HW Set 1: CH226 **Due Next Thursday**

Problem 1. This problem will familiarize you with the formulas commonly used in a nonlinear optics lab for computing powers, as well as try to give a relative scale to those powers. I encourage you to download the "APE Calculator" application so that you will always have a quick way of doing these calculations.

The following formulas are useful

Average Power, P, common units (W).
Intensity, $I = \frac{P}{area} = \frac{cn\epsilon_0}{2}|E|^2$, common units (W/cm^2)
Energy of pulse, $Energy = P/(f_{repetition\ rate})$, common units J
Peak Power or Intensity, $I_{peak}\ or\ P_{peak} = \frac{(I\ or\ P)/f_{repitition\ rate}}{\tau_{pulse}}$, common units W

Calculate the **peak power (W)**, **peak electric field (V/cm), and peak magnetic field (Tesla)** for the following cases. Assume a Ti:Sapphire amplifier operating at a 1 kHz repetition rate. Assume n=1.

(1) An average excitation power for pump probe experiments is 10 mW into a 200 um spot with a pulsed width of 50 fs.

(2) An average power for a high harmonic experiment in which electrons are tunneled from a nucleus is 0.5 W into a 100 um spot with a pulse width of 5 fs. (Note, the interatomic potential is ~10^9 V/m by using the Coulomb potential and a separation of 0.5 Angstroms, hence tunneling can be achieved)

(3) A high-field THz experiment would have an average power of 5 mW focused into 500 um and a pulse width of 1 ps. (The DC dielectric breakdown field in most insulators is 10^7 V/cm)

Problem 2. The Kramers-Kronig relationships:

(1) Define the Kramers-Kronig relationships in plain words and how they work mathematically.
(2) Why is it important to know the real or imaginary frequency components over a wide frequency range when doing a Kramers-Kronig transformation? What errors does it induce if not?
(3) For a simple harmonic oscillator, draw the real and imaginary parts of the dielectric function, and explain their curve shape in relation to the form of the Kramers-Kronig transformation.

Problem 3. This is another self-learning exercise since we do not have time in class to properly cover dispersion. Please go to https://www.newport.com/n/the-effect-of-dispersion-on-ultrashort-pulses. For this problem, draw the plot in Figure 2 and explain in words why this result exists. Note they use 20 mm of BK7, the average optic is closer to 2 mm, but they add up quickly!

Problem 1:

1) $A = \pi r^2$

$A = \pi(100\times10^{-6})^2$

$A = 3.14\times10^{-8}\,m^2$

$I = \frac{10\times10^{-3}\,W}{3.14\times10^{-8}\,m^2}$

$I = 3.18\times10^5\,W/m^2$

$\text{peak power (w)} = \frac{3.18\times10^5\,W/m^2 / 1\times10^3\,s^{-1}}{50\times10^{-15}\,s}$

$\boxed{\text{peak power (w)} = 6.369\times10^{16}\,W}$

$I = \frac{cn\epsilon_0}{2}|E|^2$

$(\text{peak electric field})\, E = \sqrt{\frac{2\times I}{\epsilon_0 c}} = \sqrt{\frac{2\times(3.18\times10^5)}{(2.25\times10^{-12})(3.0\times10^8)}} = \boxed{3.07\times10^4\,V/cm}$

$(\text{peak magnetic field})\, B = \frac{E}{c} = \frac{(3.07\times10^4\,V/cm)}{(3.0\times10^8\,m/s)} \times \left(\frac{1\,cm}{100\,m}\right) = \boxed{1.02\times10^{-6}\,T}$

2) $A = \pi r^2$

$A = \pi(50\times10^{-6})^2$

$A = 7.85\times10^{-9}\,m^2$

$I = \frac{0.5\,W}{7.85\times10^{-9}\,m^2}$

$I = 6.366\times10^7\,W/m^2$

$\text{peak power (w)} = \frac{6.366\times10^7\,W/m^2 / 1\times10^3\,s^{-1}}{5\times10^{-15}\,s}$

$\boxed{\text{peak power (w)} = 1.27\times10^{19}\,W}$

$I = \frac{cn\epsilon_0 E^2}{2}$

$E = \sqrt{\frac{2\times I}{\epsilon_0 c}} = \sqrt{\frac{2\times(6.366\times10^7)}{(2.25\times10^{-12})(3.0\times10^8)}} = \boxed{4.34\times10^5\,V/cm}$

$B = \frac{E}{c} = \frac{(4.34\times10^5\,V/cm)}{(3.0\times10^8\,m/s)} \times \left(\frac{1\,cm}{100\,m}\right) = \boxed{1.448\times10^{-5}\,T}$

3) $A = \pi r^2$

$A = \pi(250\times10^{-6})^2$

$A = 1.9634\times10^{-7}\,m^2$

$I = \frac{5\times10^{-3}\,W}{1.9634\times10^{-7}\,m^2}$

$I = 2.55\times10^4\,Wm^{-2}$

$\text{peak power (w)} = \frac{2.55\times10^4\,Wm^{-2} / 1\times10^3\,s^{-1}}{1\times10^{-12}\,s}$

$\boxed{\text{peak power (w)} = 2.55\times10^{13}\,W}$

$I = \frac{cn\epsilon_0 E^2}{2}$

$E = \sqrt{\frac{2I}{\epsilon_0 c}} = \sqrt{\frac{2(2.55\times10^4\,Wm^{-2})}{(2.25\times10^{-12})(3.0\times10^8)}} = \boxed{8.686\times10^3\,V/cm}$

$B = \frac{E}{c} = \frac{(8.686\times10^3\,V/cm)}{(3.0\times10^8\,m/s)} \times \left(\frac{1\,cm}{100\,m}\right) = \boxed{2.895\times10^{-7}\,T}$

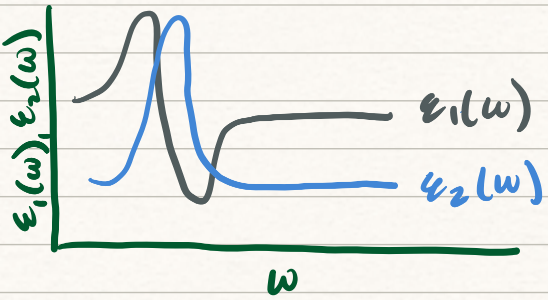
Problem 2:

1) The kramers kroner Relationship describes the relationship between the real and imaginary part of the dielectric function. This works mathematically by using the frequency from the real function and the derivative of that frequency is nearly represents the imaginary function.

2) The real and imaginary function must be over a wide range because the optical properties vary with wavelength by covering all energies you can

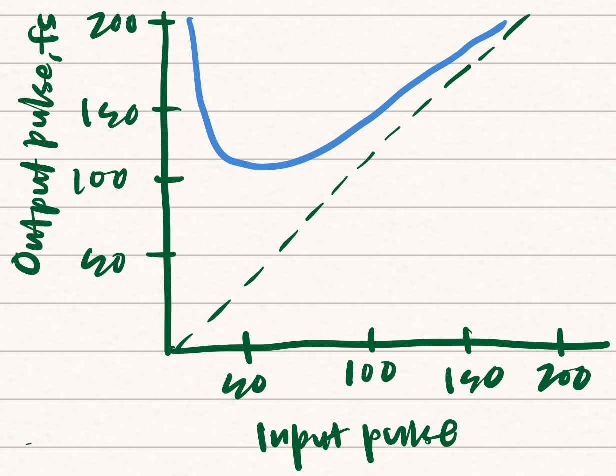
determine all absorbing and non-absorbing frequencies. If not then you are missing all absorbing points in the dielectric function, therefore we won't be able determine the relationship between the real and imaginary.

3)



The curve shape shows that they are related to one another as a derivative of the other by the kramers kronig relation. The real gives an even while the imaginary gives an odd. The actual curvature shape is given by the dampening factor in the dielectric function.

Problem 3:



In this graph we see that at very narrow input pulses we get very broad output pulse, while a wide input pulse provides a very narrow output pulse. The dispersion affects changes depending on the frequency and set on small vs. large pulses.