3- Ondas de Gravidade Inercial e distribuição de Variáveis

Nesta seção nos discutiremos o efeito da diferença centrada no espaço sobre as ondas de gravidade. Assim, nos consideramos o sistema de equação linearizada.

$$\frac{\partial u}{\partial t} = -g\frac{\partial h}{\partial x} + fv \qquad 3.1a$$

$$\frac{\partial v}{\partial t} = -g\frac{\partial h}{\partial x} - fu \qquad 3.1b$$

$$\frac{\partial h}{\partial t} = -H\nabla.\vec{V} \qquad 3.1c$$

Esta equação direre daquela da secção 2 no termo de coriolis f O termo de coriolis não contem derivadas . Entretanto, eles são difíceis de calcular sobre a grade C, que foi ideal para ondas de gravidade puras.

Assim , nos reconsideramos o problema da distribuição de variáveis.

Não é obivio como nos podemos analisar vários arranjos de variáveis. Nossa primeira opção é considerar (eq. 3.1) como parte de um sistema completo de equações primitivas. Nos estamos interessados no movimento de grande escala, por outro lado nos não devemos incluir o termo de Coriolis.

Sobre a grande escala, a equação primitiva admite dois tipos movimento distintos: baixa frequência e quase-geostrifico e escoamento quase-não divergente; e alta frequecia ondas de gravidade inercial. Ondas de gravidade inercial são continuamente excitada na atmosfera, entretanto, como elas são dispersiva, uma acumulação local de energia de ondas dispersa com o tempo. Estes processos é conhecid como ajustamento geostrofico; o movimento permanecente é um balanço aproximadamente geostrofico e muda somente lentamente com o tempo. Neste capitulo nos estaremos concentrado coma simulação correta destes processos, em que é essencialmente governado pela equação de endas de gravidade inercial(3.1).

Nos estamos interessadosem ambas ondas causadas pelo efeito físico, e que é causado por inadequados dados iniciais e procedimento numérico.

Entretanto o detalhes do processo de ajustamento não importa tanto quanto a correção do resutado do escoamento quase-geotrofico.

Nos devemos no entato investigar o efeito da distribuição do espaço de variáveis dependentes sobre a propriedade dispersiva da ondas de gravidade inercial. Este será feito usando a mais simples aproximação centrada para a derivada no espaço deixando a derivada no tempo em sua forma diferencial.

A discução é baseada sobre aquilo que Winninghoff e Arakawa como apresentado por Arakawa (Arakawa, 1972; Arakawa et al. 1974).

Nos consideramos 5 caminhos de distribuição de variáveis dependentes. No espaço.

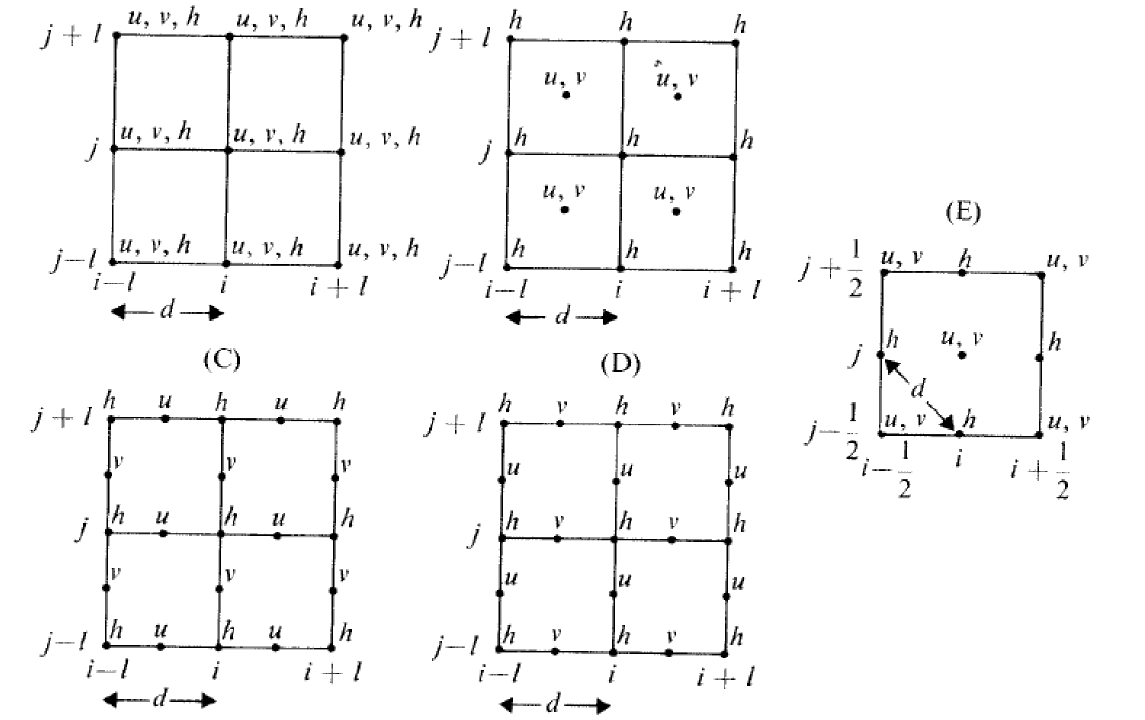

Fig 3.1

Nos Definimos d a distancia mais curta entre os pontos vizinhos carregando a mesma variável dependente. Na figura 3.1 d é o mesmo para cada uma das cinco grades. Assim, todas as grades tem o mesmo numero de variáveis dependentes por unidade de area. A tempo de computação necessário para um integração sobre cada uma das grade será sobre a mesma; propriedade da solução obtida embora , será diferente devido ao efeito do espaço de arrajamento das variáveis.

Usando o subscripts mostrado na figura 3.1, nos definimos um operador para a diferenciação no espaço centrado.

$$(\delta_x \alpha)_{i,j} \equiv \frac{1}{d'}\left(\alpha_{i+\frac{1}{2},j} - \alpha_{i-\frac{1}{2},j}\right)$$

Esta rotação é aplicável a todas as grades. Aqui d' é a distancia entre os pontos os quais a diferença finita é feita. Assim, para a grade A, embora D d' pe igual ao tamanho da grade d, e para a grade E é igual a $\sqrt{2}d$.

Nos também definimos uma media sobre o mesmo dois pontos por:

$$(\bar{\alpha}^x)_{i,j} \equiv \frac{1}{2}\left(\alpha_{i+\frac{1}{2},j} + \alpha_{i-\frac{1}{2},j}\right)$$

Assim, $\left(\delta_y \alpha\right)_{i,j}$ e $(\bar{\alpha}^y)_{i,j}$ são definido no mesmo caminho, mas com respeito ao eixo y.

Finalmente,

$$(\bar{\alpha}^{xy})_{i,j} \equiv \left(\overline{\bar{\alpha}^{x}}^{y}\right)_{i,j}$$

Para cada uma das 5 grades nos usamos uma aproximação centrada simples para a derivada no espaço e temos de coriolis (3.1). Obtemo-nos os diferentes sistemas:

GRADE A

$$\begin{aligned}
\frac{\partial u}{\partial t} = -g\overline{\delta_x h}^x + fv &= -g\left(\frac{1}{d'}\left(\bar{h}^x_{i+\frac{1}{2},j} - \bar{h}^x_{i-\frac{1}{2},j}\right)\right) + fv \\
&= -g\left(\frac{1}{d'}\left(\frac{1}{2}\left(h_{i+\frac{1}{2}+\frac{1}{2},j} + h_{i+\frac{1}{2}-\frac{1}{2},j}\right) - \frac{1}{2}\left(h_{i-\frac{1}{2}+\frac{1}{2},j} + h_{i-\frac{1}{2}-\frac{1}{2},j}\right)\right)\right) + fv \\
&= -g\left(\frac{1}{d'}\left(\frac{1}{2}\left(h_{i+1,j} + h_{i,j}\right) - \frac{1}{2}\left(h_{i,j} + h_{i-1,j}\right)\right)\right) + fv \\
&= -g\left(\frac{1}{d'}\left(\frac{1}{2}\left(h_{i+1,j} + h_{i,j} - h_{i,j} - h_{i-1,j}\right)\right)\right) + fv \\
&= -g\left(\frac{1}{2d'}\left(h_{i+1,j} - h_{i-1,j}\right)\right) + fv
\end{aligned}$$

$$\begin{aligned}
\frac{\partial v}{\partial t} = -g\overline{\delta_y h}^y - fu &= -g\left(\frac{1}{d'}\left(\bar{h}^y_{i,j+\frac{1}{2}} - \bar{h}^y_{i,j-\frac{1}{2}}\right)\right) - fu \\
&= -g\left(\frac{1}{d'}\left(\frac{1}{2}\left(h_{i,j+\frac{1}{2}+\frac{1}{2}} + h_{i,j+\frac{1}{2}-\frac{1}{2}}\right) - \frac{1}{2}\left(h_{i,j-\frac{1}{2}+\frac{1}{2}} + h_{i,j-\frac{1}{2}-\frac{1}{2}}\right)\right)\right) - fu \\
&= -g\left(\frac{1}{d'}\left(\frac{1}{2}\left(h_{i,j+1} + h_{i,j}\right) - \frac{1}{2}\left(h_{i,j} + h_{i,j-1}\right)\right)\right) - fu \\
&= -g\left(\frac{1}{d'}\left(\frac{1}{2}\left(h_{i,j+1} + h_{i,j} - h_{i,j} - h_{i,j-1}\right)\right)\right) - fu \\
&= -g\left(\frac{1}{2d'}\left(h_{i,j+1} - h_{i,j-1}\right)\right) - fu
\end{aligned}$$

$$\frac{\partial h}{\partial t} = -H\left(\overline{\delta_x u}^x + \overline{\delta_y v}^y\right) = H\left(\left(\frac{1}{d'}\left(\bar{u}^x_{i+\frac{1}{2},j} - \bar{h}^x_{i-\frac{1}{2},j}\right)\right) + \left(\frac{1}{d'}\left(\bar{v}^y_{i,j+\frac{1}{2}} - \bar{v}^y_{i,j-\frac{1}{2}}\right)\right)\right)$$

$$= H\left(\frac{1}{d'}\left(\frac{1}{2}\left(u_{i+\frac{1}{2}+\frac{1}{2},j} + u_{i+\frac{1}{2}-\frac{1}{2},j}\right) - \frac{1}{2}\left(u_{i-\frac{1}{2}+\frac{1}{2},j} + u_{i-\frac{1}{2}-\frac{1}{2},j}\right)\right)\right)$$

$$+ H\left(\frac{1}{d'}\left(\frac{1}{2}\left(v_{i,j+\frac{1}{2}+\frac{1}{2}} + v_{i,j+\frac{1}{2}-\frac{1}{2}}\right) - \frac{1}{2}\left(v_{i,j-\frac{1}{2}+\frac{1}{2}} + v_{i,j-\frac{1}{2}-\frac{1}{2}}\right)\right)\right)$$

$$= H\left(\frac{1}{d'}\left(\frac{1}{2}\left(u_{i+1,j} + u_{i,j}\right) - \frac{1}{2}\left(u_{i,j} + u_{i-1,j}\right)\right)\right)$$

$$+ H\left(\frac{1}{d'}\left(\frac{1}{2}\left(v_{i,j+1} + h_{i,j}\right) - \frac{1}{2}\left(v_{i,j} + v_{i,j-1}\right)\right)\right)$$

$$= H\left(\frac{1}{2d'}\left(u_{i+1,j} - u_{i-1,j}\right)\right) + H\left(\frac{1}{2d'}\left(v_{i,j+1} - v_{i,j-1}\right)\right)$$

GRADE B

$$\frac{\partial u}{\partial t} = -g\overline{\delta_x h}^y + fv = -g\left(\frac{1}{d'}\left(\bar{h}^y_{i+\frac{1}{2},j} - \bar{h}^y_{i-\frac{1}{2},j}\right)\right) + fv$$

$$= -g\left(\frac{1}{d'}\left(\frac{1}{2}\left(h_{i+\frac{1}{2},j+\frac{1}{2}} + h_{i+\frac{1}{2},j-\frac{1}{2}}\right) - \frac{1}{2}\left(h_{i-\frac{1}{2},j+\frac{1}{2}} + h_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right) + fv$$

$$= -g\left(\frac{1}{d'}\left(\frac{1}{2}\left(h_{i+\frac{1}{2},j+\frac{1}{2}} - h_{i-\frac{1}{2},j+\frac{1}{2}} + h_{i+\frac{1}{2},j-\frac{1}{2}} - h_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right) + fv$$

$$= -g\left(\frac{1}{2d'}\left(h_{i+\frac{1}{2},j+\frac{1}{2}} - h_{i-\frac{1}{2},j+\frac{1}{2}} + h_{i+\frac{1}{2},j-\frac{1}{2}} - h_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right) + fv$$

$$\frac{\partial v}{\partial t} = -g\overline{\delta_y h}^x - fu = -g\left(\frac{1}{d'}\left(\bar{h}^x_{i,j+\frac{1}{2}} - \bar{h}^x_{i,j-\frac{1}{2}}\right)\right) - fu$$

$$= -g\left(\frac{1}{d'}\left(\frac{1}{2}\left(h_{i+\frac{1}{2},j+\frac{1}{2}} + h_{i-\frac{1}{2},j+\frac{1}{2}}\right) - \frac{1}{2}\left(h_{i+\frac{1}{2},j-\frac{1}{2}} + h_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right) - fu$$

$$= -g\left(\frac{1}{d'}\left(\frac{1}{2}\left(h_{i+\frac{1}{2},j+\frac{1}{2}} - h_{i-\frac{1}{2},j+\frac{1}{2}} + h_{i+\frac{1}{2},j-\frac{1}{2}} - h_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right) - fu$$

$$= -g\left(\frac{1}{2d'}\left(h_{i+\frac{1}{2},j+\frac{1}{2}} - h_{i-\frac{1}{2},j+\frac{1}{2}} + h_{i+\frac{1}{2},j-\frac{1}{2}} - h_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right) - fu$$

$$\frac{\partial h}{\partial t} = -H\left(\overline{\delta_x u}^y + \overline{\delta_y v}^x\right) = H\left(\left(\frac{1}{d'}\left(\bar{u}^y_{i+\frac{1}{2},j} - \bar{h}^y_{i-\frac{1}{2},j}\right)\right) + \left(\frac{1}{d'}\left(\bar{v}^x_{i,j+\frac{1}{2}} - \bar{v}^x_{i,j-\frac{1}{2}}\right)\right)\right)$$

$$= H\left(\frac{1}{d'}\left(\frac{1}{2}\left(u_{i+\frac{1}{2},j+\frac{1}{2}} + u_{i+\frac{1}{2},j-\frac{1}{2}}\right) - \frac{1}{2}\left(u_{i-\frac{1}{2},j+\frac{1}{2}} + u_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right)$$

$$+ H\left(\frac{1}{d'}\left(\frac{1}{2}\left(v_{i+\frac{1}{2},j+\frac{1}{2}} + v_{i-\frac{1}{2},j+\frac{1}{2}}\right) - \frac{1}{2}\left(v_{i+\frac{1}{2},j-\frac{1}{2}} + v_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right)$$

$$= H\left(\frac{1}{2d'}\left(\left(u_{i+\frac{1}{2},j+\frac{1}{2}} - u_{i-\frac{1}{2},j+\frac{1}{2}} + u_{i+\frac{1}{2},j-\frac{1}{2}} - u_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right)$$

$$+ H\left(\frac{1}{2d'}\left(\left(v_{i+\frac{1}{2},j+\frac{1}{2}} - v_{i+\frac{1}{2},j-\frac{1}{2}} + v_{i-\frac{1}{2},j+\frac{1}{2}} - v_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right)$$

GRADE C

$$\frac{\partial u}{\partial t} = -g\delta_x h + f\bar{v}^{xy} = -g\left(\frac{1}{d'}\left(h_{i+\frac{1}{2},j} - h_{i-\frac{1}{2},j}\right)\right) + f\left(\frac{1}{2}\left(\bar{v}^y{}_{i+\frac{1}{2},j} + \bar{v}^y{}_{i-\frac{1}{2},j}\right)\right)$$

$$= -g\left(\frac{1}{d'}\left(h_{i+\frac{1}{2},j} - h_{i-\frac{1}{2},j}\right)\right)$$

$$+ f\left(\frac{1}{2}\left(\frac{1}{2}\left(v_{i+\frac{1}{2},j+\frac{1}{2}} + v_{i+\frac{1}{2},j-\frac{1}{2}}\right) + \frac{1}{2}\left(v_{i-\frac{1}{2},j+\frac{1}{2}} + v_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right)$$

$$= -g\left(\frac{1}{d'}\left(h_{i+\frac{1}{2},j} - h_{i-\frac{1}{2},j}\right)\right)$$

$$+ f\left(\left(\frac{1}{4}\left(v_{i+\frac{1}{2},j+\frac{1}{2}} + v_{i+\frac{1}{2},j-\frac{1}{2}} + v_{i-\frac{1}{2},j+\frac{1}{2}} + v_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right)$$

$$\frac{\partial v}{\partial t} = -g\delta_y h - f\bar{u}^{xy} = -g\left(\frac{1}{d'}\left(h_{i,j+\frac{1}{2}} - h_{i,j-\frac{1}{2}}\right)\right) - f\left(\frac{1}{2}\left(\bar{u}^y{}_{i+\frac{1}{2},j} + \bar{u}^y{}_{i-\frac{1}{2},j}\right)\right)$$

$$= -g\left(\frac{1}{d'}\left(h_{i,j+\frac{1}{2}} - h_{i,j-\frac{1}{2}}\right)\right)$$

$$- f\left(\frac{1}{2}\left(\frac{1}{2}\left(u_{i+\frac{1}{2},j+\frac{1}{2}} + u_{i+\frac{1}{2},j-\frac{1}{2}}\right) + \frac{1}{2}\left(u_{i-\frac{1}{2},j+\frac{1}{2}} + u_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right)$$

$$= -g\left(\frac{1}{d'}\left(h_{i,j+\frac{1}{2}} - h_{i,j-\frac{1}{2}}\right)\right)$$

$$- f\left(\left(\frac{1}{4}\left(u_{i+\frac{1}{2},j+\frac{1}{2}} + u_{i+\frac{1}{2},j-\frac{1}{2}} + u_{i-\frac{1}{2},j+\frac{1}{2}} + u_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right)$$

$$\frac{\partial h}{\partial t} = -H\left(\delta_x u + \delta_y v\right) = H\left(\left(\frac{1}{d'}\left(u_{i+\frac{1}{2},j} - u_{i-\frac{1}{2},j}\right)\right) + \left(\frac{1}{d'}\left(v_{i,j+\frac{1}{2}} - v_{i,j-\frac{1}{2}}\right)\right)\right)$$

$$= H\left(\frac{1}{d'}\left(\left(u_{i+\frac{1}{2},j} - u_{i-\frac{1}{2},j} + v_{i,j+\frac{1}{2}} - v_{i,j-\frac{1}{2}}\right)\right)\right)$$

GRADE D

$$\frac{\partial u}{\partial t} = -g\overline{\delta_x h}^{xy} + f\bar{v}^{xy} = -g\left(\frac{1}{d'}\left(\bar{h}^{xy}_{i+\frac{1}{2},j} - \bar{h}^{xy}_{i-\frac{1}{2},j}\right)\right) + f\left(\frac{1}{2}\left(\bar{v}^{y}{}_{i+\frac{1}{2},j} + \bar{v}^{y}{}_{i-\frac{1}{2},j}\right)\right)$$

$$= -g\left(\frac{1}{d'}\left(\frac{1}{2}\left(\bar{h}^{y}_{i+\frac{1}{2}+\frac{1}{2},j} + \bar{h}^{y}_{i+\frac{1}{2}-\frac{1}{2},j}\right) - \frac{1}{2}\left(\bar{h}^{y}_{i-\frac{1}{2}+\frac{1}{2},j} + \bar{h}^{y}_{i-\frac{1}{2}-\frac{1}{2},j}\right)\right)\right)$$

$$+ f\left(\frac{1}{2}\left(\frac{1}{2}\left(v_{i+\frac{1}{2},j+\frac{1}{2}} + v_{i+\frac{1}{2},j-\frac{1}{2}}\right) + \frac{1}{2}\left(v_{i-\frac{1}{2},j+\frac{1}{2}} + v_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right)$$

$$= -g\left(\frac{1}{d'}\left(\frac{1}{2}\left(\bar{h}^{y}_{i+1,j} + \bar{h}^{y}_{i,j}\right) - \frac{1}{2}\left(\bar{h}^{y}_{i,j} + \bar{h}^{y}_{i-1,j}\right)\right)\right)$$

$$+ f\left(\left(\frac{1}{4}\left(v_{i+\frac{1}{2},j+\frac{1}{2}} + v_{i+\frac{1}{2},j-\frac{1}{2}} + v_{i-\frac{1}{2},j+\frac{1}{2}} + v_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right) =$$

$$= -g\left(\frac{1}{d'}\left(\frac{1}{2}\left(\bar{h}^{y}_{i+1,j} - \bar{h}^{y}_{i-1,j}\right)\right)\right)$$

$$+ f\left(\left(\frac{1}{4}\left(v_{i+\frac{1}{2},j+\frac{1}{2}} + v_{i+\frac{1}{2},j-\frac{1}{2}} + v_{i-\frac{1}{2},j+\frac{1}{2}} + v_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right)$$

$$= -g\left(\frac{1}{d'}\left(\frac{1}{2}\left(\frac{1}{2}\left(h_{i+1,j+\frac{1}{2}} + h_{i+1,j-\frac{1}{2}}\right) - \frac{1}{2}\left(h_{i-1,j+\frac{1}{2}} + h_{i-1,j-\frac{1}{2}}\right)\right)\right)\right)$$

$$+ f\left(\left(\frac{1}{4}\left(v_{i+\frac{1}{2},j+\frac{1}{2}} + v_{i+\frac{1}{2},j-\frac{1}{2}} + v_{i-\frac{1}{2},j+\frac{1}{2}} + v_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right)$$

$$= -g\left(\frac{1}{4d'}\left(\left(h_{i+1,j+\frac{1}{2}} - h_{i-1,j+\frac{1}{2}} + h_{i+1,j-\frac{1}{2}} - h_{i-1,j-\frac{1}{2}}\right)\right)\right)$$

$$+ f\left(\left(\frac{1}{4}\left(v_{i+\frac{1}{2},j+\frac{1}{2}} + v_{i+\frac{1}{2},j-\frac{1}{2}} + v_{i-\frac{1}{2},j+\frac{1}{2}} + v_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right)$$

$$\frac{\partial v}{\partial t} = -g\overline{\delta_y h}^{xy} - f\bar{u}^{xy} = -g\left(\frac{1}{d'}\left(\bar{h}^{xy}_{i,j+\frac{1}{2}} - \bar{h}^{xy}_{i,j-\frac{1}{2}}\right)\right) - f\left(\frac{1}{2}\left(\bar{u}^{y}{}_{i+\frac{1}{2},j} + \bar{u}^{y}{}_{i-\frac{1}{2},j}\right)\right)$$

$$= -g\left(\frac{1}{d'}\left(\frac{1}{2}\left(\bar{h}^{x}_{i,j+\frac{1}{2}+\frac{1}{2}} + \bar{h}^{x}_{i,j+\frac{1}{2}-\frac{1}{2}}\right) - \frac{1}{2}\left(\bar{h}^{x}_{i,j-\frac{1}{2}+\frac{1}{2}} + \bar{h}^{x}_{i,j-\frac{1}{2}-\frac{1}{2}}\right)\right)\right)$$

$$- f\left(\frac{1}{2}\left(\frac{1}{2}\left(u_{i+\frac{1}{2},j+\frac{1}{2}} + u_{i+\frac{1}{2},j-\frac{1}{2}}\right) + \frac{1}{2}\left(u_{i-\frac{1}{2},j+\frac{1}{2}} + u_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right)$$

$$= -g\left(\frac{1}{d'}\left(\frac{1}{2}\left(\bar{h}^{x}_{i,j+1} + \bar{h}^{x}_{i,j}\right) - \frac{1}{2}\left(\bar{h}^{x}_{i,j} + \bar{h}^{x}_{i,j-1}\right)\right)\right)$$

$$- f\left(\left(\frac{1}{4}\left(u_{i+\frac{1}{2},j+\frac{1}{2}} + u_{i+\frac{1}{2},j-\frac{1}{2}} + u_{i-\frac{1}{2},j+\frac{1}{2}} + u_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right) =$$

$$= -g\left(\frac{1}{d'}\left(\frac{1}{2}\left(\bar{h}^{y}_{i,j+1} - \bar{h}^{y}_{i,j-1}\right)\right)\right)$$

$$- f\left(\left(\frac{1}{4}\left(u_{i+\frac{1}{2},j+\frac{1}{2}} + u_{i+\frac{1}{2},j-\frac{1}{2}} + u_{i-\frac{1}{2},j+\frac{1}{2}} + u_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right)$$

$$= -g\left(\frac{1}{d'}\left(\frac{1}{2}\left(\frac{1}{2}\left(h_{i+\frac{1}{2},j+1} + h_{i-\frac{1}{2},j+1}\right) - \frac{1}{2}\left(h_{i+\frac{1}{2},j-1} + h_{i-\frac{1}{2},j-1}\right)\right)\right)\right)$$

$$- f\left(\left(\frac{1}{4}\left(u_{i+\frac{1}{2},j+\frac{1}{2}} + u_{i+\frac{1}{2},j-\frac{1}{2}} + u_{i-\frac{1}{2},j+\frac{1}{2}} + u_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right)$$

$$= -g\left(\frac{1}{4d'}\left(\left(h_{i+\frac{1}{2},j+1} - h_{i+\frac{1}{2},j-1} + h_{i-\frac{1}{2},j+1} - h_{i-\frac{1}{2},j-1}\right)\right)\right)$$

$$- f\left(\left(\frac{1}{4}\left(u_{i+\frac{1}{2},j+\frac{1}{2}} + u_{i+\frac{1}{2},j-\frac{1}{2}} + u_{i-\frac{1}{2},j+\frac{1}{2}} + u_{i-\frac{1}{2},j-\frac{1}{2}}\right)\right)\right)$$

$$\frac{\partial h}{\partial t} = -H\left(\overline{\delta_x u}^{xy} + \overline{\delta_y v}^{xy}\right) = H\left(\left(\frac{1}{d'}\left(\bar{u}^{xy}_{i+\frac{1}{2},j} - \bar{u}^{xy}_{i-\frac{1}{2},j}\right)\right) + \left(\frac{1}{d'}\left(\bar{v}^{xy}_{i,j+\frac{1}{2}} - \bar{v}^{xy}_{i,j-\frac{1}{2}}\right)\right)\right)$$

$$= H\left(\frac{1}{d'}\left(\frac{1}{2}\left(\bar{u}^{y}_{i+\frac{1}{2}+\frac{1}{2},j} + \bar{u}^{y}_{i+\frac{1}{2}-\frac{1}{2},j}\right) - \frac{1}{2}\left(\bar{u}^{y}_{i-\frac{1}{2}+\frac{1}{2},j} + \bar{u}^{y}_{i-\frac{1}{2}-\frac{1}{2},j}\right)\right)\right)$$

$$+ H\left(\frac{1}{d'}\left(\frac{1}{2}\left(\bar{v}^{x}_{i,j+\frac{1}{2}+\frac{1}{2}} + \bar{v}^{x}_{i,j+\frac{1}{2}-\frac{1}{2}}\right) - \frac{1}{2}\left(\bar{v}^{x}_{i,j-\frac{1}{2}+\frac{1}{2}} + \bar{v}^{x}_{i,j-\frac{1}{2}-\frac{1}{2}}\right)\right)\right)$$

$$= H\left(\frac{1}{d'}\left(\frac{1}{2}\left(\bar{u}^{y}_{i+1,j} + \bar{u}^{y}_{i,j}\right) - \frac{1}{2}\left(\bar{u}^{y}_{i,j} + \bar{u}^{y}_{i-1,j}\right)\right)\right)$$

$$+ H\left(\frac{1}{d'}\left(\frac{1}{2}\left(\bar{v}^{x}_{i,j+1} + \bar{v}^{x}_{i,j}\right) - \frac{1}{2}\left(\bar{v}^{x}_{i,j} + \bar{v}^{x}_{i,j-1}\right)\right)\right)$$

$$= H\left(\frac{1}{2d'}\left(\left(\bar{u}^{y}_{i+1,j} - \bar{u}^{y}_{i-1,j}\right)\right)\right) + H\left(\frac{1}{2d'}\left(\left(\bar{v}^{x}_{i,j+1} - \bar{v}^{x}_{i,j-1}\right)\right)\right)$$

$$= H\left(\frac{1}{2d'}\left(\frac{1}{2}\left(u_{i+1,j+\frac{1}{2}} + u_{i+1,j-\frac{1}{2}}\right) - \frac{1}{2}\left(u_{i-1,j+\frac{1}{2}} + u_{i-1,j-\frac{1}{2}}\right)\right)\right)$$

$$+ H\left(\frac{1}{2d'}\left(\frac{1}{2}\left(v_{i+\frac{1}{2},j+1} + v_{i-\frac{1}{2},j+1}\right) - \frac{1}{2}\left(v_{i+\frac{1}{2},j-1} + v_{i-\frac{1}{2},j-1}\right)\right)\right)$$

$$= H\left(\frac{1}{4d'}\left(\left(u_{i+1,j+\frac{1}{2}} + u_{i+1,j-\frac{1}{2}}\right) - \left(u_{i-1,j+\frac{1}{2}} + u_{i-1,j-\frac{1}{2}}\right)\right)\right)$$

$$+ H\left(\frac{1}{4d'}\left(\left(v_{i+\frac{1}{2},j+1} + v_{i-\frac{1}{2},j+1}\right) - \left(v_{i+\frac{1}{2},j-1} + v_{i-\frac{1}{2},j-1}\right)\right)\right)$$

GRADE E

$$(\delta_x \alpha)_{i,j} \equiv \frac{1}{d'}\left(\alpha_{i+\frac{1}{2},j} - \alpha_{i-\frac{1}{2},j}\right)$$

$$(\bar{\alpha}^x)_{i,j} \equiv \frac{1}{2}\left(\alpha_{i+\frac{1}{2},j} + \alpha_{i-\frac{1}{2},j}\right)$$

$$\frac{\partial u}{\partial t} = -g\delta_x h + fv = -g\left(\frac{1}{d'}\left(h_{i+\frac{1}{2},j} - h_{i-\frac{1}{2},j}\right)\right) + fv_{i,j}$$

$$\frac{\partial v}{\partial t} = -g\delta_y h - fu = -g\left(\frac{1}{d'}\left(h_{i,j+\frac{1}{2}} - h_{i,j-\frac{1}{2}}\right)\right) - fu_{i,j}$$

$$\frac{\partial h}{\partial t} = -H\left(\delta_x u + \delta_y v\right) = H\left(\left(\frac{1}{d'}\left(u_{i+\frac{1}{2},j} - u_{i-\frac{1}{2},j}\right)\right) + \left(\frac{1}{d'}\left(v_{i,j+\frac{1}{2}} - v_{i,j-\frac{1}{2}}\right)\right)\right)$$
$$= H\left(\frac{1}{d'}\left(\left(u_{i+\frac{1}{2},j} - u_{i-\frac{1}{2},j} + v_{i,j+\frac{1}{2}} - v_{i,j-\frac{1}{2}}\right)\right)\right)$$

Nos podemos primeiro analisar um caso unidimensional , que na qual as variáveis u,v,h não variam com y, Assim nos temos

$$u, v, h = u(x,t), v(x,t), h(x,t)$$

A equação 3.1 se reduz a

$$\frac{\partial u}{\partial t} = -g\frac{\partial h}{\partial x} + fv$$ $$\frac{\partial v}{\partial t} = -fu$$ $$\frac{\partial h}{\partial t} = -H\frac{\partial u}{\partial x}$$	3.3

Substituindo a 1,2

$$u(x,t) = Re\left[\hat{u}e^{-i(kx-vt)}\right]$$

$$\frac{\partial u}{\partial t} = -iv\, Re\left[\hat{u}e^{-i(kx-vt)}\right]$$

$$\frac{\partial^2 u}{\partial^2 t} = (-iv)(-iv)Re\left[\hat{u}e^{-i(kx-vt)}\right]$$

$$\frac{\partial^2 u}{\partial^2 t} = -v^2 Re\left[\hat{u}e^{-i(kx-vt)}\right]$$

$$h(x,t) = Re\left[\hat{h}e^{-i(kx-vt)}\right]$$

$\frac{\partial^2 u}{\partial^2 t} = -g\frac{\partial}{\partial x}\frac{\partial h}{\partial t} + f\frac{\partial v}{\partial t}$ $\frac{\partial v}{\partial t} = -fu$ $\frac{\partial h}{\partial t} = -H\frac{\partial u}{\partial x}$	3.3

$\frac{\partial^2 u}{\partial^2 t} = -g\frac{\partial}{\partial x}\left(-H\frac{\partial u}{\partial x}\right) + f(-fu)$	3.3

$\frac{\partial^2 u}{\partial^2 t} = gH\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) - f^2 u$	3.3

$$u(x,t) = Re\left[\hat{u}e^{-i(kx-vt)}\right]$$

$$\frac{\partial u}{\partial t} = iv\, Re\left[\hat{u}e^{-i(kx-vt)}\right]$$

$$\frac{\partial^2 u}{\partial^2 t} = (iv)(iv)Re\left[\hat{u}e^{-i(kx-vt)}\right]$$

$$\frac{\partial^2 u}{\partial^2 t} = -v^2 Re\left[\hat{u}e^{-i(kx-vt)}\right]$$

$$\frac{\partial u}{\partial x} = -ik\, Re\left[\hat{u}e^{-i(kx-vt)}\right]$$

$$\frac{\partial^2 u}{\partial^2 x} = (-ik)(-ik)Re\left[\hat{u}e^{-i(kx-vt)}\right]$$

$$\frac{\partial^2 u}{\partial^2 x} = -k^2 Re\left[\hat{u}e^{-i(kx-vt)}\right]$$

$-v^2 Re\left[\hat{u}e^{-i(kx-vt)}\right] = -gHk^2 Re\left[\hat{u}e^{-i(kx-vt)}\right] - f^2 Re\left[\hat{u}e^{-i(kx-vt)}\right]$	3.3

$-v^2 = -gHk^2 - f^2$	3.3

$v^2 = gHk^2 + f^2$	3.3
$\left(\frac{v}{f}\right)^2 = \frac{gH}{f^2}k^2 + 1$	3.3

$\left(\frac{v}{f}\right)^2 = \frac{\lambda^2}{f}k^2 + 1$	3.3

$$\lambda = \frac{\sqrt{gH}}{f}$$