Merge Sort

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Sorting

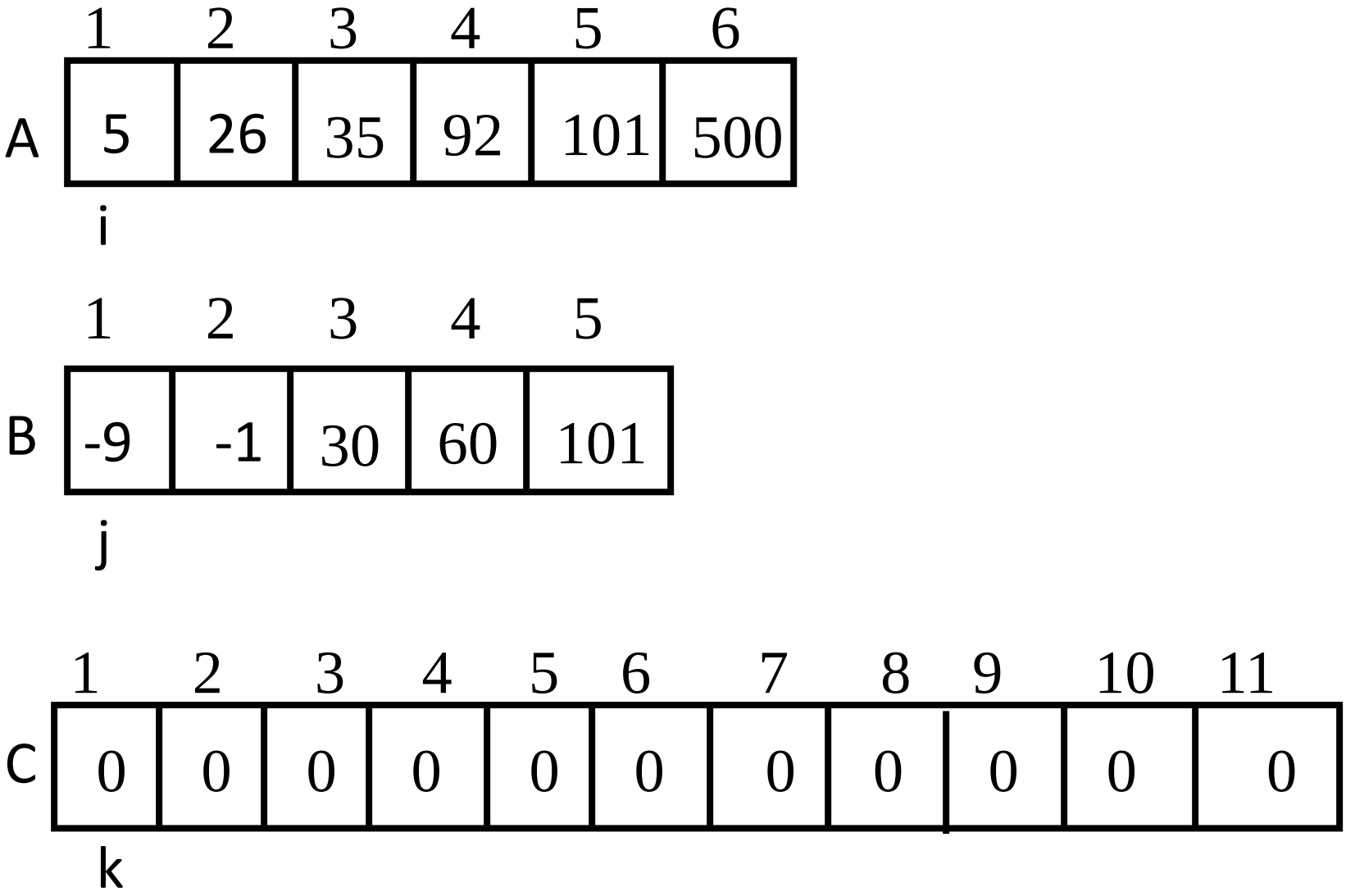
- Given an array A[] of n integers, our objective is to arrange them in non-decreasing order/non-increasing order.
- Bubble Sort/Selection Sort/Insertion Sort
- Merge Sort/Quick Sort/Heap Sort
- Counting Sort/Radix Sort
- Bucket Sort

Merge Sort

- Given two sorted arrays of length m and n, how do we merge them into a single sorted array of size m+n?

- We use an extra array of size m+n to merge the two sorted arrays.

How to merge two sorted array?



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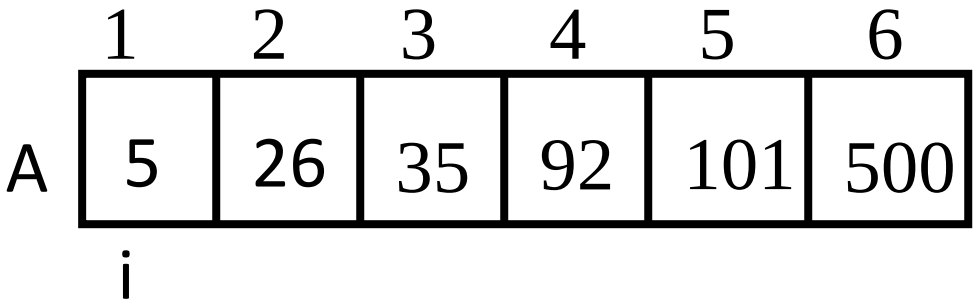

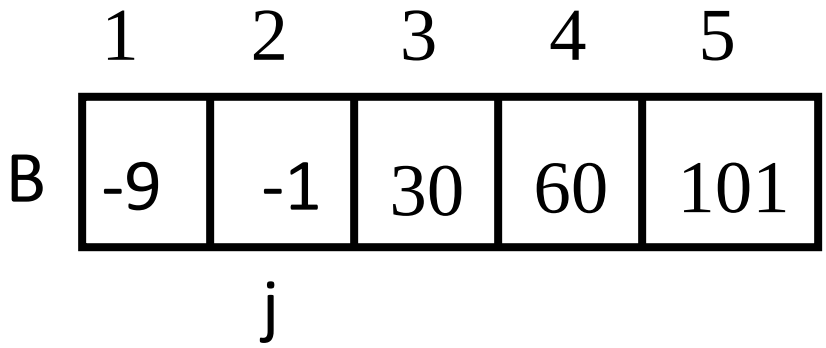

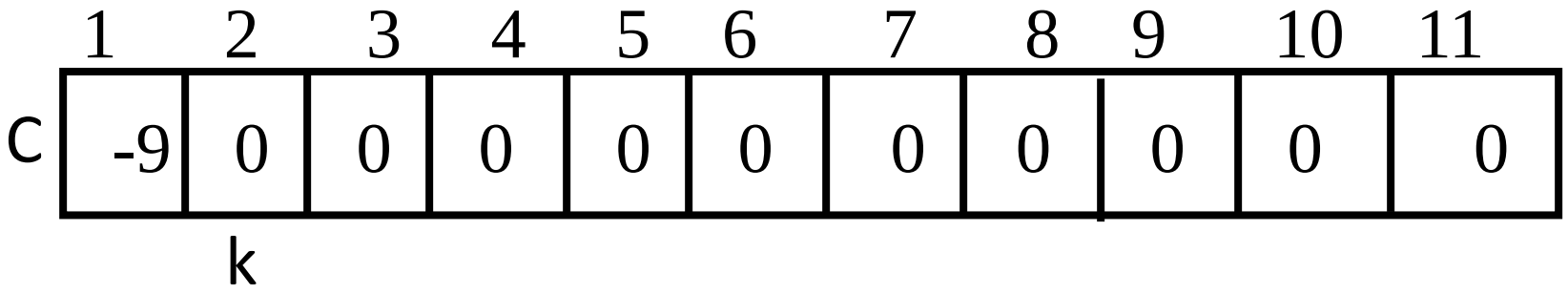

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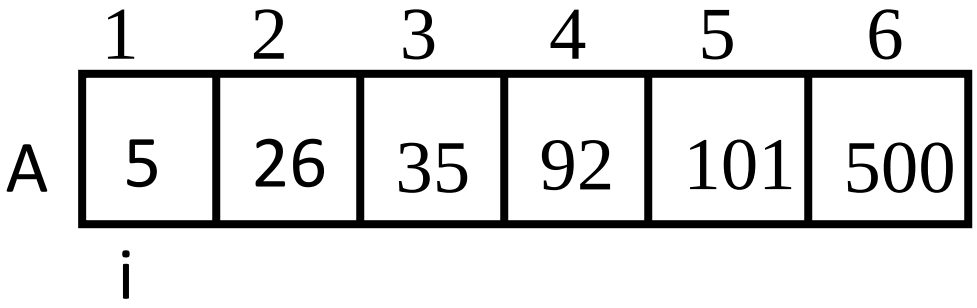

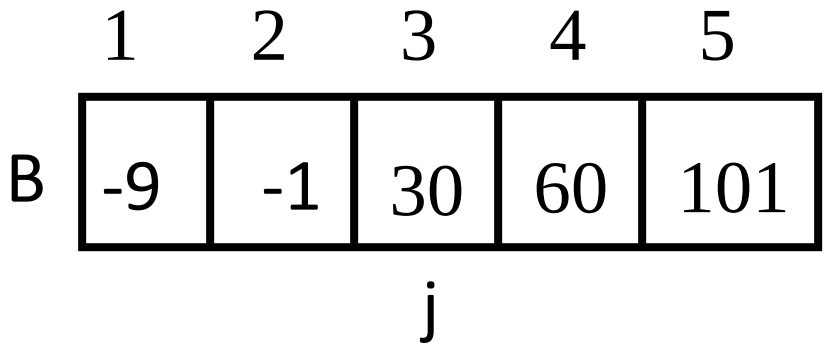

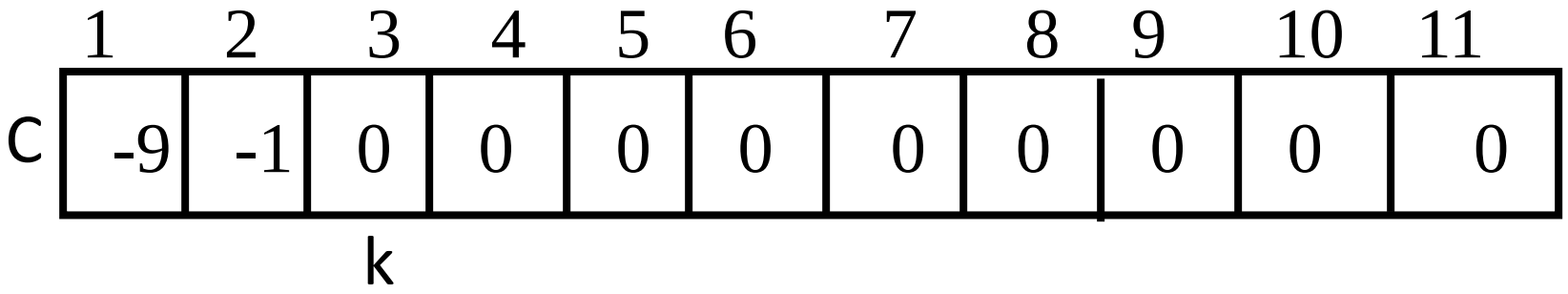

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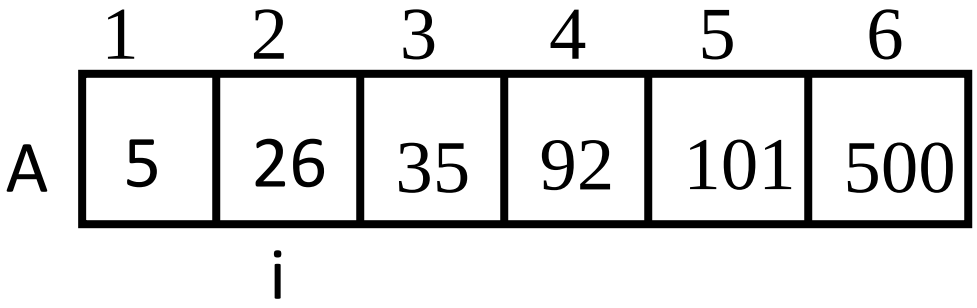

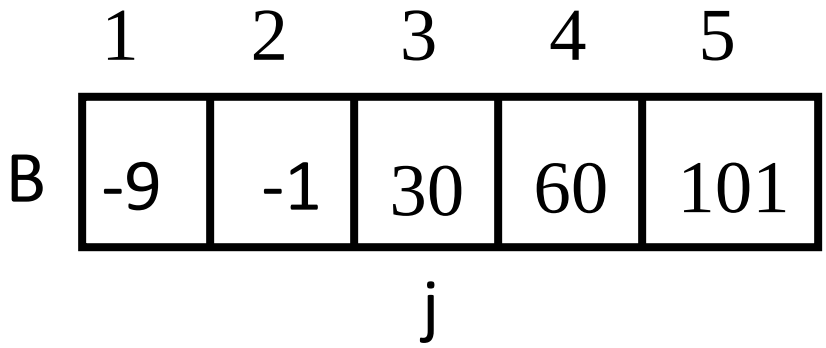

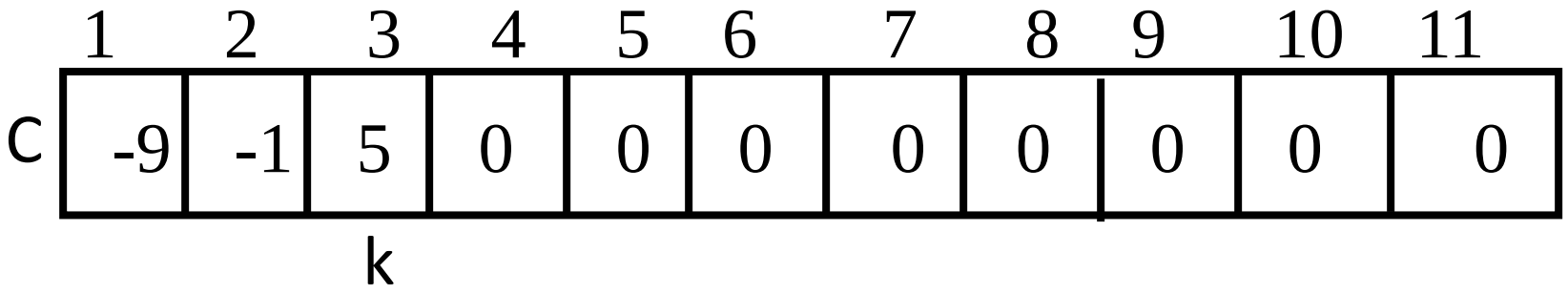

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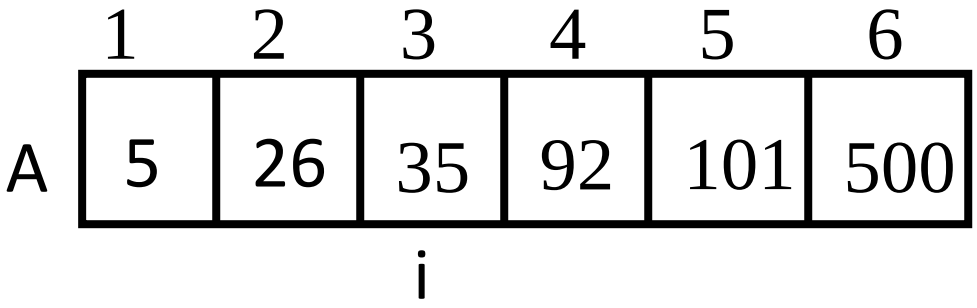

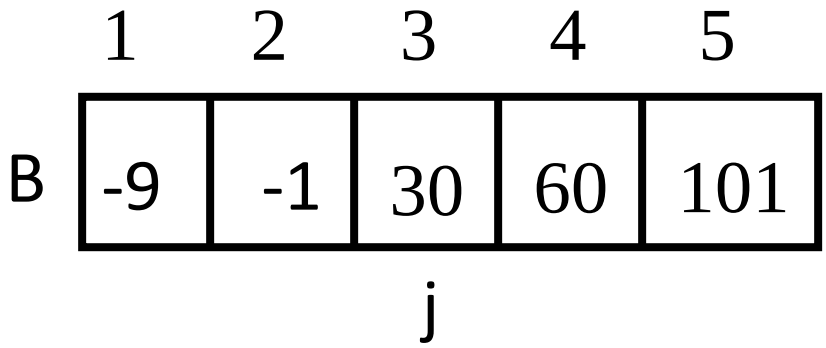

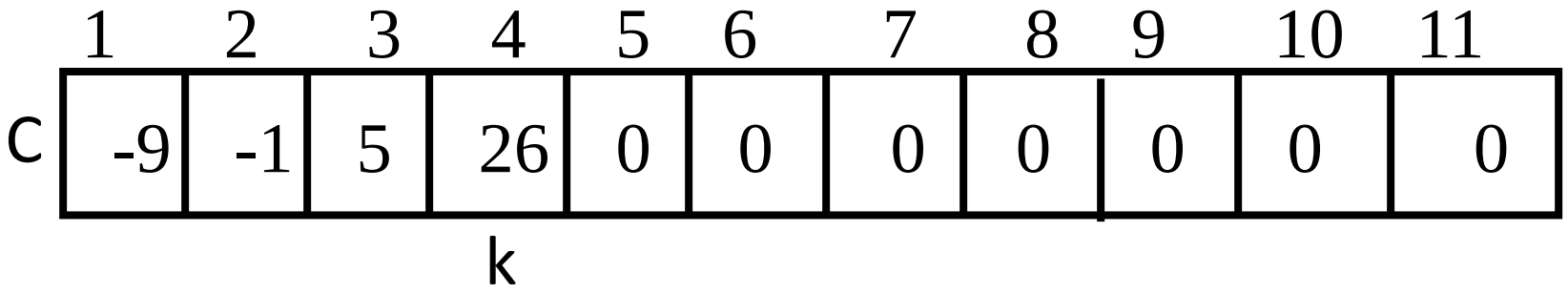

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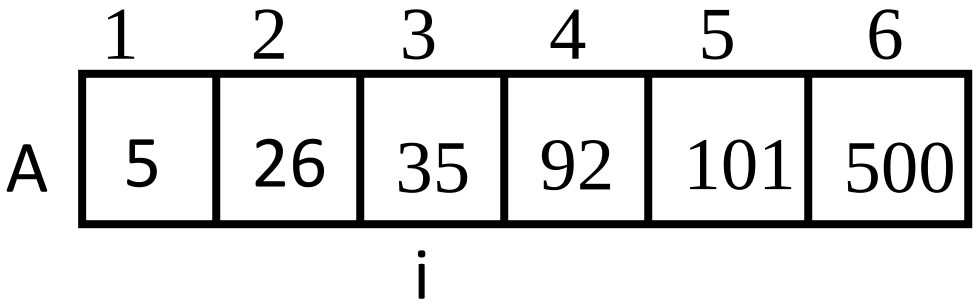

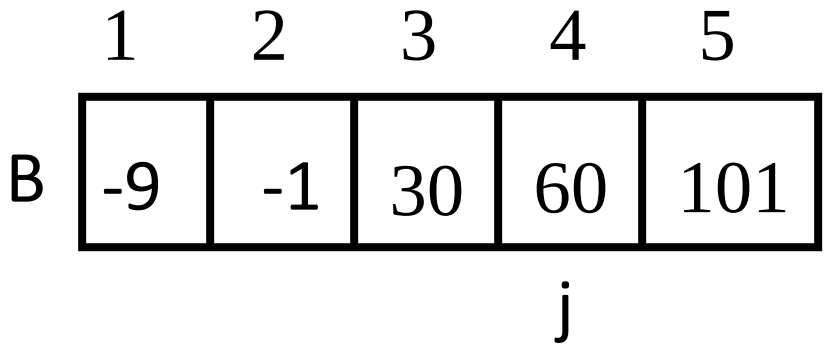

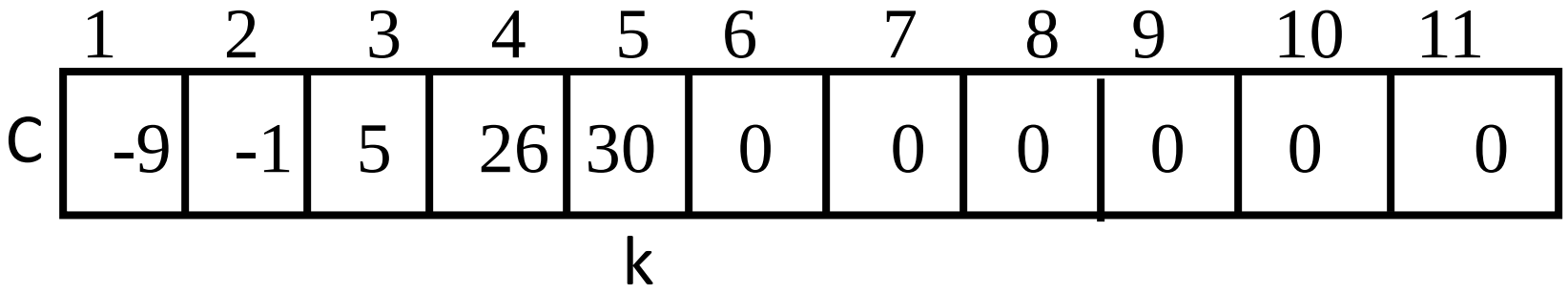

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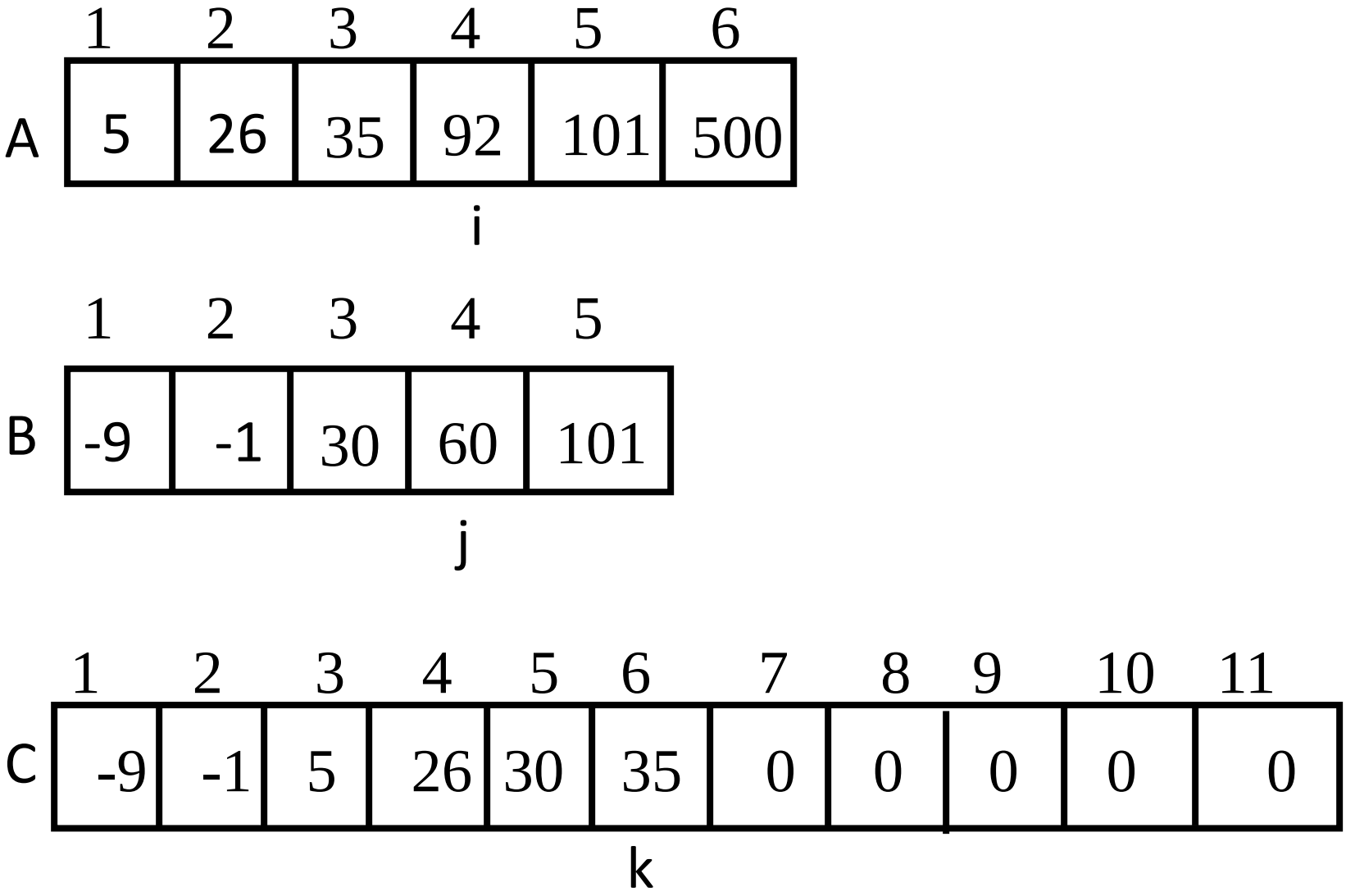

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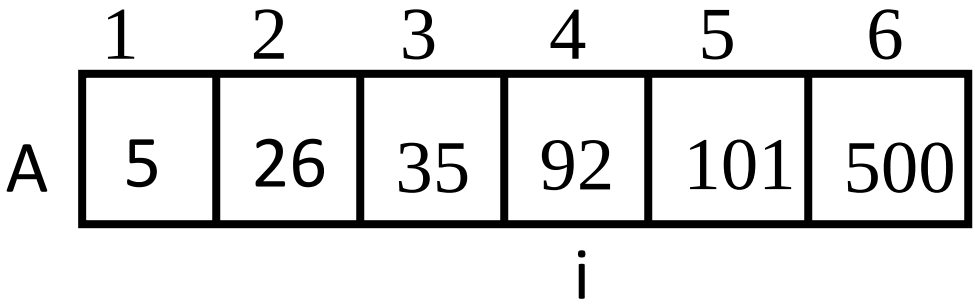

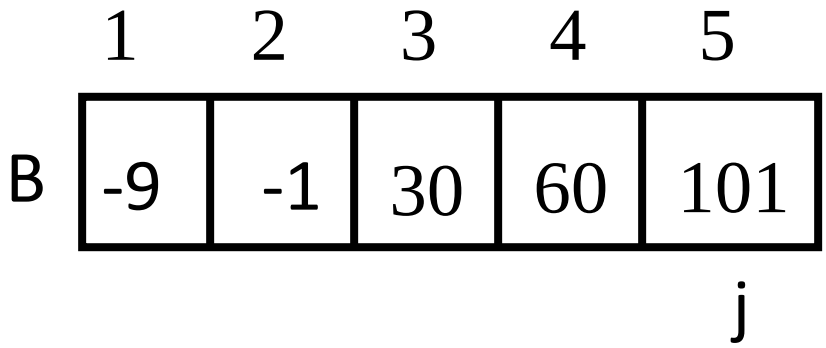

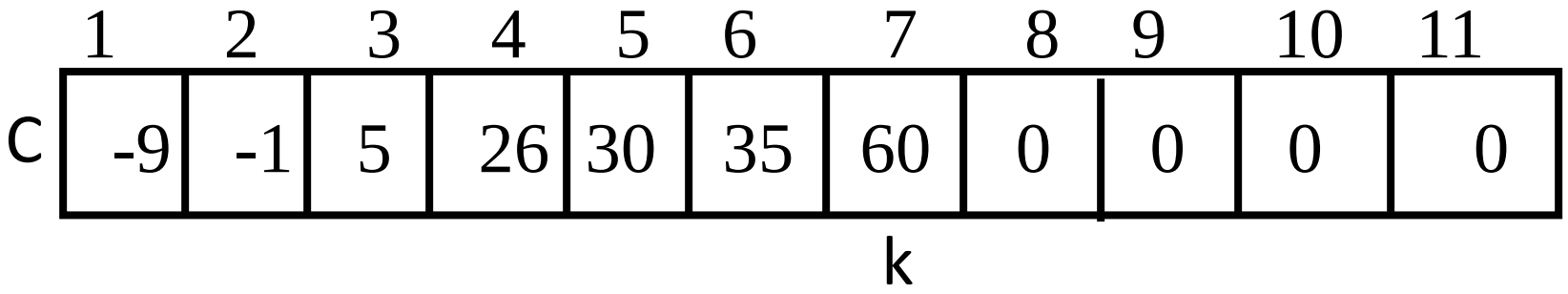

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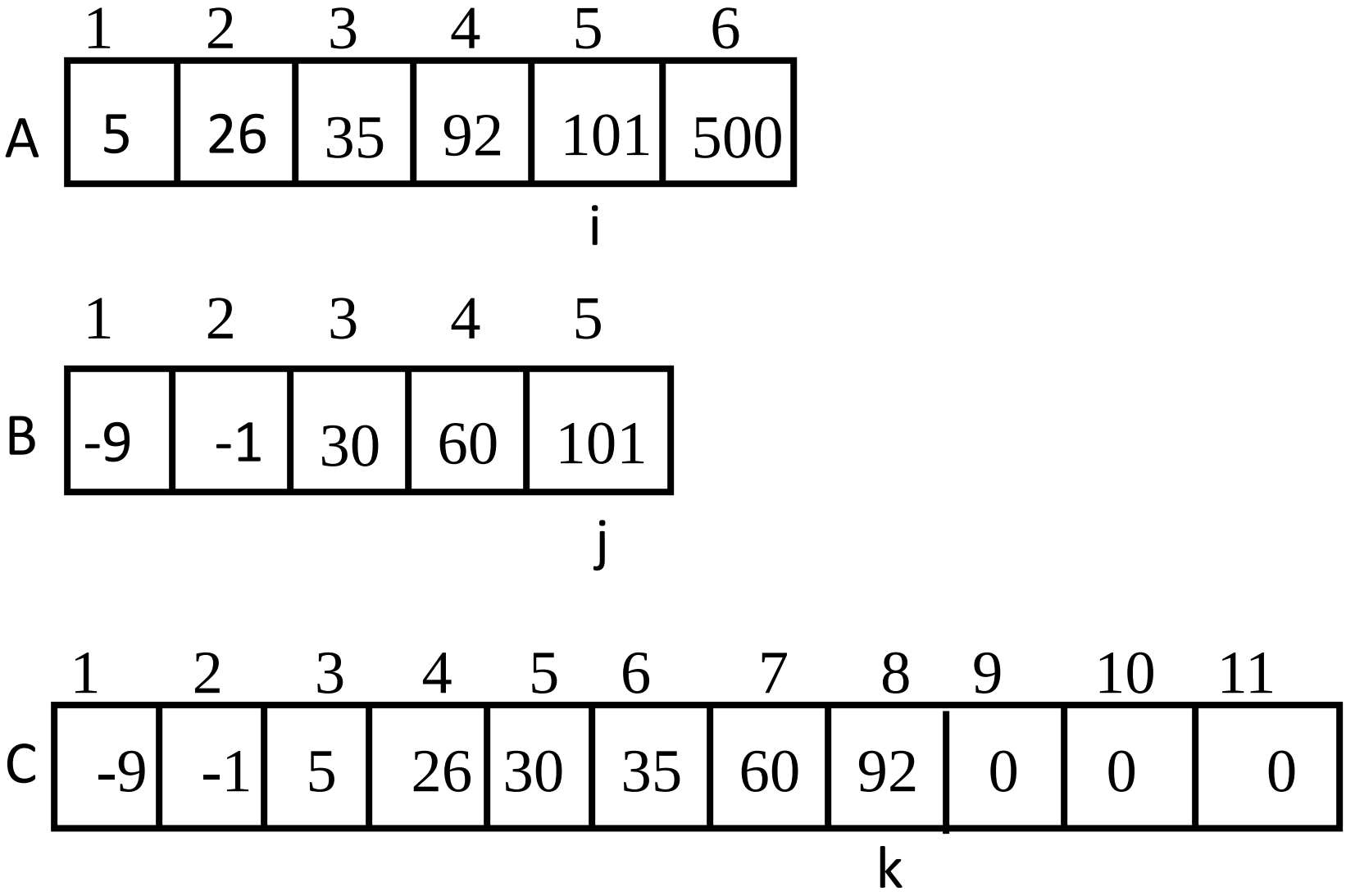

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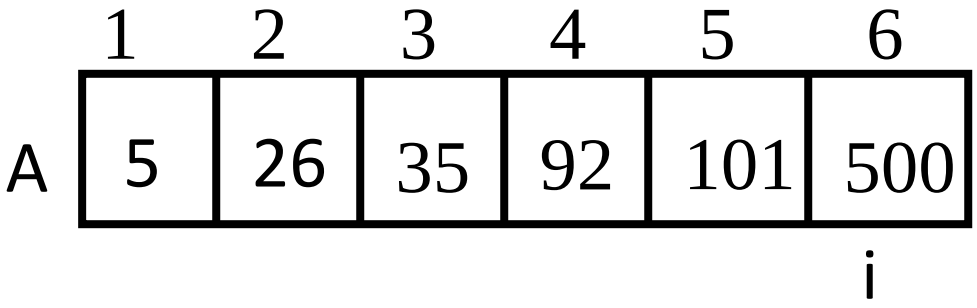

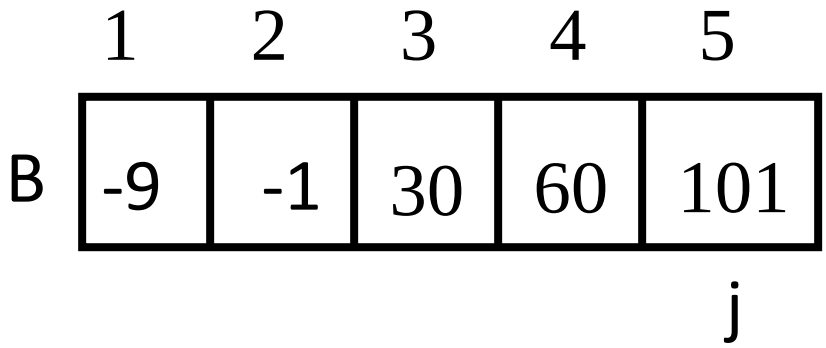

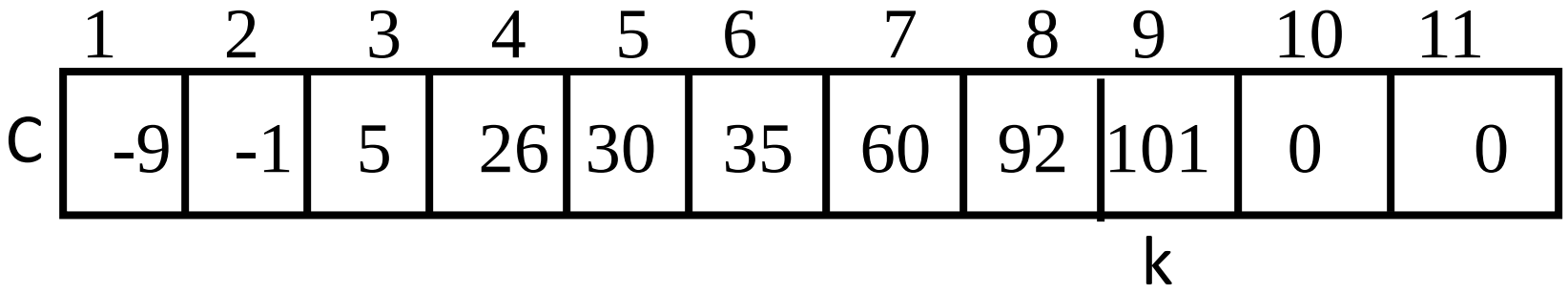

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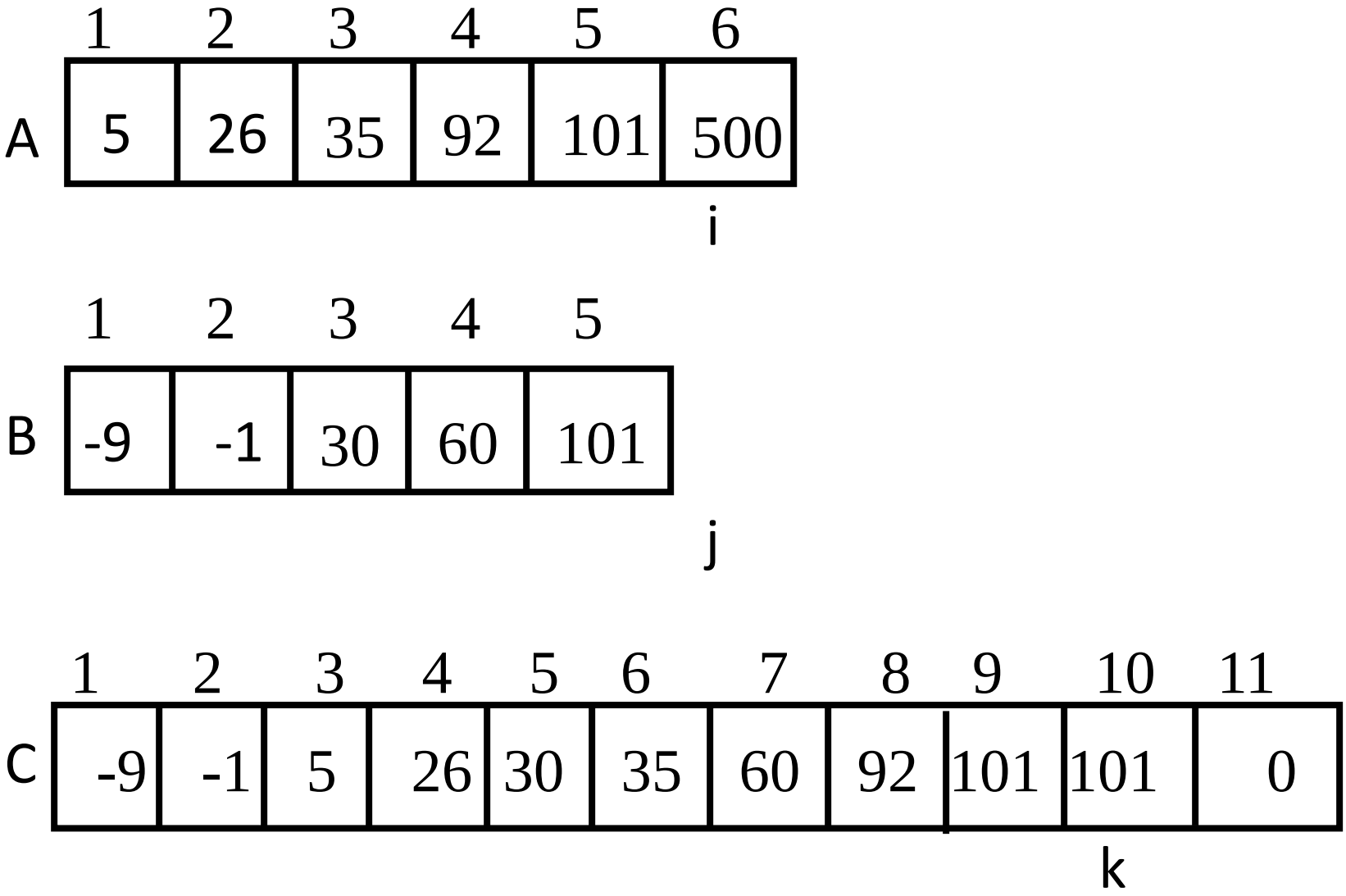

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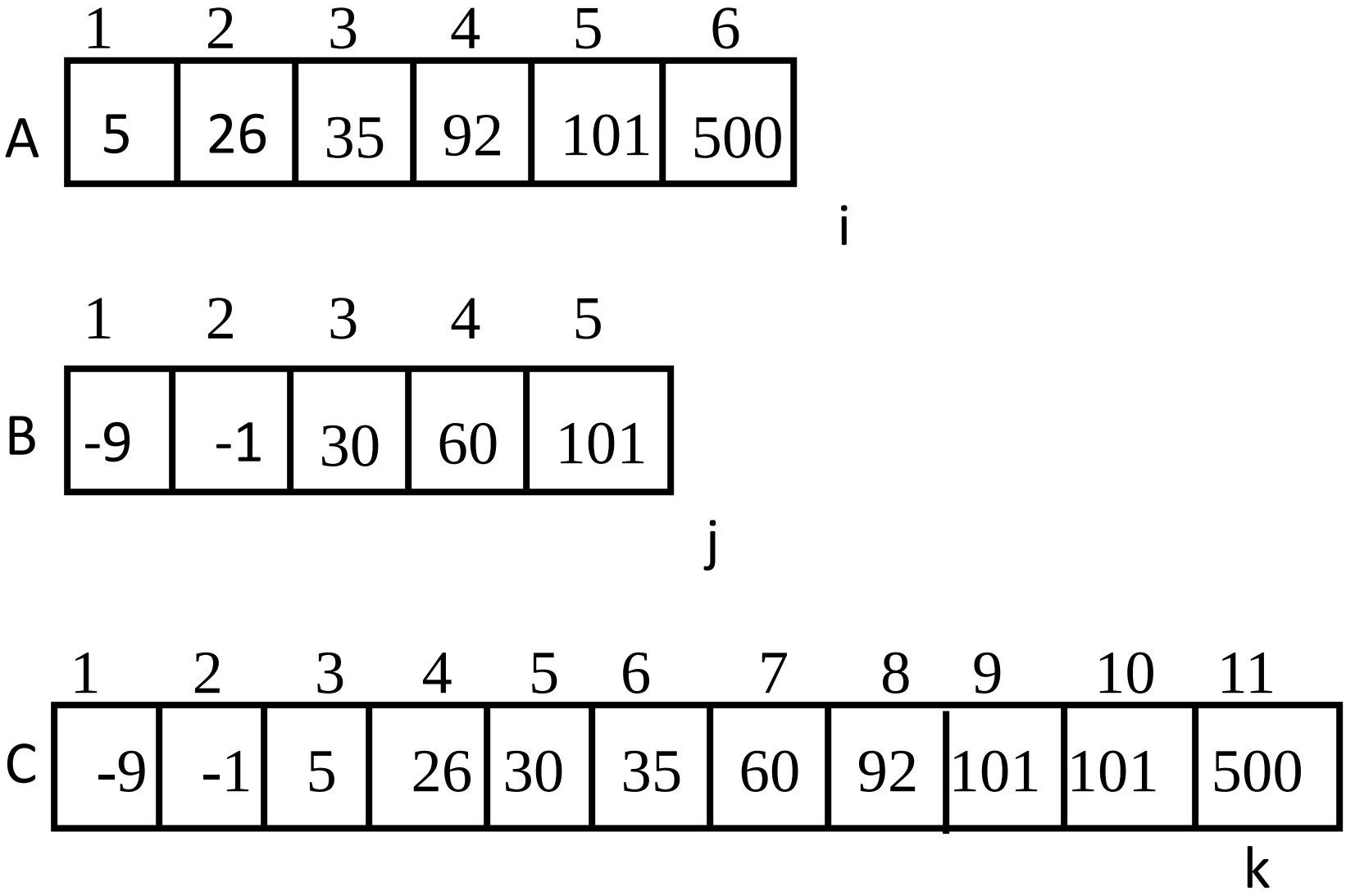
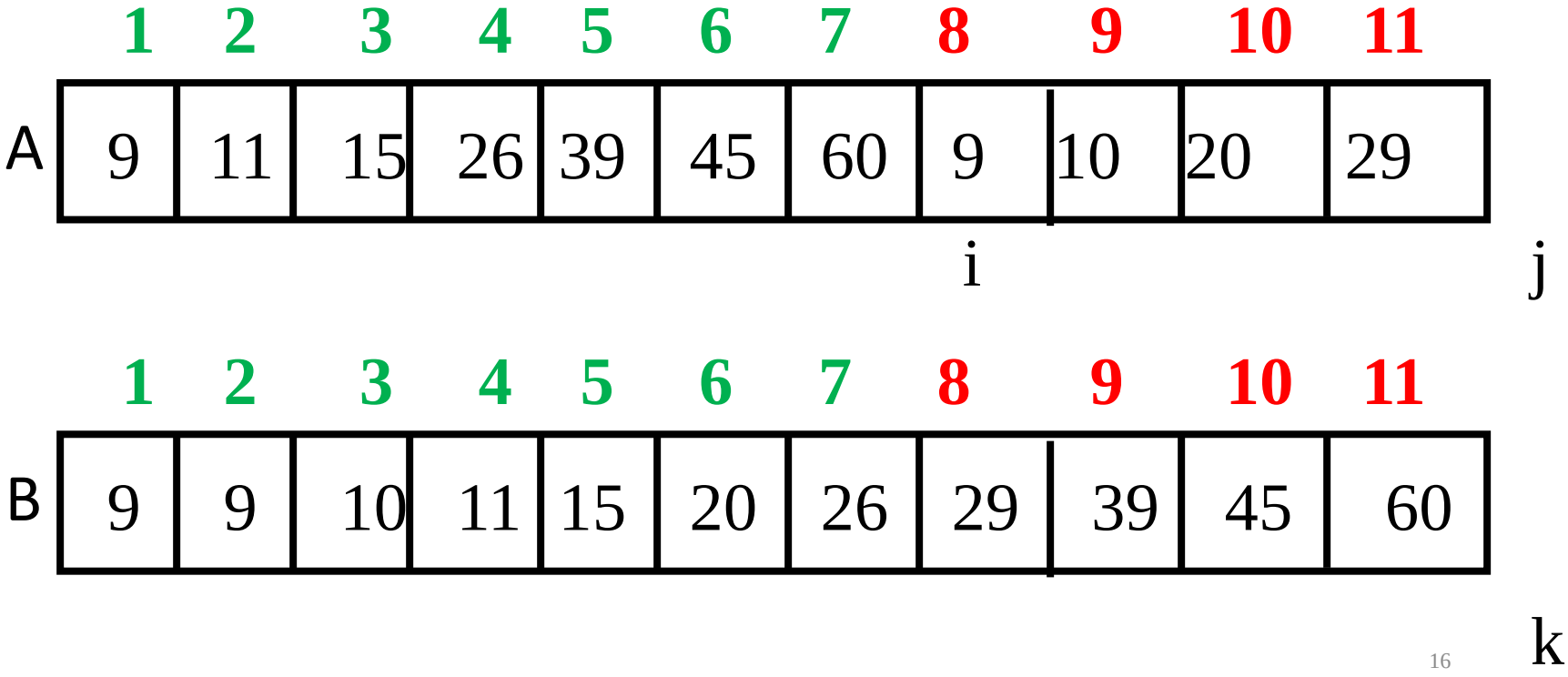


Merge

- A [low..high]
- A[low..mid] and A[mid+1..high]



Merge

```
void merge(int a[], int low, int mid,
  int high)
{
      int i = low, j = mid+1, k = 0;
      int b[high-low+1];
      while(i <= mid && j <= high) { // Both
      array are not exhausted
        if(a[i] <= a[j])
                {b[k] = a[i]; i++; k++;}
        else
                {b[k] = a[j]; j++; k++;}
      }
```

a[low] to a[mid] = A
a[mid+1] to a[high] = B
b[0] to b[high-low] = C

Merge

```
    while(j != high+1) // a[mid+1] to a[high] is not exhausted
        b[k++] = a[j++];
    while(i != mid+1) // a[low] to a[mid] is not exhausted
        b[k++] = a[i++];

    for(i = low, j = 0; i <= high; i++, j++)
        a[i] = b[j];  // Copy sorted b[0 .. High-low] to a[low to high]
}
```

Merge Sort

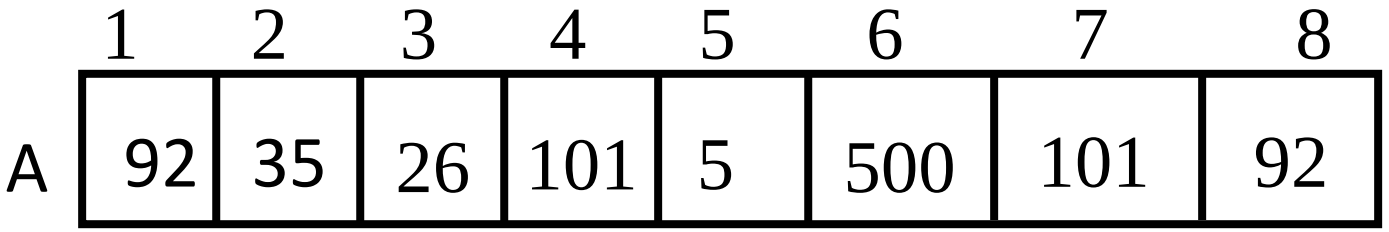
- It is a recursive function.
  - Divide the array into almost equal two halves
  - Recursively sort left half **A[low..mid]**
  - Recursively sort right half **A[mid+1..high]**
  - Merge the two halves to get the array sorted **A[low..high]**

Merge Sort

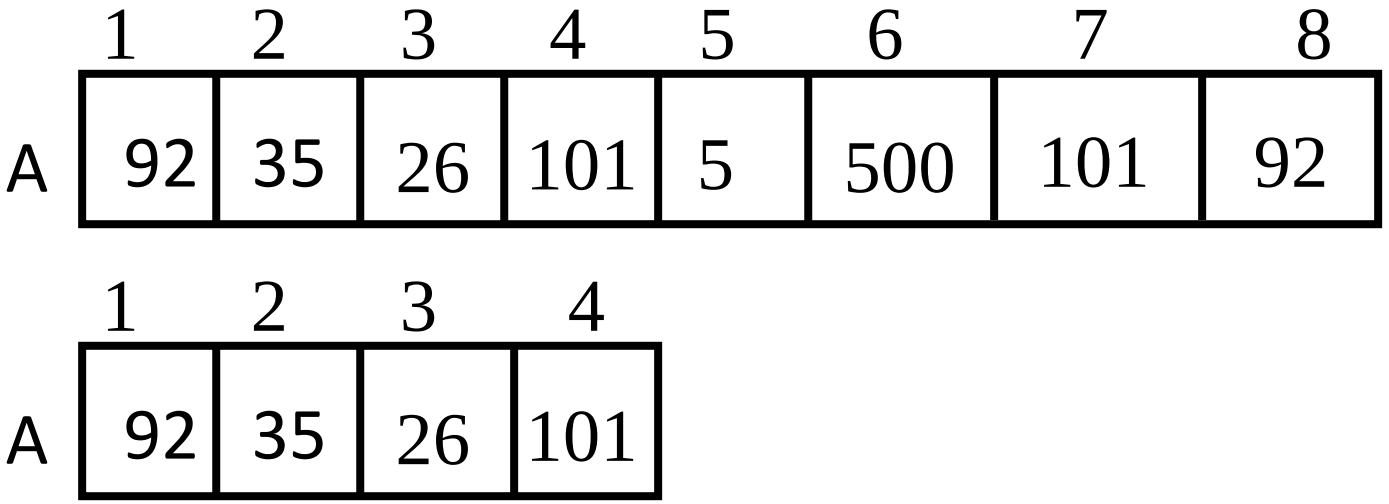
```
void mergesort(int a[], int low, int high)
{
  if(low < high) {
        int mid = (high+low)/2;
        mergesort(a, low, mid); // Sort the left half
        mergesort(a, mid+1, high); // Sort the right half
        merge(a, low, mid, high); // Merge the sorted halves
  }
}
```

From main(): mergesort(a, 0, n-1);

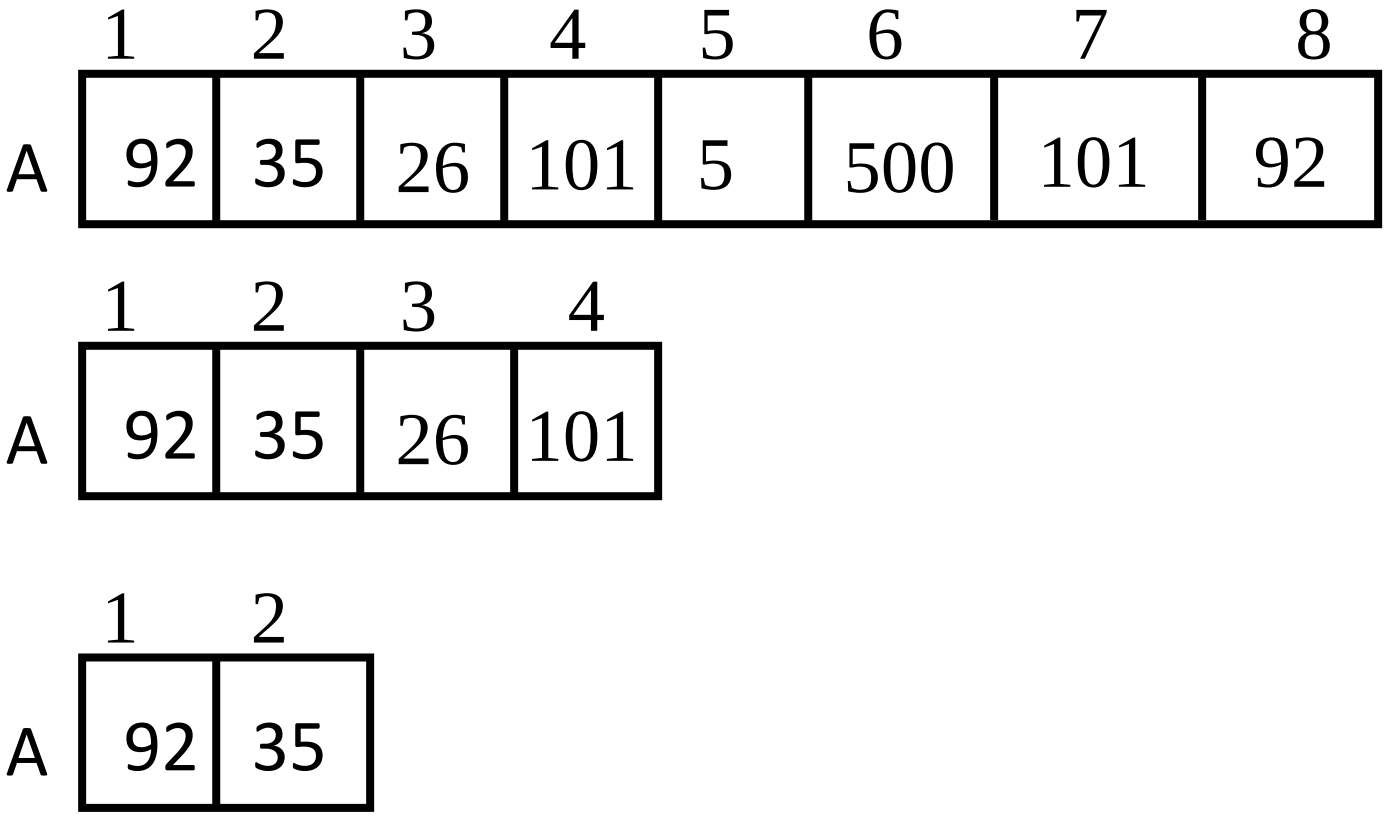
Merge Sort



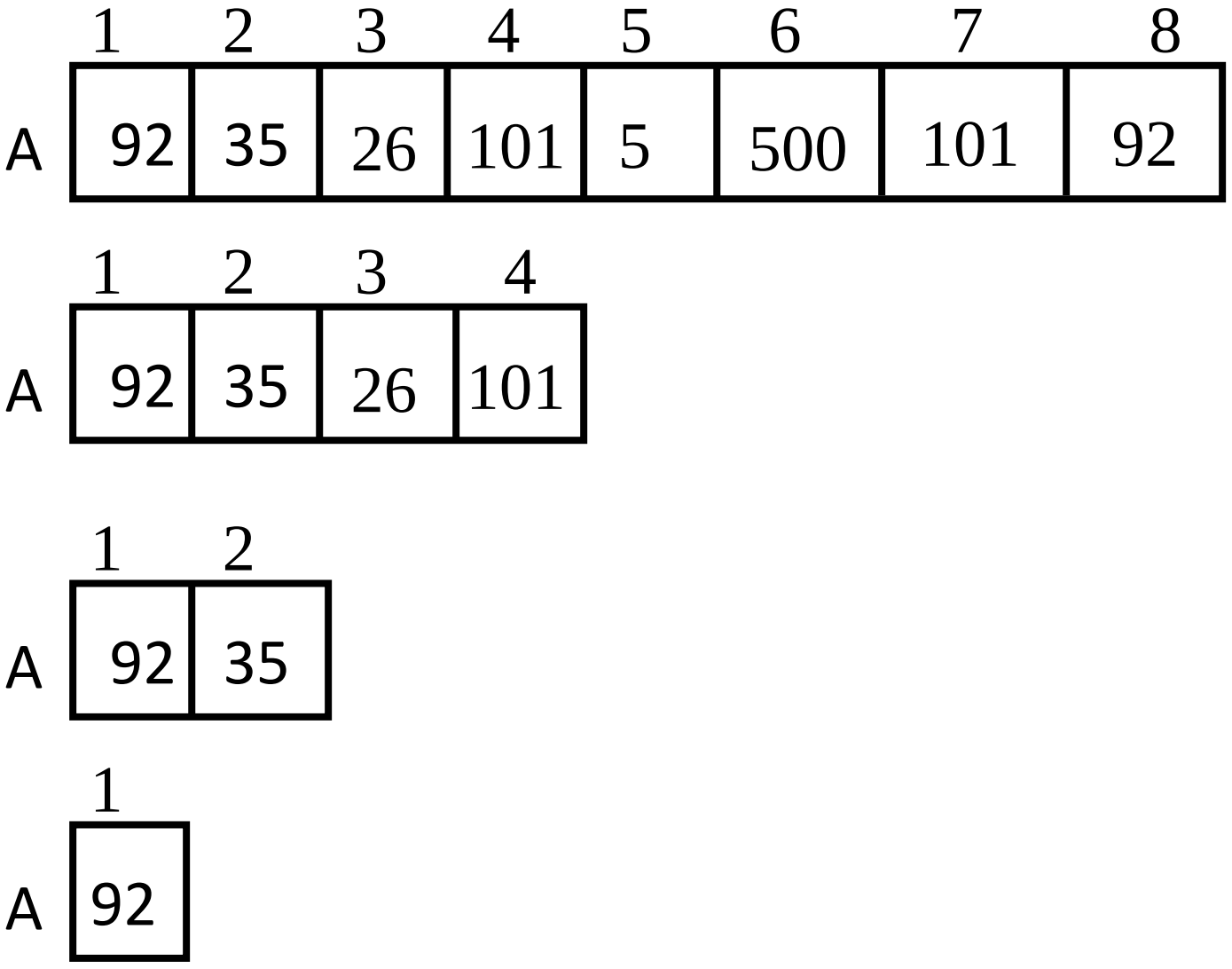
Merge Sort : Divide (Level 1)



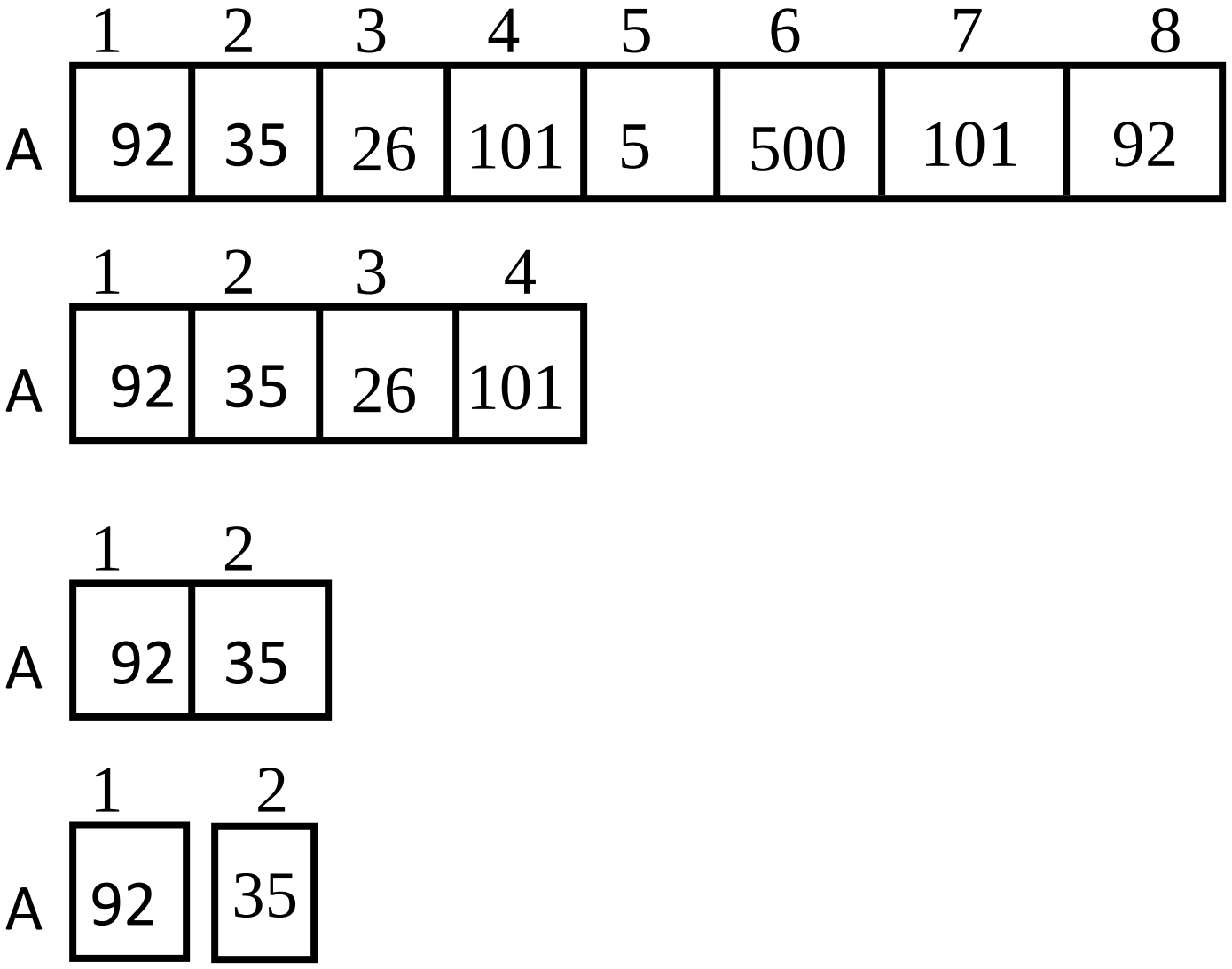
Merge Sort : Divide (Level 2)



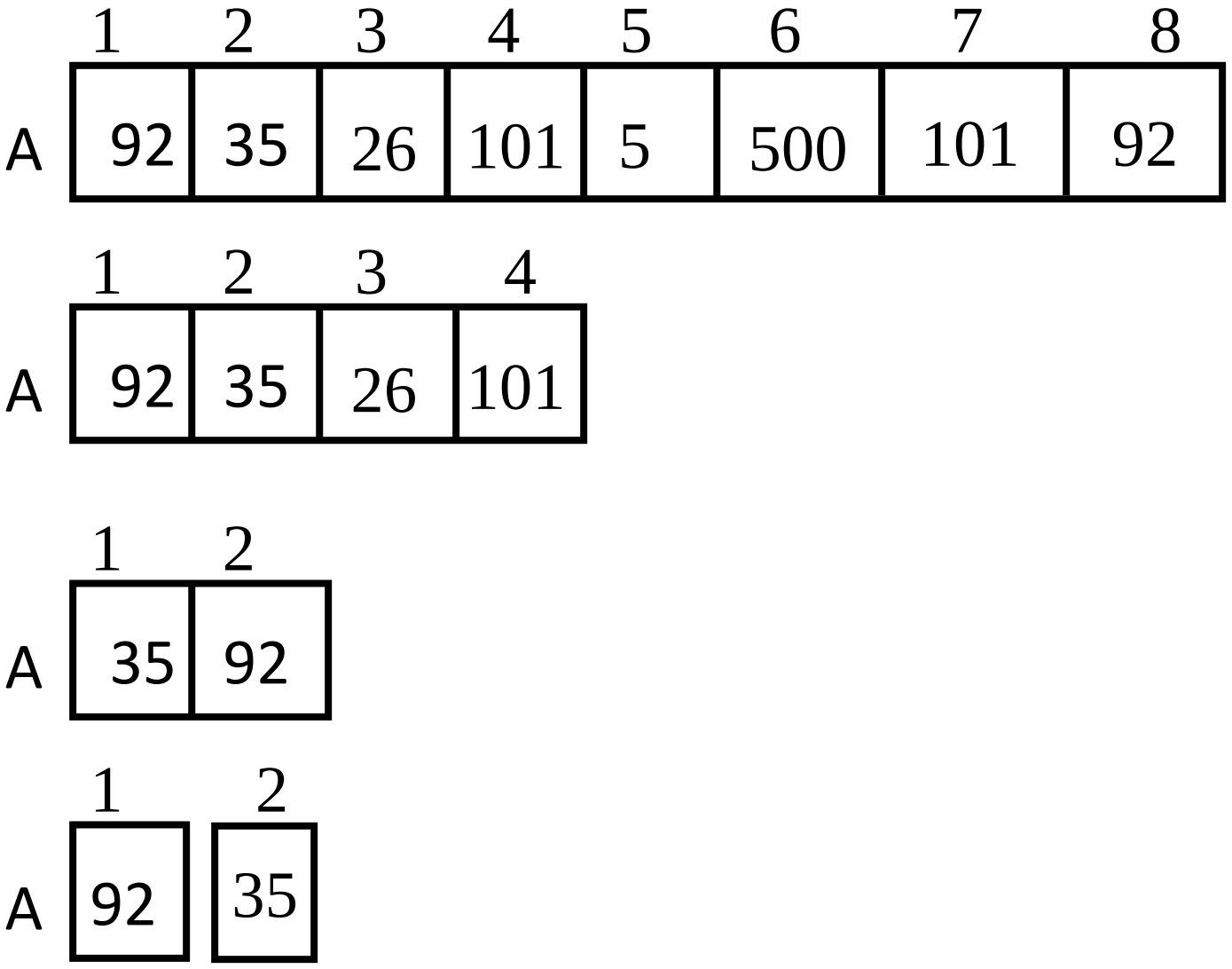
Merge Sort : Divide (Level 3) & Conquer (Level 4)



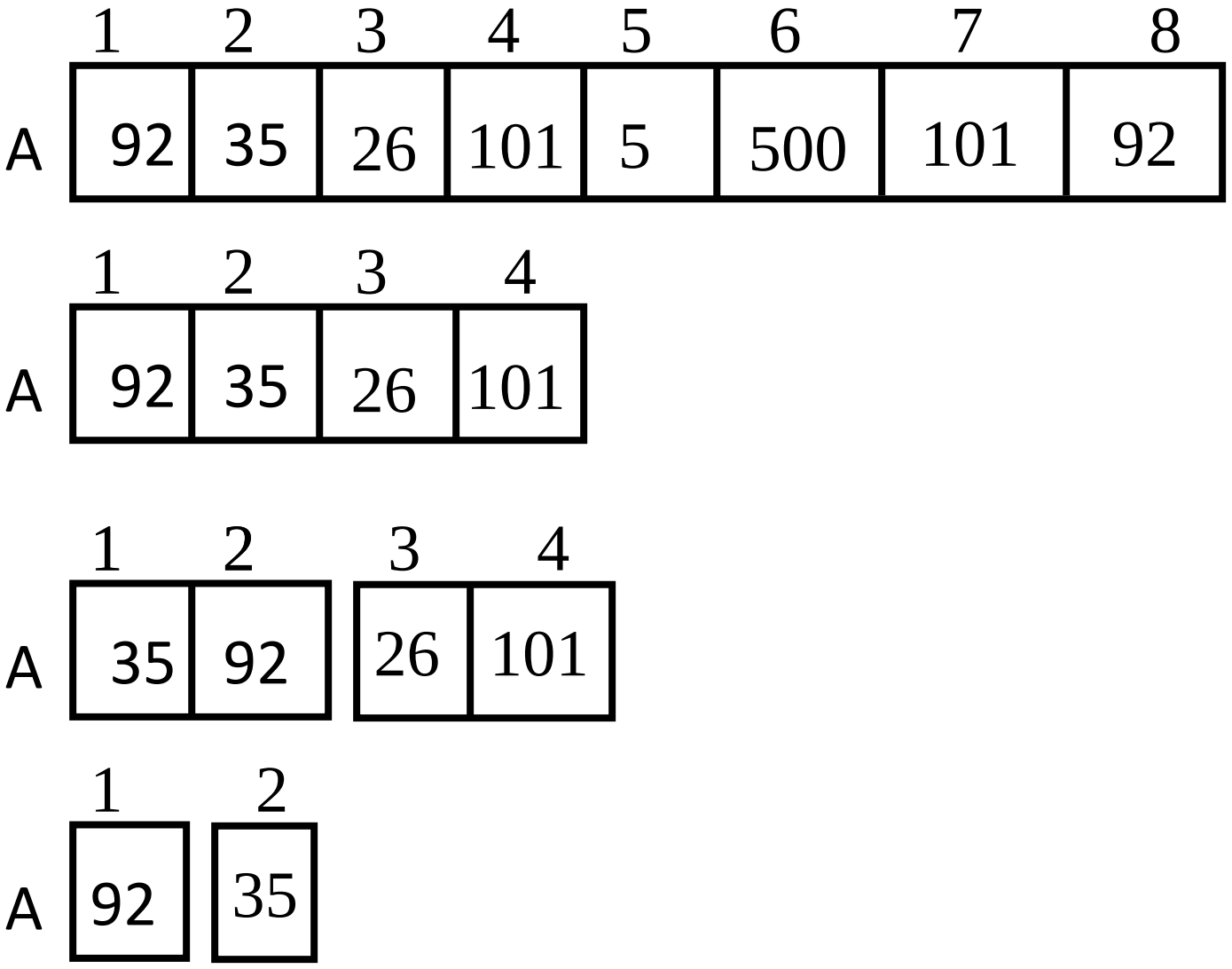
Merge Sort : Conquer (Level 4)



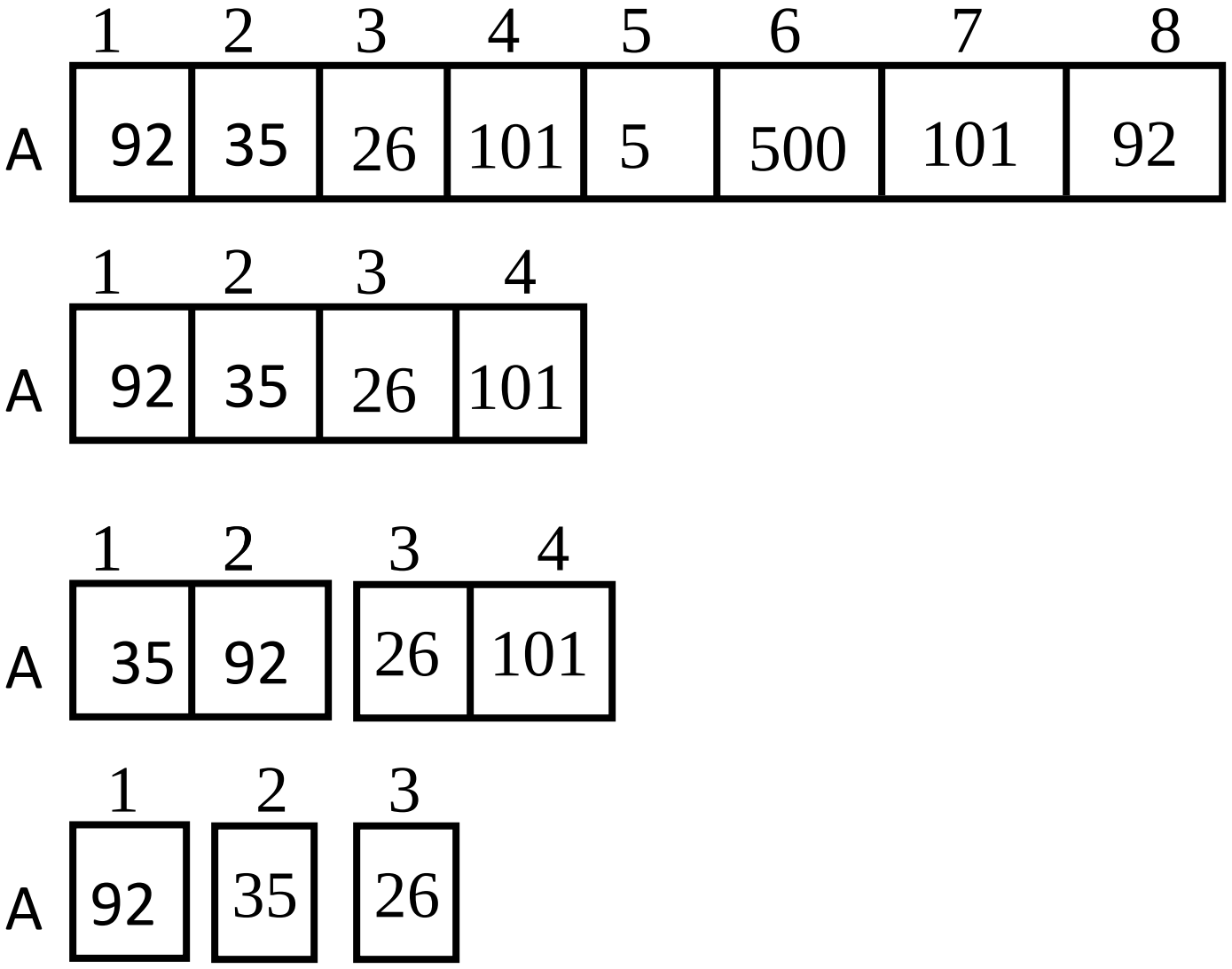
Merge Sort : Conquer (Level 3)



Merge Sort : Divide (Level 2)



Merge Sort : Divide (Level 3) & Conquer (Level 4)



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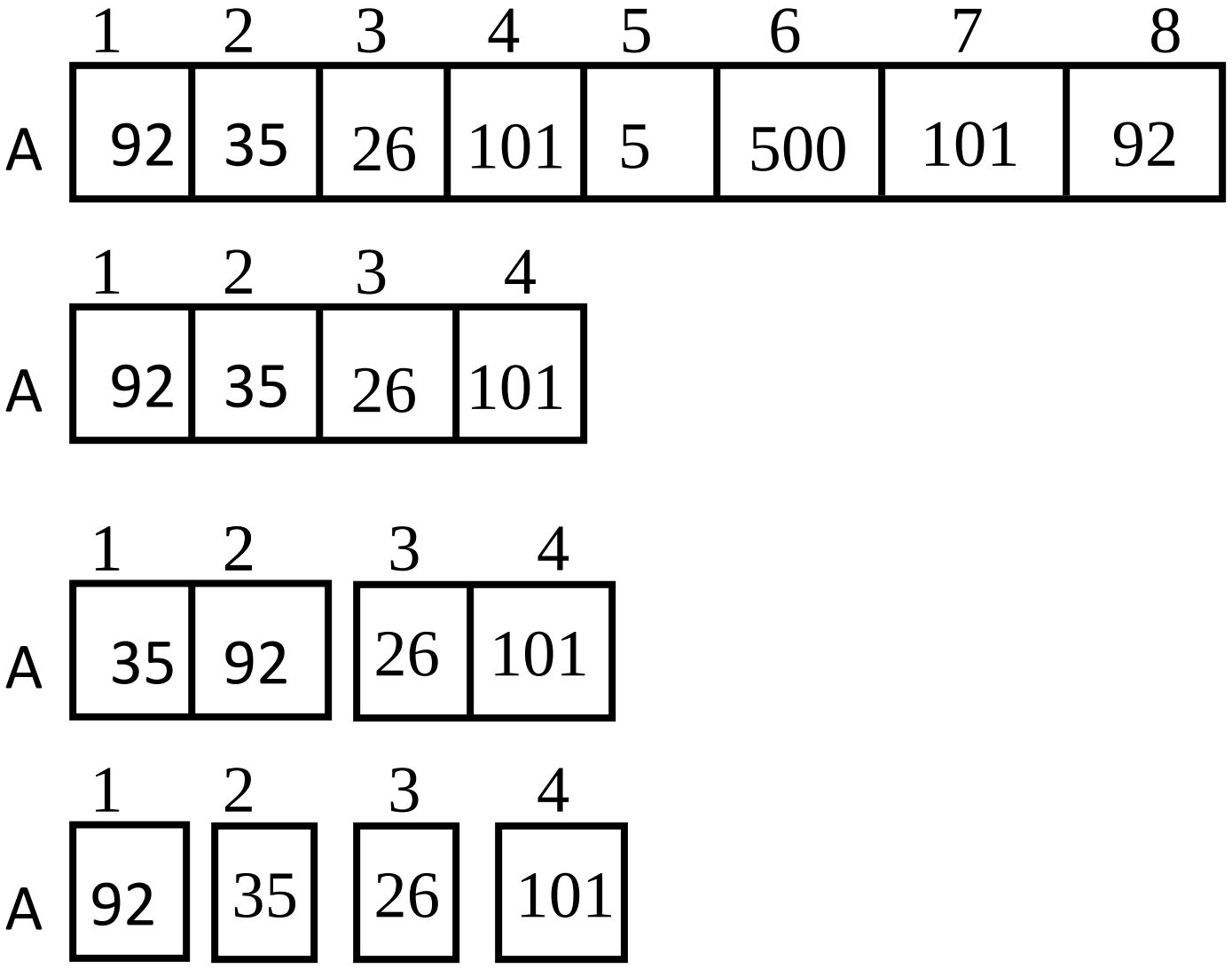

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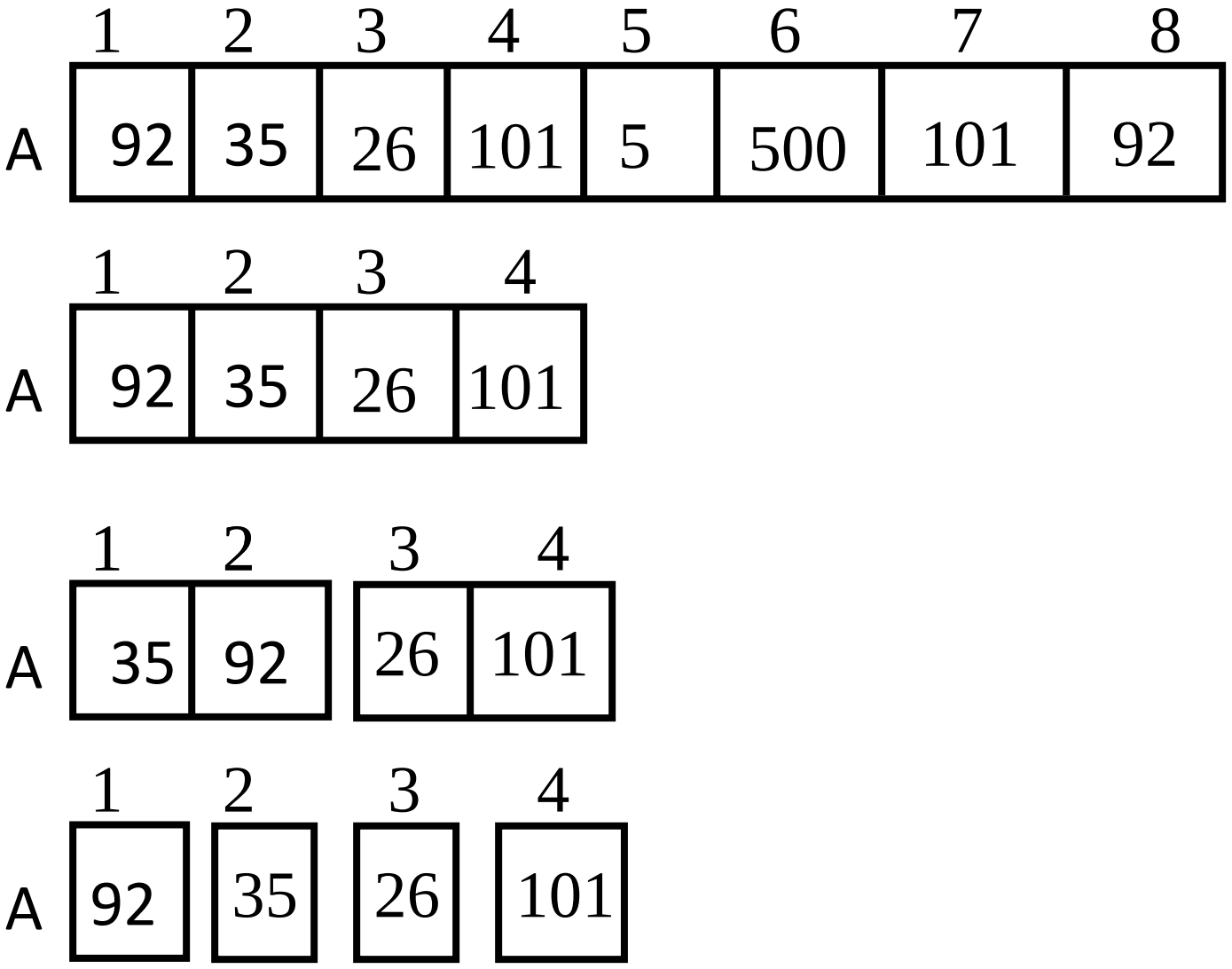

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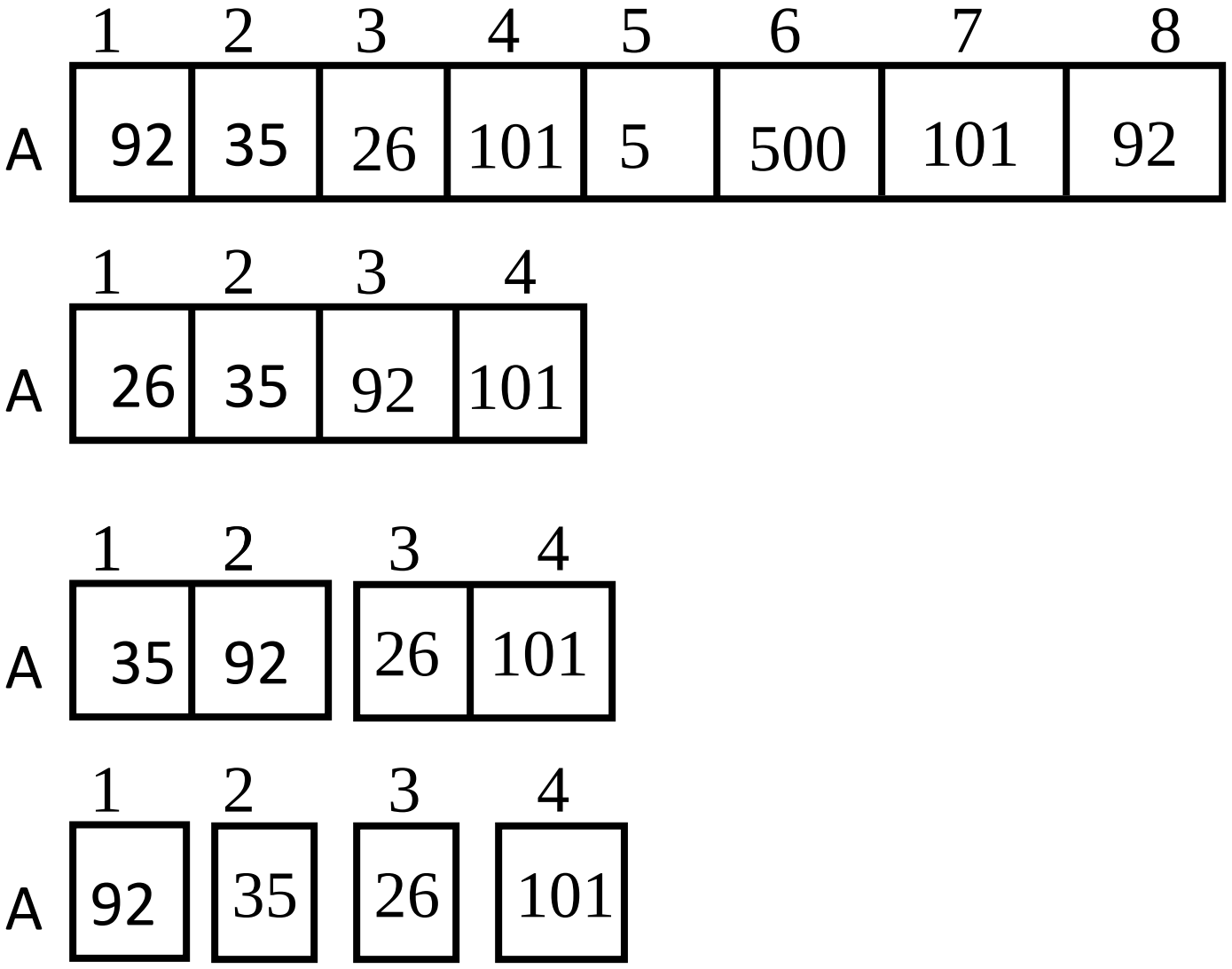

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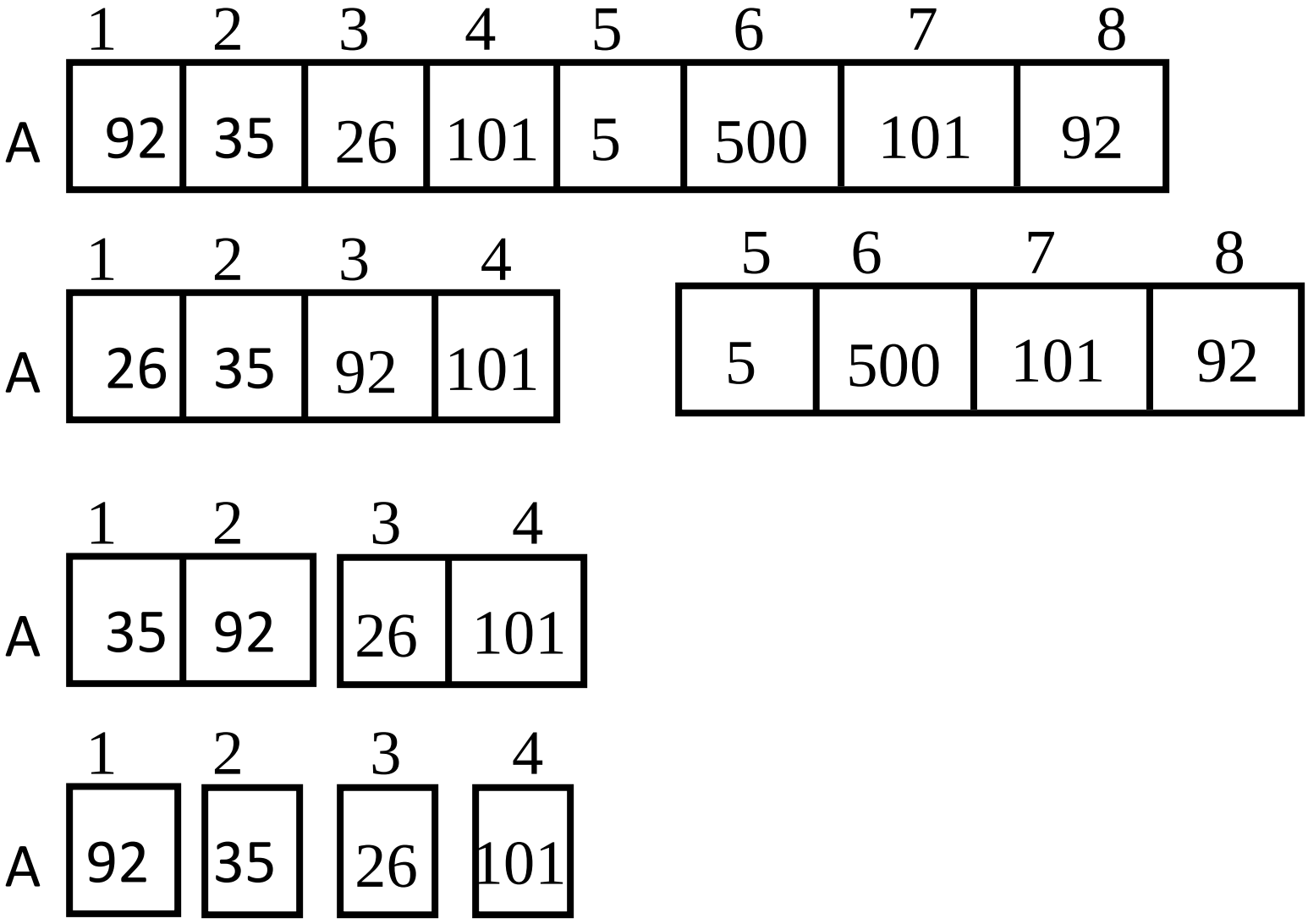
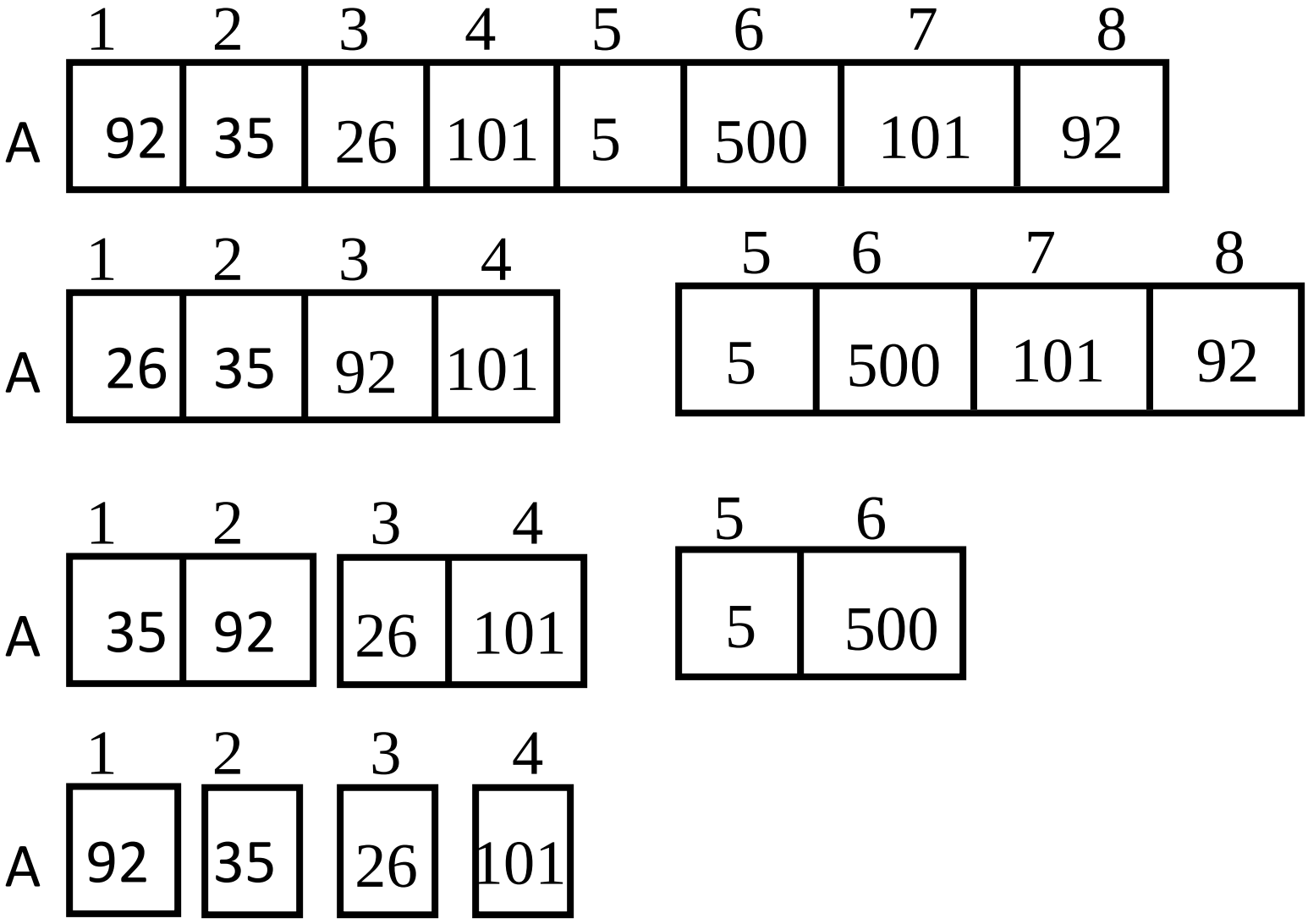
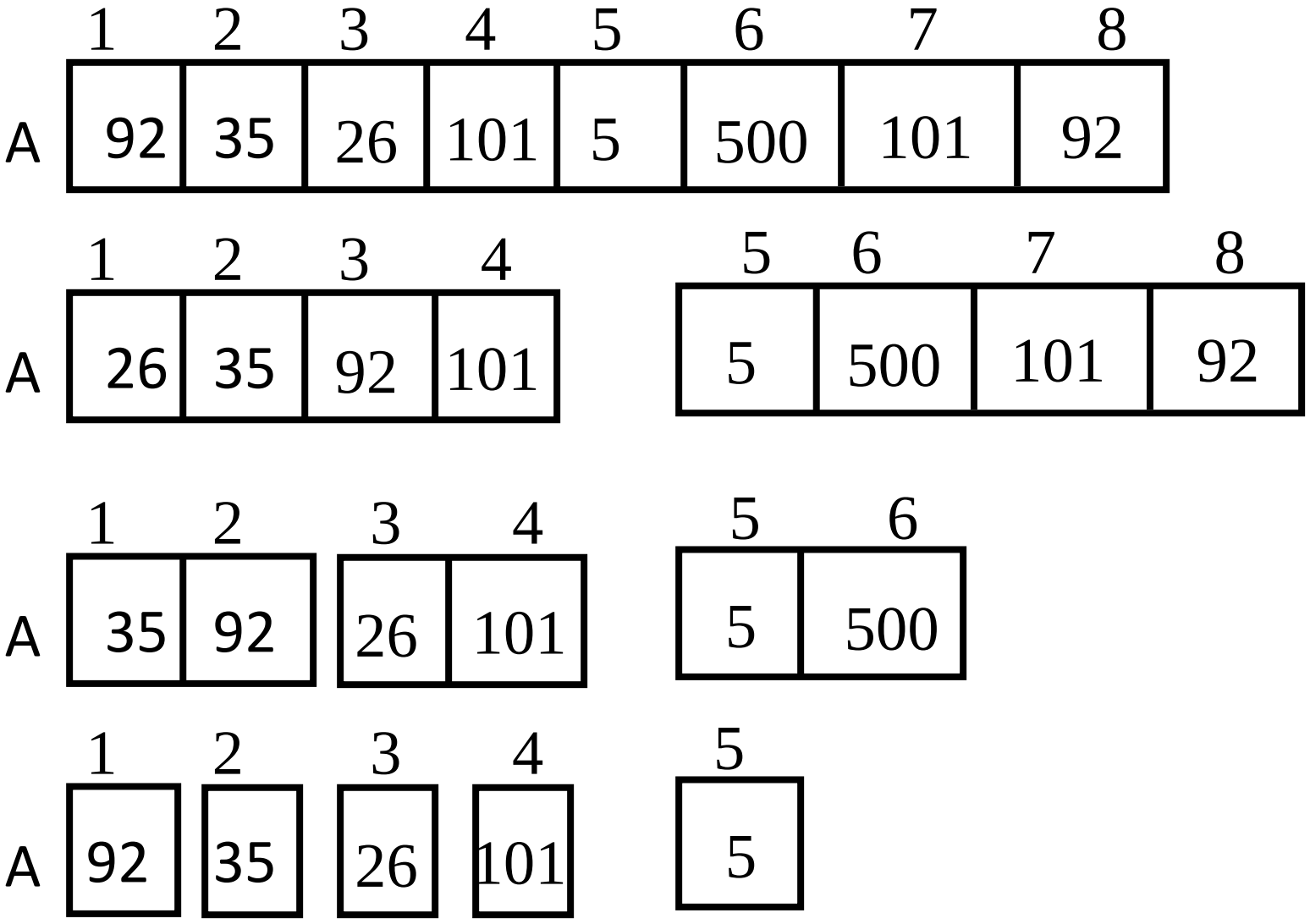


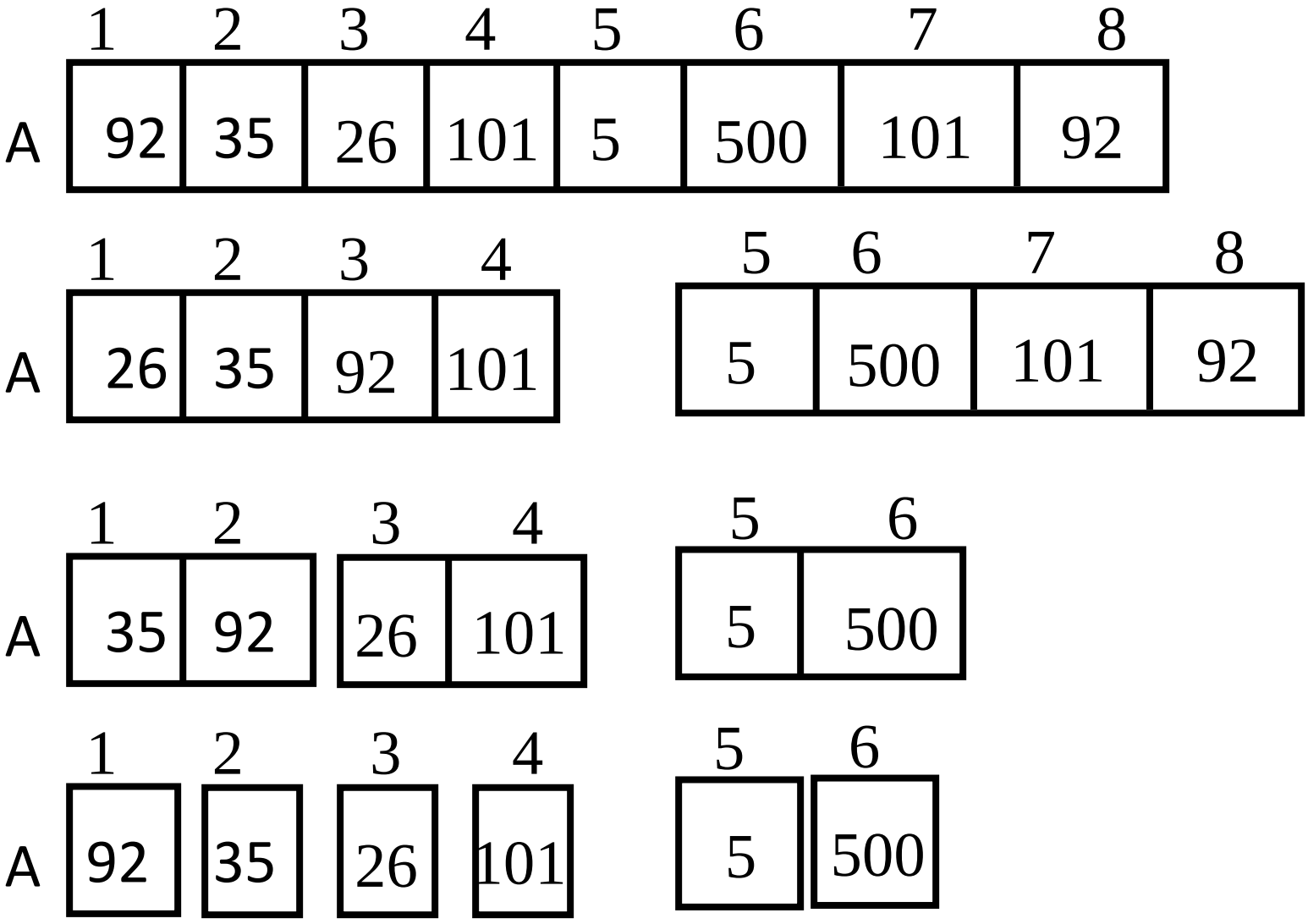
Merge Sort : Divide (Level 2)



Merge Sort : Divide (Level 3) & Conquer (Level 4)



Merge Sort : Conquer (Level 4)



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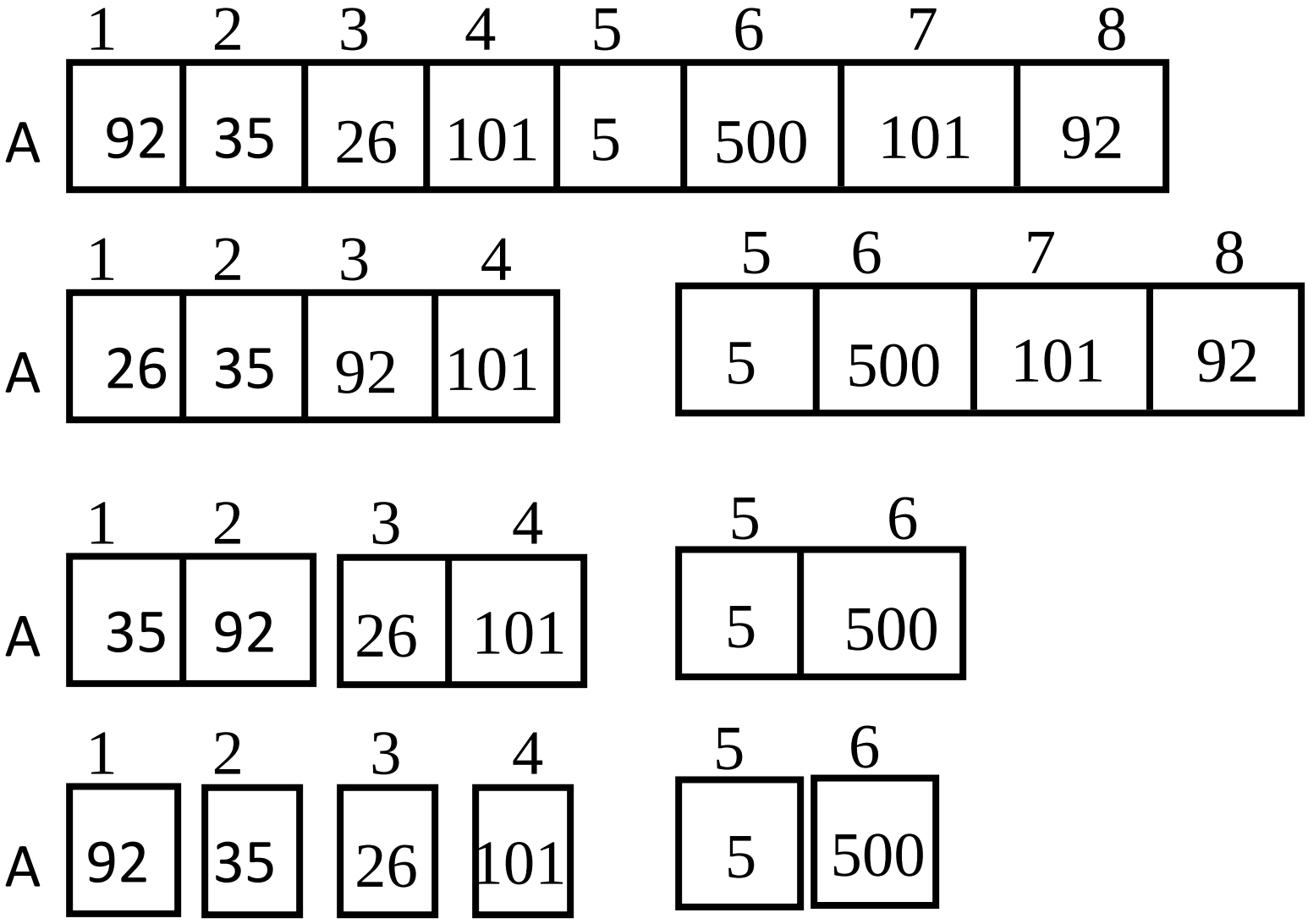

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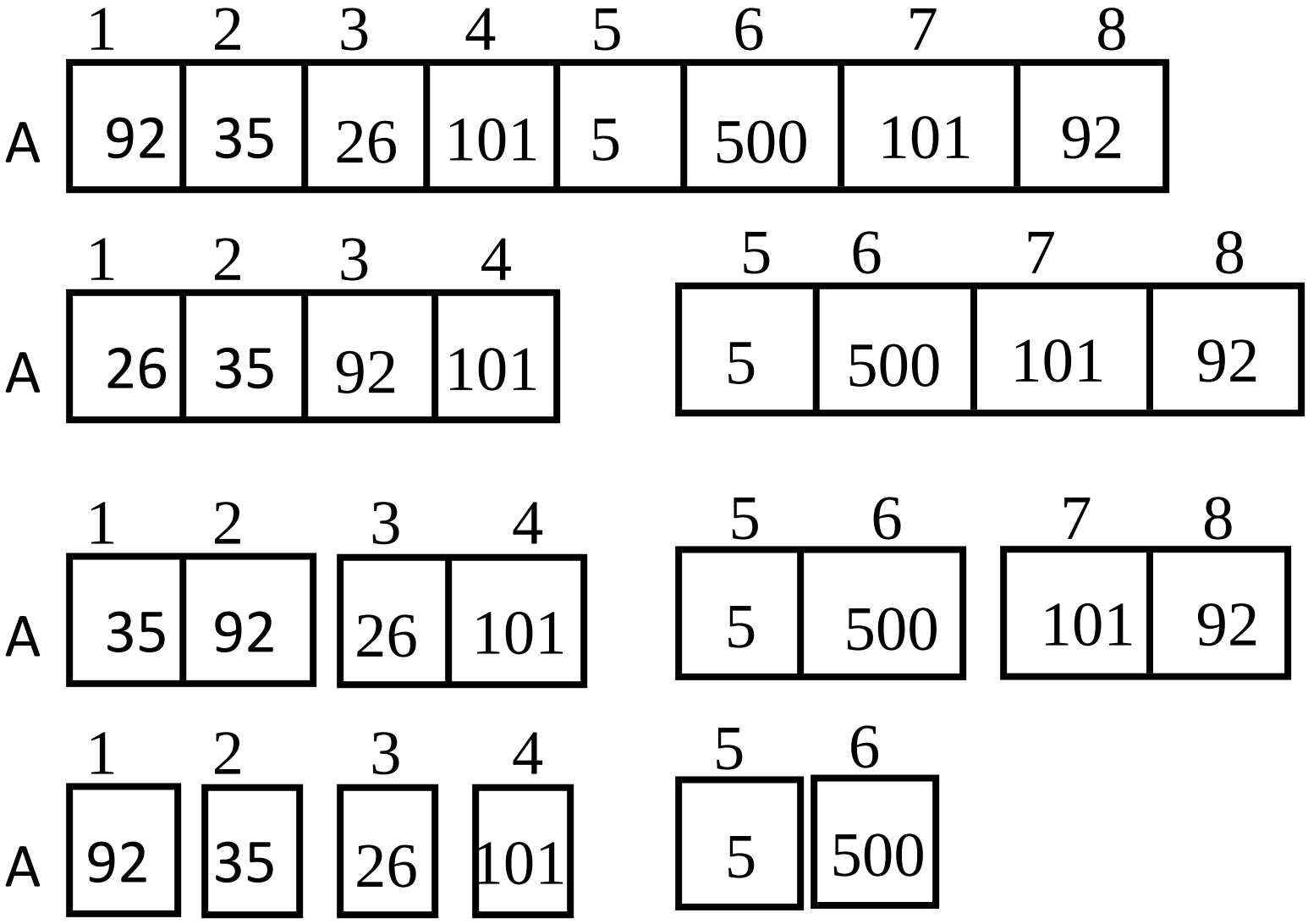

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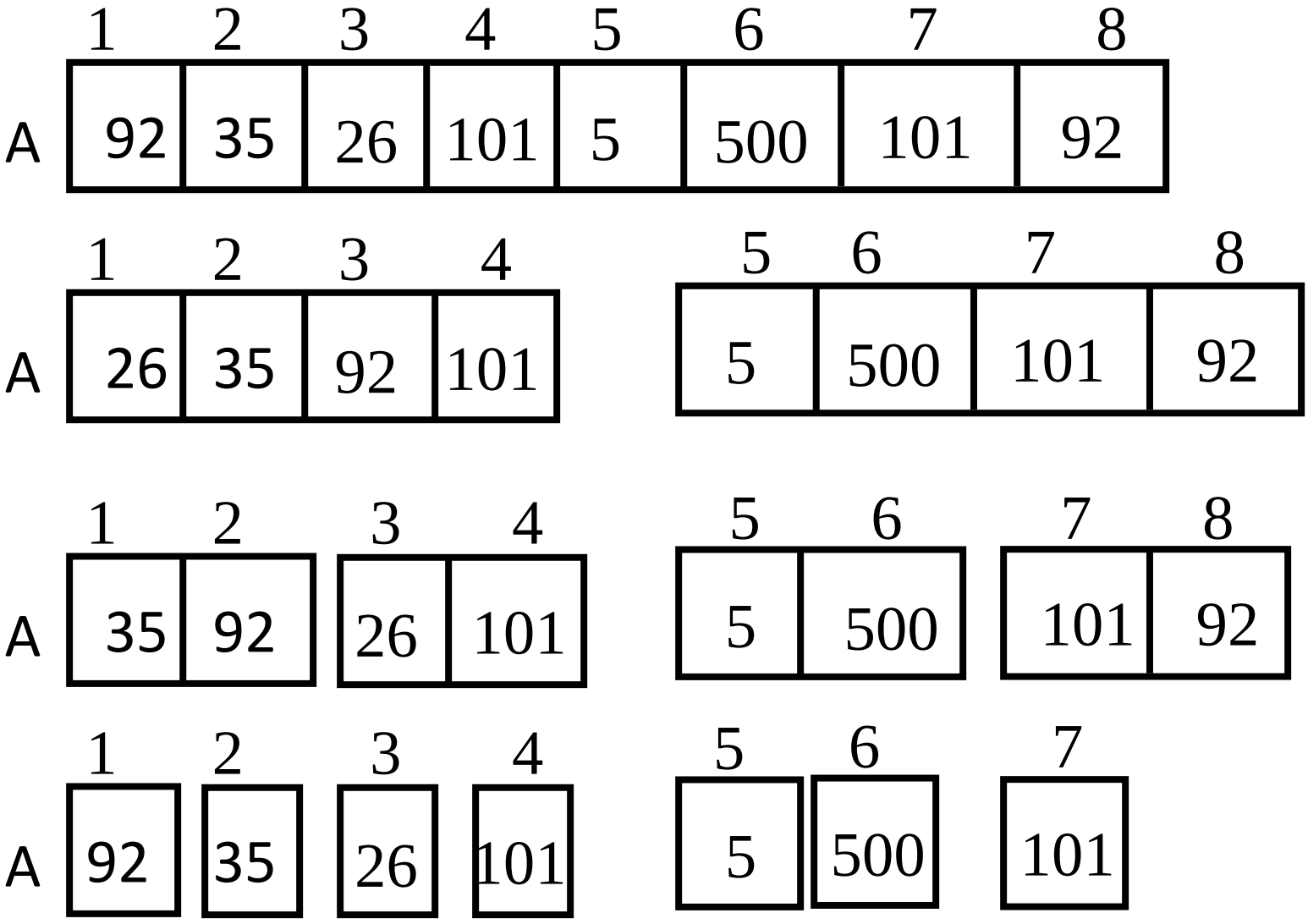
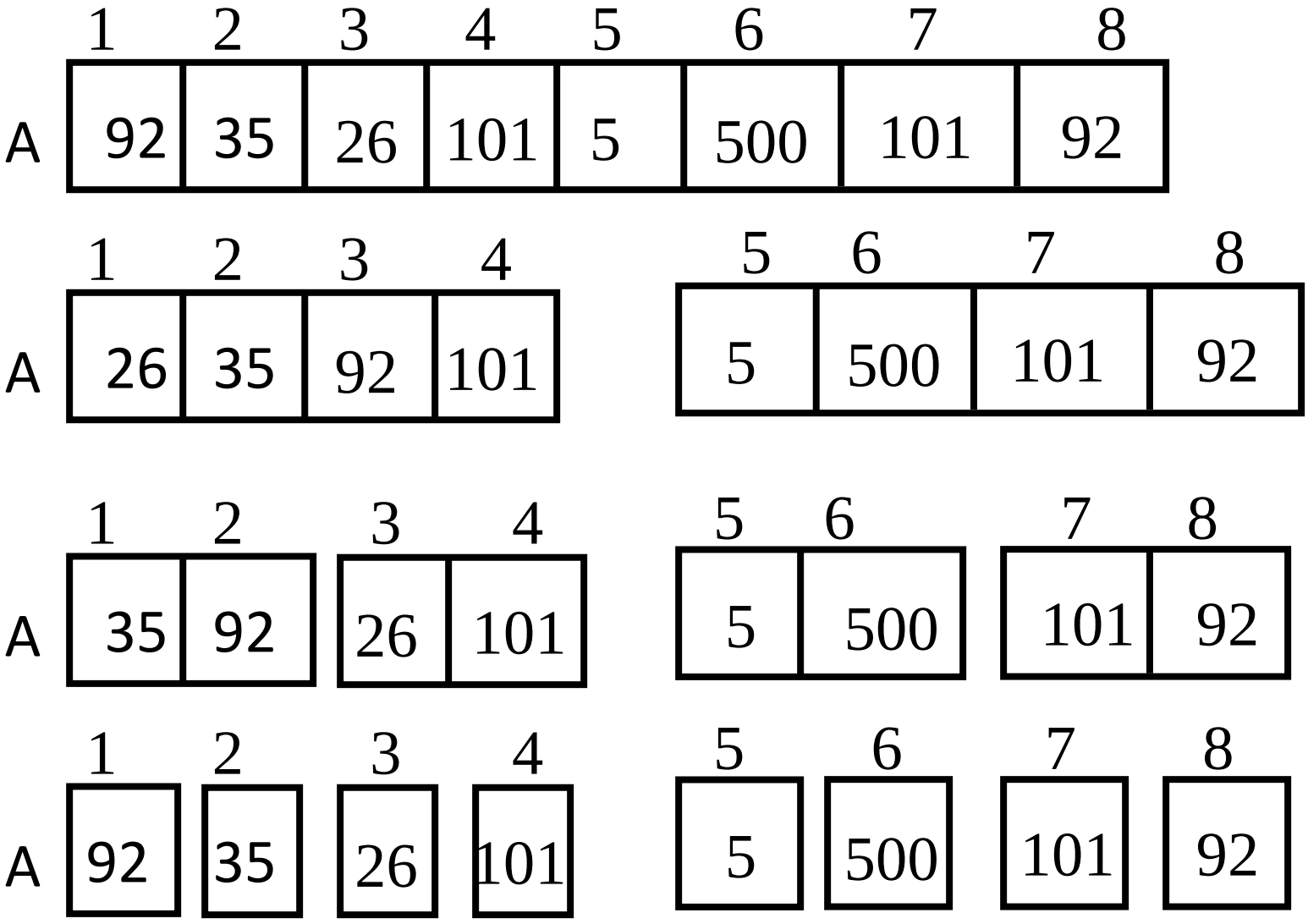


Merge Sort : Conquer (Level 4)



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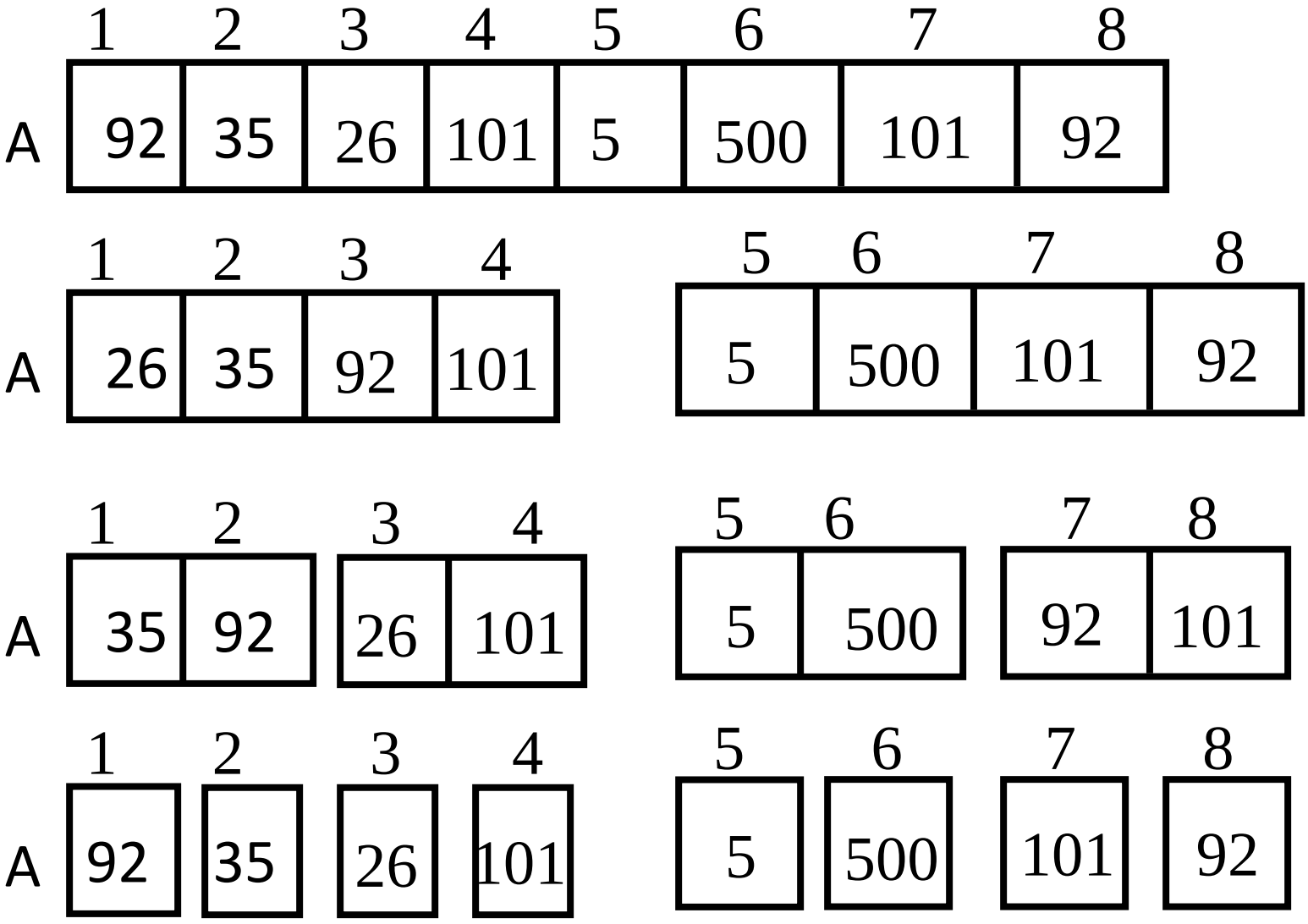

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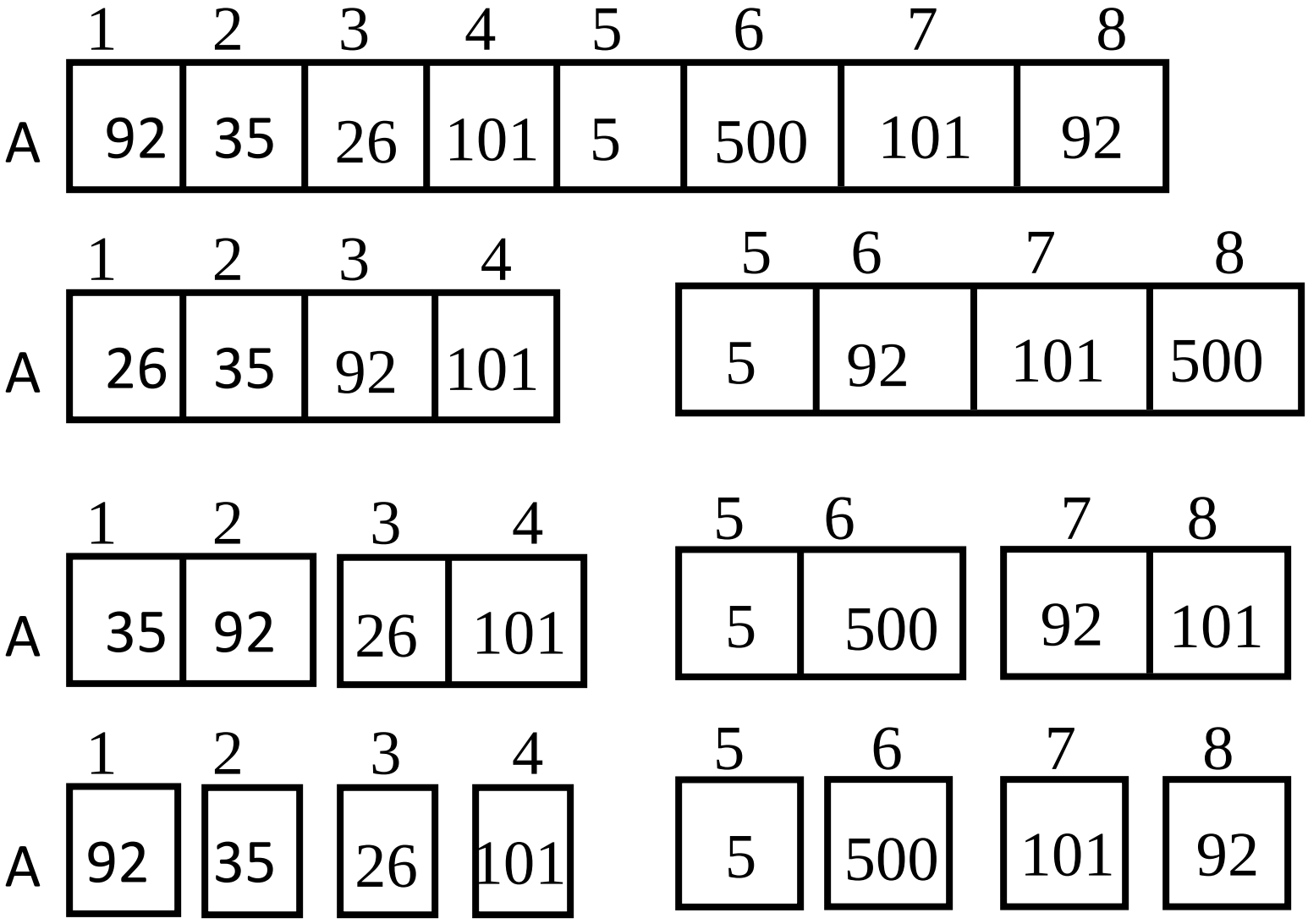
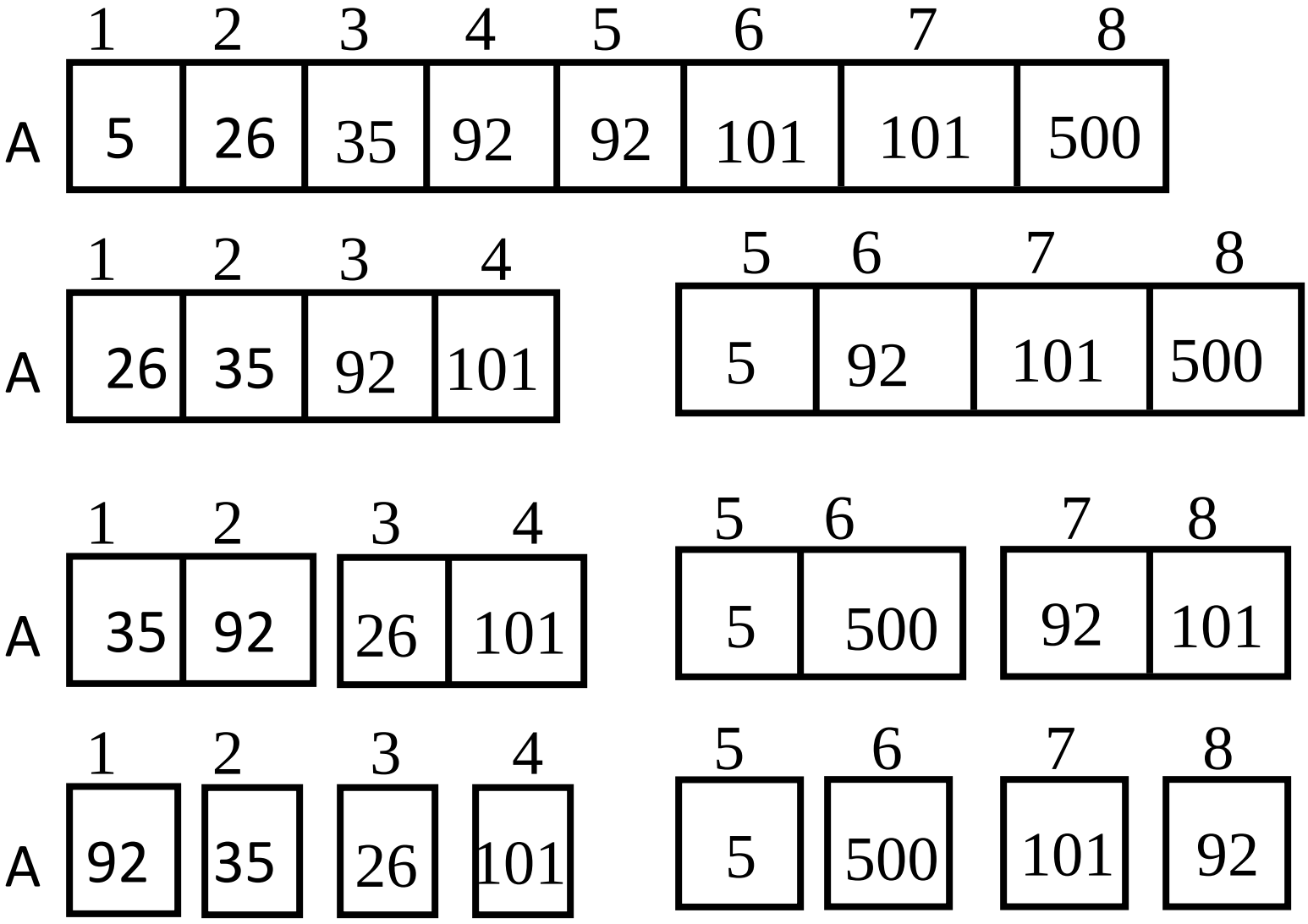


Merge Sort : Conquer (Level 1)



Merge Sort

```
void mergesort(int a[], int low, int high)
{
  if(low < high) {
        int mid = (high+low)/2;
        mergesort(a, low, mid); // Sort the left half
        mergesort(a, mid+1, high); // Sort the right half
        merge(a, low, mid, high); // Merge the sorted halves
  }
}
```

Time Complexity: Merge Sort

- T(n) = Time to merge sort array of size n = 2^k integers
- T(1) = 1

$T(n) = 2T(n/2) + cn$

$2T(n/2) = 2^2T(n/2^2) + cn$

$2^2T(n/2^2) = 2^3T(n/2^3) + cn$

.....

$2^{k-1}T(n/2^{k-1}) = 2^kT(n/2^k) + cn$

$T(n) = 2^kT(n/2^k) + cn.\ k$ (Summation of k equations)

$T(n) = nT(1) + c.n.\log_2 n = n + c.n.\log_2 n = O(n\log_2 n)$

Space Complexity: Merge Sort

- Space needed to solve merge sort
- Do not consider the space for input array
- Consider the extra space needed in terms of input size
- At the bottom level, an array of size 2 is required
- Immediately higher level, an array of size 4 is required
- In the next higher level, an array of size 8 is required
- …
- In the top level, an array of size $2^k = n$ is required
- Important to note: when array in one level is used/created, the arrays in the bottom levels have already been destroyed.
- Therefore, the space complexity = $O(\max(2, 4, 8, \ldots, 2^k = n)) = O(n)$