

# 1 problem to stag central limit theorem

## 1.1 (a)

we begin by considering that:

$$\langle (s - \langle s \rangle)^2 \rangle = \langle s^2 - 2s \langle s \rangle + \langle s^2 \rangle \rangle = \langle s^2 \rangle - 2 \langle s \langle s \rangle^2 \rangle \quad (1)$$

we can now use the fact that:  $\langle s \rangle$  and we can treat it as a constant and use the fact that expectation is a positive laner operator:

$$\langle (s - \langle s \rangle)^2 \rangle = \langle s^2 \rangle - 2 \langle s \rangle \langle s \rangle + \langle s \rangle^2 \langle 1 \rangle = \langle s^2 \rangle - \langle 2 \rangle = \sigma^2 \quad (2)$$

## 1.2 (b)

$$\langle S_n \rangle = \left\langle \sum_{i=1}^n s_i \right\rangle = \sum_{i=1}^n \langle s_i \rangle \quad (3)$$

where we have used the fact that expectation is a positive linear operator.

$$\langle S_n \rangle = \sum_{i=1}^n = \sum_{i=1}^{n-1} \langle s_i \rangle + \langle s_n \rangle = \langle S_{n-2} \rangle + 2 \langle s \rangle \quad (4)$$

where we had used the recursive definition for  $S_{n-1}$ .