FCI Questions

Patryk Kozlowski

April 21, 2023

1 one difference between two determinants

$$\langle\Psi|V|\Psi(k \to k')\rangle = v^{\alpha\beta\gamma\delta}\,(-1)^{\varepsilon\left(\kappa_1,\ldots,\kappa'_i,\ldots,\kappa_n\right)} \tag{1a}$$

$$\langle 0| \left(\prod_{\kappa=(\kappa_n\ldots\kappa_2)} a_\kappa\right) a_{\kappa_1} a^\dagger_\alpha a^\dagger_\beta a_\gamma a_\delta a^\dagger_{\kappa'_1} \left(\prod_{\kappa'=(\kappa_2\ldots\kappa_n)} a^\dagger_{\kappa'}\right)|0\rangle \tag{1b}$$

omitting into QUAL and face factor for now

$$\langle 0| \left(\prod_{\kappa=(\kappa_n\ldots\kappa_2)} a_\kappa\right) a_{\kappa_1} a^\dagger_\alpha a^\dagger_\beta a_\gamma \delta_{\delta\kappa'_1} \left(\prod_{\kappa'=(\kappa_2\ldots\kappa_n)} a^\dagger_{\kappa'}\right)|0\rangle \tag{2a}$$

$$-\langle 0| \left(\prod_{\kappa=(\kappa_n\ldots\kappa_2)} a_\kappa\right) a_{\kappa_1} a^\dagger_\alpha a^\dagger_\beta a_\gamma a^\dagger_{\kappa'_1} a_\delta \left(\prod_{\kappa'=(\kappa\ldots\kappa_n)} a^\dagger_{\kappa'}\right)|0\rangle \tag{2b}$$

$$= \langle 0| \left(\prod_{\kappa=(\kappa_n\ldots\kappa_2)} a_\kappa\right) \delta_{\delta\kappa_1'}\delta_{\alpha\kappa_1} a^\dagger_\beta a_\gamma \left(\prod_{\kappa'=(\kappa_2\ldots\kappa_n)} a^\dagger_{\kappa'}\right)|0\rangle \tag{3}$$

$$-\langle 0| \left(\prod_{\kappa=(\kappa_n\ldots\kappa_2)} a_\kappa\right) \delta_{\delta\kappa'_1} a^\dagger_\alpha a_1 a^\dagger_\beta a_\gamma \left(\prod_{\kappa'=(\kappa_2\ldots\kappa_n)} a^\dagger_{\kappa'}\right)|0\rangle \tag{4}$$

$$-\langle 0| \left(\prod_{\kappa=(\kappa_n\ldots\kappa_2)} a_\kappa\right) a_1 a^\dagger_\alpha a^\dagger_\beta \delta_{\gamma\kappa_1'} a_\delta \left(\prod_{\kappa'=(\kappa_2\ldots\kappa_n)} a^\dagger_{\kappa'}\right)|0\rangle \tag{5}$$

$$+0 \tag{6}$$

$$= \delta_{\delta\kappa_{1'}}\delta_{\alpha\kappa_1}\delta_{\beta\kappa_2}\delta_{\gamma\kappa_2} \tag{7}$$

$$-\langle 0| \left(\prod_{\kappa=(\kappa_n \ldots \kappa_2)} a_\kappa \right) \delta_{\delta\kappa_1'} a_\alpha^\dagger \delta_{\beta\kappa_1} a_\gamma \left(\prod_{\kappa'=(\kappa_2 \ldots \kappa_n)} a_{\kappa'}^\dagger \right) |0\rangle \tag{8}$$

$$+0 \tag{9}$$

$$-\langle 0| \left(\prod_{\kappa=(\kappa_n \ldots \kappa_2)} a_\kappa \right) \delta_{\alpha\kappa_1} a_\beta^\dagger \delta_{\gamma\kappa_{1'}} a_\delta \left(\prod_{\kappa'=(\kappa_2 \ldots \kappa_n)} a_{\kappa'}^\dagger \right) |0\rangle \tag{10}$$

$$+\langle 0| \left(\prod_{\kappa=(\kappa_n \ldots \kappa_2)} a_\kappa \right) a_\alpha^\dagger a_1 a_\beta^\dagger \delta_{\gamma\kappa_{1'}} a_\delta \left(\prod_{\kappa'=(\kappa_2 \ldots \kappa_n)} a_{\kappa'}^\dagger \right) |0\rangle \tag{11}$$

$$= \delta_{\delta\kappa_{1'}}\delta_{\alpha\kappa_1}\delta_{\beta\kappa_2}\delta_{\gamma\kappa_2} \tag{12}$$

$$-\delta_{\alpha\kappa_2}\delta_{\beta\kappa_1}\delta_{\gamma\kappa_2}\delta_{\delta\kappa_{1'}} \tag{13}$$

$$-\delta_{\alpha\kappa_1}\delta_{\beta\kappa_2}\delta_{\gamma\kappa_{1'}}\delta_{\delta\kappa_2} \tag{14}$$

$$+\langle 0| \left(\prod_{\kappa=(\kappa_n \ldots \kappa_2)} a_\kappa \right) a_\alpha^\dagger \delta_{\beta\kappa_1} \delta_{\gamma\kappa_{1'}} a_\delta \left(\prod_{\kappa'=(\kappa \ldots \kappa_n)} a_{\kappa'}^\dagger \right) |0\rangle \tag{15}$$

$$-0 \tag{16}$$

$$= \delta_{\delta\kappa_{1'}}\delta_{\alpha\kappa_1}\delta_{\beta\kappa_2}\delta_{\gamma\kappa_2} \tag{17}$$

$$-\delta_{\alpha\kappa_2}\delta_{\beta\kappa_1}\delta_{\gamma\kappa_2}\delta_{\delta\kappa_{1'}} \tag{18}$$

$$-\delta_{\alpha\kappa_1}\delta_{\beta\kappa_2}\delta_{\gamma\kappa_{1'}}\delta_{\delta\kappa_2} \tag{19}$$

$$+\delta_{\alpha\kappa_2}\delta_{\beta\kappa_1}\delta_{\gamma\kappa_{1'}}\delta_{\delta\kappa_2} \tag{20}$$

bringing the face falter and the integrals back.

$$= (-1)^{\varepsilon\left(\kappa_1,\ldots,\kappa_i',\ldots,\kappa_n\right)} \sum_{\kappa_2} \left(v^{1221'} - v^{2121'} - v^{121'2} + v^{211'2} \right) \tag{21}$$