Assignment: basic mathematics for TLA+

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Due: May 04, 2015

Your full name: XXX
Your login name: XXX
Your student number: xxxxxx

Note: Students who use LaTeX to write answer and submit the LaTeX Source + PDF version can get extra point as bonus in the exam. The submission should be zipped with name starting with student number, e.g. 2345678_LastName_FirstName.zip

1. Absolute Value

   Definition 1. For a number x, the **absolute value function** $|x|$ is defined by

   $$|x| := \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{otherwise} \end{cases}$$

   Prove some properties of real function. (Hint: practice case analysis)

   Theorem 1. *For every real number x, $|x| \geq 0$.*

   Theorem 2. *$|x| \geq x$ for all x.*

   Theorem 3 (Triangle Inequality)**.** *For every a, b real numbers, we have*

   $$|a + b| \leq |a| + |b|$$

2. Proof by contradiction

   Theorem 4. *For real numbers x, y, If $x > y$ then $\sqrt{x} > \sqrt{y}$*

3. Set construction:

   (1) If a set A has n members, how many members does the power set of A has?

   (2) Can you build a set of all sets? How does it look like?

4. Prove the following theory

   Theorem 5. *If $f : A \to B$ and $X, Y \subseteq A$ then*

   - $f[X \cup Y] = f[X] \cup f[Y]$
   - $f[X \cap Y] \subseteq f[X] \cap f[Y]$