

$$\langle K|O_1|K \rangle = \left(\sum_m^{\frac{N}{2}=N_\alpha} [m|h|m] + \sum_m^{\frac{N}{2}=N_\beta} [m|h|m] \right) = 2 \left(\sum_m^{\frac{N}{2}} [m|h|m] \right)$$