Derivadas de 1 orden nos espaço?
Erro nas discretizações das Derivadas espaciais

Faça a Expansão de Taylor para 1 ponto de grade

Métodos de diferenças finitas.

$$u_{j+1}^{n} = u_{j}^{n} + \Delta x \frac{\partial u}{\partial x} + \frac{\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} + \frac{\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (1B)$$

$$u_{j-1}^{n} = u_{j}^{n} - \Delta x \frac{\partial u}{\partial x} + \frac{\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} - \frac{\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (2B)$$

A partir da eq 1c e 2C, obtém-se Discretização de 1 ordem o erro

$$\frac{u_{j}^{n} - u_{j-1}^{n}}{\Delta x} = \frac{\partial u}{\partial x} - \frac{\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} - \frac{\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (BB)$$

$$\frac{u_{j+1}^{n} - u_{j}^{n}}{\Delta x} = \frac{\partial u}{\partial x} + \frac{\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (BB)$$

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Métodos de diferenças finitas.

$$\begin{cases} u_{j+1}^{n} = u_{j}^{n} + \Delta x \dfrac{\partial u}{\partial x} + \dfrac{\Delta x^2}{2}\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\Delta x^3}{6}\dfrac{\partial^3 u}{\partial x^3} + \dfrac{\Delta x^4}{24}\dfrac{\partial^4 u}{\partial x^4} \dots . & (1B) \\ u_{j-1}^{n} = u_{j}^{n} - \Delta x \dfrac{\partial u}{\partial x} + \dfrac{\Delta x^2}{2}\dfrac{\partial^2 u}{\partial x^2} - \dfrac{\Delta x^3}{6}\dfrac{\partial^3 u}{\partial x^3} + \dfrac{\Delta x^4}{24}\dfrac{\partial^4 u}{\partial x^4} \dots . & (2B) \end{cases}$$

Subtrai 1c-2C , obtem-se a Discretização de 2 ordem no erro

$$u_{j+1} - u_{j-1} = 2\Delta x \frac{\partial u}{\partial x} + \frac{2}{3!}\frac{\partial^3 u}{\partial x^3}(\Delta x)^3 + O[(\Delta x)^4] \quad (3B)$$

$$\frac{u_{j+1} - u_{j-1}}{2\Delta x} = \frac{\partial u}{\partial x} + \frac{1}{3!}\frac{\partial^3 u}{\partial x^3}(\Delta x)^2 + O[(\Delta x)^4] \quad (4B)$$

Métodos de diferenças finitas.

$$u_{j+1}^{n} = u_{j}^{n} + \Delta x\frac{\partial u}{\partial x} + \frac{\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} + \frac{\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4}\dots. \qquad (1B)$$

$$u_{j-1}^{n} = u_{j}^{n} - \Delta x\frac{\partial u}{\partial x} + \frac{\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} - \frac{\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4}\dots. \qquad (2B)$$

soma 1C+2C, , obtém-se a Discretização de 2 ordem na derivada e 2 orden no erro

$$u_{j+1} + u_{j-1} = 2u_j + \frac{\partial^2 u}{\partial x^2}\Delta x^2 + \frac{2}{4!}\frac{\partial^4 u}{\partial x^4}(\Delta x)^4 + O[(\Delta x)^4] \qquad (5B)$$

$$\frac{u_{j+1} - 2u_j + u_{j-1}}{\Delta x^2} = +\frac{\partial^2 u}{\partial x^2} + \frac{2}{4!}\frac{\partial^4 u}{\partial x^4}(\Delta x)^2 + O[(\Delta x)^4] \qquad (6B)$$

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Faça a Expansão de Taylor para 2 ponto de grade

Métodos de diferenças finitas.

$$\begin{cases} u_{j+2}^{n} = u_j^n + 2\Delta x\dfrac{\partial u}{\partial x} + \dfrac{4\Delta x^2}{2}\dfrac{\partial^2 u}{\partial x^2} + \dfrac{8\Delta x^3}{6}\dfrac{\partial^3 u}{\partial x^3} + \dfrac{16\Delta x^4}{24}\dfrac{\partial^4 u}{\partial x^4} \dots . & (1C) \\ u_{j-2}^{n} = u_j^n - 2\Delta x\dfrac{\partial u}{\partial x} + \dfrac{4\Delta x^2}{2}\dfrac{\partial^2 u}{\partial x^2} - \dfrac{8\Delta x^3}{6}\dfrac{\partial^3 u}{\partial x^3} + \dfrac{16\Delta x^4}{24}\dfrac{\partial^4 u}{\partial x^4} \dots . & (2C) \end{cases}$$

A partir da eq 1c e 2C, obtém-se Discretização de 1 ordem o erro

$$\frac{u_{j+2}^{n} - u_j^n}{2\Delta x} = \frac{\partial u}{\partial x} + \frac{2\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{6\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} + \frac{14\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (1C)$$

$$\frac{(u_j^n - u_{j-2}^{n})}{2\Delta x} = \frac{\partial u}{\partial x} - \frac{2\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{6\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} - \frac{8\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (CC)$$

Métodos de diferenças finitas.

$$u_{j+2}^{n} = u_j^n + 2\Delta x \frac{\partial u}{\partial x} + \frac{4\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} + \frac{8\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{16\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (1C)$$

$$u_{j-2}^{n} = u_j^n - 2\Delta x \frac{\partial u}{\partial x} + \frac{4\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} - \frac{8\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{16\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (2C)$$

Subtrai 1c-2C , obtem-se a Discretização de 2 ordem no erro

$$u_{j+2} - u_{j-2} = 4\Delta x \frac{\partial u}{\partial x} + \frac{16}{3!}\frac{\partial^3 u}{\partial x^3}(\Delta x)^3 + O[(\Delta x)^4] \qquad (3C)$$

$$\frac{u_{j+2} - u_{j-2}}{4\Delta x} = \frac{\partial u}{\partial x} + \frac{4}{3!}\frac{\partial^3 u}{\partial x^3}(\Delta x)^2 + O[(\Delta x)^4] \qquad (4C)$$

Métodos de diferenças finitas.

$$
\begin{cases}
u_{j+2}^{n} = u_j^n + 2\Delta x\dfrac{\partial u}{\partial x} + \dfrac{4\Delta x^2}{2}\dfrac{\partial^2 u}{\partial x^2} + \dfrac{8\Delta x^3}{6}\dfrac{\partial^3 u}{\partial x^3} + \dfrac{16\Delta x^4}{24}\dfrac{\partial^4 u}{\partial x^4}\dots. & (1C) \\
u_{j-2}^{n} = u_j^n - 2\Delta x\dfrac{\partial u}{\partial x} + \dfrac{4\Delta x^2}{2}\dfrac{\partial^2 u}{\partial x^2} - \dfrac{8\Delta x^3}{6}\dfrac{\partial^3 u}{\partial x^3} + \dfrac{16\Delta x^4}{24}\dfrac{\partial^4 u}{\partial x^4}\dots. & (2C)
\end{cases}
$$

soma 1C+2C, , obtém-se a Discretização de 2 ordem na derivada e 2 orden no erro

$$u_{j+2} + u_{j-2} = 2u_j + \frac{8\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} + \frac{32}{4!}\frac{\partial^4 u}{\partial x^4}(\Delta x)^4 + O[(\Delta x)^4] \qquad (5C)$$

$$\frac{u_{j+2} - 2u_j + u_{j-2}}{8\Delta x^2} = +\frac{1}{2}\frac{\partial^2 u}{\partial x^2} + \frac{4}{4!}\frac{\partial^4 u}{\partial x^4}(\Delta x)^2 + O[(\Delta x)^4] \qquad (6C)$$

$$\frac{u_{j+2} - 2u_j + u_{j-2}}{4\Delta x^2} = \frac{\partial^2 u}{\partial x^2} + \frac{4}{4!}\frac{\partial^4 u}{\partial x^4}(\Delta x)^2 + O[(\Delta x)^4] \qquad (6C)$$

Paulo Yoshio Kubota

Combinação das a Expansão de Taylor para1 e 2 ponto de grade

segunda ordem

Métodos de diferenças finitas.

Esquemas com diferenças centradas no espaço de segunda ordem:

$$u_{j+1}^{n} = u_{j}^{n} + \Delta x\frac{\partial u}{\partial x} + \frac{\Delta x^{2}}{2}\frac{\partial^{2} u}{\partial x^{2}} + \frac{\Delta x^{3}}{6}\frac{\partial^{3} u}{\partial x^{3}} + \frac{\Delta x^{4}}{24}\frac{\partial^{4} u}{\partial x^{4}}\ldots. \quad (1B)$$

$$u_{j-1}^{n} = u_{j}^{n} - \Delta x\frac{\partial u}{\partial x} + \frac{\Delta x^{2}}{2}\frac{\partial^{2} u}{\partial x^{2}} - \frac{\Delta x^{3}}{6}\frac{\partial^{3} u}{\partial x^{3}} + \frac{\Delta x^{4}}{24}\frac{\partial^{4} u}{\partial x^{4}}\ldots. \quad (2B)$$

$$u_{j+2}^{n} = u_{j}^{n} + 2\Delta x\frac{\partial u}{\partial x} + \frac{4\Delta x^{2}}{2}\frac{\partial^{2} u}{\partial x^{2}} + \frac{8\Delta x^{3}}{6}\frac{\partial^{3} u}{\partial x^{3}} + \frac{16\Delta x^{4}}{24}\frac{\partial^{4} u}{\partial x^{4}}\ldots. \quad (1C)$$

$$u_{j-2}^{n} = u_{j}^{n} - 2\Delta x\frac{\partial u}{\partial x} + \frac{4\Delta x^{2}}{2}\frac{\partial^{2} u}{\partial x^{2}} - \frac{8\Delta x^{3}}{6}\frac{\partial^{3} u}{\partial x^{3}} + \frac{16\Delta x^{4}}{24}\frac{\partial^{4} u}{\partial x^{4}}\ldots. \quad (2C)$$

Paulo Yoshio Kubota

Métodos de diferenças finitas.

Esquemas com diferenças centradas no espaço de segunda ordem:

$$u_{j-1}^{n} = u_j^n - \Delta x \frac{\partial u}{\partial x} + \frac{\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} - \frac{\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \ldots. \qquad (2B)$$

$$u_{j-2}^{n} = u_j^n - 2\Delta x \frac{\partial u}{\partial x} + \frac{4\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} - \frac{8\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{16\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \ldots. \qquad (2C)$$

Rearranje os termos das equações

$$\frac{u_j^n - u_{j-1}^n}{\Delta x} = \frac{\partial u}{\partial x} - \frac{\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} - \frac{\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \ldots. \qquad (BB)$$

$$\frac{(u_j^n - u_{j-2}^n)}{2\Delta x} = \frac{\partial u}{\partial x} - \frac{2\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{4\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} - \frac{8}{24}\frac{\partial^4 u}{\partial x^4} \ldots. \qquad (CC)$$

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Esquemas com diferenças centradas no espaço de segunda ordem:

$$\frac{u_j^n - u_{j-1}^n}{\Delta x} = \frac{\partial u}{\partial x} - \frac{\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} - \frac{\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \dots. \qquad (BB)$$

$$\frac{\left(u_j^n - u_{j-2}^n\right)}{2\Delta x} = \frac{\partial u}{\partial x} - \frac{2\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{4\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} - \frac{8\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \dots. \qquad (CC)$$

Mutiplica a BB por 2

$$2\frac{u_j - u_{j-1}}{\Delta x} = 2\frac{\partial u}{\partial x} + 2\frac{\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{2\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} - \frac{2\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} + O[(\Delta x)^4] \qquad (AD)$$

Soma a AD a eq. CC

$$2\frac{u_j - u_{j-1}}{\Delta x} + \frac{u_j - u_{j-2}}{2\Delta x} = 3\frac{\partial u}{\partial x} + \frac{6\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} \dots..$$

$$\frac{2}{3}\frac{u_j - u_{j-1}}{\Delta x} - \frac{1}{3}\frac{u_j - u_{j-2}}{2\Delta x} = \frac{\partial u}{\partial x} + \Delta x^2\frac{\partial^3 u}{\partial x^3} \dots.. \qquad (2D)$$

$$\left(\frac{-u_j + 2u_j - 2u_{j-1} + u_{j-2}}{6\Delta x}\right) = \frac{\partial u}{\partial x} + \Delta x^2\frac{\partial^3 u}{\partial x^3} \dots..(2D)$$

Paulo Yoshio Kubota

Combinação das a Expansão de Taylor para1 e 2 ponto de grade

2 opção segunda ordem

Métodos de diferenças finitas.

Esquemas com diferenças centradas no espaço de segunda ordem:

$$u_{j+1}^{n} = u_{j}^{n} + \Delta x \frac{\partial u}{\partial x} + \frac{\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} + \frac{\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (1B)$$

$$u_{j-1}^{n} = u_{j}^{n} - \Delta x \frac{\partial u}{\partial x} + \frac{\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} - \frac{\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (2B)$$

$$u_{j+2}^{n} = u_{j}^{n} + 2\Delta x \frac{\partial u}{\partial x} + \frac{4\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} + \frac{8\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{16\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (1C)$$

$$u_{j-2}^{n} = u_{j}^{n} - 2\Delta x \frac{\partial u}{\partial x} + \frac{4\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} - \frac{8\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{16\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (2C)$$

Paulo Yoshio Kubota

Métodos de diferenças finitas.

Esquemas com diferenças centradas no espaço de segunda ordem:

$$u_{j-1}^{n} = u_j^n - \Delta x \frac{\partial u}{\partial x} + \frac{\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} - \frac{\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (2B)$$

$$u_{j+2}^{n} = u_j^n + 2\Delta x \frac{\partial u}{\partial x} + \frac{4\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} + \frac{8\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{16\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (1C)$$

$$\frac{u_{j-1}^{n} - u_j^n}{\Delta x} = -\frac{\partial u}{\partial x} + \frac{\Delta x}{2}\frac{\partial^2 u}{\partial x^2} - \frac{\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (BB)$$

$$\frac{\left(u_{j+2}^{n} - u_j^n\right)}{2\Delta x} = \frac{\partial u}{\partial x} + \frac{2\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{4\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} - \frac{8\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (CC)$$

Métodos de diferenças finitas.

Esquemas com diferenças centradas no espaço de segunda ordem:

$$\frac{u_{j-1}^n - u_j^n}{\Delta x} = -\frac{\partial u}{\partial x} + \frac{\Delta x}{2}\frac{\partial^2 u}{\partial x^2} - \frac{\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \ldots. \qquad (BB)$$

$$\frac{\left(u_{j+2}^n - u_j^n\right)}{2\Delta x} = \frac{\partial u}{\partial x} + \frac{2\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{4\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} - \frac{8\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \ldots. \qquad (CC)$$

Mutiplica a BB por 2

$$2\frac{u_{j-1} - u_j}{\Delta x} = -2\frac{\partial u}{\partial x} + 2\frac{\Delta x}{2}\frac{\partial^2 u}{\partial x^2} - \frac{2\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} + \frac{2\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} + O[(\Delta x)^4] \qquad (AD)$$

subtrai a AD a eq. CC

$$2\frac{u_{j-1} - u_j}{\Delta x} - \frac{u_{j+2} - u_j}{2\Delta x} = -3\frac{\partial u}{\partial x} - \frac{6\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} \ldots..$$

$$\frac{2u_{j-1} - 2u_j}{3\Delta x} - \frac{u_{j+2} - u_j}{6\Delta x} = \frac{\partial u}{\partial x} - \Delta x^2\frac{\partial^3 u}{\partial x^3} \quad \ldots.. \qquad (2D)$$

$$\left(\frac{4u_{j-1} - 4u_j - u_{j+2} + u_j}{6\Delta x}\right) = \frac{\partial u}{\partial x} + \frac{\Delta x^2}{3}\frac{\partial^3 u}{\partial x^3} \quad \ldots..(2D)$$

Paulo Yoshio Kubota

Esquemas com diferenças centradas no espaço de segunda ordem:

$$\left(\frac{4u_{j-1} - 4u_j - u_{j+2} + u_j}{6\Delta x}\right) = \frac{\partial u}{\partial x} + \frac{\Delta x^2}{3}\frac{\partial^3 u}{\partial x^3} \quad \ldots\ldots (2D)$$

$$\left(\frac{4u_{j-1} - 4u_j - u_{j+2} + u_j}{6\Delta x}\right) = \frac{\partial u}{\partial x} + \frac{\Delta x^2}{3}\frac{\partial^3 u}{\partial x^3} \quad \ldots\ldots (2D)$$

$$\left(\frac{4u_{j-1} - 3u_j - u_{j+2}}{6\Delta x}\right) = \frac{\partial u}{\partial x} + \frac{\Delta x^2}{3}\frac{\partial^3 u}{\partial x^3} \quad \ldots\ldots (2D)$$

Combinação das a Expansão de Taylor para1 e 2 ponto de grade

segunda ordem

Avançado

Métodos de diferenças finitas.

Esquemas com diferenças centradas no espaço de segunda ordem:

$$u_{j+1}^{n} = u_j^n + \Delta x \frac{\partial u}{\partial x} + \frac{\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} + \frac{\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots. \qquad (1B)$$

$$u_{j+2}^{n} = u_j^n + 2\Delta x \frac{\partial u}{\partial x} + \frac{4\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} + \frac{8\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{16\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots. \qquad (1C)$$

$$\frac{u_{j+1}^{n} - u_j^n}{\Delta x} = \frac{\partial u}{\partial x} + \frac{\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \dots. \qquad (BB)$$

$$\frac{\left(u_{j+2}^{n} - u_j^n\right)}{2\Delta x} = \frac{\partial u}{\partial x} + \frac{2\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{4\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} + \frac{8\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \dots. \qquad (CC)$$

Métodos de diferenças finitas.

Esquemas com diferenças centradas no espaço de segunda ordem:

$$\frac{\left(u_{j+2}^{n}-u_{j}^{n}\right)}{2\Delta x}=\frac{\partial u}{\partial x}+\frac{2\Delta x}{2}\frac{\partial^{2}u}{\partial x^{2}}+\frac{4\Delta x^{2}}{6}\frac{\partial^{3}u}{\partial x^{3}}+\frac{8\Delta x^{3}}{24}\frac{\partial^{4}u}{\partial x^{4}}\dots. \qquad (CC)$$

$$\frac{u_{j}^{n}-u_{j-1}^{n}}{\Delta x}=-\frac{\partial u}{\partial x}+\frac{\Delta x}{2}\frac{\partial^{2}u}{\partial x^{2}}-\frac{\Delta x^{2}}{6}\frac{\partial^{3}u}{\partial x^{3}}+\frac{\Delta x^{3}}{24}\frac{\partial^{4}u}{\partial x^{4}}\dots. \qquad (BB)$$

Multiplica por 4

$$4\frac{u_{j}^{n}-u_{j-1}^{n}}{\Delta x}=-4\frac{\partial u}{\partial x}+\frac{4\Delta x}{2}\frac{\partial^{2}u}{\partial x^{2}}-\frac{4\Delta x^{2}}{6}\frac{\partial^{3}u}{\partial x^{3}}+\frac{4\Delta x^{3}}{24}\frac{\partial^{4}u}{\partial x^{4}}\dots. \qquad (DD)$$

Subtra DD da CC

$$4\frac{u_{j}^{n}-u_{j-1}^{n}}{\Delta x}-\frac{\left(u_{j+2}^{n}-u_{j}^{n}\right)}{2\Delta x}=-5\frac{\partial u}{\partial x}+\frac{2\Delta x}{2}\frac{\partial^{2}u}{\partial x^{2}}-\frac{8\Delta x^{2}}{6}\frac{\partial^{3}u}{\partial x^{3}}-\frac{4\Delta x^{3}}{24}\frac{\partial^{4}u}{\partial x^{4}}\dots \qquad (dd)$$

$$4\frac{u_{j}^{n}-u_{j-1}^{n}}{\Delta x}-\frac{\left(u_{j+2}^{n}-u_{j}^{n}\right)}{2\Delta x}-\Delta x\frac{\partial^{2}u}{\partial x^{2}}=-5\frac{\partial u}{\partial x}-\frac{8\Delta x^{2}}{6}\frac{\partial^{3}u}{\partial x^{3}}-\frac{4\Delta x^{3}}{24}\frac{\partial^{4}u}{\partial x^{4}}\dots. \qquad (dd)$$

Métodos de diferenças finitas.

Esquemas com diferenças centradas no espaço de segunda ordem:

$$4\frac{u_j^n-u_{j-1}^n}{\Delta x}-\frac{\left(u_{j+2}^n-u_j^n\right)}{2\Delta x}-\Delta x\frac{\partial^2 u}{\partial x^2}=-5\frac{\partial u}{\partial x}-\frac{8\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3}-\frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}\dots. \qquad (dd)$$

$$4\frac{u_j^n-u_{j-1}^n}{5\Delta x}-\frac{\left(u_{j+2}^n-u_j^n\right)}{10\Delta x}-\frac{\Delta x}{5}\frac{\partial^2 u}{\partial x^2}=-\frac{\partial u}{\partial x}-\frac{8\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3}-\frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}\dots. \qquad (dd)$$

Sabe-se que $\frac{\partial^2 u}{\partial x^2}\cong\frac{u_{j+2}-2u_j+u_{j-2}}{4\Delta x^2}$

$$\frac{u_{j+2}-2u_j+u_{j-2}}{4\Delta x^2}=\frac{\partial^2 u}{\partial x^2}+\frac{4}{4!}\frac{\partial^4 u}{\partial x^4}(\Delta x)^2+O[(\Delta x)^4] \qquad (6C)$$

$$4\frac{u_j^n-u_{j-1}^n}{5\Delta x}-\frac{\left(u_{j+2}^n-u_j^n\right)}{10\Delta x}-\frac{\Delta x}{5}\frac{u_{j+2}-2u_j+u_{j-2}}{4\Delta x^2}=-\frac{\partial u}{\partial x}-\frac{8\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3}-\frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}$$

$$4\frac{u_j^n-u_{j-1}^n}{5\Delta x}-\frac{\left(u_{j+2}^n-u_j^n\right)}{10\Delta x}-\frac{u_{j+2}-2u_j+u_{j-2}}{20\Delta x}=-\frac{\partial u}{\partial x}-\frac{8\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3}-\frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}$$

Métodos de diferenças finitas.

Esquemas com diferenças centradas no espaço de segunda ordem:

$$4\frac{u_j^n - u_{j-1}^n}{5\Delta x} - \frac{\left(u_{j+2}^n - u_j^n\right)}{10\Delta x} - \frac{u_{j+2} - 2u_j + u_{j-2}}{20\Delta x} = -\frac{\partial u}{\partial x} - \frac{8\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} - \frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}$$

$$\frac{4u_j^n - 4u_{j-1}^n}{5\Delta x} - \frac{\left(u_{j+2}^n - u_j^n\right)}{10\Delta x} - \frac{u_{j+2} - 2u_j + u_{j-2}}{20\Delta x} = -\frac{\partial u}{\partial x} - \frac{8\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} - \frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}$$

$$\frac{8u_j^n - 8u_{j-1}^n - u_{j+2}^n + u_j^n}{10\Delta x} - \frac{u_{j+2} - 2u_j + u_{j-2}}{20\Delta x} = -\frac{\partial u}{\partial x} - \frac{8\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} - \frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}$$

$$-\frac{8u_j^n - 8u_{j-1}^n - u_{j+2}^n + u_j^n}{10\Delta x} + \frac{u_{j+2} - 2u_j + u_{j-2}}{20\Delta x} = \frac{\partial u}{\partial x} - O[\Delta x^2]$$

Métodos de diferenças finitas.

Esquemas com diferenças centradas no espaço de segunda ordem:

$$-\frac{8u_j^n - 8u_{j-1}^n - u_{j+2}^n + u_j^n}{10\Delta x} + \frac{u_{j+2} - 2u_j + u_{j-2}}{20\Delta x} = \frac{\partial u}{\partial x} - O[\Delta x^2]$$

$$\frac{\partial u}{\partial x} = \frac{u_{j+2}^n - 9u_j^n + 8u_{j-1}^n}{10\Delta x} + \frac{u_{j+2} - 2u_j + u_{j-2}}{20\Delta x} + O[\Delta x^2]$$

Combinação das a Expansão de Taylor para1 e 2 ponto de grade

terceira ordem

Avançado

Métodos de diferenças finitas.

Esquemas com diferenças centradas no espaço de terceira ordem:

$$u_{j+1}^n = u_j^n + \Delta x \frac{\partial u}{\partial x} + \frac{\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} + \frac{\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots. \qquad (1B)$$

$$u_{j+2}^n = u_j^n + 2\Delta x \frac{\partial u}{\partial x} + \frac{4\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} + \frac{8\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{16\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots. \qquad (1C)$$

$$\frac{u_{j+1}^n - u_j^n}{\Delta x} = \frac{\partial u}{\partial x} + \frac{\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \dots. \qquad (BB)$$

$$\frac{\left(u_{j+2}^n - u_j^n\right)}{2\Delta x} = \frac{\partial u}{\partial x} + \frac{2\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{4\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} + \frac{8\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \dots. \qquad (CC)$$

Métodos de diferenças finitas.

Esquemas com diferenças centradas no espaço de terceira ordem:

$$\frac{(u_{j+2}^n - u_j^n)}{2\Delta x} = \frac{\partial u}{\partial x} + \frac{2\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{4\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} + \frac{8\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}\dots. \qquad (CC)$$

$$\frac{u_j^n - u_{j-1}^n}{\Delta x} = -\frac{\partial u}{\partial x} + \frac{\Delta x}{2}\frac{\partial^2 u}{\partial x^2} - \frac{\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}\dots. \qquad (BB)$$

Multiplica por 4

$$4\frac{u_j^n - u_{j-1}^n}{\Delta x} = -4\frac{\partial u}{\partial x} + \frac{4\Delta x}{2}\frac{\partial^2 u}{\partial x^2} - \frac{4\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} + \frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}\dots. \qquad (DD)$$

soma DD da CC

$$4\frac{u_j^n - u_{j-1}^n}{\Delta x} + \frac{(u_{j+2}^n - u_j^n)}{2\Delta x} = -3\frac{\partial u}{\partial x} + \frac{6\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{12\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}\ \dots \qquad (dd)$$

$$4\frac{u_j^n - u_{j-1}^n}{\Delta x} + \frac{(u_{j+2}^n - u_j^n)}{2\Delta x} - \frac{6\Delta x}{2}\frac{\partial^2 u}{\partial x^2} = -3\frac{\partial u}{\partial x} + \frac{12\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}\dots. \qquad (dd)$$

Métodos de diferenças finitas.

Esquemas com diferenças centradas no espaço de terceira ordem:

$$4\frac{u_j^n - u_{j-1}^n}{\Delta x} + \frac{(u_{j+2}^n - u_j^n)}{2\Delta x} - \frac{6\Delta x}{2}\frac{\partial^2 u}{\partial x^2} = -3\frac{\partial u}{\partial x} + \frac{12\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}\dots. \qquad (dd)$$

$$4\frac{u_j^n - u_{j-1}^n}{3\Delta x} + \frac{(u_{j+2}^n - u_j^n)}{6\Delta x} - \frac{6\Delta x}{6}\frac{\partial^2 u}{\partial x^2} = -\frac{\partial u}{\partial x} + \frac{12\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}\dots. \qquad (dd)$$

Sabe-se que $\frac{\partial^2 u}{\partial x^2} \cong \frac{u_{j+2} - 2u_j + u_{j-2}}{4\Delta x^2}$

$$\frac{u_{j+2} - 2u_j + u_{j-2}}{4\Delta x^2} = \frac{\partial^2 u}{\partial x^2} + \frac{4}{4!}\frac{\partial^4 u}{\partial x^4}(\Delta x)^2 + O[(\Delta x)^4] \qquad (6C)$$

$$4\frac{u_j^n - u_{j-1}^n}{3\Delta x} + \frac{(u_{j+2}^n - u_j^n)}{6\Delta x} - \frac{\Delta x}{1}\frac{u_{j+2} - 2u_j + u_{j-2}}{4\Delta x^2} = -\frac{\partial u}{\partial x} + \frac{12\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}$$

$$4\frac{u_j^n - u_{j-1}^n}{3\Delta x} + \frac{(u_{j+2}^n - u_j^n)}{6\Delta x} - \frac{u_{j+2} - 2u_j + u_{j-2}}{4\Delta x} = -\frac{\partial u}{\partial x} + \frac{12\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}$$

Paulo Yoshio Kubota

Métodos de diferenças finitas.

Esquemas com diferenças centradas no espaço de terceira ordem:

$$4\frac{u_j^n - u_{j-1}^n}{3\Delta x} + \frac{\left(u_{j+2}^n - u_j^n\right)}{6\Delta x} - \frac{u_{j+2} - 2u_j + u_{j-2}}{4\Delta x} = -\frac{\partial u}{\partial x} + \frac{12\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}$$

$$\frac{4u_j^n - 4u_{j-1}^n}{3\Delta x} + \frac{\left(u_{j+2}^n - u_j^n\right)}{6\Delta x} - \frac{u_{j+2} - 2u_j + u_{j-2}}{4\Delta x} = -\frac{\partial u}{\partial x} + \frac{12\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}$$

$$\frac{8u_j^n - 8u_{j-1}^n + u_{j+2}^n - u_j^n}{6\Delta x} - \frac{u_{j+2} - 2u_j + u_{j-2}}{4\Delta x} = -\frac{\partial u}{\partial x} + \frac{12\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}$$

$$\frac{7u_j^n - 8u_{j-1}^n + u_{j+2}^n}{6\Delta x} - \frac{u_{j+2} - 2u_j + u_{j-2}}{4\Delta x} = -\frac{\partial u}{\partial x} + \frac{12\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}$$

$$\frac{\partial u}{\partial x} = -\frac{9u_j^n - 8u_{j-1}^n - u_{j+2}^n}{6\Delta x} + \frac{u_{j+2} - 2u_j + u_{j-2}}{4\Delta x} + 0[\Delta x^3]$$

Métodos de diferenças finitas.

Esquemas com diferenças centradas no espaço de terceira ordem:

$$\frac{\partial u}{\partial x} = -\frac{9u_j^n - 8u_{j-1}^n - u_{j+2}^n}{6\Delta x} + \frac{u_{j+2} - 2u_j + u_{j-2}}{4\Delta x} + 0[\Delta x^3]$$

$$\frac{\partial u}{\partial x} = \frac{u_{j+2}^n - 9u_j^n + 8u_{j-1}^n}{6\Delta x} + \frac{u_{j+2} - 2u_j + u_{j-2}}{4\Delta x} + 0[\Delta x^3]$$

Combinação das a Expansão de Taylor para1 e 2 ponto de grade

3 opção terceira ordem

Avançado

Métodos de diferenças finitas.

Esquemas com diferenças centradas no espaço de terceira ordem:

$$u_{j+1}^{n} = u_j^n + \Delta x \frac{\partial u}{\partial x} + \frac{\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} + \frac{\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots. \qquad (1B)$$

$$u_{j-1}^{n} = u_j^n - \Delta x \frac{\partial u}{\partial x} + \frac{\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} - \frac{\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots. \qquad (2B)$$

$$u_{j+2}^{n} = u_j^n + 2\Delta x \frac{\partial u}{\partial x} + \frac{4\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} + \frac{8\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{16\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots. \qquad (1C)$$

$$u_{j-2}^{n} = u_j^n - 2\Delta x \frac{\partial u}{\partial x} + \frac{4\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} - \frac{8\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{16\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots. \qquad (2C)$$

Métodos de diferenças finitas.

Esquemas com diferenças centradas no espaço de terceira ordem:

$$u_{j-1}^{n} = u_j^n - \Delta x \frac{\partial u}{\partial x} + \frac{\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} - \frac{\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (2B)$$

$$u_{j+2}^{n} = u_j^n + 2\Delta x \frac{\partial u}{\partial x} + \frac{4\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} + \frac{8\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{16\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (1C)$$

$$\frac{u_{j-1}^{n} - u_j^n}{\Delta x} = -\frac{\partial u}{\partial x} + \frac{\Delta x}{2}\frac{\partial^2 u}{\partial x^2} - \frac{\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (BB)$$

$$\frac{\left(u_{j+2}^{n} - u_j^n\right)}{2\Delta x} = \frac{\partial u}{\partial x} + \frac{2\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{4\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} - \frac{8\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (CC)$$

Paulo Yoshio Kubota

Métodos de diferenças finitas.

Esquemas com diferenças centradas no espaço de terceira ordem:

$$\frac{u_{j-1}^n - u_j^n}{\Delta x} = -\frac{\partial u}{\partial x} + \frac{\Delta x}{2}\frac{\partial^2 u}{\partial x^2} - \frac{\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \ldots. \qquad (BB)$$

$$\frac{(u_{j+2}^n - u_j^n)}{2\Delta x} = \frac{\partial u}{\partial x} + \frac{2\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{4\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} - \frac{8\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \ldots. \qquad (CC)$$

Mutiplica a BB por 4

$$4\frac{u_{j-1} - u_j}{\Delta x} = -4\frac{\partial u}{\partial x} + 4\frac{\Delta x}{2}\frac{\partial^2 u}{\partial x^2} - \frac{4\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} + \frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} + O[(\Delta x)^4] \qquad (AD)$$

soma a AD a eq. CC

$$4\frac{u_{j-1} - u_j}{\Delta x} + \frac{u_{j+2} - u_j}{2\Delta x} = -3\frac{\partial u}{\partial x} + \frac{6\Delta x}{2}\frac{\partial^2 u}{\partial x^2} - \frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \ldots..$$

$$\frac{4u_{j-1} - 4u_j}{\Delta x} + \frac{u_{j+2} - u_j}{\Delta x} - \frac{6\Delta x}{2}\frac{\partial^2 u}{\partial x^2} = -3\frac{\partial u}{\partial x} - \frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \ldots.. \qquad (2D)$$

Paulo Yoshio Kubota

$$\frac{4u_{j-1}-4u_j}{\Delta x}+\frac{u_{j+2}-u_j}{\Delta x}-\frac{6\Delta x}{2}\frac{\partial^2 u}{\partial x^2}=-3\frac{\partial u}{\partial x}-\frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}\ldots.. \qquad (2D)$$

Sabe-se que $\frac{\partial^2 u}{\partial x^2}\cong\frac{u_{j+2}-2u_j+u_{j-2}}{4\Delta x^2}$

$$\frac{u_{j+2}-2u_j+u_{j-2}}{4\Delta x^2}=\frac{\partial^2 u}{\partial x^2}+\frac{4}{4!}\frac{\partial^4 u}{\partial x^4}(\Delta x)^2+O[(\Delta x)^4] \qquad (6C)$$

$$\frac{4u_{j-1}-4u_j}{\Delta x}+\frac{u_{j+2}-u_j}{\Delta x}-\frac{6\Delta x}{2}\left(\frac{u_{j+2}-2u_j+u_{j-2}}{4\Delta x^2}\right)=-3\frac{\partial u}{\partial x}-\frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}\ldots.. \qquad (2D)$$

$$\frac{4u_{j-1}-4u_j}{\Delta x}+\frac{u_{j+2}-u_j}{\Delta x}-\left(\frac{6u_{j+2}-12u_j+6u_{j-2}}{8\Delta x}\right)=-3\frac{\partial u}{\partial x}-\frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}\ldots.. \qquad (2D)$$

Métodos de diferenças finitas.

$$\frac{4u_{j-1}-4u_j}{\Delta x}+\frac{u_{j+2}-u_j}{\Delta x}-\frac{6u_{j+2}-12u_j+6u_{j-2}}{8\Delta x}=-3\frac{\partial u}{\partial x}-\frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}\ldots.. \qquad (2D)$$

$$\frac{4u_{j-1}-4u_j}{3\Delta x}+\frac{u_{j+2}-u_j}{3\Delta x}-\frac{2u_{j+2}-4u_j+2u_{j-2}}{8\Delta x}=-\frac{\partial u}{\partial x}-O[\Delta x^3]\ldots.. \qquad (2D)$$

$$-\frac{4u_{j-1}-4u_j}{3\Delta x}-\frac{u_{j+2}-u_j}{3\Delta x}+\frac{2u_{j+2}-4u_j+2u_{j-2}}{8\Delta x}=\frac{\partial u}{\partial x}+O[\Delta x^3]\ldots.. \qquad (2D)$$

$$\frac{-4u_{j-1}+4u_j-u_{j+2}+u_j}{3\Delta x}+\frac{u_{j+2}-2u_j+u_{j-2}}{4\Delta x}=\frac{\partial u}{\partial x}+O[\Delta x^3]\ldots.. \qquad (2D)$$

Métodos de diferenças finitas.

$$\frac{-4u_{j-1} + 4u_j - u_{j+2} + u_j}{3\Delta x} + \frac{u_{j+2} - 2u_j + u_{j-2}}{4\Delta x} = \frac{\partial u}{\partial x} + O[\Delta x^3] \ldots.. \qquad (2D)$$

$$\frac{\partial u}{\partial x} = \frac{u_{j+2} - 2u_j + u_{j-2}}{4\Delta x} + \frac{-4u_{j-1} + 4u_j - u_{j+2} + u_j}{3\Delta x} - O[\Delta x^3] \ldots.. \qquad (2D)$$

Métodos de diferenças finitas.

Esquemas com diferenças centradas no espaço de terceira ordem:

$$u_{j+1}^{n} = u_j^n + \Delta x \frac{\partial u}{\partial x} + \frac{\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} + \frac{\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots. \qquad (1B)$$

$$u_{j-1}^{n} = u_j^n - \Delta x \frac{\partial u}{\partial x} + \frac{\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} - \frac{\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots. \qquad (2B)$$

$$u_{j+2}^{n} = u_j^n + 2\Delta x \frac{\partial u}{\partial x} + \frac{4\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} + \frac{8\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{16\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots. \qquad (1C)$$

$$u_{j-2}^{n} = u_j^n - 2\Delta x \frac{\partial u}{\partial x} + \frac{4\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} - \frac{8\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{16\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots. \qquad (2C)$$

Paulo Yoshio Kubota

Métodos de diferenças finitas.

Esquemas com diferenças centradas no espaço de terceira ordem:

$$u_{j-1}^n = u_j^n - \Delta x \frac{\partial u}{\partial x} + \frac{\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} - \frac{\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots. \qquad (2B)$$

$$u_{j+2}^n = u_j^n + 2\Delta x \frac{\partial u}{\partial x} + \frac{4\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} + \frac{8\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{16\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots. \qquad (1C)$$

$$\frac{u_{j-1}^n - u_j^n}{\Delta x} = -\frac{\partial u}{\partial x} + \frac{\Delta x}{2}\frac{\partial^2 u}{\partial x^2} - \frac{\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \dots. \qquad (BB)$$

$$\frac{(u_{j+2}^n - u_j^n)}{2\Delta x} = \frac{\partial u}{\partial x} + \frac{2\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{4\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} - \frac{8\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \dots. \qquad (CC)$$

Paulo Yoshio Kubota

Métodos de diferenças finitas.

Esquemas com diferenças centradas no espaço de terceira ordem:

$$\frac{u_{j-1}^n - u_j^n}{\Delta x} = -\frac{\partial u}{\partial x} + \frac{\Delta x}{2}\frac{\partial^2 u}{\partial x^2} - \frac{\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \ldots. \qquad (BB)$$

$$\frac{\left(u_{j+2}^n - u_j^n\right)}{2\Delta x} = \frac{\partial u}{\partial x} + \frac{2\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{4\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} - \frac{8\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \ldots. \qquad (CC)$$

Mutiplica a BB por 4

$$4\frac{u_{j-1} - u_j}{\Delta x} = -4\frac{\partial u}{\partial x} + 4\frac{\Delta x}{2}\frac{\partial^2 u}{\partial x^2} - \frac{4\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} + \frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} + O[(\Delta x)^4] \qquad (AD)$$

soma a AD a eq. CC

$$4\frac{u_{j-1} - u_j}{\Delta x} + \frac{u_{j+2} - u_j}{2\Delta x} = -3\frac{\partial u}{\partial x} + \frac{6\Delta x}{2}\frac{\partial^2 u}{\partial x^2} - \frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \ldots..$$

$$\frac{4u_{j-1} - 4u_j}{\Delta x} + \frac{u_{j+2} - u_j}{\Delta x} - \frac{6\Delta x}{2}\frac{\partial^2 u}{\partial x^2} = -3\frac{\partial u}{\partial x} - \frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \ldots.. \qquad (2D)$$

Paulo Yoshio Kubota

Métodos de diferenças finitas.

$$\frac{4u_{j-1}-4u_j}{\Delta x}+\frac{u_{j+2}-u_j}{\Delta x}-\frac{6\Delta x}{2}\frac{\partial^2 u}{\partial x^2}=-3\frac{\partial u}{\partial x}-\frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}\dots\dots \qquad (2D)$$

Sabe-se que $\frac{\partial^2 u}{\partial x^2}\cong\frac{u_{j+2}-2u_j+u_{j-2}}{4\Delta x^2}$

$$\frac{u_{j+2}-2u_j+u_{j-2}}{4\Delta x^2}=\frac{\partial^2 u}{\partial x^2}+\frac{4}{4!}\frac{\partial^4 u}{\partial x^4}(\Delta x)^2+O[(\Delta x)^4] \qquad (6C)$$

$$\frac{4u_{j-1}-4u_j}{\Delta x}+\frac{u_{j+2}-u_j}{\Delta x}-\frac{6\Delta x}{2}\left(\frac{u_{j+2}-2u_j+u_{j-2}}{4\Delta x^2}\right)=-3\frac{\partial u}{\partial x}-\frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}\dots\dots \qquad (2D)$$

$$\frac{4u_{j-1}-4u_j}{\Delta x}+\frac{u_{j+2}-u_j}{\Delta x}-\left(\frac{6u_{j+2}-12u_j+6u_{j-2}}{8\Delta x}\right)=-3\frac{\partial u}{\partial x}-\frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}\dots\dots \qquad (2D)$$

Métodos de diferenças finitas.

$$\frac{4u_{j-1}-4u_j}{\Delta x}+\frac{u_{j+2}-u_j}{\Delta x}-\frac{6u_{j+2}-12u_j+6u_{j-2}}{8\Delta x}=-3\frac{\partial u}{\partial x}-\frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}\ldots.. \qquad (2D)$$

$$\frac{4u_{j-1}-4u_j}{3\Delta x}+\frac{u_{j+2}-u_j}{3\Delta x}-\frac{2u_{j+2}-4u_j+2u_{j-2}}{8\Delta x}=-\frac{\partial u}{\partial x}-O[\Delta x^3]\ldots.. \qquad (2D)$$

$$-\frac{4u_{j-1}-4u_j}{3\Delta x}-\frac{u_{j+2}-u_j}{3\Delta x}+\frac{2u_{j+2}-4u_j+2u_{j-2}}{8\Delta x}=\frac{\partial u}{\partial x}+O[\Delta x^3]\ldots.. \qquad (2D)$$

$$\frac{-4u_{j-1}+4u_j-u_{j+2}+u_j}{3\Delta x}+\frac{u_{j+2}-2u_j+u_{j-2}}{4\Delta x}=\frac{\partial u}{\partial x}+O[\Delta x^3]\ldots.. \qquad (2D)$$

Métodos de diferenças finitas.

$$\frac{-4u_{j-1} + 4u_j - u_{j+2} + u_j}{3\Delta x} + \frac{u_{j+2} - 2u_j + u_{j-2}}{4\Delta x} = \frac{\partial u}{\partial x} + O[\Delta x^3] \ldots\ldots \qquad (2D)$$

$$\frac{\partial u}{\partial x} = \frac{u_{j+2} - 2u_j + u_{j-2}}{4\Delta x} + \frac{-4u_{j-1} + 4u_j - u_{j+2} + u_j}{3\Delta x} - O[\Delta x^3] \ldots\ldots \qquad (2D)$$

Esquemas com diferenças centradas no espaço de quarta ordem:

Métodos de diferenças finitas.

Esquemas com diferenças centradas no espaço de segunda ordem:

$$\frac{u_j^n - u_{j-1}^n}{\Delta x} = \frac{\partial u}{\partial x} - \frac{\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} - \frac{\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}\dots. \qquad (BB)$$

$$\frac{\left(u_j^n - u_{j-2}^n\right)}{2\Delta x} = \frac{\partial u}{\partial x} - \frac{2\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{4\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} - \frac{8\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}\dots. \qquad (CC)$$

Mutiplica a BB por 2

$$2\frac{u_j - u_{j-1}}{\Delta x} = 2\frac{\partial u}{\partial x} + 2\frac{\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{2\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} - \frac{2\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} + O[(\Delta x)^4] \qquad (AD)$$

Soma a AD a eq. CC

$$2\frac{u_j - u_{j-1}}{\Delta x} + \frac{u_j - u_{j-2}}{2\Delta x} = 3\frac{\partial u}{\partial x} + \frac{6\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3}\dots..$$

$$\frac{2}{3}\frac{u_j - u_{j-1}}{\Delta x} - \frac{1}{3}\frac{u_j - u_{j-2}}{2\Delta x} = \frac{\partial u}{\partial x} + \Delta x^2\frac{\partial^3 u}{\partial x^3}\dots. \qquad (2D)$$

$$\left(\frac{-u_j + 2u_j - 2u_{j-1} + u_{j-2}}{6\Delta x}\right) = \frac{\partial u}{\partial x} + \Delta x^2\frac{\partial^3 u}{\partial x^3}\dots..(2D)$$

Paulo Yoshio Kubota

Métodos de diferenças finitas.

Esquemas com diferenças centradas no espaço de quarta ordem:

$$\frac{u_{j+1}-u_{j-1}}{2\Delta x}=\frac{\partial u}{\partial x}+\frac{1}{3!}\frac{\partial^3 u}{\partial x^3}(\Delta x)^2+O[(\Delta x)^4] \qquad (4B)$$

$$\frac{u_{j+2}-u_{j-2}}{4\Delta x}=\frac{\partial u}{\partial x}+\frac{4}{3!}\frac{\partial^3 u}{\partial x^3}(\Delta x)^2+O[(\Delta x)^4] \qquad (4C)$$

Mutiplica a 4B por 4

$$4\frac{u_{j+1}-u_{j-1}}{2\Delta x}=4\frac{\partial u}{\partial x}+\frac{4}{3!}\frac{\partial^3 u}{\partial x^3}(\Delta x)^2+O[(\Delta x)^4] \qquad (1D)$$

Subtaria a 1D - 4C

$$4\frac{u_{j+1}-u_{j-1}}{2\Delta x}-\frac{u_{j+2}-u_{j-2}}{4\Delta x}=3\frac{\partial u}{\partial x}+O[(\Delta x)^4]$$

$$\frac{4}{3}\frac{u_{j+1}-u_{j-1}}{2\Delta x}-\frac{1}{3}\frac{u_{j+2}-u_{j-2}}{4\Delta x}=\frac{\partial u}{\partial x} \qquad (2D)$$

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Métodos de diferenças finitas.

Esquemas com diferenças centradas no espaço de quarta ordem:

$$\frac{4}{3}\frac{u_{j+1}-u_{j-1}}{2\Delta x}-\frac{1}{3}\frac{u_{j+2}-u_{j-2}}{4\Delta x}=\frac{\partial u}{\partial x} \qquad (2D)$$

$$\left(\frac{1}{3}\right)\left(\frac{4u_{j+1}-4u_{j-1}}{2\Delta x}-\frac{u_{j+2}-u_{j-2}}{4\Delta x}\right)=\frac{\partial u}{\partial x} \qquad (2D)$$

$$\left(\frac{1}{3}\right)\left(\frac{8u_{j+1}-8u_{j-1}-u_{j+2}+u_{j-2}}{4\Delta x}\right)=\frac{\partial u}{\partial x} \qquad (2D)$$

$$\left(\frac{-u_{j+2}+8u_{j+1}-8u_{j-1}+u_{j-2}}{12\Delta x}\right)=\frac{\partial u}{\partial x} \qquad (2D)$$