Características dos Modelos MPAS e FV3 Importantes Para as Demandas do MONAN

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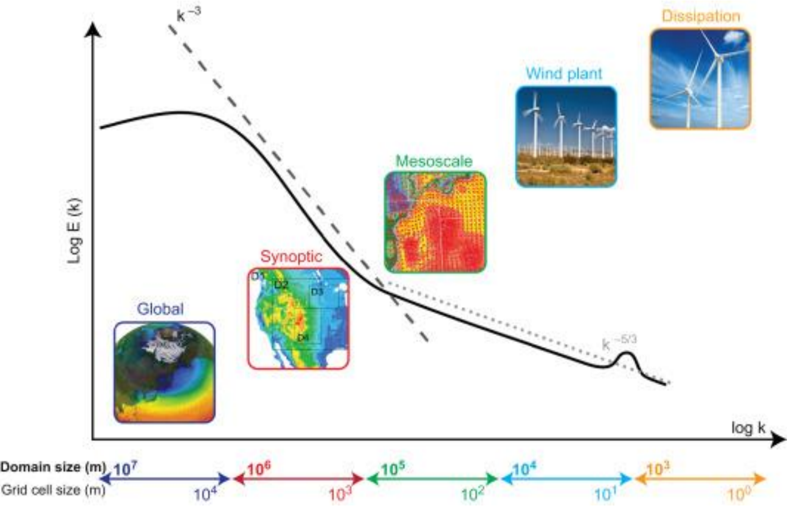
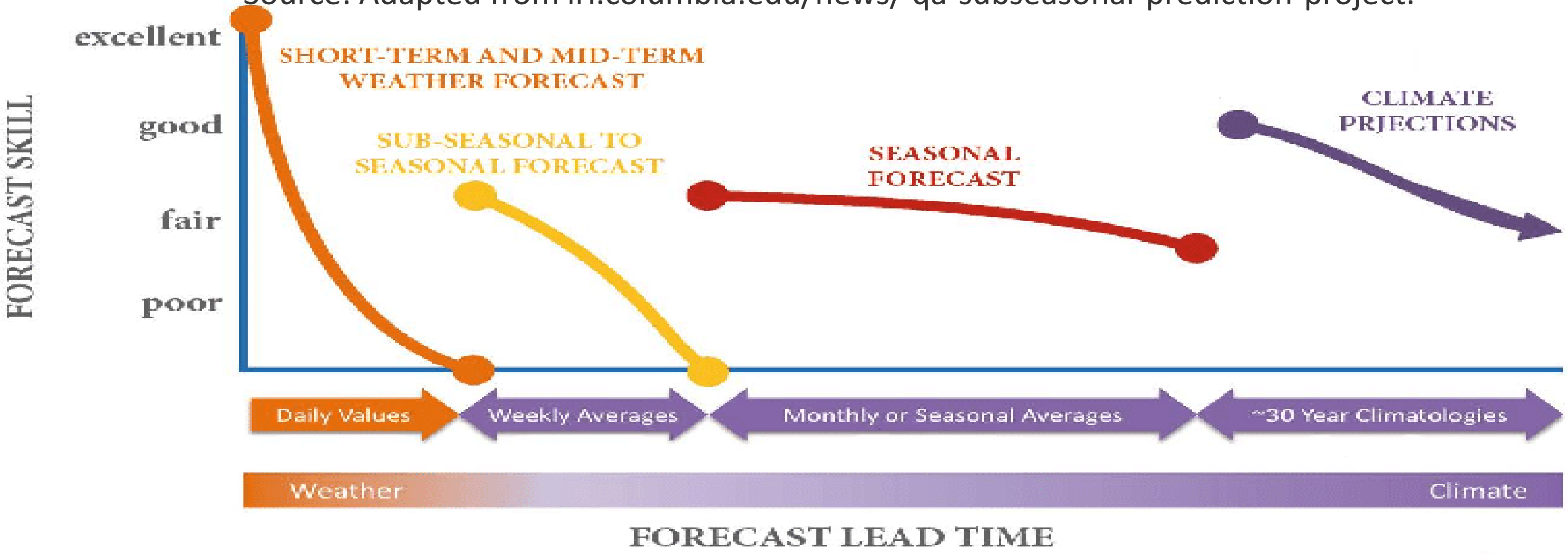
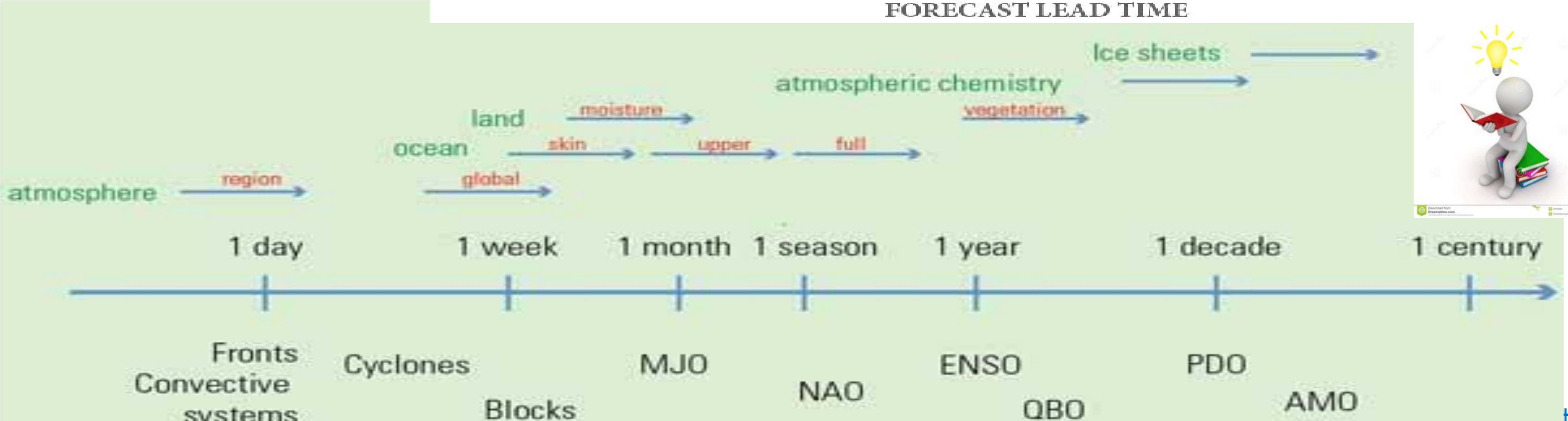
As Demandas do Modelo MONAN



Source: Adapted from iri.columbia.edu/news/ qa-subseasonal-prediction-project.

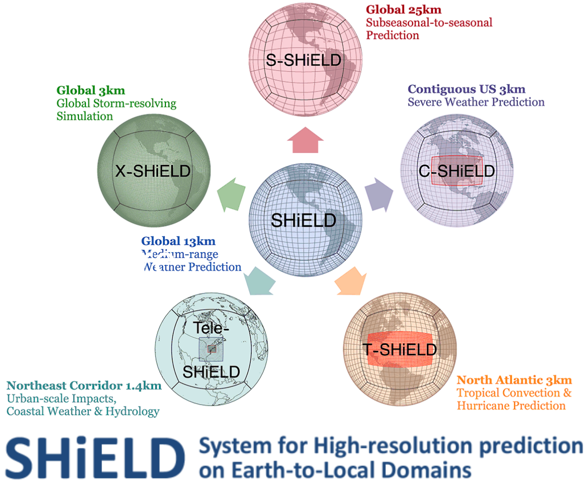




Source: Predictability Beyond the Deterministic Limit | World Meteorological Organization (wmo.int)

Considerações Técnicas Sobre os Núcleos Dinâmicos que podem ser adotados pelo Modelos MONAN

MPAS
Model for Prediction Across Scales



GEF
Global Eta FrameWork

Considerações Técnicas dos Núcleos Dinâmico dos Modelos

Formulação /Setup	MPAS	FV3-Shied	GEF
Dinâmica	Euleriana	Euleriana/ Lagrangian	Euleriana
Compressibilidade	compressible	compressible	compressible
Aproximação Vertical	nonhydrostatic	nonhydrostatic	hydrostatic
Grade	malhas de Voronoi não estruturadas (Grade-C)	cubed-sphere grid, in a version with uniform Jacobians (UJ) (Grade-C/ Grade-D)	cubed-sphere grid, in a version with uniform Jacobians (UJ) (Grade-B)
Coordenada vertical	terrain following coordinate) and a hybrid coordinate	Coordenada Lagrangiana	Coordenada Eta
Time Scheme	split-explicit time integration scheme employs a 3rd-order Runge-Kutta method	Forward e Backward dependendo da variável prognostica	Similar Ao modelo Eta Regional
Transporte de escalares	Volume finito polinomio de 3rd and 4th-order	Esquema Semi-Lagrangiano	Esquema Semi-Lagrangiano

Características Gerais que Atendem as Demandas do MONAN:

Como os Modelos consideram os seguintes itens?

1.1 Grade com refinamento variável.

1.2 Regionalização com fronteira aberta.

1.3 Dinâmica não-hidrostática.

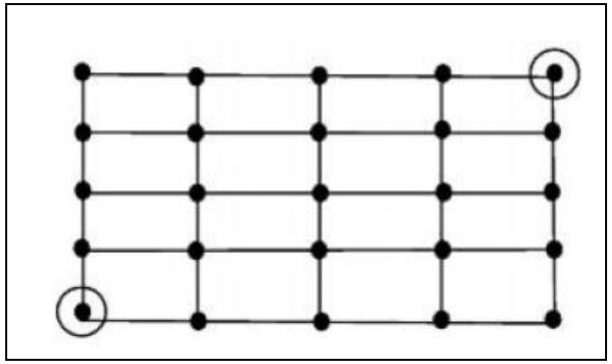
1.4 Transporte de escalares com manutenção de positividade e monotonicidade.

1.5 Suítes físicas avançadas, etc.

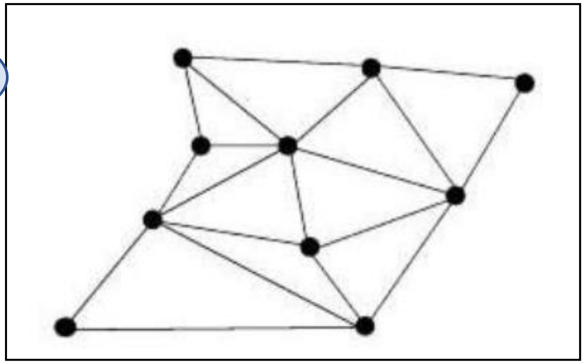
Uma visão geral da estrutura das malhas (grades)

Uma visão geral da estrutura das malhas
Tipo de Malha Numérica

ESTRUTURADA
(consistes de retângulos 2D)



NÃO-ESTRUTURADA
(consistes de polígonos não homogêneos 2D)



Como Discretizar?

Vantagens:
-A indexação sozinha, mostra uma conectividade limpa,
-Fácil de manipular os indices e armazenar na computação

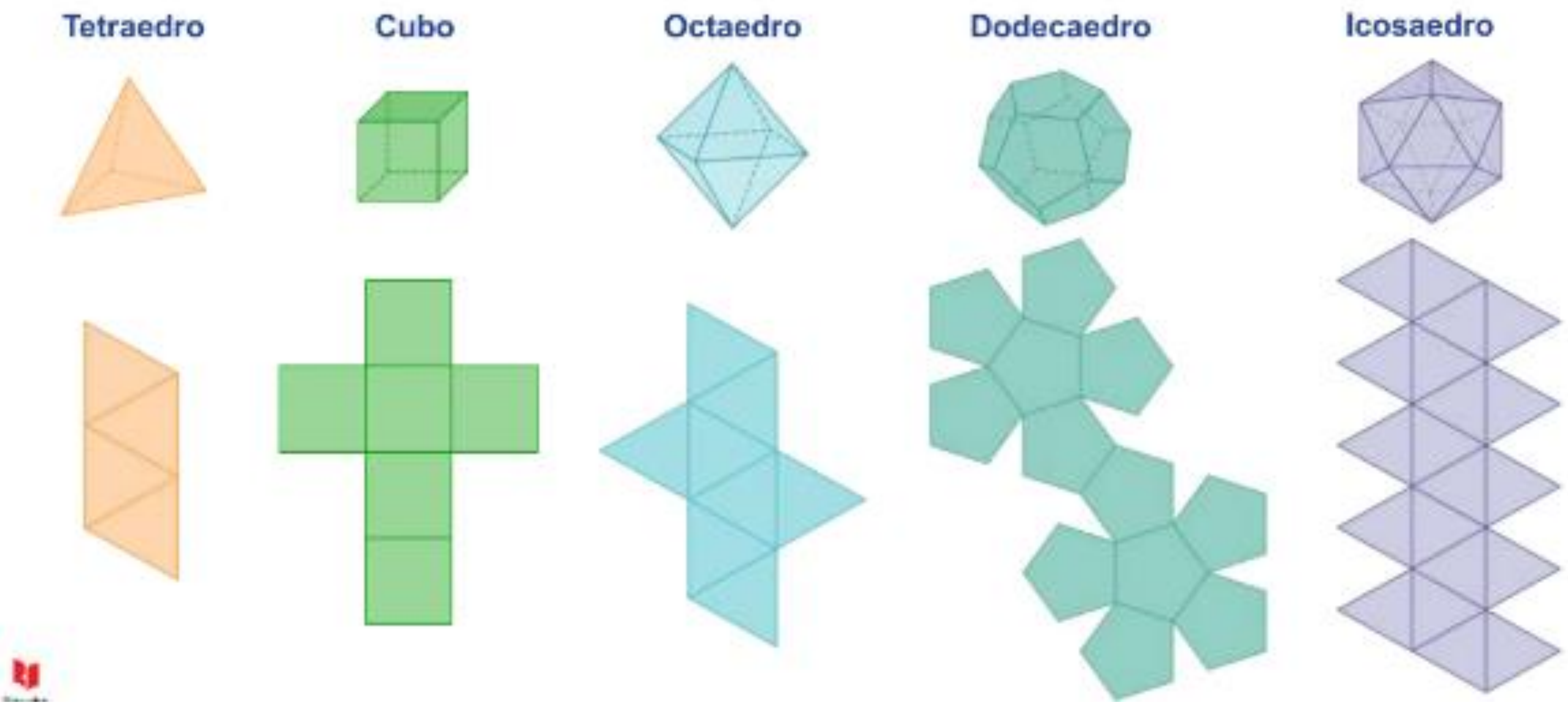
Desvantagens:
-Restrições à ortogonalidade e razão de aspecto.
-Difícil envolver geometrias complexas.
-Menos eficiente para malhas localmente refinadas.

Vantagens:
-Informações de conectividade para cada célula precisam ser armazenadas
-Difícil de armazenar e manipular dados na computação

Desvantagens:
-Mais fácil envolver forma complexas.
-Muito eficiente para malhas localmente refinadas

Uma visão geral da estrutura das malhas
Tipo de Malha Numérica

Planificações dos sólidos platónicos



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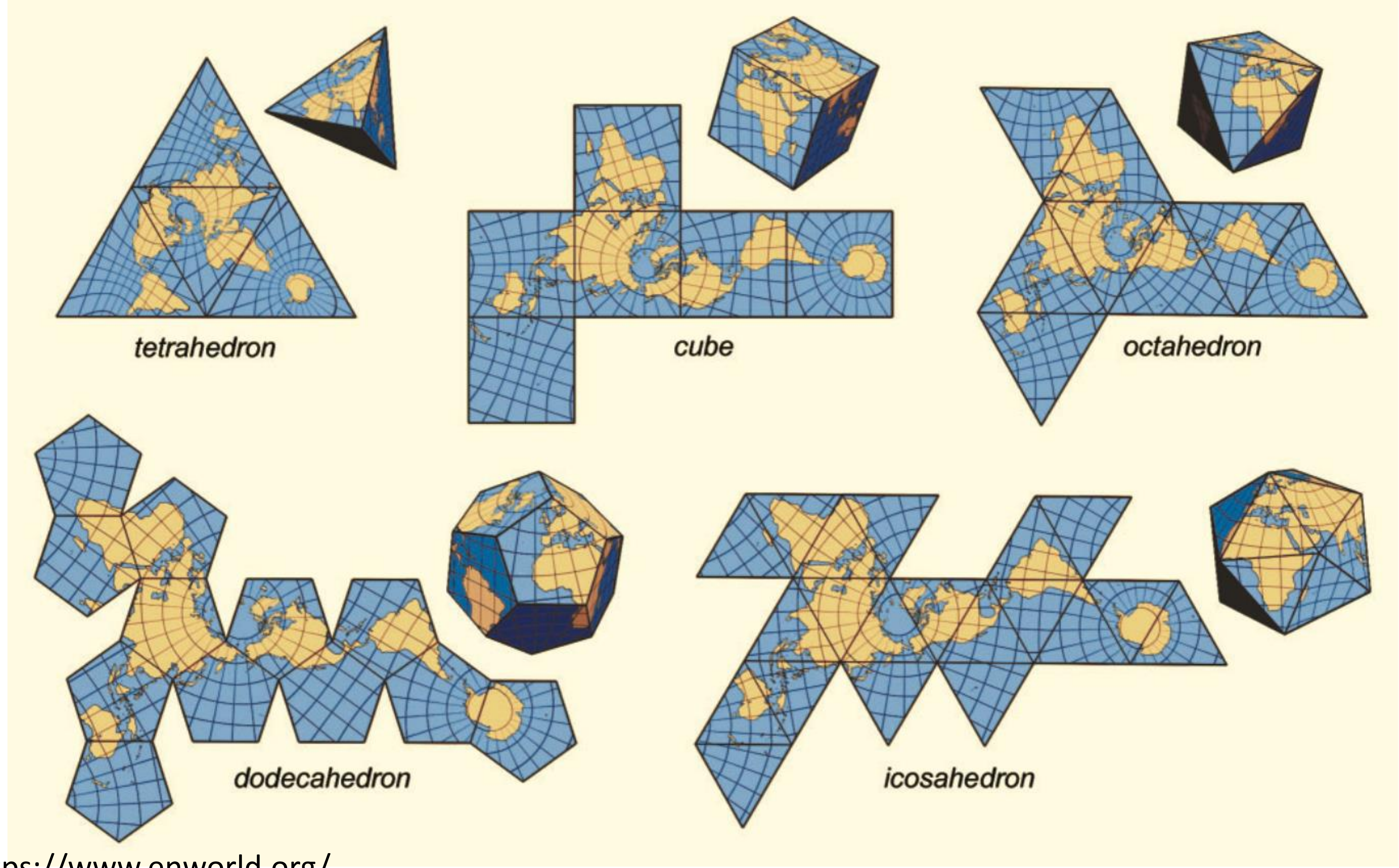
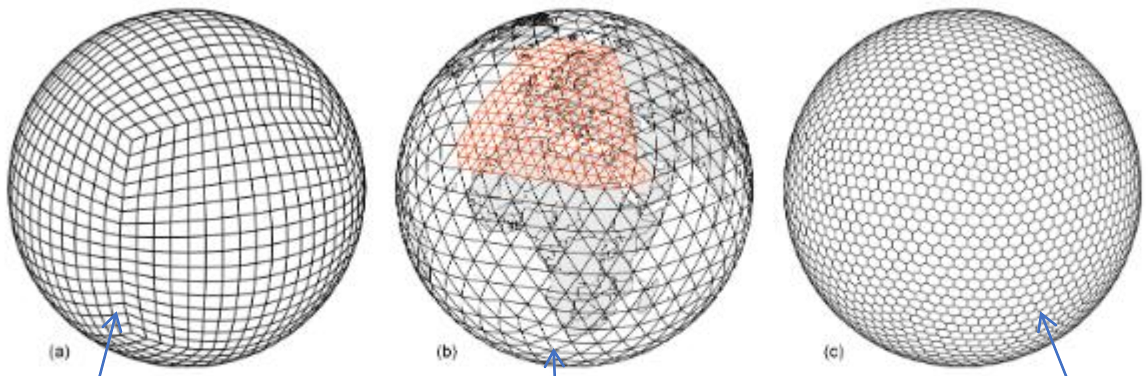
Uma visão geral da estrutura das malhas
Tipo de Malha Numérica

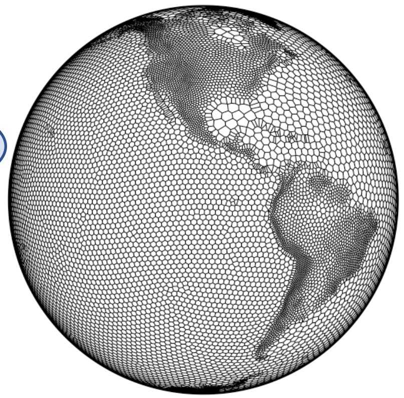


https://www.enworld.org/

Uma visão geral da estrutura das malhas MPAS



Qual é a
Melhor Malha?

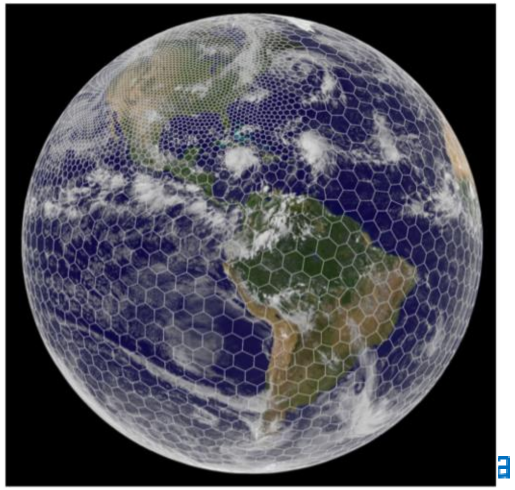


a)Esfera cubada

b) Icosaédrica triangular

c) Icosaédrica hexagonal

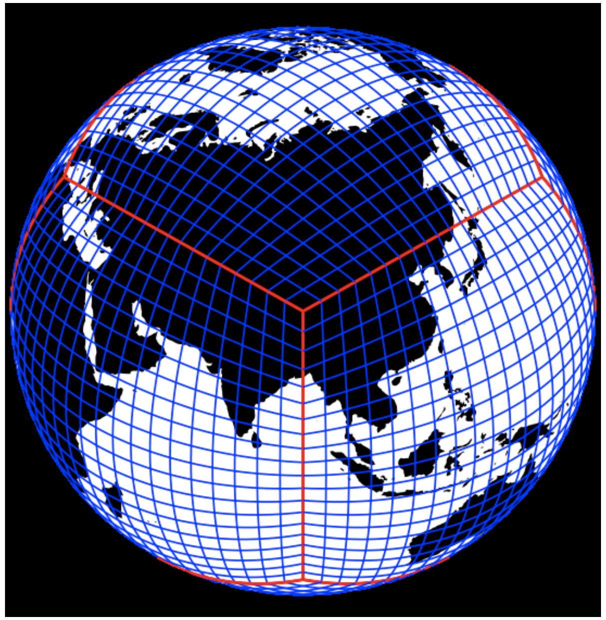
d)Tesselação centroidal de Voronoi (CVTs)

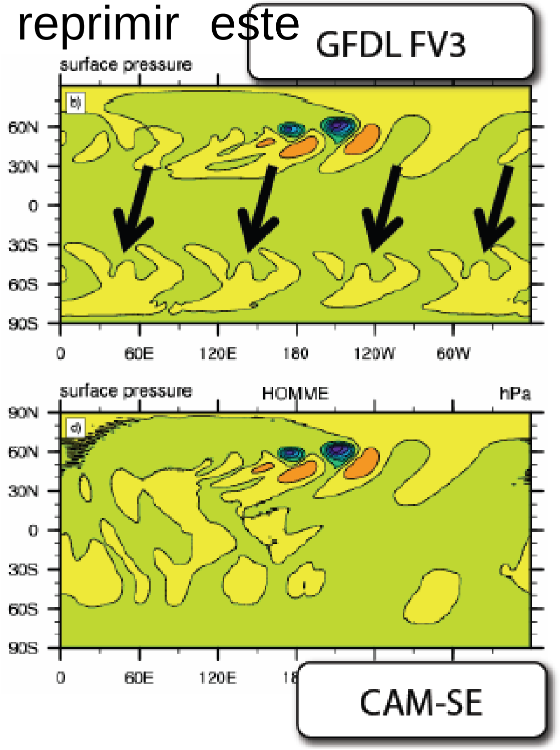


Métodos Numéricos: Problemas

Tanto o modelo **GFDL FV3** (FVcubed) como o modelo **CAM-SE** (elemento espectral) são construídos na ***esfera em cubos***. Isso leva a um **realce** do modo de onda **k = 4.**

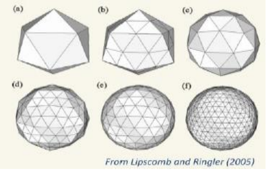
O uso de alta ordem de difusão numerica no CAM-SE é mais eficaz para reprimir este modo.

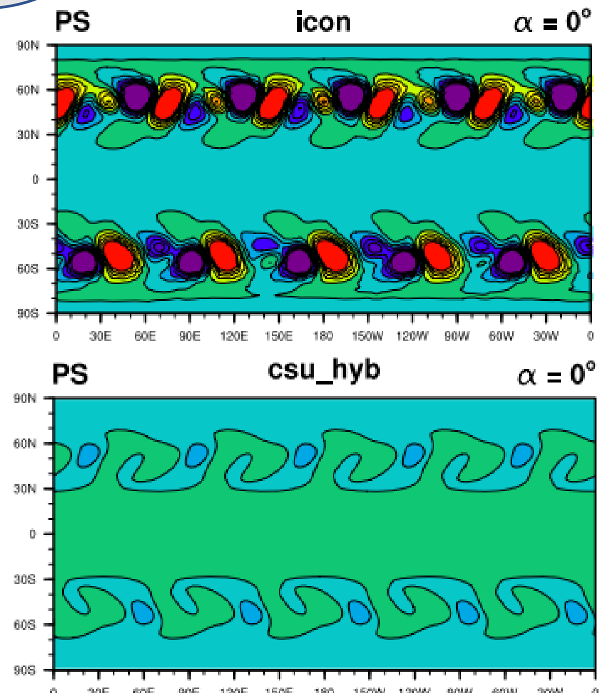




Ambos os modelos MPAS, OLAM, ICON e CSU são construídos em uma **Malha icosaédrica** (resultados da oficina de 2008). Isso leva a um **realce** do modo de onda **k = 5**.

Difusão Numérica pode filtrar ondas reais

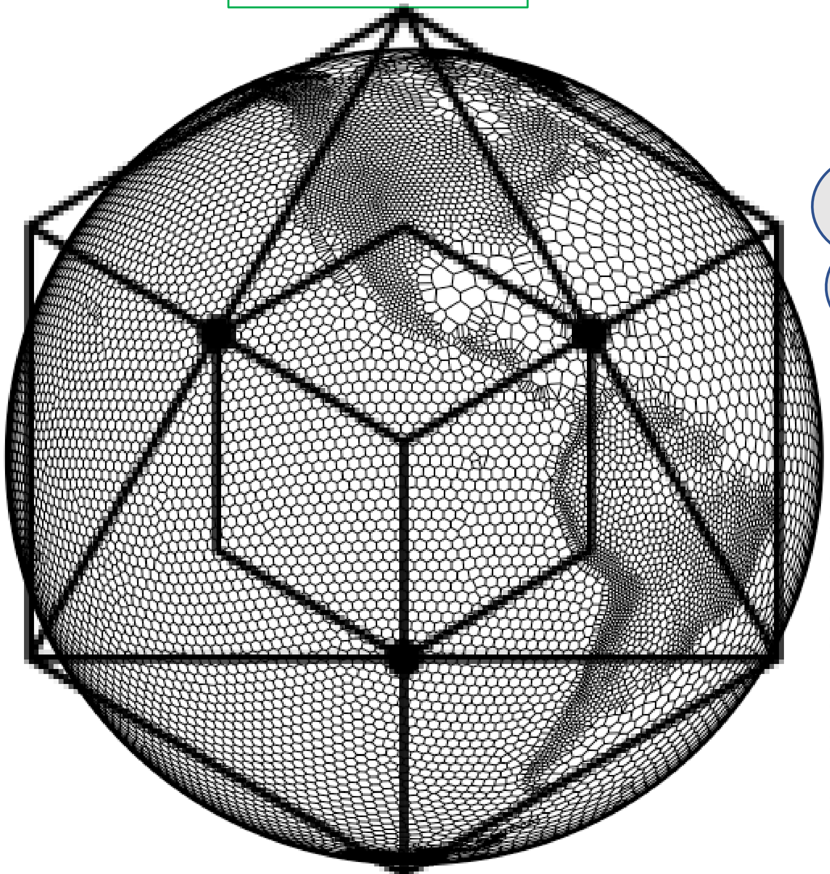




1.1 Malhas do modelos adaptadas para todas as escalas

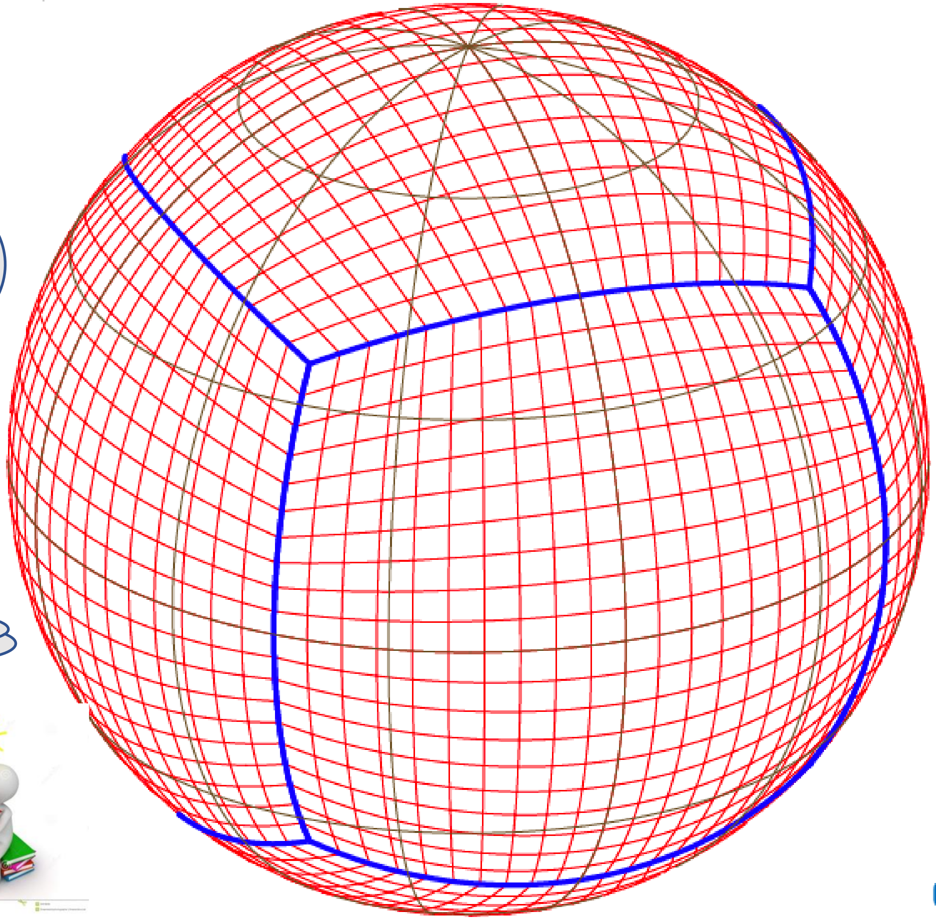
Malha
não estruturada

MPAS



Malha estruturada

FV3 e GEF



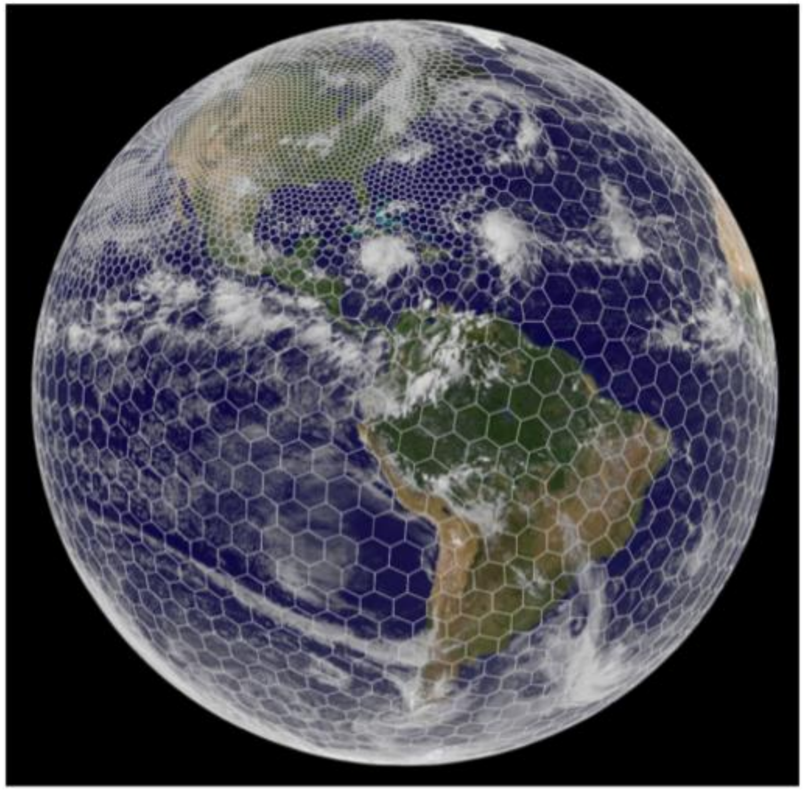
Solução-
Tesselação
centroidal de
Voronoi (CVTs)

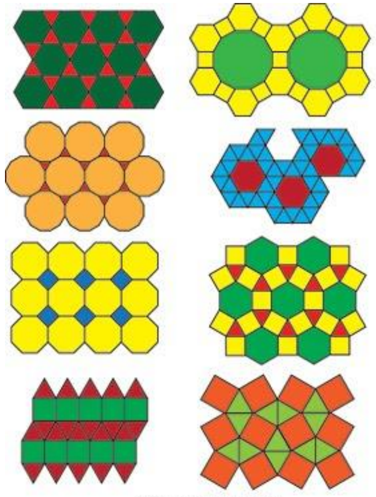
Porque?

o Kubota

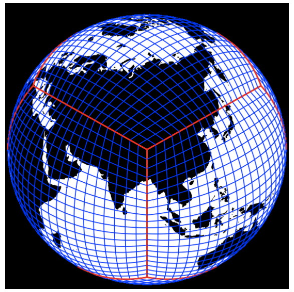
Solução (Estrutura das Malhas MPAS)

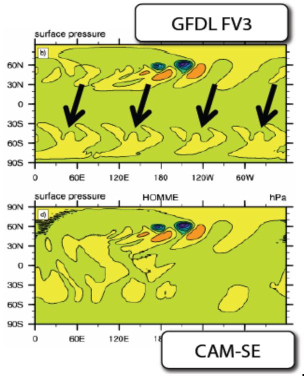
Uma característica definidora dos modelos MPAS é o uso de **Tesselação** centroidal de Voronoi (CVTs) com um deslocamento de grade C





Tesselação é o recobrimento de uma superfície bidimensional, tendo, como unidades básicas, polígonos congruentes (mesmo tamanho e a mesma forma) ou não, sem que existam espaços entre eles e de modo que a superfície total seja igual ao espaço particionado.



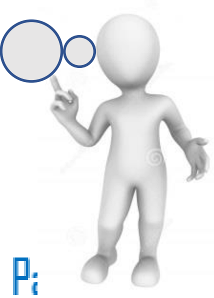


– Quando se restringe a ficar na superfície de uma esfera, geralmente os chamamos de **Tesselação** centroidal de Voronoi na esféra (SCVTs)

a) Adapta melhor a geometria esférica.

b) Pode ajudar a reduzir ondas espúrias devido ao formato da grade

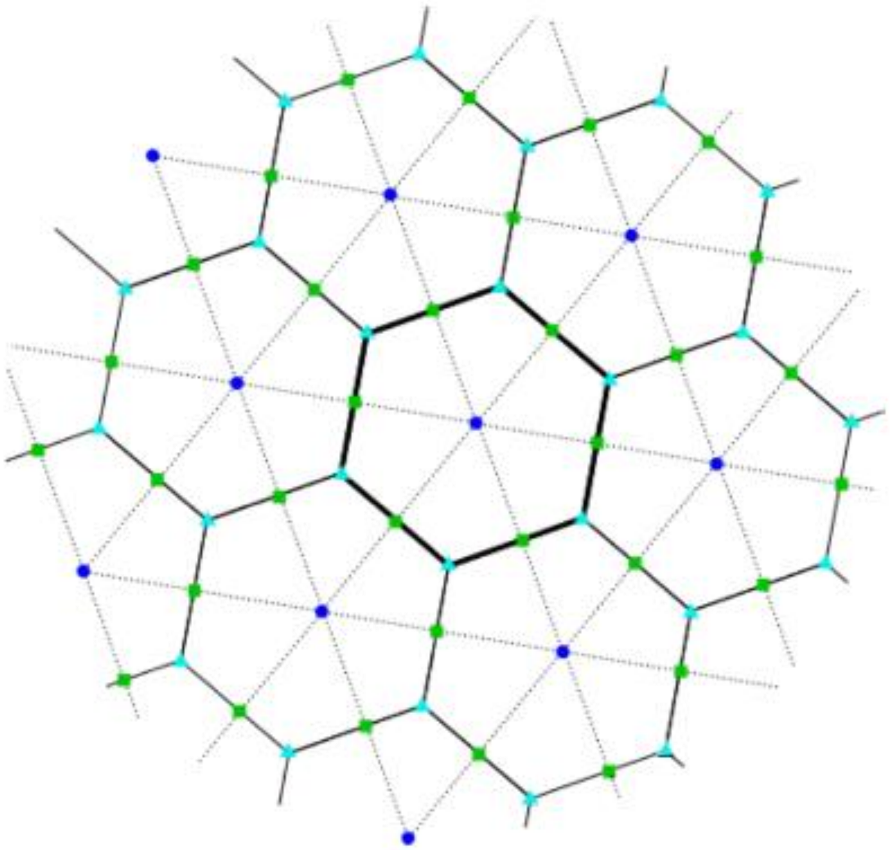
c) Menos difusão Numérica



Uma visão geral da estrutura das malhas MPAS

Como o MPAS e você acompanham essa malha Voronoi não estruturada?

Esquemas para encontrar implicitamente os índices/identidades (os “IDs”) de elementos vizinhos da malha (ou seja, células, arestas, vértices) estão fadados ao fracasso...



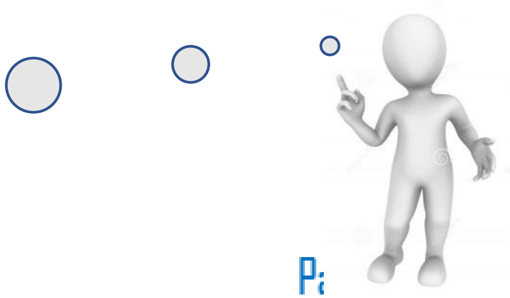
Três tipos de elementos de malha são rastreados na representação de malha:

- Localização das células (círculos azuis) - os pontos geradores da malha de Voronoi

- Locais de vértices (triângulos ciano) - os cantos das células da malha primária

- Locais de borda (quadrados verdes) - os pontos onde as bordas da malha dupla que cruzam as bordas da malha primária

Dificuldade no Mapeamento da Malha

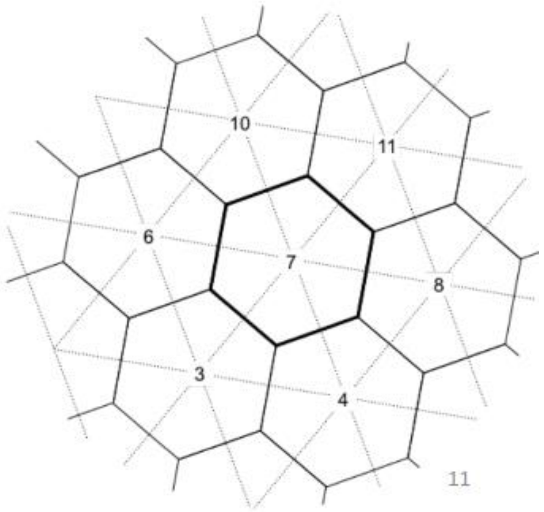


Uma visão geral da estrutura das malhas MPAS

Os campos de conectividade explícita descrevem a estrutura de uma malha

- **nEdgesOnCell** (nCells) – o número de vizinhos para cada célula
- **cellsOnCell** (maxEdges, nCells) – os índices de células vizinhas para cada célula
- **edgeOnCell** (maxEdges, nCells) – os índices de bordas delimitadoras para cada célula
- **verticesOnCell** (maxEdges, nCells) – os índices de vértices de canto para cada célula
- **edgeOnVertex** (vertexDegree,nVertices) – os índices de arestas incidentes com cada vértice
- **verticesOnEdge**(2,nEdges) – os índices de vértices de extremidade para cada aresta
- **cellsOnVertex** (vertexDegree,nVertices) – os índices de células que se encontram em cada vértice
- **cellsOnEdge** (2,nEdges) – os índices de células separadas por cada aresta

```
nEdgesOnCell(7)=6   cellsOnCell(1,7)=8
                    cellsOnCell(2,7)=11
                    cellsOnCell(3,7)=10
                    cellsOnCell(4,7)=6
                    cellsOnCell(5,7)=3
                    cellsOnCell(6,7)=4
```



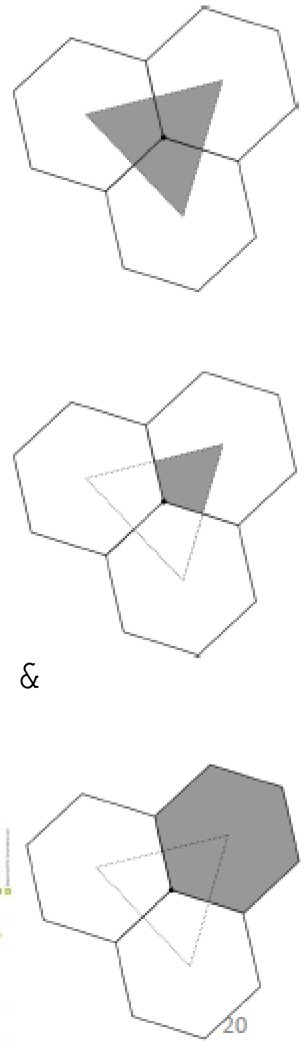
Na inicialização do modelo, todos os índices nessas matrizes são renumerados para um esquema de indexação local

Um exemplo prático de campos de malha em uso

Um exemplo de uso de campos de malha: Calculando a média de um campo baseado em vértice, vorticity(nVertLevels, nVertices), para células como vortcell(nVertLevels,nCells):

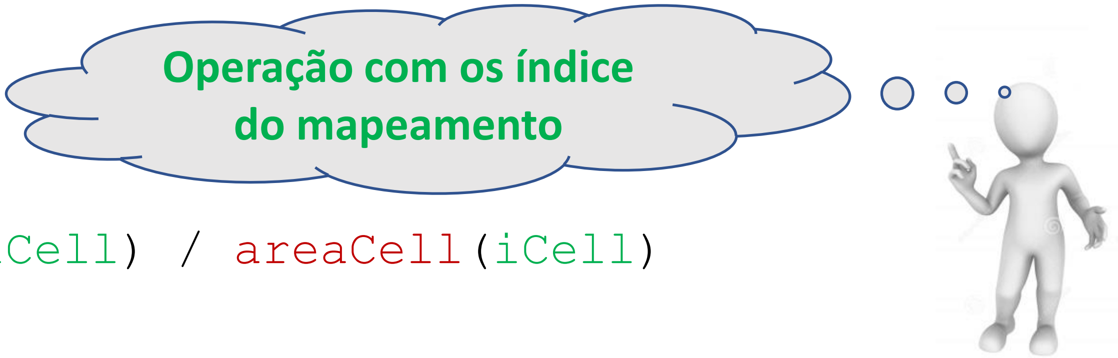
kiteAreasOnVertex(vertexDegree, nVertices)

cellsOnVertex(vertexDegree, nVertices)



```
vortcell(:,:) = 0.0
do iVtx = 1, nVertices
   do j = 1, vertexDegree
      iCell = cellsOnVertex(j, iVtx)
      vortcell(:, iCell) = vortcell(:, iCell) + kiteAreasOnVertex(j, iVtx) *  &
                           vorticity(:, iVtx)
   end do
end do

do iCell = 1, nCells
   vortcell(:,iCell) = vortcell(:,iCell) / areaCell(iCell)
end do
```



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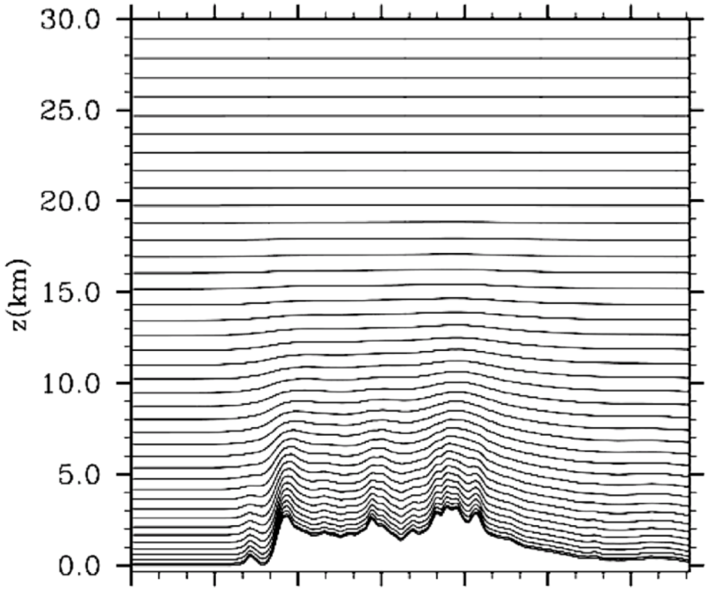
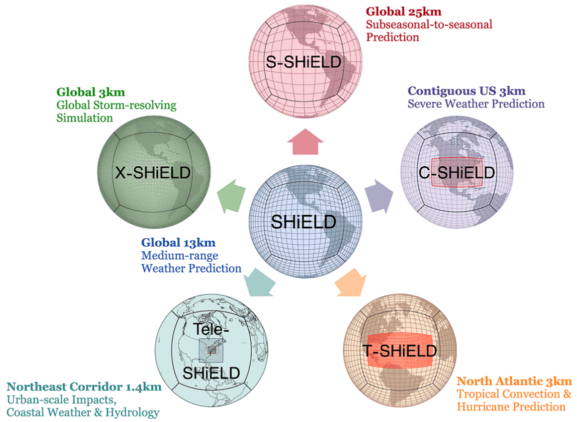
Grade Vertical

Qual a diferença?

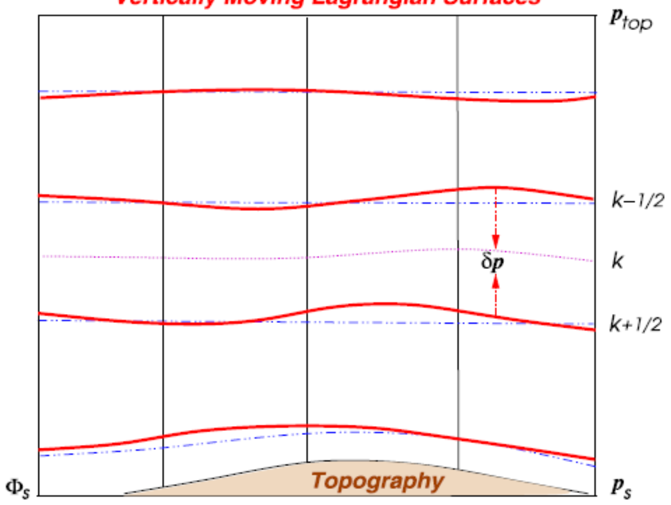
MPAS
Model for Prediction Across Scales



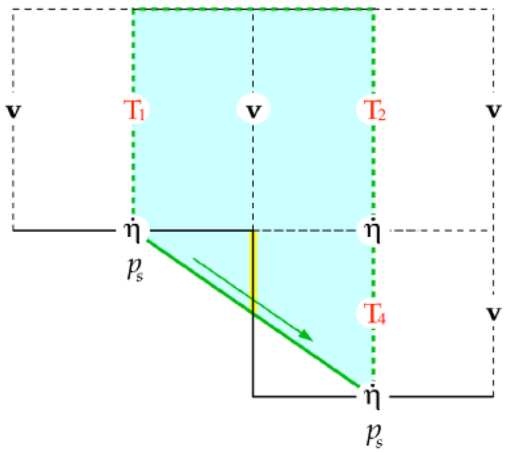


SHiELD System for High-resolution prediction on Earth-to-Local Domains

Vertically Moving Lagrangian Surfaces



GEF
Global Eta FrameWork



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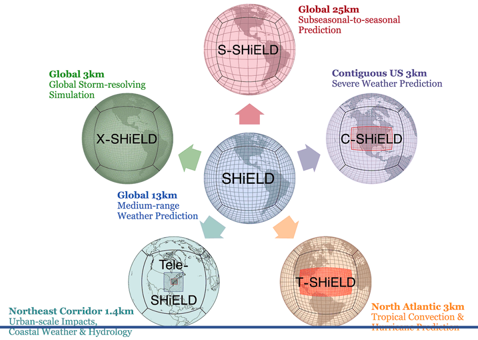
Uma visão geral da estrutura das malhas

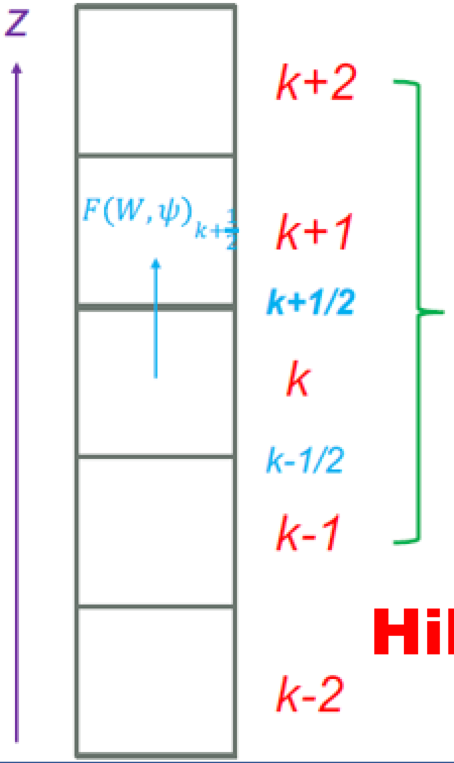
MPAS
Model for Prediction Across Scales

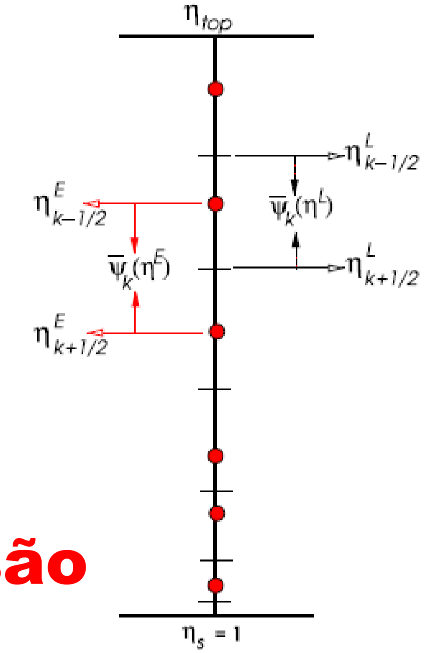


Deslocamento das malhas das variáveis de momentum em relação às variáveis de massa

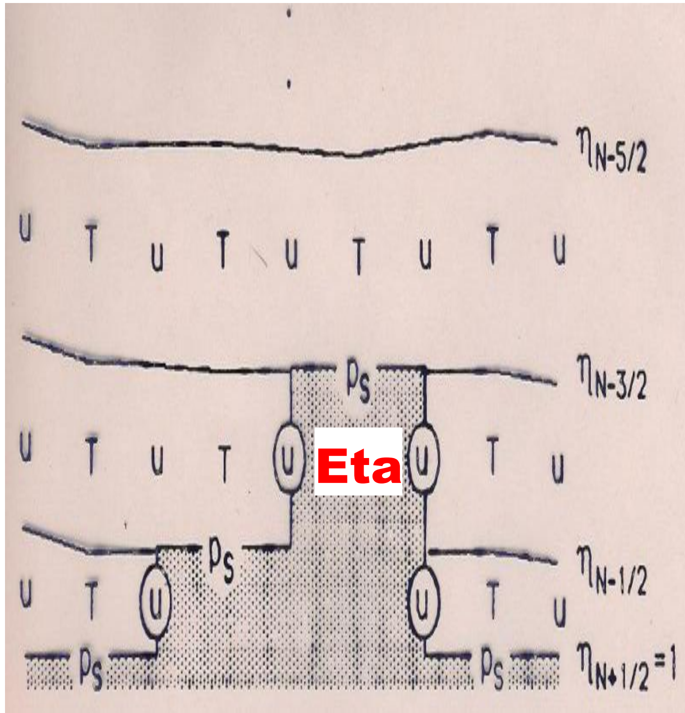
GEF
Global Eta FrameWork

SHiELD System for High-resolution prediction on Earth-to-Local Domains





Hibrido Sig-Pressão



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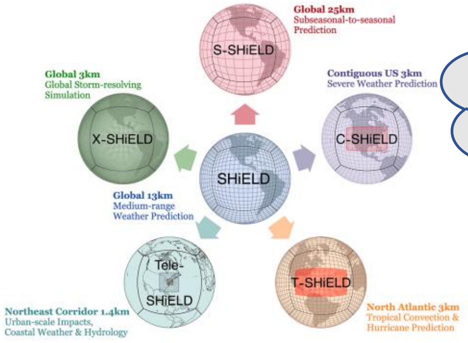
Uma visão geral da estrutura das malhas

MPAS
Model for Prediction Across Scales

3 (Volume Finito) Piecewise de 3 ou 4 ordem para a advection vertical



SHiELD System for High-resolution prediction on Earth-to-Local Domains

3 (Semi-Lagrangiano) Interpolação conservativa

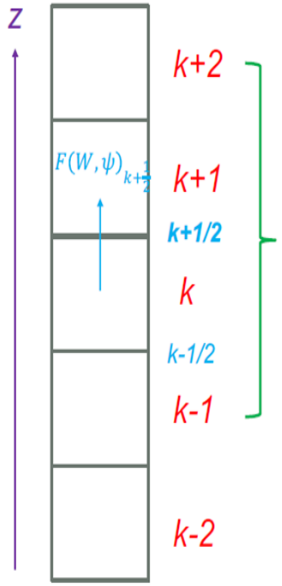
Diferenças Finitas -> Erros grandes

GEF
Global Eta FrameWork

3 (Volume Finito) Piecewise linear para a advection vertical

polinômio de ordem N=1

Vertical Discretization



$W = \rho w$

$\delta_z^2 \psi_k = \psi_{k-1} - 2\psi_k + \psi_{k+1}$

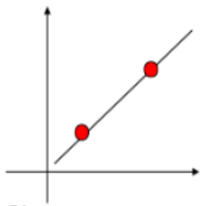
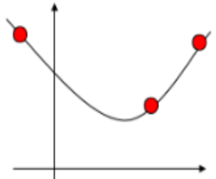
$$F(W,\psi)_{k+\frac{1}{2}} = W_{k+\frac{1}{2}}\left[\frac{1}{2}(\psi_{k+1}+\psi_k) - \frac{1}{12}(\delta_z^2\psi_{k+1}+\delta_z^2\psi_k) + sign(W)\frac{\beta}{12}(\delta_z^2\psi_{k+1}-\delta_z^2\psi_k)\right]$$

$\beta = 0$: 4th-order scheme, neutral

$\beta > 0$: 3rd-order scheme, damping

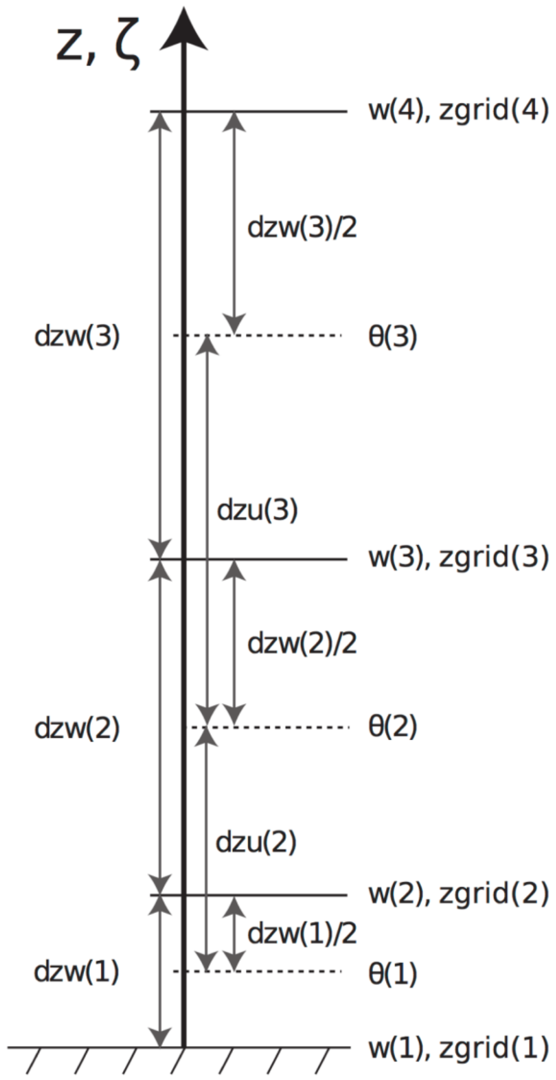
Remapeamento nas coordenadas eulerianas de referência usando o método semi-lagrangiano conservativo de integração de célula 1-D (CISL) de Nair e Machenhauer [20].

Cuidados com a conservação de Massa

Linear Interpolation	Quadratic Interpolation
	
Given any two points, there is one polynomial of order ≤ 1 that passes through the two points.	Given any three points there is one polynomial of order ≤ 2 that passes through the three points.

Uma visão geral da estrutura das malhas MPAS

Vertical grid



A grade vertical MPAS-Atmosphere também é deslocada:
- velocidades verticais em níveis w
- todos os outros campos em níveis Θ

zgrid fornece a altura geométrica nos níveis w

Os níveis Θ estão nos pontos médios dos níveis w de colchetes

Para interpolar verticalmente o campo F dos níveis theta para os níveis w:

Cuidados ao trabalhar com os índices

$$fzp(k) = 0.5 * dzw(k) / dzu(k)$$

$$fzm(k) = 0.5 * dzw(k-1) / dzu(k)$$

$$F_w(k) = fzm(k) * \mathbf{F_\Theta(k)} + fzp(k) * \mathbf{F_\Theta(k-1)}$$

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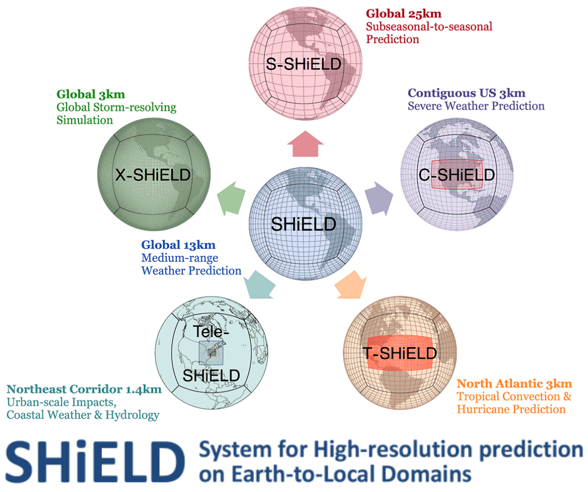
Grade com Refinamento Variável

MPAS
Model for Prediction Across Scales



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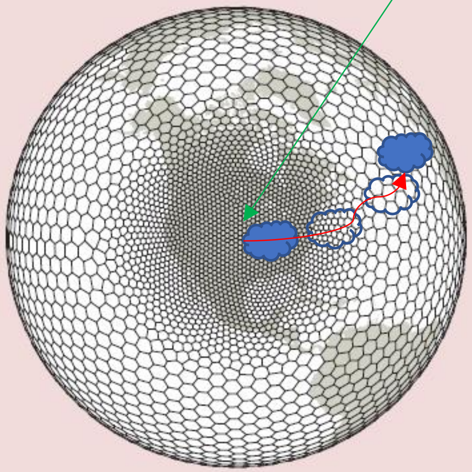
1.1 Grade com Refinamento Variável

MPAS tem maior flexibilidade no refinamento de malha

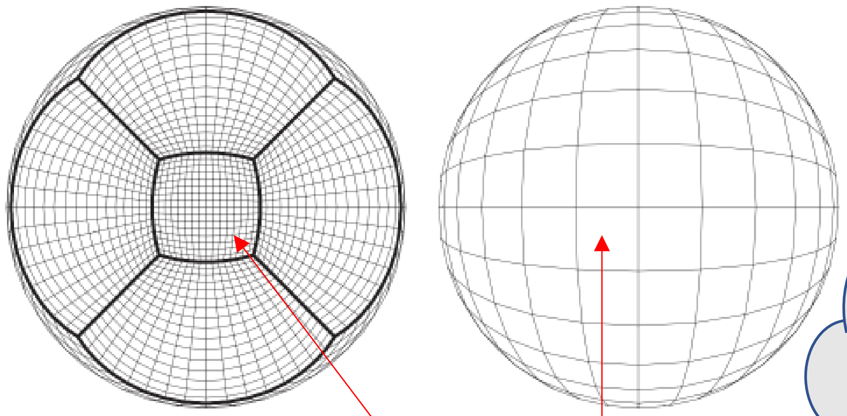
Escolha pelo Refinamento com menor distorção



MPAS
Smooth grid refinement on a conformal mesh

- Increased accuracy and flexibility for variable resolution applications
- No abrupt mesh transitions.

FV3 Mesh Refinement

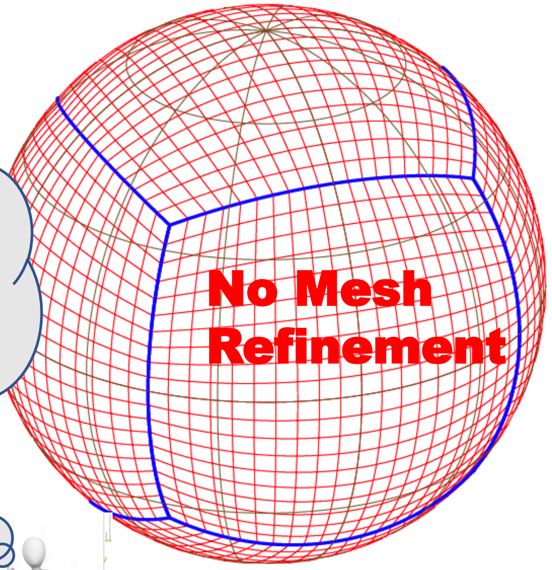


3x stretched-grid refinement from uniform

Opposite side of stretched-grid

Grandes distorções podem não resolver alguns fenômenos

GEF



FV3 Resolução muto pobre no lado oposto da área com refinamento de malha

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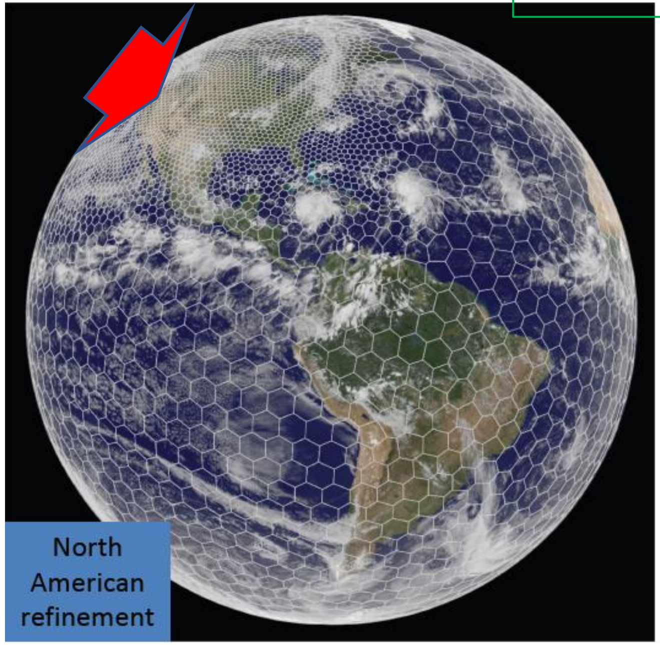
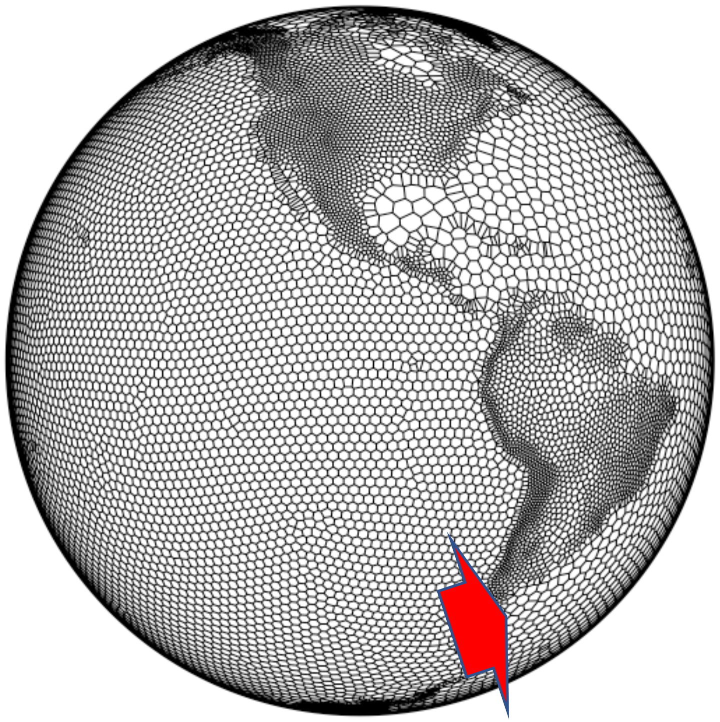
MPAS
Model for Prediction Across Scales

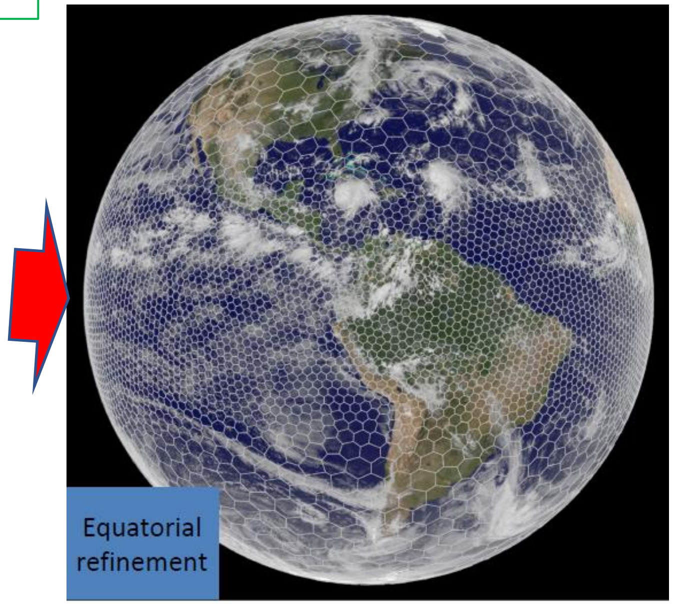
1.1 Grade com Refinamento Variável

MPAS
tem maior flexibilidade no refinamento de malha





Selective mesh refinement based on centroidal Voronoi terrain height (courtesy Michael Duda).



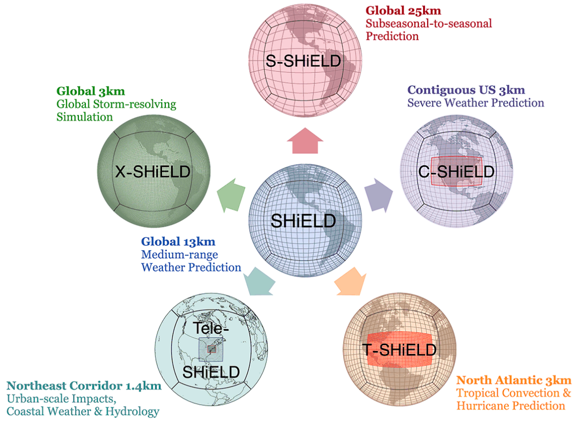
Esta Malha de Voronoi Menor Distorção

Esta Malha de Voronoi pode ser considerada a ideal para o MONAN

Paulo Yoshio

Regionalização Com Fronteira Aberta

MPAS
Model for Prediction Across Scales

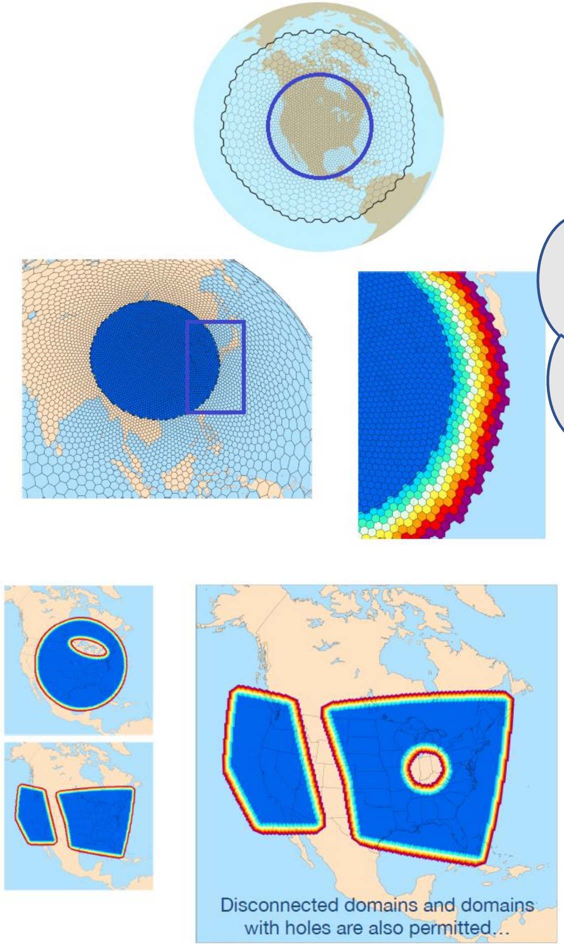


SHiELD System for High-resolution prediction on Earth-to-Local Domains

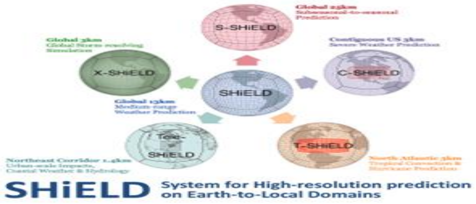
GEF
Global Eta FrameWork

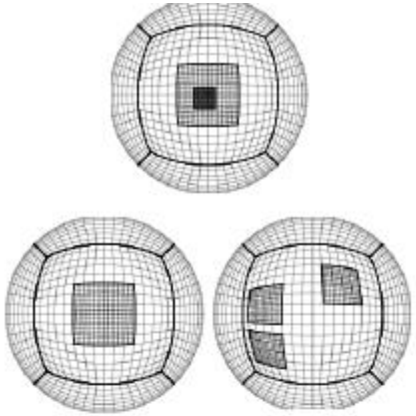
1.2 Regionalização com fronteira aberta

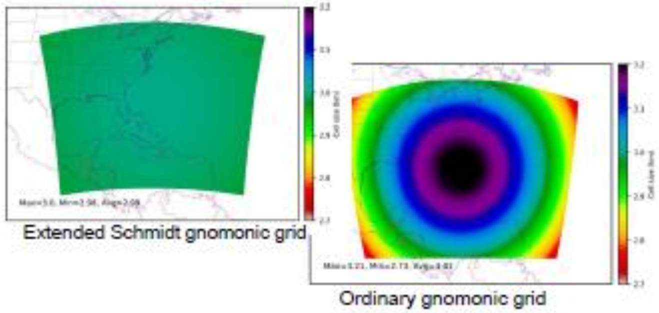
MPAS
Model for Prediction Across Scales



Flexibilidad e em configurar Domínios de fronteira aberta



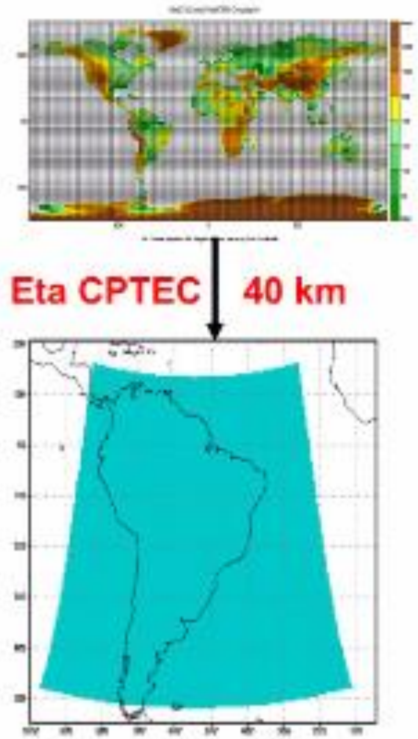




Regional-Domain Grid-cell Width
Courtesy Jim Purser and
Chan-Hoo Jeon (NCEP/EMC)

GEF
Global Eta FrameWork

Modelo Eta global

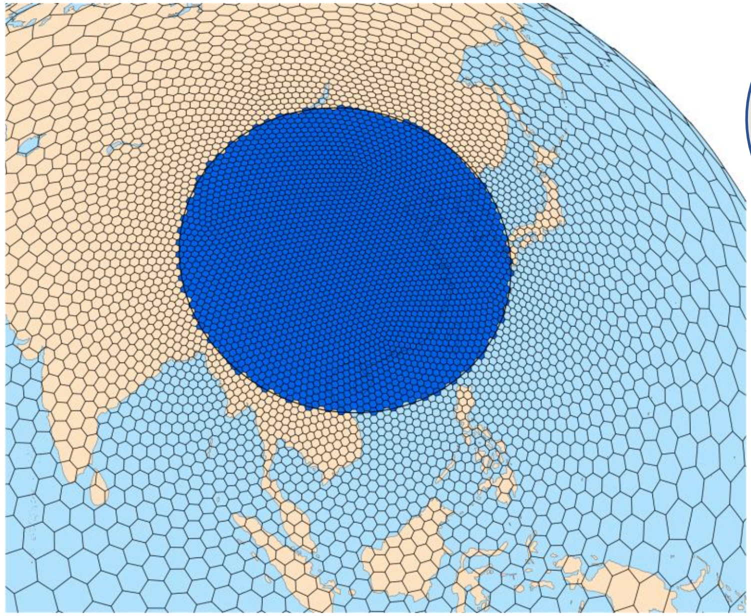


Paulo Yoshio Kubota

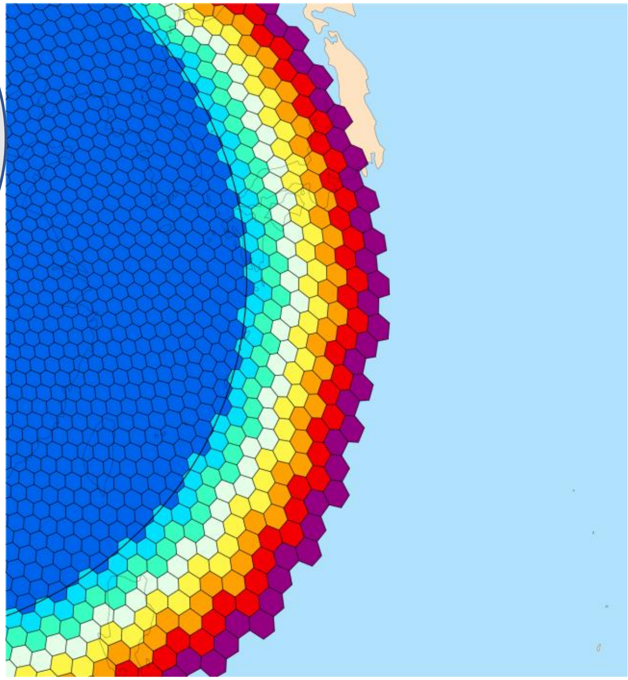
1.2 Regionalização com fronteira aberta

Comece com uma malha global, identifique o domínio regional desejado

Construir a zona de relaxamento
Construir a zona especificada
Recorte as células globais restantes



A zona de relaxamento permite uma melhor downscale da condição de contorno global



1.2 Regionalização com fronteira aberta

Especificação de Zona Fronteira Regional MPAS

Filtros na zona de relaxamento

$$\frac{\partial\psi}{\partial t} = RHS_\psi + F_1(\psi_{LS} - \psi) - F_2\Delta x^2\nabla^2(\psi_{LS} - \psi)$$

Amortecimento de Rayleigh para o valor de grande escala (LS)

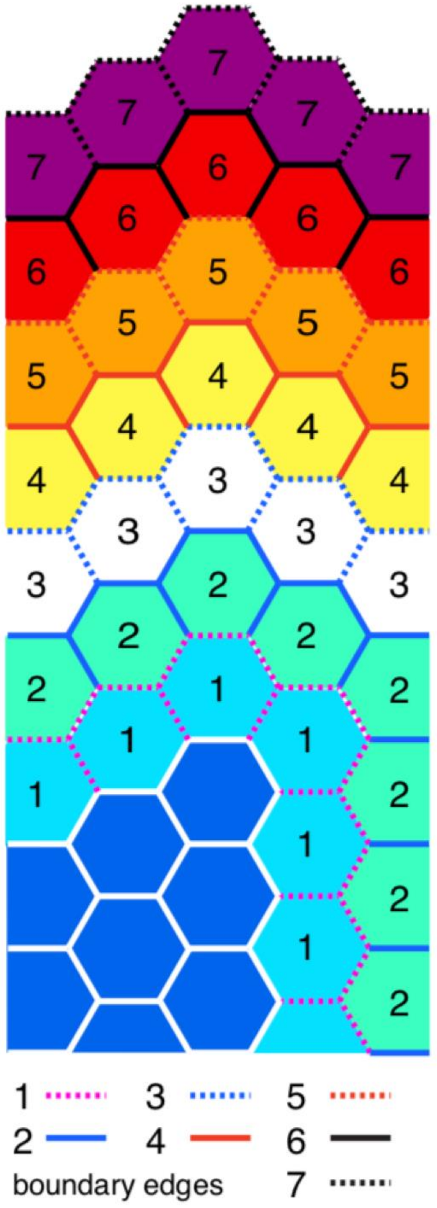
Amortecimento espacial de 2ª ordem da perturbação do valor (LS)

$$F_1 = \gamma_1\frac{(i-1)}{m} \qquad F_2 = \gamma_2\frac{(i-1)}{m}$$

m é a largura da zona de relaxamento (em # células). Exemplo (à direita), m = 5 (padrão)

$$F_1 = \gamma_1(0, 0.2, 0.4, 0.6, 0.8) \qquad F_2 = \gamma_2(0, 0.2, 0.4, 0.6, 0.8)$$

$$\gamma_1 = (0.06\Delta x)^{-1} \qquad \gamma_2 = (0.3\Delta x)^{-1}$$



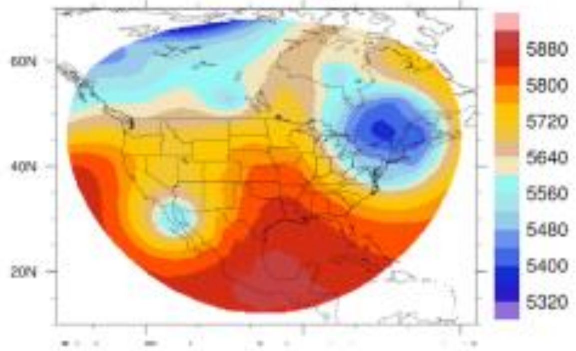
Esses valores (γ_1 e γ_2) são hardwire no MPAS

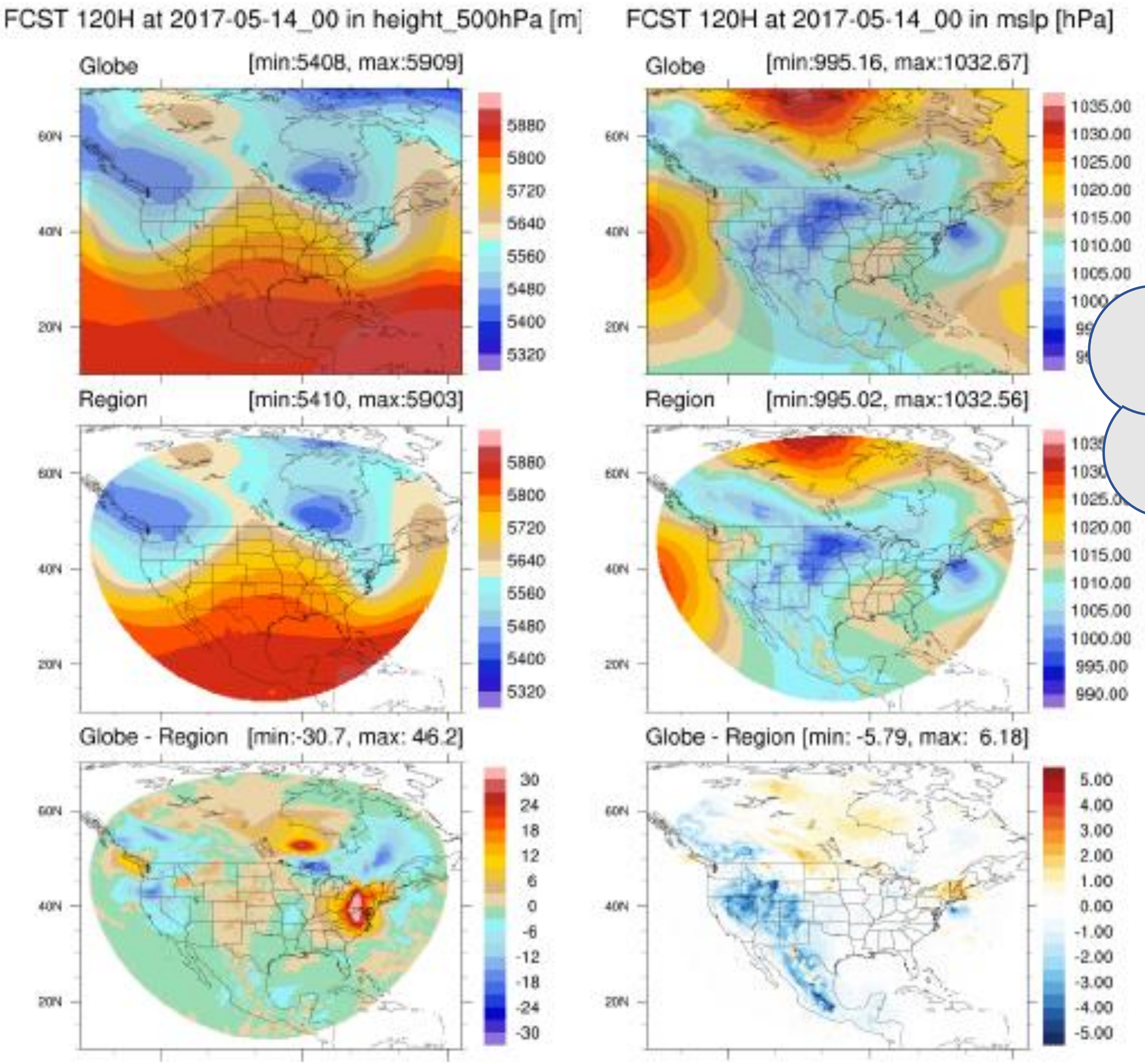
1.2 Regionalização com fronteira aberta

Regional MPAS

NCAR real-time forecasts
November 2016 - June 2017
Daily 10-day MPAS forecasts
00 UTC GFS analysis initialization

Regional initialization
500 hPa height field (m)
2017-05-09_00



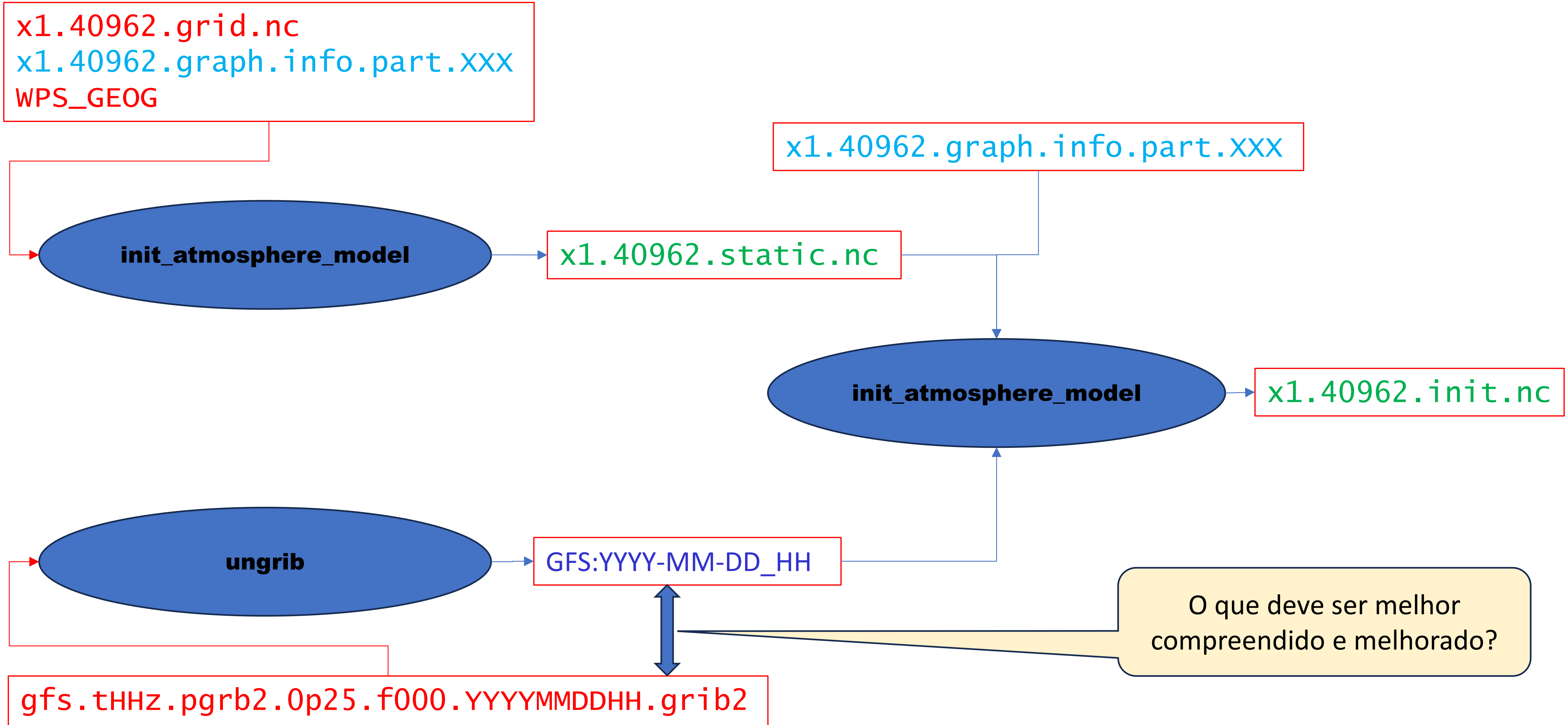


Impacto da regionalização

Global - Regional

Paulo Yoshio Kubota

Condição Inicial do modelo MONAN



Paulo Yoshio Kubota

Condição Inicial do modelo MONAN

O que se sabe até agora?

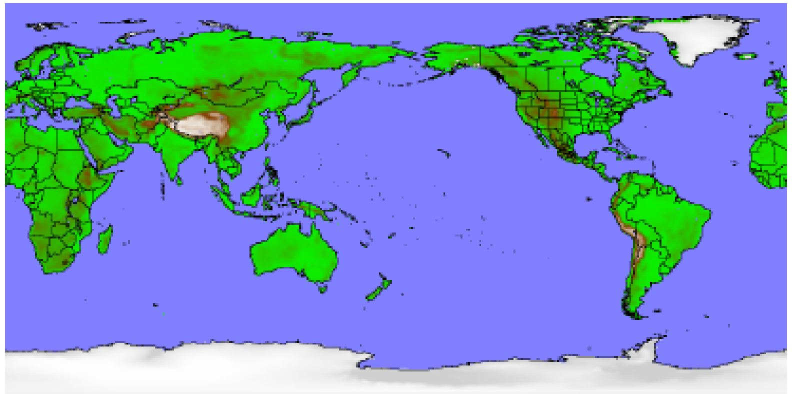
/oper/dados/ioper/tempo/GFS/..../gfs.t00z.pgrb2.0p25.anl.2024041300.grib2

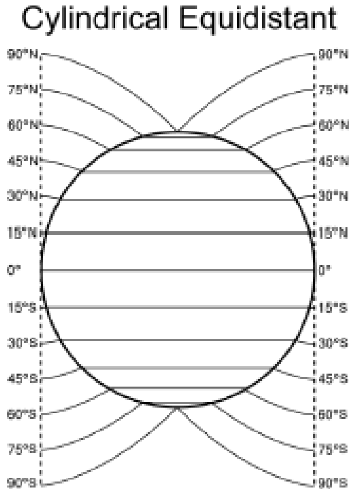
/oper/dados/ioper/tempo/GFS/..../gfs.t00z.pgrb2.0p25.f000.2024041300.grib2

Condição Inicial do modelo MONAN

/oper/dados/ioper/tempo/GFS/..../gfs.t00z.pgrb2.0p25.anl.2024041300.grib2

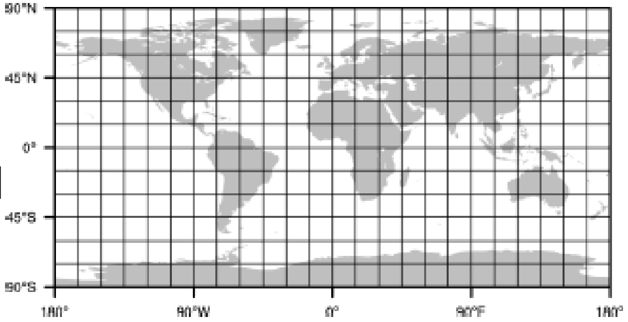
/oper/dados/ioper/tempo/GFS/..../gfs.t00z.pgrb2.0p25.f000.2024041300.grib2





Necessário para domínios globais

- Pode ser usado para domínios regionais
- Pode ser usado em sua forma normal ou rotacionada



The WRF Users' Basic Tutorial

ungrib

O que acontece neste passo?

GFS:YYYY-MM-DD_HH

Paulo Yoshio Kubota

Condição Inicial do modelo MONAN

GRIB1 Param	Level Type	From Level1	To Level2	metgrid Name	metgrid Units	metgrid Description	GRIB2 Discp	GRIB2 Catgy	GRIB2 Param	GRIB2 Level
11	100	*		TT	K	Temperature	0	0	0	100
33	100	*		UU	m s-1	U	0	2	2	100
34	100	*		VV	m s-1	V	0	2	3	100
52	100	*		RH	%	Relative Humidity	0	1	1	100
7	100	*		HGT	m	Height	0	3	5	100
11	105	2		TT	K	Temperature at 2 m	0	0	0	103
52	105	2		RH	%	Relative Humidity at 2 m	0	1	1	103
33	105	10		UU	m s-1	U at 10 m	0	2	2	103
34	105	10		VV	m s-1	V at 10 m	0	2	3	103
1	1	0		PSFC	Pa	Surface Pressure	0	3	0	1
130	102	0		PMSL	Pa	Sea-level Pressure	0	3	192	101
144	112	0	10	SM000010	fraction	Soil Moist 0-10 cm below grn layer (Up)	2	0	192	106
144	112	10	40	SM010040	fraction	Soil Moist 10-40 cm below grn layer	2	0	192	106
144	112	40	100	SM040100	fraction	Soil Moist 40-100 cm below grn layer	2	0	192	106
144	112	100	200	SM100200	fraction	Soil Moist 100-200 cm below gr layer	2	0	192	106
144	112	10	200	SM010200	fraction	Soil Moist 10-200 cm below gr layer	2	0	192	106
11	112	0	10	ST000010	K	T 0-10 cm below ground layer (Upper)	0	0	0	106
11	112	10	40	ST010040	K	T 10-40 cm below ground layer (Upper)	0	0	0	106
11	112	40	100	ST040100	K	T 40-100 cm below ground layer (Upper)	0	0	0	106
11	112	100	200	ST100200	K	T 100-200 cm below ground layer (Bottom)	0	0	0	106
85	112	0	10	ST000010	K	T 0-10 cm below ground layer (Upper)	2	0	2	106
85	112	10	40	ST010040	K	T 10-40 cm below ground layer (Upper)	2	0	2	106
85	112	40	100	ST040100	K	T 40-100 cm below ground layer (Upper)	2	0	2	106
85	112	100	200	ST100200	K	T 100-200 cm below ground layer (Bottom)	2	0	2	106
11	112	10	200	ST010200	K	T 10-200 cm below ground layer (Bottom)	0	0	0	106
91	1	0		SEAICE	proprtn	Ice flag	10	2	0	1
81	1	0		LANDSEA	proprtn	Land/Sea flag (1=land, 0 or 2=sea)	2	0	0	1
81	1	0		LANDN	proprtn		2	0	218	1
7	1	0		SOILHGT	m	Terrain field of source analysis	0	3	5	1
11	1	0		SKINTEMP	K	Skin temperature	0	0	0	1
65	1	0		SNOW	kg m-2	Water equivalent snow depth	0	1	13	1
	1	0		SNOWH	m	Physical Snow Depth	0	1		1
33	6	0		UMAXW	m s-1	U at max wind	0	2	2	6
34	6	0		VMAXW	m s-1	V at max wind	0	2	3	6
2	6	0		PMAXW	Pa	Pressure of max wind level	0	3	0	6
	6	0		PMAXWNN	Pa	PMAXW, used for nearest neighbor interp	0	3	0	6
2	6	0		TMAXW	K	Temperature at max wind level	0	0	0	6
7	6	0		HGTMAXW	m	Height of max wind level	0	3	5	6
33	7	0		UTROP	m s-1	U at tropopause	0	2	2	7
34	7	0		VTROP	m s-1	V at tropopause	0	2	3	7
2	7	0		PTROP	Pa	Pressure of tropopause	0	3	0	7
	7	0		PTROPNN	Pa	PTROP, used for nearest neighbor interp	0	3	0	7
2	7	0		TTROP	K	Temperature at tropopause	0	0	0	7
7	7	0		HGTTROP	m	Height of tropopause	0	3	5	7

gfs.t00z.pgrb2.0p25.anl.2024041300.grib2

Estas variáveis não está no arquivo de análise

Estas variáveis são necessárias para domínios globais?

gfs.t00z.pgrb2.0p25.f000.2024041300.grib2

Paulo Yoshio Kubota

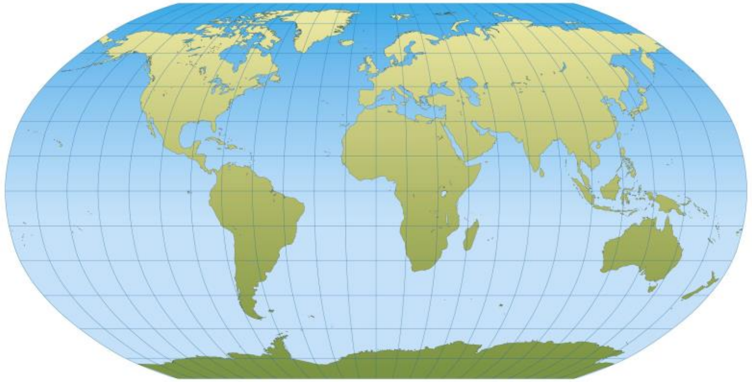
Condição Inicial do modelo MONAN

gdas.T00Z.sfcanl.netcdf.2024041300
gdas.T00Z.atmanl.netcdf.2024041300

Eccodes
Ou
wgrib2api

gdas.T00Z.anl.grib2.2024041300

GRIB1 Param	Level Type	From Level1	To Level2	metgrid Name	metgrid Units	metgrid Description	GRIB2 Discp	GRIB2 Catgy	GRIB2 Param	GRIB2 Level
11	100	*		TT	K	Temperature	0	0	0	100
33	100	*		UU	m s-1	U	0	2	2	100
34	100	*		VV	m s-1	V	0	2	3	100
52	100	*		RH	%	Relative Humidity	0	1	1	100
7	100	*		HGT	m	Height	0	3	5	100
11	105	2		TT	K	Temperature at 2 m	0	0	0	103
52	105	2		RH	%	Relative Humidity at 2 m	0	1	1	103
33	105	10		UU	m s-1	U at 10 m	0	2	2	103
34	105	10		VV	m s-1	V at 10 m	0	2	3	103
1	1	0		PSFC	Pa	Surface Pressure	0	3	0	1
130	102	0		PMSL	Pa	Sea-level Pressure	0	3	192	101
144	112	0	10	SM000010	fraction	Soil Moist 0-10 cm below grn layer (Up)	2	0	192	106
144	112	10	40	SM010040	fraction	Soil Moist 10-40 cm below grn layer	2	0	192	106
144	112	40	100	SM040100	fraction	Soil Moist 40-100 cm below grn layer	2	0	192	106
144	112	100	200	SM100200	fraction	Soil Moist 100-200 cm below gr layer	2	0	192	106
144	112	10	200	SM010200	fraction	Soil Moist 10-200 cm below gr layer	2	0	192	106
11	112	0	10	ST000010	K	T 0-10 cm below ground layer (Upper)	0	0	0	106
11	112	10	40	ST010040	K	T 10-40 cm below ground layer (Upper)	0	0	0	106
11	112	40	100	ST040100	K	T 40-100 cm below ground layer (Upper)	0	0	0	106
11	112	100	200	ST100200	K	T 100-200 cm below ground layer (Bottom)	0	0	0	106
85	112	0	10	ST000010	K	T 0-10 cm below ground layer (Upper)	2	0	2	106
85	112	10	40	ST010040	K	T 10-40 cm below ground layer (Upper)	2	0	2	106
85	112	40	100	ST040100	K	T 40-100 cm below ground layer (Upper)	2	0	2	106
85	112	100	200	ST100200	K	T 100-200 cm below ground layer (Bottom)	2	0	2	106
11	112	10	200	ST010200	K	T 10-200 cm below ground layer (Bottom)	0	0	0	106
91	1	0		SEAICE	proprtn	Ice flag	10	2	0	1
81	1	0		LANDSEA	proprtn	Land/Sea flag (1=land, 0 or 2=sea)	2	0	0	1
81	1	0		LANDN	proprtn		2	0	218	1
7	1	0		SOILHGT	m	Terrain field of source analysis	0	3	5	1
11	1	0		SKINTEMP	K	Skin temperature	0	0	0	1
65	1	0		SNOW	kg m-2	Water equivalent snow depth	0	1	13	1
	1	0		SNOWH	m	Physical Snow Depth	0	1		1



Chopping_BAM

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Paulo Yoshio Kubota

“Existem dois executáveis que precisamos inicializar e executar uma simulação do MPAS-Atmosphere:

init_atmosphere_model

- Lida com todas as etapas do processamento de condições iniciais com dados reais
- Lida com o processamento de arquivos de atualização de SST e gelo marinho
- Lida com a criação de várias condições iniciais idealizadas
- Lida com a criação de condições de contorno lateral

atmosphere_model

- O próprio modelo, responsável por realizar a integração/simulação dadas quaisquer fontes de condições iniciais"

"Como o núcleo 'init_atmosphere' consegue combinar toda essa funcionalidade em um único programa?

1. A ideia chave é que init_atmosphere_model pode ser executado em estágios usando diferentes opções"

```
&nhyd_model
config_init_case = 7
...
/
&preproc_stages
config_static_interp = true
config_native_gwd_static = true
config_vertical_grid = false
config_met_interp = false
config_input_sst = false
config_frac_seaice = false
/
```

2 = ideal baroclinic wave
4 = ideal squall line
5 = ideal supercell
6 = ideal mountain wave
7 = real-data initialization
8 = surface update file creation
9 = lateral boundary conditions

Only used for real-data cases to control, e.g., whether we interpolate static fields, compute fields for GWDO scheme, interpolate meteorological data, etc.

"Em geral, existem dois arquivos que devem ser editados toda vez que o programa init_atmosphere_model é executado:"

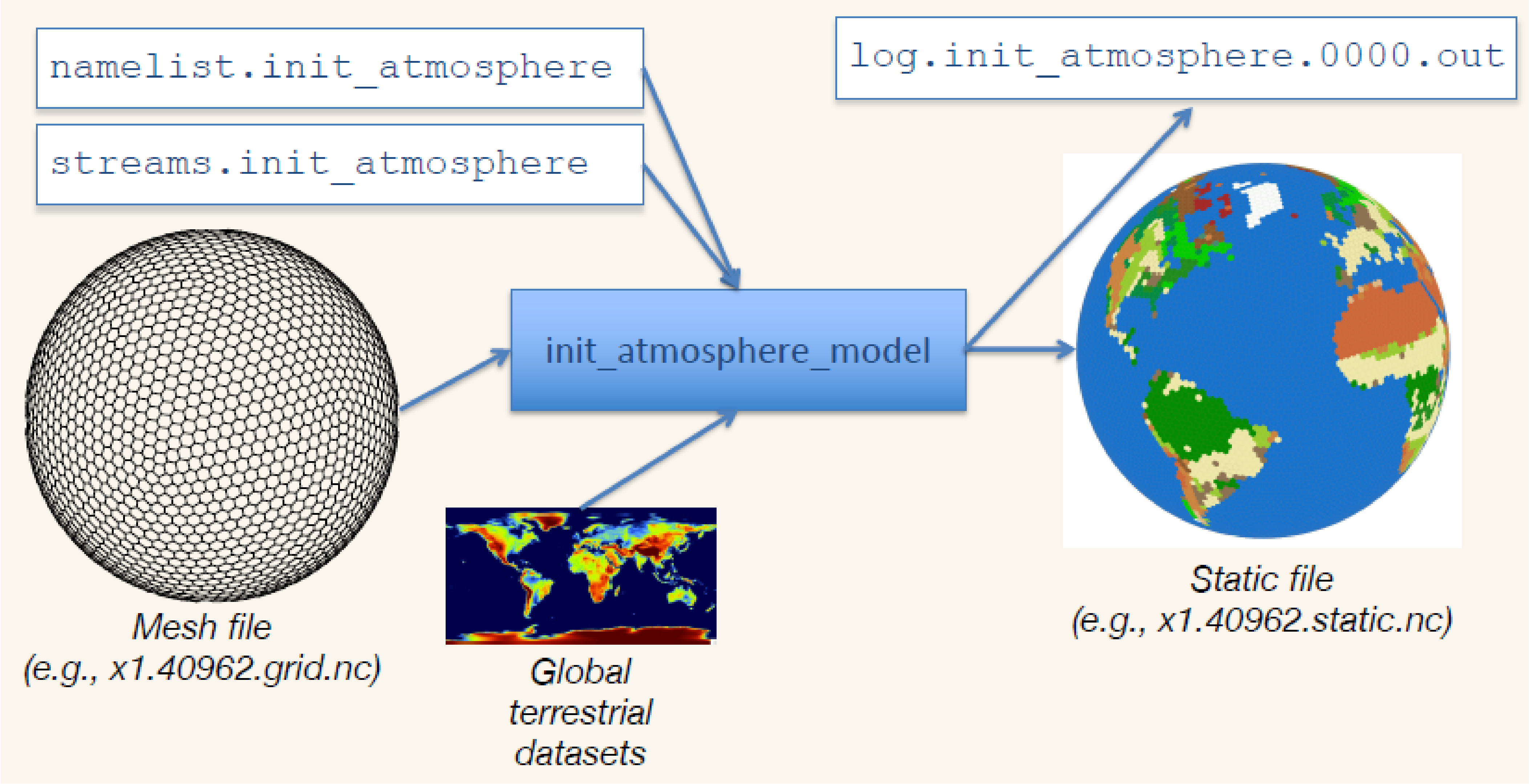
namelist.init_atmosphere

"• Arquivo de namelist do Fortran
• Determina qual "caso" será preparado (por exemplo, casos idealizados, caso de dados reais)
• Determina sub-opções para a caso da inicialização selecionada"

streams.init_atmosphere

"• Arquivo XML
• Especifica quais arquivos netCDF serão lidos e escritos pelo programa init_atmosphere_model"

"Arquivos de entrada e saída ao produzir um arquivo "estático":"



"Configurações-chave no arquivo namelist.init_atmosphere:"

```
&nhyd_model
config_init_case = 7
/
&data_sources
config_geog_data_path = '/glade/work/wrfhelp/WPS_GEOG/'
config_landuse_data = 'MODIFIED_IGBP_MODIS_NOAH'
config_topo_data = 'GMTED2010'
config_vegfrac_data = 'MODIS'
config_albedo_data = 'MODIS'
config_maxsnowalbedo_data = 'MODIS'
/
&preproc_stages
config_static_interp = true
config_native_gwd_static = true
config_vertical_grid = false
config_met_interp = false
config_input_sst = false
config_frac_seaice = false
/
```

Key settings in the streams.init_atmosphere file:

```
<immutable_stream name="input"
        type="input"
        filename_template="x1.40962.grid.nc"
        input_interval="initial_only" />


<immutable_stream name="output"
        type="output"
        filename_template="x1.40962.static.nc"
        packages="initial_conds"
        output_interval="initial_only" />
```

The result should be a “static” netCDF file with

- terrain
- land use category
- soil category
- climatological albedo
- climatological vegetation fraction
- sub-grid-scale orography statistics for the GWDO scheme

Also, the radius of the SCVT mesh should be 6371229.0 m!

"Procure por mensagens como as seguintes no arquivo log.init_atmosphere.0000.out:"

```
--- enter subroutine init_atm_static:
Using GMTED2010 terrain dataset
/shared/WPS_GEOG/topo_gmted2010_30s/00001-01200.00001-01200
/shared/WPS_GEOG/topo_gmted2010_30s/01201-02400.00001-01200
/shared/WPS_GEOG/topo_gmted2010_30s/02401-03600.00001-01200
```

```
Computing GWDO static fields on the native MPAS mesh
--- Using GMTED2010 terrain dataset for GWDO static fields
```

```
----------------------------------------
Total log messages printed:
Output messages = 3067
Warning messages = 10
Error messages = 0
Critical error messages = 0
----------------------------------------
```

"Condições iniciais com dados reais

- Processamento de campos invariáveis no tempo ("geração de arquivo estático")
- Interpolação de campos atmosféricos e de superfície terrestre
- Produção de arquivos de atualização de SST e gelo marinho

"Campos meteorológicos e de superfície terrestre variáveis no tempo no MPAS-Atmosphere são interpolados a partir de arquivos intermediários produzidos pelo componente ungrib do Sistema de Pré-processamento do WRF.

Vamos assumir neste tutorial que esses arquivos já foram preparados!

- Detalhes adicionais podem ser encontrados nos links abaixo."

WRF Model web page: http://www2.mmm.ucar.edu/wrf/users/
WRF Users' guide:https://www2.mmm.ucar.edu/wrf/users/wrf_users_guide/build/html/index.html
WPS source code: https://github.com/wrf-model/WPS
Tutorial slides for running ungrib:http://www2.mmm.ucar.edu/wrf/users/tutorial/201801/wps_general.pdf

"Configurações-chave no arquivo :"

namelist.init_atmosphere

```
&nhyd_model
config_init_case = 7
config_start_time = '2014-09-10_00:00:00'
/
&dimensions
config_nvertlevels = 55
config_nsoillevels = 4
config_nfglevels = 38
config_nfgsoillevels = 4
/
&data_sources
config_met_prefix = 'GFS'
/
```

```
&vertical_grid
config_ztop = 30000.0
config_nsmterrain = 1
config_smooth_surfaces = true
config_dzmin = 0.3
config_nsm = 30
config_tc_vertical_grid = true
config_blend_bdy_terrain = false
/&
preproc_stages
config_static_interp = false
config_native_gwd_static = false
config_vertical_grid = true
config_met_interp = true
config_input_sst = false
config_frac_seaice = true
/
```

streams.init_atmosphere

```
<immutable_stream name="input"
                  type="input"
                  filename_template="x1.40962.static.nc"
                  input_interval="initial_only" />

<immutable_stream name="output"
                  type="output"
                  filename_template="x1.40962.init.nc"
                  packages="initial_conds"
                  output_interval="initial_only" />
```

"O resultado deve ser um arquivo netCDF "init" com

- tudo do arquivo "estático"
- informações da grade vertical 3D
- temperatura potencial 3D (theta)
- ventos 3D (u e w)
- razão de mistura de vapor de água 3D (qv)
- umidade do solo 2D
- temperatura do solo 2D"

"Procure por mensagens como as seguintes no arquivo log.init_atmosphere.0000.out:"

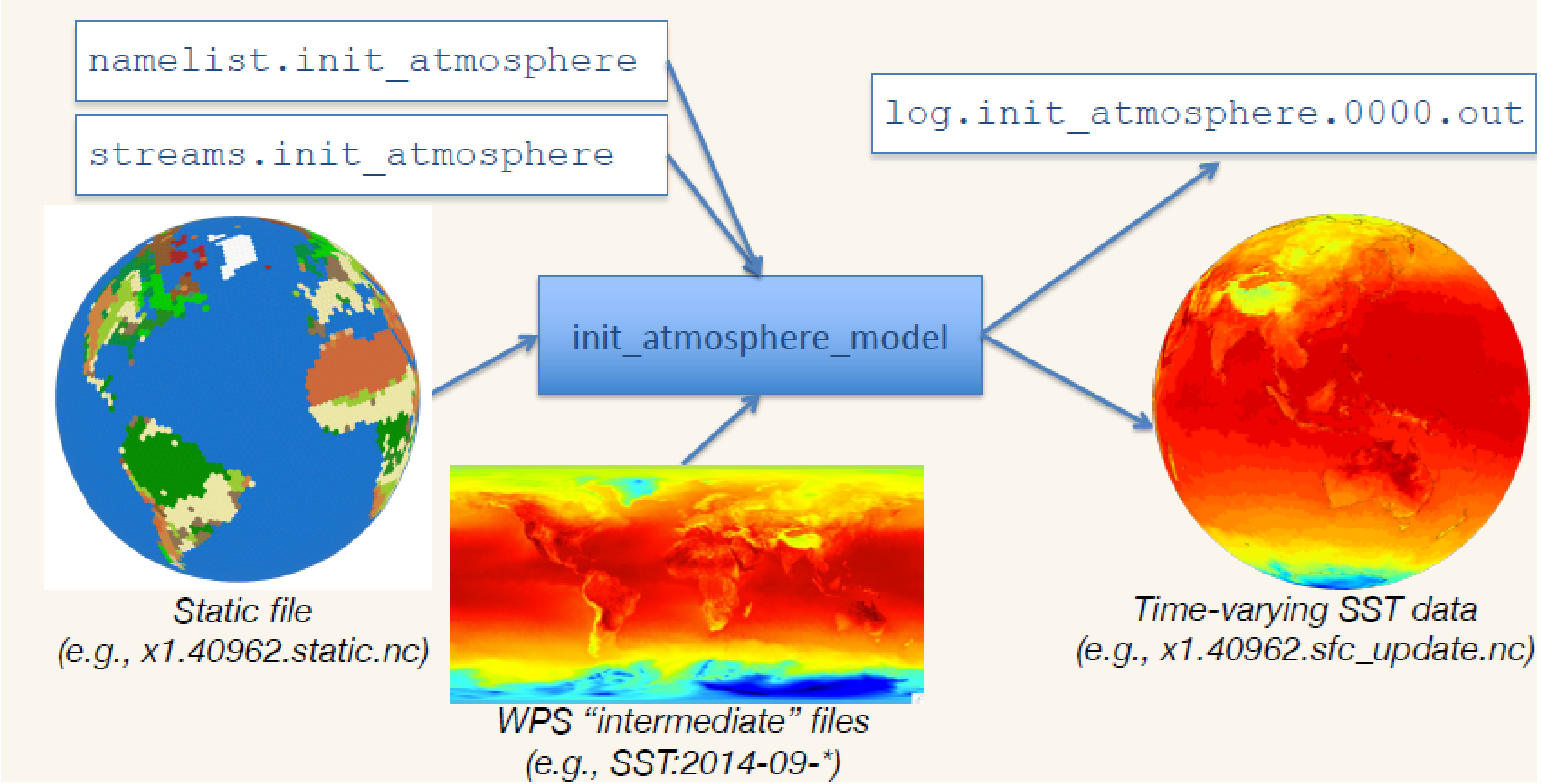
```
real-data GFS test case
Using option 'linear' for vertical extrapolation of temperature
max ter = 5393.19321458650
Setting up vertical levels as in 2014 TC experiments
--- config_tc_vertical_grid = T
--- als = 0.750000000000000E-01
--- alt = 1.70000000000000
--- zetal = 0.750000000000000
```

```
Interpolating TT at 27 1000.00000000000
Interpolating U at 27 1000.00000000000
Interpolating V at 27 1000.00000000000
Interpolating RH at 27 1000.00000000000
Interpolating GHT at 27 1000.00000000000
**************************************************
Found 27 levels in the first-guess data
**************************************************
```

"Condições iniciais com dados reais

- Processamento de campos invariáveis no tempo ("geração de arquivo estático")
- Interpolação de campos atmosféricos e de superfície terrestre
- Produção de arquivos de atualização de SST e gelo marinho

"Arquivos de entrada e saída ao produzir um arquivo de atualização de SST:"



"Configurações-chave no arquivo namelist.init_atmosphere:"

```
&nhyd_model
config_init_case = 8
config_start_time = '2014-09-10_00:00:00'
config_stop_time = '2014-09-20_00:00:00'
/
&data_sources
config_sfc_prefix = 'SST'
config_fg_interval = 86400
/
&preproc_stages
config_static_interp = false
config_native_gwd_static = false
config_vertical_grid = false
config_met_interp = false
config_input_sst = true
config_frac_seaice = true
/
```

"Configurações-chave no arquivo streams.init_atmosphere:"

```
<immutable_stream name="input"
        type="input"
        filename_template="x1.40962.static.nc"
        input_interval="initial_only" />

<immutable_stream name="surface"
        type="output"
        filename_template="x1.40962.sfc_update.nc"
        filename_interval="none"
        packages="sfc_update"
        output_interval="86400" />
```

"Procure por mensagens como as seguintes no arquivo log.init_atmosphere.0000.out:"

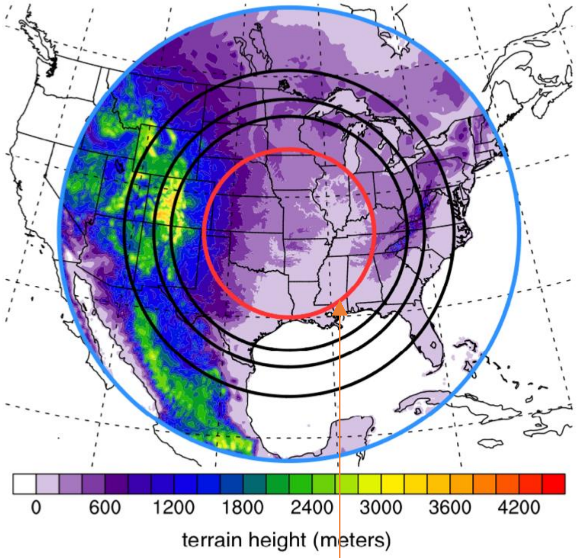
```
real-data surface (SST) update test case
Processing file SST:2014-09-10_00
Processing file SST:2014-09-11_00
Processing file SST:2014-09-12_00
Processing file SST:2014-09-13_00
Processing file SST:2014-09-14_00
Processing file SST:2014-09-15_00
Processing file SST:2014-09-16_00
Processing file SST:2014-09-17_00
Processing file SST:2014-09-18_00
Processing file SST:2014-09-19_00
Processing file SST:2014-09-20_00
```

```
----------------------------------------
Total log messages printed:
Output messages = 144
Warning messages = 0
Error messages = 0
Critical error messages = 0
----------------------------------------
```

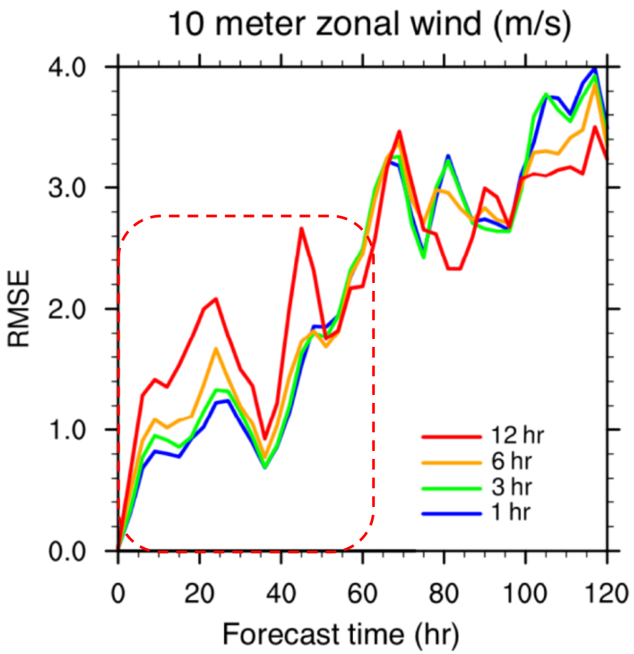
1.2 Regionalização com fronteira aberta

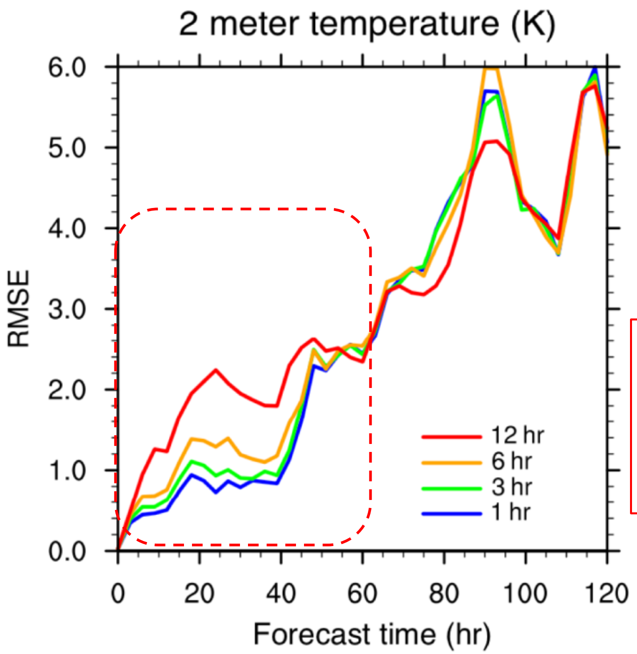
Regional MPAS

Considerações de configuração Frequência de atualização de contorno: integrações MPAS regionais de 5 dias, região do círculo vermelho (3 km de espaçamento entre células), diferentes frequências de atualização de LBC. Integração global de 3 km é verdade. Atualizações com mais frequência são melhores.



26 April 2017
00 UTC init





*Verificação sobre a pequena malha de 3 km (círculo vermelho)

Frequência das condições de contorno

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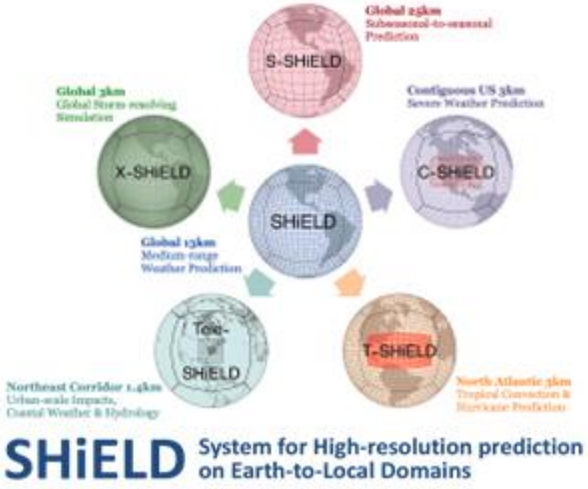
Dinâmica
Não-Hidrostática

MPAS
Model for Prediction Across Scales



GEF
Global Eta FrameWork

1.3 Dinâmica não-hidrostática,

Hydrostatic Equation

Hydrostatic approximation:

$$\frac{1}{\rho_0}\frac{\partial p}{\partial z} = b \qquad \text{H/L<<1}$$

Difference in height between upper & lower isobaric sufaces

$$\frac{\partial z}{\partial p} = -\frac{RT}{pg}$$

Mean temperature within a layer

Non-hydrostatic Equation

Vertical momentum equation for a nonhydrostatic atmosphere

Nonhydrostatic vertical pressure gradient force in a grid box, where ρ_0 is the environmental density, ρ' is the departure of pressure from the large-scale environmental hydrostatic balance

Change in vertical motion with time

$$\frac{\partial w}{\partial t} = -\left\{u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right\} \boxed{- \frac{1}{\rho_0}\frac{\partial p'}{\partial z} + gB - gq}$$

Precipitation loading, where q is specific humidity

Advection of vertical momentum by the wind field

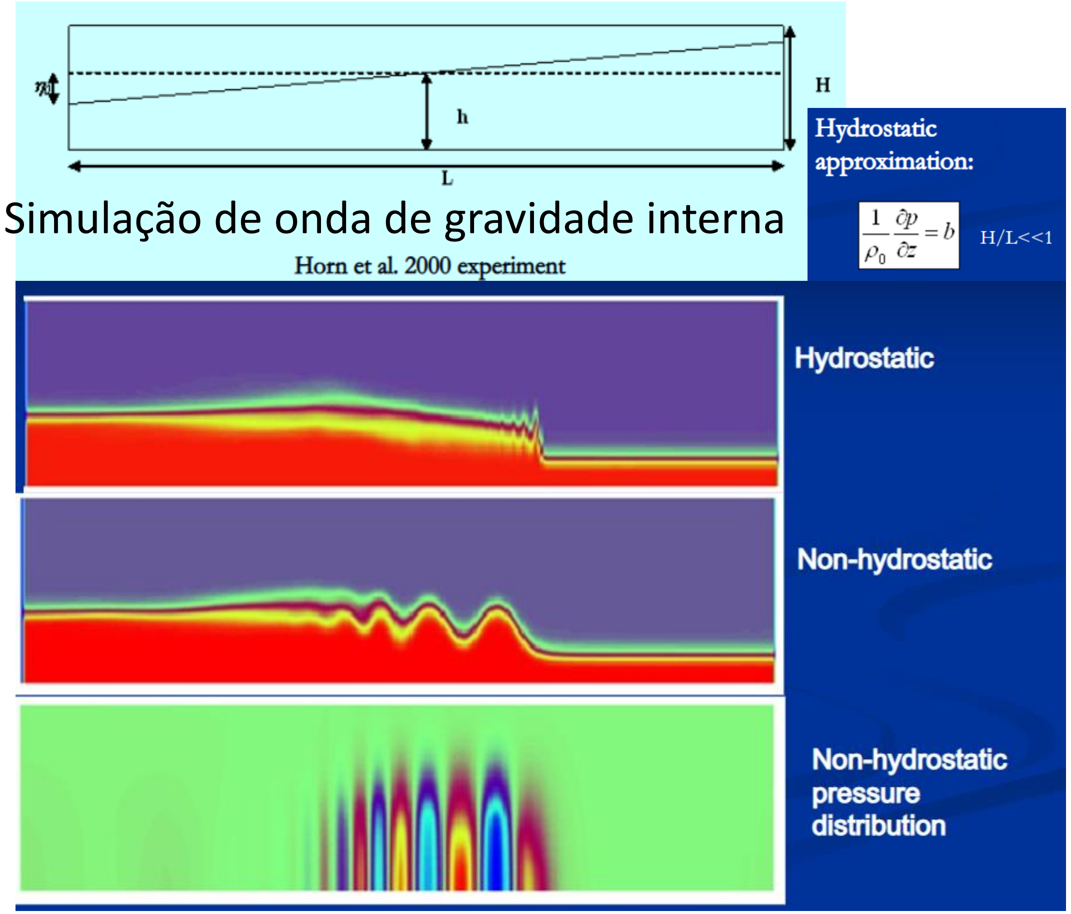
Buoyancy (B) differences from large-scale average

	Hydrostatic	Non-hydrostatic
Buoyancy	Indirect (first, horizontal pressure gradients are created, then convergence, finally vertical motion)	Direct (buoyancy - vertical motion)
Perturbation pressure acting against buoyancy	No	Yes, important for regulating convective updraft velocity and convective cloud structure and for gravity wave energy propagation
Previous vertical motion	No	Yes, vertical motion is advected
Water loading	No	Yes

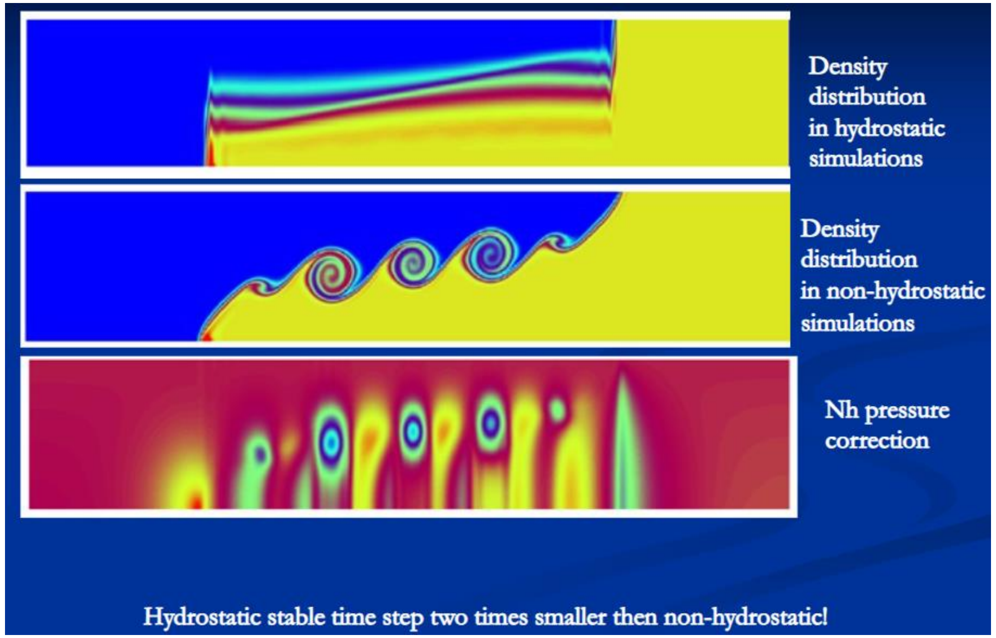
Aproximação hidrostática

$$-\frac{1}{\rho_0}\frac{\partial P'}{\partial x} = gB - gq$$

1.3 Dinâmica não-hidrostática,



Intabilidade baroclinica de Kelvin Helmoltz



Formulação /Setup	MPAS	FV3-Shied	GEF
Aproximação Vertical	nonhydrostatic	nonhydrostatic	hydrostatic

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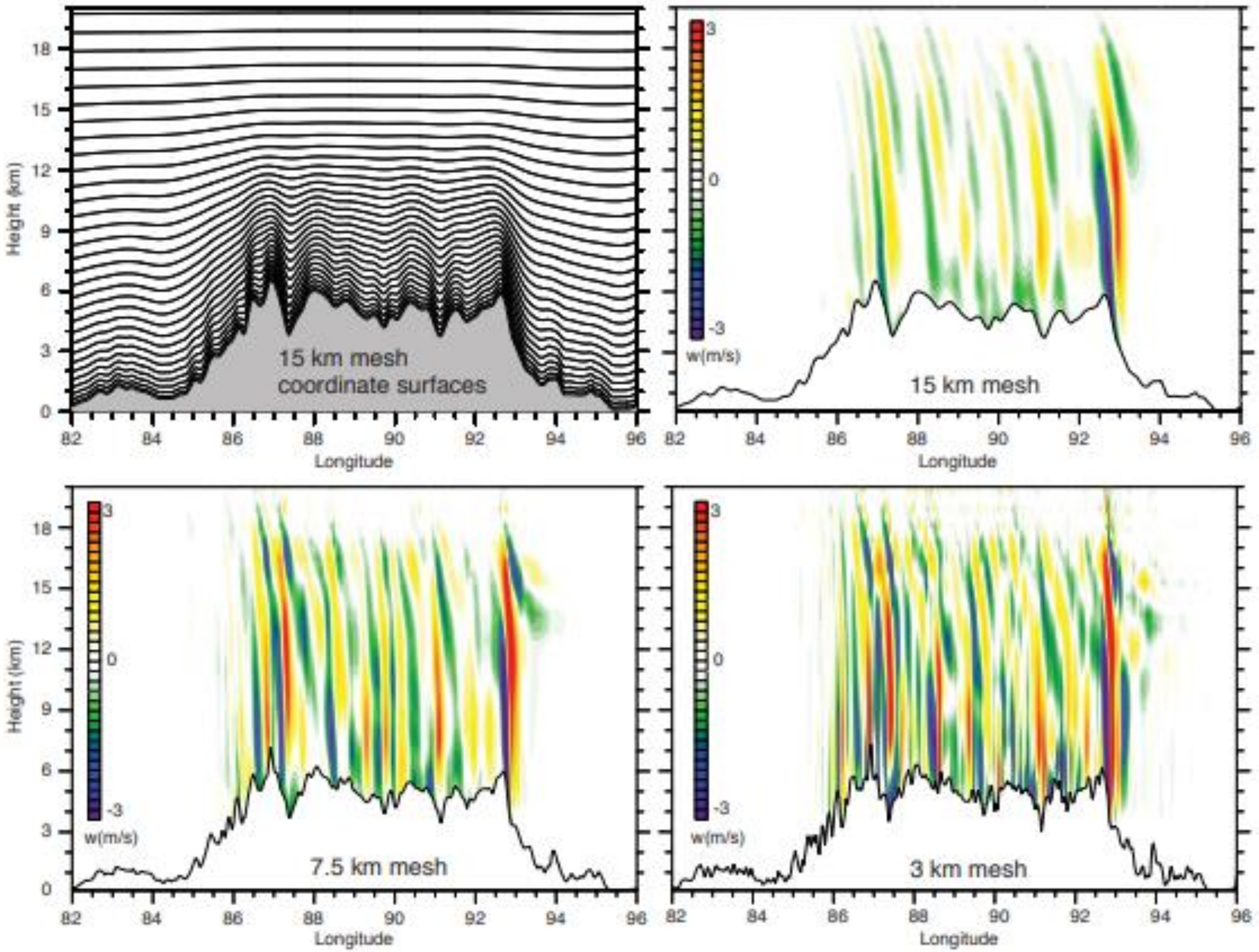
coordenada vertical híbrida MPAS-A (Klemp, 2011).

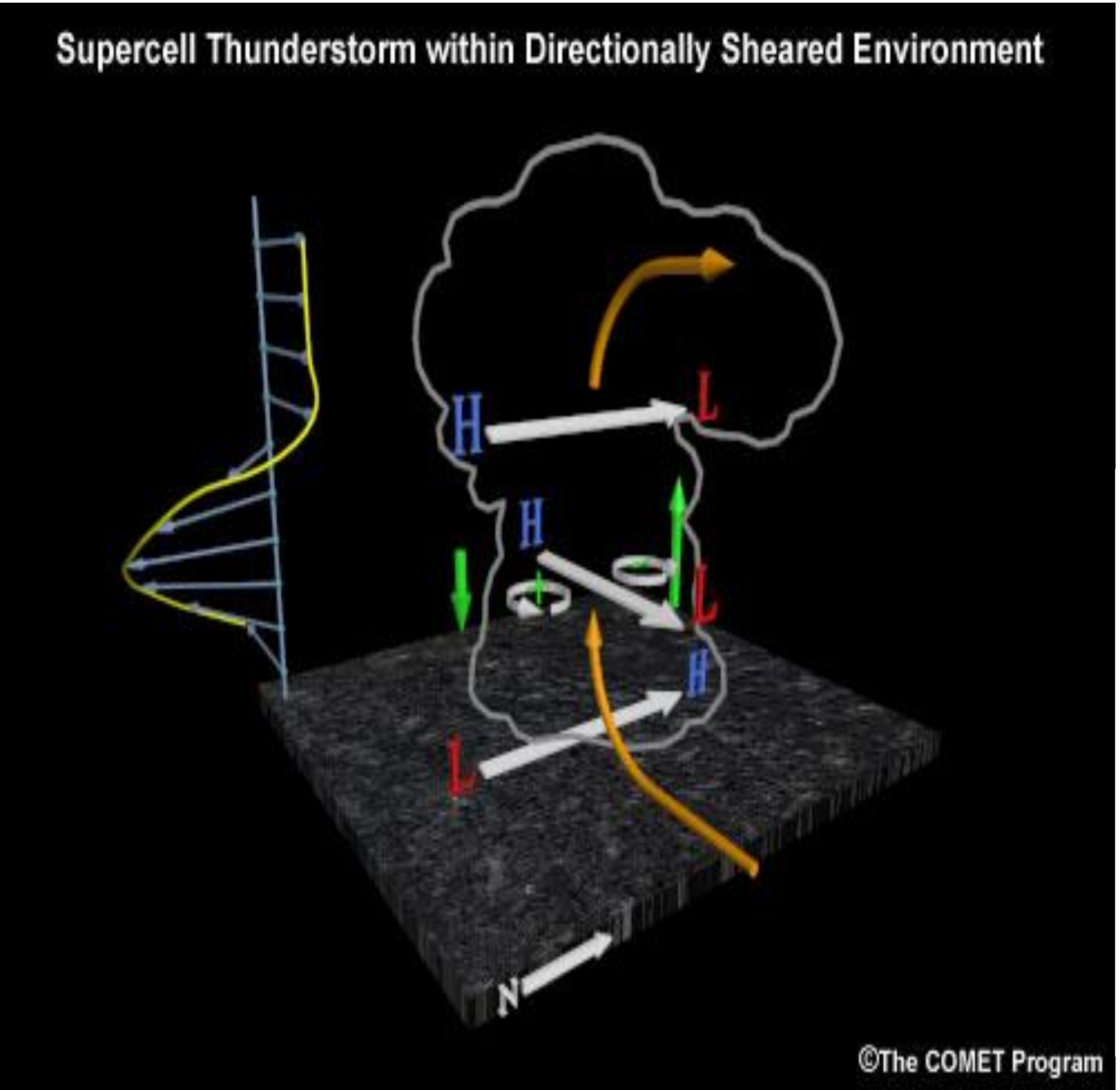


Uma das principais razões para aumentar a resolução nas escalas não hidrostáticas:

a) resolver melhor a topografia

b) começar a resolver explicitamente a estrutura e a evolução do sistema convectivo.

1.3 Dinâmica não-hidrostática,



Non-hydrostatic Equation

Vertical momentum equation for a nonhydrostatic atmosphere

Nonhydrostatic vertical pressure gradient force in a grid box, where ρ_0 is the environmental density, p' is the departure of pressure from the large-scale environmental hydrostatic balance

Change in vertical motion with time

$$\frac{\partial w}{\partial t} = -\underbrace{\left\{u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right\}}_{\mathrm{I}} \boxed{\underbrace{-\frac{1}{\rho_0}\frac{\partial p'}{\partial z}}_{\mathrm{II}} + \underbrace{gB}_{\mathrm{III}} - \underbrace{gq}_{\mathrm{VI}}}$$

Precipitation loading, where q is specific humidity

Advection of vertical momentum by the wind field

Buoyancy (B) differences from large-scale average

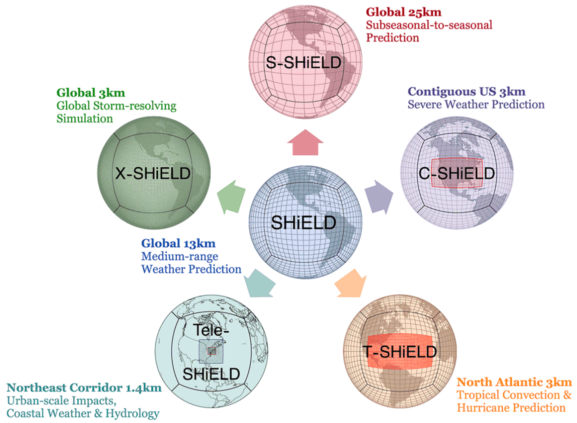
- A figura mostra um exemplo em que um modelo não hidrostático é necessário para **capturar importantes efeitos da dinâmica e da física**.

- O cisalhamento do vento dentro do ambiente da tempestade dá origem a perturbação de pressões dentro da célula da tempestade, produzindo subida e descida conforme mostrado pelas setas verdes.

- Essas **circulações induzidas pela perturbação de pressão** dão origem à divisão da tempestade.

transporte de escalares com manutenção de positividade e monotonicidade

MPAS
Model for Prediction Across Scales



GEF

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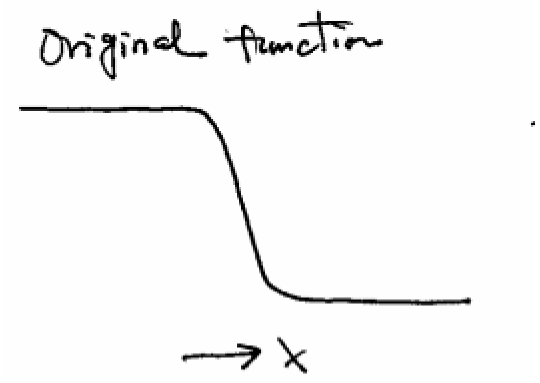
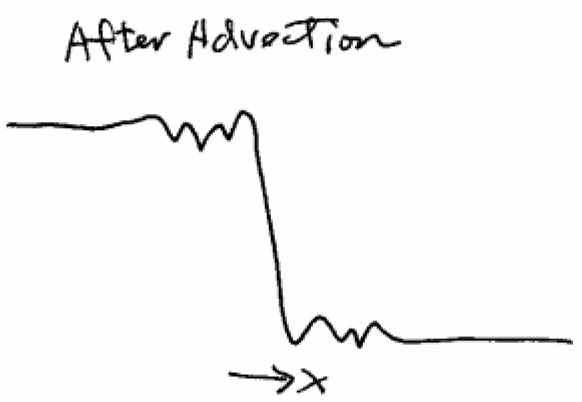
3.4.1. Conceito de Monotonicidade

Quando esquemas numéricos que são usados para **advectar** uma função monotônica, por exemplo, **uma função monotonicamente que decrescente com x**



As **soluções numéricas não necessariamente preservam a propriedade mononótica** – na verdade, **na maioria das vezes não o fazem**, e os **erros tendem a ser grandes perto de gradiente acentuado**



1.4 Transporte de escalares com manutenção de monotonicidade, positividade

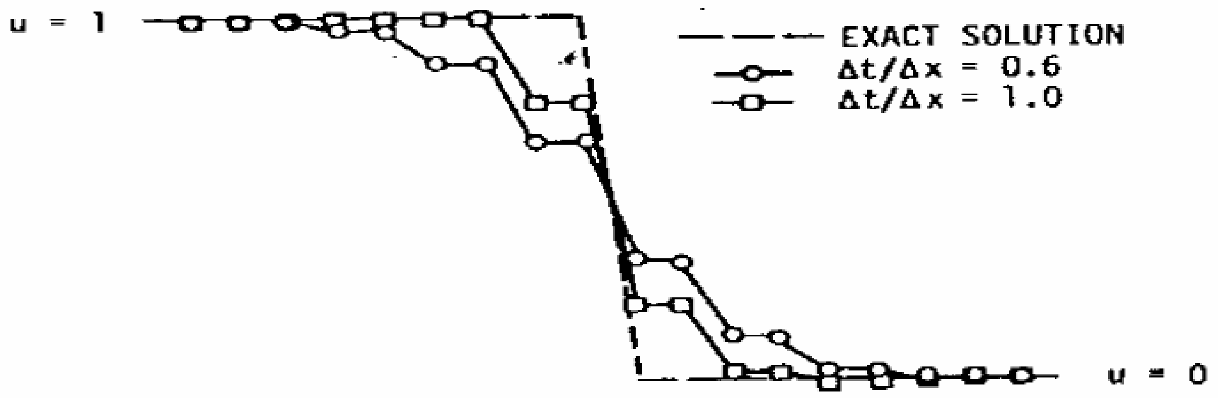


Figure 4-27 Numerical solution of Burgers' equation using Lax method.

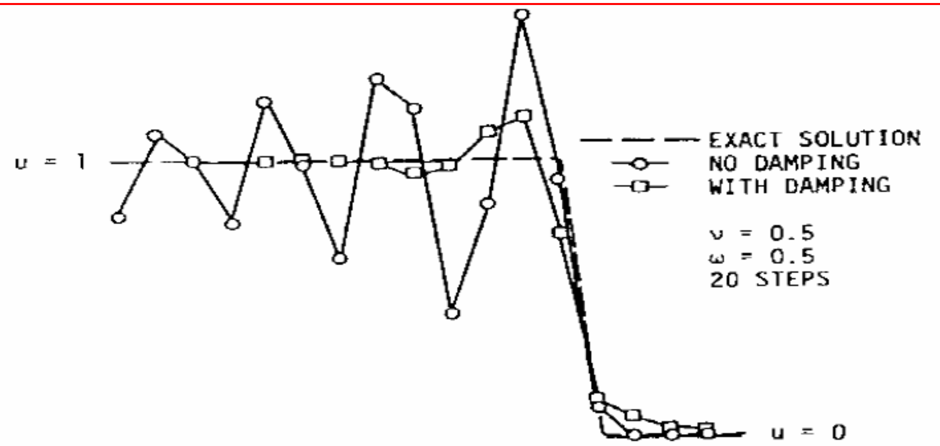


Figure 4-35 Solution of Burgers' equation using Beam-Warming (trapezoidal) method.

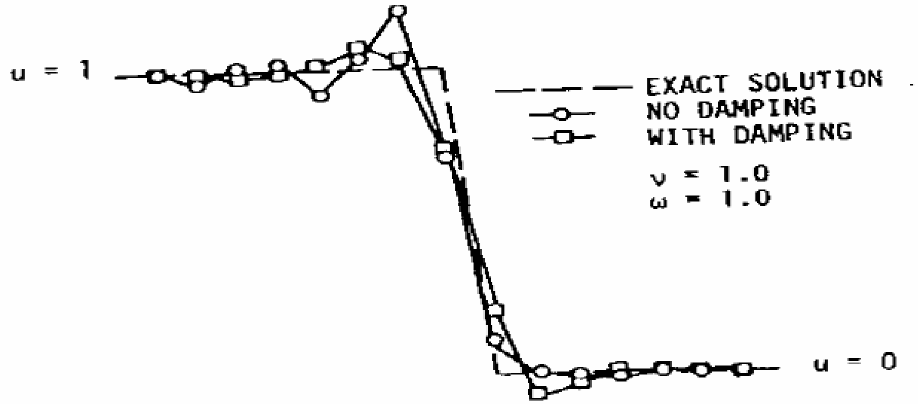


Figure 4-36 Solution for right-moving discontinuity time-centered implicit method, delta form.

Soluções Numéricas tendem a não preservar a monotonicidade

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Método de Discretização Numérica

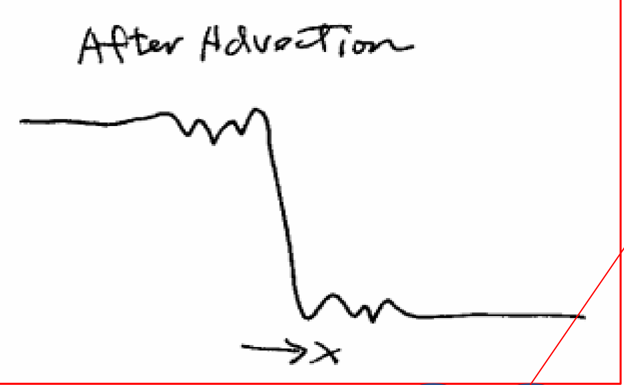
Volume Finito
MPAS

$$\frac{\partial(\rho\psi)}{\partial t} = -\nabla\cdot\mathbf{V}\rho\psi,$$

$$\frac{\partial(\psi_i)}{\partial t} = L(\mathbf{V},\psi_j),$$

$$\frac{\partial(\overline{\rho\psi})_i}{\partial t} = -\frac{1}{A_i}\int_{\Gamma_i}\rho\psi\mathbf{V}\cdot\boldsymbol{n}_i\,\partial\Gamma.$$

$$\frac{\partial(\overline{\rho\psi_i})}{\partial t} = -\frac{1}{A_i}\sum_{n_{e_i}} d_{e_i}\boldsymbol{F}_{e_i}(\mathbf{V},\psi),$$



$$F(u,\psi)_{i+1/2} = u_{i+1/2}\left[\frac{1}{2}(\psi_{i+1}+\psi_i) - \Delta x_e^2\frac{1}{12}\left\{\left(\frac{\partial^2\psi}{\partial x^2}\right)_{i+1} + \left(\frac{\partial^2\psi}{\partial x^2}\right)_i\right\} + sign(u)\,\Delta x_e^2\frac{\beta}{12}\left\{\left(\frac{\partial^2\psi}{\partial x^2}\right)_{i+1} - \left(\frac{\partial^2\psi}{\partial x^2}\right)_i\right\}\right],$$

Como Minimizar este problema? **(Volume Finito de alta ordem)**

A equação representa algoritmos de precisão de 3ª ordem ($\beta \neq 0$) ou 4ª ordem ($\beta = 0$) para malhas com espaçamento de malha constante Dx e velocidade constante.

Na **malha MPAS Voronoi**, Skamarock e Gassmann demostraram a ***convergência de 3ª ordem*** para este esquema na esfera com malhas suaves é o ideal.

Onde **recomendaram** $\beta = 0.25$ quando usado com um limitador de preservação na forma baseada em FCT.

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1.4 Transporte de escalares com manutenção de monotonicidade, positividade

Método de Discretização Numérica

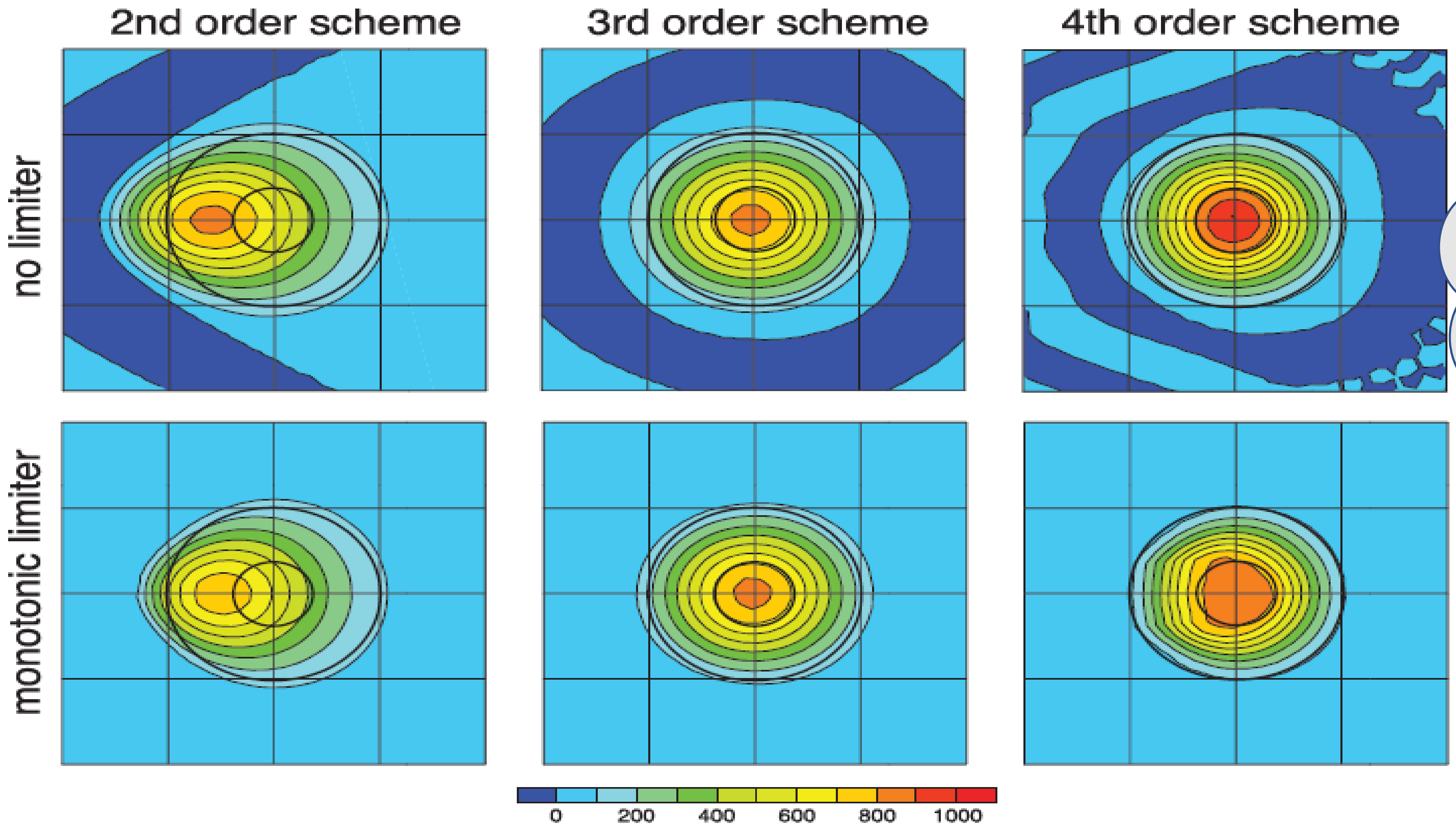

FIG. 3. Cosine bell after one revolution on the sphere. The contour interval is 100 and the thick contours show the exact solution for $\psi = 100$ and 800. The simulations were performed on the 10 242-cell mesh.

Como Minimizar este problema?

(Volume Finito de alta ordem)

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Método de Discretização Numérica

Interpolação usado no método Semi-Lagrangiano

Como Minimizar este problema?
(Metodo Semi-Lagrangiano)

Importância da qualidade da interpolação

2.2 Interpolation Procedures

Inverse Distance Weighted (IDW) scheme
(Esquema de Distância Inversa Ponderada (IDW)

Least Squares (LS) scheme
Esquema de mínimos quadrados (LS)

Thin-Plate Spline (TPS) scheme
Esquema Thin-Plate Spline (TPS)

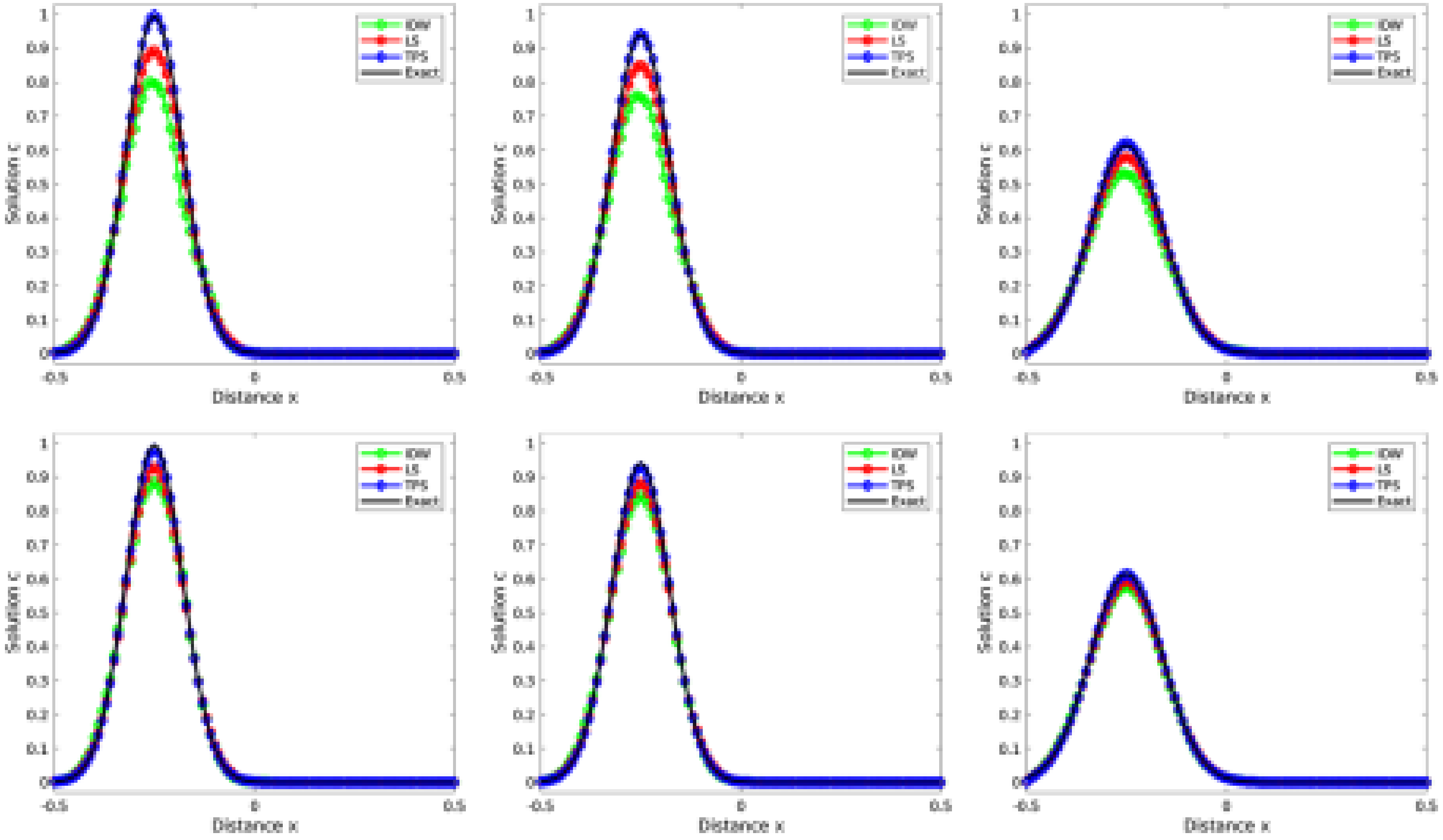


Fig. 6 Cross-sections of the results for the advection–diffusion problem with $\nu = 10^{-5}$ (first column), $\nu = 10^{-4}$ (second column) and $\nu = 10^{-3}$ (third column) using a mesh with 64 × 64 control volumes (first row) and 128 × 128 control volumes (second row)

$\nu = coeficinte\ de\ difussão$

1.4 Transporte de escalares com manutenção de monotonicidade, positividade

Método de Discretização Numérica

2.2 Interpolation Procedures

Inverse Distance Weighted (IDW) scheme
(Esquema de Distância Inversa Ponderada (IDW))

Least Squares (LS) scheme
Esquema de mínimos quadrados (LS)

Thin-Plate Spline (TPS) scheme
Esquema Thin-Plate Spline (TPS)

Importância da qualidade da interpolação

Exata

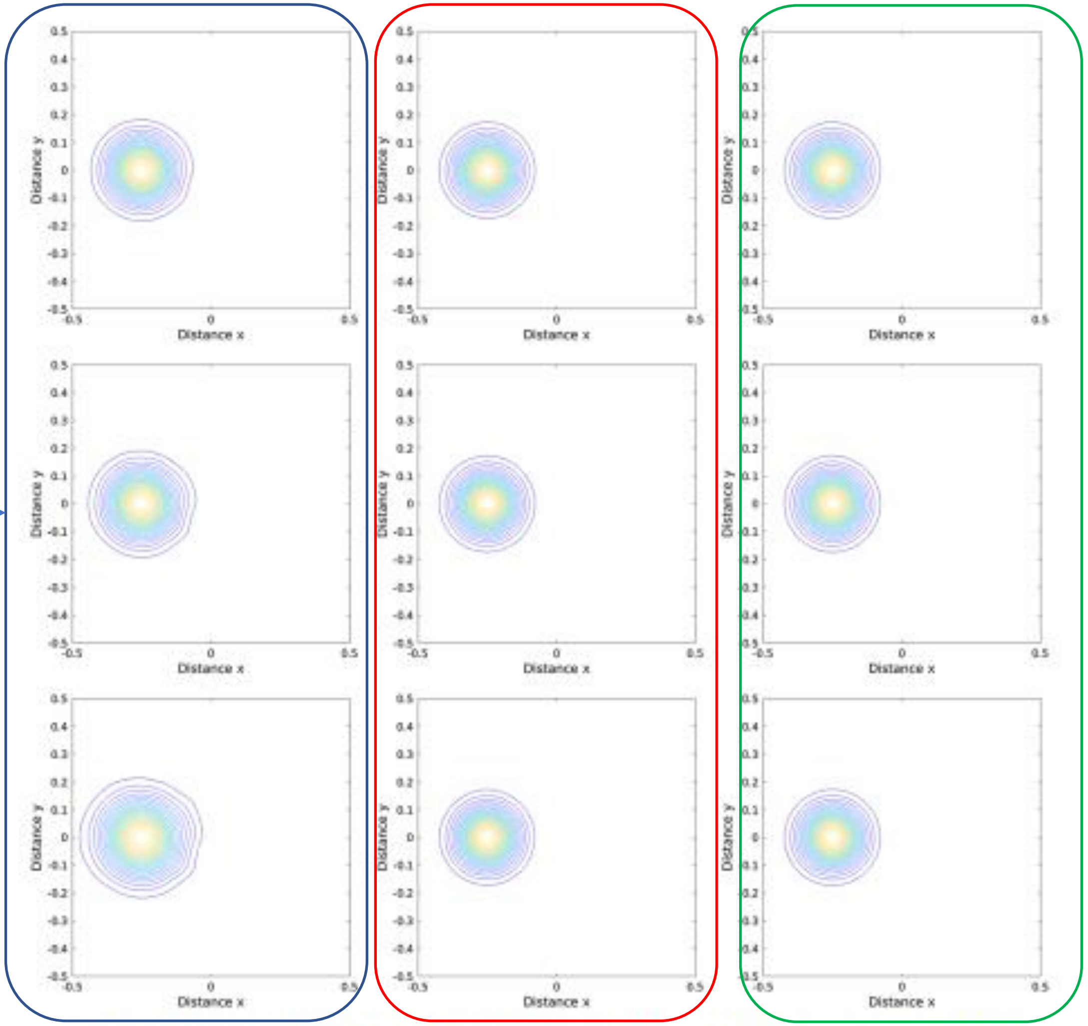


Fig. 4 Numerical solutions obtained using LS (first column), TPS (second column) and exact solution (third column) for $\nu = 0$ on a mesh with 64 × 64 control volumes after one rotation (first row) and 2 rotations (second row) and 5 rotations (third row)

Nota

1.4 Transporte de escalares com manutenção de monotonicidade, positividade

Interpolação não conservativa X conservativa

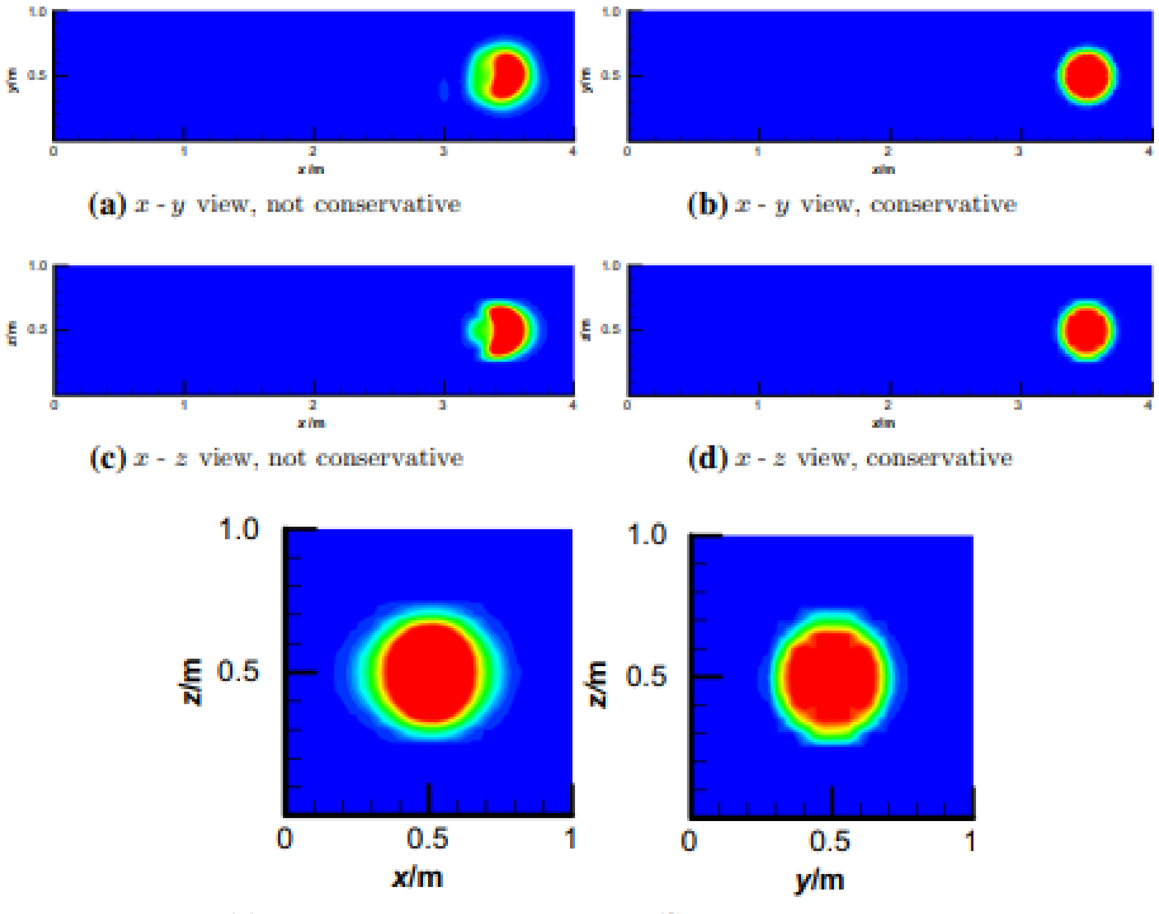
2.2 Interpolation Procedures

Inverse Distance Weighted (IDW) scheme
(Esquema de Distância Inversa Ponderada (IDW))

Least Squares (LS) scheme
Esquema de mínimos quadrados (LS)

Thin-Plate Spline (TPS) scheme
Esquema Thin-Plate Spline (TPS)

Importância da qualidade da interpolação conservativa



(a) x - y view, not conservative (b) x - y view, conservative

(c) x - z view, not conservative (d) x - z view, conservative

(e) y - z view, not conservative (f) y - z view, conservative

Fig. 14 Solution at time $t = 3$ for the advection of a sphere of contaminant problem

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1.4 Transporte de escalares com manutenção de positividade e monotonicidade,

Formulação /Setup	MPAS	FV3-Shied	GEF
Transporte de escalares	Volume finito polinômio de 3rd and 4th-order	Esquema Semi-Lagrangiano	Esquema Semi-Lagrangiano

Semi-Lagrangian Advection on a Cubic Gnomonic Projection of the Sphere

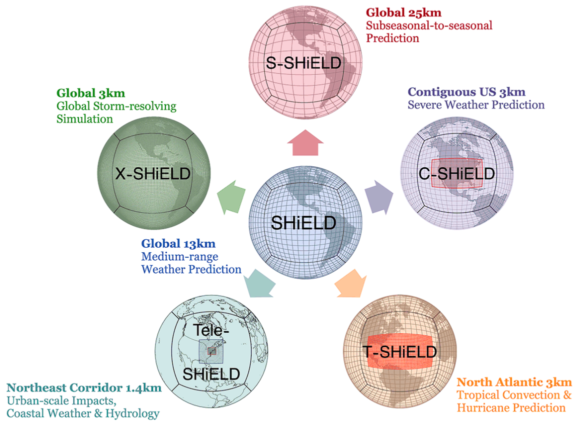
John L. McGregor (1997) Semi-Lagrangian Advection on a Cubic Gnomonic Projection of the Sphere, Atmosphere-Ocean, 35:sup1, 153-169, DOI:10.1080/07055900.1997.9687346

Higher-order finite-volume flux operators for transport algorithms:

Skamarock, W. C., and A. Gassmann, 2011: Conservative Transport Schemes for Spherical Geodesic Grids: High-Order Flux Operators for ODE-Based Time Integration. *Mon. Wea. Rev.*, **139**, 2962–2975, https://doi.org/10.1175/MWR-D-10-05056.1.

físicas avançadas

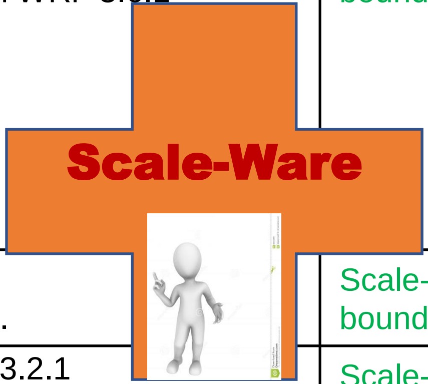
MPAS
Model for Prediction Across Scales



GEF

1.5 Suítes físicas avançadas, etc.

Formulação /Setup	MPAS	FV3-Shied	GEF
Gravity Wave Drag	YSU GWDO as in WRF 3.6.1.	GFS (orographic and convective) gravity wave drag scheme	??
Land Surface Model	Noah (4-layers) as in WRF 3.3.1	Noah land surface model	Noah land surface model
Surface Layer	module_sf_sfclay.F (Monin-Obukhov) as in WRF 3.8.1; MYNN as in WRF 3.6.1	Scale-aware TKE-EDMF planetary boundary layer scheme	Monin–Obukhov similarity theory is combined with Paulson stability functions (Paulson, 1970) and applied to the surface layer which includes a molecular viscous sublayer over land according to Zilitinkevich (1995), and over water according to Janjic (1994).
PBL	YSU as in WRF 3.8.1; MYNN as in WRF 3.6.1.	Scale-aware TKE-EDMF planetary boundary layer scheme	Mellor-Yamada 2.5 closure (Mellor and Yamada, 1982;Janjic, 1990).
Convection	Kain-Fritsch as in WRF 3.2.1 Tiedtke as in WRF 3.3.1 New Tiedtke as in WRF 3.8.1 version of scale-aware Grell-Freitas as in WRF 3.6.1	Scale-aware SAS (shallow and deep) convection scheme	Betts-Miller-Janjic (Bettsand Miller, 1986; Janjic, 1994) and Kain-Fritsch (Kain,2004),
Microphysics	WSM6 as in WRF 3.8.1 Thompson (non-aerosol aware) as in WRF 3.8.1	GFDL cloud microphysics v3	Zhao (Zhao et al., 1997) and Ferrier (Ferrier et al., 2002).
Radiation	RRTMG sw as in WRF 3.8.1 RRTMG lw as in WRF 3.8.1 CAM radiation as in WRF 3.3.1	RRTM radiation with GFDL cloud-radiation interaction scheme	radiation package (GFDL), which includes the short-wave radiation scheme of Lacis and Hansen (1974) and the long-wave radiation scheme of Schwarzkopf and Fels (1991),



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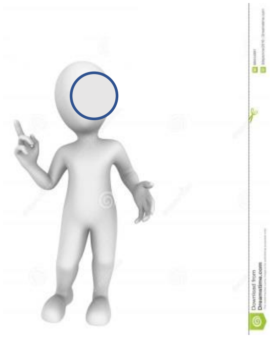
1.5 Suítes físicas avançadas, etc.

O Modelo MONAN
Necessita de
Parametrizações Físicas
Com o Tratamento
de Scale-Ware



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Pre_process_geogrid.x

2. Dados Estáticos [Nedilson Junior-PGMET /Dra. Aline -UFPA]

/mnt/beegfs/paulo.kubota/monan_project/MPAS_Model_Regional//pre/databcs/WPS_GEOG

modis_landuse_20class_30s

```
00001-01200.00001-01200
08401-09600.10801-12000
18001-19200.00001-01200
26401-27600.10801-12000
36001-37200.00001-01200
....
```

Arquivos intermediários

```
00001-01200.00001-01200.bin
08401-09600.10801-12000.bin
18001-19200.00001-01200.bin
26401-27600.10801-12000.bin
36001-37200.00001-01200.bin
....
```

modis_landuse_new_20class_30s

```
00001-01200.00001-01200.new
08401-09600.10801-12000.new
18001-19200.00001-01200.new
26401-27600.10801-12000.new
36001-37200.00001-01200.new
....
```

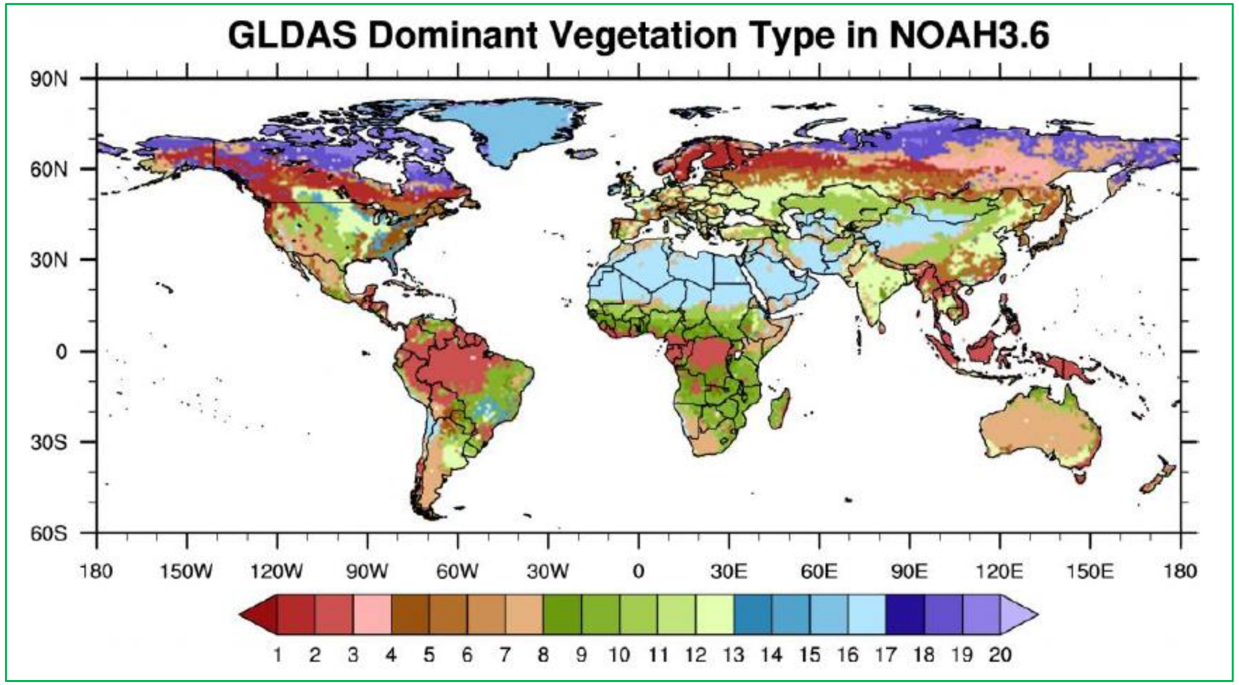
2. Dados Estáticos [Nedilson Junior-PGMET/Dra. Aline-UFPA]

LDAS
Land Data Assimilation System

GLDAS NLDAS NCA-LDAS FLDAS WLDAS FAQ Get Data

Model	Noah	Catchment	VIC, Catchment (daily)
Index	Modified MODIS IGBP Vegetation Type	Mosaic Vegetation Type	UMD Vegetation Type
0	missing value	missing value	missing value
1	Evergreen Needleleaf Forest	Broadleaf Evergreen Trees	Evergreen Needleleaf Forest
2	Evergreen Broadleaf Forest	Broadleaf Deciduous Trees	Evergreen Broadleaf Forest
3	Deciduous Needleleaf Forest	Needleleaf Trees	Deciduous Needleleaf Forest
4	Deciduous Broadleaf Forest	Grassland	Deciduous Broadleaf Forest
5	Mixed Forest	Broadleaf Shrubs, Bare Soil, Desert Soil	Mixed Cover
6	Closed Shrublands	Dwarf Trees	Woodland
7	Open Shrublands		Wooded Grasslands
8	Woody Savannas		Closed Shrublands
9	Savannas		Open Shrublands
10	Grassland		Grasslands
11	Permanent Wetland		Cropland
12	Cropland		Barren
13	Urban and Built-Up		Urban and Built-Up
14	Cropland/Natural Vegetation Mosaic		
15	Snow and Ice		
16	Barren or Sparsely Vegetated		
17	Ocean		
18	Wooded Tundra		
19	Mixed Tundra		
20	Bare Ground Tundra		

- GLDAS2/Noah Dominant Vegetation Type Data (NetCDF): 0.25 degree, 1 degree
- GLDAS2.0 & GLDAS2.1/Catchment Dominant Vegetation Type Data (NetCDF): 1 degree
- GLDAS2.0 & GLDAS2.2/Catchment Dominant Vegetation Type Data (NetCDF): 0.25 degree
- GLDAS2/VIC Dominant Vegetation Type Data (NetCDF): 1 degree



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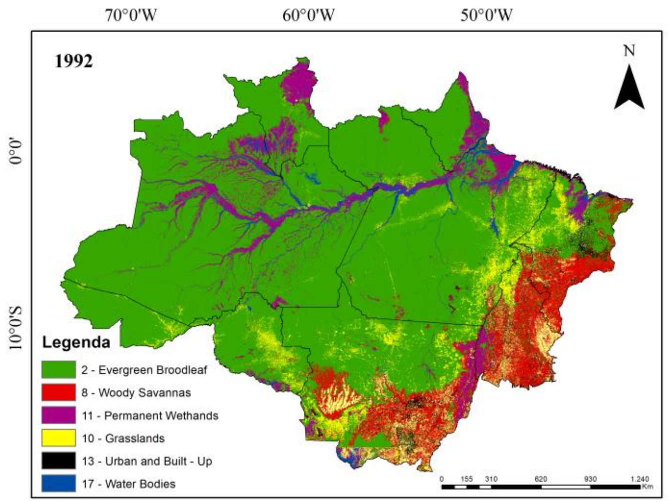
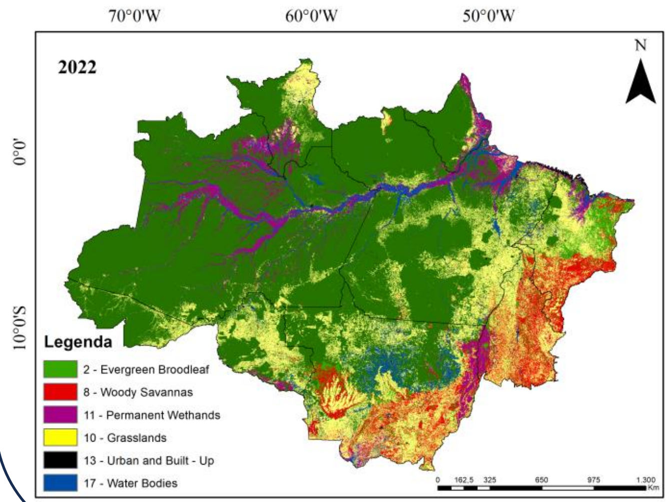
2. Dados Estáticos [Nedilson Junior-PGMET/Dra. Aline-UFPA]

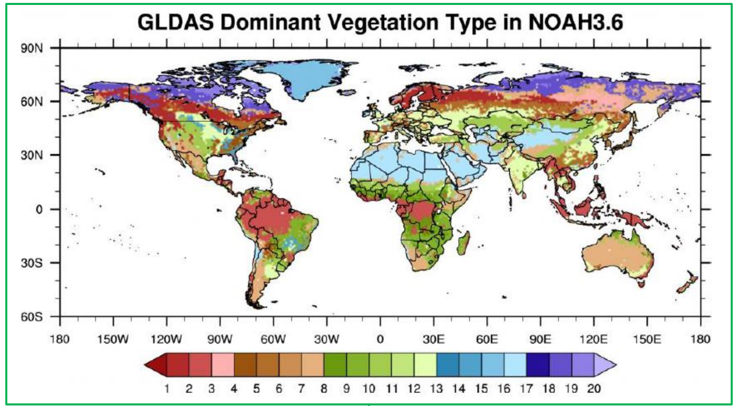




1992

QGIS

2022

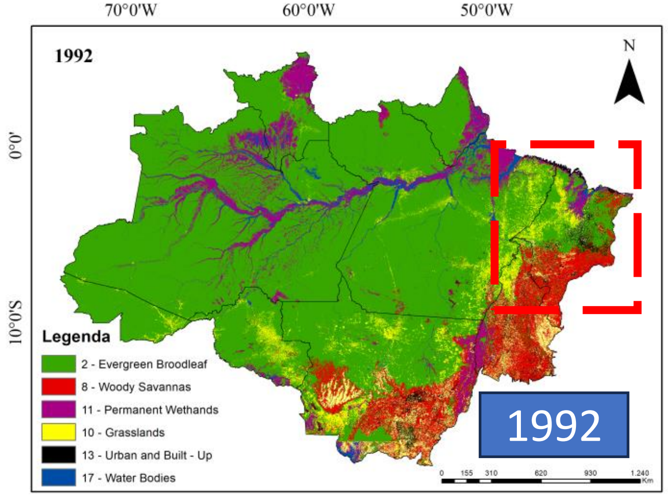


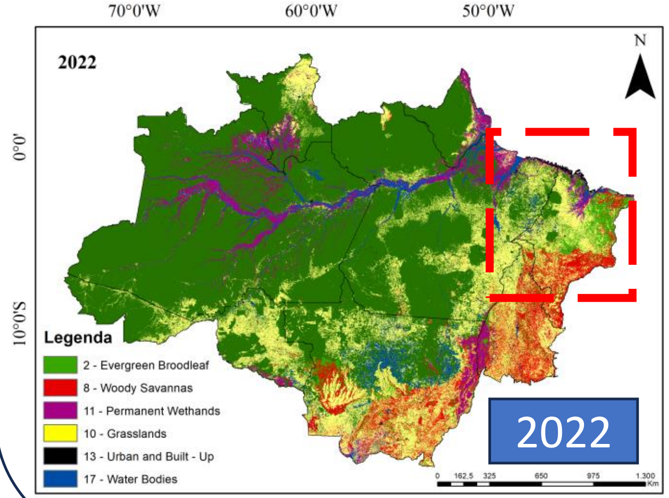
Base de Dados MapBiomas

Paulo Yoshio Kubota

2. Dados Estáticos [Nedilson Junior-PGMET /Dra. Aline -UFPA]

Classfificação MapBiomas





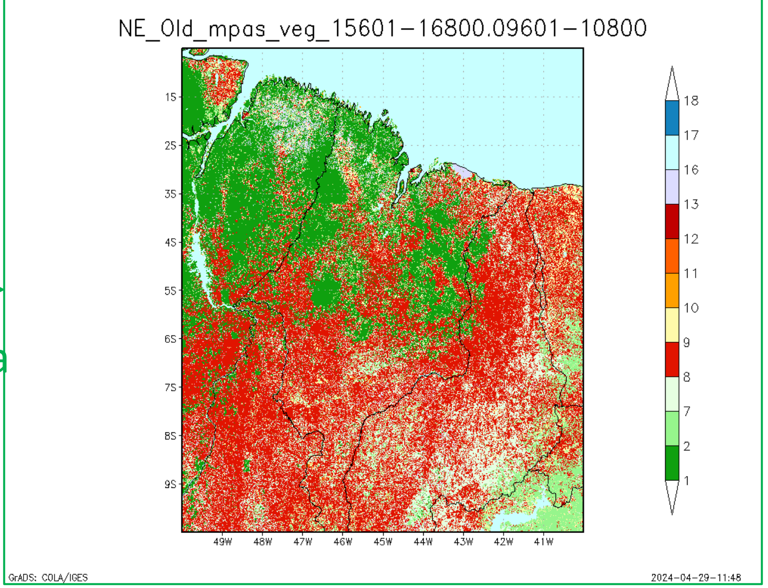
modis_landuse_20class_30s

Arquivos intermediários

modis_landuse_new_20class_30s

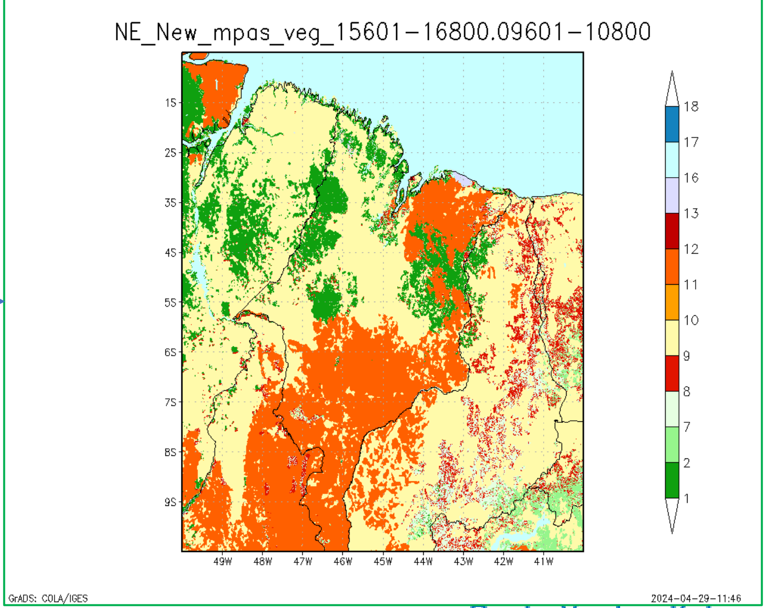
Floresta Amazonica

Wood Savannas



Area pastagem

Area agricola



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3. Dados Oceânicos

pre_mpas_sst

3. Dados Oceânicos

sst.day.mean.1997.nc

ice.day.mean.1997.nc

```
dimensions:
        time = UNLIMITED ; // (365 currently)
        lat = 720 ;
        lon = 1440 ;
variables:
        double time(time) ;
                time:long_name = "Time" ;
                time:units = "days since 1800-01-01 00:00:00" ;
                time:delta_t = "0000-00-01 00:00:00" ;
                time:avg_period = "0000-00-01 00:00:00" ;
                time:axis = "T" ;
                time:actual_range = 71953., 72317. ;
        float lat(lat) ;
                lat:long_name = "Latitude" ;
                lat:standard_name = "latitude" ;
                lat:units = "degrees_north" ;
                lat:actual_range = -89.875f, 89.875f ;
                lat:axis = "Y" ;
        float lon(lon) ;
                lon:long_name = "Longitude" ;
                lon:standard_name = "longitude" ;
                lon:units = "degrees_east" ;
                lon:actual_range = 0.125f, 359.875f ;
                lon:axis = "X" ;
        float sst(time, lat, lon) ;
                sst:long_name = "Daily Sea Surface Temperature" ;
                sst:units = "degC" ;
                sst:valid_range = -3.f, 45.f ;
                sst:missing_value = -9.96921e+36f ;
                sst:precision = 2.f ;
                sst:dataset = "NOAA High-resolution Blended Analysis" ;
                sst:var_desc = "Sea Surface Temperature" ;
                sst:level_desc = "Surface" ;
                sst:statistic = "Mean" ;
                sst:parent_stat = "Individual Observations" ;
                sst:actual_range = -1.8f, 34.56f ;
```

3. Dados Oceânicos

sst.day.mean.1997.nc

ice.day.mean.1997.nc

```
dimensions:
        time = UNLIMITED ; // (365 currently)
        lat = 720 ;
        lon = 1440 ;
variables:
        double time(time) ;
                time:long_name = "Time" ;
                time:units = "days since 1800-01-01 00:00:00" ;
                time:delta_t = "0000-00-01 00:00:00" ;
                time:avg_period = "0000-00-01 00:00:00" ;
                time:axis = "T" ;
                time:actual_range = 71953., 72317. ;
        float lat(lat) ;
                lat:long_name = "Latitude" ;
                lat:standard_name = "latitude" ;
                lat:units = "degrees_north" ;
                lat:actual_range = -89.875f, 89.875f ;
                lat:axis = "Y" ;
        float lon(lon) ;
                lon:long_name = "Longitude" ;
                lon:standard_name = "longitude" ;
                lon:units = "degrees_east" ;
                lon:actual_range = 0.125f, 359.875f ;
                lon:axis = "X" ;
        float icec(time, lat, lon) ;
                icec:long_name = "Daily Sea Ice Concentration" ;
                icec:units = "percent" ;
                icec:valid_range = 0.f, 1.f ;
                icec:missing_value = -9.96921e+36f ;
                icec:precision = 2.f ;
                icec:dataset = "NOAA High-resolution Blended Analysis" ;
                icec:var_desc = "Sea Ice Concentration" ;
                icec:level_desc = "Surface" ;
                icec:statistic = "Mean" ;
```

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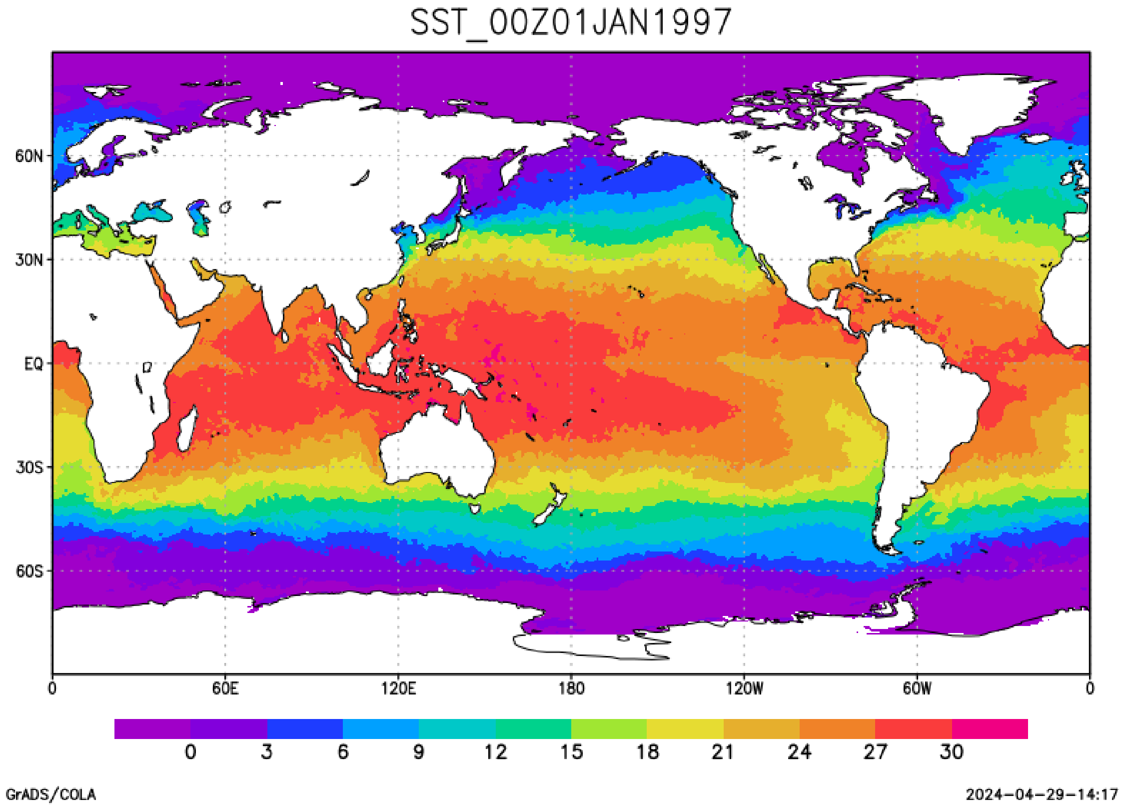
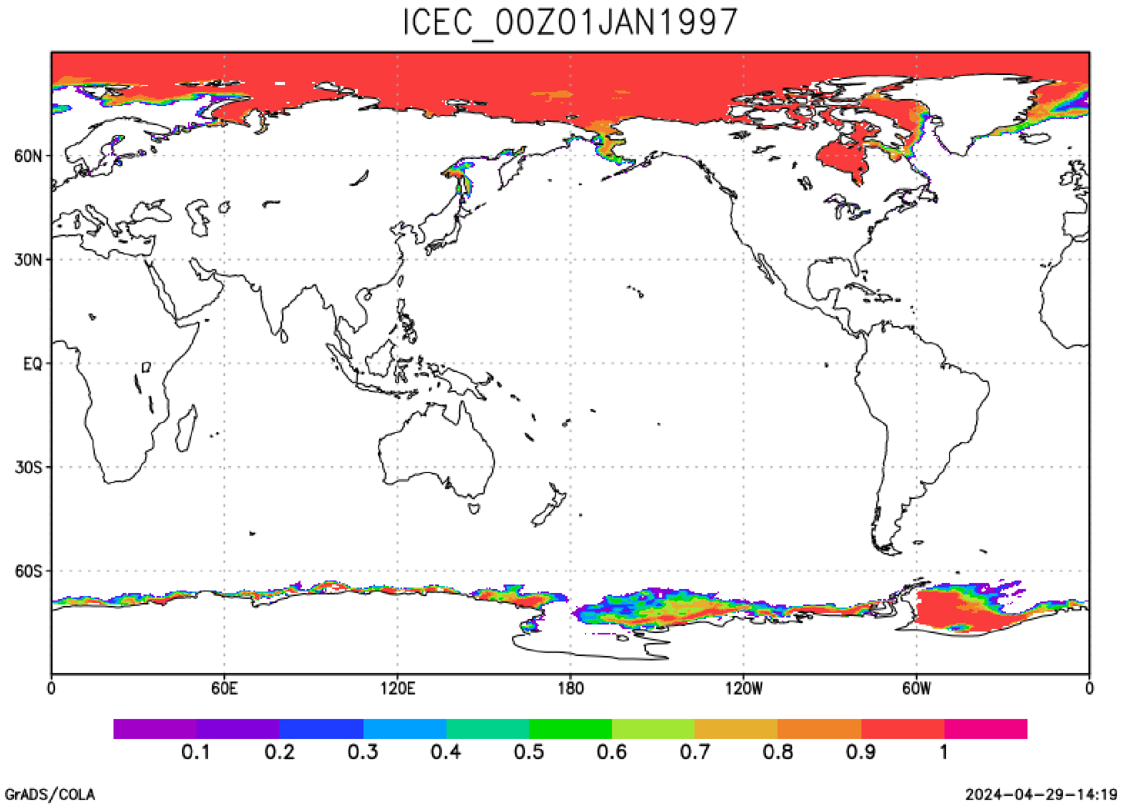
3. Dados Oceânicos

sst.day.mean.1997.nc

ice.day.mean.1997.nc





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3. Dados Oceânicos

sst.day.mean.1997.nc

ice.day.mean.1997.nc

pre_mpas_sst

SST:1997-01-01_00
...
SST:1997-07-20_00
.....
SST:1998-12-19_00

init_atmosphere_model

x1.40962.sfc_update.nc

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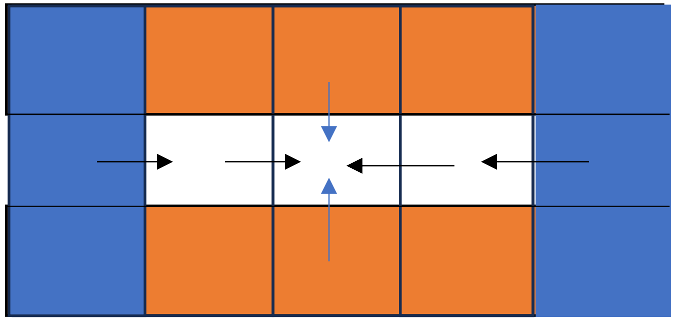
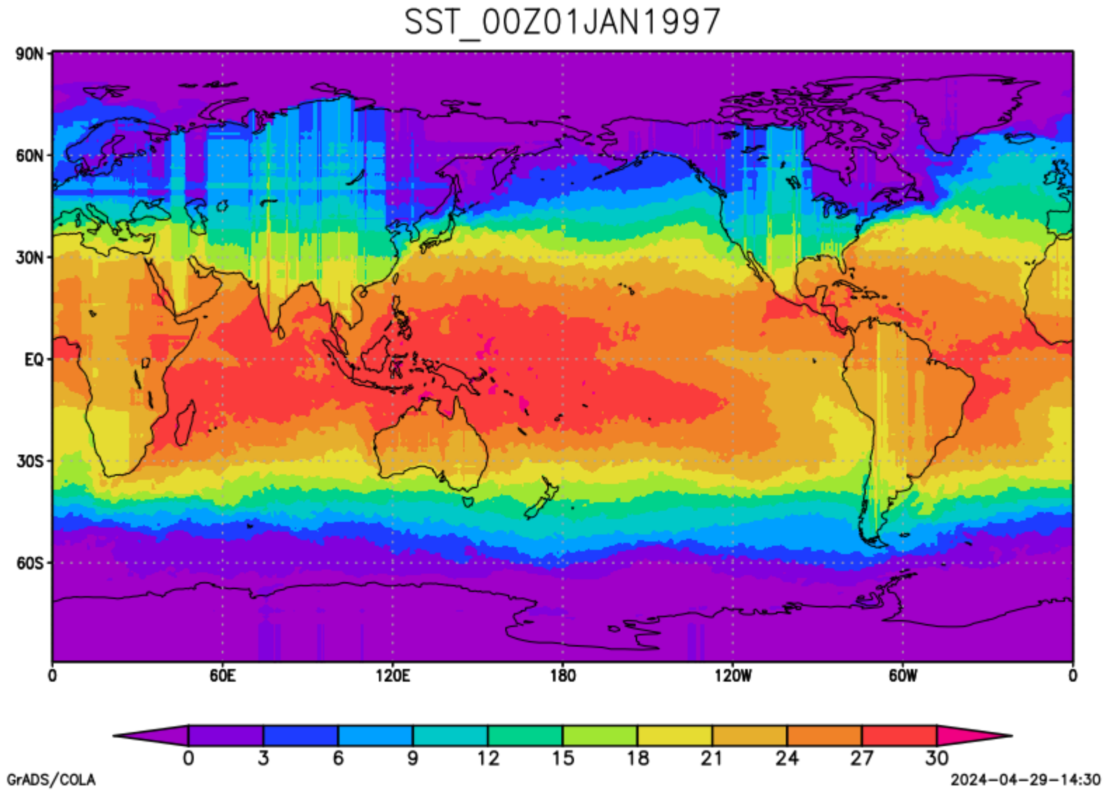
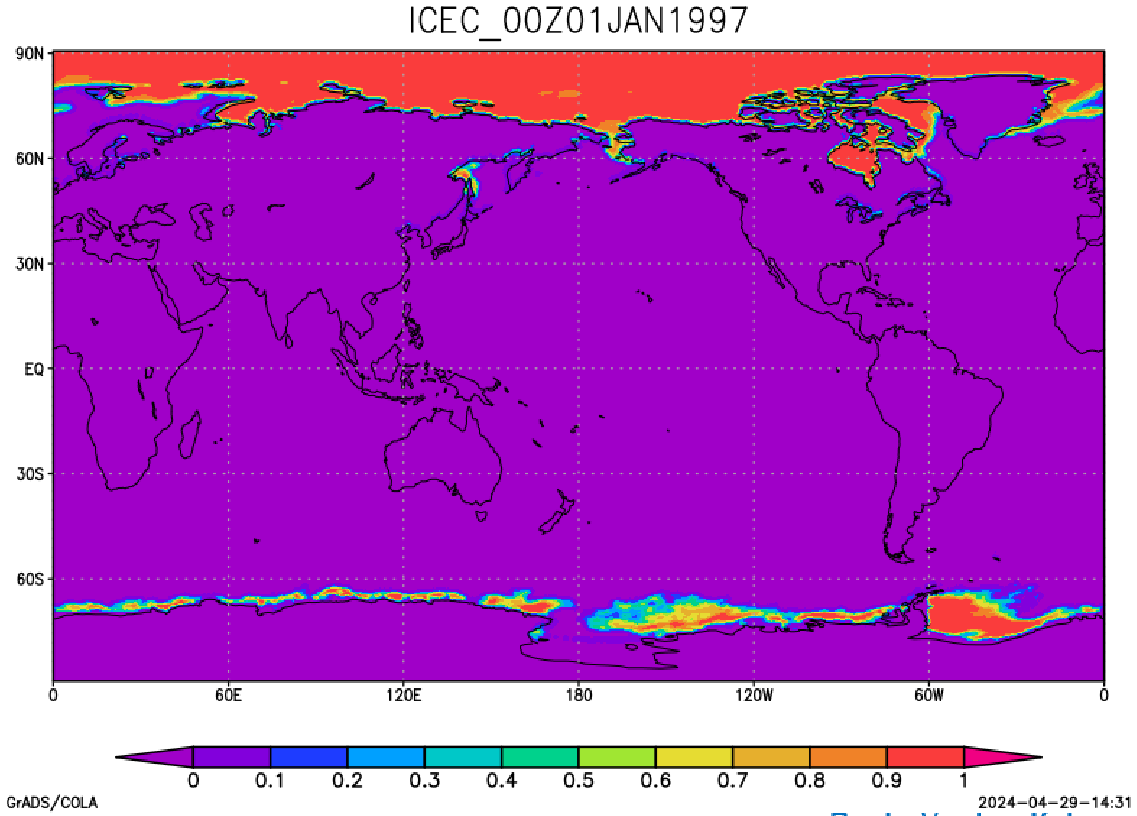
3. Dados Oceânicos



```
SST:1997-01-01_00
. . .
SST:1997-07-20_00
.....
SST:1998-12-19_00
```





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3. Dados Oceânicos

sst.day.mean.1997.nc

ice.day.mean.1997.nc

pre_mpas_sst

SST:1997-01-01_00
...
SST:1997-07-20_00
.....
SST:1998-12-19_00

init_atmosphere_model

x1.40962.sfc_update.nc

3. Dados Oceânicos

x1.40962.sfc_update.nc

```
dimensions:
        StrLen = 64 ;
        Time = UNLIMITED ; // (1 currently)
        nCells = 40962 ;
variables:
        char xtime(Time, StrLen) ;
                xtime:units = "YYYY-MM-DD_hh:mm:ss" ;
                xtime:long_name = "Model valid time" ;
        float Time(Time) ;
                Time:units = "seconds since 1997-01-01 00:00:00" ;
                Time:long_name = "CF-compliant valid time" ;
                Time:standard_name = "time" ;
        float sst(Time, nCells) ;
                sst:units = "K" ;
                sst:long_name = "sea-surface temperature" ;
        float xice(Time, nCells) ;
                xice:units = "unitless" ;
                xice:long_name = "fractional area coverage of sea-ice" ;
```

```
ncdump -v sst x1.40962.sfc_update.nc
```

```
ncdump -v icec x1.40962.sfc_update.nc
```

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3. Dados Oceânicos

```
ncdump -v sst x1.40962.sfc_update.nc
```

```
ncdump -v icec x1.40962.sfc_update.nc
```

```
data:

 sst =
  295.0165, 294.2157, 271.43, 295.2659,
297.7201, 290.1828, 295.6052,
    297.5935, 271.35, 296.3419,
297.4984, 293.4944, 290.1667, 301.98,
    302.1353, 289.4696, 275.1285,
300.3037, 296.7001, 292.1894, 273.2,
    274.5934, 278.1795, 280.2252,
300.3073, 302.9193, 295.4616, 273.3606,
    296.52, 302.99, 273.8472, 295.4381,
303.5397, 294.2111, 276.9515,
    302.0513, 291.3456, 273.8145,
295.7006, 302.9274, 276.1938, 294.1543,
```

```
data:

 xice =
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
```

Check se realmente tem valores de icec.
Não pode ser somente zeros

3. Dados Meteorológicos

Chopping_Monan

3. Dados Meteorológicos

gdas1.T00Z.sfcanl.netcdf.2022080300

gdas1.T00Z.atmanl.netcdf.2022080300

Chopping_Monan

FILE3:2024-04-03_00

init_atmosphere_model

x1.40962.init.nc

3. Dados Meteorológicos

gdas1.T00Z.sfcanl.netcdf.2022080300

gdas1.T00Z.atmanl.netcdf.2022080300

Adaptação para a grade gaussiana

Surf_Netcdf_Parameter.f90

Atmos_Netcdf_Parameter.f90

Chopping_Monan

Níveis isobáricos

mpas_VerticalInterpolation.F

Níveis em coordenada hibrida sigma/pressão

VerticalInterpolation.f90

FILE3:2024-04-03_00

3. Dados Meteorológicos

```
gdas1.T00Z.sfcanl.netcdf.2022080300
```

```
gdas1.T00Z.atmanl.netcdf.2022080300
```

Chopping_Monan

Status Atual
Checando a decomposição
de domínio MPI.

```
FILE3:2024-04-03_00
```

3. Dados Meteorológicos

gdas1.T00Z.sfcanl.netcdf.2022080300

gdas1.T00Z.atmanl.netcdf.2022080300

Chopping_Monan

FILE3:2024-04-03_00

```fortran
module met_data_module

   ! Derived types

   type met_data
      integer                         :: version
      integer                         :: nx
      integer                         :: ny
      integer                         :: iproj
      real                            :: xfcst
      real                            :: xlvl
      real                            :: startlat
      real                            :: startlon
      real                            :: starti
      real                            :: startj
      real                            :: deltalat
      real                            :: deltalon
      real                            :: dx
      real                            :: dy
      real                            :: xlonc
      real                            :: centerlat
      real                            :: centerlon
      real                            :: pole_lat
      real                            :: pole_lon
      real                            :: truelat1
      real                            :: truelat2
      real                            :: earth_radius
      real, pointer, dimension(:,:) :: slab
      logical                         :: is_wind_grid_rel
      character (len=9)               :: field
      character (len=24)              :: hdate
      character (len=25)              :: units
      character (len=32)              :: map_source
      character (len=46)              :: desc
   end type met_data

end module met_data_module
```

4. Padronização das saídas no modelo MONAN

Diagnostics/

```
Makefile                                README
mpas_atm_diagnostics_manager.F          Registry_diagnostics.xml
mpas_atm_diagnostic_template.F          Registry_template.xml
mpas_isobaric_diagnostics.F             Registry_isobaric.xml
mpas_lsm_diagnostics.F                  Registry_lsm.xml
mpas_sfclayer_diagnostics.F             Registry_sfclayer.xml
mpas_pbl_diagnostics.F                  Registry_pbl.xml
mpas_cloud_diagnostics.F                Registry_cloud.xml
mpas_radiation_diagnostics.F            Registry_radiation.xml
mpas_convective_diagnostics.F           Registry_convective.xml
mpas_microphysics_diagnostics.F         Registry_microphysics.xml
mpas_pv_diagnostics.F                   Registry_pv.xml
mpas_soundings.F                        Registry_soundings.xml
mpas_VerticalInterpolation.F
mpas_atm_diagnostics_utils.F
```

4. Padronização das saídas no modelo MONAN

```
 !Changed by RRB to add more isobaric levels
!            t_iso_levels(:) = (/10000.0, &
!                                12500.0, &
!                                15000.0, &
!                                17500.0, &
!                                20000.0, &
!                                22500.0, &
!                                25000.0, &
!                                30000.0, &
!                                35000.0, &
!                                40000.0, &
!                                45000.0, &
!                                50000.0, &
!                                55000.0, &
!                                60000.0, &
!                                65000.0, &
!                                70000.0, &
!                                75000.0, &
!                                77500.0, &
!                                80000.0, &
!                                82500.0, &
!                                85000.0, &
!                                87500.0, &
!                                90000.0, &
!                                92500.0, &
!                                95000.0, &
!                                97500.0, &
!                               100000.0/)
```

```
  t_iso_levels(:) = (/100000.0, &
                       97500.0, &
                       95000.0, &
                       92500.0, &
                       90000.0, &
                       87500.0, &
                       85000.0, &
                       82500.0, &
                       80000.0, &
                       77500.0, &
                       75000.0, &
                       70000.0, &
                       65000.0, &
                       60000.0, &
                       55000.0, &
                       50000.0, &
                       45000.0, &
                       40000.0, &
                       35000.0, &
                       30000.0, &
                       25000.0, &
                       22500.0, &
                       20000.0, &
                       17500.0, &
                       15000.0, &
                       12500.0, &
                       10000.0/)
```

Atrapalha o pós-processamento. É invertido em relação aos níveis do modelo

Ideal para o pós-processamento. Os níveis verticais (modelo e isobárico) tem a mesma distribuição

Paulo Yoshio Kubota

4. Padronização das saídas no modelo MONAN

```
do iCell=1,nCells
   do k=1,nVertLevels
      kk = nVertLevels+1-k
      press_in(iCell,kk) = pressure(k,iCell) * 100.0
   end do
end do

if (need_t_isobaric .or. need_qv_isobaric .or. need_rh_isobaric) then
   ! Additional temperature levels for vortex tracking
   do iCell=1,nCells
      do k=1,nVertLevels
         kk = nVertLevels+1-k
         field_in(iCell,kk) = temperature(k,iCell)
      end do
   end do

   call interp_tofixed_pressure(nCells, nVertLevels, nIsoLevelsT,&
                press_in, field_in, press_interp, field_interp)

   do k=1,nIsoLevelsT
      t_isobaric(k,1:nCells) = field_interp(1:nCells,k)
   end do
```

```
do iCell=1,nCells
   do k=1,nVertLevels

      press_in(iCell,k) = pressure(k,iCell) * 100.0
   end do
end do

if (need_t_isobaric .or. need_qv_isobaric .or. need_rh_isobaric) then
   ! Additional temperature levels for vortex tracking
   do iCell=1,nCells
      do k=1,nVertLevels

         field_in(iCell,k) = temperature(k,iCell)
      end do
   end do

   call interp_tofixed_pressure(nCells, nVertLevels, nIsoLevelsT,&
                press_in, field_in, press_interp, field_interp)

   do k=1,nIsoLevelsT
      t_isobaric(k,1:nCells) = field_interp(1:nCells,k)
   end do
```

Lembrando que nos arquivos history.xxxxx e diag.xxxxx podem conter variáveis em níveis do modelo e níveis isobáricos.

Então é importante que a distribuição dos níveis (sfc-top) seja igual para os níveis isobáricos e os níveis do modelo

5. Impacto das saídas do modelo no convert_mpas

convert_mpas

5. Impacto das saídas do modelo no convert_mpas

- **Padronização do convert_mpas**
- **Níveis isobáricos e níveis do modelo com a mesma distribuição (sfc-top)**

Pauta da 3 Reunião Geral do Modelo MONAN

- Avanços na Análise Modal no MONAN [Bonatti].
- Avanços na modelagem de Superfície [A. Manzi].
- Parametrização de Turbulência [Paulo Kubota]
- Parametrização de Radiação [Paulo Kubota]
- Parametrização de Microfísica [Enver/Jorge Gomes]
- LES_SAM/Convecção Rasa [Silvio Nilo/Jhonatan]
- Convecção Grell-Freitas [Saulo Freitas]

Slides Grupo Atmos

Slides Grupo Superfície

Implementação da parametrização YSU BL

(1)Quais são as fontes de incerteza do modelo?

Processos físicos da turbulência na camada limite

Implementation of a revised SBL scheme in the YSU BL package

“Propoem-se a adição da contribuição do fluxo de calor devido o entranhamento no topo da Camada Limite Planetária (PBL) "

Noh et al 2003, modificaram o esquema original

$$\frac{\partial C}{\partial t} = \frac{\partial[-\overline{w'c'}]}{\partial z}$$

$$-\overline{w'c'} = K_c\left(\frac{\partial C}{\partial z} - \gamma_c\right) - \overline{(w'c')}_h \left(\frac{z}{h}\right)^n$$

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial z}\left[K_c\left(\frac{\partial C}{\partial z} - \gamma_c\right)\right]$$

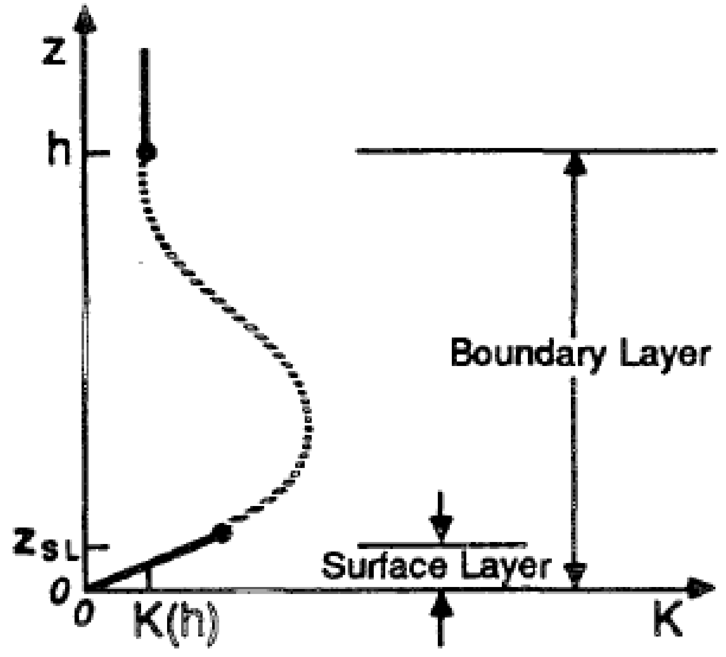

FIG. 1. Typical variation of eddy viscosity K with height in the boundary layer proposed by O'Brien (1970). Adopted from Stull (1988).

"Aqui obtivemos a constante empírica n como $n = 3$ com base na comparação entre os resultados do modelo e dados de LES, mas descobriu-se que os resultados são altamente insensíveis à escolha de n. Por exemplo, simulações com valores substancialmente diferentes de n, como $n = 1\ ou\ 5$, também apresentam resultados semelhantes, pois a contribuição do novo termo é desprezado $\left(\frac{z}{h}\right)^n$, exceto próximo ao topo da Camada Limite Planetária (PBL $z = h \Rightarrow \frac{z}{h} = 1$)."

(1)Quais são as fontes de incerteza do modelo?

Processos físicos da turbulência na camada limite

Implementation of a revised SBL scheme in the YSU BL package

"O esquema YSU antigo usava uma teoria K modificada, com um termo de contra-gradiente adicional que incorpora a contribuição de vórtices de grande escala ao fluxo total."

$$\gamma_c = b\frac{\overline{(w'c')_0}}{w_{s0}}$$

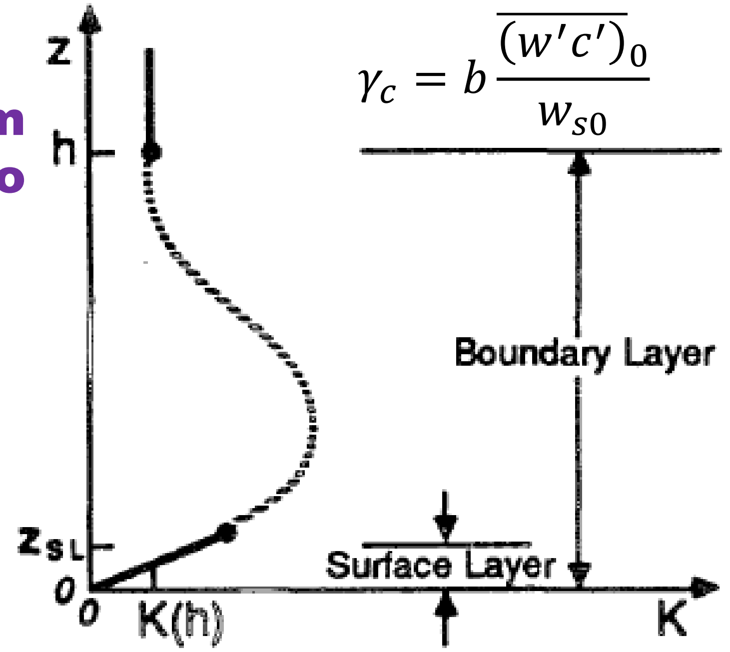

FIG. 1. Typical variation of eddy viscosity K with height in the boundary layer proposed by O'Brien (1970). Adopted from Stull (1988).

Hong SY et al 2006. Noh et al 2003, modificaram o esquema original introduzindo o efeito de entranhamento

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial z}\left\{K_c\left(\frac{\partial C}{\partial z} - \gamma_c\right) - \overline{(w'c')}_h\left(\frac{z}{h}\right)^3\right\}$$

$$\frac{\partial C}{\partial t} = \left\{\frac{\partial}{\partial z}K_c\frac{\partial C}{\partial z} - \frac{\partial}{\partial z}\left(K_c\gamma_c + \overline{(w'c')}_h\left(\frac{z}{h}\right)^3\right)\right\}$$

$\overline{(w'c')}_h$ **é o fluxo na camada de inversão**

"A fórmula mantém o conceito básico de Hong SY et al 1996, mas inclui um termo assintótico de fluxo de entrada na camada de inversão $\overline{(w'c')}_h\left(\frac{z}{h}\right)^3$.

Neste caso a altura da (PBL) h é definida como o nível em que o fluxo mínimo ocorre na camada de inversão, enquanto em Hong SY et al 1996 é definida como o nível em que a mistura turbulenta da camada limite diminui.

Implementation of a revised SBL scheme in the YSU BL package

$$\frac{\partial C}{\partial t} = \left\{ \frac{\partial}{\partial z} K_c \frac{\partial C}{\partial z} - \frac{\partial}{\partial z} \left(K_c \gamma_c + \overline{(w'c')}_h \left(\frac{z}{h}\right)^3 \right) \right\}$$

$$\int_k^{k+1} \frac{\partial C}{\partial t} dz = \left\{ \int_k^{k+1} \frac{\partial}{\partial z} K_c \frac{\partial C}{\partial z} \partial z - \int_k^{k+1} \frac{\partial}{\partial z} \left(K_c \gamma_c + \overline{(w'c')}_h \left(\frac{z}{h}\right)^3 \right) \partial z \right\}$$

$$\frac{\partial}{\partial t} \int_k^{k+1} C\, dz = \left\{ K_c \frac{\partial C}{\partial z} \Big|_k^{k+1} - \left(K_c \gamma_c + \overline{(w'c')}_h \left(\frac{z}{h}\right)^3 \right) \Big|_k^{k+1} \right\}$$

$$\frac{\partial \bar{C}}{\partial t} = \left\{ \left(K_c \frac{\partial C}{\partial z} \right)^{k+1} - \left(K_c \frac{\partial C}{\partial z} \right)^{k} - \left(K_c \gamma_c + \overline{(w'c')}_h \left(\frac{z}{h}\right)^3 \right)^{k+1} + \left(K_c \gamma_c + \overline{(w'c')}_h \left(\frac{z}{h}\right)^3 \right)^{k} \right\}$$

$$\frac{\partial \bar{C}}{\partial t} - \left(K_c \frac{\partial C}{\partial z} \right)^{k+1} + \left(K_c \frac{\partial C}{\partial z} \right)^{k} = \left\{ - \left(K_c \gamma_c + \overline{(w'c')}_h \left(\frac{z}{h}\right)^3 \right)^{k+1} + \left(K_c \gamma_c + \overline{(w'c')}_h \left(\frac{z}{h}\right)^3 \right)^{k} \right\}$$

Implementation of a revised SBL scheme in the YSU BL package

$$\frac{\partial \bar{C}}{\partial t} - \left(K_c \frac{\partial C}{\partial z}\right)^{k+1} + \left(K_c \frac{\partial C}{\partial z}\right)^{k} = -\left(K_c\gamma_c + \overline{(w'c')}_h \left(\frac{z}{h}\right)^3\right)^{k+1} + \left(K_c\gamma_c + \overline{(w'c')}_h \left(\frac{z}{h}\right)^3\right)^{k}$$

$$\frac{C_k^{n+1}-C_k^{n-1}}{2\Delta t} - \left({K_c}_{k+1}^{\,n} \frac{C_{k+1}^{n+1}-C_k^{n+1}}{\Delta z}\right)^{k+1} + \left({K_c}_k^{\,n} \frac{C_k^{n+1}-C_{k-1}^{n+1}}{\Delta z}\right)^{k} = -\left(K_c\gamma_c + \overline{(w'c')}_h \left(\frac{z}{h}\right)^3\right)^{k+1} + \left(K_c\gamma_c + \overline{(w'c')}_h \left(\frac{z}{h}\right)^3\right)^{k}$$

$$\frac{C_k^{n+1}-C_k^{n-1}}{2\Delta t} - {K_c}_{k+1}^{\,n} \frac{C_{k+1}^{n+1}-C_k^{n+1}}{\Delta z} + {K_c}_k^{\,n} \frac{C_k^{n+1}-C_{k-1}^{n+1}}{\Delta z} = -\left(K_c\gamma_c + \overline{(w'c')}_h \left(\frac{Z}{h}\right)^3\right)^{k+1} + \left(K_c\gamma_c + \overline{(w'c')}_h \left(\frac{Z}{h}\right)^3\right)^{k}$$

$$C_k^{n+1} - C_k^{n-1} - \frac{{K_c}_{k+1}^{\,n}2\Delta t}{\Delta z}\left(C_{k+1}^{n+1} - C_k^{n+1}\right) + \frac{{K_c}_k^{\,n}2\Delta t}{\Delta z}\left(C_k^{n+1} - C_{k-1}^{n+1}\right) = -2\Delta t\left(K_c\gamma_c + \overline{(w'c')}_h \left(\frac{z}{h}\right)^3\right)^{k+1} + 2\Delta t\left(K_c\gamma_c + \overline{(w'c')}_h \left(\frac{z}{h}\right)^3\right)^{k}$$

$$C_k^{n+1} - C_k^{n-1} + \left(-\frac{{K_c}_{k+1}^{\,n}2\Delta t}{\Delta z}C_{k+1}^{n+1} + \frac{{K_c}_{k+1}^{\,n}2\Delta t}{\Delta z}C_k^{n+1}\right) + \left(\frac{{K_c}_k^{\,n}2\Delta t}{\Delta z}C_k^{n+1} - \frac{{K_c}_k^{\,n}2\Delta t}{\Delta z}C_{k-1}^{n+1}\right) = -2\Delta t\left(K_c\gamma_c + \overline{(w'c')}_h \left(\frac{z}{h}\right)^3\right)^{k+1} + 2\Delta t\left(K_c\gamma_c + \overline{(w'c')}_h \left(\frac{z}{h}\right)^3\right)^{k}$$

$$-\frac{{K_c}_{k+1}^{\,n}2\Delta t}{\Delta z}C_{k+1}^{n+1} + \frac{{K_c}_{k+1}^{\,n}2\Delta t}{\Delta z}C_k^{n+1} + \frac{{K_c}_k^{\,n}2\Delta t}{\Delta z}C_k^{n+1} + C_k^{n+1} + \left(-\frac{{K_c}_k^{\,n}2\Delta t}{\Delta z}C_{k-1}^{n+1}\right) = C_k^{n-1} - 2\Delta t\left(K_c\gamma_c + \overline{(w'c')}_h \left(\frac{z}{h}\right)^3\right)^{k+1} + 2\Delta t\left(K_c\gamma_c + \overline{(w'c')}_h \left(\frac{z}{h}\right)^3\right)^{k}$$

(1)Quais são as fontes de incerteza do modelo?

Processos físicos da turbulência na camada limite

Implementation of a revised SBL scheme in the YSU BL package

$$-\frac{Kc_{k+1}^{n}2\Delta t}{\Delta z}C_{k+1}^{n+1}+\frac{Kc_{k+1}^{n}2\Delta t}{\Delta z}C_{k}^{n+1}+\frac{Kc_{k}^{n}2\Delta t}{\Delta z}C_{k}^{n+1}+C_{k}^{n+1}+\left(-\frac{Kc_{k}^{n}2\Delta t}{\Delta z}C_{k-1}^{n+1}\right)=C_{k}^{n-1}-2\Delta t\left(K_c\gamma_c+\overline{(w'c')}_h\left(\frac{z}{h}\right)^3\right)^{k+1}+2\Delta t\left(K_c\gamma_c+\overline{(w'c')}_h\left(\frac{z}{h}\right)^3\right)^{k}$$

$$-\frac{Kc_{k+1}^{n}2\Delta t}{\Delta z}C_{k+1}^{n+1}+\frac{Kc_{k+1}^{n}2\Delta t}{\Delta z}C_{k}^{n+1}+\frac{Kc_{k}^{n}2\Delta t}{\Delta z}C_{k}^{n+1}+C_{k}^{n+1}+\left(-\frac{Kc_{k}^{n}2\Delta t}{\Delta z}C_{k-1}^{n+1}\right)=C_{k}^{n-1}-2\Delta t\left(K_c\gamma_c+\overline{(w'c')}_h\left(\frac{z}{h}\right)^3\right)^{k+1}+2\Delta t\left(K_c\gamma_c+\overline{(w'c')}_h\left(\frac{z}{h}\right)^3\right)^{k}$$

$$\left(-\frac{Kc_{k}^{n}2\Delta t}{\Delta z}\right)C_{k-1}^{n+1}+\left(\frac{Kc_{k+1}^{n}2\Delta t}{\Delta z}+\frac{Kc_{k}^{n}2\Delta t}{\Delta z}+1\right)C_{k}^{n+1}-\frac{Kc_{k+1}^{n}2\Delta t}{\Delta z}C_{k+1}^{n+1}=C_{k}^{n-1}-2\Delta t\left(K_c\gamma_c+\overline{(w'c')}_h\left(\frac{z}{h}\right)^3\right)^{k+1}+2\Delta t\left(K_c\gamma_c+\overline{(w'c')}_h\left(\frac{z}{h}\right)^3\right)^{k}$$

$$(AA)C_{k-1}^{n+1}+BBC_{k}^{n+1}-DDC_{k+1}^{n+1}=C_{k}^{n-1}+f1_{k}-f1_{k+1}$$

$$(AA)C_{k-1}^{n+1}+BBC_{k}^{n+1}-DDC_{k+1}^{n+1}=FF_{k}^{n-1}$$

$$\begin{bmatrix} BB & DD & 0 & 0 & 0 & 0\\ AA & BB & DD & 0 & 0 & 0\\ 0 & AA & BB & DD & 0 & 0\\ 0 & 0 & AA & BB & DD & 0\\ 0 & 0 & 0 & AA & BB & DD\\ 0 & 0 & 0 & 0 & AA & BB \end{bmatrix} * \begin{bmatrix} C_1^{n+1}\\ C_2^{n+1}\\ C_3^{n+1}\\ C_4^{n+1}\\ C_5^{n+1}\\ C_6^{n+1}\end{bmatrix} = \begin{bmatrix} FF_1^{n-1}\\ FF_2^{n-1}\\ FF_3^{n-1}\\ FF_4^{n-1}\\ FF_5^{n-1}\\ FF_6^{n-1}\end{bmatrix}$$

$$\frac{\partial C}{\partial t}=\frac{\partial[-\overline{w'c'}]}{\partial z}$$

Implementação da parametrização de Melor Yamada Nino Nakanishi com fluxo de Massa

$$\overline{w'\phi'} = -K\frac{\partial \phi}{\partial z} + M_{\phi,u}(\phi_u - \phi) - M_{\phi,d}(\phi_d - \phi)$$

$$K_\phi = c_\phi L_k \sqrt{\bar{e}}$$

$$K_\phi = S_\phi L_k \sqrt{2\bar{e}}$$

5.2 TURBULENT KINETIC ENERGY **A equação de Navier Stokes: conservação de momentum**

i=1,2,3
J=1,2,3
K=1,2,3

$$\frac{Du_i}{Dt} = \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho_0}\frac{\partial P}{\partial x_i} - \frac{\rho}{\rho_0} g\delta_{i3} - 2\Omega\varepsilon_{ijk}\eta_j u_k + \nu\left(\frac{\partial^2 u_i}{\partial {x_j}^2}\right)$$

$u_i = \overline{u_i} + u_i{}'$

Aplique a Média de Reynolds na Variáveis

$$\frac{\partial(\overline{u_i} + u_i{}')}{\partial t} + (\overline{u_j} + u_j{}')\frac{\partial(\overline{u_i} + u_i{}')}{\partial x_j} = -\frac{1}{\rho_0}\frac{\partial(\bar{P} + P')}{\partial x_i} - g\frac{(\rho + \rho')}{\rho_0}\delta_{i3} - 2\Omega\varepsilon_{ijk}\eta_j(\overline{u_k} + u_k{}') + \nu\left(\frac{\partial^2(\overline{u_i} + u_i{}')}{\partial {x_j}^2}\right)$$

Expanda os termos

$$\frac{\partial(\overline{u_i})}{\partial t} + \frac{\partial(u_i{}')}{\partial t} + (\overline{u_j})\frac{\partial(\overline{u_i})}{\partial x_j} + (\overline{u_j})\frac{\partial(u_i{}')}{\partial x_j} + (u_j{}')\frac{\partial(\overline{u_i})}{\partial x_j} + (u_j{}')\frac{\partial(u_i{}')}{\partial x_j} =$$
$$-\frac{1}{\rho_0}\frac{\partial(\bar{P})}{\partial x_i} - \frac{1}{\rho_0}\frac{\partial(P')}{\partial x_i} - g\frac{\rho}{\rho_0}\delta_{i3} - g\frac{\rho'}{\rho_0}\delta_{i3} - 2\Omega\varepsilon_{ijk}\eta_j(\overline{u_k}) - 2\Omega\varepsilon_{ijk}\eta_j(u_k{}') + \nu\frac{\partial^2(\overline{u_i})}{\partial {x_j}^2} + \nu\frac{\partial^2(u_i{}')}{\partial {x_j}^2}$$

A equação de Navier Stokes: conservação de momentum

$$\frac{\partial(\overline{u_i})}{\partial t}+\frac{\partial(u_i')}{\partial t}+(\overline{u_j})\frac{\partial(\overline{u_i})}{\partial x_j}+(\overline{u_j})\frac{\partial(u_i')}{\partial x_j}+(u_j')\frac{\partial(\overline{u_i})}{\partial x_j}+(u_j')\frac{\partial(u_i')}{\partial x_j}=$$
$$-\frac{1}{\rho_0}\frac{\partial(\bar{P})}{\partial x_i}-\frac{1}{\rho_0}\frac{\partial(P')}{\partial x_i}-g\frac{\rho}{\rho_0}\delta_{i3}-g\frac{\rho'}{\rho_0}\delta_{i3}-2\Omega\varepsilon_{ijk}\eta_j(\overline{u_k})-2\Omega\varepsilon_{ijk}\eta_j(u_k')+\nu\frac{\partial^2(\overline{u_i})}{\partial {x_j}^2}+\nu\frac{\partial^2(u_i')}{\partial {x_j}^2}$$

Separa os termos na equação acima

$$\frac{\partial(\overline{u_i})}{\partial t}+(\overline{u_j})\frac{\partial(\overline{u_i})}{\partial x_j}+(\overline{u_j})\frac{\partial(u_i')}{\partial x_j}+(u_j')\frac{\partial(\overline{u_i})}{\partial x_j}+(u_j')\frac{\partial(u_i')}{\partial x_j}=$$
$$-\frac{1}{\rho_0}\frac{\partial(\bar{P})}{\partial x_i}-g\frac{\rho}{\rho_0}\delta_{i3}-2\Omega\varepsilon_{ijk}\eta_j(\overline{u_k})+\nu\frac{\partial^2(\overline{u_i})}{\partial {x_j}^2}$$

A equação de Navier Stokes: conservação de momentum

$$\frac{\partial(\overline{u}_i)}{\partial t}+(\overline{u}_j)\frac{\partial(\overline{u}_i)}{\partial x_j}+(u_j')\frac{\partial(\overline{u}_i)}{\partial x_j}+\boxed{(u_j')\frac{\partial(u_i')}{\partial x_j}}=$$
$$-\frac{1}{\rho_0}\frac{\partial(\overline{P})}{\partial x_i}-g\frac{\rho}{\rho_0}\delta_{i3}-2\Omega\varepsilon_{ijk}\eta_j(\overline{u_k})+\nu\frac{\partial^2(\overline{u}_i)}{\partial {x_j}^2}$$

$$\frac{\partial(\overline{u}_i)}{\partial t}+(\overline{u}_j)\frac{\partial(\overline{u}_i)}{\partial x_j}+(u_j')\frac{\partial(\overline{u}_i)}{\partial x_j}+\boxed{\frac{\partial\left(u_j'u_i'\right)}{\partial x_j}-(u_i')\frac{\partial\left(u_j'\right)}{\partial x_j}}$$
$$=-\frac{1}{\rho_0}\frac{\partial(\overline{P})}{\partial x_i}-g\frac{\rho}{\rho_0}\delta_{i3}-2\Omega\varepsilon_{ijk}\eta_j(\overline{u_k})+\nu\frac{\partial^2(\overline{u}_i)}{\partial {x_j}^2}$$

Aplique as media de Reynolds

$$\frac{\partial(\overline{\overline{u}_i})}{\partial t}+(\overline{\overline{u}_j})\frac{\partial(\overline{\overline{u}_i})}{\partial x_j}+\frac{\partial\left(\overline{u_j'u_i'}\right)}{\partial x_j}-(\overline{u_i'})\frac{\partial\left(\overline{u_j'}\right)}{\partial x_j}+(\overline{u_j'})\frac{\partial(\overline{\overline{u}_i})}{\partial x_j}$$
$$=-\frac{1}{\rho_0}\frac{\partial(\overline{\overline{P}})}{\partial x_i}-g\frac{\overline{\rho}}{\rho_0}\delta_{i3}-2\Omega\varepsilon_{ijk}\eta_j(\overline{\overline{u_k}})+\nu\frac{\partial^2(\overline{\overline{u}_i})}{\partial {x_j}^2}$$

Linearização da Equações de Navier Stokes

A equação de Navier Stokes: conservação de momentum

$$\frac{\partial(\bar{\bar{u}}_i)}{\partial t} + (\bar{\bar{u}}_j)\frac{\partial(\bar{\bar{u}}_i)}{\partial x_j} + \frac{\partial\left(\overline{u_j{}'u_i{}'}\right)}{\partial x_j} - (\overline{u_i{}'})\frac{\partial\left(\overline{u_j{}'}\right)}{\partial x_j} + (\overline{u_j{}'})\frac{\partial(\overline{u_i})}{\partial x_j}$$
$$= -\frac{1}{\rho_0}\frac{\partial\left(\bar{\bar{P}}\right)}{\partial x_i} - g\frac{\bar{\rho}}{\rho_0}\delta_{i3} - 2\Omega\varepsilon_{ijk}\eta_j(\overline{\overline{u_k}}) + \nu\frac{\partial^2(\bar{\bar{u}}_i)}{\partial {x_j}^2}$$

Aplique as considerações da media de Reynolds

$\bar{\bar{u}}_i = \bar{u}_i$ $\overline{u_j{}'u_i{}'} \neq 0$ $\overline{u_j{}'} = 0$ $\overline{u_i{}'} = 0$

$$\frac{\partial(\bar{u}_i)}{\partial t} + (\bar{u}_j)\frac{\partial(\bar{u}_i)}{\partial x_j} = -\frac{\partial\left(\overline{u_j{}'u_i{}'}\right)}{\partial x_j} - \frac{1}{\rho_0}\frac{\partial(\bar{P})}{\partial x_i} - g\frac{\bar{\rho}}{\rho_0}\delta_{i3} - 2\Omega\varepsilon_{ijk}\eta_j(\overline{u_k}) + \nu\frac{\partial^2(\bar{u}_i)}{\partial {x_j}^2}$$

- ***Equação primitiva não linear para o escoamento da Atmosfera***

$$\frac{Du_i}{Dt} = \frac{\partial u_i}{\partial t} + u_j\frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho_0}\frac{\partial P}{\partial x_i} - \frac{\rho}{\rho_0}g\delta_{i3} - 2\Omega\varepsilon_{ijk}\eta_j u_k + \nu\left(\frac{\partial^2 u_i}{\partial {x_j}^2}\right)$$

- ***Equação Governante Linearizada do Estado Médio do escoamento da Atmosfera***

$$\frac{\partial(\overline{u_i})}{\partial t} + (\overline{u_j})\frac{\partial(\overline{u_i})}{\partial x_j} = -\frac{\partial\left(\overline{u_j{}'u_i{}'}\right)}{\partial x_j} - \frac{1}{\rho_0}\frac{\partial(\bar{P})}{\partial x_i} - g\frac{\bar{\rho}}{\rho_0}\delta_{i3} - 2\Omega\varepsilon_{ijk}\eta_j(\overline{u_k}) + \nu\frac{\partial^2(\overline{u_i})}{\partial {x_j}^2}$$

Paulo Yoshio Kubota

5.2 TURBULENT KINETIC ENERGY **A equação de Navier Stokes: conservação de momentum**

$$\frac{\partial(\overline{u_i})}{\partial t} + \frac{\partial(u_i')}{\partial t} + (\overline{u_j})\frac{\partial(\overline{u_i})}{\partial x_j} + (\overline{u_j})\frac{\partial(u_i')}{\partial x_j} + (u_j')\frac{\partial(\overline{u_i})}{\partial x_j} + (u_j')\frac{\partial(u_i')}{\partial x_j} =$$
$$-\frac{1}{\rho_0}\frac{\partial(\overline{P})}{\partial x_i} - \frac{1}{\rho_0}\frac{\partial(P')}{\partial x_i} - g\frac{\rho}{\rho_0}\delta_{i3} - g\frac{\rho'}{\rho_0}\delta_{i3} - 2\Omega\varepsilon_{ijk}\eta_j(\overline{u_k}) - 2\Omega\varepsilon_{ijk}\eta_j(u_k') + \nu\frac{\partial^2(\overline{u_i})}{\partial {x_j}^2} + \nu\frac{\partial^2(u_i')}{\partial {x_j}^2}$$

Separe os termos com perturbação que se cancelariam com a media de Reynolds

$$\frac{\partial(u_i')}{\partial t} + (\overline{u_j})\frac{\partial(u_i')}{\partial x_j} + \boxed{(u_j')\frac{\partial(\overline{u_i})}{\partial x_j}} + \boxed{(u_j')\frac{\partial(u_i')}{\partial x_j}}$$
$$= -\frac{1}{\rho_0}\frac{\partial(P')}{\partial x_i} - g\frac{\rho'}{\rho_0}\delta_{i3} - 2\Omega\varepsilon_{ijk}\eta_j(u_k') + \nu\frac{\partial^2(u_i')}{\partial {x_j}^2}$$

$$\frac{\partial(u_i')}{\partial t} + (\overline{u_j})\frac{\partial(u_i')}{\partial x_j} + \boxed{\frac{\partial(u_j'\overline{u_i})}{\partial x_j} - (\overline{u_i})\frac{\partial(u_j')}{\partial x_j}} + \boxed{\frac{\partial(u_j'u_i')}{\partial x_j} - (u_i')\frac{\partial(u_j')}{\partial x_j}}$$
$$= -\frac{1}{\rho}\frac{\partial(P')}{\partial x_i} - g\frac{\rho'}{\rho_0}\delta_{i3} - 2\Omega\varepsilon_{ijk}\eta_j(u_k') + \nu\frac{\partial^2(u_i')}{\partial {x_j}^2}$$

Paulo Yoshio Kubota

5.2 TURBULENT KINETIC ENERGY A equação de Navier Stokes: conservação de momentum

$$\frac{\partial(u_i')}{\partial t} + (\overline{u_j})\frac{\partial(u_i')}{\partial x_j} + \boxed{(u_j')\frac{\partial(\overline{u_i})}{\partial x_j}} + \boxed{(u_j')\frac{\partial(u_i')}{\partial x_j}} = -\frac{1}{\rho_0}\frac{\partial(P')}{\partial x_i} - g\frac{\rho'}{\rho_0}\delta_{i3} - 2\Omega\varepsilon_{ijk}\eta_j(u_k') + \nu\frac{\partial^2(u_i')}{\partial {x_j}^2}$$

Aplique a derivada do produto nos termos em destaque:

$$\frac{\partial(u_i')}{\partial t} + (\overline{u_j})\frac{\partial(u_i')}{\partial x_j} + \boxed{\frac{\partial(u_j'\overline{u_i})}{\partial x_j} - (\overline{u_i})\frac{\partial(u_j')}{\partial x_j}} + \boxed{\frac{\partial(u_j'u_i')}{\partial x_j} - (u_i')\frac{\partial(u_j')}{\partial x_j}}$$
$$= -\frac{1}{\rho}\frac{\partial(P')}{\partial x_i} - g\frac{\rho'}{\rho_0}\delta_{i3} - 2\Omega\varepsilon_{ijk}\eta_j(u_k') + \nu\frac{\partial^2(u_i')}{\partial {x_j}^2}$$

5.2 TURBULENT KINETIC ENERGY **A equação de Navier Stokes: conservação de momentum**

$$\frac{\partial(u_i')}{\partial t} + (\overline{u_j})\frac{\partial(u_i')}{\partial x_j} + \frac{\partial(u_j'\overline{u_i})}{\partial x_j} - (\overline{u_i})\frac{\partial(u_j')}{\partial x_j} + \frac{\partial(u_j'u_i')}{\partial x_j} - (u_i')\frac{\partial(u_j')}{\partial x_j}$$
$$= -\frac{1}{\rho}\frac{\partial(P')}{\partial x_i} - g\frac{\rho'}{\rho_0}\delta_{i3} - 2\Omega\varepsilon_{ijk}\eta_j(u_k') + \nu\frac{\partial^2(u_i')}{\partial {x_j}^2}$$

Considere que a propriedade da equação da continuidade para o fluxo $\nabla.\vec{V} = 0$

$$\nabla.\vec{V'} = \frac{\partial(u_j')}{\partial x_j} = 0, \qquad j = 1,2,3$$

$$\frac{\partial(u_i')}{\partial t} + (\overline{u_j})\frac{\partial(u_i')}{\partial x_j} + \frac{\partial(u_j'\overline{u_i})}{\partial x_j} + \frac{\partial(u_j'u_i')}{\partial x_j} = -\frac{1}{\rho_0}\frac{\partial(P')}{\partial x_i} - g\frac{\rho'}{\rho_0}\delta_{i3} - 2\Omega\varepsilon_{ijk}\eta_j(u_k') + \nu\frac{\partial^2(u_i')}{\partial {x_j}^2}$$

5.2 TURBULENT KINETIC ENERGY

A equação de Navier Stokes: conservação de momentum

$$\boxed{\frac{\partial(u_i')}{\partial t} + (\overline{u_j})\frac{\partial(u_i')}{\partial x_j} + \frac{\partial\left(u_j'\overline{u_i}\right)}{\partial x_j} + \frac{\partial\left(u_j'u_i'\right)}{\partial x_j} = -\frac{1}{\rho_0}\frac{\partial(P')}{\partial x_i} - g\frac{\rho'}{\rho_0}\delta_{i3} - 2\Omega\varepsilon_{ijk}\eta_j(u_k') + \nu\frac{\partial^2(u_i')}{\partial x_j{}^2}}$$

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1.1 A Teoria de Similaridade de Monin-Obukhov

As equações são apresentadas de uma forma geral considerável para que, em princípio, possam ser integradas para simular, por exemplo, uma camada limite planetária completa.

Aqui, no entanto, restringimos a atenção à região da superfície de fluxo constante, evitando assim, por enquanto, o considerável esforço computacional necessário para a camada completa.

No entanto, este é um primeiro passo lógico, uma vez que é possível comparar diretamente com os dados de fluxo constante de Businger et al. (1971) na forma de variáveis de similaridade de Monin-Obukhoff.

1.1 A Teoria de Similaridade de Monin-Obukhov

2. As equações básicas

As equações de movimento para a velocidade média $\bar{u}_j$; e a temperatura potencial média $\bar{\theta}$, As barras superiores representam as médias do conjunto e os termos minúsculos, u'_k e θ', são os componentes flutuantes da velocidade e da temperatura e são governados por são

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0 \qquad (1)$$

$$\frac{\partial \bar{u}_j}{\partial t} + \bar{u}_k \frac{\partial (\bar{u}_j)}{\partial x_k} + \bar{u}_j \left[\frac{\partial (\bar{u}_k)}{\partial x_k}\right] + \frac{\partial \left(\overline{u'_k u'_j}\right)}{\partial x_k} + \epsilon_{j,k,l} f_k \bar{u}_l = -\frac{\partial P}{\partial x_j} - g_j \beta \bar{\theta} + \nu \nabla^2 \bar{u}_j \qquad (2)$$

$$\frac{\partial \bar{u}_j}{\partial t} + \frac{\partial (\bar{u}_k \bar{u}_j)}{\partial x_k} + \frac{\partial \left(\overline{u'_k u'_j}\right)}{\partial x_k} + \epsilon_{j,k,l} f_k \bar{u}_l = -\frac{\partial P}{\partial x_j} - g_j \beta \bar{\theta} + \nu \nabla^2 \bar{u}_j \qquad (2)$$

$$\boxed{\frac{\partial \bar{u}_j}{\partial t} + \frac{\partial}{\partial x_k}\left(\bar{u}_k \bar{u}_j + \overline{u'_k u'_j}\right) + \epsilon_{j,k,l} f_k \bar{u}_l = -\frac{\partial P}{\partial x_j} - g_j \beta \bar{\theta} + \nu \nabla^2 \bar{u}_j \qquad (2)}$$

$$\boxed{\frac{\partial \bar{\theta}}{\partial t} + \frac{\partial}{\partial \bar{u}_k}\left(\bar{u}_k \bar{\theta} + \overline{u'_k \theta'}\right) = \alpha \nabla^2 \bar{\theta} \qquad (3)}$$

onde P é a pressão cinemática média, g_j = (0, 0, −g)vetor de gravidade, f_j = (0,f_y,f) o parâmetro de Coriolis (o componente vertical de não terá subscrito), $\beta = \left(\frac{\partial \rho}{\partial T}\right)_P / \rho$ o coeficiente de expansão térmica, ν é a viscosidade cinemática e α é a condutividade térmica cinemática (ou difusividade térmica).

2. As equações básicas

$$\frac{\partial u_i'}{\partial x_i} = 0 \qquad (4)$$

$$\boxed{\frac{\partial(u_j')}{\partial t} + (\overline{u_k})\frac{\partial(u_j')}{\partial x_k} + \frac{\partial(u_k'\overline{u_j})}{\partial x_k} + \frac{\partial(u_k'u_j')}{\partial x_k} = -\frac{1}{\rho_0}\frac{\partial(P')}{\partial x_j} - g\frac{\rho'}{\rho_0}\delta_{j3} - 2\Omega\varepsilon_{jki}\eta_k(u_k') + \nu\frac{\partial^2(u_j')}{\partial {x_k}^2}}$$

Aplique a media de reynolds

$$\boxed{\frac{\partial(\overline{u_j'})}{\partial t} + (\overline{\overline{u_k}})\frac{\partial(\overline{u_j'})}{\partial x_k} + \frac{\partial(\overline{u_k'\overline{u_j}})}{\partial x_k} + \frac{\partial(\overline{u_k'u_j'})}{\partial x_k} = -\frac{1}{\rho_0}\frac{\partial(\overline{P'})}{\partial x_j} - g\frac{\overline{\rho'}}{\rho_0}\delta_{j3} - 2\Omega\varepsilon_{jki}\eta_k(\overline{u_k'}) + \nu\frac{\partial^2(\overline{u_j'})}{\partial {x_k}^2}}$$

$$\boxed{\frac{\partial(\overline{u_k'u_j'})}{\partial x_k} = 0}$$

$$\frac{\partial u_j'}{\partial t} + \left(\bar{u}_k\frac{\partial u_j'}{\partial x_k} + u_j'\frac{\partial \bar{u}_k}{\partial x_k} + \frac{\partial(\bar{u}_j u_k')}{\partial x_k} + \frac{\partial(u_k'u_j')}{\partial x_k} - \frac{\partial(\overline{u_k'u_j'})}{\partial x_k}\right) + \epsilon_{j,k,l}f_k u_l' = -\frac{\partial P'}{\partial x_j} - g_j\beta\theta' + \nu\nabla^2 u_j' \quad (5)$$

Como as equações médias (1), (2) e (3) envolvem a tensão de Reynolds $\overline{u_i'u_j'}$ e os momentos de condução de calor, $u_i'\theta'$, obtemos suas equações governantes de (5) e (6) usando a (4)

1.1 A Teoria de Similaridade de Monin-Obukhov

2. As equações básicas

$$\frac{\partial u_i'}{\partial x_i} = 0 \qquad (4)$$

$$\boxed{\frac{\partial (u_j{}')}{\partial t} + (\overline{u_k})\frac{\partial (u_j{}')}{\partial x_k} + \frac{\partial (u_k{}'\overline{u_j})}{\partial x_k} + \frac{\partial (u_k{}'u_j{}')}{\partial x_k} = -\frac{1}{\rho_0}\frac{\partial (P')}{\partial x_j} - g\frac{\rho'}{\rho_0}\delta_{j3} - 2\Omega\varepsilon_{jki}\eta_k(u_k{}') + \nu\frac{\partial^2 (u_j{}')}{\partial x_k{}^2}}$$

$$\frac{\partial u_j'}{\partial t} + \left(\bar{u}_k\frac{\partial u_j'}{\partial x_k} + u_j'\frac{\partial \bar{u}_k}{\partial x_k} + \frac{\partial (\bar{u}_j u_k{}')}{\partial x_k} + \frac{\partial (u_k{}'u_j{}')}{\partial x_k} - \frac{\partial (\overline{u_k{}'u_j{}'})}{\partial x_k}\right) + \epsilon_{j,k,l}f_k u_l' = -\frac{\partial P'}{\partial x_j} - g_j\beta\theta' + \nu\nabla^2 u_j'$$

$$\frac{\partial u_j'}{\partial t} + \left(\frac{\partial \bar{u}_k u_j'}{\partial x_k} + \frac{\partial \bar{u}_j u_k'}{\partial x_k} + \frac{\partial u_k' u_j'}{\partial x_k} - \frac{\partial \overline{u_k' u_j'}}{\partial x_k}\right) + \epsilon_{j,k,l}f_k u_l' = -\frac{\partial P'}{\partial x_j} - g_j\beta\theta' + \nu\nabla^2 u_j' \qquad (5)$$

$$\frac{\partial u_j'}{\partial t} + \frac{\partial}{\partial x_k}\left(\bar{u}_k u_j' + \bar{u}_j u_k' + u_k' u_j' - \overline{u_k' u_j'}\right) + \epsilon_{j,k,l}f_k u_l' = -\frac{\partial P'}{\partial x_j} - g_j\beta\theta' + \nu\nabla^2 u_j' \qquad (5)$$

$$\frac{\partial \theta'}{\partial t} + \frac{\partial}{\partial x_k}\left(\bar{\theta}u_k' + \bar{u}_k\theta' + u_k'\theta' + \overline{u_k'\theta'}\right) = \alpha\nabla^2\theta' \qquad (6)$$

Como as equações médias (1), (2) e (3) envolvem a tensão de Reynolds $\overline{u_i'u_j'}$ e os momentos de condução de calor, $u_i'\theta'$, obtemos suas equações governantes de (5) e (6) usando a (4)

2. As equações básicas

$$\frac{\partial u_j'}{\partial t} + \frac{\partial}{\partial x_k}\left(\bar{u}_k u_j' + \bar{u}_j u_k' + u_k' u_j' - \overline{u_k' u_j'}\right) + \epsilon_{j,k,l} f_k u_l' = -\frac{\partial P'}{\partial x_j} - g_j \beta \theta' + \nu \nabla^2 u_j' \qquad (5)$$

$$u_i' \frac{\partial u_j'}{\partial t} + u_i' \frac{\partial}{\partial x_k}\left(\bar{u}_k u_j' + \bar{u}_j u_k' + u_k' u_j' - \overline{u_k' u_j'}\right) + \epsilon_{j,k,l} f_k u_l' u_i' = -u_i' \frac{\partial P'}{\partial x_j} - g_j \beta u_i' \theta' + \nu u_i' \nabla^2 u_j' \qquad (5a)$$

$$u_j' \frac{\partial u_i'}{\partial t} + u_j' \frac{\partial}{\partial x_k}\left(\bar{u}_k u_i' + \bar{u}_i u_k' + u_k' u_i' - \overline{u_k' u_i'}\right) + \epsilon_{i,k,l} f_k u_l' u_j' = -u_j' \frac{\partial P'}{\partial x_i} - g_i \beta u_j' \theta' + \nu u_j' \nabla^2 u_i' \qquad (5b)$$

$$\left(u_i' \frac{\partial u_j'}{\partial t} + u_j' \frac{\partial u_i'}{\partial t}\right) + \left(u_i' \frac{\partial \bar{u}_k u_j'}{\partial x_k} + u_j' \frac{\partial \bar{u}_k u_i'}{\partial x_k}\right) + \left(u_i' \frac{\partial \bar{u}_j u_k'}{\partial x_k} + u_j' \frac{\partial \bar{u}_i u_k'}{\partial x_k}\right) + \left(u_i' \frac{\partial u_k' u_j'}{\partial x_k} + u_j' \frac{\partial u_k' u_i'}{\partial x_k}\right) - \left(u_i' \frac{\partial \overline{u_k' u_j'}}{\partial x_k} + u_j' \frac{\partial \overline{u_k' u_i'}}{\partial x_k}\right)$$
$$+ \epsilon_{j,k,l} f_k u_l' u_i' + \epsilon_{i,k,l} f_k u_l' u_j' = -u_i' \frac{\partial P'}{\partial x_j} - u_j' \frac{\partial P'}{\partial x_i} - g_j \beta u_i' \theta' - g_i \beta u_j' \theta' + \nu u_i' \nabla^2 u_j' + \nu u_j' \nabla^2 u_i' \, (5c)$$

2. As equações básicas

$$\left(u_i'\frac{\partial u_j'}{\partial t}+u_j'\frac{\partial u_i'}{\partial t}\right)+\left(u_i'\frac{\partial \bar{u}_k u_j'}{\partial x_k}+u_j'\frac{\partial \bar{u}_k u_i'}{\partial x_k}\right)+\left(u_i'\frac{\partial \bar{u}_j u_k'}{\partial x_k}+u_j'\frac{\partial \bar{u}_i u_k'}{\partial x_k}\right)+\left(u_i'\frac{\partial u_k' u_j'}{\partial x_k}+u_j'\frac{\partial u_k' u_i'}{\partial x_k}\right)-\left(u_i'\frac{\partial \overline{u_k' u_j'}}{\partial x_k}+u_j'\frac{\partial \overline{u_k' u_i'}}{\partial x_k}\right)$$
$$+\epsilon_{j,k,l}f_k u_l' u_i'+\epsilon_{i,k,l}f_k u_l' u_j'=-u_i'\frac{\partial P'}{\partial x_j}-u_j'\frac{\partial P'}{\partial x_i}-g_j\beta u_i'\theta'-g_i\beta u_j'\theta'+\nu u_i'\nabla^2 u_j'\ +\nu u_j'\nabla^2 u_i'$$

$$\left(u_i'\frac{\partial u_j'}{\partial t}+u_j'\frac{\partial u_i'}{\partial t}\right)+\left(u_i'\frac{\partial \bar{u}_k u_j'}{\partial x_k}+u_j'\frac{\partial \bar{u}_k u_i'}{\partial x_k}\right)+\left(u_i'\frac{\partial u_k' u_j'}{\partial x_k}+u_j'\frac{\partial u_k' u_i'}{\partial x_k}\right)+\left(u_i'\frac{\partial \bar{u}_j u_k'}{\partial x_k}+u_j'\frac{\partial \bar{u}_i u_k'}{\partial x_k}\right)-\left(u_i'\frac{\partial \overline{u_k' u_j'}}{\partial x_k}+u_j'\frac{\partial \overline{u_k' u_i'}}{\partial x_k}\right)$$
$$+\epsilon_{j,k,l}f_k u_l' u_i'+\epsilon_{i,k,l}f_k u_l' u_j'=-u_i'\frac{\partial P'}{\partial x_j}-u_j'\frac{\partial P'}{\partial x_i}-g_j\beta u_i'\theta'-g_i\beta u_j'\theta'+\nu u_i'\nabla^2 u_j'\ +\nu u_j'\nabla^2 u_i'$$

$$\left(\frac{\partial u_i' u_j'}{\partial t}\right)+\left(\left[\frac{\partial \bar{u}_k u_i' u_j'}{\partial x_k}-u_j'\frac{\partial \bar{u}_k u_i'}{\partial x_k}\right]+\left[\frac{\partial \bar{u}_k u_j' u_i'}{\partial x_k}-u_i'\frac{\partial \bar{u}_k u_j'}{\partial x_k}\right]\right)+\left(\left[\frac{\partial u_k' u_i' u_j'}{\partial x_k}-u_j'\frac{\partial u_k' u_i'}{\partial x_k}\right]+\left[\frac{\partial u_k' u_j' u_i'}{\partial x_k}-u_i'\frac{\partial u_k' u_j'}{\partial x_k}\right]\right)$$
$$+\left(\left[\frac{\partial \bar{u}_j u_i' u_k'}{\partial x_k}-u_k'\frac{\partial \bar{u}_j u_i'}{\partial x_k}\right]+\left[\frac{\partial \bar{u}_i u_j' u_k'}{\partial x_k}-u_k'\frac{\partial \bar{u}_i u_j'}{\partial x_k}\right]\right)-\left(\left[\frac{\partial u_i'\overline{u_k' u_j'}}{\partial x_k}-\overline{u_k' u_j'}\frac{\partial u_i'}{\partial x_k}\right]+\left[\frac{\partial u_j'\overline{u_k' u_i'}}{\partial x_k}-\overline{u_k' u_i'}\frac{\partial u_j'}{\partial x_k}\right]\right)$$
$$+f_k\left(\epsilon_{j,k,l}u_l' u_i'+\epsilon_{i,k,l}u_l' u_j'\right)=-\left[\frac{\partial u_i' P'}{\partial x_j}-P'\frac{\partial u_i'}{\partial x_j}\right]-\left[\frac{\partial u_j' P'}{\partial x_i}-P'\frac{\partial u_j'}{\partial x_i}\right]-\beta\left(g_j u_i'\theta'-g_i u_j'\theta'\right)+\nu u_i'\nabla^2 u_j'\ +\nu u_j'\nabla^2 u_i'$$

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2. As equações básicas

$$\left(\frac{\partial u_i'u_j'}{\partial t}\right)+\left(\left[\frac{\partial \bar{u}_k u_i'u_j'}{\partial x_k}-u_j'\frac{\partial \bar{u}_k u_i'}{\partial x_k}\right]+\left[\frac{\partial \bar{u}_k u_j'u_i'}{\partial x_k}-u_i'\frac{\partial \bar{u}_k u_j'}{\partial x_k}\right]\right)+\left(\left[\frac{\partial u_k' u_i'u_j'}{\partial x_k}-u_j'\frac{\partial u_k' u_i'}{\partial x_k}\right]+\left[\frac{\partial u_k' u_j'u_i'}{\partial x_k}-u_i'\frac{\partial u_k' u_j'}{\partial x_k}\right]\right)$$

$$+\left(\left[\frac{\partial \bar{u}_j u_i'u_k'}{\partial x_k}-u_k'\frac{\partial \bar{u}_j u_i'}{\partial x_k}\right]+\left[\frac{\partial \bar{u}_i u_j'u_k'}{\partial x_k}-u_k'\frac{\partial \bar{u}_i u_j'}{\partial x_k}\right]\right)-\left(\left[\frac{\partial u_i'\overline{u_k'u_j'}}{\partial x_k}-\overline{u_k'u_j'}\frac{\partial u_i'}{\partial x_k}\right]+\left[\frac{\partial u_j'\overline{u_k'u_i'}}{\partial x_k}-\overline{u_k'u_i'}\frac{\partial u_j'}{\partial x_k}\right]\right)$$

$$+f_k\left(\epsilon_{j,k,l}u_l'u_i'+\epsilon_{i,k,l}u_l'u_j'\right)=-\left[\frac{\partial u_i'P'}{\partial x_j}-P'\frac{\partial u_i'}{\partial x_j}\right]-\left[\frac{\partial u_j'P'}{\partial x_i}-P'\frac{\partial u_j'}{\partial x_i}\right]-\beta\left(g_j u_i'\theta'-g_i u_j'\theta'\right)+\nu u_i'\nabla^2 u_j'+\nu u_j'\nabla^2 u_i'$$

$$\left(\frac{\partial u_i'u_j'}{\partial t}\right)+\left(\left[\frac{\partial \bar{u}_k u_i'u_j'}{\partial x_k}+\frac{\partial \bar{u}_k u_j'u_i'}{\partial x_k}\right]+\left[-u_j'\frac{\partial \bar{u}_k u_i'}{\partial x_k}-u_i'\frac{\partial \bar{u}_k u_j'}{\partial x_k}\right]\right)+\left(\left[\frac{\partial u_k' u_i'u_j'}{\partial x_k}+\frac{\partial u_k' u_j'u_i'}{\partial x_k}\right]+\left[-u_j'\frac{\partial u_k' u_i'}{\partial x_k}-u_i'\frac{\partial u_k' u_j'}{\partial x_k}\right]\right)$$

$$+\left(\left[\frac{\partial \bar{u}_j u_i'u_k'}{\partial x_k}+\frac{\partial \bar{u}_i u_j'u_k'}{\partial x_k}\right]+\left[-u_k'\frac{\partial \bar{u}_j u_i'}{\partial x_k}-u_k'\frac{\partial \bar{u}_i u_j'}{\partial x_k}\right]\right)-\left(\left[\frac{\partial u_i'\overline{u_k'u_j'}}{\partial x_k}\right]+\left[\frac{\partial u_j'\overline{u_k'u_i'}}{\partial x_k}\right]\right)-\left(\left[-\overline{u_k'u_j'}\frac{\partial u_i'}{\partial x_k}\right]+\left[-\overline{u_k'u_i'}\frac{\partial u_j'}{\partial x_k}\right]\right)$$

$$+f_k\left(\epsilon_{j,k,l}u_l'u_i'+\epsilon_{i,k,l}u_l'u_j'\right)=-\left[\frac{\partial u_i'P'}{\partial x_j}+\frac{\partial u_j'P'}{\partial x_i}\right]+\left[P'\frac{\partial u_i'}{\partial x_j}+P'\frac{\partial u_j'}{\partial x_i}\right]-\beta\left(g_j u_i'\theta'-g_i u_j'\theta'\right)+\nu u_i'\nabla^2 u_j'+\nu u_j'\nabla^2 u_i'$$

2. As equações básicas

$$\left(\frac{\partial u_i' u_j'}{\partial t}\right) + \left(\left[\frac{\partial \bar{u}_k u_i' u_j'}{\partial x_k} + \frac{\partial \bar{u}_k u_j' u_i'}{\partial x_k}\right] + \left[-u_j' \frac{\partial \bar{u}_k u_i'}{\partial x_k} - u_i' \frac{\partial \bar{u}_k u_j'}{\partial x_k}\right]\right) + \left(\left[\frac{\partial u_k' u_i' u_j'}{\partial x_k} + \frac{\partial u_k' u_j' u_i'}{\partial x_k}\right] + \left[-u_j' \frac{\partial u_k' u_i'}{\partial x_k} - u_i' \frac{\partial u_k' u_j'}{\partial x_k}\right]\right)$$
$$+ \left(\left[\frac{\partial \bar{u}_j u_i' u_k'}{\partial x_k} + \frac{\partial \bar{u}_i u_j' u_k'}{\partial x_k}\right] + \left[-u_k' \frac{\partial \bar{u}_j u_i'}{\partial x_k} - u_k' \frac{\partial \bar{u}_i u_j'}{\partial x_k}\right]\right) - \left(\left[\frac{\partial u_i' \overline{u_k' u_j'}}{\partial x_k}\right] + \left[\frac{\partial u_j' \overline{u_k' u_i'}}{\partial x_k}\right]\right) - \left(\left[-\overline{u_k' u_j'} \frac{\partial u_i'}{\partial x_k}\right] + \left[-\overline{u_k' u_i'} \frac{\partial u_j'}{\partial x_k}\right]\right)$$
$$+ f_k\left(\epsilon_{j,k,l} u_l' u_i' + \epsilon_{i,k,l} u_l' u_j'\right) = -\left[\frac{\partial u_i' P'}{\partial x_j} + \frac{\partial u_j' P'}{\partial x_i}\right] + \left[P' \frac{\partial u_i'}{\partial x_j} + P' \frac{\partial u_j'}{\partial x_i}\right] - \beta\left(g_j u_i' \theta' - g_i u_j' \theta'\right) + \nu u_i' \nabla^2 u_j' + \nu u_j' \nabla^2 u_i'$$

$$\left(\frac{\partial u_i' u_j'}{\partial t}\right) + \left(\left[\frac{\partial \bar{u}_k u_i' u_j'}{\partial x_k} + \frac{\partial \bar{u}_k u_i' u_j'}{\partial x_k}\right] + \left[-u_j' \frac{\partial \bar{u}_k u_i'}{\partial x_k} - u_i' \frac{\partial \bar{u}_k u_j'}{\partial x_k}\right]\right) + \left(\left[\frac{\partial u_k' u_i' u_j'}{\partial x_k} + \frac{\partial u_k' u_i' u_j'}{\partial x_k}\right] + \left[-u_j' \frac{\partial u_k' u_i'}{\partial x_k} - u_i' \frac{\partial u_k' u_j'}{\partial x_k}\right]\right)$$
$$+ \left(\left[\frac{\partial \bar{u}_j u_i' u_k'}{\partial x_k} + \frac{\partial \bar{u}_i u_j' u_k'}{\partial x_k}\right] + \left[-u_k' \frac{\partial \bar{u}_j u_i'}{\partial x_k} - u_k' \frac{\partial \bar{u}_i u_j'}{\partial x_k}\right]\right) - \left(\left[\frac{\partial u_i' \overline{u_k' u_j'}}{\partial x_k}\right] + \left[\frac{\partial u_j' \overline{u_k' u_i'}}{\partial x_k}\right]\right) - \left(\left[-\overline{u_k' u_j'} \frac{\partial u_i'}{\partial x_k}\right] + \left[-\overline{u_k' u_i'} \frac{\partial u_j'}{\partial x_k}\right]\right)$$
$$+ f_k\left(\epsilon_{j,k,l} u_l' u_i' + \epsilon_{i,k,l} u_l' u_j'\right) = -\left[\frac{\partial u_i' P'}{\partial x_j} + \frac{\partial u_j' P'}{\partial x_i}\right] + \left[P' \frac{\partial u_i'}{\partial x_j} + P' \frac{\partial u_j'}{\partial x_i}\right] - \beta\left(g_j u_i' \theta' - g_i u_j' \theta'\right) + \nu u_i' \nabla^2 u_j' + \nu u_j' \nabla^2 u_i'$$

2. As equações básicas

$$\left(\frac{\partial u_i' u_j'}{\partial t}\right) + \left(\left[\frac{\partial \bar{u}_k u_i' u_j'}{\partial x_k} + \frac{\partial \bar{u}_k u_i' u_j'}{\partial x_k}\right] + \left[-u_j' \frac{\partial \bar{u}_k u_i'}{\partial x_k} - u_i' \frac{\partial \bar{u}_k u_j'}{\partial x_k}\right]\right) + \left(\left[\frac{\partial u_k' u_i' u_j'}{\partial x_k} + \frac{\partial u_k' u_i' u_j'}{\partial x_k}\right] + \left[-u_j' \frac{\partial u_k' u_i'}{\partial x_k} - u_i' \frac{\partial u_k' u_j'}{\partial x_k}\right]\right)$$
$$+ \left(\left[\frac{\partial \bar{u}_j u_i' u_k'}{\partial x_k} + \frac{\partial \bar{u}_i u_j' u_k'}{\partial x_k}\right] + \left[-u_k' \frac{\partial \bar{u}_j u_i'}{\partial x_k} - u_k' \frac{\partial \bar{u}_i u_j'}{\partial x_k}\right]\right) - \left(\left[\frac{\partial u_i' \overline{u_k' u_j'}}{\partial x_k}\right] + \left[\frac{\partial u_j' \overline{u_k' u_i'}}{\partial x_k}\right]\right) - \left(\left[-\overline{u_k' u_j'} \frac{\partial u_i'}{\partial x_k}\right] + \left[-\overline{u_k' u_i'} \frac{\partial u_j'}{\partial x_k}\right]\right)$$
$$+ f_k\left(\epsilon_{j,k,l} u_l' u_i' + \epsilon_{i,k,l} u_l' u_j'\right) = -\left[\frac{\partial u_i' P'}{\partial x_j} + \frac{\partial u_j' P'}{\partial x_i}\right] + \left[P' \frac{\partial u_i'}{\partial x_j} + P' \frac{\partial u_j'}{\partial x_i}\right] - \beta\left(g_j u_i' \theta' - g_i u_j' \theta'\right) + \nu u_i' \nabla^2 u_j' + \nu u_j' \nabla^2 u_i'$$

$$\left(\frac{\partial u_i' u_j'}{\partial t}\right) + \left(2\left[\frac{\partial \bar{u}_k u_i' u_j'}{\partial x_k}\right] - \left[\frac{\partial \bar{u}_k u_i' u_j'}{\partial x_k}\right]\right) + \left(2\left[\frac{\partial u_k' u_i' u_j'}{\partial x_k}\right] - \left[\frac{\partial u_k' u_i' u_j'}{\partial x_k}\right]\right) + \left(\left[\frac{\partial \bar{u}_j u_i' u_k'}{\partial x_k} + \frac{\partial \bar{u}_i u_j' u_k'}{\partial x_k}\right] + \left[-u_k' \frac{\partial \bar{u}_j u_i'}{\partial x_k} - u_k' \frac{\partial \bar{u}_i u_j'}{\partial x_k}\right]\right)$$
$$- \left(\left[\frac{\partial u_i' \overline{u_k' u_j'}}{\partial x_k}\right] + \left[\frac{\partial u_j' \overline{u_k' u_i'}}{\partial x_k}\right]\right) - \left(\left[-\overline{u_k' u_j'} \frac{\partial u_i'}{\partial x_k} - \overline{u_k' u_i'} \frac{\partial u_j'}{\partial x_k}\right]\right) + f_k\left(\epsilon_{j,k,l} u_l' u_i' + \epsilon_{i,k,l} u_l' u_j'\right)$$
$$= -\left[\frac{\partial u_i' P'}{\partial x_j} + \frac{\partial u_j' P'}{\partial x_i}\right] + P'\left[\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right] - \beta\left(g_j u_i' \theta' - g_i u_j' \theta'\right) + \nu u_i' \frac{\partial}{\partial x_k}\frac{\partial u_j'}{\partial x_k} + \nu u_j' \frac{\partial}{\partial x_k}\frac{\partial u_i'}{\partial x_k}$$

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2. As equações básicas

$$
\begin{aligned}
&\left(\frac{\partial u_i' u_j'}{\partial t}\right) + \left(2\left[\frac{\partial \bar{u}_k u_i' u_j'}{\partial x_k}\right] - \left[\frac{\partial \bar{u}_k u_i' u_j'}{\partial x_k}\right]\right) + \left(2\left[\frac{\partial u_k' u_i' u_j'}{\partial x_k}\right] - \left[\frac{\partial u_k' u_i' u_j'}{\partial x_k}\right]\right) + \left(\left[\frac{\partial \bar{u}_j u_i' u_k'}{\partial x_k} + \frac{\partial \bar{u}_i u_j' u_k'}{\partial x_k}\right] + \left[-u_k' \frac{\partial \bar{u}_j u_i'}{\partial x_k} - u_k' \frac{\partial \bar{u}_i u_j'}{\partial x_k}\right.\right. \\
&- \left(\left[\frac{\partial u_i' \overline{u_k' u_j'}}{\partial x_k}\right] + \left[\frac{\partial u_j' \overline{u_k' u_i'}}{\partial x_k}\right]\right) - \left(\left[-\overline{u_k' u_j'} \frac{\partial u_i'}{\partial x_k} - \overline{u_k' u_i'} \frac{\partial u_j'}{\partial x_k}\right]\right) + f_k\left(\epsilon_{j,k,l} u_l' u_i' + \epsilon_{i,k,l} u_l' u_j'\right) \\
&= -\left[\frac{\partial u_i' P'}{\partial x_j} + \frac{\partial u_j' P'}{\partial x_i}\right] + P'\left[\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right] - \beta\left(g_j u_i' \theta' - g_i u_j' \theta'\right) + \nu u_i' \frac{\partial}{\partial x_k}\frac{\partial u_j'}{\partial x_k} + \nu u_j' \frac{\partial}{\partial x_k}\frac{\partial u_i'}{\partial x_k}
\end{aligned}
$$

$$
\begin{aligned}
&\left(\frac{\partial u_i' u_j'}{\partial t}\right) + \left(\left[\frac{\partial \bar{u}_k u_i' u_j'}{\partial x_k}\right]\right) + \left(\left[\frac{\partial u_k' u_i' u_j'}{\partial x_k}\right]\right) + \left(\left[u_k' \frac{\partial \bar{u}_j u_i'}{\partial x_k} + u_i' \frac{\partial \bar{u}_j u_k'}{\partial x_k} + u_k' \frac{\partial \bar{u}_i u_j'}{\partial x_k} + u_j' \frac{\partial \bar{u}_i u_k'}{\partial x_k}\right] + \left[-u_k' \frac{\partial \bar{u}_j u_i'}{\partial x_k} - u_k' \frac{\partial \bar{u}_i u_j'}{\partial x_k}\right]\right) \\
&- \left(\left[\frac{\partial u_i' \overline{u_k' u_j'}}{\partial x_k}\right] + \left[\frac{\partial u_j' \overline{u_k' u_i'}}{\partial x_k}\right]\right) - \left(\left[-\overline{u_k' u_j'} \frac{\partial u_i'}{\partial x_k} - \overline{u_k' u_i'} \frac{\partial u_j'}{\partial x_k}\right]\right) + f_k\left(\epsilon_{j,k,l} u_l' u_i' + \epsilon_{i,k,l} u_l' u_j'\right) \\
&= -\left[\frac{\partial u_i' P'}{\partial x_j} + \frac{\partial u_j' P'}{\partial x_i}\right] + P'\left[\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right] - \beta\left(g_j u_i' \theta' - g_i u_j' \theta'\right) + \nu\left[\frac{\partial}{\partial x_k}\left(u_i' \frac{\partial u_j'}{\partial x_k}\right) - \left(\frac{\partial u_i'}{\partial x_k}\frac{\partial u_j'}{\partial x_k}\right)\right] \\
&+ \nu\left[\frac{\partial}{\partial x_k}\left(u_j' \frac{\partial u_i'}{\partial x_k}\right) - \left(\frac{\partial u_j'}{\partial x_k}\frac{\partial u_i'}{\partial x_k}\right)\right]
\end{aligned}
$$

2. As equações básicas

$$
\begin{aligned}
&\left(\frac{\partial u_i' u_j'}{\partial t}\right) + \left(\left[\frac{\partial \bar{u}_k u_i' u_j'}{\partial x_k}\right]\right) + \left(\left[\frac{\partial u_k' u_i' u_j'}{\partial x_k}\right]\right) + \left(\left[+u_i' \frac{\partial \bar{u}_j u_k'}{\partial x_k} + u_j' \frac{\partial \bar{u}_i u_k'}{\partial x_k}\right] + \left[u_k' \frac{\partial \bar{u}_j u_i'}{\partial x_k} - u_k' \frac{\partial \bar{u}_j u_i'}{\partial x_k} + u_k' \frac{\partial \bar{u}_i u_j'}{\partial x_k} - u_k' \frac{\partial \bar{u}_i u_j'}{\partial x_k}\right]\right) \\
&- \left(\left[\frac{\partial u_i' \overline{u_k' u_j'}}{\partial x_k}\right] + \left[\frac{\partial u_j' \overline{u_k' u_i'}}{\partial x_k}\right]\right) - \left(\left[-\overline{u_k' u_j'} \frac{\partial u_i'}{\partial x_k} - \overline{u_k' u_i'} \frac{\partial u_j'}{\partial x_k}\right]\right) + f_k\left(\epsilon_{j,k,l} u_l' u_i' + \epsilon_{i,k,l} u_l' u_j'\right) \\
&= -\left[\frac{\partial u_i' P'}{\partial x_j} + \frac{\partial u_j' P'}{\partial x_i}\right] + P'\left[\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right] - \beta\left(g_j u_i' \theta' - g_i u_j' \theta'\right) + \nu\left[\frac{\partial}{\partial x_k}\left(u_i' \frac{\partial u_j'}{\partial x_k}\right) - \left(\frac{\partial u_i'}{\partial x_k}\frac{\partial u_j'}{\partial x_k}\right)\right] \\
&+ \nu\left[\frac{\partial}{\partial x_k}\left(u_j' \frac{\partial u_i'}{\partial x_k}\right) - \left(\frac{\partial u_j'}{\partial x_k}\frac{\partial u_i'}{\partial x_k}\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
&\left(\frac{\partial u_i' u_j'}{\partial t}\right) + \left(\left[\frac{\partial \bar{u}_k u_i' u_j'}{\partial x_k}\right]\right) + \left(\left[\frac{\partial u_k' u_i' u_j'}{\partial x_k}\right]\right) + \left(\left[u_i' \frac{\partial \bar{u}_j u_k'}{\partial x_k} + u_j' \frac{\partial \bar{u}_i u_k'}{\partial x_k}\right]\right) - \left(\left[\frac{\partial u_i' \overline{u_k' u_j'}}{\partial x_k}\right] + \left[\frac{\partial u_j' \overline{u_k' u_i'}}{\partial x_k}\right]\right) \\
&- \left(\left[-\overline{u_k' u_j'} \frac{\partial u_i'}{\partial x_k} - \overline{u_k' u_i'} \frac{\partial u_j'}{\partial x_k}\right]\right) + f_k\left(\epsilon_{j,k,l} u_l' u_i' + \epsilon_{i,k,l} u_l' u_j'\right) \\
&= -\left[\frac{\partial u_i' P'}{\partial x_j} + \frac{\partial u_j' P'}{\partial x_i}\right] + P'\left[\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right] - \beta\left(g_j u_i' \theta' - g_i u_j' \theta'\right) + \nu \frac{\partial}{\partial x_k}\left[\left(u_i' \frac{\partial u_j'}{\partial x_k}\right) + \left(u_j' \frac{\partial u_i'}{\partial x_k}\right)\right] - 2\nu\left(\frac{\partial u_j'}{\partial x_k}\frac{\partial u_i'}{\partial x_k}\right)
\end{aligned}
$$

1.1 A Teoria de Similaridade de Monin-Obukhov

2. As equações básicas

$$
\begin{aligned}
&\left(\frac{\partial u_i' u_j'}{\partial t}\right) + \left(\left[\frac{\partial \bar{u}_k u_i' u_j'}{\partial x_k}\right]\right) + \left(\left[\frac{\partial u_k' u_i' u_j'}{\partial x_k}\right]\right) + \left(\left[u_i' \frac{\partial \bar{u}_j u_k'}{\partial x_k} + u_j' \frac{\partial \bar{u}_i u_k'}{\partial x_k}\right]\right) - \left(\left[\frac{\partial u_i' \overline{u_k' u_j'}}{\partial x_k}\right] + \left[\frac{\partial u_j' \overline{u_k' u_i'}}{\partial x_k}\right]\right) \\
&- \left(\left[-\overline{u_k' u_j'} \frac{\partial u_i'}{\partial x_k} - \overline{u_k' u_i'} \frac{\partial u_j'}{\partial x_k}\right]\right) + f_k\left(\epsilon_{j,k,l} u_l' u_i' + \epsilon_{i,k,l} u_l' u_j'\right) \\
&= -\left[\frac{\partial u_i' P'}{\partial x_j} + \frac{\partial u_j' P'}{\partial x_i}\right] + P'\left[\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right] - \beta\left(g_j u_i' \theta' - g_i u_j' \theta'\right) + \nu \frac{\partial}{\partial x_k}\left[\left(u_i' \frac{\partial u_j'}{\partial x_k}\right) + \left(u_j' \frac{\partial u_i'}{\partial x_k}\right)\right] - 2\nu\left(\frac{\partial u_j'}{\partial x_k} \frac{\partial u_i'}{\partial x_k}\right)
\end{aligned}
$$

$$
\begin{aligned}
&\left(\frac{\partial u_i' u_j'}{\partial t}\right) + \left(\left[\frac{\partial \bar{u}_k u_i' u_j'}{\partial x_k}\right]\right) + \left(\left[\frac{\partial u_k' u_i' u_j'}{\partial x_k}\right]\right) + \left(\left[u_i' u_k' \frac{\partial \bar{u}_j}{\partial x_k} + u_i' \bar{u}_j \frac{\partial u_k'}{\partial x_k} + u_j' u_k' \frac{\partial \bar{u}_i}{\partial x_k} + u_j' \bar{u}_i \frac{\partial u_k'}{\partial x_k}\right]\right) - \left(\left[\frac{\partial u_i' \overline{u_k' u_j'}}{\partial x_k}\right] + \left[\frac{\partial u_j' \overline{u_k' u_i'}}{\partial x_k}\right]\right) \\
&- \left(\left[-\overline{u_k' u_j'} \frac{\partial u_i'}{\partial x_k} - \overline{u_k' u_i'} \frac{\partial u_j'}{\partial x_k}\right]\right) + f_k\left(\epsilon_{j,k,l} u_l' u_i' + \epsilon_{i,k,l} u_l' u_j'\right) \\
&= -\left[\frac{\partial u_i' P'}{\partial x_j} + \frac{\partial u_j' P'}{\partial x_i}\right] + P'\left[\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right] - \beta\left(g_j u_i' \theta' - g_i u_j' \theta'\right) + \nu \frac{\partial}{\partial x_k}\left[\left(u_i' \frac{\partial u_j'}{\partial x_k}\right) + \left(u_j' \frac{\partial u_i'}{\partial x_k}\right)\right] - 2\nu\left(\frac{\partial u_j'}{\partial x_k} \frac{\partial u_i'}{\partial x_k}\right)
\end{aligned}
$$

2. As equações básicas

$$\begin{aligned}
&\left(\frac{\partial u'_i u'_j}{\partial t}\right) + \left(\left[\frac{\partial \bar{u}_k u'_i u'_j}{\partial x_k}\right]\right) + \left(\left[\frac{\partial u'_k u'_i u'_j}{\partial x_k}\right]\right) + \left(\left[u'_i u'_k \frac{\partial \bar{u}_j}{\partial x_k} + u'_i \bar{u}_j \frac{\partial u'_k}{\partial x_k} + u'_j u'_k \frac{\partial \bar{u}_i}{\partial x_k} + u'_j \bar{u}_i \frac{\partial u'_k}{\partial x_k}\right]\right) - \left(\left[\frac{\partial u'_i \overline{u'_k u'_j}}{\partial x_k}\right] + \left[\frac{\partial u'_j \overline{u'_k u'_i}}{\partial x_k}\right]\right) \\
&- \left(\left[-\overline{u'_k u'_j}\frac{\partial u'_i}{\partial x_k} - \overline{u'_k u'_i}\frac{\partial u'_j}{\partial x_k}\right]\right) + f_k\left(\epsilon_{j,k,l} u'_l u'_i + \epsilon_{i,k,l} u'_l u'_j\right) \\
&= -\left[\frac{\partial u'_i P'}{\partial x_j} + \frac{\partial u'_j P'}{\partial x_i}\right] + P'\left[\frac{\partial u'_i}{\partial x_j} + \frac{\partial u'_j}{\partial x_i}\right] - \beta\left(g_j u'_i \theta' - g_i u'_j \theta'\right) + \nu \frac{\partial}{\partial x_k}\left[\left(u'_i \frac{\partial u'_j}{\partial x_k}\right) + \left(u'_j \frac{\partial u'_i}{\partial x_k}\right)\right] - 2\nu\left(\frac{\partial u'_j}{\partial x_k}\frac{\partial u'_i}{\partial x_k}\right)
\end{aligned}$$

$$\begin{aligned}
&\left(\frac{\partial u'_i u'_j}{\partial t}\right) + \left(\left[\frac{\partial \bar{u}_k u'_i u'_j}{\partial x_k}\right]\right) + \left(\left[\frac{\partial u'_k u'_i u'_j}{\partial x_k}\right]\right) + \left(\left[u'_i u'_k \frac{\partial \bar{u}_j}{\partial x_k} + u'_j u'_k \frac{\partial \bar{u}_i}{\partial x_k} + u'_i \bar{u}_j \frac{\partial u'_k}{\partial x_k} + u'_j \bar{u}_i \frac{\partial u'_k}{\partial x_k}\right]\right) - \left(\left[\frac{\partial u'_i \overline{u'_k u'_j}}{\partial x_k}\right] + \left[\frac{\partial u'_j \overline{u'_k u'_i}}{\partial x_k}\right]\right) \\
&- \left(\left[-\overline{u'_k u'_j}\frac{\partial u'_i}{\partial x_k} - \overline{u'_k u'_i}\frac{\partial u'_j}{\partial x_k}\right]\right) + f_k\left(\epsilon_{j,k,l} u'_l u'_i + \epsilon_{i,k,l} u'_l u'_j\right) \\
&= -\left[\frac{\partial u'_i P'}{\partial x_j} + \frac{\partial u'_j P'}{\partial x_i}\right] + P'\left[\frac{\partial u'_i}{\partial x_j} + \frac{\partial u'_j}{\partial x_i}\right] - \beta\left(g_j u'_i \theta' - g_i u'_j \theta'\right) + \nu \frac{\partial}{\partial x_k}\left[\left(u'_i \frac{\partial u'_j}{\partial x_k}\right) + \left(u'_j \frac{\partial u'_i}{\partial x_k}\right)\right] - 2\nu\left(\frac{\partial u'_j}{\partial x_k}\frac{\partial u'_i}{\partial x_k}\right)
\end{aligned}$$

1.1 A Teoria de Similaridade de Monin-Obukhov

2. As equações básicas

$$
\begin{aligned}
&\left(\frac{\partial u'_i u'_j}{\partial t}\right) + \left(\left[\frac{\partial \bar{u}_k u'_i u'_j}{\partial x_k}\right]\right) + \left(\left[\frac{\partial u'_k u'_i u'_j}{\partial x_k}\right]\right) + \left(\left[u'_i u'_k \frac{\partial \bar{u}_j}{\partial x_k} + u'_j u'_k \frac{\partial \bar{u}_i}{\partial x_k} + u'_i \bar{u}_j \cancel{\frac{\partial u'_k}{\partial x_k}} + u'_j \bar{u}_i \cancel{\frac{\partial u'_k}{\partial x_k}}\right]\right) - \left(\left[\frac{\partial u'_i \overline{u'_k u'_j}}{\partial x_k}\right] + \left[\frac{\partial u'_j \overline{u'_k u'_i}}{\partial x_k}\right]\right) \\
&- \left(\left[-\overline{u'_k u'_j}\frac{\partial u'_i}{\partial x_k} - \overline{u'_k u'_i}\frac{\partial u'_j}{\partial x_k}\right]\right) + f_k\left(\epsilon_{j,k,l} u'_l u'_i + \epsilon_{i,k,l} u'_l u'_j\right) \\
&= -\left[\frac{\partial u'_i P'}{\partial x_j} + \frac{\partial u'_j P'}{\partial x_i}\right] + P'\left[\frac{\partial u'_i}{\partial x_j} + \frac{\partial u'_j}{\partial x_i}\right] - \beta\left(g_j u'_i \theta' - g_i u'_j \theta'\right) + \nu \frac{\partial}{\partial x_k}\left[\left(u'_i \frac{\partial u'_j}{\partial x_k}\right) + \left(u'_j \frac{\partial u'_i}{\partial x_k}\right)\right] - 2\nu\left(\frac{\partial u'_j}{\partial x_k}\frac{\partial u'_i}{\partial x_k}\right)
\end{aligned}
$$

$$
\begin{aligned}
&\left(\frac{\partial u'_i u'_j}{\partial t}\right) + \left(\left[\frac{\partial \bar{u}_k u'_i u'_j}{\partial x_k}\right]\right) + \left(\left[\frac{\partial u'_k u'_i u'_j}{\partial x_k}\right]\right) + \left(\left[u'_i u'_k \frac{\partial \bar{u}_j}{\partial x_k} + u'_j u'_k \frac{\partial \bar{u}_i}{\partial x_k}\right]\right) - \left(\left[\frac{\partial u'_i \overline{u'_k u'_j}}{\partial x_k}\right] + \left[\frac{\partial u'_j \overline{u'_k u'_i}}{\partial x_k}\right]\right) \\
&- \left(\left[-\overline{u'_k u'_j}\frac{\partial u'_i}{\partial x_k} - \overline{u'_k u'_i}\frac{\partial u'_j}{\partial x_k}\right]\right) + f_k\left(\epsilon_{j,k,l} u'_l u'_i + \epsilon_{i,k,l} u'_l u'_j\right) \\
&= -\left[\frac{\partial u'_i P'}{\partial x_j} + \frac{\partial u'_j P'}{\partial x_i}\right] + P'\left[\frac{\partial u'_i}{\partial x_j} + \frac{\partial u'_j}{\partial x_i}\right] - \beta\left(g_j u'_i \theta' - g_i u'_j \theta'\right) + \nu \frac{\partial}{\partial x_k}\left[\left(u'_i \frac{\partial u'_j}{\partial x_k}\right) + \left(u'_j \frac{\partial u'_i}{\partial x_k}\right)\right] - 2\nu\left(\frac{\partial u'_j}{\partial x_k}\frac{\partial u'_i}{\partial x_k}\right)
\end{aligned}
$$

2. As equações básicas

$$\left(\frac{\partial u_i' u_j'}{\partial t}\right) + \left(\left[\frac{\partial \bar{u}_k u_i' u_j'}{\partial x_k}\right]\right) + \left(\left[\frac{\partial u_k' u_i' u_j'}{\partial x_k}\right]\right) + \left(\left[u_i' u_k' \frac{\partial \bar{u}_j}{\partial x_k} + u_j' u_k' \frac{\partial \bar{u}_i}{\partial x_k}\right]\right) - \left(\left[\frac{\partial u_i' \overline{u_k' u_j'}}{\partial x_k}\right] + \left[\frac{\partial u_j' \overline{u_k' u_i'}}{\partial x_k}\right]\right)$$
$$- \left(\left[-\overline{u_k' u_j'} \frac{\partial u_i'}{\partial x_k} - \overline{u_k' u_i'} \frac{\partial u_j'}{\partial x_k}\right]\right) + f_k\left(\epsilon_{j,k,l} u_l' u_i' + \epsilon_{i,k,l} u_l' u_j'\right)$$
$$= -\left[\frac{\partial u_i' P'}{\partial x_j} + \frac{\partial u_j' P'}{\partial x_i}\right] + P'\left[\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right] - \beta\left(g_j u_i' \theta' - g_i u_j' \theta'\right) + \nu \frac{\partial}{\partial x_k}\left[\left(u_i' \frac{\partial u_j'}{\partial x_k}\right) + \left(u_j' \frac{\partial u_i'}{\partial x_k}\right)\right] - 2\nu\left(\frac{\partial u_j'}{\partial x_k} \frac{\partial u_i'}{\partial x_k}\right)$$

$$\left(\frac{\partial u_i' u_j'}{\partial t}\right) + \left(\left[\frac{\partial \bar{u}_k u_i' u_j'}{\partial x_k}\right]\right) + \left(\left[\frac{\partial u_k' u_i' u_j'}{\partial x_k}\right]\right) + \left(\left[u_i' u_k' \frac{\partial \bar{u}_j}{\partial x_k} + u_j' u_k' \frac{\partial \bar{u}_i}{\partial x_k}\right]\right) - \left(\left[\frac{\partial u_i' \overline{u_k' u_j'}}{\partial x_k}\right] + \left[\frac{\partial u_j' \overline{u_k' u_i'}}{\partial x_k}\right]\right)$$
$$- \left(\left[-\overline{u_k' u_j'} \frac{\partial u_i'}{\partial x_k} - \overline{u_k' u_i'} \frac{\partial u_j'}{\partial x_k}\right]\right) + f_k\left(\epsilon_{j,k,l} u_l' u_i' + \epsilon_{i,k,l} u_l' u_j'\right)$$
$$= -\left[\frac{\partial u_i' P'}{\partial x_j} + \frac{\partial u_j' P'}{\partial x_i}\right] + P'\left[\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right] - \beta\left(g_j u_i' \theta' - g_i u_j' \theta'\right) + \nu \frac{\partial}{\partial x_k}\left(\frac{\partial u_i' u_j'}{\partial x_k}\right) - 2\nu\left(\frac{\partial u_j'}{\partial x_k} \frac{\partial u_i'}{\partial x_k}\right)$$

2. As equações básicas

$$\left(\frac{\partial u_i' u_j'}{\partial t}\right) + \left(\left[\frac{\partial \bar{u}_k u_i' u_j'}{\partial x_k}\right]\right) + \left(\left[\frac{\partial u_k' u_i' u_j'}{\partial x_k}\right]\right) + \left(\left[u_i' u_k' \frac{\partial \bar{u}_j}{\partial x_k} + u_j' u_k' \frac{\partial \bar{u}_i}{\partial x_k}\right]\right) - \left(\left[\frac{\partial u_i' \overline{u_k' u_j'}}{\partial x_k}\right] + \left[\frac{\partial u_j' \overline{u_k' u_i'}}{\partial x_k}\right]\right)$$

$$- \left(\left[-\overline{u_k' u_j'} \frac{\partial u_i'}{\partial x_k} - \overline{u_k' u_i'} \frac{\partial u_j'}{\partial x_k}\right]\right) + f_k\left(\epsilon_{j,k,l} u_l' u_i' + \epsilon_{i,k,l} u_l' u_j'\right)$$

$$= -\left[\frac{\partial u_i' P'}{\partial x_j} + \frac{\partial u_j' P'}{\partial x_i}\right] + P'\left[\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right] - \beta\left(g_j u_i' \theta' - g_i u_j' \theta'\right) + \nu \frac{\partial}{\partial x_k}\left(\frac{\partial u_i' u_j'}{\partial x_k}\right) - 2\nu\left(\frac{\partial u_j'}{\partial x_k}\frac{\partial u_i'}{\partial x_k}\right)$$

$$\left(\frac{\partial \overline{u_i' u_j'}}{\partial t}\right) + \left(\left[\frac{\partial \overline{\bar{u}_k u_i' u_j'}}{\partial x_k}\right]\right) + \left(\left[\frac{\partial \overline{u_k' u_i' u_j'}}{\partial x_k}\right]\right) + \left(\left[\overline{u_i' u_k'} \frac{\partial \bar{\bar{u}}_j}{\partial x_k} + \overline{u_j' u_k'} \frac{\partial \bar{\bar{u}}_i}{\partial x_k}\right]\right) - \left(\left[\frac{\partial \overline{u_i'}\,\overline{\overline{u_k' u_j'}}}{\partial x_k}\right] + \left[\frac{\partial \overline{u_j'}\,\overline{\overline{u_k' u_i'}}}{\partial x_k}\right]\right)$$

$$+ \left(\left[\overline{\overline{u_k' u_j'}} \frac{\partial \overline{u_i'}}{\partial x_k} + \overline{\overline{u_k' u_i'}} \frac{\partial \overline{u_j'}}{\partial x_k}\right]\right) + f_k\left(\epsilon_{j,k,l} \overline{u_l' u_i'} + \epsilon_{i,k,l} \overline{u_l' u_j'}\right)$$

$$= -\left[\frac{\partial \overline{u_i' P'}}{\partial x_j} + \frac{\partial \overline{u_j' P'}}{\partial x_i}\right] + \bar{P}'\left[\frac{\partial \overline{u_i'}}{\partial x_j} + \frac{\partial \overline{u_j'}}{\partial x_i}\right] - \beta\left(g_j \overline{u_i' \theta'} - g_i \overline{u_j' \theta'}\right) + \nu \frac{\partial}{\partial x_k}\left(\frac{\partial \overline{u_i' u_j'}}{\partial x_k}\right) - 2\nu\left(\frac{\partial \overline{u_j'}}{\partial x_k}\frac{\partial \overline{u_i'}}{\partial x_k}\right)$$

2. As equações básicas

$$\left(\frac{\partial \overline{u_i'u_j'}}{\partial t}\right) + \left(\left[\frac{\partial \overline{\bar{u}_k u_i'u_j'}}{\partial x_k}\right]\right) + \left(\left[\frac{\partial \overline{u_k'u_i'u_j'}}{\partial x_k}\right]\right) + \left(\left[\overline{u_i'u_k'}\frac{\partial \bar{\bar{u}}_j}{\partial x_k} + \overline{u_j'u_k'}\frac{\partial \bar{\bar{u}}_i}{\partial x_k}\right]\right) - \left(\left[\cancel{\frac{\partial \overline{u_i'}\,\overline{\overline{u_k'u_j'}}}{\partial x_k}}\right] + \left[\cancel{\frac{\partial \overline{u_j'}\,\overline{\overline{u_k'u_i'}}}{\partial x_k}}\right]\right)$$

$$+ \left(\left[\cancel{\overline{\overline{u_k'u_j'}}\frac{\partial \overline{u_i'}}{\partial x_k}} + \cancel{\overline{\overline{u_k'u_i'}}\frac{\partial \overline{u_j'}}{\partial x_k}}\right]\right) + f_k\left(\epsilon_{j,k,l}\overline{u_l'u_i'} + \epsilon_{i,k,l}\overline{u_l'u_j'}\right)$$

$$= -\left[\frac{\partial \overline{u_i'P'}}{\partial x_j} + \frac{\partial \overline{u_j'P'}}{\partial x_i}\right] + \overline{P'}\left[\frac{\partial \overline{u_i'}}{\partial x_j} + \frac{\partial \overline{u_j'}}{\partial x_i}\right] - \beta\left(g_j\overline{u_i'\theta'} - g_i\overline{u_j'\theta'}\right) + \nu\frac{\partial}{\partial x_k}\left(\frac{\partial \overline{u_i'u_j'}}{\partial x_k}\right) - 2\nu\left(\frac{\partial \overline{u_j'}}{\partial x_k}\frac{\partial \overline{u_i'}}{\partial x_k}\right)$$

$$\left(\frac{\partial \overline{u_i'u_j'}}{\partial t}\right) + \left(\left[\frac{\partial \overline{\bar{u}_k u_i'u_j'}}{\partial x_k}\right]\right) + \left(\left[\frac{\partial \overline{u_k'u_i'u_j'}}{\partial x_k}\right]\right) + \left(\left[\overline{u_i'u_k'}\frac{\partial \bar{\bar{u}}_j}{\partial x_k} + \overline{u_j'u_k'}\frac{\partial \bar{\bar{u}}_i}{\partial x_k}\right]\right) + f_k\left(\epsilon_{j,k,l}\overline{u_l'u_i'} + \epsilon_{i,k,l}\overline{u_l'u_j'}\right)$$

$$= -\left[\frac{\partial \overline{u_i'P'}}{\partial x_j} + \frac{\partial \overline{u_j'P'}}{\partial x_i}\right] + \overline{P'}\left[\frac{\partial \overline{u_i'}}{\partial x_j} + \frac{\partial \overline{u_j'}}{\partial x_i}\right] - \beta\left(g_j\overline{u_i'\theta'} - g_i\overline{u_j'\theta'}\right) + \nu\frac{\partial}{\partial x_k}\left(\frac{\partial \overline{u_i'u_j'}}{\partial x_k}\right) - 2\nu\left(\frac{\partial \overline{u_j'}}{\partial x_k}\frac{\partial \overline{u_i'}}{\partial x_k}\right)$$

2. As equações básicas

$$\left(\frac{\partial \overline{u_i' u_j'}}{\partial t}\right) + \left(\left[\frac{\partial \bar{u}_k \overline{u_i' u_j'}}{\partial x_k}\right]\right) + \left(\left[\frac{\partial \overline{u_k' u_i' u_j'}}{\partial x_k}\right]\right) + \left(\left[\overline{u_i' u_k'}\frac{\partial \bar{u}_j}{\partial x_k} + \overline{u_j' u_k'}\frac{\partial \bar{u}_i}{\partial x_k}\right]\right) + f_k\left(\epsilon_{j,k,l}\overline{u_l' u_i'} + \epsilon_{i,k,l}\overline{u_l' u_j'}\right)$$
$$= -\left[\frac{\partial \overline{u_i' P'}}{\partial x_j} + \frac{\partial \overline{u_j' P'}}{\partial x_i}\right] + \overline{P'}\left[\frac{\partial \overline{u_i'}}{\partial x_j} + \frac{\partial \overline{u_j'}}{\partial x_i}\right] - \beta\left(g_j \overline{u_i' \theta'} - g_i \overline{u_j' \theta'}\right) + \nu \frac{\partial}{\partial x_k}\left(\frac{\partial \overline{u_i' u_j'}}{\partial x_k}\right) - 2\nu\left(\frac{\partial \overline{u_j'}}{\partial x_k}\frac{\partial \overline{u_i'}}{\partial x_k}\right)$$

$$\frac{\partial \overline{u_i' u_j'}}{\partial t} + \frac{\partial}{\partial x_k}\left[\bar{u}_k \overline{u_i' u_j'} + \overline{u_k' u_i' u_j'} - \nu \frac{\partial \overline{u_i' u_j'}}{\partial x_k}\right] + \frac{\partial \overline{P' u_i'}}{\partial x_j} + \frac{\partial \overline{P' u_j'}}{\partial x_i} + f_k\left(\epsilon_{j,k,l} u_l' u_i' + \epsilon_{i,k,l} u_l' u_j'\right)$$
$$= -\overline{u_k' u_i'}\frac{\partial \bar{u}_j}{\partial x_k} - \overline{u_k' u_j'}\frac{\partial \bar{u}_i}{\partial x_k} - \beta\left(g_j \overline{u_i' \theta'} + g_i \overline{u_j' \theta'}\right) + \overline{P'\left(\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right)} - 2\nu \overline{\left(\frac{\partial u_i'}{\partial x_k}\frac{\partial u_j'}{\partial x_k}\right)}$$

2. As equações básicas

$$\frac{\partial u_j'}{\partial t} + \frac{\partial}{\partial x_k}\left(\bar{u}_k u_j' + \bar{u}_j u_k' + u_k' u_j' - \overline{u_k' u_j'}\right) + \epsilon_{j,k,l} f_k u_l' = -\frac{\partial P'}{\partial x_j} - g_j \beta \theta' + \nu \nabla^2 u_j' \qquad (5)$$

$$\frac{\partial \theta'}{\partial t} + \frac{\partial}{\partial x_k}\left(\bar{\theta} u_k' + \bar{u}_k \theta' + u_k' \theta' + \overline{u_k' \theta'}\right) = \alpha \nabla^2 \theta' \qquad (6)$$

$$\frac{\partial u_j' \theta'}{\partial t} = u_j' \frac{\partial \theta'}{\partial t} + \theta' \frac{\partial u_j'}{\partial t}$$

$$\frac{\partial \overline{u_j' \theta'}}{\partial t} + \frac{\partial}{\partial x_k}\left[\bar{u}_k \overline{\theta' u_j'} + \overline{u_k' u_j' \theta'} - \alpha \overline{u_j' \frac{\partial \theta'}{\partial x_k}} - \nu \overline{\theta' \frac{\partial u_j'}{\partial x_k}}\right] + \frac{\partial}{\partial x_j} \overline{P' \theta'} + \epsilon_{j,k,l} f_k \overline{u_l' \theta'}$$
$$= -\overline{u_j' u_k'} \frac{\partial \bar{\theta}}{\partial x_k} - \overline{\theta' u_k'} \frac{\partial \bar{u}_j}{\partial x_k} - \beta g_j \overline{\theta'^2} + \overline{P' \frac{\partial \theta'}{\partial x_j}} - (\alpha + \nu) \overline{\frac{\partial u_j'}{\partial x_k} \frac{\partial \theta'}{\partial x_k}} \qquad (8)$$

2. As equações básicas

A equação 8 envolve $\overline{\theta^2}$, na equação . Onde é necessário obter uma equação para este termo da equação 6

$$\frac{\partial \theta'}{\partial t} + \frac{\partial}{\partial x_k}\left(\bar{\theta} u_k' + \bar{u}_k \theta' + u_k' \theta' + \overline{u_k' \theta'}\right) = \alpha \nabla^2 \theta' \qquad (6)$$

$$\frac{\partial \overline{\theta'^1}}{\partial t} = + \frac{\partial}{\partial x_k}\left[\bar{u}_k \overline{\theta'^1} + \overline{k_k' \theta'^1} - \alpha \frac{\partial \overline{\theta'^1}}{\partial x_k}\right] = 2\overline{k_k' \theta'} - 2\alpha \overline{\frac{\partial \theta'}{\partial x_k} \frac{\partial \theta'}{\partial x_k}} \qquad (9)$$

2. As equações básicas

Suposições de modelagem

. A principal contribuição de Rotta (1951) foi sugerir uma suposição para o termo , $\overline{P'\left(\frac{\partial u_i'}{\partial x_j}+\frac{\partial u_j'}{\partial x_i}\right)}$, que ele chamou de "***termo de redistribuição de energia***", **uma vez que uma de suas funções é particionar a energia entre os três componentes de energia sem contribuir para o total**. contração, o termo sai da Ed.7. Com base nas relações integrais obtidas da Eq.5 no caso neutro onde $-\beta\left(g_j u_i'\theta'\right)$ não é significante, Rotta sugeriu que o termo poderia ser razoavelmente proporcional a $u_i'u_j'$ e $\frac{\partial \bar{u}_i}{\partial x_j}$. Por isso,

$$\overline{P'\left(\frac{\partial u_i'}{\partial x_j}+\frac{\partial u_j'}{\partial x_i}\right)} = C_{ijkm}\overline{u_k'u_m'} + C_{ijkm}'\frac{\partial \bar{u}_k}{\partial x_m}$$

2. As equações básicas

Suposições de modelagem

aqui assumimos que os coeficientes constitutivos são tensores isotrópicos, ou seja

$$C_{ijkm} = C_1\delta_{i,j}\delta_{k,m} + C_2\delta_{i,k}\delta_{j,m} + C_3\delta_{j,k}\delta_{i,m}$$

Da eq. Da continuidade obtém-se $C_{\prime} = \frac{(C_2+C_3)}{3}$raciocínio semelhante se aplica ao C_{ijkm}. Obtém-se portanto

$$\overline{P'\left(\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right)} = C_{ijkm}\overline{u_k' u_m'} + C_{ijkm}'\frac{\partial \bar{u}_k}{\partial x_m}$$

$$\overline{P'\left(\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right)} = -\frac{q}{3l_1}\left(\overline{u_i' u_j'} - \frac{\delta_{i,j}}{3}q^2 + Cq^2\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right)\right)$$

Onde $-\frac{q}{3l_1}$ e Cq^2 the sido substituído por coeficientes escalares sobreviventes $q \equiv \left(\overline{u_i'^2}\right)^{1/2}$.O comprimento l_1 e a constante C devem ser determinadas empiricamente.

Paulo Yoshio Kubota

5.2 TURBULENT KINETIC ENERGY **A equação de Navier Stokes: conservação de momentum**

$$\frac{\partial(u_i'u_k')}{\partial t}+(\overline{u_j})\frac{\partial(u_i'u_k')}{\partial x_j}$$
$$=-(u_j'u_k')\frac{\partial(\overline{u_i})}{\partial x_j}-(u_j'\overline{u_i})\frac{\partial(u_k')}{\partial x_j}-\overline{u_i}u_k'\frac{\partial(u_j')}{\partial x_j}-\frac{\partial(u_j'u_i'u_k')}{\partial x_j}-\frac{1}{\rho_0}\frac{\partial(P'u_k')}{\partial x_i}-g\frac{\rho'u_k'}{\rho_0}\delta_{i3}-2\Omega\varepsilon_{ijk}\eta_j(u_k'u_k')+\nu\frac{\partial^2(u_i'u_k')}{\partial {x_j}^2}$$

Considere que a propriedade da equação da continuidade para o fluxo $\nabla.\vec{V}=0$

$$\nabla.\vec{V'}=\frac{\partial(u_j')}{\partial x_j}=0,\qquad j=1,2,3$$

$$\frac{\partial(u_i'u_k')}{\partial t}+(\overline{u_j})\frac{\partial(u_i'u_k')}{\partial x_j}=-(u_j'u_k')\frac{\partial(\overline{u_i})}{\partial x_j}-(u_j'\overline{u_i})\frac{\partial(u_k')}{\partial x_j}-\frac{\partial(u_j'u_i'u_k')}{\partial x_j}-\frac{1}{\rho_0}\frac{\partial(P'u_k')}{\partial x_i}-g\frac{\rho'u_k'}{\rho_0}\delta_{i3}-2\Omega\varepsilon_{ijk}\eta_j(u_k'u_k')+\nu\frac{\partial^2(u_i'u_k')}{\partial {x_j}^2}$$

$$\frac{\partial \overline{u_j'\theta'}}{\partial t} + \frac{\partial}{\partial x_k}\left[\bar{u}_k\overline{\theta' u_j'} + \overline{u_k' u_j'\theta'} - \alpha\overline{u_j'\frac{\partial \theta'}{\partial x_k}} - \nu\overline{\theta'\frac{\partial u_j'}{\partial x_k}}\right] + \frac{\partial}{\partial x_j}\overline{P'\theta'} + \epsilon_{j,k,l}f_k\overline{u_l'\theta'}$$
$$= -\overline{u_j' u_k'}\frac{\partial \bar{\theta}}{\partial x_k} - \overline{\theta' u_k'}\frac{\partial \bar{u}_j}{\partial x_k} - \beta g_j\overline{\theta'^2} + \overline{P'\frac{\partial \theta'}{\partial x_j}} - (\alpha+\nu)\overline{\frac{\partial u_j'}{\partial x_k}\frac{\partial \theta'}{\partial x_k}} \qquad (8)$$

$$\frac{\partial \overline{u_i' u_j'}}{\partial t} + \frac{\partial}{\partial x_k}\left[\bar{u}_k\overline{u_i' u_j'} + \overline{u_k' u_i' u_j'} - \nu\frac{\partial \overline{u_i' u_j'}}{\partial x_k}\right] + \frac{\partial \overline{P' u_i'}}{\partial x_j} + \frac{\partial \overline{P' u_j'}}{\partial x_i} + f_k\left(\epsilon_{j,k,l}u_l' u_i' + \epsilon_{i,k,l}u_l' u_j'\right)$$
$$= -\overline{u_k' u_i'}\frac{\partial \bar{u}_j}{\partial x_k} - \overline{u_k' u_j'}\frac{\partial \bar{u}_i}{\partial x_k} - \beta\left(g_j\overline{u_i'\theta'} + g_i\overline{u_j'\theta'}\right) + \overline{P'\left(\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right)} - 2\nu\overline{\left(\frac{\partial u_i'}{\partial x_k}\frac{\partial u_j'}{\partial x_k}\right)}$$

Linearização da Equações de Navier Stokes

A equação de Navier Stokes: conservação de momentum

$$\frac{\partial \overline{u_i' u_j'}}{\partial t} + \frac{\partial}{\partial x_k}\left[\bar{u}_k \overline{u_i' u_j'}\right]$$

$$= -\overline{u_k' u_i'}\frac{\partial \bar{u}_j}{\partial x_k} - \overline{u_k' u_j'}\frac{\partial \bar{u}_i}{\partial x_k} - \frac{\partial \overline{u_k' u_i' u_j'}}{\partial x_k} - \beta\left(g_j \overline{u_i' \theta'} + g_i \overline{u_j' \theta'}\right) - f_k\left(\epsilon_{j,k,l} u_l' u_i' + \epsilon_{i,k,l} u_l' u_j'\right) - \left[\frac{\partial \overline{P' u_i'}}{\partial x_j} + \frac{\partial \overline{P' u_j'}}{\partial x_i}\right] - 2\nu\overline{\left(\frac{\partial u_i'}{\partial x_k}\frac{\partial u_j'}{\partial x_k}\right)}$$

$$+ \nu\frac{\partial}{\partial x_k}\left[\frac{\partial \overline{u_i' u_j'}}{\partial x_k}\right] + \overline{\boldsymbol{P'}\left(\frac{\boldsymbol{\partial u_i'}}{\boldsymbol{\partial x_j}} + \frac{\boldsymbol{\partial u_j'}}{\boldsymbol{\partial x_i}}\right)}$$

O termo do lado esquerdo é a taxa temporal local de mudança e advecção de $\overline{(u_i' u_k')}$

O 1 e termo do lado direito são os termos de produção resultante da interação da turbulência e o escoamento médio

O 2 termo (terceiro momento) correlação tripla pode ser interpretado como transporte de turbulência (segundo momento) pela flutuação turbulenta com o ganho ou perda devido a divergência do fluxo turbulento

O 3 termo representa a produção e destruição da flutuabilidade (conversão da energia cinética turbulenta para a energia potencial turbulenta)

O termo 4 é a rotação e pode ser desprezado para média temporal menor do que 1 hora

O termo 5 é a interação da flutuação de pressão e do campo de velocidade

O termo 6 é a dissipação molecular

O termo 7 é a redistribuição de energia, **uma vez que uma de suas funções é particionar a energia entre os três componentes de energia sem contribuir para o total**

Linearização da Equações de Navier Stokes

Para caso de homogeneidade horizontal

$$\frac{\partial\overline{(u_i'u_k')}}{\partial t}+(\overline{\overline{u_j}})\frac{\partial\overline{(u_i'u_k')}}{\partial x_j}$$
$$=-\overline{(u_j'u_k')}\frac{\partial(\overline{\overline{u_i}})}{\partial x_j}-\frac{\partial\overline{(u_j'u_i'u_k')}}{\partial x_j}-g\frac{\overline{\rho' u_k'}}{\rho_0}\delta_{i3}-2\Omega\varepsilon_{ijk}\eta_j\overline{(u_k'u_k')}-\frac{1}{\rho_0}\frac{\partial\overline{(P'u_k')}}{\partial x_i}+\nu\frac{\partial^2\overline{(u_i'u_k')}}{\partial {x_j}^2}$$

O termo molecular $\nu\frac{\partial^2\overline{(u_i'u_k')}}{\partial {x_j}^2}\rightarrow 0$ **é desprezado no caso da covariância, porque a viscosidade é dominante somente em numero de ondas grandes.**

Porém, neste caso a turbulência é isotrópica e assim a covariância é zero na horizontal.

(j=k) $$\frac{\partial\overline{u'w'}}{\partial t}=-\overline{w'^2}\frac{\partial\bar{u}}{\partial z}+\frac{g}{\overline{\theta_v}}\overline{u'\theta_v'}-\frac{\partial\overline{u'w'^2}}{\partial z}-\frac{1}{\rho}\left(\overline{u'\frac{\partial P'}{\partial z}}+\overline{w'\frac{\partial P'}{\partial x}}\right)$$

Isotrópico é a caracterização de uma substância que possui as mesmas propriedades físicas, independentemente da direção considerada.

Para caso de homogeneidade horizontal e o estado básico em condições neutra $\frac{\partial \overline{u'w'}}{\partial t} = 0$ **e** $\frac{g}{\overline{\theta_v}}\overline{u'\theta_v'} = 0$

(j=k) $$\frac{\partial \overline{u'w'}}{\partial t} = -\overline{w'^2}\frac{\partial \bar{u}}{\partial z} + \frac{g}{\overline{\theta_v}}\overline{u'\theta_v'} - \frac{\partial \overline{u'w'^2}}{\partial z} - \frac{1}{\rho}\left(\overline{u'\frac{\partial P'}{\partial z}} + \overline{w'\frac{\partial P'}{\partial x}}\right)$$

Isto mostra que o termo de correlação de pressão-velocidade destrói o stress na mesma taxa como ela é produzida

$$0 = -\overline{w'^2}\frac{\partial \bar{u}}{\partial z} - \frac{\partial \overline{u'w'^2}}{\partial z} - \frac{1}{\rho}\left(\overline{u'\frac{\partial P'}{\partial z}} + \overline{w'\frac{\partial P'}{\partial x}}\right)$$

$$+\overline{w'^2}\frac{\partial \bar{u}}{\partial z} + \frac{\partial \overline{u'w'^2}}{\partial z} = -\frac{1}{\rho}\left(\overline{u'\frac{\partial P'}{\partial z}} + \overline{w'\frac{\partial P'}{\partial x}}\right)$$

Equação
da
Energia Cinética Turbulenta

5.2 Energia Cinética Turbulenta

$$\frac{\partial\overline{(u_i'u_k')}}{\partial t}+(\bar{\bar{u}}_j)\frac{\partial\overline{(u_i'u_k')}}{\partial x_j}$$

$$=-\overline{(u_j'u_k')}\frac{\partial(\bar{\bar{u}}_i)}{\partial x_j}-\frac{\partial\overline{(u_j'u_i'u_k')}}{\partial x_j}-g\frac{\overline{\rho'u_k'}}{\rho_0}\delta_{i3}-2\Omega\varepsilon_{ijk}\eta_j\overline{(u_k'u_k')}-\frac{1}{\rho_0}\frac{\partial\overline{(P'u_k')}}{\partial x_i}+\nu\frac{\partial^2\overline{(u_i'u_k')}}{\partial {x_j}^2}$$

$$\overline{u_k'u_k'}=\overline{{u_k'}^2}=\overline{u_k'}^2=0$$

$$\bar{e}=\frac{\overline{u_i'^2}}{2}=\frac{\overline{\left(u'^2+v'^2+w'^2\right)}}{2} \qquad i=k=1,2,3$$

$$\frac{\partial\bar{e}}{\partial t}+(\bar{\bar{u}}_j)\frac{\partial\bar{e}}{\partial x_j}=-\overline{(u_j'u_i')}\frac{\partial(\bar{\bar{u}}_i)}{\partial x_j}-g\frac{\overline{u_i'\rho'}}{\rho_0}\delta_{i3}-\frac{\partial\overline{(eu_j')}}{\partial x_j}-\frac{1}{\rho_0}\frac{\partial\overline{(P'u_i')}}{\partial x_i}+\nu\frac{\partial^2\overline{(u_i'u_k')}}{\partial {x_j}^2}$$

$$\boxed{\frac{\partial\bar{e}}{\partial t}+(\bar{\bar{u}}_j)\frac{\partial\bar{e}}{\partial x_j}=-\frac{\overline{(u_j'u_i')}}{2}\frac{\partial(\bar{\bar{u}}_i)}{\partial x_j}-g\frac{\overline{u_i'\rho'}}{2\rho_0}\delta_{i3}-\frac{\partial\overline{(eu_j')}}{\partial x_j}-\frac{1}{2\rho_0}\frac{\partial\overline{(P'u_i')}}{\partial x_i}-\epsilon}$$

5.2 Energia Cinética Turbulenta

$$\frac{\partial \bar{e}}{\partial t} + \left(\bar{\bar{u}}_j\right)\frac{\partial \bar{e}}{\partial x_j} = -\frac{\overline{\left(u_j{}'u_i{}'\right)}}{2}\frac{\partial\left(\bar{\bar{u}}_i\right)}{\partial x_j} - g\,\frac{\overline{u_i{}'\rho'}}{2\rho_0}\delta_{i3} - \frac{\partial\overline{\left(e u_j{}'\right)}}{\partial x_j} - \frac{1}{2\rho_0}\frac{\partial\left(\overline{P'u_i{}'}\right)}{\partial x_i} - \epsilon$$

A quantidade ϵ é um parâmetro significante para a atmosfera desde que seja relacionado a dissipação da energia cinética turbulenta de todos os movimentos atmosféricos

5.2 Energia Cinética Turbulenta

A essência da equação da energia cinética turbulenta pode ser expressa pela equação:

$$\frac{D\bar{e}}{Dt} = -\frac{\overline{\left(u_j{}'u_i{}'\right)}}{2}\frac{\partial(\bar{\bar{u}}_i)}{\partial x_j} - g\frac{\overline{u_i{}'\rho'}}{2\rho_0}\delta_{i3} - \frac{\partial\overline{\left(eu_j{}'\right)}}{\partial x_j} - \frac{1}{2\rho_0}\frac{\partial(\overline{P'u_i{}'})}{\partial x_i} - \epsilon$$

$$\frac{\overline{D}(TKE)}{Dt} = MP + BPL + TR - \varepsilon$$

MP **é a produção mecânica**

BPL **é a produção e perda por flutuabilidade**

TR **redistribuição de tke por transporte e força de pressão**

ε **dissipação por atrito**

5.2 Energia Cinética Turbulenta

$$BPL \equiv \overline{w'\theta' \frac{g}{\theta_0}}$$

É a conversão da energia potencial do escoamento médio e a energia cinética turbulenta:
É positivo para movimentos que baixa o centro de massa da atmosfera
É negativo para movimentos que aumenta o centro de massa da atmosfera

Correlação positiva(fonte tke)
Atms. instável

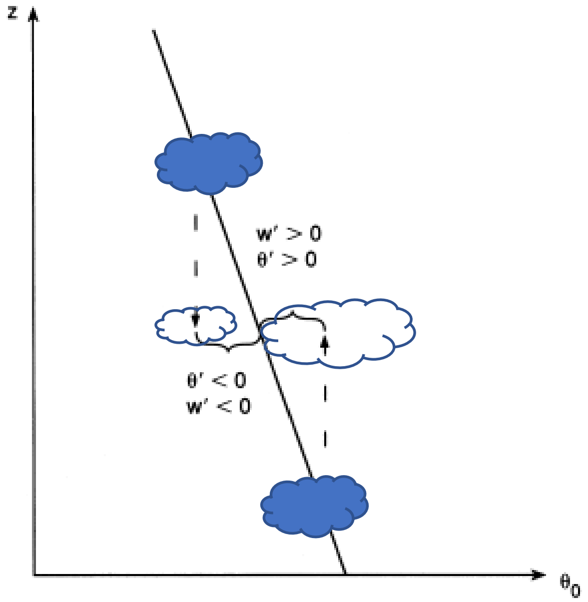
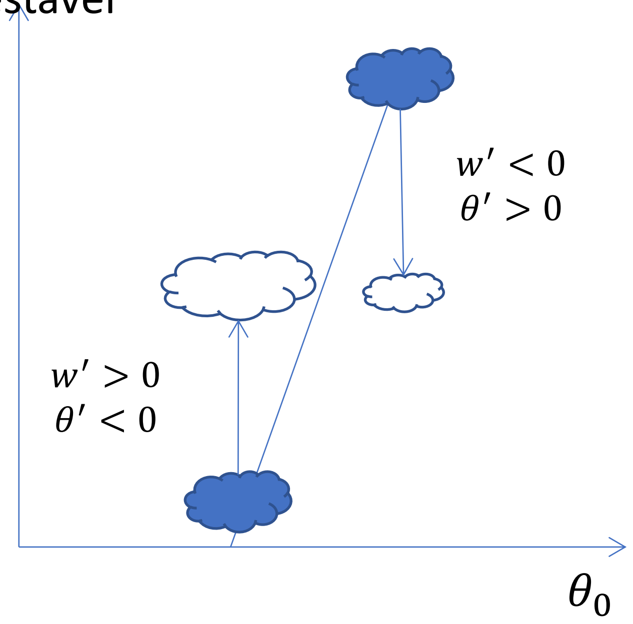


Fig. 5.1 Correlation between vertical velocity and potential temperature perturbations for upward or downward parcel displacements when the mean potential temperature $\theta_0(z)$ decreases with height.

Correlação negativa (destroi tke)
Atms. estável



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5.2 Energia Cinética Turbulenta

Para ambos as condições estáveis e instáveis da CLP a turbulência pode ser produzida mecanicamente pela instabilidade dinâmica através do cisalhamento. Conversão de energia entre o escoamento médio e a flutuação turbulenta.

$$MP \equiv -\frac{\overline{u'w'}}{2}\frac{\partial \bar{u}}{\partial z} - \frac{\overline{v'w'}}{2}\frac{\partial \bar{v}}{\partial z}$$

$MP > 0$ **quando o fluxo de momentum (**$\overline{u'w'}$**) é direcionado para baixo e o gradiente vertical é positivo**

5.2 Energia Cinética Turbulenta

Estatisticamente na camada limite estável a turbulência pode existir somente se a produção mecânica for grande o suficiente para superar o efeito de supressão da estabilidade e da viscosidade

Esta condição é medida pelo numero de Richardson de fluxo

$$Rf \equiv -\frac{BPL}{MP} \equiv \frac{\overline{w'\theta'\frac{g}{\theta_0}}}{-\overline{u'w'}\frac{\partial \bar{u}}{\partial z} - \overline{v'w'}\frac{\partial \bar{v}}{\partial z}}$$

5.2 Energia Cinética Turbulenta

$$Rf \equiv -\frac{BPL}{MP} \equiv \frac{\overline{w'\theta'}\,\frac{g}{\theta_0}}{-\overline{u'w'}\frac{\partial \bar{u}}{\partial z} - \overline{v'w'}\frac{\partial \bar{v}}{\partial z}}$$

- **Se** $Rf < 0$ **a CLP é estatisticamente instável (a turbulências é sustentada pela convecção)**
- $Rf > 0$ **a CLP é estatisticamente estável**
- $Rf < 0.25$ **(a produção mecânica excede a produção por flutuabilidade por um fato de 4)**

5.2 Energia Cinética Turbulenta

5.3 Equações de momentum da camada limite Planetária

$$\frac{\partial(\overline{u_i})}{\partial t} + (\overline{u_j})\frac{\partial(\overline{u_i})}{\partial x_j} = -\frac{\partial\left(\overline{u_j{}'u_i{}'}\right)}{\partial x_j} - \frac{1}{\rho_0}\frac{\partial(\bar{P})}{\partial x_i} - g\frac{\bar{\rho}}{\rho_0}\delta_{i3} - 2\Omega\varepsilon_{ijk}\eta_j(\overline{u_k}) + \nu\frac{\partial^2(\overline{u_i})}{\partial x_j{}^2}$$

$$\frac{\partial\left(\overline{u_i{}'u_k{}'}\right)}{\partial t} + (\overline{\overline{u_j}})\frac{\partial\left(\overline{u_i{}'u_k{}'}\right)}{\partial x_j}$$

$$= -\left(\overline{u_j{}'u_k{}'}\right)\frac{\partial(\overline{\overline{u_i}})}{\partial x_j} - g\frac{\overline{\rho' u_k{}'}}{\rho_0}\delta_{i3} - 2\Omega\varepsilon_{ijk}\eta_j\overline{(u_k{}'u_k{}')} - \frac{\partial\left(\overline{u_j{}'u_i{}'u_k{}'}\right)}{\partial x_j} - \frac{1}{\rho_0}\frac{\partial\left(\overline{P' u_k{}'}\right)}{\partial x_i} + \nu\frac{\partial^2\left(\overline{u_i{}'u_k{}'}\right)}{\partial x_j{}^2}$$

$$\boxed{\frac{\partial\bar{e}}{\partial t} + (\overline{\overline{u_j}})\frac{\partial\bar{e}}{\partial x_j} = -\frac{\overline{\left(u_j{}'u_i{}'\right)}}{2}\frac{\partial(\overline{\overline{u_i}})}{\partial x_j} - g\frac{\overline{u_i{}'\rho'}}{2\rho_0}\delta_{i3} - \frac{\partial\left(\overline{e u_j{}'}\right)}{\partial x_j} - \frac{1}{2\rho_0}\frac{\partial\left(\overline{P' u_i{}'}\right)}{\partial x_i} - \epsilon}$$

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5.2 Energia Cinética Turbulenta

5.3 Equações de momentum da camada limite Planetária

$$\frac{\partial(\overline{u_i})}{\partial t} + (\overline{u_j})\frac{\partial(\overline{u_i})}{\partial x_j} = -\frac{\partial\left(\overline{u_j{}'u_i{}'}\right)}{\partial x_j} - \frac{1}{\rho_0}\frac{\partial(\bar{P})}{\partial x_i} - g\frac{\rho}{\rho_0}\delta_{i3} - 2\Omega\varepsilon_{ijk}\eta_j(\overline{u_k}) + \nu\frac{\partial^2(\overline{u_i})}{\partial {x_j}^2}$$

Para o caso especial de turbulência horizontalmente homogênea:

-> A camada viscosa, a viscosidade molecular e o temo da divergência horizontal do fluxo de momentum turbulento podem ser desprezados.

$$\frac{\bar{D}\bar{u}}{Dt} = -\frac{1}{\rho_0}\frac{\partial\bar{P}}{\partial x} + f\bar{v} - \frac{\partial\overline{u'w'}}{\partial z}$$

$$\frac{\bar{D}\bar{v}}{Dt} = -\frac{1}{\rho_0}\frac{\partial\bar{P}}{\partial y} - f\bar{u} - \frac{\partial\overline{v'w'}}{\partial z}$$

Só pode ser resolvida se conhecermos a distribuição vertical do fluxo de momentum

Implementação da parametrização de Radiação ECRAD do ECMWF

Slides Grupo Atmos

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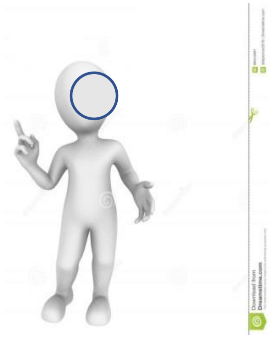
1.5 Suítes físicas avançadas, etc.

Obrigado Pela Atenção de Todos



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