Ch/ChE 164 Winter 2024
Homework Problem Set #1
Due Date: Friday, January 19, 2024 by 11:59pm

Homework Policy

- You may not refer to homework or exams from previous years.
- You may consult with one another; however, the work you submit must represent your own understanding.
- Please write clearly; points will be deducted if the homework is unclear or illegible.
- Homework must be turned in by the posted due date. Late homework will be penalized 10% per day unless prior arrangements have been made with the instructor.

1. Various six-digit numbers can be formed by permuting the digits 666655. All arrangements are equally likely. Given that a number is even, what is the probability that two fives are together?
2. An urn contains seven red marbles, four white marbles, and five blue marbles. If three marbles are drawn in succession (each being replaced before the next is drawn), what is the probability that the first marble is red, the second is white, and the third is blue?
3. The probability of an archer hitting his target is one in four.
   (a) If he shoots five times, what is the probability of hitting the target at least three times?
   (b) How many times must he shoot so that the probability of hitting the target at least once is more than 80%?
4. Consider the Poisson distribution $P(n) = \frac{\lambda^n}{n!}e^{-\lambda} \quad (n = 0, 1, 2, \cdots)$.
   (a) Calculate the mean $\langle n \rangle$ and the variance $\langle \Delta n^2 \rangle \equiv \left\langle (n - \langle n \rangle)^2 \right\rangle$.
   (b) The moment generating function $g(k)$ for a distribution $P(n)$ can be defined as

$$g(k) = \sum_{n=0}^{\infty} e^{kn} P(n).$$

   Find the moment generating function for the Poisson distribution. Calculate the mean and the variance from the generating function.

5. Consider a box of volume V_T which contains N_T non-interacting molecules. Assume that the N_T molecules have an equally likely chance of being anywhere in the box. Now take a sub-volume V of the box.

   (a) What is the probability that a given molecule is found to be within V? What is the average (mean) number $\langle N \rangle$ of molecules located within V?

   (b) What is the standard deviation (square root of the variance)? Express your answer in terms of $\langle N \rangle$, V, and V_T.

   (c) What does the answer to part (b) become as the volume, V_T becomes very large with respect to the fixed sub-volume, V? What does the probability distribution ($P(N; N_T, V/V_T)$) for the number of molecules, N, in V become in this limit? (Keep in mind that the box is at constant density.)

   Hint:

$$\lim_{N_T \to \infty} \frac{N_T!}{(N_T - N)!} = N_T^N$$

   (Note: this question is not thermodynamic in nature, it merely illustrates properties of statistical distributions)