FCI Questions

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April 15, 2023

1 0 differences between two determines

$$\langle \Psi | V | \Psi \rangle = v^{\alpha\beta\gamma\delta} \langle 0 | \left(\prod_{\kappa=(\kappa_n...\kappa_1)} a_\kappa \right) a_\alpha^\dagger a_\beta^\dagger a_\gamma a_\delta \left(\prod_{\kappa'=(\kappa_1...\kappa_n)} a_{\kappa'}^\dagger \right) |0\rangle \qquad (1)$$

$$= \langle 0 | \left(\prod_{\kappa=(\kappa_n...\kappa_1)} a_\kappa \right) a_\alpha^\dagger a_\beta^\dagger a_\gamma \delta_{\delta\kappa_1} \left(\prod_{\kappa'=(\kappa_2...\kappa_n)} a_{\kappa'}^\dagger \right) |0\rangle \qquad (2a)$$

$$- \langle 0 | \left(\prod_{\kappa=(\kappa_n...\kappa_1)} a_\kappa \right) a_\alpha^\dagger a_\beta^\dagger a_\gamma a_{\kappa_1}^\dagger a_\delta \left(\prod_{\kappa'=(\kappa_2...\kappa_n)} a_{\kappa'}^\dagger \right) |0\rangle \qquad (2b)$$

$$= \langle 0 | \left(\prod_{\kappa=(\kappa_n...\kappa_1)} a_\kappa \right) a_\alpha^\dagger a_\beta^\dagger \delta_{\delta\kappa_1} \delta_{\gamma\kappa_2} \left(\prod_{\kappa'=(\kappa_3...\kappa_n)} a_{\kappa'}^\dagger \right) |0\rangle \qquad (3aa)$$

$$- \langle 0 | \left(\prod_{\kappa=(\kappa_n...\kappa_1)} a_\kappa \right) a_\alpha^\dagger a_\beta^\dagger \delta_{\delta_{\kappa_1}} a_{\kappa_2}^\dagger a_\gamma \left(\prod_{\kappa'=(\kappa_3...\kappa_n)} a_{\kappa'}^\dagger \right) |0\rangle \qquad (3ab)$$

$$- \langle 0 | \left(\prod_{\kappa=(\kappa_n...\kappa_1)} a_\kappa \right) a_l^\dagger a_\beta^\dagger \delta_{\gamma\kappa_1} a_\delta \left(\prod_{\kappa'=(\kappa_2...\kappa_n)} a_{\kappa'}^\dagger \right) |0\rangle \qquad (3ba)$$

$$+ \langle 0| \left(\prod_{\kappa=(\kappa_n...\kappa_1)} a_\kappa \right) a_\alpha^\dagger a_\beta^\dagger a_{\kappa_1}^\dagger a_\gamma a_\delta \left(\prod_{\kappa'=(\kappa_2...\kappa_n)} a_{\kappa'}^\dagger \right) |0\rangle \qquad \text{(3bb)}$$

$$= \delta_{\alpha\kappa_1} \delta_{\beta\kappa_2} \delta_{\gamma\kappa_2} \delta_{\delta\kappa_1} \qquad \text{(3aa)}$$

$$- \langle 0| \left(\prod_{\kappa=(\kappa_n...\kappa_3)} a_\kappa \right) a_{\kappa_2} a_{\kappa_1} a_{\kappa_2}^\dagger a_\alpha^\dagger a_\beta^\dagger \delta_{\delta\kappa_1} a_\gamma \left(\prod_{\kappa'=(\kappa_3...\kappa_n)} a_{\kappa'}^\dagger \right) |0\rangle \qquad \text{(3ab)}$$

$$- \langle 0| \left(\prod_{\kappa=(\kappa_n...\kappa_1)} a_\kappa \right) \delta_{\gamma\kappa_1} a_\alpha^\dagger a_\beta^\dagger \delta_{\delta\kappa_2} \left(\prod_{\kappa'=(\kappa_3...\kappa_n)} a_{\kappa'}^\dagger \right) |0\rangle \qquad \text{(3ba)}$$

$$+ \langle 0| \left(\prod_{\kappa=(\kappa_n...\kappa_1)} a_\kappa \right) \delta_{\gamma\kappa_1} a_\alpha^\dagger a_\beta^\dagger a_{\kappa_2}^\dagger a_\delta \left(\prod_{\kappa'=(\kappa_3...\kappa_n)} a_{\kappa'}^\dagger \right) |0\rangle \qquad \text{(3bb)}$$

$$= \delta_{\alpha\kappa_1} \delta_{\beta\kappa_2} \delta_{\gamma\kappa_2} \delta_{\delta\kappa_1} \qquad \text{(3aa)}$$

$$+ \langle 0| \left(\prod_{\kappa=(\kappa_n...\kappa_3)} a_\kappa \right) \delta_{\delta\kappa_1} \delta_{\alpha\kappa_1} a_\beta^\dagger a_\gamma \left(\prod_{\kappa'=(\kappa_3...\kappa_n)} a_{\kappa'}^\dagger \right) |0\rangle \qquad \text{(3aba)}$$

$$- \langle 0| \left(\prod_{\kappa=(\kappa_n...\kappa_3)} a_\kappa \right) \delta_{\delta\kappa_1} a_\alpha^\dagger a_{\kappa_1} a_\beta^\dagger a_\gamma \left(\prod_{\kappa'=(\kappa_3...\kappa_n)} a_{\kappa'}^\dagger \right) |0\rangle \qquad \text{(3abb)}$$

$$- \delta_{\alpha\kappa_1} \delta_{\beta\kappa_2} \delta_{\gamma\kappa_1} \delta_{\delta\kappa_2} \qquad \text{(3ca)}$$

$$- \langle 0| \left(\prod_{\kappa=(\kappa_n...\kappa_3)} a_\kappa \right) \delta_{\gamma\kappa_1} a_{\kappa_1} a_\alpha^\dagger a_\beta^\dagger a_\delta \left(\prod_{\kappa'=(\kappa_3...\kappa_n)} a_{\kappa'}^\dagger \right) |0\rangle \qquad \text{(3cb)}$$

$$= \delta_{\alpha\kappa_1}\delta_{\beta\kappa_2}\delta_{\gamma\kappa_2}\delta_{\delta\kappa_1} \tag{4aa}$$

$$+\delta_{\delta\kappa_1}\delta_{\alpha\kappa_1}\langle 0|\left(\prod_{\kappa=(\kappa_n...\kappa_3)}a_\kappa\right)a_\beta^\dagger a_\gamma\left(\prod_{\kappa'=(\kappa_3...\kappa_n)}a_{\kappa'}^\dagger\right)|0\rangle \tag{4aba}$$

$$-\langle 0|\left(\prod_{\kappa=(\kappa_n...\kappa_3)}a_\kappa\right)\delta_{\delta\kappa_1}a_\alpha^\dagger\delta_{\beta\kappa_1}a_\gamma\left(\prod_{\kappa'=(\kappa_3...\kappa_n)}a_{\kappa'}^\dagger\right)|0\rangle \tag{4abba}$$

$$+\langle 0|\left(\prod_{\kappa=(\kappa_n...\kappa_3)}a_\kappa\right)\delta_{\delta\kappa_1}a_\alpha^\dagger a_{\kappa_1}a_\beta^\dagger a_\gamma\left(\prod_{\kappa'=(\kappa_3...\kappa_n)}a_{\kappa'}^\dagger\right)|0\rangle \tag{4abbb}$$

$$-\delta_{\alpha\kappa_1}\delta_{\beta\kappa_2}\delta_{\gamma\kappa_1}\delta_{\delta\kappa_2} \tag{4ca}$$

$$-\langle 0|\left(\prod_{\kappa=(\kappa_n...\kappa_3)}a_\kappa\right)\delta_{\gamma\kappa_1}\delta_{\alpha\kappa_1}a_\beta^\dagger a_\gamma\left(\prod_{\kappa'=(\kappa_3...\kappa_n)}a_{\kappa'}^\dagger\right)|0\rangle \tag{4cba}$$

$$+\langle 0|\left(\prod_{\kappa=(\kappa_n...\kappa_3)}a_\kappa\right)\delta_{\gamma\kappa_1}a_\alpha^\dagger a_{\kappa_1}a_\beta^\dagger a_\gamma\left(\prod_{\kappa'=(\kappa_3...\kappa_n)}a_{\kappa'}^\dagger\right)|0\rangle \tag{4cbb}$$

$$= \delta_{\alpha\kappa_1}\delta_{\beta\kappa_2}\delta_{\gamma\kappa_2}\delta_{\delta\kappa_1} \tag{3aa}$$

$$+\delta_{\alpha\kappa_1}\delta_{\delta\kappa_1}\langle\Psi|a_{\kappa_1}^\dagger a_{\kappa_2}^\dagger H_0 a_{\kappa_2}a_{\kappa_1}|\Psi\rangle \tag{3aba}$$

$$-\delta_{\beta\kappa_1}\delta_{\delta\kappa_1}\langle\Psi|a_{\kappa_1}^\dagger a_2^\dagger H_0 a_2 a_1 a_{\kappa_2}|\Psi\rangle \tag{3abba}$$

$$+\langle 0|\left(\prod_{\kappa=(\kappa_n...\kappa_3)}a_\kappa\right)\delta_{\delta\kappa_1}a_\alpha^\dagger\delta_{\beta\kappa_1}a_\gamma\left(\prod_{\kappa'=(\kappa_3...\kappa_n)}a_{\kappa'}^\dagger\right)|0\rangle \tag{3abbba}$$

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$$- \langle 0| \left(\prod_{\kappa=(\kappa_n \dots \kappa_3)} a_\kappa \right) \delta_{\delta\kappa_1} a_\alpha^\dagger a_\beta^\dagger a_{\kappa_1} a_\gamma \left(\prod_{\kappa'=(\kappa_3 \dots \kappa_n)} a_{\kappa'}^\dagger \right) |0\rangle \qquad \text{(3abbbb)}$$

$$-\delta_{\alpha\kappa_1} \delta_{\beta\kappa_2} \delta_{\gamma\kappa_1} \delta_{\delta\kappa_2} \qquad \text{(3c)}$$

$$-\delta_{\gamma\kappa_1} \delta_{\alpha\kappa_1} \langle\Psi| a_{\kappa_1}^\dagger a_{\kappa_2}^\dagger H_0 a_{\kappa_2} a_{\kappa_1} |\Psi\rangle \qquad \text{(3da)}$$

$$+\delta_{\gamma\kappa_1} \langle 0| \left(\prod_{\kappa=(\kappa_n \dots \kappa_3)} a_\kappa \right) a_\alpha^\dagger \delta_{\beta\kappa_1} a_\gamma \left(\prod_{\kappa'=(\kappa \dots \kappa_n)} a_{\kappa'}^\dagger \right) |0\rangle \qquad \text{(3dba)}$$

$$-0 \qquad \text{(3dbb)}$$

$$= \delta_{\alpha\kappa_1} \delta_{\beta\kappa_2} \delta_{\gamma\kappa_2} \delta_{\delta\kappa_1} \qquad \text{(3aa)}$$

$$+\delta_{\alpha\kappa_1} \delta_{\delta\kappa_1} \langle\Psi| a_{\kappa_1}^\dagger a_{\kappa_2}^\dagger H_0 a_{\kappa_2} a_{\kappa_1} |\Psi\rangle \qquad \text{(3aba)}$$

$$-\delta_{\beta\kappa_1} \delta_{\delta\kappa_1} \langle\Psi| a_{\kappa_1}^\dagger a_2^\dagger H_0 a_2 a_1 a_{\kappa_2} |\Psi\rangle \qquad \text{(3abba)}$$

$$+\delta_{\beta\kappa_1} \delta_{\delta\kappa_1} \langle\Psi| a_{\kappa_1}^\dagger a_{\kappa_2}^\dagger H_0 a_{\kappa_2} a_{\kappa_1} |\Psi\rangle \qquad \text{(3abbba)}$$

$$-0 \qquad \text{(3abbbb)}$$

$$-\delta_{\alpha\kappa_1} \delta_{\beta\kappa_2} \delta_{\gamma\kappa_1} \delta_{\delta\kappa_2} \qquad \text{(3c)}$$

$$-\delta_{\gamma\kappa_1} \delta_{\alpha\kappa_1} \langle\Psi| a_{\kappa_1}^\dagger a_{\kappa_2}^\dagger H_0 a_{\kappa_2} a_{\kappa_1} |\Psi\rangle \qquad \text{(3da)}$$

$$+\delta_{\gamma\kappa_1} \delta_{\beta\kappa_1} \langle\Psi| a_{\kappa_1 a_{\kappa_2}^\dagger}^\dagger H_0 a_{\kappa_2} a_{\kappa_1} |\Psi\rangle \qquad \text{(3dba)}$$

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$$-0 \tag{3dbb}$$

$$= \delta_{\alpha\kappa_1}\delta_{\beta\kappa_2}\delta_{\gamma\kappa_2}\delta_{\delta\kappa_1} \tag{3aa}$$

$$-\delta_{\alpha\kappa_1}\delta_{\beta\kappa_2}\delta_{\gamma\kappa_1}\delta_{\delta\kappa_2} \tag{3ab}$$

$$+\delta_{\alpha\kappa_1}\delta_{\delta\kappa_1}\langle\Psi|a_{\kappa_1}^\dagger a_{\kappa_2}^\dagger H_0 a_{\kappa_2} a_{\kappa_1}|\Psi\rangle \tag{3ba}$$

$$+\delta_{\gamma\kappa_1}\delta_{\beta\kappa_1}\langle\Psi|a_{\kappa_1}^\dagger a_{\kappa_2}^\dagger H_0 a_{\kappa_2} a_{\kappa_1}|\Psi\rangle \tag{3bb}$$

$$-\delta_{\gamma\kappa_1}\delta_{\alpha\kappa_1}\langle\Psi|a_{\kappa_1}^\dagger a_{\kappa_2}^\dagger H_0 a_{\kappa_2} a_{\kappa_1}|\Psi\rangle \tag{3bc}$$

adding the integrals back in.

$$= v^{\kappa_1\kappa_2\kappa_2\kappa_1} \tag{3aa}$$

$$-v^{\kappa_1\kappa_2\kappa_1\kappa_2} \tag{3ab}$$

$$+v^{\kappa_1\beta\gamma\kappa_1} \tag{3ba}$$

$$+v^{\alpha\kappa_1\kappa_1\delta} \tag{3bb}$$

$$-v^{\kappa_1\beta\kappa_1\delta} \tag{3bc}$$

equation 3 part b similar to the Condon roles. not sure were to go with this, or with part a

2 math˙drafts.pdf

I'm confused about the steps you took to get from

$$\sum_\kappa h^{\kappa\kappa} \tag{3}$$

$$\sum_\kappa h^{(\kappa)(\kappa)}\delta_{[\kappa],[\kappa]} \tag{4}$$