

Formal Specification and Verification of Object-Oriented Programs

Proof Obligations



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making the connection between

JML

and

Dynamic Logic / KeY

- ▶ generating,
- ▶ understanding,
- ▶ and proving

DL proof obligations from JML specifications

From JML Contracts to DL Contracts to Proof Obligations (PO)

```
public class A {  
  /*@ public normal_behavior  
    @ requires <Precondition>;  
    @ ensures <Postcondition>;  
    @ assignable <locations>;  
    @*/  
  public int m(params) {...}  
}
```

Translation

Functional JavaDL method contract

$F = (pre, post, div, var, mod)$

PO Generation

Proof obligation as DL formula

$$pre \rightarrow$$
$$\langle this.m(params); \rangle$$
$$(post \ \& \ frame)$$



Normalization of JML Contracts

1. Flattening of nested specifications
2. Making implicit specifications explicit
3. Processing of modifiers
4. Adding of default clauses if not present
5. Contraction of several clauses

We look only at some aspects of this process

Normalisation:

Making Implicit Specifications Explicit



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Implicit Specifications

- ▶ `non_null` by default
- ▶ Implicit `\invariant_for(this)` as `requires`, `ensures` & `signals` clause
- ▶ Kind of behavior

Making `non_null` explicit for method specifications

1. Deactivate implicit `non_null` by adding `nullable` to parameter and return type declarations, if of reference type and `nullable` not already present
2. Add explicit non null specifications to preconditions (for parameters) and postcondition (for return value)
E.g., for a parameter $T\ p$ add `requires p != null`; if T reference type, but not an array over reference types (more complicated in those cases)

Normalisation:

Making Implicit Specifications Explicit



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Implicit Specifications

- ▶ `non_null` by default
- ▶ Implicit `\invariant_for(this)` as `requires`, `ensures` & `signals` clause
- ▶ Kind of behavior

Making `\invariant_for(this)` explicit for method specifications

1. Deactivate implicit `\invariant_for(this)` by adding `helper` modifier to method (if not already present)
2. Add explicit `\invariant_for(this)` as
 - ▶ `requires \invariant_for(this);` (same for `ensures`) and
 - ▶ `signals (Throwable t)\invariant_for(this);`

Normalisation: Example



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```
/*@ public normal_behavior
   @ requires c.id >= 0;
   @ ensures \result == ( ... );
   @*/
   public boolean addCategory(Category c) { ... }
```

becomes

```
/*@ public normal_behavior
   @ requires c.id >= 0;
   @ requires c != null;
   @ requires \invariant_for(this);
   @ ensures \result == (...);
   @ ensures \invariant_for(this);
   @ signals (Throwable exc) \invariant_for(this);
   @*/
   public boolean addCategory(/*@ nullable @*/Category c) { ... }
```

Normalisation:

Making Implicit Specifications Explicit



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Implicit Specifications

- ▶ `non_null` by default
- ▶ Implicit `\invariant_for(this)` as `requires`, `ensures` & `signals` clause
- ▶ **Kind of behavior**

Making 'kind of behavior' explicit

1. Deactivate implicit behavior specification by replacing `normal_behavior` (`exceptional_behavior`) by `behavior`
2. Add in case of replaced
 - ▶ `normal_behavior` the clause `signals (Throwable t) false;`
 - ▶ `exceptional_behavior` the clause `ensures false;`



```
/*@ public behavior
  @ requires c.id >= 0;
  @ requires c != null;
  @ requires \invariant_for(this);
  @ ensures \result == (...);
  @ ensures \invariant_for(this);
  @ signals (Throwable exc) \invariant_for(this);
  @ signals (Throwable exc) false;
  @*/
public boolean addCategory(/*@ nullable */Category c) { ... }
```



Implicit Specifications

- ▶ `non_null` by default
- ▶ Implicit `\invariant_for(this)` as `requires`, `ensures` & `signals` clause
- ▶ Kind of behavior

Next Normalisation Steps (Not detailed)

- ▶ Expanding `pure` modifier
- ▶ Adding default clauses (e.g., for `diverges`, `assignable`) if clause not present



Merge multiple clauses of the same kind into a single one of that kind.

For instance,

```
/*@ public behavior
```

```
  @ requires R1;
```

```
  @ requires R2;
```

```
  @ ensures E1;
```

```
  @ ensures E2;
```

```
  @ signals (T1 exc) ExcPost1
```

```
;
```

```
  @ signals (T2 exc) ExcPost2
```

```
;
```

```
@*/
```

```
/*@ public behavior
```

```
  @ requires R1 && R2;
```

```
  @ ensures E1 && E2;
```

```
  @ signals (Throwable exc)
```

```
  @ (exc instanceof T1 ==> ExcPost1)
```

```
  @ &&
```

```
  @ (exc instanceof T2 ==> ExcPost2);
```

```
@*/
```



Other not considered steps (here):

- ▶ Separating functional from dependency contracts (later lecture)
- ▶ Separating diverging contracts: We consider here only contracts with either
 - ▶ **diverges false** (default for exceptional and normal behavior) or
 - ▶ **diverges true**

Functional DL contract F for a method m

$$F = (pre, post, div, var, mod)$$

with

- ▶ a precondition DL formula pre ,
- ▶ a postcondition DL formula $post$,
- ▶ a divergence indicator $div \in \{TOTAL, PARTIAL\}$,
- ▶ a variant var a term of type `any`,
- ▶ and a modifies set mod is either a term of type `LocSet` or `\strictly_nothing`

Translating JML Expressions to DL-Terms: Arithmetic Expressions



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Translation replaces arithmetic JAVA operators by generalized operators

Generic towards various integer semantics (JAVA, Math).

Example:

“+” becomes “`javaAddInt`” or “`javaAddLong`”

“-” becomes “`javaSubInt`” or “`javaSubLong`”

...

Translating JML Expressions to DL-Terms:

The `this` Reference

The `this` reference

- ▶ explicit or
- ▶ implicit

has only a meaning within a program (refers to currently executing instance).

On logic level (outside the modalities) no such context exists.

this reference translated to a program variable (named by convention) `self`

e.g., given class

```
public class MyClass {  
    private int f;  
}
```

In JML expressions

- ▶ `f` or **this.f** translated to `select(heap, self, f)`



First-order logic treated fundamentally different in JML and KeY logic

JML

- ▶ Formulas no separate syntactic category
- ▶ Instead: JAVA's `boolean` expressions extended with first-order concepts (i.p. quantifiers)

Dynamic Logic

- ▶ **Formulas** and **expressions** completely separate
- ▶ Truth constants **true**, **false** are formulas, `boolean` constants **TRUE**, **FALSE** are terms
- ▶ Atomic formulas take terms as arguments; e.g.:
 - ▶ $x - y < 5$
 - ▶ $b = \text{TRUE}$

Translating Boolean JML Expressions



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$\mathcal{F}(v)$	=	$v = \text{TRUE}$
$\mathcal{F}(o.f)$	=	$\mathcal{E}(o.f) = \text{TRUE}$
$\mathcal{F}(m())$	=	$\mathcal{E}(m)() = \text{TRUE}$
$\mathcal{F}(!b_0)$	=	$!\mathcal{F}(b_0)$
$\mathcal{F}(b_0 \ \&\& \ b_1)$	=	$\mathcal{F}(b_0) \ \& \ \mathcal{F}(b_1)$
$\mathcal{F}(b_0 \ \ b_1)$	=	$\mathcal{F}(b_0) \ \ \mathcal{F}(b_1)$
$\mathcal{F}(b_0 \ ==> \ b_1)$	=	$\mathcal{F}(b_0) \ \rightarrow \ \mathcal{F}(b_1)$
$\mathcal{F}(b_0 \ <==> \ b_1)$	=	$\mathcal{F}(b_0) \ \leftrightarrow \ \mathcal{F}(b_1)$
$\mathcal{F}(e_0 == e_1)$	=	$\mathcal{E}(e_0) \doteq \mathcal{E}(e_1)$
$\mathcal{F}(e_0 != e_1)$	=	$!\mathcal{E}(e_0) \doteq \mathcal{E}(e_1)$
$\mathcal{F}(e_0 >= e_1)$	=	$\mathcal{E}(e_0) \succcurlyeq \mathcal{E}(e_1)$

$v/f/m()$ **boolean** variables/fields/pure methods

b_0, b_1 **boolean** JML expressions, e_0, e_1 JML expressions

\mathcal{E} translates JML expressions to **DL terms**

\mathcal{F} Translates boolean JML Expressions to Formulas



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Quantified formulas over reference types:

$$\begin{aligned} \mathcal{F}((\text{\texttt{\textbackslashforall}} \text{ T } x; e_0; e_1)) = \\ \text{\texttt{\textbackslashforall}} \text{ T } x; (\\ (!x = \text{null} \ \& \ \text{boolean:select}(\text{heap}, x, \text{\texttt{<created>}}) \ \& \ \mathcal{F}(e_0)) \rightarrow \mathcal{F}(e_1)) \end{aligned}$$
$$\begin{aligned} \mathcal{F}((\text{\texttt{\textbackslashexists}} \text{ T } x; e_0; e_1)) = \\ \text{\texttt{\textbackslashexists}} \text{ T } x; (\\ !x = \text{null} \ \& \ \text{boolean:select}(\text{heap}, x, \text{\texttt{<created>}}) \ \& \ \mathcal{F}(e_0) \ \& \ \mathcal{F}(e_1)) \end{aligned}$$

\mathcal{F} Translates boolean JML Expressions to Formulas



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Quantified formulas over primitive types, e.g., int

$$\mathcal{F}((\backslash\text{forall int } x; e_0; e_1)) = \\ \backslash\text{forall } T \ x; (\text{inInt}(x) \ \& \ \mathcal{F}(e_0) \rightarrow \mathcal{F}(e_1))$$

$$\mathcal{F}((\backslash\text{exists } T \ x; e_0; e_1)) = \\ \backslash\text{exists } T \ x; (\text{inInt}(x) \ \& \ \mathcal{F}(e_0) \ \& \ \mathcal{F}(e_1))$$

inInt (similar inLong, inByte):

Predefined predicate symbol with fixed interpretation

Meaning: Argument is within the range of the Java int datatype.



$\mathcal{F}(\backslash\text{invariant_for}(e)) = \text{java.lang.Object} :: \langle \text{inv} \rangle (\text{heap}, \mathcal{E}(e))$

Later in detail, here only:

- ▶ `\invariant_for` JML expressions are translated to formulas using placeholder predicates that are abbreviations for the translated invariants using \mathcal{F}, \mathcal{E}
- ▶ Pretty printed as:

$e. \langle \text{inv} \rangle ()$



Functional DL contract F for a method m

$$F = (pre, post, div, var, mod)$$

with

- ▶ a precondition DL formula pre ✓ ,
- ▶ a postcondition DL formula $post$ ✓ ?almost ,
- ▶ a divergence indicator $div \in \{TOTAL, PARTIAL\}$,
- ▶ a variant var a term of type any,
- ▶ and a modifies set mod is either a term of type LocSet or \strictly_nothing

What is missing for `ensures` clauses?

- ▶ Translation of `\result`
- ▶ Translation of `\old(.)` expressions

Translating `\result`

The JML expression `\result` used in an `ensures` clause of a method $T\ m(params)$ is translated to:

$$\mathcal{E}(\text{\code{\result}}) = res$$

where $res \in PVar$ of type T not occurring in the program.



`\old(e)` evaluates e in the prestate of the method

Accesses to heap must be evaluated w.r.t. to the 'old' heap

1. Introduce a global program variables `heapAtPre` of type `Heap`
(Intention: `heapAtPre` refers to the heap in the method's pre-state)
2. Define: $\mathcal{E}(\text{\code{\old}(e)}) = \mathcal{E}_{\text{heap}}^{\text{heapAtPre}}(e)$
($\mathcal{E}_{\text{heap}}^{\text{heapAtPre}}$ uses `heapAtPre` instead of `heap` for heap-sensitive expressions)

Example

$$\mathcal{E}(\text{\code{o.f == \old(o.f) + 1}}) =$$
$$\text{int}::\text{select}(\text{heap}, \text{o}, \text{f}) = \text{int}::\text{select}(\text{heapAtPre}, \text{o}, \text{f}) + 1$$

```
signals (Throwable exc)  
  (exc instanceof ExcType1 ==> ExcPost1) && ...;
```

Translation using \mathcal{F}, \mathcal{E} as normal

A global fresh program variable `exc` of type `Throwable` is used from `PVar`



The DL formula *post* is then defined as

$$(exc \doteq \text{null} \rightarrow \mathcal{F}(\text{ensures})) \ \& \ (exc \neq \text{null} \rightarrow \mathcal{F}(\text{signals}))$$

Note: A normal behavior contract has normalized signals `(Throwable exc>false;`
As expected we get:

$$\begin{aligned} & (exc \doteq \text{null} \rightarrow \mathcal{F}(\text{ensures})) \ \& \ (exc \neq \text{null} \rightarrow \mathcal{F}(\text{false})) \\ \Leftrightarrow & (exc \doteq \text{null} \rightarrow \mathcal{F}(\text{ensures})) \ \& \ (exc \neq \text{null} \rightarrow \text{false}) \\ \Leftrightarrow & (exc \doteq \text{null} \rightarrow \mathcal{F}(\text{ensures})) \ \& \ exc = \text{null} \end{aligned}$$

Translating JML into Functional DL Contracts

Functional DL contract F for a method m

$$F = (pre, post, div, var, mod)$$

with

- ▶ a precondition DL formula pre ✓ ,
- ▶ a postcondition DL formula $post$ ✓ ,
- ▶ a divergence indicator $div \in \{TOTAL, PARTIAL\}$, ✓
- ▶ a variant var a term of type any (postponed to later lecture),
- ▶ and a modifies set mod is either a term of type `LocSet` or `\strictly_nothing`

The Divergence Indicator

$$div = \begin{cases} TOTAL & \text{if normalised JML contract contains clause } \texttt{diverges false}; \\ PARTIAL & \text{if normalised JML contract contains clause } \texttt{diverges true}; \end{cases}$$

Translating Assignable Clauses: The DL Type

LocSet



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Assignable clauses are translated to

a term of type `LocSet` or the special value `\strictly_nothing`

Intention: A term of type `LocSet` represents a set of locations

Definition (Locations)

A location is a tuple (o, f) with $o \in D^{\text{Object}}$, $f \in D^{\text{Field}}$

Note: Location is a **semantic** and not a syntactic entity.



The Data Type LocSet

Predefined type with $D(\text{LocSet}) = 2^{\text{Location}}$ and the functions (incomplete):

`unique LocSet empty`

`unique LocSet allLocs`

`LocSet singleton(Object, Field)`

`LocSet union(LocSet, LocSet)`

`LocSet intersect(LocSet, LocSet)`

`LocSet allFields(Object)`

`LocSet allObjects(Field)`

`LocSet arrayRange(Object, int, int)`

empty set of locations: $l(\text{empty}) = \emptyset$

set of all locations, i.e., $l(\text{allLocs}) = \{(d, f) \mid f.a. d \in D^{\text{Object}}, f \in D^{\text{Field}}\}$

singleton set

set of all locations for the given object

set of all locations for the given field;

e.g., $\{(d, f) \mid f.a. d \in D^{\text{Object}}\}$

set representing all array locations in the specified range (both inclusive)

Translating Assignable Clauses—Example



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Example

```
assignable \everything;
```

is translated into the DL term

```
allLocs
```

Example

```
assignable this.next, this.content[0..this.content.length-1];
```

is translated into the DL term

```
union(singleton(self,next)  
      arrayRange(0, javaSubInt(length(self.content), 1))
```



Functional DL contract F for a method m

$$F = (pre, post, div, var, mod)$$

with

- ▶ a precondition DL formula pre ✓ ,
- ▶ a postcondition DL formula $post$ ✓ ,
- ▶ a divergence indicator $div \in \{TOTAL, PARTIAL\}$ ✓ ,
- ▶ a variant var a term of type any (postponed),
- ▶ a modifies set mod is either a term of type LocSet or \strictly_nothing ✓

From JML Contracts to DL Contracts to Proof Obligations (PO)

```
public class A {  
  /*@ public normal_behavior  
    @ requires <Precondition>;  
    @ ensures <Postcondition>;  
    @ assignable <locations>;  
    @*/  
  public int m(params) {...}  
}
```

Translation

Functional JavaDL method contract

$F = (pre, post, div, var, mod)$

PO Generation

Proof obligation as DL formula

$$pre \rightarrow$$
$$\langle this.m(params); \rangle$$
$$(post \ \& \ frame)$$

Generating a PO from a Functional DL Contract: Idea

Given functional contract F_m for a method m implemented in class C :

$$F_m = (pre, post, TOTAL, var, mod)$$



$$pre \rightarrow \langle self.m(args) \rangle (post \ \& \ \underbrace{frame}_{\text{correctness of assignable}})$$

(in case of $div = \text{PARTIAL}$ box modality is used)

Generating a PO from a Functional DL Contract: Method Identification



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$$pre \rightarrow \langle self.m(args) \rangle (post \ \& \ frame)$$

- ▶ Dynamic dispatch: `self.m(...)` causes split into all possible implementations
- ▶ Special statement **Method Body Statement**:

`m(args)@package.Class`

Meaning: Placeholder for the method body of class `package.Class`

Generating a PO from a Functional DL Contract: Exceptions



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$$pre \rightarrow \langle self.m(args)@package.Class \rangle (post \ \& \ frame)$$

Postcondition *post* states either

- ▶ that no exception is thrown or
- ▶ that in case of an exception the exceptional postcondition holds

but: $\langle \text{throw } exc; \rangle \varphi$ is trivially false

How to refer to an exception in post-state?

$$pre \rightarrow$$
$$\left\langle \begin{array}{l} exc = \text{null}; \\ \text{try } \{ \\ \quad self.m(args)@package.Class \\ \} \text{ catch } (Throwable \ t) \{ exc = t; \} \end{array} \right\rangle (post \ \& \ frame)$$

(reminder: Normalisation uses program variable *exc* when translating signals)

$$pre \rightarrow \langle \text{exc}=\text{null}; \text{ try } \{ \text{self.m}(\text{args}) \} \text{ catch } \dots \rangle (\text{post} \ \& \ \text{frame})$$

Additional properties (known to hold in Java, but not formalized), e.g.,

- ▶ **this** is not null
- ▶ created objects cannot reference not yet created objects (dangling references)
- ▶ integer parameters have correct range
- ▶ ...

& need to refer to prestate in post, e.g. for old-expressions

Need to make these assumption on initial state explicit in DL! (Why?)

Idea: Formalise assumption as additional precondition *freePre* (free precondition)

Extend general shape:

$$(\text{freePre} \wedge pre) \rightarrow \{ \text{heapAtPre} := \text{heap} \} \\ \langle \text{exc}=\text{null}; \text{ try } \{ \text{self.m}(\text{args}) \} \text{ catch } \dots \rangle (\text{post} \ \& \ \text{frame})$$



```
freePre :=  wellFormed(heap)
           ∧ paramsInRange
           ∧ self! = null
           ∧ boolean :: select(heap, self, <created>) = TRUE
           ∧ package.Class :: exactInstance(self)
           ∧ exc = null
```

- ▶ *wellFormed*: predefined predicate; true iff. given heap is regular Java heap
- ▶ *paramsInRange* formula stating that the method arguments are in range
- ▶ *C :: exactInstance*: predefined predicate; true iff. given argument has *C* as exact type (i.e., is not of a subtype)

Generating a PO from a Functional DL Contract:

The *frame* DL Formula

$(freePre \wedge pre) \rightarrow$
 $\{heapAtPre := heap\} \langle exc = null; \text{ try } \{ self.m(args) \} \text{ catch } \dots \rangle$
(*post* & *frame*)

If *mod* = \strictly_nothing then *frame* is defined

$$\forall o; \forall f; (any :: select(heapAtPre, o, f) = any :: select(heap, o, f))$$

If *mod* is a location set, then *frame* is defined as:

$$\begin{aligned} \forall o; \forall f; (& \text{ boolean} :: select(heapAtPre, o, \langle created \rangle) = \text{FALSE} \\ & \vee any :: select(heapAtPre, o, f) = any :: select(heap, o, f) \\ & \vee (o, f) \in \{ heap := heapAtPre \} mod) \end{aligned}$$

States that any location (o, f)

- ▶ belongs to an object that is not (yet) created before the method invocation or
- ▶ holds the same value after the invocation as before the invocation, or
- ▶ belongs to the modifies set (evaluated in the pre-state).

Examples



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Demo