DS-630: HW1
Assigned Sept. 12, 2017
Due Sept. 19, 2017

1. Given the distribution $p(x|\theta)$, assume that N samples $x_1, \ldots, x_N$ are drawn independently according to $p(x|\theta)$. Find the maximum-likelihood estimate of θ for the following cases of $p(x|\theta)$:

   (a) $p(x|\theta) = \theta e^{-\theta x}$ for $x \geq 0$

   (b) $p(x|\theta) = \dfrac{1}{\theta}$ for $0 \leq x \leq \theta$

   (c) $p(x|\theta) = \dfrac{\theta^x e^{-\theta}}{x!}$ for $x = 0, 1, 2, \ldots$

   (d) $p(x|\theta) = \alpha \dfrac{\beta^\alpha}{x^{\alpha+1}}$ for $x \geq \beta$ and $\theta = \{\alpha, \beta\}$

2. Linear regression questions:

   (a) For the data given in the text files (xdata.txt and ydata.txt) run in R a simple linear regression of y on x. Report the summary data supplied by R for your regression.

   (b) Given an $n \times d$ matrix X of N data points, each point d-dimensional, an $n \times 1$ vector Y of observed values, and a $d \times d$ diagonal weight matrix A, deduce the closed form expression for the coefficients in the weighted linear least squares regression problem given below. In weighted linear least squares, the term minimized is the weighted sum of squared distances

   $$\sum_{i=1}^{N} a_i (y_i - \sum_{j=1}^{d} X_{ij}\beta_j - \beta_0)^2 \tag{1}$$

   where a_i is the i^{th} diagonal element of A.

   For the same X and Y data in the text files and the corresponding weights (100 single values) given in the text file wdata.txt, run a weighted least squares model in R to calculate the corresponding coefficients. Report the summary data supplied by R for your regression.

   (c) Consider a form of regression which minimizes the sum of the *perpendicular* distances from the data points to the regression line, instead of the *vertical* distances used in ordinary least squares. For n one-dimensional data points $x_1, x_2, \ldots, x_n$ and corresponding observed values $y_1, y_2, \ldots, y_n$, deduce the solution for this form of linear regression.