第四节　定积分的近似计算

习题　5-4

1. 用三种近似计算法计算$\int_0^1 \mathrm{e}^{-x^2}\mathrm{d}x$.　（取$n=10$，被积函数值取五位小数）.

解　将$[0,1]$ 10 等分得

$$x_0=0,\ x_1=0.1,\ x_2=0.2,\ x_3=0.3,\ x_4=0.4,\ x_5=0.5,$$

$$x_6=0.6,\ x_7=0.7,\ x_8=0.8,\ x_9=0.9,\ x_{10}=1.$$

令函数$y=\mathrm{e}^{-x^2}$，相应地有

$$y_i=y(x_i)=\mathrm{e}^{-x_i^2},\ (i=0,\ 1,\ 2,\ \cdots,\ 10),$$

于是得

$$y_0=1,\ y_1=0.99004,\ y_2=0.96077,\ y_3=0.91388,\ y_4=0.85206,\ y_5=0.77868,$$

$$y_6=0.69752,\ y_7=0.61244,\ y_8=0.52708,\ y_9=0.44463,\ y_{10}=0.36792.$$

1°　矩形法:

$$\int_0^1 \mathrm{e}^{-x^2}\mathrm{d}x\approx\frac{1}{10}(y_1+y_2+\cdots+y_{10})=0.71461.$$

2°　梯形法:

$$\int_0^1 \mathrm{e}^{-x^2}\mathrm{d}x=\frac{2-1}{10}[(y_0+y_{10})\cdot\frac{1}{2}+y_1+y_2+\cdots+y_9]=0.74621.$$

3°　抛物线法:

$$\int_0^1 \mathrm{e}^{-x^2}\mathrm{d}x\approx\frac{2-1}{10}(y_1+y_2+\cdots+y_{10})=0.74683.$$

1. 用三种近似计算法计算$\int_1^2\frac{\mathrm{d}x}{x}$以求$\ln 2$的近似值（取$n=10$，被积函数值取四位小数）.

解　将$[1,2]$ 10 等分得

$$x_0 = 1,\ \ x_1 = 1.1,\ \ x_2 = 1.2,\ \ x_3 = 1.3,\ \ x_4 = 1.4,\ \ x_5 = 1.5,$$

$$x_6 = 1.6,\ \ x_7 = 1.7,\ \ x_8 = 1.8,\ \ x_9 = 1.9,\ \ x_{10} = 2.$$

令函数 $y = \dfrac{1}{x}$，相应地有

$$y_i = y(x_i) = \frac{1}{x_i},\ \ (i = 0,\ 1,\ 2, \cdots,\ 10),$$

于是得

$$y_0 = 1,\ \ y_1 = 0.9091,\ \ y_2 = 0.8333,\ \ y_3 = 0.7692,\ \ y_4 = 0.7143,\ \ y_5 = 0.6667,$$

$$y_6 = 0.6250,\ \ y_7 = 0.5882,\ \ y_8 = 0.5556,\ \ y_9 = 0.5263,\ \ y_{10} = 0.5000.$$

1°　矩形法:

$$\ln 2 = \int_1^2 \frac{\mathrm{d}x}{x} \approx \frac{2-1}{10}(y_1 + y_2 + \cdots + y_{10}) = 0.6688.$$

2°　梯形法:

$$\ln 2 \approx \int_1^2 \frac{\mathrm{d}x}{x} = \frac{2-1}{10}[(y_0 + y_{10}) \cdot \frac{1}{2} + y_1 + y_2 + \cdots + y_9] = 0.6938.$$

3°　抛物线法:

$$\ln 2 = \int_1^2 \frac{\mathrm{d}x}{x} \approx \frac{2-1}{10}(y_1 + y_2 + \cdots + y_{10}) = 0.6688.$$