Selection Assignment @ MathFinance2024

Procedure: Submit the solution files and your CV by Oct 28 noon. The subject of the email should be 'Selection Assignment @ MathFinance2024'. Also mention if you are planning to attend the 4 weeks of onsite sessions to be held in Bengaluru. This will be done in two parts, tentatively (a) last two weeks of December and (b) first two weeks of March. We will finalize the list of selected participants by November 2nd.

Since we can only select a limited number of participants, we want to ensure that people are serious about the program, and don't end up wasting someone else's slot. So, if you are selected, and have opted in for the onsite session, we will ask you to deposit Rs.10,000 by Nov 4th. This will be adjusted against the accommodation costs for the onsite. If you end up finding your own accommodation, we will return this deposit amount.

Probability

Submit your solution as a latex file.

1. Take a 'fair' coin with $P(H) = P(T) = 0.5$. We keep tossing the coin till we see the first head. The number of tosses, say N, is then a random variable. What is the distribution of N. Find its expectation and standard deviation.

2. Suppose X is a random variable distributed uniformly over the interval $[0, 5]$. Find the probability density functions of (a) $Y = 5X + 10$, (b) $Z = X^3 + 2X$ and (c) $U = \sin(2\pi X)$.

3. You are given two envelopes with each envelope containing a positive integer that can take values from 1 to 1000. You are told that one number is three times the other number. You can open exactly one envelope to see the number inside, and then choose either the open envelope or the closed one as your final envelope. You win if the final envelope that you chose contains a larger number than the other envelope. If you see the following numbers on the open envelope, enumerate if you will choose the open or the closed envelope as your final envelope and compute your probability of winning: (a) 20, (b)150, (c) 300 and (d) 600.

4. Take three independent normal random variables $X \sim N(0, 1)$, $Y \sim N(0, 1)$ and $Z \sim N(0, 2)$ where $N(a, b)$ denotes a normal distribution with mean a and variance b. Compute the following probabilities:

   (a) $P(X > Y) :=$ the probability that $X > Y$.

   (b) $P(|X| > |Y|)$

   (c) $P(X > Z)$

   (d) $P(|X| > |Z|)$

5. We keep tossing a fair dice till we get the same number on consecutive tosses. What is the expected number of tosses?

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Statistics

Submit the Jupyter notebook, with your interpretations as comments in the file. If you have accompanying notes, submit it as a latex file. You can refer to the book 'Applied Linear Statistical Models' by Kutner, Nachtsheim, Neter and Li for this section.

1. Install Jupyter notebook on your laptop.
2. You have been provided two text files named 'dataset-one' and 'dataset-two' containing 10,000 and 100 real numbers respectively. Import them to the Jupyter notebook and save them to two 'numpy' arrays.
3. Test if the numbers have been sampled from normal distributions for each of the data sets. They can be from two different distributions. You can do this in various ways, and it is recommended to do so as no single method is fool proof. So, more tests conducted with *correct interpretations* will get extra marks.
4. Assuming that both the distributions have been sampled from normal distributions, estimate the population mean from these samples.
5. Do you have more confidence in your prediction for dataset-one or dataset-two ? Can you explain this intuitively and then try to quantify it?

Code of Conduct:

- You are allowed to refer to books and online materials.
- You can use chatgpt, but write the answers in your own language.
- You are NOT allowed to discuss with other people.