Chapter 11 Subsonic Compressible Flow over Airfoils: Linear Theory

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Chapter 11 绕翼型的亚声速可压缩流：线化理论

11.1 引言P712

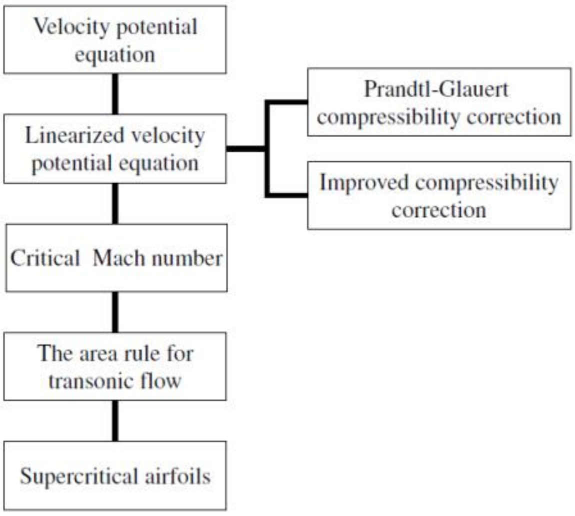
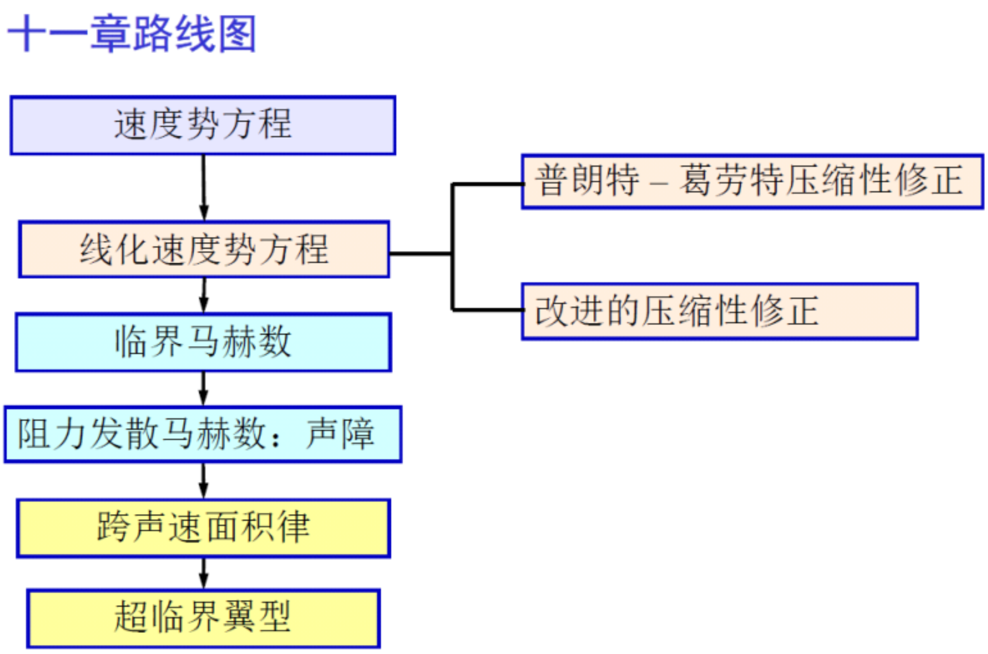


Figure 11.1 Road map for Chapter 11.



11.2 速度势方程P714

Substituting Equations (11.10) and (11.11) into (11.5), canceling the ρ which appears in each term, and factoring out the second derivatives of ϕ, we obtain

$$\boxed{\begin{aligned}&\left[1-\frac{1}{a^2}\left(\frac{\partial\phi}{\partial x}\right)^2\right]\frac{\partial^2\phi}{\partial x^2}+\left[1-\frac{1}{a^2}\left(\frac{\partial\phi}{\partial y}\right)^2\right]\frac{\partial^2\phi}{\partial y^2}\\&\qquad-\frac{2}{a^2}\left(\frac{\partial\phi}{\partial x}\right)\left(\frac{\partial\phi}{\partial y}\right)\frac{\partial^2\phi}{\partial x\,\partial y}=0\end{aligned}} \tag{11.12}$$

which is called the *velocity potential equation*. It is almost completely in terms of ϕ; only the speed of sound appears in addition to ϕ. However, a can be readily expressed in terms of ϕ as follows. From Equation (8.33), we have

$$\begin{aligned}a^2&=a_0^2-\frac{\gamma-1}{2}V^2=a_0^2-\frac{\gamma-1}{2}(u^2+v^2)\\&=a_0^2-\frac{\gamma-1}{2}\left[\left(\frac{\partial\phi}{\partial x}\right)^2+\left(\frac{\partial\phi}{\partial y}\right)^2\right]\end{aligned} \tag{11.13}$$

Since a_0 is a known constant of the flow, Equation (11.13) gives the speed of sound a as a function of ϕ. Hence, substitution of Equation (11.13) into (11.12) yields a single partial differential equation in terms of the unknown ϕ. This equation represents a combination of the continuity, momentum, and energy equations. In principle, it can be solved to obtain ϕ for the flow field around any two-dimensional shape, subject of course to the usual boundary conditions at infinity and along the body surface. These boundary conditions on ϕ are detailed in Section 3.7, and are given by Equations (3.47*a* and *b*) and (3.48*b*).

Because Equation (11.12) [along with Equation (11.13)] is a single equation in terms of one dependent variable ϕ, the analysis of isentropic, irrotational, steady, compressible flow is greatly simplified—we only have to solve one equation instead of three or more. Once ϕ is known, all the other flow variables are directly obtained as follows:

1. Calculate u and v from Equations (11.2*a* and *b*).
2. Calculate a from Equation (11.13).
3. Calculate $M = V/a = \sqrt{u^2+v^2}/a$.
4. Calculate T, p, and ρ from Equations (8.40), (8.42), and (8.43), respectively. In these equations, the total conditions T_0, p_0, and ρ_0 are known quantities; they are constant throughout the flow field and hence are obtained from the given freestream conditions.

11.3 线化速度势方程P717

$$\boxed{(1 - M_\infty^2)\frac{\partial^2 \hat{\phi}}{\partial x^2} + \frac{\partial^2 \hat{\phi}}{\partial y^2} = 0} \qquad (11.18)$$

$$\boxed{C_p = \frac{2}{\gamma M_\infty^2}\left(\frac{p}{p_\infty} - 1\right)} \qquad (11.22)$$

$$\hat{v} = V_\infty \tan\theta$$

or

$$\boxed{\frac{\partial \hat{\phi}}{\partial y} = V_\infty \tan\theta} \qquad (11.34)$$

11.4 普朗特—格劳厄特压缩性修正P722

$$\boxed{C_p = \frac{C_{p,0}}{\sqrt{1 - M_\infty^2}}} \qquad (11.51)$$

$$c_l = \frac{c_{l,0}}{\sqrt{1 - M_\infty^2}} \qquad [11.52]$$

$$c_m = \frac{c_{m,0}}{\sqrt{1 - M_\infty^2}} \qquad [11.53]$$

11.5 改进的压缩性修正P727

The importance of accurate compressibility corrections reached new highs during the rapid increase in airplane speeds spurred by World War II. Efforts were made to improve upon the Prandtl-Glauert rule discussed in Section 11.4. Several of the more popular formulas are given below.

The Karman-Tsien rule states

$$C_p = \frac{C_{p,0}}{\sqrt{1-M_\infty^2} + \left[M_\infty^2/\left(1+\sqrt{1-M_\infty^2}\right)\right] C_{p,0}/2} \tag{11.54}$$

This formula, derived in References 27 and 28, has been widely adopted by the aeronautical industry since World War II.

Laitone's rule states

$$C_p = \frac{C_{p,0}}{\sqrt{1-M_\infty^2} + \left(M_\infty^2\{1+[(\gamma-1)/2]M_\infty^2\}/2\sqrt{1-M_\infty^2}\right) C_{p,0}} \tag{11.55}$$

This formula is more recent than either the Prandtl-Glauert or the Karman-Tsien rule; it is derived in Reference 29.

11.6 临界马赫数P728

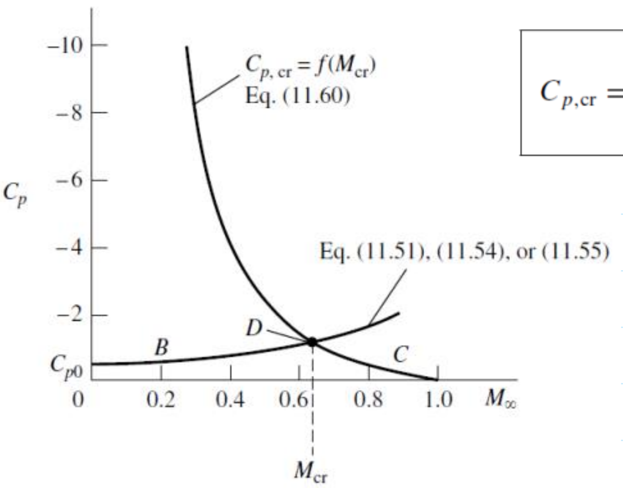


Figure 11.6 Estimation of critical Mach number.

$$\boxed{C_{p,\mathrm{cr}} = \frac{2}{\gamma M_{\mathrm{cr}}^2}\left[\left(\frac{1+[(\gamma-1)/2]M_{\mathrm{cr}}^2}{1+(\gamma-1)/2}\right)^{\gamma/(\gamma-1)} - 1\right]} \tag{11.60}$$

Equation (11.60), in conjunction with any one of the compressibility corrections given by Equations (11.51), (11.54), or (11.55), allows us to estimate the critical Mach number for a given airfoil as follows:

1. By some means, either experimental or theoretical, obtain the low-speed incompressible value of the pressure coefficient $C_{p,0}$ at the minimum pressure point on the given airfoil.
2. Using any of the compressibility corrections, Equation (11.51), (11.54), or (11.55), plot the variation of C_p with M_∞. This is represented by curve B in Figure 11.6.
3. Somewhere on curve B, there will be a single point where the pressure coefficient corresponds to locally sonic flow. Indeed, this point must coincide with Equation (11.60), represented by curve C in Figure 11.6. Hence, the *intersection* of curves B and C represents the point corresponding to sonic flow at the minimum pressure location on the airfoil. In turn, the value of M_∞ at this intersection is, by definition, the critical Mach number, as shown in Figure 11.6.

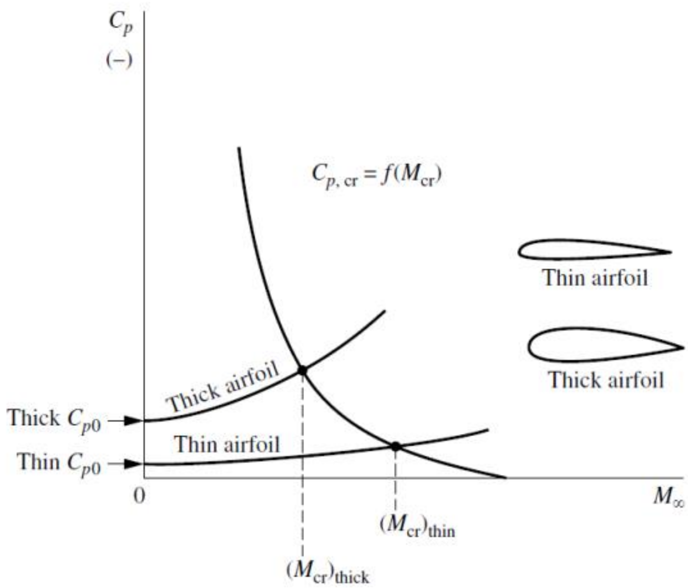


Figure 11.7 Effect of airfoil thickness on critical Mach number.

11.6.1 最小压力（最大速度）位置的讨论P737

11.7 阻力发散马赫数：声障P737

11.8 面积律P745

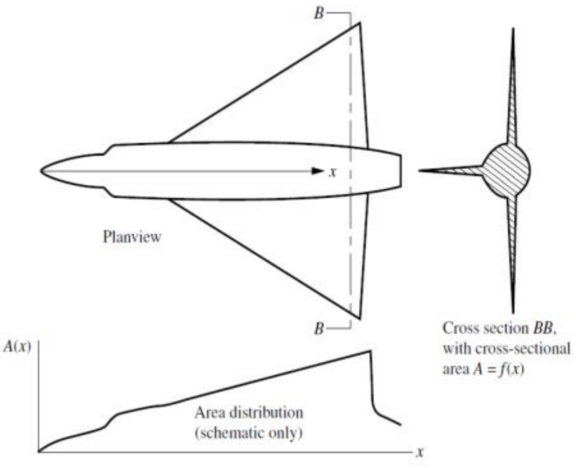


Figure 11.16 A schematic of a non-area-ruled aircraft.

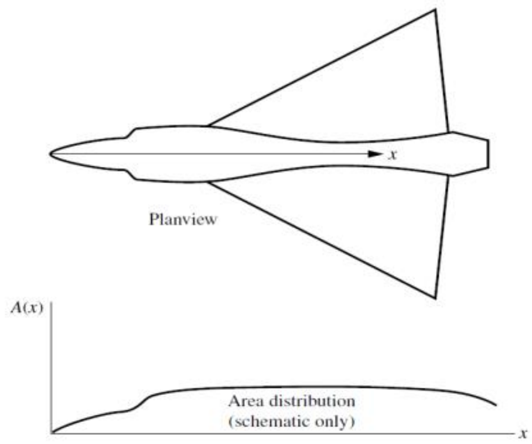


Figure 11.17 A schematic of an area-ruled aircraft.

11.9 超临界翼型P747

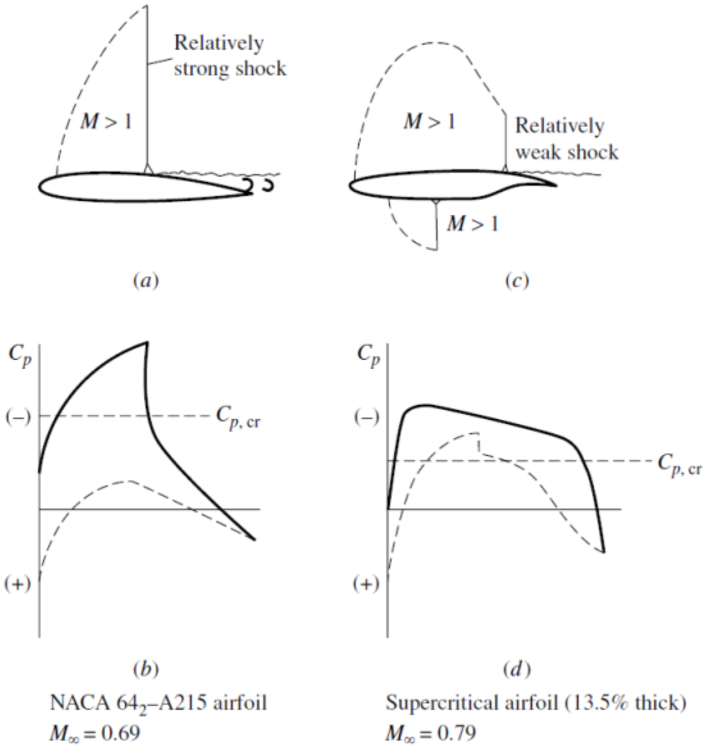


Figure 11.19 Standard NACA 64-series airfoil compared with a supercritical airfoil at cruise lift conditions. *(From Reference 32.)*

11.10 CFD的应用：跨声速翼型和机翼P749

11.11 应用空气动力学：翼身融合体P754

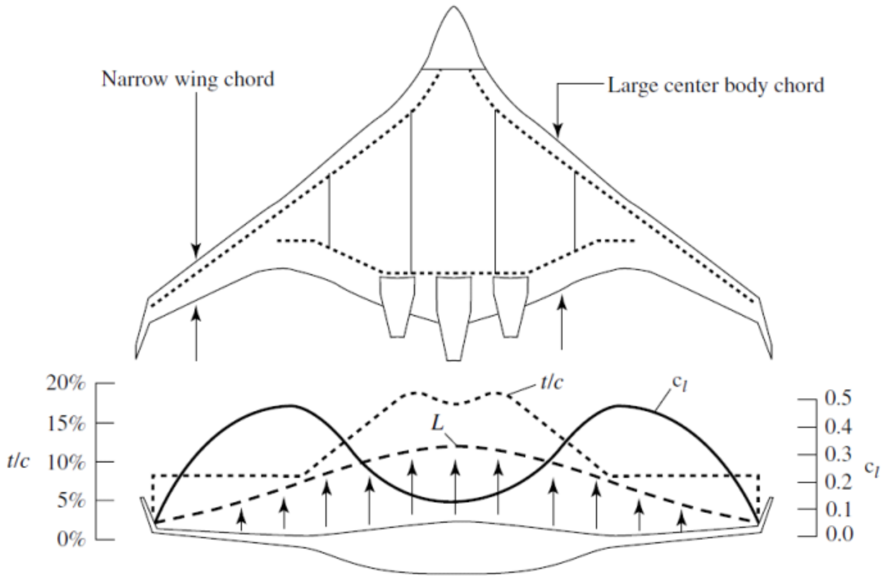


Figure 11.25 Typical spanwise distribution of lift L, lift coefficient c_l, and thickness-to-chord ratio for the blended wing body.

The BWB is a high-speed subsonic airplane intended to fly at the lower end of the transonic flight regime. Hence, major efforts are made to obtain as high a drag-divergence Mach number as possible. Toward that end, the BWB incorporates two design features, both of which deal with aerodynamic fundamentals discussed in this chapter.

1. *Supercritical airfoils.* The function of a supercritical airfoil is discussed in Section 11.9. The outer portions of the BWB wing incorporate a modern supercritical airfoil section with aft camber, similar to that shown in Figure 11.19*c*. The center body profile is also an airfoil section. In the first generation of the BWB development, the airfoil shape chosen was a Liebeck LW102A airfoil (Ref. 93) point designed for $c_l = 0.25$ at a Mach number of 0.7. A side view of the resulting center body profile is shown in Figure 11.26*a*. The new-generation BWB uses an advanced customized transonic airfoil design for the center body profile. Taking into account the constraints in cross-sectional area required to effectively hold passengers, baggage, and cargo, the new transonic airfoil design dealt with a careful three-dimensional contouring of the center body smoothly blending into the outer wing panels. The resulting center body profile is shown in Figure 11.26*b*, giving a cleaner, more streamlined appearance than the original profile in Figure 11.26*a*, and providing a higher critical Mach number. Indeed, the new centerbody profile in Figure 11.2*b* increased the BWB lift-to-drag ratio by 4 percent.
2. *Area rule.* The notion of the area rule is discussed in Section 11.8. The blended wing body, with its smooth contours and smoothly varying cross section, is almost naturally area-ruled. Figure 11.27 compares the cross-sectional area distributions as a function of longitudinal coordinate for the BWB (solid curve) and a conventional subsonic transport, the MD-11 (dashed curve). Clearly, the BWB area distribution is much smoother than that of the MD-11, thus exhibiting good area-rule qualities. Liebeck (Ref. 94) states that for the BWB "there appears to be no explicit boundary for increasing the cruise Mach number beyond 0.88." Indeed, a set of blended wing bodies have been designed for Mach numbers of 0.85, 0.9., 0.93, and 0.95.

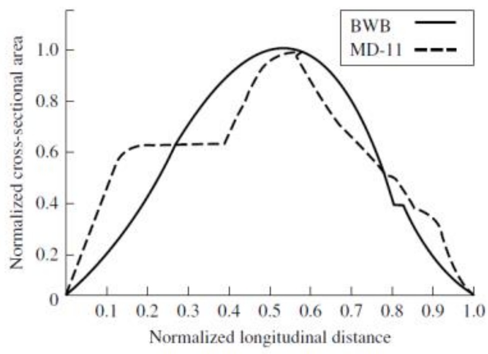


Figure 11.27 Longitudinal distributions of cross-sectional area comparing the blended wing body with a conventional wide-body civil transport, the McDonnell-Douglas MD-11.

11.12 历史摘记：高速翼型—早期研究和发展P760

11.13 历史摘记：后掠翼概念的起源P764

11.14 历史摘记：理查特·T·惠特科姆—面积律的建立和超临界翼型P773

11.15 总结P774

For two-dimensional, irrotational, isentropic, steady flow of a compressible fluid, the exact velocity potential equation is

$$\left[1-\frac{1}{a^2}\left(\frac{\partial\phi}{\partial x}\right)^2\right]\frac{\partial^2\phi}{\partial x^2}+\left[1-\frac{1}{a^2}\left(\frac{\partial\phi}{\partial y}\right)^2\right]\frac{\partial^2\phi}{\partial y^2} -\frac{2}{a^2}\left(\frac{\partial\phi}{\partial x}\right)\left(\frac{\partial\phi}{\partial y}\right)\frac{\partial^2\phi}{\partial x\,\partial y}=0 \tag{11.12}$$

where

$$a^2=a_0^2-\frac{\gamma-1}{2}\left[\left(\frac{\partial\phi}{\partial x}\right)^2+\left(\frac{\partial\phi}{\partial y}\right)^2\right] \tag{11.13}$$

This equation is exact, but it is nonlinear and hence difficult to solve. At present, no general analytical solution to this equation exists.

For the case of small perturbations (slender bodies at low angles of attack), the exact velocity potential equation can be approximated by

$$(1 - M_\infty^2)\frac{\partial^2 \hat{\phi}}{\partial x^2} + \frac{\partial^2 \hat{\phi}}{\partial y^2} = 0 \tag{11.18}$$

This equation is approximate, but linear, and hence more readily solved. This equation holds for subsonic ($0 \le M_\infty \le 0.8$) and supersonic ($1.2 \le M_\infty \le 5$) flows; it does not hold for transonic ($0.8 \le M_\infty \le 1.2$) or hypersonic ($M_\infty > 5$) flows.

The Prandtl-Glauert rule is a compressibility correction that allows the modification of existing incompressible flow data to take into account compressibility effects:

$$C_p = \frac{C_{p,0}}{\sqrt{1 - M_\infty^2}} \tag{11.51}$$

Also,

$$c_l = \frac{c_{l,0}}{\sqrt{1 - M_\infty^2}} \tag{11.52}$$

and

$$c_m = \frac{c_{m,0}}{\sqrt{1 - M_\infty^2}} \tag{11.53}$$

The critical Mach number is that freestream Mach number at which sonic flow is first obtained at some point on the surface of a body. For thin airfoils, the critical Mach number can be estimated as shown in Figure 11.6.

The drag-divergence Mach number is that freestream Mach number at which a large rise in the drag coefficient occurs, as shown in Figure 11.11.

The area rule for transonic flow states that the cross-sectional area distribution of an airplane, including fuselage, wing, and tail, should have a smooth distribution along the axis of the airplane.

Supercritical airfoils are specially designed profiles to increase the drag-divergence Mach number.

11.16 作业题P776