Derivadas de 1 orden nos espaço?
Erro nas discretizações das Derivadas espaciais

Faça a Expansão de Taylor para 1 ponto de grade

Paulo Yoshio Kubota

Faça a Expansão de Taylor para 1 ponto de grade

Métodos de diferenças finitas.

$$u_{j+1}^{n} = u_j^n + \Delta x \frac{\partial u}{\partial x} + \frac{\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} + \frac{\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots. \qquad (1B)$$

$$u_{j-1}^{n} = u_j^n - \Delta x \frac{\partial u}{\partial x} + \frac{\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} - \frac{\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots. \qquad (2B)$$

A partir da eq 1c e 2C, obtém-se Discretização de 1 ordem o erro

$$\frac{u_j^n - u_{j-1}^n}{\Delta x} = \frac{\partial u}{\partial x} - \frac{\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} - \frac{\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \dots. \qquad (BB)$$

$$\frac{u_{j+1}^n - u_j^n}{\Delta x} = \frac{\partial u}{\partial x} + \frac{\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \dots. \qquad (BB)$$

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Métodos de diferenças finitas.

$$u_{j+1}^{n} = u_{j}^{n} + \Delta x \frac{\partial u}{\partial x} + \frac{\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} + \frac{\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (1B)$$

$$u_{j-1}^{n} = u_{j}^{n} - \Delta x \frac{\partial u}{\partial x} + \frac{\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} - \frac{\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (2B)$$

Subtrai 1c-2C , obtem-se a Discretização de 2 ordem no erro

$$u_{j+1} - u_{j-1} = 2\Delta x \frac{\partial u}{\partial x} + \frac{2}{3!}\frac{\partial^3 u}{\partial x^3}(\Delta x)^3 + O[(\Delta x)^4] \qquad (3B)$$

$$\frac{u_{j+1} - u_{j-1}}{2\Delta x} = \frac{\partial u}{\partial x} + \frac{1}{3!}\frac{\partial^3 u}{\partial x^3}(\Delta x)^2 + O[(\Delta x)^4] \qquad (4B)$$

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Métodos de diferenças finitas.

$$u_{j+1}^{n} = u_j^n + \Delta x \frac{\partial u}{\partial x} + \frac{\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} + \frac{\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (1B)$$

$$u_{j-1}^{n} = u_j^n - \Delta x \frac{\partial u}{\partial x} + \frac{\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} - \frac{\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (2B)$$

soma 1C+2C, , obtém-se a Discretização de 2 ordem na derivada e 2 orden no erro

$$u_{j+1} + u_{j-1} = 2u_j + \frac{\partial^2 u}{\partial x^2}\Delta x^2 + \frac{2}{4!}\frac{\partial^4 u}{\partial x^4}(\Delta x)^4 + O[(\Delta x)^4] \qquad (5B)$$

$$\frac{u_{j+1} - 2u_j + u_{j-1}}{\Delta x^2} = +\frac{\partial^2 u}{\partial x^2} + \frac{2}{4!}\frac{\partial^4 u}{\partial x^4}(\Delta x)^2 + O[(\Delta x)^4] \qquad (6B)$$

Faça a Expansão de Taylor para 2 ponto de grade

Métodos de diferenças finitas.

$$u_{j+2}^{n} = u_j^n + 2\Delta x\frac{\partial u}{\partial x} + \frac{4\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} + \frac{8\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{16\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (1C)$$

$$u_{j-2}^{n} = u_j^n - 2\Delta x\frac{\partial u}{\partial x} + \frac{4\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} - \frac{8\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{16\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (2C)$$

A partir da eq 1c e 2C, obtém-se Discretização de 1 ordem o erro

$$\frac{u_{j+2}^{n} - u_j^n}{2\Delta x} = \frac{\partial u}{\partial x} + \frac{2\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{6\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} + \frac{14\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (1C)$$

$$\frac{\left(u_j^n - u_{j-2}^{n}\right)}{2\Delta x} = \frac{\partial u}{\partial x} - \frac{2\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{6\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} - \frac{8\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (CC)$$

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Métodos de diferenças finitas.

$$u_{j+2}^{n} = u_j^n + 2\Delta x \frac{\partial u}{\partial x} + \frac{4\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} + \frac{8\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{16\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4}\dots. \qquad (1C)$$

$$u_{j-2}^{n} = u_j^n - 2\Delta x \frac{\partial u}{\partial x} + \frac{4\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} - \frac{8\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{16\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4}\dots. \qquad (2C)$$

Subtrai 1c-2C , obtem-se a Discretização de 2 ordem no erro

$$u_{j+2} - u_{j-2} = 4\Delta x \frac{\partial u}{\partial x} + \frac{16}{3!}\frac{\partial^3 u}{\partial x^3}(\Delta x)^3 + O[(\Delta x)^4] \qquad (3C)$$

$$\frac{u_{j+2} - u_{j-2}}{4\Delta x} = \frac{\partial u}{\partial x} + \frac{4}{3!}\frac{\partial^3 u}{\partial x^3}(\Delta x)^2 + O[(\Delta x)^4] \qquad (4C)$$

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Métodos de diferenças finitas.

$$u_{j+2}^{n} = u_j^n + 2\Delta x\frac{\partial u}{\partial x} + \frac{4\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} + \frac{8\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{16\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4}\dots. \qquad (1C)$$

$$u_{j-2}^{n} = u_j^n - 2\Delta x\frac{\partial u}{\partial x} + \frac{4\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} - \frac{8\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{16\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4}\dots. \qquad (2C)$$

soma 1C+2C, , obtém-se a Discretização de 2 ordem na derivada e 2 orden no erro

$$u_{j+2} + u_{j-2} = 2u_j + \frac{8\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} + \frac{32}{4!}\frac{\partial^4 u}{\partial x^4}(\Delta x)^4 + O[(\Delta x)^4] \qquad (5C)$$

$$\frac{u_{j+2} - 2u_j + u_{j-2}}{8\Delta x^2} = +\frac{1}{2}\frac{\partial^2 u}{\partial x^2} + \frac{4}{4!}\frac{\partial^4 u}{\partial x^4}(\Delta x)^2 + O[(\Delta x)^4] \qquad (6C)$$

$$\frac{u_{j+2} - 2u_j + u_{j-2}}{4\Delta x^2} = \frac{\partial^2 u}{\partial x^2} + \frac{4}{4!}\frac{\partial^4 u}{\partial x^4}(\Delta x)^2 + O[(\Delta x)^4] \qquad (6C)$$

Combinação das a Expansão de Taylor para1 e 2 ponto de grade

segunda ordem

Esquemas com diferenças centradas no espaço de segunda ordem:

$$u_{j+1}^{n} = u_j^n + \Delta x \frac{\partial u}{\partial x} + \frac{\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} + \frac{\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots. \quad (1B)$$

$$u_{j-1}^{n} = u_j^n - \Delta x \frac{\partial u}{\partial x} + \frac{\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} - \frac{\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots. \quad (2B)$$

$$u_{j+2}^{n} = u_j^n + 2\Delta x \frac{\partial u}{\partial x} + \frac{4\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} + \frac{8\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{16\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots. \quad (1C)$$

$$u_{j-2}^{n} = u_j^n - 2\Delta x \frac{\partial u}{\partial x} + \frac{4\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} - \frac{8\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{16\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots. \quad (2C)$$

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Métodos de diferenças finitas.

Esquemas com diferenças centradas no espaço de segunda ordem:

$$u_{j-1}^{n} = u_{j}^{n} - \Delta x\frac{\partial u}{\partial x} + \frac{\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} - \frac{\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots. \qquad (2B)$$

$$u_{j-2}^{n} = u_{j}^{n} - 2\Delta x\frac{\partial u}{\partial x} + \frac{4\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} - \frac{8\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{16\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots. \qquad (2C)$$

Rearranje os termos das equações

$$\frac{u_{j}^{n} - u_{j-1}^{n}}{\Delta x} = \frac{\partial u}{\partial x} - \frac{\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} - \frac{\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \dots. \qquad (BB)$$

$$\frac{\left(u_{j}^{n} - u_{j-2}^{n}\right)}{2\Delta x} = \frac{\partial u}{\partial x} - \frac{2\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{4\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} - \frac{8}{24}\frac{\partial^4 u}{\partial x^4} \dots. \qquad (CC)$$

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Esquemas com diferenças centradas no espaço de segunda ordem:

$$\frac{u_j^n - u_{j-1}^n}{\Delta x} = \frac{\partial u}{\partial x} - \frac{\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} - \frac{\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \ldots. \qquad (BB)$$

$$\frac{\left(u_j^n - u_{j-2}^n\right)}{2\Delta x} = \frac{\partial u}{\partial x} - \frac{2\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{4\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} - \frac{8\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \ldots. \qquad (CC)$$

Mutiplica a BB por 2

$$2\frac{u_j - u_{j-1}}{\Delta x} = 2\frac{\partial u}{\partial x} + 2\frac{\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{2\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} - \frac{2\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} + O[(\Delta x)^4] \qquad (AD)$$

Soma a AD a eq. CC

$$2\frac{u_j - u_{j-1}}{\Delta x} + \frac{u_j - u_{j-2}}{2\Delta x} = 3\frac{\partial u}{\partial x} + \frac{6\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} \ldots..$$

$$\frac{2}{3}\frac{u_j - u_{j-1}}{\Delta x} - \frac{1}{3}\frac{u_j - u_{j-2}}{2\Delta x} = \frac{\partial u}{\partial x} + \Delta x^2 \frac{\partial^3 u}{\partial x^3} \ldots.. \qquad (2D)$$

$$\left(\frac{-u_j + 2u_j - 2u_{j-1} + u_{j-2}}{6\Delta x}\right) = \frac{\partial u}{\partial x} + \Delta x^2 \frac{\partial^3 u}{\partial x^3} \ldots..(2D)$$

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Combinação das a Expansão de Taylor para1 e 2 ponto de grade

2 opção segunda ordem

Métodos de diferenças finitas.

Esquemas com diferenças centradas no espaço de segunda ordem:

$$u_{j+1}^{n} = u_j^n + \Delta x \frac{\partial u}{\partial x} + \frac{\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} + \frac{\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (1B)$$

$$u_{j-1}^{n} = u_j^n - \Delta x \frac{\partial u}{\partial x} + \frac{\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} - \frac{\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (2B)$$

$$u_{j+2}^{n} = u_j^n + 2\Delta x \frac{\partial u}{\partial x} + \frac{4\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} + \frac{8\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{16\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (1C)$$

$$u_{j-2}^{n} = u_j^n - 2\Delta x \frac{\partial u}{\partial x} + \frac{4\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} - \frac{8\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{16\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (2C)$$

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Métodos de diferenças finitas.

Esquemas com diferenças centradas no espaço de segunda ordem:

$$u_{j-1}^n = u_j^n - \Delta x \frac{\partial u}{\partial x} + \frac{\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} - \frac{\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots. \qquad (2B)$$

$$u_{j+2}^n = u_j^n + 2\Delta x \frac{\partial u}{\partial x} + \frac{4\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} + \frac{8\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{16\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots. \qquad (1C)$$

$$\frac{u_{j-1}^n - u_j^n}{\Delta x} = -\frac{\partial u}{\partial x} + \frac{\Delta x}{2}\frac{\partial^2 u}{\partial x^2} - \frac{\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \dots. \qquad (BB)$$

$$\frac{(u_{j+2}^n - u_j^n)}{2\Delta x} = \frac{\partial u}{\partial x} + \frac{2\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{4\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} - \frac{8\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \dots. \qquad (CC)$$

Métodos de diferenças finitas.

Esquemas com diferenças centradas no espaço de segunda ordem:

$$\frac{u_{j-1}^n - u_j^n}{\Delta x} = -\frac{\partial u}{\partial x} + \frac{\Delta x}{2}\frac{\partial^2 u}{\partial x^2} - \frac{\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \dots. \qquad (BB)$$

$$\frac{(u_{j+2}^n - u_j^n)}{2\Delta x} = \frac{\partial u}{\partial x} + \frac{2\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{4\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} - \frac{8\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \dots. \qquad (CC)$$

Mutiplica a BB por 2

$$2\frac{u_{j-1} - u_j}{\Delta x} = -2\frac{\partial u}{\partial x} + 2\frac{\Delta x}{2}\frac{\partial^2 u}{\partial x^2} - \frac{2\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} + \frac{2\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} + O[(\Delta x)^4] \qquad (AD)$$

subtrai a AD a eq. CC

$$2\frac{u_{j-1} - u_j}{\Delta x} - \frac{u_{j+2} - u_j}{2\Delta x} = -3\frac{\partial u}{\partial x} - \frac{6\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} \dots..$$

$$\frac{2u_{j-1} - 2u_j}{3\Delta x} - \frac{u_{j+2} - u_j}{6\Delta x} = \frac{\partial u}{\partial x} - \Delta x^2\frac{\partial^3 u}{\partial x^3} \dots.. \qquad (2D)$$

$$\left(\frac{4u_{j-1} - 4u_j - u_{j+2} + u_j}{6\Delta x}\right) = \frac{\partial u}{\partial x} + \frac{\Delta x^2}{3}\frac{\partial^3 u}{\partial x^3} \dots.. (2D)$$

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Esquemas com diferenças centradas no espaço de segunda ordem:

$$\left(\frac{4u_{j-1} - 4u_j - u_{j+2} + u_j}{6\Delta x}\right) = \frac{\partial u}{\partial x} + \frac{\Delta x^2}{3}\frac{\partial^3 u}{\partial x^3} \quad \ldots\ldots(2D)$$

$$\left(\frac{4u_{j-1} - 4u_j - u_{j+2} + u_j}{6\Delta x}\right) = \frac{\partial u}{\partial x} + \frac{\Delta x^2}{3}\frac{\partial^3 u}{\partial x^3} \quad \ldots\ldots(2D)$$

$$\left(\frac{4u_{j-1} - 3u_j - u_{j+2}}{6\Delta x}\right) = \frac{\partial u}{\partial x} + \frac{\Delta x^2}{3}\frac{\partial^3 u}{\partial x^3} \quad \ldots\ldots(2D)$$

Métodos de diferenças finitas.

Combinação das a Expansão de Taylor para1 e 2 ponto de grade

segunda ordem

Avançado

Métodos de diferenças finitas.

Esquemas com diferenças centradas no espaço de segunda ordem:

$$u_{j+1}^{n} = u_j^n + \Delta x \frac{\partial u}{\partial x} + \frac{\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} + \frac{\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (1B)$$

$$u_{j+2}^{n} = u_j^n + 2\Delta x \frac{\partial u}{\partial x} + \frac{4\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} + \frac{8\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{16\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (1C)$$

$$\frac{u_{j+1}^{n} - u_j^n}{\Delta x} = \frac{\partial u}{\partial x} + \frac{\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (BB)$$

$$\frac{(u_{j+2}^{n} - u_j^n)}{2\Delta x} = \frac{\partial u}{\partial x} + \frac{2\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{4\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} + \frac{8\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (CC)$$

Métodos de diferenças finitas.

Esquemas com diferenças centradas no espaço de segunda ordem:

$$\frac{(u_{j+2}^n - u_j^n)}{2\Delta x} = \frac{\partial u}{\partial x} + \frac{2\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{4\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} + \frac{8\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (CC)$$

$$\frac{u_j^n - u_{j-1}^n}{\Delta x} = -\frac{\partial u}{\partial x} + \frac{\Delta x}{2}\frac{\partial^2 u}{\partial x^2} - \frac{\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (BB)$$

Multiplica por 4

$$4\frac{u_j^n - u_{j-1}^n}{\Delta x} = -4\frac{\partial u}{\partial x} + \frac{4\Delta x}{2}\frac{\partial^2 u}{\partial x^2} - \frac{4\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} + \frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (DD)$$

Subtra DD da CC

$$4\frac{u_j^n - u_{j-1}^n}{\Delta x} - \frac{(u_{j+2}^n - u_j^n)}{2\Delta x} = -5\frac{\partial u}{\partial x} + \frac{2\Delta x}{2}\frac{\partial^2 u}{\partial x^2} - \frac{8\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} - \frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \dots \qquad (dd)$$

$$4\frac{u_j^n - u_{j-1}^n}{\Delta x} - \frac{(u_{j+2}^n - u_j^n)}{2\Delta x} - \Delta x\frac{\partial^2 u}{\partial x^2} = -5\frac{\partial u}{\partial x} - \frac{8\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} - \frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (dd)$$

Métodos de diferenças finitas.

Esquemas com diferenças centradas no espaço de segunda ordem:

$$4\frac{u_j^n - u_{j-1}^n}{\Delta x} - \frac{\left(u_{j+2}^n - u_j^n\right)}{2\Delta x} - \Delta x\frac{\partial^2 u}{\partial x^2} = -5\frac{\partial u}{\partial x} - \frac{8\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} - \frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}\ldots. \qquad (dd)$$

$$4\frac{u_j^n - u_{j-1}^n}{5\Delta x} - \frac{\left(u_{j+2}^n - u_j^n\right)}{10\Delta x} - \frac{\Delta x}{5}\frac{\partial^2 u}{\partial x^2} = -\frac{\partial u}{\partial x} - \frac{8\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} - \frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}\ldots. \qquad (dd)$$

Sabe-se que $\frac{\partial^2 u}{\partial x^2} \cong \frac{u_{j+2} - 2u_j + u_{j-2}}{4\Delta x^2}$

$$\frac{u_{j+2} - 2u_j + u_{j-2}}{4\Delta x^2} = \frac{\partial^2 u}{\partial x^2} + \frac{4}{4!}\frac{\partial^4 u}{\partial x^4}(\Delta x)^2 + O[(\Delta x)^4] \qquad (6C)$$

$$4\frac{u_j^n - u_{j-1}^n}{5\Delta x} - \frac{\left(u_{j+2}^n - u_j^n\right)}{10\Delta x} - \frac{\Delta x}{5}\frac{u_{j+2} - 2u_j + u_{j-2}}{4\Delta x^2} = -\frac{\partial u}{\partial x} - \frac{8\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} - \frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}$$

$$4\frac{u_j^n - u_{j-1}^n}{5\Delta x} - \frac{\left(u_{j+2}^n - u_j^n\right)}{10\Delta x} - \frac{u_{j+2} - 2u_j + u_{j-2}}{20\Delta x} = -\frac{\partial u}{\partial x} - \frac{8\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} - \frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}$$

Métodos de diferenças finitas.

Esquemas com diferenças centradas no espaço de segunda ordem:

$$4\frac{u_j^n - u_{j-1}^n}{5\Delta x} - \frac{\left(u_{j+2}^n - u_j^n\right)}{10\Delta x} - \frac{u_{j+2} - 2u_j + u_{j-2}}{20\Delta x} = -\frac{\partial u}{\partial x} - \frac{8\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} - \frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}$$

$$\frac{4u_j^n - 4u_{j-1}^n}{5\Delta x} - \frac{\left(u_{j+2}^n - u_j^n\right)}{10\Delta x} - \frac{u_{j+2} - 2u_j + u_{j-2}}{20\Delta x} = -\frac{\partial u}{\partial x} - \frac{8\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} - \frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}$$

$$\frac{8u_j^n - 8u_{j-1}^n - u_{j+2}^n + u_j^n}{10\Delta x} - \frac{u_{j+2} - 2u_j + u_{j-2}}{20\Delta x} = -\frac{\partial u}{\partial x} - \frac{8\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} - \frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}$$

$$-\frac{8u_j^n - 8u_{j-1}^n - u_{j+2}^n + u_j^n}{10\Delta x} + \frac{u_{j+2} - 2u_j + u_{j-2}}{20\Delta x} = \frac{\partial u}{\partial x} - O[\Delta x^2]$$

Paulo Yoshio Kubota

Métodos de diferenças finitas.

Esquemas com diferenças centradas no espaço de segunda ordem:

$$-\frac{8u_j^n - 8u_{j-1}^n - u_{j+2}^n + u_j^n}{10\Delta x} + \frac{u_{j+2} - 2u_j + u_{j-2}}{20\Delta x} = \frac{\partial u}{\partial x} - O[\Delta x^2]$$

$$\frac{\partial u}{\partial x} = \frac{u_{j+2}^n - 9u_j^n + 8u_{j-1}^n}{10\Delta x} + \frac{u_{j+2} - 2u_j + u_{j-2}}{20\Delta x} + O[\Delta x^2]$$

Combinação das a Expansão de Taylor para1 e 2 ponto de grade

terceira ordem

Avançado

Esquemas com diferenças centradas no espaço de terceira ordem:

$$u_{j+1}^{n} = u_j^n + \Delta x \frac{\partial u}{\partial x} + \frac{\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} + \frac{\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots. \qquad (1B)$$

$$u_{j+2}^{n} = u_j^n + 2\Delta x \frac{\partial u}{\partial x} + \frac{4\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} + \frac{8\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{16\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots. \qquad (1C)$$

$$\frac{u_{j+1}^{n} - u_j^n}{\Delta x} = \frac{\partial u}{\partial x} + \frac{\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \dots. \qquad (BB)$$

$$\frac{\left(u_{j+2}^{n} - u_j^n\right)}{2\Delta x} = \frac{\partial u}{\partial x} + \frac{2\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{4\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} + \frac{8\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \dots. \qquad (CC)$$

Métodos de diferenças finitas.

Esquemas com diferenças centradas no espaço de terceira ordem:

$$\frac{(u_{j+2}^n - u_j^n)}{2\Delta x} = \frac{\partial u}{\partial x} + \frac{2\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{4\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} + \frac{8\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}\dots. \qquad (CC)$$

$$\frac{u_j^n - u_{j-1}^n}{\Delta x} = -\frac{\partial u}{\partial x} + \frac{\Delta x}{2}\frac{\partial^2 u}{\partial x^2} - \frac{\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}\dots. \qquad (BB)$$

Multiplica por 4

$$4\frac{u_j^n - u_{j-1}^n}{\Delta x} = -4\frac{\partial u}{\partial x} + \frac{4\Delta x}{2}\frac{\partial^2 u}{\partial x^2} - \frac{4\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} + \frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}\dots. \qquad (DD)$$

soma DD da CC

$$4\frac{u_j^n - u_{j-1}^n}{\Delta x} + \frac{(u_{j+2}^n - u_j^n)}{2\Delta x} = -3\frac{\partial u}{\partial x} + \frac{6\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{12\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}\dots \qquad (dd)$$

$$4\frac{u_j^n - u_{j-1}^n}{\Delta x} + \frac{(u_{j+2}^n - u_j^n)}{2\Delta x} - \frac{6\Delta x}{2}\frac{\partial^2 u}{\partial x^2} = -3\frac{\partial u}{\partial x} + \frac{12\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}\dots. \qquad (dd)$$

Paulo Yoshio Kubota

Métodos de diferenças finitas.

Esquemas com diferenças centradas no espaço de terceira ordem:

$$4\frac{u_j^n - u_{j-1}^n}{\Delta x} + \frac{\left(u_{j+2}^n - u_j^n\right)}{2\Delta x} - \frac{6\Delta x}{2}\frac{\partial^2 u}{\partial x^2} = -3\frac{\partial u}{\partial x} + \frac{12\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}\dots \qquad (dd)$$

$$4\frac{u_j^n - u_{j-1}^n}{3\Delta x} + \frac{\left(u_{j+2}^n - u_j^n\right)}{6\Delta x} - \frac{6\Delta x}{6}\frac{\partial^2 u}{\partial x^2} = -\frac{\partial u}{\partial x} + \frac{12\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}\dots \qquad (dd)$$

Sabe-se que $\frac{\partial^2 u}{\partial x^2} \cong \frac{u_{j+2} - 2u_j + u_{j-2}}{4\Delta x^2}$

$$\frac{u_{j+2} - 2u_j + u_{j-2}}{4\Delta x^2} = \frac{\partial^2 u}{\partial x^2} + \frac{4}{4!}\frac{\partial^4 u}{\partial x^4}(\Delta x)^2 + O[(\Delta x)^4] \qquad (6C)$$

$$4\frac{u_j^n - u_{j-1}^n}{3\Delta x} + \frac{\left(u_{j+2}^n - u_j^n\right)}{6\Delta x} - \frac{\Delta x}{1}\frac{u_{j+2} - 2u_j + u_{j-2}}{4\Delta x^2} = -\frac{\partial u}{\partial x} + \frac{12\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}$$

$$4\frac{u_j^n - u_{j-1}^n}{3\Delta x} + \frac{\left(u_{j+2}^n - u_j^n\right)}{6\Delta x} - \frac{u_{j+2} - 2u_j + u_{j-2}}{4\Delta x} = -\frac{\partial u}{\partial x} + \frac{12\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}$$

Métodos de diferenças finitas.

Esquemas com diferenças centradas no espaço de terceira ordem:

$$4\frac{u_j^n - u_{j-1}^n}{3\Delta x} + \frac{\left(u_{j+2}^n - u_j^n\right)}{6\Delta x} - \frac{u_{j+2} - 2u_j + u_{j-2}}{4\Delta x} = -\frac{\partial u}{\partial x} + \frac{12\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}$$

$$\frac{4u_j^n - 4u_{j-1}^n}{3\Delta x} + \frac{\left(u_{j+2}^n - u_j^n\right)}{6\Delta x} - \frac{u_{j+2} - 2u_j + u_{j-2}}{4\Delta x} = -\frac{\partial u}{\partial x} + \frac{12\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}$$

$$\frac{8u_j^n - 8u_{j-1}^n + u_{j+2}^n - u_j^n}{6\Delta x} - \frac{u_{j+2} - 2u_j + u_{j-2}}{4\Delta x} = -\frac{\partial u}{\partial x} + \frac{12\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}$$

$$\frac{7u_j^n - 8u_{j-1}^n + u_{j+2}^n}{6\Delta x} - \frac{u_{j+2} - 2u_j + u_{j-2}}{4\Delta x} = -\frac{\partial u}{\partial x} + \frac{12\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}$$

$$\frac{\partial u}{\partial x} = -\frac{9u_j^n - 8u_{j-1}^n - u_{j+2}^n}{6\Delta x} + \frac{u_{j+2} - 2u_j + u_{j-2}}{4\Delta x} + 0[\Delta x^3]$$

Métodos de diferenças finitas.

Esquemas com diferenças centradas no espaço de terceira ordem:

$$\frac{\partial u}{\partial x} = -\frac{9u_j^n - 8u_{j-1}^n - u_{j+2}^n}{6\Delta x} + \frac{u_{j+2} - 2u_j + u_{j-2}}{4\Delta x} + 0[\Delta x^3]$$

$$\frac{\partial u}{\partial x} = \frac{u_{j+2}^n - 9u_j^n + 8u_{j-1}^n}{6\Delta x} + \frac{u_{j+2} - 2u_j + u_{j-2}}{4\Delta x} + 0[\Delta x^3]$$

Combinação das a Expansão de Taylor para1 e 2 ponto de grade

3 opção terceira ordem

Avançado

Métodos de diferenças finitas.

Esquemas com diferenças centradas no espaço de terceira ordem:

$$u_{j+1}^{n} = u_{j}^{n} + \Delta x \frac{\partial u}{\partial x} + \frac{\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} + \frac{\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (1B)$$

$$u_{j-1}^{n} = u_{j}^{n} - \Delta x \frac{\partial u}{\partial x} + \frac{\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} - \frac{\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (2B)$$

$$u_{j+2}^{n} = u_{j}^{n} + 2\Delta x \frac{\partial u}{\partial x} + \frac{4\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} + \frac{8\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{16\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (1C)$$

$$u_{j-2}^{n} = u_{j}^{n} - 2\Delta x \frac{\partial u}{\partial x} + \frac{4\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} - \frac{8\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{16\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (2C)$$

Paulo Yoshio Kubota

Métodos de diferenças finitas.

Esquemas com diferenças centradas no espaço de terceira ordem:

$$u_{j-1}^n = u_j^n - \Delta x \frac{\partial u}{\partial x} + \frac{\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} - \frac{\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (2B)$$

$$u_{j+2}^n = u_j^n + 2\Delta x \frac{\partial u}{\partial x} + \frac{4\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} + \frac{8\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{16\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (1C)$$

$$\frac{u_{j-1}^n - u_j^n}{\Delta x} = -\frac{\partial u}{\partial x} + \frac{\Delta x}{2}\frac{\partial^2 u}{\partial x^2} - \frac{\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (BB)$$

$$\frac{(u_{j+2}^n - u_j^n)}{2\Delta x} = \frac{\partial u}{\partial x} + \frac{2\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{4\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} - \frac{8\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (CC)$$

Métodos de diferenças finitas.

Esquemas com diferenças centradas no espaço de terceira ordem:

$$\frac{u_{j-1}^n - u_j^n}{\Delta x} = -\frac{\partial u}{\partial x} + \frac{\Delta x}{2}\frac{\partial^2 u}{\partial x^2} - \frac{\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}\ldots. \qquad (BB)$$

$$\frac{(u_{j+2}^n - u_j^n)}{2\Delta x} = \frac{\partial u}{\partial x} + \frac{2\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{4\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} - \frac{8\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}\ldots. \qquad (CC)$$

Mutiplica a BB por 4

$$4\frac{u_{j-1} - u_j}{\Delta x} = -4\frac{\partial u}{\partial x} + 4\frac{\Delta x}{2}\frac{\partial^2 u}{\partial x^2} - \frac{4\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} + \frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} + O[(\Delta x)^4] \qquad (AD)$$

soma a AD a eq. CC

$$4\frac{u_{j-1} - u_j}{\Delta x} + \frac{u_{j+2} - u_j}{2\Delta x} = -3\frac{\partial u}{\partial x} + \frac{6\Delta x}{2}\frac{\partial^2 u}{\partial x^2} - \frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}\ldots..$$

$$\frac{4u_{j-1} - 4u_j}{\Delta x} + \frac{u_{j+2} - u_j}{\Delta x} - \frac{6\Delta x}{2}\frac{\partial^2 u}{\partial x^2} = -3\frac{\partial u}{\partial x} - \frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}\ldots.. \qquad (2D)$$

Paulo Yoshio Kubota

Métodos de diferenças finitas.

$$\frac{4u_{j-1}-4u_j}{\Delta x}+\frac{u_{j+2}-u_j}{\Delta x}-\frac{6\Delta x}{2}\frac{\partial^2 u}{\partial x^2}=-3\frac{\partial u}{\partial x}-\frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}\ldots.. \qquad (2D)$$

Sabe-se que $\frac{\partial^2 u}{\partial x^2}\cong\frac{u_{j+2}-2u_j+u_{j-2}}{4\Delta x^2}$

$$\frac{u_{j+2}-2u_j+u_{j-2}}{4\Delta x^2}=\frac{\partial^2 u}{\partial x^2}+\frac{4}{4!}\frac{\partial^4 u}{\partial x^4}(\Delta x)^2+O[(\Delta x)^4] \qquad (6C)$$

$$\frac{4u_{j-1}-4u_j}{\Delta x}+\frac{u_{j+2}-u_j}{\Delta x}-\frac{6\Delta x}{2}\left(\frac{u_{j+2}-2u_j+u_{j-2}}{4\Delta x^2}\right)=-3\frac{\partial u}{\partial x}-\frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}\ldots.. \qquad (2D)$$

$$\frac{4u_{j-1}-4u_j}{\Delta x}+\frac{u_{j+2}-u_j}{\Delta x}-\left(\frac{6u_{j+2}-12u_j+6u_{j-2}}{8\Delta x}\right)=-3\frac{\partial u}{\partial x}-\frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}\ldots.. \qquad (2D)$$

Métodos de diferenças finitas.

$$\frac{4u_{j-1} - 4u_j}{\Delta x} + \frac{u_{j+2} - u_j}{\Delta x} - \frac{6u_{j+2} - 12u_j + 6u_{j-2}}{8\Delta x} = -3\frac{\partial u}{\partial x} - \frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \ldots.. \qquad (2D)$$

$$\frac{4u_{j-1} - 4u_j}{3\Delta x} + \frac{u_{j+2} - u_j}{3\Delta x} - \frac{2u_{j+2} - 4u_j + 2u_{j-2}}{8\Delta x} = -\frac{\partial u}{\partial x} - O[\Delta x^3] \ldots.. \qquad (2D)$$

$$-\frac{4u_{j-1} - 4u_j}{3\Delta x} - \frac{u_{j+2} - u_j}{3\Delta x} + \frac{2u_{j+2} - 4u_j + 2u_{j-2}}{8\Delta x} = \frac{\partial u}{\partial x} + O[\Delta x^3] \ldots.. \qquad (2D)$$

$$\frac{-4u_{j-1} + 4u_j - u_{j+2} + u_j}{3\Delta x} + \frac{u_{j+2} - 2u_j + u_{j-2}}{4\Delta x} = \frac{\partial u}{\partial x} + O[\Delta x^3] \ldots.. \qquad (2D)$$

Métodos de diferenças finitas.

$$\frac{-4u_{j-1} + 4u_j - u_{j+2} + u_j}{3\Delta x} + \frac{u_{j+2} - 2u_j + u_{j-2}}{4\Delta x} = \frac{\partial u}{\partial x} + O[\Delta x^3] \ldots.. \qquad (2D)$$

$$\frac{\partial u}{\partial x} = \frac{u_{j+2} - 2u_j + u_{j-2}}{4\Delta x} + \frac{-4u_{j-1} + 4u_j - u_{j+2} + u_j}{3\Delta x} - O[\Delta x^3] \ldots.. \qquad (2D)$$

Métodos de diferenças finitas.

Esquemas com diferenças centradas no espaço de terceira ordem:

$$u_{j+1}^n = u_j^n + \Delta x \frac{\partial u}{\partial x} + \frac{\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} + \frac{\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots. \quad (1B)$$

$$u_{j-1}^n = u_j^n - \Delta x \frac{\partial u}{\partial x} + \frac{\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} - \frac{\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots. \quad (2B)$$

$$u_{j+2}^n = u_j^n + 2\Delta x \frac{\partial u}{\partial x} + \frac{4\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} + \frac{8\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{16\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots. \quad (1C)$$

$$u_{j-2}^n = u_j^n - 2\Delta x \frac{\partial u}{\partial x} + \frac{4\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} - \frac{8\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{16\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4} \dots. \quad (2C)$$

Métodos de diferenças finitas.

Esquemas com diferenças centradas no espaço de terceira ordem:

$$u_{j-1}^{n} = u_j^n - \Delta x\frac{\partial u}{\partial x} + \frac{\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} - \frac{\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4}\dots. \qquad (2B)$$

$$u_{j+2}^{n} = u_j^n + 2\Delta x\frac{\partial u}{\partial x} + \frac{4\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} + \frac{8\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{16\Delta x^4}{24}\frac{\partial^4 u}{\partial x^4}\dots. \qquad (1C)$$

$$\frac{u_{j-1}^{n} - u_j^n}{\Delta x} = -\frac{\partial u}{\partial x} + \frac{\Delta x}{2}\frac{\partial^2 u}{\partial x^2} - \frac{\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}\dots. \qquad (BB)$$

$$\frac{\left(u_{j+2}^{n} - u_j^n\right)}{2\Delta x} = \frac{\partial u}{\partial x} + \frac{2\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{4\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} - \frac{8\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}\dots. \qquad (CC)$$

Esquemas com diferenças centradas no espaço de terceira ordem:

$$\frac{u_{j-1}^n - u_j^n}{\Delta x} = -\frac{\partial u}{\partial x} + \frac{\Delta x}{2}\frac{\partial^2 u}{\partial x^2} - \frac{\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} + \frac{\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (BB)$$

$$\frac{(u_{j+2}^n - u_j^n)}{2\Delta x} = \frac{\partial u}{\partial x} + \frac{2\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{4\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} - \frac{8\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \dots . \qquad (CC)$$

Mutiplica a BB por 4

$$4\frac{u_{j-1} - u_j}{\Delta x} = -4\frac{\partial u}{\partial x} + 4\frac{\Delta x}{2}\frac{\partial^2 u}{\partial x^2} - \frac{4\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} + \frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} + O[(\Delta x)^4] \qquad (AD)$$

soma a AD a eq. CC

$$4\frac{u_{j-1} - u_j}{\Delta x} + \frac{u_{j+2} - u_j}{2\Delta x} = -3\frac{\partial u}{\partial x} + \frac{6\Delta x}{2}\frac{\partial^2 u}{\partial x^2} - \frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \dots ..$$

$$\frac{4u_{j-1} - 4u_j}{\Delta x} + \frac{u_{j+2} - u_j}{\Delta x} - \frac{6\Delta x}{2}\frac{\partial^2 u}{\partial x^2} = -3\frac{\partial u}{\partial x} - \frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \dots .. \qquad (2D)$$

Métodos de diferenças finitas.

$$\frac{4u_{j-1}-4u_j}{\Delta x}+\frac{u_{j+2}-u_j}{\Delta x}-\frac{6\Delta x}{2}\frac{\partial^2 u}{\partial x^2}=-3\frac{\partial u}{\partial x}-\frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}\dots.. \qquad (2D)$$

Sabe-se que $\frac{\partial^2 u}{\partial x^2}\cong\frac{u_{j+2}-2u_j+u_{j-2}}{4\Delta x^2}$

$$\frac{u_{j+2}-2u_j+u_{j-2}}{4\Delta x^2}=\frac{\partial^2 u}{\partial x^2}+\frac{4}{4!}\frac{\partial^4 u}{\partial x^4}(\Delta x)^2+O[(\Delta x)^4] \qquad (6C)$$

$$\frac{4u_{j-1}-4u_j}{\Delta x}+\frac{u_{j+2}-u_j}{\Delta x}-\frac{6\Delta x}{2}\left(\frac{u_{j+2}-2u_j+u_{j-2}}{4\Delta x^2}\right)=-3\frac{\partial u}{\partial x}-\frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}\dots.. \qquad (2D)$$

$$\frac{4u_{j-1}-4u_j}{\Delta x}+\frac{u_{j+2}-u_j}{\Delta x}-\left(\frac{6u_{j+2}-12u_j+6u_{j-2}}{8\Delta x}\right)=-3\frac{\partial u}{\partial x}-\frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}\dots.. \qquad (2D)$$

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$$\frac{4u_{j-1}-4u_j}{\Delta x}+\frac{u_{j+2}-u_j}{\Delta x}-\frac{6u_{j+2}-12u_j+6u_{j-2}}{8\Delta x}=-3\frac{\partial u}{\partial x}-\frac{4\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4}\ldots.. \qquad (2D)$$

$$\frac{4u_{j-1}-4u_j}{3\Delta x}+\frac{u_{j+2}-u_j}{3\Delta x}-\frac{2u_{j+2}-4u_j+2u_{j-2}}{8\Delta x}=-\frac{\partial u}{\partial x}-O[\Delta x^3]\ldots.. \qquad (2D)$$

$$-\frac{4u_{j-1}-4u_j}{3\Delta x}-\frac{u_{j+2}-u_j}{3\Delta x}+\frac{2u_{j+2}-4u_j+2u_{j-2}}{8\Delta x}=\frac{\partial u}{\partial x}+O[\Delta x^3]\ldots.. \qquad (2D)$$

$$\frac{-4u_{j-1}+4u_j-u_{j+2}+u_j}{3\Delta x}+\frac{u_{j+2}-2u_j+u_{j-2}}{4\Delta x}=\frac{\partial u}{\partial x}+O[\Delta x^3]\ldots.. \qquad (2D)$$

Métodos de diferenças finitas.

$$\frac{-4u_{j-1}+4u_j-u_{j+2}+u_j}{3\Delta x}+\frac{u_{j+2}-2u_j+u_{j-2}}{4\Delta x}=\frac{\partial u}{\partial x}+O[\Delta x^3]\ldots\ldots \qquad (2D)$$

$$\frac{\partial u}{\partial x}=\frac{u_{j+2}-2u_j+u_{j-2}}{4\Delta x}+\frac{-4u_{j-1}+4u_j-u_{j+2}+u_j}{3\Delta x}-O[\Delta x^3]\ldots\ldots \qquad (2D)$$

Esquemas com diferenças centradas no espaço de quarta ordem:

Métodos de diferenças finitas.

Esquemas com diferenças centradas no espaço de segunda ordem:

$$\frac{u_j^n - u_{j-1}^n}{\Delta x} = \frac{\partial u}{\partial x} - \frac{\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} - \frac{\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \dots. \qquad (BB)$$

$$\frac{\left(u_j^n - u_{j-2}^n\right)}{2\Delta x} = \frac{\partial u}{\partial x} - \frac{2\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{4\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} - \frac{8\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} \dots. \qquad (CC)$$

Mutiplica a BB por 2

$$2\frac{u_j - u_{j-1}}{\Delta x} = 2\frac{\partial u}{\partial x} + 2\frac{\Delta x}{2}\frac{\partial^2 u}{\partial x^2} + \frac{2\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} - \frac{2\Delta x^3}{24}\frac{\partial^4 u}{\partial x^4} + O[(\Delta x)^4] \qquad (AD)$$

Soma a AD a eq. CC

$$2\frac{u_j - u_{j-1}}{\Delta x} + \frac{u_j - u_{j-2}}{2\Delta x} = 3\frac{\partial u}{\partial x} + \frac{6\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} \dots..$$

$$\frac{2}{3}\frac{u_j - u_{j-1}}{\Delta x} - \frac{1}{3}\frac{u_j - u_{j-2}}{2\Delta x} = \frac{\partial u}{\partial x} + \Delta x^2\frac{\partial^3 u}{\partial x^3} \dots.. \qquad (2D)$$

$$\left(\frac{-u_j + 2u_j - 2u_{j-1} + u_{j-2}}{6\Delta x}\right) = \frac{\partial u}{\partial x} + \Delta x^2\frac{\partial^3 u}{\partial x^3} \dots..(2D)$$

Paulo Yoshio Kubota

Métodos de diferenças finitas.

Esquemas com diferenças centradas no espaço de quarta ordem:

$$\frac{u_{j+1}-u_{j-1}}{2\Delta x}=\frac{\partial u}{\partial x}+\frac{1}{3!}\frac{\partial^3 u}{\partial x^3}(\Delta x)^2+O[(\Delta x)^4] \qquad (4B)$$

$$\frac{u_{j+2}-u_{j-2}}{4\Delta x}=\frac{\partial u}{\partial x}+\frac{4}{3!}\frac{\partial^3 u}{\partial x^3}(\Delta x)^2+O[(\Delta x)^4] \qquad (4C)$$

Mutiplica a 4B por 4

$$4\frac{u_{j+1}-u_{j-1}}{2\Delta x}=4\frac{\partial u}{\partial x}+\frac{4}{3!}\frac{\partial^3 u}{\partial x^3}(\Delta x)^2+O[(\Delta x)^4] \qquad (1D)$$

Subtaria a 1D - 4C

$$4\frac{u_{j+1}-u_{j-1}}{2\Delta x}-\frac{u_{j+2}-u_{j-2}}{4\Delta x}=3\frac{\partial u}{\partial x}+O[(\Delta x)^4]$$

$$\frac{4}{3}\frac{u_{j+1}-u_{j-1}}{2\Delta x}-\frac{1}{3}\frac{u_{j+2}-u_{j-2}}{4\Delta x}=\frac{\partial u}{\partial x} \qquad (2D)$$

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Métodos de diferenças finitas.

Esquemas com diferenças centradas no espaço de quarta ordem:

$$\frac{4}{3}\frac{u_{j+1}-u_{j-1}}{2\Delta x}-\frac{1}{3}\frac{u_{j+2}-u_{j-2}}{4\Delta x}=\frac{\partial u}{\partial x} \qquad (2D)$$

$$\left(\frac{1}{3}\right)\left(\frac{4u_{j+1}-4u_{j-1}}{2\Delta x}-\frac{u_{j+2}-u_{j-2}}{4\Delta x}\right)=\frac{\partial u}{\partial x} \qquad (2D)$$

$$\left(\frac{1}{3}\right)\left(\frac{8u_{j+1}-8u_{j-1}-u_{j+2}+u_{j-2}}{4\Delta x}\right)=\frac{\partial u}{\partial x} \qquad (2D)$$

$$\left(\frac{-u_{j+2}+8u_{j+1}-8u_{j-1}+u_{j-2}}{12\Delta x}\right)=\frac{\partial u}{\partial x} \qquad (2D)$$

Resumo

$$\frac{\partial u}{\partial x} = \frac{u_{j+1} - u_{j-1}}{2\Delta x} - O[\Delta x^2]$$

$$\frac{\partial u}{\partial x} = \frac{u_j^n - u_{j-1}^n}{\Delta x} + O[\Delta x^1]$$

$$\frac{\partial u}{\partial x} = \frac{u_{j+1}^n - u_j^n}{\Delta x} + O[\Delta x^1]$$

$$\frac{\partial u}{\partial x} = \left(\frac{-2u_{j-1} + u_j + u_{j-2}}{6\Delta x}\right) - O[\Delta x^2]$$

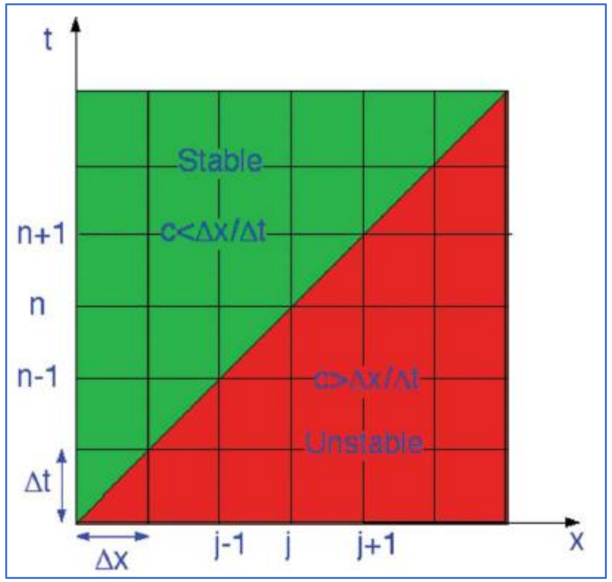
$$\frac{\partial u}{\partial x} = \left(\frac{4u_{j-1} - 3u_j - u_{j+2}}{6\Delta x}\right) - O[\Delta x^2]$$

$$\frac{\partial u}{\partial x} = \frac{u_{j+2}^n - 9u_j^n + 8u_{j-1}^n}{10\Delta x} + \frac{u_{j+2} - 2u_j + u_{j-2}}{20\Delta x} + O[\Delta x^2]$$

$$\frac{\partial u}{\partial x} = \frac{u_{j+2}^n - 9u_j^n + 8u_{j-1}^n}{6\Delta x} + \frac{u_{j+2} - 2u_j + u_{j-2}}{4\Delta x} + 0[\Delta x^3]$$

$$\frac{\partial u}{\partial x} = \frac{u_{j+2} - 2u_j + u_{j-2}}{4\Delta x} + \frac{-4u_{j-1} + 4u_j - u_{j+2} + u_j}{3\Delta x} - O[\Delta x^3] \ldots.. \quad (2D)$$

$$\frac{\partial u}{\partial x} = \left(\frac{-u_{j+2} + 8u_{j+1} - 8u_{j-1} + u_{j-2}}{12\Delta x}\right) + O[\Delta x^4] \ldots..(2D)$$



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Resumo

```
Module ModClassDerivada
 IMPLICIT NONE
 INTEGER              :: xdim
 REAL, ALLOCATABLE        :: f(:)
 REAL, ALLOCATABLE        :: ff(:)
 REAL, ALLOCATABLE        :: x(:)
 REAL, ALLOCATABLE        :: df2dx(:)
 REAL   :: dx
 REAL   :: Length
 REAL, PARAMETER   :: Pi=3.14
 PUBLIC :: InitClass,runDeriv,finalizeClassDerivada
CONTAINS
SUBROUTINE InitClass
 INTEGER :: i,xb,xc,xf
 Length=2*Pi
 xdim  =100
 Dx= Length/REAL(xdim)
 allocate(ff(0:xdim)); ff(0:xdim) =0.0
 allocate(df2dx(0:xdim)); df2dx(0:xdim) =0.0
 allocate(f(0:xdim)); f(0:xdim) =0.0
 allocate(x(0:xdim)); x(0:xdim) =0.0
 DO i=1,xdim
    CALL index(i,xdim,xb,xc,xf)
    x(xc) = x(xb) + Dx
 END DO

 DO i=0,xdim
    CALL index(i,xdim,xb,xc,xf)
    f (xc) = sin(x(xc))
    ff(xc) = cos(x(xc))
 END DO
END SUBROUTINE InitClass
```

```
SUBROUTINE runDeriv()
 INTEGER :: i,xb,xc,xf
 DO i=0,xdim
    CALL index(i,xdim,xb,xc,xf)
    ! df2dx(i) =  (( F(xf) - F(xb) )/(2.0*Dx))
     df2dx(i) =  (( F(xf) - F(xc) )/(Dx))
    !df2dx(i) =  (( F(xb) - F(xc) )/(Dx))
 END DO
  PRINT*,'Effective Error=',SUM(df2dx(1:xdim-1)-ff(1:xdim-1))/SUM(ff(1:xdim-1))
 CALL WriteBinary()
 CALL WriteCTL()
END SUBROUTINE runDeriv

SUBROUTINE WriteBinary()
  INTEGER :: lrec
  INQUIRE(IOLENGTH=lrec)df2dx(1:xdim-1)
  OPEN(1,FILE='runDeriv.bin',ACCESS='DIRECT',FORM='UNFORMATTED', RECL=lrec,&
       STATUS='UNKNOWN',ACTION='WRITE')
  WRITE(1,rec=1)df2dx(1:xdim-1)
  WRITE(1,rec=2)ff   (1:xdim-1)
  WRITE(1,rec=3)f    (1:xdim-1)
  CLOSE(1,STATUS='KEEP')
END SUBROUTINE WriteBinary
```

Resumo

```
SUBROUTINE WriteCTL()
 INTEGER :: i
  OPEN(1,FILE='runDeriv.ctl',ACCESS='SEQUENTIAL',FORM='FORMATTED', &
       STATUS='UNKNOWN',ACTION='WRITE')

  WRITE(1,'(A6,A12        )')'dset ^','runDeriv.bin'
  WRITE(1,'(A             )')'*options big_endian'
  WRITE(1,'(A6,A10        )')'undef ','-999.99999'
  WRITE(1,'(A6,I5,A8      )')'xdef  ',xdim-1,' levels '
  WRITE(1,'(10F15.5)')(x(i),i=1,xdim-1)
  WRITE(1,'(A6,A3,A8,A7 ,A4)')'ydef  ',' 1 ',' linear ',' -90.0 ',' 1.0'
  WRITE(1,'(A6,A3,A8,A14,A4)')'tdef  ',' 1 ',' linear ',' 00z01apr2014 ',' 1hr'
  WRITE(1,'(A6,A3,A8,A4   )')'zdef  ',' 1 ',' levels ','1000'
  WRITE(1,'(A6,A3         )')'VARS  ',' 3 '
  WRITE(1,'(A6,A23        )')'Fnum  ','0 99 derivada numerica '
  WRITE(1,'(A6,A23        )')'Fana  ','0 99 derivada analitica'
  WRITE(1,'(A6,A23        )')'Fori  ','0 99 funcao original'
  WRITE(1,'(A7            )')'ENDVARS'
END SUBROUTINE WriteCTL
```

```
SUBROUTINE index(i,Idim,xb,xc,xf)
  IMPLICIT NONE
  INTEGER, INTENT(IN          ) :: i
  INTEGER, INTENT(IN          ) :: Idim
  INTEGER, INTENT(OUT  ) :: xb,xc,xf
  IF(i==0) THEN
    xb=Idim
    xc=i
    xf=i+1
  ELSE IF(i==Idim)THEN
    xb=Idim-1
    xc=Idim
    xf=0
  ELSE
    xb=i-1
    xc=i
    xf=i+1
  END IF
END SUBROUTINE index

SUBROUTINE finalizeClassDerivada()
  DEALLOCATE(F)
  DEALLOCATE(FF)
  DEALLOCATE(df2dx)
  DEALLOCATE(x)

END SUBROUTINE finalizeClassDerivada

END Module ModClassDerivada
!=========================================================
```

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Resumo

```fortran
!=======================================================
PROGRAM Main
  USE ModClassDerivada, Only: InitClass,runDeriv,&
  finalizeClassDerivada
 IMPLICIT NONE

 CALL Init()
 CALL run()
 CALL finalize()

CONTAINS

SUBROUTINE Init()
    CALL InitClass()
END SUBROUTINE Init

SUBROUTINE run()
  CALL runDeriv()
END SUBROUTINE run

SUBROUTINE finalize()
  CALL finalizeClassDerivada()
END SUBROUTINE finalize

END PROGRAM Main
!=======================================================
```