Large-scale visual recognition
II – Efficient matching

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CVPR tutorial: Large-Scale Visual Recognition (LSVR)
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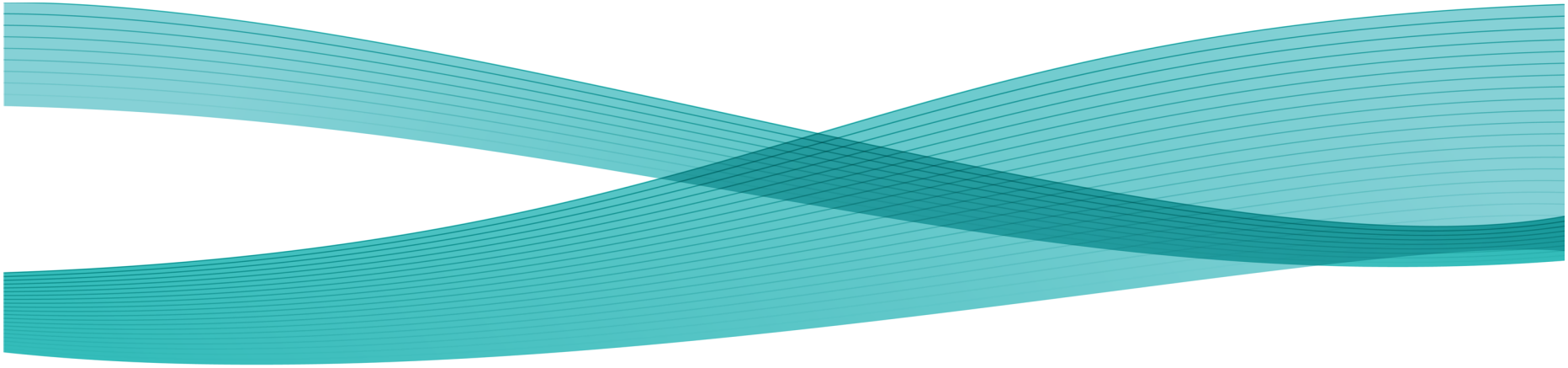

Outline

- **Preliminary**

- Locality Sensitive Hashing: the two modes
  - Hashing
  - Embedding

- Searching with Product Quantization

Finding neighbors

- Nearest neighbor search is a critical step in object recognition
  - To compute the image descriptor itself
    E.g., assignment with k-means to a large vocabulary
  - To find the most similar images/patches in a database
  - For instance, the closest one w.r.t to Euclidean distance:

$$\mathrm{NN}(x) = \arg\min_{y \in \mathcal{Y}} ||x - y||^2$$

- Problem: Exhaustive search has complexity O(n*d)

The cost of (efficient) exact matching

- But what about the actual timings ? With an efficient implementation!
  - Yael NN search: http://gforge.inria.fr/projects/yael
  - On a typical machine (this laptop)

- BoW with large vocabulary
  - 2,000 SIFTs (d=128)
  - 100,000 visual words
  - i.e., **100 million L2-distances**

- VLAD – dense
  - 20,000 SIFTs (d=128)
  - 512 centroids

```
%--- BOW assignment

d = 128;          % vector dimensionality
n = 100000;     % number of visual words
nq = 2000 ;       % number of query vectors
X = single (randn(d, n));
Y = single (randn(d, nq));

tic
[idx, dis] = yael_nn (X, Y);
toc



% VLAD assignment (dense)

d = 128;          % vector dimensionality
n = 512 ;         % number of centroids
nq = 20000 ;      % number of query vectors
X = single (randn(d, n));
Y = single (randn(d, nq));

tic
[idx, dis] = yael_nn (X, Y, 1);
toc
```

The cost of (efficient) exact matching

- But what about the actual timings ? With an efficient implementation!
  - Yael NN search: http://gforge.inria.fr/projects/yael
  - On a typical machine (this laptop)

- BoW with large vocabulary
  - 2,000 SIFTs (d=128)
  - 100,000 visual words
  - i.e., **100 million L2-distances**
  - **About 2 seconds**

- VLAD – dense
  - 20,000 SIFTs (d=128)
  - 512 centroids
  - **About 100 milliseconds**

```
%--- BOW assignment

d = 128;          % vector dimensionality
n = 100000;     % number of visual words
nq = 2000 ;       % number of query vectors
X = single (randn(d, n));
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X = single (randn(d, n));
Y = single (randn(d, nq));

tic
[idx, dis] = yael_nn (X, Y, 1);
toc
```

Need for approximate nearest neighbors

- BOW & VLAD are already (bad) **approximation** of direct matching

- 1 million images, 1000 descriptors per image
  - 1 billion distances per local descriptor
  - 10^{12} distances in total
  - 1 hour 30 minutes to perform the query for Euclidean vectors

- To improve the scalability,
  - We allow to find the nearest neighbors in probability only:
    Approximate nearest neighbor (ANN) search

- Three (contradictory) performance criteria for ANN schemes
  - search quality (retrieved vectors are actual nearest neighbors)
  - speed
  - memory usage

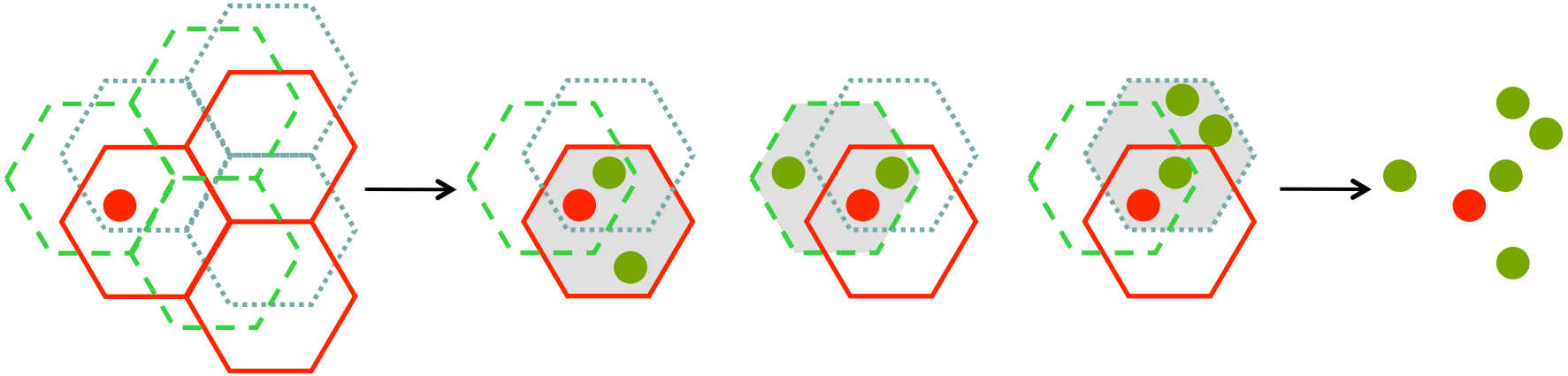
Outline

- Preliminary
- **Locality Sensitive Hashing: the two modes**
  - Hashing
  - Embedding
- Searching with Product Quantization

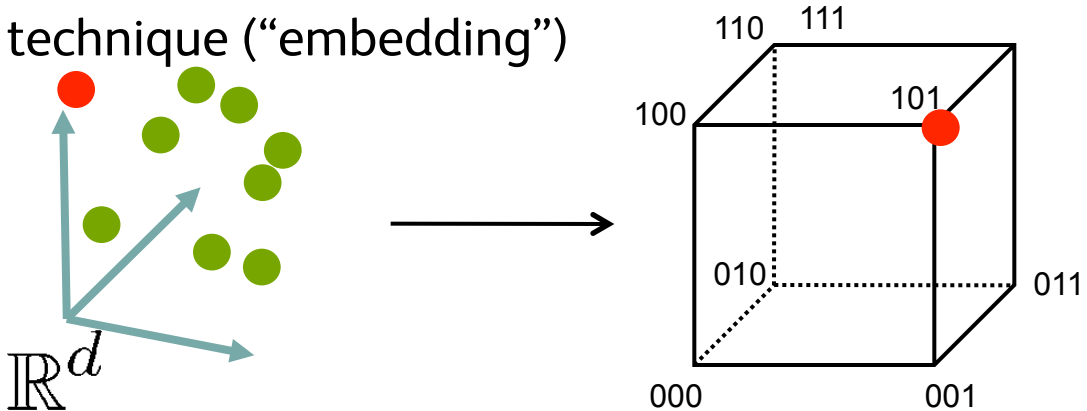
Locality Sensitive Hashing (LSH)

- Most known ANN technique
  [Charikar 98, Gionis 99, Datar 04, Indyk'04, Andoni'06, …]

- But "LSH" is associated with two distinct search algorithms
  - As an indexing technique involving several hash functions



  - As a binarization technique ("embedding")



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- Preliminary
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  - **Hashing**
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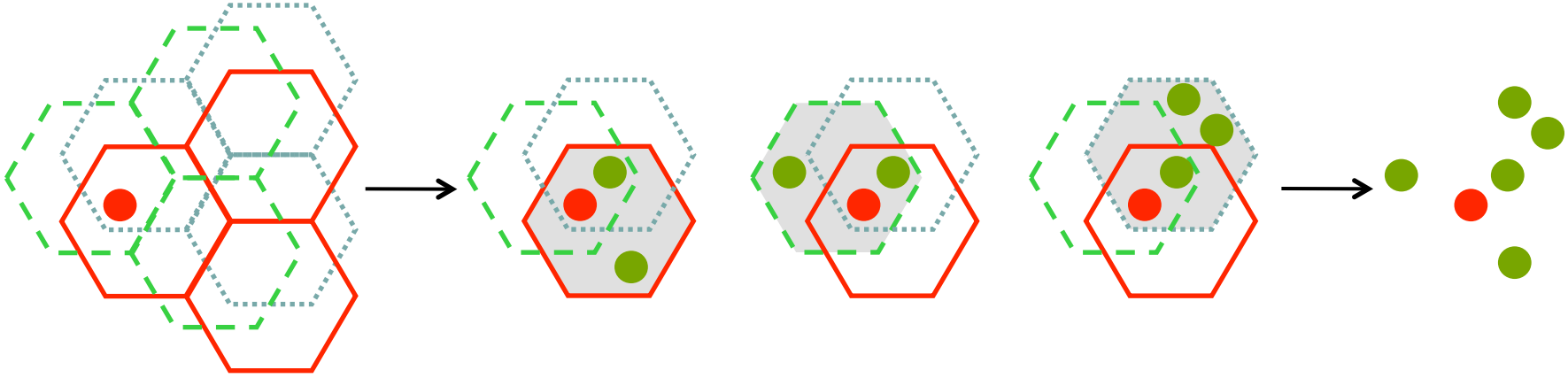
LSH – partitioning technique

- General idea:
  - Define m hash functions in parallel
  - Each vector: associated with m distinct hash keys
  - Each hash key is associated with a hash table

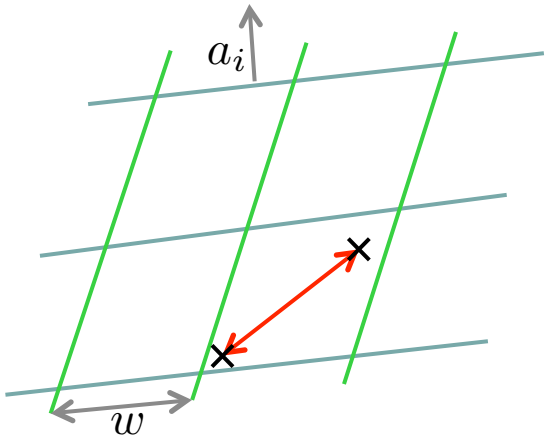


- At query time:
  - Compute the hash keys associated with the query
  - For each hash function, retrieve all the database vectors assigned to the same key (for this hash function)
  - Compute the exact distance on this short-list

Hash functions

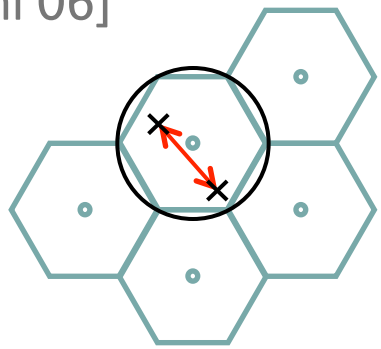
- Typical choice: use random projections

$$h_i(x) = \left\lfloor \frac{a_i^\top x - b_i}{w} \right\rfloor$$



- Why not directly using a structured quantizer?
  - Vector quantizers: better compression performance than scalar ones

- Structured vector quantizer: Lattice quantizers [Andoni 06]
  - Hexagonal (d=2), E_8 (d=8), Leech (d=24)



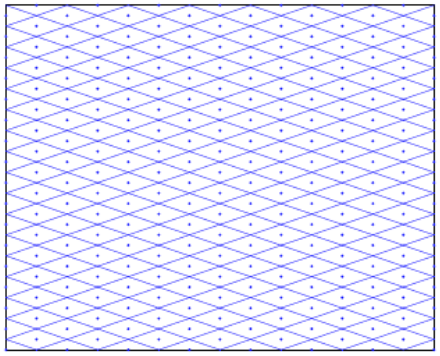
- But still: **lack of adaptation to the data**

Alternative hash functions – Learned

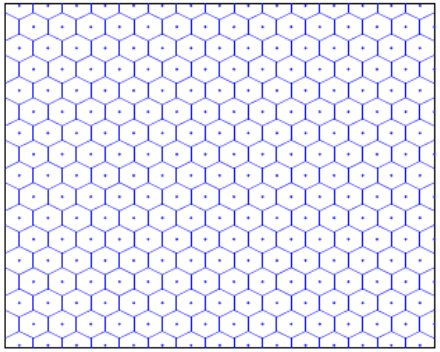
- Any hash function can be used in LSH/partitioning
  - Just need a set of functions $f_j : \mathbb{R}^d \rightarrow \mathbb{K}$
  - Therefore, could be learned on sample examples

- In particular: k-means, Hierarchical k-means, KD-trees

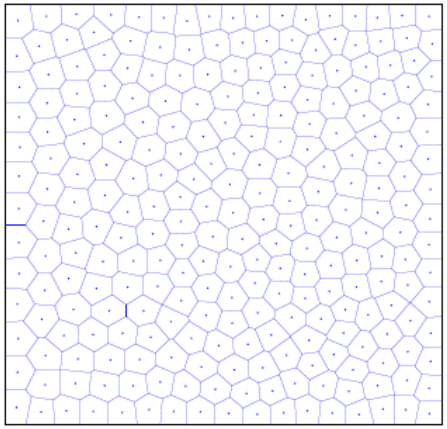
From [Pauleve 10]



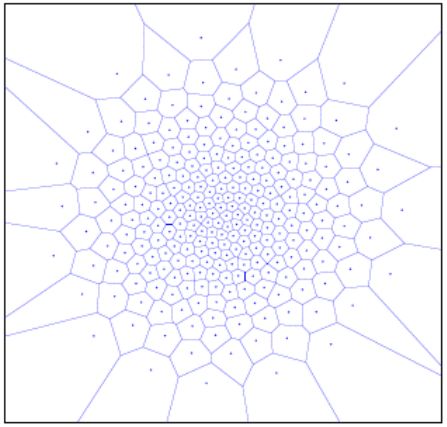
(a) Random projections



(b) A_2 lattice



(c) k-means
Uniform distribution

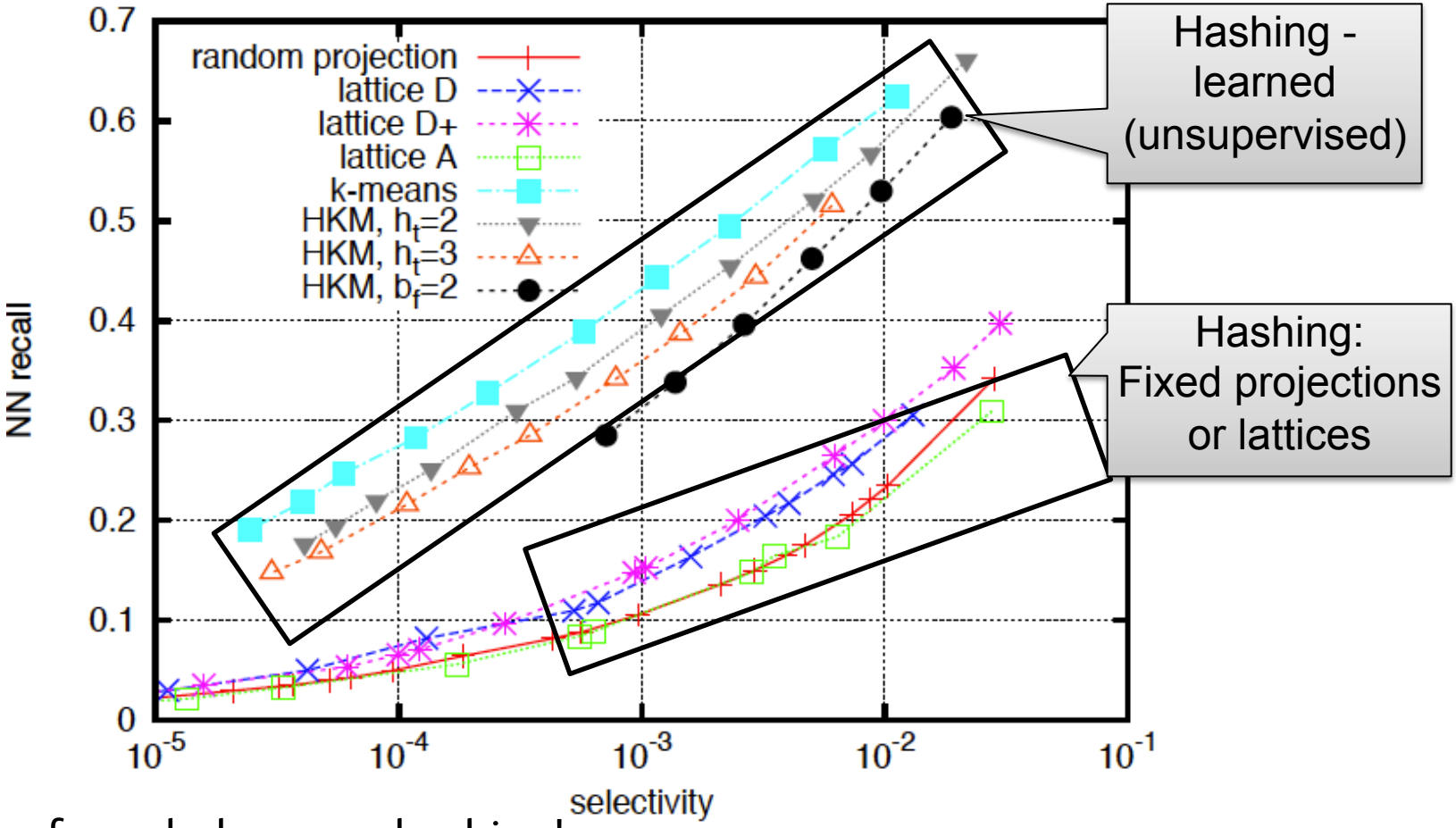


(d) k-means
Gaussian distribution

- Better data adaptation than with structured quantizers

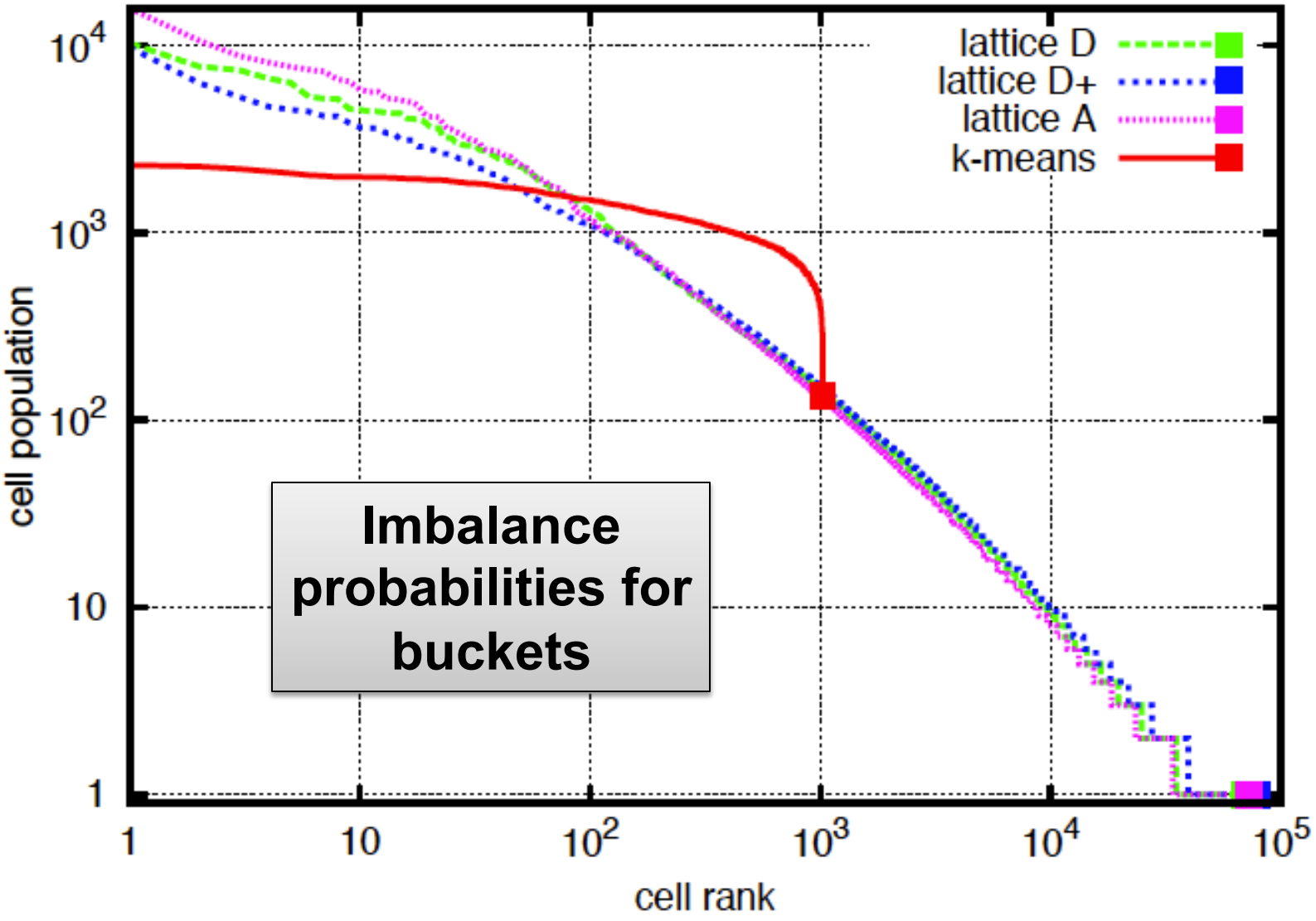
Alternative hash functions – Learned

- Example of search: quality for a **single** hash function



- Bag-of-words: k-means hashing!
- HKM: loss compared with k-means [Nister 06]

The problem with regular hashing

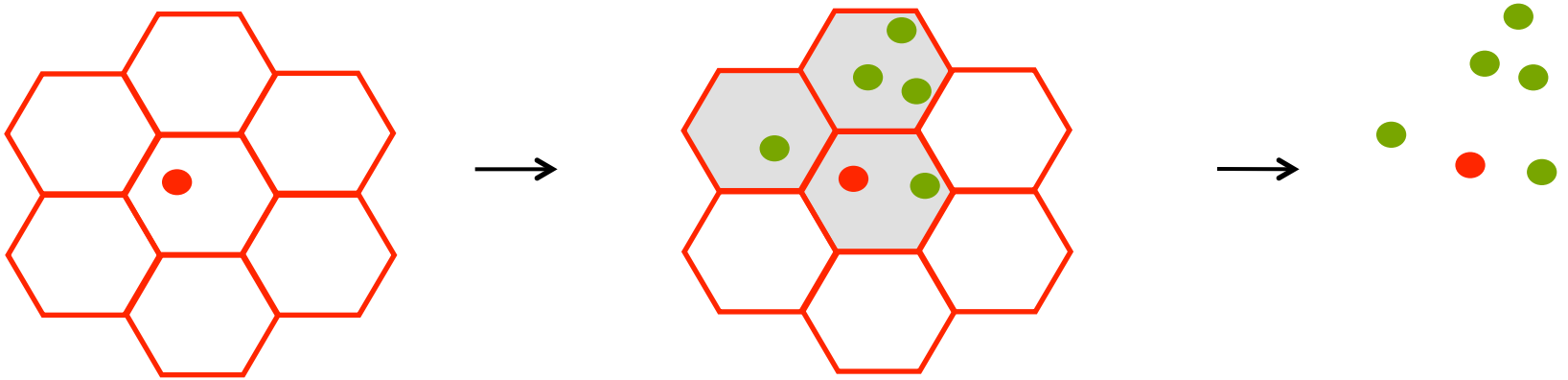


Multi-probe LSH [Lv 07]

- Multiple hash functions use a lot of memory
  - Per vector and per hash table: at least an id

- Multi-probe LSH [Lv 07]: Use less hash functions (possibly 1)
  - Probe several (closest) cells per hash function

    ⇨ save a lot of memory
  - Similar to multiple/soft-assignment with BOV [Jegou'07, Philbin'08]



- See also: [Pauleve'10]

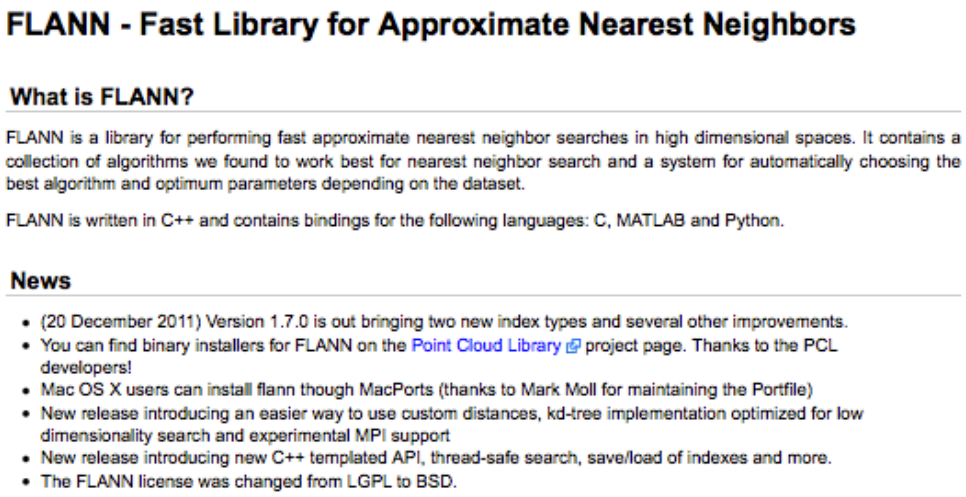
FLANN

- ANN package described in Muja's VISAPP paper [Muja 09]
  - Multiple kd-tree or k-means tree
  - With auto-tuning under given constraints

    Remark: self-tuned LSH proposed in [Dong 07, Slaney 12]

- **Excellent package**: high integration quality and interface!
- Still **high memory requirement** for large vector sets

- See http://www.cs.ubc.ca/~mariusm/index.php/FLANN/FLANN



FLANN - Fast Library for Approximate Nearest Neighbors

What is FLANN?

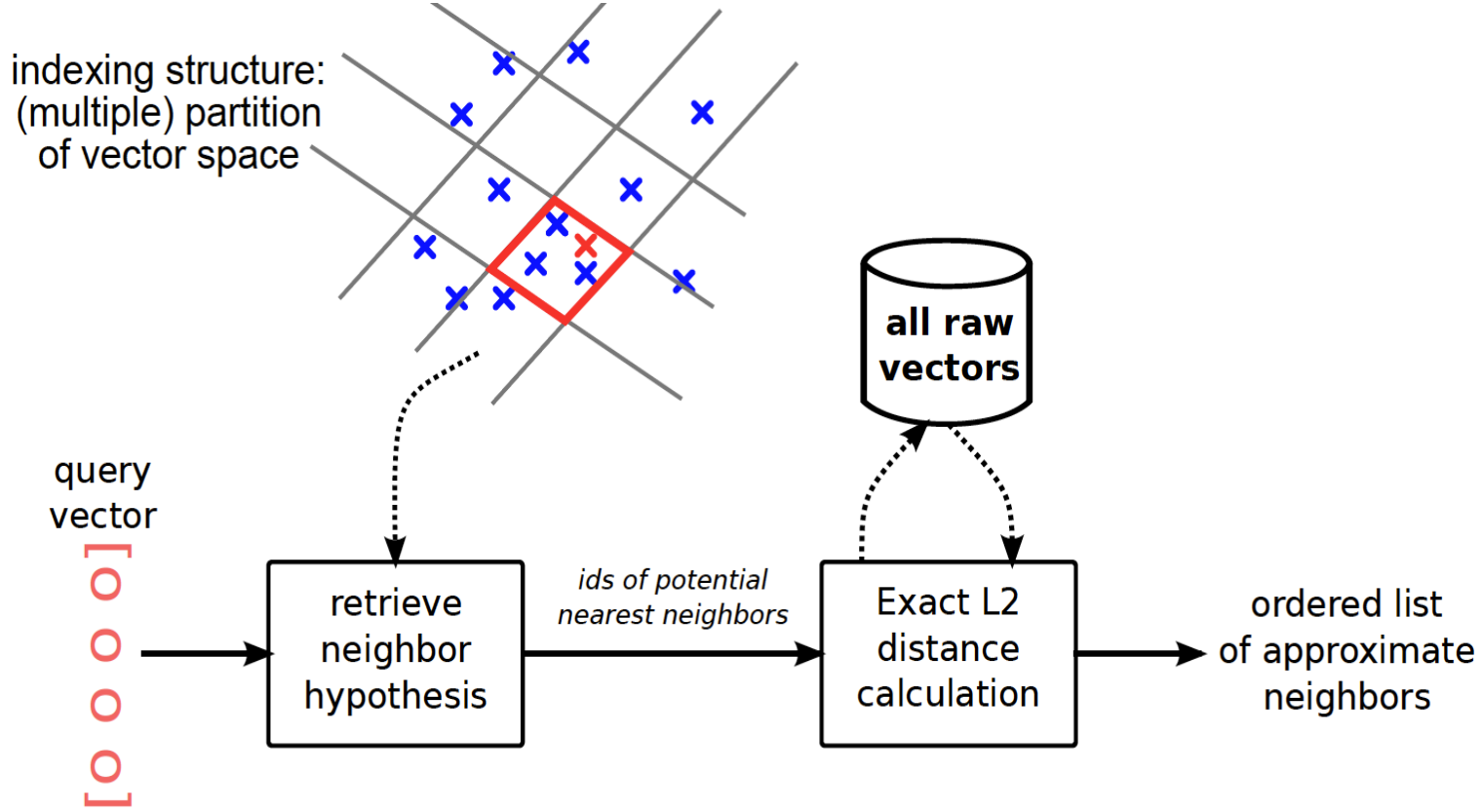
FLANN is a library for performing fast approximate nearest neighbor searches in high dimensional spaces. It contains a collection of algorithms we found to work best for nearest neighbor search and a system for automatically choosing the best algorithm and optimum parameters depending on the dataset.

FLANN is written in C++ and contains bindings for the following languages: C, MATLAB and Python.

News

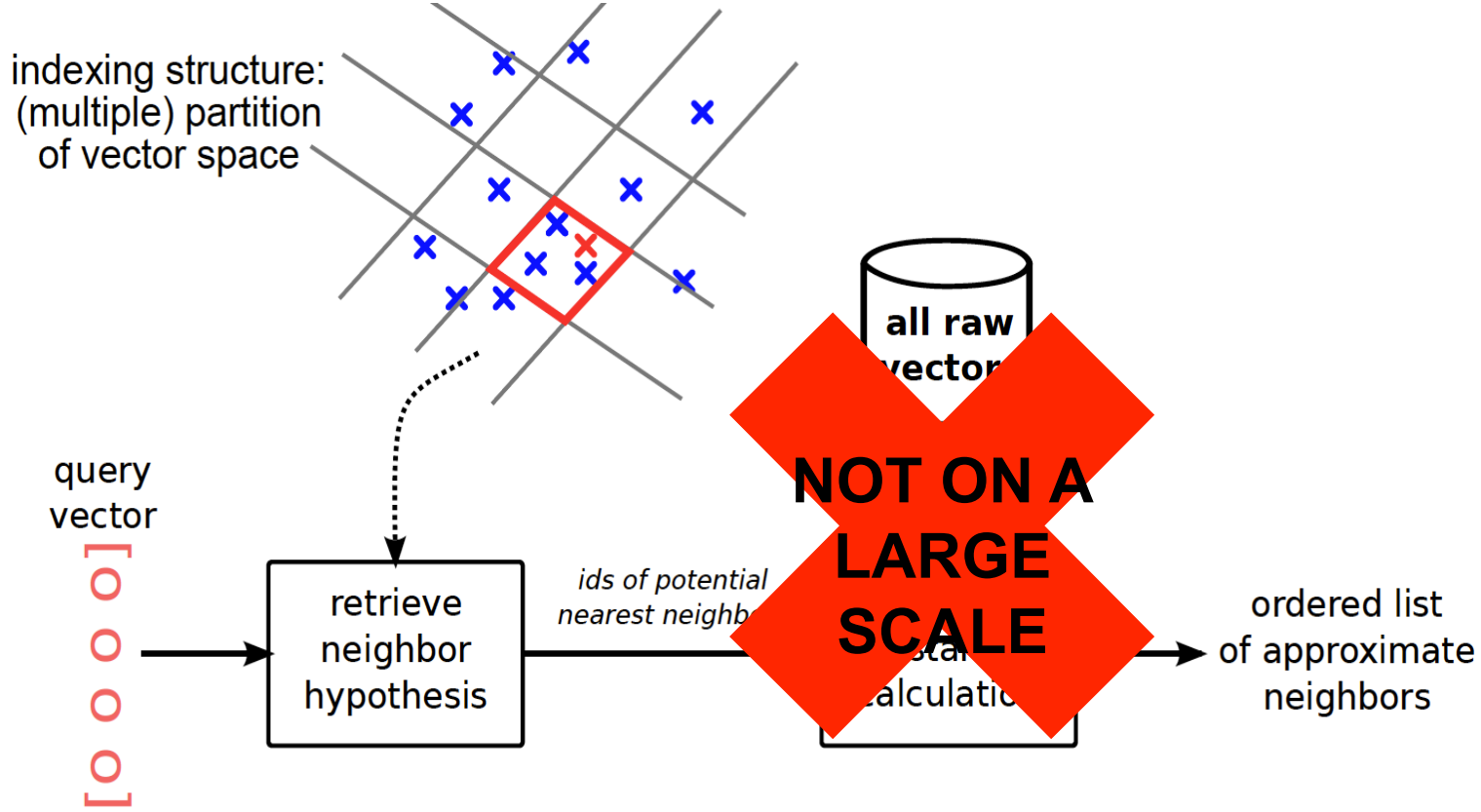
- (20 December 2011) Version 1.7.0 is out bringing two new index types and several other improvements.
- You can find binary installers for FLANN on the Point Cloud Library project page. Thanks to the PCL developers!
- Mac OS X users can install flann though MacPorts (thanks to Mark Moll for maintaining the Portfile)
- New release introducing an easier way to use custom distances, kd-tree implementation optimized for low dimensionality search and experimental MPI support
- New release introducing new C++ templated API, thread-safe search, save/load of indexes and more.
- The FLANN license was changed from LGPL to BSD.

Issue for large scale: final verification



- For this second ("re-ranking") stage, we need raw descriptors, i.e.,
  - either huge amount of memory → 128GB for 1 billion SIFTs
  - either to perform disk accesses → severely impacts efficiency

Issue for large scale: final verification



- Some techniques –like BOV– keep all vectors (no verification)

- Better: use very short codes for the filtering stage
  - See later in this presentation

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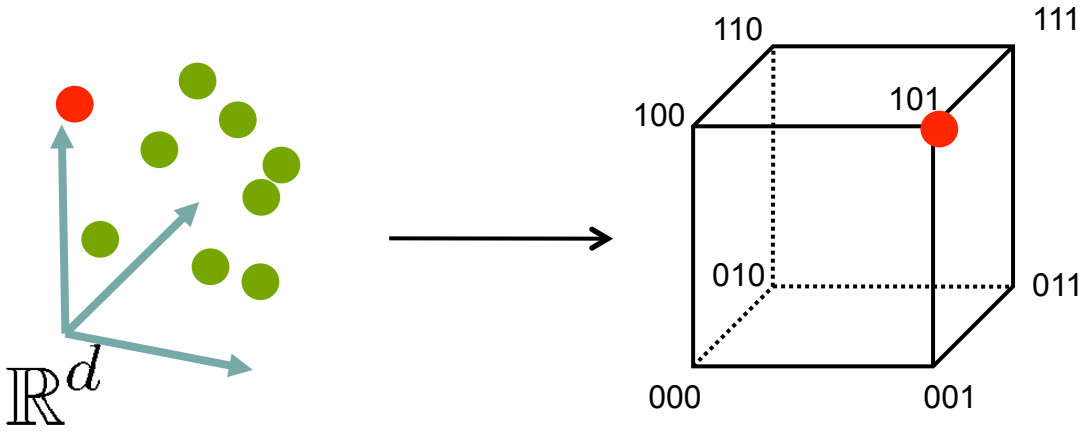
LSH for binarization



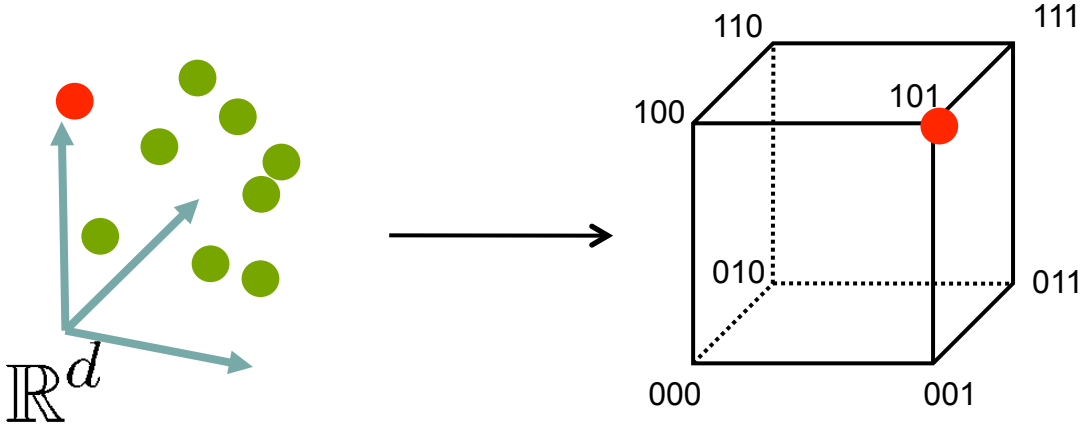
- Idea: design/learn a function mapping the original space into the compact Hamming space: $e : \mathbb{R}^d \rightarrow \{0,1\}^D$

  $x \rightarrow e(x)$

- Objective: neighborhood in the Hamming space try to reflect original neighborhood $\arg\min_i h\left(e(x), e(y_i)\right) \approx \arg\min_i d(x, y)$

- Advantages: compact descriptor, fast distance computation

LSH for binarization



- Given B random projection directions a_i
- For a given vector x, compute a bit for each direction, as $b_i(x) = \text{sign } a_i^\top x$

$$b(x) = (b_1(x), ..., b_B(x))$$

- Property: For two normalized vectors x and x':

$$\mathbb{P}\left(b_i(x) = b_i(x')\right) = 1 - \frac{1}{\pi} \cos^{-1} x^\top x'$$

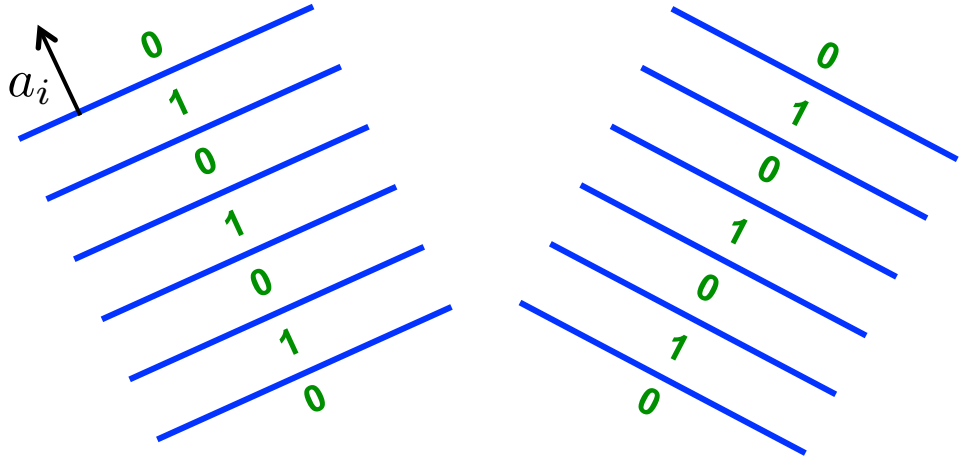
- The Hamming distance is related *in expectation* to the angle by

$$\mathbb{E}\left(b_i(x) = b_i(x')\right) \approx 1 - \frac{b(x)^\top b(x')}{B}$$

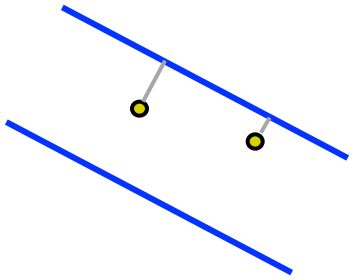
Binary sketches and asymmetric scheme

[Charikar 02, Dong'08]

- Hash functions more focused on smaller distances



- Combined with asymmetric method [Dong'08]
  - database vectors are binarized, not the query
  - Significant improvement (reduce noise on query)



- See also
  - Spectral hashing [Weiss'08],
  - 1-bit compressive sensing and Boufonos' "Universal quantization"
  - Locality-Sensitive Binary Codes from Shift-invariant Kernels [Raginsky'09]
  - Asymmetric distances for binary embeddings [Gordo'11]

Other topics on LSH and related topics

- Optimized random projections
  - Variance balancing on components [Jegou 10]
  - ITQ [Gong 11] to minimize quantization error

Remark: orthogonality is a key factor for performance

- More general and learned metrics, e.g, :
  - Kernelized LSH [Kulis 09]
  - Semantic hashing [Salakhutdinov'07,09]
  - Semi-supervised hashing for large scale search [Belkin'03, Wang'12]

- Binary LSH (=searching binary vectors)
  - Like E2LSH but: random subset of bits instead of projections
  - **Exact** binary search variant [Nourouzi and Fleet 12]

Anti-sparse coding: spread representations

- Fuchs, “Spread representations”, ASILOMAR’11

- Consider $A = [\mathbf{a}_1 | \dots | \mathbf{a}_m]$: $d \times n$ full rank matrix
- The system $A\mathbf{x} = \mathbf{y}$ admits a infinite number of solutions

- Add one constraint to single out a unique solution, e.g.:
  - Typical choice: solution of lowest energy
  - Sparse coding: minimize L_0 norm of the solution
    → or, in practice, minimal L_1 norm

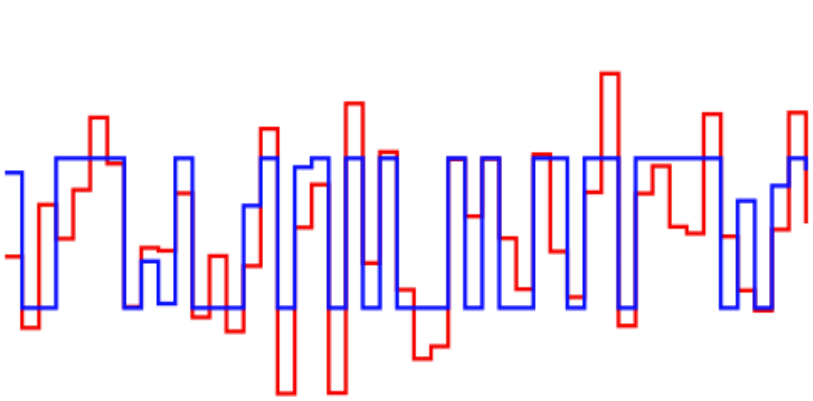
- Anti-sparse coding minimizes instead the L_∞ norm

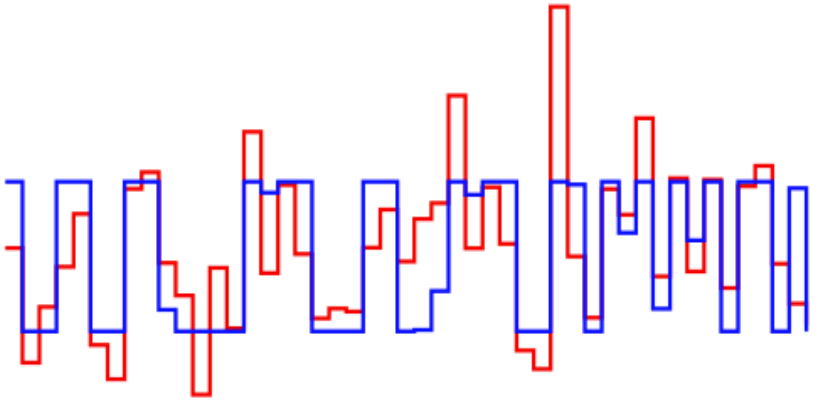
$$\mathbf{x}^\star = \min_{\mathbf{x}:\, A\mathbf{x}=\mathbf{y}} \|\mathbf{x}\|_\infty$$

Properties

Examples of encoded vectors:

- **Simple frame projection** (as in LSH)
- **Anti-sparse coding**: spread representation





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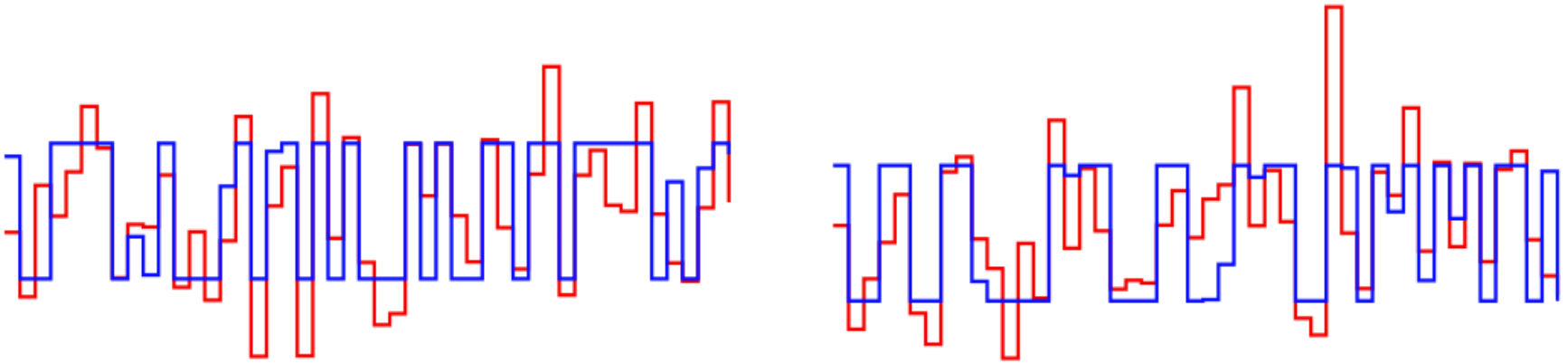


Most components are equal to $-||\mathbf{x}||_\infty$ or $+||\mathbf{x}||_\infty$

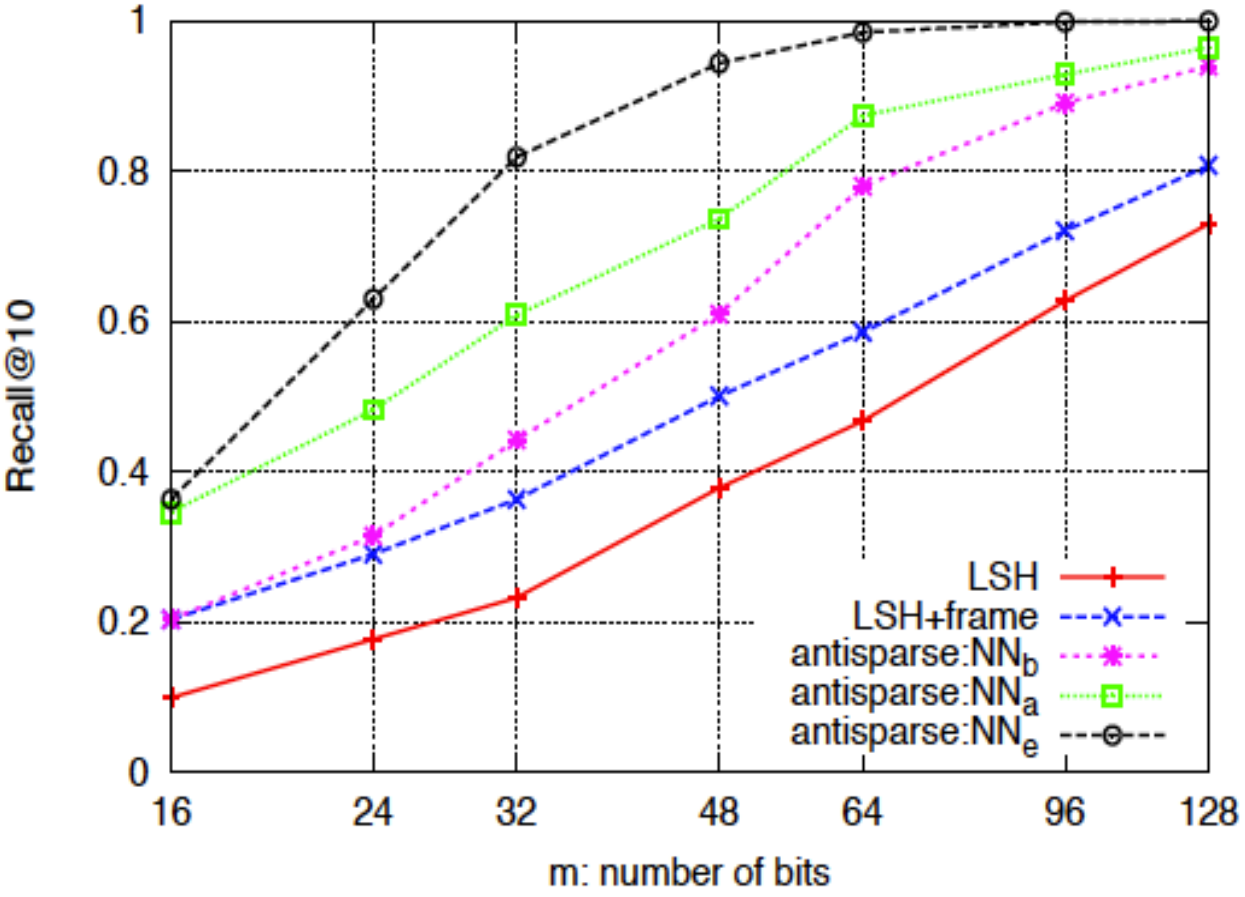
- At least n-d+1 components are stuck to the limit → “natural” binarization

- The original vector is reconstructed up to a scaling factor
  - **The quantization is error is significantly reduced**

Anti-sparse coding for ANN

Three comparison strategies with anti-sparse coding [J. Furon Fuchs, Icassp'12]

- N_b: Use binary vectors and Hamming distance
- N_a: Asymmetric comparison (similar to [Dong'08])
- N_e: Comparison after explicit reconstruction



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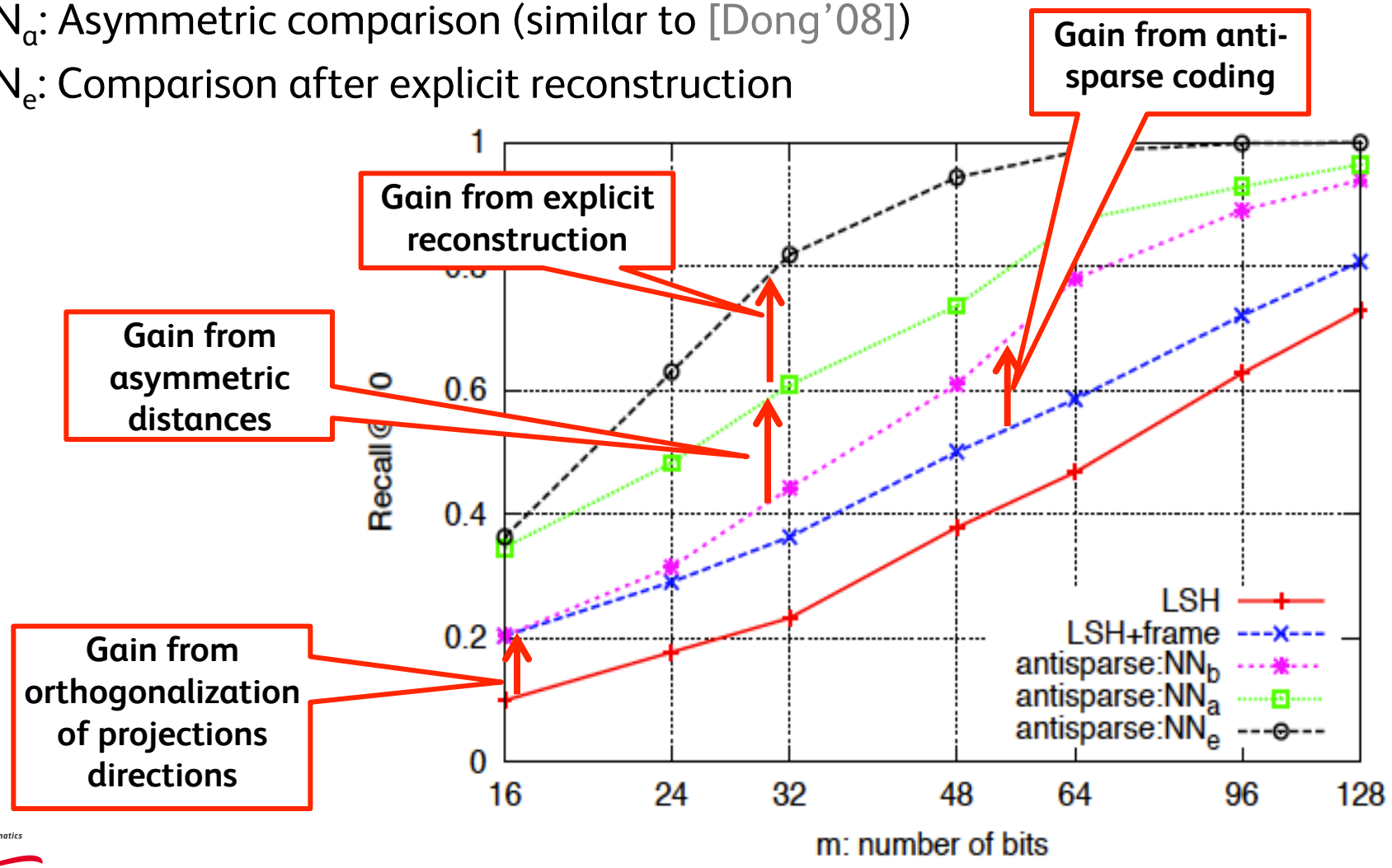


LSH: the two modes – approximate guidelines

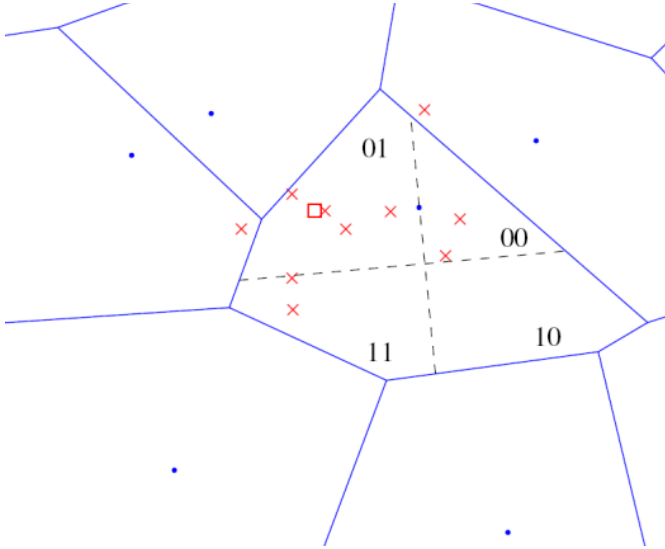
Partitioning technique

- **Sublinear** search
- Several hash indexes (integer)
- **Large memory overhead**
  - Hash table overhead (store ids)
- Need original vectors for re-ranking
  - **Need a lot of memory**
  - **Or to access the disk**
- Interesting when (e.g., FLANN)
  - Not too large dimensionality
  - Dataset small enough (memory)
- Very good variants/software (FLANN)

Binarization technique

- **Linear** search
- Produce a binary code per vector
- **Very compact**
  - bit-vectors, concatenated (no ids)
- **Very fast comparison**
  - Hamming distance (popcnt SSE4)
  - 1 billion comparisons/second
- Interesting
  - For **very high-dimensional** vectors
  - When memory is critical
- Simple to implement and extend. Very active problem with many variants

Hybrid scheme: Hamming Embedding



- Introduced as an extension of BOV [Jegou 08]

- Combination of
  - A partitioning technique (k-means)
  - A binary code that refine the descriptor

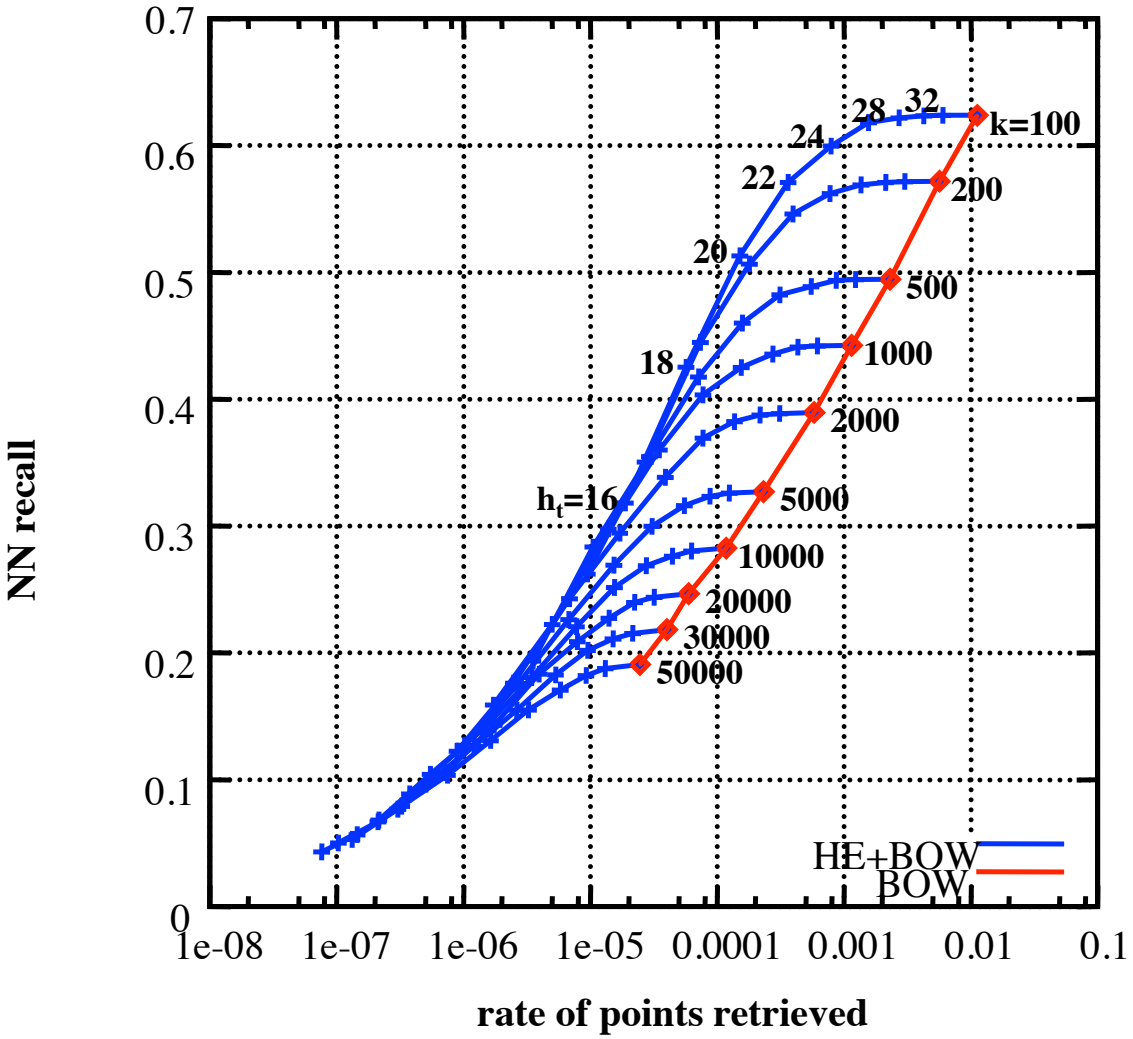
Representation of a descriptor *x*
- Vector-quantized to *q(x)* as in standard BOV
- \+ short binary vector *b(x)* for an additional localization in the Voronoi cell

- Two descriptors x and y match iif

$$f_{\mathrm{HE}}(x,y) = \begin{cases} (\text{tf-idf}(q(x)))^2 & \text{if } q(x) = q(y) \\ & \quad \text{and } h\left(b(x), b(y)\right) \leq h_t \\ 0 & \text{otherwise} \end{cases}$$

Where h(.,.) denotes the Hamming distance

ANN evaluation of Hamming Embedding



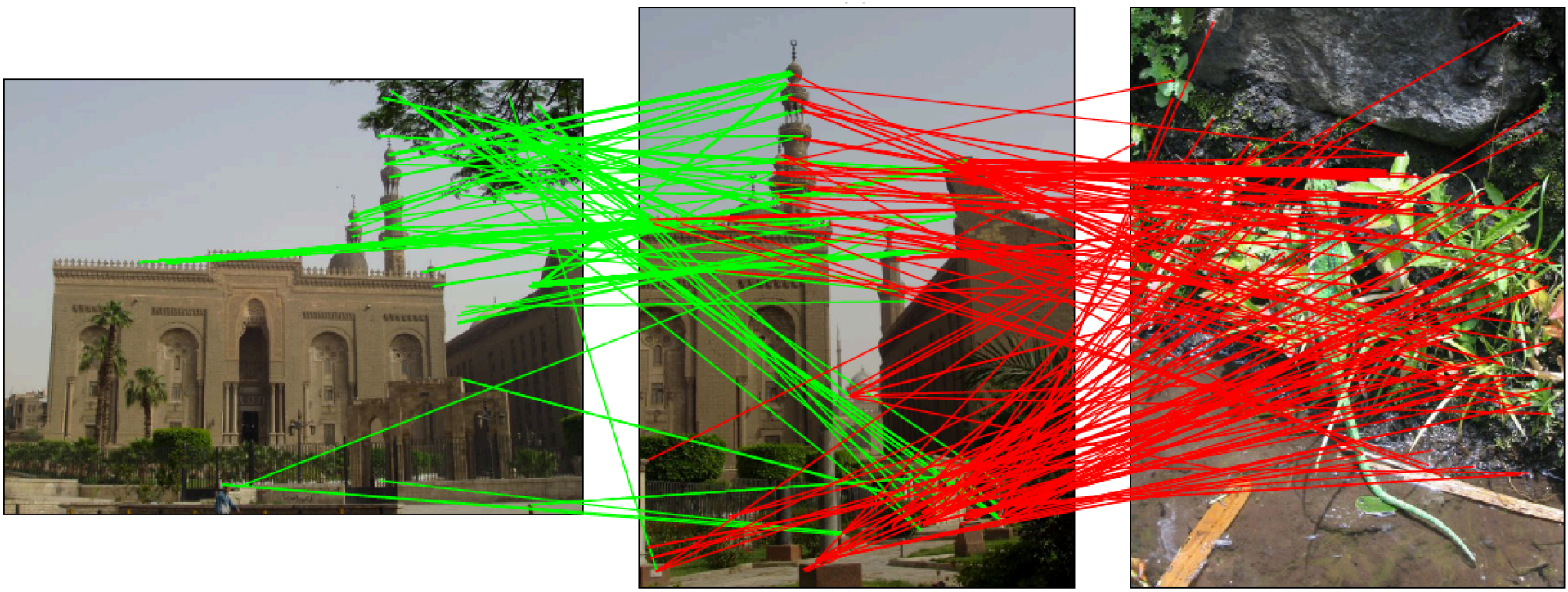
compared to BOW: at least 10 times less points in the short-list for the same level of accuracy

Hamming Embedding provides a much better trade-off between recall and remove false positives

Matching points - 20k centroids

201 matches

240 matches



Many matches with the non-corresponding image

Matching points - 200k centroids

69 matches

35 matches

Still many matches with the non-corresponding images

Matching points - 20k centroids + HE filter

83 matches

8 matches

10x more matches with the corresponding image

Efficient matlab implementation will be provided in next release of Yael (09/2013)

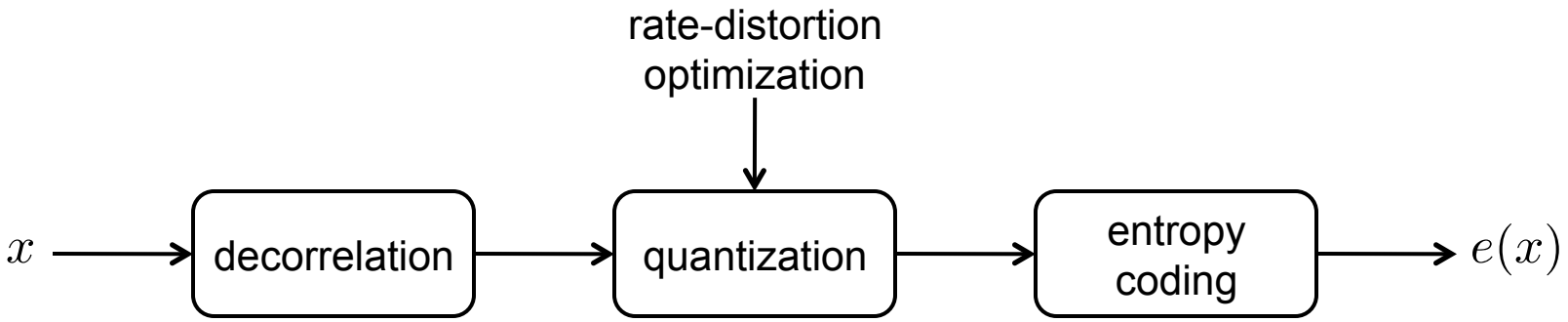
Outline

- Preliminary

- Locality Sensitive Hashing: the two modes
  - Hashing
  - Embedding

- **Searching with Product Quantization**

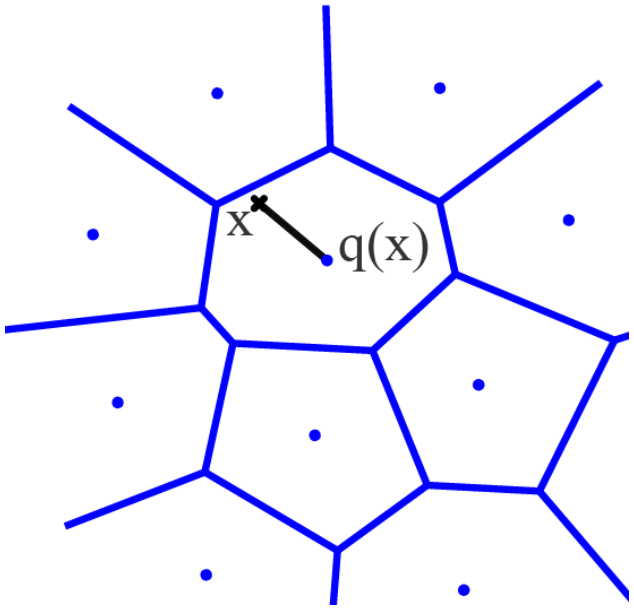
A typical source coding system



- Simple source coding system:
  - Decorrelation, e.g., PCA
  - Quantization
  - Entropy coding

- To a code e(x) is associated a unique reconstruction value q(x)
  ⇨ i.e., the visual word

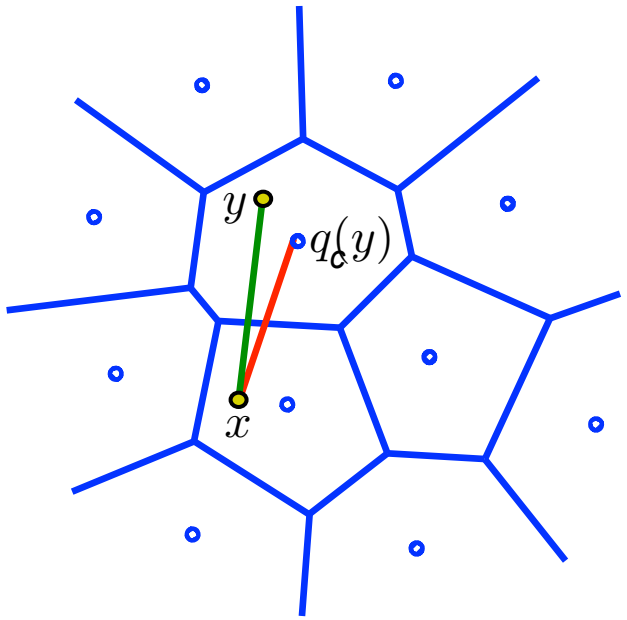
- Focus on quantization (lossy step)



Relationship between Reconstruction and Distance estimation

- Assume y quantized to $q_c(y)$
  x is a query vector

- If we estimate the distance by
$$d(x, y) \approx d(x, q_c(y))$$



- Then we can show that:

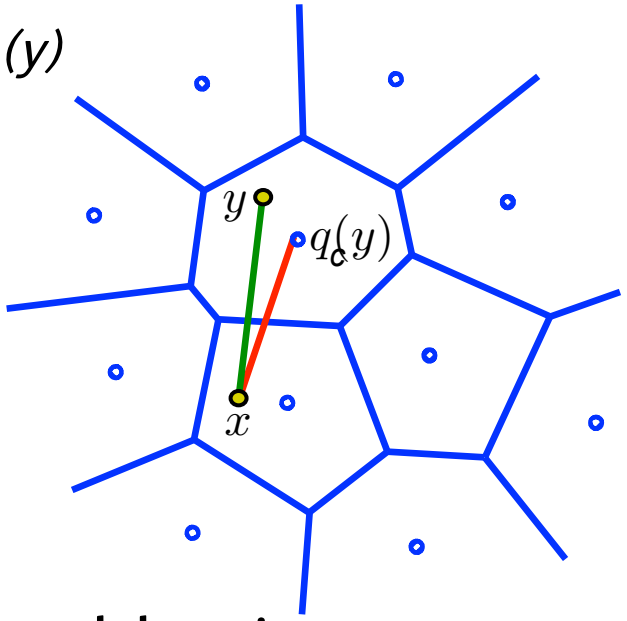
$$\mathbb{E}_Y[(d(x,y) - d(x,q_c(y)))^2] \leq \mathbb{E}_Y[(y - q_c(y))^2] = \mathrm{MSE}$$

i.e., the error on the square distance is statistically bounded by the quantization error

Searching with quantization [Jegou 11]

- Main idea: compressed representation of the database vectors
  - Each database vector *y* is represented by $q_c(y)$ where $q_c(.)$ is a **product quantizer**

$$d(x, y) \approx d(x, q_c(y))$$



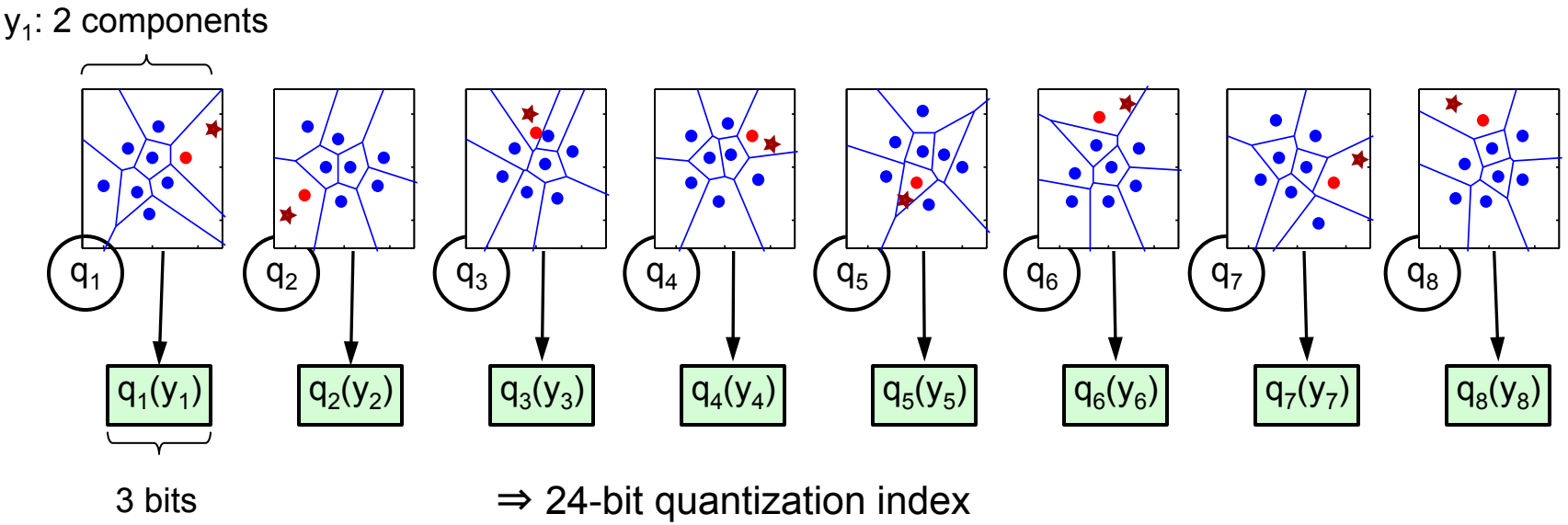
- Search = distance approximation problem

- **The key**: Estimate the distances **in the compressed domain** such that
  - Quantization is fast enough
  - Quantization is precise, i.e., many different possible indexes (ex: 2^{64})

- Regular k-means is not appropriate: not for k=2^{64} centroids

Product Quantizer

- Vector split into m subvectors: $y \rightarrow \left[y_1 | \ldots | y_m\right]$
- Subvectors are quantized separately
- Example: y = 16-dim vector split in 8 subvectors of dimension 16



- In practice: 8 bits/subquantizer (256 centroids),
  - SIFT: m=4-16
  - VLAD/Fisher: 4-128 bytes per indexed vector

Asymmetric distance computation (ADC)

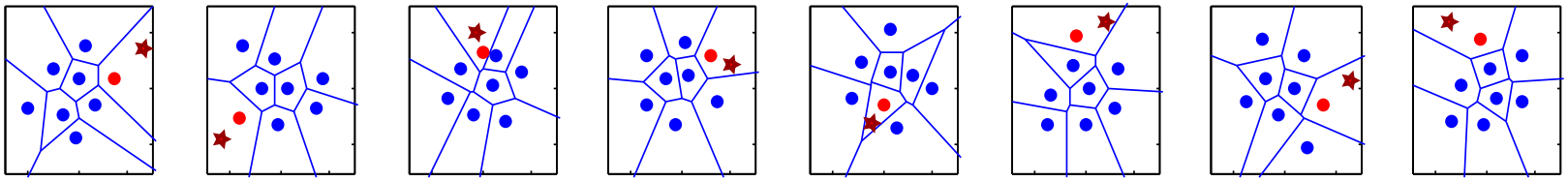
- Compute the square distance approximation in the compressed domain

$$d\left(x, y\right)^2 \approx \sum_{i=1}^{m} d\left(x_i, q_i(y_i)\right)^2$$

- To compute distance between query x and **many** codes
  - compute $d\left(x_i, c_{i,j}\right)^2$ for each subvector x_i and all possible centroids

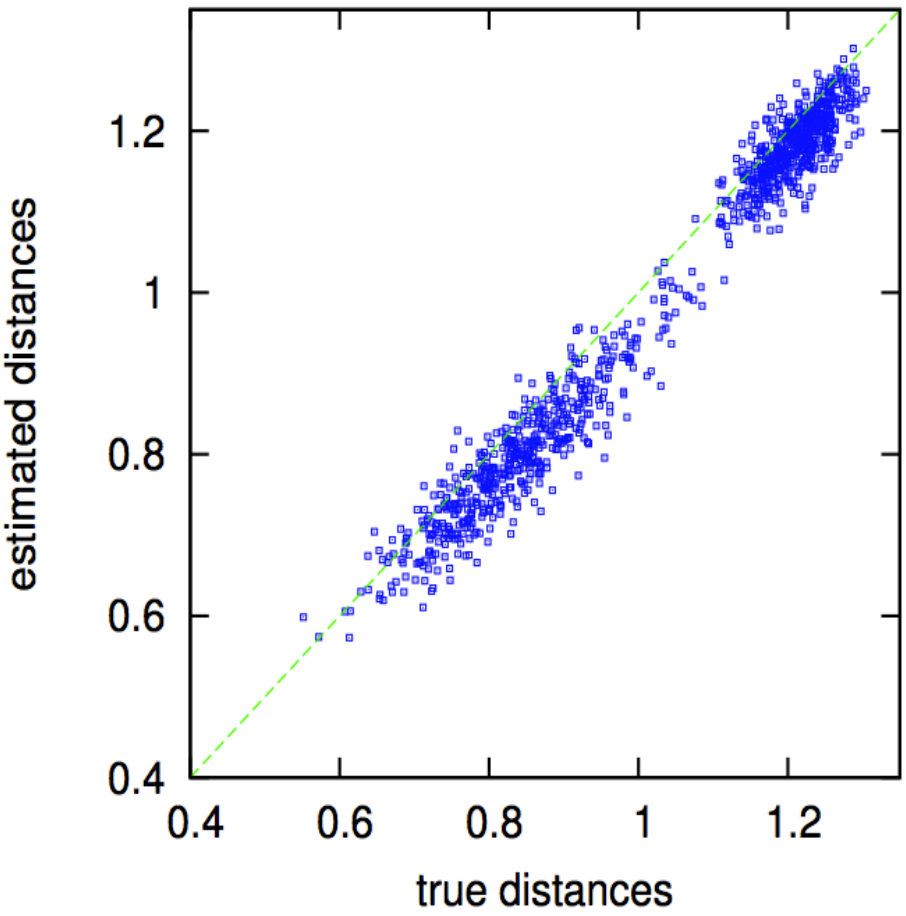
  → stored in look-up tables

  → fixed cost for quantization
  - for each database code: sum the elementary square distances



- Each 8x8=64-bits code requires only **m=8 additions per distance**
- IVFADC: combination with an inverted file to avoid exhaustive search

Estimated distances versus true distances



- The estimator $d(X,q(Y))^2$ of $d(X,Y)^2$ is biased:
  - The bias can removed by quantization error terms
  - but does not improve the NN search quality

Combination with an inverted file system

ALGORITHM

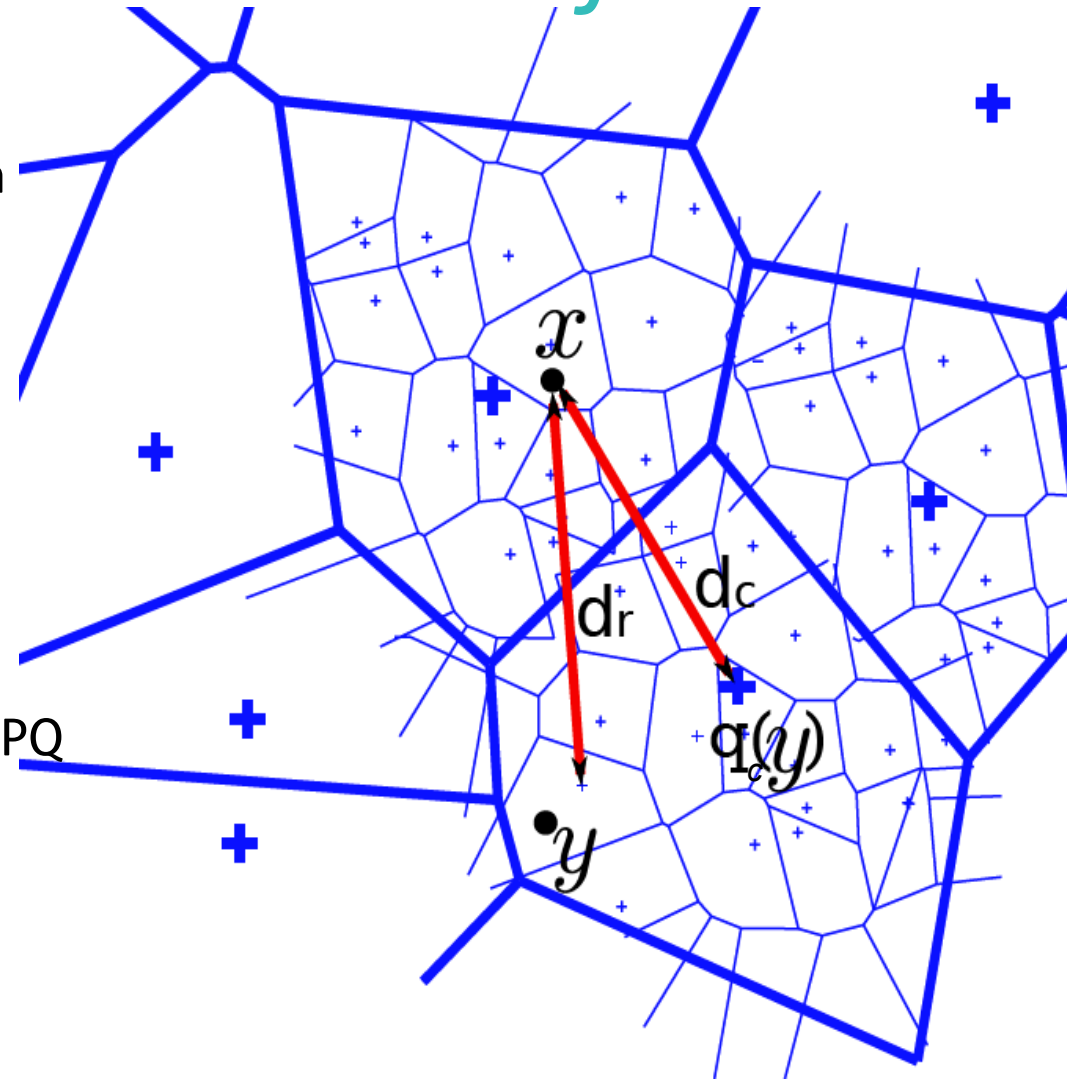
① Coarse k-means hash function

Select k' closest centroids c_i and corresponding cells

② Compute the **residual vector** $x-c_i$ of the query vector

③ Encode the residual vector by PQ

④ Apply the PQ search method. Distance is approximated by $d(x,y) = d(x-c_i, q(y-c_i))$

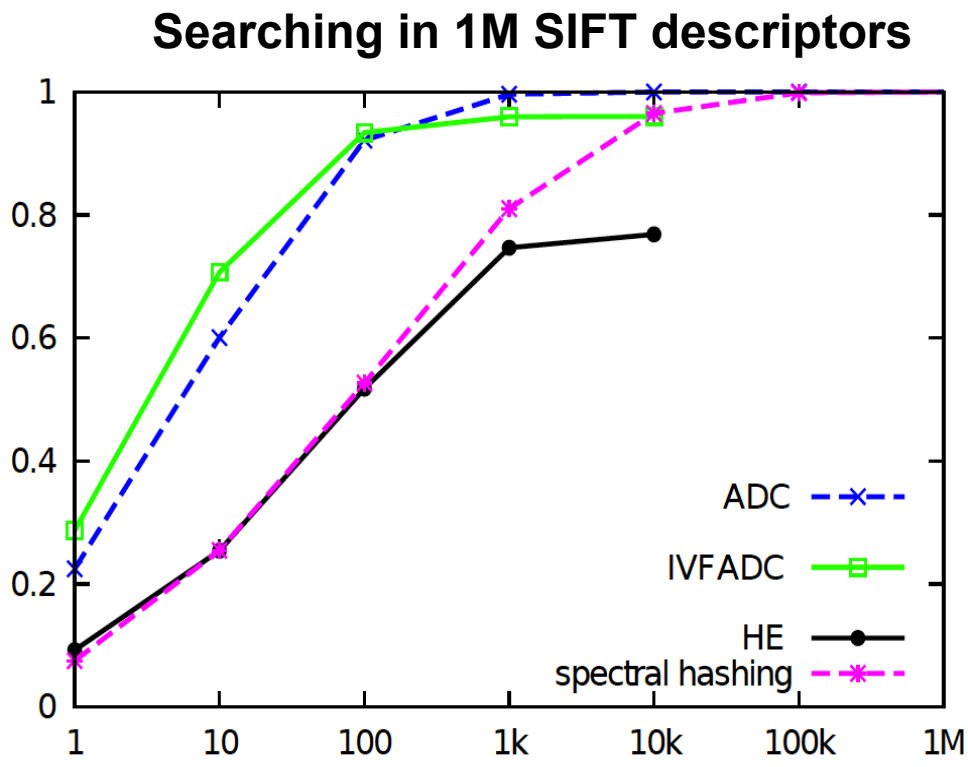


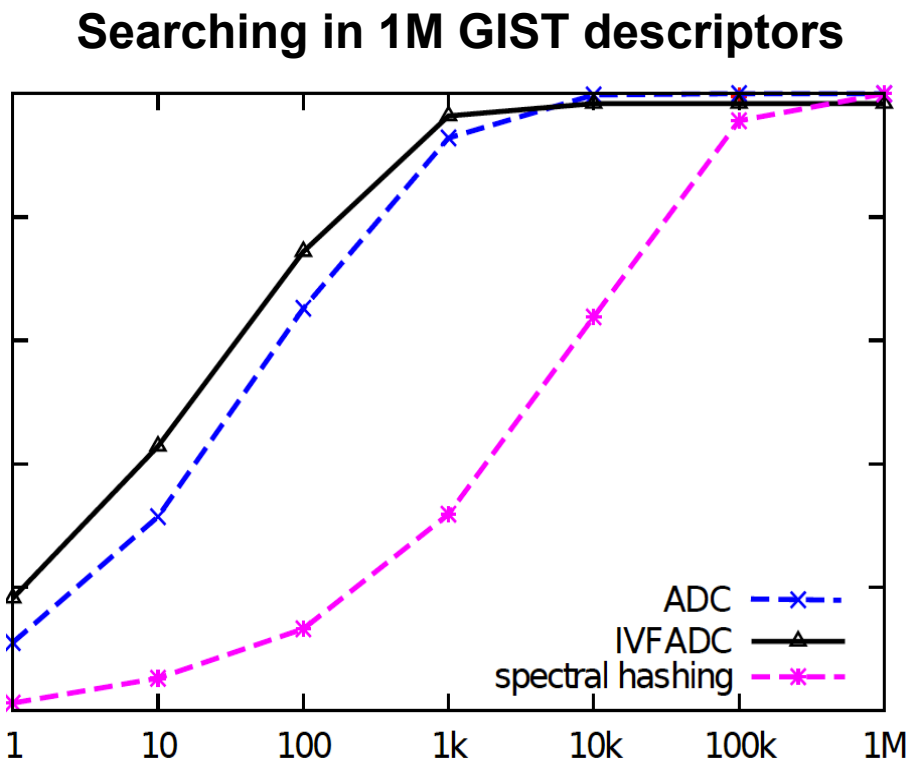
Example timing: 3.5 ms per vector for a search in 2 billion vectors

Performance evaluation

- Comparison with other *memory efficient* approximate neighbor search techniques, i.e., binarization techniques
  - Spectral Hashing [Weiss 09] – exhaustive search
  - Hamming Embedding [Jegou 08] – non exhaustive search

- Performance measured by searching 1M vector (recall@R, varying R)





Product Quantization: some applications

- PQ search was first proposed for searching local descriptors [Jegou'09-11] i.e., to replace bag-of-words or Hamming Embedding

- [Jegou 10]: Encoding a global image representation (Vlad/Fisher)

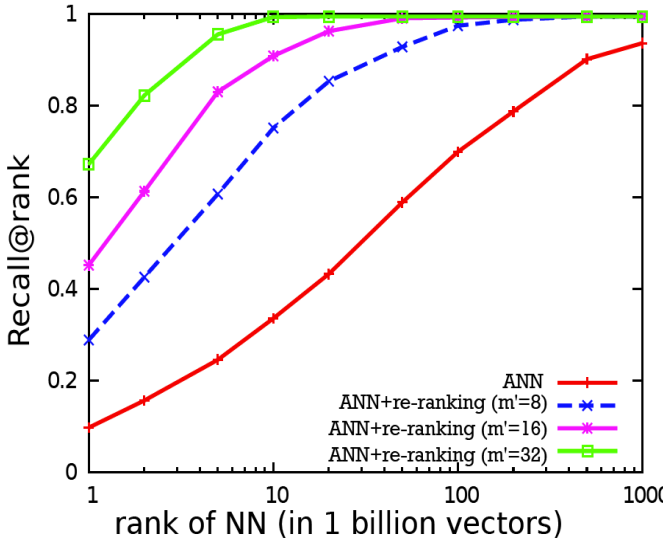
- [Gammeter et al'10]: Fast geometrical re-ranking with local descriptors

- [Perronnin et al.'11]: Large scale classification (Imagenet)
  - Combined with Stochastic Gradient Descent SVM
  - Decompression on-the-fly when feeding the classifier

- Learning with in the PQ-compressed domain
  - With Matrix/vector multiplication [Harchaoui'12]
  - Sparse coding interpretation and lazy expansion [Vedaldi'12]
  - See later in this tutorial

Variants / Extensions

- Adapted codebook for residual vectors [Uchida 11]
  - Learn the product quantizer separately in the different cells



- Re-ranking with source coding [Jegou 11]
  - Exploit the explicit reconstruction of PQ
  - Refine the database vector by a short code

$$\hat{y} = \mathrm{q_c}(y) + \mathrm{q_r}(\mathrm{r}(y))$$

- The "multiple inverted index" [Babenko 12]
  - Replace the coarse k-means by a product quantizer

- In this CVPR,
  - "Cartesian k-means", Norouzi and Fleet
  - "Optimized product quantization for NN search", Ge, He, Qe and Sun
  - "Circulant temporal encoding for event retrieval", Revaud *et al.*

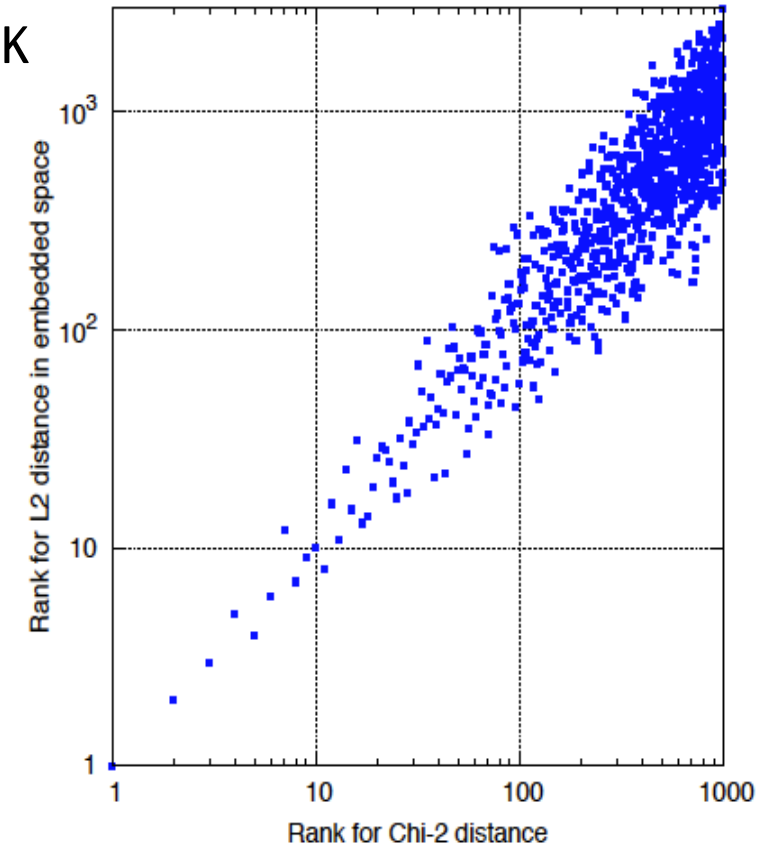
PQ with other distances/kernels

- Approximate search for more general kernels
  - Kernelized LSH [Kulis 09], RMMH [Joly 11], …
  - These techniques rely on **implicit embeddings** (and the kernel trick)

- Alternative: explicit embeddings of a kernel K
  - Idea: implicit feature space projected on a finite-dimensional subspace
  - Done with KPCA

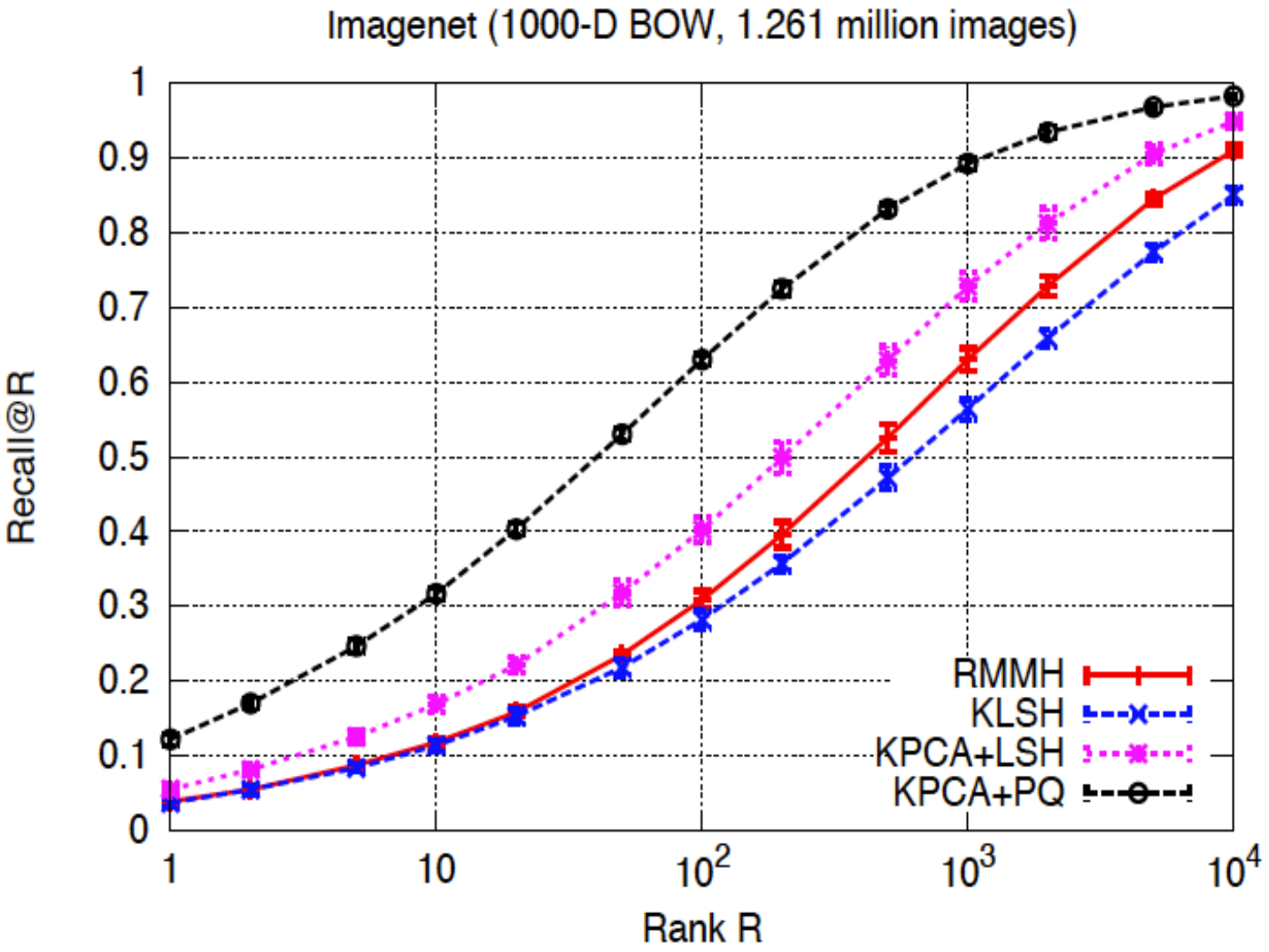
  → or approximation [Vedaldi'10, Perronnin'10]

  - Cosine in the embedded space ≈ K



Searching with explicit embeddings [Bourrier'12]

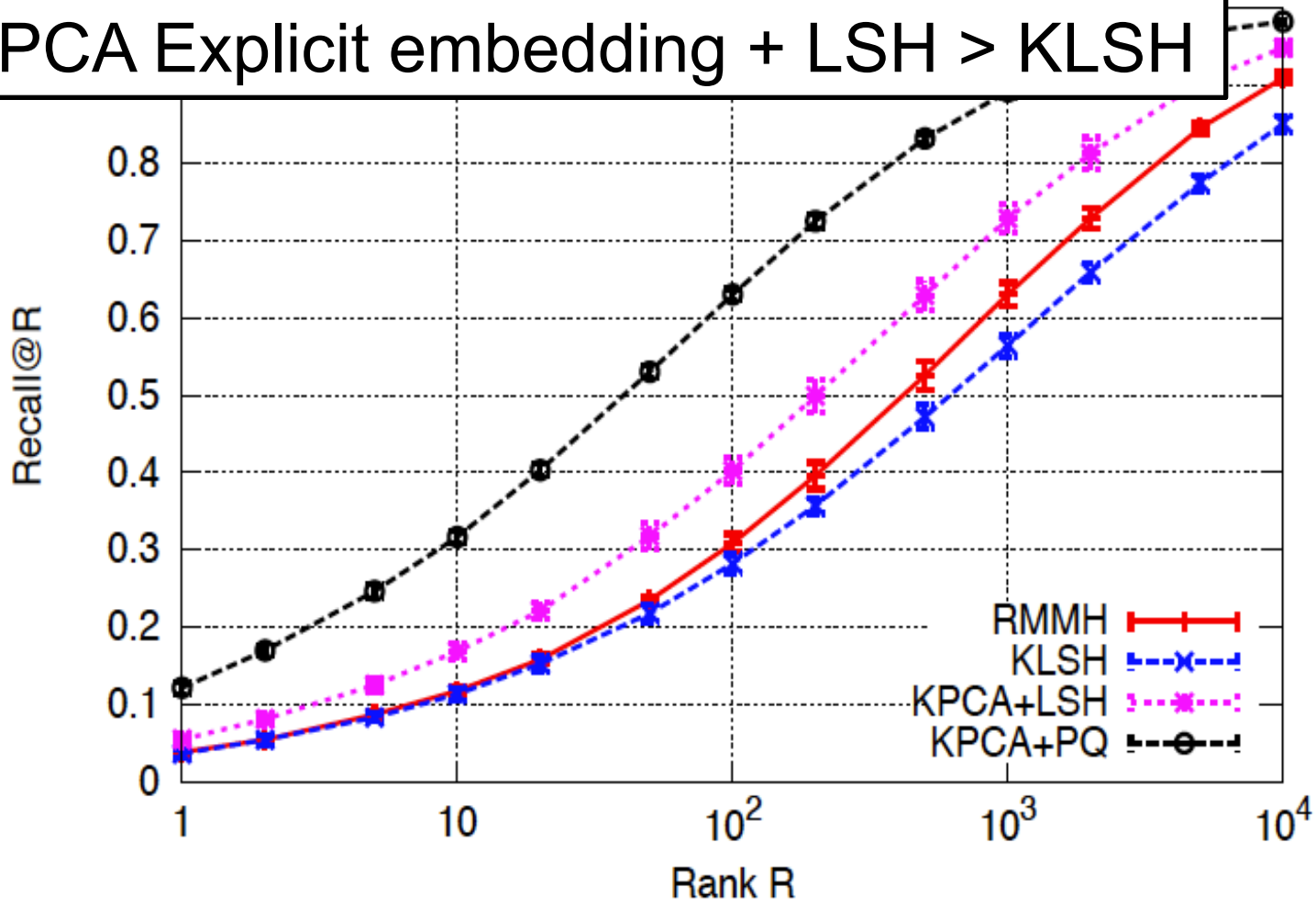
- L2/Cosine-search in the embedded space ≈ search for kernel K !



Searching with explicit embeddings [Bourrier'12]

- L2/Cosine-search in the embedded space ≈ search for kernel K !

Remark:
KPCA Explicit embedding + LSH > KLSH



Conclusion

Nearest neighbor and compact coding is a sustained area of research

Most CV papers on binary embeddings – but inherent limitations

Product quantization-based approach offers

- Competitive search accuracy with very compact vectors
- Tested on 220 million video frames, 100 million still images
  extrapolation for 1 billion images: 20GB RAM, query < 1s on 8 cores

PQ codes: Toy Matlab package

- Not an efficient implementation
- Yet reproduce paper's results

```
% vlearn: learning set
% vbase:  set of vectors to be searched in
% vquery: query vectors

k = 100;  % k NN to be returned
nsq = 8;  % number of subquantizers

% Learn the PQ code structure
pq = pq_new (nsq, vtrain)

% encode the database vectors
cbase = pq_assign (pq, vbase);

% perform the search + return distance estimates
[ids, dis] = pq_search (pq, cbase, vquery, k);
```

http://people.rennes.inria.fr/Herve.Jegou/projects/ann.html