Fixed Income Derivatives: Managing Delta Risk

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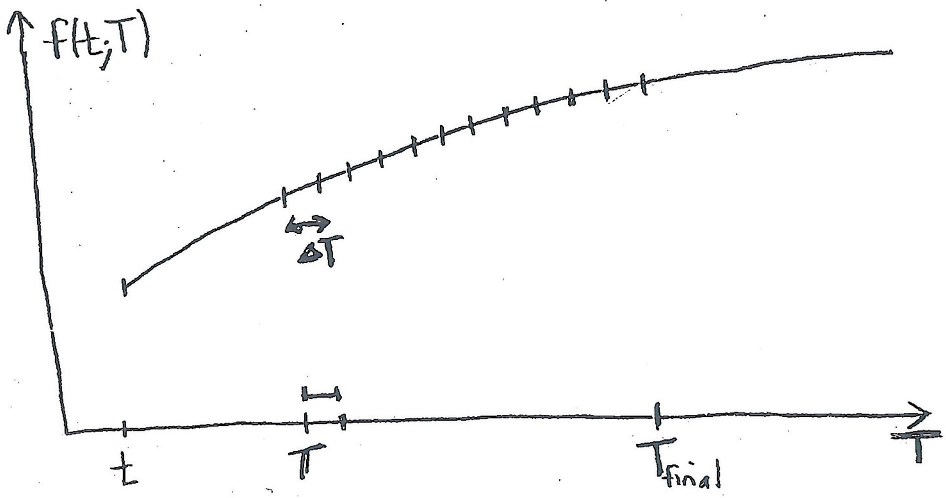
Notation



- At t agree to: pay \$1 at T, receive $e^{f(t,T)\Delta T}$ at $T + \Delta T$

  $f(t,T)$ = (instantaneous) forward rate

  $r(t) = f(t,t)$ = short rate

- FRAs: at t agree to:

  pay \$1 at T_0, receive $e^{\int_{T_0}^{T} f(t,T')dT'}$ at T

  or

  pay $e^{-\int_{T_0}^{T} f(t,T')dT'}$ at T_0, receive \$1 at T

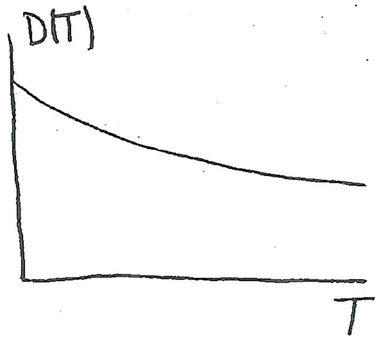
- Zeroes:

  $Z(t,T) = e^{-\int_t^T f(t,T')dT'}$ = value at t of \$1 paid at T

- Discount factors:

  $D(T) = Z(0,T)$ = today's value of \$1 paid at T

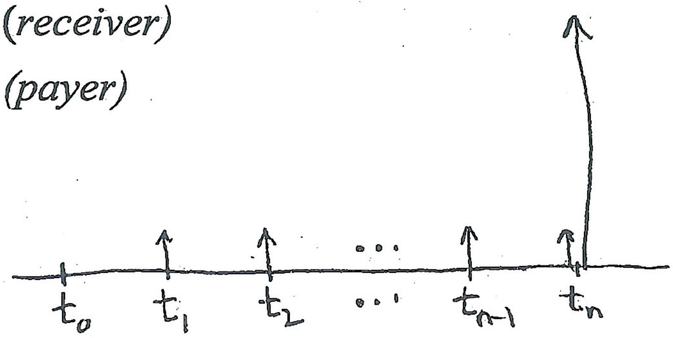
Swaps



$D(T) = Z(0,T) =$ value of \$1 paid at T

• Rec fixed leg, pay floating leg (*receiver*)

(*payer*)



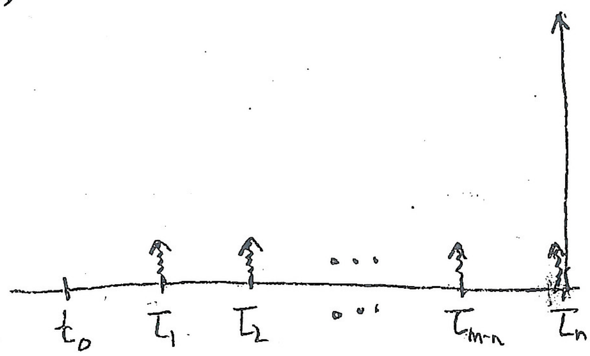
– fixed leg:

$\alpha_j R_f$ paid at t_j

$1 + a_n R_f$ paid at t_n

$\alpha_j =$ *"coverages"* = *"year fracs"* $\quad$ $\alpha_j \approx \frac{1}{2}$ for semi ; $\alpha_j \approx 1$ for annual

$$V_{fix}(t) = R_f \sum_{j=1}^{n} \alpha_j Z(t,t_j) + Z(t,t_n)$$



– floating leg:

$$V_{flt}(t) = \{\$1 \text{ paid at } T_0\} = Z(t,t_0) + \underline{\text{basis spread}}$$

Swap rates

- $V_{sw}(t) = R_f \sum_{j=1}^{n} \alpha_j Z(t,t_j) + Z(t,t_n) - Z(t,t_0)$ receiver

  (fixed leg) (flt leg)

- $V_{sw}(0) = R_f \sum_{j=1}^{n} \alpha_j D(t_j) + D(t_n) - D(t_0)$ ← today's value

- swap rate:

$$R_s(t) = \frac{Z(t,t_0) - Z(t,t_n)}{\sum_{j=1}^{n} \alpha_j Z(t,t_j)}; \qquad R_s^0 = \frac{D(t_0) - D(t_n)}{\sum_{j=1}^{n} \alpha_j D(t_j)}$$

today's fwd swap rate

- $V_{sw}(t) = [R_f - R_s(t)] \sum_{j=1}^{n} \alpha_j Z(t,t_j) = [R_f - R_s(t)]L(t)$

$$L(t) = \sum_{j=1}^{n} \alpha_j Z(t,t_j) = level = \frac{\partial V_{sw}}{\partial R_f}$$

"level", "DV01," "PV01", "annuity", "numerical duration"

- MTM/hedging the swap book:
- a) strip yield curve, find ΔV_{book}
- b) bump instrument, re-strip, find ΔV_{book}
- c) buy/sell instrument so $\Delta V_{book} = 0$

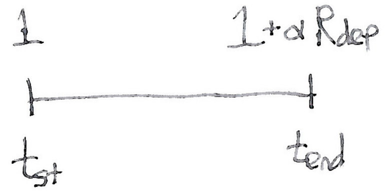
O/N
T/N
1m
EDF_1
EDF_2
⋮
EDF_{20}
R_{sw}^{5y}
R_{sw}^{7y}
⋮
R_{sw}^{30y}

$\xrightarrow{\text{strip}} D(T)$

Yield Curve ID	USDSwap.001				
	YCName	USDSwap		*[tOrigin]*	10/24/2001
	[Method]	MCSpline		*|| Shift:*	0
Use	**Insts**	**Rate/Price**	**[RateAdj]**	**[RateShift]**	**EffRate**
1	O/N	3.51			3.5100
1	T/N	3.57			3.5700
1	1w	3.57406			3.5741
1	stub	3.575			3.5750
1	1m	3.57			3.5700
1	2m	3.52			3.5200
1	3m	3.48625			3.4863
1	U01	96.66			3.3400
1	Z01	96.725	0.0008		3.2742
1	H02	96.645	0.0032		3.3518
1	M02	96.37	0.0072		3.6228
1	U02	95.995	0.0128		3.9922
1	Z02	95.585	0.02		4.3950
1	H03	95.33	0.0288		4.6412
1	M03	95.05	0.0392		4.9108
1	U03	94.825	0.0512		5.1238
1	Z03	94.605	0.0648		5.3302
1	H04	94.535	0.08		5.3850
1	M04	94.4	0.0968		5.5032
1	U04	94.29	0.1152		5.5948
1	Z04	94.145	0.1352		5.7198
1	H05	94.125	0.1568		5.7182
1	M05	94.055	0.18		5.7650
1	U05	93.97	0.2048		5.8252
1	Z05	93.855	0.2312		5.9138
1	H06	93.88	0.2592		5.8608
1	M06	93.835	0.2888		5.8762
1	5Y	5.028			5.0280
1	6Y	5.191			5.1910
1	7Y	5.325			5.3250
1	8Y	5.423			5.4230
1	9Y	5.499			5.4990
1	10Y	5.576			5.5760
1	15Y	5.823			5.8230
1	20Y	5.941			5.9410
1	30Y	5.989			5.9890

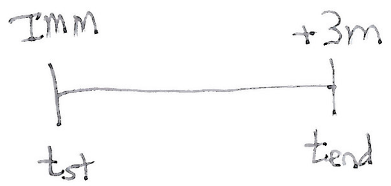
Stripping Instruments

• Deposits:



$$(1 + \alpha R_{dep})D(t_{end}) - D(t_{st}) = 0$$

• Futures:

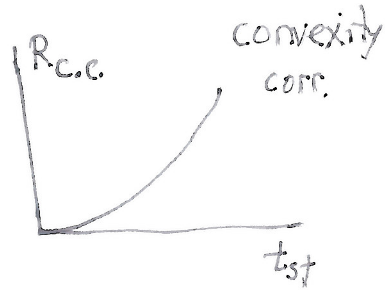


$$R_{fut} = 1 - P_{fut}/100$$

$$P_{fut} = 96.75 \Rightarrow R_{fut} = 3.75\%$$

– futures settle daily:
receive money high rates / pay money low rates

$$R_{fwd} = R_{fut} + R_{c.c.}$$



$$(1 + \alpha R_{fwd})D(t_{end}) - D(t_{st}) = 0$$

• Swaps:

$$R_{sw} \sum_{j=1}^{n} \alpha_j D(t_j) + D(t_n) - D(t_0) = 0$$

Stripping: Interpolation method

$$D(T) = e^{-\int_0^T f(T')dT'}$$

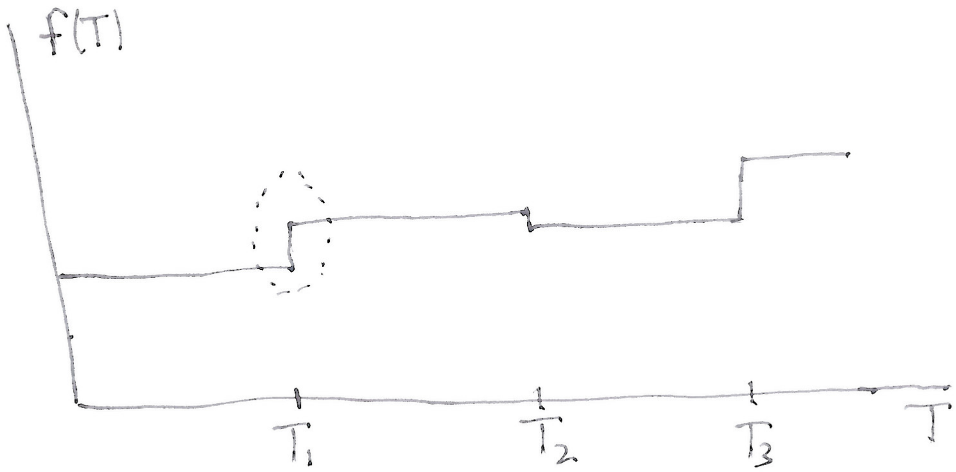
maturity

$$R_1 \sum_{j=1}^{n} \alpha_j D(t_j) + D(t_n) - D(t_0) = 0 \qquad T_1$$

$$R_2 \sum_{j=1}^{n} \alpha_j D(t_j) + D(t_n) - D(t_0) = 0 \qquad T_2$$

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

- Interpolation: pw constant forwards

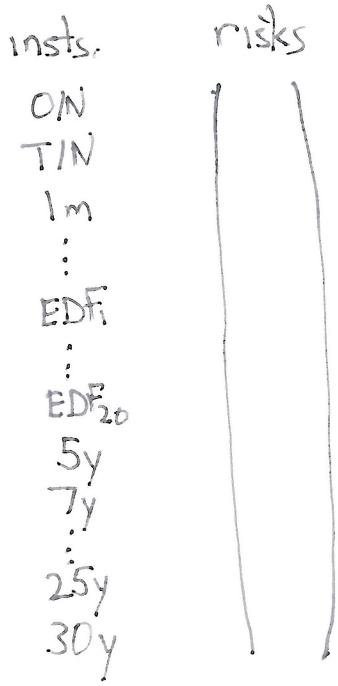


– better: pw linear? cubic spline? *unstable!* X

– smart quadratics, smart quartic, monotone convex splines, cubic splines under tension

– *West & Hagan*

Delta risks by bumping



- MTM / hedging

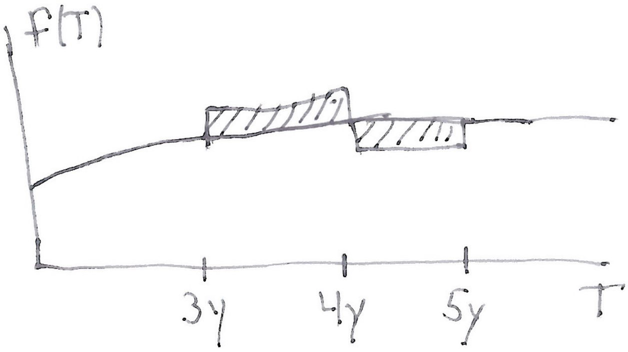
– strip yield curve; find V_{book}

– bump instrument j; re-strip; find ΔV_{book}

– buy / sell instruments so $\Delta V_{book} = 0$

– modern software carries the Jacobian

vector of risks $\Rightarrow$ vector of hedges



- Problems:

– bleeding

2y position may have 30y risks

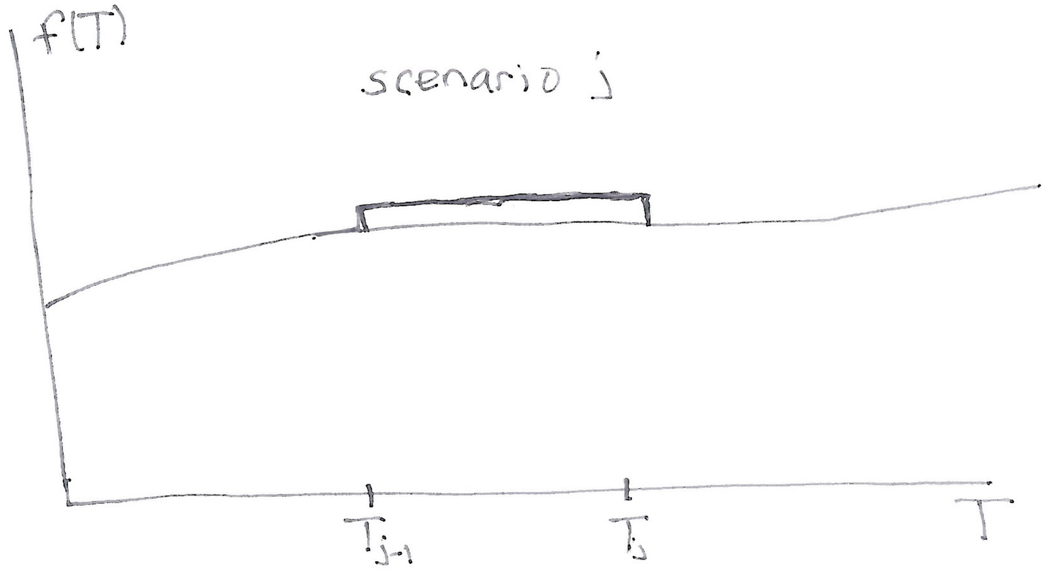
– excessive risks

yield curve may have 43 instruments

– non-intuitive risks

4y swap bumped; 1y, 2y, 3y, 5y unchanged

Risks: Wave method



- Pick J hedging instruments $H_1, H_2, \ldots, H_J$

– maturities T_1, T_2, …, T_J

– scenario j :
$$f_j(T) = f_{base}(T) + \begin{cases} 0 & T < T_{j-1} \\ \delta & T_{j-1} < T < T_j \\ 0 & T_j < T \end{cases}$$

ΔH_j^k = change in inst k, scene j

ΔV_j = change in book value, scene j

hedges: $\sum a_k H^k + \Delta V_k = 0$

- Intuitive risks, no bleeding, choose level of detail