A advecção regime não-linear

Ondas de Gravidade

Modelagem gravity waves

Considere as equações da água rasa uni-dimensional , com A força de Coriolis despresadas, e linearizado sobre um estado de repouso $U = 0,\ \Phi = \Phi_o = gH$

$$\frac{\partial u}{\partial t} = -g\frac{\partial h}{\partial x} \qquad \textbf{Dinâmica} \qquad (53)$$

$$\frac{\partial h}{\partial t} = -H\frac{\partial u}{\partial x} \qquad \textbf{Massa} \qquad (54)$$

Estas equações dão suporte para as soluções da equação

$$u(x,t) = R_e\left[\hat{u}e^{-i(kx-\nu t)}\right] \qquad \mathrm{h}(x,t) = R_e\left[\hat{h}e^{-i(kx-\nu t)}\right]$$

Estas ondas são Não-dispersivas e todos elas têm a mesma velocidade de fase. Por exemplo considere a solução para as condições iniciais de h(x,0)=f(x) e h(x,0)=0, e o domínio periódico

CTCS as solução ondas de gravidade

Discretize utilizando equações da água rasa de segunda ordem com diferenças centrada no espaço e no tempo.

$$\frac{u_j^{n+1} - u_j^{n-1}}{2\Delta t} + g\frac{h_{j+1}^n - h_{j-1}^n}{2\Delta x} = 0 \qquad (55)$$

$$\frac{h_j^{n+1} - h_j^{n-1}}{2\Delta t} + H\frac{u_{j+1}^n - u_{j-1}^n}{2\Delta x} \qquad (56)$$

Dado

$$u_j^{n+1} = u_j^{n-1} - \frac{\Delta t}{\Delta x} g\left(h_{j+1}^n - h_{j-1}^n\right) \qquad (57)$$

$$h_j^{n+1} = h_j^{n-1} - \frac{H\Delta t}{\Delta x}\left(u_{j+1}^n - u_{j-1}^n\right) \qquad (58)$$

Como de costume, podemos analisar as propriedades do presente regime, a Von Neumann. Olhar para as soluções de forma

$$U_j^n = A_n e^{ikj\delta x} \qquad \Phi_j^n = BA_n e^{ikj\delta x}$$

Onde ambas A e B podem ser constantes Complexas.

CTCS as solução ondas de gravidade

$$u_j^{n+1} = u_j^{n-1} - \frac{\Delta t}{\Delta x} g\left(h_{j+1}^n - h_{j-1}^n\right)$$

$$h_j^{n+1} = h_j^{n-1} - \frac{H\Delta t}{\Delta x}\left(u_{j+1}^n - u_{j-1}^n\right)$$

$$u_j^{n+1} = A^{n+1} e^{ikj\Delta x}$$

$$u_j^{n-1} = A^{n-1} e^{ikj\Delta x}$$

$$h_{j+1}^n = A^n e^{ik(j+1)\Delta x}$$

$$h_{j-1}^n = BA^n e^{ik(j-1)\Delta x}$$

$$A^{n+1} e^{ikj\Delta x} = A^{n-1} e^{ikj\Delta x} - \frac{gB\Delta t}{\Delta x} g\left(A^n e^{ik(j+1)\Delta x} - A^n e^{ik(j-1)\Delta x}\right)$$

$$BA^{n+1} e^{ikj\Delta x} = BA^{n-1} e^{ikj\Delta x} - \frac{H\Delta t}{\Delta x}\left(A^n e^{ik(j+1)\Delta x} - A^n e^{ik(j-1)\Delta x}\right)$$

CTCS as solução ondas de gravidade

$$A^n A e^{ikj\Delta x} = A^n A^{-1} e^{ikj\Delta x} - \frac{gB\Delta t}{\Delta x}\left(A^n e^{ik(j)\Delta x} e^{ik\Delta x} - A^n e^{ik(j)\Delta x} e^{-ik\Delta x}\right)$$

$$A^n A B e^{ikj\Delta x} = A^n A^{-1} B e^{ikj\Delta x} - \frac{H\Delta t}{\Delta x}\left(A^n e^{ik(j)\Delta x} e^{ik\Delta x} - A^n e^{ik(j)\Delta x} e^{-ik\Delta x}\right)$$

$$A^n A e^{ikj\Delta x} = A^n A^{-1} e^{ikj\Delta x} - \frac{gB\Delta t}{\Delta x} A^n e^{ikj\Delta x}\left(e^{ik\Delta x} - e^{-ik\Delta x}\right)$$

$$A^n A B e^{ikj\Delta x} = A^n A^{-1} B e^{ikj\Delta x} - \frac{H\Delta t}{\Delta x} A^n e^{ikj\Delta x}\left(e^{ik\Delta x} - e^{-ik\Delta x}\right)$$

$$A = A^{-1} - \frac{gB\Delta t}{\Delta x}\left(e^{ik\Delta x} - e^{-ik\Delta x}\right)$$

$$AB = BA^{-1} - \frac{H\Delta t}{\Delta x}\left(e^{ik\Delta x} - e^{-ik\Delta x}\right)$$

CTCS as solução ondas de gravidade

$$A^2 = 1 - \frac{gAB\Delta t}{\Delta x}\left(e^{ik\Delta x} - e^{-ik\Delta x}\right)$$

$$BA^2 = B - \frac{HA\Delta t}{\Delta x}\left(e^{ik\Delta x} - e^{-ik\Delta x}\right)$$

$$A^2 + A\frac{gB\Delta t}{\Delta x}2isin(k\Delta x) - 1 = 0$$

$$A^2 + A\frac{H\Delta t}{B\Delta x}2isin(k\Delta x) - 1 = 0$$

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

CTCS as solução ondas de gravidade

$$A^2 + A\frac{gB\Delta t}{\Delta x}2isin(k\Delta x) - 1 = 0$$

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = \frac{-\frac{gB\Delta t}{\Delta x}2isin(k\Delta x) \pm \sqrt{\left(\frac{gB\Delta t}{\Delta x}2isin(k\Delta x)\right)^2 + 4}}{2}$$

$$A = -\frac{gB\Delta t}{\Delta x}isin(k\Delta x) \pm \sqrt{\frac{4}{4}\left(\frac{gB\Delta t}{\Delta x}isin(k\Delta x)\right)^2 + 1}$$

$$A = -\frac{gB\Delta t}{\Delta x}isin(k\Delta x) \pm \sqrt{-\left(\frac{gB\Delta t}{\Delta x}sin(k\Delta x)\right)^2 + 1}$$

CTCS as solução ondas de gravidade

$$|A+|^2$$

CTCS as solução ondas de gravidade

$$A = -\frac{gB\Delta t}{\Delta x} isin(k\Delta x) \pm \sqrt{-\left(\frac{gB\Delta t}{\Delta x} sin(k\Delta x)\right)^2 + 1}$$

$$AA^* = \left(-\frac{gB\Delta t}{\Delta x} isin(k\Delta x) \pm \sqrt{-\left(\frac{gB\Delta t}{\Delta x} sin(k\Delta x)\right)^2 + 1}\right)\left(\frac{gB\Delta t}{\Delta x} isin(k\Delta x) \pm \sqrt{-\left(\frac{gB\Delta t}{\Delta x} sin(k\Delta x)\right)^2 + 1}\right)$$

$$|A+|^2 = \left(-\frac{gB\Delta t}{\Delta x} isin(k\Delta x) + \sqrt{-\left(\frac{gB\Delta t}{\Delta x} sin(k\Delta x)\right)^2 + 1}\right)\left(\frac{gB\Delta t}{\Delta x} isin(k\Delta x) + \sqrt{-\left(\frac{gB\Delta t}{\Delta x} sin(k\Delta x)\right)^2 + 1}\right)$$

$$\begin{aligned} |A+|^2 &= \left(\left(\frac{gB\Delta t}{\Delta x} sin(k\Delta x)\right)^2 - \frac{gB\Delta t}{\Delta x} isin(k\Delta x)\sqrt{-\left(\frac{gB\Delta t}{\Delta x} sin(k\Delta x)\right)^2 + 1}\right) \\ &+ \left(\frac{gB\Delta t}{\Delta x} isin(k\Delta x)\sqrt{-\left(\frac{gB\Delta t}{\Delta x} sin(k\Delta x)\right)^2 + 1}\right) + \left(\sqrt{-\left(\frac{gB\Delta t}{\Delta x} sin(k\Delta x)\right)^2 + 1}\right)^2 \end{aligned}$$

CTCS as solução ondas de gravidade

$$|A+|^2 = \left(\left(\frac{gB\Delta t}{\Delta x}sin(k\Delta x)\right)^2 - \frac{gB\Delta t}{\Delta x}isin(k\Delta x)\sqrt{-\left(\frac{gB\Delta t}{\Delta x}sin(k\Delta x)\right)^2 + 1}\right) + \left(\frac{gB\Delta t}{\Delta x}isin(k\Delta x)\sqrt{-\left(\frac{gB\Delta t}{\Delta x}sin(k\Delta x)\right)^2 + 1}\right) + \left(\sqrt{-\left(\frac{gB\Delta t}{\Delta x}sin(k\Delta x)\right)^2 + 1}\right)^2$$

$$|A+|^2 = \left(\frac{gB\Delta t}{\Delta x}sin(k\Delta x)\right)^2 - \left(\frac{gB\Delta t}{\Delta x}sin(k\Delta x)\right)^2 + 1$$

$$|A+|^2 = +1$$

CTCS as solução ondas de gravidade

$$|A -|^2$$

CTCS as solução ondas de gravidade

$$A = -\frac{gB\Delta t}{\Delta x} isin(k\Delta x) \pm \sqrt{-\left(\frac{gB\Delta t}{\Delta x} sin(k\Delta x)\right)^2 + 1}$$

$$AA^* = \left(-\frac{gB\Delta t}{\Delta x} isin(k\Delta x) \pm \sqrt{-\left(\frac{gB\Delta t}{\Delta x} sin(k\Delta x)\right)^2 + 1}\right)\left(\frac{gB\Delta t}{\Delta x} isin(k\Delta x) \pm \sqrt{-\left(\frac{gB\Delta t}{\Delta x} sin(k\Delta x)\right)^2 + 1}\right)$$

$$|A -|^2 = \left(-\frac{gB\Delta t}{\Delta x} isin(k\Delta x) - \sqrt{-\left(\frac{gB\Delta t}{\Delta x} sin(k\Delta x)\right)^2 + 1}\right)\left(\frac{gB\Delta t}{\Delta x} isin(k\Delta x) - \sqrt{-\left(\frac{gB\Delta t}{\Delta x} sin(k\Delta x)\right)^2 + 1}\right)$$

$$\begin{aligned} &|A -|^2 \\ &= \left(\left(\frac{gB\Delta t}{\Delta x} sin(k\Delta x)\right)^2 + \frac{gB\Delta t}{\Delta x} isin(k\Delta x)\sqrt{-\left(\frac{gB\Delta t}{\Delta x} sin(k\Delta x)\right)^2 + 1}\right) \\ &- \left(\frac{gB\Delta t}{\Delta x} isin(k\Delta x)\sqrt{-\left(\frac{gB\Delta t}{\Delta x} sin(k\Delta x)\right)^2 + 1}\right) + \left(\sqrt{-\left(\frac{gB\Delta t}{\Delta x} sin(k\Delta x)\right)^2 + 1}\right)^2 \end{aligned}$$

CTCS as solução ondas de gravidade

$$|A-|^2 = \left(\left(\frac{gB\Delta t}{\Delta x} sin(k\Delta x)\right)^2 + \frac{gB\Delta t}{\Delta x} isin(k\Delta x)\sqrt{-\left(\frac{gB\Delta t}{\Delta x} sin(k\Delta x)\right)^2 + 1}\right) - \left(\frac{gB\Delta t}{\Delta x} isin(k\Delta x)\sqrt{-\left(\frac{gB\Delta t}{\Delta x} sin(k\Delta x)\right)^2 + 1}\right) + \left(\sqrt{-\left(\frac{gB\Delta t}{\Delta x} sin(k\Delta x)\right)^2 + 1}\right)^2$$

$$|A-|^2 = \left(\frac{gB\Delta t}{\Delta x} sin(k\Delta x)\right)^2 - \left(\frac{gB\Delta t}{\Delta x} sin(k\Delta x)\right)^2 + 1$$

$$|A-|^2 = +1$$

CTCS as solução ondas de gravidade

$$A^2 + A\frac{H\Delta t}{B\Delta x}2isin(k\Delta x) - 1 = 0$$

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = \frac{-\frac{H\Delta t}{B\Delta x}2isin(k\Delta x) \pm \sqrt{\left(\frac{H\Delta t}{B\Delta x}2isin(k\Delta x)\right)^2 + 4}}{2}$$

$$A = -\frac{H\Delta t}{B\Delta x}isin(k\Delta x) \pm \sqrt{-\left(\frac{H\Delta t}{B\Delta x}sin(k\Delta x)\right)^2 + 1}$$

CTCS as solução ondas de gravidade

$$|A+|^2$$

CTCS as solução ondas de gravidade

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = -\frac{H\Delta t}{B\Delta x} isin(k\Delta x) \pm \sqrt{-\left(\frac{H\Delta t}{B\Delta x} sin(k\Delta x)\right)^2 + 1}$$

$$AA^* = \left(-\frac{H\Delta t}{B\Delta x} isin(k\Delta x) \pm \sqrt{-\left(\frac{H\Delta t}{B\Delta x} sin(k\Delta x)\right)^2 + 1}\right)\left(\quad \right)\frac{H\Delta t}{B\Delta x} isin(k\Delta x) \pm \sqrt{-\left(\frac{H\Delta t}{B\Delta x} sin(k\Delta x)\right)^2 + 1}$$

$$|A+|^2 = \left(-\frac{H\Delta t}{B\Delta x} isin(k\Delta x) + \sqrt{-\left(\frac{H\Delta t}{B\Delta x} sin(k\Delta x)\right)^2 + 1}\right)\left(\frac{H\Delta t}{B\Delta x} isin(k\Delta x) + \sqrt{-\left(\frac{H\Delta t}{B\Delta x} sin(k\Delta x)\right)^2 + 1}\right)$$

$$\begin{aligned} &|A+|^2 \\ &= \left(\frac{H\Delta t}{B\Delta x} sin(k\Delta x)\right)^2 - \left(\frac{H\Delta t}{B\Delta x} isin(k\Delta x)\sqrt{-\left(\frac{H\Delta t}{B\Delta x} sin(k\Delta x)\right)^2 + 1}\right) \\ &+ \left(\frac{H\Delta t}{B\Delta x} sin(k\Delta x)\sqrt{-\left(\frac{H\Delta t}{B\Delta x} sin(k\Delta x)\right)^2 + 1}\right) + \left(\sqrt{-\left(\frac{H\Delta t}{B\Delta x} sin(k\Delta x)\right)^2 + 1}\right)^2 \end{aligned}$$

CTCS as solução ondas de gravidade

$$|A+|^2$$
$$= \left(\frac{H\Delta t}{B\Delta x} sin(k\Delta x)\right)^2 - \left(\frac{H\Delta t}{B\Delta x} isin(k\Delta x)\sqrt{-\left(\frac{H\Delta t}{B\Delta x} sin(k\Delta x)\right)^2 + 1}\right)$$
$$+ \left(\frac{H\Delta t}{B\Delta x} isin(k\Delta x)\sqrt{-\left(\frac{H\Delta t}{B\Delta x} sin(k\Delta x)\right)^2 + 1}\right) + \left(\sqrt{-\left(\frac{H\Delta t}{B\Delta x} sin(k\Delta x)\right)^2 + 1}\right)^2$$

$$|A+|^2 = \left(\frac{H\Delta t}{B\Delta x} sin(k\Delta x)\right)^2 + \left(\sqrt{-\left(\frac{H\Delta t}{B\Delta x} sin(k\Delta x)\right)^2 + 1}\right)^2$$

$$|A+|^2 = -\left(\frac{H\Delta t}{B\Delta x} sin(k\Delta x)\right)^2 - \left(\frac{H\Delta t}{B\Delta x} sin(k\Delta x)\right)^2 +1$$

$$|A+|^2 = +1$$

Critério de Estabilidade para a CTC esquema

Resultando em

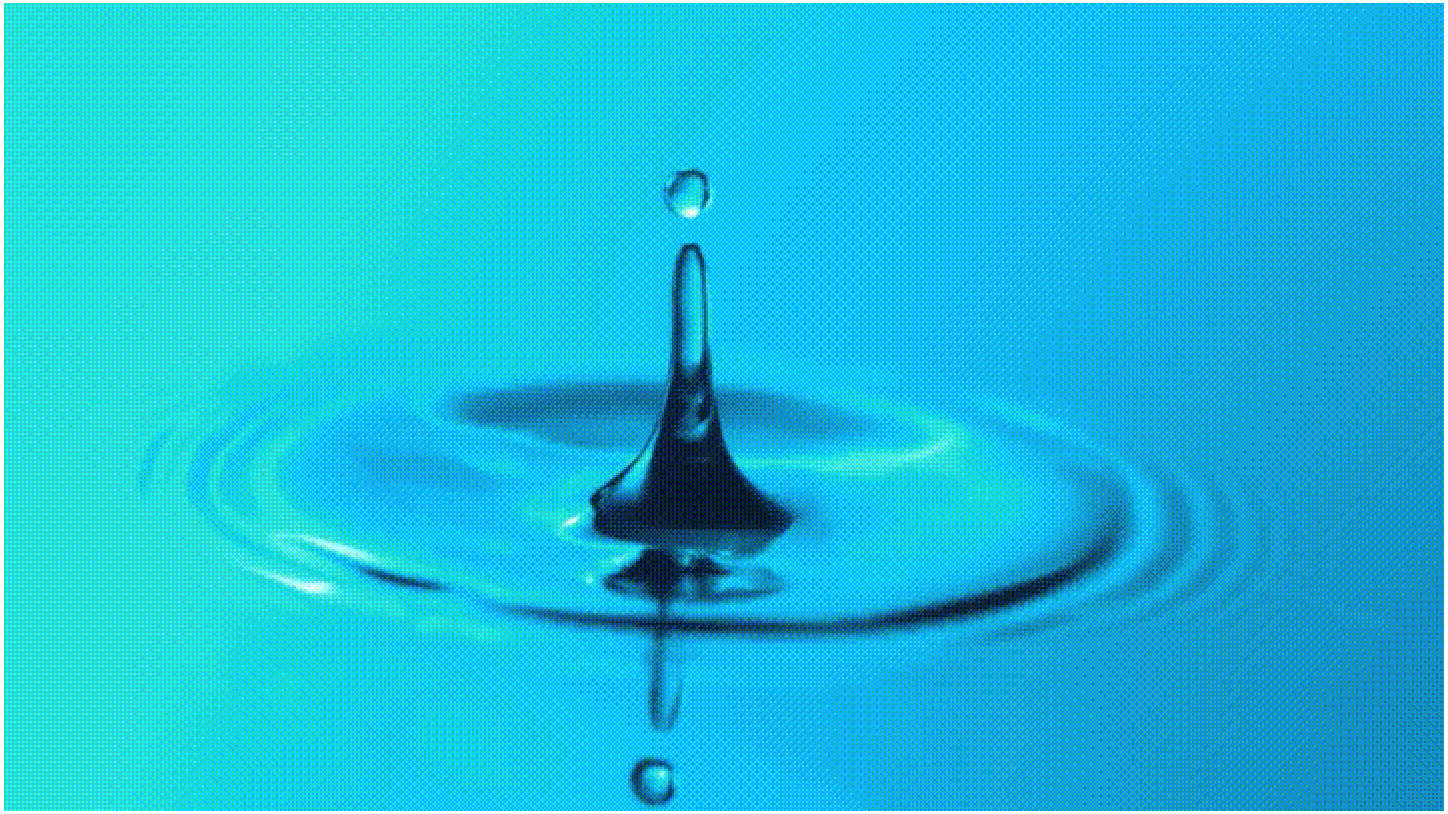
$$A = -\frac{gB\Delta t}{\Delta x} isin(k\Delta x) \pm \sqrt{-\left(\frac{gB\Delta t}{\Delta x} sin(k\Delta x)\right)^2 + 1} \quad (59)$$

$$A = -\frac{H\Delta t}{B\Delta x} isin(k\Delta x) \pm \sqrt{-\left(\frac{H\Delta t}{B\Delta x} sin(k\Delta x)\right)^2 + 1} \quad (60)$$

$$u_j^{n+1} = u_j^{n-1} - \frac{\Delta t}{\Delta x} g\left(h_{j+1}^n - h_{j-1}^n\right)$$

$$h_j^{n+1} = h_j^{n-1} - \frac{H\Delta t}{\Delta x}\left(u_{j+1}^n - u_{j-1}^n\right)$$

Critério de Estabilidade para a CTC esquema



Gravidade onda: Exercício

- Resolver numericamente a onda de gravidade das equações 55 e 56 no domínio $0 \leq X \leq 1000$m. Defina $\Delta x = 1$ m, assumir Condições de limite periódica, Suponha que a altura média dos Sistema é tal que $gH = 1\ \frac{m^2}{s^2}$ ou H=1/g. Defina o estado inicial Para h como sendo um triângulo

$$h(x,0) = \begin{cases} 0 & \text{Para } X < 400 \\ 0.001\,(X - 400) & \text{Para } 400 \leq X \leq 500 \\ 0,2 - 0,001\,(x - 400) & \text{Para } 500 \leq X \leq 600 \\ 0 & \text{Para } X > 600 \end{cases}$$

A condição inicial para o velocity é $u(x, 0) = 0$. Escolha o Intervalo de tempo tal que o sistema seja estável. Integrar para o t=2000, E mostrar as soluções (Φ e U Para T = 0s T = 200s, T = 400s T = 600s, T = 800s, T = 1000s, T = 1200s, T = 1400s T = 1600s, T = 1800S, T = 2000S.