Fachbereich Mathematik
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TECHNISCHE UNIVERSITÄT DARMSTADT

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Exercise Sheet no. 3 – Formal Foundations of Computer Science

(E3.1) [minimisation of DFAs]

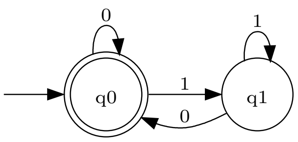
Let the DFA $A = (\{a, \ldots, h\}, \{0, 1\}, \delta, a, \{c\})$ have transition function δ given by

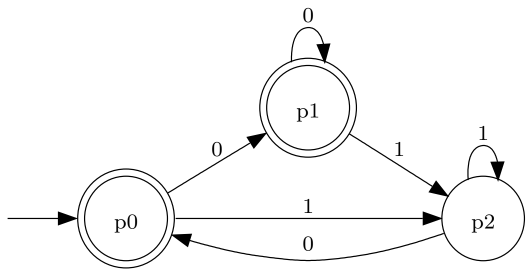
	a	b	c	d	e	f	g	h
0	b	g	a	c	h	c	g	g
1	f	c	c	g	f	g	e	c

Find a minimal DFA accepting the same language as A.

(E3.2) [equivalence of automata]

In the lecture we saw a method for finding pairs of equivalent states of a DFA. We used it to minimise automata, but it can also be applied to testing whether two DFAs A and B are equivalent or not, i.e. whether $L(A) = L(B)$: Compute pairs of equivalent states for the automaton $A \oplus B$, which is just the two automata A and B put next to each other. If the initial state of A and the initial state of B are equivalent, then $L(A) = L(B)$, otherwise $L(A) \neq L(B)$. Use this to check whether the following two automata are equivalent:





(E3.3) [reverse and half languages]

(i) For a language $L \subseteq \Sigma^*$, the *reverse language* L^{rev} is defined to be

$$L^{\text{rev}} := \{w = a_1 \dots a_\ell \mid a_\ell \dots a_1 \in L\}.$$

Show that if L is regular, then L^{rev} is regular as well.

Hint: This is easy if L is accepted by an NFA with just one accepting state. How can you get the result for general NFA from this?

(ii) For a language $L \subseteq \Sigma^*$ let $\frac{1}{2}L$ be the language

$$\{x \in \Sigma^* \mid xy \in L \text{ for some } y \in \Sigma^* \text{ with } |x| = |y|\}.$$

Prove that if L is regular, then $\frac{1}{2}L$ is regular as well.

Hint: Combine an NFA for L and an NFA for L^{rev}.

(E3.4) [regular or not?]

Which of these languages are regular? Justify your answers, e.g. by giving an automaton/a regular expression or by using the Pumping Lemma or the theorem of Myhill and Nerode.

- $L_1 := \{a^k b^\ell c^m \mid m = k + \ell\}$
- $L_2 := \{a^k b^\ell c^m \mid m \equiv k + \ell \pmod 5\}$, where $m \equiv k + \ell \pmod 5$ means that m and $k + \ell$ leave the same remainder after division by 5.
- $L_3 := \{w \in \{a, b\}^* \mid |w|_a = |w|_b\}$
- $L_4 := \{w \in \{a, b\}^* \mid w \text{ contains a substring } abba \text{ but not } baba\}$

(E3.5) [context-free languages]

Show that the following languages are context-free by giving a context-free grammar for each of them. Also, bring your grammars into Chomsky normal form.

$$\begin{aligned} L_1 :=& \{w \in \{a, b\}^* \mid |w|_a = |w|_b \geq 1 \text{ and } |w'|_a \geq |w'|_b \text{ for all prefixes } w' \text{ of } w\} \\ =& \{ab, aabb, abab, aaabbb, aabbab, ababab, abaabb, \dots\} \\ L_2 :=& \{a^k b^\ell c^m \mid m = k + \ell\} \end{aligned}$$