

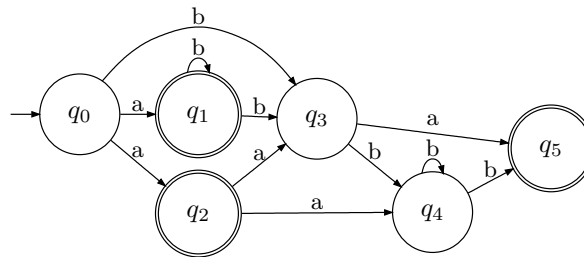


## Extra Exercises – Formal Foundations of Computer Science

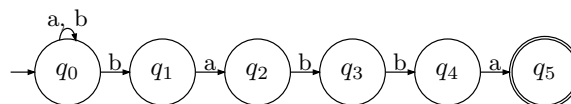
### (EE.1) [DFAs from NFAs, and minimisation]

Convert the following NFAs to a DFAs and minimise the resulting DFAs:

(i)



(ii)



### (EE.2) [regular and context-free languages]

Which of the following languages are regular? Which ones are context-free? Which one are neither? For regular languages, give either DFA, and NFA, or a regular expression. For non-regular context-free languages, give a context-free grammar or a PDA and show that the language is not regular (e.g. using Myhill-Nerode or the Pumping Lemma for regular languages). Otherwise, use the Pumping Lemma for context-free languages to show that the language is not context-free.

- $L_1 := \{w \in \{0,1\}^* \mid 2 \cdot |w|_0 = |w|_1\}$
- $L_2 := \{a^n b^m c^n d^m \mid n, m \geq 0\}$
- $L_3 := \{a^n b^m c^m d^n \mid n, m \geq 0\}$
- $L_4 := \{w \in \{0,1\}^* \mid w \text{ does not contain } 010 \text{ as a subword}\}$

### (EE.3) [context-free languages]

Consider the context-free grammar  $G = (\Sigma, V, P, I)$  with  $\Sigma = \{b, e, f, i, s\}$ ,  $V = \{I, L\}$ , and production rules

$$\begin{aligned} P : \quad I &\rightarrow bLe \mid i \mid fI \\ L &\rightarrow i \mid isL \end{aligned}$$

- Give an equivalent grammar in Chomsky Normal Form for  $G$ .
- Use the CYK algorithm to check that  $bisfbisiee \in L(G)$ , and find a syntax tree for this word.