1

We consider the quantum mechanics of a particle in the earth's gravitational field:

$$V(r) = -\frac{GMm}{r} \tag{1}$$
$$= -\frac{GMm}{R+z} \tag{2}$$
$$\approx -\frac{GMm}{R} + mgz \tag{3}$$

where

- M = mass of earth
- m = mass of particle
- r = distance from center of earth
- G = Newton's gravitational constant
- R = radius of earth
- z = height of particle above surface of earth
- $g = \frac{GM}{R^2}$

We may drop the constant term in our discussion, and consider only the mgz piece, with $z \ll R$. We further assume that no angular momentum is involved, and treat this as a one-dimensional problem. Finally, assume that the particle is unable to penetrate the earth's surface.

1.1 Question

(a) Make a WKB calculation for the energy spectrum of the particle.

1.2 Answer

First, we want to figure out our classical turning points. Since the particle can't penetrate the earth's surface, we have:

$$V(z) = \begin{cases} mgz, & z \geq 0 \\ \infty, & z < 0 \end{cases} \tag{1}$$

Then, we also have

$$p = \sqrt{2m(E - mgz)} \tag{2}$$

So, the $f(E)$ is given by:

$$f(E) = \int_{z_1}^{z_2} \sqrt{2m(E - mgz)}dz = \int_{0}^{\infty} \sqrt{2m(E - mgz)}dz \tag{3}$$