

Causal Inference

Patrick Lam

Reference on History of Causal Inference

Holland, Paul W. 1986. "Statistics and Causal Inference." *Journal of the American Statistical Association* 81(396): 945-960.

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The approach we will discuss is known as the *Rubin Causal Model*.

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- ▶ SUTVA
- ▶ unconfoundedness/ignorability

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- ▶ Job training for too many people may flood the market with qualified job applicants (interference)
- ▶ Some patients get extra-strength aspirin (variation in treatment)

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Violations:

- ▶ Omitted Variable Bias

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Ignorability: controlled for by random treatment assignment

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 - ▶ Problem goes away as sample size gets large.

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 - ▶ We try to alleviate the curse of dimensionality and problem of continuous covariates by specifying a model.
 - ▶ Estimates of ATE or ATT may differ depending on the model you specify.

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Goal: **BALANCE** on covariates

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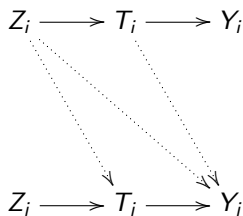
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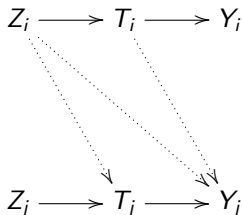


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Example: The veteran status of any man at risk of being drafted in the lottery was not affected by the draft status of others at risk of being drafted, and, similarly, that the civilian mortality of any such man was not affected by the draft status of others.

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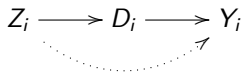
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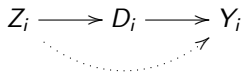


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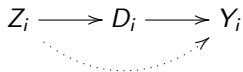
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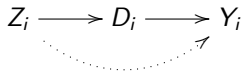
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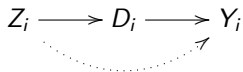
5. Monotonicity:

2. Random (Ignorable) Assignment of the Instrument Z

Example: Assignment of draft status was random.

3. Exclusion Restriction: Any effect of Z on Y must be via an effect of Z on T .

Figure: Exclusion assumption implies absence of dotted arrow.



Example: Civilian mortality risk was not affected by draft status once veteran status is taken into account.

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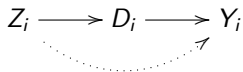
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5. Monotonicity: No Defiers

Example: There is no one who would have served if given a high lottery number, but not if given a low lottery number.

If all the assumptions hold, then the **Local Average Treatment Effect (LATE)** of T on Y is

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It is only a local average treatment effect because it's the effect of T on Y for the subpopulation of compliers, and not the whole population.

Angrist, Joshua D., Guido W. Imbens and Donald B. Rubin. 1996.
“Identification of Causal Effects Using Instrumental Variables.”
Journal of the American Statistical Association 91(434):444-455.

Describes instrumental variables in more detail and compares it to the econometric treatment.