Surprise Test 21BCE11066.

(1)

$$\begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix}$$

$$S = A^T A = \begin{bmatrix} 2 & 0 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix}$$

$$S = \begin{bmatrix} 4 & 6 \\ 6 & 13 \end{bmatrix}.$$

Eigenvalues $\lambda_1 7$ λ_2

$$\begin{bmatrix} 4-\lambda & 6 \\ 6 & 13-\lambda \end{bmatrix}$$

$$(4-\lambda)(13-\lambda)-(6).(6) = \lambda^2 - 17\lambda + 16$$

$$= \lambda^2 - 17\lambda + 16$$

$$= (\lambda-16)(\lambda-1)$$

$$(\lambda-16)(\lambda-1) = 0$$

$$\boxed{\lambda_1 = 16} \quad \boxed{\lambda_2 = 1}$$

$\sigma_1 = 4 \qquad \sigma_2 = 1$

$$\Sigma = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}$$

eigenvectors

$$V_1 = \begin{bmatrix} 1/2 \\ 1 \end{bmatrix} \qquad V_2 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

Normalise

$$V_1 = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1/2 \\ 1 \end{pmatrix} = V_1'$$

$$V_2 = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \end{pmatrix} = V_2'$$

$$\|V_1\|' = \sqrt{\langle V_1, V_1 \rangle}$$

$$= \sqrt{\left\langle \begin{pmatrix} 1/2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1/2 \\ 1 \end{pmatrix} \right\rangle} = \frac{\sqrt{5}}{2}$$

$$\|V_2\|' = \sqrt{\langle V_2 , V_2 \rangle}$$

$(-2)^2+(1)^2$

$4+1$

$$= \sqrt{\left\langle \begin{pmatrix}-2\\1\end{pmatrix}, \begin{pmatrix}-2\\1\end{pmatrix} \right\rangle}$$

$$\|V_2\|' = \sqrt{5}$$

$$V = \left[V_1' : V_2' \right]$$

$$\cancel{V = \begin{bmatrix} 1/2 & 1 \\ -2 & 1 \end{bmatrix}} \qquad V = \begin{bmatrix} 1/2 & -2 \\ 1 & 1 \end{bmatrix}$$

$$U_1 = \frac{1}{\sigma_1} A V_1'$$

$$= \frac{1}{4} \begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1/2 \\ 1 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1/2 \end{bmatrix} = U_1$$

Scanned with CamScanner

$$V_2 = \sigma_2 A V_2'$$

$$= 1\begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$V_2 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$\|U_1\| = \sqrt{\langle U_1, U_1 \rangle}$$

$$= \sqrt{\left\langle \begin{matrix} 1 \\ 1/2 \end{matrix}, \begin{matrix} 1 \\ 1/2 \end{matrix} \right\rangle}$$

$$\|U_1\| = \frac{\sqrt{5}}{2}$$

$$\|U_2\| = \sqrt{\langle U_2, U_2 \rangle}$$

$$= \sqrt{\left\langle \begin{matrix} -1 \\ 2 \end{matrix}, \begin{matrix} -1 \\ 2 \end{matrix} \right\rangle} = \sqrt{5}$$

Scanned with CamScanner

$$u_1' = \frac{u_1}{\|u_1\|}$$

$$= \cancel{\frac{1}{2}\begin{bmatrix}1\\ 1/2\end{bmatrix}_{2\times1}\begin{bmatrix}2\\ \sqrt5\end{bmatrix}_{2\times1}}$$

$$= \frac{\begin{bmatrix}1\\ 1/2\end{bmatrix}}{\frac{\sqrt5}{2}}$$

$$= \frac{2}{\sqrt5}\begin{bmatrix}1\\ 1/2\end{bmatrix}$$

$\cancel{\frac{2\sqrt5}{\sqrt5}}$ $\cancel{2\times5}$ $\frac{2\times5}{\sqrt5}$ $\frac{1}{\sqrt5}$

$$u_1' = \begin{bmatrix}2/\sqrt5\\ 1/\sqrt5\end{bmatrix}$$

$\frac{2\sqrt5}{5}$ $\frac{2\sqrt5}{}$

$$u_2' = \frac{u_2}{\|u_2\|}$$

$$= \frac{\begin{bmatrix}-1\\ 2\end{bmatrix}}{\sqrt5} = \sqrt5\begin{bmatrix}-1\\ 2\end{bmatrix} = \begin{bmatrix}-\sqrt5\\ 2\sqrt5\end{bmatrix}$$

$$u_2' = \begin{bmatrix}-\sqrt5\\ 2\sqrt5\end{bmatrix}$$

$$U = (u_1 : u_2)$$

$$U = \begin{pmatrix}2/\sqrt5 & -\sqrt5\\ 1/\sqrt5, & 2\sqrt5\end{pmatrix}$$

$$U\Sigma V^T$$

$$= \begin{pmatrix}2/\sqrt5 & -\sqrt5\\ 1/\sqrt5 & 2\sqrt5\end{pmatrix}\begin{pmatrix}4 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}1/2 & 1\\ -2 & 1\end{pmatrix}$$

$$\frac{1}{\sqrt5}\begin{pmatrix}2 & -5\\ 1 & 10\end{pmatrix}\begin{pmatrix}4 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}1/2 & 1\\ -2 & 1\end{pmatrix}$$

Scanned with CamScanner

$$U \Sigma V^T = \frac{1}{\sqrt{5}} \begin{pmatrix} 2 & -5 \\ 1 & 10 \end{pmatrix} \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1/2 \\ -2 \end{pmatrix}$$

$$\frac{1}{\sqrt{5}} \begin{pmatrix} 8 & -5 \\ 4 & 10 \end{pmatrix} \begin{pmatrix} 1/2 & 1 \\ -2 & 1 \end{pmatrix}$$

$$= \begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix} = A$$

2

$$\begin{bmatrix} 4 & 1 \\ 3 & -5 \end{bmatrix}$$

$A = QR$

$R = Q^T A$

(3)

$(-2,3,1), (-1,2,0), (-1,4,2)$

$\cos\theta = \frac{v \cdot v}{|v|}$

$(-1,2,0)$

$(-1,4,2)$

$= (-1,2,0) - \frac{(-1,2,0)(-1,4,2)}{(-1,4,2)(-1,4,2)}(-1,4,2)$

$= (-1,2,0) - \frac{(1,8,0)}{(1,16,4)}(-1,4,2)$

$= (-1,2,0) - (1,2,0)(-1,4,2)$

$(-1,2,0) - (-1,8,0)$

$= (0,-6,0)$

$w = (-2,-3,1) - \frac{(-2,-3,1)(-1,4,2)}{(-1,4,2)(-1,4,2)}(-1,4,2)$

$w = -\frac{(-2,-3,1)(0,-6,0)}{(0,-6,0)(0,-6,0)}(0,-6,0)$

Scanned with CamScanner

$$W = (-2,-3,1) - \frac{(-2,-3,1)\cdot(-1,4,0)}{(-1,4,2)(-1,4,2)}(-1,4,2)$$

$$= -\frac{(-2,-3,1)(0,-6,0)}{(0,-6,0)(0,-6,0)}(0,-6,0)$$

$$\therefore W = (-2,-3,1) - \frac{(2,-12,2)}{(-1,\cancel{16},\cancel{4})}\cancel{(-1,4,2)}$$

$$4 \quad 2$$

$$-\frac{(0,18,0)}{(0,-6,0)}(0,-6,0)$$

$$W = (-2,-3,1) - \frac{(2,-12,2)}{(-1,4,2)} - \frac{(0,10,0)}{(0,-6,0)}$$

$$\therefore W = (-2,-3,-1) - (-2,-3,1) - (0,-3,0)$$

$$= (0,9,-2)$$