Credit Default Swaps

Abstract

In this lecture we will study the most common type of credit derivative: credit default swaps (CDS). We will first review some instances of CDS in the media. Then, we will study the characteristics of CDS contracts and their pricing in the context of the intensity (reduced form) approach. We will conclude by studying some numerical examples and creating a CDS pricing in an Excel Workshop.

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Contents of this Lecture

The Story begins...

Musée du Louvre, Paris

Code of Hammurabi
1754 BC

First regulations in history about interest, forgiveness of debt and extension of credit

Payments through a local banker or by written draft against deposit

Law 117: Collateral

"If any one fail to meet a claim for debt, and sell himself, his wife, his son, and daughter for money or give them away to forced labor: they shall work for three years in the house of the man who bought them and in the fourth year they shall be set free."

March 24, 1989
The Exxon Valdez was an oil tanker that gained notoriety after running aground in Prince William Sound spilling hundreds of thousands of barrels of crude oil in Alaska.

In 1994 J.P. Morgan extended a $4.8 billion credit line to Exxon, which faced the threat of $5 billion in punitive damages for the Exxon Valdez oil spill. A team of J.P. Morgan bankers led by Blythe Masters then sold the credit risk from the credit line to the European Bank of Reconstruction and Development in order to cut the reserves that J.P. Morgan was required to hold against Exxon's default, thus improving its own balance sheet.

2015

Derivatives statistics

Updated 30 Apr 2015

The BIS compiles and publishes three sets of statistics on derivatives markets:

- notional amounts outstanding and gross market values of OTC derivatives, from the semiannnual survey and Triennial Survey,
- turnover of OTC derivatives, from the Triennial Survey, and
- turnover and open interest of exchange-traded derivatives, from commercial data sources

The objective of the semiannual survey is to obtain comprehensive and internationally consistent information on the size and structure of the largest OTC derivatives markets. They provide data on notional amounts outstanding and gross market values and permit the evolution of particular market segments to be monitored. In conjunction with the banking and securities statistics, they offer a more comprehensive picture of activity in global financial markets as well.

Following the initiative from the Committee on the Global Financial System (CGFS), central banks of the G10 countries started in June 1998 reporting to the BIS semiannual OTC derivatives statistics on forwards, swaps and options of foreign exchange, interest rate, equity and commodity derivatives. As of end-June 2004, the BIS also releases statistics on concentration measures, back to June 1998. The data include concentration measures for foreign exchange, interest rate and equity-linked derivatives. Finally, as of end-December 2004 the BIS releases semiannual data on credit default swaps (CDS) including notional amounts outstanding and gross market values for single- and multi-name instruments. Additional information on CDS by counterparty, sector and rating has been made available as of December 2005.

As of end-June 2010 more granular information is published on CDS counterparties (eg CCPs, SPVs and Hedge Funds as well as on Index products in the multi-name CDS instruments).

From December 2011, Australia and Spain began contributing to the semiannual survey, bringing the number of reporting countries to 13.

Table		Data		
		PDF	CSV	BIS WebStats
19	**Amounts outstanding of over-the-counter (OTC) derivatives by risk category and instrument**	PDF	CSV	-
20	**Amounts outstanding of OTC foreign exchange derivatives**			
20A	By instrument and counterparty	PDF	CSV	-
20B	By currency	PDF	CSV	-
20C	By instrument, maturity and counterparty	PDF	CSV	-
21	**Amounts outstanding of OTC single-currency interest rate derivatives**			

http://www.bis.org/statistics/derstats.htm

Table 19: Amounts outstanding of over-the-counter (OTC) derivatives

By risk category and instrument

In billions of US dollars

Risk Category / Instrument	Notional amounts outstanding					Gross market values				
	Dec 2012	Jun 2013	Dec 2013	Jun 2014	Dec 2014	Dec 2012	Jun 2013	Dec 2013	Jun 2014	Dec 2014
Total contracts	**635,685**	**696,408**	**710,633**	**691,640**	**630,150**	**24,953**	**20,245**	**18,825**	**17,438**	**20,880**
Foreign exchange contracts	**67,358**	**73,121**	**70,553**	**74,782**	**75,879**	**2,313**	**2,427**	**2,284**	**1,724**	**2,944**
Forwards and forex swaps	31,718	34,421	33,218	35,190	37,076	806	957	824	572	1,205
Currency swaps	25,420	24,654	25,448	26,141	24,204	1,259	1,131	1,186	939	1,351
Options	10,220	14,046	11,886	13,451	14,600	249	339	273	213	389
Interest rate contracts	**492,605**	**564,673**	**584,799**	**563,290**	**505,454**	**19,038**	**15,238**	**14,200**	**13,461**	**15,608**
Forward rate agreements	71,960	86,892	78,810	92,575	80,836	48	168	108	126	145
Interest rate swaps	372,293	428,385	456,725	421,273	381,028	17,285	13,745	12,919	12,042	13,946
Options	48,351	49,396	49,264	49,442	43,591	1,706	1,325	1,174	1,292	1,517
Equity-linked contracts	**6,251**	**6,821**	**6,560**	**7,084**	**7,941**	**600**	**692**	**700**	**678**	**615**
Forwards and swaps	2,045	2,321	2,277	2,505	2,495	157	206	202	199	178
Options	4,207	4,501	4,284	4,579	5,446	443	486	498	479	437
Commodity contracts	**2,587**	**2,458**	**2,204**	**2,206**	**1,868**	**347**	**384**	**264**	**269**	**317**
Gold	486	461	341	319	300	42	80	47	32	32
Other commodities	2,101	1,997	1,863	1,887	1,568	304	304	217	237	285
Forwards and swaps	1,363	1,327	1,260	1,283	1,053					
Options	739	670	603	604	515					
Credit default swaps	**25,068**	**24,349**	**21,020**	**19,462**	**16,399**	**848**	**725**	**653**	**635**	**593**
Single-name instruments	14,309	13,135	11,324	10,845	9,041	527	430	369	368	366
Multi-name instruments	10,760	11,214	9,696	8,617	7,358	321	295	284	266	227
of which index products	9,656	10,163	8,746	7,939	6,747					
Unallocated	**41,815**	**24,986**	**25,496**	**24,815**	**22,609**	**1,808**	**779**	**724**	**671**	**803**
Memorandum Item:										
Gross Credit Exposure						3,612	3,784	3,033	2,826	3,358

http://www.bis.org/statistics/derstats.htm

ISDA
CDS Marketplace℠

ABOUT CDS | DAILY PRICES | EXPOSURES & ACTIVITY | MARKET OVERVIEW | RESEARCH & TOOLS

Credit default swaps are important risk transfer instruments in today's global economy. ISDA CDS Marketplace℠ brings together information, data and statistics to help you better understand the CDS business.

About CDS

A credit default swap (CDS) is a bilateral agreement designed to transfer risk from one party to another. A growing number of firms rely on these instruments to efficiently manage their risks.

Learn More

Daily Prices

How is the CDS market trading? Daily price changes for a range of industry indices and single name reference entities are provided here to enable readers to see and understand current trends in the CDS business.

Learn More

Exposures & Activity

Which reference entities were most actively traded during the past week, in terms of number of contracts and notional exposures? And which reference entities have the highest level of protection sold on them?

Learn More

Market Overview

How large is the CDS market? ISDA, along with other institutions such as the Bank for International Settlements, periodically survey this global business to measure its growth and size.

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http://www.isdacdsmarketplace.com/

Part 1

An Introduction to CDS

CDS Definition

$$\text{CDS} = \text{CREDIT} + \text{DEFAULT} + \text{SWAP}$$

with

CREDIT: debt-linked instrument

DEFAULT: sensitive to default

SWAP: exchange of cashflows

CDS Definition

A credit default swap is a financial derivative which offers protection* against the default of an underlying instrument.

In a CDS the protection* buyer makes a series of payments to the protection* seller and, in exchange, receives a payoff if a credit instrument (e.g. bond, loan) goes into default.

In some contracts, the credit event that triggers the payoff can be a company undergoing restructuring, bankruptcy or even just having its credit rating downgraded.

*Note the word "Protection". Can this be interpreted as a form of insurance?

CDS Are Not Insurance

In a CDS the seller need not be a regulated entity.

In a CDS the seller is not required to maintain any reserves to pay off that particular buyer (apart from bank capital requirements).

Insurers manage risk primarily by setting loss reserves, while dealers in CDS manage risk primarily by means of hedging CDS with other dealers.

The buyer of a CDS does not need to own the underlying security or other form of credit exposure; in fact the buyer does not even have to suffer a loss from the default event. In contrast, to purchase insurance the insured is generally expected to have an insurable interest such as owning a debt.

CDS Example

Consider that an investor buys a CDS from Bank of America, where the reference entity is General Motors.

The investor will make regular payments to Bank of America, and if General Motors defaults on its debt, the investor will receive a one-off payment from Bank of America and the CDS contract is terminated.

If the investor actually owns General Motors debt, the CDS can be thought of as **hedging**. But investors can also buy CDS contracts referencing General Motors debt, without actually owning any General Motors debt. This may be done for **speculative** purposes, to bet against the solvency of General Motors in a gamble to make money if it fails, or to hedge investments in other companies whose fortunes are expected to be similar to those of General Motors.

CDS Example

If the reference entity (General Motors) defaults, one of two things can happen:

Physical settlement: The investor delivers a defaulted asset to Bank of America for a payment of the par value.

Cash settlement: Bank of America pays the investor the difference between the par value and the market price of a specified debt obligation (even if General Motors defaults, there is usually some recovery).

CDS Example

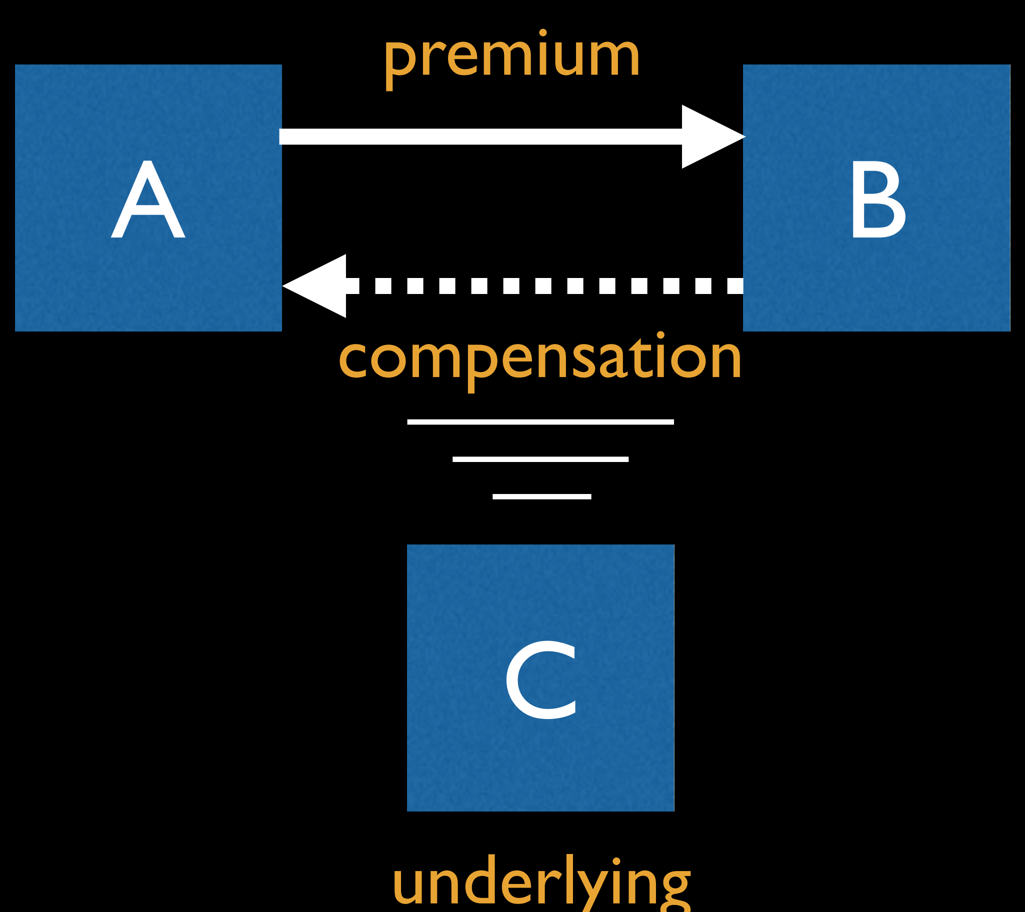
The **spread** of a CDS is the annual amount the protection buyer must pay the protection seller over the length of the contract, expressed as a percentage of the notional amount. For example, if the CDS spread of General Motors is 50 basis points or 0.5%, then an investor buying $10 million worth of protection from Bank of America must pay the bank $50,000 per year. These payments continue until either the CDS contract expires or General Motors defaults.

All things being equal, at any given time, if the **maturity** of two credit default swaps is the same, then the CDS associated with a company with a higher CDS spread is considered more likely to default by the market, since a higher fee is being charged to protect against this happening. However, factors such as liquidity and estimated loss given default can impact the comparison.

The CDS concept

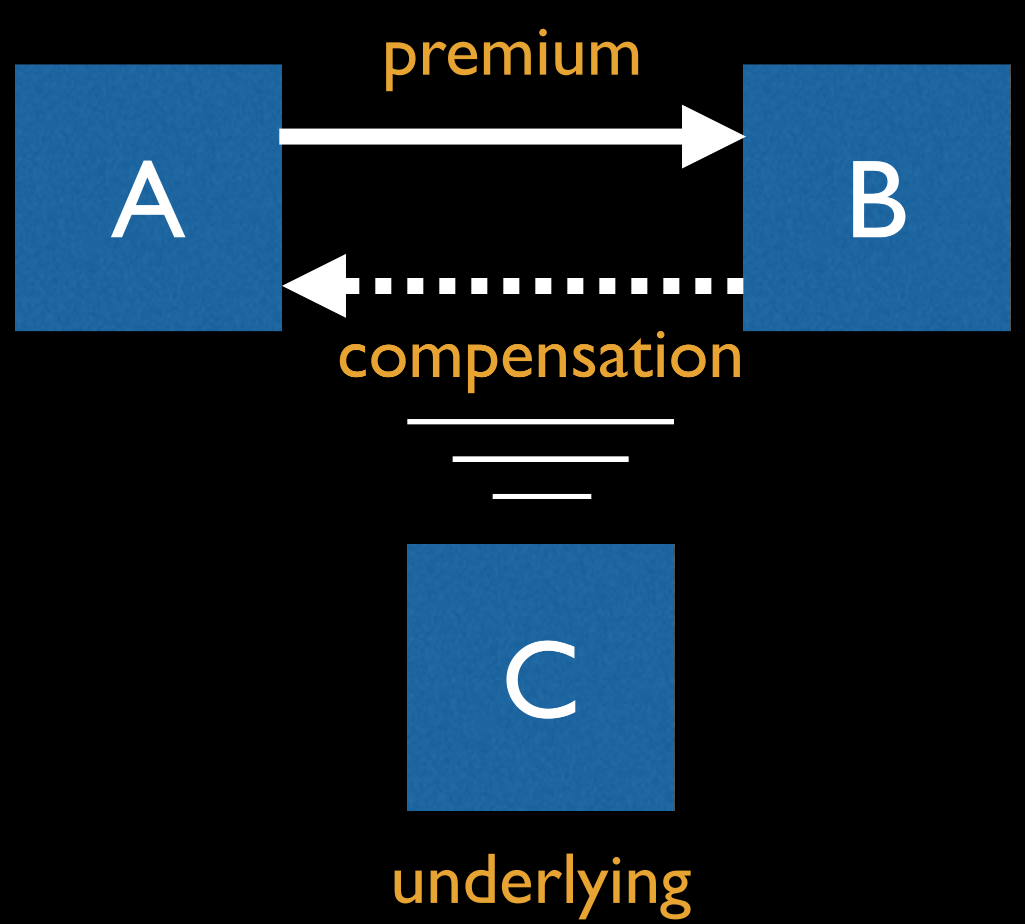
Definition

A CDS is a financial contract between two counterparties A and B, in which one party pays to the other party a regular premium to buy credit protection against the possible default of an underlying C.



Definition

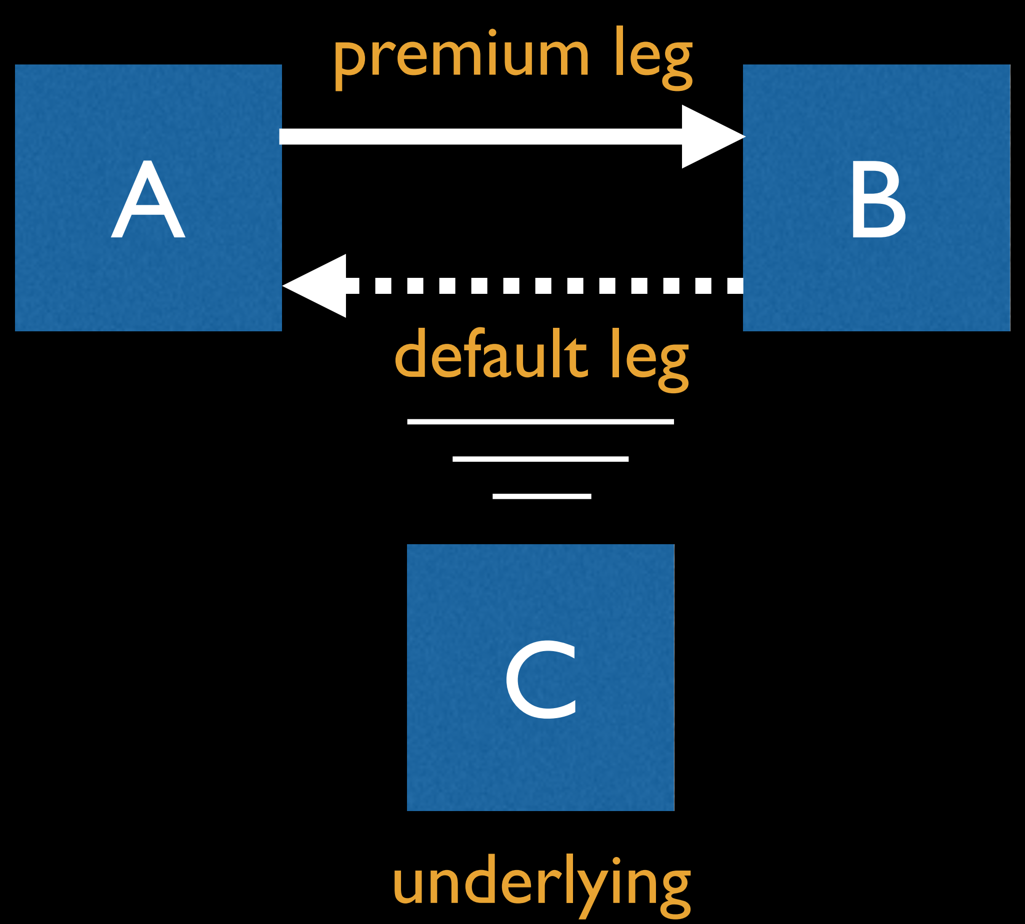
In structure, the CDS is similar to the plain vanilla IRS, as it can be considered as an exchange of cash flows between the parties.



Definition

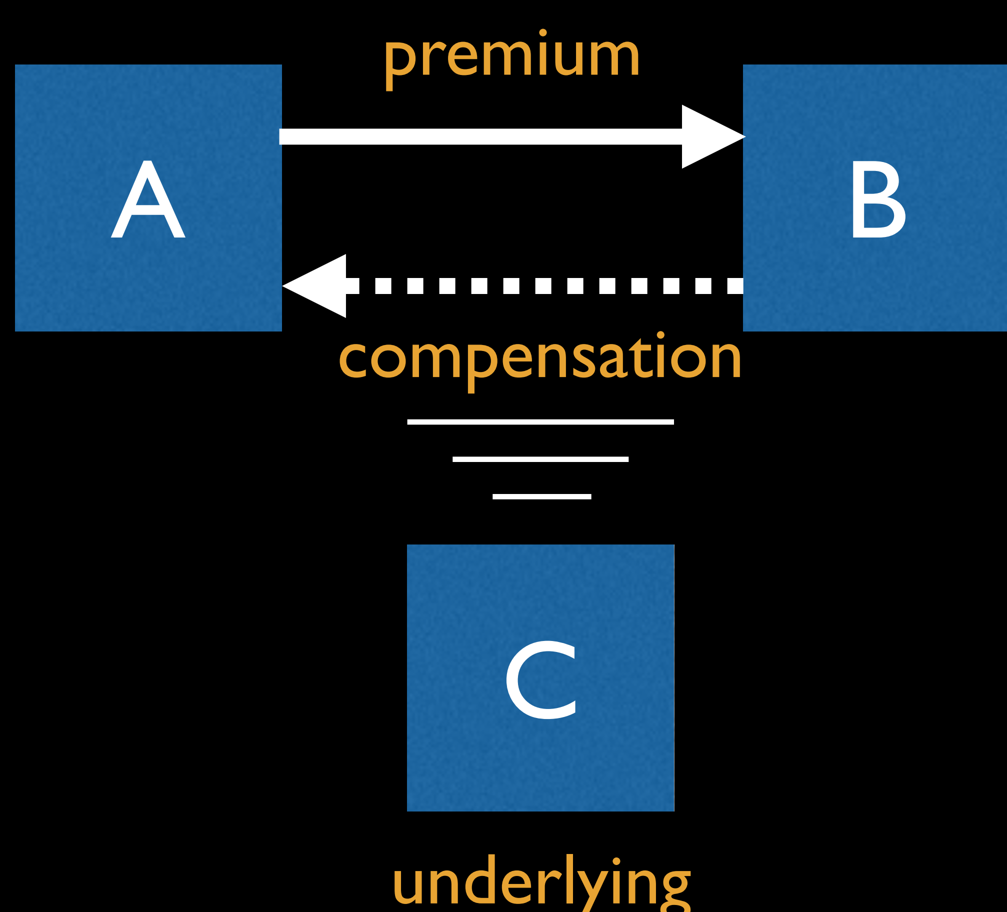
premium leg = the stream of cashflows that A pays B.

default leg = the protection payment paid by B to A in case of default of the underlying C.



Definition

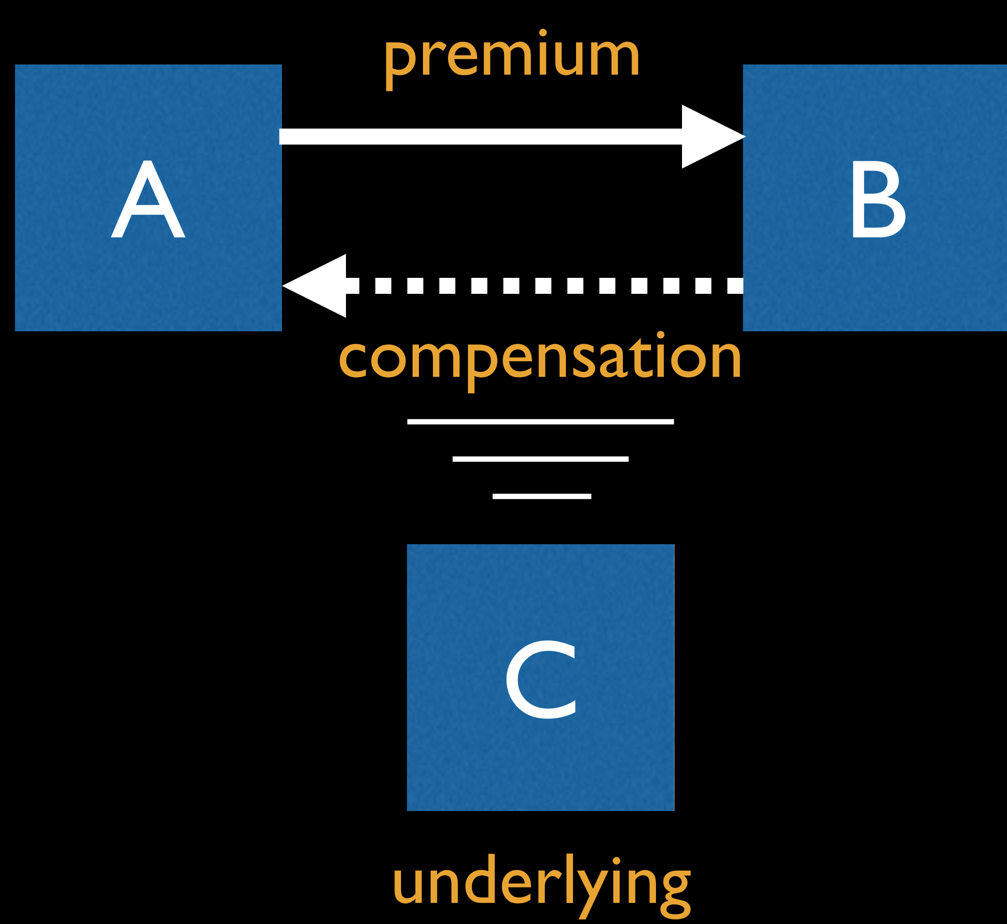
In a typical CDS with duration of five years (T=5), counterparty A pays B a series of premium payments at regular intervals (every 3M) upon an agreed notional (N).



Definition

The payments from A to B will be made as long as underlying C doesn't default (i.e. survives).

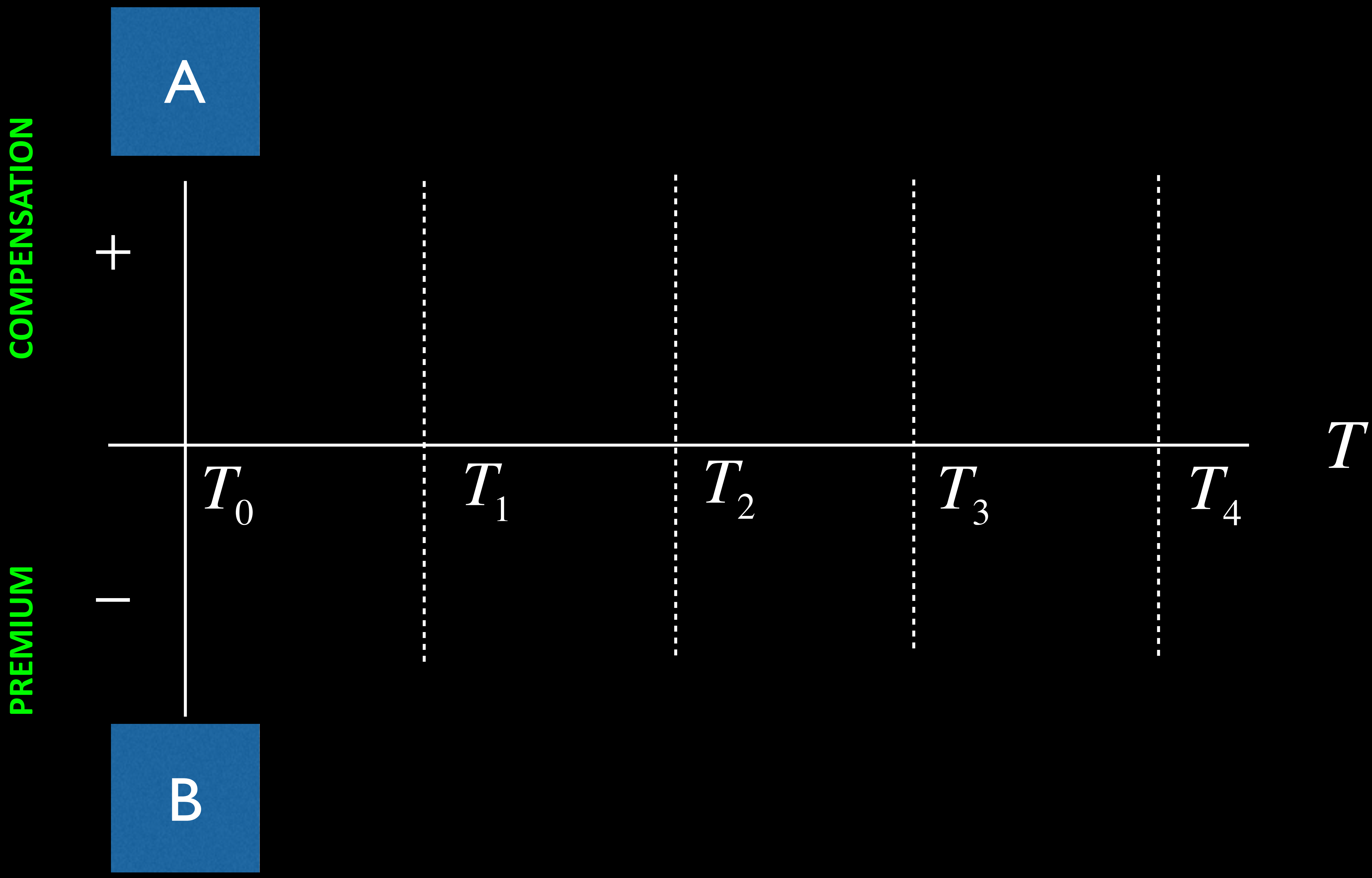
When default happens a single protection payment is made from B to A, and the contract ends.



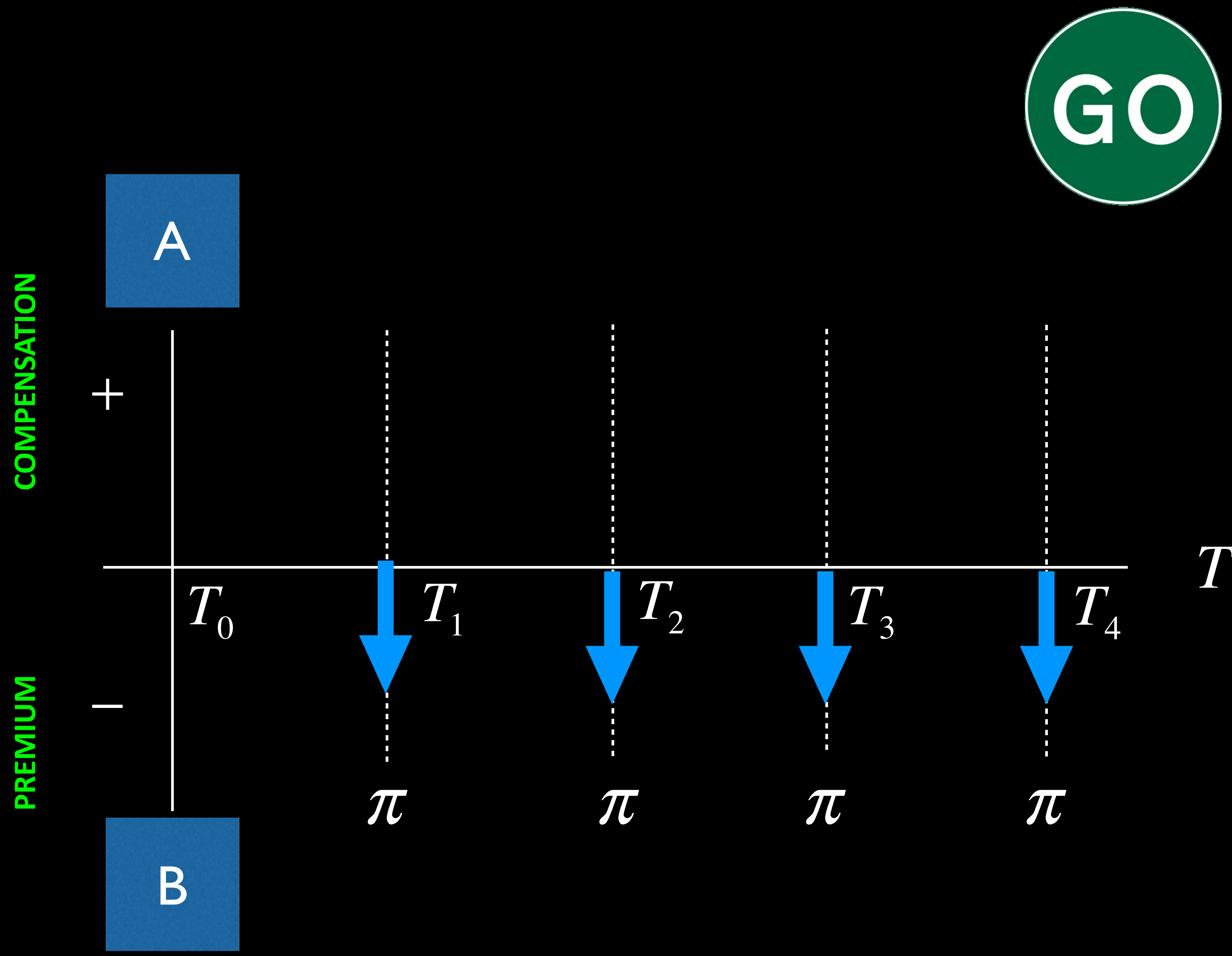
To default or not default, that is the question

Scenarios

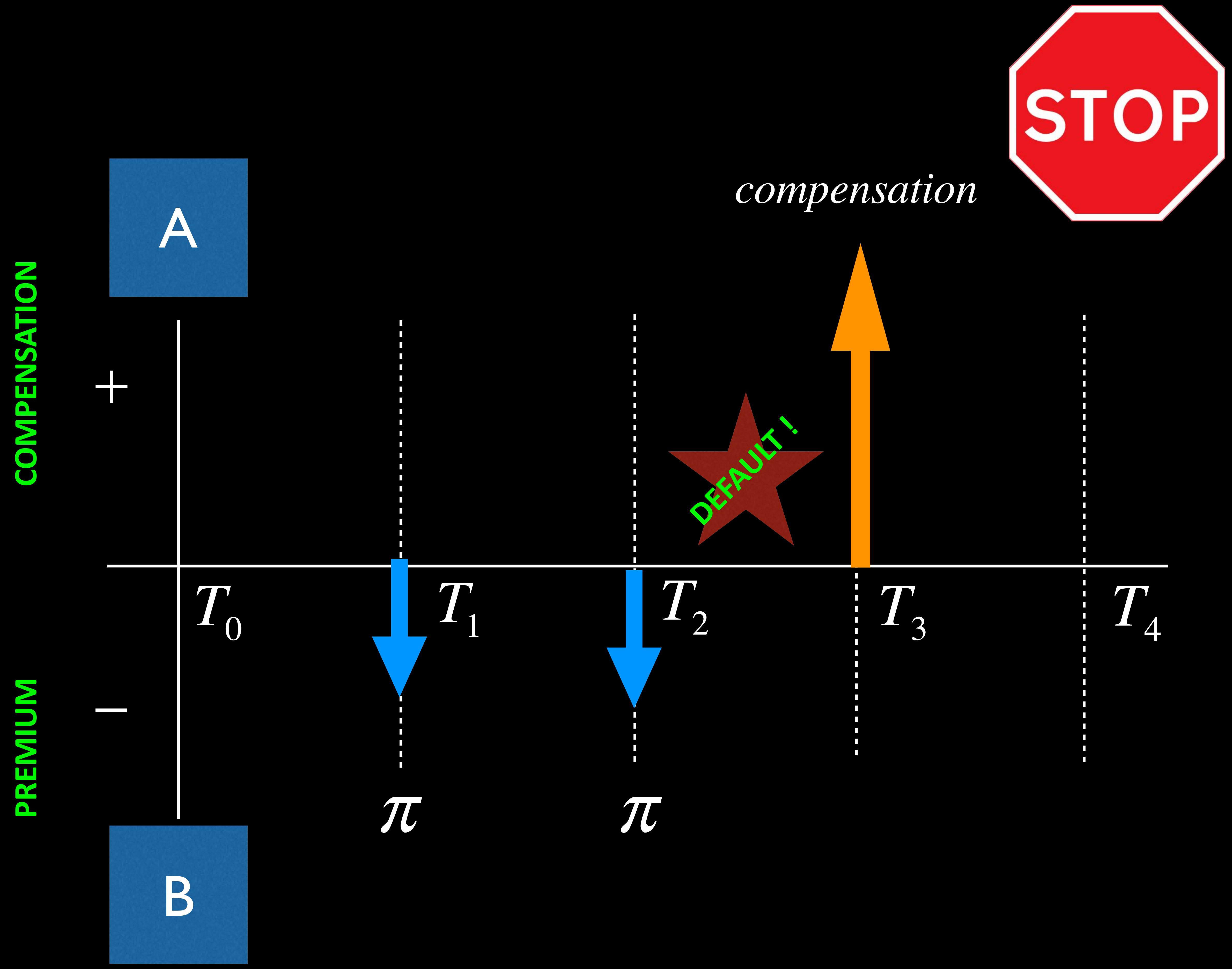
credit default swap



scenario = no default


GO
A
COMPENSATION
+
PREMIUM
−
B
T
T_0
T_1
T_2
T_3
T_4
π
π
π
π

scenario = default


STOP
compensation
A
COMPENSATION
+
DEFAULT !
T
T_0
T_1
T_2
T_3
T_4
PREMIUM
−
π
π
B

CDS Applications

- Hedging

- Speculation

- Arbitrage

CDS Applications: Hedging

The holder of a corporate bond may hedge their exposure by entering into a CDS contract as the buyer of protection. If the bond goes into default, the proceeds from the CDS contract will cancel out the losses on the underlying bond.

CDS Applications: Speculation

CDS allow investors to speculate on changes in credit spreads. An investor might believe that an entity's CDS spreads are either too high or too low relative to the entity's bond yields and attempt to profit from that view by entering into a trade, known as a basis trade, that combines a CDS with a cash bond and an interest rate swap.

Also, an investor might speculate on an entity's credit quality, since generally CDS spreads will increase as credit-worthiness declines, and decline as credit-worthiness increases. The investor might therefore buy CDS protection on a company in order to speculate that the company is about to default.

Alternatively, the investor might sell protection if they think that the company's creditworthiness might improve.

CDS Applications: Arbitrage

Capital Structure Arbitrage (CSA) is an example of an arbitrage strategy which utilises CDS transactions. This technique relies on the fact that a company's stock price and its CDS spread should exhibit negative correlation; i.e. if the outlook for a company improves then its share price should go up and its CDS spread should tighten, since it is less likely to default on its debt. However if its outlook worsens then its CDS spread should widen and its stock price should fall.

Techniques reliant on this are known as capital structure arbitrage because they exploit market inefficiencies between different parts of the same company's capital structure; i.e. mis-pricings between a company's debt and equity.

Part 2
Default Modelling Toolkit

2.1 Mathematical Setup

2.2 Stopping Times

2.3 Poisson Processes

2.4 Inhomogeneous Poisson Processes

Setup

All processes and random variables we introduce are defined on a complete filtered probability space $(\Omega, (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$, where Ω is the set of possible states of nature, the filtration $(\mathcal{F}_t)_{t\geq 0}$ represents the information structure of the setup and $\mathbb{P}$ is the probability measure that attaches probabilities to the events in Ω.

Stopping times

To model the *arrival* of a credit event we need to model an unknown random point in time $\tau \in \mathbb{R}_+$.

For default risk modeling we use the *default indicator function* (the indicator function of the default event) and *survival indicator function* (one minus the default indicator function).

Survival indicator function

$$I(t) = 1_{\{\tau > T\}} = \begin{cases} 1 \; if \tau > nT \\ 0 \; if \tau \leq T \end{cases}$$

Poisson Processes

Poisson processes are usually used to model rare events and discretely countable events such as defaults. Usually one models the time of default of a firm as *the time of the first jump of a Poisson process*. The parameter λ in the construction of the Poisson process is called the *intensity* of the process.

- The Poisson process has no memory.

- The inter-arrival times of a Poisson process $(\tau_{n+1} - \tau_n)$ are exponentially distributed

- Two or more jumps at exactly the same time have probability zero.

Poisson Processes

In this framework we can model the arrival of default as the first jump of a Poisson process with intensity λ. Its associated survival probability $P(t,T)$ can be obtained as:

$$P(t,T) = exp\left(-\lambda(T-t)\right)$$

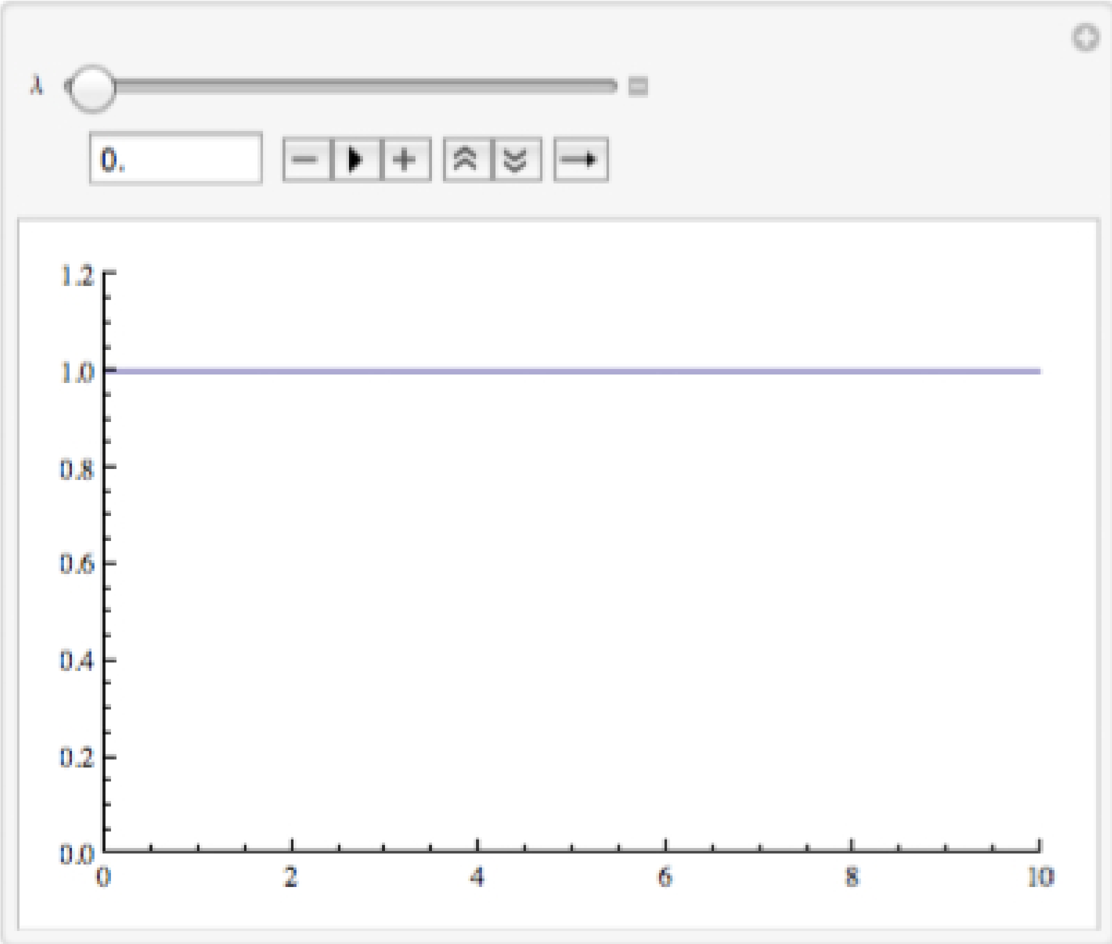
Note that the intensity λ is constant, i.e. does not depend on time.

Inhomogeneous Poisson Processes

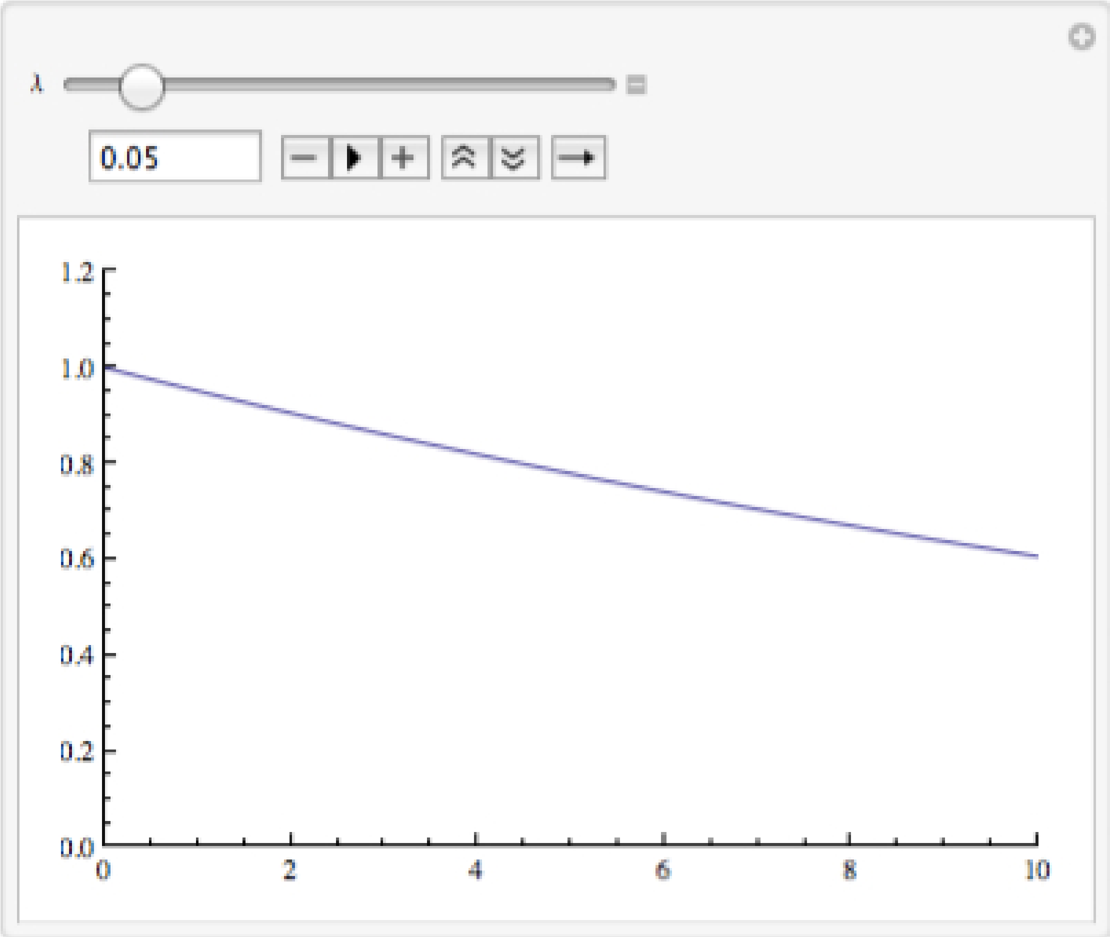
If the intensity $\lambda(t)$ is a non-negative function of time, we can derive the survival probability for an inhomogeneous Poisson process as:

$$P(t,T) = exp\left(-\int_t^T \lambda(s)ds\right)$$

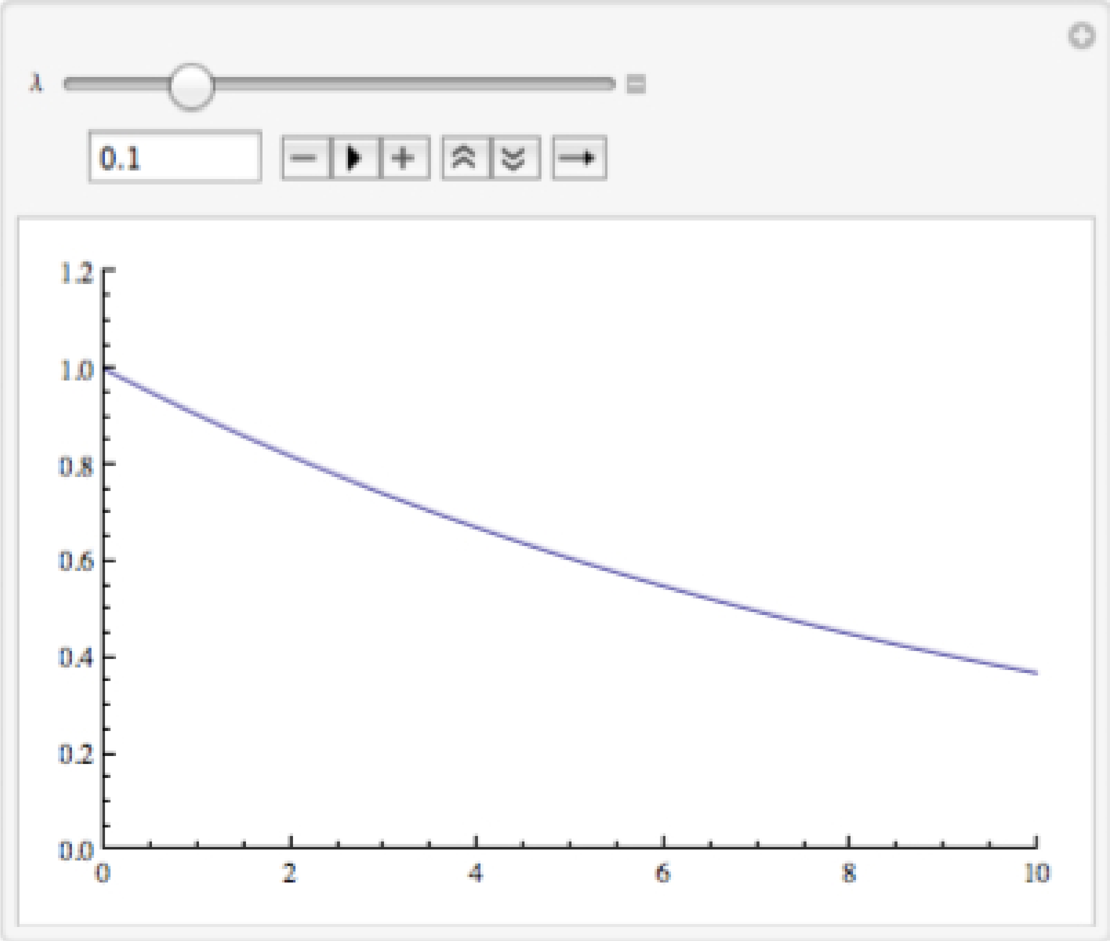
Now the default hazard rate depends on the time horizon T: the term structure of hazard rates is not flat, but given by $\lambda(T)$, so we can reach every term structure we desire so as to match the market exactly.



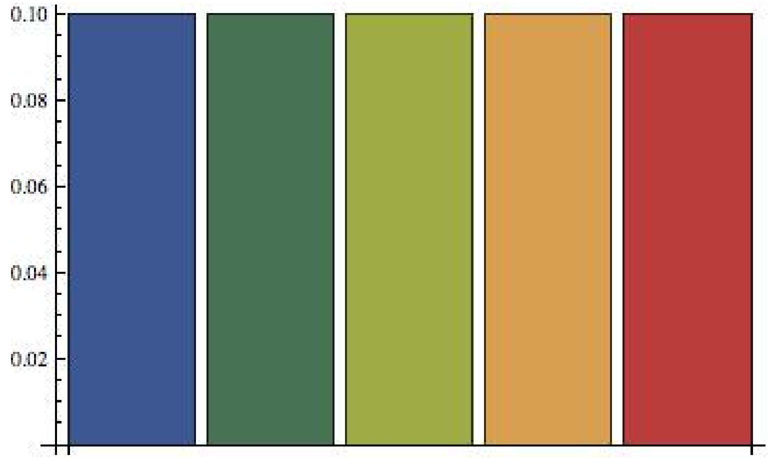
Survival probability $P(t,T) = exp(-\lambda T)$ with hazard rate $\lambda = 0\%$

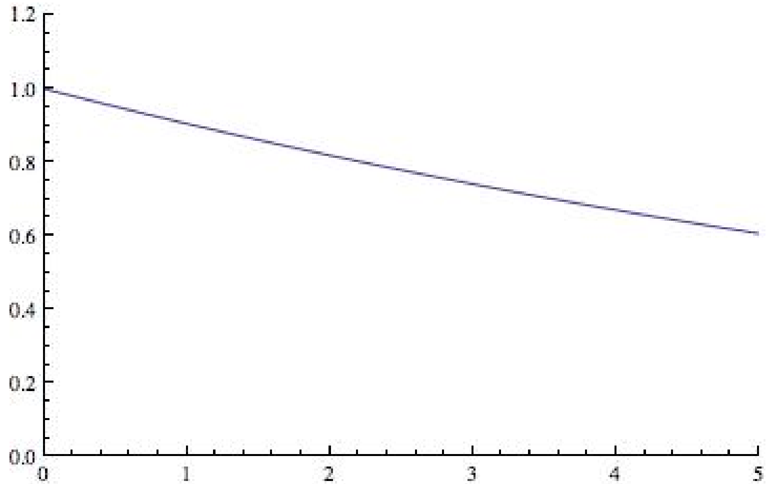


Survival probability $P(t,T) = exp(-\lambda T)$ with hazard rate $\lambda = 5\%$

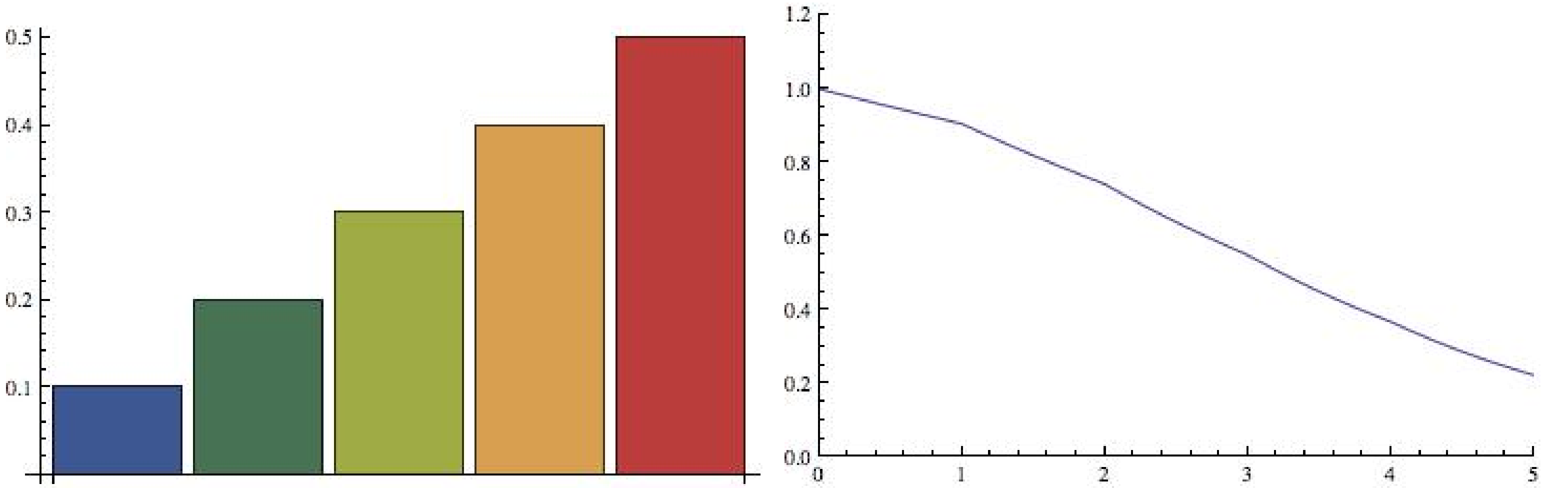


Survival probability $P(t,T) = exp(-\lambda T)$ with hazard rate $\lambda = 10\%$

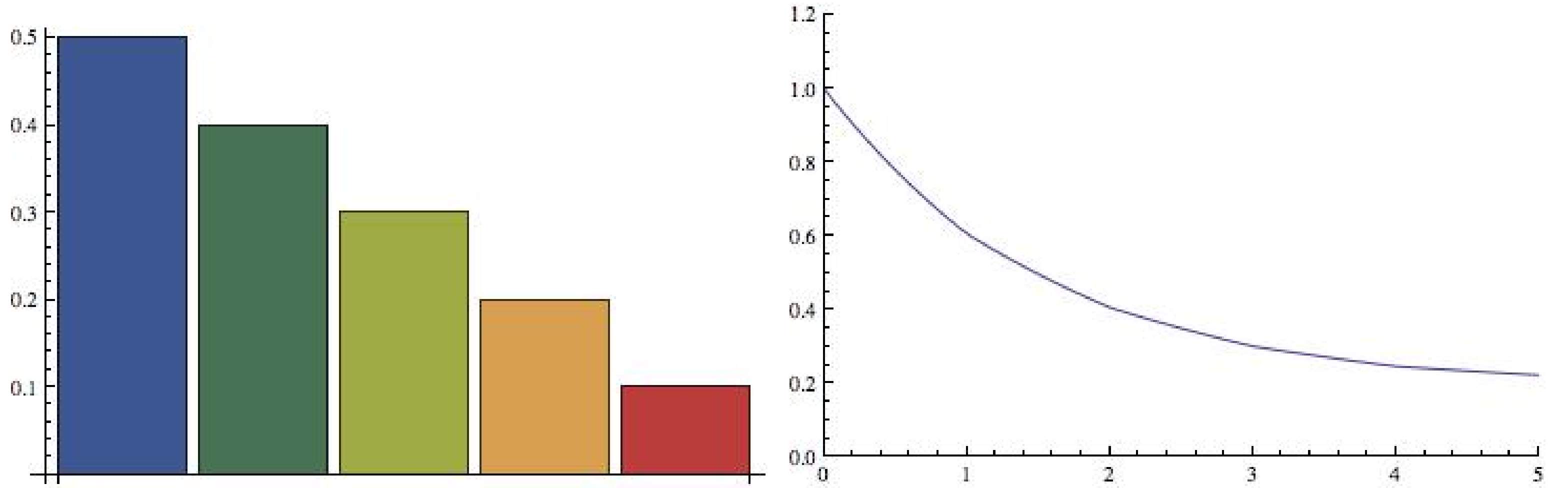




FLAT TERM STRUCTURE Term structure of hazard rates $\lambda(t)$ (*left*) and its associated survival probability $P(t,T)$ (*right*).



INCREASING TERM STRUCTURE Term structure of hazard rates $\lambda(t)$ (*left*) and its associated survival probability $P(t,T)$ (*right*).



DECREASING TERM STRUCTURE Term structure of hazard rates $\lambda(t)$ (*left*) and its associated survival probability $P(t, T)$ (*right*).

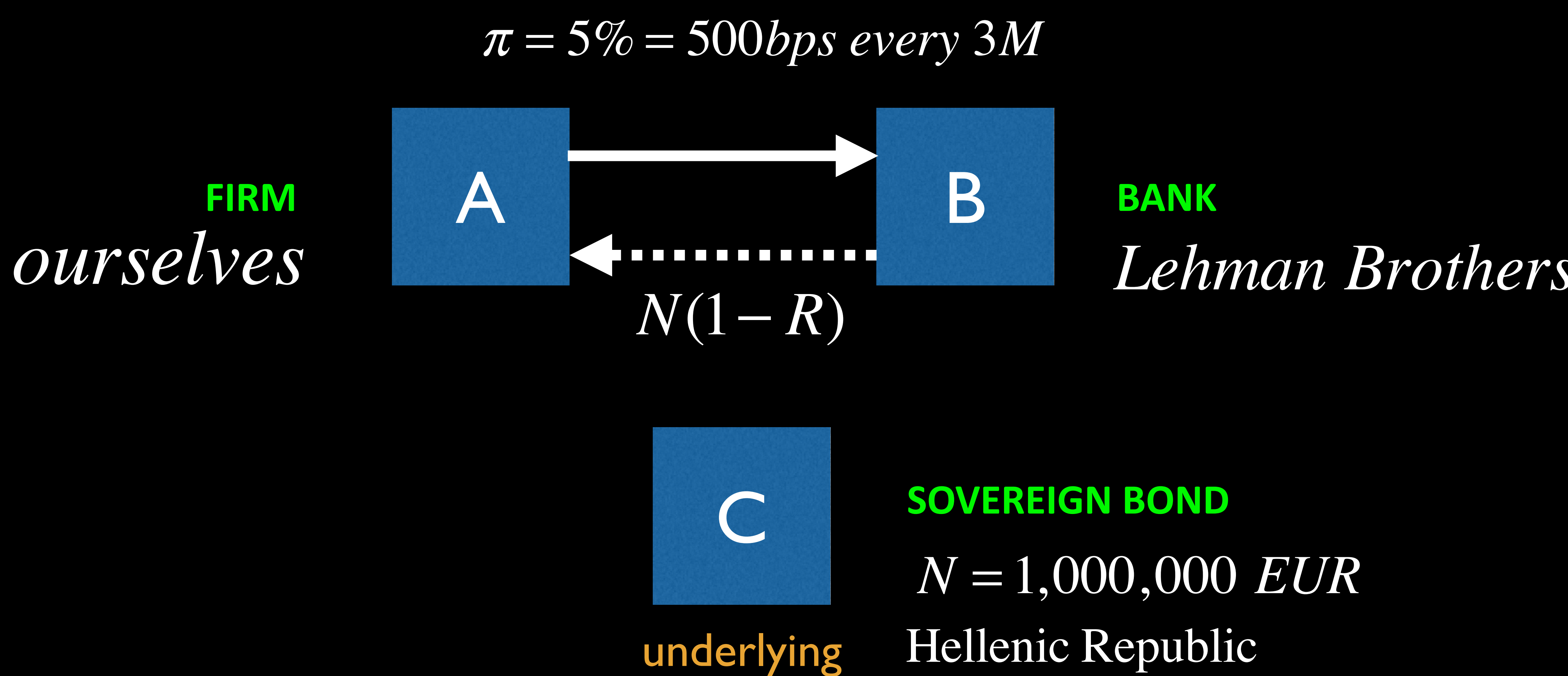
Part 3

CDS Pricing: Basic Model

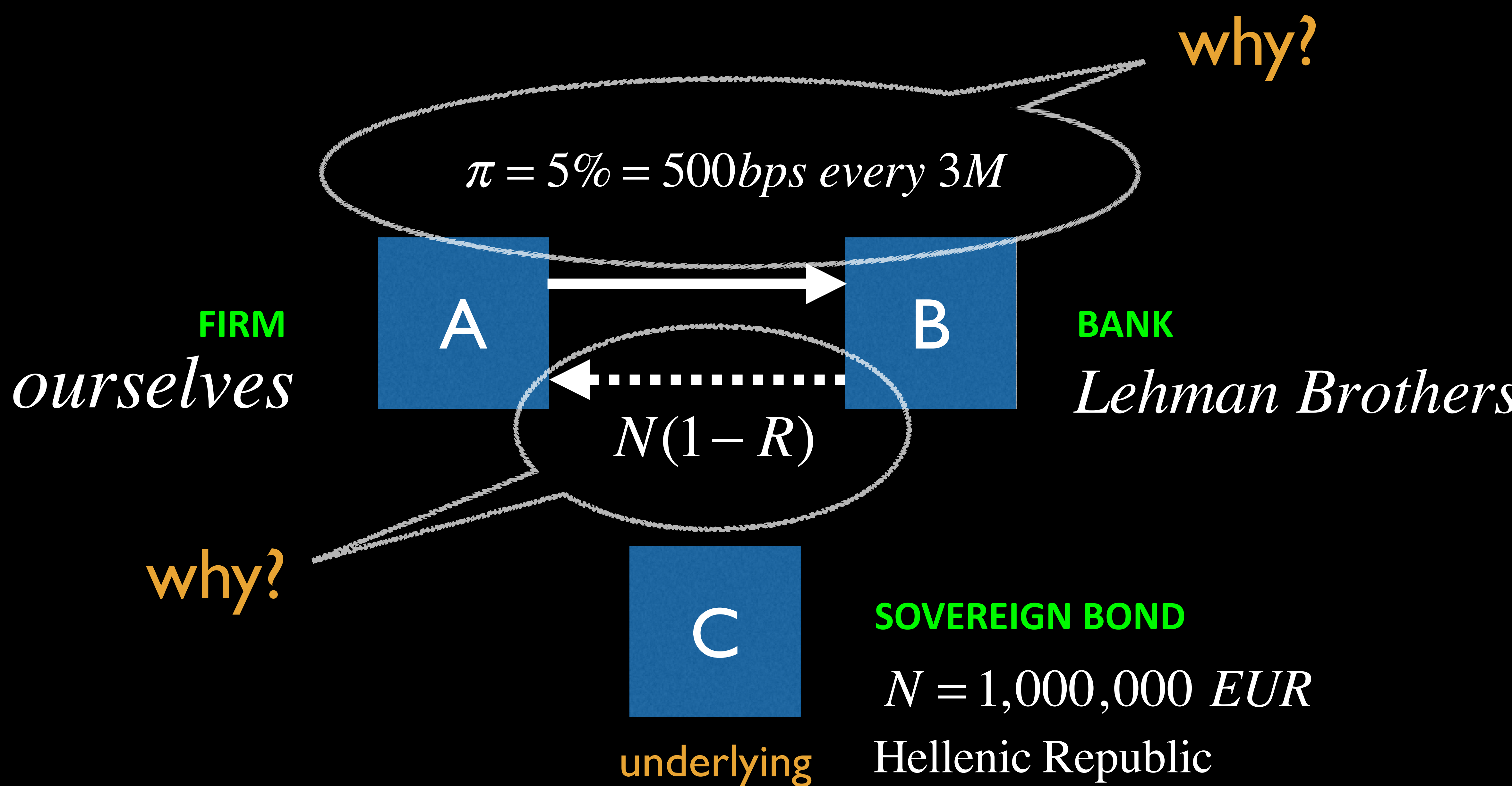
Pricing a CDS

π

EXAMPLE: CDS contract duration is one year (T=1), quarterly payments (@3M), notional N= 1 million EUR.

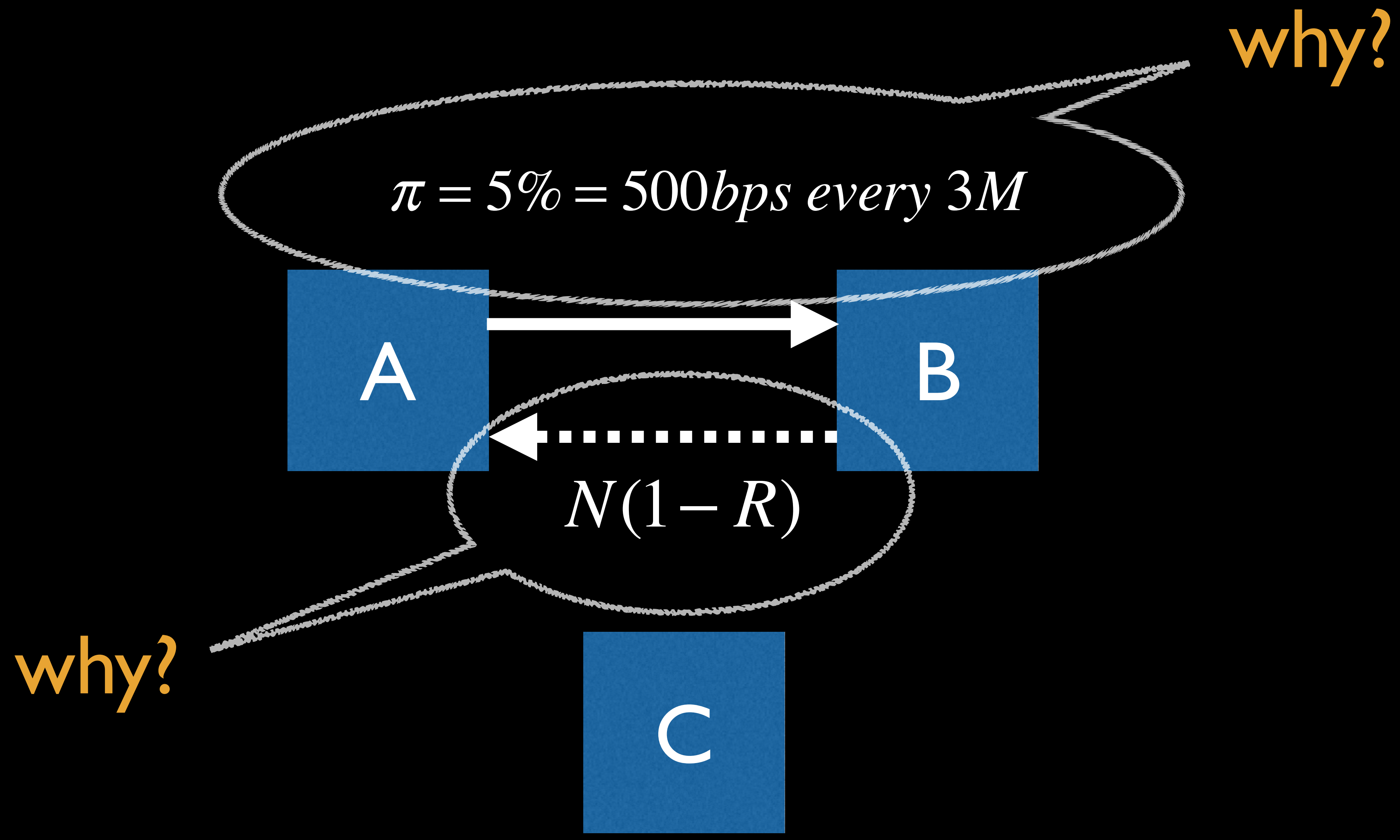


EXAMPLE: CDS contract duration is one year (T=1), quarterly payments (@3M), notional N= 1 million EUR.



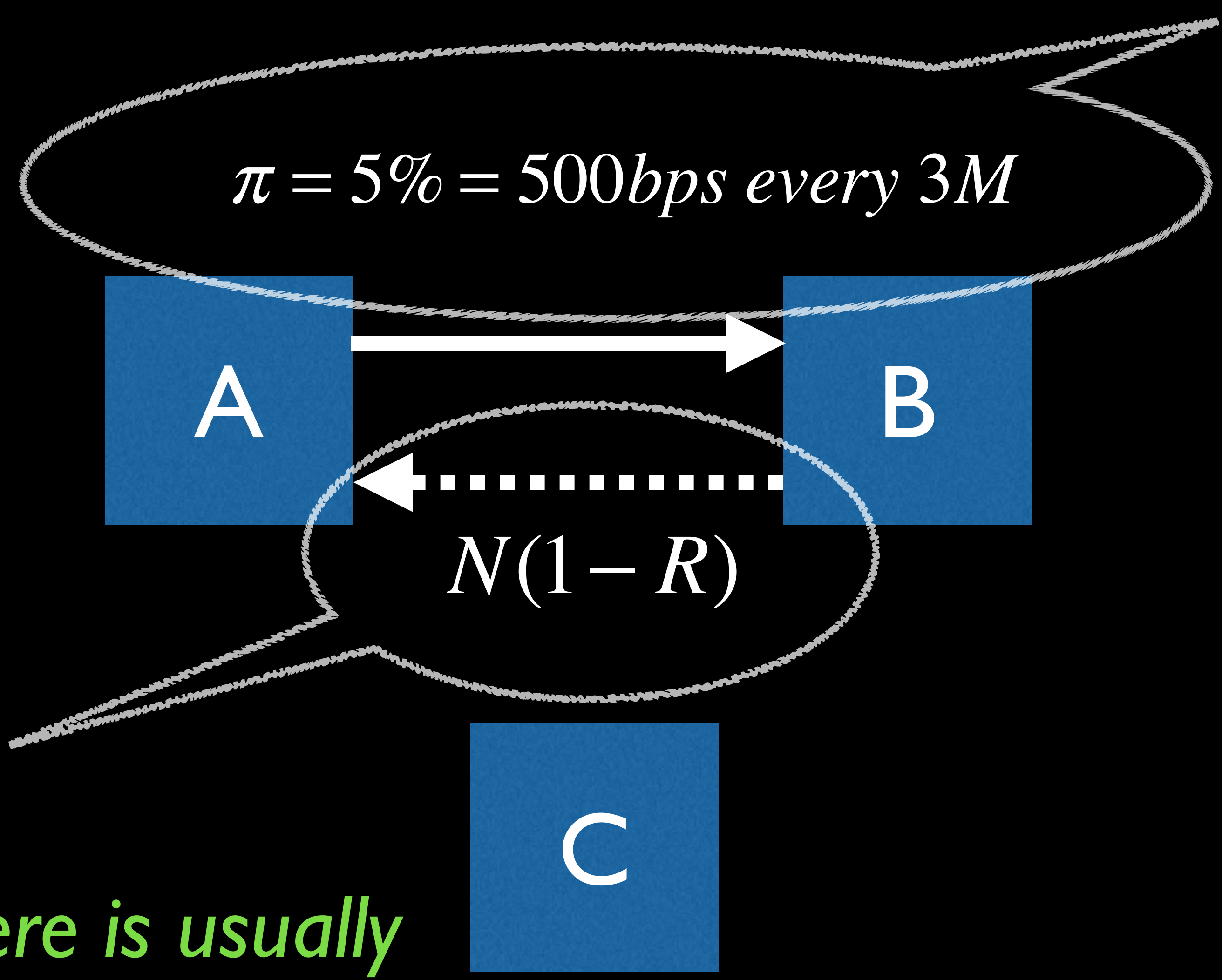
Pricing

credit default swap



the premium is the price of the CDS,
should depend on the risk of default of C…

why?



why?

upon default there is usually
some recovery rate (R)…

Pricing

In summary to determine the premium (price) of a CDS contract we need:

1. maturity (T)
2. notional (N)
3. frequency payments (dt)
4. recovery rate (R)
5. interest rates (r)
6. the "risk of default" of C

how?

Q: how do we model the risk of default of C ?

Pricing

A: The risk of default is modelled using the survival probability (P) and its complement the probability of default (PD).

$$P(t) = \exp(-\lambda \times t)$$

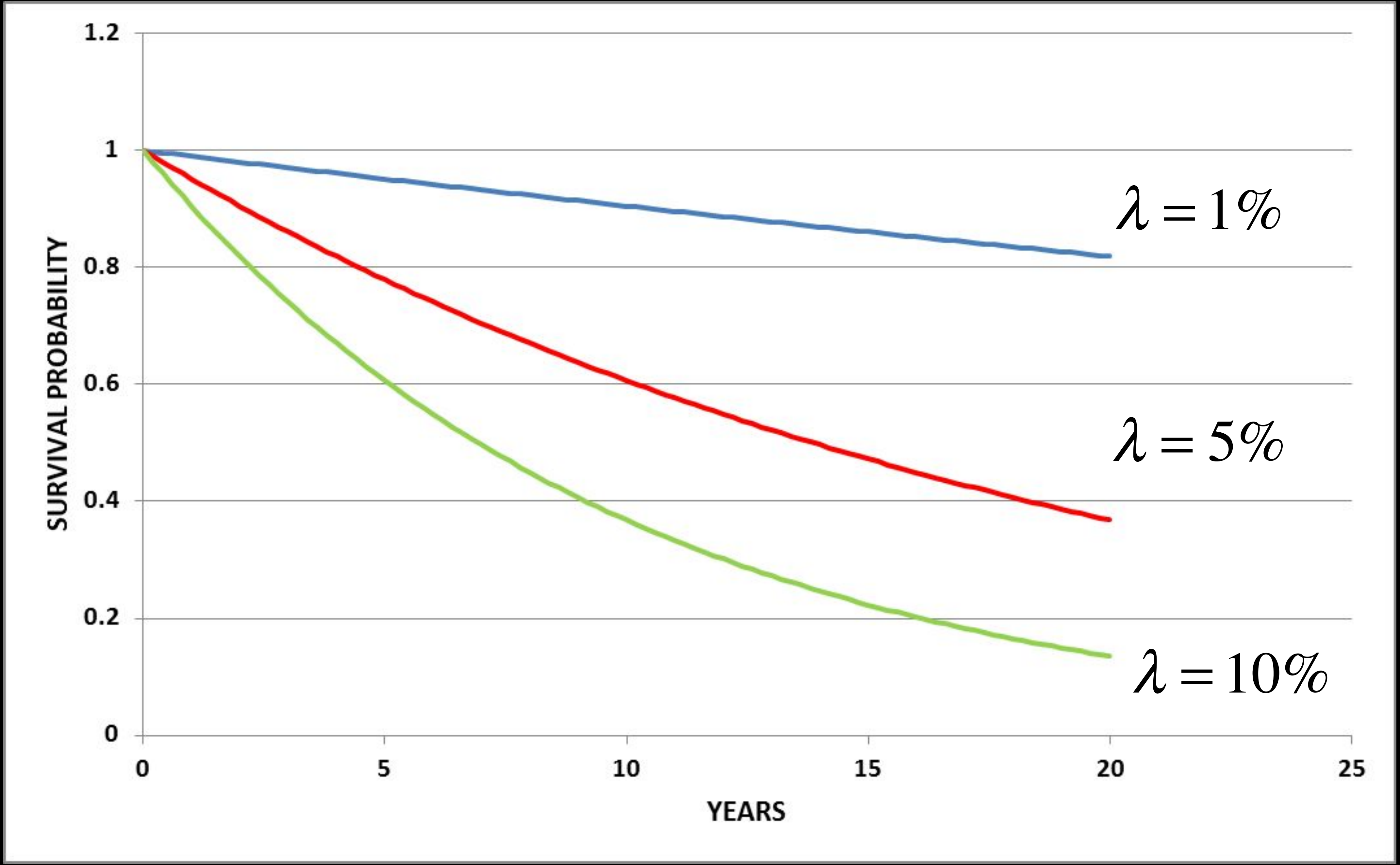
survival probability
(function of time)

hazard rate

time

Pricing

$$P(t) = \exp(-\lambda \times t)$$



Pricing

We are going to use the survival probability to estimate the likelihood of the various cashflows in the CDS.

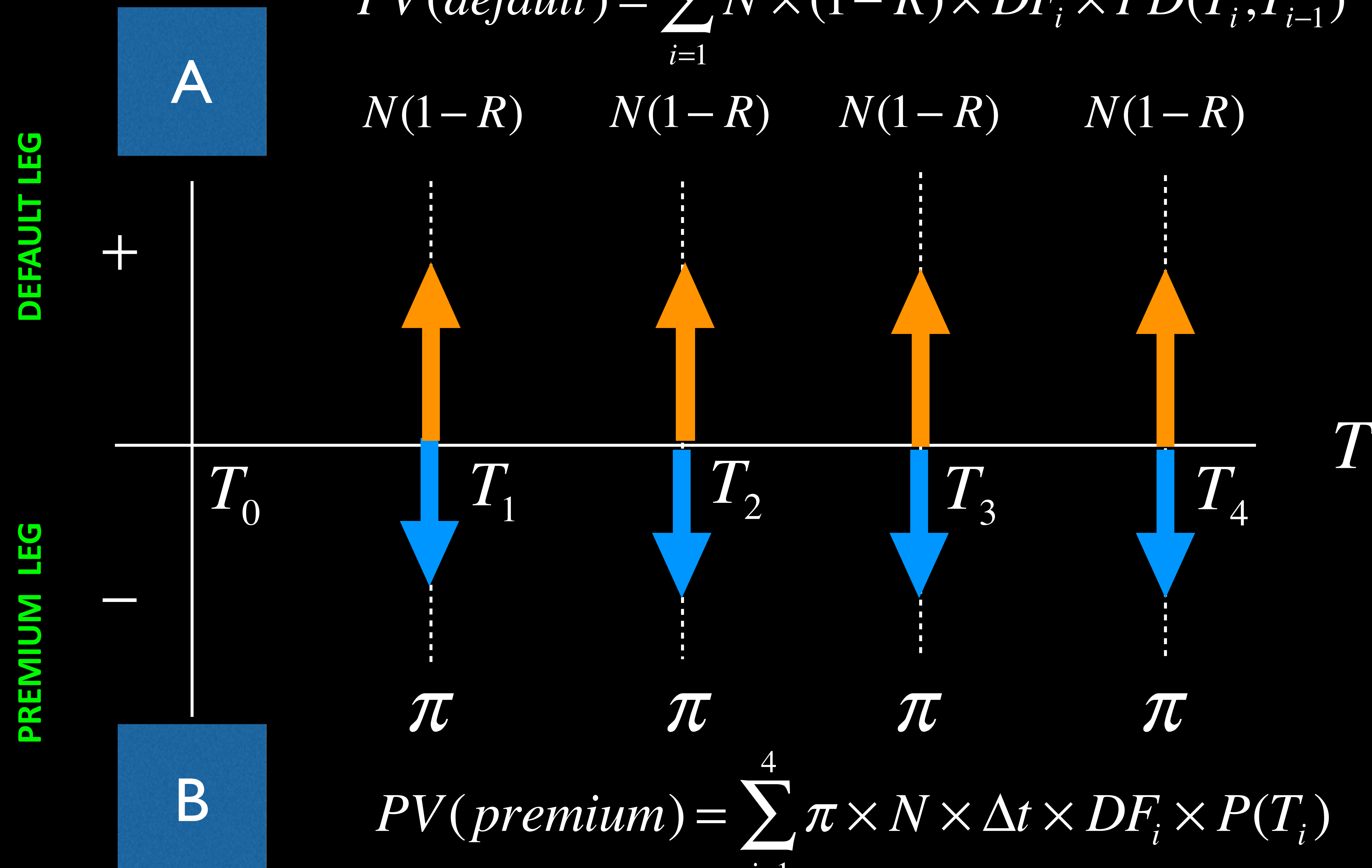
For the premium payments ->
P(t): survival

For the compensation (default) payment(s) ->
PD(t): probability of default

Pricing

credit default swap

$$PV(default)=\sum_{i=1}^{4} N\times(1-R)\times DF_i \times PD(T_i,T_{i-1})$$



$$PV(premium)=\sum_{i=1}^{4} \pi\times N\times \Delta t\times DF_i \times P(T_i)$$

Pricing

From the point of view of counterpart A:

A

THIS IS RECEIVED… THUS POSITIVE…

$$PV(default) = \sum_{i=1}^{4} N \times (1-R) \times DF_i \times PD(T_i, T_{i-1})$$

THIS IS PAID… THUS NEGATIVE…

$$PV(premium) = \sum_{i=1}^{4} \pi \times N \times \Delta t \times DF_i \times P(T_i)$$

THE MARK TO MARKET IS THE SUM…

$$MTM = PV(default) - PV(premium)$$

Pricing

for fair pricing the MTM=0, thus

$$0 = PV(default) - PV(premium)$$

WE CAN EQUATE THE LEGS…

$$PV(premium) = PV(default)$$

$$\sum_{i=1}^{4} \pi \times N \times \Delta t \times DF_i \times P(T_i) = \sum_{i=1}^{4} N \times (1-R) \times DF_i \times PD(T_i, T_{i-1})$$

AND FINALLY ISOLATE THE PREMIUM…

$$\pi = \frac{\sum_{i=1}^{4} N \times (1-R) \times DF_i \times PD(T_i, T_{i-1})}{\sum_{i=1}^{4} N \times \Delta t \times DF_i \times P(T_i)}$$

Pricing

in terms of the survival probability only, the fair price (premium) of the CDS is:

$$\pi = \frac{\sum_{i=1}^{4} (1-R) \times DF_i \times \left[P(T_{i-1}) - P(T_i) \right]}{\sum_{i=1}^{4} \Delta t \times DF_i \times P(T_i)}$$

Mathematical Setup

In this section we provide a simplified pricing model for credit default swaps (a.k.a. in the literature as "Standard Model" or JP Morgan Approach).

Let us suppose that there are N periods, indexed by $n = 1, \ldots, N$. Without loss of generality, each period is of length Δt, expressed in units of years. Thus, time intervals are:

$$\{(0, \Delta t), (\Delta t, 2\Delta t), \ldots, ((N-1)\Delta t, N\Delta t)\}$$

The corresponding end of period maturities are:

$$T_n = n\Delta t$$

.

Mathematical Setup

Risk free forward interest rates are denoted $r((n-1)\Delta t, n\Delta t) \equiv r(T_{n-1}, T_n)$, i.e. the rate over the n^{th} period. We write these one-period forward rates in short form as r_n, as the forward rate applicable to the n^{th} time interval. The discount factors may be written as functions of forward rates, i.e.

$$D(0, T_n) = \exp\left(-\sum_{k=1}^{n} r_k \Delta t\right).$$

Mathematical Setup

For a given obligor, we suppose that default is likely with an hazard rate $\lambda_n \equiv \lambda(T_{n-1}, T_n)$, constant over forward period n. Given these default intensities, the survival function of the obligor is defined as

$$P(T_n) = \exp\left(-\sum_{k=1}^{n} \lambda_k \Delta t \right),$$

assuming that at time zero the obligor is solvent, i.e.

$$P(T_0) = P(0) = 1$$

Mathematical Setup

In our framework, the buyer of the CDS (**A**) purchases credit protection against the default of the reference security and, in return, pays a periodic payment to the seller (**B**) (*Premium leg*). These periodic payments continue until maturity or until the reference instrument default, in which event the seller pays to the buyer the loss on default of the reference security (*Default leg*).

CDS Pricing: Premium Leg

We denote the $N-$period CDS spread as S_N, stated as an annualized percentage of the nominal value of the contract. Without loss of generality, we set the nominal value to 1 €. We assume that defaults occur only at the end of the period, so the premiums will be paid until the end of the period. Since the premium payments are made as long as the reference security survives, the expected present value of the premium leg (PL_N) is:

$$PL_N = S_N \sum_{n=1}^{N} D(0, T_n)P(T_n)\,(\Delta t_n)$$

where Δ_n is the year fraction corresponding to $T_{n-1} - T_n$ and $P(T_n)$ is the survival probability up to time T_n. This accounts for the expected present value of payments made from the buyer **A** to the seller **B**.

CDS Pricing: Default Leg

The other possible payment of the CDS arises in the event of default, and goes from the seller to the buyer. The expected present value of this payment depends on the recovery rate in the event of default, which we denote as R. The loss payment on default is then equal to $(1-R)$ for every 1 € of notional principal. This implicitly assumes that the recovery of par convention is used.

CDS Pricing: Default Leg

The expected loss payment in period n is based on the probability of default in period n, conditional on no default in a prior period. This probability is given by the probability of surviving until period $n-1$ and then defaulting in period n.

Therefore, the expected present value of loss payments (DL_N) is:

$$DL_N = (1-R)\sum_{n=1}^{N} D(0,T_n)\left(P(T_{n-1}) - P(T_n)\right)$$

CDS Pricing: Fair Spread

The fair pricing of the $N-$period CDS, i.e. the fair quote of the spread S_N, must be such that the expected present value of payments made by buyer and seller are equal, i.e. $PL_N = DL_N$. Thus we obtain

$$S_N = \frac{(1-R)\sum_{n=1}^{N} D(0,T_n)\left(P(T_{n-1}) - P(T_n)\right)}{\sum_{n=1}^{N} D(0,T_n)P(T_n)\left(\Delta t_n\right)}.$$

Bootstrapping Hazard Rates

In the previous equations the spread S_N and the discount factors $D(0, T_n)$ are observable in the default risk and government bond markets, respectively. However, the survival probabilities $P(T_n)$ are not directly observed and need to be inferred from the observable variables.

Bootstrapping Hazard Rates

Since there are N periods, we may use N CDSs of increasing maturity, each with spread S_n, and impose $PL_n = DL_n$, $n = 1, \ldots, N$. Thus we have N equations with as many unknowns, which can be identified in a recursive manner using bootstrapping.

FIRST STEP $(N = 1)$

Starting with the one-period $(N = 1)$ CDS with a spread S_1 per annum, for T_1 we equate payments on the swap as follows:

$$
\begin{aligned}
PL_1 &= DL_1 \\
S_1 D(0,T_1) P(T_1) \Delta t_1 &= (1-R) D(0,T_1)(P(T_0) - P(T_1)) \\
&\dots \\
D(0,T_1) P(T_1)(S_1 \Delta t_1 + L) &= L D(0,T_1) P(T_0) \\
&\dots
\end{aligned}
$$

after some algebra we obtain...

an identification the survival probability $P(T_1)$ for the first period, i.e.

$$P(T_1) = \frac{L}{L + \Delta t_1 S_1}$$

where $L = (1 - R)$ and using the fact that $P(T_0) = 1$.

SECOND STEP $(N = 2)$

We now use the 2−period CDS to extract the survival probability for the second period, whose spread is denoted as S_2. We set $PL_2 = DL_2$ and obtain the following equation which can be solved algebraically for $P(T_2)$ as:

$$P(T_2) = \frac{D(0,T_1)\,[L(1) - (L + \Delta t_1 S_2)P(T_1)]}{D(0,T_2)(L + \Delta t_2 S_2)} + \frac{P(T_1)L}{L + \Delta t_2 S_2}$$

SUBSEQUENT STEPS ($n = N$)

In general, we can write down the expression for $P(T_N)$ as

$$P(T_N) = \frac{\sum_{n=1}^{N-1} D(0,T_n)\left[LP(T_{n-1}) - (L + \Delta t_n S_N)P(T_n)\right]}{D(0,T_N)(L + \Delta t_n S_N)}$$

$$+\frac{P(T_{N-1})L}{(L + \Delta t_N S_N)}.$$

Thus, we begin with $P(T_1)$ and, through a process of bootstrapping, we arrive at all $P(T_n)$, $n = 1, \ldots, N$.

Part 4

CDS Pricing: Examples

Example 1: Hedging

A pension fund owns \$10 million of a five-year bond issued by General Motors. In order to manage the risk of losing money if General Motors defaults on its debt, the pension fund buys a CDS from Bank of America in a notional amount of \$10 million. The CDS trades at 200 basis points. In return for this credit protection, the pension fund pays 2% of 10 million (\$200,000) pa in quarterly installments of \$50,000 to Bank of America.

Example 2: Speculation

A hedge fund believes that General Motors will soon default on its debt. Therefore it buys $10 million worth of CDS protection for 2 years from Bank of America, with General Motors as the reference entity, at a spread of 500 bps pa.

Scenario 1: No Default if General Motors does not default, then the CDS contract will run for 2 years, and the hedge fund will have ended up paying $1 million, without any return, thereby making a loss.

Scenario 2: Default if General Motors does indeed default after, say, one year, then the hedge fund will have paid $500,000 to Bank of America, but will then receive $10 million (assuming zero recovery rate, and that Bank of America has the liquidity to cover the loss), thereby making a tidy profit. Bank of America, and its investors, will incur a $9.5 million loss unless the bank has somehow offset the position before the default.

Example 3: Arbitrage

Consider a company which has announced some bad news and its share price has dropped by 25%, but its CDS spread has remained unchanged, then an investor might expect the CDS spread to increase relative to the share price.

Therefore a basic strategy would be to go long on the CDS spread (by buying CDS protection) while simultaneously hedging oneself by buying the underlying stock.

This technique would benefit in the event of the CDS spread widening relative to the equity price, but would lose money if the company's CDS spread tightened relative to its equity.

Part 5

CDS Pricing: Other Models

5.1 Basic Model with Accruals

5.2 ISDA Standard Model

5.3 Upfront CDS

5.1 Basic Model WITH accruals

An extended formulation of the Basic CDS pricing model (JP Morgan) can be found *Par Credit Default Swap Approximation from Default Probabilities* published by JP Morgan.

In this methodology payments can occur anytime (not only on payment dates) and takes into account payment accruals in the premium leg.

5.1 Basic Model WITH accruals

The PV of the premium leg is now

$$PL_N = S_N \sum_{n=1}^{N} D(0,T_n)P(T_n)\,(\Delta t_n) \\ + S_N \sum_{n=1}^{N} D(0,T_n)\,(P(T_{n-1}) - P(T_n))\,\frac{(\Delta t_n)}{2}$$

which includes an extra payment accrual term.

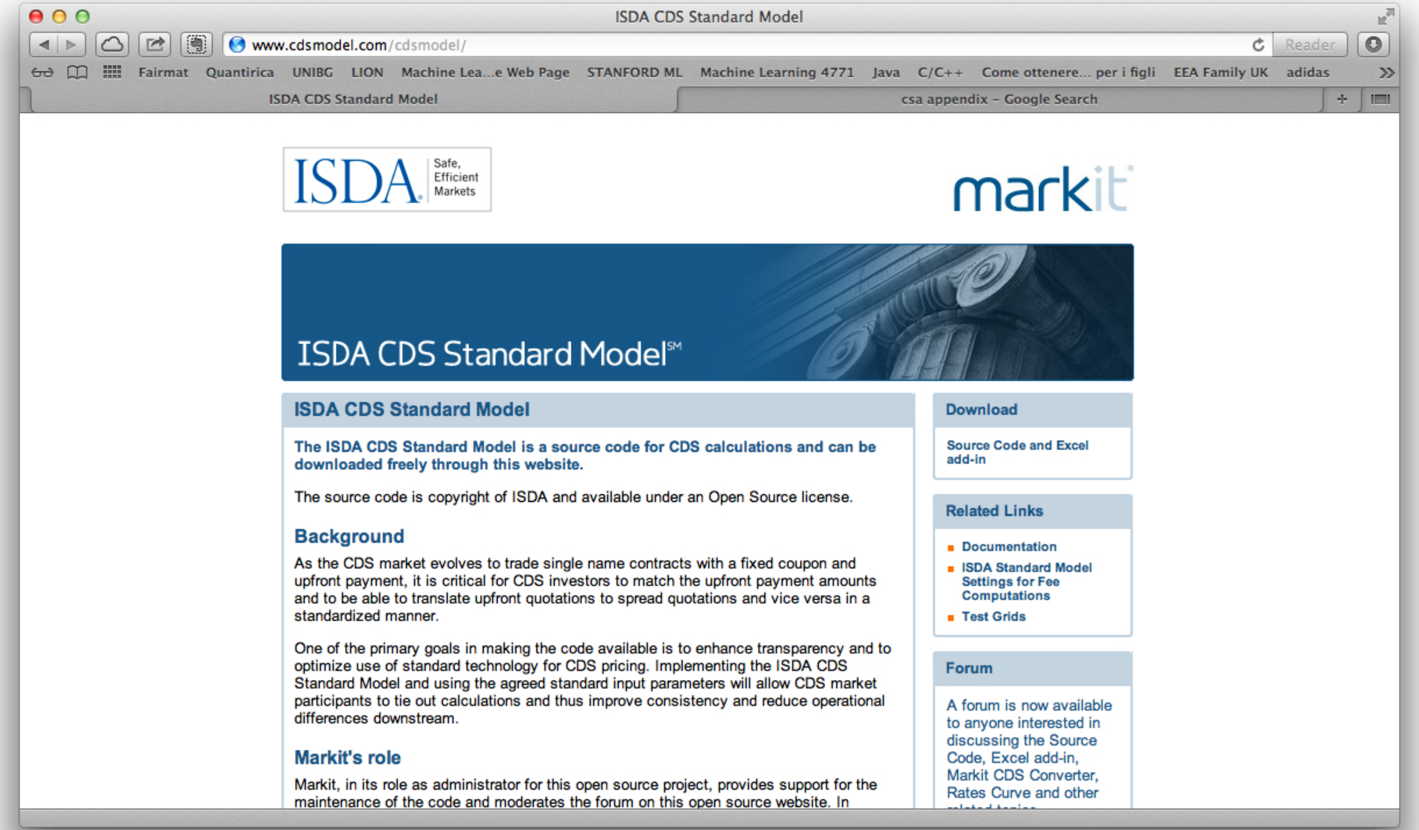
5.1 Basic Model WITH accruals

The PV of the default leg is the same as before.

Therefore the par credit default swap spread S_N is given by

$$S_N = \frac{(1-R)\sum_{n=1}^{N} D(0,T_n)\left(P(T_{n-1}) - P(T_n)\right)}{\sum_{n=1}^{N} D(0,T_n)P(T_n)\left(\Delta t_n\right) + D(0,T_n)\left(P(T_{n-1}) - P(T_n)\right)\frac{(\Delta t_n)}{2}}$$

You can find these formulas implemented in the accompanying Excel Spreadsheet *CDSJPMExample.xls* , available from the course website.



CDS Standard Model: http://www.cdsmodel.com/cdsmodel/

5.2 ISDA "Standard" Model

- The International Swaps and Derivatives Association (ISDA) CDS Standard Model is a source code for CDS calculations.

- Can be downloaded freely through ISDA website (XLL, C++).

- The source code is copyright of ISDA and available under an Open Source license.

Source: www.isda.org

ISDA Safe, Efficient Markets

NEWS RELEASE
For Immediate Release

ISDA Leads Industry Effort to Standardize the Credit Support Annex

NEW YORK, Thursday, November 3, 2011 – The International Swaps and Derivatives Association, Inc. (ISDA) today outlined key provisions to the Standard Credit Support Annex (SCSA) proposal as part of its continuing efforts to increase efficiency and improve standardization in the over-the-counter (OTC) derivatives markets.

The SCSA proposal addresses three primary objectives. The SCSA seeks to standardize market practices by removing embedded optionality in the existing CSA, promote the adoption of overnight index swap (OIS) discounting for derivatives, and align the mechanics and economics of collateralization between the bilateral and cleared OTC derivative markets. In addition, the SCSA seeks to create a homogeneous valuation framework, reducing current barriers to novation and valuation disputes.

"ISDA will continue to lead standardization initiatives in an effort to make global derivatives

Credit Support Annex: http://www2.isda.org

5.3 Upfront CDS

The PV of the premium leg in this case is now

$$
\begin{aligned}
PL_N = UF(0)P(T_0) + S_{fixed} \sum_{n=1}^{N} D(0, T_n)P(T_n)\left(\Delta t_n\right) \\
+S_{fixed} \sum_{n=1}^{N} D(0, T_n)\left(P(T_{n-1}) - P(T_n)\right)\frac{(\Delta t_n)}{2}
\end{aligned}
$$

where $UF(0)$ corresponds to the upfront paid at time T_0 and S_{fixed} is the fixed coupon.

5.3 Upfront CDS

The PV of the default leg is the same as before.

Therefore the par credit default swap spread S_N is given by

$$
\begin{aligned}
UF(0)P(T_0) = & -S_{fixed} \sum_{n=1}^{N} D(0,T_n)P(T_n)\,(\Delta t_n) \\
& -S_{fixed} \sum_{n=1}^{N} D(0,T_n)\,(P(T_{n-1}) - P(T_n))\,\frac{(\Delta t_n)}{2} \\
& +(1-R) \sum_{n=1}^{N} D(0,T_n)(P(T_{n-1}) - P(T_n))
\end{aligned}
$$

Where you now solve for $UF(0)$ given a fixed spread S_{fixed} and recovery rate R values.

5.4 CDS variations: Forward Credit Default Swap

In the standard forward CDS contract, a protection buyer agrees to enter into a contract to buy protection at a forward date t_F and at a contractual spread $S(t, t_F, T)$, which is agreed today. This protection mature on some maturity date T which clearly $T > t_F$. Between today and the forward date there are no payments. If a credit event occurs before the forward date the standard contract cancels at no cost to either party.

5.5 CDS variations: Default Swaption

The default swaption is an over-the-counter product which grants the holder the right, but not the obligation, to enter into a credit default swap on some future date at a spread fixed today. Almost, all default swaptions are European style - that is,the holder can exercise the option on one specific date known as the option expiry date t_E.

5.6 CDS variations: CMCDS

Constant maturity credit default swap (CMCDS). This is a variation of the standard CDS contract in which the floating leg pays a variable spread, linked to an observation of some M-year tenor spread made on the previous coupon date. It is similar to the way in which the Libor payment on a floating leg is set in advance and paid in arrears. Typically,the CDS spread used is the five-year CDS spread of the reference credit as observed on the previous coupon payment date.

Excel Workshop

Implementing CDS Pricing and Bootstrapping

Compute the price (premium) of the following CDS:

Contract duration is one year, quarterly payments (that is, four payments per year), notional = 100 million USD, risk-free rate = 5 % pa, hazard rate of underlying = 1 % pa, recovery rate = 50 %.

To be submitted at the end of the lecture, in <u>either</u> paper format (hand calculations) or Excel spreadsheet (via email).

Appendix:
The ABC of Finance... revisited

Probability of Survival (from t=0 to t=T)

$$P(\mathrm{t},T) = \exp\left[-\lambda(T-t)\right]$$

Probability of Default (between t=S and t=T)

$$PD(\mathrm{t},\mathrm{S},T) = \exp\left[-\lambda(\mathrm{S}-t)\right] - \exp\left[-\lambda(T-t)\right]$$

Cashflows with Credit Risk

$$PV = \exp(-rT) \times E[FV]$$

$$PV = \exp(-rT) \times FV \times P(0,T)$$

$$PV = \exp(-rT) \times FV \times \exp(-\lambda T)$$

$$\boxed{PV = \exp(-(r+\lambda)T) \times FV}$$

References

Schönbucher PJ, *Credit Derivatives Pricing Models*, Wiley Finance, 2003.

Bomfim AN, Understanding Credit Derivatives and Related Instruments, Academic Press, 2004.

Bielecki TR, Rutkowski M, *Credit Risk: Modeling, Valuation and Hedging*, Springer Finance, 2002.

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