# Equação da Quantidade de Movimento

Sabemos a *equação da quantidade de movimento na sua forma integral*.

$$\vec{F} = \vec{F}_s + \vec{F}_B = \frac{\partial}{\partial t}\int_{VC} \vec{V}\rho d\forall + \int_{SC} \vec{V}\rho\vec{V} d\vec{A}$$

Na forma diferencial expressamos as equações para um sistema infinitesimal de massa *dm*, para a qual a segunda lei de Newton pode ser expressa como:

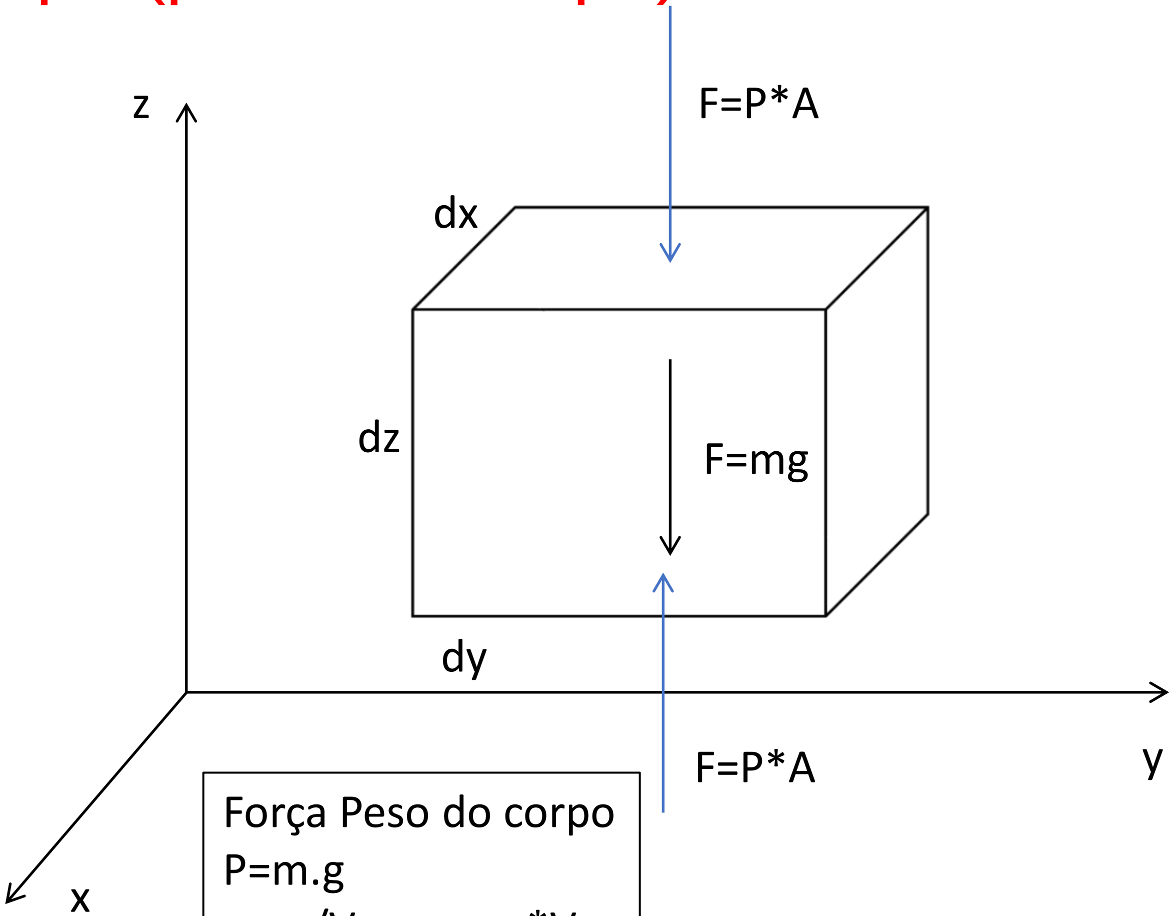
$$d\vec{F} = dm \left.\frac{d\vec{V}}{dt}\right)_{sistema}$$

o termo *dm* é facilmente determinado pelo produto entre a massa específica do fluido dentro do V.C. e o volume diferencial.

$$\left.\frac{d\vec{V}}{dt}\right)_{sistema} = \frac{D\vec{V}}{Dt}$$

# Equação da Quantidade de Movimento

## Força Campo (peso do Corpo)




Força Peso do corpo
P=m.g
ρ=m/V => m= ρ*V
P=ρg*V
P=ρg*(dx*dy*dz)
P=γ*(dx*dy*dz)

$$F = Força\ Peso = P$$

$$m\frac{d\vec{V}}{dt} = \rho g*(dx*dy*dz)$$

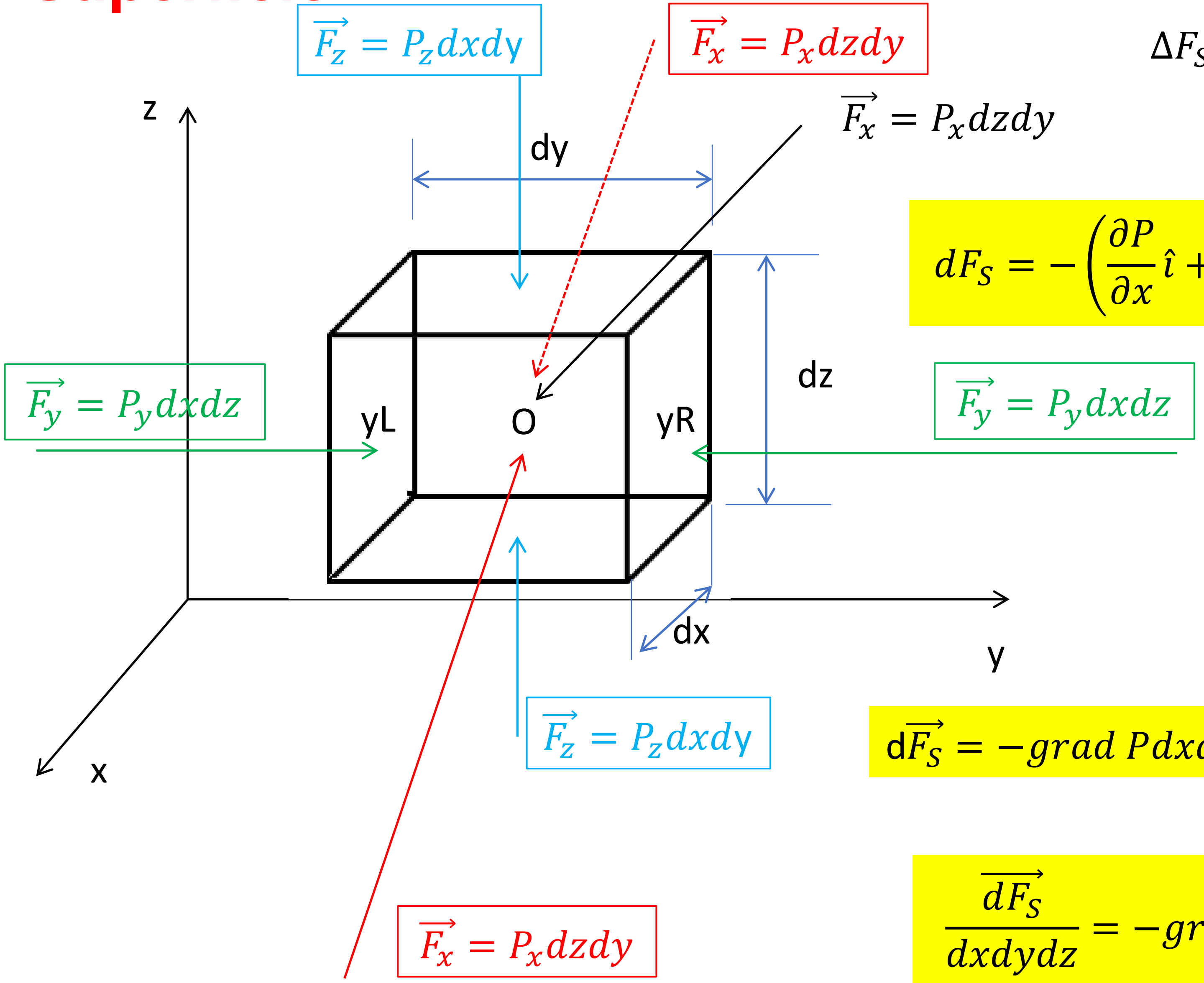
$$\rho dxdydz\frac{d\vec{V}}{dt} = \rho g*(dx*dy*dz)$$

$$\frac{d\vec{V}}{dt} = \vec{g}$$

$$dF_b = \vec{\gamma}dxdydz$$

# Equação da Quantidade de Movimento

## Força de Superfície




$$\Delta F_S = \Delta \vec{F_x} + \Delta \vec{F_y} + \Delta \vec{F_z}$$

$$dF_S = -\left(\frac{\partial P}{\partial x}\hat{\imath} + \frac{\partial P}{\partial y}\hat{\jmath} + \frac{\partial P}{\partial z}\hat{k}\right) \mathrm{dxdydz}$$

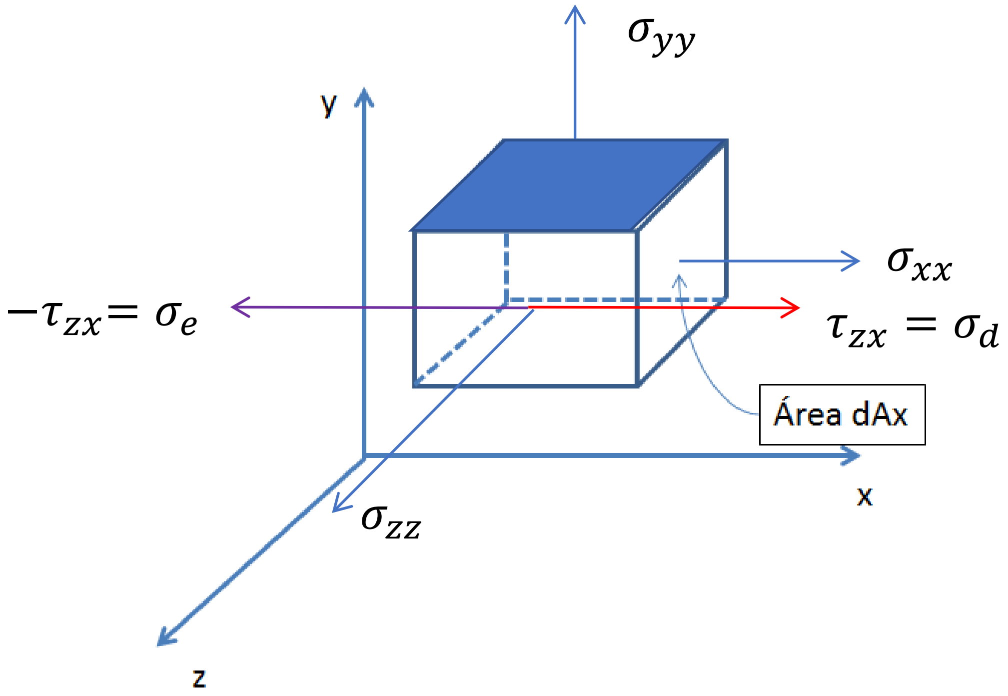
$$\mathrm{d}\vec{F_S} = -grad\, P dxdydz = -\nabla P dxdydz$$

$$\frac{\vec{dF_S}}{dxdydz} = -grad\, P = -\nabla P$$

# FÍSICA DOS FLUÍDOS

## Tensões normais e tangenciais num elemento de fluido

### Tensões de Superfície num Elemento de Fluido

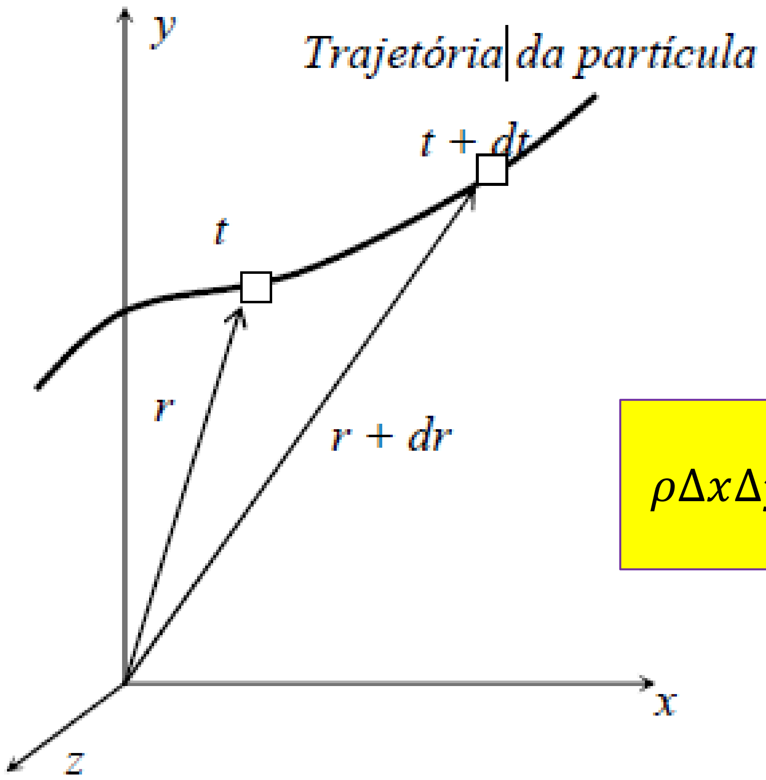



$$\Delta F_{sx} = \left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right)\Delta x \Delta y \Delta z$$

$$\Delta F_{sy} = \left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z}\right)\Delta x \Delta y \Delta z$$

$$\Delta F_{sy} = \left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z}\right)\Delta x \Delta y \Delta z$$

# CINEMÁTICA DE UM ELEMENTO DE FLUIDO




$$\Delta\vec{F} = \Delta m\vec{a} = \rho\Delta x\Delta y\Delta z\vec{a}$$

$$\Delta\vec{F} = \Delta m\vec{a} = \rho\Delta x\Delta y\Delta z\frac{d\vec{V}}{dt}$$

$$\boxed{\vec{a}_p = \frac{D\vec{V}}{Dt} = u\frac{\partial\vec{V}}{\partial x} + v\frac{\partial\vec{V}}{\partial y} + w\frac{\partial\vec{V}}{\partial z} + \frac{\partial\vec{V}}{\partial t}}$$

$$\boxed{\rho\Delta x\Delta y\Delta z\frac{D\vec{V}}{Dt} = \left(u\frac{\partial\vec{V}}{\partial x} + v\frac{\partial\vec{V}}{\partial y} + w\frac{\partial\vec{V}}{\partial z} + \frac{\partial\vec{V}}{\partial t}\right)\rho\Delta x\Delta y\Delta z}$$

# CINEMÁTICA DE UM ELEMENTO DE FLUIDO

$$\vec{F}_{total} = \vec{F}_{pressão}+\vec{F}_{gravidade}+\vec{F}_{superficie}+\dots+\vec{F}_{n}$$

$$\vec{F}_{pressão} = -grad\ Pdxdydz = -\vec{\nabla}P\Delta x\Delta y\Delta z$$

$$\vec{F}_{gravidade} = \rho dxdydz\frac{d\vec{V}}{dt} = \rho\vec{g}*(\Delta x*\Delta y*\Delta z)$$

$$\vec{F}_{superficie} = \left(\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\tau_{yx}}{\partial y}+\frac{\partial\tau_{zx}}{\partial z}\right)\Delta x\Delta y\Delta z + \left(\frac{\partial\tau_{xy}}{\partial x}+\frac{\partial\sigma_{yy}}{\partial y}+\frac{\partial\tau_{zy}}{\partial z}\right)\Delta x\Delta y\Delta z + \left(\frac{\partial\tau_{xz}}{\partial x}+\frac{\partial\tau_{yz}}{\partial y}+\frac{\partial\sigma_{zz}}{\partial z}\right)\Delta x\Delta y\Delta z$$

$$\vec{F}_{total} = \left(u\frac{\partial\vec{V}}{\partial x}+v\frac{\partial\vec{V}}{\partial y}+w\frac{\partial\vec{V}}{\partial z}+\frac{\partial\vec{V}}{\partial t}\right)\rho\Delta x\Delta y\Delta z$$

$$\vec{F}_{total} = \vec{F}_{pressão}+\vec{F}_{gravidade}+\vec{F}_{superficie}+\dots+\vec{F}_{n}$$

# CINEMÁTICA DE UM ELEMENTO DE FLUIDO

$$\vec{F}_{total} = \vec{F}_{pressão} + \vec{F}_{gravidade} + \vec{F}_{superficie} + \dots + \vec{F}_n$$

$$\left(u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z} + \frac{\partial \vec{V}}{\partial t}\right)\rho\Delta x\Delta y\Delta z$$
$$= -\vec{\nabla}P\Delta x\Delta y\Delta z + \rho\vec{g}*(\Delta x*\Delta y*\Delta z) + \left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right)\Delta x\Delta y\Delta z$$
$$+ \left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z}\right)\Delta x\Delta y\Delta z + \left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z}\right)\Delta x\Delta y\Delta z$$

$$\left(u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z} + \frac{\partial \vec{V}}{\partial t}\right)\rho$$
$$= -\vec{\nabla}P + \rho\vec{g} + \left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right) + \left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z}\right) + \left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z}\right)$$

# CINEMÁTICA DE UM ELEMENTO DE FLUIDO

$$\left(u\frac{\partial \vec{V}}{\partial x}+v\frac{\partial \vec{V}}{\partial y}+w\frac{\partial \vec{V}}{\partial z}+\frac{\partial \vec{V}}{\partial t}\right)\rho$$
$$=-\vec{\nabla}P+\rho\vec{g}+\left(\frac{\partial \sigma_{xx}}{\partial x}+\frac{\partial \tau_{yx}}{\partial y}+\frac{\partial \tau_{zx}}{\partial z}\right)+\left(\frac{\partial \tau_{xy}}{\partial x}+\frac{\partial \sigma_{yy}}{\partial y}+\frac{\partial \tau_{zy}}{\partial z}\right)$$
$$+\left(\frac{\partial \tau_{xz}}{\partial x}+\frac{\partial \tau_{yz}}{\partial y}+\frac{\partial \sigma_{zz}}{\partial z}\right)$$

Separando nas componentes x,y,z

$$\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}+\frac{\partial u}{\partial t}\right)\rho=-\frac{\partial P}{\partial x}+\cancel{\rho\overrightarrow{g_x}}+\left(\frac{\partial \sigma_{xx}}{\partial x}+\frac{\partial \tau_{yx}}{\partial y}+\frac{\partial \tau_{zx}}{\partial z}\right)$$

$$\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}+\frac{\partial v}{\partial t}\right)\rho=-\frac{\partial P}{\partial y}+\cancel{\rho\overrightarrow{g_y}}+\left(\frac{\partial \tau_{xy}}{\partial x}+\frac{\partial \sigma_{yy}}{\partial y}+\frac{\partial \tau_{zy}}{\partial z}\right)$$

$$\left(u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}+\frac{\partial w}{\partial t}\right)\rho=-\frac{\partial P}{\partial z}+\rho\overrightarrow{g_z}+\left(\frac{\partial \tau_{xz}}{\partial x}+\frac{\partial \tau_{yz}}{\partial y}+\frac{\partial \sigma_{zz}}{\partial z}\right)$$

# CINEMÁTICA DE UM ELEMENTO DE FLUIDO

$$\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right)\rho = -\frac{\partial P}{\partial x} + \left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right)$$

$$\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right)\rho = -\frac{\partial P}{\partial y} + \left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z}\right)$$

$$\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right)\rho = -\frac{\partial P}{\partial z} + \rho\overrightarrow{g_z} + \left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z}\right)$$

Considerando que as variações na densidade é pequena, a densidade $\boldsymbol{\rho}$ pode ser substituído em todos os lugares pela constante (referência) densidade $\boldsymbol{\rho_0}$, com uma grande exceção.

Esta exceção é o **termo gravitacional**, através do qual, mesmo uma pequena variação da densidade pode causar um **efeito de flutuação importante** na maioria das aplicações ambientais.

A simplificação resultante é conhecida como a **aproximação Boussinesq**.

# CINEMÁTICA DE UM ELEMENTO DE FLUIDO

$$\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)\rho_0=-\frac{\partial P}{\partial x}+\left(\frac{\partial \sigma_{xx}}{\partial x}+\frac{\partial \tau_{yx}}{\partial y}+\frac{\partial \tau_{zx}}{\partial z}\right)$$

$$\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)\rho_0=-\frac{\partial P}{\partial y}+\left(\frac{\partial \tau_{xy}}{\partial x}+\frac{\partial \sigma_{yy}}{\partial y}+\frac{\partial \tau_{zy}}{\partial z}\right)$$

$$\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)\rho_0=-\frac{\partial P}{\partial z}+\boldsymbol{\rho}\vec{g_z}+\left(\frac{\partial \tau_{xz}}{\partial x}+\frac{\partial \tau_{yz}}{\partial y}+\frac{\partial \sigma_{zz}}{\partial z}\right)$$

Dividindo por $\rho_0$

$$\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{1}{\rho_0}\frac{\partial P}{\partial x}+\frac{1}{\rho_0}\left(\frac{\partial \sigma_{xx}}{\partial x}+\frac{\partial \tau_{yx}}{\partial y}+\frac{\partial \tau_{zx}}{\partial z}\right)$$

$$\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=-\frac{1}{\rho_0}\frac{\partial P}{\partial y}+\frac{1}{\rho_0}\left(\frac{\partial \tau_{xy}}{\partial x}+\frac{\partial \sigma_{yy}}{\partial y}+\frac{\partial \tau_{zy}}{\partial z}\right)$$

$$\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=-\frac{1}{\rho_0}\frac{\partial P}{\partial z}+\frac{\boldsymbol{\rho}}{\rho_0}\vec{g_z}+\frac{1}{\rho_0}\left(\frac{\partial \tau_{xz}}{\partial x}+\frac{\partial \tau_{yz}}{\partial y}+\frac{\partial \sigma_{zz}}{\partial z}\right)$$

# CINEMÁTICA DE UM ELEMENTO DE FLUIDO

$$\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{1}{\rho_0}\frac{\partial P}{\partial x} + \vec{\tau}_x$$

$$\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = -\frac{1}{\rho_0}\frac{\partial P}{\partial y} + \vec{\tau}_y$$

$$\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = -\frac{1}{\rho_0}\frac{\partial P}{\partial z} + \frac{\boldsymbol{\rho}}{\rho_0}\overrightarrow{g_z} + \vec{\tau}_z$$

Na Forma Vetorial

$$\frac{\partial \vec{V}}{\partial t} + (\vec{V}.\nabla)\vec{V} = -\frac{1}{\rho_0}\vec{\nabla}P + \frac{\boldsymbol{\rho}}{\rho_0}\vec{g} + \vec{F}_{tensão}$$

# CINEMÁTICA DE UM ELEMENTO DE FLUIDO

# Equações de Navier-Stokes

Para ***fluidos newtonianos***, as tensões podem ser expressas em termos de **gradientes de velocidades** e propriedades dos fluidos:

Sabemos que p/ **um fluido newtoniano** a tensão viscosa é proporcional a taxa de deformação por cisalhamento (**taxa de deformação angular**). As tensões podem ser expressas em termos dos gradientes de velocidade e das propriedades dos fluidos como:

$$\tau_{xy} = \tau_{yx} = \mu\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)$$

$$\tau_{yz} = \tau_{zy} = \mu\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)$$

$$\tau_{zx} = \tau_{xz} = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)$$

$$\sigma_{xx} = -p - \frac{2}{3}\mu\nabla\vec{V} + 2\mu\frac{\partial u}{\partial x}$$

$$\sigma_{yy} = -p - \frac{2}{3}\mu\nabla\vec{V} + 2\mu\frac{\partial v}{\partial y}$$

$$\sigma_{zz} = -p - \frac{2}{3}\mu\nabla\vec{V} + 2\mu\frac{\partial w}{\partial z}$$

Onde $p$ é a pressão termodinâmica local.

# Equações de Navier Stokes

No caso de ***fluido incompressível***, $\nabla \cdot V = 0$ , e a equação acima pode ser simplificada.
Fazendo desprezíveis as forças de campo ( $B = 0$ ) se obtém **as *Equações de Navier Stokes*.**

$$\rho \frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left(2\mu \frac{\partial u}{\partial x} - \frac{2}{3}\mu \nabla \vec{V}\right) + \frac{\partial}{\partial y}\left[\mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right] + \frac{\partial}{\partial z}\left[\mu\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right)\right]$$

$$\rho \frac{Dv}{Dt} = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial x}\left[\mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right] + \frac{\partial}{\partial y}\left(2\mu \frac{\partial v}{\partial y} - \frac{2}{3}\mu \nabla \vec{V}\right) + \frac{\partial}{\partial z}\left[\mu\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)\right]$$

$$\rho \frac{Dw}{Dt} = -\frac{\partial p}{\partial z} + \frac{\partial}{\partial x}\left[\mu\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right)\right] + \frac{\partial}{\partial y}\left[\mu\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)\right] + \frac{\partial}{\partial z}\left(2\mu \frac{\partial w}{\partial z} - \frac{2}{3}\mu \nabla \vec{V}\right)$$

no caso de **escoamento incompressível** **permanente** com viscosidade constante e incluindo as forças de campo

$$\rho \frac{Du}{Dt} = \rho g_x - \frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)$$

$$\rho \frac{Dv}{Dt} = \rho g_y - \frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right)$$

$$\rho \frac{Dw}{Dt} = \rho g_z - \frac{\partial p}{\partial z} + \mu \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right)$$

Em forma vetorial pode ser representada como

$$\rho \frac{D\vec{V}}{Dt} = \rho \vec{g} - \nabla p + \mu \nabla^2 \vec{V}$$

# Equações de Euler

Quando os termos viscosos são pequenos e podem ser desprezíveis ($\mu=0$) as equações resultantes são conhecidas como ***Equações de Euler***, que podem ser representas na forma vetorial como:

$$\rho \frac{D\vec{V}}{Dt} = \rho\vec{g} - \nabla p$$