

# Large-scale Geometry

geometric relations between corresponding features  
in (pairs of) images for large scale retrieval

Ondra Chum

CMP

Czech Technical University in Prague

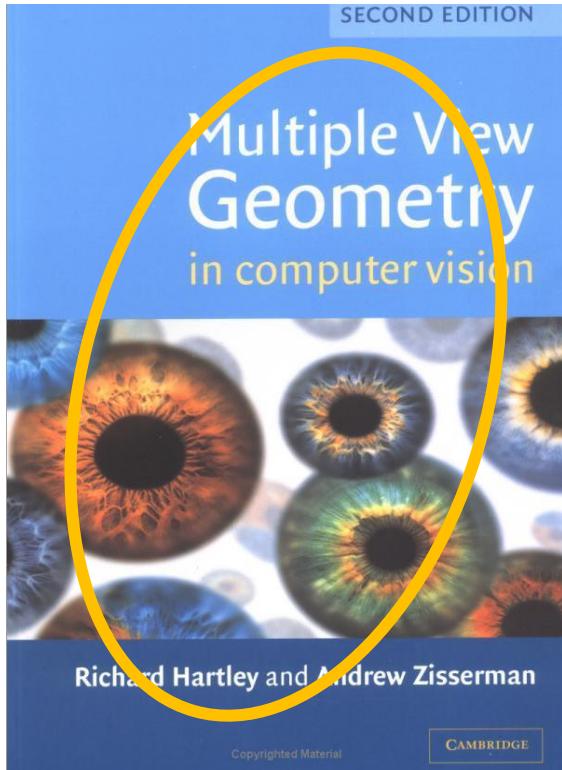
# Outline

- Semi-local constraints
- Global constraints
  - Voting
  - RANSAC
- Applications
  - Query expansion
  - Browsing image collections
- Storing the geometry

# Semi-local Geometry

- Tentative correspondence verification
- Geometric min-Hash

# Semi-local Geometry vs Large Features

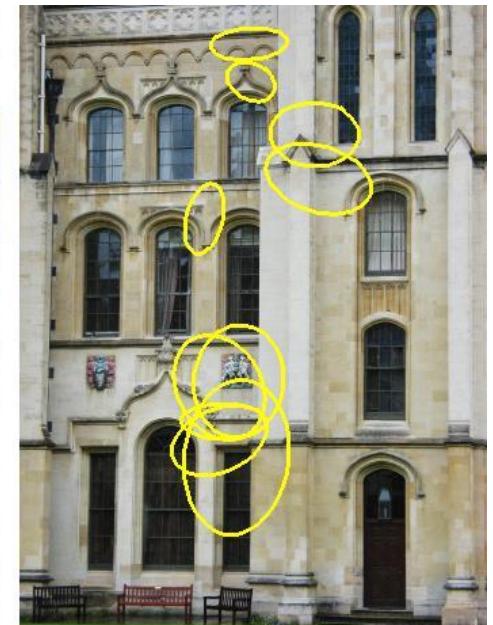


Good for:

- Large planar objects
- No occlusions

Robustness to:

- viewpoint change
- depth discontinuities
- background change
- partial occlusion



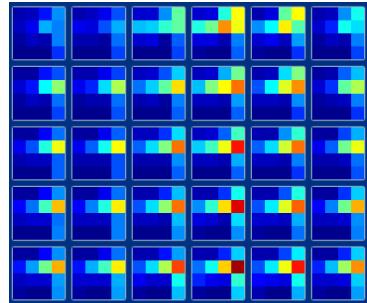
# Repeatability of Feature Sets: Translation and Occlusion



Probabilities that features ‘survive’  
viewpoint change, occlusion, etc are not  
independent.



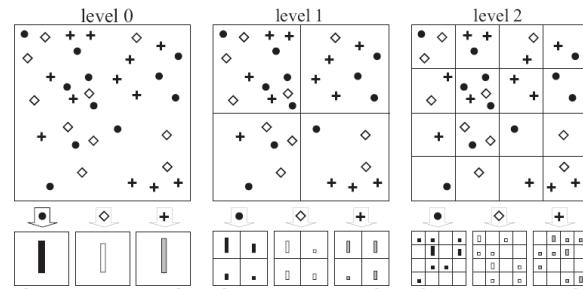
# Global Descriptors over Region of Interest



GIST

Oliva, Torralba: Modeling the shape of the scene: a holistic representation of the spatial envelope, IJCV 2001

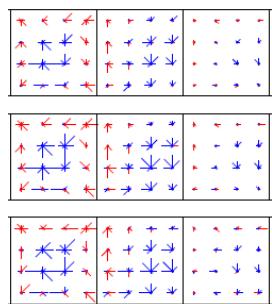
Deselaers, Alexe, Ferrari: Weakly Supervised Localization and Learning with Generic Knowledge, IJCV 2012



Spatial pyramid

Lazebnik, Schmid, Ponce: Beyond BoF: Spatial Pyramid Matching for Recognizing Natural Scene Categories. CVPR'06

Chum and Zisserman: An Exemplar Model for Learning Object Classes, CVPR 2007

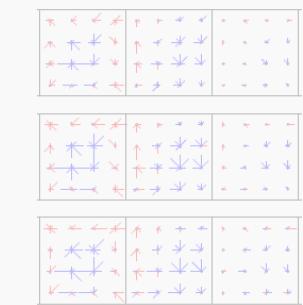
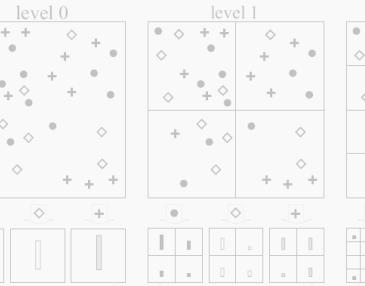
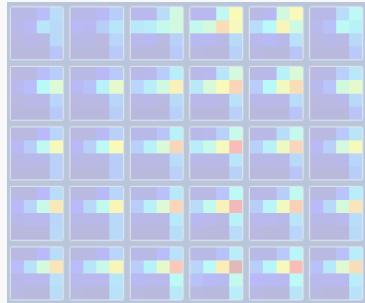


VLAD

Jégou, Douze, Schmid, and Pérez : Aggregating local descriptors into a compact image representation, CVPR'10

Arandjelovic and Zisserman: All about VLAD, CVPR'13

# Global Descriptors over Region of Interest



of the scene: a  
envelope, IJCV 2001

ervised Localization  
e, IJCV 2012

E: Spatial Pyramid  
ne Categories.

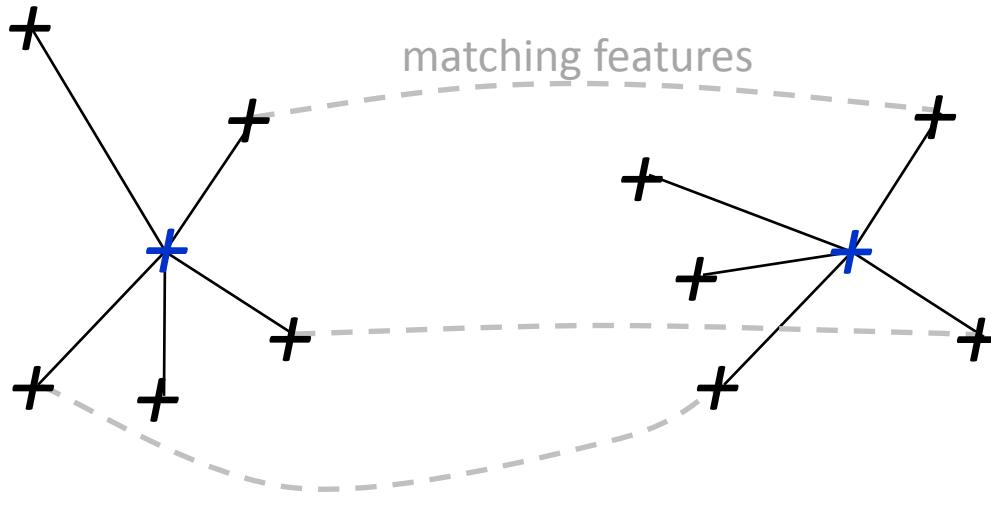
Model for Learning

gregating local  
resentation, CVPR'10

t VLAD, CVPR'13

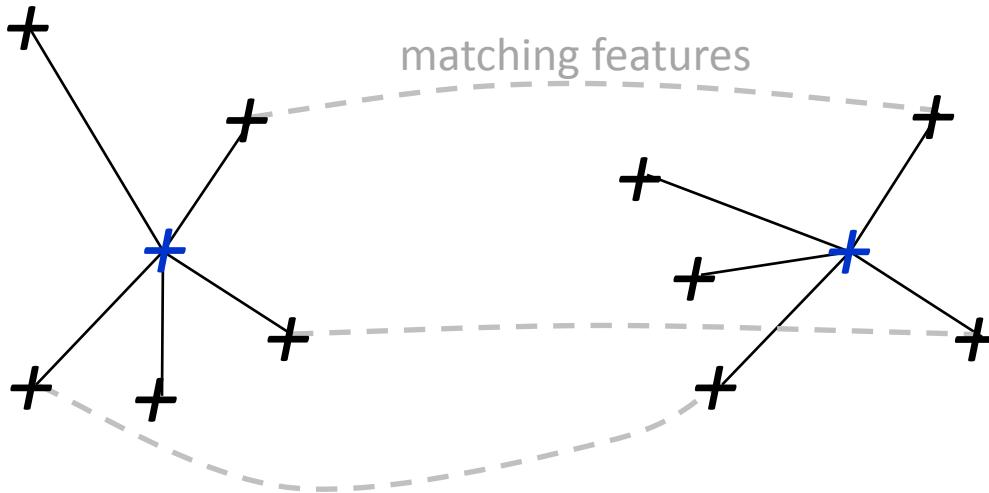
# Local Geometric Constraints

Idea: verify a tentative match “ $+$ ” by comparing neighboring features  
[Schmid and Mohr: Local Greyvalue Invariants for Image Retrieval. PAMI 1997]



matching appearance and locality  
used in [Sivic and Zisserman: Video Google, ICCV 2003]

# Local Geometric Constraints



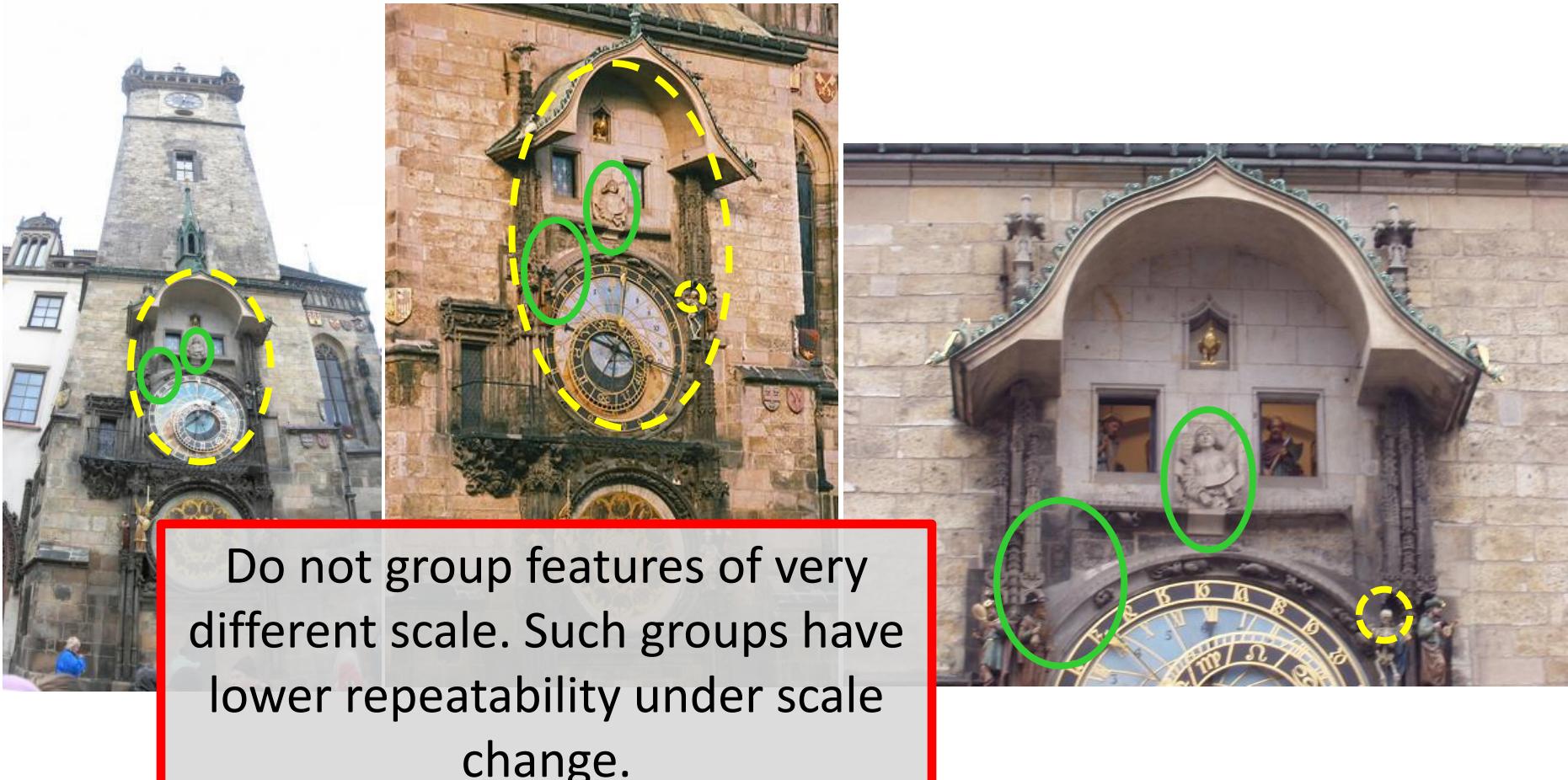
Neighbouring features:

- n nearest
- within an absolute distance
- within a relative distance
- also limited to a similar scale

Constraints (rigidity):

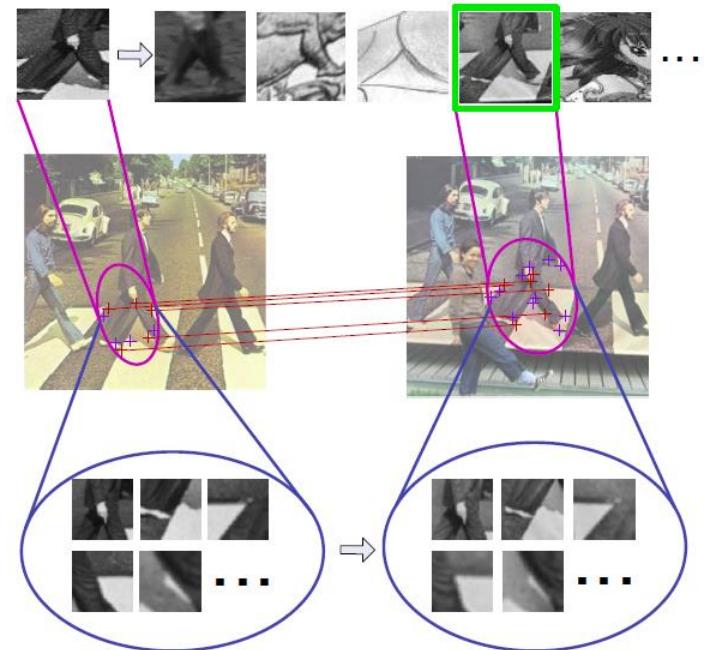
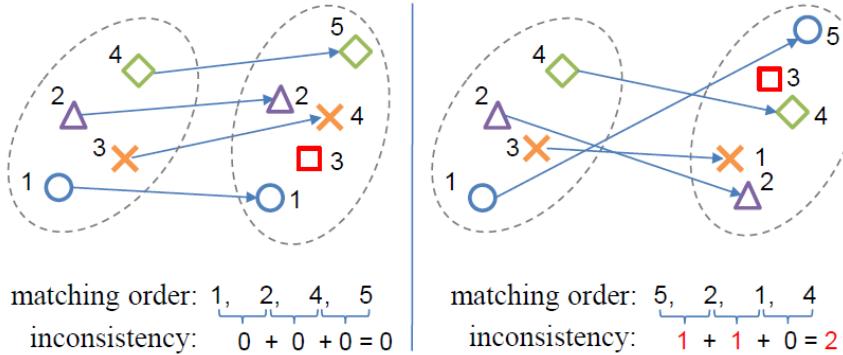
- presence (local BoF)
- similar angles
- similar distances
- combination

# Repeatability of Feature Sets: Scale Change



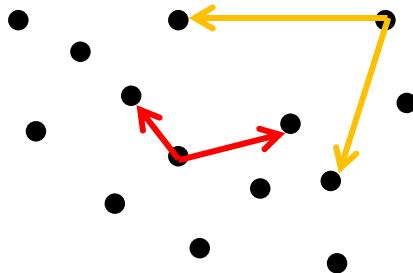
# Data driven neighbourhood

- Segments / regions (MSER) define neighbourhood
- Feature points (DoG)
- Order of projection to x and y
- Efficiently in the posting list



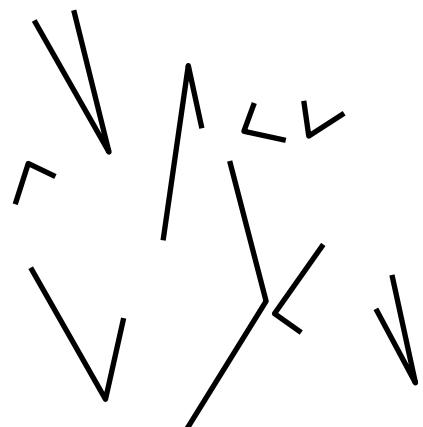
Zhong Wu, Qifa Ke, Michael Isard, and Jian Sun:  
 Bundling Features for Large Scale Partial-Duplicate Web Image Search, CVPR '09

# Geometric Hashing (the geometry is the descriptor)



- define coordinate system by (triplets of) points
- express other points in the coordinate system
- combinatorially many

Lamdan, Wolfson: Geometric hashing: A general and efficient model-based recognition scheme, ICCV 1988



- use regions that define a coordinate system
- express other regions in the coordinate system
- use spatial and scale neighbourhood

Chum and Matas: Geometric Hashing with LAFs, CVPR 2006



# Geometric min-Hash

# min-Hash

min-Hash is an efficient representation of a set  $A_i$

$$m(\mathcal{A}_i, f) = \arg \min_{X \in \mathcal{A}_i} f(X)$$

↑      ↑      ←  
set of visual words    hash function      visual word

min-Hash is a locality sensitive hashing (LSH) function  $m$  that selects an element (visual word)  $m(A_i)$  from each set  $A_i$  of visual words detected in image  $i$  so that

$$P\{m(\mathcal{I}_1) == m(\mathcal{I}_2)\} = \frac{|\mathcal{I}_1 \cap \mathcal{I}_2|}{|\mathcal{I}_1 \cup \mathcal{I}_2|}$$

Image similarity  $\text{sim}(\mathcal{I}_1, \mathcal{I}_2) = \frac{|\mathcal{I}_1 \cap \mathcal{I}_2|}{|\mathcal{I}_1 \cup \mathcal{I}_2|}$

# min-Hash

Vocabulary

vocabulary of 1 M  
visual words

Random orderings  
(independent)

3	6	2	5	4	1
1	2	6	3	5	4
3	2	1	6	4	5

Set  $I_1$

$\sim 2000$  words  
per image

Set  $I_2$

B C D F

min-Hash

$m_1 :$

F

$m_2 :$

A

$m_3 :$

C

fixed size  
image  
representation  
 $\sim 100$

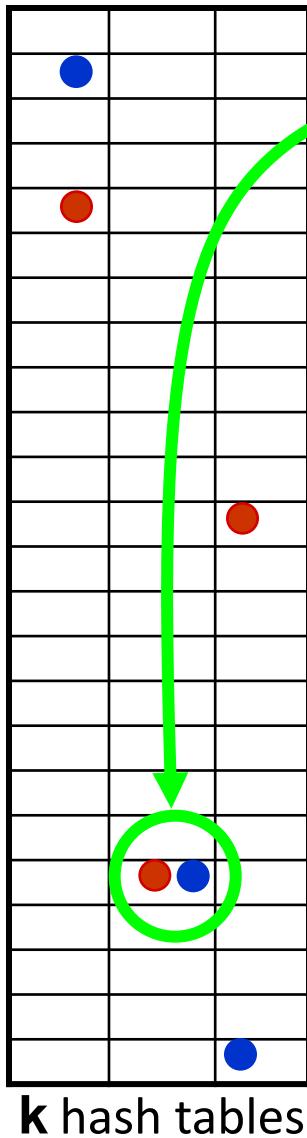
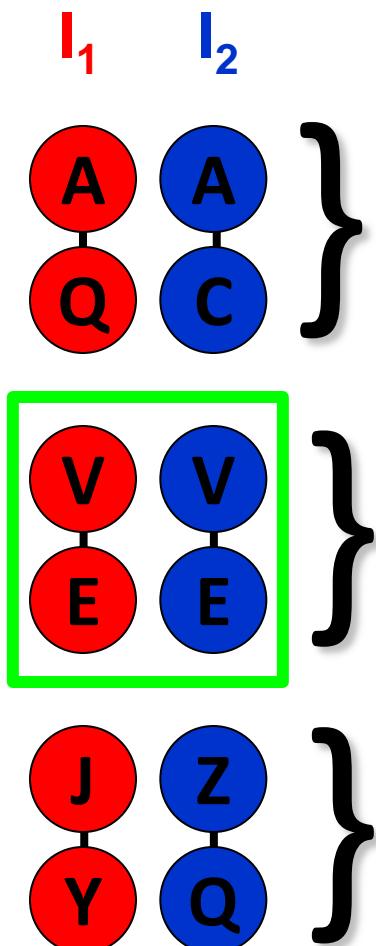
F  
B  
C

$$\text{sim } (I_1, I_2) = 1/2$$

Estimated similarity of  $I_1$  and  $I_2$  from 3 min-Hashes = 2/3

# Seed Generation: min-Hash Retrieval

a sketch =  $s$ -tuple of min-Hashes



**Sketch collision:**  
all  $s$  min-Hashes must agree

$$P\{\text{collision}\} = \text{sim}(I_1, I_2)^s$$

**retrieval:**

1. generate  $k$  sketches
2. at least one of  $k$  sketches must collide

$$P\{\text{retrieval}\} = 1 - (1 - \text{sim}(I_1, I_2)^s)^k$$

# Probability of Retrieving an Image Pair

Images of the same object  
and unrelated images



13.9 % (sim = 0,066)



8.9 % (sim = 0.057)



5.1% (sim = 0.047)

$s = 3, k = 512$

Near duplicate Images



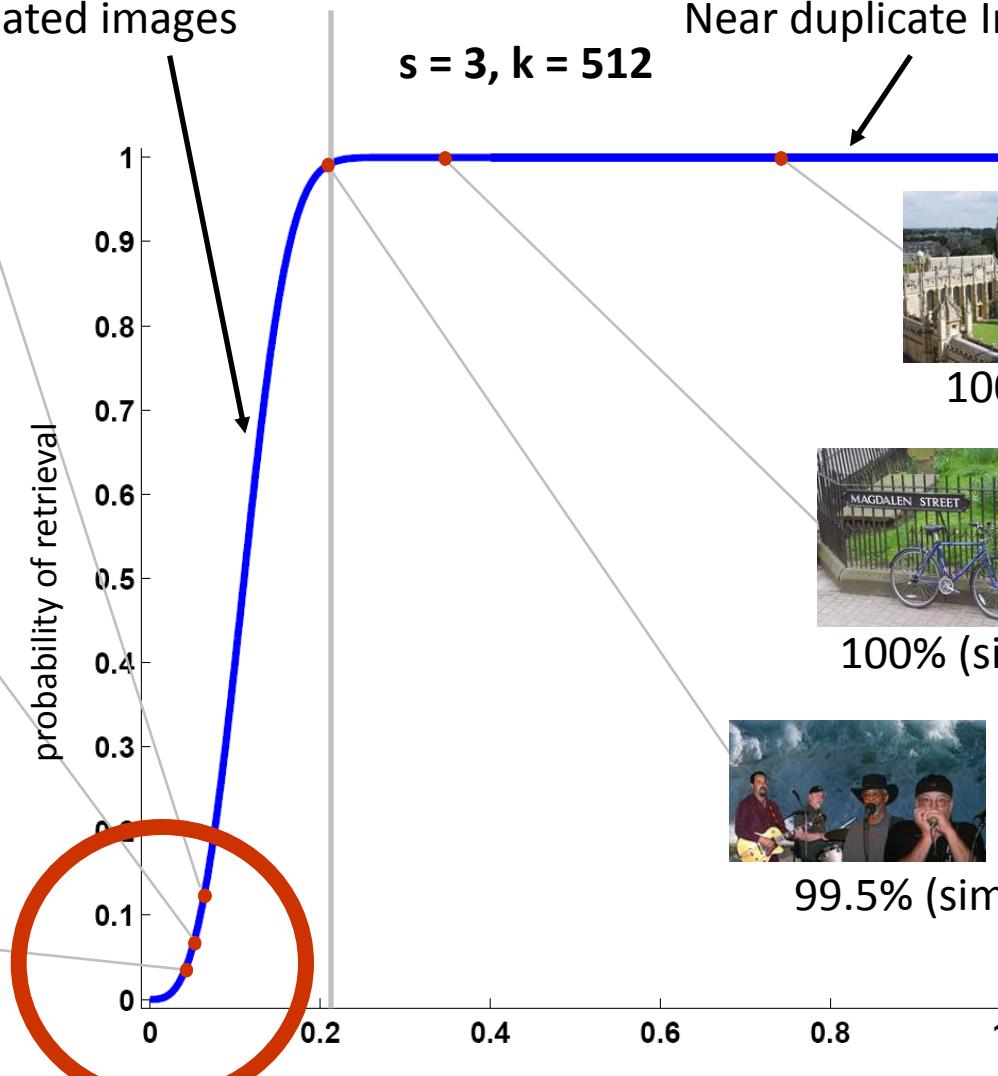
100% (sim = 0.746)



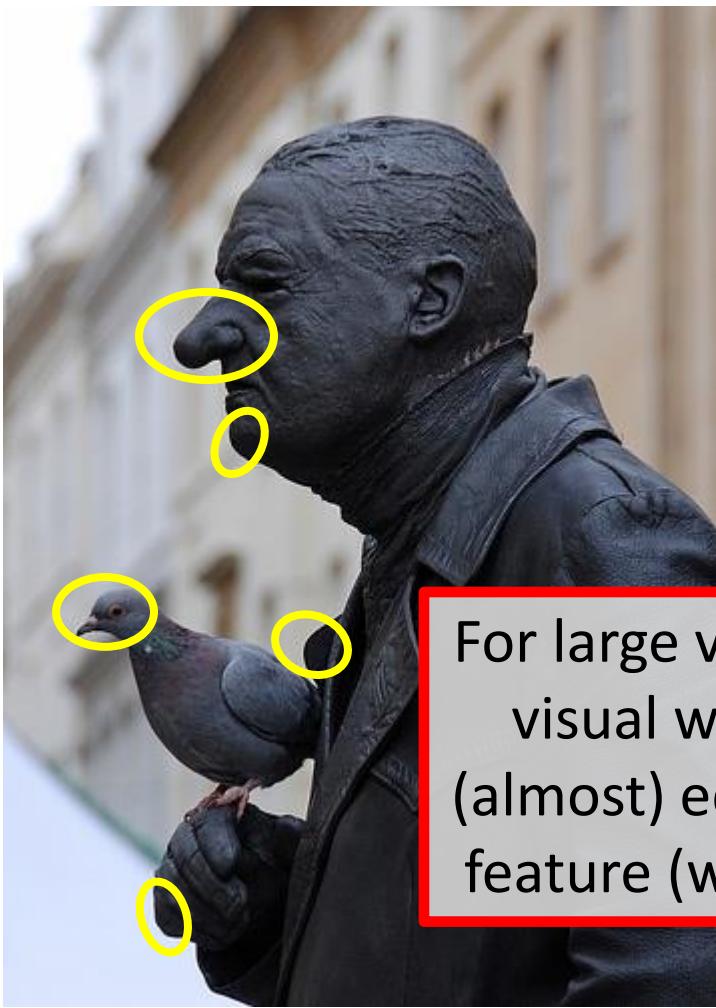
100% (sim = 0.322)



99.5% (sim = 0,217)



# Vocabulary Size and Set Representation



For large vocabularies selecting a visual word from an image is (almost) equivalent to selecting a feature (with location and scale)

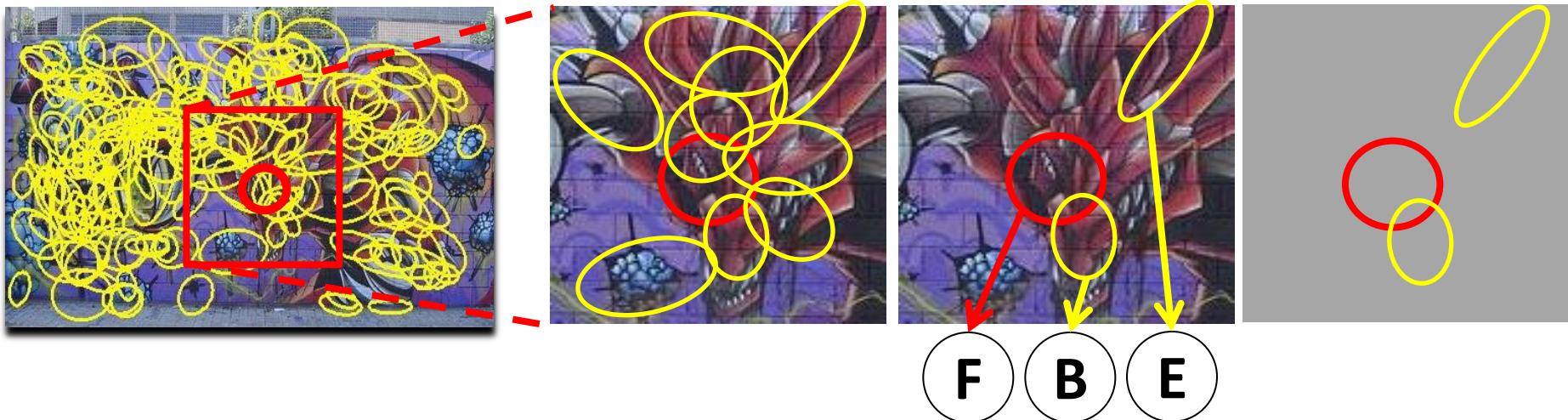


On a 100k dataset of images, 95% of features have a unique visual word in an image

# Geometric min-Hash algorithm

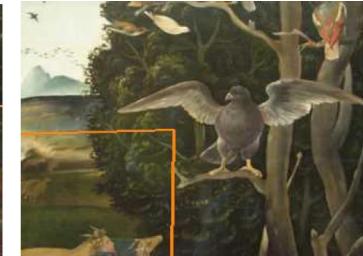
1. Keep features with unique visual word in the image
2. Obtain the “central feature” by min-Hash
3. Select scale and spatial neighbourhood of the central feature
4. Select secondary min-Hash(es) from the neighbourhood
5. Relative pose of the sketch features is a geometric invariant (as in geometric hashing)

Sketch of GmH: s-tuple of visual words + geometric invariant



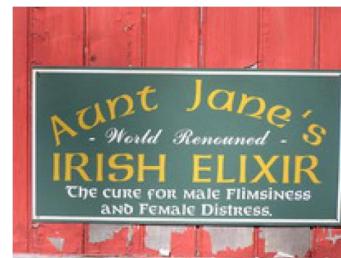
# Geometric vs. Standard min-Hash

- Higher true positives
  - View point change
  - Severe occlusion
  - Scale change
  - Object on a different background
- Lower false positives
  - Additional geometric invariant (part of the hash key or verification)
  - Lower probability random sketch collisions (next slide)
- Faster spatial verification
  - Sketch collision defines geometric transformation



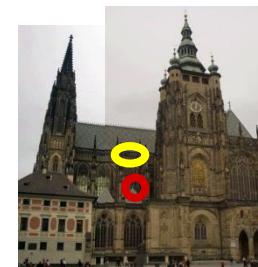
Sketches S=2, k =5000

GmH: 37 collisions      mH: 4 collisions



Sketches S=2, k =5000

GmH: 0 collisions      mH: 4 collisions



# Overlap of Random Sets

False positive = sketch collision of two random images

The probability of two random sets  $I_1$  and  $I_2$  having a common min-Hash (*i.e.* the average overlap of two random sets)

$$\frac{\min(|I_1|, |I_2|)}{2w} \leq E\left(\frac{|I_1 \cap I_2|}{|I_1 \cup I_2|}\right) \leq \frac{\min(|I_1|, |I_2|)}{w}$$

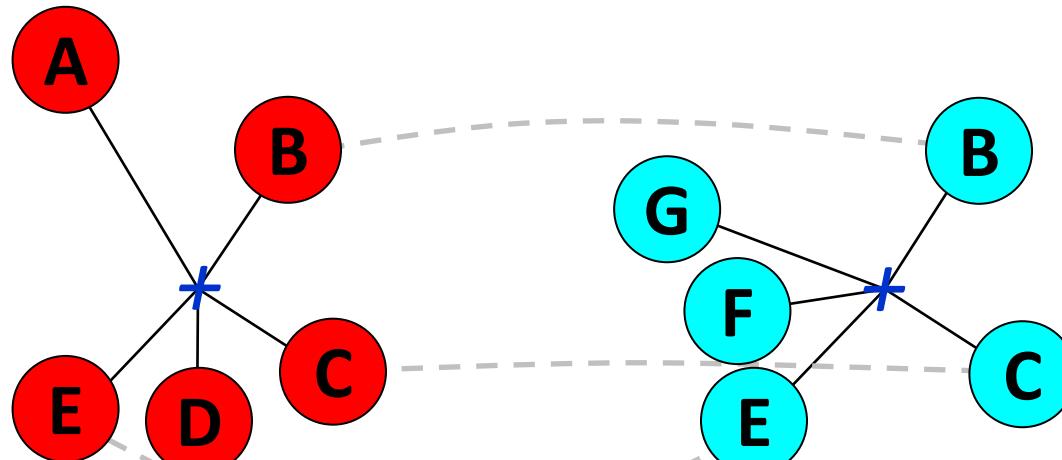
where  $w$  is the size of the vocabulary

**The smaller the sets, the smaller probability of random collision**

- Min-Hash: features in the whole image
- Geometric min-Hash: only small subset of the image

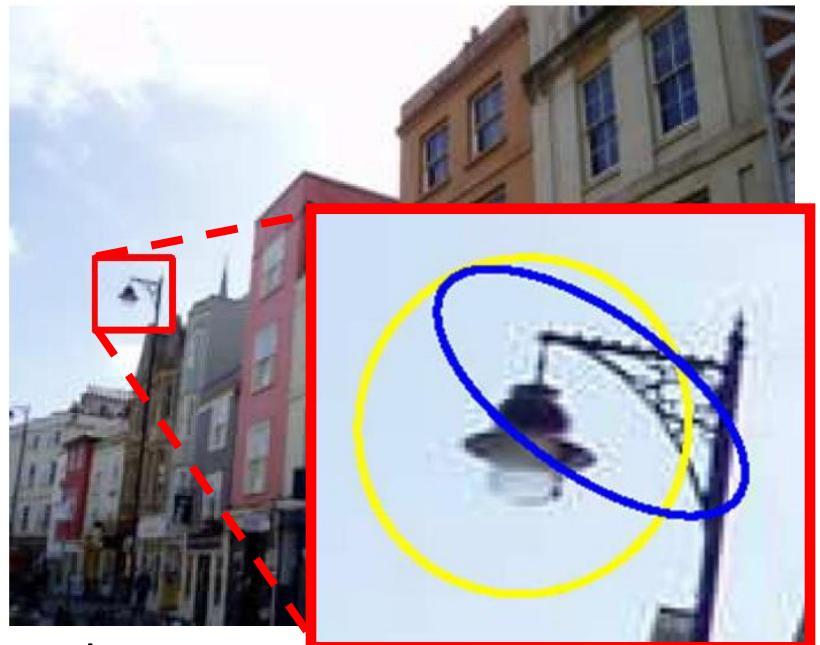
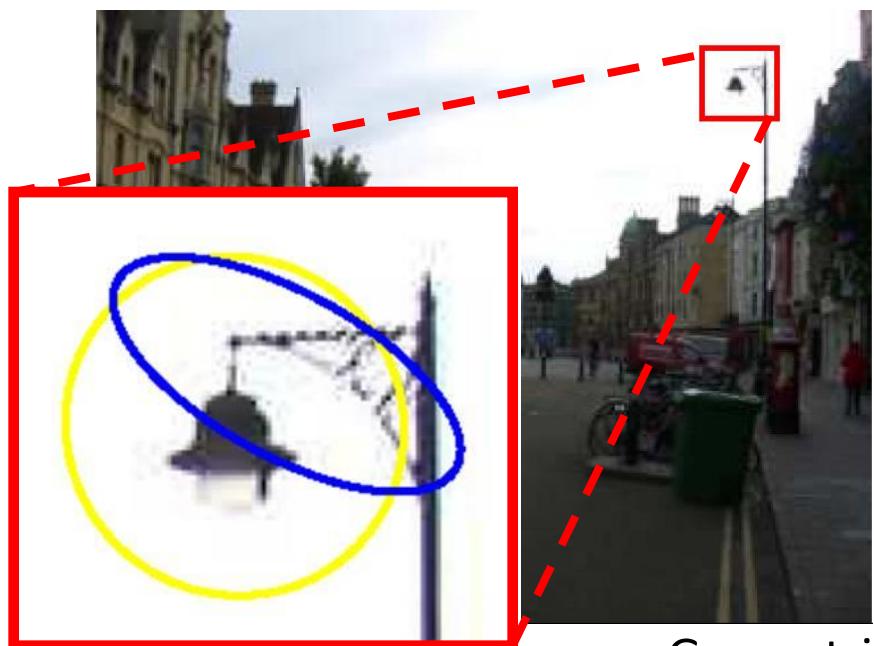
# Geometric min-Hash as Local Geometric Constraints

Idea: verify a tentative match “+” by comparing neighboring features  
[Schmid and Mohr: Local Greyvalue Invariants for Image Retrieval. PAMI 1997]



**Geometric min-Hash is an  
implicit, robust, constant-time matching of local neighborhood**

# Object Discovery



Geometric min-Hash  
sketch collision  
 $s = 2, k = 256$

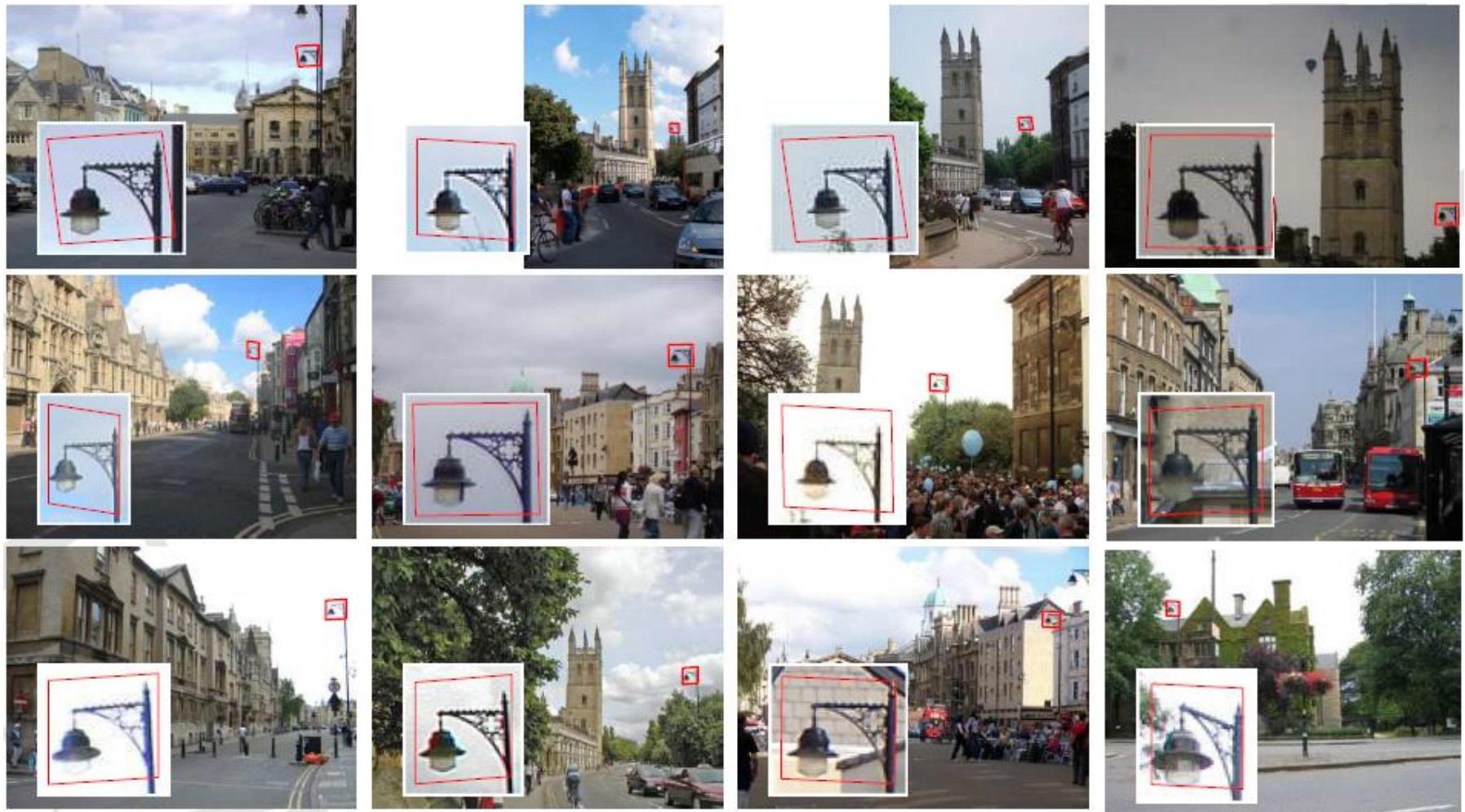
**Verification by co-segmentation  
critical for small objects**



[Cech, Matas, Perdoch CVPR 08], code available on WWW  
[Ferrari, Tuytelaars, Van Gool, ECCV 2004]

# Object Discovery

Other instances of the discovered object by (sub)image retrieval



# Global Geometry

- Voting in the parameter space
- RANSAC

# Robust Estimation: Hough vs. RANSAC

## Voting:

- discretized parameter space
  - votes for parameters consistent with the measurements
  - more votes higher support
- 
- + multiple models
  - + can be very fast
  - memory demanding
  - distances measured in the parameter space

## RANSAC:

- hypothesize and verify loop
  - randomized (unless you try it all)
  - typically slower than voting
  - + no extra memory required
  - + measures distances in pixels!

# Geometric Re-ranking

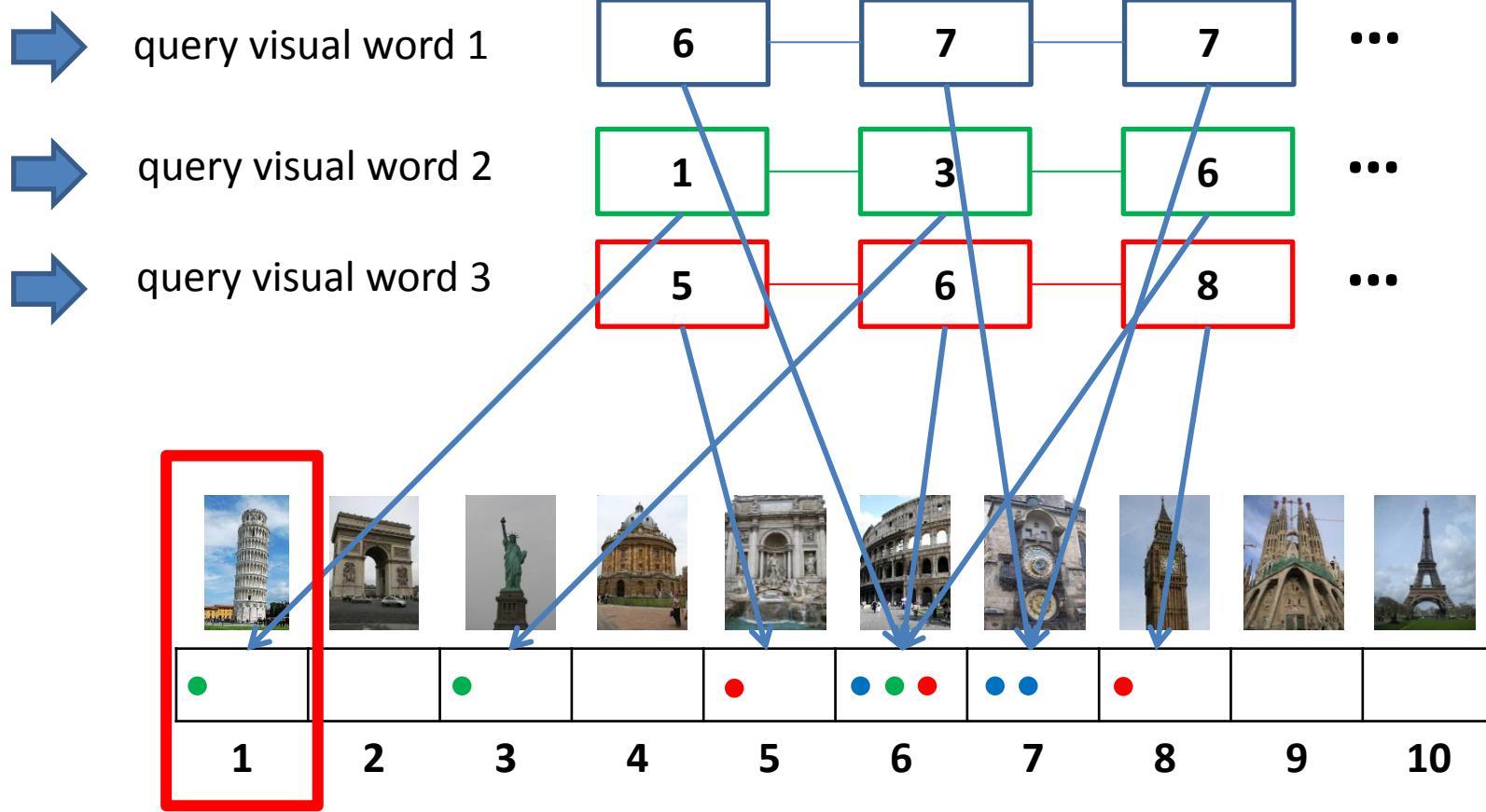
1. Perform ranking without geometric information
  - BoW
  - VLAD
2. Re-rank top ranked images (removing false positives)
  - RANSAC

Sivic, Zisserman: Video Google, ICCV 2003

Philbin, Chum, Isard, Sivic, Zisserman: Object retrieval with large vocabularies and fast spatial matching, CVPR'07

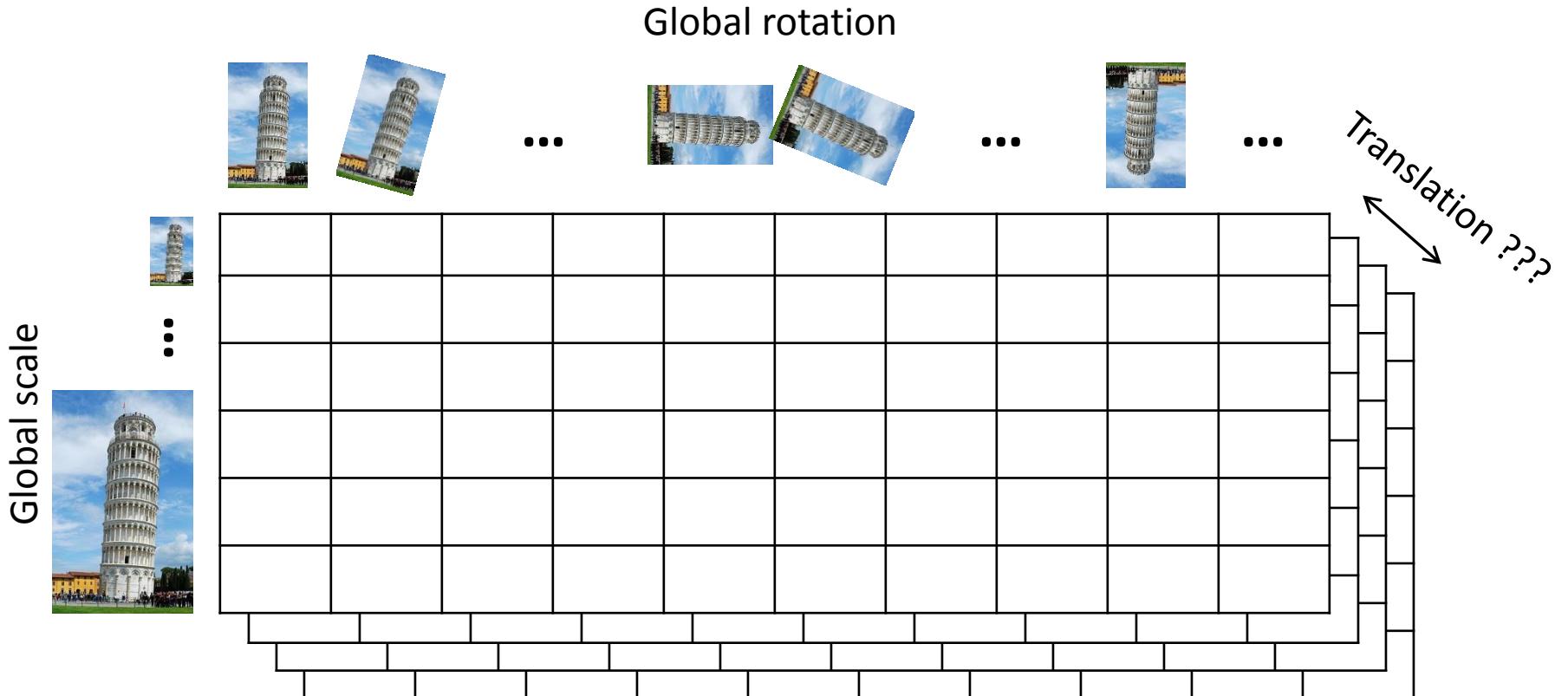
# BoW and Inverted File

$$\text{score} = \frac{\mathbf{q}^T \mathbf{x}}{\|\mathbf{x}\|}$$



# Weak Geometry Constraints

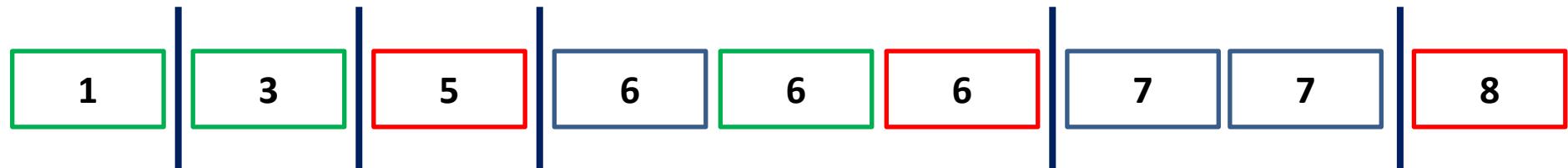
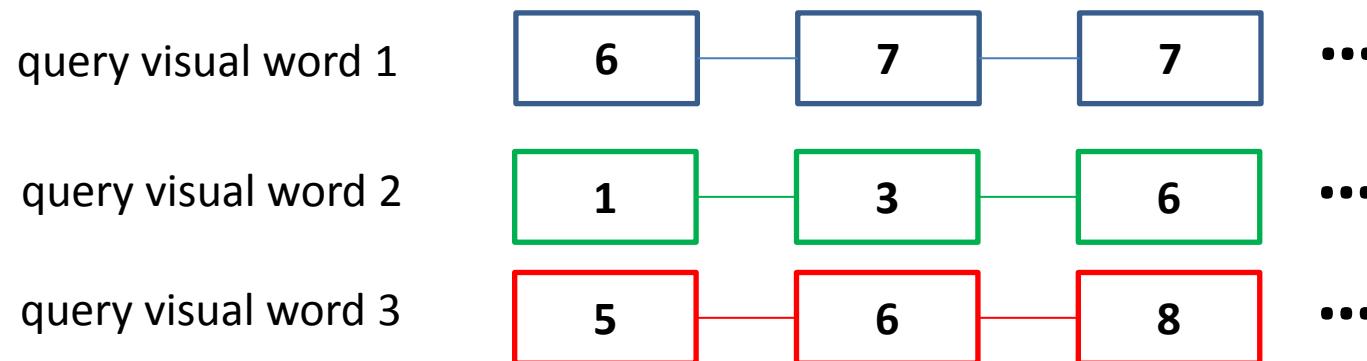
Jégou, Douze, Schmid: Hamming Embedding and  
Weak Geometric consistency for large-scale image search, ECCV'08, IJCV'10;  
(use margins over rotation and scale rather than the whole table)



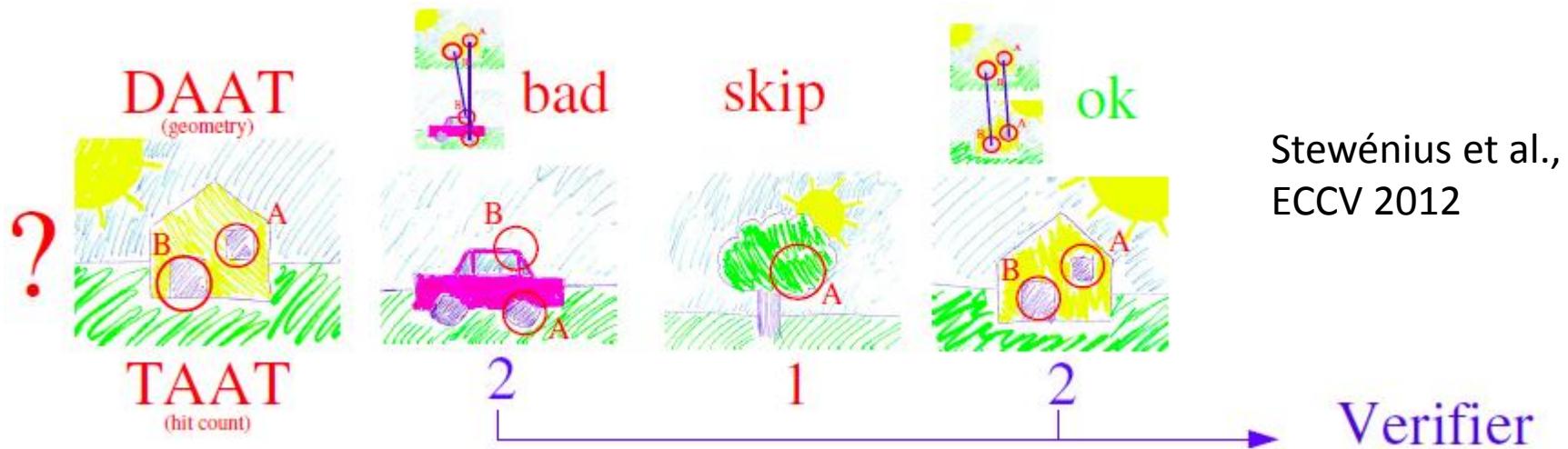
# Exhaustive Geometric Verification

Stewénius, Gunderson, Pilet:

Size matters: exhaustive geometric verification for image retrieval, ECCV 2012

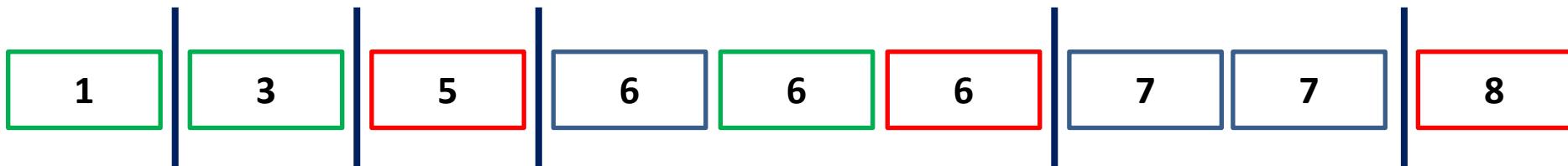


# Exhaustive Geometric Verification



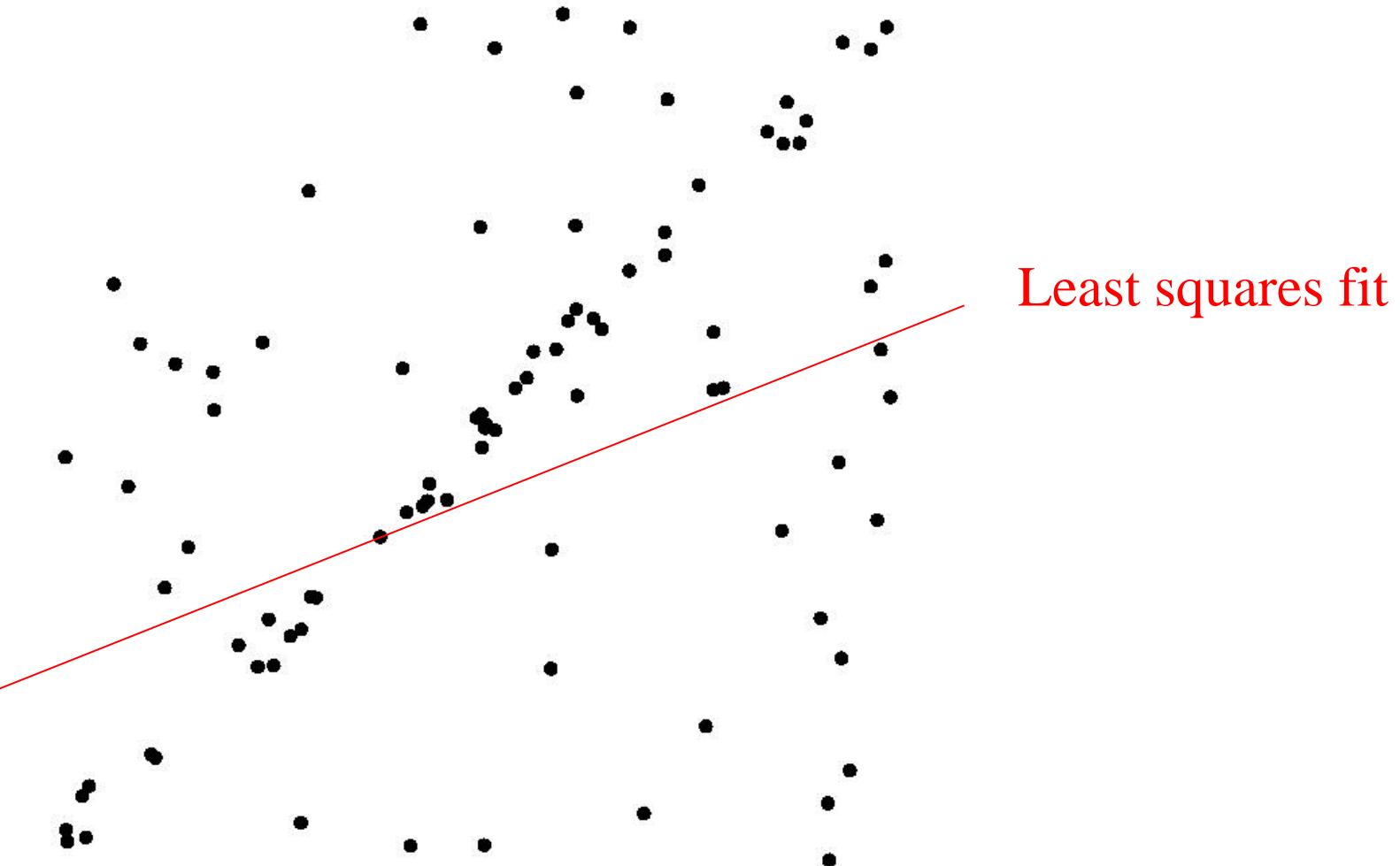
- Costs time:  $\log$  (number of posting lists involved)
- Efficient merging with fixed tree structure
- Any kind of spatial verification can be used
- Images with too few matches are rejected directly

Combined with small number of features per image and very large vocabularies gives impressive massive-scale results

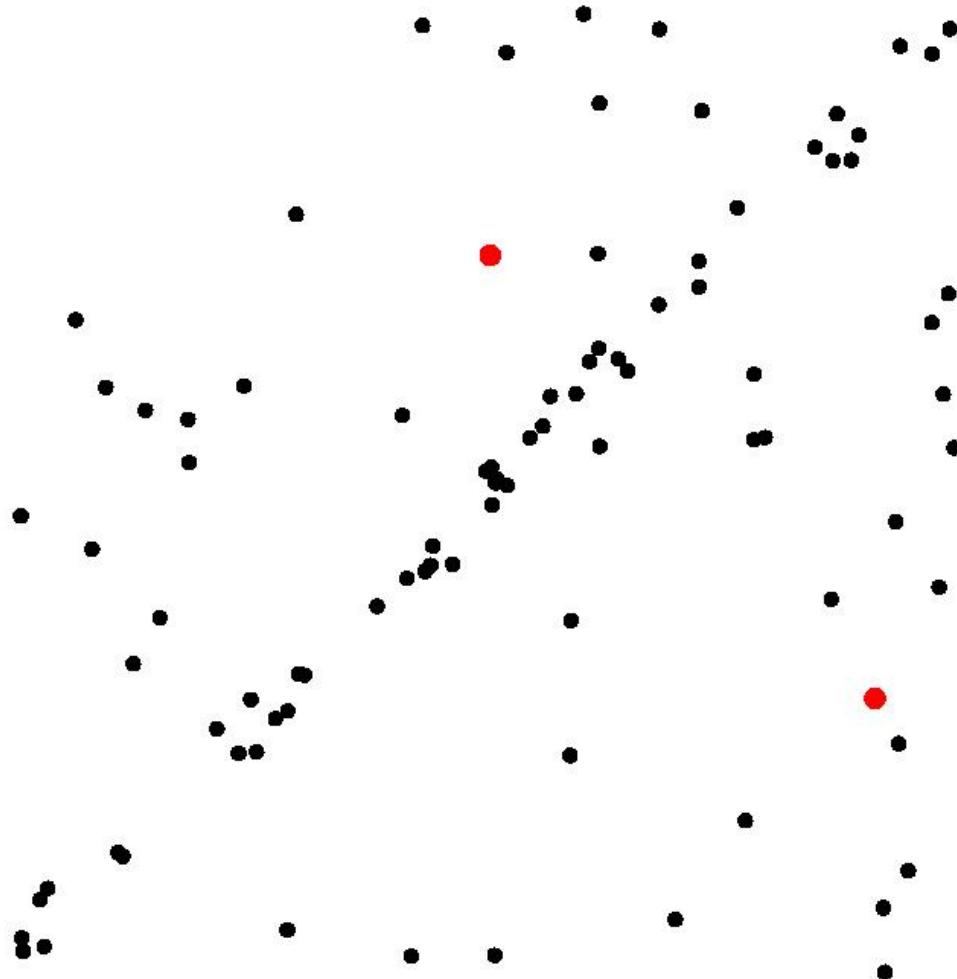


# RANSAC

# Fitting a Line

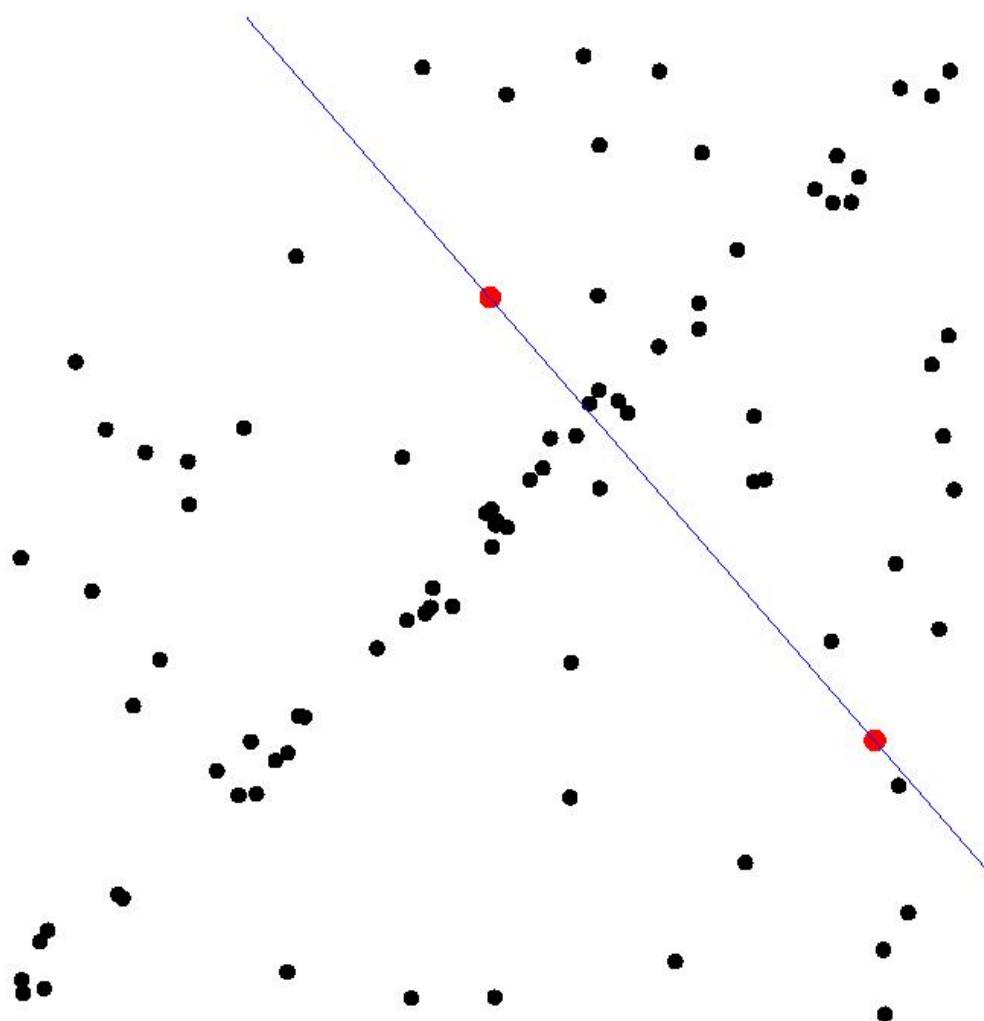


# RANSAC



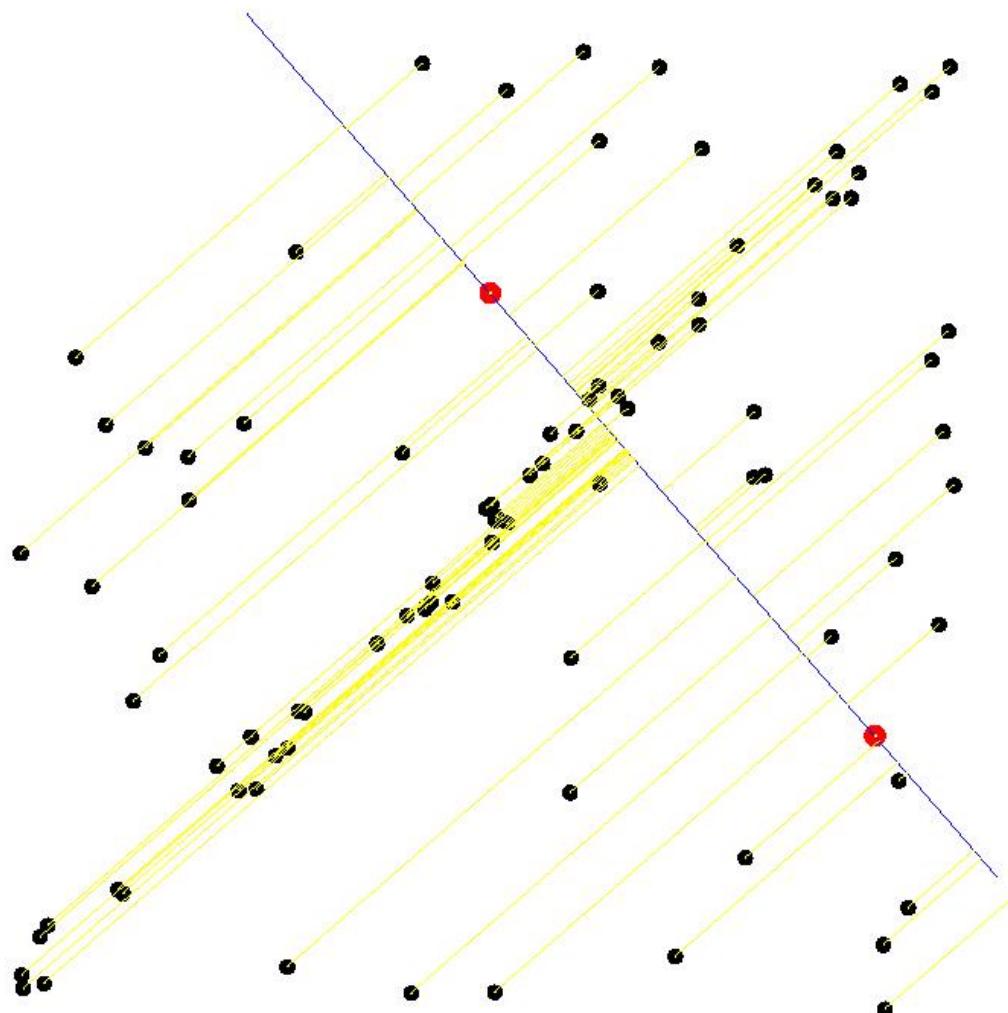
- Select sample of  $m$  points at random

# RANSAC



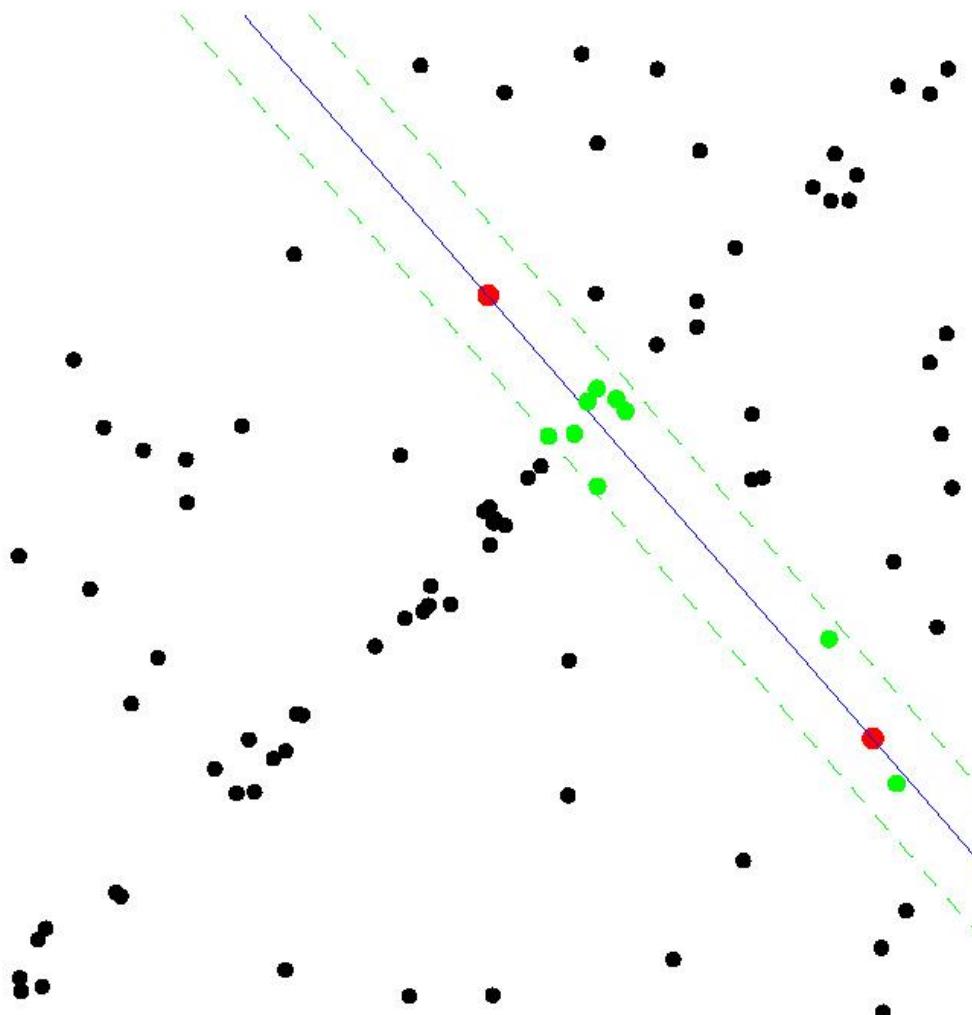
- Select sample of  $m$  points at random
- Calculate model parameters that fit the data in the sample

# RANSAC



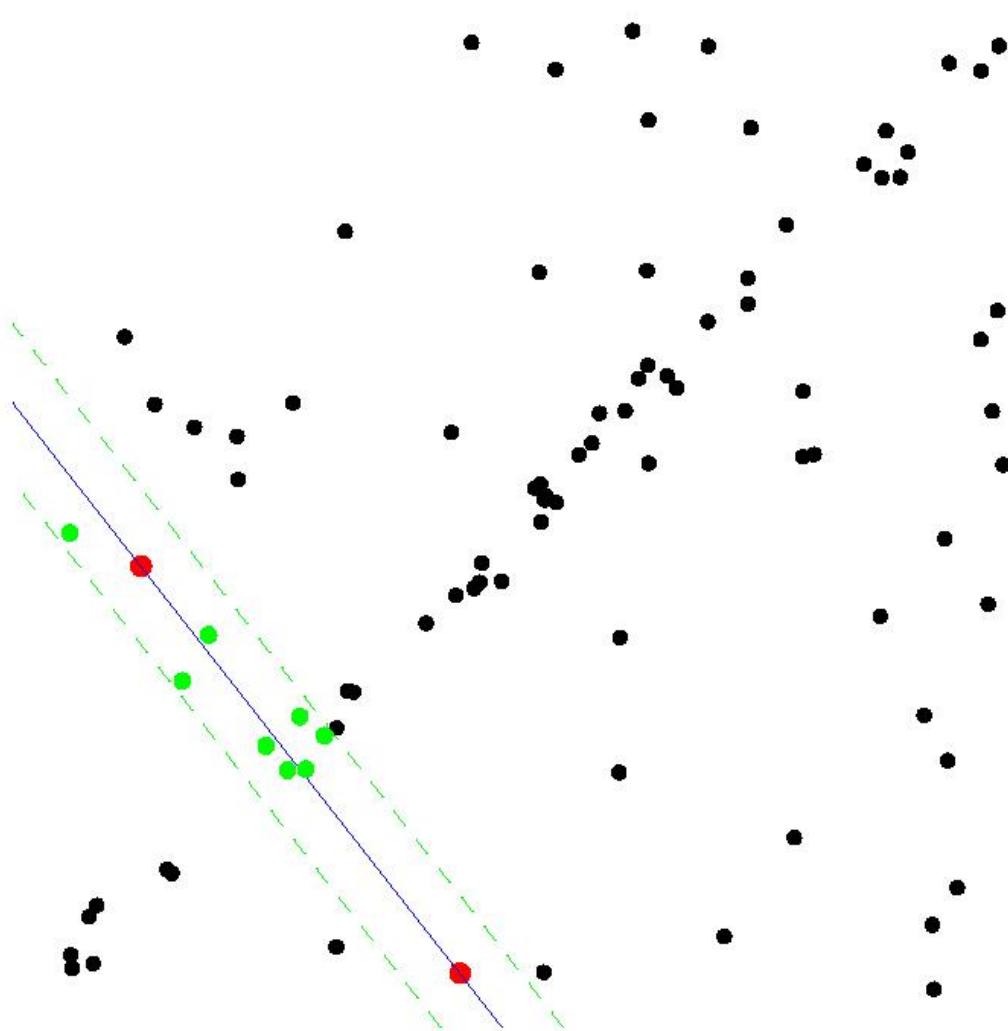
- Select sample of  $m$  points at random
- Calculate model parameters that fit the data in the sample
- **Calculate error function for each data point**

# RANSAC



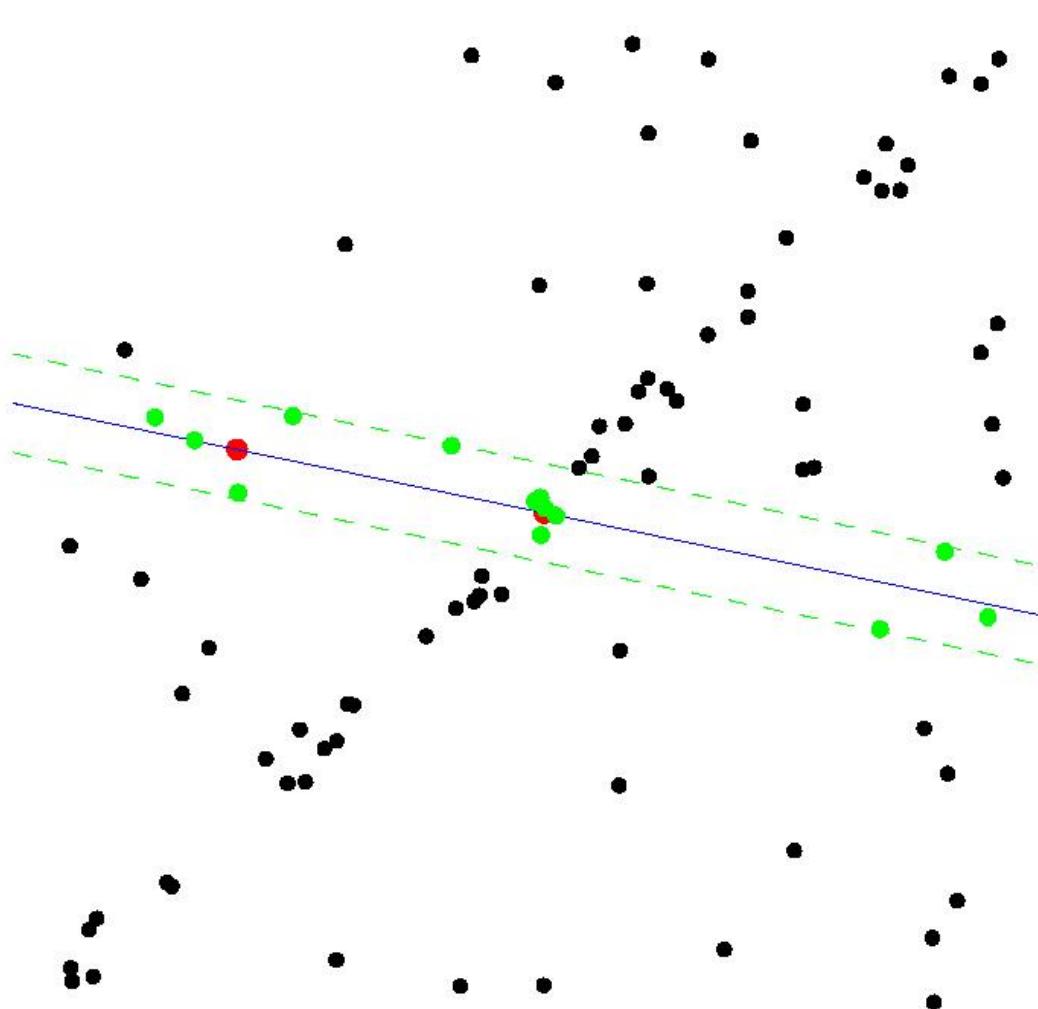
- Select sample of  $m$  points at random
- Calculate model parameters that fit the data in the sample
- Calculate error function for each data point
- **Select data that support current hypothesis**

# RANSAC



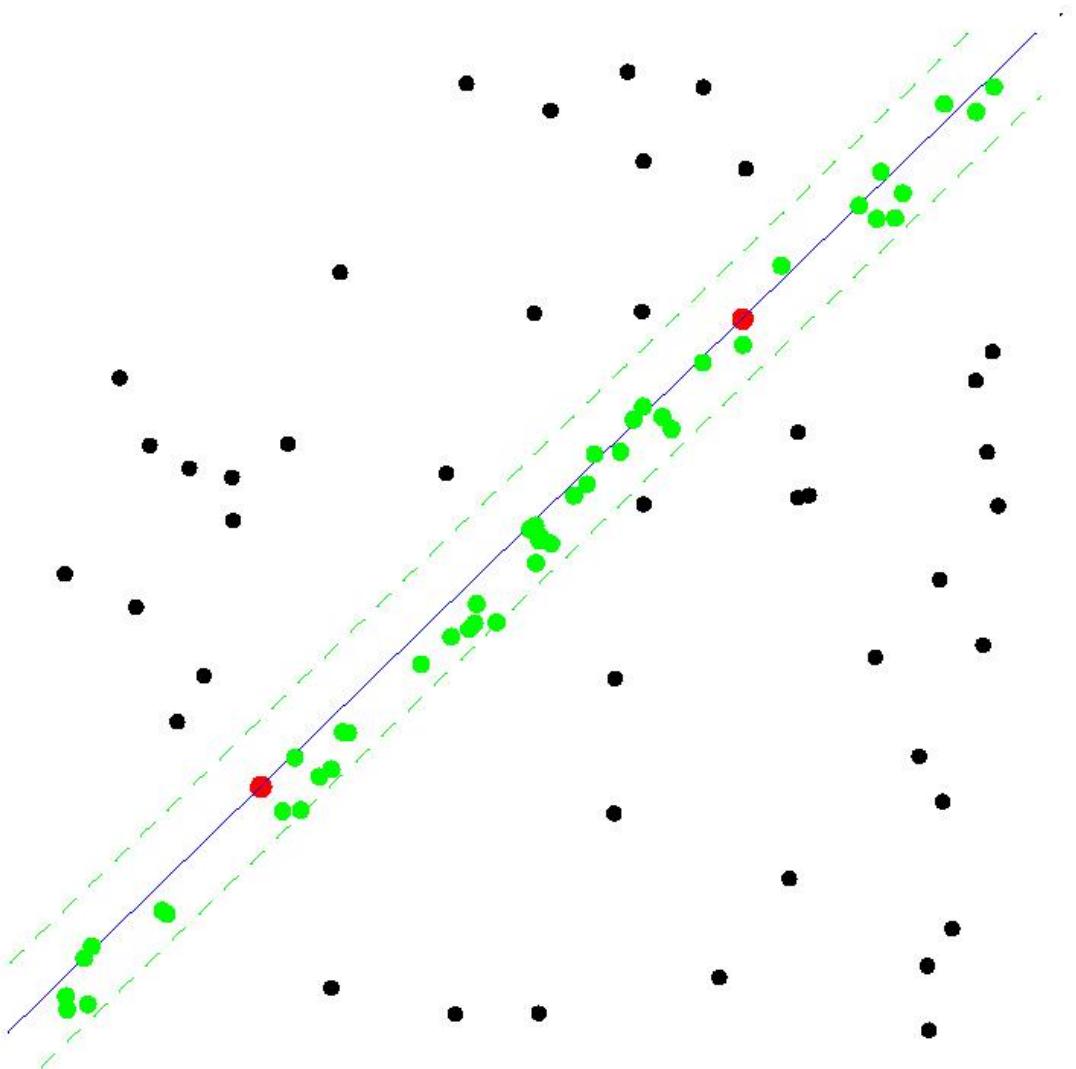
- Select sample of  $m$  points at random
- Calculate model parameters that fit the data in the sample
- Calculate error function for each data point
- Select data that support current hypothesis
- **Repeat sampling**

# RANSAC



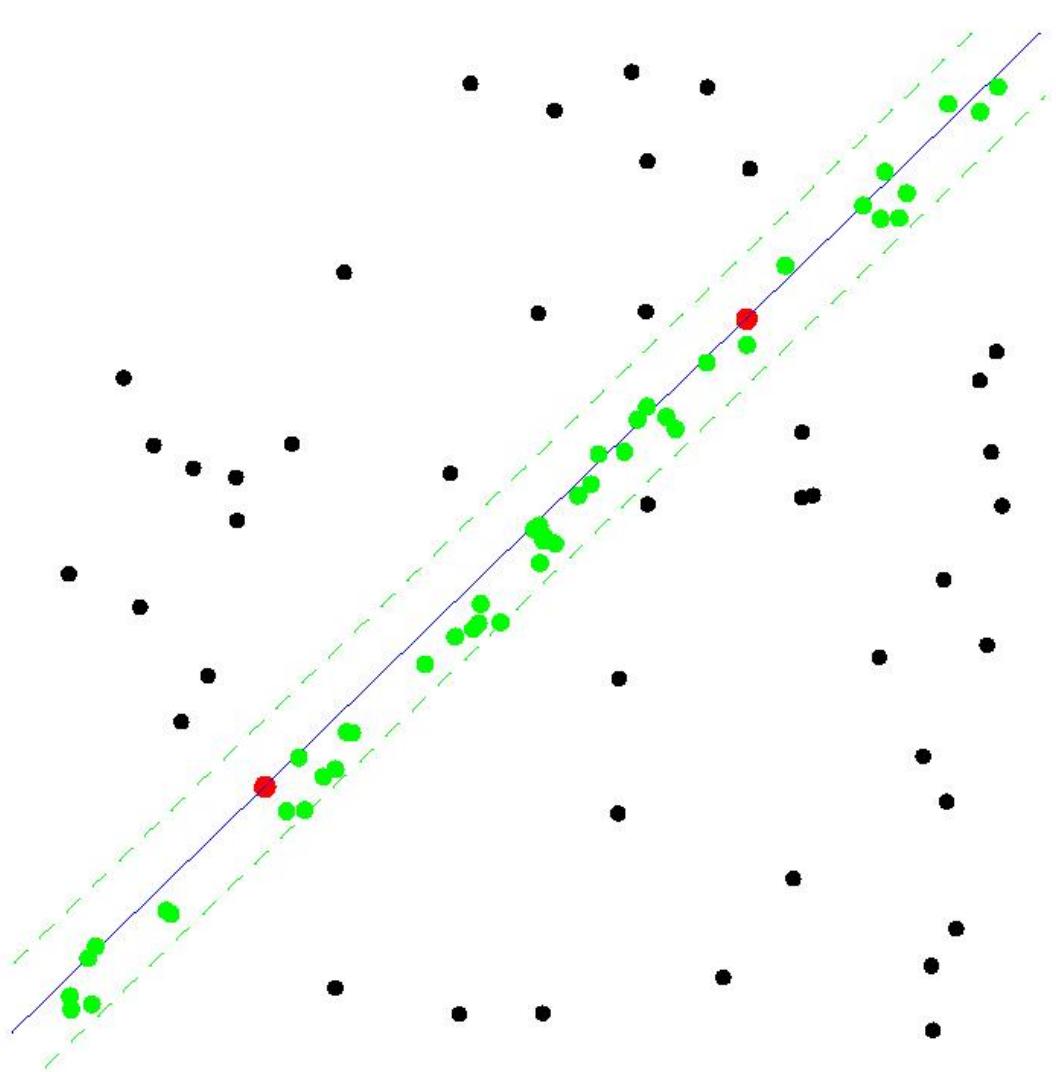
- Select sample of  $m$  points at random
- Calculate model parameters that fit the data in the sample
- Calculate error function for each data point
- Select data that support current hypothesis
- **Repeat sampling**

# RANSAC



- Select sample of  $m$  points at random
- Calculate model parameters that fit the data in the sample
- Calculate error function for each data point
- Select data that support current hypothesis
- **Repeat sampling**

# RANSAC



$$k = \frac{\log(1 - p)}{\log \left( 1 - \frac{I^m}{N^m} \right)}$$

- $k$  ... number of samples drawn
- $m$  ... minimal sample size
- $N$  ... number of data points
- $I$  ... time to compute a single model
- $p$  ... confidence in the solution (.95)

# How Many Samples

$I / N [\%]$

Size of the sample  $m$

	15%	20%	30%	40%	50%	70%
2	132	73	32	17	10	4
4	5916	1871	368	116	46	11
7	$1.75 \cdot 10^6$	$2.34 \cdot 10^5$	$1.37 \cdot 10^4$	1827	382	35
8	$1.17 \cdot 10^7$	$1.17 \cdot 10^6$	$4.57 \cdot 10^4$	4570	765	50
12	$2.31 \cdot 10^{10}$	$7.31 \cdot 10^8$	$5.64 \cdot 10^6$	$1.79 \cdot 10^5$	$1.23 \cdot 10^4$	215
18	$2.08 \cdot 10^{15}$	$1.14 \cdot 10^{13}$	$7.73 \cdot 10^9$	$4.36 \cdot 10^7$	$7.85 \cdot 10^5$	1838
30	$\infty$	$\infty$	$1.35 \cdot 10^{16}$	$2.60 \cdot 10^{12}$	$3.22 \cdot 10^9$	$1.33 \cdot 10^5$
40	$\infty$	$\infty$	$\infty$	$2.70 \cdot 10^{16}$	$3.29 \cdot 10^{12}$	$4.71 \cdot 10^6$

# RANSAC [Fischler, Bolles '81]

**In:**  $U = \{x_i\}$  set of **data points**,  $|U| = N$

$f(S) : S \rightarrow p$  function  $f$  computes **model parameters**  $p$  given a sample  $S$  from  $U$

$\rho(p, x)$  the **cost function** for a single data point  $x$

**Out:**  $p^*$   $p^*$ , parameters of the model maximizing the cost function

$k := 0$

Repeat until  $P\{\text{better solution exists}\} < \eta$  (a function of  $C^*$  and no. of steps  $k$ )

$k := k + 1$

## I. Hypothesis

(1) select randomly set  $S_k \subset U$ , **sample size**  $|S_k| = m$

(2) compute parameters  $p_k = f(S_k)$

## II. Verification

(3) compute cost  $C_k = \sum_{x \in U} \rho(p_k, x)$

(4) if  $C^* < C_k$  then  $C^* := C_k$ ,  $p^* := p_k$

end

# Advanced RANSAC

In:  $U = \{x_i\}$

set of **data points**,  $|U| = N$

$f(S) : S \rightarrow p$

function  $f$  computes **model parameters**  $p$  given a sample  $S$  from  $U$

$\rho(p, x)$

the **cost function** for a single data point  $x$

Out:  $p^*$

$p^*$ , parameters of the model maximizing the cost function

$k := 0$

Preemptive scoring

Repeat until  $P\{\text{better solution exists}\} < \eta$  (a function of  $C^*$  and no. of steps  $k$ )

$k := k + 1$

Non-uniform sampling

I. Hypothesis

(1) select randomly set  $S_k \subset U$ , **sample size**  $|S_k| = m$

(2) compute parameters  $p_k = f(S_k)$

Error scale estimation

II. Verification

(3) compute cost  $C_k = \sum_{x \in U} \rho(p_k, x)$

Randomized verification

(4) if  $C^* < C_k$  then  $C^* := C_k$ ,  $p^* := p_k$

end

Potential degeneracy tests

Improving precision

# \*SAC

**RANSAC** [Fischler'81], **MLESAC** [Torr'00], **R-RANSAC** [Chum'02], **NAPSAC** [Myatt'02], **Guided MLESAC** [Tordoff'02], **LO-RANSAC** [Chum'03], **Preemptive RANSAC** [Nister'03], **PROSAC** [Chum'05], **RANSAC with bail-out** [Capel'05], **DegenSAC** [Chum'05], **WaldSAC** [Matas'05], **QDEGSAC** [Frahm'06], **GASAC** [Rodehorst'06], **ARRSAC** [Raguram'08] **GroupSAC** [Ni'09], **Cov-RANSAC** [Raguram'09], ...

Lebeda, Matas, and Chum: **Fixing the Locally Optimized RANSAC**, BMVC 2012

images, data, executables:

<http://cmp.felk.cvut.cz/software/LO-RANSAC/index.xhtml>

Raguram, Chum, Pollefeys, Matas, Frahm:

**"USAC: A Universal Framework for Random Sample Consensus"**, PAMI 2013

code, data:

<http://cs.unc.edu/~rraguram/usac/>

# Minimal Samples

# Single Correspondence Models

Point

Translation

Point + scale

Translation and isotropic scale

Point + scale + orientation

Similarity

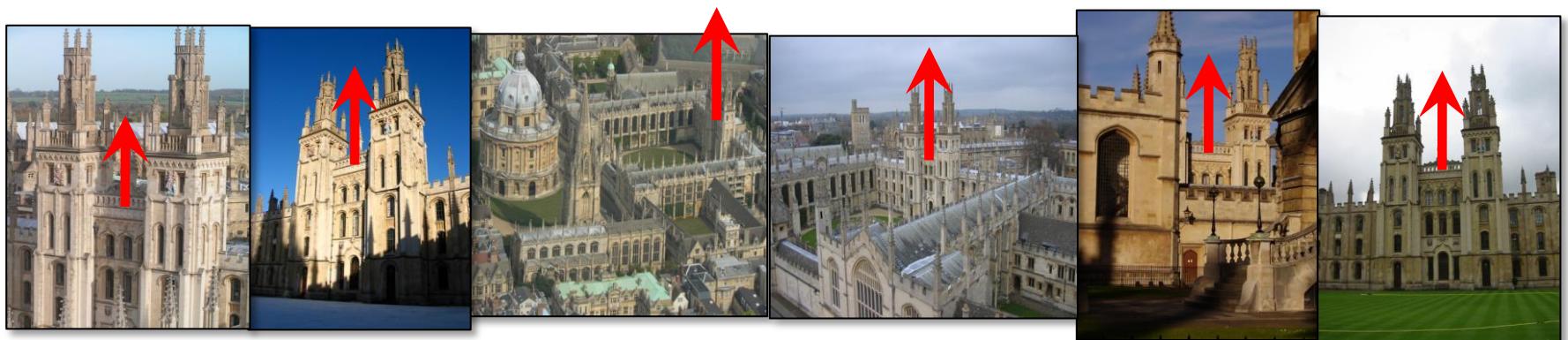
Ellipse + orientation

Affine transformation

Ellipse correspondence and a gravity vector

Philbin et al.: Object retrieval with large vocabularies and fast spatial matching,  
CVPR'07 (using rotationally invariant descriptors)

Perdoch, Chum, Matas: Efficient Representation of Local Geometry for Large Scale  
Object Retrieval, CVPR 2009 (using rotationally variant descriptors)



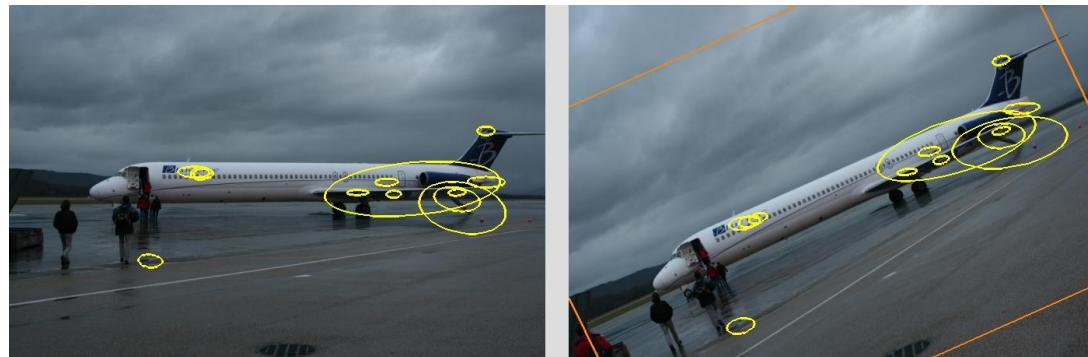
# Gravity Vector Assumption

- Performance comparison

mAP	Oxford5K vocab.		Paris vocab.	
	Ox5K	Ox105K	Ox5K	Ox105K
Dominant orient.	0.772 0.887*	0.687 0.844*	0.592 0.733*	0.501 0.637*
Gravity vector	0.786 0.900*	0.723 0.852*	0.635 0.782*	0.572 0.725*

\*with query expansion.

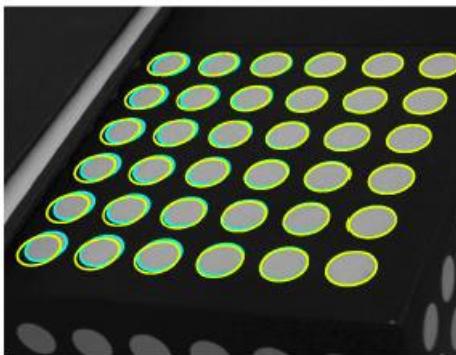
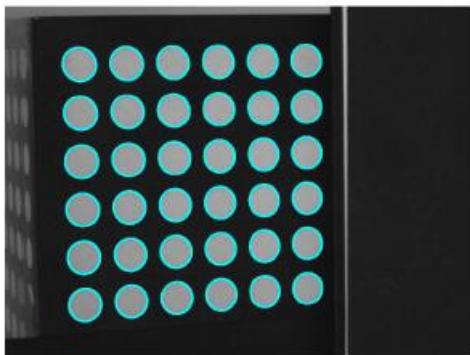
- SIFT dominant orientation [Lowe'04] produces 50% more descriptors.
- Robustness of gravity vector assumption



- Robustness of SIFT to small imprecision in orientation.
- Correct final geometry due to LO-step in RANSAC.

# Planar Homography

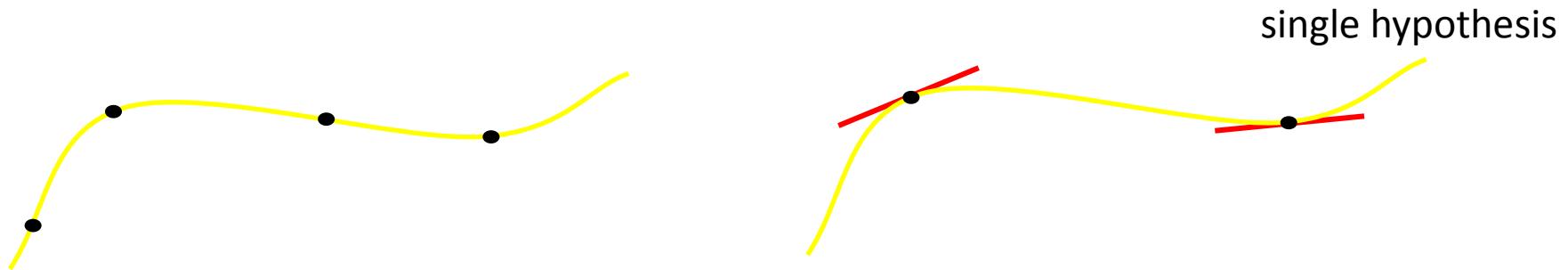
from Two or More Elliptical Correspondences



four hypotheses

Kannala, Salo and Heikkilä: Algorithms for computing a planar homography from conics in correspondence, BMVC 2006

Matlab code available: <http://www.ee.oulu.fi/~jkannala/bmvc.html>



Chum, Matas: Homography Estimation from Correspondence of Two Local Elliptical Features, ICPR 2012

Matlab code available: <http://cmp.felk.cvut.cz/~chum/code/ell2h.html>

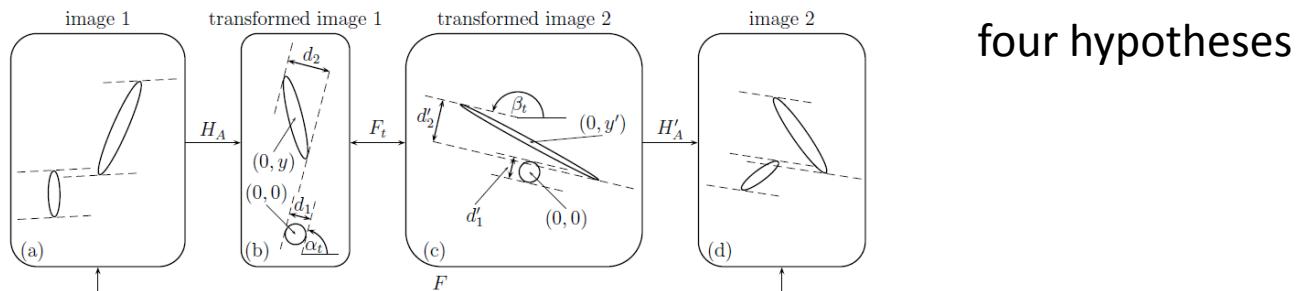
# More Models ...

- Camera rotation around optical center
- Homography 7 DoF
- Fully defined by a derivative in a point
- Derivative approx. by a single ellipse + orientation correspondence



Koeser, Beder, Koch: Conjugate Rotation: Parameterization and Estimation from an Affine Feature Correspondence, CVPR 2008

Affine epipolar geometry estimated from two elliptical correspondences



Arandjelovic, Zisserman: Efficient Image Retrieval for 3D Structures , BMVC 2010

# Points vs. Regions

## Points

- more precise models from good all-inlier samples
- more often exhibit degenerate cases
- lower probability of hitting a all-inlier sample (longer RANSAC)

## Regions

- less precise models from good all-inlier samples  
(model from a single correspondence extrapolates everywhere)
- less often generates degenerate model
- much faster RANSAC (especially with loose thresholds)

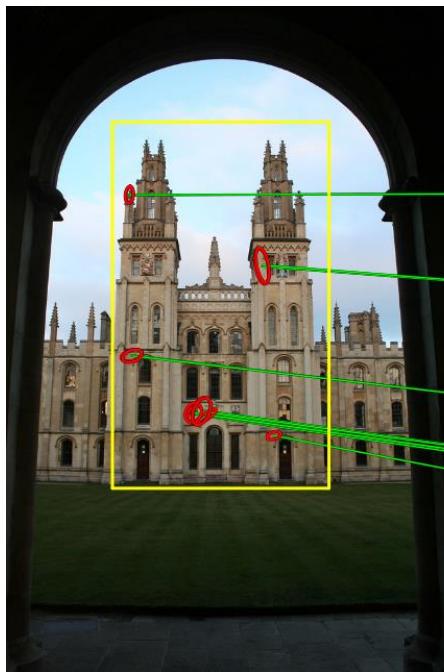
# Application Query Expansion

# Query Expansion

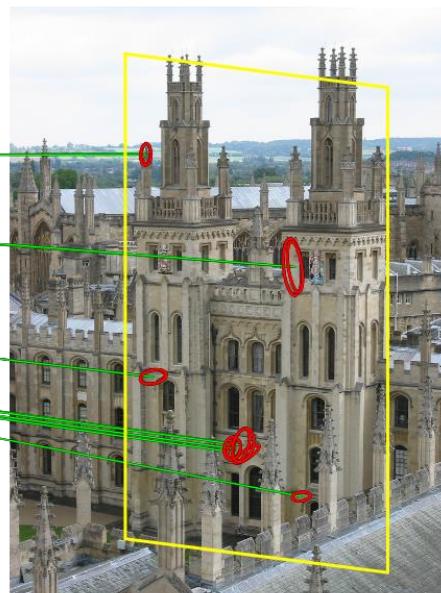


Chum, Philbin, Sivic, Isard, Zisserman: Total Recall..., ICCV 2007

# Query Expansion Step by Step



Query Image

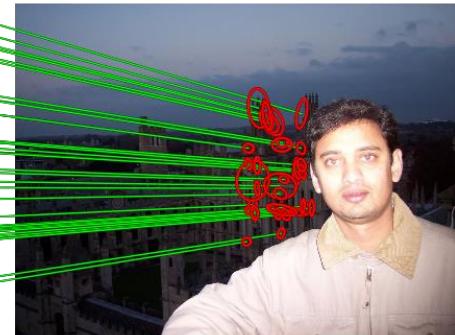
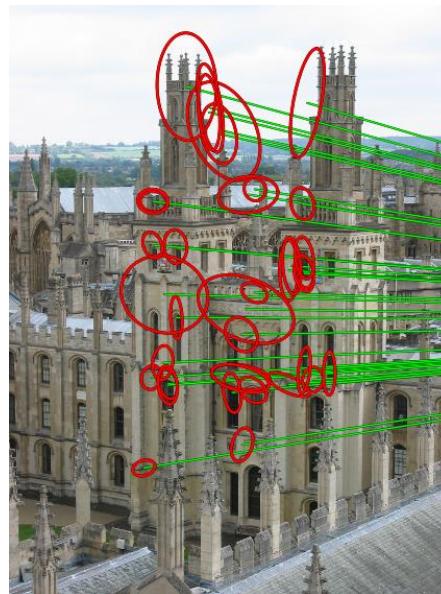
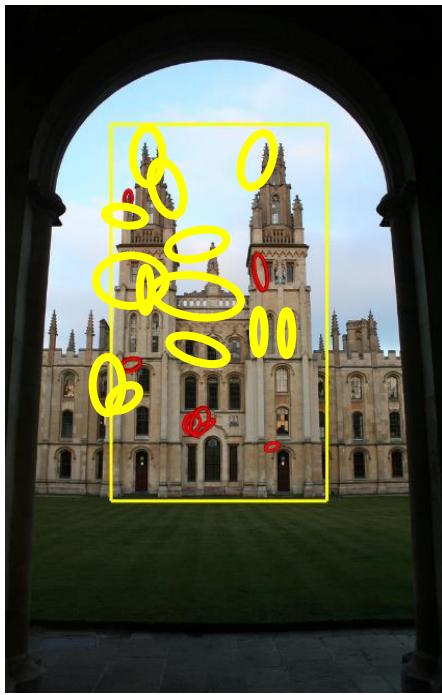


Retrieved image

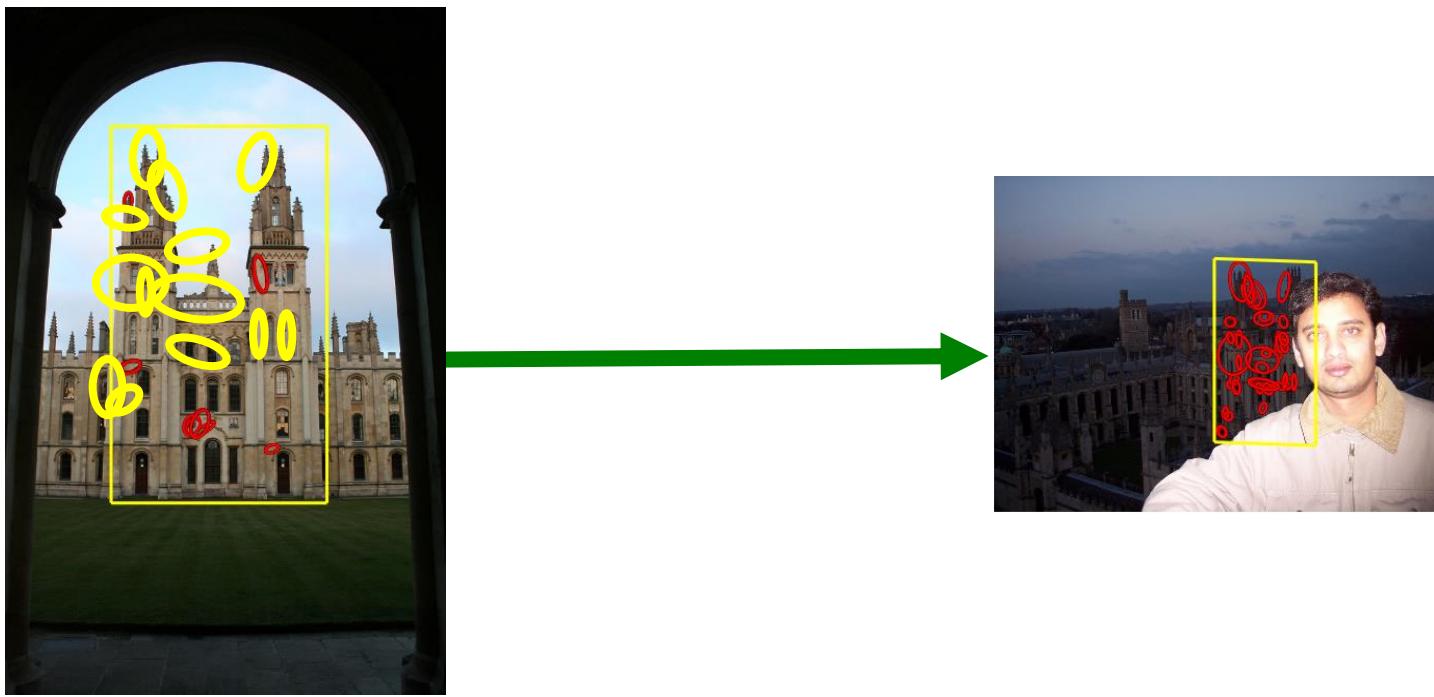


Originally not retrieved

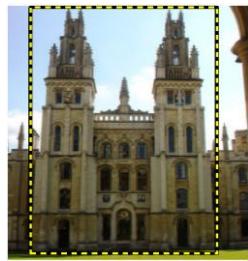
# Query Expansion Step by Step



# Query Expansion Step by Step

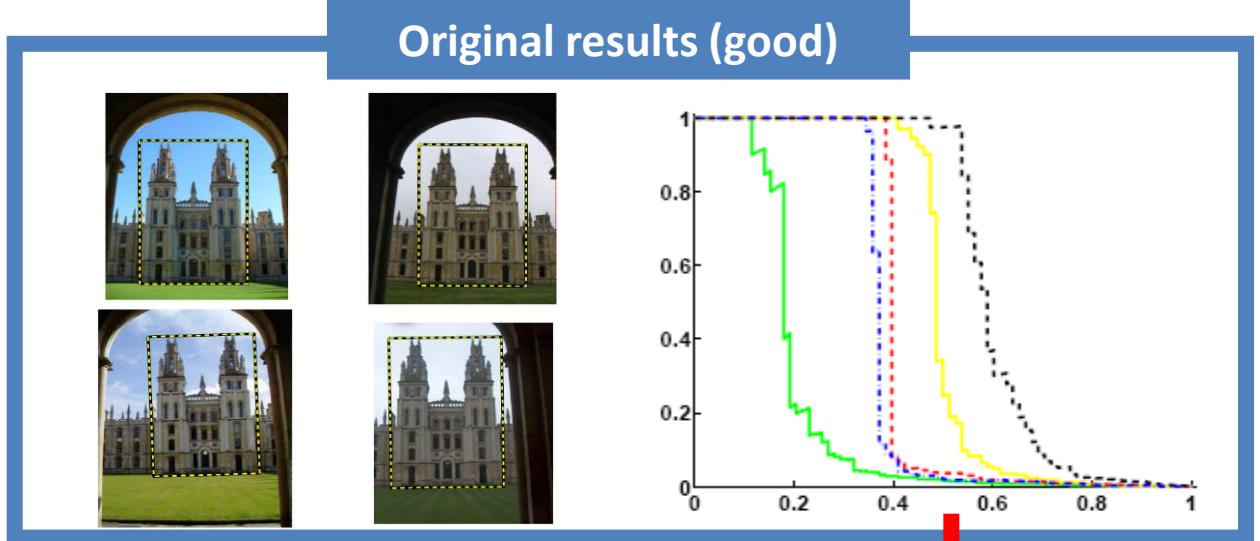


# Query Expansion Results

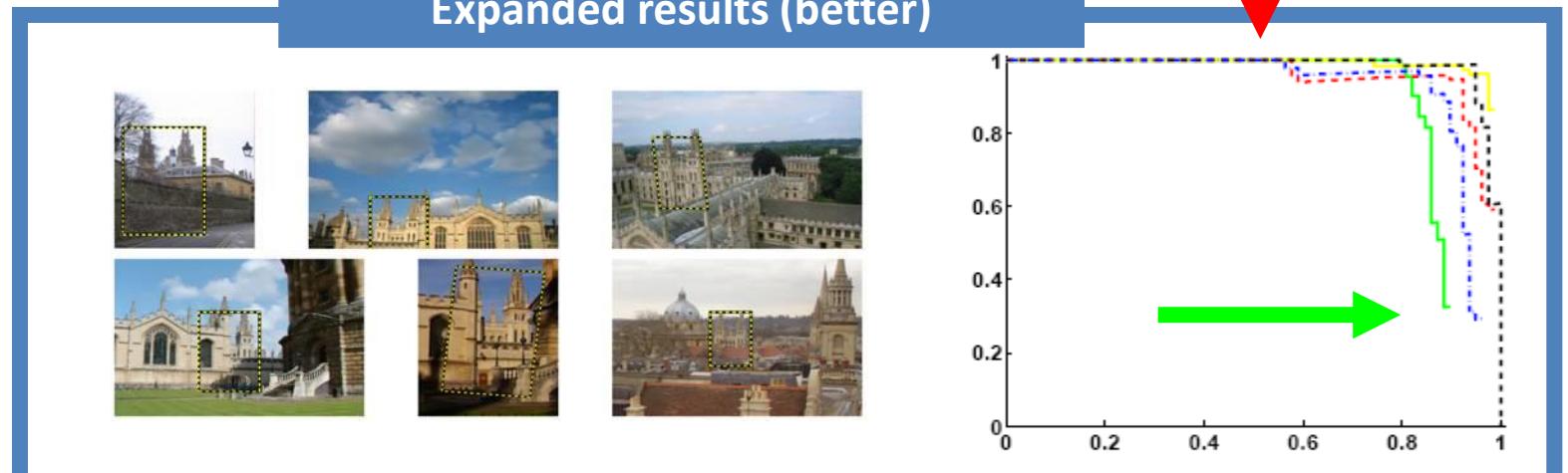


Query  
image

Original results (good)

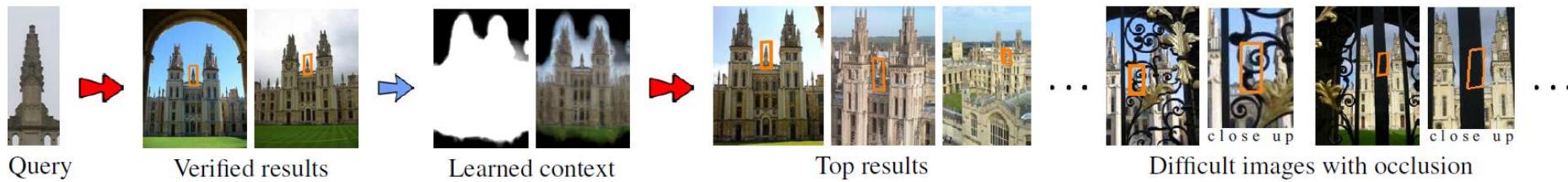


Expanded results (better)



# Context expansion

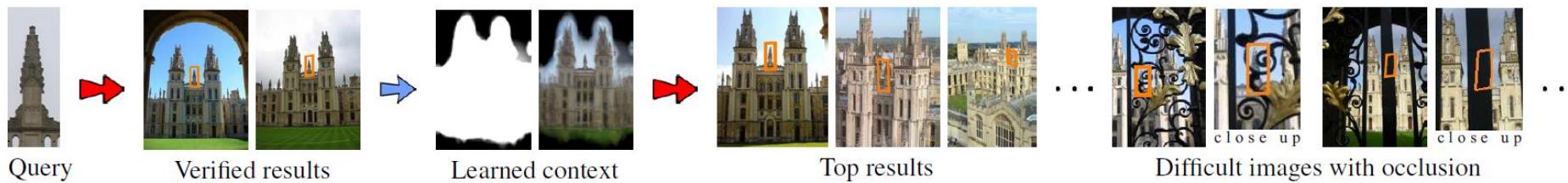
- the model of the object is grown beyond the boundaries of the initial query,
- a feature added into the model that is not inside the context is inactive until confirmed by feature(s) from another image with the same visual word and similar geometry.
- Once a feature is confirmed, it adds the neighbourhood around its center to the context.



Chum, Mikulik, Perdoch, Matas: Total Recall II: Query Expansion Revisited, CVPR 2011

# Context expansion

- the model object is growing into the model
  - initial query
  - a feature is inactive until the same word and similar confirmed, it is und
  - Once a feature is confirmed, its center t
- 
- object is growing into the model
- initial query
- a feature is inactive until the same word and similar confirmed, it is und
- Once a feature is confirmed, its center t

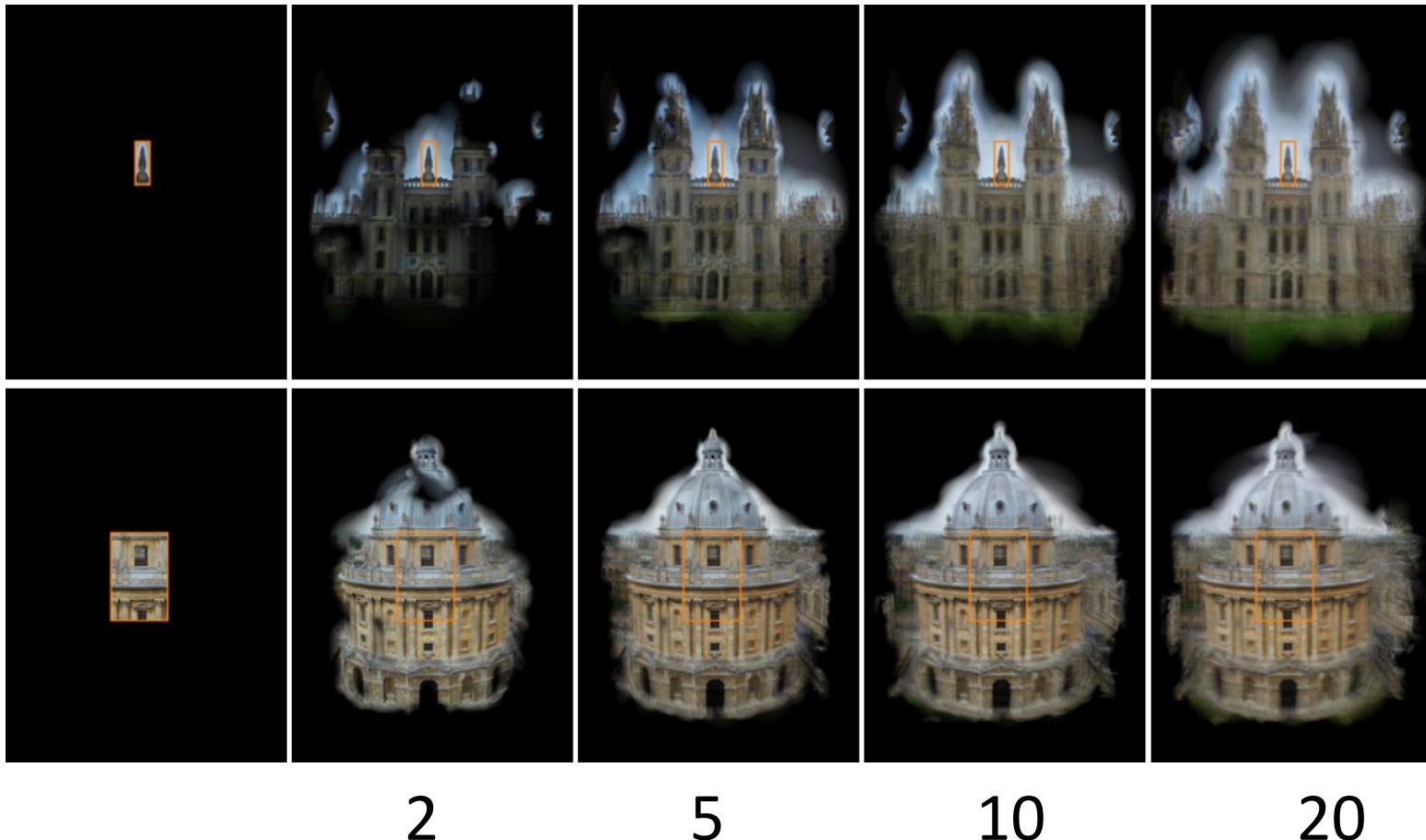


Chum, Mikulik, Perdoch, Matas: Total Recall II: Query Expansion Revisited, CVPR 2011

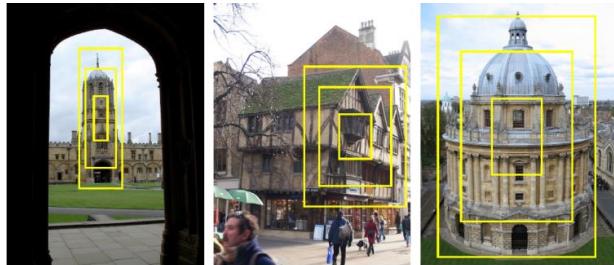
# Learning the Context

Feature patches back-projected into the context from spatially verified images.

The query



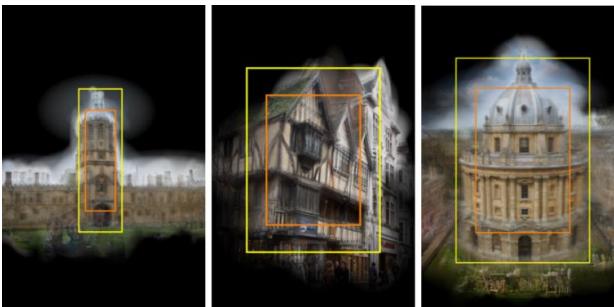
# How Much Do We Need to See?



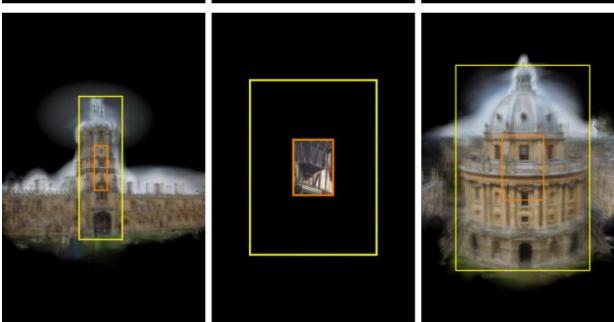
Oxford landmarks – 3 queries  
100%, 50%, and 10% of the query bounding box



Context learned from the full bounding box

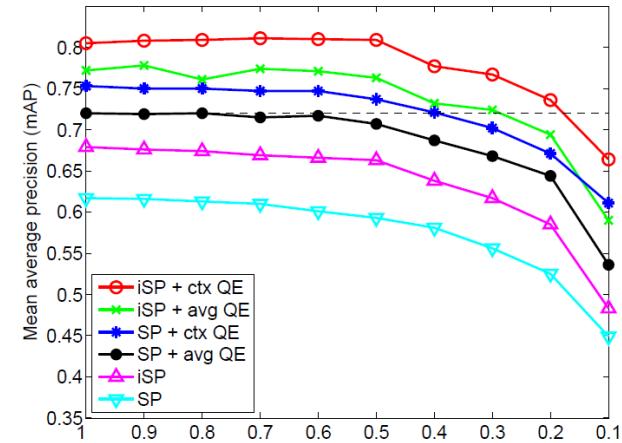


Context learned from 50% of the bounding box

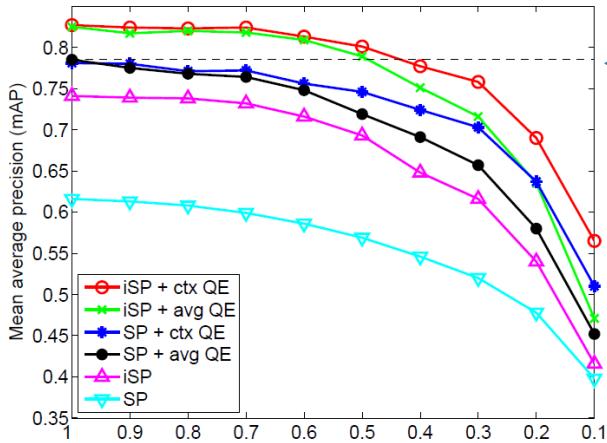


Context learned from 10% of the bounding box

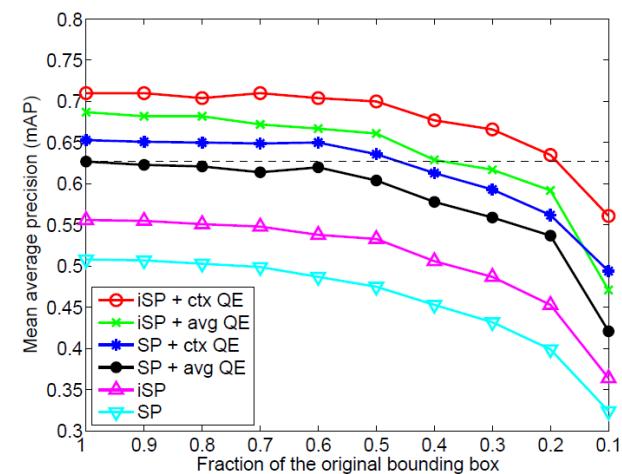
# Effects of decreasing the query bounding-box size



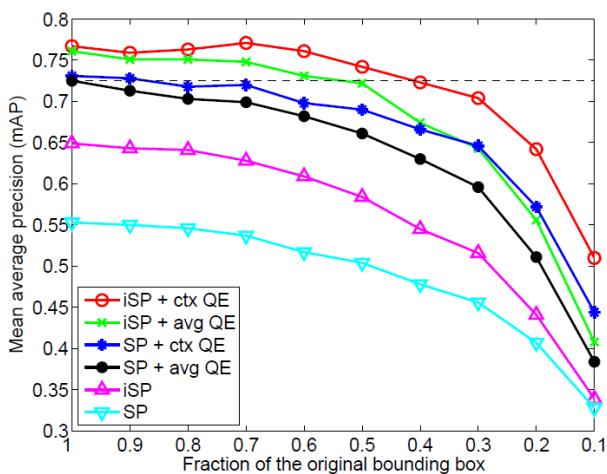
(a) Paris 6k



(b) Oxford 5k



(c) Paris 106k



(d) Oxford 105k

Baseline:  
spatial verification +  
full bounding box

Context QE at the baseline  
performance needs only:

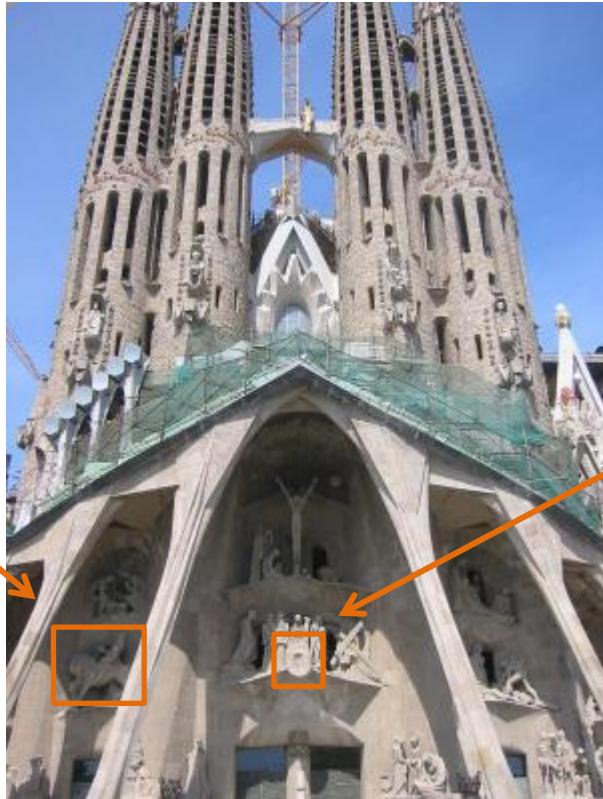
- 20% of the BB on the Paris dataset
- 40% of the BB on the Oxford dataset

# Application Browsing Image Collections

# Retrieval for Browsing

What is this?

... and what is that?



Let's query!

# Retrieval for Browsing



Query 1



Query 2



**GOOD! ... to maximize mAP**

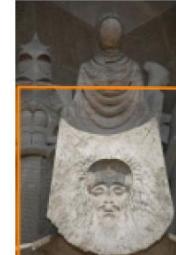
# Retrieval for Browsing



Query 1



Query 2

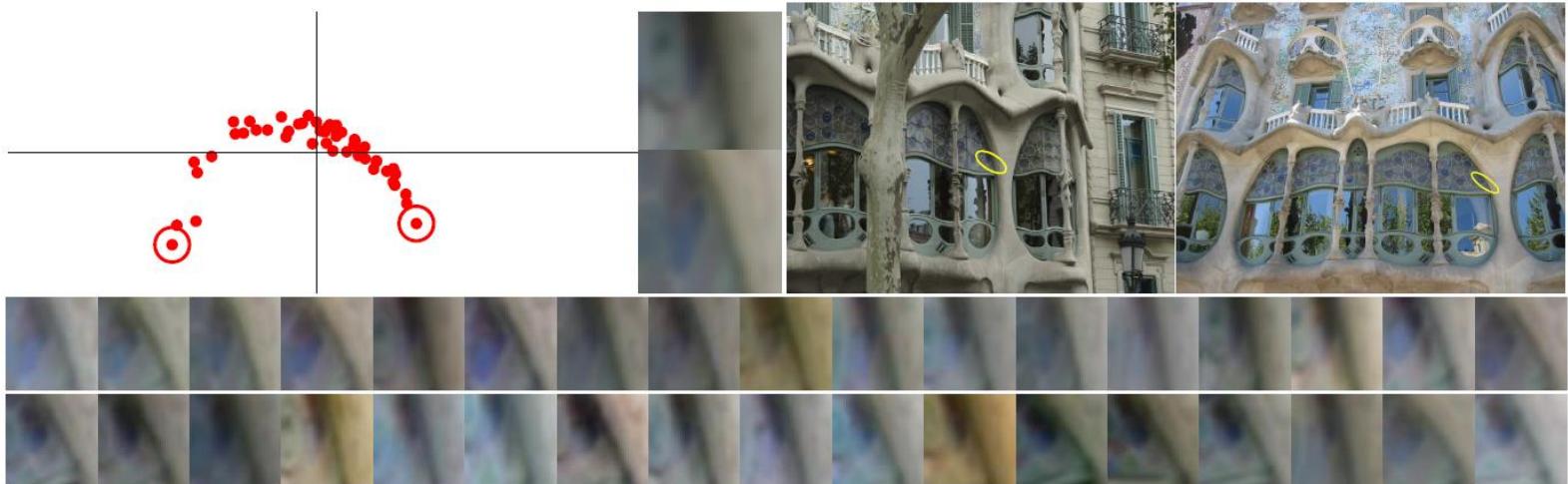


Mikulik, Chum, Matas

# Application

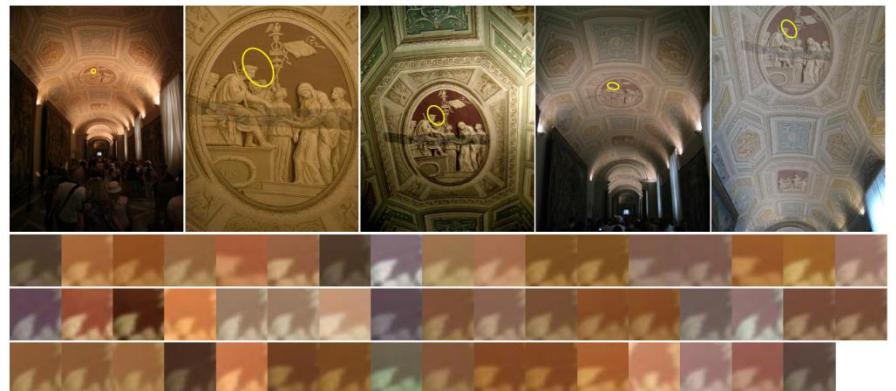
## Surface Patch Appearance Mining

# Appearance Variance of a Single Feature



Mikulik, Perdoch, Chum, Matas: Learning Vocabularies over a Fine Quantization, IJCV 2012

- over 5 million images
- almost 20k clusters of 750k images (visual word based)
- 733k successfully matched in WBS matching (raw descriptor based)
- over 111 M feature tracks established (12.3 M with 6+ features)
- 564 M features in the tracks (319.5 M in tracks of 6+ features)



<http://cmp.felk.cvut.cz/~qqmikula/publications/ijcv2012/index.html>

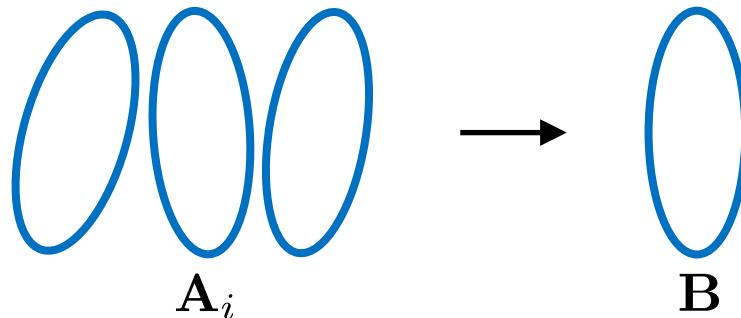
# Storing Geometry (ellipse compression)

# Geometry compression

- Ellipses + gravity vector used to hypothesize affine transformations
- Ellipses represented as an affine transform

$$\mathbf{A} = \begin{pmatrix} a & 0 \\ b & c \end{pmatrix},$$

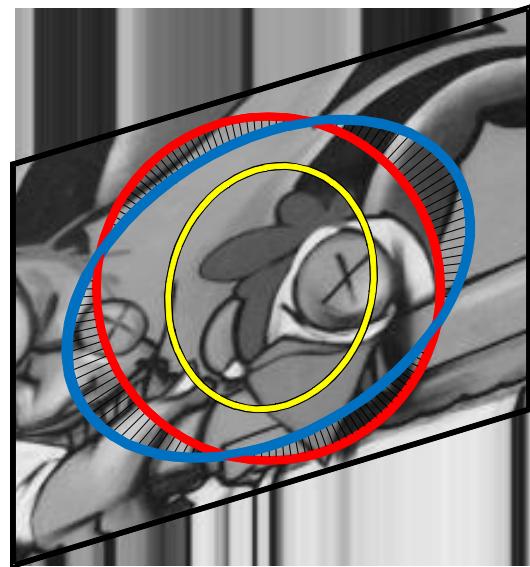
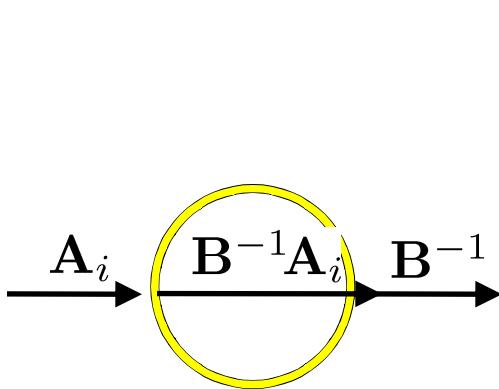
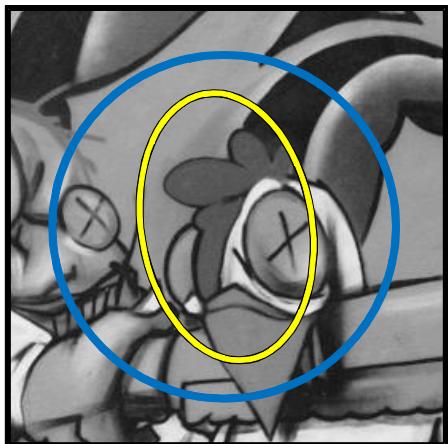
- Vector quantization, minimizing error of the generated model



$$\mathbf{H} = \mathbf{B}^{-1} \mathbf{A}_i = \mathbf{I} + \mathcal{E}.$$

# Minimizing Geometric Error of H

- $\mathbf{H} = \mathbf{B}^{-1} \mathbf{A}_i$



- We capture the quality of  $\mathbf{H}$  by integrating reprojection error over unit circle

$$e = \int_{\|\mathbf{x}\|^2=1} \|\mathbf{I}\mathbf{x} - (\mathbf{I} + \mathcal{E})\mathbf{x}\|^2 = \int_{\|\mathbf{x}\|^2=1} \|\mathcal{E}\mathbf{x}\|^2,$$

using simple manipulations

$$e = \int_{\alpha=0}^{2\pi} \left\| \mathcal{E} \begin{pmatrix} \cos \alpha \\ \sin \alpha \end{pmatrix} \right\|^2 = \pi \cdot \|\mathcal{E}\|_F^2, \quad \mathcal{E} = \mathbf{B}^{-1} \mathbf{A}_i - \mathbf{I}.$$

- Frobenius norm can be used in space of transformations as a similarity.

# K-means-like Clustering

- For a set of transformations  $\mathbf{A}$ , we are looking for a best set  $\mathbf{B}$  of  $K$  representative candidates  $\mathbf{B}_j$

$$\mathcal{A} = \{\mathbf{A}_1, \dots, \mathbf{A}_N\}, \quad \mathcal{B} = \{\mathbf{B}_1, \dots, \mathbf{B}_K\}, \quad N \ll K$$

- assignment step

$$f(i) = \operatorname{argmin}_j \|\mathbf{B}_j^{-1} \mathbf{A}_i - \mathbf{I}\|_F^2 \quad \mathcal{A}_j = \{\mathbf{A}_i, f(i) = j\}$$

- refinement step

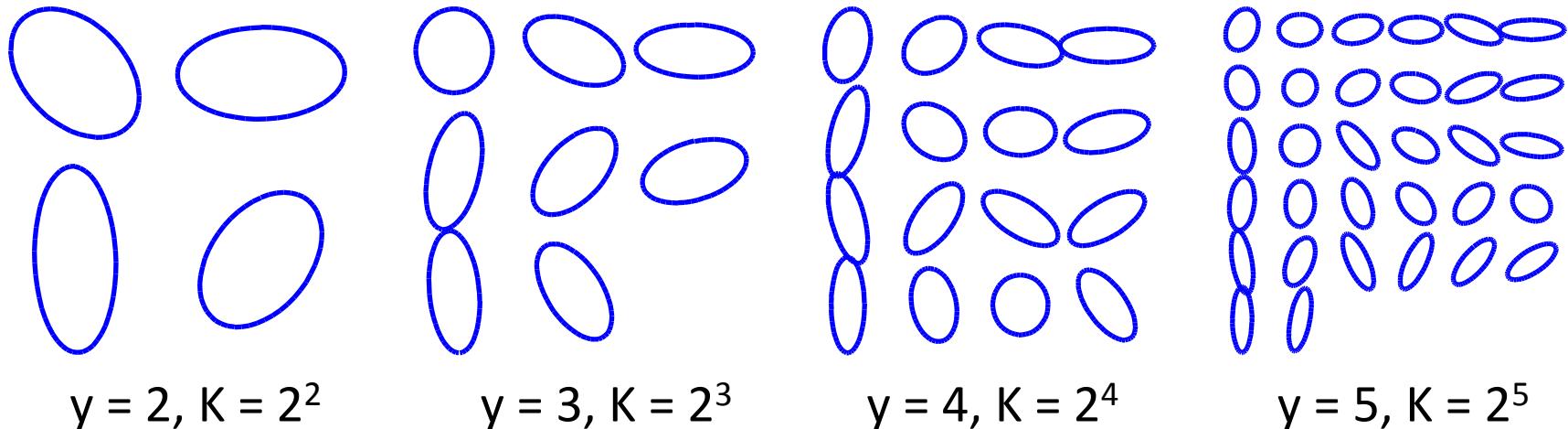
$$\mathbf{B}_j = \operatorname{argmin}_{\mathbf{B}} \sum_{\mathbf{A}_i \in \mathcal{A}_j} \|\mathbf{B}^{-1} \mathbf{A}_i - \mathbf{I}\|_F^2$$

- which leads to a **closed form** solution

$$\begin{aligned} \mathbf{B}_j &= \begin{pmatrix} -\frac{\sum_{k=1}^n a_k \sum_{k=1}^n c_k^2}{D} & 0 \\ \frac{\sum_{k=1}^n c_k b_k \sum_{k=1}^n a_k}{D} & \sum_{k=1}^n c_k / \sum_{k=1}^n c_k^2 \end{pmatrix} \\ D &= \sum_{k=1}^n c_k b_k - \sum_{k=1}^n c_k^2 \sum_{k=1}^n a_k^2 - \sum_{k=1}^n c_k^2 \sum_{k=1}^n b_k^2 \end{aligned}$$

# Geometric “vocabularies”

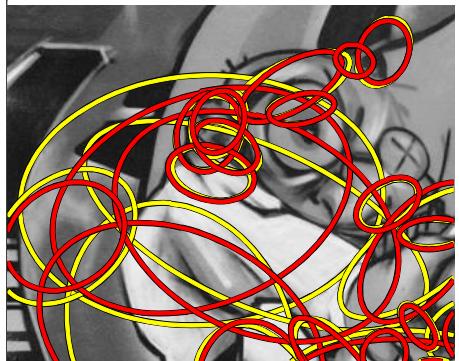
- Assuming that the **scale** and **shape** of the ellipse are **independent**, scale can be extracted and **quantized separately**.
- Each geometric vocabulary is characterized by two numbers ( $x$ ,  $y$ ) and denoted  $SxEy$ 
  - $x$  – number of bits for encoding scale.
  - $y$  – number of bits for encoding ellipse shape.
- Geometric vocabularies learnt on a set of local geometries of 10M affine covariant points (for visualisation scale was quantized separately).



# Geometric “vocabularies”

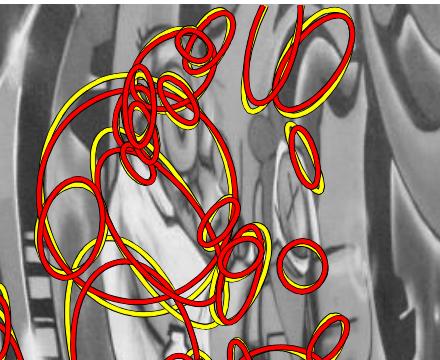
SOE8

256 ellipses



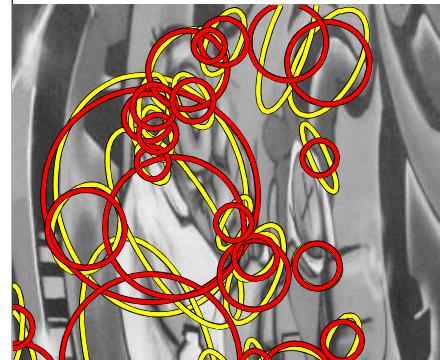
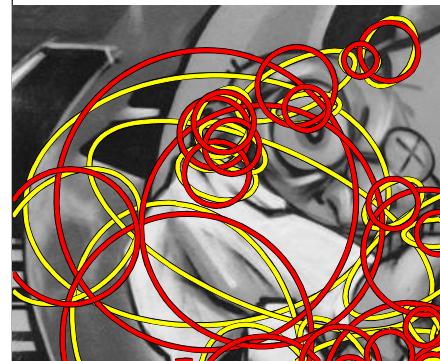
S4E4

16 scales, 16 ellipses



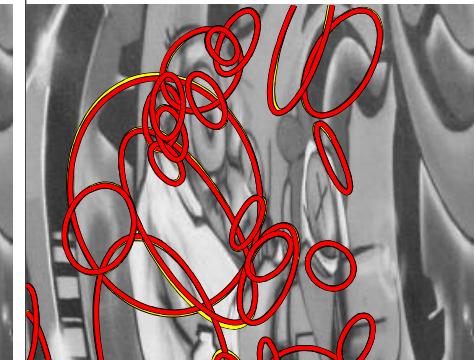
S8E0

256 scales



S4E12

16 scales, 4k ellipses



# Summary

## Geometric constraints

- semi-local
- global

are useful for

- filtering out false positives
- object localization by robust model estimation
- number of other applications

# Advertisement

**PhD and post-doc** positions available  
on large scale object and category retrieval

chum@cmp.felk.cvut.cz

<http://cmp.felk.cvut.cz/~chum>

