Causal Inference

Patrick Lam

Reference on History of Causal Inference

Holland, Paul W. 1986. "Statistics and Causal Inference." *Journal of the American Statistical Association* 81(396): 945-960.

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The approach we will discuss is known as the *Rubin Causal Model*.

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- SUTVA
- unconfoundedness/ignorability

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- Some patients get extra-strength aspirin (variation in treatment)

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- Omitted Variable Bias

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Ignorability: controlled for by random treatment assignment

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  - Problem goes away as sample size gets large.

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  - We try to alleviate the curse of dimensionality and problem of continuous covariates by specifying a model.
  - Estimates of ATE or ATT may differ depending on the model you specify.

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Goal: **BALANCE** on covariates

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Since we don't have the true propensity scores, we need to check for balance on our covariates at the end.

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Example: The veteran status of any man at risk of being drafted in the lottery was not affected by the draft status of others at risk of being drafted, and, similarly, that the civilian mortality of any such man was not affected by the draft status of others.

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\end{tikzcd}
$$

   Example: Civilian mortality risk was not affected by draft status once veteran status is taken into account.

4. Nonzero Average Causal Effect of Z on T.

   Example: Having a low lottery number increases the average probability of service.

5. Monotonicity: No Defiers

   Example: There is no one who would have served if given a high lottery number, but not if given a low lottery number.

If all the assumptions hold, then the **Local Average Treatment Effect (LATE)** of T on Y is

$$\text{LATE} = \frac{\text{Effect of Z on Y}}{\text{Effect of Z on T}}$$

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It is only a local average treatment effect because it's the effect of T on Y for the subpopulation of compliers, and not the whole population.

Angrist, Joshua D., Guido W. Imbens and Donald B. Rubin. 1996. "Identification of Causal Effects Using Instrumental Variables." *Journal of the American Statistical Association* 91(434):444-455.

Describes instrumental variables in more detail and compares it to the econometric treatment.