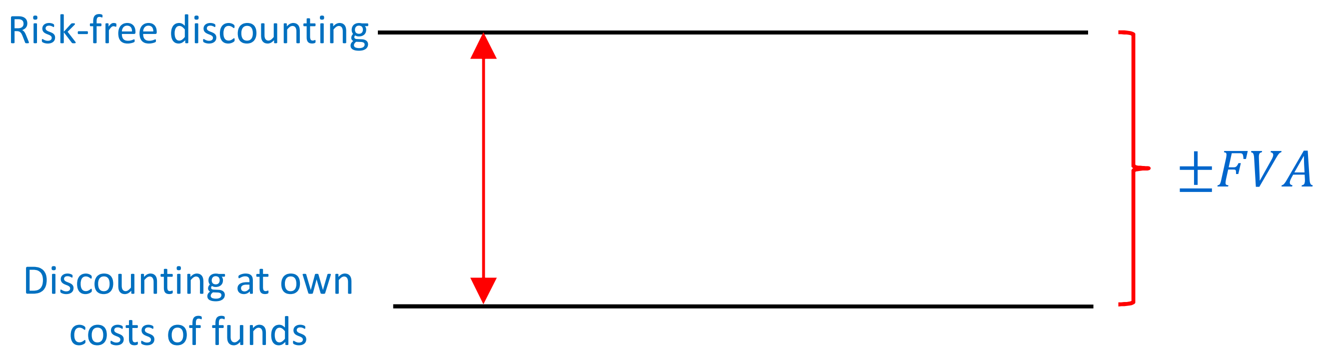
$$FVA = -\int_t^T EFV(u)FS(u)S_{CI}(u)du$$

Forward funding spread (with respect to OIS)

Survival probability

- **How to calculate**
  - Can be implemented as a CVA style calculation with LGD = 100% and funding spread replacing counterparty credit spread
  - NOTE: the same impact can be achieved by discounting with funding costs included

Discounting Approach or FVA formula?



- **Why not just discount uncollateralised trades at our own funding cost?**
- **Not wrong but this is not generic and the following situations are problematic:**
  - One-way collateral agreements
  - Partially collateralised transactions
  - Where funding costs and benefits are not symmetric
- **But we can split the funding profile into positive and negative components**

$$EFV = EE + NEE$$

FVA Formula – General Approach

- Separate formula into cost and benefit terms (survival probability left out)

$$FVA = -\int_t^T EE(u)\underbrace{FS_B(u)}_{\substack{\text{Funding spread when}\\ \text{derivatives desk}\\ \underline{\text{requires}}\text{ funding}}}du - \int_t^T NEE(u)\underbrace{FS_L(u)}_{\substack{\text{Funding spread when}\\ \text{derivatives desk}\\ \underline{\text{generates}}\text{ funding}}}du = FCA + FBA$$

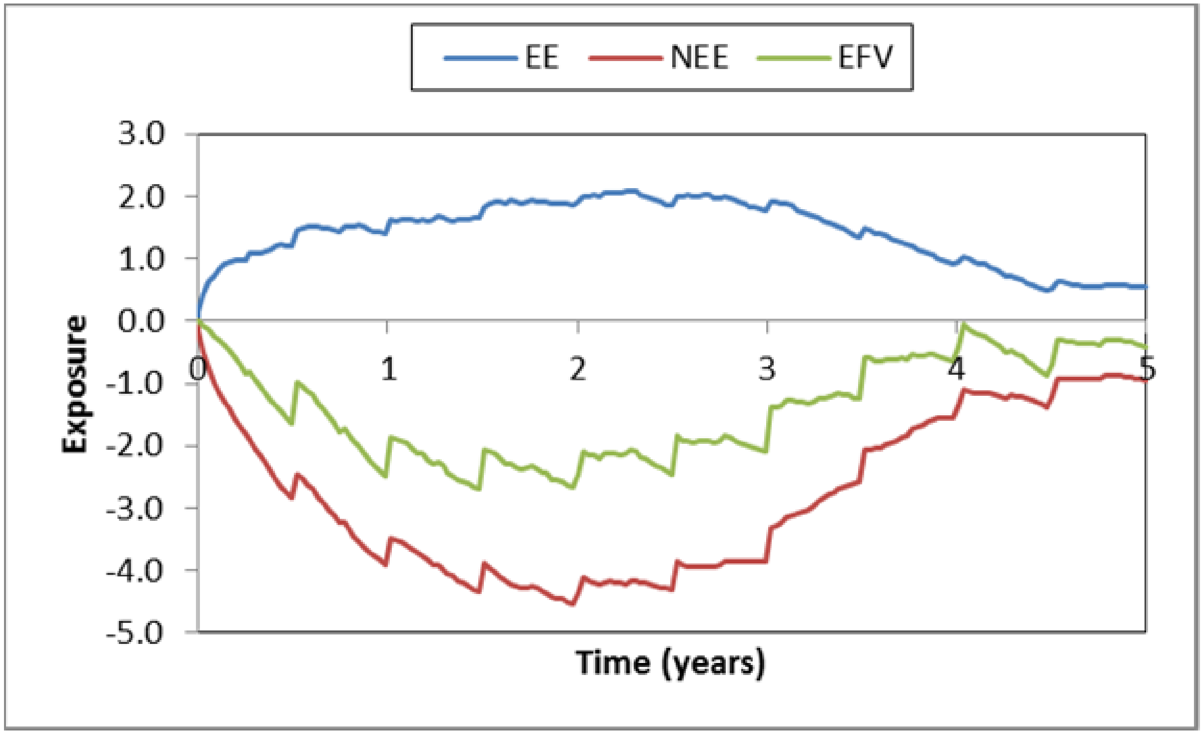
- The funding benefit is a more economically pleasing version of DVA

$$FVA = FCA + \boxed{FBA}$$

$$BCVA = CVA + \boxed{DVA}$$

FVA Example

- **Swap portfolio, cost of funding = 100 bps**



FCA = -0.0690

FVA = +0.0687

FBA = +0.1377

- **Overall FVA is a profit due to expected future value being negative**

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Basel III Capital Requirements

- **Prior to Basel III, there was a CCR capital charge covering potential defaults**
- **BCBS Consultative document (December 2009)**
  - "Roughly ***two-thirds of CCR losses were due to CVA losses*** and only about one-third were due to actual defaults. The current framework addresses CCR as a default and credit migration risk, but does not fully account for market value losses short of default."
  - "Banks will be subject to a ***capital charge for potential mark-to-market losses (CVA***) associated with a deterioration in the credit worthiness of a counterparty."
- **There are now two capital charges (see Chapter 8 of my book)**
  - CCR capital charge
  - CVA capital charge

KVA (Capital Value Adjustment) Formula

$$KVA = -\int_t^T ECP(u)CC(u)S_{CI}(u)du$$

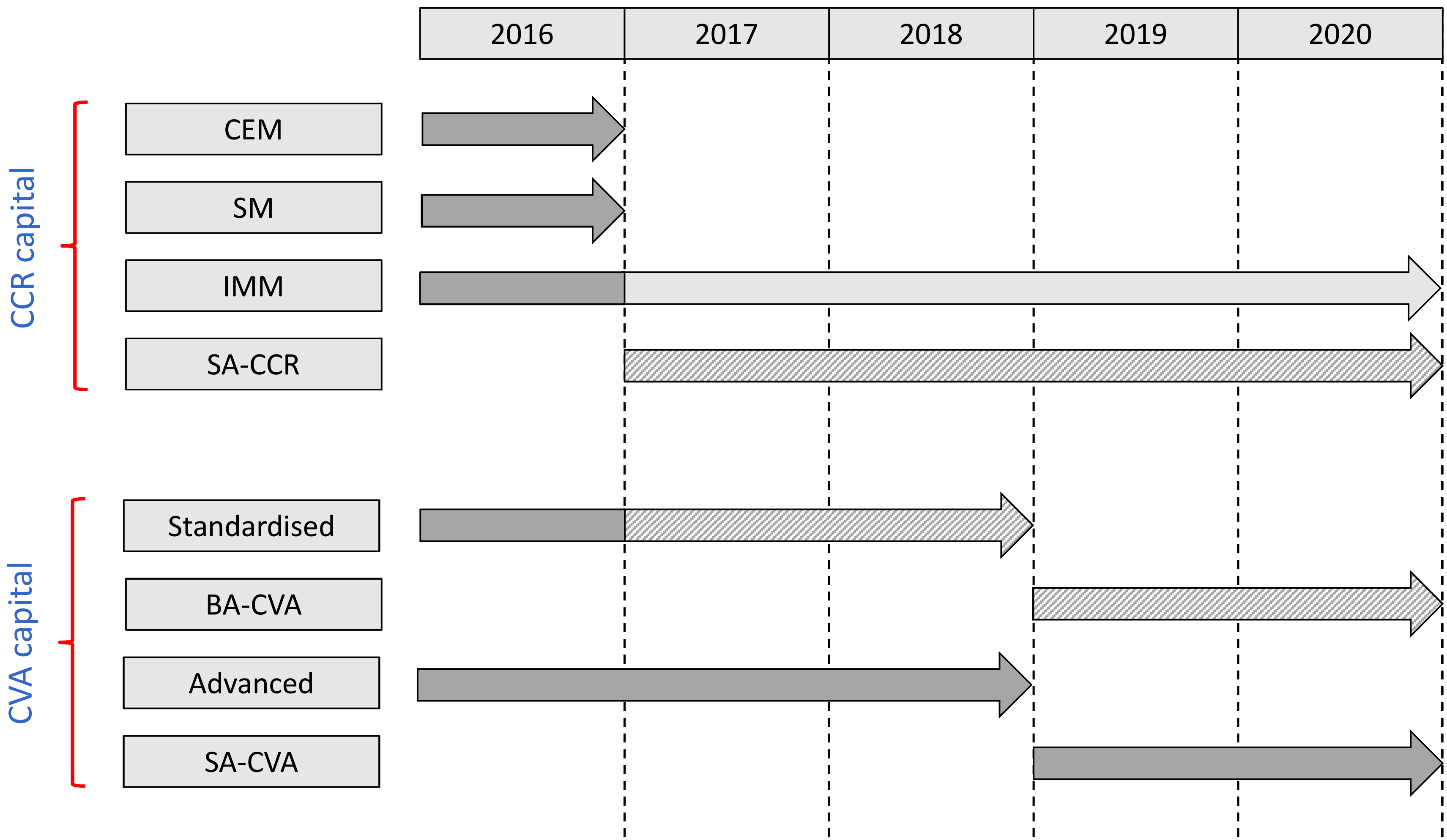
Discounted expected capital profile

Cost of capital

Probability of no defaults

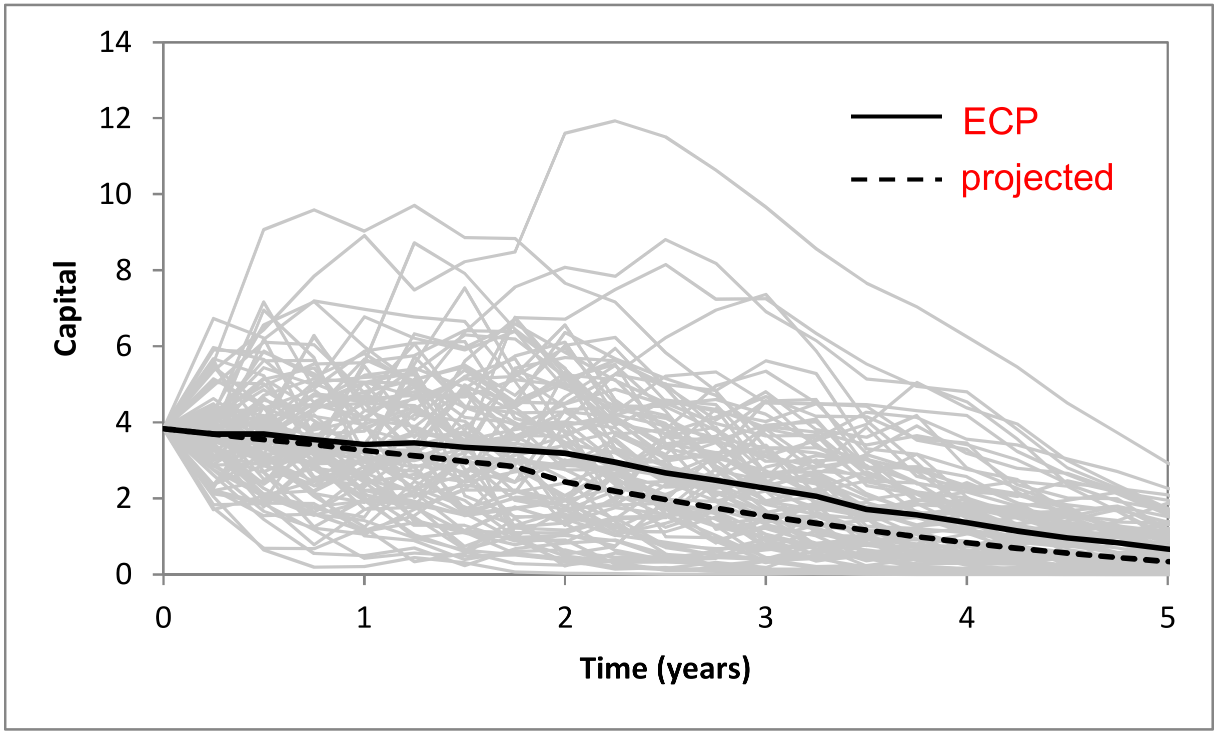
- **Aim of KVA**
  - To provide a profit that can be released over time and matches the cost of regulatory capital requirements
  - ECP depends on our model for regulatory capital and has two components (CCR and CVA)
  - Should we discount also at cost of capital?

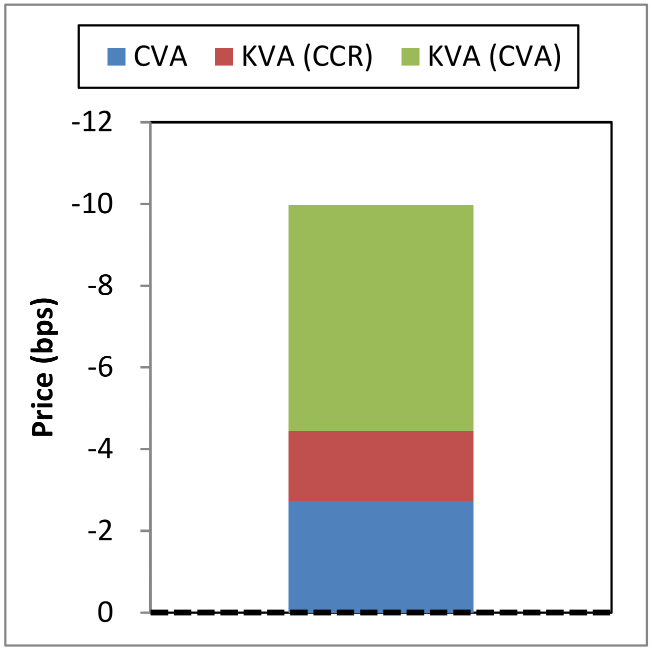
Methodologies and Timescales



Projected Capital Profile

- **Projection is much easier to calculate**
- **ECP may require a Monte Carlo within a Monte Carlo**





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Clearing and Margining Requirements

- **Initial margin (independent amount) is an extra collateral posting**
  - Very uncommon in bilateral OTC derivatives in the past
  - But this is going to change for two reasons (except for end-users who are exempt)
- **Clearing Mandate**
  - Standardised OTC derivatives must be cleared directly or indirectly
  - CCPs require significant <u>initial margin</u>
- **Bilateral margin requirements**
  - Applies to bilateral (non-clearable) OTC derivatives
  - Variation margin (already quite common)
  - <u>Initial margin</u> (uncommon in bilateral markets) phased in from September 2016
- **This leads to margin value adjustment (MVA)**

MVA Formula

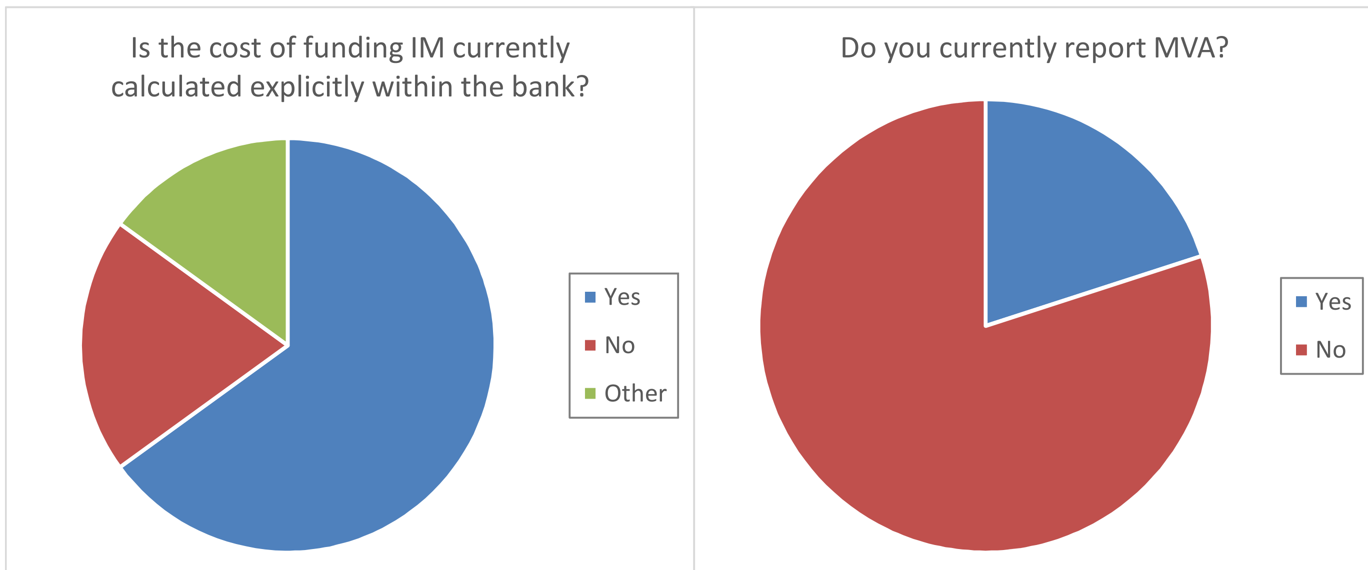
$$MVA = \int_t^T EIM(u) \times (FC(u) - R_{IM})\, S(u)\, du$$

Expected IM profile

Cost of funding IM

Remuneration of IM

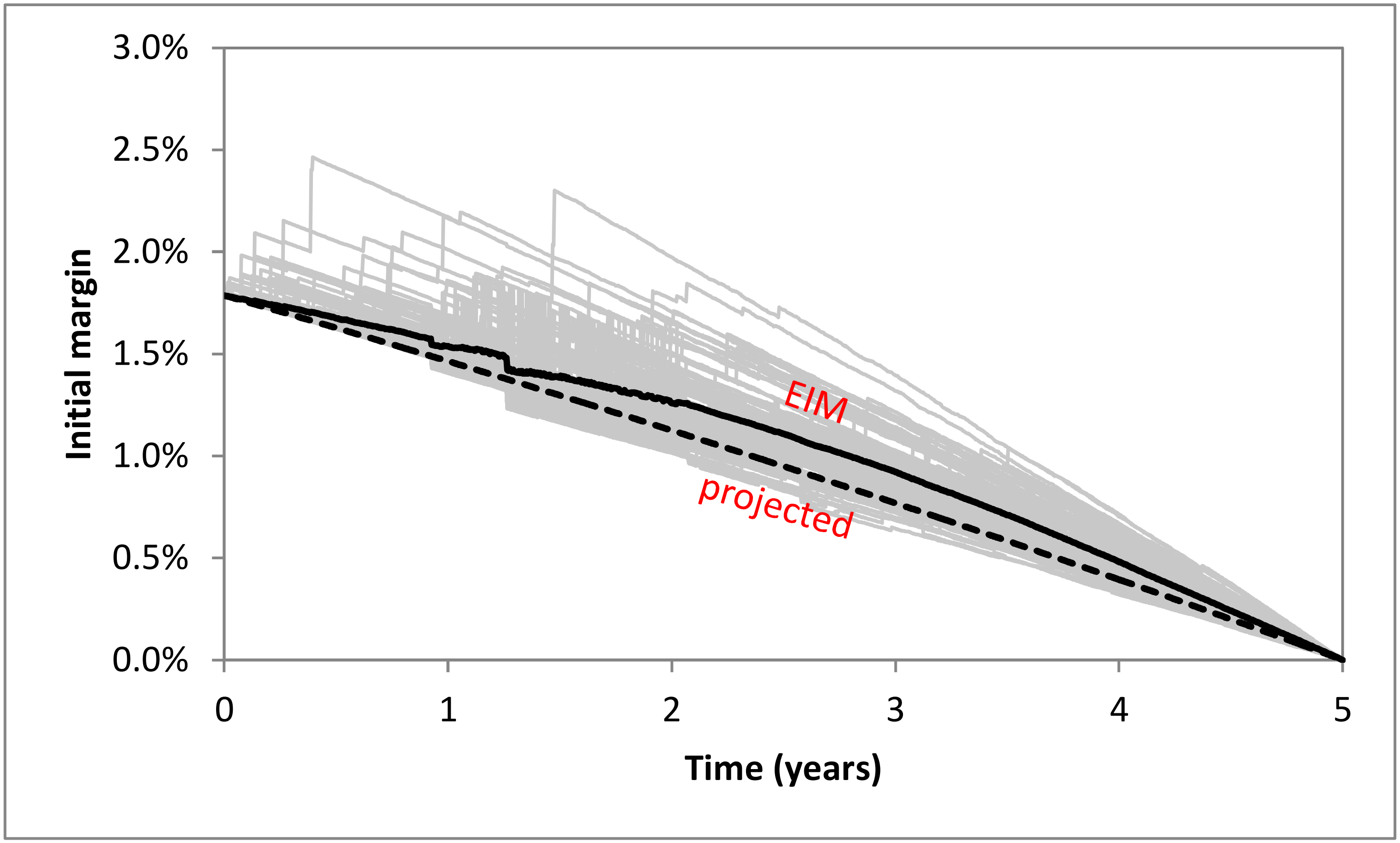
Probability of no defaults



Source: Solum Collateral Survey 2015

Projected IM and Expected IM Costs

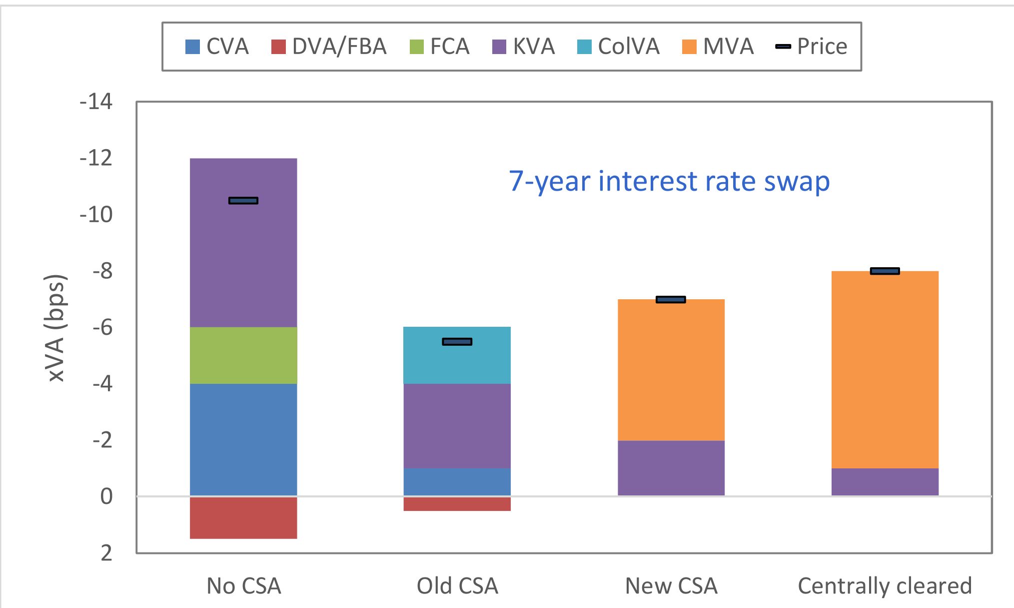
- **Like KVA, MVA is complex to compute but projection is easier**



Important Components – High Level

	Uncollateralised	Collateralised	Overcollateralised	
	No CSA	Old CSA	New CSA (initial margin)	Central Clearing
CVA and DVA	✓✓	✓		
FVA	✓✓			
KVA	✓✓	✓✓	✓	✓
ColVA		✓✓		
MVA			✓✓	✓✓[1]

[1] Plus default fund contributions etc



A Final Word

- **Calculating the xVA terms is quite challenging**
  - Portfolio effects, time consuming simulations etc.
  - Precise definitions of the xVA terms
- **But there are more complexities that need to be considered also**
- **xVA terms are not actually distinct**
  - There is acknowledged overlap between CVA/KVA, FVA/KVA under debate
- **And xVA will gradually be a consideration in any valuation problem**
  - For example, we should consider xVA when we exercise a physically settled option
- **There is an enormous amount of quant work to be done in the coming years**