

Additional Exercises



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Problem 1 First-Order Logics

Problem 1.1 FOL Calculus

Prove the validity of the following untyped first-order formula

$$\begin{aligned} & (\forall x; \forall y; \forall z; (p(x, y) \wedge p(y, z) \rightarrow p(x, z)) \wedge \\ & \quad \forall x; \forall y; (p(x, y) \rightarrow \neg p(y, x)) \wedge \\ & \quad \exists x; (p(a, x) \wedge p(x, b)) \\ &) \rightarrow \neg p(b, a) \end{aligned}$$

(with a, b being constant symbols and p a binary predicate symbol) using the sequent calculus. You are only allowed to use the rules presented in the lecture.

Provide the name of each rule used in your proof as well as the resulting sequent and make clear on which sequent you have applied the rule. For the quantifier rules, justify that the side conditions on substituted terms are fulfilled.

To save space and time you can introduce abbreviations for some formulas.

Solution:

See file `fol.key.proof` Justifications: `exists` introduces new symbols (here: x_0) and `forall` uses existing terms (here: a, b, x_0)

Hint: If you want to generate more assignments of this type, try to formalize a (mathematical) property e.g. about relations in FOL and then verify it. You can check your solution by writing a `.key` file with your problem and by replaying your proof within KeY.

Problem 1.2 FOL Formalisation

People from the town of Liarsville always tell lies. Of the following people below, only one is not from Liarsville. Can you tell which one is the outsider based on the following statements made?

Mr. Applebee: I am very honest.

Mrs. Beatle: Dr. Doodle is not from Liarsville.

Ms. Casey: I am a liar.

Dr. Doodle: Mrs. Beatle does not lie.

Mr. Eastwood: I am from the East.

Formalize the above riddle as a set of axioms Ax as well as your solution Sol as a formula in first-order logic (with sort `Person` as only additional sort next to `⊤`) such that $(\bigwedge_{\phi \in Ax} \phi) \rightarrow Sol$ is valid if and only if your solution is correct.

Try to prove your solution using KeY.

Solution:

see files `liarsvilleXXX.key` for possible solutions.

Problem 2 Dynamic Logics

Problem 2.1 Formalisation

Formalise the following statement in DL:

Let A be a class with an integer instance field a and let o be a program variable of type A.

- Execution of the program p terminates and in its final state the following holds: If for all created objects of type A their field a has a positive value then program q terminates.

Solution:

- $\langle p \rangle ((\forall A o; o.a > 0) \rightarrow \langle q \rangle \text{true})$

Problem 2.2

Are the following rules sound and/or complete? Give a counterexample if the rule is unsound.

a)

$$\frac{\Gamma, A \Rightarrow \Delta}{\Gamma, A \wedge (B \vee C) \Rightarrow \Delta}$$

b)

$$\frac{\Gamma, A \wedge B \Rightarrow \Delta}{\Gamma, A \wedge (B \vee C) \Rightarrow \Delta}$$

Solution:

- a) Sound but not complete.
- b) Unsound. Application of the rule allows to prove the sequent

$$isANumber(4) \wedge (odd(4) \vee even(4)) \Rightarrow odd(4)$$

(with isANumber, odd and even are predicates), which is obviously not valid, e.g., when interpreting even, odd, isANumber as implied by name.

Problem 2.3

Simplify the following formula (P is a binary predicate) as far as possible by only using the update simplification rules from the lecture. Provide each intermediate step.

$$\{i := j\} \{j := j + i \mid i := 4\} P(i, j)$$

Solution:

Several solutions either merge updates first to a parallel update and apply update then or apply update successively.

See e.g. file `updateSimp.key.proof` (note: using the rules on the slides, the nodes 9,10 as well as 11,12 are done in one step using the last rule of "Definition (Composition Sequential Updates/Conflict Resolution)")

Problem 3 Java Modelling Language

All specifications have to be given in the Java Modeling Language (JML).

Consider the Java classes `Interval` and `IntervalSeq`:

```
public class Interval {
    private final int start, end;

    public Interval(int start, int end) {
        this.start = start;
        this.end = end;
    }

    public int getStart() {
        return start;
    }

    public int getEnd() {
        return end;
    }

    public boolean contains(int e) {
        // ...
    }
}

/** Class to represent sequence of intervals. */
public class IntervalSeq {

    protected int size = 0;
    protected Interval[] contents = new Interval[1000];

    /** Insert a new element in the sequence;
     *  it is not specified in which place the element will be inserted
     */
    public void insert(Interval iv) {
        // ...
    }
}
```

In the following, observe the usual restrictions under which Java elements can be used in JML specifications.

Problem 3.1

Specify in JML that for class `Interval` the value returned by `getEnd()` is always greater than the value returned by method `getStart()`.

Problem 3.2

Method `contains` shall return `true` if and only if the given element lies within the interval. An interval contains all elements which are between `start` (inclusive) and `end` (exclusive). The method terminates always normally and is side-effect-free.

Problem 3.3

In class `IntervalSeq`, the field `size` holds the number of `Interval` objects which have yet been inserted into the `IntervalSeq` object. All inserted `Interval` objects are stored contiguously from the beginning of the array. The remaining cells of the array are `null`.

Augment class `IntervalSeq` with a JML specification stating the following:

- The `size` field is never negative and smaller than the length of the `contents` array.
- The cells of the array `contents` which are stored below index `size` are never `null`.
- Each `IntervalSeq` object has a different `contents` array.
- If the value of `size` is strictly smaller than `contents.length`, then all of the following must hold:
 - `insert` terminates normally
 - `insert` increases `size` by one
 - After `insert(iv)`, the interval `iv` is stored in `contents` at some index `i` below `size`. Below index `i`, the array `contents` is unchanged. The elements stored in between `i` and `size` were shifted one index upwards (as compared to the old `contents`).
- If the `size` has reached `contents.length`, `insert` will throw an `IndexOutOfBoundsException`.
- Provide also the assignable clause as precise as possible.

Solution:

(a)

```
public class Interval {  
  
    private /*@ spec_public @*/ final int start, end;  
  
    /*@ public invariant getEnd() > getStart();  
       @*/  
  
    public Interval(int start, int end) {  
        this.start = start;  
        this.end = end;  
    }  
  
    public /*@ pure @*/ int getStart() {  
        return start;  
    }  
  
    public /*@ pure @*/ int getEnd() {  
        return end;  
    }  
}
```

```

    /*@ public normal_behavior
        @ requires true;
        @ ensures e >= getStart() && e < getEnd();
        @*/
    public /*@ pure @*/ boolean contains(int e) {
        // ...
    }
}

(b)
/**
 * Class to represent sequence of intervals.
 */
public class IntervalSeq {

    /*@ public invariant size >= 0;

    protected /*@ spec_public @*/ int size = 0;

    /*@ public invariant contents.length >= size;
    /*@ public invariant contents != null;
    /*@ public invariant (\forallall int i;
    /*@
        0 <= i && i < size;
    /*@
        contents[i] != null);

    protected /*@ nullable spec_public @*/ Interval[] contents
        = new Interval[1000];

    /**
     * Insert a new element in the sequence; it is not specified
     * in which place the element will be inserted
     */
    /*@
        @ public normal_behavior
        @ requires size < contents.length;
        @ ensures size == \old(size) + 1;
        @ ensures (\exists int i;
        @
            0 <= i && i < size;
        @
            contents[i] == iv
        @
            && (\forallall int j; 0 <= j && j < i;
        @
            contents[j] == \old(contents[j]))
        @
            && (\forallall int k; i < k && k < size;
        @
            contents[k] == \old(contents[k-1]));
        @ assignable contents, contents[*, size];
        @
        @ also
        @
        @ public exceptional_behavior
        @ requires contents.length == size;
        @ signals_only IndexOutOfBoundsException;
        @*/
    public void insert(Interval iv) {
        // ...
    }
}

```

Problem 4 Loop Invariants

Problem 4.1

The file `Snippet.java` contains a method with a loop. Provide a loop invariant, decreases term (variant) and assignable clause that are strong enough to prove the method contract.

Hint: `\old(e)` can be used in an invariant. It evaluates `e` always in the prestate of the method.

Creating your own examples: If you want to exercise more loops, write your own small examples and use KeY to ensure the correctness of your solution.

Problem 4.2

Given the following sequent with `i` denoting an integer typed program variable:

$$\Gamma, i > 0 \Rightarrow \{i := 10\}[\text{while } (i > 0) \{i = i - 1;\}]i \dot{=} 0, \Delta$$

Let $i \geq 0$ be the invariant. Give the sequent for the preserves case of the improved invariant, as it looks directly after application of the loop invariant rule.

Solution:

$$\Gamma, i > 0, \{i := 10\}\{i := c\}(i \geq 0 \wedge i > 0) \Rightarrow \{i := 10\}\{i := c\}[\{i = i - 1;\}]i \geq 0, \Delta$$

with `c` is a fresh constant of the same static type as `i`

Problem 4.3 Recursion & Linked Data Structure

File `Tree` contains an implementation of a *binary* tree data structure, i.e., both children are either null or non-null. Specify all methods as complete as possible (incl. assignable). Specify as part of the invariant that the tree is actually a tree (i.e., no cycles). In case of recursion specify also a `measured_by` clause. Provide accessible clauses for the invariant and all methods.

Hint: Proving this specification is hard and requires some non-trivial interaction. So just check your spec against the provided solution and remember there is more than one way to express the same property. In doubt, use the forum to get opinions on your solution.

Solution:

see file `Tree.java`

Problem 4.4

Explain in your own words under which circumstances and why using a dependency contract might ease verification that a method `m` preserves the invariant of its class if an accessible clause for the invariant is given.

Solution:

If `m` does not change any locations mentioned in the accessible clause of the invariant, it can obviously not invalidate the invariant. This line of reasoning is provided when using dependency contracts, it is not necessary to actually unpack the invariant and prove that it is re-established. Instead simplify the heap expression provided as an argument to the invariant predicate using a dep. contract. This achieves that on both sides of the sequent the same formula e.g. `java.lang.Object :: <inv> (heap, self)` occurs and the goal can be closed directly.

Additional exercise: Create such an example and verify it in KeY using dependency contracts.
