to electron case with one electron operator
$\sum_{ij} < |a_2 a_1 a_i^\dagger a_j a_1^\dagger a_2^\dagger| >$
$\delta_{i1} \sum_{ij} < |a_2 a_1 a_i^\dagger a_j a_1^\dagger a_2^\dagger| > - \sum_{ij} < |a_2 a_i^\dagger a_1 a_j a_1^\dagger a_2^\dagger| >$
$\sum_{j} < |a_2 a_1 a_1^\dagger a_j a_1^\dagger a_2^\dagger| > -\delta_{i2} \sum_{ij} < |a_2 a_i^\dagger a_1 a_j a_1^\dagger a_2^\dagger| >$
$\sum_{j} < vac|a_2 a_j a_1^\dagger a_2^\dagger|vac > - \sum_{j} < vac|a_1 a_j a_1^\dagger a_2^\dagger|vac > -0$
$- \sum_{j} < vac|a_2 a_1^\dagger a_j a_2^\dagger|vac > +\delta_{j1} \sum_{j} < vac|a_2 a_j a_1^\dagger a_2^\dagger|vac >$
to be continued...
6 electron case with one electron or provider
$\sum_{ij} < vac|a_f a_e a_d a_c a_b a_a a_i^\dagger a_j^\dagger a_k a_l a_u^\dagger a_v^\dagger a_w^\dagger a_x^\dagger a_y^\dagger a_z^\dagger|vac >$
well evaluating such a second quantization operator string help me with determining the face factorand help me with using the einstein summation convention?