Let us consider the particle and the box in three dimensions: So, the hambletonian is given by:

$$\hat{H} = \frac{\hat{p}_x^2 + \hat{p}_y^2 + \hat{p}_z^2}{2m} \tag{1}$$

and the solutions are given by:

$$\psi_{n_x,n_y,n_z}(x,y,z) = \sqrt{\frac{8}{L_x L_y L_z}} \sin\left(\frac{n_x \pi x}{L_x}\right) \sin\left(\frac{n_y \pi y}{L_y}\right) \sin\left(\frac{n_z \pi z}{L_z}\right) \tag{2}$$

with the energy:

$$E_{n_x,n_y,n_z} = \frac{\hbar^2 \pi^2}{2m} \left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2}\right) \tag{3}$$

Now, we want to find the expectation value of the position squared:

$$\langle \hat{x}^2 \rangle = \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} \psi^*_{n_x,n_y,n_z}(x,y,z) \hat{x}^2 \psi_{n_x,n_y,n_z}(x,y,z) \,\mathrm{d}x \,\mathrm{d}y \,\mathrm{d}z \tag{4}$$

We can use the fact that the wavefunction is zero outside the box, so we can integrate from $-\infty$ to ∞:

$$\langle \hat{x}^2 \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \psi^*_{n_x,n_y,n_z}(x,y,z) \hat{x}^2 \psi_{n_x,n_y,n_z}(x,y,z) \,\mathrm{d}x \,\mathrm{d}y \,\mathrm{d}z \tag{5}$$

Then we change to spherical coordinates:

$$\langle \hat{x}^2 \rangle = \int_0^{\infty} \int_0^{\pi} \int_0^{2\pi} \psi^*_{n_x,n_y,n_z}(r,\theta,\phi) \hat{x}^2 \psi_{n_x,n_y,n_z}(r,\theta,\phi) r^2 \sin\theta \,\mathrm{d}r \,\mathrm{d}\theta \,\mathrm{d}\phi \tag{6}$$