Concepts of Programming Languages

Functions

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A Taxonomy of Functions

- **First-class functions**
  - Functions are values/objects with all the rights of other values
    - Can be constructed at runtime
    - Can be passed as arguments to other functions, returned by other functions, stored in data structures etc
  - No first-class functions => functions can only be defined in designated portions of the program, where they are given names for use in the rest of the program
- **Higher-order functions**
  - Functions that return and/or take other functions as parameters
  - Parameterize computations over other computations
- **First-order Functions**
  - Functions that neither return nor take other functions as parameters
  - Parameterize computations over data

F1WAE

- A language with **first-order** functions
  - Function applications are expressions
- No first-class/higher-order functions:
  - Function definitions are **not** expressions

⇒ separate definitions from expressions

- predefined functions given to interpreter as an argument

Concrete & Abstract Syntax for F1WAE

```
<F1WAE> ::= <num>
          | {+ <F1WAE> <F1WAE>}
          | {- <F1WAE> <F1WAE>}
          | {let {<id> <F1WAE>} <F1WAE>}
          | <id>
          | ???
```

how does the concrete syntax change?

```
sealed abstract class F1WAE
case class Num(n: Int) extends F1WAE
case class Add(lhs: F1WAE, rhs: F1WAE) extends F1WAE
case class Sub(lhs: F1WAE, rhs: F1WAE) extends F1WAE
case class Let(name: Symbol, namedExpr: F1WAE, body: F1WAE) extends F1WAE
case class Id(name: Symbol) extends F1WAE
???
```

how does the abstract syntax change?

Concrete & Abstract Syntax for F1WAE

```
<F1WAE> ::= <num>
          | {+ <F1WAE> <F1WAE>}
          | {- <F1WAE> <F1WAE>}
          | {let {<id> <F1WAE>} <F1WAE>}
          | <id>
          | {<id> <F1WAE>}
```

concrete syntax for function application

What does this tell us about valid F1WAE programs?

```
sealed abstract class F1WAE
case class Num(n: Int) extends F1WAE
case class Add(lhs: F1WAE, rhs: F1WAE) extends F1WAE
case class Sub(lhs: F1WAE, rhs: F1WAE) extends F1WAE
case class Let(name: Symbol, namedExpr: F1WAE, body: F1WAE) extends F1WAE
case class Id(name: Symbol) extends F1WAE
case class App(funName: Symbol, arg: F1WAE) extends F1WAE
```

abstract syntax for function application

Function Definitions in F1WAE

- Cannot define functions in F1WAE
  - More strict than necessary for first-orderness
- Predefined functions are passed to the interpreter
- How to represent functions?

```
case class FunDef(argName: Symbol, body: F1WAE)
type FunDefs = Map[Symbol, FunDef]
```

- Example

```
FunDef('n, Add('n, 1))
```

- Class **`FunDef`** does not extend class **`F1WAE`** => no syntax for function definitions

Example Interpreter Calls

scala.Symbol: tick (‘) followed by identifier

```
interp(
   App('f, 10),
   Map('f -> FunDef('n, App('n, 'n)))
)
```

```
interp(
   App('f, 10),
   Map(
     'f -> FunDef('x, App('g, Add('x, 3))),
     'g -> FunDef('y, Sub('y, 1)))
```

Discussion

What is needed to interpret F1WAE expressions?

<Interpreter>

Discussion on Scoping

What is the result of interpreting `{f 10}` where `{f n} = {n n}`?

```
interp(
   App('f, 10),
   Map('f -> FunDef('n, App('n, 'n)))
)
```

- Should the interpreter try to substitute the `n` in function position of App with `10`? What would happen if this is the case?
- Or should function names and function arguments live in separate "spaces" ==> an identifier in function position within an App is not replaced.

Let's see what our interpreter does…

Discussion on Scoping

What is the result of `{f 10}` where `{f x} = {g {+ x 3}}`
`{g y} = {- y 1}`

```
val funDefs = Map(
    'f -> FunDef('x, App('g, Add('x, 3))),
    'g -> FunDef('y, Sub('y, 1))
)

interp(App('f, 10), funDefs)
```

- Should `f` be able to invoke `g` or should the invocation fail because `g` is defined after `f`?
- What if there are multiple bindings for the same name?
- If a function can invoke every defined function, it can also invoke itself. Do we have recursion in F1WAE?

Outline

- Implementing first-order functions
- **Environments (static vs. dynamic scoping)**
- About first-class functions
- Functional decomposition and recursion patterns
- Implementing first-class functions

`let` and Substitution

- In `{let {x e} t}` we immediately replace free identifiers `x` in expression `t` with the value expression `e` evaluates to
- `id` expressions left in `t` after substitution denote free identifiers
- If the interpreter encounters an `id` expression → error

Quiz
Do you see any problems with this strategy?

`let` and Environments

- The interpreter receives a store called **environment,** which maps identifiers to values
- **`{let {x e} t}`** simply stores a mapping from **`x`** to the value **`e`** evaluates to in the **environment**
- When the interpreter encounters an **`id`** expression, it looks up the corresponding value in the environment
- Free variables not in environment → error

- We represent environments as values of the type **`Env`**:

```
type Env = Map[Symbol, Int]
```

Here is F1WAE. How Does it Change?

```
def interp(expr: F1WAE, funDefs: FunDefs): Int = expr match {
    case Num(n) =>
      n
    case Add(lhs, rhs) =>
      interp(lhs, funDefs) + interp(rhs, funDefs)
    case Sub(lhs, rhs) =>
      interp(lhs, funDefs) - interp(rhs, funDefs)

    case Let(id, expr, body) =>
      val body = subst(body, id, Num(interp(expr, funDefs)))
      interp(body, funDefs)

    case Id(name) =>
      sys.error("found unbound id " + name)

    case App(fun, arg) => funDefs(fun) match {
      case FunDef(param, body) =>
        interp(subst(body, param, Num(interp(arg, funDefs))), funDefs)
    }
  }
```

Is this our F1WAE with Environments?

```
def interp(expr: F1WAE, funDefs: FunDefs, env: Env): Int = expr match {
  case Num(n) =>
    n
  case Add(lhs, rhs) =>
    interp(lhs, funDefs, env) + interp(rhs, funDefs, env)
  case Sub(lhs, rhs) =>
    interp(lhs, funDefs, env) - interp(rhs, funDefs, env)

  case Let(id, expr, body) =>
    val newEnv = env + (id -> interp(expr, funDefs, env))
    interp(body, funDefs, newEnv)

  case Id(name) =>
    env(name)

  case App(fun, arg) => funDefs(fun) match {
    case FunDef(param, body) =>
      val funEnv = env + (param -> interp(arg, funDefs, env))
      interp(body, funDefs, funEnv)
  }
}
```

What's the result of evaluating
`{let {n 5} {f 10}}`
where `{f x} = {n}`?

What is the answer when using F1WAE with substitution?

Static Versus Dynamic Scoping

Definition Scope (of a name binding):
The scope of a name binding is the part of the program where the binding is in effect.

Definition Static/Lexical Scoping:
The scope of a name binding is determined syntactically (at compile-time).

Definition Dynamic Scoping:
The scope of a name binding is determined by the execution context (at runtime).

F1WAE with Environments

```
def interp(expr: F1WAE, funDefs: FunDefs, env: Env): Int = expr match {
  case Num(n) =>
    n
  case Add(lhs, rhs) =>
    interp(lhs, funDefs, env) + interp(rhs, funDefs, env)
  case Sub(lhs, rhs) =>
    interp(lhs, funDefs, env) - interp(rhs, funDefs, env)

  case Let(id, expr, body) =>
    val newEnv = env + (id -> interp(expr, funDefs, env))
    interp(body, funDefs, newEnv)

  case Id(name) =>
    env(name)

  case App(fun, arg) => funDefs(fun) match {
    case FunDef(param, body) =>
      val funEnv = env + Map(param -> interp(arg, funDefs, env))
      interp(body, funDefs, funEnv)
  }
}
```

Static Scoping!

Outline

- Implementing first-order functions
- Environments (static vs. dynamic scoping)
- **About first-class functions**
- Functional decomposition and recursion patterns
- Implementing first-class functions

Abstracting over Computations

```
def filter[A, B] (relOp: (A, B) => Boolean, b: B, list: List[A]): List[A] =
 list match {
  case Nil => Nil
  case x :: xs =>
   val filteredRest = filter(relOp, b, xs)
   if (relOp(x, b)) x :: filteredRest
   else filteredRest
 }
```

`relOp` parameter stands for any relational operation to apply

```
def <(a: Int, b: Int) = a < b
def >(a: Int, b: Int) = a > b

def below(thres: Int, l: List[Int]) = filter(<, thres, l)
def above(thres: Int, l: List[Int]) = filter(>, thres, l)

println(below(4, List(1, 2, 3, 4, 5)))
println(above(4, List(1, 2, 3, 4, 5)))

def squaredGt(x: Int, c: Int) = x * x > c
println(filter(squaredGt, 10, List(1, 2, 3, 4, 5)))
```

Functions that Return Functions

Expressions in functional lanuages (Scheme, Haskell, Scala) can evaluate to functions.

The body of a function is an expression → a function can return a function.

Especially useful when produced function “remembers” arguments …

```
def add(x: Int) = {
  def xAdder(y: Int) = x + y
  xAdder _
}
```

Functions that Return Functions

```
def filter2[A, B](relOp: (A, B) => Boolean) = {

  def innerFilter(b: B, list: List[A]): List[A] = list match {
    case Nil => Nil
    case x :: xs => {
      val filteredRest = innerFilter(b, xs)
      if (relOp(x, b)) x :: filteredRest
      else filteredRest
    }
  }

  innerFilter _
}
```

filter2 consumes **relOp**, defines **innerFilter**, and returns it. **innerFilter** remembers **relOp**

Anonymous Functions

- If auxiliary functions are only used as arguments to some abstract function `f`, we use an inner `def`.

```
case class Person(name: Symbol)

def findAnon(list: List[Person], name: Symbol) = {
  def hasName(p: Person, name: Symbol) = p.name == name
  filter(hasName, name, list)
}
```

- Anonymous functions provide a short-hand for this frequent use of locally defined functions → `lambda`-expressions

Syntax of lambdas in Scala

```
(<var> ... <var>) => <exp>
```

- the sequence of variables in parentheses are the function's parameters
- component on the right-hand side is the function's body

```
(x-1 ... x-n) => exp
```

```
def plus4(x: Int) = x + 4
def plus4 = (x: Int) => x + 4
```

```
def aNewName(x-1 ... x-n) exp
aNewName
```

aNewName may not occur in **exp**

Syntax of lambdas in Scala

```
case class Person(name: Symbol)

def findAnon(list: List[Person], name: Symbol) = {
  def hasName(p: Person, name: Symbol) = p.name == name
  filter(hasName, name, list)
}
```

```
def findAnon(list: List[Person], name: Symbol) = {
  filter((p: Person, name: Symbol) => { p.name == name }, name, list)
}
```

Functions that Return Functions

```
def filter2[A, B](relOp: (A, B) => Boolean) = {

  def innerFilter(b: B, list: List[A]): List[A] = list match {
    case Nil => Nil
    case x :: xs =>
      val filteredRest = innerFilter(b, xs)
      if (relOp(x, b)) x :: filteredRest
      else filteredRest
  }

  innerFilter _
}
```

Can we use an anonymous function here?

Outline

- Implementing first-order functions
- Environments (static vs. dynamic scoping)
- About first-class functions
- **Functional decomposition and recursion patterns**
- Implementing first-class functions

Recursion Operators

- Many recursive programs share a **common pattern of recursion**.
- Repeating the same patterns again and again is tedious, time consuming, error prone.
- Such repetition can be avoided by introducing special *recursion operators.*
- Recursion operators encapsulate common patterns allowing to concentrate on parts that are different for each application.

Any Problems with this `list-of-squares`?

```
def listOfSquares(list: List[Int]): List[Int] = list match {
    case Nil => Nil
    case x :: xs => square(x) :: listOfSquares(xs)
}
```

- This definition draws attention to the **element-by-element processing** of the input list.
- The operation applied to elements (`square`) is hard-coded.
- How the result is constructed from the input is hard-coded (coupled to a particular implementation of the list).

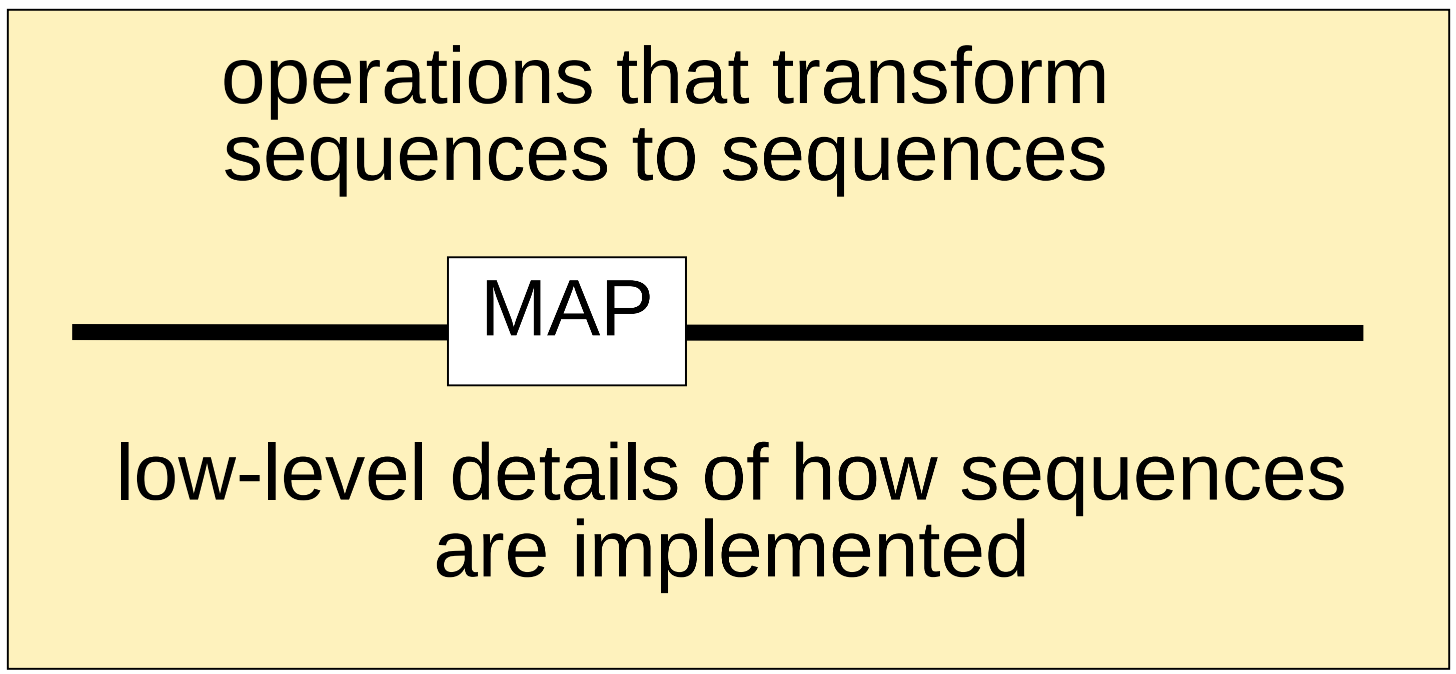
What about this Definition?

```
def map[A, B](f: A => B, list: List[A]): List[B] = list match {
    case Nil => Nil
    case x :: xs => f(x) :: map(f, xs)
}

def listOfSquares2(list: List[Int]) = map(square, list)
```

This version emphasizes squaring as a transformation of a list to another list

`map`: an Abstraction Barrier

- **<u>Abstraction barrier</u>**: `map` supports a higher-level of abstraction by isolating the implementation of procedures that transform lists from the details of how list elements are extracted and combined.



- One can vary the implementation of the list independent of the mapping function applied to each element.

- `map` encapsulates a recursion pattern.

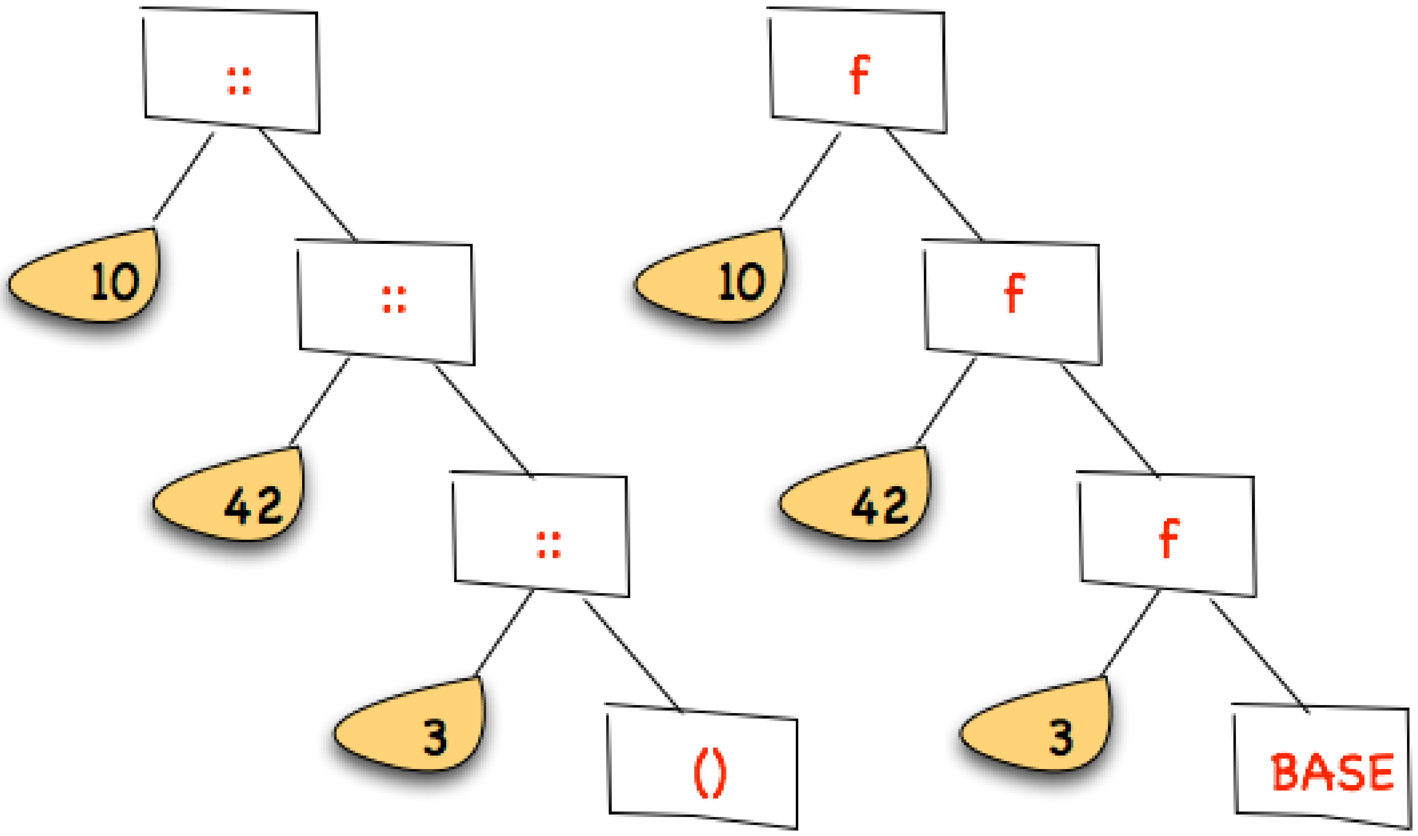
Quiz
Can you think of map as an instance of another recursion operator?

The `fold`*) Recursion Operator

*) aka **reduce**, **accumulate**, **compress** or **inject**

In Scala:

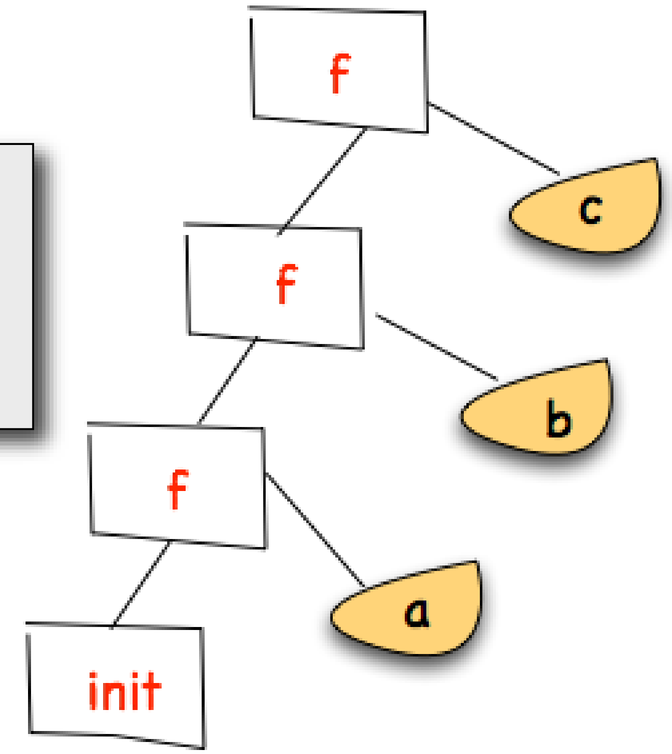
```
def fold[A, B](init: B, combine: (A, B) => B, l: List[A]): B =
 l match {
   case Nil => init
   case x :: xs => combine(x, fold(init, combine, xs))
 }
```

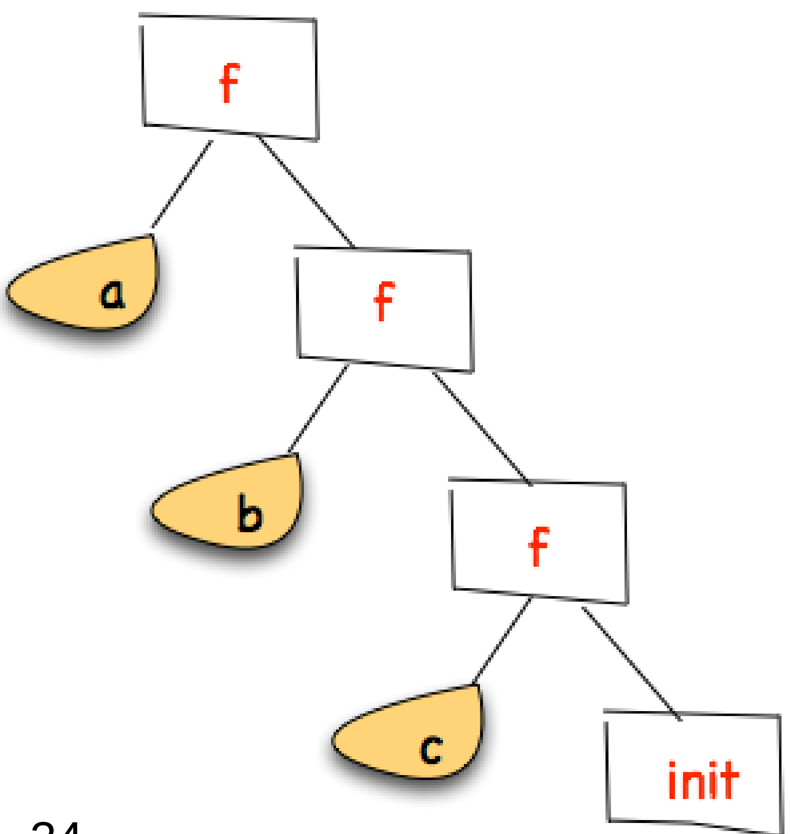


Folding in Scala

Two predefined operations: `foldLeft, foldRight:`

```
List(a, b, c).foldLeft(init)(f)
==
f(f(f(init a), b), c)
```






```
List(a, b, c).foldRight(init)(f)
==
f(a, f(b, f(c, init)))
```

Some Instantiations of `fold`

```
package templates

object FoldingTemp extends App {
  def summing(list: List[Int]): Int = ???
  def product(list: List[Int]): Int = ???
  def length[A](list: List[A]): Int = ???
  def reverse[A](list: List[A]): List[A] = ???
  def myMap[A, B] (f: A => B, list: List[A]): List[B] = ???
  def myFilter[A](p: A => Boolean, list: List[A]): List[A] = ???

  println(summing(List(1, 4, 5)))
  println(product(List(1, 4, 5)))
  println(length(List(1, 4, 5)))
  println(reverse(List(1, 4, 5)))
  println(myMap( (x:Int) => 2*x, List(1, 4, 5) ))
  println(myFilter( (x: Int) => x > 2, List(1, 4, 5) ) )
}
```

When Process Structure Follows Data …

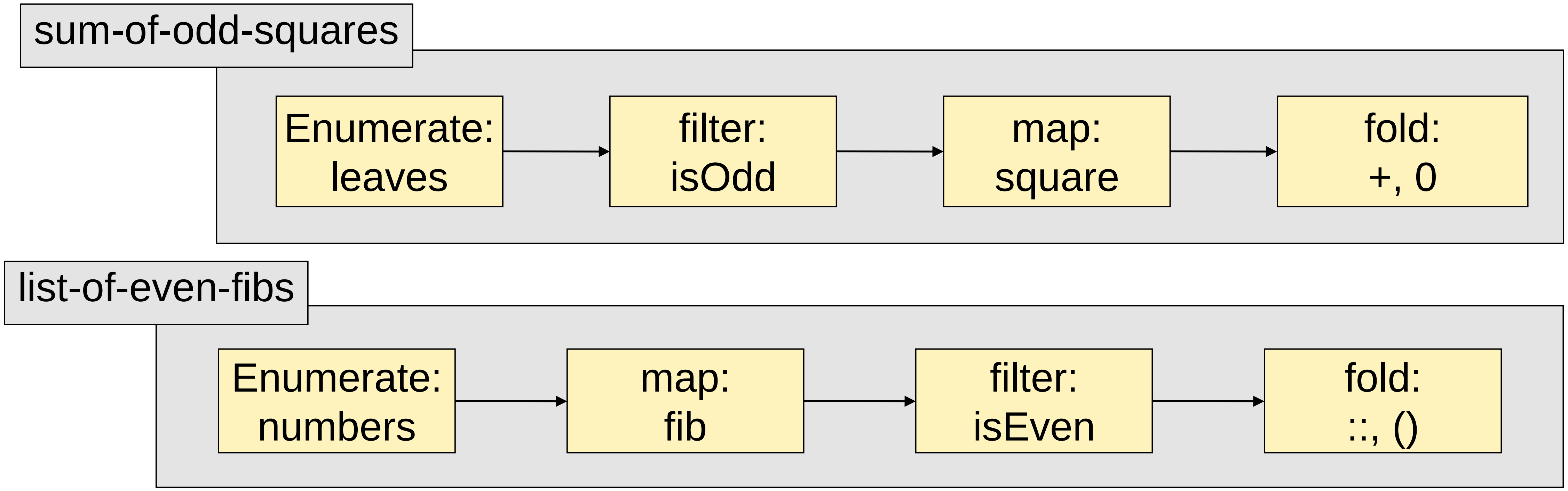
```
case class Tree(value: Int, branches: List[Tree])

def sumOfOddSquares(forest: List[Tree]): Int = forest match {
  case Nil => 0
  case x :: xs => x match {
    case Tree(value, branches) =>
      val restSum = sumOfOddSquares(branches) + sumOfOddSquares(xs)
      if (isOdd(value)) square(value) + restSum
      else restSum
  }
}
```

```
def evenFibs(n: Int): List[Int] = {
  def process(k: Int): List[Int] = {
    if (k > n) Nil
    else {
      val f = fib(k)
      if (isEven(f)) f :: process(k + 1)
      else process(k + 1)
    }
  }
  process(0)
}
```

Do you recognize any similarity between these two processes?

Focusing on the Process Structure…

- **`sumOfOddSquares`**:
  - enumerates the leaves of a tree
  - filters them to select the odd ones — **generic filter**
  - process each of the selected ones by squaring — **generic map**
  - folds the result list using +, starting with 0 — **generic fold**
- **`evenFibs`**:
  - enumerates the integers from 0 to n
  - process each integer by computing its fibonacci — **generic map**
  - filters them to select the even ones — **generic filter**
  - folds the result list using ::, starting with empty list — **generic fold**



Advantages of Functional Decomposition

Functional decomposition supports **program designs that are modular** by combining independent components, instantiated from recursion operators that can be provided in a library of standard components.

Modular construction is a powerful strategy for **controlling complexity** in engineering design

Outline

- Implementing first-order functions
- Environments (static vs. dynamic scoping)
- About first-class functions
- Functional decomposition and recursion patterns
- **Implementing first-class functions**

FWAE: A Language with First-Class Functions

- **Function definitions are expressions in FWAE:**
  - They can appear everywhere within other compound expressions
  - Functions are values just like numbers
  - Function values can be passed to and returned by other functions

- Examples of FWAE programs:

```
{{fun {x} {+ x x}} 3}

{let {inc {fun {x} {+ x 1}}}
    {+ {inc 4} {inc 5}}}

{let {x 3} {fun {y} {+ x y}}}
```

Concrete and Abstract Syntax of FWAE

```
<F1WAE> ::= <num>
          | {+ <F1WAE> <F1WAE>}
          | {- <F1WAE> <F1WAE>}
          | {let {<id> <F1WAE>} <F1WAE>}
          | <id>
          | {<id> <F1WAE>}
```

How will the concrete and abstract syntax of FWAE differ?

```scala
sealed abstract class F1WAE
case class Num(n: Int) extends F1WAE
case class Add(lhs: F1WAE, rhs: F1WAE) extends F1WAE
case class Sub(lhs: F1WAE, rhs: F1WAE) extends F1WAE
case class Let(name: Symbol, namedExpr: F1WAE, body: F1WAE) extends F1WAE
case class Id(name: Symbol) extends F1WAE
case class App(funName: Symbol, arg: F1WAE) extends F1WAE
```

Concrete and Abstract Syntax of FWAE

- Concrete syntax

```
<FWAE> ::= <num>
         | {+ <FWAE> <FWAE>}
         | {- <FWAE> <FWAE>}
         | {let {<id> <FWAE>} <FWAE>}
         | <id>
         | {fun {<id>} <FWAE>}
         | {<FWAE> <FWAE>}
```

- Abstract syntax

```
sealed abstract class FWAE
  case class Num(n: Int) extends FWAE
  case class Add(lhs: FWAE, rhs: FWAE) extends FWAE
  case class Sub(lhs: FWAE, rhs: FWAE) extends FWAE
  case class Let(name: Symbol, namedExpr: FWAE, body: FWAE) extends FWAE
  case class Id(name: Symbol) extends FWAE
  case class Fun(param: Symbol, body: FWAE) extends FWAE
  case class App(funExpr: FWAE, arg: FWAE) extends FWAE
```

Interpreting FWAE

- We first implement an interpreter for FWAE that employs substitution
  - To facilitate the comparison to WAE and F1WAE
  - To define a „reference“ specification of the semantics

- Next, we will replace substitutions with environments

Interpreting FWAE

- What does the interpreter produce, i.e., what are the values of FWAE?
- What needs to be done to turn WAE into FWAE?

FWAE with Substitution

1. Extend the class of values
   - Interpreters so far produced Scala integers.
   - The interpretation of FWAE expressions can also produce functions
   - First try: Return values of FWAE are **`Num`** or **`Fun`**

2. Add clauses for function definition expressions to the substitution procedure and the interpreter

3. Modify substitution and interpretation of application expressions

<fill in holes in the templates …>

FWAE with Environments

- Replace substitution with environment lookup
- Preserve static scoping

Question
To avoid dynamic scoping, in F1WAE with environments we used an empty environment for evaluating function applications.

Can we apply the same trick here?

FWAE with Environments

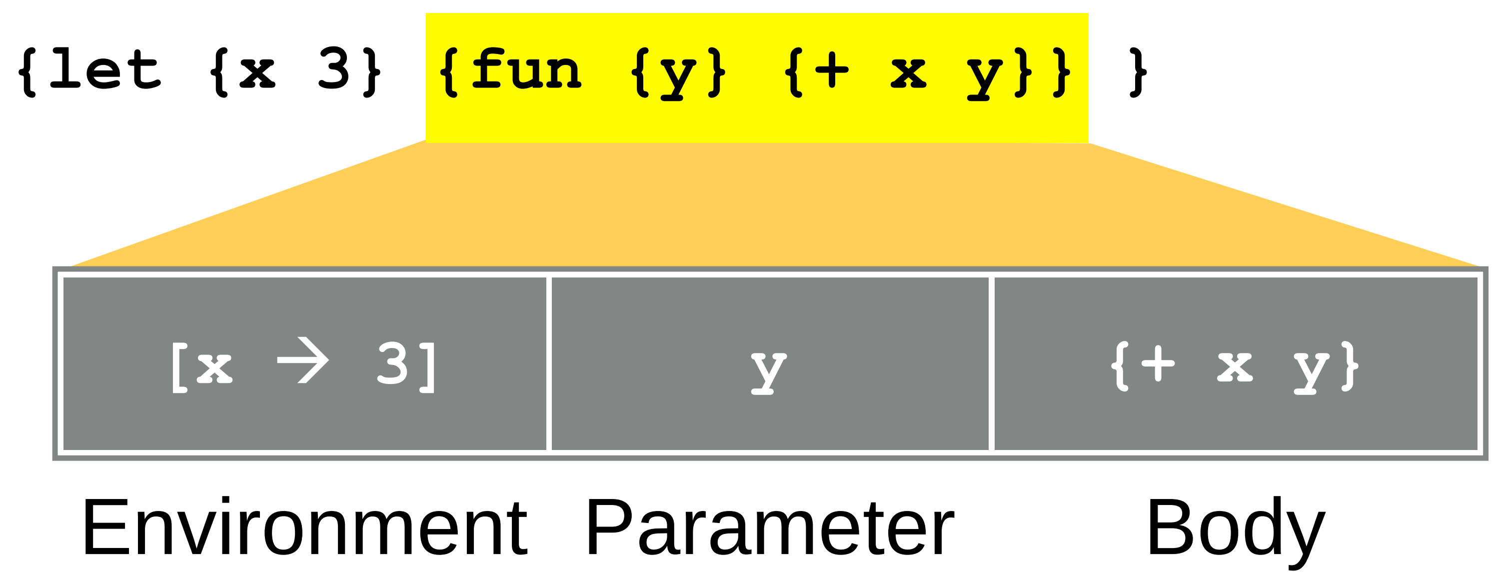
- If uncertain, compare manual evaluations of the following…

```
interp(App('f, Num(4)),
       Map('f -> FunDef('y, Add('x 'y))))
```

```
interp(Let('x, Num(3),
               App(Fun('y, Add('x 'y)) Num(4))),
       Map())
```

FWAE with Environments

- Remember environment at function-definition time, so that it can be used for binding free identifiers in function bodies when applying the function.
- Function + environment = **closure**



type Env = Map[Symbol, Value]

FWAE with Environments

1. Define a data type for representing FWAE values (closures)

2. Modify the definition of environments to use the new values

3. Modify the interpreter to:
   - use the environment for deferring substitutions
   - return closures as the result of evaluating function definitions
   - use the environment of the closure returned by evaluating the function sub-expression when evaluating function applications

FWAE with Environments

```
def interp(expr: FWAE, env: Env = Map()): Value = expr match {

...

}
```

<fill in holes in the template …>

Quiz

Do we really need **`let`** expressions in a language with first-class functions?