Assignment: basic mathematics for TLA+: continued

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Due: May 17 , 2015

Your full name: XXX
Your login name: XXX
Your student number: xxxxxx

Note: Students who use LaTeX to write answer and submit the LaTeX Source + PDF version can get extra point as bonus in the exam. The submission should be zipped with name starting with student number, e.g. 2345678_LastName_FirstName.zip

1. Propositional logic

   Definition 1. For given boolean variables x and y, the **de morgan rule** can be expressed as

   - $\neg(x \wedge y) \equiv (\neg x) \vee (\neg y)$
   - $\neg(x \vee y) \equiv (\neg x) \wedge (\neg y)$

   One can rewrite the expression $x \to y$ to $\neg x \vee y$. The unary boolean operator $\neg$ binds stronger than binary operators like $\wedge$ or *vee*.

   A formula is in *Negation Normal Form (NNF)* if the negation operator ($\neg$, not) is only applied to variables and the only other allowed Boolean operators are conjunction ($\wedge$, and) and disjunction ($\vee$, or).

   Transform the following boolean expression to Negation Normal Form

   (a) $\neg(x \to (x \to y))$
   (b) $\neg(x \wedge \neg y) \to \neg(x \leftrightarrow \neg y)$

2. First order logic

   Definition 2. $\forall$ is a conjunction over the universe of objects and $\exists$ is a disjunction over the universe of objects. Therefore, de morgan rule can be applied on first order quantifiers $\forall$, $\exists$.

   Please transform the following first order expression to Negation Normal Form Let $f : X \mapsto X$, and $p : X \mapsto \{\text{TRUE}, \text{FALSE}\}$

   - $\neg(\forall x \in X.p(f(x)) \to p(x))$
   - $\neg(\exists x \in X.p(x) \to (p(x) \to p(x)))$

3. Syntax

   - Please write down the first order logic expression as semantic of the syntactic sugar "IF p then x ELSE y ".
   - Please write down the semantic of a function in TLA$^+$
     $[f \text{ EXCEPT } ![e_1] = e_2]$
   - Please write down the equivalent logic expression using bracket for the following TLA+ expression.

     $$\begin{array}{l} \lor\ d_1 \\ \lor\ \land\ b_1 \\ \quad \land\ b_2 \end{array}$$

4. TLA modeling: specify your own clock showing the actions of hour, minute and second, use "PrintT" from module TLC to print out each variable changes.

5. (Optional) Mathematical puzzle modeling

   A farmer has a 40 pound stone and a balance scale. How can he break the stone into 4 pieces so that, using those pieces and the balance scale, he can weigh out any integral number of pounds of corn from 1 pound through 40 pounds.

   Try to use TLA+ to specify the problem and use TLC to find the solution. Student who can successfully finish this task on his own and handed in the hand written model will get one extra point for the final exam.