FCI Questions

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1. second quantization rules

$$a^{\dagger}_{\kappa_1} \equiv a^{\dagger}_1 ... a^{\dagger}_{\kappa_n} \equiv a^{\dagger}_n$$

$$a_{\kappa_1} \equiv a_1 \ldots a_{\kappa_n} \equiv a_n$$

also, I frequently use these interchangeably

$$\kappa_1 \equiv 1 \ldots \kappa_n \equiv n$$

so

$$\delta_{greekNumber,1} \equiv \delta_{GreekNumber,\kappa_1}$$

2. 2 determinants with no differences

$$\langle \Psi | V | \Psi \rangle = v^{\alpha\beta\gamma\delta} \langle 0 | \left(\prod_{\kappa} a_{\kappa} \right) a^{\dagger}_{\alpha} a^{\dagger}_{\beta} a_{\gamma} a_{\delta} \left(\prod_{\kappa'} a^{\dagger}_{\kappa'} \right) |0\rangle \tag{1}$$

$$= v^{\alpha\beta\gamma\delta} \langle 0 | \left(\prod_{\kappa} a_{\kappa} \right) \left(a^{\dagger}_{\alpha} a^{\dagger}_{\beta} a_{\gamma} a_{\delta} \left(a^{\dagger}_1 a_1 - a_1 a^{\dagger}_1 \right) \right) \left(\prod_{\kappa'} a^{\dagger}_{\kappa'} \right) |0\rangle \tag{2}$$

$$= v^{\alpha\beta\gamma\delta} \langle 0 | \left(\prod_{\kappa} a_{\kappa} \right) \left(a^{\dagger}_{\alpha} a^{\dagger}_{\beta} a_{\gamma} \delta_{\delta 1} a_1 - a^{\dagger}_{\alpha} a^{\dagger}_{\beta} a_{\gamma} a^{\dagger}_1 a_{\delta} a_1 \right) \left(\prod_{\kappa'} a^{\dagger}_{\kappa'} \right) |0\rangle \tag{3}$$

$$
\begin{aligned}
&= v^{\alpha\beta\gamma\delta} \langle 0| \left(\prod_{\kappa} a_{\kappa} \right) \left(\delta_{\delta 1} a^{\dagger}_{\alpha} a^{\dagger}_{\beta} a_{\gamma} \left(a^{\dagger}_{2} a_{2} - a_{2} a^{\dagger}_{2} \right) a_{1} \right. \\
&\qquad \left. - a^{\dagger}_{\alpha} a^{\dagger}_{\beta} \delta_{\gamma 1} a_{\delta} a_{1} + a^{\dagger}_{\alpha} a^{\dagger}_{\beta} a^{\dagger}_{1} a_{\gamma} a_{\delta} a_{1} \right) \left(\prod_{\kappa'} a^{\dagger}_{\kappa'} \right) |0\rangle
\end{aligned} \tag{4}
$$

$$
\begin{aligned}
&= v^{\alpha\beta\gamma\delta} \langle 0| \left(\prod_{\kappa} a_{\kappa} \right) \left(\delta_{\delta 1} a^{\dagger}_{\alpha} a^{\dagger}_{\beta} \delta_{\gamma 2} a_{2} a_{1} - \delta_{\delta 1} a^{\dagger}_{2} a^{\dagger}_{\alpha} a^{\dagger}_{\beta} a_{\gamma} a_{2} a_{1} \right. \\
&\qquad - \delta_{\gamma 1} a^{\dagger}_{\alpha} a^{\dagger}_{\beta} a_{\delta} \left(a^{\dagger}_{2} a_{2} - a_{2} a^{\dagger}_{2} \right) a_{1} \\
&\qquad \left. + a^{\dagger}_{1} a^{\dagger}_{\alpha} a^{\dagger}_{\beta} a_{\gamma} a_{\delta} a_{1} \right) \left(\prod_{\kappa'} a^{\dagger}_{\kappa'} \right) |0\rangle
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
&= v^{\alpha\beta\gamma\delta} \langle 0| \left(\prod_{\kappa} a_{\kappa} \right) \left(\delta_{\delta 1} \delta_{\gamma 2} a^{\dagger}_{\alpha} a^{\dagger}_{\beta} a_{2} a_{1} - \delta_{\gamma 1} a^{\dagger}_{\alpha} a^{\dagger}_{\beta} \delta_{\delta 2} a_{2} a_{1} \right. \\
&\qquad \left. + \delta_{\gamma 1} a^{\dagger}_{\alpha} a^{\dagger}_{\beta} a^{\dagger}_{2} a_{\delta} a_{2} a_{1} + a^{\dagger}_{1} a^{\dagger}_{\alpha} a^{\dagger}_{\beta} a_{\gamma} a_{\delta} a_{1} \right) \left(\prod_{\kappa'} a^{\dagger}_{\kappa'} \right) |0\rangle
\end{aligned} \tag{6}
$$

$$
\begin{aligned}
&= v^{\alpha\beta\gamma\delta} \left(\delta_{\delta 1} \delta_{\gamma 2} \delta_{\beta 2} \delta_{\alpha 1} - \delta_{\gamma 1} \delta_{\delta 2} \delta_{\beta 2} \delta_{\alpha 1} \right) \langle 0| \left(\prod_{\kappa} a_{\kappa} \right) \left(\prod_{\kappa'} a^{\dagger}_{\kappa'} \right) |0\rangle \\
&\quad + v^{\alpha\beta\gamma\delta} \langle 0| \left(\prod_{\kappa} a_{\kappa} \right) \left(-\delta_{\delta 1} a^{\dagger}_{2} a^{\dagger}_{\alpha} a^{\dagger}_{\beta} a_{\gamma} a_{2} a_{1} \right. \\
&\qquad \left. + \delta_{\gamma 1} a^{\dagger}_{2} a^{\dagger}_{\alpha} a^{\dagger}_{\beta} a_{\delta} a_{2} a_{1} \right) \left(\prod_{\kappa'} a^{\dagger}_{\kappa'} \right) |0\rangle \\
&\quad + v^{\alpha\beta\gamma\delta} \langle 0| \left(\prod_{\kappa=(n\ldots 2)} a_{\kappa} \right) \left(a^{\dagger}_{\alpha} a^{\dagger}_{\beta} a_{\gamma} a_{\delta} \right) \left(\prod_{\kappa'=(2\ldots n)} a^{\dagger}_{\kappa'} \right) |0\rangle
\end{aligned} \tag{7}
$$

$$= \sum_{\kappa_1,\kappa_2} (v^{\kappa_1\kappa_2\kappa_2\kappa_1} - v^{\kappa_1\kappa_2\kappa_1\kappa_2}) \tag{8}$$

+(these two terms same except for greek delta and gamma switch places, so can I cancel?)
+(I notice the recursive pattern here, but I'm not sure what it means.)