Fachbereich Mathematik
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TECHNISCHE UNIVERSITÄT DARMSTADT

Winter Term 2015/16

Mock Exam – Formal Foundations of Computer Science

(EM.1) [multiple choice, 8 credits]

Decide whether the following statements are true or false in general. No justification is needed.

(i) For regular languages $L_1, L_2, L_3 \subseteq \{0,1\}^*$ one can algorithmically check if $L_1 = L_2 \cap L_3$.

(ii) If L is a regular language and $M \subseteq L$, then M is regular.

(iii) If L is a regular language and $M \subseteq L$, then M is context-free.

(iv) If $L_1, L_2 \subseteq \Sigma^*$ are context-free, then $L_1 \cap L_2$ is decidable (i.e. one can algorithmically check for $w \in \Sigma^*$ whether $w \in L_1 \cap L_2$).

(v) If $L_1, L_2 \subseteq \Sigma^*$ are context-free, then $L_1 \cap L_2$ is also context-free.

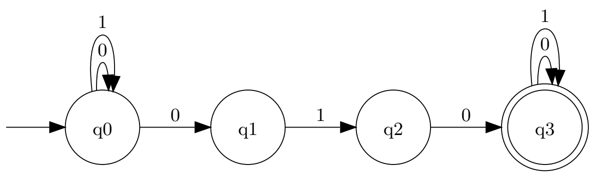
(vi) Every context-free grammar is context-sensitive.

(vii) Every context-free language is context-sensitive.

(viii) If L is context-sensitive, then L^* is context-sensitive.

(EM.2) [minimisation/conversion, 12 credits]

Consider the following NFA with four states:



(i) Convert this NFA into a DFA.

(ii) Minimise the resulting DFA.

(EM.3) [regular languages, 12 credits]

For a word $a_1a_2\dots a_\ell \in \{0,1\}^*$ let

$$f(a_1a_2\dots a_\ell) := \sum_{i=1}^{\ell} a_i 2^{i-1},$$

and $f(\epsilon) := 0$. In other words, $f(w)$ is the numerical value of w if it is interpreted as a natural number in binary representation, with the least significant bit first.

(i) Show that the language

$$L_1 := \{a_1b_1c_1a_2b_2c_2\dots a_\ell b_\ell c_\ell \mid f(a_1\dots a_\ell) + f(b_1\dots b_\ell) = f(c_1\dots c_\ell)\} \subseteq \{0,1\}^*$$

is regular by giving a DFA that accepts L_1.

(ii) Show that the language

$$L_2 := \{uvw \mid u,v,w \in \{0,1\}^n \text{ for some } n \geq 0 \text{ and } f(u) + f(v) = f(w)\} \subseteq \{0,1\}^*$$

is *not* regular.

(EM.4) [rotated regular language, 10 credits]

For a language $L \subseteq \Sigma^*$ we define $\mathrm{Rot}(L)$ as

$$\mathrm{Rot}(L) := \{a_2a_3\dots a_\ell a_1 \mid a_1a_2\dots a_\ell \in L\}.$$

Show that if L is regular, then $\mathrm{Rot}(L)$ is regular as well.

(EM.5) [context-free grammar, 12 credits]

Let $G = (\{f,g,x\},\{T\},P,T)$ be the context-free grammar with production rules

$$T \to x \mid fT \mid gTT$$

(i) Convert this grammar into Chomsky normal form.

(ii) Use the CYK algorithm to check that $fgfxx \in L(G)$.