FCI Questions

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1. second quantization rules

$$a^\dagger_{\kappa_1} \equiv a^\dagger_1 ... a^\dagger_{\kappa_n} \equiv a^\dagger_n$$

$$a_{\kappa_1} \equiv a_1 \ldots a_{\kappa_n} \equiv a_n$$

also, I frequently use these interchangeably

$$\kappa_1 \equiv 1 \ldots \kappa_n \equiv n$$

so

$$\delta_{greekNumber,1} \equiv \delta_{GreekNumber,\kappa_1}$$

2. 2 determinants with no differences

$$\langle \Psi | V | \Psi \rangle = v^{\alpha\beta\gamma\delta} \langle 0| \left(\prod_\kappa a_\kappa \right) a^\dagger_\alpha a^\dagger_\beta a_\gamma a_\delta \left(\prod_{\kappa'} a^\dagger_{\kappa'} \right) |0\rangle \quad (1)$$

$$= v^{\alpha\beta\gamma\delta} \langle 0| \left(\prod_\kappa a_\kappa \right) \left(a^\dagger_\alpha a^\dagger_\beta a_\gamma a_\delta \left(a^\dagger_1 a_1 - a_1 a^\dagger_1 \right) \right) \left(\prod_{\kappa'} a^\dagger_{\kappa'} \right) |0\rangle \, (2)$$

$$= v^{\alpha\beta\gamma\delta} \langle 0| \left(\prod_\kappa a_\kappa \right) \left(a^\dagger_\alpha a^\dagger_\beta a_\gamma \delta_{\delta 1} a_1 - a^\dagger_\alpha a^\dagger_\beta a_\gamma a^\dagger_1 a_\delta a_1 \right) \left(\prod_{\kappa'} a^\dagger_{\kappa'} \right) |0\rangle \, (3)$$

$$= v^{\alpha\beta\gamma\delta} \langle 0| \left(\prod_{\kappa} a_{\kappa} \right) \left(\delta_{\delta 1} a_{\alpha}^{\dagger} a_{\beta}^{\dagger} a_{\gamma} \left(a_2^{\dagger} a_2 - a_2 a_2^{\dagger} \right) a_1 \right.$$
$$\left. - a_{\alpha}^{\dagger} a_{\beta}^{\dagger} \delta_{\gamma 1} a_{\delta} a_1 + a_{\alpha}^{\dagger} a_{\beta}^{\dagger} a_1^{\dagger} a_{\gamma} a_{\delta} a_1 \right) \left(\prod_{\kappa'} a_{\kappa'}^{\dagger} \right) |0\rangle \tag{4}$$

$$= v^{\alpha\beta\gamma\delta} \langle 0| \left(\prod_{\kappa} a_{\kappa} \right) \left(\delta_{\delta 1} a_{\alpha}^{\dagger} a_{\beta}^{\dagger} \delta_{\gamma 2} a_2 a_1 - \delta_{\delta 1} a_2^{\dagger} a_{\alpha}^{\dagger} a_{\beta}^{\dagger} a_{\gamma} a_2 a_1 \right.$$
$$- \delta_{\gamma 1} a_{\alpha}^{\dagger} a_{\beta}^{\dagger} a_{\delta} \left(a_2^{\dagger} a_2 - a_2 a_2^{\dagger} \right) a_1$$
$$\left. + a_1^{\dagger} a_{\alpha}^{\dagger} a_{\beta}^{\dagger} a_{\gamma} a_{\delta} a_1 \right) \left(\prod_{\kappa'} a_{\kappa'}^{\dagger} \right) |0\rangle \tag{5}$$

$$= v^{\alpha\beta\gamma\delta} \langle 0| \left(\prod_{\kappa} a_{\kappa} \right) \left(\delta_{\delta 1} \delta_{\gamma 2} a_{\alpha}^{\dagger} a_{\beta}^{\dagger} a_2 a_1 - \delta_{\gamma 1} a_{\alpha}^{\dagger} a_{\beta}^{\dagger} \delta_{\delta 2} a_2 a_1 \right.$$
$$\left. + \delta_{\gamma 1} a_{\alpha}^{\dagger} a_{\beta}^{\dagger} a_2^{\dagger} a_{\delta} a_2 a_1 + a_1^{\dagger} a_{\alpha}^{\dagger} a_{\beta}^{\dagger} a_{\gamma} a_{\delta} a_1 \right) \left(\prod_{\kappa'} a_{\kappa'}^{\dagger} \right) |0\rangle \tag{6}$$

$$= v^{\alpha\beta\gamma\delta} \left(\delta_{\delta 1} \delta_{\gamma 2} \delta_{\beta 2} \delta_{\alpha 1} - \delta_{\gamma 1} \delta_{\delta 2} \delta_{\beta 2} \delta_{\alpha 1} \right) \langle 0| \left(\prod_{\kappa} a_{\kappa} \right) \left(\prod_{\kappa'} a_{\kappa'}^{\dagger} \right) |0\rangle$$
$$+ v^{\alpha\beta\gamma\delta} \langle 0| \left(\prod_{\kappa} a_{\kappa} \right) \left(-\delta_{\delta 1} a_2^{\dagger} a_{\alpha}^{\dagger} a_{\beta}^{\dagger} a_{\gamma} a_2 a_1 \right.$$
$$\left. + \delta_{\gamma 1} a_2^{\dagger} a_{\alpha}^{\dagger} a_{\beta}^{\dagger} a_{\delta} a_2 (-1)_1^a \right) \left(\prod_{\kappa'} a_{\kappa'}^{\dagger} \right) |0\rangle$$
$$+ v^{\alpha\beta\gamma\delta} \langle 0| \left(\prod_{\kappa=(n...2)} a_{\kappa} \right) \left(a_{\alpha}^{\dagger} a_{\beta}^{\dagger} a_{\gamma} a_{\delta} \right) \left(\prod_{\kappa'=(2...n)} a_{\kappa'}^{\dagger} \right) |0\rangle \tag{7}$$

$$= \sum_{\kappa_1, \kappa_2} \left(v^{\kappa_1 \kappa_2 \kappa_2 \kappa_1} - v^{\kappa_1 \kappa_2 \kappa_1 \kappa_2} \right) \tag{8}$$

$+($ these two terms same except for greek delta and gamma switch places, so can I cancel?$)$
$+($I notice the recursive pattern here, but I'm not sure what it means.$)$