

#### Mixture of Experts Meets Prompt-Based Continual Learning

#### **Multi-head Self Attention & MoE**

$$\operatorname{Attention}(oldsymbol{Q}, oldsymbol{K}, oldsymbol{V}) = \operatorname{softmax}(rac{oldsymbol{Q} oldsymbol{K}^ op}{\sqrt{d_k}}) oldsymbol{V}$$

$$MSA(\boldsymbol{X}^{Q}, \boldsymbol{X}^{K}, \boldsymbol{X}^{V}) := Concat(\boldsymbol{h}_{1}, ..., \boldsymbol{h}_{m})W^{O} \in \mathbb{R}^{N \times d},$$
$$\boldsymbol{h}_{i} := Attention(\boldsymbol{X}^{Q}W_{i}^{Q}, \boldsymbol{X}^{K}W_{i}^{K}, \boldsymbol{X}^{V}W_{i}^{V}), \ i \in [m].$$

$$\mathbf{y} := \sum_{j=1}^{N} G(\boldsymbol{h})_{j} \cdot f_{j}(\boldsymbol{h}) := \sum_{j=1}^{N} \frac{\exp\left(s_{j}(\boldsymbol{h})\right)}{\sum_{\ell=1}^{N} \exp\left(s_{\ell}(\boldsymbol{h})\right)} \cdot f_{j}(\boldsymbol{h}),$$

## **Prefix Tuning**

$$\boldsymbol{h}_{l,i} = \sum_{j=1}^{N} \frac{\exp\left(\frac{\boldsymbol{x}_{i}^{\top} W_{l}^{Q} W_{l}^{K^{\top}} \boldsymbol{x}_{j}}{\sqrt{d_{v}}}\right)}{\sum_{k=1}^{N} \exp\left(\frac{\boldsymbol{x}_{i}^{\top} W_{l}^{Q} W_{l}^{K^{\top}} \boldsymbol{x}_{k}}{\sqrt{d_{v}}}\right)} W_{l}^{V^{\top}} \boldsymbol{x}_{j} = \sum_{j=1}^{N} \frac{\exp(s_{i,j}(\boldsymbol{X}))}{\sum_{k=1}^{N} \exp(s_{i,k}(\boldsymbol{X}))} f_{j}(\boldsymbol{X}),$$

$$\tilde{m{h}}_l = \operatorname{Attention}\left(m{X}^Q W_l^Q, \begin{bmatrix}m{p}^K \\ m{X}^K\end{bmatrix} W_l^K, \begin{bmatrix}m{p}^V \\ m{X}^V\end{bmatrix} W_l^V
ight) = \left[\tilde{m{h}}_{l,1}, \dots, \tilde{m{h}}_{l,N}\right]^{ op} \in \mathbb{R}^{N imes d_v},$$

$$\tilde{\boldsymbol{h}}_{l,i} = \sum_{j=1}^{N} \frac{\exp(s_{i,j}(\boldsymbol{X}))}{\sum_{k=1}^{N} \exp(s_{i,k}(\boldsymbol{X})) + \sum_{k'=1}^{L} \exp(s_{i,N+k'}(\boldsymbol{X}))} f_{j}(\boldsymbol{X})$$

$$+ \sum_{j'=1}^{L} \frac{\exp(s_{i,N+j'}(\boldsymbol{X}))}{\sum_{k=1}^{N} \exp(s_{i,k}(\boldsymbol{X})) + \sum_{k'=1}^{L} \exp(s_{i,N+k'}(\boldsymbol{X}))} f_{N+j'}(\boldsymbol{X})$$

$$\mathcal{O}\left(\frac{1}{\log^{\tau} n}\right)$$

## Non-linear Residual Gate Meet Prefix Tuning

$$\hat{s}_{i,N+j}(\boldsymbol{X}) := \frac{\boldsymbol{X}^{\top} E_i^{\top} W_l^Q W_l^{K^{\top}} \boldsymbol{p}_j^K}{\sqrt{d_v}} + \alpha \cdot \sigma \left( \tau \cdot \frac{\boldsymbol{X}^{\top} E_i^{\top} W_l^Q W_l^{K^{\top}} \boldsymbol{p}_j^K}{\sqrt{d_v}} \right)$$
$$= s_{i,N+j}(\boldsymbol{X}) + \alpha \cdot \sigma (\tau \cdot s_{i,N+j}(\boldsymbol{X})), \ i \in [N], \ j \in [L],$$

$$\begin{split} g_{G_*}(\boldsymbol{X}) := \sum_{j=1}^N \frac{\exp(\boldsymbol{X}^\top B_j^0 \boldsymbol{X} + c_j^0)}{T(\boldsymbol{X})} \cdot h(\boldsymbol{X}, \eta_j^0) \\ + \sum_{j'=1}^L \frac{\exp((\beta_{1j'}^*)^\top \boldsymbol{X} + \alpha \sigma(\tau(\beta_{1j'}^*)^\top \boldsymbol{X}) + \beta_{0j'}^*)}{T(\boldsymbol{X})} \cdot h(\boldsymbol{X}, \eta_{j'}^*), \end{split}$$

#### **Non-linear Residual Gate Meet Prefix Tuning**

**Theorem 4.1** (Regression Estimation Rate). Equipped with a least squares estimator  $\widehat{G}_n$  given in equation (15), the model estimation  $g_{\widehat{G}_n}(\cdot)$  converges to the true model  $g_{G_*}(\cdot)$  at the following rate:

$$\|g_{\widehat{G}_n} - g_{G_*}\|_{L_2(\mu)} = \mathcal{O}_P(\sqrt{\log(n)/n}).$$
 (16)

**Theorem 4.3.** Assume that the expert function  $h(x, \eta)$  and the activation  $\sigma(\cdot)$  are algebraically independent, then we achieve the following lower bound for any  $G \in \mathcal{G}_{L'}(\Theta)$ :

$$||g_G - g_{G_*}||_{L_2(\mu)} \gtrsim \mathcal{L}_1(G, G_*),$$

which together with Theorem 4.1 indicates that  $\mathcal{L}_1(\widehat{G}_n, G_*) = \widetilde{\mathcal{O}}_P(n^{-1/2})$ .

# **Non-linear Residual Gate Meet Prefix Tuning**

What do you see?



#### **Experiment**

Table 1: Overall performance comparison on Split CIFAR-100 and Split ImageNet-R. We present Final Average Accuracy (FA), Cumulative Average Accuracy (CA), and Average Forgetting Measure (FM) of all methods under different pre-trained models.

PTM	Method	Split CIFAR-100			Split Imagenet-R		
		<b>FA</b> (↑)	CA(†)	FM(↓)	<b>FA</b> (↑)	CA(↑)	FM(↓)
Sup-21K	L2P	$83.06 \pm 0.17$	$88.27 \pm 0.71$	$5.61 \pm 0.32$	$67.53 \pm 0.44$	$71.98 \pm 0.52$	$5.84 \pm 0.38$
	DualPrompt	$87.30 \pm 0.27$	$91.23 \pm 0.65$	$3.87 \pm 0.43$	$70.93 \pm 0.08$	$75.67 \pm 0.52$	$5.47 \pm 0.19$
	S-Prompt	$87.57 \pm 0.42$	$91.38 \pm 0.69$	$3.63 \pm 0.41$	$69.88 \pm 0.51$	$74.25 \pm 0.55$	$4.73 \pm 0.47$
	CODA-Prompt	$86.94 \pm 0.63$	$91.57 \pm 0.75$	$4.04 \pm 0.18$	$70.03 \pm 0.47$	$74.26 \pm 0.24$	$5.17 \pm 0.22$
	HiDe-Prompt	$92.61 \pm 0.28$	$94.03 \pm 0.01$	$1.50 \pm 0.28$	$75.06 \pm 0.12$	$76.60 \pm 0.01$	$4.09 \pm 0.13$
	NoRGa (Ours)	$94.48 \pm 0.13$	$95.83 \pm 0.37$	$1.44 \pm 0.27$	$75.40 \pm 0.39$	$79.52 \pm 0.07$	$4.59 \pm 0.07$
iBOT-21K	L2P	$79.13 \pm 1.25$	$85.13 \pm 0.05$	$7.50 \pm 1.21$	$61.31 \pm 0.50$	$68.81 \pm 0.52$	$10.72 \pm 0.40$
	DualPrompt	$78.84 \pm 0.47$	$86.16 \pm 0.02$	$8.84 \pm 0.67$	$58.69 \pm 0.61$	$66.61 \pm 0.67$	$11.75 \pm 0.92$
	S-Prompt	$79.14 \pm 0.65$	$85.85 \pm 0.17$	$8.23 \pm 1.15$	$57.96 \pm 1.10$	$66.42 \pm 0.71$	$11.27 \pm 0.72$
	CODA-Prompt	$80.83 \pm 0.27$	$87.02 \pm 0.20$	$7.50 \pm 0.25$	$61.22 \pm 0.35$	$66.76 \pm 0.37$	$9.66 \pm 0.20$
	HiDe-Prompt	$93.02 \pm 0.15$	$94.56 \pm 0.05$	$1.26 \pm 0.13$	$70.83 \pm 0.17$	$73.23 \pm 0.08$	$6.77 \pm 0.23$
	NoRGa (Ours)	<b>94.76</b> $\pm$ 0.15	$95.86 \pm 0.31$	$1.34 \pm 0.14$	$73.06 \pm 0.26$	$77.46 \pm 0.42$	$6.88 \pm 0.49$
iBOT-1K	L2P	$75.51 \pm 0.88$	$82.53 \pm 1.10$	$6.80 \pm 1.70$	$59.43 \pm 0.28$	$66.83 \pm 0.92$	$11.33\pm1.25$
	DualPrompt	$76.21 \pm 1.00$	$83.54 \pm 1.23$	$9.89 \pm 1.81$	$60.41 \pm 0.76$	$66.87 \pm 0.41$	$9.21 \pm 0.43$
	S-Prompt	$76.60 \pm 0.61$	$82.89 \pm 0.89$	$8.60 \pm 1.36$	$59.56 \pm 0.60$	$66.60 \pm 0.13$	$8.83 \pm 0.81$
	CODA-Prompt	$79.11 \pm 1.02$	$86.21 \pm 0.49$	$7.69 \pm 1.57$	$66.56 \pm 0.68$	$73.14 \pm 0.57$	$7.22 \pm 0.38$
	HiDe-Prompt	$93.48 \pm 0.11$	$95.02 \pm 0.01$	$1.63 \pm 0.10$	$71.33 \pm 0.21$	$73.62 \pm 0.13$	$7.11 \pm 0.02$
	NoRGa (Ours)	$94.01 \pm 0.04$	$95.11 \pm 0.35$	$1.61 \pm 0.30$	$72.77 \pm 0.20$	$76.55 \pm 0.46$	$7.10 \pm 0.39$
DINO-1K	L2P	$72.23 \pm 0.35$	$79.71 \pm 1.26$	$8.37 \pm 2.30$	$57.21 \pm 0.69$	$64.09 \pm 0.74$	$7.47 \pm 0.96$
	DualPrompt	$73.95 \pm 0.49$	$81.85 \pm 0.59$	$9.32 \pm 1.42$	$57.98 \pm 0.71$	$65.39 \pm 0.27$	$9.32 \pm 0.69$
	S-Prompt	$74.39 \pm 0.17$	$81.60 \pm 0.74$	$9.07 \pm 1.13$	$57.55 \pm 0.72$	$64.90 \pm 0.13$	$8.73 \pm 0.56$
	CODA-Prompt	$77.50 \pm 0.64$	$84.81 \pm 0.30$	$8.10 \pm 0.01$	$63.15 \pm 0.39$	$69.73 \pm 0.25$	$6.86 \pm 0.11$
	HiDe-Prompt	$92.51 \pm 0.11$	$94.25 \pm 0.01$	$1.67 \pm 0.20$	$68.11 \pm 0.18$	$71.70 \pm 0.01$	$6.45 \pm 0.58$
	NoRGa (Ours)	$93.43 \pm 0.33$	$94.65 \pm 0.62$	$1.65 \pm 0.25$	$71.77 \pm 0.44$	$75.76 \pm 0.49$	$6.42 \pm 0.68$
MoCo-1K	L2P	$77.24 \pm 0.69$	$83.73 \pm 0.70$	$5.57 \pm 0.75$	$54.13 \pm 0.67$	$62.09 \pm 0.76$	$\textbf{4.88} \pm 0.42$
	DualPrompt	$77.56 \pm 0.63$	$84.37 \pm 0.51$	$6.54 \pm 0.50$	$54.45 \pm 0.30$	$62.92 \pm 0.41$	$5.34 \pm 0.41$
	S-Prompt	$77.20 \pm 0.39$	$84.47 \pm 0.37$	$7.00 \pm 0.62$	$53.94 \pm 0.32$	$62.42 \pm 0.51$	$5.16 \pm 0.48$
	CODA-Prompt	$77.83 \pm 0.34$	$84.97 \pm 0.23$	$12.60 \pm 0.02$	$55.75 \pm 0.26$	$65.49 \pm 0.36$	$10.46 \pm 0.04$
	HiDe-Prompt	$91.57 \pm 0.20$	$93.70 \pm 0.01$	$1.51 \pm 0.17$	$63.77 \pm 0.49$	$68.26 \pm 0.01$	$9.37 \pm 0.71$
	NoRGa (Ours)	$93.52 \pm 0.06$	$94.94 \pm 0.29$	$1.63 \pm 0.13$	$64.52 \pm 0.16$	$70.21 \pm 0.64$	$9.06 \pm 0.19$

## **Experiment**

Table 2: Final average accuracy (FA) on Split CUB-200 and 5-Datasets.

Method	Split C	CUB-200	5-Datasets		
	Sup-21K	iBOT-21K	Sup-21K	iBOT-21K	
L2P	75.46	46.60	81.84	82.25	
DualPrompt	77.56	45.93	77.91	68.03	
S-Prompt	77.13	44.22	86.06	77.20	
CODA-Prompt	74.34	47.79	64.18	51.65	
HiDe-Prompt	86.56	78.23	93.83	94.88	
NoRGa (Ours)	90.90	80.69	94.16	94.92	

Table 3: Ablation study of different activation functions, measured by final average accuracy (FA).

Method	Split CI	FAR-100	Split CUB-200		
	Sup-21K	iBOT-21K	Sup-21K	iBOT-21K	
HiDe-Prompt	92.61	93.02	86.56	78.23	
NoRGa tanh	94.36	94.76	90.87	80.69	
NoRGa sigmoid	94.48	94.69	90.90	80.18	
NoRGa GELU	94.05	94.63	90.74	80.54	

# **Thanks**