Formal Specification and Verification of Object-Oriented Programs

Dynamic Logic

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(JAVA) Dynamic Logic

Typed FOL (with JAVA-like type system)

- + (JAVA) programs `p`
- + modalities $\langle p \rangle \phi$, $[p]\phi$ (`p` program, ϕ DL formula)
- + . . . (later)

Proving Validity of DL Formulas

An Example

$$
\begin{array}{l}
\forall\ \mathbf{int}\ x; \\
\quad (x \doteq \mathtt{n} \land x >= 0 \rightarrow \\
\qquad [\ \mathtt{i\ =\ 0;\ r\ =\ 0;} \\
\qquad\ \ \mathbf{while}\ \mathtt{(i<n)\{\ i\ =\ i\ +\ 1;\ r\ =\ r\ +\ i;\}} \\
\qquad\ \ \mathtt{r=r+r-n;} \\
\qquad]\mathtt{r} \doteq XXX)
\end{array}
$$

Proving Validity of DL Formulas

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An Example

$\forall$ **int** x;
  ($x \doteq$ `n` $\land\ x$ >= 0 $\rightarrow$
    [`i = 0; r = 0;`
     while `(i<n){ i = i + 1; r = r + i;}`
     `r=r+r-n;`
    ]`r` $\doteq x * x$)

Proving Validity of DL Formulas

An Example

$\forall$ **int** x;
  ($x \doteq \mathtt{n} \land x >= 0 \rightarrow$
    [`i = 0; r = 0;`
     while `(i<n){ i = i + 1; r = r + i;}`
     `r=r+r-n;`
    ]$\mathtt{r} \doteq x * x$)

How can we prove that the above formula is valid
(i.e., true in all program states)?

Semantics of DL Sequents

$\Gamma = \{\phi_1, \ldots, \phi_n\}$ and $\Delta = \{\psi_1, \ldots, \psi_m\}$ sets of DL formulas where all logical variables occur bound

Recall: $\mathcal{S} \models (\Gamma \Rightarrow \Delta)$ iff $\mathcal{S} \models (\phi_1 \wedge \cdots \wedge \phi_n) \rightarrow (\psi_1 \vee \cdots \vee \psi_m)$

Define semantics of DL sequents identical to semantics of FOL sequents

Definition (Validity of Sequents over DL Formulas)

A sequent $\Gamma \Rightarrow \Delta$ over DL formulas is valid iff

$$\mathcal{S} \models (\Gamma \Rightarrow \Delta) \text{ in all states } \mathcal{S}$$

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Consequence for program variables

Initial value of program variables implicitly "universally quantified"

Symbolic Execution of Programs

FOL sequent calculus decomposes top-level operator of a formula
What is “top-level” in a sequential program `p; q; r;` ?

Symbolic Execution

- Follow the natural control flow when analysing a program
- Values of some variables unknown: symbolic state representation

Symbolic Execution of Programs

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Example

Compute the final state after termination for initial state $\mathtt{x} = x_0$, $\mathtt{y} = y_0$

```
x=x+y; y=x-y; x=x-y;
```

Symbolic Execution of Programs Cont'd

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General form of rule conclusions in symbolic execution calculus

$$\langle \mathit{stmt}; \mathit{rest} \rangle \phi, \qquad [\mathit{stmt}; \mathit{rest}]\phi$$

- Rules symbolically execute *first* statement ("active statement")
- Repeated application of such rules corresponds to symbolic program execution

Symbolic Execution of Programs Cont'd

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- Rules symbolically execute *first* statement ("active statement")
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Example (`symbolicExecution/simpleIf.key`, Demo, active statement only)

```
\programVariables { int x; int y; boolean b; }

\problem {
 \<{ if (b) { x = 1; } else { x = 2; } y = 3; }\> y > x
}
```

Symbolic Execution of Programs Cont'd

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Symbolic execution of conditional

$$\text{if} \; \frac{\Gamma, \mathtt{b} \doteq \mathtt{TRUE} \Rightarrow \langle \mathit{p}; \mathit{rest} \rangle \phi, \Delta \qquad \Gamma, \mathtt{b} \doteq \mathtt{FALSE} \Rightarrow \langle \mathit{q}; \mathit{rest} \rangle \phi, \Delta}{\Gamma \Rightarrow \langle \mathit{if}\ (\mathit{b})\ \{\ \mathit{p}\ \}\ \mathit{else}\ \{\ \mathit{q}\ \}; \mathit{rest} \rangle \phi, \Delta}$$

Symbolic execution must consider all possible execution branches

Symbolic Execution of Programs Cont'd

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Symbolic execution of conditional

$$\text{if}\ \frac{\Gamma, \mathtt{b} \doteq \mathtt{TRUE} \Rightarrow \langle p; rest\rangle\phi, \Delta \qquad \Gamma, \mathtt{b} \doteq \mathtt{FALSE} \Rightarrow \langle q; rest\rangle\phi, \Delta}{\Gamma \Rightarrow \langle \mathit{if}\ (b)\ \{\ p\ \}\ \mathit{else}\ \{\ q\ \}; rest\rangle\phi, \Delta}$$

Symbolic execution must consider all possible execution branches

Symbolic execution of loops: unwind (simplified version)

$$\text{unwindLoop}\ \frac{\Gamma \Rightarrow \langle \mathbf{if}(b)\ \{\ p;\ \mathbf{while}(b)\ p\ \};\ \mathtt{rest}\rangle\phi, \Delta}{\Gamma \Rightarrow \langle \mathbf{while}(b)\ \{\ p\ \}; \mathtt{rest}\rangle\phi, \Delta}$$

Updates for KeY-Style Symbolic Execution

Need to model control flow and state changes

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Need to model control flow and state changes

Explicit Notation for Symbolic State Changes: Requirements

- Symbolic execution should "walk" through program in natural forward direction
- Need succint representation of state changes effected by each symbolic execution step
- Want to simplify effects of program execution early
- Want to apply state changes late (to post condition)

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We use dedicated notation for state changes: updates

Explicit State Updates

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Definition (Syntax of Updates, Updated Terms/Formulas)

If $\mathtt{v}$ is program variable, t FOL term type-conformant to $\mathtt{v}$,
t' any FOL term, and ϕ any DL formula, then

- $\mathtt{v} := t$ is an update
- $\{\mathtt{v} := t\}t'$ is DL term
- $\{\mathtt{v} := t\}\phi$ is DL formula

Definition (Semantics of Updates)

State $\mathcal{S}$ interprets program variables v with $\mathcal{I}_{\mathcal{S}}(v)$
β variable assignment for logical variables in t, define semantics ρ as:

$\rho(\mathtt{v} := t)(\mathcal{S}) = \mathcal{S}'$ where $\mathcal{S}'$ identical to $\mathcal{S}$ except $\mathcal{I}_{\mathcal{S}'}(\mathtt{v}) = val_{\mathcal{S},\beta}(t)$

Explicit State Updates Cont'd

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Facts about updates $\mathtt{v} := t$

- Update semantics almost identical to that of assignment
- Value of update also depends on $\mathcal{S}$ and logical variables in t, i.e., β
- Updates are not assignments: right-hand side is FOL term

  $\{\mathtt{x} := n\}\phi$ cannot be turned into assignment (n logical variable)

  $\langle x = i++;\rangle\phi$ cannot (immediately) be turned into an update
- Updates are not equations: they change value of $\mathtt{v}$

Computing Effect of Updates (Automatic)

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Rewrite rules for update followed by . . .

program variable	$\begin{cases} \{\mathtt{x} := t\}\mathtt{y} & \rightsquigarrow & \mathtt{y} & \mathtt{x} \neq \mathtt{y} \\ \{\mathtt{x} := t\}\mathtt{x} & \rightsquigarrow & t & \text{update happens} \end{cases}$
logical variable	$\{\mathtt{x} := t\}w \rightsquigarrow w$
complex term	$\{\mathtt{x} := t\}f(t_1, \ldots, t_n) \rightsquigarrow f(\{\mathtt{x} := t\}t_1, \ldots, \{\mathtt{x} := t\}t_n)$ (f function symbol)
FOL formula	$\begin{cases} \{\mathtt{x} := t\}(\phi \ \& \ \psi) \rightsquigarrow \{\mathtt{x} := t\}\phi \ \& \ \{\mathtt{x} := t\}\psi \\ \cdots \\ \{\mathtt{x} := t\}(\forall\, T\ y;\ \phi) \rightsquigarrow \forall\, T\ y;\ (\{\mathtt{x} := t\}\phi) \end{cases}$
program formula	No rewrite rule for $\{\mathtt{x} := t\}(\langle p \rangle \phi)$ unchanged!

Update rewriting delayed until $\mathtt{p}$ symbolically executed

Assignment Rule Using Updates

Symbolic execution of assignment using updates

$$\text{assign}\ \frac{\Gamma \Rightarrow \{\mathtt{x} := \mathtt{t}\}\langle \mathit{rest}\rangle\phi, \Delta}{\Gamma \Rightarrow \langle x = t; \mathit{rest}\rangle\phi, \Delta}$$

- Simple! No variable renaming, etc.
- Works as long as `t` has no side effects
- Special cases needed for `x` $= t_1 + t_2$, etc.

Demo

`updates/assignmentToUpdate.key` (complete)

Parallel Updates

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How to apply updates on updates?

Example

Symbolic execution of

```
x=x+y; y=x-y; x=x-y;
```

yields:

```
{x := x+y}{y := x-y}{x := x-y}
```

Need to compose three sequential state changes into a single one!

Parallel Updates Cont'd

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Definition (Parallel Update)

A parallel update is an expression of the form $v_1 := r_1 || \cdots || v_n := r_n$ where each $v_i := r_i$ is simple update

- All r_i computed in old state before update is applied
- Updates of all program variables v_i executed simultaneously
- Upon conflict $v_i = v_j, r_i \neq r_j$ later update ($\max\{i, j\}$) wins

Parallel Updates Cont'd

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Definition (Composition Sequential Updates/Conflict Resolution)

$\{v_1 := r_1\}\{u\}\xi \rightsquigarrow \{v_1 := r_1 || \{v_1 := r_1\}u\}\xi$ (ξ is a DL formula or DL term)

$\{v_1 := r_1\}v_2 := r_2 \rightsquigarrow v_2 := \{v_1 := r_1\}r_2$

$$\{v_1 := r_1 || \cdots || v_n := r_n\}\mathtt{x} \rightsquigarrow \begin{cases} \mathtt{x} & \text{if } \mathtt{x} \notin \{v_1, \ldots, v_n\} \\ r_k & \text{if } \mathtt{x} = v_k,\ \mathtt{x} \notin \{v_{k+1}, \ldots, v_n\} \end{cases}$$

Parallel Updates Cont'd

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Example

$$
\begin{array}{l}
(\{\mathtt{x} := \mathtt{x+y}\}\{\mathtt{y} := \mathtt{x-y}\})\{\mathtt{x} := \mathtt{x-y}\}\ \phi = \\
\{\mathtt{x} := \mathtt{x+y} \mid\mid \mathtt{y} := \mathtt{(x+y)-y}\}\{\mathtt{x} := \mathtt{x-y}\}\ \phi = \\
\{\mathtt{x} := \mathtt{x+y} \mid\mid \mathtt{y} := \mathtt{(x+y)-y} \mid\mid \mathtt{x} := \mathtt{(x+y)-((x+y)-y)}\}\ \phi = \\
\{\mathtt{x} := \mathtt{x+y} \mid\mid \mathtt{y} := \mathtt{x} \mid\mid \mathtt{x} := \mathtt{y}\}\ \phi = \\
\{\mathtt{y} := \mathtt{x} \mid\mid \mathtt{x} := \mathtt{y}\}\ \phi
\end{array}
$$

KeY automatically deletes overwritten (unnecessary) updates

Demo

`updates/swap2.key`

Parallel Updates Cont'd

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Example

```
({x := x+y}{y := x-y}){x := x-y} φ =
{x := x+y || y := (x+y)-y}{x := x-y} φ =
{x := x+y || y := (x+y)-y || x := (x+y)-((x+y)-y)} φ =
{x := x+y || y := x || x := y} φ =
{y := x || x := y} φ
```

KeY automatically deletes overwritten (unnecessary) updates

Demo

`updates/swap2.key`

Parallel updates to store intermediate state of symbolic computation

Another use of Updates

If you would like to quantify over a program variable ...

Another use of Updates

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If you would like to quantify over a program variable ...

Not allowed: $\forall\, T\ \texttt{i};\ \langle ...i... \rangle \phi$ (program $\neq$ logical variable)

Another use of Updates

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If you would like to quantify over a program variable ...

Not allowed: $\forall\, T\; \mathtt{i};\ \langle ...i... \rangle \phi$ (program $\neq$ logical variable)

Instead

Quantify over value, and assign it to program variable:

$\forall\, T\; i_0;\ \{\mathtt{i} := i_0\} \langle ...i... \rangle \phi$

Modelling JAVA in FOL: Fixing a Type Hierarchy

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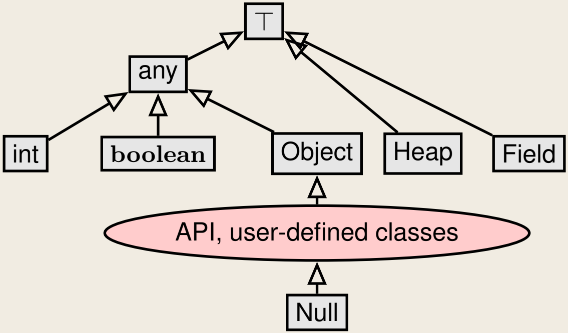
Signature based on JAVA's type hierarchy



Each interface and class in API and in target program becomes type with appropriate subtype relation

Modelling the Heap in FOL

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The JAVA Heap

Values of reference types live on the heap

- Can dynamically change during execution
- In each program state (model) associates objects, fields, values

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The JAVA Heap Model of KeY

Data type Heap models content of heap in a state (model)

Write Heap `store(Heap, Object, Field, any);`
Modifies field of object to have value in 4th argument

Read any `select(Heap, Object, Field);`
Selects value of field of object

Modelling Fields in FOL

Modelling instance fields

Person
int age int id
int setAge(int newAge) int getId()

- for each JAVA reference type C there is a logical type C, for example, Person

Modelling Fields in FOL

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- for each JAVA reference type C there is a logical type C, for example, Person
- for each field `f` there is a unique constant `f` of type Field, for example, `age`

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- a heap relates objects and fields to values

Modelling Fields in FOL

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- domain of all Person objects: D^{Person}
- a heap relates objects and fields to values

Reading Fields (Simplified)

Signature F_Σ: any `select`(Heap, Object, Field);

JAVA expression `p.age >= 0`

Typed FOL $\mathtt{select}(\mathtt{heap}, \mathtt{p}, \mathtt{age}) >= 0$

`heap` is special program variable for “current” heap

Modelling Fields in FOL—The Full Story

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Reading Fields

Signature F_Σ: any `select(Heap, Object, Field);`

$\texttt{select}(\texttt{heap}, \texttt{p}, \texttt{age}) >= 0$ well-formed?

Modelling Fields in FOL—The Full Story

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- Return type is any—need to cast to **int**
- There can be many fields with name $\mathtt{age}$

Modelling Fields in FOL—The Full Story

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$\mathtt{select}(\mathrm{heap}, \mathrm{p}, \mathrm{age}) >= 0$ well-formed?

- Return type is any—need to cast to $\mathbf{int}$
- There can be many fields with name age

Use function $\mathbf{int}::\mathtt{select}(\mathrm{heap}, \mathrm{p}, \text{Person}::\$\mathrm{age})$

($\mathbf{int}::\mathtt{select}$ has same meaning as $(\mathbf{int})\mathtt{select}$)

Modelling Fields in FOL—The Full Story

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Reading Fields

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Use function $\mathbf{int}::\mathtt{select}(\mathrm{heap}, \mathrm{p}, \text{Person}::\$\mathrm{age})$

($\mathbf{int}::\mathtt{select}$ has same meaning as $\mathtt{(}\mathbf{int}\mathtt{)select}$)

Writing to Fields

Signature F_Σ: Heap `store(Heap, Object, Field, any);`

Use function $\mathtt{store}(\mathrm{heap}, \mathrm{p}, \text{Person}::\$\mathrm{age}, 42)$

Modelling Fields in FOL—The Full Story: Semantics

`Heap` is a predefined type with predefined functions `T` :: `select`, `store` and more

Semantics

Given a Kripke structure K and a first order state $S = (D, \rho, I)$

- $D^{\mathtt{Heap}} = \{h \mid h : D^{\mathtt{Object}} \times D^{\mathtt{Field}} \to D^{\mathtt{any}}\}$
  (domain of `Heap` is set of functions associating objects and fields with a value)
- $I(\mathtt{select})(h, o, f) = h(o, f)$ with $h \in D^{\mathtt{Heap}}, o \in D^{\mathtt{Object}}, f \in D^{\mathtt{Field}}$
- $I(\mathtt{store})(h, o, f, v) = h'$ with $h \in D^{\mathtt{Heap}}, o \in D^{\mathtt{Object}}, f \in D^{\mathtt{Field}}, v \in D^{\mathtt{any}}$ and

$$h'(u, g) = \begin{cases} v & \text{if } u = o \text{ and } g = f \\ h(u, g) & \text{otherwise} \end{cases}$$

Field Updates

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The Global Program Variable `heap`

The dynamic logic contains a distinguished program variable Heap `heap`. The heap stored in this variable is used by the Java program for read/write field accesses.

Changing the value of fields

How to translate assignment to field, for example, `p.age=17;` ?

$$\text{assign}\ \frac{\Gamma \Rightarrow \{\mathtt{l} := t\}\langle \mathit{rest}\rangle\phi, \Delta}{\Gamma \Rightarrow \langle l = t; \mathit{rest}\rangle\phi, \Delta}$$

Admit on left-hand side of update JAVA location expressions

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Changing the value of fields

How to translate assignment to field, for example, `p.age=17;` ?

$$\text{assign}\ \frac{\Gamma \Rightarrow \{\mathtt{heap} := \mathtt{store}(\mathtt{heap}, \mathtt{p}, \mathtt{age}, 17)\}\langle \mathit{rest} \rangle \phi, \Delta}{\Gamma \Rightarrow \langle \mathit{p.age}\ = 17; \mathit{rest} \rangle \phi, \Delta}$$

Admit on left-hand side of update JAVA location expressions

Evaluating a field access

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Evaluation

Computing the value of a field *a* of an object *o* of type *T* in a given heap, performs a lookup in the heap using the pair (*o*, *a*) as key/index.

Example

$\mathtt{int}::\mathtt{select}(\mathtt{store}(\mathtt{heap}, \mathtt{o}, \mathtt{f}, 15), \mathtt{o}, \mathtt{f}) \leadsto$

Evaluating a field access

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$$\mathtt{int}::\mathtt{select}(\mathtt{store}(\mathtt{heap}, \mathtt{o}, \mathtt{f}, 15), \mathtt{o}, \mathtt{g}) \rightsquigarrow$$

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$$\mathtt{int}::\mathtt{select}(\mathtt{store}(\mathtt{heap},\mathtt{o},\mathtt{f},15),\mathtt{u},\mathtt{f}) \rightsquigarrow$$

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Example

$$\texttt{int::select}(\texttt{store}(\texttt{heap}, \texttt{o}, \texttt{f}, 15), \texttt{o}, \texttt{f}) \rightsquigarrow 15$$

$$\texttt{int::select}(\texttt{store}(\texttt{heap}, \texttt{o}, \texttt{f}, 15), \texttt{o}, \texttt{g}) \rightsquigarrow \texttt{int::select}(\texttt{heap}, \texttt{o}, \texttt{g})$$

$$\texttt{int::select}(\texttt{store}(\texttt{heap}, \texttt{o}, \texttt{f}, 15), \texttt{u}, \texttt{f}) \rightsquigarrow \text{if } (\texttt{o} \doteq \texttt{u}) \text{ then } (15) \text{ else } (\texttt{int::select}(\texttt{heap}, \texttt{u}, \texttt{f}))$$

Evaluating a field access

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Computing the value of a field *a* of an object *o* of type *T* in a given heap, performs a lookup in the heap using the pair (o, a) as key/index.

Example

$$\mathtt{int::select}(\mathtt{store}(\mathtt{heap},\mathtt{o},\mathtt{f},15),\mathtt{o},\mathtt{f}) \rightsquigarrow 15$$

$$\mathtt{int::select}(\mathtt{store}(\mathtt{heap},\mathtt{o},\mathtt{f},15),\mathtt{o},\mathtt{g}) \rightsquigarrow \mathtt{int::select}(\mathtt{heap},\mathtt{o},\mathtt{g})$$

$$\mathtt{int::select}(\mathtt{store}(\mathtt{heap},\mathtt{o},\mathtt{f},15),\mathtt{u},\mathtt{f}) \rightsquigarrow$$
$$\text{if } (\mathtt{o} \doteq \mathtt{u}) \text{ then } (15) \text{ else } (\mathtt{int::select}(\mathtt{heap},\mathtt{u},\mathtt{f}))$$

Pretty Printing

$\mathtt{T} :: \mathtt{select}(\mathtt{heap},\mathtt{o},\mathtt{f})$ is shown as $\mathtt{o.f}$

$\mathtt{select}(\mathtt{store}(\mathtt{heap},\mathtt{o},\mathtt{f},17),\mathtt{u},\mathtt{f})$ is shown as $\mathtt{u.f@heap[o.f} := 17]$

Evaluating a field access

Evaluation

Computing the value of a field *a* of an object *o* of type *T* in a given heap, performs a lookup in the heap using the pair (o, a) as key/index.

Example

On the following slides we often use the pretty printed version and omit the T :: prefix for ease of presentation.

$\mathtt{int::select(store(heap, o, f, 15), u, f)} \rightsquigarrow$

$$\text{if } (\mathtt{o} \doteq \mathtt{u}) \text{ then } (15) \text{ else } (\mathtt{int::select(heap, u, f)})$$

Pretty Printing

$\mathtt{T :: select(heap, o, f)}$ is shown as $\mathtt{o.f}$

$\mathtt{select(store(heap, o, f, 17), u, f)}$ is shown as $\mathtt{u.f@heap[o.f := 17]}$

Dynamic Logic: KeY input file


```
\javaSource "path to source code referenced in problem";

\programVariables { Person p; }

\problem {
       \<{   p.age = 18;  }\> p.age = 18
}
```

KeY reads in all source files and creates automatically
the necessary signature (types, program variables, field constants)

Dynamic Logic: KeY input file

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\programVariables { Person p; }

\problem {
       \<{   p.age = 18;  }\> p.age = 18
}
```

KeY reads in all source files and creates automatically
the necessary signature (types, program variables, field constants)

Demo

`updates/firstAttributeExample.key`

Semantics of Abrupt Termination

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Does abrupt termination count as normal termination?
No! Need to distinguish normal and exceptional termination

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- $[p]\phi$: If $\mathtt{p}$ terminates normally then formula ϕ holds in final state (partial correctness)

Abrupt termination counts as non-termination!

Example Reconsidered: Exception Handling


```
\javaSource "path to source code";

\programVariables {
  ...
}

\problem {
       p != null -> \<{   p.age = 18;  }\> p.age = 18
}
```

Only provable when no top-level exception thrown

Demo

`updates/secondAttributeExample.key`

Null Pointers

Null pointer exceptions

There are no “exceptions” in FOL: $\mathcal{I}$ total on FSym

Need to model possibility that $\mathtt{o} \doteq \mathbf{null}$ in $\mathtt{o.a}$

- KeY branches over $\mathtt{o}$ $!\!= \mathbf{null}$ upon each field access

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Assignment Rule for Fields

$$\text{assign}_{\text{Field}} \quad \frac{\begin{array}{l}\Gamma, \{u\}(o \mathrel{!=} \mathbf{null}) \Rightarrow \{u\}\{\mathtt{heap} := \mathtt{store}(\mathtt{heap}, o, f, v)\}\langle rest\rangle\phi, \Delta \\ \Gamma, \{u\}(o \doteq \mathbf{null}) \Rightarrow \{u\}\langle \mathbf{throw\ new}\ \mathtt{NullPointerException();}\, rest\rangle\phi, \Delta\end{array}}{\Gamma \Rightarrow \{u\}\langle o.f \;=\; v;\, rest\rangle\phi, \Delta}$$

o, *v* schemavariables matching prog. variables; *f* schemavariable matching fields

Aliasing

Demo

`aliasing/attributeAlias1.key`

Aliasing

Demo

`aliasing/attributeAlias1.key`

Reference Aliasing

Naive alias resolution causes proof split (on $\mathtt{o} \doteq \mathtt{u}$) at each access

$$\Rightarrow \mathtt{o.age} \doteq 1 \quad \text{-->} \quad \langle \mathit{u.age} = 2; \rangle \mathtt{o.age} \doteq \mathtt{u.age}$$

Aliasing Cont'd

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Unnecessary case analyses

$$\Rightarrow \mathtt{o.age} \doteq 1 \;\; \texttt{->} \;\; \langle u.age = 2; o.age = 2; \rangle \mathtt{o.age} \doteq \mathtt{u.age}$$

$$\Rightarrow \mathtt{o.age} \doteq 1 \;\; \texttt{->} \;\; \langle u.age = 2; \rangle \mathtt{u.age} \doteq 2$$

Aliasing Cont'd

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Avoiding case analyses— Demo

`aliasing/avoidingCaseAnalysis2.key`

- Delayed state computation until clear what is required
- Simplification of heap terms

Modeling Static Fields in FOL

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Modeling class (static) fields which are not compile-time constants

For each class C with static field `a` of type T that is not a compile time constant, there is

- a unique constant $C.a$ of type Field
- value v is stored on heap as $\mathtt{store}(\mathtt{heap}, \mathtt{null}, \mathtt{C.a}, \mathtt{v})$
- C is the fully qualified class name

Value stored on the heap.

Modeling class (static) fields which are compile-time constants

For each class C with final static field `a` of type T that is a compile time constant, there is

- a constant $C.a$ of type T whose interpretation $I(C.a)$ is fixed, e.g., $I(\mathtt{java.util.Byte.MAX\_VALUE}) = 127$

Value not stored on the heap

`this` **and** `self`

Modeling reference **this** to the receiving object

Special name for the object whose JAVA code is currently executed:

in JML: `Object` **this**;

in JAVA: `Object` **this**;

in KeY: Arbitrary, but by convention `Object self;`
Name is arbitrary as long as unique: only a program variable

Default assumption in JML-KeY translation: `self` != **null**

Extending Dynamic Logic to Java

KeY admits any syntactically correct JAVA with some extensions:

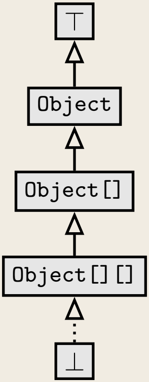
- Needs not be compilable unit
- Permit externally declared, non-initialized variables
- All referenced class definitions loaded in background

And some limitations . . .

- Limited concurrency (some support on side-branches)
- No generics
- No I/O
- No floats
- No dynamic class loading or reflexion
- API method calls: need either JML contract or implementation

JAVA Features in Dynamic Logic: Arrays

Arrays



- JAVA type hierarchy includes array types that occur in given program (for finiteness)
- Types ordered according to JAVA subtyping rules
- Value of entry in array `T[] a;` defined in class `C` depends on reference `a` to array in `C` and index
- Function $\mathtt{arr} : \mathrm{int} \to \mathrm{Field}$ injective mapping from indices to fields
- Store array elements on heap, e.g., the value of $\mathtt{a}[\mathtt{i}]$ on the heap $\mathtt{store}(\mathtt{heap}, \mathtt{a}, \mathtt{arr}(\mathtt{i}), 17)$ is 17
- Arrays `a` and `b` can refer to same object (aliases)
- KeY implements simplification and evaluation rules for array locations

JAVA Features in Dynamic Logic: Complex Expressions

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Complex expressions with side effects

- JAVA expressions may contain assignment operator with side effect
- JAVA expressions can be complex, nested, have method calls
- FOL terms have no side effect on the state

Example (Complex expression with side effects in JAVA)

`int i = 0; if ((i=2)>= 2) i++;` value of `i` ?

Complex Expressions Cont'd

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Decomposition of complex terms by symbolic execution

Follow the rules laid down in JAVA Language Specification

Local code transformations

$$\text{evalOrderIteratedAssgnmt}\quad \frac{\Gamma \Rightarrow \langle y = t; x = y;\ \omega\rangle\phi, \Delta}{\Gamma \Rightarrow \langle x = y = t;\ \omega\rangle\phi, \Delta}\quad \texttt{t}\ \text{simple}$$

Temporary variables store result of evaluating subexpression

$$\text{ifEval}\quad \frac{\Gamma \Rightarrow \langle \mathbf{boolean}\ v0; v0\ = b; \mathbf{if}\ (v0)\ p; \omega\rangle\phi, \Delta}{\Gamma \Rightarrow \langle \mathbf{if}\ (b)\ p;\ \omega\rangle\phi, \Delta}\quad \texttt{b}\text{complex}$$

Guards of conditionals/loops always evaluated (hence: side effect-free) before conditional/unwind rules applied

JAVA Features in Dynamic Logic: Abrupt Termination

Abrupt Termination: Exceptions and Jumps

Redirection of control flow via **return**, **break**, **continue**, exceptions

$$\langle\pi\ \mathbf{try}\ \{p\}\ \mathbf{catch}(e)\ \{q\}\ \mathbf{finally}\ \{r\}\ \omega\rangle\phi$$

Rules ignore inactive prefix, work on **active statement**, leave postfix

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Rule tryThrow matches $\mathbf{try}$–$\mathbf{catch}$ in pre-/postfix and active $\mathbf{throw}$

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Demo: `exceptions/try-catch.key`

Which Objects do Exist?

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How to model object creation with **new** ?

Which Objects do Exist?

How to model object creation with $\mathbf{new}$?

Constant Domain Assumption

Assume that domain $\mathcal{D}$ is the same in all states of LTS $K = (S, \rho)$

Desirable consequence:
Validity of rigid FOL formulas unaffected by programs containing $\mathbf{new}$()

$$\models \forall\, T\ x;\ \phi \ {->}\ [p](\forall\, T\ x;\ \phi) \qquad \text{is valid for rigid } \phi$$

Object Creation

Realizing Constant Domain Assumption

- Implicitly declared field `boolean <created>` in class `java.lang.Object`
- Equal to **true** iff argument object has been created
- Object creation modeled as $\{$`heap` := `create(heap, o)`$\}$ for next “free” o (essentially sets `<created>` field of o to **true**)
- Normal heap function *store* “cannot” set value of field `<created>`

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ObjectCreation(simplified)

$$\frac{\Gamma,\ \{u\}(\texttt{select}(\texttt{heap},\texttt{newObj},\texttt{Object}::\texttt{<created>}) = \texttt{FALSE}) \Rightarrow \{u\}(\{\texttt{heap} := \texttt{create}(\texttt{heap},\texttt{newObj})\}\{\texttt{o}:=\texttt{newObj}\}\langle\pi\ \texttt{o.<init>(}\mathit{selist}\texttt{);}\ \omega\rangle\phi),\ \Delta}{\{u\}(\langle\pi\ \texttt{o = } \mathbf{new}\ \texttt{T(}\mathit{selist}\texttt{);}\ \omega\rangle\phi)}$$

`newObj` is a fresh program variable

Summary

- Most JAVA features covered in KeY
- Several of remaining features available in experimental version
  - Simplified multi-threaded JMM
  - Floats
- Degree of automation for loop-free programs is very high
- Symbolic execution paradigm lets you use KeY w/o understanding details of logic

Remaining Questions

- How to deal with method calls?
  - Inlining method bodies vs. using contracts
  - Proof obligations from method contracts
- Proving loops

Literature for this Lecture

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Essential

KeY Book Verification of Object-Oriented Software (see course web page), Chapter 10: Using KeY

KeY Book Verification of Object-Oriented Software (see course web page), Chapter 3: Dynamic Logic, Sections 3.1, 3.2, 3.4, 3.5, 3.6.1, 3.6.2, 3.6.3, 3.6.4, 3.6.7