Tree: A Non-Linear Data Structure

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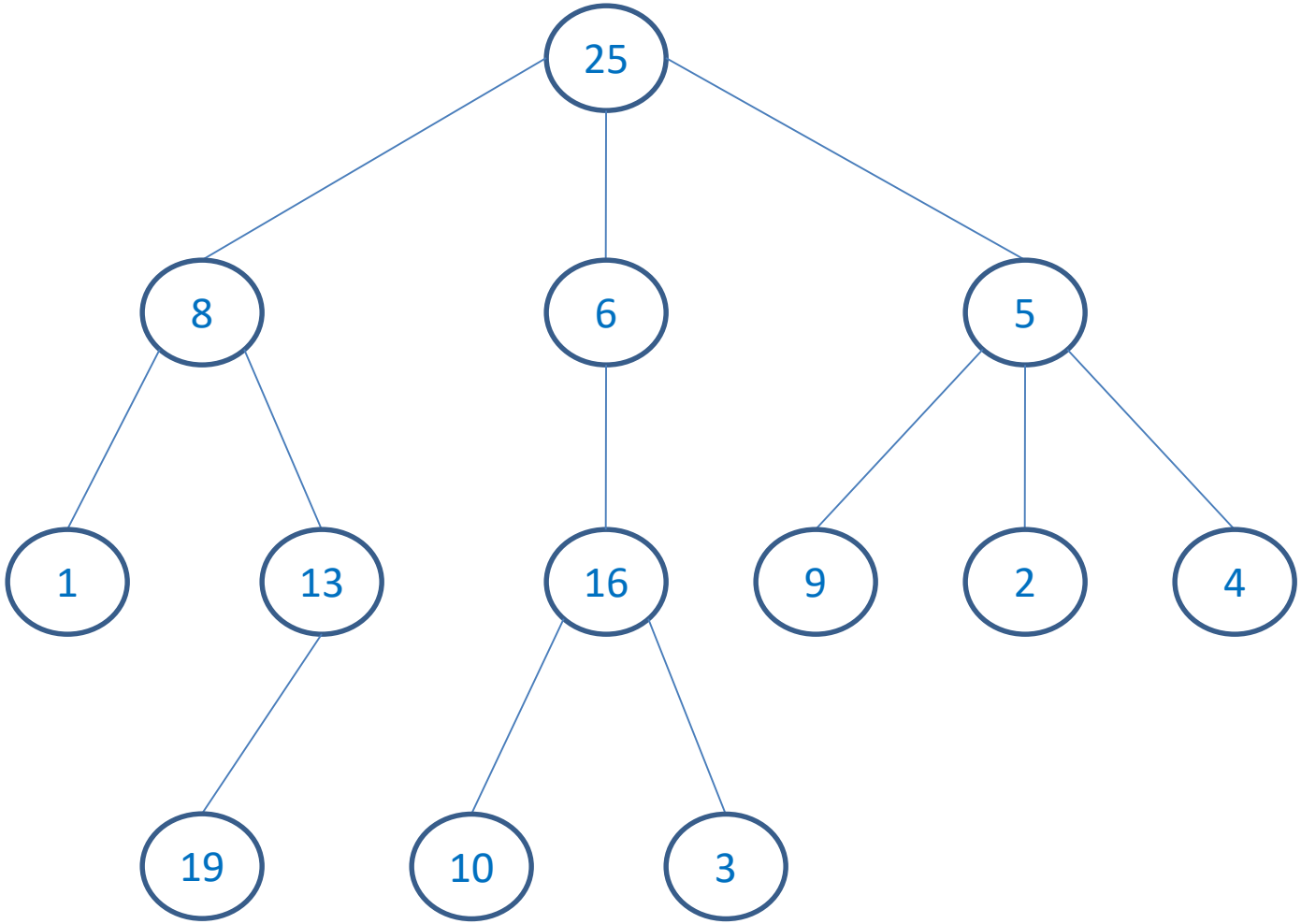
Example of a Tree



Tree

- A tree with n nodes has n-1 edges
- A tree does not have a cycle
- A tree is connected
- There exists one and only one path between any two nodes

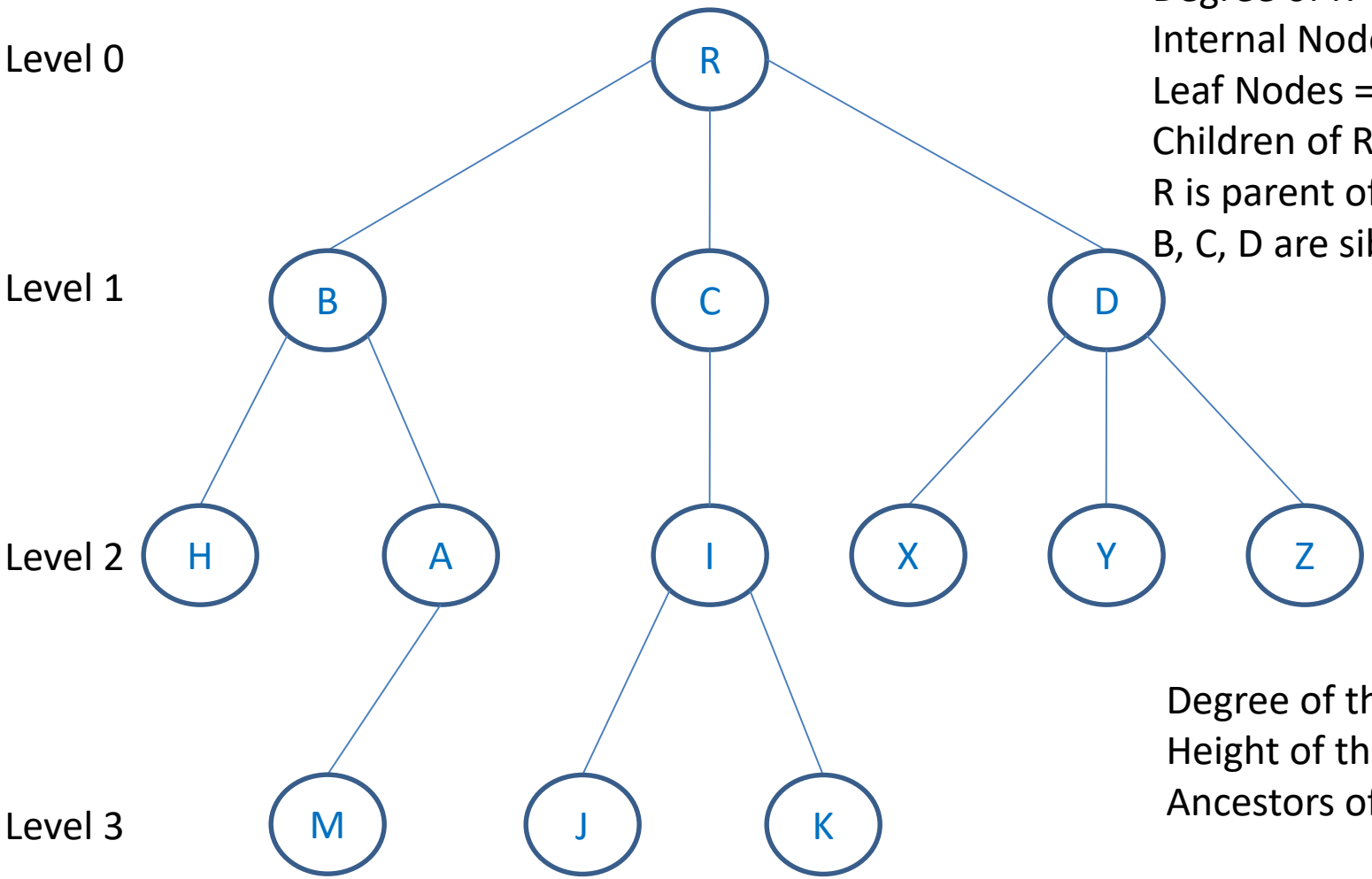
Definitions

- **Degree of a node**: No of subtrees of the node
- **Leaf Node**: Node having degree zero
- **Internal Node**: Node having degree at least one
- **Children of a node**: Roots of the subtrees of the node
- **Siblings**: Children of the same parent.
- **Degree of a tree**: Maximum degree of any node in the tree

Definitions

- **Ancestors of a node**: All the nodes along the path from the root to that node.
- **Level of root =** 0.
- **Level of a child** = 1+Level of its parent
- **Height/Depth of a tree**: Maximum level of any node in the tree

Example



Degree of R = 3
Internal Nodes = R, B,C,D,A, I
Leaf Nodes = H, M, J,K, X, Y, Z
Children of R = B, C, D
R is parent of B, C, D
B, C, D are siblings

Degree of the tree = 3
Height of the tree = 3
Ancestors of K = R, C, I

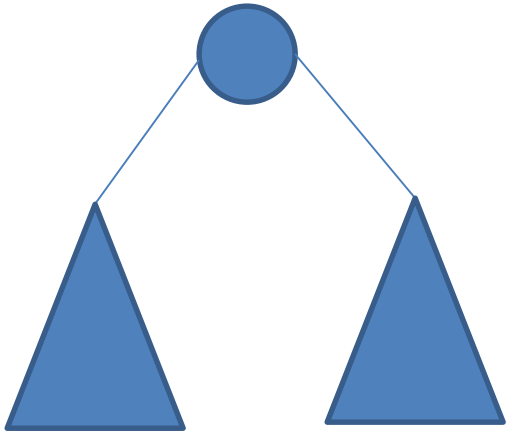
Binary Tree

Binary Tree

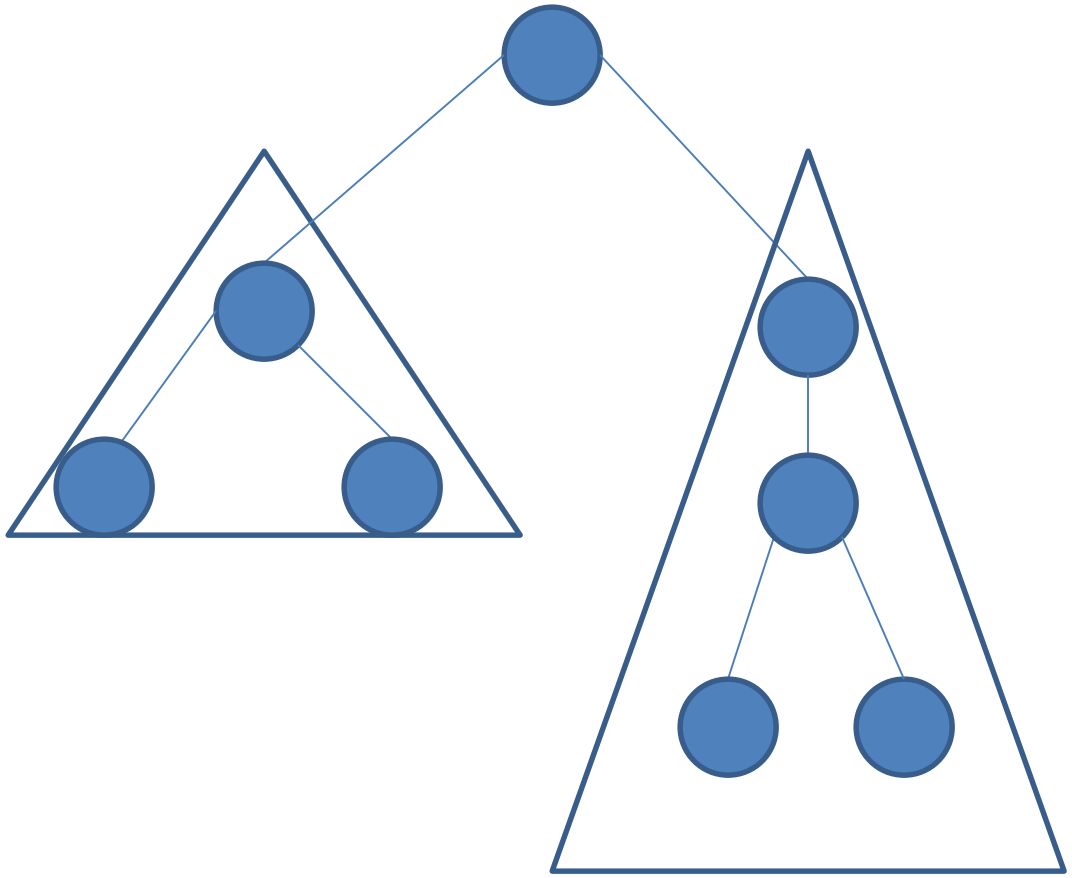
- A binary tree is a finite set of nodes that is either empty or consists of a root and two disjoint binary trees called the left subtree and the right subtree.
- Every node has at most two children.
- Maximum no of nodes at level x = 2^x

Binary Tree: Recursive Definition

- An empty node is a binary tree
- A single node, called root, is a binary tree
- Every node has at most two disjoint subtrees which are also binary trees.
- Disjointness ensures that the tree is acyclic and there is a unique path between any two nodes of the tree.



Binary Tree



Binary Tree

- Each node has either 0, 1 or 2 children.
- Maximum no of nodes in a binary tree of height h is n = $2^0 + 2^1 + 2^2 + \ldots + 2^h = 2^{h+1} - 1$
- Given a binary tree with n nodes, the minimum height of the binary tree $h_{min} = \log_2(n+1) - 1$ (Each node has 2 children)
- Given a binary tree with n nodes, the maximum height of the binary tree $h_{max} = n - 1$ (Each node has a single child)

Binary Tree

- For any non-empty binary tree, if N_0 be the no of leaf nodes, and N_2 be the no of nodes having degree 2, then $N_0 = N_2 + 1$.

Binary Tree Data Structure

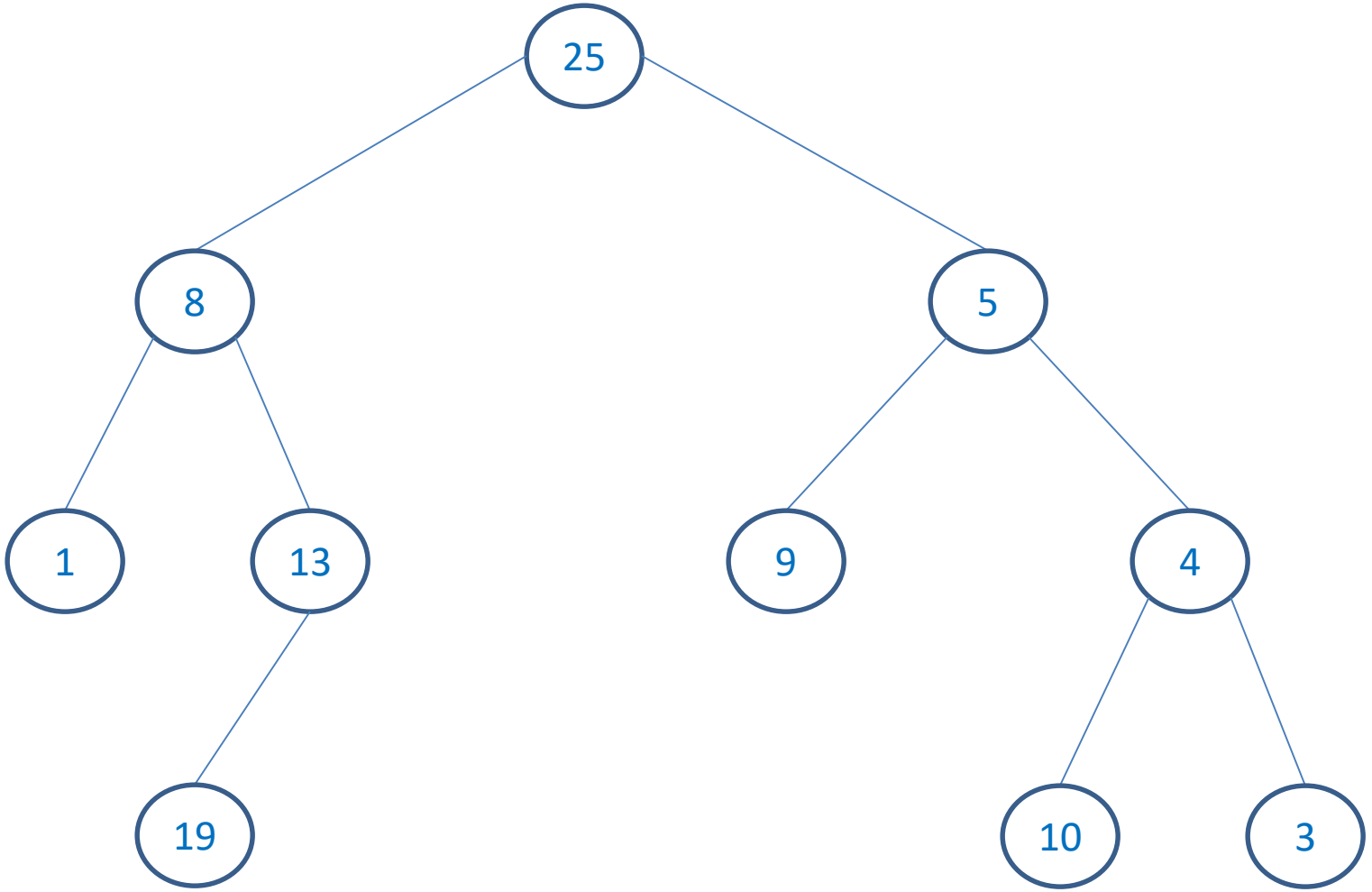
```
typedef struct node {
   int data;
   struct node *left;
   struct node *right;
}node;

void preorder(node *);
void inorder(node *);
void postorder(node *);
```

```
typedef struct node {
   int data;
   struct node *next;
}node;

void linearTraversal(node *h)
{
        while(h!= NULL) {
          printf("%d", h->data);
          h = h->next;
        }
}
```

Example



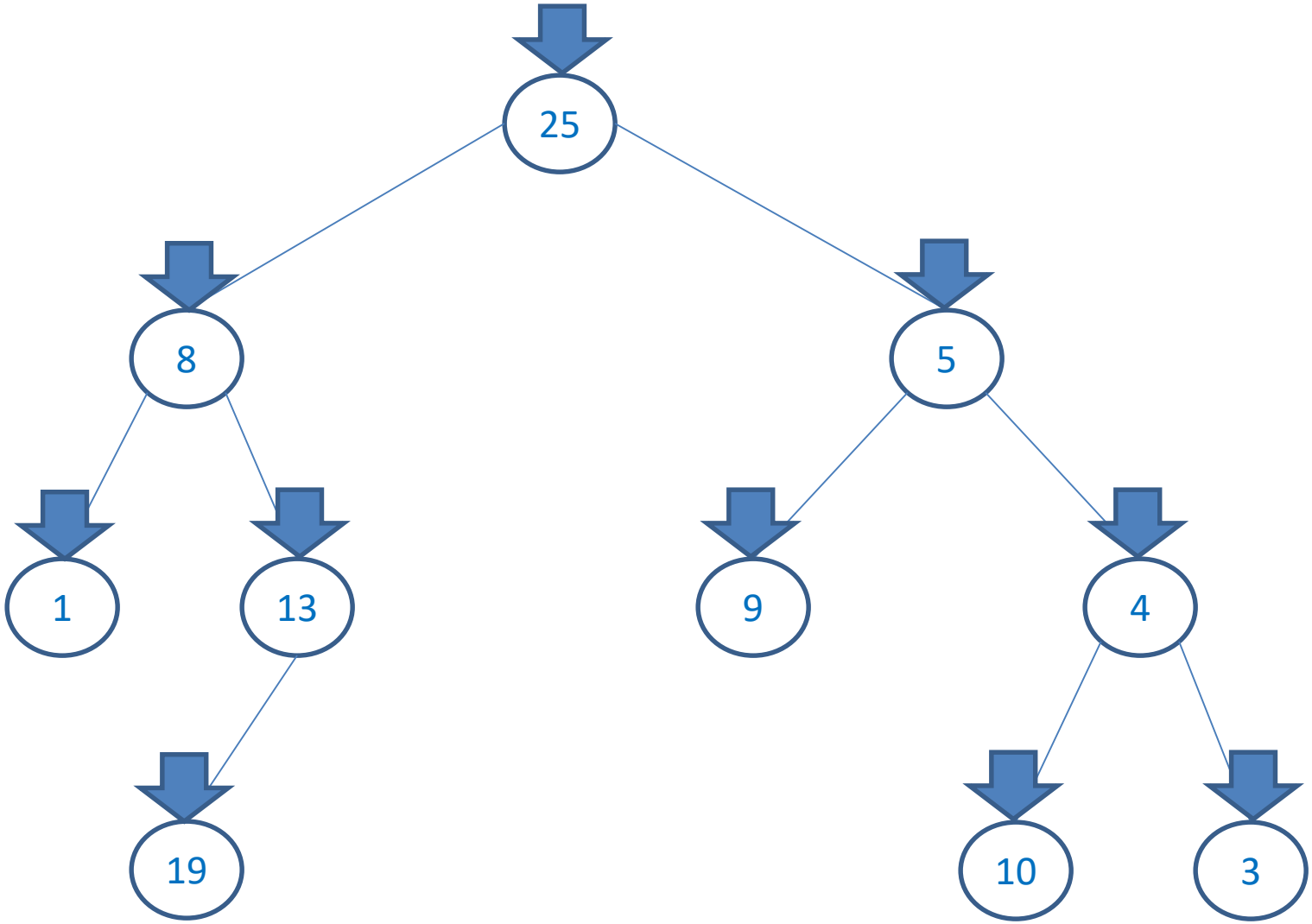
Binary Tree Traversals

- **Preorder**: Print Root -> Left Subtree -> Right Subtree
- **Inorder**: Left Subtree -> Print Root -> Right Subtree
- **Postorder**: Left Subtree -> Right Subtree-> Print Root
- In all the traversals, Left subtree is traversed before right subtree. (Left -> Right)

Preorder Traversal

```
void preorder(node *root)
{
   if(root != NULL) {
      printf("%4d", root->data);
      preorder(root->left);
      preorder(root->right);
   }
}
```

Preorder Traversal

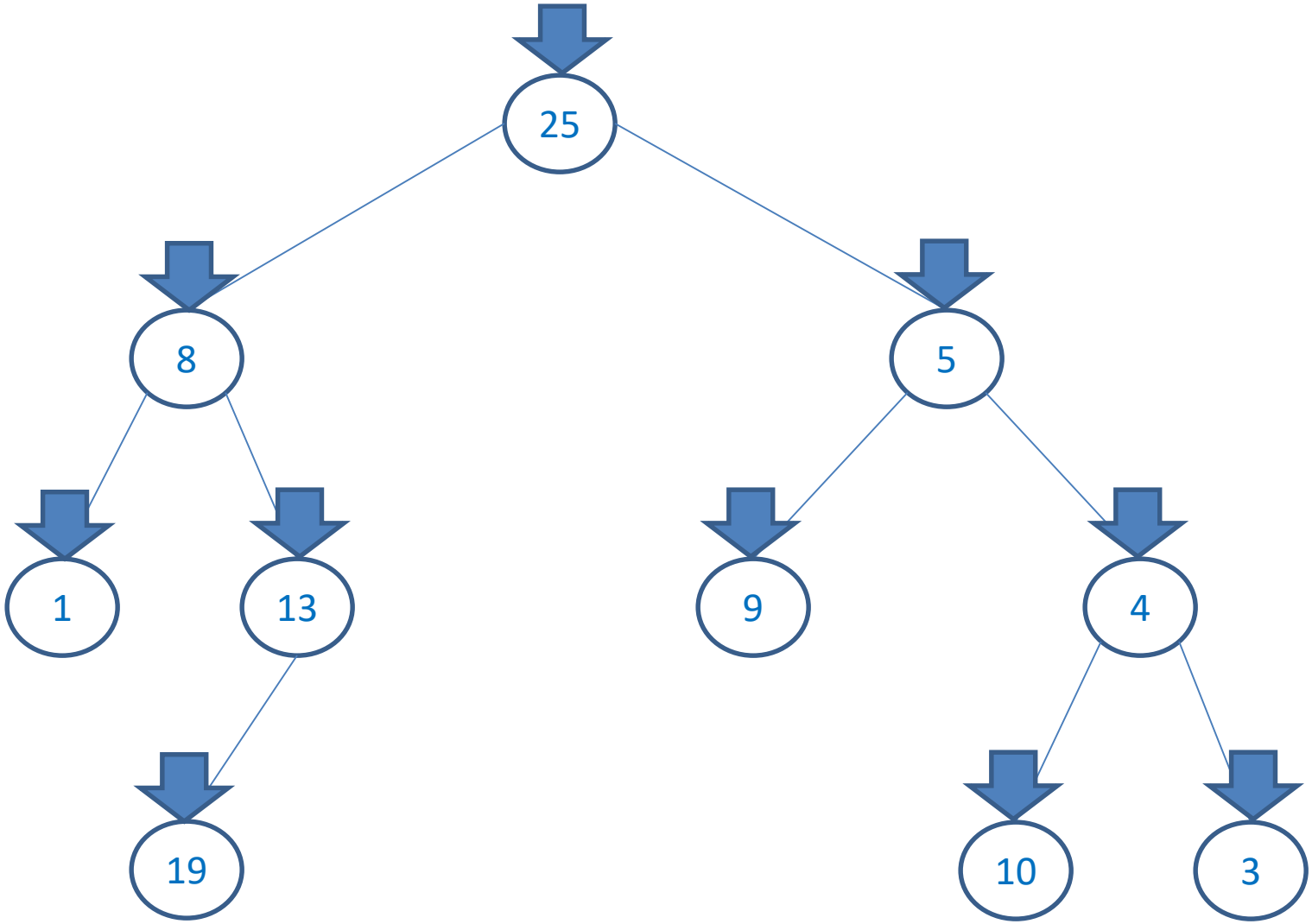


25, 8, 1, 13, 19, 5, 9, 4, 10, 3

Inorder Traversal

```
void inorder(node *root)
{
    if(root != NULL) {
        inorder(root->left);
        printf("%4d", root->data);
         inorder(root->right);
     }
}
```

Inorder Traversal

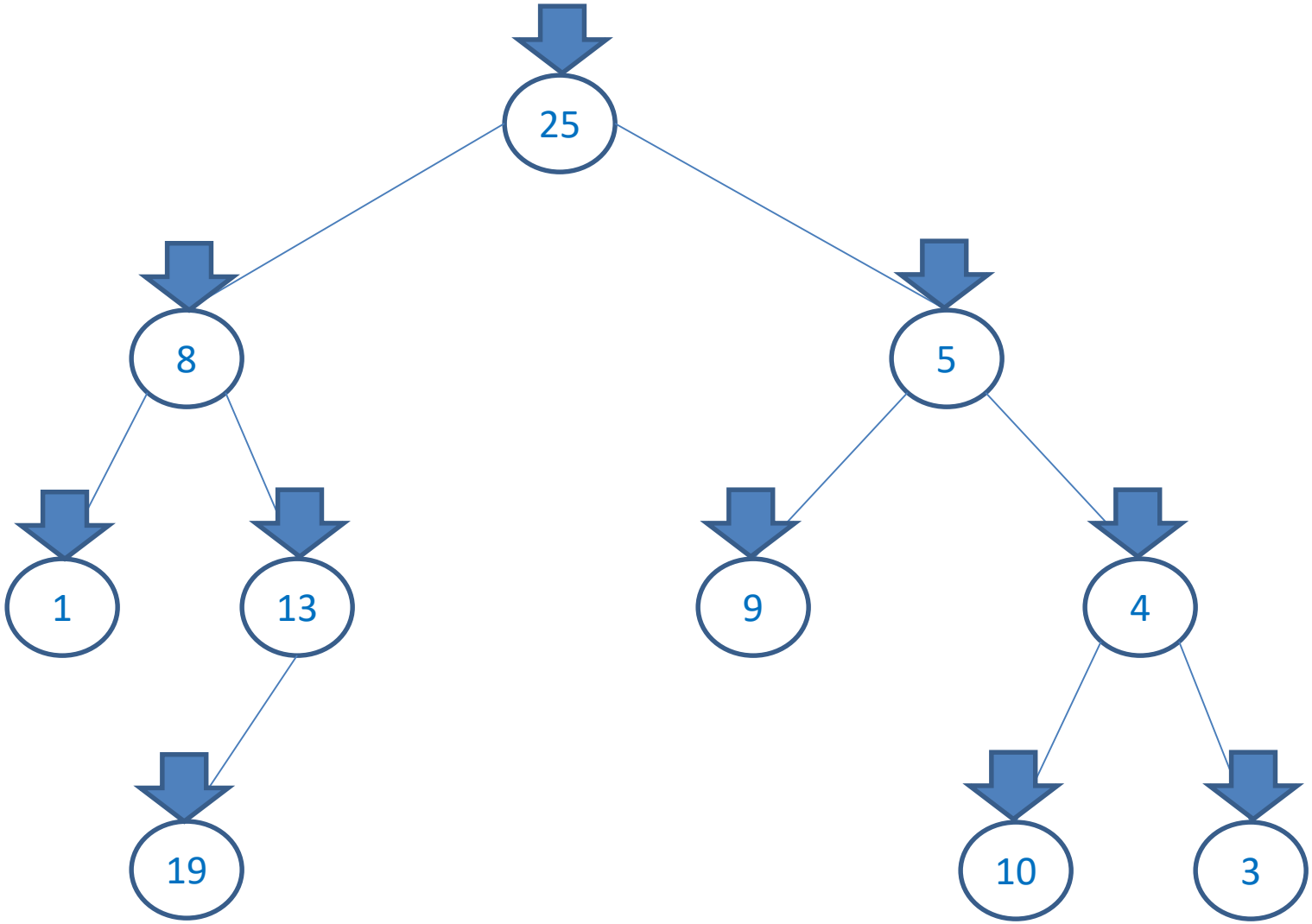


1, 8, 19, 13, 25, 9, 5, 10, 4,3

Postorder Traversal

```
void postorder(node *root)
{
    if(root != NULL) {
        postorder(root->left);
        postorder(root->right);
        printf("%4d", root->data);
    }
}
```

Postorder Traversal



1, 19, 13, 8, 9, 10, 3, 4, 5, 25

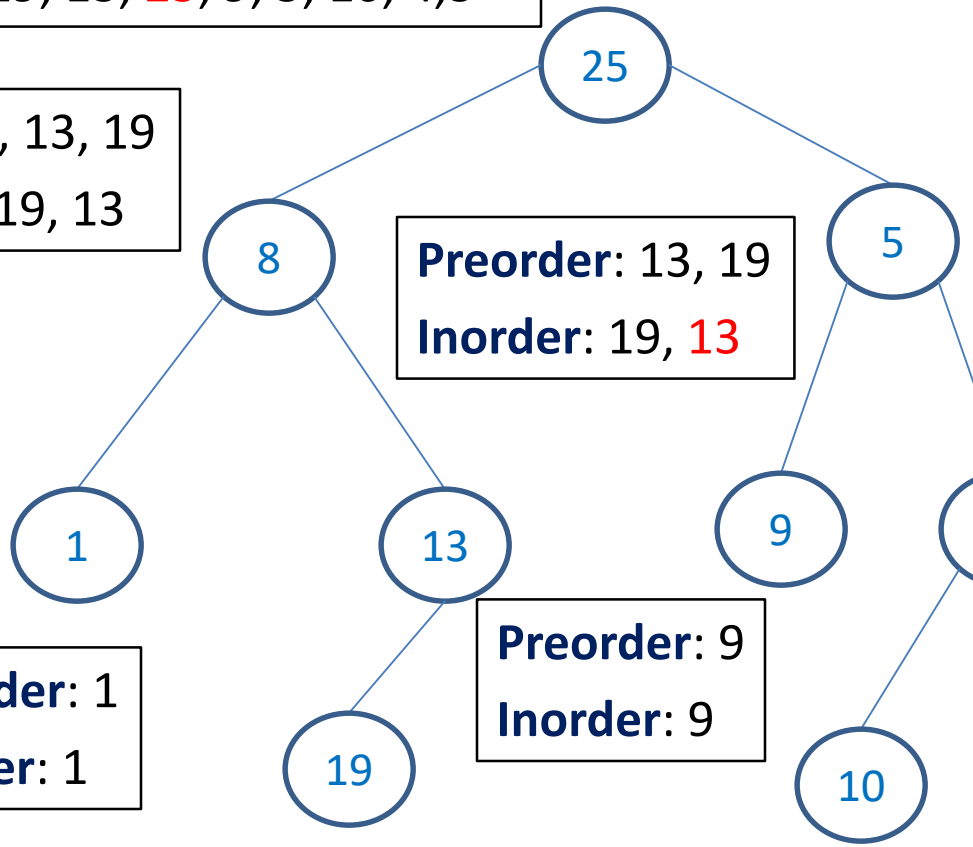
Binary Tree Construction

- Inorder + Preorder -> Unique Binary Tree
- Inorder + Postorder -> Unique Binary Tree
- Preorder + Postorder -> Unique Binary Tree construction is not possible, in general.

- Preorder: Root -> Left -> Right (Root is my first node)
- Inorder: Left -> Root-> Right (Differentiate b/w left & right subtrees)
- Postorder: Left -> Right -> Root(Root is my last node)

BT Construction from Preorder + Inorder

Preorder: 25, 8, 1, 13, 19, 5, 9, 4, 10, 3
Inorder: 1, 8, 19, 13, 25, 9, 5, 10, 4,3



Preorder: 8, 1, 13, 19
Inorder: 1, 8, 19, 13

Preorder: 5, 9, 4, 10, 3
Inorder: 9, 5, 10, 4,3

Preorder: 13, 19
Inorder: 19, 13

Preorder: 4, 10, 3
Inorder: 10, 4,3

Preorder: 1
Inorder: 1

Preorder: 9
Inorder: 9

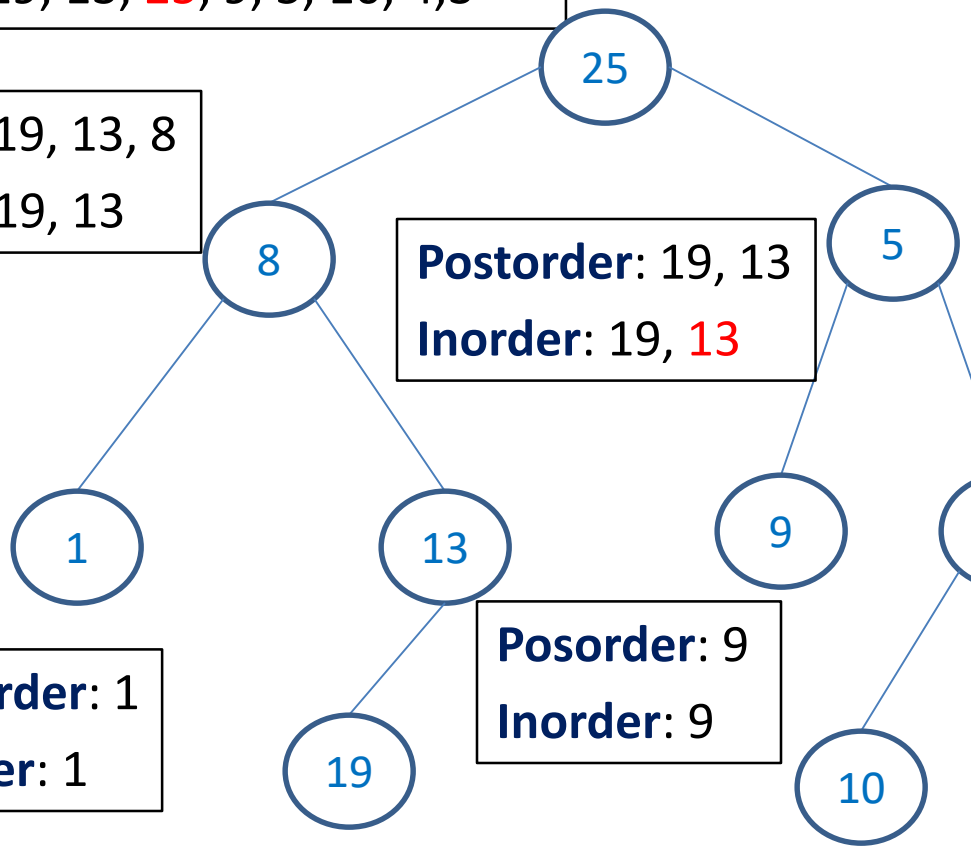
Preorder: 19
Inorder: 19

Preorder: 10
Inorder: 10

Preorder: 3
Inorder: 3

BT Construction from Postorder + Inorder

Postorder: 1, 19, 13, 8, 9, 10, 3, 4, 5, 25
Inorder: 1, 8, 19, 13, 25, 9, 5, 10, 4,3



Postorder: 1, 19, 13, 8
Inorder: 1, 8, 19, 13

Postorder: 19, 13
Inorder: 19, 13

Postorder: 9, 10, 3, 4, 5
Inorder: 9, 5, 10, 4,3

Postorder: 10, 3, 4
Inorder: 10, 4,3

Postorder: 1
Inorder: 1

Posorder: 9
Inorder: 9

Posorder: 19
Inorder: 19

Posorder: 10
Inorder: 10

Posorder: 3
Inorder: 3

Binary Search Tree

Binary Search Tree

- A Binary Search Tree (BST) is a binary tree. It may be empty. If it is not empty, then it satisfies the following properties:

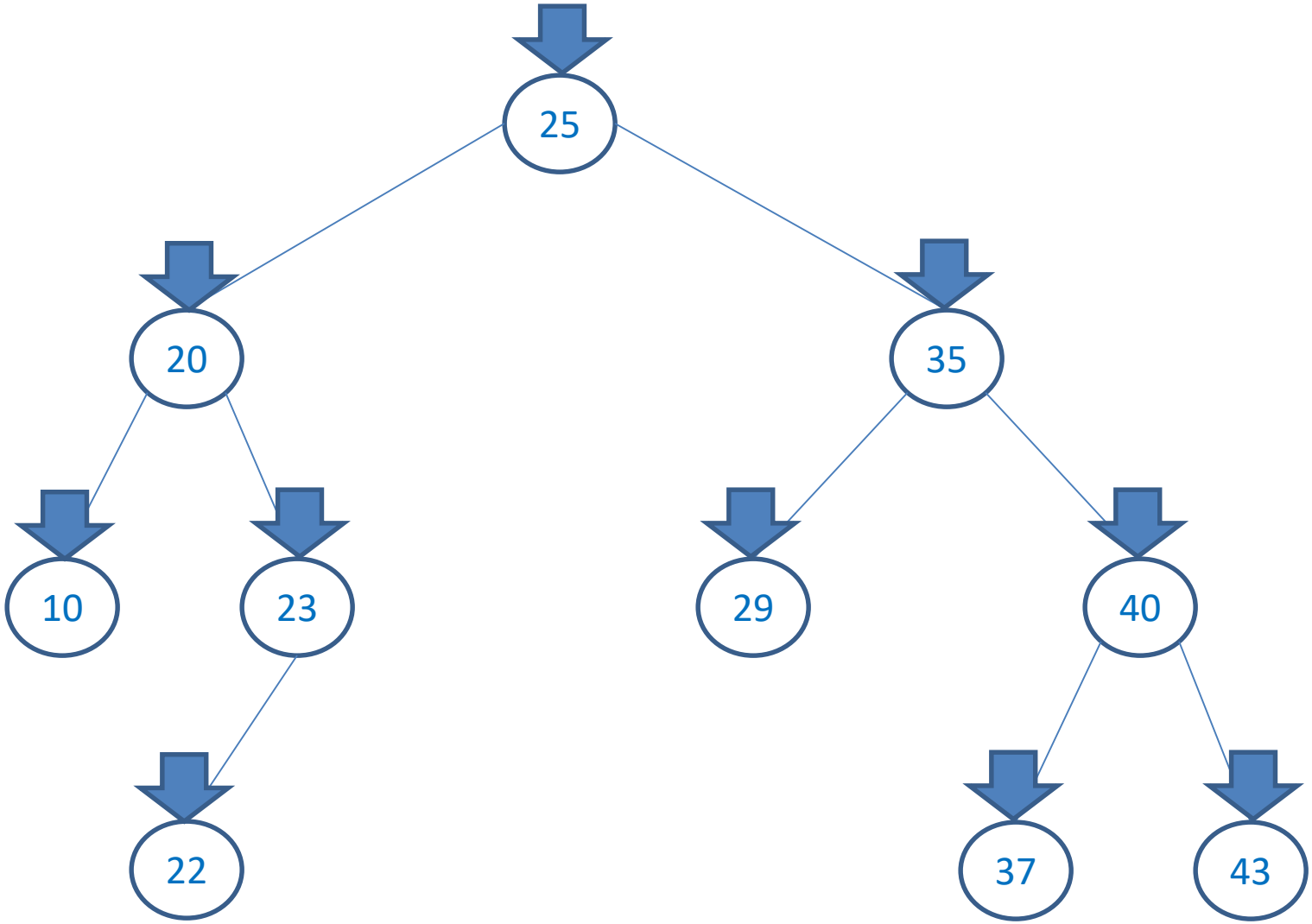
i. The root has a key.

ii. The keys (if any) in the left subtree are smaller than the key in the root.

iii. The keys (if any) in the right subtree are larger than the key in the root.

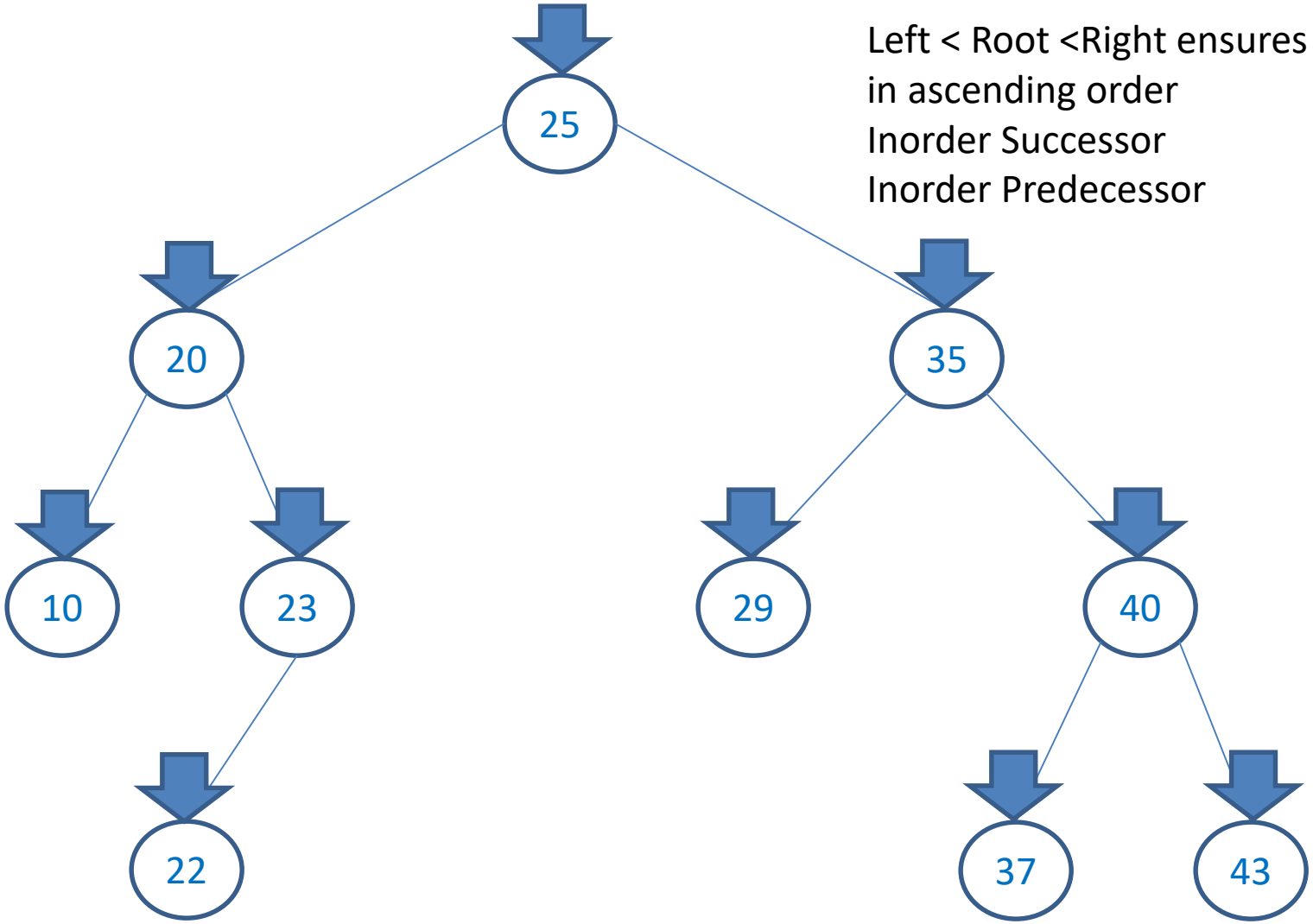
iv. The left and right subtrees are also BSTs.

Preorder Traversal



25, 20, 10, 23, 22, 35, 29, 40, 37, 43

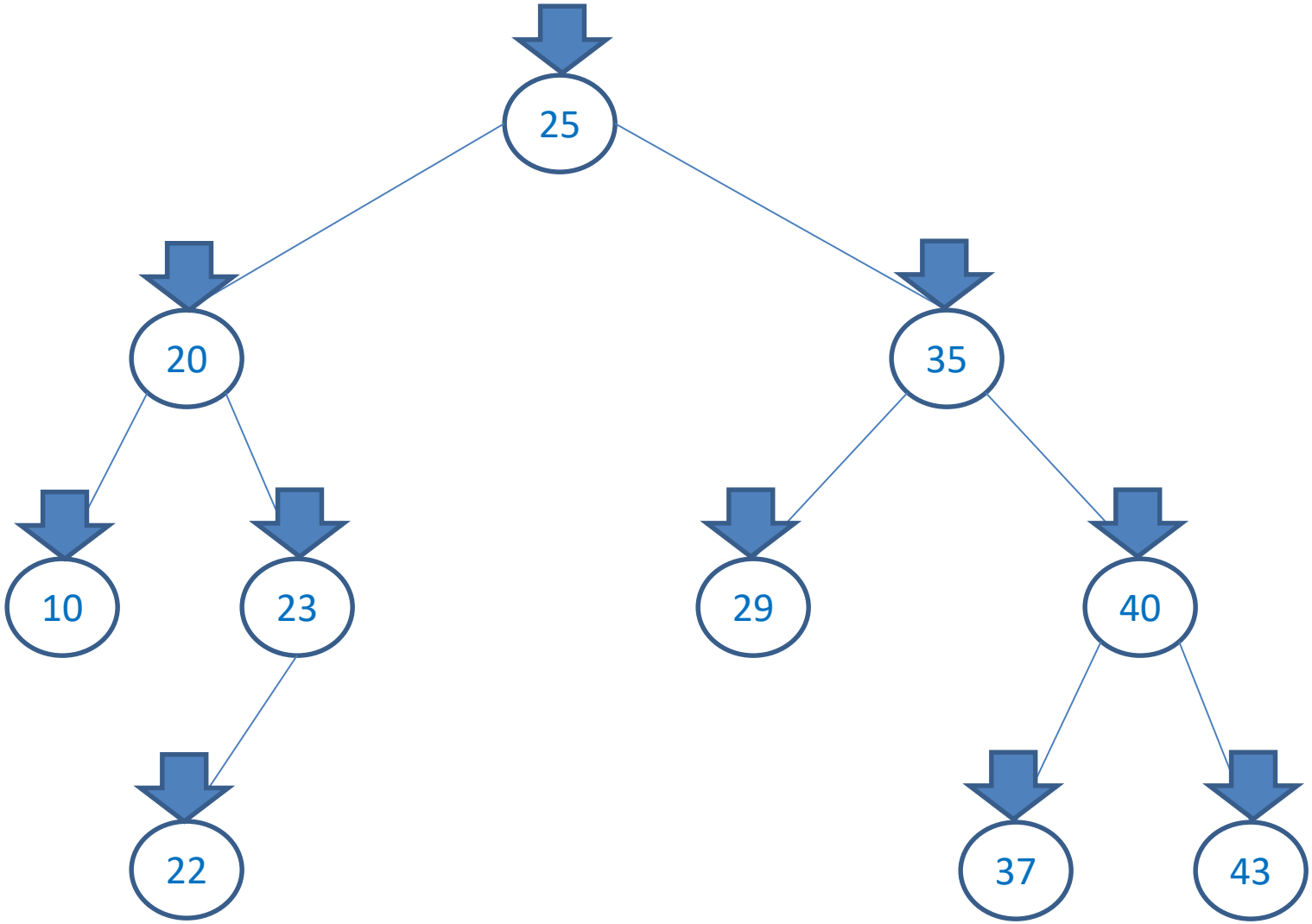
Inorder Traversal



Left < Root <Right ensures sorting of data in ascending order
Inorder Successor
Inorder Predecessor

10, 20, 22, 23, 25, 29, 35, 37, 40, 43

Postorder Traversal

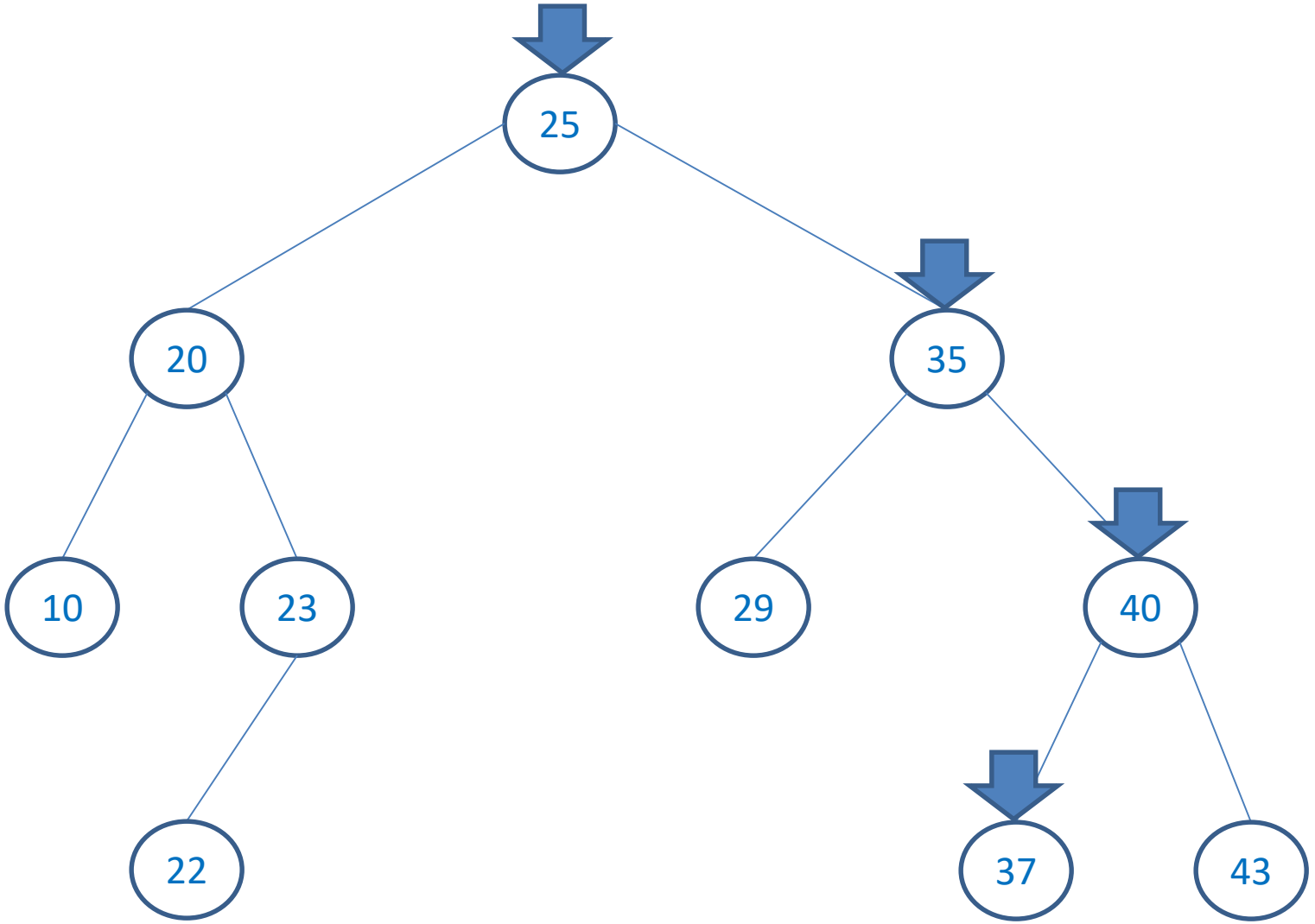


10, 22, 23, 20, 29, 37, 43, 40, 35, 25

Search in BST

1. If the BST is empty, then search is unsuccessful
2. If search_key is in the root node, then return root
3. If search_key < Key in the root node, then search_key is searched in root->left
4. If search_key > Key in the root node, then search_key is searched in root->right

Search (root, 37)



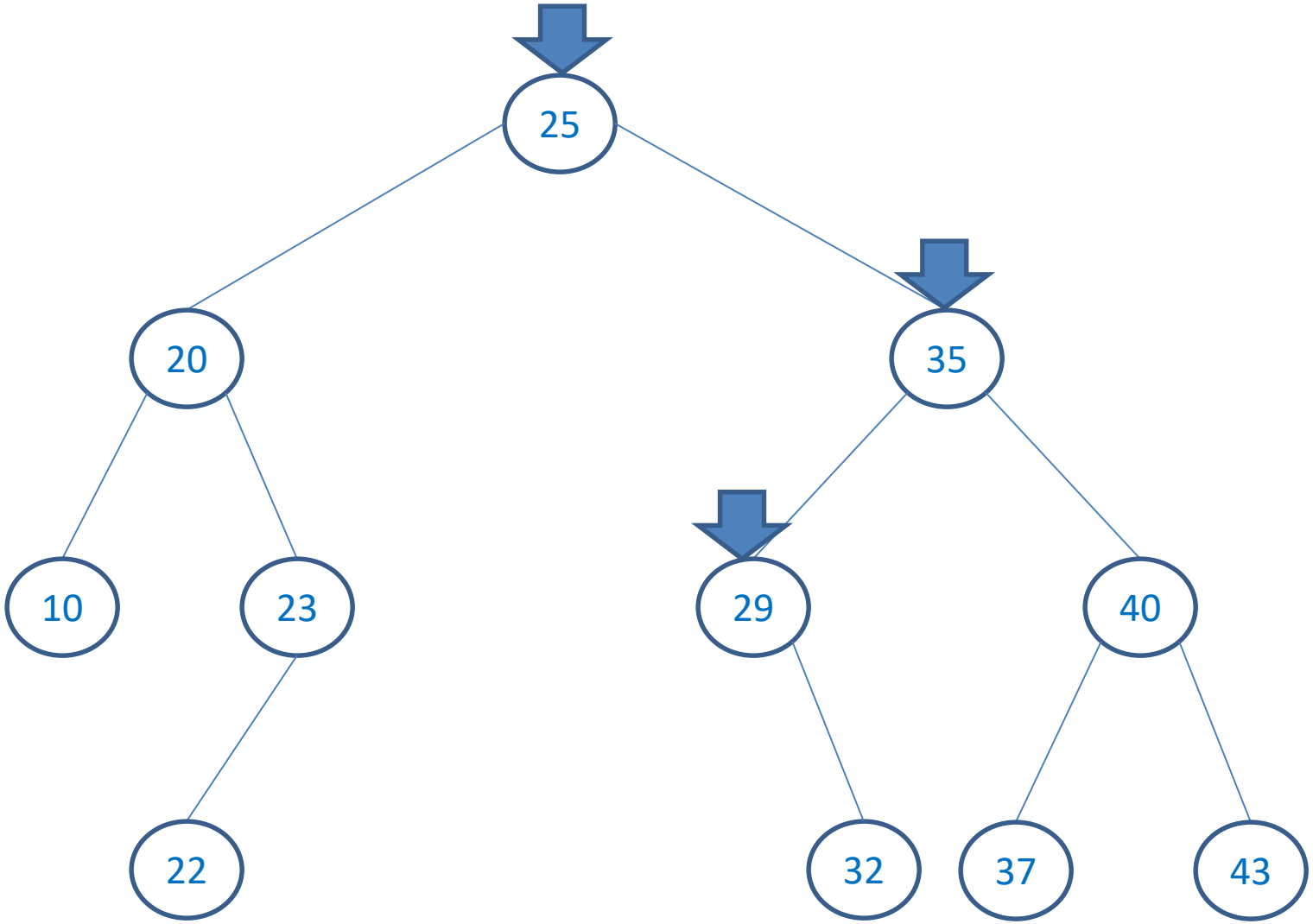
Search in BST

```
node *search(node *root, int data)
{
   if(root == NULL || root->data == data)
                return root;
   else if(data < root->data)
                return search(root->left, data);
   else
                return search(root->right, data);
}
```

Insertion in BST

1. If the BST is empty, then create a node and it will be the root
2. If insert_key < Key in the root node, then insert_key will be inserted in root->left
3. If insert_key > Key in the root node, then insert_key will be inserted in root->right

Insert(root, 32)



Insertion in BST

```
node *createNode(int
    data)
{
    node *n = (node
    *)malloc(sizeof(node));
    n->data = data;
    n->left = NULL;
    n->right = NULL;
    return n;
}
```

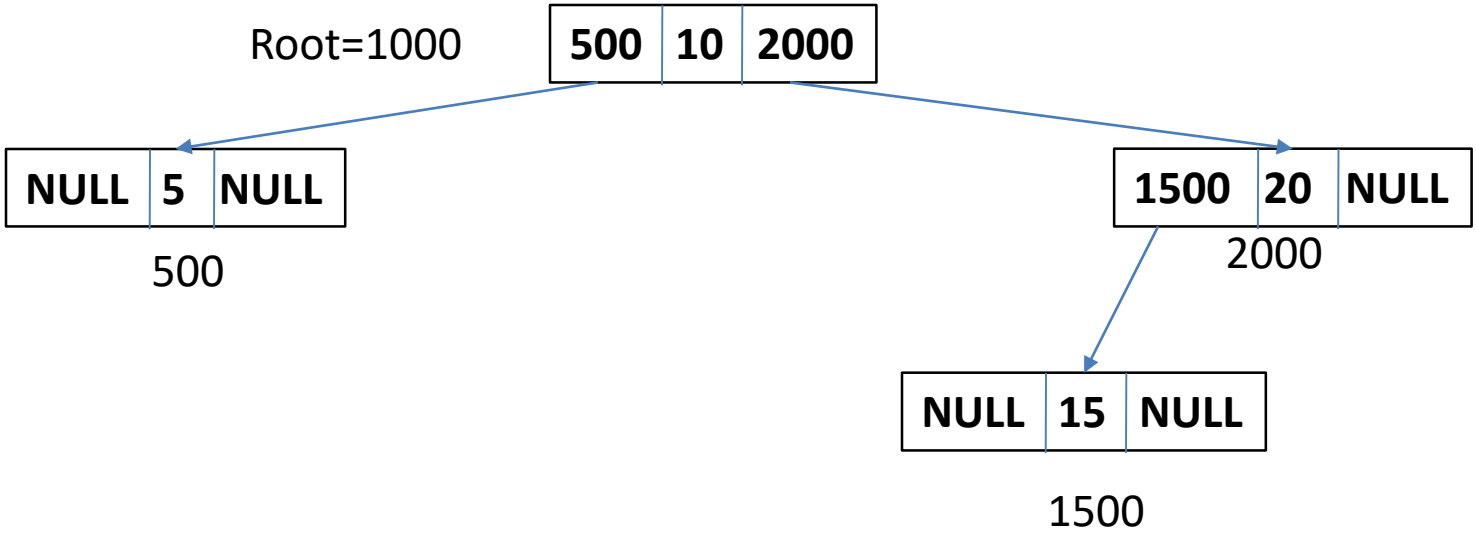
Insertion in BST

```
node *insert(node *root, int data)
{
    if(root == NULL)   root = createNode(data);
    else if(data < root->data) root->left = insert(root-> left, data);
    else if(data > root->data) root->right = insert(root->right, data);
    return root;
}
```

From main():

```
node *root = NULL;
for(i = 0; i < n; i++)
    root = insert(root, a[i]);
```

Insertion at BST: 10, 20, 5, 15



Deletion in BST

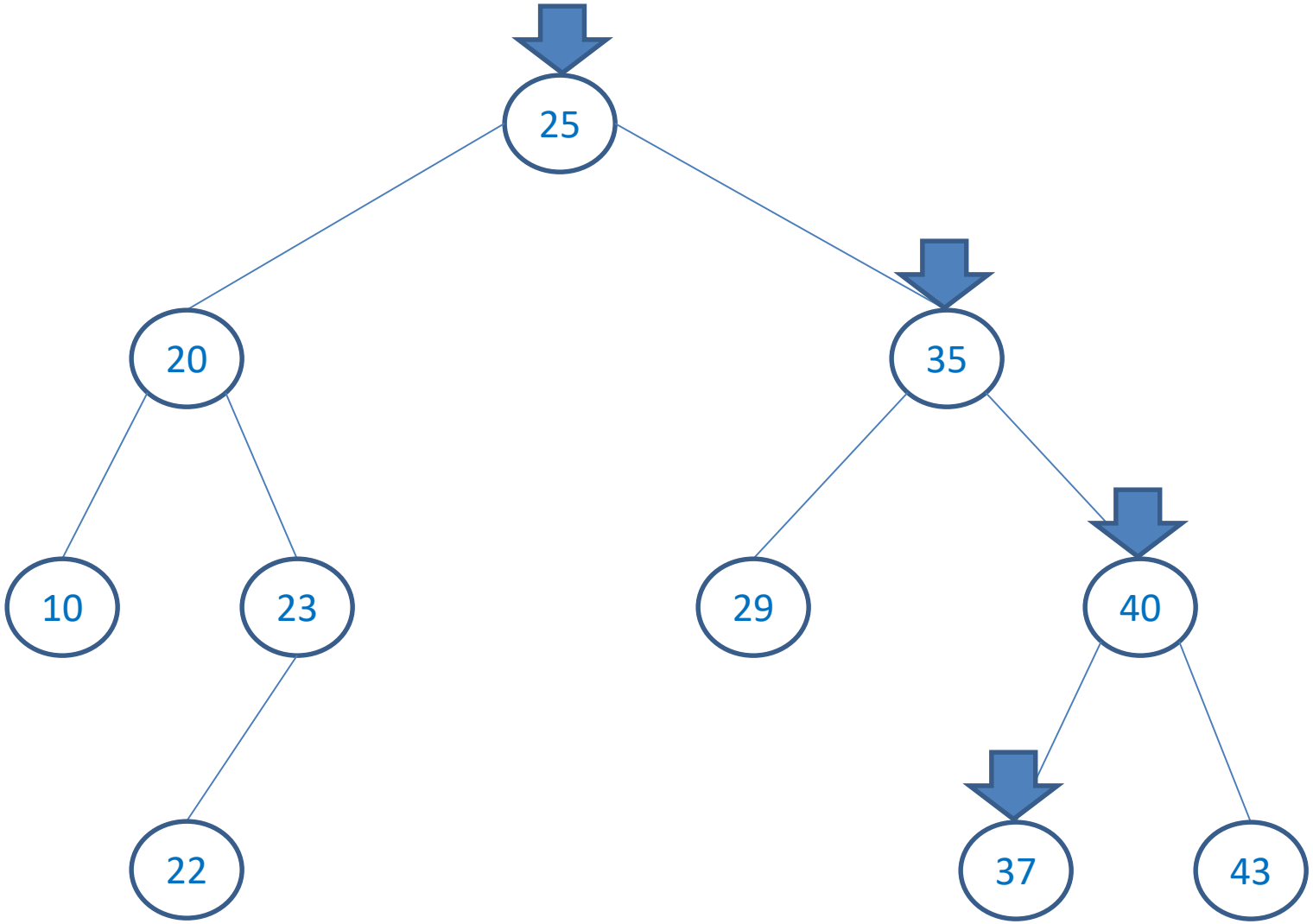
```
node *delete(node *root, int data)
{
   if(root == NULL)      return NULL;
   else if(data < root->data)  root->left = delete(root->left, data);
   else if(data > root->data) root->right = delete(root->right, data);
   else { // data == root->data
        Actual Code for Deletion
        Case 1: The node to be deleted is a leaf node
        Case 2: The node to be deleted has a single child
        Case 3: The node to be deleted has two children
    }
    return root;
}
```

Deletion in BST

Case 1: Node is a leaf node:

1. Temp = leaf node
2. If temp is the left child of its parent, then set the left child of the parent NULL
3. If temp is the right child of its parent, then set the right child of the parent NULL
4. Delete temp

Delete(root, 37)

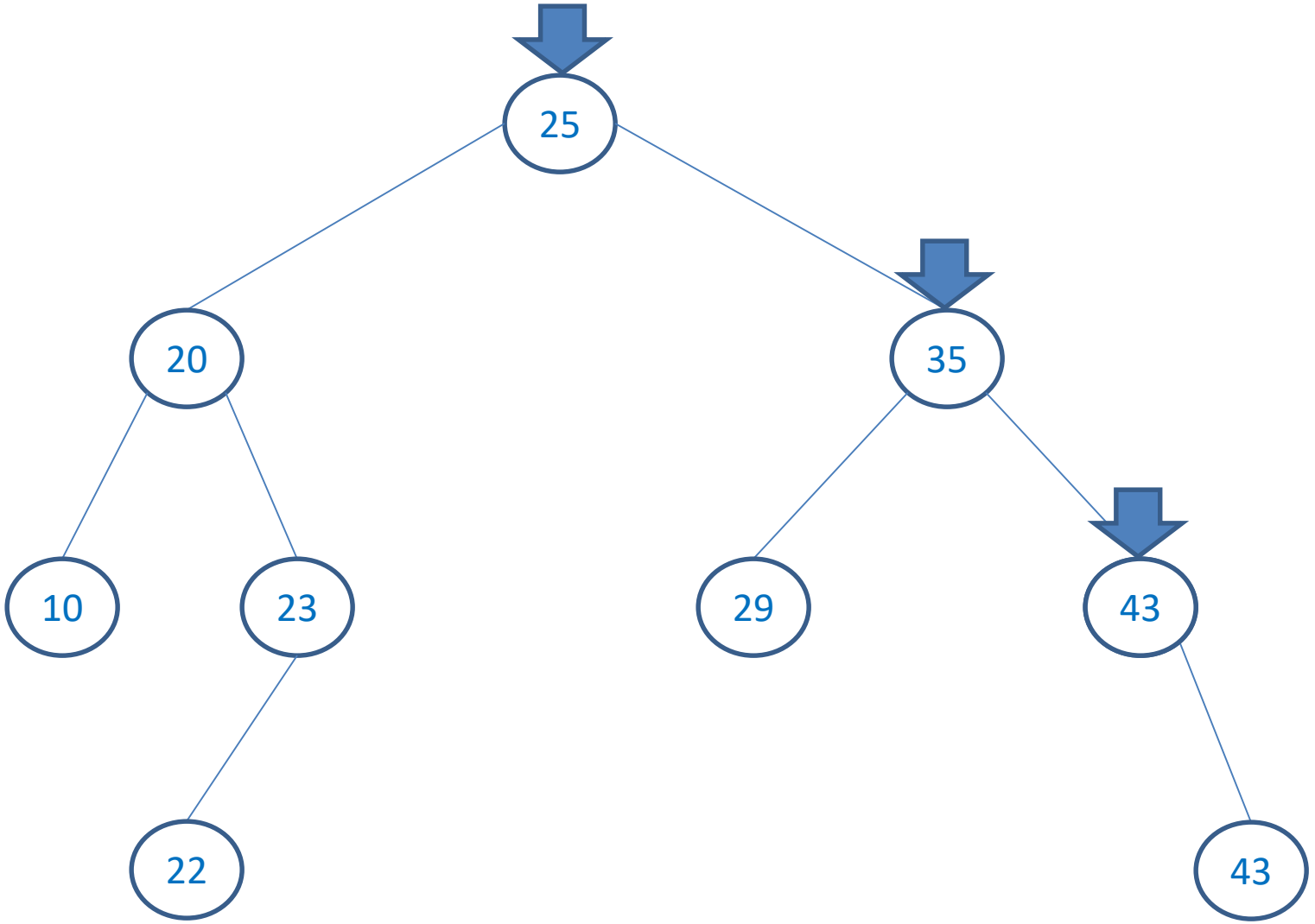


Deletion in BST

Case 2: Node has a left/right child C:

1. Temp = node to be deleted
2. If temp is the left child of its parent, then set the left child of the parent to C
3. If temp is the right child of its parent, then set the right child of the parent to C
4. Delete temp

Delete(root, 40)

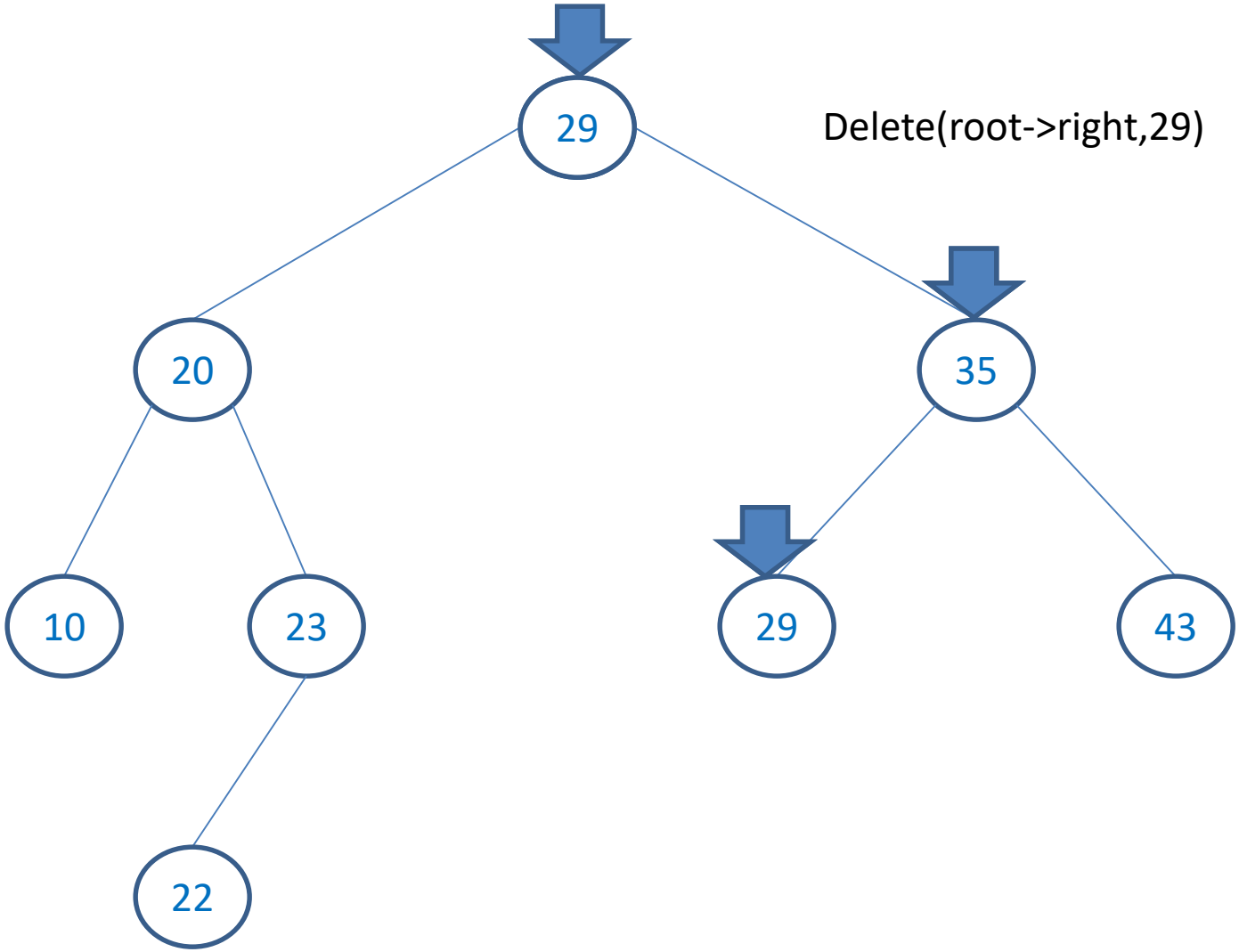


Deletion in BST

Case 3: Node have a left child L and a right child R:

1. Temp = node to be deleted
2. Find the minimum key in temp->right subtree
3. Copy that minimum key to temp->key
4. Delete the node with the minimum key in its right subtree

Delete(root, 25)

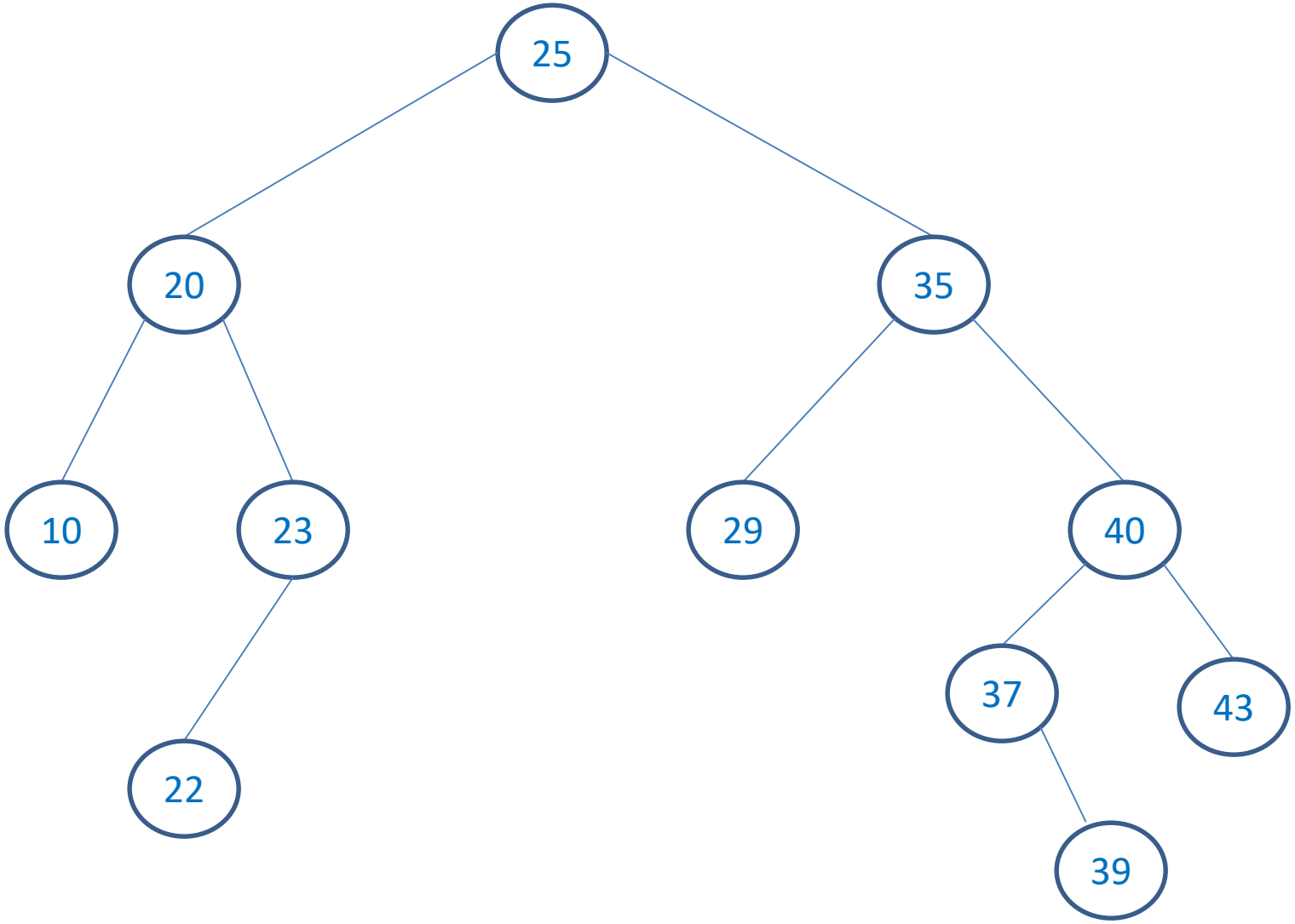


Delete(root->right,29)

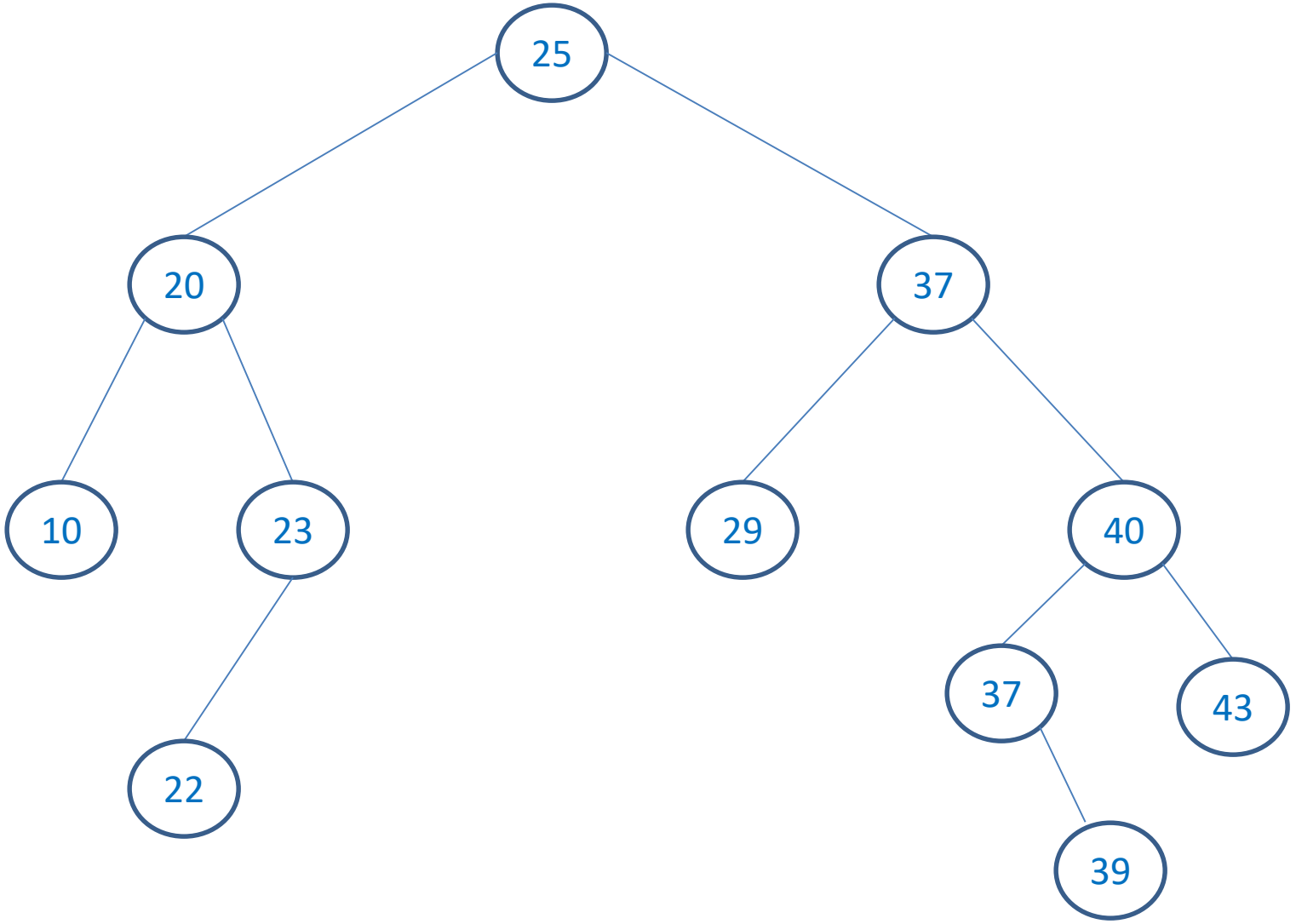
Deletion in BST

```
node *temp = NULL;
if(root->left != NULL && root->right != NULL) {
        temp = root->right;
        while(temp->left != NULL)
                        temp = temp->left;
        root->data = temp->data;
        root->right = delete(root->right, temp->data);
        return root;
}
temp = (root->left != NULL) ? root->left : root->right;
free(root);
root = NULL;
return temp;
```

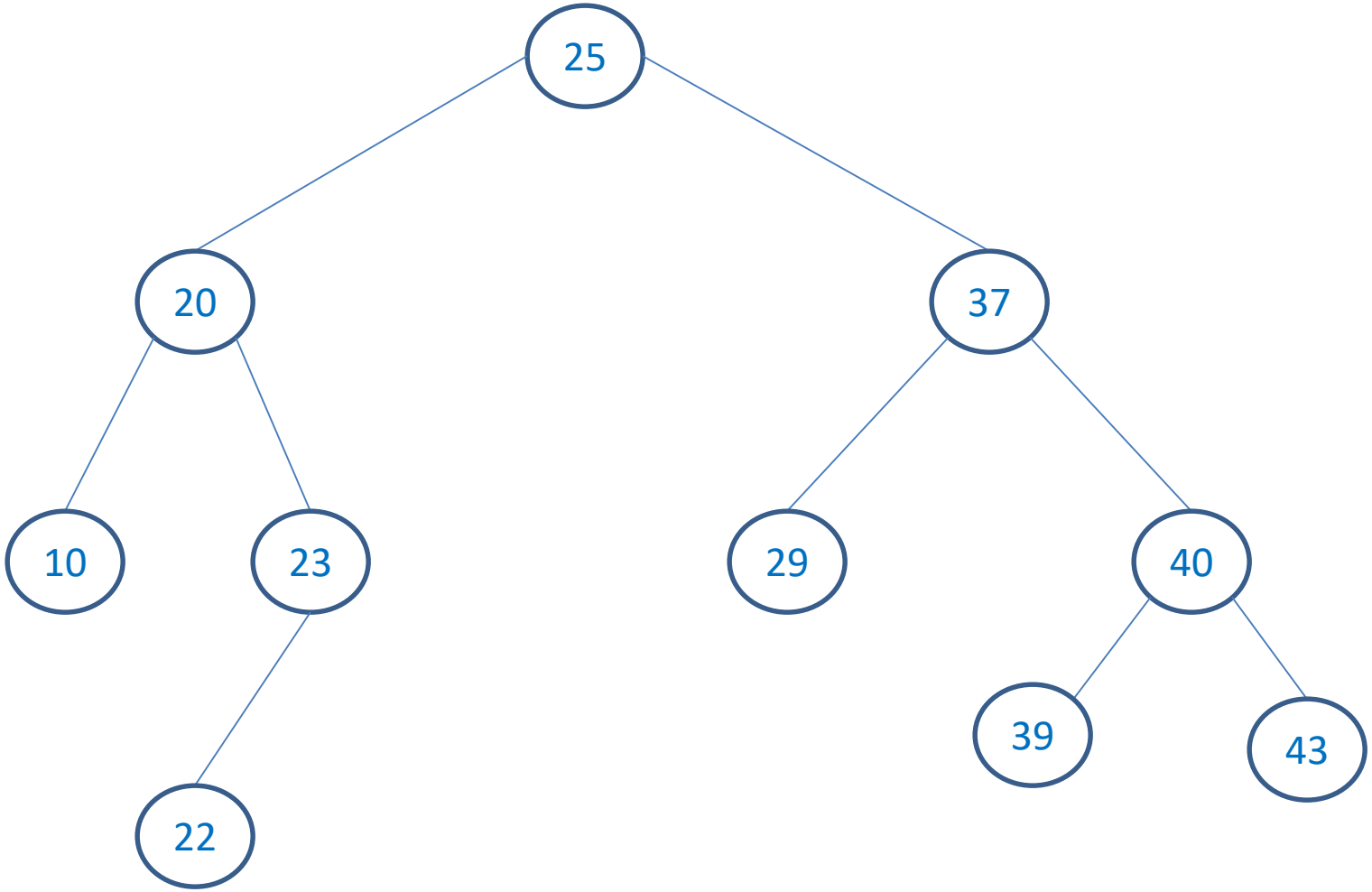
Delete(root, 35)



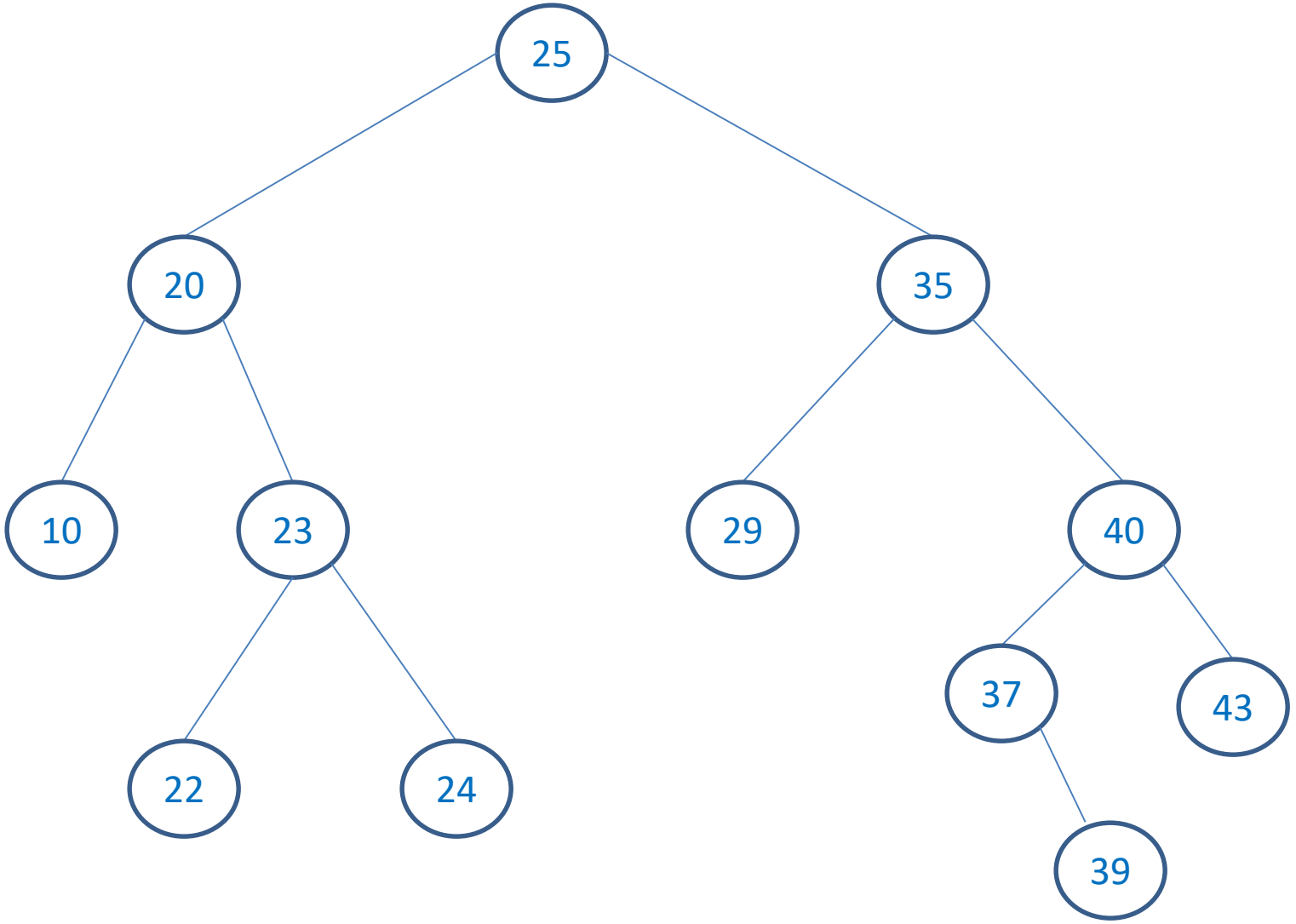
Delete(root->right, 37)



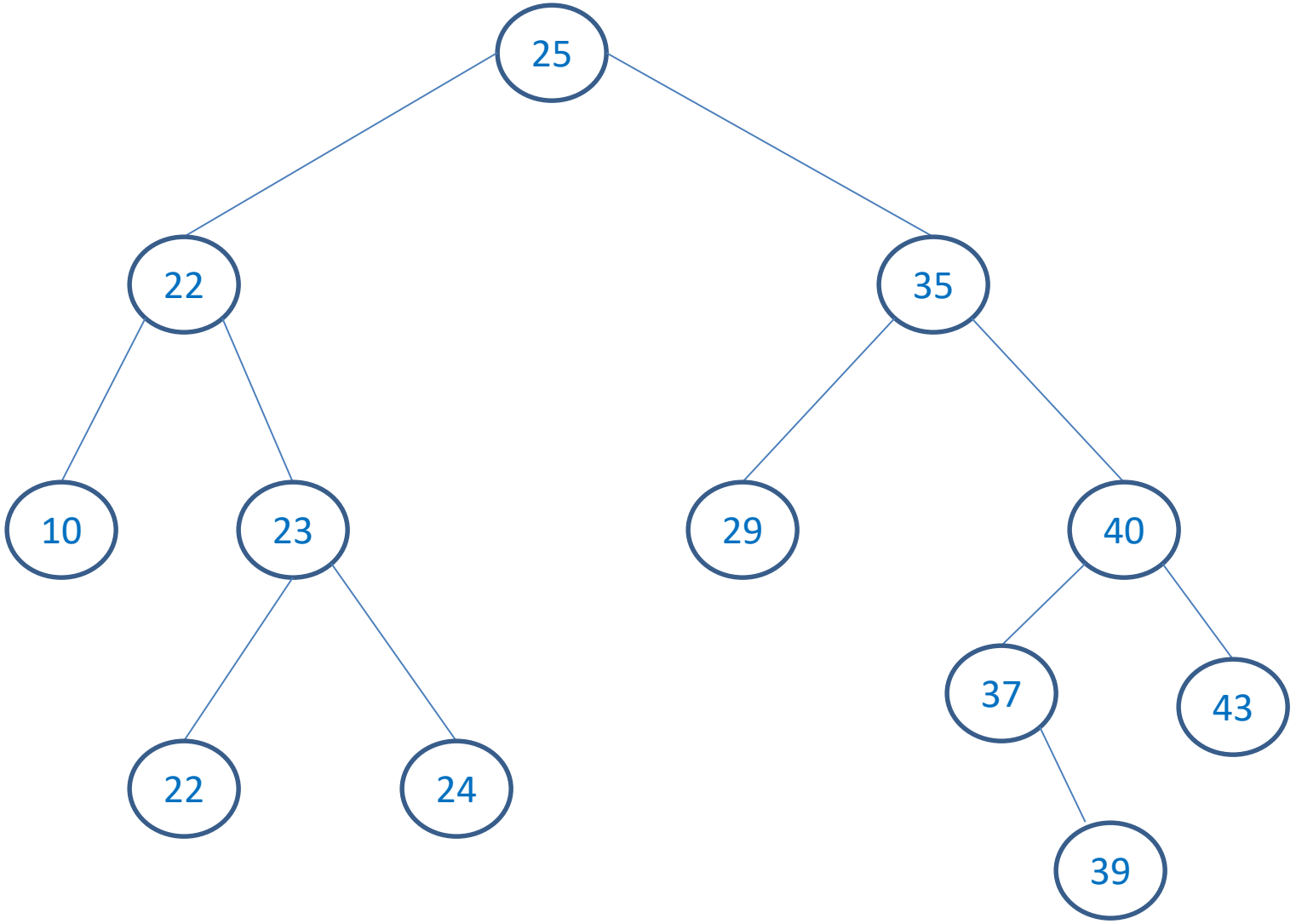
Delete(root->right, 37)



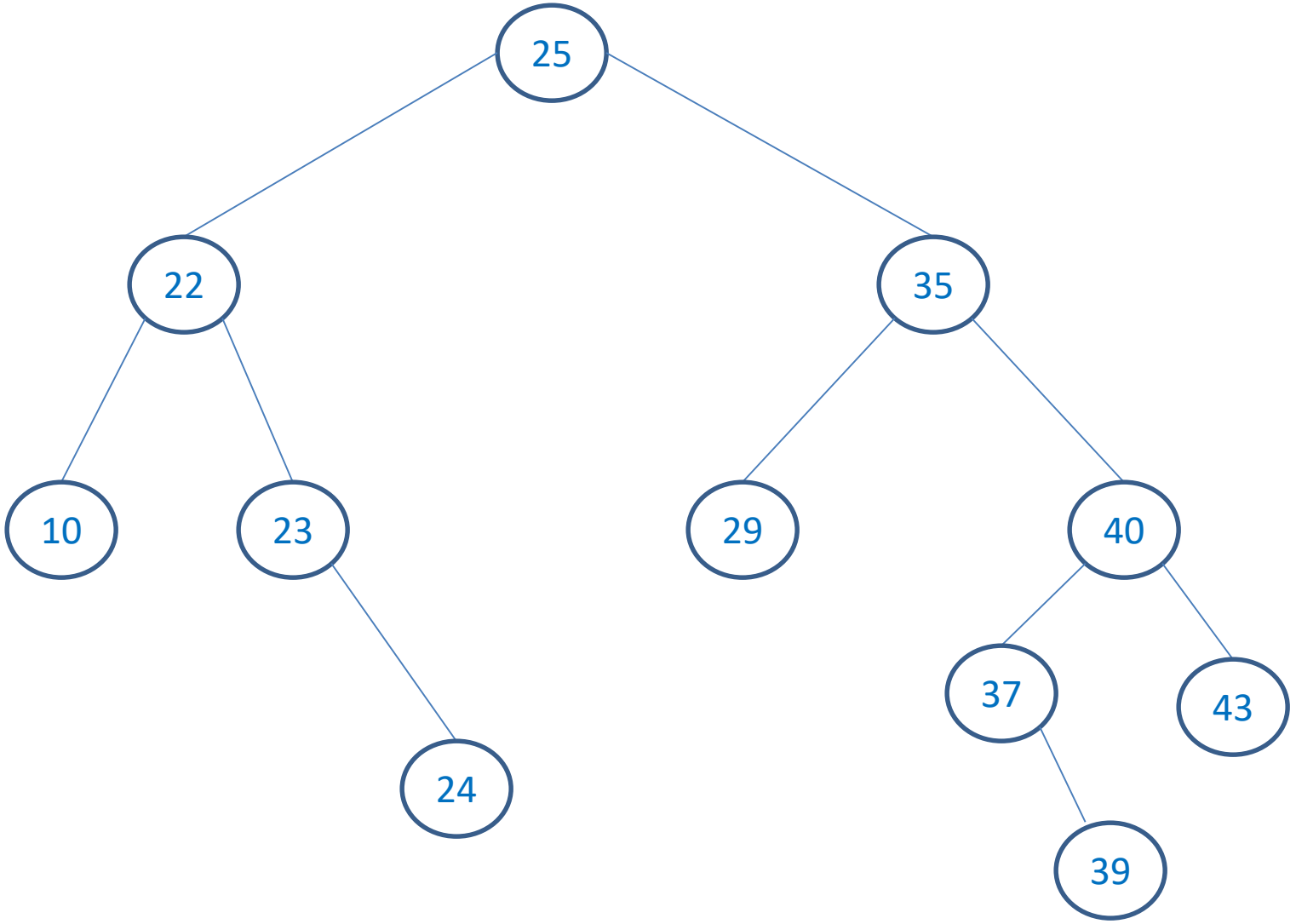
Delete(root, 20)



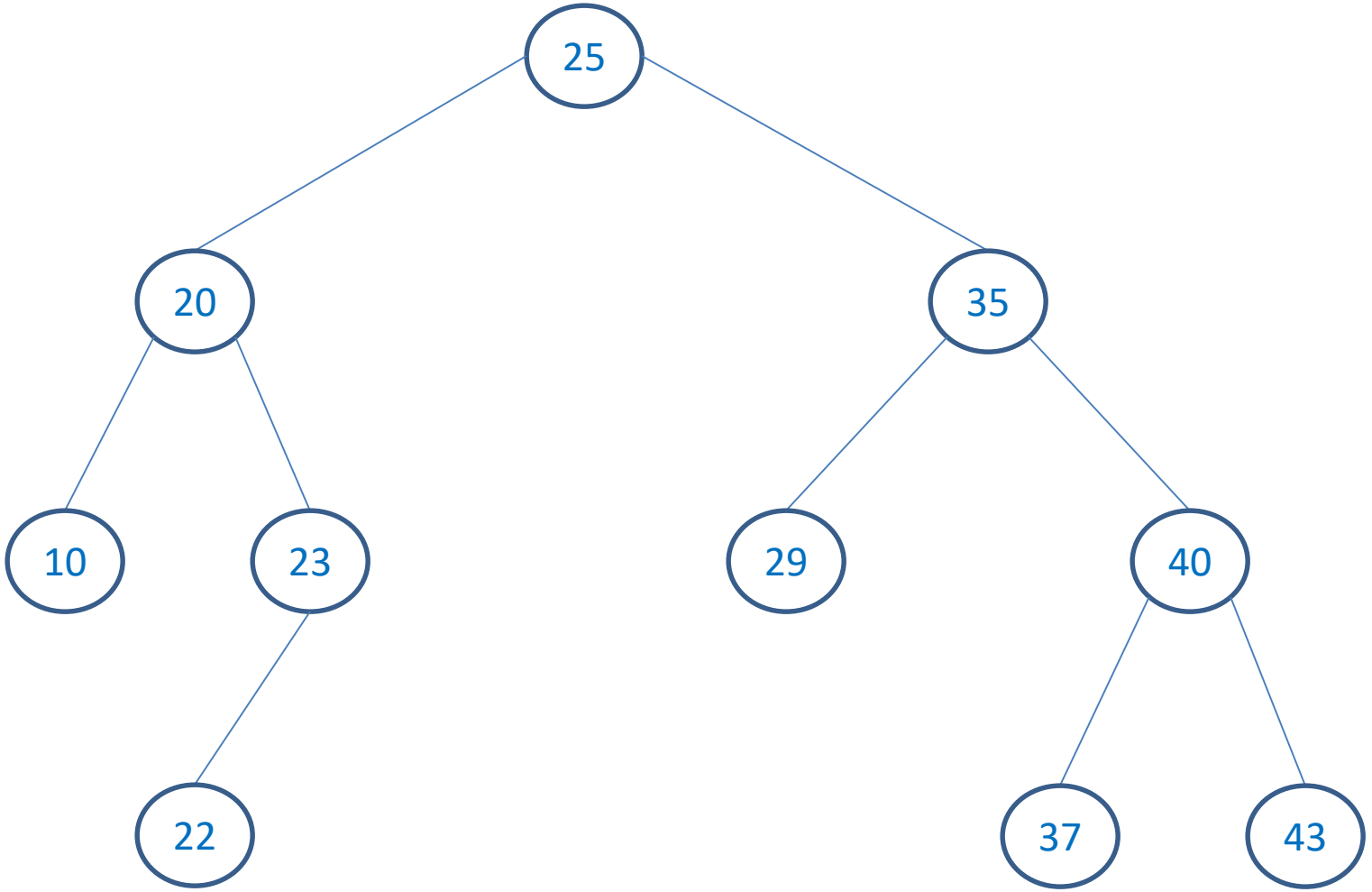
Delete(root->right, 22)



Delete(root->right, 22)

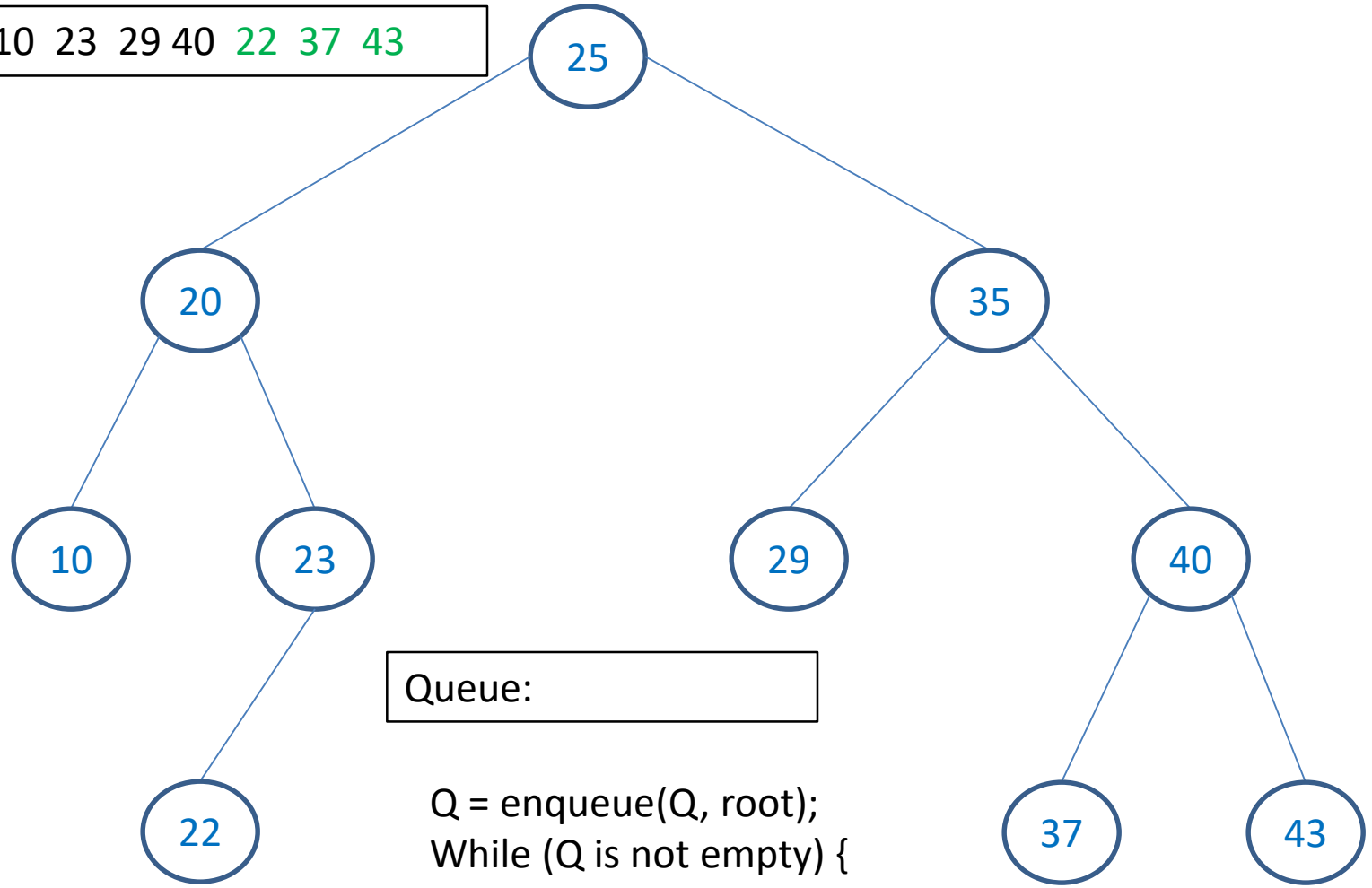


Application of Queue: Levelorder Traversal



25, 20, 35, 10, 23, 29, 40, 22, 37, 43

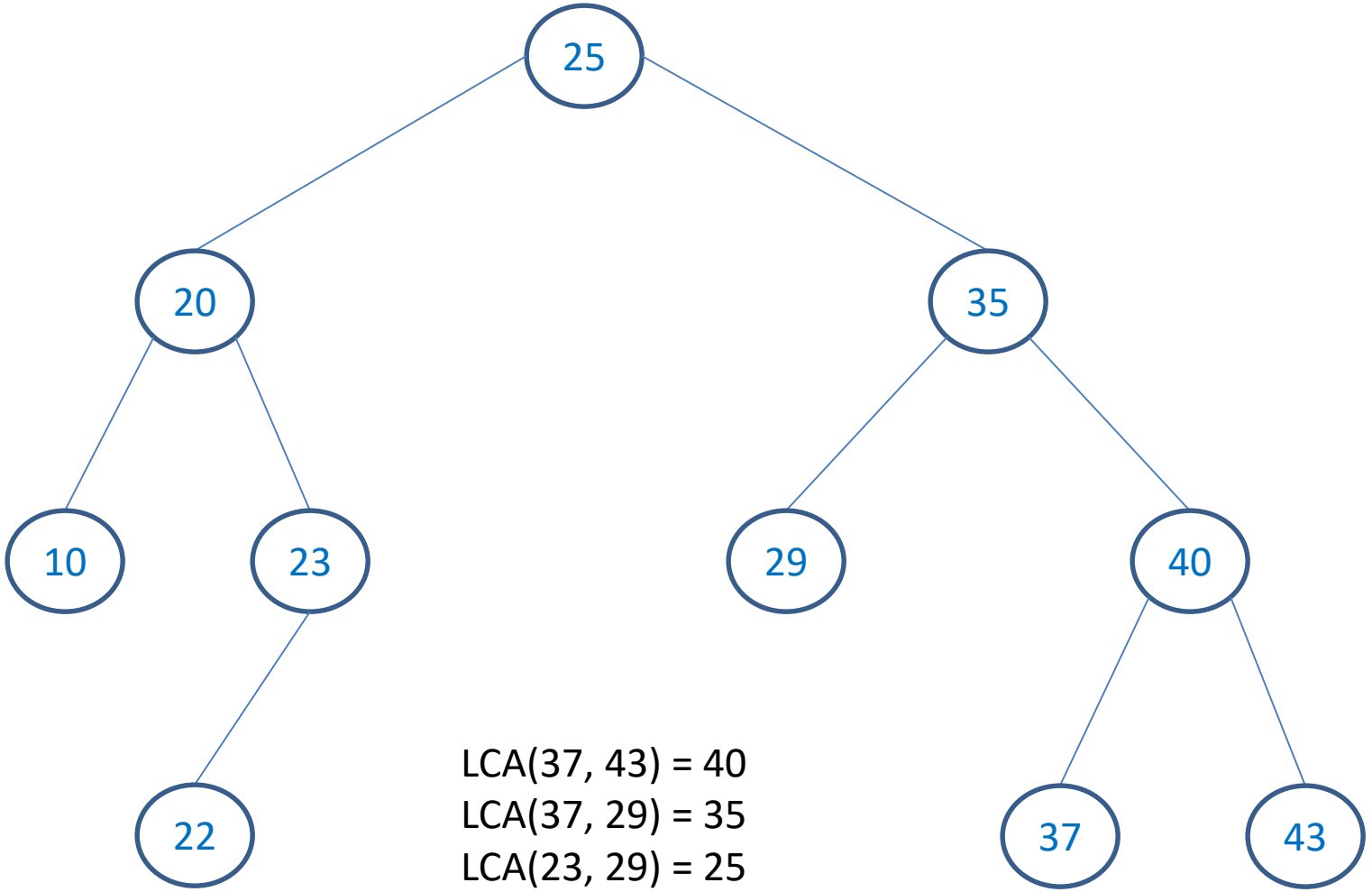
25 20 35 10 23 29 40 22 37 43



Queue:

```
Q = enqueue(Q, root);
While (Q is not empty) {
  x = dequeue(Q);
  print(x->data);
  if(x->left != NULL)
        Q = enqueue(Q, x->left);
  if(x->right != NULL)
        Q = enqueue(Q, x->right);
}
```

Find Lowest Common Ancestor



LCA(37, 43) = 40
LCA(37, 29) = 35
LCA(23, 29) = 25
LCA(22, 20) = 25
LCA(25, 29) = undefined