Camada Limite Planetária

Yonsei University (YSU)

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Equações governantes para o escoamento médio da atmosférico

Equações Conservação de Momentum

$$\frac{\partial(\overline{u})}{\partial t} + (\overline{u})\frac{\partial(\overline{u})}{\partial x} + (\overline{v})\frac{\partial(\overline{u})}{\partial y} + (\overline{w})\frac{\partial(\overline{u})}{\partial z} + \frac{1}{\rho_0}\frac{\partial(\overline{P})}{\partial x} - 2\Omega\eta_3(\overline{v}) - \nu\frac{\partial^2(\overline{u})}{\partial x^2} - \nu\frac{\partial^2(\overline{u})}{\partial y^2} - \nu\frac{\partial^2(\overline{u})}{\partial z^2} = -\frac{\partial(\overline{u'u'})}{\partial x} - \frac{\partial(\overline{v'u'})}{\partial y} - \frac{\partial(\overline{w'u'})}{\partial z}$$

$$\frac{\partial(\overline{v})}{\partial t} + (\overline{u})\frac{\partial(\overline{v})}{\partial x} + (\overline{v})\frac{\partial(\overline{v})}{\partial y} + (\overline{w})\frac{\partial(\overline{v})}{\partial z} + \frac{1}{\rho_0}\frac{\partial(\overline{P})}{\partial y} + 2\Omega\eta_3(u) - \nu\frac{\partial^2(\overline{v})}{\partial x^2} - \nu\frac{\partial^2(\overline{v})}{\partial y^2} - \nu\frac{\partial^2(v)}{\partial z^2} = -\frac{\partial(\overline{u'v'})}{\partial x} - \frac{\partial(\overline{v'v'})}{\partial y} - \frac{\partial(\overline{w'v'})}{\partial z}$$

$$\frac{\partial(\overline{w})}{\partial t} + (\overline{u})\frac{\partial(\overline{w})}{\partial x} + (\overline{v})\frac{\partial(\overline{w})}{\partial y} + (\overline{w})\frac{\partial(\overline{w})}{\partial z} + \frac{1}{\rho_0}\frac{\partial(\overline{P})}{\partial z} + g\frac{\overline{\rho}}{\rho_0} - \nu\frac{\partial^2(\overline{w})}{\partial x^2} - \nu\frac{\partial^2(\overline{w})}{\partial y^2} - \nu\frac{\partial^2(\overline{w})}{\partial z^2} = -\frac{\partial(\overline{u'w'})}{\partial x} - \frac{\partial(\overline{v'w'})}{\partial y} - \frac{\partial(\overline{w'w'})}{\partial z}$$

Equação Conservação de Energia

$$\frac{\partial(\overline{T})}{\partial t} + (\overline{u})\frac{\partial(\overline{T})}{\partial x} + (\overline{v})\frac{\partial(\overline{T})}{\partial y} + (\overline{w})\frac{\partial(\overline{T})}{\partial z} - S_P\overline{\omega} = -\frac{\partial(\overline{u'T'})}{\partial x} - \frac{\partial(\overline{v'T'})}{\partial y} - \frac{\partial(\overline{w'T'})}{\partial z} + \frac{\overline{J}}{C_p}$$

Equação Conservação de Massa

$$\frac{\partial(\overline{c})}{\partial t} + (\overline{u})\frac{\partial(\overline{c})}{\partial x} + (v)\frac{\partial(\overline{c})}{\partial y} + (\overline{w})\frac{\partial(\overline{c})}{\partial z} = -\frac{\partial(\overline{u'c'})}{\partial x} - \frac{\partial(\overline{u'c'})}{\partial y} - \frac{\partial(\overline{w'c'})}{\partial z} + \overline{S}_c$$

$$u_3 = w$$
$$x_3 = z$$
$$\nu\frac{\partial^2(\overline{u_3})}{\partial {x_3}^2} \cong 0$$

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Equações governantes para o escoamento médio da atmosférico

$$\frac{\partial(\bar{u})}{\partial t} = -\frac{\partial(\overline{w'u'})}{\partial z}$$

$$\frac{\partial(\bar{v})}{\partial t} = -\frac{\partial(\overline{w'v'})}{\partial z}$$

$$\frac{\partial(\bar{w})}{\partial t} = -\frac{\partial(\overline{w'w'})}{\partial z} = -\frac{\partial\left(\overline{w'^2}\right)}{\partial z} = -\frac{\partial(TKE)_k}{\partial z}$$

$$\frac{\partial(\bar{T})}{\partial t} = -\frac{\partial\left(\overline{w'T'}\right)}{\partial z}$$

$$\frac{\partial(\bar{c})}{\partial t} = -\frac{\partial\left(\overline{w'c'}\right)}{\partial z}$$

$$\frac{\partial\left(\overline{w'X'}\right)}{\partial z}$$

Mudança de fase e processos tubulentos presentes na atmosfera

$$\frac{D\bar{e}}{Dt} = -\frac{\overline{(u_j'u_i')}}{2}\frac{\partial(\bar{\bar{u}}_i)}{\partial x_j} - g\frac{\overline{u_i'\rho'}}{2\rho_0}\delta_{i3} - \frac{\partial\left(\overline{eu_j'}\right)}{\partial x_j} - \frac{1}{2\rho_0}\frac{\partial(\overline{P'u_i'})}{\partial x_i} - \epsilon$$

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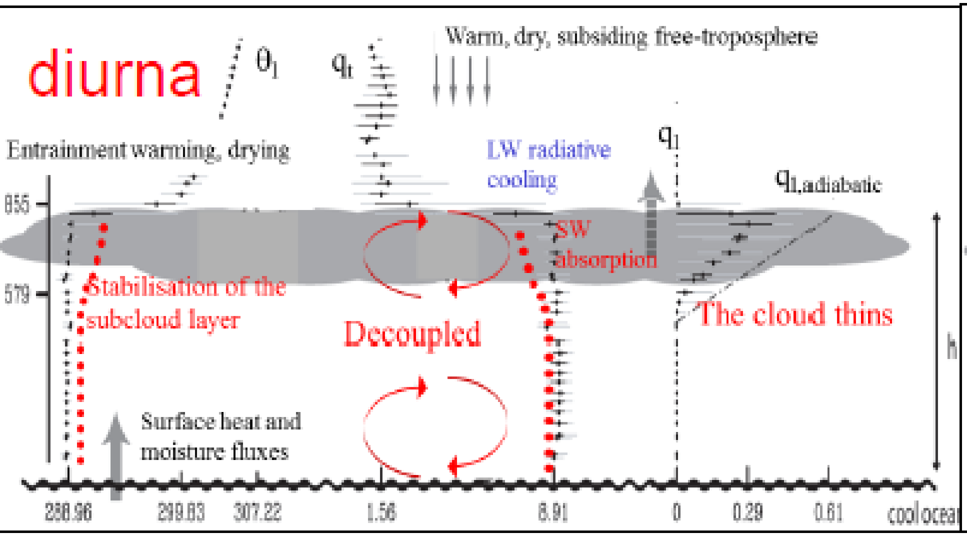
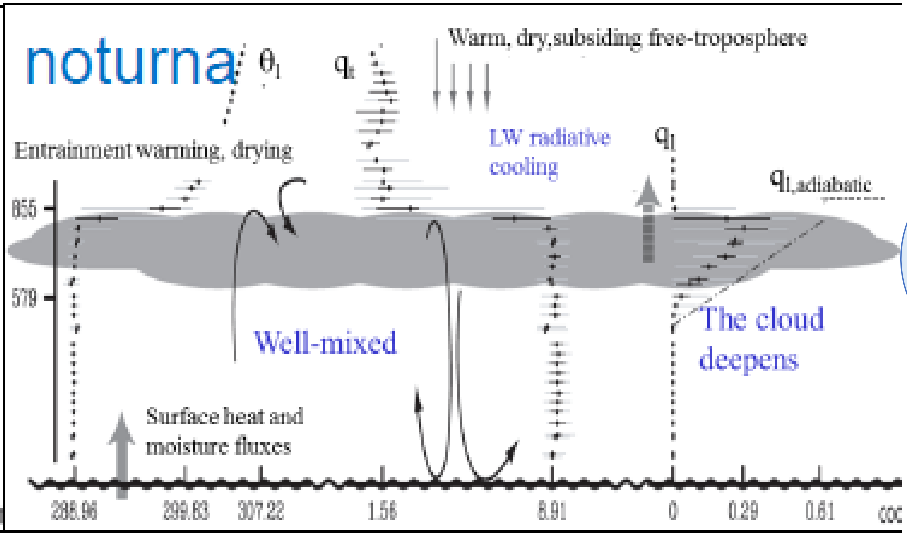
(1)Quais são as fontes de incerteza do modelo?

Processos físicos da turbulência na camada limite

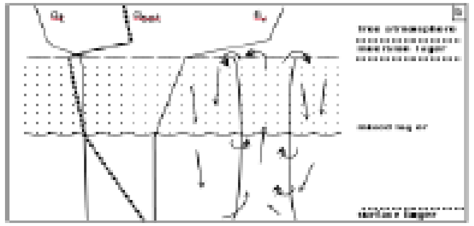


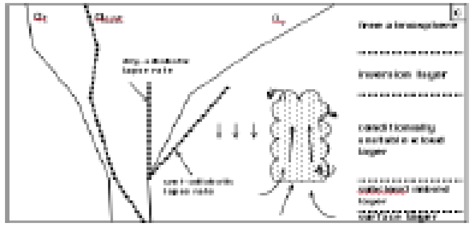


Como calcular o coeficiente d difusão turbulenta?

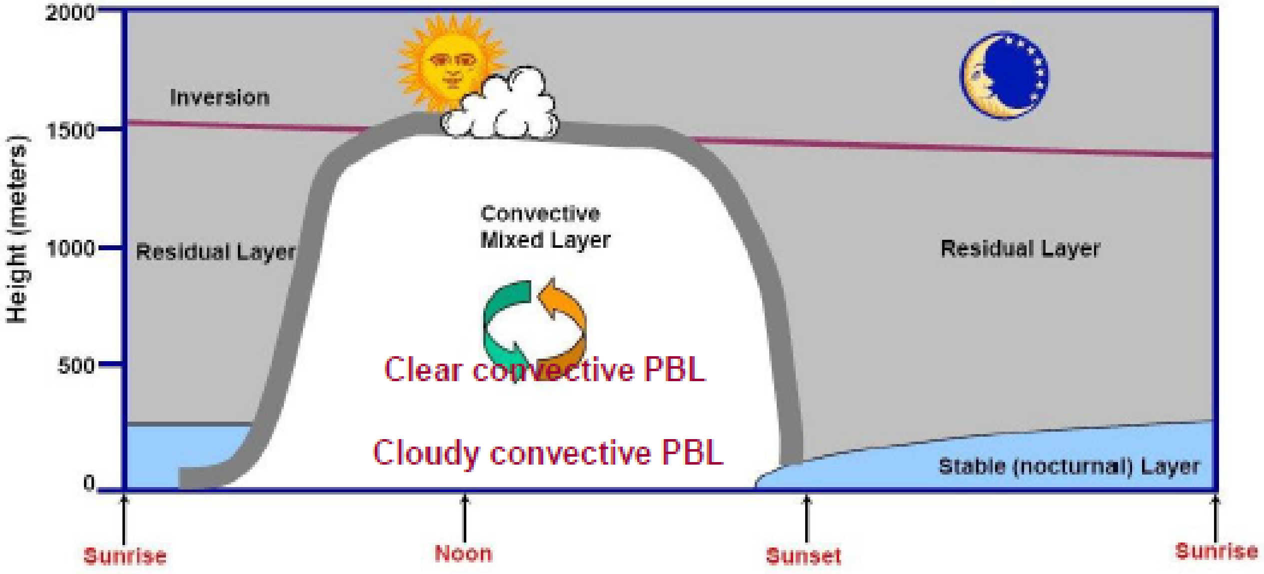
$K(x, y, z, ri, cloud, rad, entr, .)$

Estratocumulos PBL





Cumulos PBL



Adapted from Introduction to Boundary Layer Meteorology -R.B. Stull, 1988

Paulo Yoshio Kubota

Camada Limite Planetária
Yonsei University (YSU)

"**Fechamento de primeira ordem**;

Conceito de Troen e Mahrt (1986) de incorporar um termo de correção contra o gradiente na difusão descendente expressa exclusivamente pela mistura local.

O YSU representa explicitamente a entranhamento no topo da CAMADA LIMITE PLANETÁRIA."

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Vantagens

"Simula de forma mais precisa a profundidade a mistura vertical em PBL impulsionadas pela flutuabilidade, e mistura mais rasa em regimes de ventos fortes em comparação com o (Hong et al. 2006).“

Desvantagens

"Ainda foi constatado que aprofunda excessivamente a PBL em ambientes de convecção profunda na primavera, resultando em ar muito seco próximo à superfície e subestimação da CAPE em relação a ambientes de convecção profunda (Coniglio et al. 2013)."

Esquema de difusão vertical da camada limite NonLocal

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial z}\left[K_c\left(\frac{\partial C}{\partial z} - \gamma_c\right)\right]$$

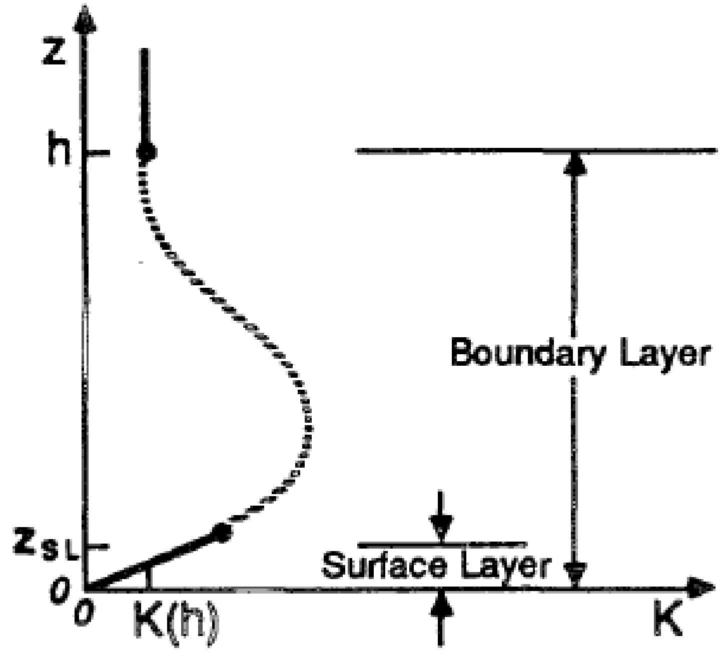

FIG. 1. Typical variation of eddy viscosity K with height in the boundary layer proposed by O'Brien (1970). Adopted from Stull (1988).

1. K_c é **coeficiente de difusividade** de Eddy.

2. γ_c é uma **correção para o gradiente local** que incorpora a contribuição de eddies de grande escala ao fluxo total

Deardorff (1973), Troen and Mahrt (1986) Holtslag and Moeng (1991), Holtslag and Boville (1993)

(1)Quais são as fontes de incerteza do modelo?

Processos físicos da turbulência na camada limite

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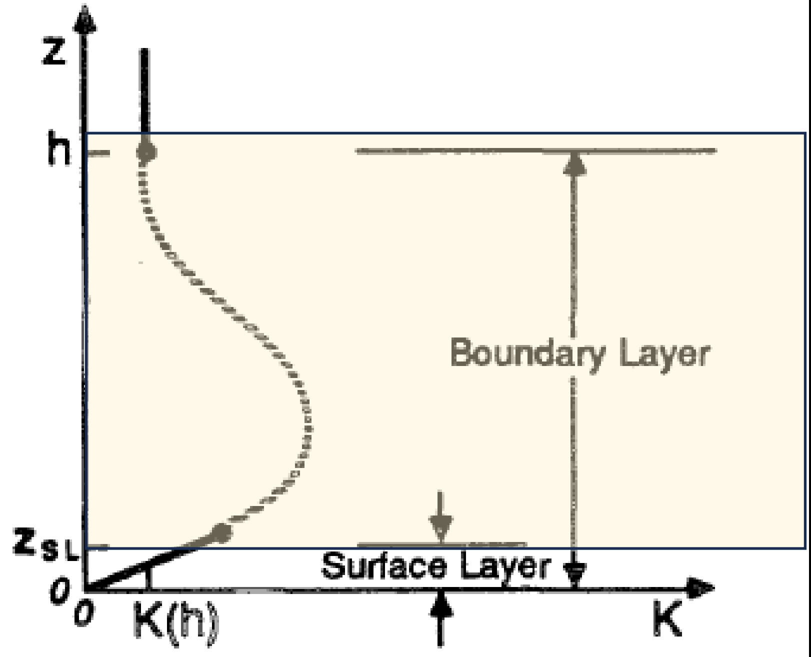
Difusão da camada de mistura



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Difusão da camada de mistura

Deardorff (1973), Troen and Mahrt (1986) Holtslag and Moeng (1991), Holtslag and Boville (1993)

$$K_{zm} = k w_s z \left(1 - \frac{z}{h}\right)^p$$

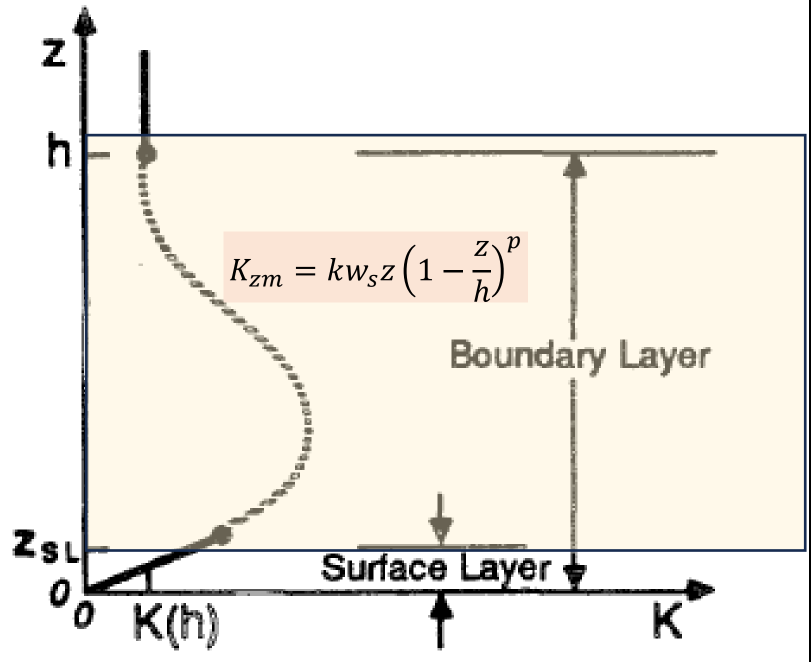


FIG. 1. Typical variation of eddy viscosity K with height in the boundary layer proposed by O'Brien (1970). Adopted from Stull (1988).

"onde:

1. p **é o expoente da forma do perfil, considerado como 2.**
2. k **é a constante de von Kármán (= 0.4),**
3. z **é a altura a partir da superfície**
4. h **é a altura da PBL.**
5. w_s **é A escala de velocidade da camada mistura é representada como"**

Difusão da camada de mistura

Deardorff (1973), Troen and Mahrt (1986) Holtslag and Moeng (1991), Holtslag and Boville (1993)

Escala de velocidade da camada mistura

$$w_s = \left(u_*^3 + \frac{8kw_{*b}^3 z}{h}\right)^{\frac{1}{3}}$$

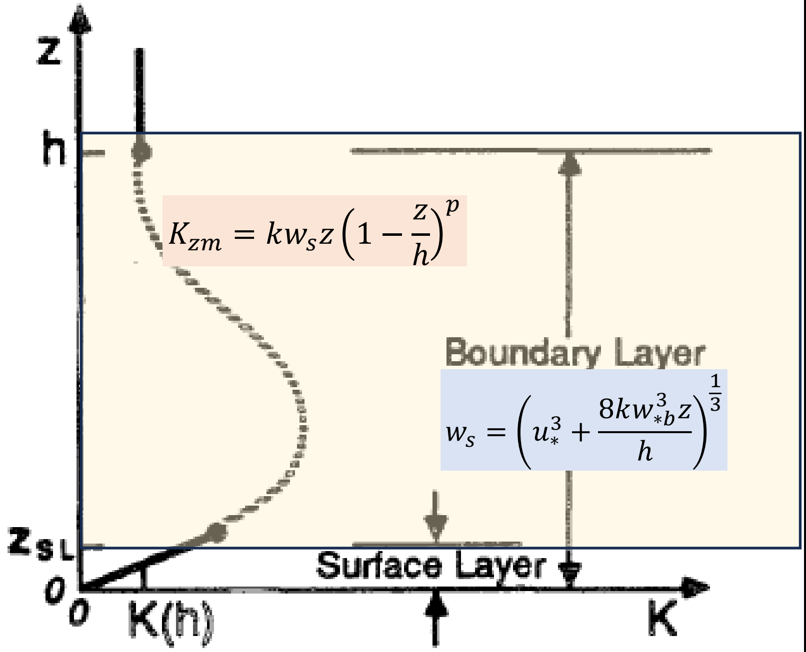
Onde:

u_* **é a escala de velocidade de fricção na superfície**

w_{*b} **é a escala da velocidade convectiva**

Escala da velocidade convectiva

$$w_{*b} = \left\{\frac{g}{\theta_{va}}(w'\theta_v')_0 h\right\}^{\frac{1}{3}}$$



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Difusão da camada de mistura

Deardorff (1973), Troen and Mahrt (1986) Holtslag and Moeng (1991), Holtslag and Boville (1993)

O termo de contra-gradiente para θe momentum´:

$$\gamma_c = b\frac{\overline{(w'c')}_0}{w_{s0}}$$

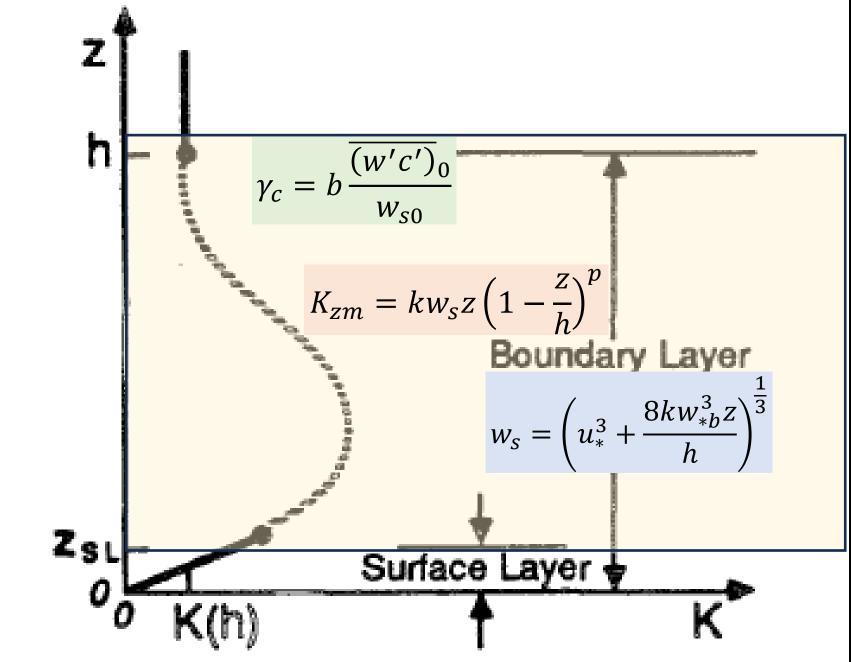


FIG. 1. Typical variation of eddy viscosity K with height in the boundary layer proposed by O'Brien (1970). Adopted from Stull (1988).

Onde:

$\overline{(w'c')}_0$ **é fluxo de superfície correspondente a θ e a u e v.**

b **é um coeficiente de proporcionalidade**

w_{s0} **é a escala de velocidade na camada de mistura, é definida como a velocidade em $z = 0.5h$ na equação:**

$$w_{s0} = w_s = \left(u_*^3 + \frac{8kw_{*b}^3 z}{h}\right)^{\frac{1}{3}} = \left(u_*^3 + \frac{8kw_{*b}^3 0.5h}{h}\right)^{\frac{1}{3}} = \left(u_*^3 + 4kw_{*b}^3\right)^{\frac{1}{3}}$$

Difusão da camada de mistura

Deardorff (1973), Troen and Mahrt (1986) Holtslag and Moeng (1991), Holtslag and Boville (1993)

"A difusividade turbulenta para temperatura e umidade K_h é calculada a partir de K_{zm} na equação (2) usando a relação do número de Prandtl."

$$P_r = \left(\frac{\phi_t}{\phi_m} + bk\frac{0.1h}{h}\right)$$

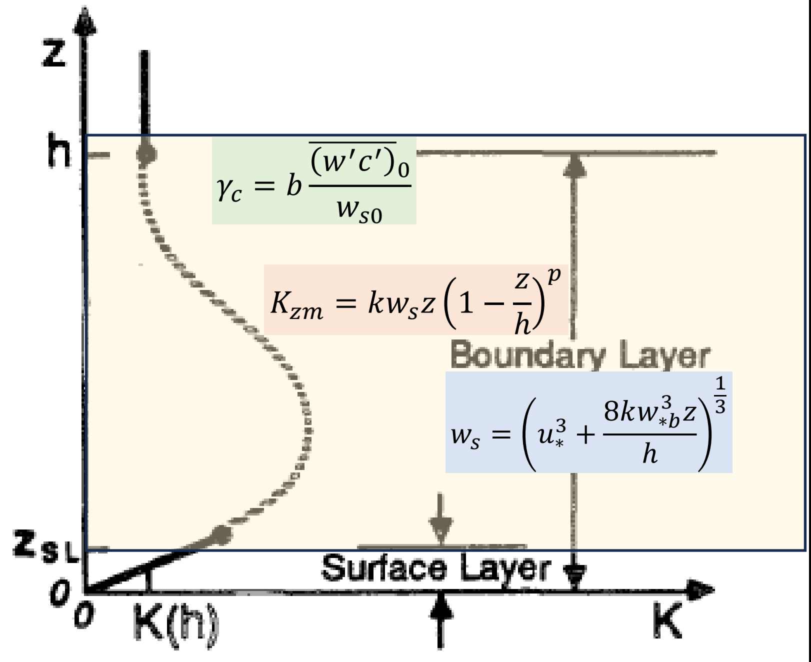


FIG. 1. Typical variation of eddy viscosity K with height in the boundary layer proposed by O'Brien (1970). Adopted from Stull (1988).

"onde:

P_r é uma constante dentro de toda a camada limite de mistura."

$$K_h \sim P_r K_{zm}$$

Difusão da camada de mistura

Deardorff (1973), Troen and Mahrt (1986) Holtslag and Moeng (1991), Holtslag and Boville (1993)

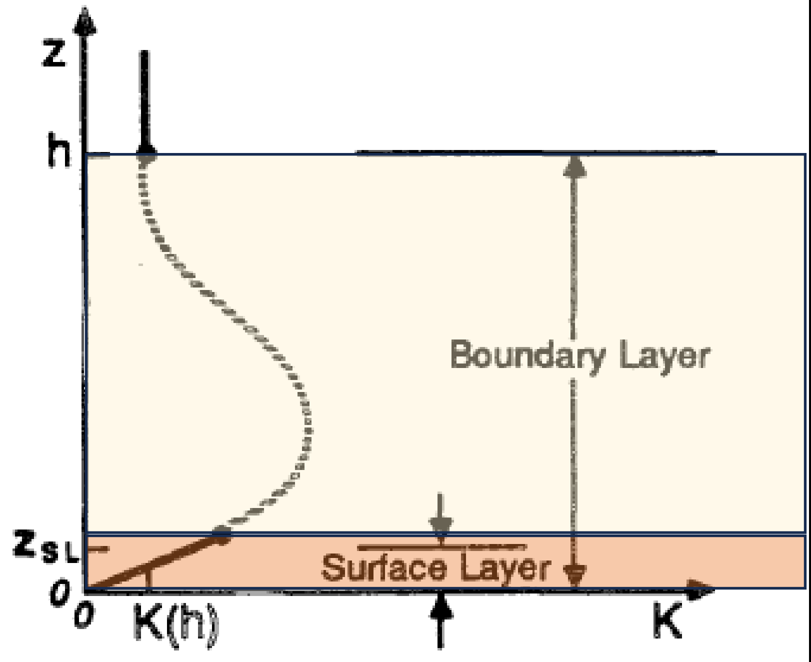


FIG. 1. Typical variation of eddy viscosity K with height in the boundary layer proposed by O'Brien (1970). Adopted from Stull (1988).

Acoplamento da Camada superficial e a camada de mistura

Camada superficial $\overline{(w'C')}_0 \sim cte$ **entre 0** $\leq z \leq Z_{SL}$

Como parameterizar os momentum de 2 ordem $w'\varphi'$ na CL Superficial

$$F = \rho K(z)\frac{d\varphi}{dz} \qquad (F = \overline{w'\varphi'})$$

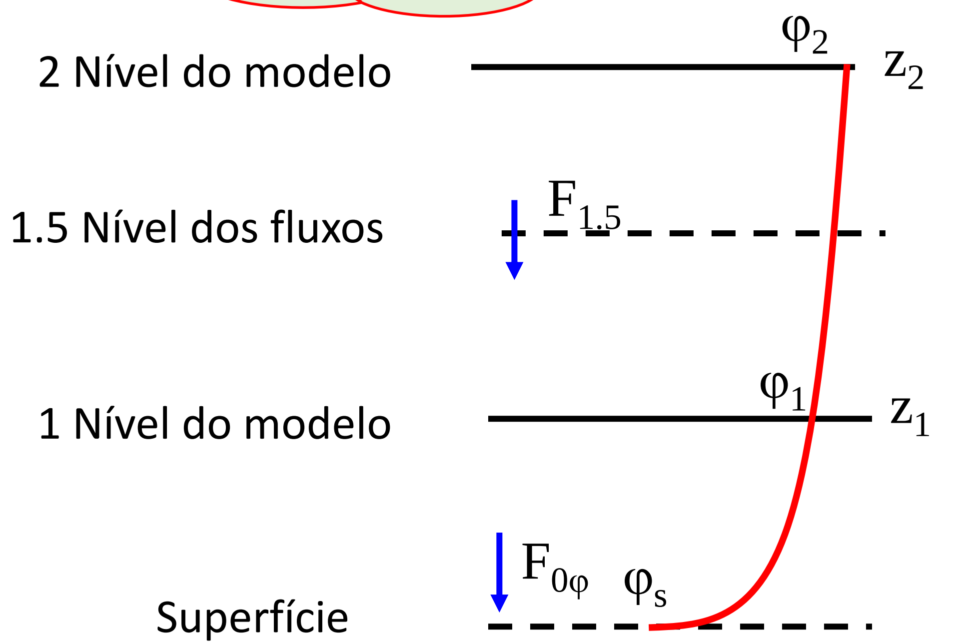
Calcular $K(z_{1.5})$???

Formulação em diferenças finitas

$$F_{1.5} = \rho K(z_{1.5}) \frac{\varphi_2 - \varphi_1}{z_2 - z_1}$$

Integral da camada superfície :

$$\varphi_1 - \varphi_s = \int_{z_{o\varphi}}^{z_1} \frac{F_{0\varphi}}{\rho K(z)} dz$$



Camada de fluxo constante:

$$\varphi_1 - \varphi_s \approx \frac{F_0}{\rho} \int_{z_{0\varphi}}^{z_1} \frac{1}{K(z)} dz$$

Escoamento neutro:

$$K(z) = \kappa u_* z$$

$$\varphi_1 - \varphi_s \approx \frac{F_{0\varphi}}{\rho \kappa u_*} \int_{z_{0\varphi}}^{z_1} \frac{dz}{z} \quad \Rightarrow \quad \varphi_1 - \varphi_s = \frac{F_{0\varphi}}{\rho \kappa u_*} \ln\left(\frac{z_1}{z_{o\varphi}}\right)$$

u,v,T,q

κ : Von Karman constant (0.4)
u_* : Friction velocity
ρ : Density

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(1)Quais são as fontes de incerteza do modelo?

Processos físicos da turbulência na camada limite

Deardorff (1973), Troen and Mahrt (1986) Holtslag and Moeng (1991), Holtslag and Boville (1993)

Função universal integrada para a camada limite superficial

$(z_0 \leq z \leq z_{SL})$

when $\zeta < 0$ (unstable)

$$\Psi_M = \ln\left(\frac{z_m - d_0}{z_0}\right) + \ln\frac{(x_0^2+1)(x_0+1)^2}{(x^2+1)(x+1)^2} + 2(\tan^{-1}x - \tan^{-1}x_0) \quad (2.59)$$

$$\Psi_H = \ln\left(\frac{z_m - d_0}{z_0}\right) + 2\ln\left(\frac{y_0+1}{y+1}\right) \quad (2.60)$$

$$x = (1-16\zeta)^{1/4},\ x_0 = (1-16\zeta_0)^{1/4},\ y = (1-16\zeta)^{1/2},\ y_0 = (1-16\zeta_0)^{1/2} \quad (2.61)$$

when $\zeta \geq 0$ (stable)

$$\Psi_M = \ln\left(\frac{z_m - d_0}{z_0}\right) + \frac{7}{3}\ln\frac{1+3\zeta+10\zeta^3}{1+3\zeta_0+10\zeta_0^3} \quad (2.62)$$

$$\Psi_H = \ln\left(\frac{z_m - d_0}{z_0}\right) + 400\ln\frac{1+7/400\zeta+0.005\zeta^2}{1+7/400\zeta_0+0.005\zeta_0^2} \quad (2.63)$$

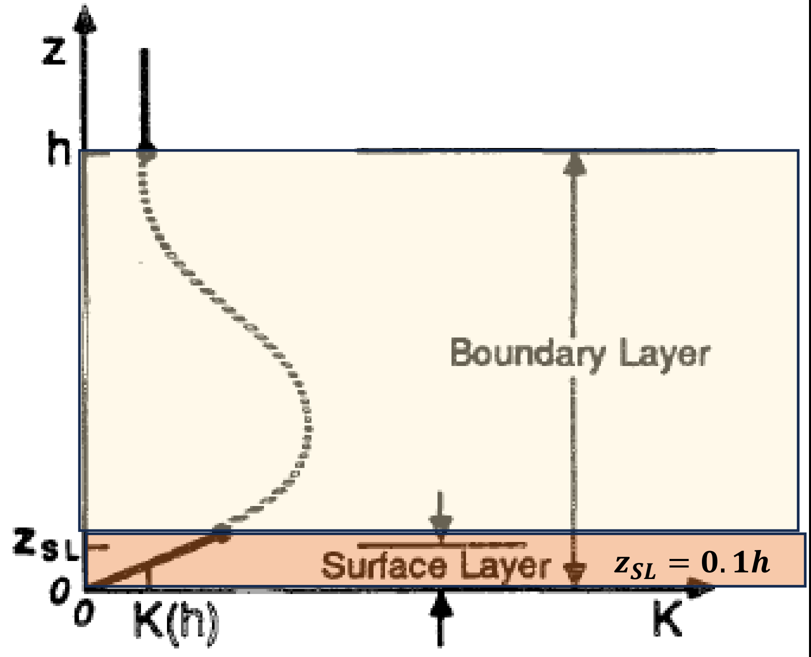

FIG. 1. Typical variation of eddy viscosity K with height in the boundary layer proposed by O'Brien (1970). Adopted from Stull (1988).

$$H = \rho_a C_p (T_{au} - T_m)\kappa u_*/\Psi_H \quad (2.56)$$

$$L = -\rho_a C_p T_m u_*^3/\kappa g H \quad (2.57)$$

$$\zeta = z_m/L \quad (2.58)$$

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Processos físicos da turbulência na camada limite

Deardorff (1973), Troen and Mahrt (1986) Holtslag and Moeng (1991), Holtslag and Boville (1993)

"Para satisfazer a compatibilidade entre o topo da camada superficial ($z = 0.1h$) e a base da PBL .

$$C_{DN} = \frac{k^2}{\left[\ln\left(\frac{z_r}{z_0}\right)\right]^2}$$

condições neutras

$$F = \rho K(z)\frac{d\varphi}{dz} \qquad (F = \overline{w'\varphi'})$$

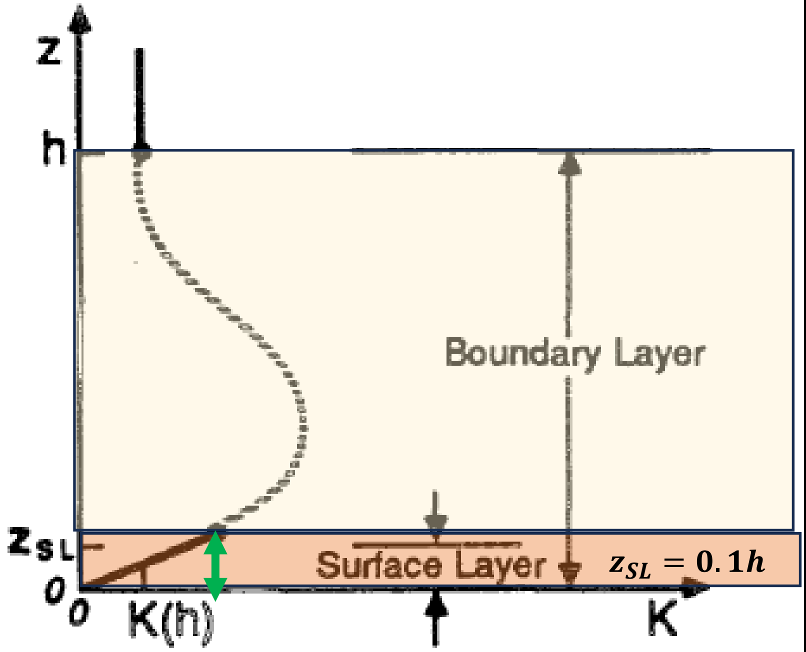

FIG. 1. Typical variation of eddy viscosity K with height in the boundary layer proposed by O'Brien (1970). Adopted from Stull (1988).

$$F_{0\varphi} = \tau = H = LE$$
$$\tau = \frac{1}{r_{aw}}\rho \mathbf{u_r}$$
$$H = \frac{1}{r_{aw}}\rho c_p(T - T_r)$$
$$LE = \frac{1}{(r_{aw} + r_{soil})}(\rho c_p/\gamma)[he_s(T) - e_r]$$

$$\therefore \frac{1}{r_{aw}} = C_D U_r$$

$$\varphi_1 - \varphi_s = \frac{F_{0\varphi}}{\rho\kappa u_*}\ln\left(\frac{Z_1}{Z_{o\varphi}}\right)$$

u,v,T,q

$$\tau_{wb} = \rho u_*^2$$
$$r_{aw} = \frac{\Psi_H}{\kappa u_*} = \frac{\Psi_M \Psi_H}{\kappa^2 u_m}$$

$$F_{0\varphi} = \tau = H = LE$$
$$\tau = C_D \rho \mathbf{u_r}$$
$$H = C_D \rho c_p(T - T_r)$$
$$LE = C_{Dh}(\rho c_p/\gamma)[he_s(T) - e_r]$$

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"Para **satisfazer a compatibilidade entre o topo da camada superficial** ($z = 0.1h$) **e a base da PBL .**

Função universal integrada para a camada limite de mistura

$$(z_{SL} \leq z \leq h)$$

Primeiro para condições instáveis e neutras $\overline{(w'\theta_v')}_0 \leq 0$

$$\phi_m = \left(1 - 16\frac{Z}{L}\right)^{-\frac{1}{4}} = \left(1 - 16\frac{0.1h}{L}\right)^{-\frac{1}{4}}, para\ u\ e\ v$$

$$\phi_t = \left(1 - 16\frac{Z}{L}\right)^{-\frac{1}{2}} = \left(1 - 16\frac{0.1h}{L}\right)^{-\frac{1}{2}}, para\ \theta\ e\ q$$

Emquanto para regime estável $\overline{(w'\theta_v')}_0 > 0$

$$\phi_m = \phi_t = \left(1 - 5\frac{Z}{L}\right) = \left(1 - 5\frac{0.1h}{L}\right)$$

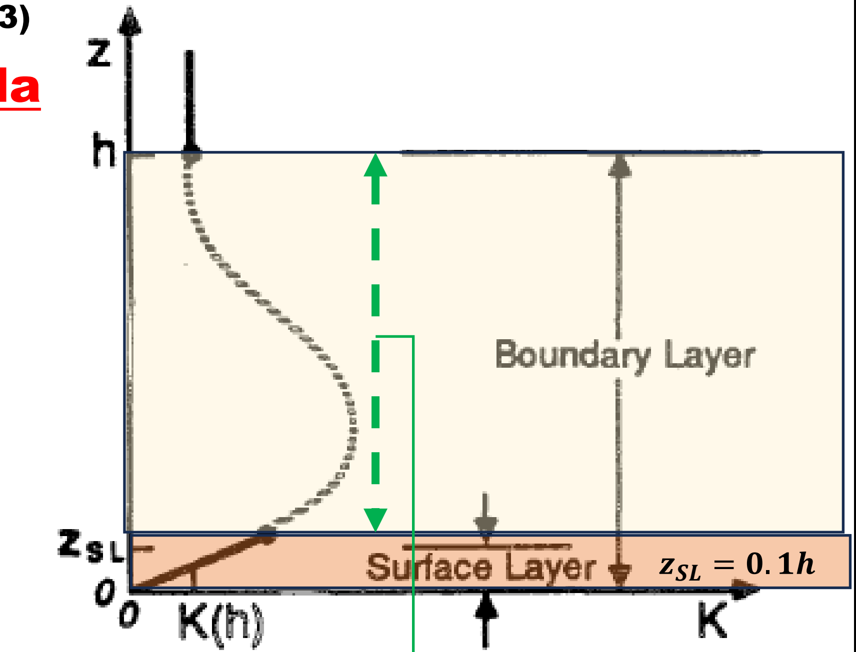

FIG. 1. Typical variation of eddy viscosity K with height in the boundary layer proposed by O'Brien (1970). Adopted from Stull (1988).

As funções universais com perfil simulares (regime de estabilidade) às da física da camada superficial devem ser usadas

(1)Quais são as fontes de incerteza do modelo?

Processos físicos da turbulência na camada limite

O topo da camada superficial é estimada em $0.1h$ para estimar o fator b da equação do contra-gradiente, o expoente $-\frac{1}{3}$ é escolhido para assegurar o limite de convecção livre. Portanto, utiliza-se a aproximação:

$$\phi_m = \left(1 - 16\frac{0.1h}{L}\right)^{-\frac{1}{4}} \approx \left(1 - 12\frac{0.1h}{L}\right)^{-\frac{1}{3}}, para\ u\ e\ v$$

$$w_{*b} = \left\{\frac{g}{\theta_{va}}(w'\theta_v')_0 h\right\}^{\frac{1}{3}}$$

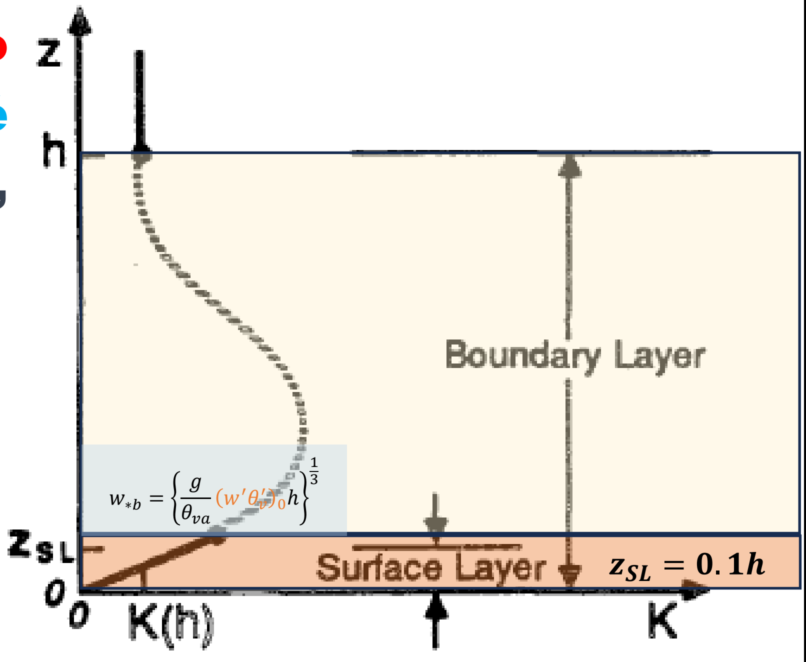

FIG. 1. Typical variation of eddy viscosity K with height in the boundary layer proposed by O'Brien (1970). Adopted from Stull (1988).

Baseado na derivação de Troen and Mahrt (1986) Holtslag (1990), encontra-se

$$b = 7.8$$

$$P_r = \left(\frac{\phi_t}{\phi_m} + bk\frac{0.1h}{h}\right)$$

$$K_h \sim P_r K_{zm}$$

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Processos físicos da turbulência na camada limite

Deardorff (1973), Troen and Mahrt (1986) Holtslag and Moeng (1991), Holtslag and Boville (1993)

A altura da Camada Limite Planetária é definida:

$$h = Rib_{cr}\frac{\theta_{va}|U(h)|^2}{g(\theta_v(h)-\theta_s)}$$

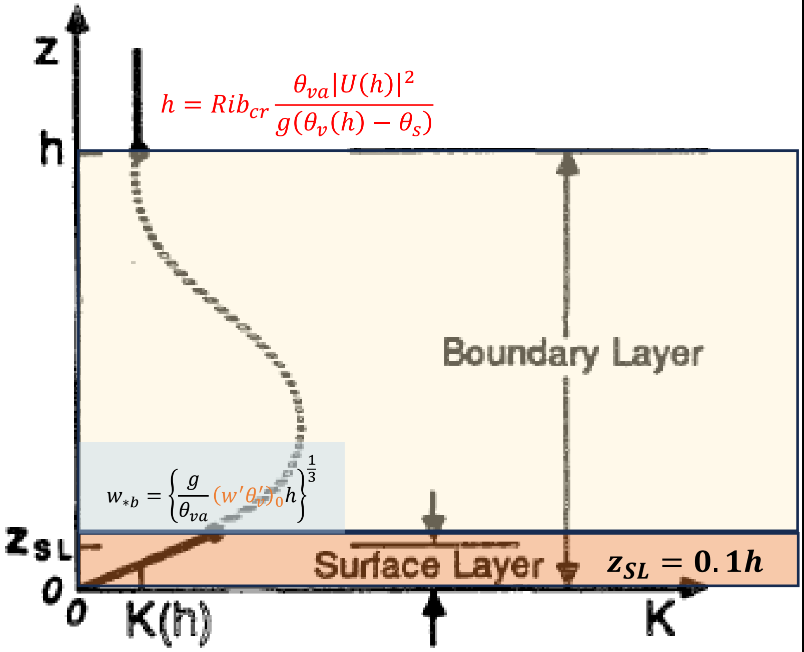

FIG. 1. Typical variation of eddy viscosity K with height in the boundary layer proposed by O'Brien (1970). Adopted from Stull (1988).

Onde:

Rib_{cr} **é o numero de Richardson bulk crítico**

$U(h)$ **é a velocidade do vento horizontal em** h

θ_{va} **é a temperatura virtual no nível mais baixo do modelo (30-50 m) da superfície**

$\theta_v(h)$ **é a temperatura potencial virtual em h**

θ_s **é a temperatura potencial apropriada próxima a superfície**

h é a altura da camada limite

L é a escala de comprimento de MONIN – OBUKHOV

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Processos físicos da turbulência na camada limite

Deardorff (1973), Troen and Mahrt (1986) Holtslag and Moeng (1991), Holtslag and Boville (1993)

A temperatura potencial apropriada próxima a superfície é definida:

$$\theta_s = \theta_{va} + \theta_T \left[= b\frac{\overline{(w'\theta_v{}')_0}}{w_s h} \right]$$

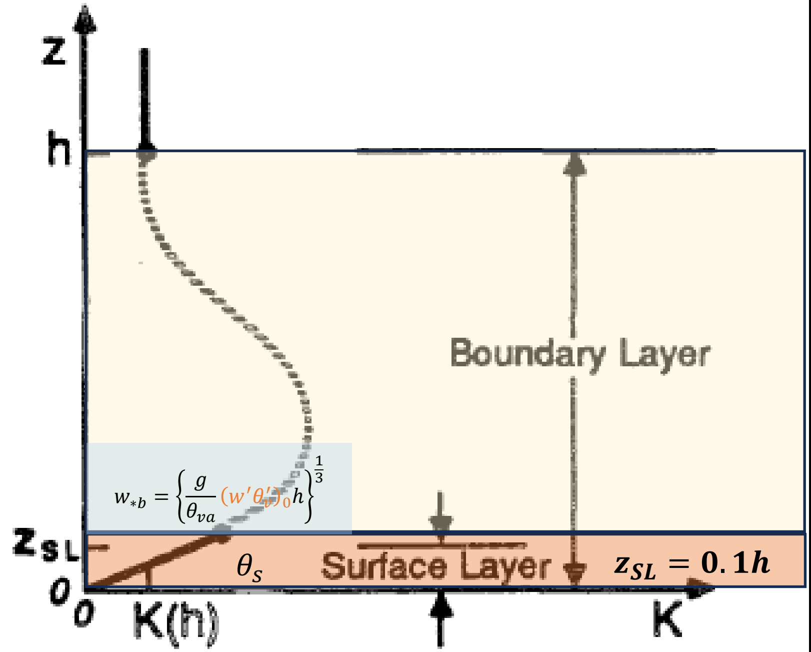


FIG. 1. Typical variation of eddy viscosity K with height in the boundary layer proposed by O'Brien (1970). Adopted from Stull (1988).

Onde:

θ_T **é o a escala de excesso de temperatura virtual próxima a superfície**

Testes preliminares indica que θ_T alguma vezes torna muito grande quando vento de superfície é muito baixo, resultando em h irrealista . "*Este alto valor de h devido ao irrealismo de θ_T não prejudica o resultado porque resulta em um coeficiente de difusividade muito pequeno nesta situações*. "mas isso não é desejável para fins de diagnóstico“. Por estas razões definí-se um limite máximo de $\theta_T = 3K$ "

(1)Quais são as fontes de incerteza do modelo?

Processos físicos da turbulência na camada limite

Louis (1979)

Difusão na atmosfera livre

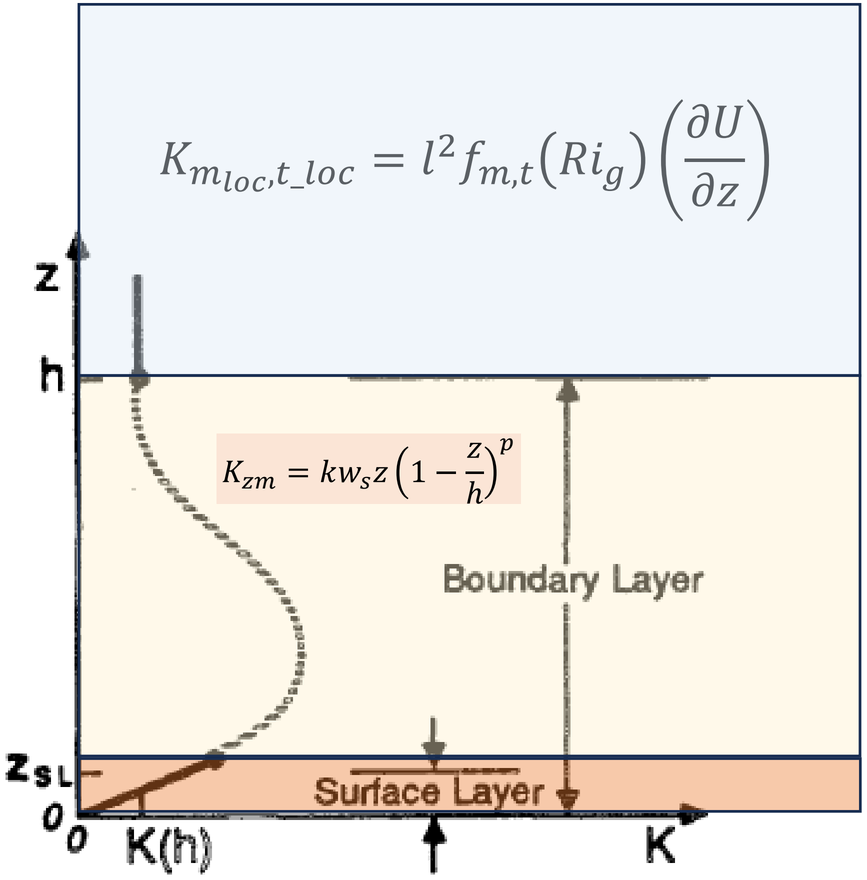


FIG. 1. Typical variation of eddy viscosity K with height in the boundary layer proposed by O'Brien (1970). Adopted from Stull (1988).

(1)Quais são as fontes de incerteza do modelo?

Processos físicos da turbulência na camada limite

"Os coeficientes de difusividade vertical para momentum (m; u, v) e escalares (t; θ, q), seguindo Louis (1979) acima de h,"

Louis (1979)

$$K_{m_{loc},t\_loc} = l^2 f_{m,t}(Ri_g)\left(\frac{\partial U}{\partial z}\right)$$

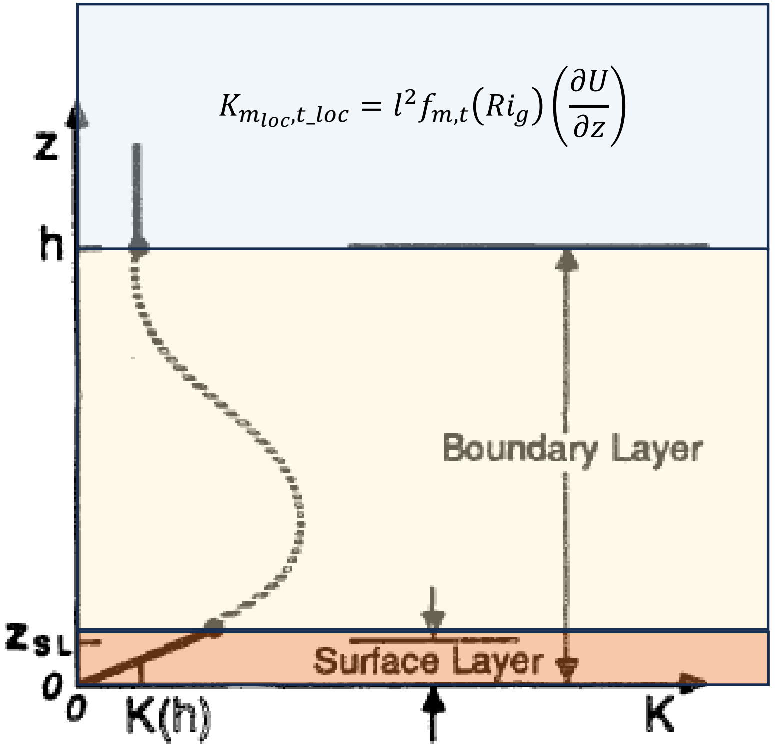


FIG. 1. Typical variation of eddy viscosity K with height in the boundary layer proposed by O'Brien (1970). Adopted from Stull (1988).

Onde:

l **é o comprimento de mistura.**

$f_{m,t}(Ri_g)$ **função de estabilidade dependente do numero de Richardson bulk.**

$\frac{\partial U}{\partial z}$ **cisalhamento vertical do vento**

h é a altura da camada limite

L *é a escala de comprimento de MONIN − OBUKHOV*

(1)Quais são as fontes de incerteza do modelo?

Processos físicos da turbulência na camada limite

Louis (1979)

"O comprimento de mistura l é dado por:,"

$$\frac{1}{l} = \frac{1}{kz} + \frac{1}{\lambda_0} \qquad \frac{1}{l} = \frac{\lambda_0 + kz}{\lambda_0 kz}$$

Onde:

z é a altura acima da superfície.

λ_0 é a escala de comprimento assintótico (λ_0=30m)

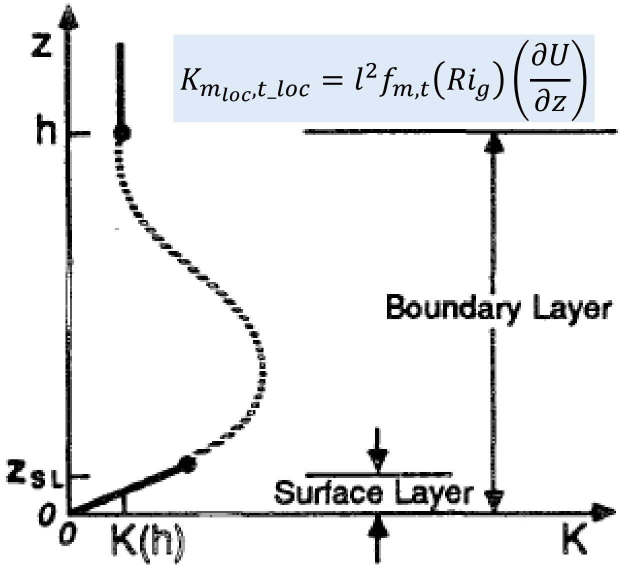
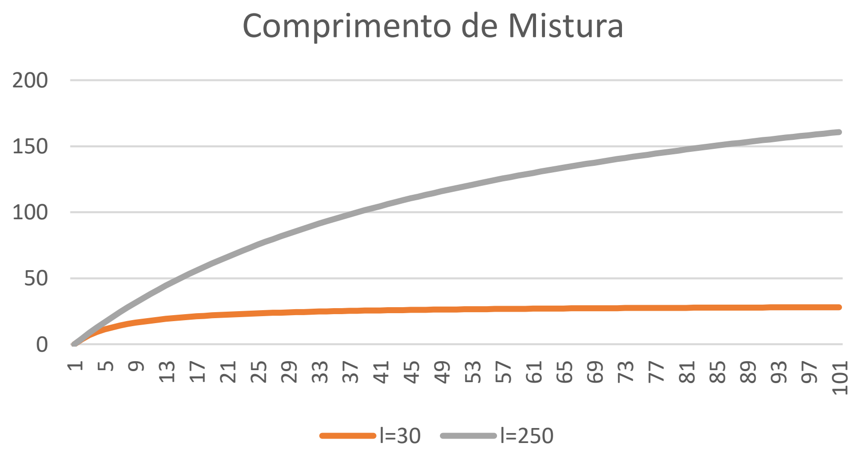


FIG. 1. Typical variation of eddy viscosity K with height in the boundary layer proposed by O'Brien (1970). Adopted from Stull (1988).

Kim (1991) propôs o comprimento de mistura de 30m para a atmosfera livre neutra através de observações por avião

No modelo operacional utiliza-se (λ_0=250 m)

$$l = \frac{\lambda_0 kz}{\lambda_0 + kz}$$



"$\frac{1}{\lambda_0} = \frac{1}{250}$ Isto é reduzido porque o regime instável dentro da camada de mistura agora é levado em consideração pelo esquema de difusão não local."

A função de estabilidade $f_{m,t}(Ri_g)$ são diferentes para regimes estável e instável.

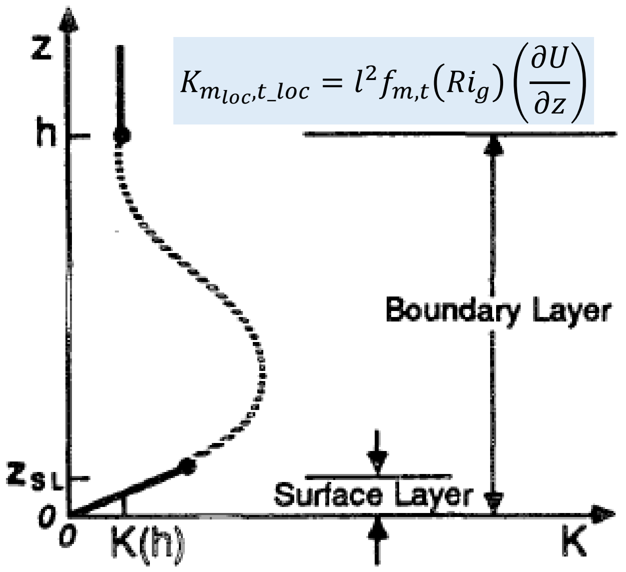


FIG. 1. Typical variation of eddy viscosity K with height in the boundary layer proposed by O'Brien (1970). Adopted from Stull (1988).

Para um atmosfera livre estavelmente estratificada $Ri_g > 0$ adota-se a equação de kim(1991)

$$f_{m,t}(Ri_g) = e^{-8.5Ri_g} + \frac{0.15}{Ri_g + 3.0}$$

Para um atmosfera livre neutralmente e instavelmente estratificada $Ri_g \le 0$ adota-se a equação para a superfície, substituindo $\frac{0.1h}{L}$ por Ri_g

Primeiro para condições instáveis e neutras $\overline{(w'\theta_v')}_0 \le 0$

$$\phi_m = \left(1 - 16\frac{0.1h}{L}\right)^{-\frac{1}{4}}, para\ u\ e\ v \qquad f_m(Ri_g) = (1 - 16Ri_g)^{-\frac{1}{4}}$$

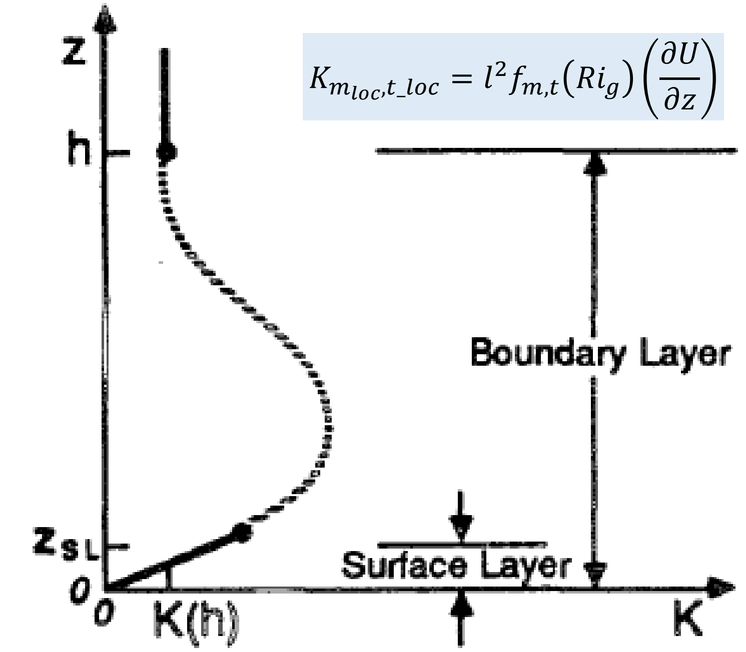
$$\phi_t = \left(1 - 16\frac{0.1h}{L}\right)^{-\frac{1}{2}}, para\ \theta\ e\ q \qquad f_t(Ri_g) = (1 - 16Ri_g)^{-\frac{1}{2}}$$

(1)Quais são as fontes de incerteza do modelo?

Processos físicos da turbulência na camada limite

"O esquema YSU também considera o fluxo de entranhamento acima de h , que expressa a penetração do fluxo de entranhamento acima de h, independentemente da estabilidade local.

O algoritmo YSU-06 não contém uma formulação específica para a Camada Limite Superficial (SBL).



$$K_{m_{loc},t\_loc} = l^2 f_{m,t}(Ri_g)\left(\frac{\partial U}{\partial z}\right)$$

FIG. 1. Typical variation of eddy viscosity K with height in the boundary layer proposed by O'Brien (1970). Adopted from Stull (1988).

Em outras palavras, a mistura de turbulência dentro da SBL é tratada como uma difusão para uma atmosférica livre, calculando os coeficientes de difusão com:

$$\frac{1}{l} = \frac{1}{kz} + \frac{1}{\lambda_0}$$

$$K_{m_{loc},t\_loc} = l^2 f_{m,t}(Ri_g)\left(\frac{\partial U}{\partial z}\right)$$

h é a altura da camada limite

L é a escala de comprimento de MONIN-OBUKHOV

(1)Quais são as fontes de incerteza do modelo?

Processos físicos da turbulência na camada limite

$$K_{m_{loc},t\_loc} = l^2 f_{m,t}(Ri_g)\left(\frac{\partial U}{\partial z}\right)$$

$K_h \sim P_r K_{zm}$

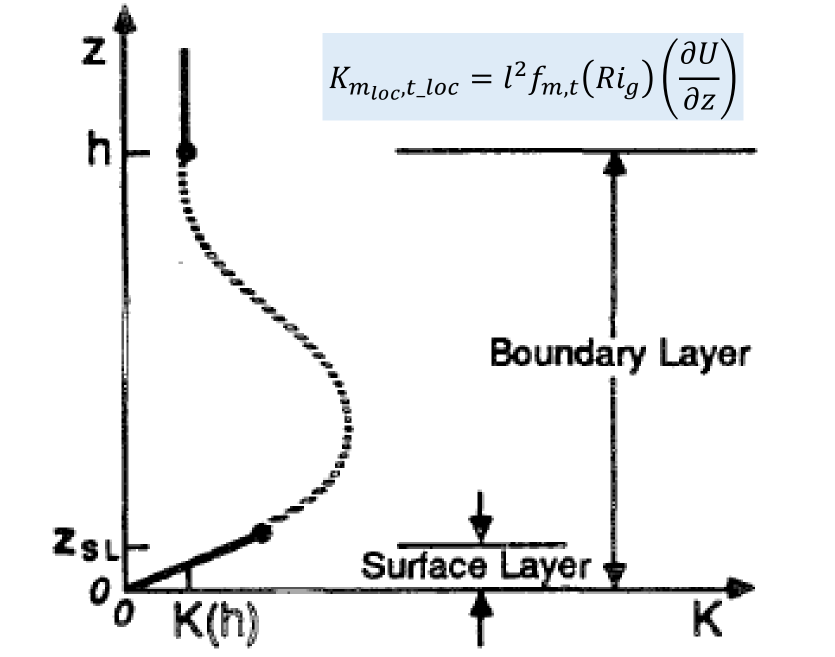


FIG. 1. Typical variation of eddy viscosity K with height in the boundary layer proposed by O'Brien (1970). Adopted from Stull (1988).

Introduz-se uma difusão de fundo (background) $K_{z,0} = 1\frac{m^2}{s}$

"para compensar a difusão numérica, devido ao modo como K_{m_{loc},t_loc} é calculado" Limita-se o valor entre $1\frac{m^2}{s}$ **a 1000** $\frac{m^2}{s}$

E o número de Prant deve ser definido entre (0.24 \`a 4):

$$P_r = 1.5 + 3.08Ri_g$$

h é a altura da camada limite

L *é a escala de comprimento de MONIN-OBUKHOV*

(1)Quais são as fontes de incerteza do modelo?

Processos físicos da turbulência na camada limite

Implementation of a revised SBL scheme in the YSU BL package

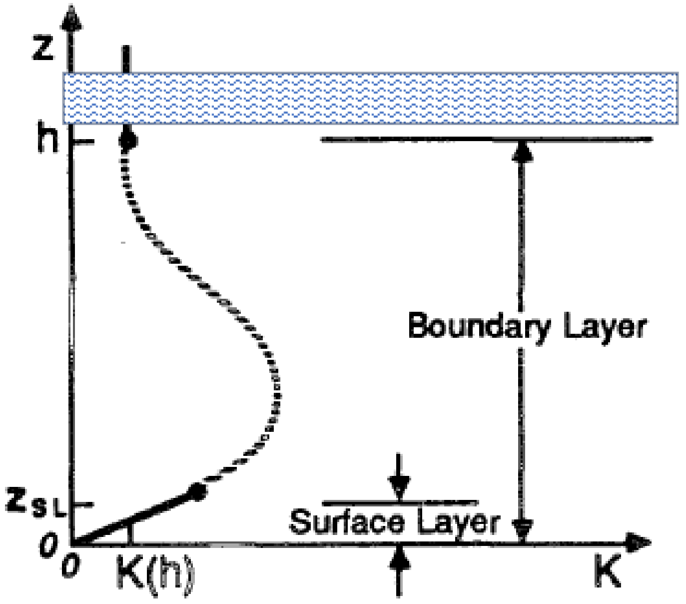


FIG. 1. Typical variation of eddy viscosity K with height in the boundary layer proposed by O'Brien (1970). Adopted from Stull (1988).

Noh et al 2003,

"Propõem-se a adição da contribuição do fluxo de calor devido o entranhamento no topo da Camada Limite Planetária (PBL) "

(1)Quais são as fontes de incerteza do modelo?

Processos físicos da turbulência na camada limite

“Propõem-se a adição da contribuição do fluxo de calor devido o entranhamento no topo da Camada Limite Planetária (PBL) "

Noh et al 2003, modificaram o esquema original

$$-\overline{w'c'} = K_c\left(\frac{\partial C}{\partial z} - \gamma_c\right) - \overline{(w'c')}_h \left(\frac{z}{h}\right)^n$$

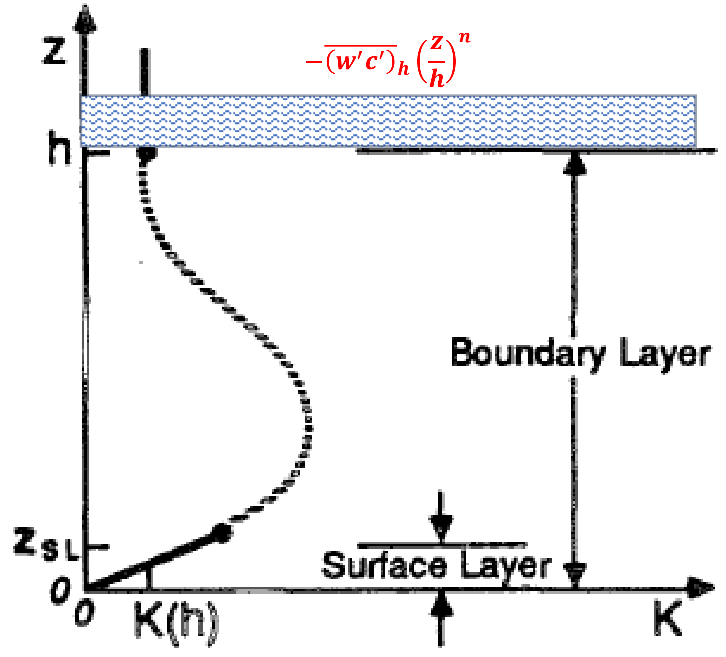


FIG. 1. Typical variation of eddy viscosity K with height in the boundary layer proposed by O'Brien (1970). Adopted from Stull (1988).

"Aqui obtivemos a constante empírica n como $n = 3$ com base na comparação entre os resultados do modelo e dados de LES, mas descobriu-se que os resultados são altamente insensíveis à escolha de n. Por exemplo, simulações com valores substancialmente diferentes de n, como $n = 1\ ou\ 5$, também apresentam resultados semelhantes, pois a contribuição do novo termo é desprezado $\left(\frac{z}{h}\right)^n$, exceto próximo ao topo da Camada Limite Planetária (PBL $z = h \Rightarrow \frac{z}{h} = 1$)."

(1)Quais são as fontes de incerteza do modelo?

Processos físicos da turbulência na camada limite

"O **esquema YSU antigo** usava uma teoria K modificada, **com um termo de contra-gradiente adicional que incorpora a contribuição de vórtices de grande escala ao fluxo total.**"

Hong SY et al 2006. Noh et al 2003, modificaram o esquema original introduzindo o efeito de entranhamento

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial z}\left\{K_c\left(\frac{\partial C}{\partial z} - \gamma_c\right) - \overline{(w'c')}_h\left(\frac{z}{h}\right)^3\right\}$$

$\overline{(w'c')}_h$ **é o fluxo na camada de inversão**

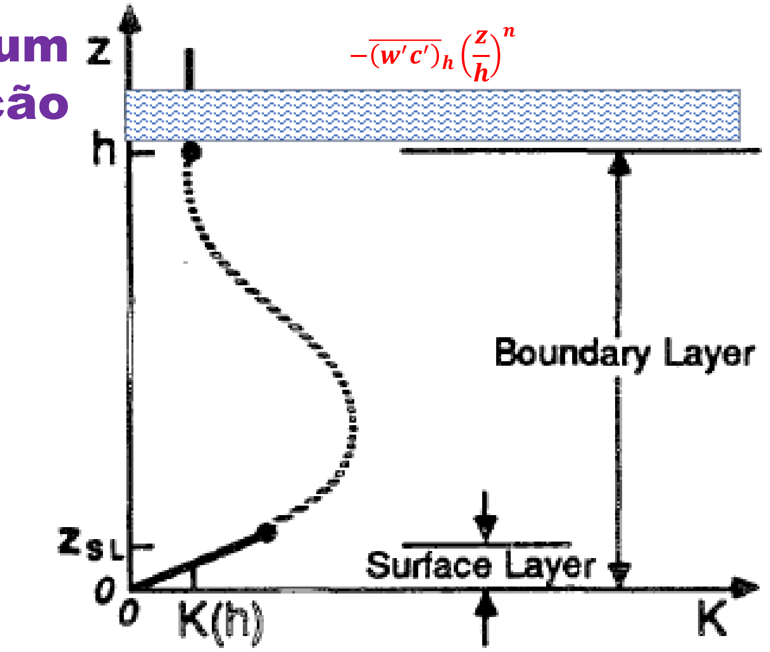


FIG. 1. Typical variation of eddy viscosity K with height in the boundary layer proposed by O'Brien (1970). Adopted from Stull (1988).

"A fórmula mantém o conceito básico de **Hong SY et al 1996**, mas inclui um **termo assintótico de fluxo de entrada na camada de inversão** $\overline{(w'c')}_h\left(\frac{z}{h}\right)^3$."

Neste caso a altura da (PBL) h é definida como o nível em que o fluxo mínimo ocorre na camada de inversão, enquanto em Hong SY et al 1996 é definida como o nível em que a mistura turbulenta da camada limite diminui.

(1)Quais são as fontes de incerteza do modelo?

Processos físicos da turbulência na camada limite

$\overline{(w'c')}_h$ **é o fluxo na camada de inversão "em condições de convecção livre, geralmente é estimado por"**

$$\frac{\overline{w'\theta'_h}}{\overline{w'\theta'_0}} = -A_R \qquad \overline{w'\theta'_h} = -A_R\overline{w'\theta'_0}$$

$$\overline{w'\theta'_h} = -A\frac{w_*^3}{h}$$

"e o valor apropriado para A_R $\left(=\left(\frac{g}{T0}\right)A\right)$ é sugerido estar na faixa entre 0,1 e 0,3 (Ball, 1960)."

$\overline{(w'c')}_h$

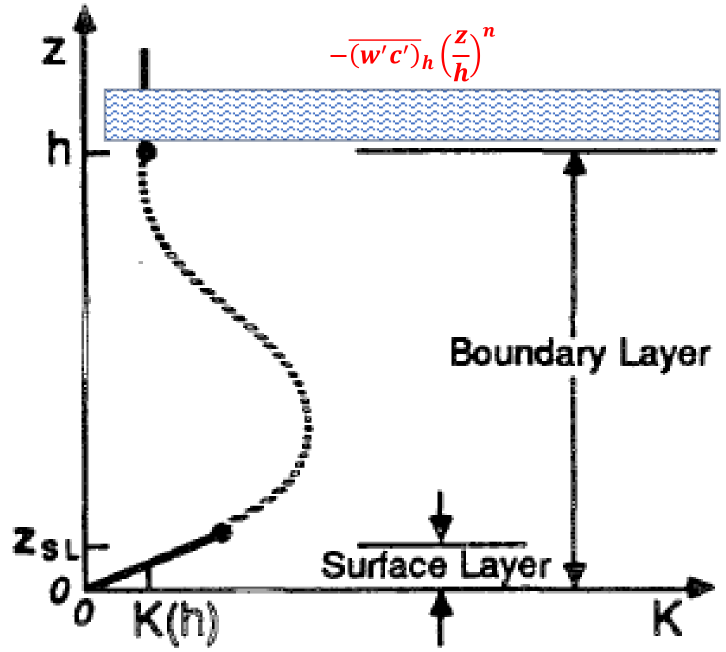


FIG. 1. Typical variation of eddy viscosity K with height in the boundary layer proposed by O'Brien (1970). Adopted from Stull (1988).

"Enquanto isso, na presença de cisalhamento, Moeng e Sullivan (1994) e Driedonks (1982) estenderam a fórmula "

$$\overline{w'\theta'_h} = -A\frac{w_m^3}{h} \qquad w_m^3 = w_*^3 + Bu_*^3$$

"com o valor sugerido B = 5 (Moeng e Sullivan, 1994) ou B = 25 (Driedonks, 1982).

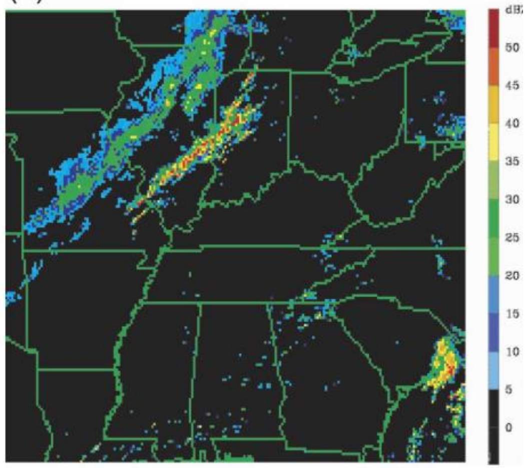
$$w_s = \left(u_*^3 + \frac{8kw_{*b}^3 z}{h}\right)^{\frac{1}{3}}$$

Note que a escala de velocidade w_s não pode ser aplicada aqui, pois a eficiência do entranhamento é diferente entre a turbulência induzida por cisalhamento e a turbulência convectiva."

(1)Quais são as fontes de incerteza do modelo?

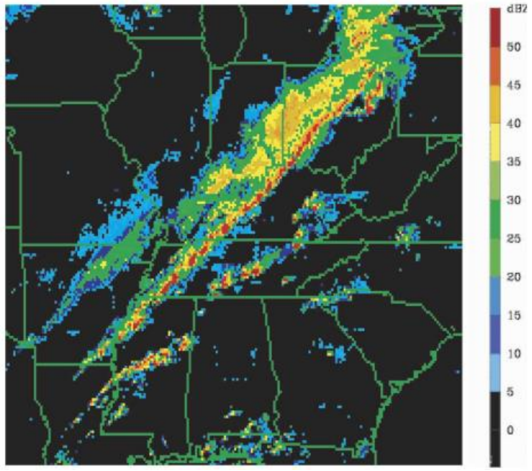
Processos físicos da turbulência na camada limite

(a) *observado*



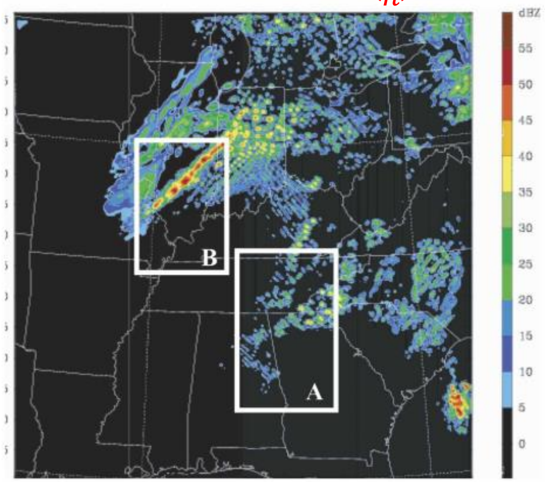
maximum (dBZ) at (a) 1800 UTC
10 Nov 2002

(b)



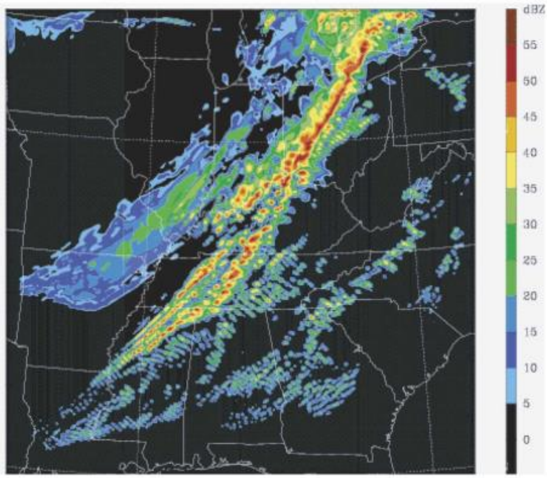
maximum (dBZ) at (a)0000 UTC
11 Nov 2002

(a) *com o* $\overline{(w'c')}_h \left(\frac{z}{h}\right)^3$



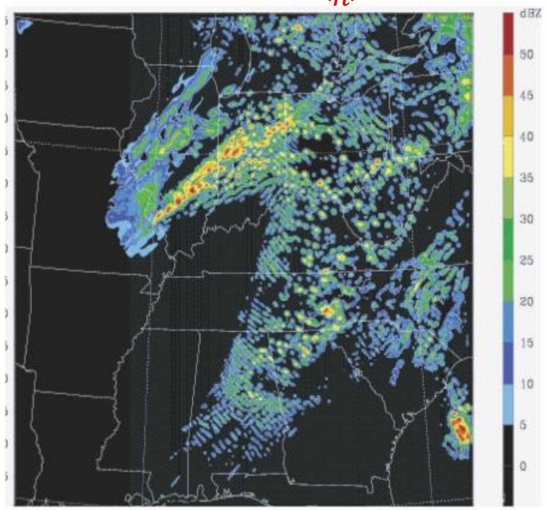
maximum (dBZ) at (a) 1800 UTC
10 Nov 2002

(b)



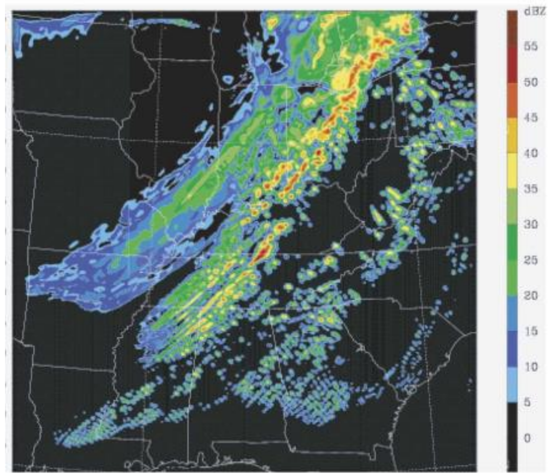
maximum (dBZ) at (a)0000 UTC
11 Nov 2002

(c) *sem o* $\overline{(w'c')}_h \left(\frac{z}{h}\right)^3$



maximum (dBZ) at (a) 1800 UTC
10 Nov 2002

(d)



maximum (dBZ) at (a)0000 UTC
11 Nov 2002

(1)Quais são as fontes de incerteza do modelo?

Processos físicos da turbulência na camada limite

$\overline{(w'c')}_h$ é o fluxo na camada de inversão

1. "O esquema YSU de (PBL) aumenta a mistura na camada limite durante regimes de convecção livre induzida termicamente e a diminui a mistura durante regimes de convecção forçada induzida mecanicamente.
2. A mistura excessiva na camada de mistura na presença de ventos fortes é resolvida.
3. O crescimento excessivamente rápido da PBL no caso de Hong e Pan também é corrigido.
4. .Consequentemente, o novo esquema reproduz melhor a inibição convectiva.
5. Isso ocorre porque a camada limite do esquema YSU PBL permanece menos diluída pelo entranhamento, deixando mais energia para a convecção severa quando a frente a dispara."

(1)Quais são as fontes de incerteza do modelo?

Processos físicos da turbulência na camada limite

Erros

incerteza processo físico

erro estrutural

$$\frac{\partial u}{\partial t} = K \frac{\partial^2 u}{\partial^2 z}$$

$$K(x, y, z, ri, cloud, rad, entr, etc)$$

- explícito
- implícito
- Volume finito?

Incerteza na observação, interpretação

Processo relacionados ao regime de turbulência e estabilidade da CLP

Limitação da Teoria de Monin-Obukov

incerteza parâmetro

- Ric =>critico (regime de estabilidade)
- Comprimento de mistura mínimo
- Limitação de hpbl com tke.
- Funções adimensionais
- Os parâmetros não são considerações universais

Implementação da parametrização YSU BL

(1)Quais são as fontes de incerteza do modelo?

Processos físicos da turbulência na camada limite

Implementation of a revised SBL scheme in the YSU BL package

“Propoem-se a adição da contribuição do fluxo de calor devido o entranhamento no topo da Camada Limite Planetária (PBL) "

Noh et al 2003, modificaram o esquema original

$$\frac{\partial C}{\partial t} = \frac{\partial[-\overline{w'c'}]}{\partial z}$$

$$-\overline{w'c'} = K_c\left(\frac{\partial C}{\partial z} - \gamma_c\right) - \overline{(w'c')}_h\left(\frac{z}{h}\right)^n$$

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial z}\left[K_c\left(\frac{\partial C}{\partial z} - \gamma_c\right)\right]$$

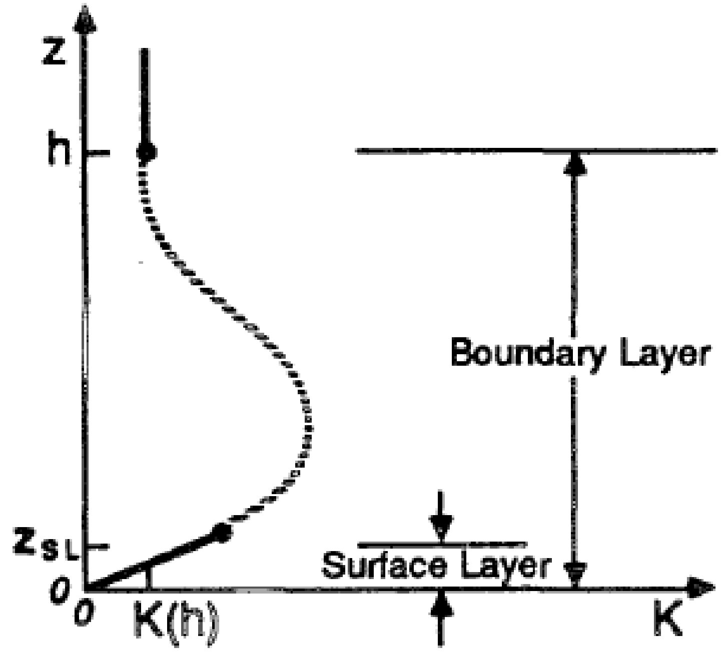


FIG. 1. Typical variation of eddy viscosity K with height in the boundary layer proposed by O'Brien (1970). Adopted from Stull (1988).

"Aqui obtivemos a constante empírica n como $n = 3$ com base na comparação entre os resultados do modelo e dados de LES, mas descobriu-se que os resultados são altamente insensíveis à escolha de n. Por exemplo, simulações com valores substancialmente diferentes de n, como $n = 1 \text{ ou } 5$, também apresentam resultados semelhantes, pois a contribuição do novo termo é desprezado $\left(\frac{z}{h}\right)^n$, exceto próximo ao topo da Camada Limite Planetária (PBL $z = h \Rightarrow \frac{z}{h} = 1$)."

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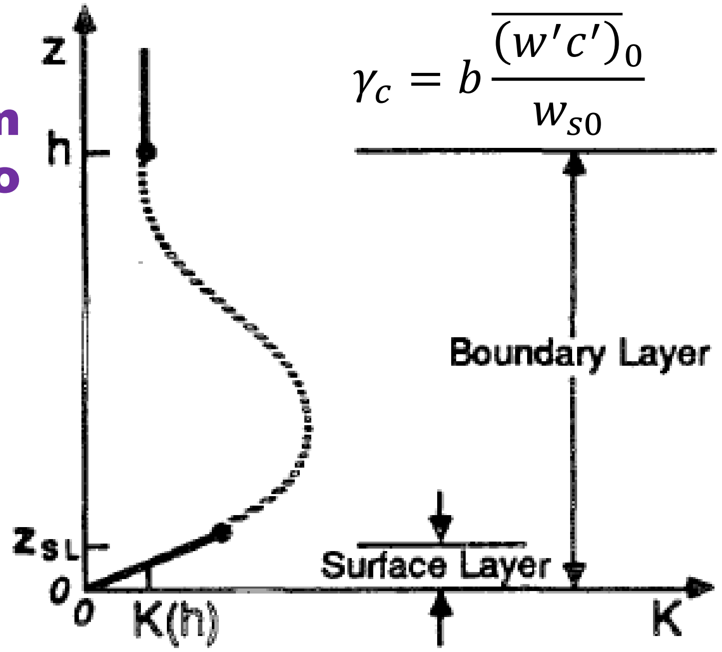
Processos físicos da turbulência na camada limite

Implementation of a revised SBL scheme in the YSU BL package

"O esquema YSU antigo usava uma teoria K modificada, com um termo de contra-gradiente adicional que incorpora a contribuição de vórtices de grande escala ao fluxo total."



$$\gamma_c = b\frac{\overline{(w'c')_0}}{w_{s0}}$$

FIG. 1. Typical variation of eddy viscosity K with height in the boundary layer proposed by O'Brien (1970). Adopted from Stull (1988).

Hong SY et al 2006. Noh et al 2003, modificaram o esquema original introduzindo o efeito de entranhamento

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial z}\left\{K_c\left(\frac{\partial C}{\partial z} - \gamma_c\right) - \overline{(w'c')_h}\left(\frac{z}{h}\right)^3\right\}$$

$$\frac{\partial C}{\partial t} = \left\{\frac{\partial}{\partial z}K_c\frac{\partial C}{\partial z} - \frac{\partial}{\partial z}\left(K_c\gamma_c + \overline{(w'c')_h}\left(\frac{z}{h}\right)^3\right)\right\}$$

$\overline{(w'c')_h}$ **é o fluxo na camada de inversão**

"A fórmula mantém o conceito básico de Hong SY et al 1996, mas inclui um termo assintótico de fluxo de entrada na camada de inversão $\overline{(w'c')_h}\left(\frac{z}{h}\right)^3$.

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Implementation of a revised SBL scheme in the YSU BL package

$$\frac{\partial C}{\partial t} = \left\{\frac{\partial}{\partial z} K_c \frac{\partial C}{\partial z} - \frac{\partial}{\partial z}\left(K_c \gamma_c + \overline{(w'c')}_h \left(\frac{z}{h}\right)^3\right)\right\}$$

$$\int_k^{k+1} \frac{\partial C}{\partial t} dz = \left\{\int_k^{k+1} \frac{\partial}{\partial z} K_c \frac{\partial C}{\partial z} \partial z - \int_k^{k+1} \frac{\partial}{\partial z}\left(K_c \gamma_c + \overline{(w'c')}_h \left(\frac{z}{h}\right)^3\right) \partial z\right\}$$

$$\frac{\partial}{\partial t} \int_k^{k+1} C\, dz = \left\{ K_c \frac{\partial C}{\partial z}\Big|_k^{k+1} - \left(K_c \gamma_c + \overline{(w'c')}_h \left(\frac{z}{h}\right)^3\right)\Big|_k^{k+1} \right\}$$

$$\frac{\partial \bar{C}}{\partial t} = \left\{\left(K_c \frac{\partial C}{\partial z}\right)^{k+1} - \left(K_c \frac{\partial C}{\partial z}\right)^{k} - \left(K_c \gamma_c + \overline{(w'c')}_h \left(\frac{z}{h}\right)^3\right)^{k+1} + \left(K_c \gamma_c + \overline{(w'c')}_h \left(\frac{z}{h}\right)^3\right)^{k}\right\}$$

$$\frac{\partial \bar{C}}{\partial t} - \left(K_c \frac{\partial C}{\partial z}\right)^{k+1} + \left(K_c \frac{\partial C}{\partial z}\right)^{k} = \left\{-\left(K_c \gamma_c + \overline{(w'c')}_h \left(\frac{z}{h}\right)^3\right)^{k+1} + \left(K_c \gamma_c + \overline{(w'c')}_h \left(\frac{z}{h}\right)^3\right)^{k}\right\}$$

Paulo Yoshio Kubota

(1)Quais são as fontes de incerteza do modelo?

Processos físicos da turbulência na camada limite

Implementation of a revised SBL scheme in the YSU BL package

$$\frac{\partial \bar{C}}{\partial t} - \left(K_c \frac{\partial C}{\partial z}\right)^{k+1} + \left(K_c \frac{\partial C}{\partial z}\right)^{k} = -\left(K_c\gamma_c + \overline{(w'c')}_h \left(\frac{z}{h}\right)^3\right)^{k+1} + \left(K_c\gamma_c + \overline{(w'c')}_h \left(\frac{z}{h}\right)^3\right)^{k}$$

$$\frac{C_k^{n+1} - C_k^{n-1}}{2\Delta t} - \left({K_c}_{k+1}^{n} \frac{C_{k+1}^{n+1} - C_k^{n+1}}{\Delta z}\right)^{k+1} + \left({K_c}_{k}^{n} \frac{C_{k}^{n+1} - C_{k-1}^{n+1}}{\Delta z}\right)^{k} = -\left(K_c\gamma_c + \overline{(w'c')}_h \left(\frac{z}{h}\right)^3\right)^{k+1} + \left(K_c\gamma_c + \overline{(w'c')}_h \left(\frac{z}{h}\right)^3\right)^{k}$$

$$\frac{C_k^{n+1} - C_k^{n-1}}{2\Delta t} - {K_c}_{k+1}^{n} \frac{C_{k+1}^{n+1} - C_k^{n+1}}{\Delta z} + {K_c}_{k}^{n} \frac{C_{k}^{n+1} - C_{k-1}^{n+1}}{\Delta z} = -\left(K_c\gamma_c + \overline{(w'c')}_h \left(\frac{Z}{h}\right)^3\right)^{k+1} + \left(K_c\gamma_c + \overline{(w'c')}_h \left(\frac{Z}{h}\right)^3\right)^{k}$$

$$C_k^{n+1} - C_k^{n-1} - \frac{{K_c}_{k+1}^{n} 2\Delta t}{\Delta z}\left(C_{k+1}^{n+1} - C_k^{n+1}\right) + \frac{{K_c}_{k}^{n} 2\Delta t}{\Delta z}\left(C_{k}^{n+1} - C_{k-1}^{n+1}\right) = -2\Delta t\left(K_c\gamma_c + \overline{(w'c')}_h \left(\frac{z}{h}\right)^3\right)^{k+1} + 2\Delta t\left(K_c\gamma_c + \overline{(w'c')}_h \left(\frac{z}{h}\right)^3\right)^{k}$$

$$C_k^{n+1} - C_k^{n-1} + \left(-\frac{{K_c}_{k+1}^{n} 2\Delta t}{\Delta z} C_{k+1}^{n+1} + \frac{{K_c}_{k+1}^{n} 2\Delta t}{\Delta z} C_{k}^{n+1}\right) + \left(\frac{{K_c}_{k}^{n} 2\Delta t}{\Delta z} C_{k}^{n+1} - \frac{{K_c}_{k}^{n} 2\Delta t}{\Delta z} C_{k-1}^{n+1}\right) = -2\Delta t\left(K_c\gamma_c + \overline{(w'c')}_h \left(\frac{z}{h}\right)^3\right)^{k+1} + 2\Delta t\left(K_c\gamma_c + \overline{(w'c')}_h \left(\frac{z}{h}\right)^3\right)^{k}$$

$$-\frac{{K_c}_{k+1}^{n} 2\Delta t}{\Delta z} C_{k+1}^{n+1} + \frac{{K_c}_{k+1}^{n} 2\Delta t}{\Delta z} C_{k}^{n+1} + \frac{{K_c}_{k}^{n} 2\Delta t}{\Delta z} C_{k}^{n+1} + C_k^{n+1} + \left(-\frac{{K_c}_{k}^{n} 2\Delta t}{\Delta z} C_{k-1}^{n+1}\right) = C_k^{n-1} - 2\Delta t\left(K_c\gamma_c + \overline{(w'c')}_h \left(\frac{z}{h}\right)^3\right)^{k+1} + 2\Delta t\left(K_c\gamma_c + \overline{(w'c')}_h \left(\frac{z}{h}\right)^3\right)^{k}$$

Paulo Yoshio Kubota

(1)Quais são as fontes de incerteza do modelo?

Processos físicos da turbulência na camada limite

Implementation of a revised SBL scheme in the YSU BL package

$$-\frac{K{c}_{k+1}^{n}2\Delta t}{\Delta z}C_{k+1}^{n+1}+\frac{K{c}_{k+1}^{n}2\Delta t}{\Delta z}C_{k}^{n+1}+\frac{K{c}_{k}^{n}2\Delta t}{\Delta z}C_{k}^{n+1}+C_{k}^{n+1}+\left(-\frac{K{c}_{k}^{n}2\Delta t}{\Delta z}C_{k-1}^{n+1}\right)=C_{k}^{n-1}-2\Delta t\left(K_c\gamma_c+\overline{(w'c')}_h\left(\frac{z}{h}\right)^3\right)^{k+1}+2\Delta t\left(K_c\gamma_c+\overline{(w'c')}_h\left(\frac{z}{h}\right)^3\right)^{k}$$

$$-\frac{K{c}_{k+1}^{n}2\Delta t}{\Delta z}C_{k+1}^{n+1}+\frac{K{c}_{k+1}^{n}2\Delta t}{\Delta z}C_{k}^{n+1}+\frac{K{c}_{k}^{n}2\Delta t}{\Delta z}C_{k}^{n+1}+C_{k}^{n+1}+\left(-\frac{K{c}_{k}^{n}2\Delta t}{\Delta z}C_{k-1}^{n+1}\right)=C_{k}^{n-1}-2\Delta t\left(K_c\gamma_c+\overline{(w'c')}_h\left(\frac{z}{h}\right)^3\right)^{k+1}+2\Delta t\left(K_c\gamma_c+\overline{(w'c')}_h\left(\frac{z}{h}\right)^3\right)^{k}$$

$$\left(-\frac{K{c}_{k}^{n}2\Delta t}{\Delta z}\right)C_{k-1}^{n+1}+\left(\frac{K{c}_{k+1}^{n}2\Delta t}{\Delta z}+\frac{K{c}_{k}^{n}2\Delta t}{\Delta z}+1\right)C_{k}^{n+1}-\frac{K{c}_{k+1}^{n}2\Delta t}{\Delta z}C_{k+1}^{n+1}=C_{k}^{n-1}-2\Delta t\left(K_c\gamma_c+\overline{(w'c')}_h\left(\frac{z}{h}\right)^3\right)^{k+1}+2\Delta t\left(K_c\gamma_c+\overline{(w'c')}_h\left(\frac{z}{h}\right)^3\right)^{k}$$

$$(AA)C_{k-1}^{n+1}+BBC_{k}^{n+1}-DDC_{k+1}^{n+1}=C_{k}^{n-1}+f1_{k}-f1_{k+1}$$

$$(AA)C_{k-1}^{n+1}+BBC_{k}^{n+1}-DDC_{k+1}^{n+1}=FF_{k}^{n-1}$$

$$\begin{bmatrix} BB & DD & 0 & 0 & 0 & 0 \\ AA & BB & DD & 0 & 0 & 0 \\ 0 & AA & BB & DD & 0 & 0 \\ 0 & 0 & AA & BB & DD & 0 \\ 0 & 0 & 0 & AA & BB & DD \\ 0 & 0 & 0 & 0 & AA & BB \end{bmatrix} * \begin{bmatrix} C_1^{n+1} \\ C_2^{n+1} \\ C_3^{n+1} \\ C_4^{n+1} \\ C_5^{n+1} \\ C_6^{n+1} \end{bmatrix} = \begin{bmatrix} FF_1^{n-1} \\ FF_2^{n-1} \\ FF_3^{n-1} \\ FF_4^{n-1} \\ FF_5^{n-1} \\ FF_6^{n-1} \end{bmatrix} \longleftarrow \frac{\partial C}{\partial t}=\frac{\partial[-\overline{w'c'}]}{\partial z}$$

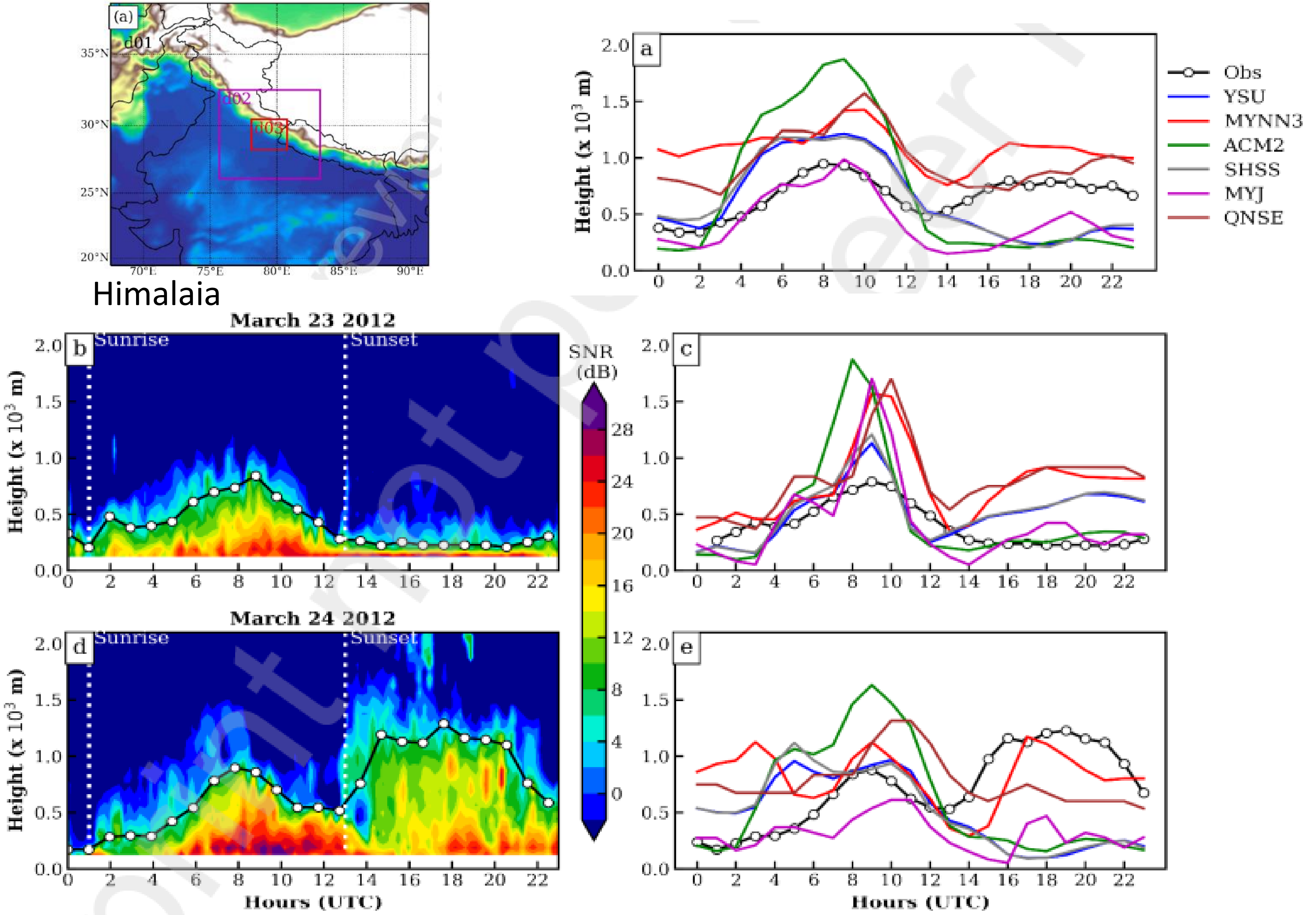
Paulo Yoshio Kubota

Alguns resultados interessantes

Atmosphere Numerical Modeling Group

(1)Quais são as fontes de incerteza do modelo?

Processos físicos da turbulência na camada limite



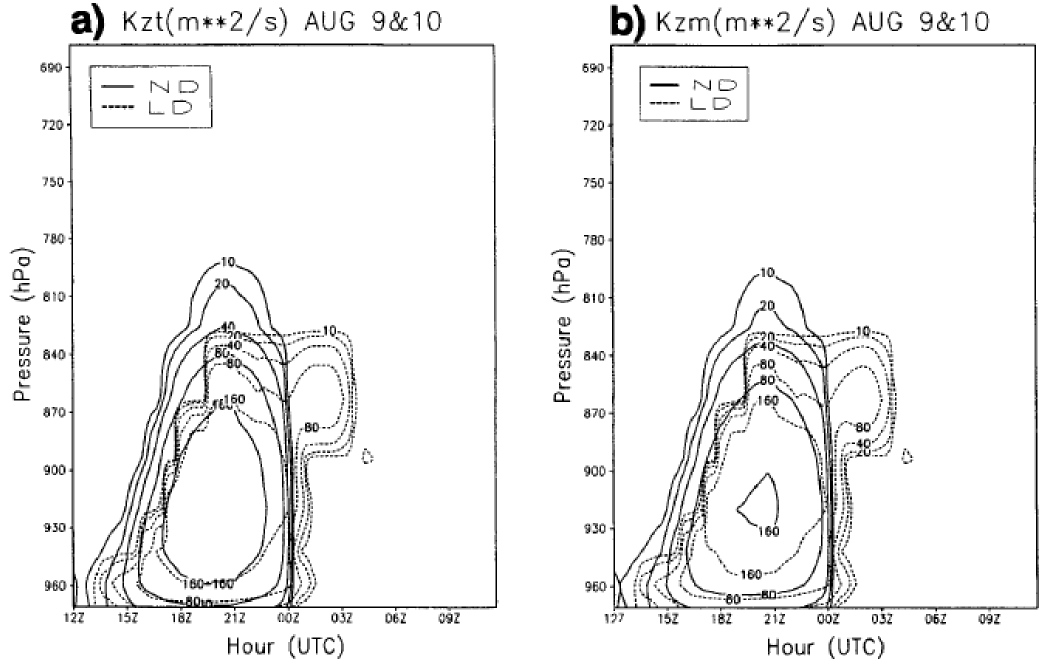
"Variação diurna da altura da (PBLH) (a) simulada usando diferentes esquemas de PBL e observações de RWP.
A variação da relação sinal-ruído (SNR) junto com a PBLH para

(b) 23 de março de 2012, com baixa PBLH durante a noite.

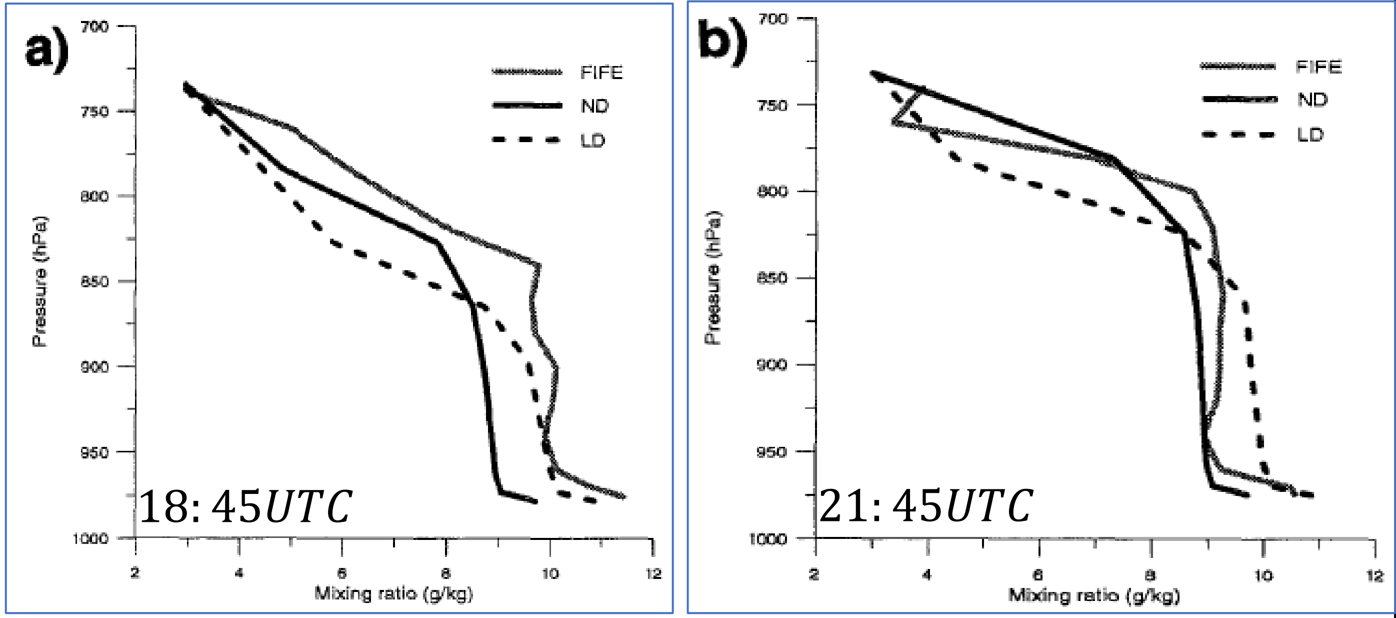
(d) 24 de março de 2012, com PBLH mais alta durante a noite."

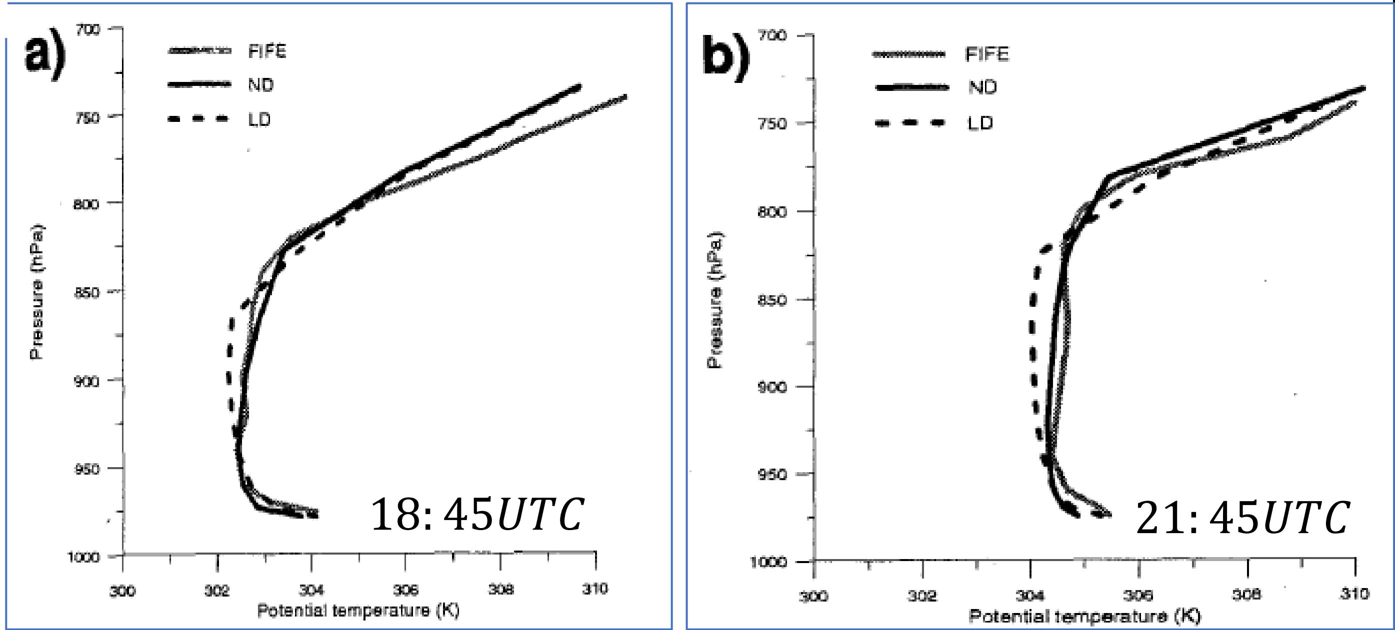
(1)Quais são as fontes de incerteza do modelo?

Processos físicos da turbulência na camada limite



Perfil de difusividade de Eddies ($\frac{m^2}{s}$) linha pontilhada (local) e linhas continuas não local





(1)Quais são as fontes de incerteza do modelo?

Processos físicos da turbulência na camada limite

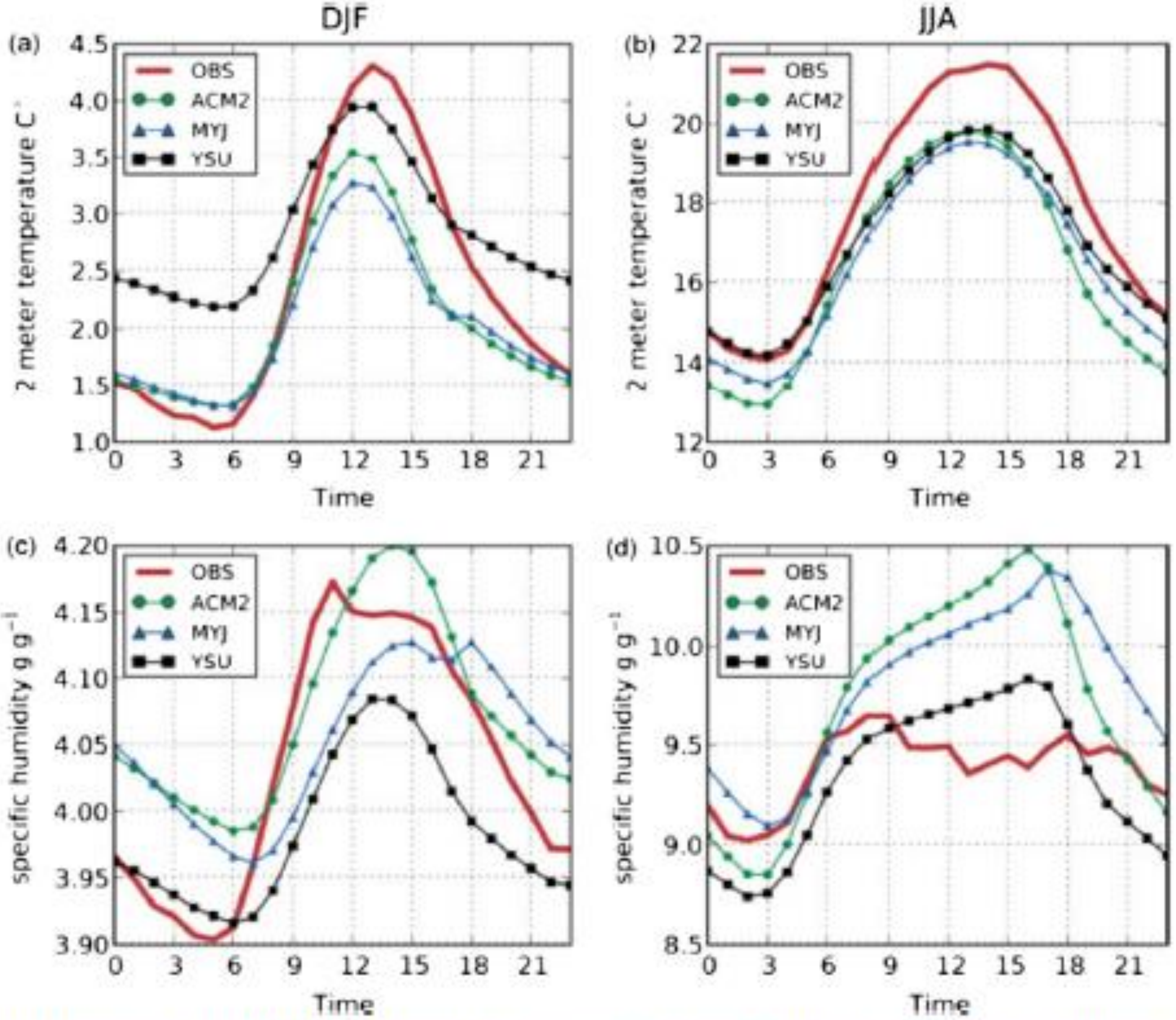


FIG. 3. Comparisons of WRF PBL schemes with European synoptic observations for temperatures during the (a) winter and (b) summer and for specific humidity during the (c) winter and (d) summer [from García-Díez et al. (2013)].

(1)Quais são as fontes de incerteza do modelo?

Processos físicos da turbulência na camada limite

near-surface

balloon sounding

celiometer

Evaluation

Meteorlogy

$PM_{2.5}$

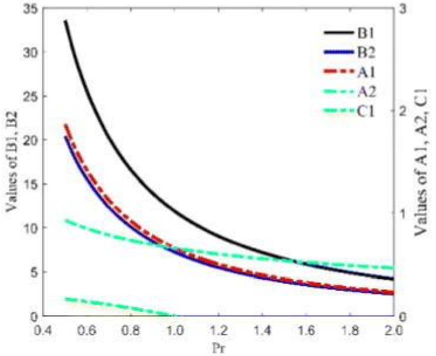
K_m

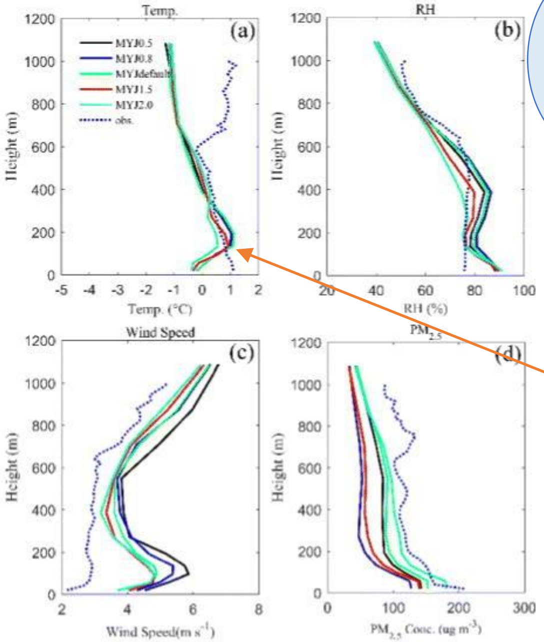
PBL parameterization

- YSU
- ACM2
- MYJ
- MYNN
- BouLac

Winter Haze Simulation

Parametric Sensitivity





A formulação do calculo de K não incorpora corretamente os processos físicos. Os parâmetros podem estar mal calibrados

$$K(x, y, z, ri, cloud, rad, entr, ..)$$

Implementação da parametrização de Melor Yamada Nino Nakanishi com fluxo de Massa

$$K_\phi = c_\phi L_k \sqrt{\bar{e}}$$

$$\overline{w'\phi'} = -K\frac{\partial \phi}{\partial z} + M_{\phi,u}(\phi_u - \phi) - M_{\phi,d}(\phi_d - \phi)$$

$$K_\phi = S_\phi L_k \sqrt{2\bar{e}}$$

Linearização da Equações de Navier Stokes

5.2 TURBULENT KINETIC ENERGY **A equação de Navier Stokes: conservação de momentum**

i=1,2,3
J=1,2,3
K=1,2,3

$$\frac{Du_i}{Dt} = \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho_0}\frac{\partial P}{\partial x_i} - \frac{\rho}{\rho_0} g\delta_{i3} - 2\Omega\varepsilon_{ijk}\eta_j u_k + \nu\left(\frac{\partial^2 u_i}{\partial {x_j}^2}\right)$$

$u_i = \overline{u_i} + u_i{}'$

Aplique a Média de Reynolds na Variáveis

$$\frac{\partial(\overline{u_i} + u_i{}')}{\partial t} + (\overline{u_j} + u_j{}')\frac{\partial(\overline{u_i} + u_i{}')}{\partial x_j} = -\frac{1}{\rho_0}\frac{\partial(\bar{P} + P')}{\partial x_i} - g\frac{(\rho + \rho')}{\rho_0}\delta_{i3} - 2\Omega\varepsilon_{ijk}\eta_j(\overline{u_k} + u_k{}') + \nu\left(\frac{\partial^2(\overline{u_i} + u_i{}')}{\partial {x_j}^2}\right)$$

Expanda os termos

$$\frac{\partial(\overline{u_i})}{\partial t} + \frac{\partial(u_i{}')}{\partial t} + (\overline{u_j})\frac{\partial(\overline{u_i})}{\partial x_j} + (\overline{u_j})\frac{\partial(u_i{}')}{\partial x_j} + (u_j{}')\frac{\partial(\overline{u_i})}{\partial x_j} + (u_j{}')\frac{\partial(u_i{}')}{\partial x_j} =$$
$$-\frac{1}{\rho_0}\frac{\partial(\bar{P})}{\partial x_i} - \frac{1}{\rho_0}\frac{\partial(P')}{\partial x_i} - g\frac{\rho}{\rho_0}\delta_{i3} - g\frac{\rho'}{\rho_0}\delta_{i3} - 2\Omega\varepsilon_{ijk}\eta_j(\overline{u_k}) - 2\Omega\varepsilon_{ijk}\eta_j(u_k{}') + \nu\frac{\partial^2(\overline{u_i})}{\partial {x_j}^2} + \nu\frac{\partial^2(u_i{}')}{\partial {x_j}^2}$$

A equação de Navier Stokes: conservação de momentum

$$\frac{\partial(\overline{u_i})}{\partial t}+\frac{\partial(u_i{}')}{\partial t}+(\overline{u_j})\frac{\partial(\overline{u_i})}{\partial x_j}+(\overline{u_j})\frac{\partial(u_i{}')}{\partial x_j}+(u_j{}')\frac{\partial(\overline{u_i})}{\partial x_j}+(u_j{}')\frac{\partial(u_i{}')}{\partial x_j}=$$
$$-\frac{1}{\rho_0}\frac{\partial(\bar{P})}{\partial x_i}-\frac{1}{\rho_0}\frac{\partial(P')}{\partial x_i}-g\frac{\rho}{\rho_0}\delta_{i3}-g\frac{\rho'}{\rho_0}\delta_{i3}-2\Omega\varepsilon_{ijk}\eta_j(\overline{u_k})-2\Omega\varepsilon_{ijk}\eta_j(u_k{}')+\nu\frac{\partial^2(\overline{u_i})}{\partial x_j{}^2}+\nu\frac{\partial^2(u_i{}')}{\partial x_j{}^2}$$

Separa os termos na equação acima

$$\frac{\partial(\overline{u_i})}{\partial t}+(\overline{u_j})\frac{\partial(\overline{u_i})}{\partial x_j}+(\overline{u_j})\frac{\partial(u_i{}')}{\partial x_j}+(u_j{}')\frac{\partial(\overline{u_i})}{\partial x_j}+(u_j{}')\frac{\partial(u_i{}')}{\partial x_j}=$$
$$-\frac{1}{\rho_0}\frac{\partial(\bar{P})}{\partial x_i}-g\frac{\rho}{\rho_0}\delta_{i3}-2\Omega\varepsilon_{ijk}\eta_j(\overline{u_k})+\nu\frac{\partial^2(\overline{u_i})}{\partial x_j{}^2}$$

A equação de Navier Stokes: conservação de momentum

$$\frac{\partial(\bar{u}_i)}{\partial t} + (\bar{u}_j)\frac{\partial(\bar{u}_i)}{\partial x_j} + (u_j')\frac{\partial(\bar{u}_i)}{\partial x_j} + \boxed{(u_j')\frac{\partial(u_i')}{\partial x_j}} = -\frac{1}{\rho_0}\frac{\partial(\bar{P})}{\partial x_i} - g\frac{\rho}{\rho_0}\delta_{i3} - 2\Omega\varepsilon_{ijk}\eta_j(\bar{u}_k) + \nu\frac{\partial^2(\bar{u}_i)}{\partial {x_j}^2}$$

$$\frac{\partial(\bar{u}_i)}{\partial t} + (\bar{u}_j)\frac{\partial(\bar{u}_i)}{\partial x_j} + (u_j')\frac{\partial(\bar{u}_i)}{\partial x_j} + \boxed{\frac{\partial(u_j'u_i')}{\partial x_j} - (u_i')\frac{\partial(u_j')}{\partial x_j}} = -\frac{1}{\rho_0}\frac{\partial(\bar{P})}{\partial x_i} - g\frac{\rho}{\rho_0}\delta_{i3} - 2\Omega\varepsilon_{ijk}\eta_j(\bar{u}_k) + \nu\frac{\partial^2(\bar{u}_i)}{\partial {x_j}^2}$$

Aplique as media de Reynolds

$$\frac{\partial(\bar{\bar{u}}_i)}{\partial t} + (\bar{\bar{u}}_j)\frac{\partial(\bar{\bar{u}}_i)}{\partial x_j} + \frac{\partial(\overline{u_j'u_i'})}{\partial x_j} - (\overline{u_i'})\frac{\partial(\overline{u_j'})}{\partial x_j} + (\overline{u_j'})\frac{\partial(\bar{u}_i)}{\partial x_j} = -\frac{1}{\rho_0}\frac{\partial(\bar{\bar{P}})}{\partial x_i} - g\frac{\bar{\rho}}{\rho_0}\delta_{i3} - 2\Omega\varepsilon_{ijk}\eta_j(\overline{\bar{u}_k}) + \nu\frac{\partial^2(\bar{\bar{u}}_i)}{\partial {x_j}^2}$$

Paulo Yoshio Kubota

Linearização da Equações de Navier Stokes

A equação de Navier Stokes: conservação de momentum

$$\frac{\partial(\overline{\overline{u}}_i)}{\partial t} + (\overline{\overline{u}}_j)\frac{\partial(\overline{\overline{u}}_i)}{\partial x_j} + \frac{\partial\left(\overline{u_j' u_i'}\right)}{\partial x_j} - (\overline{u_i'})\frac{\partial\left(\overline{u_j'}\right)}{\partial x_j} + (\overline{u_j'})\frac{\partial(\overline{u_i})}{\partial x_j} = -\frac{1}{\rho_0}\frac{\partial\left(\overline{\overline{P}}\right)}{\partial x_i} - g\frac{\bar{\rho}}{\rho_0}\delta_{i3} - 2\Omega\varepsilon_{ijk}\eta_j(\overline{\overline{u_k}}) + \nu\frac{\partial^2(\overline{\overline{u}}_i)}{\partial {x_j}^2}$$

Aplique as considerações da media de Reynolds

$\overline{\overline{u}}_i = \overline{u}_i$

$\overline{u_j' u_i'} \neq 0$

$\overline{u_j'} = 0$

$\overline{u_i'} = 0$

$$\frac{\partial(\overline{u}_i)}{\partial t} + (\overline{u}_j)\frac{\partial(\overline{u}_i)}{\partial x_j} = -\frac{\partial\left(\overline{u_j' u_i'}\right)}{\partial x_j} - \frac{1}{\rho_0}\frac{\partial(\bar{P})}{\partial x_i} - g\frac{\bar{\rho}}{\rho_0}\delta_{i3} - 2\Omega\varepsilon_{ijk}\eta_j(\overline{u_k}) + \nu\frac{\partial^2(\overline{u}_i)}{\partial {x_j}^2}$$

- ***Equação primitiva não linear para o escoamento da Atmosfera***

$$\frac{Du_i}{Dt} = \frac{\partial u_i}{\partial t} + u_j\frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho_0}\frac{\partial P}{\partial x_i} - \frac{\rho}{\rho_0}g\delta_{i3} - 2\Omega\varepsilon_{ijk}\eta_j u_k + \nu\left(\frac{\partial^2 u_i}{\partial {x_j}^2}\right)$$

- ***Equação Governante Linearizada do Estado Médio do escoamento da Atmosfera***

$$\frac{\partial(\overline{u_i})}{\partial t} + (\overline{u_j})\frac{\partial(\overline{u_i})}{\partial x_j} = -\frac{\partial\left(\overline{u_j' u_i'}\right)}{\partial x_j} - \frac{1}{\rho_0}\frac{\partial(\bar{P})}{\partial x_i} - g\frac{\bar{\rho}}{\rho_0}\delta_{i3} - 2\Omega\varepsilon_{ijk}\eta_j(\overline{u_k}) + \nu\frac{\partial^2(\overline{u_i})}{\partial {x_j}^2}$$

Paulo Yoshio Kubota

5.2 TURBULENT KINETIC ENERGY **A equação de Navier Stokes: conservação de momentum**

$$\frac{\partial(\overline{u_i})}{\partial t}+\frac{\partial(u_i')}{\partial t}+(\overline{u_j})\frac{\partial(\overline{u_i})}{\partial x_j}+(\overline{u_j})\frac{\partial(u_i')}{\partial x_j}+(u_j')\frac{\partial(\overline{u_i})}{\partial x_j}+(u_j')\frac{\partial(u_i')}{\partial x_j}=$$
$$-\frac{1}{\rho_0}\frac{\partial(\overline{P})}{\partial x_i}-\frac{1}{\rho_0}\frac{\partial(P')}{\partial x_i}-g\frac{\rho}{\rho_0}\delta_{i3}-g\frac{\rho'}{\rho_0}\delta_{i3}-2\Omega\varepsilon_{ijk}\eta_j(\overline{u_k})-2\Omega\varepsilon_{ijk}\eta_j(u_k')+\nu\frac{\partial^2(\overline{u_i})}{\partial {x_j}^2}+\nu\frac{\partial^2(u_i')}{\partial {x_j}^2}$$

Separe os termos com perturbação que se cancelariam com a media de Reynolds

$$\frac{\partial(u_i')}{\partial t}+(\overline{u_j})\frac{\partial(u_i')}{\partial x_j}+\boxed{(u_j')\frac{\partial(\overline{u_i})}{\partial x_j}}+\boxed{(u_j')\frac{\partial(u_i')}{\partial x_j}}$$
$$=-\frac{1}{\rho_0}\frac{\partial(P')}{\partial x_i}-g\frac{\rho'}{\rho_0}\delta_{i3}-2\Omega\varepsilon_{ijk}\eta_j(u_k')+\nu\frac{\partial^2(u_i')}{\partial {x_j}^2}$$

$$\frac{\partial(u_i')}{\partial t}+(\overline{u_j})\frac{\partial(u_i')}{\partial x_j}+\boxed{\frac{\partial(u_j'\overline{u_i})}{\partial x_j}-(\overline{u_i})\frac{\partial(u_j')}{\partial x_j}}+\boxed{\frac{\partial(u_j'u_i')}{\partial x_j}-(u_i')\frac{\partial(u_j')}{\partial x_j}}$$
$$=-\frac{1}{\rho}\frac{\partial(P')}{\partial x_i}-g\frac{\rho'}{\rho_0}\delta_{i3}-2\Omega\varepsilon_{ijk}\eta_j(u_k')+\nu\frac{\partial^2(u_i')}{\partial {x_j}^2}$$

Paulo Yoshio Kubota

Linearização da Equações de Navier Stokes

5.2 TURBULENT KINETIC ENERGY   A equação de Navier Stokes: conservação de momentum

$$\frac{\partial(u_i')}{\partial t}+(\overline{u}_j)\frac{\partial(u_i')}{\partial x_j}+\boxed{(u_j')\frac{\partial(\overline{u}_i)}{\partial x_j}}+\boxed{(u_j')\frac{\partial(u_i')}{\partial x_j}}=-\frac{1}{\rho_0}\frac{\partial(P')}{\partial x_i}-g\frac{\rho'}{\rho_0}\delta_{i3}-2\Omega\varepsilon_{ijk}\eta_j(u_k')+\nu\frac{\partial^2(u_i')}{\partial x_j{}^2}$$

Aplique a derivada do produto nos termos em destaque:

$$\frac{\partial(u_i')}{\partial t}+(\overline{u}_j)\frac{\partial(u_i')}{\partial x_j}+\boxed{\frac{\partial(u_j'\overline{u}_i)}{\partial x_j}-(\overline{u}_i)\frac{\partial(u_j')}{\partial x_j}}+\boxed{\frac{\partial(u_j'u_i')}{\partial x_j}-(u_i')\frac{\partial(u_j')}{\partial x_j}}$$
$$=-\frac{1}{\rho}\frac{\partial(P')}{\partial x_i}-g\frac{\rho'}{\rho_0}\delta_{i3}-2\Omega\varepsilon_{ijk}\eta_j(u_k')+\nu\frac{\partial^2(u_i')}{\partial x_j{}^2}$$

5.2 TURBULENT KINETIC ENERGY A equação de Navier Stokes: conservação de momentum

$$\frac{\partial(u_i')}{\partial t}+(\overline{u_j})\frac{\partial(u_i')}{\partial x_j}+\frac{\partial(u_j'\overline{u_i})}{\partial x_j}-(\overline{u_i})\frac{\partial(u_j')}{\partial x_j}+\frac{\partial(u_j'u_i')}{\partial x_j}-(u_i')\frac{\partial(u_j')}{\partial x_j}$$
$$=-\frac{1}{\rho}\frac{\partial(P')}{\partial x_i}-g\frac{\rho'}{\rho_0}\delta_{i3}-2\Omega\varepsilon_{ijk}\eta_j(u_k')+\nu\frac{\partial^2(u_i')}{\partial {x_j}^2}$$

Considere que a propriedade da equação da continuidade para o fluxo $\nabla.\vec{V}=0$

$$\nabla.\vec{V'}=\frac{\partial(u_j')}{\partial x_j}=0, \qquad j=1,2,3$$

$$\frac{\partial(u_i')}{\partial t}+(\overline{u_j})\frac{\partial(u_i')}{\partial x_j}+\frac{\partial(u_j'\overline{u_i})}{\partial x_j}+\frac{\partial(u_j'u_i')}{\partial x_j}=-\frac{1}{\rho_0}\frac{\partial(P')}{\partial x_i}-g\frac{\rho'}{\rho_0}\delta_{i3}-2\Omega\varepsilon_{ijk}\eta_j(u_k')+\nu\frac{\partial^2(u_i')}{\partial {x_j}^2}$$

5.2 TURBULENT KINETIC ENERGY

A equação de Navier Stokes: conservação de momentum

$$\boxed{\frac{\partial(u_i')}{\partial t} + (\overline{u_j})\frac{\partial(u_i')}{\partial x_j} + \frac{\partial\left(u_j'\overline{u_i}\right)}{\partial x_j} + \frac{\partial\left(u_j'u_i'\right)}{\partial x_j} = -\frac{1}{\rho_0}\frac{\partial(P')}{\partial x_i} - g\frac{\rho'}{\rho_0}\delta_{i3} - 2\Omega\varepsilon_{ijk}\eta_j(u_k') + \nu\frac{\partial^2(u_i')}{\partial {x_j}^2}}$$

Gorge_mellor_Analytic Prediction of the Properties of Stratified Planetary Surface Layers_atsc-1520-0469_1973

1.1 A Teoria de Similaridade de Monin-Obukhov

As equações são apresentadas de uma forma geral considerável para que, em princípio, possam ser integradas para simular, por exemplo, uma camada limite planetária completa.

Aqui, no entanto, restringimos a atenção à região da superfície de fluxo constante, evitando assim, por enquanto, o considerável esforço computacional necessário para a camada completa.

No entanto, este é um primeiro passo lógico, uma vez que é possível comparar diretamente com os dados de fluxo constante de Businger et al. (1971) na forma de variáveis de similaridade de Monin-Obukhoff.

1.1 A Teoria de Similaridade de Monin-Obukhov

2. As equações básicas

As equações de movimento para a velocidade média $\bar{u}_j$; e a temperatura potencial média $\bar{\theta}$, As barras superiores representam as médias do conjunto e os termos minúsculos, u_k' e θ', são os componentes flutuantes da velocidade e da temperatura e são governados por são

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0 \qquad (1)$$

$$\frac{\partial \bar{u}_j}{\partial t} + \bar{u}_k \frac{\partial (\bar{u}_j)}{\partial x_k} + \bar{u}_j \left[\frac{\partial (\bar{u}_k)}{\partial x_k}\right] + \frac{\partial \left(\overline{u_k' u_j'}\right)}{\partial x_k} + \epsilon_{j,k,l} f_k \bar{u}_l = -\frac{\partial P}{\partial x_j} - g_j \beta \bar{\theta} + \nu \nabla^2 \bar{u}_j \qquad (2)$$

$$\frac{\partial \bar{u}_j}{\partial t} + \frac{\partial (\bar{u}_k \bar{u}_j)}{\partial x_k} + \frac{\partial \left(\overline{u_k' u_j'}\right)}{\partial x_k} + \epsilon_{j,k,l} f_k \bar{u}_l = -\frac{\partial P}{\partial x_j} - g_j \beta \bar{\theta} + \nu \nabla^2 \bar{u}_j \qquad (2)$$

$$\boxed{\frac{\partial \bar{u}_j}{\partial t} + \frac{\partial}{\partial x_k}\left(\bar{u}_k \bar{u}_j + \overline{u_k' u_j'}\right) + \epsilon_{j,k,l} f_k \bar{u}_l = -\frac{\partial P}{\partial x_j} - g_j \beta \bar{\theta} + \nu \nabla^2 \bar{u}_j \qquad (2)}$$

$$\boxed{\frac{\partial \bar{\theta}}{\partial t} + \frac{\partial}{\partial \bar{u}_k}\left(\bar{u}_k \bar{\theta} + \overline{u_k' \theta'}\right) = \alpha \nabla^2 \bar{\theta} \qquad (3)}$$

onde P é a pressão cinemática média, $g_j = (0, 0, -\mathrm{g})$vetor de gravidade, $f_j = (0, f_y, \mathrm{f})$ o parâmetro de Coriolis (o componente vertical de não terá subscrito), $\beta = \left(\frac{\partial \rho}{\partial T}\right)_P / \rho$ o coeficiente de expansão térmica, ν é a viscosidade cinemática e α é a condutividade térmica cinemática (ou difusividade térmica).

Paulo Yoshio Kubota

1.1 A Teoria de Similaridade de Monin-Obukhov

2. As equações básicas

$$\frac{\partial u_i'}{\partial x_i} = 0 \qquad (4)$$

$$\boxed{\frac{\partial(u_j')}{\partial t} + (\overline{u_k})\frac{\partial(u_j')}{\partial x_k} + \frac{\partial(u_k'\overline{u_j})}{\partial x_k} + \frac{\partial(u_k'u_j')}{\partial x_k} = -\frac{1}{\rho_0}\frac{\partial(P')}{\partial x_j} - g\frac{\rho'}{\rho_0}\delta_{j3} - 2\Omega\varepsilon_{jki}\eta_k(u_k') + \nu\frac{\partial^2(u_j')}{\partial {x_k}^2}}$$

Aplique a media de reynolds

$$\boxed{\frac{\partial(\overline{u_j'})}{\partial t} + (\overline{\overline{u_k}})\frac{\partial(\overline{u_j'})}{\partial x_k} + \frac{\partial(\overline{u_k'\overline{\overline{u_j}}})}{\partial x_k} + \frac{\partial(\overline{u_k'u_j'})}{\partial x_k} = -\frac{1}{\rho_0}\frac{\partial(\overline{P'})}{\partial x_j} - g\frac{\overline{\rho'}}{\rho_0}\delta_{j3} - 2\Omega\varepsilon_{jki}\eta_k(\overline{u_k'}) + \nu\frac{\partial^2(\overline{u_j'})}{\partial {x_k}^2}}$$

$$\boxed{\frac{\partial(\overline{u_k'u_j'})}{\partial x_k} = 0}$$

$$\frac{\partial u_j'}{\partial t} + \left(\bar{u}_k\frac{\partial u_j'}{\partial x_k} + u_j'\frac{\partial \bar{u}_k}{\partial x_k} + \frac{\partial(\bar{u}_j u_k')}{\partial x_k} + \frac{\partial(u_k'u_j')}{\partial x_k} - \frac{\partial(\overline{u_k'u_j'})}{\partial x_k}\right) + \epsilon_{j,k,l}f_k u_l' = -\frac{\partial P'}{\partial x_j} - g_j\beta\theta' + \nu\nabla^2 u_j' \quad (5)$$

Como as equações médias (1), (2) e (3) envolvem a tensão de Reynolds $\overline{u_i'u_j'}$ e os momentos de condução de calor, $u_i'\theta'$, obtemos suas equações governantes de (5) e (6) usando a (4)

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2. As equações básicas

$$\frac{\partial u_i'}{\partial x_i} = 0 \qquad (4)$$

$$\boxed{\frac{\partial(u_j')}{\partial t} + (\overline{u_k})\frac{\partial(u_j')}{\partial x_k} + \frac{\partial(u_k'\overline{u_j})}{\partial x_k} + \frac{\partial(u_k'u_j')}{\partial x_k} = -\frac{1}{\rho_0}\frac{\partial(P')}{\partial x_j} - g\frac{\rho'}{\rho_0}\delta_{j3} - 2\Omega\varepsilon_{jki}\eta_k(u_k') + \nu\frac{\partial^2(u_j')}{\partial {x_k}^2}}$$

$$\frac{\partial u_j'}{\partial t} + \left(\bar{u}_k\frac{\partial u_j'}{\partial x_k} + u_j'\frac{\partial \bar{u}_k}{\partial x_k} + \frac{\partial(\bar{u}_j u_k')}{\partial x_k} + \frac{\partial(u_k'u_j')}{\partial x_k} - \frac{\partial(\overline{u_k'u_j'})}{\partial x_k}\right) + \epsilon_{j,k,l}f_k u_l' = -\frac{\partial P'}{\partial x_j} - g_j\beta\theta' + \nu\nabla^2 u_j'$$

$$\frac{\partial u_j'}{\partial t} + \left(\frac{\partial \bar{u}_k u_j'}{\partial x_k} + \frac{\partial \bar{u}_j u_k'}{\partial x_k} + \frac{\partial u_k' u_j'}{\partial x_k} - \frac{\partial \overline{u_k' u_j'}}{\partial x_k}\right) + \epsilon_{j,k,l}f_k u_l' = -\frac{\partial P'}{\partial x_j} - g_j\beta\theta' + \nu\nabla^2 u_j' \qquad (5)$$

$$\frac{\partial u_j'}{\partial t} + \frac{\partial}{\partial x_k}\left(\bar{u}_k u_j' + \bar{u}_j u_k' + u_k' u_j' - \overline{u_k' u_j'}\right) + \epsilon_{j,k,l}f_k u_l' = -\frac{\partial P'}{\partial x_j} - g_j\beta\theta' + \nu\nabla^2 u_j' \qquad (5)$$

$$\frac{\partial \theta'}{\partial t} + \frac{\partial}{\partial x_k}\left(\bar{\theta}u_k' + \bar{u}_k\theta' + u_k'\theta' + \overline{u_k'\theta'}\right) = \alpha\nabla^2\theta' \qquad (6)$$

Como as equações médias (1), (2) e (3) envolvem a tensão de Reynolds $\overline{u_i'u_j'}$ e os momentos de condução de calor, $u_i'\theta'$, obtemos suas equações governantes de (5) e (6) usando a (4)

2. As equações básicas

$$\frac{\partial u_j'}{\partial t} + \frac{\partial}{\partial x_k}\left(\bar{u}_k u_j' + \bar{u}_j u_k' + u_k' u_j' - \overline{u_k' u_j'}\right) + \epsilon_{j,k,l} f_k u_l' = -\frac{\partial P'}{\partial x_j} - g_j \beta \theta' + \nu \nabla^2 u_j' \qquad (5)$$

$$u_i' \frac{\partial u_j'}{\partial t} + u_i' \frac{\partial}{\partial x_k}\left(\bar{u}_k u_j' + \bar{u}_j u_k' + u_k' u_j' - \overline{u_k' u_j'}\right) + \epsilon_{j,k,l} f_k u_l' u_i' = -u_i' \frac{\partial P'}{\partial x_j} - g_j \beta u_i' \theta' + \nu u_i' \nabla^2 u_j' \qquad (5a)$$

$$u_j' \frac{\partial u_i'}{\partial t} + u_j' \frac{\partial}{\partial x_k}\left(\bar{u}_k u_i' + \bar{u}_i u_k' + u_k' u_i' - \overline{u_k' u_i'}\right) + \epsilon_{i,k,l} f_k u_l' u_j' = -u_j' \frac{\partial P'}{\partial x_i} - g_i \beta u_j' \theta' + \nu u_j' \nabla^2 u_i' \qquad (5b)$$

$$\left(u_i' \frac{\partial u_j'}{\partial t} + u_j' \frac{\partial u_i'}{\partial t}\right) + \left(u_i' \frac{\partial \bar{u}_k u_j'}{\partial x_k} + u_j' \frac{\partial \bar{u}_k u_i'}{\partial x_k}\right) + \left(u_i' \frac{\partial \bar{u}_j u_k'}{\partial x_k} + u_j' \frac{\partial \bar{u}_i u_k'}{\partial x_k}\right) + \left(u_i' \frac{\partial u_k' u_j'}{\partial x_k} + u_j' \frac{\partial u_k' u_i'}{\partial x_k}\right) - \left(u_i' \frac{\partial \overline{u_k' u_j'}}{\partial x_k} + u_j' \frac{\partial \overline{u_k' u_i'}}{\partial x_k}\right)$$
$$+ \epsilon_{j,k,l} f_k u_l' u_i' + \epsilon_{i,k,l} f_k u_l' u_j' = -u_i' \frac{\partial P'}{\partial x_j} - u_j' \frac{\partial P'}{\partial x_i} - g_j \beta u_i' \theta' - g_i \beta u_j' \theta' + \nu u_i' \nabla^2 u_j' + \nu u_j' \nabla^2 u_i' \qquad (5c)$$

2. As equações básicas

$$\left(u_i'\frac{\partial u_j'}{\partial t}+u_j'\frac{\partial u_i'}{\partial t}\right)+\left(u_i'\frac{\partial \bar{u}_k u_j'}{\partial x_k}+u_j'\frac{\partial \bar{u}_k u_i'}{\partial x_k}\right)+\left(u_i'\frac{\partial \bar{u}_j u_k'}{\partial x_k}+u_j'\frac{\partial \bar{u}_i u_k'}{\partial x_k}\right)+\left(u_i'\frac{\partial u_k' u_j'}{\partial x_k}+u_j'\frac{\partial u_k' u_i'}{\partial x_k}\right)-\left(u_i'\frac{\partial \overline{u_k' u_j'}}{\partial x_k}+u_j'\frac{\partial \overline{u_k' u_i'}}{\partial x_k}\right)$$

$$+\epsilon_{j,k,l}f_k u_l' u_i' + \epsilon_{i,k,l} f_k u_l' u_j' = -u_i'\frac{\partial P'}{\partial x_j} - u_j'\frac{\partial P'}{\partial x_i} - g_j\beta u_i'\theta' - g_i\beta u_j'\theta' + \nu u_i'\nabla^2 u_j' + \nu u_j'\nabla^2 u_i'$$

$$\left(u_i'\frac{\partial u_j'}{\partial t}+u_j'\frac{\partial u_i'}{\partial t}\right)+\left(u_i'\frac{\partial \bar{u}_k u_j'}{\partial x_k}+u_j'\frac{\partial \bar{u}_k u_i'}{\partial x_k}\right)+\left(u_i'\frac{\partial u_k' u_j'}{\partial x_k}+u_j'\frac{\partial u_k' u_i'}{\partial x_k}\right)+\left(u_i'\frac{\partial \bar{u}_j u_k'}{\partial x_k}+u_j'\frac{\partial \bar{u}_i u_k'}{\partial x_k}\right)-\left(u_i'\frac{\partial \overline{u_k' u_j'}}{\partial x_k}+u_j'\frac{\partial \overline{u_k' u_i'}}{\partial x_k}\right)$$

$$+\epsilon_{j,k,l}f_k u_l' u_i' + \epsilon_{i,k,l} f_k u_l' u_j' = -u_i'\frac{\partial P'}{\partial x_j} - u_j'\frac{\partial P'}{\partial x_i} - g_j\beta u_i'\theta' - g_i\beta u_j'\theta' + \nu u_i'\nabla^2 u_j' + \nu u_j'\nabla^2 u_i'$$

$$\left(\frac{\partial u_i' u_j'}{\partial t}\right)+\left(\left[\frac{\partial \bar{u}_k u_i' u_j'}{\partial x_k}-u_j'\frac{\partial \bar{u}_k u_i'}{\partial x_k}\right]+\left[\frac{\partial \bar{u}_k u_j' u_i'}{\partial x_k}-u_i'\frac{\partial \bar{u}_k u_j'}{\partial x_k}\right]\right)+\left(\left[\frac{\partial u_k' u_i' u_j'}{\partial x_k}-u_j'\frac{\partial u_k' u_i'}{\partial x_k}\right]+\left[\frac{\partial u_k' u_j' u_i'}{\partial x_k}-u_i'\frac{\partial u_k' u_j'}{\partial x_k}\right]\right)$$

$$+\left(\left[\frac{\partial \bar{u}_j u_i' u_k'}{\partial x_k}-u_k'\frac{\partial \bar{u}_j u_i'}{\partial x_k}\right]+\left[\frac{\partial \bar{u}_i u_j' u_k'}{\partial x_k}-u_k'\frac{\partial \bar{u}_i u_j'}{\partial x_k}\right]\right)-\left(\left[\frac{\partial u_i'\overline{u_k' u_j'}}{\partial x_k}-\overline{u_k' u_j'}\frac{\partial u_i'}{\partial x_k}\right]+\left[\frac{\partial u_j'\overline{u_k' u_i'}}{\partial x_k}-\overline{u_k' u_i'}\frac{\partial u_j'}{\partial x_k}\right]\right)$$

$$+f_k\left(\epsilon_{j,k,l}u_l' u_i' + \epsilon_{i,k,l}u_l' u_j'\right) = -\left[\frac{\partial u_i' P'}{\partial x_j}-P'\frac{\partial u_i'}{\partial x_j}\right]-\left[\frac{\partial u_j' P'}{\partial x_i}-P'\frac{\partial u_j'}{\partial x_i}\right]-\beta\left(g_j u_i'\theta' - g_i u_j'\theta'\right)+\nu u_i'\nabla^2 u_j' + \nu u_j'\nabla^2 u_i'$$

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2. As equações básicas

$$
\left(\frac{\partial u_i' u_j'}{\partial t}\right) + \left(\left[\frac{\partial \bar{u}_k u_i' u_j'}{\partial x_k} - u_j' \frac{\partial \bar{u}_k u_i'}{\partial x_k}\right] + \left[\frac{\partial \bar{u}_k u_j' u_i'}{\partial x_k} - u_i' \frac{\partial \bar{u}_k u_j'}{\partial x_k}\right]\right) + \left(\left[\frac{\partial u_k' u_i' u_j'}{\partial x_k} - u_j' \frac{\partial u_k' u_i'}{\partial x_k}\right] + \left[\frac{\partial u_k' u_j' u_i'}{\partial x_k} - u_i' \frac{\partial u_k' u_j'}{\partial x_k}\right]\right)
$$

$$
+ \left(\left[\frac{\partial \bar{u}_j u_i' u_k'}{\partial x_k} - u_k' \frac{\partial \bar{u}_j u_i'}{\partial x_k}\right] + \left[\frac{\partial \bar{u}_i u_j' u_k'}{\partial x_k} - u_k' \frac{\partial \bar{u}_i u_j'}{\partial x_k}\right]\right) - \left(\left[\frac{\partial u_i' \overline{u_k' u_j'}}{\partial x_k} - \overline{u_k' u_j'} \frac{\partial u_i'}{\partial x_k}\right] + \left[\frac{\partial u_j' \overline{u_k' u_i'}}{\partial x_k} - \overline{u_k' u_i'} \frac{\partial u_j'}{\partial x_k}\right]\right)
$$

$$
+ f_k\left(\epsilon_{j,k,l} u_l' u_i' + \epsilon_{i,k,l} u_l' u_j'\right) = -\left[\frac{\partial u_i' P'}{\partial x_j} - P' \frac{\partial u_i'}{\partial x_j}\right] - \left[\frac{\partial u_j' P'}{\partial x_i} - P' \frac{\partial u_j'}{\partial x_i}\right] - \beta\left(g_j u_i' \theta' - g_i u_j' \theta'\right) + \nu u_i' \nabla^2 u_j' + \nu u_j' \nabla^2 u_i'
$$

$$
\left(\frac{\partial u_i' u_j'}{\partial t}\right) + \left(\left[\frac{\partial \bar{u}_k u_i' u_j'}{\partial x_k} + \frac{\partial \bar{u}_k u_j' u_i'}{\partial x_k}\right] + \left[-u_j' \frac{\partial \bar{u}_k u_i'}{\partial x_k} - u_i' \frac{\partial \bar{u}_k u_j'}{\partial x_k}\right]\right) + \left(\left[\frac{\partial u_k' u_i' u_j'}{\partial x_k} + \frac{\partial u_k' u_j' u_i'}{\partial x_k}\right] + \left[-u_j' \frac{\partial u_k' u_i'}{\partial x_k} - u_i' \frac{\partial u_k' u_j'}{\partial x_k}\right]\right)
$$

$$
+ \left(\left[\frac{\partial \bar{u}_j u_i' u_k'}{\partial x_k} + \frac{\partial \bar{u}_i u_j' u_k'}{\partial x_k}\right] + \left[-u_k' \frac{\partial \bar{u}_j u_i'}{\partial x_k} - u_k' \frac{\partial \bar{u}_i u_j'}{\partial x_k}\right]\right) - \left(\left[\frac{\partial u_i' \overline{u_k' u_j'}}{\partial x_k}\right] + \left[\frac{\partial u_j' \overline{u_k' u_i'}}{\partial x_k}\right]\right) - \left(\left[-\overline{u_k' u_j'} \frac{\partial u_i'}{\partial x_k}\right] + \left[-\overline{u_k' u_i'} \frac{\partial u_j'}{\partial x_k}\right]\right)
$$

$$
+ f_k\left(\epsilon_{j,k,l} u_l' u_i' + \epsilon_{i,k,l} u_l' u_j'\right) = -\left[\frac{\partial u_i' P'}{\partial x_j} + \frac{\partial u_j' P'}{\partial x_i}\right] + \left[P' \frac{\partial u_i'}{\partial x_j} + P' \frac{\partial u_j'}{\partial x_i}\right] - \beta\left(g_j u_i' \theta' - g_i u_j' \theta'\right) + \nu u_i' \nabla^2 u_j' + \nu u_j' \nabla^2 u_i'
$$

1.1 A Teoria de Similaridade de Monin-Obukhov

2. As equações básicas

$$
\left(\frac{\partial u_i' u_j'}{\partial t}\right) + \left(\left[\frac{\partial \bar{u}_k u_i' u_j'}{\partial x_k} + \frac{\partial \bar{u}_k u_j' u_i'}{\partial x_k}\right] + \left[-u_j' \frac{\partial \bar{u}_k u_i'}{\partial x_k} - u_i' \frac{\partial \bar{u}_k u_j'}{\partial x_k}\right]\right) + \left(\left[\frac{\partial u_k' u_i' u_j'}{\partial x_k} + \frac{\partial u_k' u_j' u_i'}{\partial x_k}\right] + \left[-u_j' \frac{\partial u_k' u_i'}{\partial x_k} - u_i' \frac{\partial u_k' u_j'}{\partial x_k}\right]\right)
$$

$$
+ \left(\left[\frac{\partial \bar{u}_j u_i' u_k'}{\partial x_k} + \frac{\partial \bar{u}_i u_j' u_k'}{\partial x_k}\right] + \left[-u_k' \frac{\partial \bar{u}_j u_i'}{\partial x_k} - u_k' \frac{\partial \bar{u}_i u_j'}{\partial x_k}\right]\right) - \left(\left[\frac{\partial u_i' \overline{u_k' u_j'}}{\partial x_k}\right] + \left[\frac{\partial u_j' \overline{u_k' u_i'}}{\partial x_k}\right]\right) - \left(\left[-\overline{u_k' u_j'} \frac{\partial u_i'}{\partial x_k}\right] + \left[-\overline{u_k' u_i'} \frac{\partial u_j'}{\partial x_k}\right]\right)
$$

$$
+ f_k\left(\epsilon_{j,k,l} u_l' u_i' + \epsilon_{i,k,l} u_l' u_j'\right) = -\left[\frac{\partial u_i' P'}{\partial x_j} + \frac{\partial u_j' P'}{\partial x_i}\right] + \left[P' \frac{\partial u_i'}{\partial x_j} + P' \frac{\partial u_j'}{\partial x_i}\right] - \beta\left(g_j u_i' \theta' - g_i u_j' \theta'\right) + \nu u_i' \nabla^2 u_j' + \nu u_j' \nabla^2 u_i'
$$

$$
\left(\frac{\partial u_i' u_j'}{\partial t}\right) + \left(\left[\frac{\partial \bar{u}_k u_i' u_j'}{\partial x_k} + \frac{\partial \bar{u}_k u_i' u_j'}{\partial x_k}\right] + \left[-u_j' \frac{\partial \bar{u}_k u_i'}{\partial x_k} - u_i' \frac{\partial \bar{u}_k u_j'}{\partial x_k}\right]\right) + \left(\left[\frac{\partial u_k' u_i' u_j'}{\partial x_k} + \frac{\partial u_k' u_i' u_j'}{\partial x_k}\right] + \left[-u_j' \frac{\partial u_k' u_i'}{\partial x_k} - u_i' \frac{\partial u_k' u_j'}{\partial x_k}\right]\right)
$$

$$
+ \left(\left[\frac{\partial \bar{u}_j u_i' u_k'}{\partial x_k} + \frac{\partial \bar{u}_i u_j' u_k'}{\partial x_k}\right] + \left[-u_k' \frac{\partial \bar{u}_j u_i'}{\partial x_k} - u_k' \frac{\partial \bar{u}_i u_j'}{\partial x_k}\right]\right) - \left(\left[\frac{\partial u_i' \overline{u_k' u_j'}}{\partial x_k}\right] + \left[\frac{\partial u_j' \overline{u_k' u_i'}}{\partial x_k}\right]\right) - \left(\left[-\overline{u_k' u_j'} \frac{\partial u_i'}{\partial x_k}\right] + \left[-\overline{u_k' u_i'} \frac{\partial u_j'}{\partial x_k}\right]\right)
$$

$$
+ f_k\left(\epsilon_{j,k,l} u_l' u_i' + \epsilon_{i,k,l} u_l' u_j'\right) = -\left[\frac{\partial u_i' P'}{\partial x_j} + \frac{\partial u_j' P'}{\partial x_i}\right] + \left[P' \frac{\partial u_i'}{\partial x_j} + P' \frac{\partial u_j'}{\partial x_i}\right] - \beta\left(g_j u_i' \theta' - g_i u_j' \theta'\right) + \nu u_i' \nabla^2 u_j' + \nu u_j' \nabla^2 u_i'
$$

2. As equações básicas

$$\left(\frac{\partial u_i' u_j'}{\partial t}\right) + \left(\left[\frac{\partial \bar{u}_k u_i' u_j'}{\partial x_k} + \frac{\partial \bar{u}_k u_i' u_j'}{\partial x_k}\right] + \left[-u_j' \frac{\partial \bar{u}_k u_i'}{\partial x_k} - u_i' \frac{\partial \bar{u}_k u_j'}{\partial x_k}\right]\right) + \left(\left[\frac{\partial u_k' u_i' u_j'}{\partial x_k} + \frac{\partial u_k' u_i' u_j'}{\partial x_k}\right] + \left[-u_j' \frac{\partial u_k' u_i'}{\partial x_k} - u_i' \frac{\partial u_k' u_j'}{\partial x_k}\right]\right)$$
$$+ \left(\left[\frac{\partial \bar{u}_j u_i' u_k'}{\partial x_k} + \frac{\partial \bar{u}_i u_j' u_k'}{\partial x_k}\right] + \left[-u_k' \frac{\partial \bar{u}_j u_i'}{\partial x_k} - u_k' \frac{\partial \bar{u}_i u_j'}{\partial x_k}\right]\right) - \left(\left[\frac{\partial u_i' \overline{u_k' u_j'}}{\partial x_k}\right] + \left[\frac{\partial u_j' \overline{u_k' u_i'}}{\partial x_k}\right]\right) - \left(\left[-\overline{u_k' u_j'} \frac{\partial u_i'}{\partial x_k}\right] + \left[-\overline{u_k' u_i'} \frac{\partial u_j'}{\partial x_k}\right]\right)$$
$$+ f_k\left(\epsilon_{j,k,l} u_l' u_i' + \epsilon_{i,k,l} u_l' u_j'\right) = -\left[\frac{\partial u_i' P'}{\partial x_j} + \frac{\partial u_j' P'}{\partial x_i}\right] + \left[P' \frac{\partial u_i'}{\partial x_j} + P' \frac{\partial u_j'}{\partial x_i}\right] - \beta\left(g_j u_i' \theta' - g_i u_j' \theta'\right) + \nu u_i' \nabla^2 u_j' + \nu u_j' \nabla^2 u_i'$$

$$\left(\frac{\partial u_i' u_j'}{\partial t}\right) + \left(2\left[\frac{\partial \bar{u}_k u_i' u_j'}{\partial x_k}\right] - \left[\frac{\partial \bar{u}_k u_i' u_j'}{\partial x_k}\right]\right) + \left(2\left[\frac{\partial u_k' u_i' u_j'}{\partial x_k}\right] - \left[\frac{\partial u_k' u_i' u_j'}{\partial x_k}\right]\right) + \left(\left[\frac{\partial \bar{u}_j u_i' u_k'}{\partial x_k} + \frac{\partial \bar{u}_i u_j' u_k'}{\partial x_k}\right] + \left[-u_k' \frac{\partial \bar{u}_j u_i'}{\partial x_k} - u_k' \frac{\partial \bar{u}_i u_j'}{\partial x_k}\right]\right)$$
$$- \left(\left[\frac{\partial u_i' \overline{u_k' u_j'}}{\partial x_k}\right] + \left[\frac{\partial u_j' \overline{u_k' u_i'}}{\partial x_k}\right]\right) - \left(\left[-\overline{u_k' u_j'} \frac{\partial u_i'}{\partial x_k} - \overline{u_k' u_i'} \frac{\partial u_j'}{\partial x_k}\right]\right) + f_k\left(\epsilon_{j,k,l} u_l' u_i' + \epsilon_{i,k,l} u_l' u_j'\right)$$
$$= -\left[\frac{\partial u_i' P'}{\partial x_j} + \frac{\partial u_j' P'}{\partial x_i}\right] + P'\left[\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right] - \beta\left(g_j u_i' \theta' - g_i u_j' \theta'\right) + \nu u_i' \frac{\partial}{\partial x_k} \frac{\partial u_j'}{\partial x_k} + \nu u_j' \frac{\partial}{\partial x_k} \frac{\partial u_i'}{\partial x_k}$$

2. As equações básicas

$$
\begin{aligned}
&\left(\frac{\partial u'_i u'_j}{\partial t}\right) + \left(2\left[\frac{\partial \bar{u}_k u'_i u'_j}{\partial x_k}\right] - \left[\frac{\partial \bar{u}_k u'_i u'_j}{\partial x_k}\right]\right) + \left(2\left[\frac{\partial u'_k u'_i u'_j}{\partial x_k}\right] - \left[\frac{\partial u'_k u'_i u'_j}{\partial x_k}\right]\right) + \left(\left[\frac{\partial \bar{u}_j u'_i u'_k}{\partial x_k} + \frac{\partial \bar{u}_i u'_j u'_k}{\partial x_k}\right] + \left[-u'_k \frac{\partial \bar{u}_j u'_i}{\partial x_k} - u'_k \frac{\partial \bar{u}_i u'_j}{\partial x_k}\right.\right. \\
&- \left(\left[\frac{\partial u'_i \overline{u'_k u'_j}}{\partial x_k}\right] + \left[\frac{\partial u'_j \overline{u'_k u'_i}}{\partial x_k}\right]\right) - \left(\left[-\overline{u'_k u'_j}\frac{\partial u'_i}{\partial x_k} - \overline{u'_k u'_i}\frac{\partial u'_j}{\partial x_k}\right]\right) + f_k\left(\epsilon_{j,k,l} u'_l u'_i + \epsilon_{i,k,l} u'_l u'_j\right) \\
&= -\left[\frac{\partial u'_i P'}{\partial x_j} + \frac{\partial u'_j P'}{\partial x_i}\right] + P'\left[\frac{\partial u'_i}{\partial x_j} + \frac{\partial u'_j}{\partial x_i}\right] - \beta\left(g_j u'_i \theta' - g_i u'_j \theta'\right) + \nu u'_i \frac{\partial}{\partial x_k}\frac{\partial u'_j}{\partial x_k} + \nu u'_j \frac{\partial}{\partial x_k}\frac{\partial u'_i}{\partial x_k}
\end{aligned}
$$

$$
\begin{aligned}
&\left(\frac{\partial u'_i u'_j}{\partial t}\right) + \left(\left[\frac{\partial \bar{u}_k u'_i u'_j}{\partial x_k}\right]\right) + \left(\left[\frac{\partial u'_k u'_i u'_j}{\partial x_k}\right]\right) + \left(\left[u'_k \frac{\partial \bar{u}_j u'_i}{\partial x_k} + u'_i \frac{\partial \bar{u}_j u'_k}{\partial x_k} + u'_k \frac{\partial \bar{u}_i u'_j}{\partial x_k} + u'_j \frac{\partial \bar{u}_i u'_k}{\partial x_k}\right] + \left[-u'_k \frac{\partial \bar{u}_j u'_i}{\partial x_k} - u'_k \frac{\partial \bar{u}_i u'_j}{\partial x_k}\right]\right) \\
&- \left(\left[\frac{\partial u'_i \overline{u'_k u'_j}}{\partial x_k}\right] + \left[\frac{\partial u'_j \overline{u'_k u'_i}}{\partial x_k}\right]\right) - \left(\left[-\overline{u'_k u'_j}\frac{\partial u'_i}{\partial x_k} - \overline{u'_k u'_i}\frac{\partial u'_j}{\partial x_k}\right]\right) + f_k\left(\epsilon_{j,k,l} u'_l u'_i + \epsilon_{i,k,l} u'_l u'_j\right) \\
&= -\left[\frac{\partial u'_i P'}{\partial x_j} + \frac{\partial u'_j P'}{\partial x_i}\right] + P'\left[\frac{\partial u'_i}{\partial x_j} + \frac{\partial u'_j}{\partial x_i}\right] - \beta\left(g_j u'_i \theta' - g_i u'_j \theta'\right) + \nu\left[\frac{\partial}{\partial x_k}\left(u'_i \frac{\partial u'_j}{\partial x_k}\right) - \left(\frac{\partial u'_i}{\partial x_k}\frac{\partial u'_j}{\partial x_k}\right)\right] \\
&+ \nu\left[\frac{\partial}{\partial x_k}\left(u'_j \frac{\partial u'_i}{\partial x_k}\right) - \left(\frac{\partial u'_j}{\partial x_k}\frac{\partial u'_i}{\partial x_k}\right)\right]
\end{aligned}
$$

2. As equações básicas

$$\left(\frac{\partial u_i' u_j'}{\partial t}\right)+\left(\left[\frac{\partial \bar{u}_k u_i' u_j'}{\partial x_k}\right]\right)+\left(\left[\frac{\partial u_k' u_i' u_j'}{\partial x_k}\right]\right)+\left(\left[+u_i'\frac{\partial \bar{u}_j u_k'}{\partial x_k}+u_j'\frac{\partial \bar{u}_i u_k'}{\partial x_k}\right]+\left[u_k'\frac{\partial \bar{u}_j u_i'}{\partial x_k}-u_k'\frac{\partial \bar{u}_j u_i'}{\partial x_k}+u_k'\frac{\partial \bar{u}_i u_j'}{\partial x_k}-u_k'\frac{\partial \bar{u}_i u_j'}{\partial x_k}\right]\right)$$
$$-\left(\left[\frac{\partial u_i'\overline{u_k' u_j'}}{\partial x_k}\right]+\left[\frac{\partial u_j'\overline{u_k' u_i'}}{\partial x_k}\right]\right)-\left(\left[-\overline{u_k' u_j'}\frac{\partial u_i'}{\partial x_k}-\overline{u_k' u_i'}\frac{\partial u_j'}{\partial x_k}\right]\right)+f_k\left(\epsilon_{j,k,l}u_l' u_i'+\epsilon_{i,k,l}u_l' u_j'\right)$$
$$=-\left[\frac{\partial u_i' P'}{\partial x_j}+\frac{\partial u_j' P'}{\partial x_i}\right]+P'\left[\frac{\partial u_i'}{\partial x_j}+\frac{\partial u_j'}{\partial x_i}\right]-\beta\left(g_j u_i'\theta'-g_i u_j'\theta'\right)+\nu\left[\frac{\partial}{\partial x_k}\left(u_i'\frac{\partial u_j'}{\partial x_k}\right)-\left(\frac{\partial u_i'}{\partial x_k}\frac{\partial u_j'}{\partial x_k}\right)\right]$$
$$+\nu\left[\frac{\partial}{\partial x_k}\left(u_j'\frac{\partial u_i'}{\partial x_k}\right)-\left(\frac{\partial u_j'}{\partial x_k}\frac{\partial u_i'}{\partial x_k}\right)\right]$$

$$\left(\frac{\partial u_i' u_j'}{\partial t}\right)+\left(\left[\frac{\partial \bar{u}_k u_i' u_j'}{\partial x_k}\right]\right)+\left(\left[\frac{\partial u_k' u_i' u_j'}{\partial x_k}\right]\right)+\left(\left[u_i'\frac{\partial \bar{u}_j u_k'}{\partial x_k}+u_j'\frac{\partial \bar{u}_i u_k'}{\partial x_k}\right]\right)-\left(\left[\frac{\partial u_i'\overline{u_k' u_j'}}{\partial x_k}\right]+\left[\frac{\partial u_j'\overline{u_k' u_i'}}{\partial x_k}\right]\right)$$
$$-\left(\left[-\overline{u_k' u_j'}\frac{\partial u_i'}{\partial x_k}-\overline{u_k' u_i'}\frac{\partial u_j'}{\partial x_k}\right]\right)+f_k\left(\epsilon_{j,k,l}u_l' u_i'+\epsilon_{i,k,l}u_l' u_j'\right)$$
$$=-\left[\frac{\partial u_i' P'}{\partial x_j}+\frac{\partial u_j' P'}{\partial x_i}\right]+P'\left[\frac{\partial u_i'}{\partial x_j}+\frac{\partial u_j'}{\partial x_i}\right]-\beta\left(g_j u_i'\theta'-g_i u_j'\theta'\right)+\nu\frac{\partial}{\partial x_k}\left[\left(u_i'\frac{\partial u_j'}{\partial x_k}\right)+\left(u_j'\frac{\partial u_i'}{\partial x_k}\right)\right]-2\nu\left(\frac{\partial u_j'}{\partial x_k}\frac{\partial u_i'}{\partial x_k}\right)$$

1.1 A Teoria de Similaridade de Monin-Obukhov

2. As equações básicas

$$\left(\frac{\partial u_i' u_j'}{\partial t}\right) + \left(\left[\frac{\partial \bar{u}_k u_i' u_j'}{\partial x_k}\right]\right) + \left(\left[\frac{\partial u_k' u_i' u_j'}{\partial x_k}\right]\right) + \left(\left[u_i' \frac{\partial \bar{u}_j u_k'}{\partial x_k} + u_j' \frac{\partial \bar{u}_i u_k'}{\partial x_k}\right]\right) - \left(\left[\frac{\partial u_i' \overline{u_k' u_j'}}{\partial x_k}\right] + \left[\frac{\partial u_j' \overline{u_k' u_i'}}{\partial x_k}\right]\right)$$
$$- \left(\left[-\overline{u_k' u_j'} \frac{\partial u_i'}{\partial x_k} - \overline{u_k' u_i'} \frac{\partial u_j'}{\partial x_k}\right]\right) + f_k\left(\epsilon_{j,k,l} u_l' u_i' + \epsilon_{i,k,l} u_l' u_j'\right)$$
$$= -\left[\frac{\partial u_i' P'}{\partial x_j} + \frac{\partial u_j' P'}{\partial x_i}\right] + P'\left[\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right] - \beta\left(g_j u_i' \theta' - g_i u_j' \theta'\right) + \nu \frac{\partial}{\partial x_k}\left[\left(u_i' \frac{\partial u_j'}{\partial x_k}\right) + \left(u_j' \frac{\partial u_i'}{\partial x_k}\right)\right] - 2\nu\left(\frac{\partial u_j'}{\partial x_k}\frac{\partial u_i'}{\partial x_k}\right)$$

$$\left(\frac{\partial u_i' u_j'}{\partial t}\right) + \left(\left[\frac{\partial \bar{u}_k u_i' u_j'}{\partial x_k}\right]\right) + \left(\left[\frac{\partial u_k' u_i' u_j'}{\partial x_k}\right]\right) + \left(\left[u_i' u_k' \frac{\partial \bar{u}_j}{\partial x_k} + u_i' \bar{u}_j \frac{\partial u_k'}{\partial x_k} + u_j' u_k' \frac{\partial \bar{u}_i}{\partial x_k} + u_j' \bar{u}_i \frac{\partial u_k'}{\partial x_k}\right]\right) - \left(\left[\frac{\partial u_i' \overline{u_k' u_j'}}{\partial x_k}\right] + \left[\frac{\partial u_j' \overline{u_k' u_i'}}{\partial x_k}\right]\right)$$
$$- \left(\left[-\overline{u_k' u_j'} \frac{\partial u_i'}{\partial x_k} - \overline{u_k' u_i'} \frac{\partial u_j'}{\partial x_k}\right]\right) + f_k\left(\epsilon_{j,k,l} u_l' u_i' + \epsilon_{i,k,l} u_l' u_j'\right)$$
$$= -\left[\frac{\partial u_i' P'}{\partial x_j} + \frac{\partial u_j' P'}{\partial x_i}\right] + P'\left[\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right] - \beta\left(g_j u_i' \theta' - g_i u_j' \theta'\right) + \nu \frac{\partial}{\partial x_k}\left[\left(u_i' \frac{\partial u_j'}{\partial x_k}\right) + \left(u_j' \frac{\partial u_i'}{\partial x_k}\right)\right] - 2\nu\left(\frac{\partial u_j'}{\partial x_k}\frac{\partial u_i'}{\partial x_k}\right)$$

Paulo Yoshio Kubota

2. As equações básicas

$$
\left(\frac{\partial u_i' u_j'}{\partial t}\right) + \left(\left[\frac{\partial \bar{u}_k u_i' u_j'}{\partial x_k}\right]\right) + \left(\left[\frac{\partial u_k' u_i' u_j'}{\partial x_k}\right]\right) + \left(\left[u_i' u_k' \frac{\partial \bar{u}_j}{\partial x_k} + u_i' \bar{u}_j \frac{\partial u_k'}{\partial x_k} + u_j' u_k' \frac{\partial \bar{u}_i}{\partial x_k} + u_j' \bar{u}_i \frac{\partial u_k'}{\partial x_k}\right]\right) - \left(\left[\frac{\partial u_i' \overline{u_k' u_j'}}{\partial x_k}\right] + \left[\frac{\partial u_j' \overline{u_k' u_i'}}{\partial x_k}\right]\right)
$$
$$
- \left(\left[-\overline{u_k' u_j'} \frac{\partial u_i'}{\partial x_k} - \overline{u_k' u_i'} \frac{\partial u_j'}{\partial x_k}\right]\right) + f_k\left(\epsilon_{j,k,l} u_l' u_i' + \epsilon_{i,k,l} u_l' u_j'\right)
$$
$$
= -\left[\frac{\partial u_i' P'}{\partial x_j} + \frac{\partial u_j' P'}{\partial x_i}\right] + P'\left[\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right] - \beta\left(g_j u_i' \theta' - g_i u_j' \theta'\right) + \nu \frac{\partial}{\partial x_k}\left[\left(u_i' \frac{\partial u_j'}{\partial x_k}\right) + \left(u_j' \frac{\partial u_i'}{\partial x_k}\right)\right] - 2\nu\left(\frac{\partial u_j'}{\partial x_k} \frac{\partial u_i'}{\partial x_k}\right)
$$

$$
\left(\frac{\partial u_i' u_j'}{\partial t}\right) + \left(\left[\frac{\partial \bar{u}_k u_i' u_j'}{\partial x_k}\right]\right) + \left(\left[\frac{\partial u_k' u_i' u_j'}{\partial x_k}\right]\right) + \left(\left[u_i' u_k' \frac{\partial \bar{u}_j}{\partial x_k} + u_j' u_k' \frac{\partial \bar{u}_i}{\partial x_k} + u_i' \bar{u}_j \frac{\partial u_k'}{\partial x_k} + u_j' \bar{u}_i \frac{\partial u_k'}{\partial x_k}\right]\right) - \left(\left[\frac{\partial u_i' \overline{u_k' u_j'}}{\partial x_k}\right] + \left[\frac{\partial u_j' \overline{u_k' u_i'}}{\partial x_k}\right]\right)
$$
$$
- \left(\left[-\overline{u_k' u_j'} \frac{\partial u_i'}{\partial x_k} - \overline{u_k' u_i'} \frac{\partial u_j'}{\partial x_k}\right]\right) + f_k\left(\epsilon_{j,k,l} u_l' u_i' + \epsilon_{i,k,l} u_l' u_j'\right)
$$
$$
= -\left[\frac{\partial u_i' P'}{\partial x_j} + \frac{\partial u_j' P'}{\partial x_i}\right] + P'\left[\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right] - \beta\left(g_j u_i' \theta' - g_i u_j' \theta'\right) + \nu \frac{\partial}{\partial x_k}\left[\left(u_i' \frac{\partial u_j'}{\partial x_k}\right) + \left(u_j' \frac{\partial u_i'}{\partial x_k}\right)\right] - 2\nu\left(\frac{\partial u_j'}{\partial x_k} \frac{\partial u_i'}{\partial x_k}\right)
$$

2. As equações básicas

$$\left(\frac{\partial u_i' u_j'}{\partial t}\right)+\left(\left[\frac{\partial \bar{u}_k u_i' u_j'}{\partial x_k}\right]\right)+\left(\left[\frac{\partial u_k' u_i' u_j'}{\partial x_k}\right]\right)+\left(\left[u_i' u_k' \frac{\partial \bar{u}_j}{\partial x_k}+u_j' u_k' \frac{\partial \bar{u}_i}{\partial x_k}+u_i' \bar{u}_j \cancel{\frac{\partial u_k'}{\partial x_k}}+u_j' \bar{u}_i \cancel{\frac{\partial u_k'}{\partial x_k}}\right]\right)-\left(\left[\frac{\partial u_i' \overline{u_k' u_j'}}{\partial x_k}\right]+\left[\frac{\partial u_j' \overline{u_k' u_i'}}{\partial x_k}\right]\right)$$
$$-\left(\left[-\overline{u_k' u_j'} \frac{\partial u_i'}{\partial x_k}-\overline{u_k' u_i'} \frac{\partial u_j'}{\partial x_k}\right]\right)+f_k\left(\epsilon_{j,k,l} u_l' u_i'+\epsilon_{i,k,l} u_l' u_j'\right)$$
$$=-\left[\frac{\partial u_i' P'}{\partial x_j}+\frac{\partial u_j' P'}{\partial x_i}\right]+P'\left[\frac{\partial u_i'}{\partial x_j}+\frac{\partial u_j'}{\partial x_i}\right]-\beta\left(g_j u_i' \theta'-g_i u_j' \theta'\right)+\nu \frac{\partial}{\partial x_k}\left[\left(u_i' \frac{\partial u_j'}{\partial x_k}\right)+\left(u_j' \frac{\partial u_i'}{\partial x_k}\right)\right]-2 \nu\left(\frac{\partial u_j'}{\partial x_k} \frac{\partial u_i'}{\partial x_k}\right)$$

$$\left(\frac{\partial u_i' u_j'}{\partial t}\right)+\left(\left[\frac{\partial \bar{u}_k u_i' u_j'}{\partial x_k}\right]\right)+\left(\left[\frac{\partial u_k' u_i' u_j'}{\partial x_k}\right]\right)+\left(\left[u_i' u_k' \frac{\partial \bar{u}_j}{\partial x_k}+u_j' u_k' \frac{\partial \bar{u}_i}{\partial x_k}\right]\right)-\left(\left[\frac{\partial u_i' \overline{u_k' u_j'}}{\partial x_k}\right]+\left[\frac{\partial u_j' \overline{u_k' u_i'}}{\partial x_k}\right]\right)$$
$$-\left(\left[-\overline{u_k' u_j'} \frac{\partial u_i'}{\partial x_k}-\overline{u_k' u_i'} \frac{\partial u_j'}{\partial x_k}\right]\right)+f_k\left(\epsilon_{j,k,l} u_l' u_i'+\epsilon_{i,k,l} u_l' u_j'\right)$$
$$=-\left[\frac{\partial u_i' P'}{\partial x_j}+\frac{\partial u_j' P'}{\partial x_i}\right]+P'\left[\frac{\partial u_i'}{\partial x_j}+\frac{\partial u_j'}{\partial x_i}\right]-\beta\left(g_j u_i' \theta'-g_i u_j' \theta'\right)+\nu \frac{\partial}{\partial x_k}\left[\left(u_i' \frac{\partial u_j'}{\partial x_k}\right)+\left(u_j' \frac{\partial u_i'}{\partial x_k}\right)\right]-2 \nu\left(\frac{\partial u_j'}{\partial x_k} \frac{\partial u_i'}{\partial x_k}\right)$$

2. As equações básicas

$$
\begin{aligned}
&\left(\frac{\partial u_i' u_j'}{\partial t}\right) + \left(\left[\frac{\partial \bar{u}_k u_i' u_j'}{\partial x_k}\right]\right) + \left(\left[\frac{\partial u_k' u_i' u_j'}{\partial x_k}\right]\right) + \left(\left[u_i' u_k' \frac{\partial \bar{u}_j}{\partial x_k} + u_j' u_k' \frac{\partial \bar{u}_i}{\partial x_k}\right]\right) - \left(\left[\frac{\partial u_i' \overline{u_k' u_j'}}{\partial x_k}\right] + \left[\frac{\partial u_j' \overline{u_k' u_i'}}{\partial x_k}\right]\right) \\
&- \left(\left[-\overline{u_k' u_j'} \frac{\partial u_i'}{\partial x_k} - \overline{u_k' u_i'} \frac{\partial u_j'}{\partial x_k}\right]\right) + f_k\left(\epsilon_{j,k,l} u_l' u_i' + \epsilon_{i,k,l} u_l' u_j'\right) \\
&= -\left[\frac{\partial u_i' P'}{\partial x_j} + \frac{\partial u_j' P'}{\partial x_i}\right] + P'\left[\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right] - \beta\left(g_j u_i' \theta' - g_i u_j' \theta'\right) + \nu \frac{\partial}{\partial x_k}\left[\left(u_i' \frac{\partial u_j'}{\partial x_k}\right) + \left(u_j' \frac{\partial u_i'}{\partial x_k}\right)\right] - 2\nu\left(\frac{\partial u_j'}{\partial x_k}\frac{\partial u_i'}{\partial x_k}\right)
\end{aligned}
$$

$$
\begin{aligned}
&\left(\frac{\partial u_i' u_j'}{\partial t}\right) + \left(\left[\frac{\partial \bar{u}_k u_i' u_j'}{\partial x_k}\right]\right) + \left(\left[\frac{\partial u_k' u_i' u_j'}{\partial x_k}\right]\right) + \left(\left[u_i' u_k' \frac{\partial \bar{u}_j}{\partial x_k} + u_j' u_k' \frac{\partial \bar{u}_i}{\partial x_k}\right]\right) - \left(\left[\frac{\partial u_i' \overline{u_k' u_j'}}{\partial x_k}\right] + \left[\frac{\partial u_j' \overline{u_k' u_i'}}{\partial x_k}\right]\right) \\
&- \left(\left[-\overline{u_k' u_j'} \frac{\partial u_i'}{\partial x_k} - \overline{u_k' u_i'} \frac{\partial u_j'}{\partial x_k}\right]\right) + f_k\left(\epsilon_{j,k,l} u_l' u_i' + \epsilon_{i,k,l} u_l' u_j'\right) \\
&= -\left[\frac{\partial u_i' P'}{\partial x_j} + \frac{\partial u_j' P'}{\partial x_i}\right] + P'\left[\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right] - \beta\left(g_j u_i' \theta' - g_i u_j' \theta'\right) + \nu \frac{\partial}{\partial x_k}\left(\frac{\partial u_i' u_j'}{\partial x_k}\right) - 2\nu\left(\frac{\partial u_j'}{\partial x_k}\frac{\partial u_i'}{\partial x_k}\right)
\end{aligned}
$$

2. As equações básicas

$$\left(\frac{\partial u_i' u_j'}{\partial t}\right) + \left(\left[\frac{\partial \bar{u}_k u_i' u_j'}{\partial x_k}\right]\right) + \left(\left[\frac{\partial u_k' u_i' u_j'}{\partial x_k}\right]\right) + \left(\left[u_i' u_k' \frac{\partial \bar{u}_j}{\partial x_k} + u_j' u_k' \frac{\partial \bar{u}_i}{\partial x_k}\right]\right) - \left(\left[\frac{\partial u_i' \overline{u_k' u_j'}}{\partial x_k}\right] + \left[\frac{\partial u_j' \overline{u_k' u_i'}}{\partial x_k}\right]\right)$$

$$-\left(\left[-\overline{u_k' u_j'}\frac{\partial u_i'}{\partial x_k} - \overline{u_k' u_i'}\frac{\partial u_j'}{\partial x_k}\right]\right) + f_k\left(\epsilon_{j,k,l} u_l' u_i' + \epsilon_{i,k,l} u_l' u_j'\right)$$

$$= -\left[\frac{\partial u_i' P'}{\partial x_j} + \frac{\partial u_j' P'}{\partial x_i}\right] + P'\left[\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right] - \beta\left(g_j u_i'\theta' - g_i u_j'\theta'\right) + \nu\frac{\partial}{\partial x_k}\left(\frac{\partial u_i' u_j'}{\partial x_k}\right) - 2\nu\left(\frac{\partial u_j'}{\partial x_k}\frac{\partial u_i'}{\partial x_k}\right)$$

$$\left(\frac{\partial \overline{u_i' u_j'}}{\partial t}\right) + \left(\left[\frac{\partial \overline{\bar{u}_k u_i' u_j'}}{\partial x_k}\right]\right) + \left(\left[\frac{\partial \overline{u_k' u_i' u_j'}}{\partial x_k}\right]\right) + \left(\left[\overline{u_i' u_k'} \frac{\partial \bar{\bar{u}}_j}{\partial x_k} + \overline{u_j' u_k'} \frac{\partial \bar{\bar{u}}_i}{\partial x_k}\right]\right) - \left(\left[\frac{\partial \overline{u_i'}\,\overline{\overline{u_k' u_j'}}}{\partial x_k}\right] + \left[\frac{\partial \overline{u_j'}\,\overline{\overline{u_k' u_i'}}}{\partial x_k}\right]\right)$$

$$+\left(\left[\overline{\overline{u_k' u_j'}}\frac{\partial \overline{u_i'}}{\partial x_k} + \overline{\overline{u_k' u_i'}}\frac{\partial \overline{u_j'}}{\partial x_k}\right]\right) + f_k\left(\epsilon_{j,k,l} \overline{u_l' u_i'} + \epsilon_{i,k,l} \overline{u_l' u_j'}\right)$$

$$= -\left[\frac{\partial \overline{u_i' P'}}{\partial x_j} + \frac{\partial \overline{u_j' P'}}{\partial x_i}\right] + \bar{P}'\left[\frac{\partial \overline{u_i'}}{\partial x_j} + \frac{\partial \overline{u_j'}}{\partial x_i}\right] - \beta\left(g_j \overline{u_i'\theta'} - g_i \overline{u_j'\theta'}\right) + \nu\frac{\partial}{\partial x_k}\left(\frac{\partial \overline{u_i' u_j'}}{\partial x_k}\right) - 2\nu\left(\frac{\partial \overline{u_j'}}{\partial x_k}\frac{\partial \overline{u_i'}}{\partial x_k}\right)$$

2. As equações básicas

$$\left(\frac{\partial \overline{u_i' u_j'}}{\partial t}\right) + \left(\left[\frac{\partial \bar{u}_k \overline{u_i' u_j'}}{\partial x_k}\right]\right) + \left(\left[\frac{\partial \overline{u_k' u_i' u_j'}}{\partial x_k}\right]\right) + \left(\left[\overline{u_i' u_k'}\frac{\partial \bar{\bar{u}}_j}{\partial x_k} + \overline{u_j' u_k'}\frac{\partial \bar{\bar{u}}_i}{\partial x_k}\right]\right) - \left(\cancel{\left[\frac{\partial \overline{u_i'}\,\overline{\overline{u_k' u_j'}}}{\partial x_k}\right]} + \cancel{\left[\frac{\partial \overline{u_j'}\,\overline{\overline{u_k' u_i'}}}{\partial x_k}\right]}\right)$$

$$+ \left(\left[\cancel{\overline{\overline{u_k' u_j'}}\frac{\partial \overline{u_i'}}{\partial x_k}} + \cancel{\overline{\overline{u_k' u_i'}}\frac{\partial \overline{u_j'}}{\partial x_k}}\right]\right) + f_k\left(\epsilon_{j,k,l}\overline{u_l' u_i'} + \epsilon_{i,k,l}\overline{u_l' u_j'}\right)$$

$$= -\left[\frac{\partial \overline{u_i' P'}}{\partial x_j} + \frac{\partial \overline{u_j' P'}}{\partial x_i}\right] + \bar{P}'\left[\frac{\partial \overline{u_i'}}{\partial x_j} + \frac{\partial \overline{u_j'}}{\partial x_i}\right] - \beta\left(g_j \overline{u_i' \theta'} - g_i \overline{u_j' \theta'}\right) + \nu \frac{\partial}{\partial x_k}\left(\frac{\partial \overline{u_i' u_j'}}{\partial x_k}\right) - 2\nu\left(\frac{\partial \overline{u_j'}}{\partial x_k}\frac{\partial \overline{u_i'}}{\partial x_k}\right)$$

$$\left(\frac{\partial \overline{u_i' u_j'}}{\partial t}\right) + \left(\left[\frac{\partial \bar{u}_k \overline{u_i' u_j'}}{\partial x_k}\right]\right) + \left(\left[\frac{\partial \overline{u_k' u_i' u_j'}}{\partial x_k}\right]\right) + \left(\left[\overline{u_i' u_k'}\frac{\partial \bar{\bar{u}}_j}{\partial x_k} + \overline{u_j' u_k'}\frac{\partial \bar{\bar{u}}_i}{\partial x_k}\right]\right) + f_k\left(\epsilon_{j,k,l}\overline{u_l' u_i'} + \epsilon_{i,k,l}\overline{u_l' u_j'}\right)$$

$$= -\left[\frac{\partial \overline{u_i' P'}}{\partial x_j} + \frac{\partial \overline{u_j' P'}}{\partial x_i}\right] + \bar{P}'\left[\frac{\partial \overline{u_i'}}{\partial x_j} + \frac{\partial \overline{u_j'}}{\partial x_i}\right] - \beta\left(g_j \overline{u_i' \theta'} - g_i \overline{u_j' \theta'}\right) + \nu \frac{\partial}{\partial x_k}\left(\frac{\partial \overline{u_i' u_j'}}{\partial x_k}\right) - 2\nu\left(\frac{\partial \overline{u_j'}}{\partial x_k}\frac{\partial \overline{u_i'}}{\partial x_k}\right)$$

2. As equações básicas

$$\left(\frac{\partial \overline{u_i' u_j'}}{\partial t}\right) + \left(\left[\frac{\partial \overline{\bar{u}_k u_i' u_j'}}{\partial x_k}\right]\right) + \left(\left[\frac{\partial \overline{u_k' u_i' u_j'}}{\partial x_k}\right]\right) + \left(\left[\overline{u_i' u_k'}\frac{\partial \bar{\bar{u}}_j}{\partial x_k} + \overline{u_j' u_k'}\frac{\partial \bar{\bar{u}}_i}{\partial x_k}\right]\right) + f_k\left(\epsilon_{j,k,l}\overline{u_l' u_i'} + \epsilon_{i,k,l}\overline{u_l' u_j'}\right)$$

$$= -\left[\frac{\partial \overline{u_i' P'}}{\partial x_j} + \frac{\partial \overline{u_j' P'}}{\partial x_i}\right] + \overline{P'}\left[\frac{\partial \overline{u_i'}}{\partial x_j} + \frac{\partial \overline{u_j'}}{\partial x_i}\right] - \beta\left(g_j\overline{u_i'\theta'} - g_i\overline{u_j'\theta'}\right) + \nu\frac{\partial}{\partial x_k}\left(\frac{\partial \overline{u_i' u_j'}}{\partial x_k}\right) - 2\nu\left(\frac{\partial \overline{u_j'}}{\partial x_k}\frac{\partial \overline{u_i'}}{\partial x_k}\right)$$

$$\frac{\partial \overline{u_i' u_j'}}{\partial t} + \frac{\partial}{\partial x_k}\left[\bar{u}_k\overline{u_i' u_j'} + \overline{u_k' u_i' u_j'} - \nu\frac{\partial \overline{u_i' u_j'}}{\partial x_k}\right] + \frac{\partial \overline{P' u_i'}}{\partial x_j} + \frac{\partial \overline{P' u_j'}}{\partial x_i} + f_k\left(\epsilon_{j,k,l}u_l' u_i' + \epsilon_{i,k,l}u_l' u_j'\right)$$

$$= -\overline{u_k' u_i'}\frac{\partial \bar{u}_j}{\partial x_k} - \overline{u_k' u_j'}\frac{\partial \bar{u}_i}{\partial x_k} - \beta\left(g_j\overline{u_i'\theta'} + g_i\overline{u_j'\theta'}\right) + \overline{P'\left(\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right)} - 2\nu\overline{\left(\frac{\partial u_i'}{\partial x_k}\frac{\partial u_j'}{\partial x_k}\right)}$$

2. As equações básicas

$$\frac{\partial u_j'}{\partial t}+\frac{\partial}{\partial x_k}\left(\bar{u}_k u_j'+\bar{u}_j u_k'+u_k' u_j'-\overline{u_k' u_j'}\right)+\epsilon_{j,k,l} f_k u_l'=-\frac{\partial P'}{\partial x_j}-g_j\beta\theta'+\nu\nabla^2 u_j' \qquad (5)$$

$$\frac{\partial \theta'}{\partial t}+\frac{\partial}{\partial x_k}\left(\bar{\theta} u_k'+\bar{u}_k\theta'+u_k'\theta'+\overline{u_k'\theta'}\right)=\alpha\nabla^2\theta' \qquad (6)$$

$$\frac{\partial u_j'\theta'}{\partial t}=u_j'\frac{\partial \theta'}{\partial t}+\theta'\frac{\partial u_j'}{\partial t}$$

$$\frac{\partial \overline{u_j'\theta'}}{\partial t}+\frac{\partial}{\partial x_k}\left[\bar{u}_k\overline{\theta' u_j'}+\overline{u_k' u_j'\theta'}-\alpha\overline{u_j'\frac{\partial \theta'}{\partial x_k}}-\nu\overline{\theta'\frac{\partial u_j'}{\partial x_k}}\right]+\frac{\partial}{\partial x_j}\overline{P'\theta'}+\epsilon_{j,k,l} f_k\overline{u_l'\theta'}$$
$$=-\overline{u_j' u_k'}\frac{\partial \bar{\theta}}{\partial x_k}-\overline{\theta' u_k'}\frac{\partial \bar{u}_j}{\partial x_k}-\beta g_j\overline{\theta'^2}+\overline{P'\frac{\partial \theta'}{\partial x_j}}-(\alpha+\nu)\overline{\frac{\partial u_j'}{\partial x_k}\frac{\partial \theta'}{\partial x_k}} \qquad (8)$$

2. As equações básicas

A equação 8 envolve $\overline{\theta^2}$, na equação . Onde é necessário obter uma equação para este termo da equação 6

$$\frac{\partial \theta'}{\partial t} + \frac{\partial}{\partial x_k}\left(\bar{\theta} u_k' + \bar{u}_k \theta' + u_k' \theta' + \overline{u_k' \theta'}\right) = \alpha \nabla^2 \theta' \qquad (6)$$

$$\frac{\partial \overline{\theta'^1}}{\partial t} = + \frac{\partial}{\partial x_k}\left[\bar{u}_k \overline{\theta'^1} + \overline{k_k' \theta'^1} - \alpha \frac{\partial \overline{\theta'^1}}{\partial x_k}\right] = 2\overline{k_k' \theta'} - 2\alpha \overline{\frac{\partial \theta'}{\partial x_k} \frac{\partial \theta'}{\partial x_k}} \qquad (9)$$

2. As equações básicas

Suposições de modelagem

. A principal contribuição de **Rotta (1951)** foi sugerir ***uma suposição para o termo***, $\overline{P'\left(\frac{\partial u_i'}{\partial x_j}+\frac{\partial u_j'}{\partial x_i}\right)}$, que ele chamou de "***termo de redistribuição de energia***", **uma vez que uma de suas funções é particionar a energia entre os três componentes de energia sem contribuir para o total**. contração, o termo sai da Ed.7. Com base nas relações integrais obtidas da Eq.5 no caso neutro onde $-\beta\left(g_j u_i'\theta'\right)$ não é significante, Rotta sugeriu que o termo poderia ser razoavelmente proporcional a $u_i'u_j'$ e $\frac{\partial \bar{u}_i}{\partial x_j}$. Por isso,

$$\overline{P'\left(\frac{\partial u_i'}{\partial x_j}+\frac{\partial u_j'}{\partial x_i}\right)} = C_{ijkm}\overline{u_k'u_m'} + C_{ijkm}'\frac{\partial \bar{u}_k}{\partial x_m}$$

2. As equações básicas

Suposições de modelagem

aqui assumimos que os coeficientes constitutivos são tensores isotrópicos, ou seja

$$C_{ijkm} = C_1\delta_{i,j}\delta_{k,m} + C_2\delta_{i,k}\delta_{j,m} + C_3\delta_{j,k}\delta_{i,m}$$

Da eq. Da continuidade obtém-se $C, = \frac{(C_2+C_3)}{3}$ raciocínio semelhante se aplica ao C_{ijkm}. Obtém-se portanto

$$\overline{P'\left(\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right)} = C_{ijkm}\overline{u_k'u_m'} + C_{ijkm}'\frac{\partial \bar{u}_k}{\partial x_m}$$

$$\overline{P'\left(\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right)} = -\frac{q}{3l_1}\left(\overline{u_i'u_j'} - \frac{\delta_{i,j}}{3}q^2 + Cq^2\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right)\right)$$

Onde $-\frac{q}{3l_1}$ e Cq^2 the sido substituído por coeficientes escalares sobreviventes $q \equiv \left(\overline{u_i'^2}\right)^{1/2}$.O comprimento l_1 e a constante C devem ser determinadas empiricamente.

5.2 TURBULENT KINETIC ENERGY A equação de Navier Stokes: conservação de momentum

$$\frac{\partial(u_i'u_k')}{\partial t}+(\overline{u_j})\frac{\partial(u_i'u_k')}{\partial x_j}$$
$$=-(u_j'u_k')\frac{\partial(\overline{u_i})}{\partial x_j}-(u_j'\overline{u_i})\frac{\partial(u_k')}{\partial x_j}-\overline{u_i}u_k'\frac{\partial(u_j')}{\partial x_j}-\frac{\partial(u_j'u_i'u_k')}{\partial x_j}-\frac{1}{\rho_0}\frac{\partial(P'u_k')}{\partial x_i}-g\frac{\rho'u_k'}{\rho_0}\delta_{i3}-2\Omega\varepsilon_{ijk}\eta_j(u_k'u_k')+\nu\frac{\partial^2(u_i'u_k')}{\partial x_j{}^2}$$

Considere que a propriedade da equação da continuidade para o fluxo $\nabla.\vec{V}=0$

$$\nabla.\vec{V'}=\frac{\partial(u_j')}{\partial x_j}=0,\qquad j=1,2,3$$

$$\frac{\partial(u_i'u_k')}{\partial t}+(\overline{u_j})\frac{\partial(u_i'u_k')}{\partial x_j}=-(u_j'u_k')\frac{\partial(\overline{u_i})}{\partial x_j}-(u_j'\overline{u_i})\frac{\partial(u_k')}{\partial x_j}-\frac{\partial(u_j'u_i'u_k')}{\partial x_j}-\frac{1}{\rho_0}\frac{\partial(P'u_k')}{\partial x_i}-g\frac{\rho'u_k'}{\rho_0}\delta_{i3}-2\Omega\varepsilon_{ijk}\eta_j(u_k'u_k')+\nu\frac{\partial^2(u_i'u_k')}{\partial x_j{}^2}$$

Linearização da Equações de Navier Stokes

$$\frac{\partial \overline{u_j'\theta'}}{\partial t} + \frac{\partial}{\partial x_k}\left[\bar{u}_k\overline{\theta' u_j'} + \overline{u_k' u_j' \theta'} - \alpha\overline{u_j'\frac{\partial \theta'}{\partial x_k}} - \nu\overline{\theta'\frac{\partial u_j'}{\partial x_k}}\right] + \frac{\partial}{\partial x_j}\overline{P'\theta'} + \epsilon_{j,k,l} f_k \overline{u_l'\theta'}$$
$$= -\overline{u_j' u_k'}\frac{\partial \bar{\theta}}{\partial x_k} - \overline{\theta' u_k'}\frac{\partial \bar{u}_j}{\partial x_k} - \beta g_j \overline{\theta'^2} + \overline{P'\frac{\partial \theta'}{\partial x_j}} - (\alpha+\nu)\overline{\frac{\partial u_j'}{\partial x_k}\frac{\partial \theta'}{\partial x_k}} \qquad (8)$$

$$\frac{\partial \overline{u_i' u_j'}}{\partial t} + \frac{\partial}{\partial x_k}\left[\bar{u}_k\overline{u_i' u_j'} + \overline{u_k' u_i' u_j'} - \nu\frac{\partial \overline{u_i' u_j'}}{\partial x_k}\right] + \frac{\partial \overline{P' u_i'}}{\partial x_j} + \frac{\partial \overline{P' u_j'}}{\partial x_i} + f_k\left(\epsilon_{j,k,l} u_l' u_i' + \epsilon_{i,k,l} u_l' u_j'\right)$$
$$= -\overline{u_k' u_i'}\frac{\partial \bar{u}_j}{\partial x_k} - \overline{u_k' u_j'}\frac{\partial \bar{u}_i}{\partial x_k} - \beta\left(g_j\overline{u_i'\theta'} + g_i\overline{u_j'\theta'}\right) + \overline{P'\left(\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right)} - 2\nu\overline{\left(\frac{\partial u_i'}{\partial x_k}\frac{\partial u_j'}{\partial x_k}\right)}$$

Linearização da Equações de Navier Stokes

A equação de Navier Stokes: conservação de momentum

$$\frac{\partial \overline{u_i' u_j'}}{\partial t} + \frac{\partial}{\partial x_k}\left[\bar{u}_k \overline{u_i' u_j'}\right] = -\overline{u_k' u_i'}\frac{\partial \bar{u}_j}{\partial x_k} - \overline{u_k' u_j'}\frac{\partial \bar{u}_i}{\partial x_k} - \frac{\partial \overline{u_k' u_i' u_j'}}{\partial x_k} - \beta\left(g_j \overline{u_i'\theta'} + g_i \overline{u_j'\theta'}\right) - f_k\left(\epsilon_{j,k,l} u_l' u_i' + \epsilon_{i,k,l} u_l' u_j'\right) - \left[\frac{\partial \overline{P' u_i'}}{\partial x_j} + \frac{\partial \overline{P' u_j'}}{\partial x_i}\right] - 2\nu\overline{\left(\frac{\partial u_i'}{\partial x_k}\frac{\partial u_j'}{\partial x_k}\right)} + \nu\frac{\partial}{\partial x_k}\left[\frac{\partial \overline{u_i' u_j'}}{\partial x_k}\right] + \overline{\boldsymbol{P'}\left(\frac{\partial \boldsymbol{u_i'}}{\partial \boldsymbol{x_j}} + \frac{\partial \boldsymbol{u_j'}}{\partial \boldsymbol{x_i}}\right)}$$

O termo do lado esquerdo é a taxa temporal local de mudança e advecção de $\overline{(u_i' u_k')}$

O 1 e termo do lado direito são os termos de produção resultante da interação da turbulência e o escoamento médio

O 2 termo (terceiro momento) correlação tripla pode ser interpretado como transporte de turbulência (segundo momento) pela flutuação turbulenta com o ganho ou perda devido a divergência do fluxo turbulento

O 3 termo representa a produção e destruição da flutuabilidade (conversão da energia cinética turbulenta para a energia potencial turbulenta)

O termo 4 é a rotação e pode ser desprezado para média temporal menor do que 1 hora

O termo 5 é a interação da flutuação de pressão e do campo de velocidade

O termo 6 é a dissipação molecular

O termo 7 é a redistribuição de energia, **uma vez que uma de suas funções é particionar a energia entre os três componentes de energia sem contribuir para o total**

Linearização da Equações de Navier Stokes

Para caso de homogeneidade horizontal

$$\frac{\partial\overline{(u_i'u_k')}}{\partial t}+(\overline{\overline{u_j}})\frac{\partial\overline{(u_i'u_k')}}{\partial x_j}$$
$$=-\overline{(u_j'u_k')}\frac{\partial(\overline{\overline{u_i}})}{\partial x_j}-\frac{\partial\overline{\left(u_j'u_i'u_k'\right)}}{\partial x_j}-g\frac{\overline{\rho'u_k'}}{\rho_0}\delta_{i3}-2\Omega\varepsilon_{ijk}\eta_j\overline{(u_k'u_k')}-\frac{1}{\rho_0}\frac{\partial\overline{(P'u_k')}}{\partial x_i}+\nu\frac{\partial^2\overline{(u_i'u_k')}}{\partial {x_j}^2}$$

O termo molecular $\nu\frac{\partial^2\overline{(u_i'u_k')}}{\partial {x_j}^2}\rightarrow 0$ **é desprezado no caso da covariância, porque a viscosidade é dominante somente em numero de ondas grandes.**

Porém, neste caso a turbulência é isotrópica e assim a covariância é zero na horizontal.

(j=k) $$\frac{\partial\overline{u'w'}}{\partial t}=-\overline{w'^2}\frac{\partial\bar{u}}{\partial z}+\frac{g}{\overline{\theta_v}}\overline{u'\theta_v'}-\frac{\partial\overline{u'w'^2}}{\partial z}-\frac{1}{\rho}\left(\overline{u'\frac{\partial P'}{\partial z}}+\overline{w'\frac{\partial P'}{\partial x}}\right)$$

Isotrópico é a caracterização de uma substância que possui as mesmas propriedades físicas, independentemente da direção considerada.

Linearização da Equações de Navier Stokes

Para caso de homogeneidade horizontal e o estado básico em condições neutra $\frac{\partial \overline{u'w'}}{\partial t} = 0$ **e** $\frac{g}{\overline{\theta_v}}\overline{u'\theta_v'} = 0$

(j=k) $$\frac{\partial \overline{u'w'}}{\partial t} = -\overline{w'^2}\frac{\partial \bar{u}}{\partial z} + \frac{g}{\overline{\theta_v}}\overline{u'\theta_v'} - \frac{\partial \overline{u'w'^2}}{\partial z} - \frac{1}{\rho}\left(\overline{u'\frac{\partial P'}{\partial z}} + \overline{w'\frac{\partial P'}{\partial x}}\right)$$

Isto mostra que o termo de correlação de pressão-velocidade destrói o stress na mesma taxa como ela é produzida

$$0 = -\overline{w'^2}\frac{\partial \bar{u}}{\partial z} - \frac{\partial \overline{u'w'^2}}{\partial z} - \frac{1}{\rho}\left(\overline{u'\frac{\partial P'}{\partial z}} + \overline{w'\frac{\partial P'}{\partial x}}\right)$$

$$+\overline{w'^2}\frac{\partial \bar{u}}{\partial z} + \frac{\partial \overline{u'w'^2}}{\partial z} = -\frac{1}{\rho}\left(\overline{u'\frac{\partial P'}{\partial z}} + \overline{w'\frac{\partial P'}{\partial x}}\right)$$

Equação
da
Energia Cinética Turbulenta

Linearização da Equações de Navier Stokes

5.2 Energia Cinética Turbulenta

$$\frac{\partial\overline{(u_i'u_k')}}{\partial t}+(\bar{\bar{u}}_j)\frac{\partial\overline{(u_i'u_k')}}{\partial x_j}$$
$$=-\overline{(u_j'u_k')}\frac{\partial(\bar{\bar{u}}_i)}{\partial x_j}-\frac{\partial\overline{(u_j'u_i'u_k')}}{\partial x_j}-g\frac{\overline{\rho'u_k'}}{\rho_0}\delta_{i3}-2\Omega\varepsilon_{ijk}\eta_j\overline{(u_k'u_k')}-\frac{1}{\rho_0}\frac{\partial\overline{(P'u_k')}}{\partial x_i}+\nu\frac{\partial^2\overline{(u_i'u_k')}}{\partial {x_j}^2}$$

$$\overline{u_k'u_k'}=\overline{u_k'^2}=\overline{u_k'}^2=0$$

$$\bar{e}=\frac{\overline{u_i'^2}}{2}=\frac{\overline{\left(u'^2+v'^2+w'^2\right)}}{2} \qquad i=k=1{,}2{,}3$$

$$\frac{\partial\bar{e}}{\partial t}+(\bar{\bar{u}}_j)\frac{\partial\bar{e}}{\partial x_j}=-\overline{(u_j'u_i')}\frac{\partial(\bar{\bar{u}}_i)}{\partial x_j}-g\frac{\overline{u_i'\rho'}}{\rho_0}\delta_{i3}-\frac{\partial\overline{(eu_j')}}{\partial x_j}-\frac{1}{\rho_0}\frac{\partial\overline{(P'u_i')}}{\partial x_i}+\nu\frac{\partial^2\overline{(u_i'u_k')}}{\partial {x_j}^2}$$

$$\boxed{\frac{\partial\bar{e}}{\partial t}+(\bar{\bar{u}}_j)\frac{\partial\bar{e}}{\partial x_j}=-\frac{\overline{(u_j'u_i')}}{2}\frac{\partial(\bar{\bar{u}}_i)}{\partial x_j}-g\frac{\overline{u_i'\rho'}}{2\rho_0}\delta_{i3}-\frac{\partial\overline{(eu_j')}}{\partial x_j}-\frac{1}{2\rho_0}\frac{\partial\overline{(P'u_i')}}{\partial x_i}-\epsilon}$$

5.2 Energia Cinética Turbulenta

$$\frac{\partial \bar{e}}{\partial t} + \left(\bar{\bar{u}}_j\right)\frac{\partial \bar{e}}{\partial x_j} = -\frac{\overline{\left(u_j{'}u_i{'}\right)}}{2}\frac{\partial\left(\bar{\bar{u}}_i\right)}{\partial x_j} - g\,\frac{\overline{u_i{'}\rho'}}{2\rho_0}\delta_{i3} - \frac{\partial\overline{\left(eu_j{'}\right)}}{\partial x_j} - \frac{1}{2\rho_0}\frac{\partial\left(\overline{P'u_i{'}}\right)}{\partial x_i} - \epsilon$$

A quantidade ϵ é um parâmetro significante para a atmosfera desde que seja relacionado a dissipação da energia cinética turbulenta de todos os movimentos atmosféricos

5.2 Energia Cinética Turbulenta

A essência da equação da energia cinética turbulenta pode ser expressa pela equação:

$$\frac{D\bar{e}}{Dt} = -\frac{\overline{(u_j{}'u_i{}')}}{2}\frac{\partial(\bar{\bar{u}}_i)}{\partial x_j} - g\frac{\overline{u_i{}'\rho'}}{2\rho_0}\delta_{i3} - \frac{\partial\overline{(eu_j{}')}}{\partial x_j} - \frac{1}{2\rho_0}\frac{\partial(\overline{P'u_i{}'})}{\partial x_i} - \epsilon$$

$$\frac{\bar{D}(TKE)}{Dt} = MP + BPL + TR - \varepsilon$$

MP **é a produção mecânica**

BPL **é a produção e perda por flutuabilidade**

TR **redistribuição de tke por transporte e força de pressão**

ε **dissipação por atrito**

5.2 Energia Cinética Turbulenta

$$BPL \equiv \overline{w'\theta' \frac{g}{\theta_0}}$$

É a conversão da energia potencial do escoamento médio e a energia cinética turbulenta:
É positivo para movimentos que baixa o centro de massa da atmosfera
É negativo para movimentos que aumenta o centro de massa da atmosfera

Correlação positiva(fonte tke)
Atms. instável

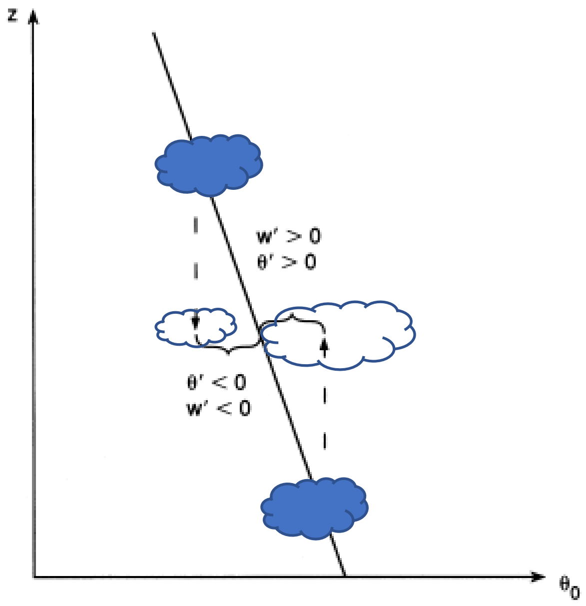
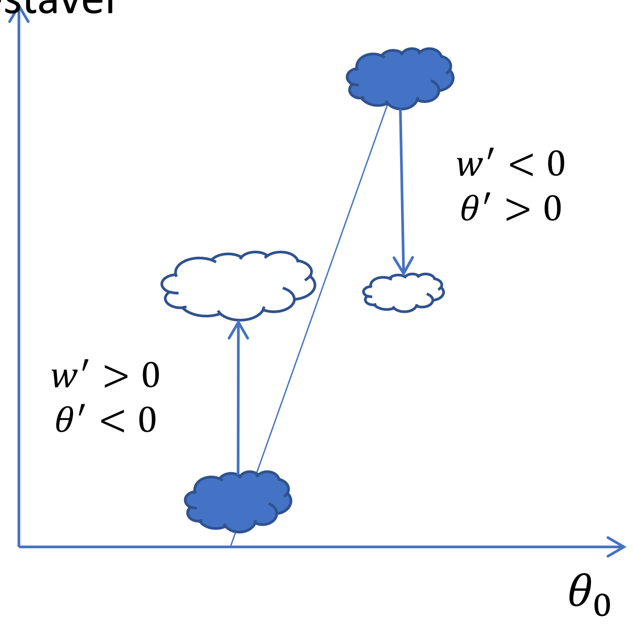


Fig. 5.1 Correlation between vertical velocity and potential temperature perturbations for upward or downward parcel displacements when the mean potential temperature $\theta_0(z)$ decreases with height.

Correlação negativa (destroi tke)
Atms. estável



5.2 Energia Cinética Turbulenta

Para ambos as condições estáveis e instáveis da CLP a turbulência pode ser produzida mecanicamente pela instabilidade dinâmica através do cisalhamento. Conversão de energia entre o escoamento médio e a flutuação turbulenta.

$$MP \equiv -\frac{\overline{u'w'}}{2}\frac{\partial \bar{u}}{\partial z} - \frac{\overline{v'w'}}{2}\frac{\partial \bar{v}}{\partial z}$$

$MP > 0$ **quando o fluxo de momentum (**$\overline{u'w'}$**) é direcionado para baixo e o gradiente vertical é positivo**

5.2 Energia Cinética Turbulenta

Estatisticamente na camada limite estável a turbulência pode existir somente se a produção mecânica for grande o suficiente para superar o efeito de supressão da estabilidade e da viscosidade

Esta condição é medida pelo numero de Richardson de fluxo

$$Rf \equiv -\frac{BPL}{MP} \equiv \frac{\overline{w'\theta'\frac{g}{\theta_0}}}{-\overline{u'w'}\frac{\partial \bar{u}}{\partial z} - \overline{v'w'}\frac{\partial \bar{v}}{\partial z}}$$

5.2 Energia Cinética Turbulenta

$$Rf \equiv -\frac{BPL}{MP} \equiv \frac{\overline{w'\theta'}\frac{g}{\theta_0}}{-\overline{u'w'}\frac{\partial \bar{u}}{\partial z} - \overline{v'w'}\frac{\partial \bar{v}}{\partial z}}$$

- **Se** $Rf < 0$ **a CLP é estatisticamente instável (a turbulências é sustentada pela convecção)**
- $Rf > 0$ **a CLP é estatisticamente estável**
- $Rf < 0.25$ **(a produção mecânica excede a produção por flutuabilidade por um fato de 4)**

5.2 Energia Cinética Turbulenta

5.3 Equações de momentum da camada limite Planetária

$$\frac{\partial(\overline{u_i})}{\partial t}+(\overline{u_j})\frac{\partial(\overline{u_i})}{\partial x_j}=-\frac{\partial\left(\overline{\boldsymbol{u_j}'\boldsymbol{u_i}'}\right)}{\partial x_j}-\frac{1}{\rho_0}\frac{\partial(\bar{P})}{\partial x_i}-g\frac{\bar{\rho}}{\rho_0}\delta_{i3}-2\Omega\varepsilon_{ijk}\eta_j(\overline{u_k})+\nu\frac{\partial^2(\overline{u_i})}{\partial {x_j}^2}$$

$$\frac{\partial\left(\overline{\boldsymbol{u_i}'\boldsymbol{u_k}'}\right)}{\partial t}+(\bar{\bar{u}}_j)\frac{\partial\left(\overline{u_i'u_k'}\right)}{\partial x_j}$$
$$=-\left(\overline{u_j'u_k'}\right)\frac{\partial(\bar{\bar{u}}_i)}{\partial x_j}-g\frac{\overline{\rho'u_k'}}{\rho_0}\delta_{i3}-2\Omega\varepsilon_{ijk}\eta_j\overline{(u_k'u_k')}-\frac{\partial\left(\overline{u_j'u_i'u_k'}\right)}{\partial x_j}-\frac{1}{\rho_0}\frac{\partial(\overline{P'u_k'})}{\partial x_i}+\nu\frac{\partial^2\overline{(u_i'u_k')}}{\partial {x_j}^2}$$

$$\frac{\partial\bar{e}}{\partial t}+(\bar{\bar{u}}_j)\frac{\partial\bar{e}}{\partial x_j}=-\frac{\left(\overline{u_j'u_i'}\right)}{2}\frac{\partial(\bar{\bar{u}}_i)}{\partial x_j}-g\frac{\overline{u_i'\rho'}}{2\rho_0}\delta_{i3}-\frac{\partial\left(\overline{eu_j'}\right)}{\partial x_j}-\frac{1}{2\rho_0}\frac{\partial(\overline{P'u_i'})}{\partial x_i}-\epsilon$$

Paulo Yoshio Kubota

5.2 Energia Cinética Turbulenta

5.3 Equações de momentum da camada limite Planetária

$$\frac{\partial(\overline{u_i})}{\partial t} + (\overline{u_j})\frac{\partial(\overline{u_i})}{\partial x_j} = -\frac{\partial\left(\overline{u_j{}'u_i{}'}\right)}{\partial x_j} - \frac{1}{\rho_0}\frac{\partial(\bar{P})}{\partial x_i} - g\frac{\rho}{\rho_0}\delta_{i3} - 2\Omega\varepsilon_{ijk}\eta_j(\overline{u_k}) + \nu\frac{\partial^2(\overline{u_i})}{\partial x_j{}^2}$$

Para o caso especial de turbulência horizontalmente homogênea:

-> A camada viscosa, a viscosidade molecular e o temo da divergência horizontal do fluxo de momentum turbulento podem ser desprezados.

$$\frac{\bar{D}\bar{u}}{Dt} = -\frac{1}{\rho_0}\frac{\partial\bar{P}}{\partial x} + f\bar{v} - \frac{\partial\overline{u'w'}}{\partial z}$$

$$\frac{\bar{D}\bar{v}}{Dt} = -\frac{1}{\rho_0}\frac{\partial\bar{P}}{\partial y} - f\bar{u} - \frac{\partial\overline{v'w'}}{\partial z}$$

Só pode ser resolvida se conhecermos a distribuição vertical do fluxo de momentum