Review of 5th course/第五次课复习

CHAPTER 8 NORMAL SHOCK WAVES AND RELATED TOPICS
第八章 正激波及有关问题

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第八章 路线图

理论部分

8.2 正激波基本控制方程推导

8.3 声速

8.4 能量方程特殊形式

8.5 什么情况下流动可压？

计算方法

8.6 通过正激波气体特性变化的方程的详细推导; 物理特性变化趋势讨论

实际应用

8.7 用皮托管测量可压缩流的流动速度

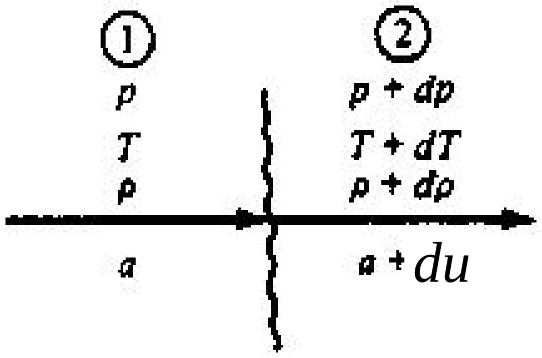
8.3 Speed of Sound/声速（音速）

- Key points of this section / 本节复习要点:
  - Physical mechanism of propagation of sound waves
    声波传播的物理机理
  - Calculation of the speed of sound/
    声速的计算
  - Speed of sound depends on local temperature of gas
    声速由气体的当地温度决定
  - Relationship between speed of sound and compressibility
    声速与压缩性的关系
  - Physical meaning of Mach number
    马赫数的物理意义?

1）Physical mechanism of the propagation of sound waves声波传播的物理机理

- **Summary**
  - The physical mechanism of sound propagation in a gas is based on molecular motion.
  - 声音在气体中的传播是基于分子的运动，是由分子碰撞引起的。

2）Calculation of Sound of Speed/声速的计算



(b) A stationary sound wave in a moving gas; the upstream velocity relative to the wave is a

图8.4b 运动气体中的静止声波

声波波动的特点

- 首先，声波波动的过程可视为绝热过程；
- 其次，声波波动过程可以进一步视为等熵过程；

声速计算公式

$$\boxed{a^2 = \frac{dp}{d\rho}} \qquad \boxed{or \quad a = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_s}} \qquad \boxed{a = \sqrt{\frac{\gamma p}{\rho}}}$$

3）What properties of the gas does it depend on? 声速由气体的什么特性决定？

$$\boxed{a = \sqrt{\gamma RT}} \qquad (8.25)$$

- Eq. 8.25 is our final expression for the speed of sound; it clearly states that *the speed of sound in a calorically perfect gas is a function of temperature* only.
- 我们得到的声速计算公式的表达式清楚地表明，对于量热完全气体，声速是温度的唯一函数。

4）Speed of Sound and Compressibility
声速与压缩性的关系?

$$a=\sqrt{\frac{1}{\rho\tau_s}} \qquad (8.27)$$

- The lower of compressibility, the higher the speed of sound/ 压缩性越小，声速越大;
  (空气约340m/s, 海水约1500m/s, 钢铁约5200m/s)
- The speed of sound in the theoretically incompressible flow is infinite/ 理论上不可缩流动中声速为无穷大（不可压流动Ma=V/a=0只是一种合理近似而已）。

5）Physical Meaning of Mach Number

马赫数的物理意义？

$$\frac{V^2/2}{e}=\frac{V^2/2}{c_vT}=\frac{V^2/2}{RT/(\gamma-1)}=\frac{(\gamma/2)V^2}{a^2/(\gamma-1)}=\frac{\gamma(\gamma-1)}{2}M^2$$

- Square of Mach number is proportional to the ratio of kinetic and interval energies.
- 马赫数的平方正比于气体动能与内能之比。

$M=0.3 \Longrightarrow \frac{V^2/2}{e}=2.52\%$

$M=2.0 \Longrightarrow \frac{V^2/2}{e}=1.12$

$M=7.0 \Longrightarrow \frac{V^2/2}{e}=13.72$

- 马赫数的平方为气体惯性力与弹性力之比。

8.4 Special Form of Energy Equation
能量方程特殊形式

- Key points of this section/ 本节复习要点:
  - 能量方程的各种特殊表达形式
  - 总温的计算公式
  - 总压、总密度的计算公式
  - 临界参数的定义与计算公式
  - 特征马赫数（速度系数）M^*的定义及计算公式

8.4 Special Form of Energy Equation 能量方程特殊形式

- 写成温度形式

$$\boxed{c_p T_1 + \frac{u_1^2}{2} = c_p T_2 + \frac{u_2^2}{2}} \qquad (8.30)$$

- 写成声速形式

$$\boxed{\frac{a_1^2}{\gamma - 1} + \frac{u_1^2}{2} = \frac{a_2^2}{\gamma - 1} + \frac{u_2^2}{2}} \qquad (8.32)$$

8. 4 Special Form of Energy Equation
能量方程特殊形式（续）

引入a_0、a^*的定义

滞止声速定义

$$\frac{a^2}{\gamma-1}+\frac{u^2}{2}=\frac{a_0{}^2}{(\gamma-1)} \tag{8.33}$$

临界声速定义

$$\frac{a^2}{\gamma-1}+\frac{u^2}{2}=\frac{(\gamma+1)a^{*2}}{2(\gamma-1)} \tag{8.35}$$

- 写成滞止声速形式

$$\frac{a_1{}^2}{\gamma-1}+\frac{u_1{}^2}{2}=\frac{a_2{}^2}{\gamma-1}+\frac{u_2{}^2}{2}=\frac{a_0{}^2}{\gamma-1}=\mathrm{const} \tag{8.34}$$

- 写成临界声速形式

$$\frac{a_1{}^2}{\gamma-1}+\frac{u_1{}^2}{2}=\frac{a_2{}^2}{\gamma-1}+\frac{u_2{}^2}{2}=\frac{(\gamma+1)a^{*2}}{2(\gamma-1)}=\mathrm{const} \tag{8.36}$$

多选题 1分

设置

对于定常、无粘、绝热、忽略体积力的流动，沿流线成立的能量方程可以用如下方程表示：

A h_0=常数

B a_0=常数

C $$\frac{a_1^2}{\gamma-1}+\frac{u_1^2}{2}=\frac{a_2^2}{\gamma-1}+\frac{u_2^2}{2}=\frac{(\gamma+1)}{2(\gamma-1)}a^{*2}=const.$$

D $$c_pT_1+\frac{u_1^2}{2}=c_pT_2+\frac{u_2^2}{2}$$

E a^*=常数

提交

Definition of Total Properties & Physical Meanings
总参数定义及其物理意义(复习)

- T_0定义:
  - 假想流体微团被*绝热地*减速为静止所对应的温度
- T_0的物理意义:
  - 表征了流体具有的总能量的大小，同时也代表了绝热流动中流体微团可能出现的最高温度
- p_0和ρ_0的定义
  - 当流体微元被*等熵地*减速至静止时对应的压强和密度
- p_0和ρ_0的物理意义
  - p_0代表了流体做有用功的能力。p_0，ρ_0分别代表了等熵流动中流体微团可能出现的最大压强和最大密度。

Calculation of Total Temperature
总温计算

- 总温计算

$$\boxed{\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2} \tag{8.40}$$

- 总压和总密度计算

$$\frac{p_0}{p} = (1 + \frac{\gamma - 1}{2} M^2)^{\gamma/(\gamma-1)} \tag{8.42}$$

$$\frac{\rho_0}{\rho} = (1 + \frac{\gamma - 1}{2} M^2)^{1/(\gamma-1)} \tag{8.43}$$

参照总参数定义，临界参数T^*、p^*和ρ^*的定义为：

- T^*定义：
  - 假想亚声速运动流体微团被*绝热地*加速到声速、或超声速运动流体微团被*绝热地*减速到声速所对应的温度

- p^*、ρ^*定义：
  - 假想亚声速运动流体微团被*等熵地*加速到声速、或超声速运动流体微团被*等熵地*减速到声速所对应的压强、密度

Calculation of critical properties
临界参数的定量值计算

$$\gamma = 1.4$$

$$\frac{T^*}{T_0} = \frac{2}{\gamma + 1} = 0.833 \quad (8.44)$$

$$\frac{p^*}{p_0} = \left(\frac{2}{\gamma + 1}\right)^{\gamma/(\gamma - 1)} = 0.528 \quad (8.45)$$

$$\frac{\rho^*}{\rho_0} = \left(\frac{2}{\gamma + 1}\right)^{1/(\gamma - 1)} = 0.634 \quad (8.46)$$

特征马赫数（速度系数）*M**的定义及计算公式

- “Characteristic” Mach number, M*, defined as：特征马赫数(也被内流计算人员称为速度系数），其定义如下：

$$M^* = \frac{u}{a^*}$$

- 特征马赫数计算：

$$\boxed{M^2 = \frac{2}{(\gamma+1)/M^{*2} - (\gamma-1)}} \qquad (8.47)$$

$$\boxed{M^{*2} = \frac{(\gamma+1)M^2}{2+(\gamma-1)M^2}} \qquad (8.48)$$

例8.7 用本节推导出的公式解例7.6

例7.6：气流中一点处的压强、温度和速度分别为1atm, 320K,1000m/s。计算这一点的总温和总压。例8.2算出 M=2.79）

$$\frac{T_0}{T}=1+\frac{\gamma-1}{2}M^2=1+\frac{0.4}{2}(2.79)^2=2.557$$

$$\Longrightarrow \quad T_0=2.557T=2.557\times 320(K)=818(K)$$

$$\frac{p_0}{p}=(1+\frac{\gamma-1}{2}M^2)^{\gamma/(\gamma-1)}=(2.557)^{\frac{1.4}{0.4}}=26.7$$

$$\Longrightarrow \quad p_0=26.7p=26.7\times 1(atm)=26.7(atm)$$

Example 8.8 **Consider a point in an airflow where local Mach number, static pressure, static temperature are 3.5, 0.3atm, and 180K, respectively. Calculate the local values of p_0, T_0, T^*, a^*, and M^* at this point.**

$$p_0 = (1+\frac{\gamma-1}{2}M^2)^{\frac{\gamma}{\gamma-1}} p = 22.9atm \qquad T_0 = (1+\frac{\gamma-1}{2}M^2)T = 621K$$

$$T^* = \frac{2}{\gamma+1}T_0 = 517.5K \qquad a^* = \sqrt{\gamma R T^*} = 456m/s$$

$$a = \sqrt{\gamma R T} = 268.9m/s \qquad V = M \cdot a = 941m/s \qquad M^* = \frac{V}{a^*} = 2.06$$

也可以用公式（8.48）计算M^*：

$$M^{*2} = \frac{(\gamma+1)M^2}{2+(\gamma-1)M^2} = \frac{2.4(3.5)^2}{2+0.4(3.5)^2} = 4.26 \qquad M^* = \sqrt{4.26} = 2.06$$

Example 8.9 如图8.5所示翼型流动，假设流动为等熵流动，计算点1处的当地马赫数。

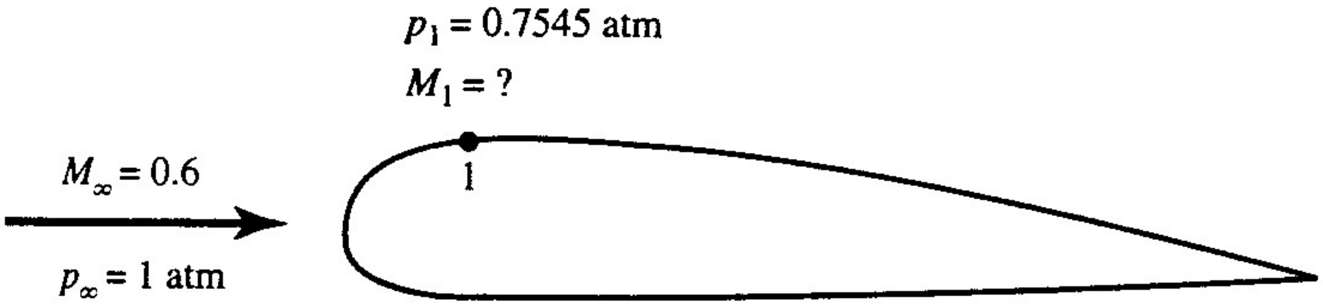


Figure 8.5 Figure for Example 8.2.

$$p_{0,\infty} = \frac{p_{0,\infty}}{p_\infty} p_\infty = 1.276 \times 1 = 1.276 atm$$

$$\because \quad p_{0,1} = p_{0,\infty}$$

$$\therefore \quad \frac{p_{0,1}}{p_1} = \frac{1.276}{0.7545} = 1.691$$

查表A：得 $M = 0.9$

Example 8.10 如图8.5所示翼型流动，假设流动为等熵流动，当自由来流的温度T_∞=59°F时，计算点1处的速度。

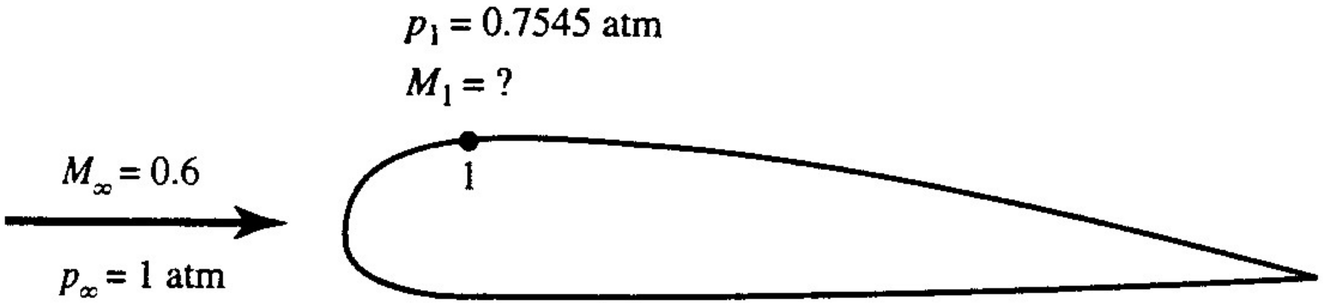


Figure 8.5 Figure for Example 8.2.

$$T_\infty = 460 + 59 = 519^0 R = (519/1.8)K = 288.33K$$

$$\because \frac{p_1}{p_\infty} = \left(\frac{T_1}{T_\infty}\right)^{\frac{\gamma}{\gamma-1}} \qquad \therefore T_1 = T_\infty\left(\frac{p_1}{p_\infty}\right)^{\frac{\gamma-1}{\gamma}} = 288.33\times 0.7545^{\frac{1.4-1}{1.4}} = 265.11K$$

$$a_1 = \sqrt{\gamma R T_1} = \sqrt{1.4(287)(265.11)} = 326.4m/s$$

$$V_1 = M_1 a_1 = 0.9\times 326.4 = 293.76m/s$$

投票(匿名) 最多可选1项

设置

前一次课的掌握情况投票

- A 完全掌握了这部分知识内容
- B 掌握了大部分
- C 掌握了一小部分
- D 完全不懂

提交

End of Review of 5th course!
Thank you for your attention!

Lecture #6

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Road Map

8.2 Derivation of the basic normal shock equations

8.3 Speed of sound

8.4 Special form of the energy equation

8.5 When is a flow compressible

8.6 Derivation of detailed equations for the calculation of changes across a normal shock wave: discussion of physical trends

8.7 Compressible airspeed measurements by means of a Pitot tube

第八章 路线图

理论部分

8.2 正激波基本控制方程推导

8.3 声速

8.4 能量方程特殊形式

8.5 什么情况下流动可压？

计算方法

8.6 通过正激波气体特性变化的方程的详细推导; 物理特性变化趋势讨论

实际应用

8.7 用皮托管测量可压缩流的流动速度

8.5 WHEN IS A FLOW COMPRESSIBLE?
什么条件下流动是可压缩的？

We have stated several times in the preceding that a flow can be reasonably assumed to be incompressible when M<0.3, whereas it should be considered compressible when M>0.3. Why?

$$\frac{\rho_0}{\rho} = \left(1+\frac{\gamma-1}{2}M^2\right)^{1/(\gamma-1)} \quad \text{即} \quad \frac{\rho}{\rho_0} = \left(1+\frac{\gamma-1}{2}M^2\right)^{-1/(\gamma-1)}$$

8.5 WHEN IS A FLOW COMPRESSIBLE?
什么条件下流动是可压缩的？

- 结论：

$$If \quad M < 0.32, \quad \frac{\Delta\rho}{\rho_0} < 5\%$$

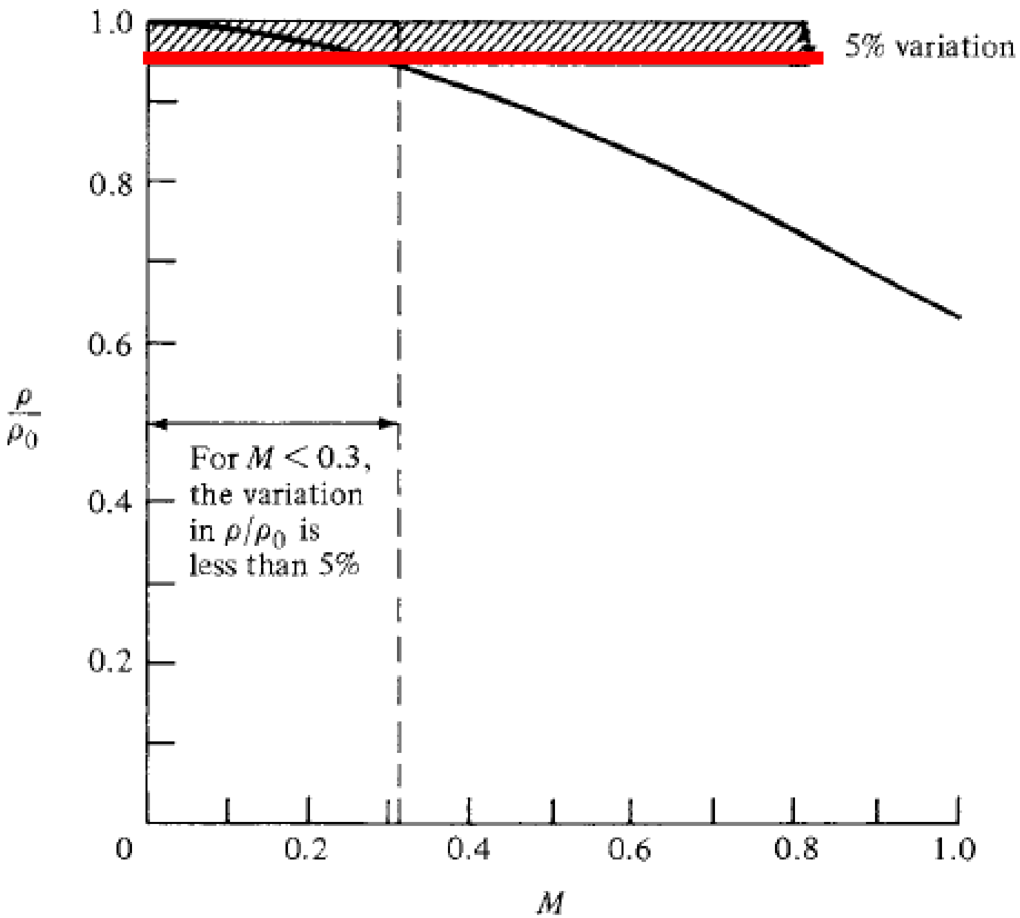


Figure 8.6 Isentropic variation of density with Mach number.

对于一个给定的速度变化，$\frac{\rho_0}{\rho}$ 的变化对压强的影响：

$$dp = -\rho V dV \qquad (3.12)$$

$$\frac{dp}{p} = -\frac{\rho}{p} V^2 \frac{dV}{V}$$

$$\left(\frac{dp}{p}\right)_0 = -\frac{\rho_0}{p} V^2 \frac{dV}{V}$$

$$\frac{dp/p}{(dp/p)_0} = \frac{\rho}{\rho_0}$$

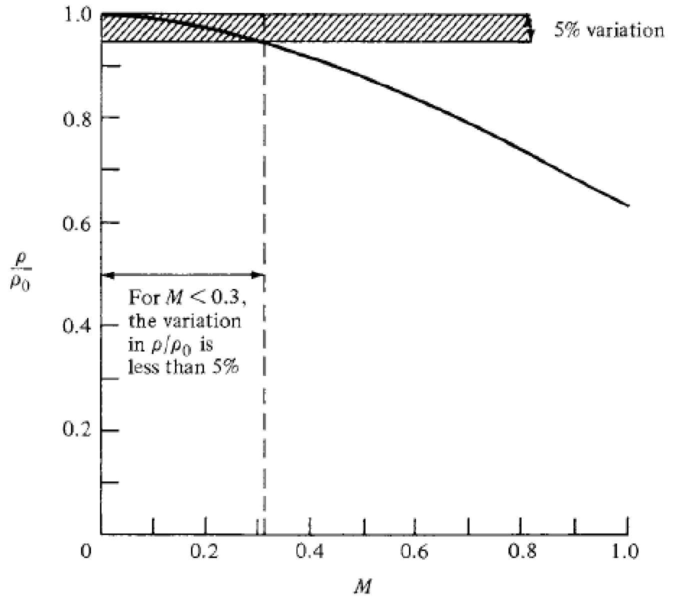


Figure 8.6 Isentropic variation of density with Mach number.

Hence, the degree by which ρ/ρ_0 deviates from unity as shown in Fig.8.6 is related to the same degree by which the fractional pressure change for a given *dV/V.*

这里指的是当地压力变化

补充：澄清一个关于可压与不可压的概念

- **流体介质的压缩性？**

流体介质自身的性质

- **流动是否可压？**

根据实际流动中密度的变化而言。

空气的不可压缩流动是一种特殊的假设，假设密度不变，内能不变

举例1：储气室中 $p_0 = 1.0132\times10^5 N/m^2$，$T_0 = 288K$ 的气体在管道出口处等熵加速到106.7m/s, 出口处压力用不可压假设和可压流假设的计算结果分别为：

- 不可压： $\rho = \rho_0 = \dfrac{p_0}{RT_0} = 1.226(Kg/m^3)$

$$p = p_0 - \frac{1}{2}\rho V^2 = 1.0132\times10^5 - 0.5(1.226)(106.7)^2 = 9.4341\times10^4 (N/m^2)$$

- 可压

$$T = T_0 - \frac{V^2}{2c_p} = 288 - \frac{106.7^2}{2\times1004.5} = 282.3(K)$$

$$p = (\frac{T}{T_0})^{\frac{\gamma}{\gamma-1}} p_0 = (\frac{282.3}{288})^{3.5}(1.0132\times10^5) = 9.4473\times10^4 (N/m^2)$$

相对误差： $\dfrac{9.4473\times10^4 - 9.4341\times10^4}{9.4473\times10^4} = 0.13\%$

此时的马赫数： $M = \dfrac{106.7}{\sqrt{1.4\times287\times282.3}} = 0.317$

举例2：储气室中 $p_0 = 1.0132\times10^5 N/m^2$，$T_0 = 288K$ **的气体在管道出口处等熵加速到274.3m/s, 出口处压力用不可压假设和可压流假设的计算结果分别为：**

- 不可压： $\rho = \rho_0 = \dfrac{p_0}{RT_0} = 1.226(Kg/m^3)$

$$p = p_0 - \frac{1}{2}\rho V^2 = 1.0132\times10^5 - 0.5(1.226)(274.3)^2 = 5.5198\times10^4 (N/m^2)$$

- 可压

$$T = T_0 - \frac{V^2}{2c_p} = 288 - \frac{274.3^2}{2\times1004.5} = 250.55(K)$$

$$p = (\frac{T}{T_0})^{\frac{\gamma}{\gamma-1}} p_0 = (\frac{250.55}{288})^{3.5}(1.0132\times10^5) = 6.2223\times10^4 (N/m^2)$$

相对误差： $\dfrac{6.2223\times10^4 - 5.5198\times10^4}{6.2223\times10^4} = 11.3\%$

此时的马赫数： $M = \dfrac{274.3}{\sqrt{1.4\times287\times250.55}} = 0.864$

8.6 Calculation of Normal Shock Wave Properties
正激波特性的计算

- 本节的要点
  - 计算通过正激波后流动参数的变化

$$M_2 = ?\quad \frac{p_2}{p_1} = ?\quad \frac{\rho_2}{\rho_1} = ?\quad \frac{u_2}{u_1} = ?\quad \frac{T_2}{T_1} = ?\quad s_2 - s_1 = ?\quad \frac{T_{0,2}}{T_{0,1}} = ?\quad \frac{p_{0,2}}{p_{0,1}} = ?$$

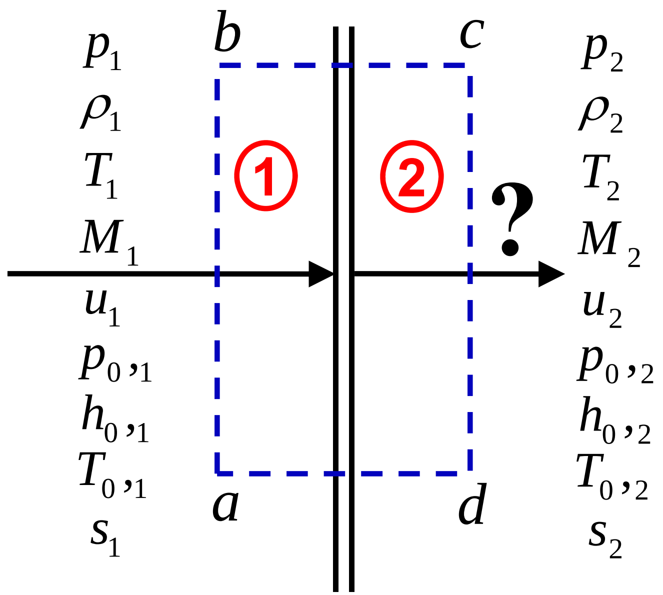


图8.3　正激波示意图

问题：　已知激波前区域1的流动参数，计算激波后区域2的流动参数。

正激波基本方程回顾：

- 连续方程： $\rho_1 u_1 = \rho_2 u_2$ (8.2)
- 动量方程： $p_1 + \rho_1 u_1{}^2 = p_2 + \rho_2 u_2{}^2$ (8.6)
- 能量方程： $h_1 + \frac{u_1{}^2}{2} = h_2 + \frac{u_2{}^2}{2}$ (8.10)

$$\frac{a_1{}^2}{\gamma - 1} + \frac{u_1{}^2}{2} = \frac{a_2{}^2}{\gamma - 1} + \frac{u_2{}^2}{2} = \frac{(\gamma + 1) a^{*2}}{2(\gamma - 1)} = const.$$

- 状态方程： $p_2 = \rho_2 R T_2$ (8.36)
- 焓： $h_2 = c_p T_2$

(8.6)式除以(8.2)式：

$$\rho_1 u_1 = \rho_2 u_2 \qquad \textbf{(8.2)}$$

$$p_1 + \rho_1 u_1^{\ 2} = p_2 + \rho_2 u_2^{\ 2} \qquad \textbf{(8.6)}$$

$$\frac{p_1}{\rho_1 u_1} + u_1 = \frac{p_2}{\rho_2 u_2} + u_2$$

$$\Longrightarrow \quad \frac{p_1}{\rho_1 u_1} - \frac{p_2}{\rho_2 u_2} = u_2 - u_1 \qquad \textbf{(8.51)}$$

因为 $a = \sqrt{\gamma p/\rho}$ **：**

$$\frac{a_1^{\ 2}}{\gamma u_1} - \frac{a_2^{\ 2}}{\gamma u_2} = u_2 - u_1 \qquad \textbf{(8.52)}$$

由公式(8.36): $\frac{a_1{}^2}{\gamma-1}+\frac{u_1{}^2}{2}=\frac{a_2{}^2}{\gamma-1}+\frac{u_2{}^2}{2}=\frac{(\gamma+1)a^{*2}}{2(\gamma-1)}=const.$ **可得：**

$$a_1{}^2=\frac{\gamma+1}{2}a^{*2}-\frac{\gamma-1}{2}u_1{}^2 \tag{8.53}$$

$$a_2{}^2=\frac{\gamma+1}{2}a^{*2}-\frac{\gamma-1}{2}u_2{}^2 \tag{8.54}$$

将(8.53),(8.54)式代入(8.52)式：

$$\frac{\gamma+1}{2}\frac{a^{*2}}{\gamma u_1}-\frac{\gamma-1}{2\gamma}u_1-\frac{\gamma+1}{2}\frac{a^{*2}}{\gamma u_2}+\frac{\gamma-1}{2\gamma}u_2=u_2-u_1$$

整理为：
$$\frac{\gamma+1}{2\gamma u_1u_2}(u_2-u_1)a^{*2}+\frac{\gamma-1}{2\gamma}(u_2-u_1)=u_2-u_1$$

两边同除以u_2-u_1：
$$\frac{\gamma+1}{2\gamma u_1u_2}a^{*2}+\frac{\gamma-1}{2\gamma}=1$$

$$\boxed{a^{*2}=u_1u_2} \qquad (8.55)$$

$$\boxed{a^{*2} = u_1 u_2} \qquad (8.55)$$

- Equation (8.55) is called the *Prandtl relation* and is a useful intermediate relation for normal shock waves. 方程(8.55)被称为*Prandtl* 关系式，是一个很有用的正激波中间关系式.

(8.55)式还可写成： $$1 = \frac{u_1}{a^*}\frac{u_2}{a^*} \qquad (8.56)$$

由特征马赫数的定义： $M^* = \dfrac{u}{a^*}$ 可得：

$$1 = M_1^* M_2^* \qquad M_2^* = \frac{1}{M_1^*} \qquad (8.57)$$

应用（8.48)式： $$M^{*2} = \frac{(\gamma+1)M^2}{2+(\gamma-1)M^2}$$

$$\frac{(\gamma+1)M_2^2}{2+(\gamma-1)M_2^2} = \left[\frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}\right]^{-1} \quad \textbf{(8.58)}$$

$$\boxed{M_2^2 = \frac{1+[(\gamma-1)/2]M_1^2}{\gamma M_1^2-(\gamma-1)/2}} \quad \textbf{(8.59)}$$

- Equation (8.59) is our first major result for a normal shock wave. Examine Eq. (8.59) closely; it states that the Mach number behind the wave, M_2, is a function only of the Mach number ahead of the wave, M_1.

方程（8.59)是我们得到的第一个主要正激波关系式，表明波后马赫数M_2是波前马赫数M_1的唯一函数.

$$\boxed{M_2^{\,2} = \frac{1+[(\gamma-1)/2]M_1^{\,2}}{\gamma M_1^{\,2}-(\gamma-1)/2}} \qquad \textbf{(8.59)}$$

- Moreover, if $M_1=1$, then $M_2=1$. This is the case of an infinitely weak normal shock wave, defined as a *Mach wave***.**
  如果$M_1=1$，则$M_2=1$。这种情况对应无限弱的正激波，我们定义为*马赫波*。

- Furthermore, if $M_1>1$, then $M_2<1$; i.e., the Mach number behind the normal shock wave is *subsonic*.
  如果$M_1>1$, 则 $M_2<1$；也就是：正激波后的流动是*亚声速*的。

$$\boxed{M_2^{\,2} = \frac{1+[(\gamma-1)/2]M_1^{\,2}}{\gamma M_1^{\,2}-(\gamma-1)/2}} \qquad \textbf{(8.59)}$$

- **As M_1 increases above 1, the normal shock wave becomes stronger, and M_2 becoming progressively less than 1.**

  当 M_1 由1逐渐增大时，正激波越来越强，激波后马赫数 M_2越来越小（在小于1的范围内）。

- **However, in the limit as $M_1 \to \infty$, M_2 approaches a finite minimum value，$M_2 \to \sqrt{(\gamma-1)/2\gamma}$， which for air is 0.378.**

  然而，当M_1趋于无穷大，M_2趋于一有限的最小值 $M_2 \to \sqrt{(\gamma-1)/2\gamma}$，对于空气，当 $\gamma=1.4$时其值为0.378。

下面我们来推导通过正激波的热力学特性,即ρ_2/ρ_1 、p_2/p_1 、T_2/T_1的表达式：

- ρ_2/ρ_1 的推导：

$$\frac{\rho_2}{\rho_1}=\frac{u_1}{u_2}=\frac{u_1^{\ 2}}{u_1u_2}=\frac{u_1^{\ 2}}{a^{*2}}=M_1^{\ *2}$$

$$\boxed{\frac{\rho_2}{\rho_1}=\frac{u_1}{u_2}=\frac{(\gamma+1)M_1^{\ 2}}{2+(\gamma-1)M_1^{\ 2}}} \qquad (8.61)$$

- p_2/p_1 的推导:

$$p_2 - p_1 = \rho_1 u_1^2 - \rho_2 u_2^2 = \rho_1 u_1 (u_1 - u_2) = \rho_1 u_1^2 (1 - \frac{u_2}{u_1}) \qquad \textbf{(8.62)}$$

$$\frac{p_2 - p_1}{p_1} = \frac{\gamma \rho_1 u_1^2}{\gamma p_1}(1 - \frac{u_2}{u_1}) = \frac{\gamma u_1^2}{a_1^2}(1 - \frac{u_2}{u_1}) = \gamma M_1^2 (1 - \frac{u_2}{u_1}) \qquad \textbf{(8.63)}$$

$$\frac{p_2 - p_1}{p_1} = \gamma M_1^2 \left[1 - \frac{2 + (\gamma - 1) M_1^2}{(\gamma + 1) M_1^2} \right] \qquad \textbf{(8.64)}$$

$$\boxed{\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma + 1}(M_1^2 - 1)} \qquad \textbf{(8.65)}$$

T_2/T_1 的推导:

$$\frac{T_2}{T_1}=\left(\frac{p_2}{p_1}\right)\left(\frac{\rho_1}{\rho_2}\right) \tag{8.66}$$

$$\boxed{\frac{T_2}{T_1}=\frac{h_2}{h_1}=\left[1+\frac{2\gamma}{\gamma+1}({M_1}^2-1)\right]\frac{2+(\gamma-1){M_1}^2}{(\gamma+1){M_1}^2}} \tag{8.67}$$

读P578 “Equations (8.61) … a calorically perfect gas

- **Equations (8.61), (8.65), and (8.67) are important. Examine them closely. Note that ρ_2/ρ_1 , p_2/p_1 *and* T_2/T_1 *are functions of the upstream Mach number M_1 only.***

 ρ_2/ρ_1, p_2/p_1 **和** T_2/T_1 **只是上游马赫数** M_1 **的函数。**

- **The upstream Mach Number M_1 is *the determining parameter* for changes across a normal shock wave in a calorically perfect gas.**

正激波上游马赫数M_1是确定量热完全气体通过正激波特性变化的*决定性参数*。

读P578 “ If M1 =1,… increase above 1

- **If M_1=1, then** $p_2/p_1 = \rho_2/\rho_1 = T_2/T_1 = 1$ **; i.e., we have the case of a normal shock wave of vanishing strength---a Mach wave.** **(如果M_1=1，那么有** $p_2/p_1 = \rho_2/\rho_1 = T_2/T_1 = 1$ **；即马赫波是无限弱的正激波。)**
- **As M_1 increases above 1,** p_2/p_1, ρ_2/ρ_1, and T_2/T_1 **progressively increase above 1. 当 M_1 大于1逐渐增加时，** p_2/p_1, ρ_2/ρ_1, 和 T_2/T_1 **也逐渐沿大于1的趋势增大。**

$$\lim_{M_1\to\infty} M_2 = \sqrt{\frac{\gamma-1}{2\gamma}} = 0.378$$

$$\lim_{M_1\to\infty} \frac{\rho_2}{\rho_1} = \frac{\gamma+1}{\gamma-1} = 6$$

$$\lim_{M_1\to\infty} \frac{p_2}{p_1} = \infty \qquad \lim_{M_1\to\infty} \frac{T_2}{T_1} = \infty$$

- **We have stated earlier that shock waves occur in supersonic flows; a stationary normal shock such as shown in Fig.8.3 does not occur in subsonic flow.**

- **That is , Eqs. (8.59),(8.61),(8.65), and (8.67), the upstream Mach number is supersonic, $M_1 \geq 1$.**

- **However, on a *mathematical basis*, these equations also allow solution for $M_1 \leq 1$. These equations embody the continuity, momentum, and energy equations, which in principle do not care whether the value of M_1 is subsonic or supersonic.**

- **Here is an ambiguity which can only be resolved by second law of thermodynamics.**

应用我们在第七章推导出的熵增公式，我们可以得到通过正激波的熵增计算公式：

$$s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1} \tag{7.25}$$

$$\boxed{\begin{aligned} s_2 - s_1 = c_p \ln\left\{\left[1+\frac{2\gamma}{\gamma+1}(M_1^{\ 2}-1)\right]\frac{2+(\gamma-1)M_1^{\ 2}}{(\gamma+1)M_1^{\ 2}}\right\} \\ - R\ln\left[1+\frac{2\gamma}{(\gamma+1)}(M_1^{\ 2}-1)\right] \end{aligned}} \tag{8.68}$$

From Eq. (8.68), we see that the entropy change s_2-s_1 across the shock is a function of M_1 only. 由方程(8.68)可以看出，通过正激波的熵增s_2-s_1 只是波前马赫数M_1的函数。

The second law dictates that

$$s_2 - s_1 \geq 0$$

In Eq. (8.68), if $M_1=1$, $s_2=s_1$, and if $M_1>1$, then $s_2-s_1>0$, both of which obey the second law. However, if $M_1<1$, then Eq.(8.68) gives $s_2-s_1<0$, which is not allowed by the second law. Consequently, in nature, only cases involving $M_1 \geq 1$ are valid, i.e., normal shock waves can occur only in supersonic flow.

译文:热力学第二定律指出：

$$s_2 - s_1 \geq 0$$

由方程(8.68)可以看出，如果$M_1=1$, $s_2=s_1$ ；如果 $M_1>1$, then $s_2-s_1>0$， 两种情况都符合热力学第二定律。然而，如果$M_1<1$， 则(8.68) 式的结果为 $s_2-s_1<0$，其不符合热力学第二定律。因此，只有$M_1 \geq 1$的情况会发生，即，正激波只能在超声速流动中发生。

- **Why does the entropy increase across the shock wave? The second law tells us that it must, but what's the mechanism?**

- **Recall that a shock wave is a very thin region (on the order of 10^{-5}cm) across which some large changes occur almost discontinuously. Therefore, within the shock wave itself, large gradients in velocity and temperature occur; i.e., the mechanisms of friction and thermal conduction are strong. These are dissipative, irreversible mechanisms that always increase the entropy.**

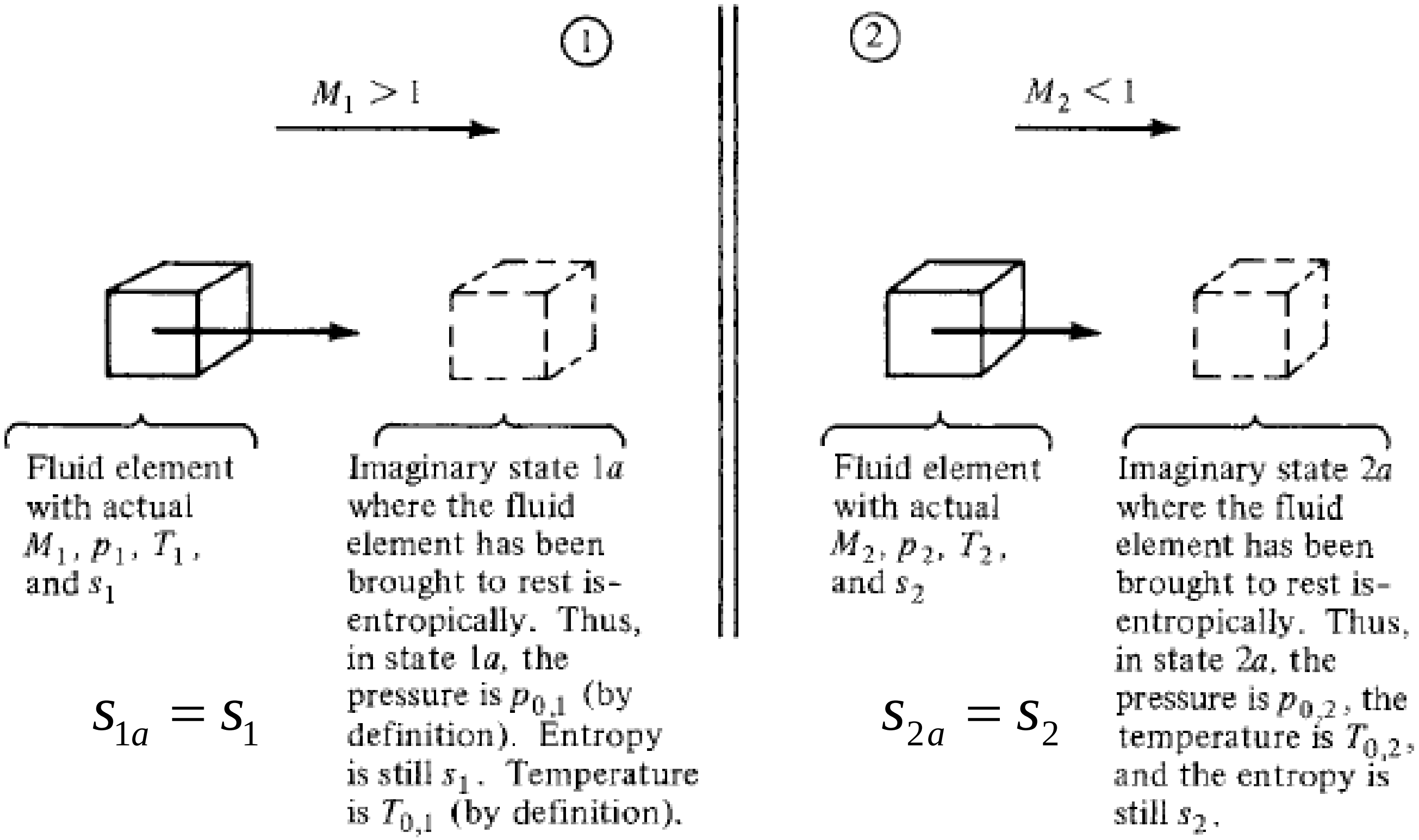
- **Therefore, the precise entropy increase predicted by Eq.(8.68) for a given supersonic M_1 is appropriately provided by nature in the form of friction and thermal conduction within the interior of the shock wave itself.**

□译文: 为什么通过激波会出现熵增？热力学第二定律告诉我们一定会有熵增，但是这个熵增产生的机理是什么呢？

□为了回答这些问题，让我们回忆第七章讨论过的内容：激波是非常薄的，厚度只有10^{-5}cm，通过激波流动性质发生剧烈变化，几乎是不连续的。因此，在激波本身内部，有很大的速度梯度和温度梯度，即摩擦和热传导的作用是非常强的。

□这些耗散的、不可逆的机制总是引起熵增，因此，(8.68)式给出的超声速波前马赫数M_1对应的熵增实际是由于激波本身内部的摩擦与热传导引起的。

总参数T_0与p_0如何变化？



$$S_{1a} = S_1$$

$$S_{2a} = S_2$$

Figure 8.7 Total conditions ahead of and behind a normal shock wave.

首先回答T_0如何变化?

能量方程:
$$\boxed{c_pT_1+\frac{u_1^2}{2}=c_pT_2+\frac{u_2^2}{2}} \tag{8.30}$$

总温定义:
$$\boxed{c_pT_0=c_pT+\frac{u^2}{2}} \tag{8.38}$$

$$\boxed{c_pT_{0,1}=c_pT_{0,2}}$$

$$\boxed{T_{0,1}=T_{0,2}} \tag{8.39}$$

- **Equation (8.39) states that *total temperature is constant across a stationary normal shock wave.*** 方程（8.39）表明：*通过静止正激波总温不变。*

总压如何变化？可借助熵增计算公式求出：

$$s_{2a} - s_{1a} = c_p \ln \frac{T_{2a}}{T_{1a}} - R \ln \frac{p_{2a}}{p_{1a}} \qquad \textbf{(8.70)}$$

$$s_2 - s_1 = c_p \ln \frac{T_{0,2}}{T_{0,1}} - R \ln \frac{p_{0,2}}{p_{0,1}} \qquad \textbf{(8.71)}$$

$$\boxed{s_2 - s_1 = -R \ln \frac{p_{0,2}}{p_{0,1}}} \qquad \textbf{(8.72)}$$

$$\boxed{\frac{p_{0,2}}{p_{0,1}} = e^{-(s_2 - s_1)/R}} \qquad \textbf{(8.73)}$$

□ **From Eq. (8.68), we know that s_2-s_1>0 for a normal shock wave. Hence, Eq. (8.73) states that $p_{0,2}<p_{0,1}$.**

由公式（8.68），我们知道对于正激波s_2-s_1>0，因此，式（8.73)表明：$p_{0,2}<p_{0,1}$。

□ ***The total pressure decreases across a shock wave. The total pressure ratio $p_{0,2}/p_{0,1}$ across a normal shock wave is a function of M_1 only.***

通过正激波总压降低，且正激波的波后波前总压比 $p_{0,2}/p_{0,1}$ 只是波前马赫数 M_1 的函数。

至此，我们已经全部回答了本节开始提出的问题：

$$M_2 = ? \quad \frac{p_2}{p_1} = ? \quad \frac{\rho_2}{\rho_1} = ? \quad \frac{u_2}{u_1} = ? \quad \frac{T_2}{T_1} = ? \quad s_2 - s_1 = ? \quad \frac{T_{0,2}}{T_{0,1}} = ? \quad \frac{p_{0,2}}{p_{0,1}} = ?$$

$$M_2^{\ 2} = \frac{1 + [(\gamma-1)/2]M_1^{\ 2}}{\gamma M_1^{\ 2} - (\gamma-1)/2}$$

$$\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} = \frac{(\gamma+1)M_1^{\ 2}}{2 + (\gamma-1)M_1^{\ 2}}$$

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}(M_1^{\ 2} - 1)$$

$$\frac{T_2}{T_1} = \frac{h_2}{h_1} = \left[1 + \frac{2\gamma}{\gamma+1}(M_1^{\ 2} - 1)\right]\frac{2 + (\gamma-1)M_1^{\ 2}}{(\gamma+1)M_1^{\ 2}}$$

$$s_2 - s_1 = c_p \ln\left\{\left[1 + \frac{2\gamma}{\gamma+1}(M_1^{\ 2} - 1)\right]\frac{2 + (\gamma-1)M_1^{\ 2}}{(\gamma+1)M_1^{\ 2}}\right\} - R\ln\left[1 + \frac{2\gamma}{(\gamma+1)}(M_1^{\ 2} - 1)\right]$$

$$T_{0,1} = T_{0,2}$$

$$\frac{p_{0,2}}{p_{0,1}} = e^{-(s_2 - s_1)/R}$$

这些关系式在附录B中以列表形式给出。

◻ In summary, we have now verified the qualitative changes across a normal shock wave as sketched in Fig.7.5b and as originally discussed in Sec. 7.6.

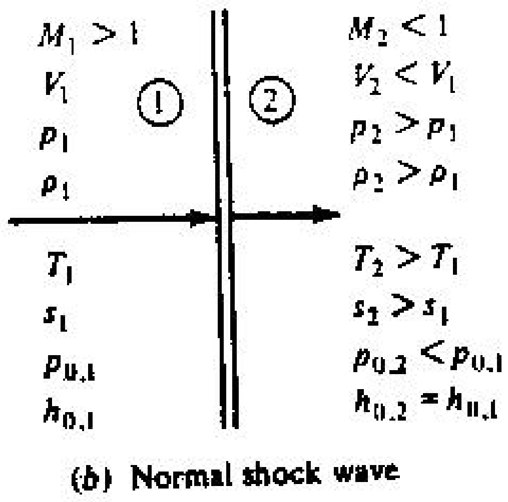


图7.5b

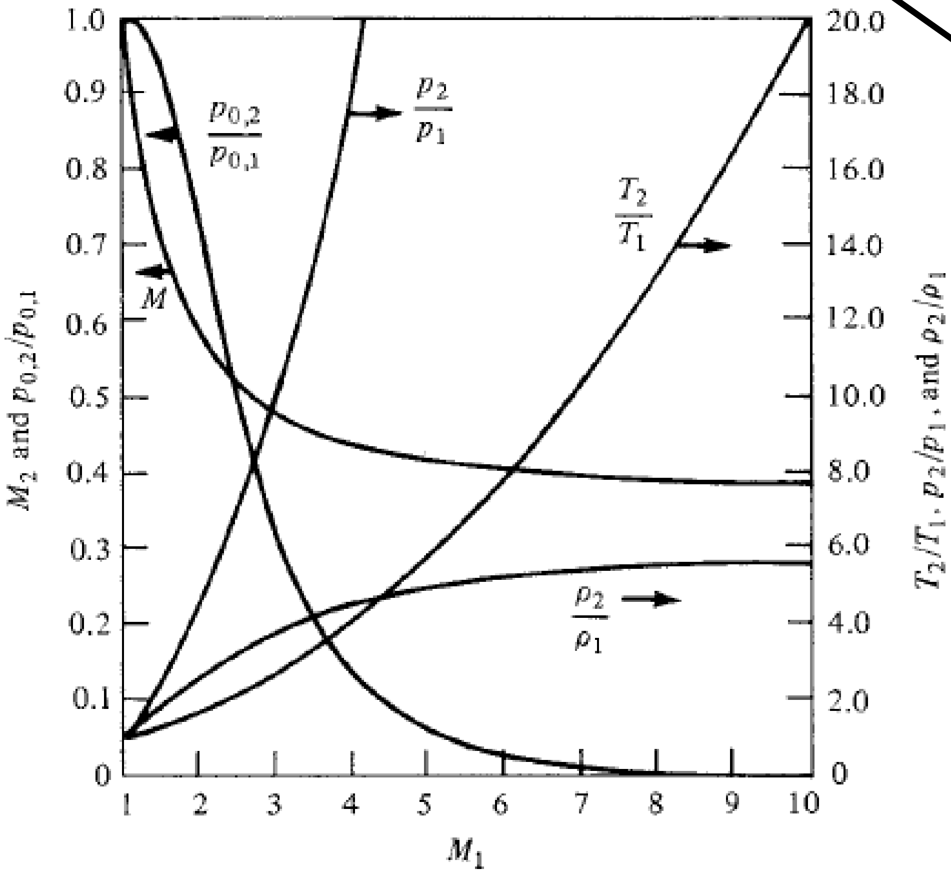


Figure 8.8 The variation of properties across a normal shock wave as a function of upstream Mach number: $\gamma = 1.4$.

书上p581
写错成
7. 4b

多选题 1分

设置

下列表述正确的是

A 正激波上游马赫数M_1是确定量热完全气体通过正激波特性变化的决定性参数。

B 正激波只能在超声速流动中发生，正激波后的流动是亚声速的。

C 通过静止正激波总温不变。

D 通过正激波熵增大、总压减小

提交

Example 8.11 : **Consider a normal shock wave in air where the upstream flow properties are u_1=680m/s, T_1=288k, and p_1=1atm. Calculate the velocity, temperature, and pressure downstream of the shock.**

Solution

$$a_1 = \sqrt{\gamma RT} = \sqrt{1.4(287)(288)} = 340(m/s)$$

$$M_1 = \frac{u_1}{a_1} = \frac{680}{340} = 2$$

查表B：对于M_1=2, 有:

$$\frac{p_2}{p_1} = 4.5, \quad \frac{T_2}{T_1} = 1.687, \quad M_2 = 0.5774$$

所以:

$$p_2 = \frac{p_2}{p_1} p_1 = 4.5(1\text{atm}) = 4.5\text{atm}$$

$$\mathrm{T}_2 = \frac{T_2}{T_1} T_1 = 1.687(288K) = 486K$$

$$a_2 = \sqrt{\gamma R T_2} = \sqrt{1.4(287)(486)} = 442(m/s)$$

$$u_2 = M_2 a_2 = 0.5774(442m/s) = 255m/s$$

例 8.12 超音速流中的正激波上游压强为1atm，计算在上游马赫数分别为(a)M_1=2和(b)M_1=4时，通过正激波的总压损失。比较两种结果并讨论

(a)

$$p_{0,1} = \frac{p_{0,1}}{p_1} p_1 = 7.824 \times 1atm = 7.824atm$$

$$p_{0,2} = \frac{p_{0,2}}{p_{0,1}} p_{0,1} = 0.7209 \times 7.824atm = 5.64atm$$

$$p_{0,1} - p_{0,2} = 7.824atm - 5.64atm = 2.184atm$$

(b) M_1=4

$$p_{0,1} = \frac{p_{0,1}}{p_1} p_1 = 151.8 \times 1 atm = 151.8 atm$$

$$p_{0,2} = \frac{p_{0,2}}{p_{0,1}} p_{0,1} = 0.1388 \times 151.8 atm = 21.07 atm$$

$$p_{0,1} - p_{0,2} = 151.8 atm - 21.07 atm = 130.7 atm$$

Note: 在任何流动中，总压是宝贵的资源。流动总压的损失意味着流体做功能力的损失。总压的损失降低流体机械的性能。本例说明，当流动中出现正激波时，在其他条件相同的情况下，正激波前马赫数越小越好。

Problem 8.5, 8.6, 8.7, 8.8, 8.9, 8.10

The End!
Thank you for your attention!

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