



## Review of 1st course/第一课复习

# Chapter 7 Compressible Flow: Some Preliminary Aspects

## 可压缩流动基础

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Introduction: 可压缩流动的基本特征:

1)The pivotal aspect of high -speed flow is that **the density is a variable**——密度是变量.

2)Another pivotal aspect of high-speed compressible flow is energy. **A high-speed flow is a high energy flow.**——是一个高能量的流动。

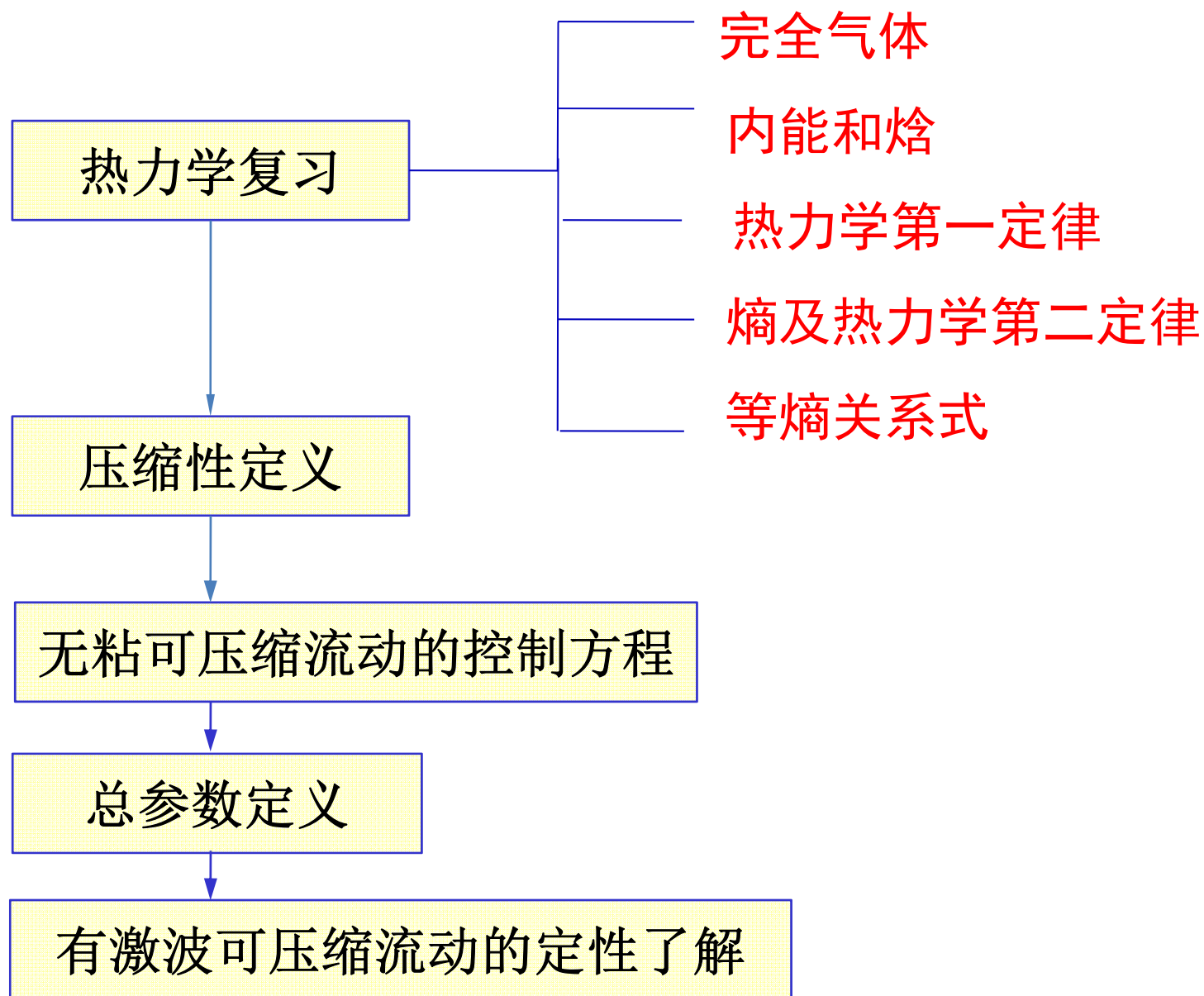
3)Energy transformation and temperature changes are important considerations.——必须考虑能量转换与温度变化。

因此,研究可压缩流必须引入重要的热力学(Thermodynamics)。

热力学——研究能量及其转换规律的基础科学。



## 第七章路线图



## 7.2 热力学简要复习

### 7.2.1 Perfect gas -完全气体

**定义：** 忽略分子间作用力的气体定义为完全气体

完全气体满足状态方程： (*equation of state*)

$$p = \rho RT \quad (7.1)$$

$$pv = RT \quad (7.2)$$

$R$ 为气体常数 (specific gas constant)  $R = 287 \text{ J}/(\text{kg} \cdot \text{K})$



## 7.2.2 内能和焓 (Internal Energy and Enthalpy)

### ➤ 内能的定义

- 一个给定分子的能量是其平动动能、转动动能、振动能和电子能的总和
- 所有分子所具有的能量总和称为气体的内能
- 单位质量气体的内能称为气体的比内能, 用  $e$  表示

注意内能与宏观动能的区别!

### ➤ 焓的定义:

$$h = e + pv = e + \frac{p}{\rho}$$

(7.3)

焓可以理解为由流体携带的取决于热力学状态参数的能量



## 内能和焓的计算

$$e = c_v T$$

(7.6a)

$$h = c_p T$$

(7.6b)

$$c_p = \frac{\gamma R}{\gamma - 1}$$

(7.9)

$$c_v = \frac{R}{\gamma - 1}$$

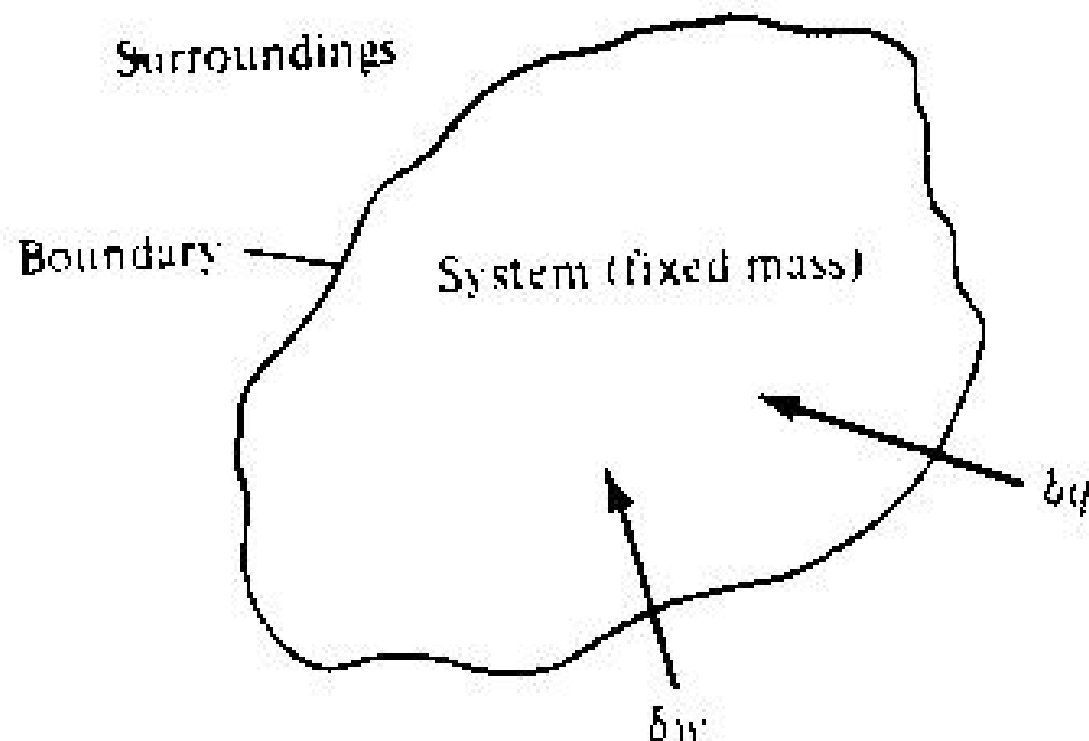
(7.10)

量热完全气体：  $c_p$  和  $c_v$  为常数



## 7.2.3 First Law of Thermodynamics (热力学第一定律)

系统—system    环境—surroundings    边界—boundary



**FIGURE 7.2**  
Thermodynamic system.



## 热力学第一定律

$$\delta q$$

: 外界通过边界加于系统的热增量（与过程有关，用  $\delta$  作增量符号）；

$$\delta w$$

: 外界对系统做的功（与过程有关，用  $\delta$  作增量符号）；

$$de$$

: 系统内能的增量（与过程无关，用全微分符号  $d$  作增量符号）；

Assume this system is stationary: (假定系统为静止的)

$$\delta q + \delta w = de$$

(7.11)





## 三个常见的热力学过程：

### 1 . *Adiabatic process* （绝热过程）

在过程中与外界没有热传递。  $\delta q = 0$

### 2 . *Reversible process* （可逆过程）

没有耗散现象、内部热传导发生。  $\delta w = -p dv$

### 3 . *Isentropic process* （等熵过程）

绝热、可逆。  $\delta w = -p dv$  且  $\delta q = 0$

对于可逆过程，有：

$$\delta q - p dv = de \quad (7.12)$$



绝热过程的热力学第一定律表达:  $de = \delta w$

可逆过程的热力学第一定律表达:  $de = \delta q - p dv$

等熵过程的热力学第一定律表达:  $de = -p dv$

热力学第一定律小结:

热力学第一定律就是能量守恒原理的一种表达,说明能量既不能产生也不能消失,只能从一种形式转换为另一种形式。



内能与定容比热的关系式推导：

定容比热定义：
$$c_v = \frac{(\delta q)_v}{dT}$$

热力学第一定律：
$$\delta q - p dv = de$$

$$dv = 0 \rightarrow c_v = \frac{de}{dT}$$



焓与定压比热的关系式推导：

定压比热定义：
$$c_p = \frac{(\delta q)_p}{dT}$$

$$dh = de + d(pv) = de + pdv + vdp$$

$$\rightarrow de = dh - pdv - vdp$$

热力学第一定律：
$$\delta q - pdv = de$$

即：
$$\delta q = dh - vdp$$

$$dp = 0 \rightarrow \boxed{c_p = \frac{dh}{dT}}$$



定容比热、定压比热与内能、焓的关系的更严谨的推导：

定容比热：

$$e = e(T, v)$$

$$de = \left(\frac{\partial e}{\partial T}\right)_v dT + \left(\frac{\partial e}{\partial v}\right)_T dv$$

$$\delta q = de + p dv = \left(\frac{\partial e}{\partial T}\right)_v dT + \left[\left(\frac{\partial e}{\partial v}\right)_T + p\right] dv$$

$$\because dv = 0 \quad \therefore c_v = \frac{(\delta q)_v}{dT} = \left(\frac{\partial e}{\partial T}\right)_v$$



定压比热:

$$h = h(T, p)$$

$$dh = \left(\frac{\partial h}{\partial T}\right)_p dT + \left(\frac{\partial h}{\partial p}\right)_T dp$$

$$\delta q = dh - v dp = \left(\frac{\partial h}{\partial T}\right)_p dT + \left[ \left(\frac{\partial h}{\partial p}\right)_T - v \right] dp$$

$$\because dp = 0 \quad \therefore c_p = \frac{(\delta q)_p}{dT} = \left(\frac{\partial h}{\partial T}\right)_p$$



**此处添加投票内容**

- ☐ **A 完全掌握了这部分知识内容**
- ☐ **B 掌握了大部分**
- ☐ **C 掌握了一小部分**
- ☐ **D 完全不懂**

**提交**



## Lecture #2

# Chapter 7 Compressible Flow: Some Preliminary Aspects

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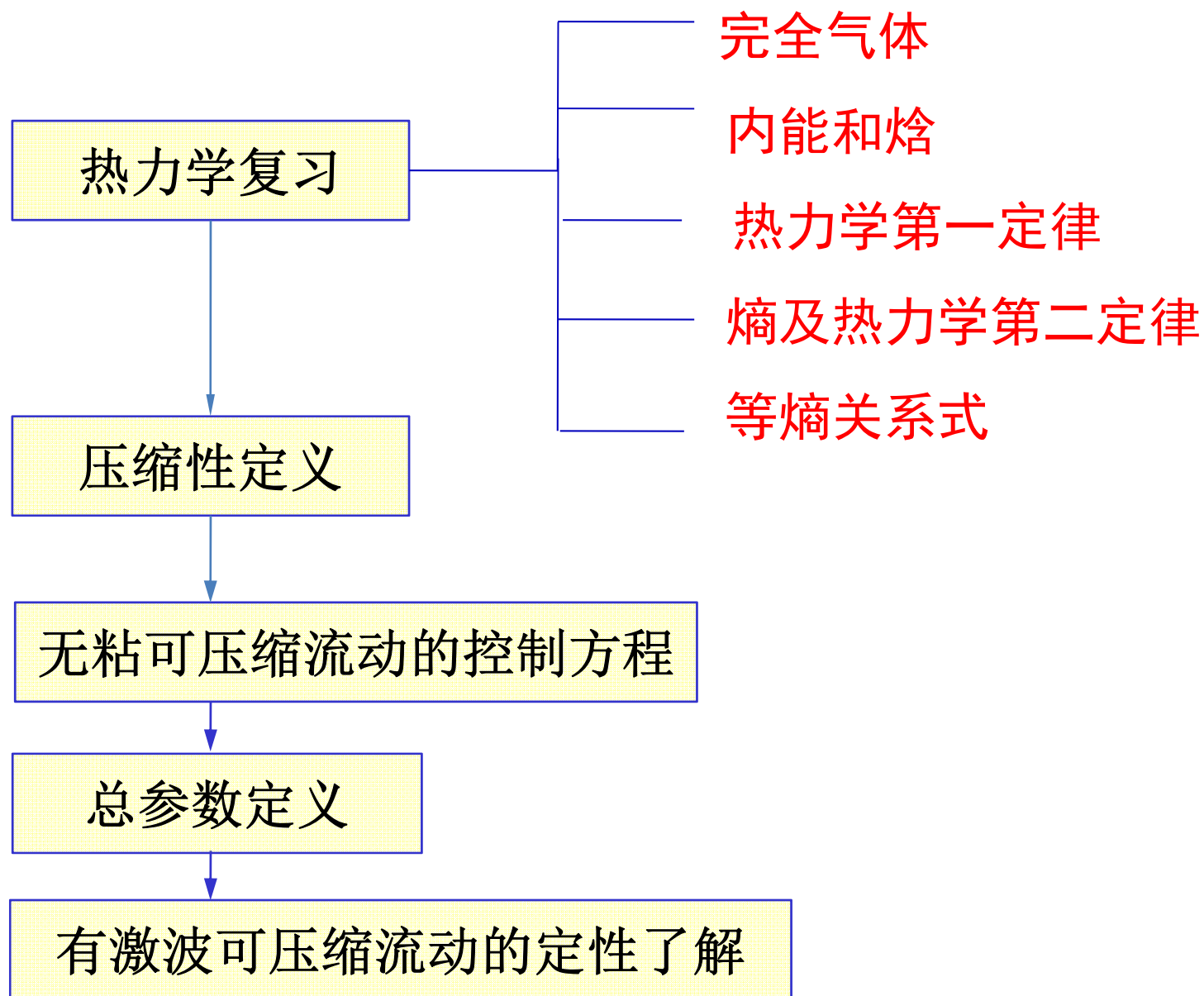
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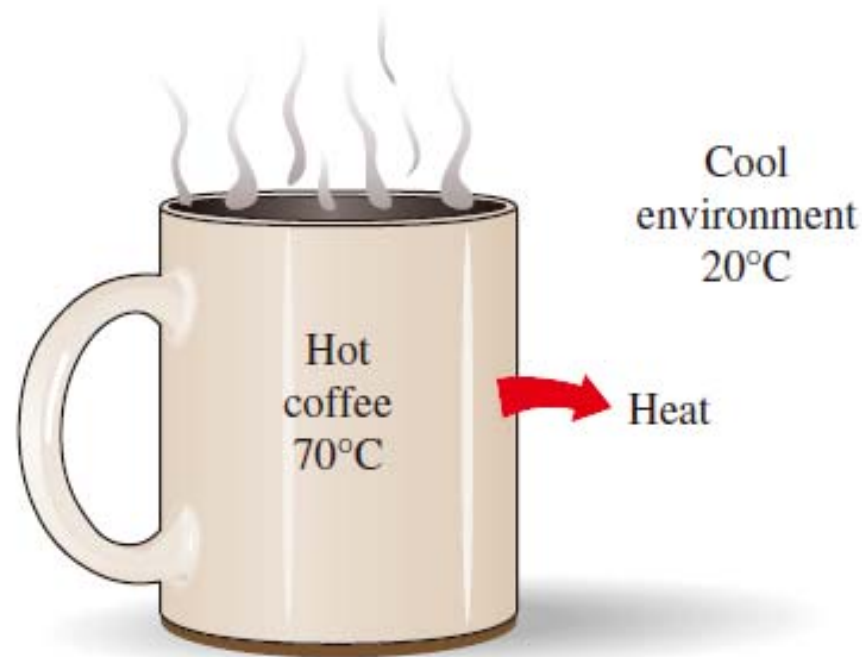




## 第七章路线图



**Q: Is the first law of thermodynamic sufficient for determining a process ?**

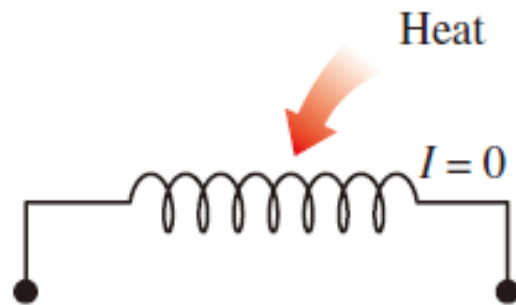


**FIGURE 1–3**

Heat flows in the direction of decreasing temperature.

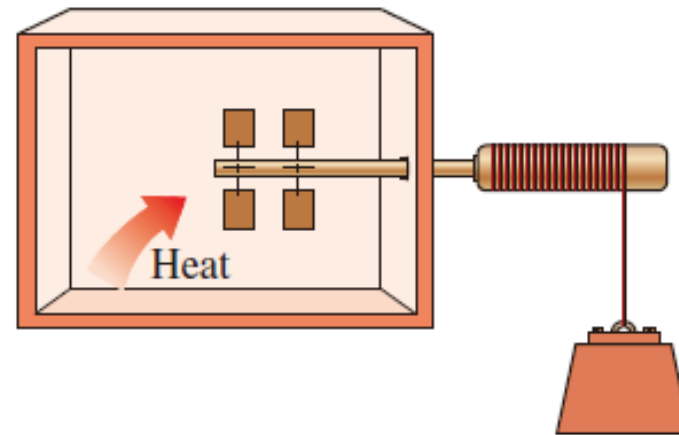


**Q: Is the first law of thermodynamic sufficient for determining a process ?**



**FIGURE 6-2**

Transferring heat to a wire will not generate electricity.



**FIGURE 6-3**

Transferring heat to a paddle wheel will not cause it to rotate.



## 7.2.4 熵及热力学第二定律 (Entropy and the Second Law of Thermodynamics)

- 自发过程：不需要任何外界作用而自动进行的过程
- 自发过程具有方向性、条件、限度

问题：能否有判断自然界自发过程方向性共同规律的判据？

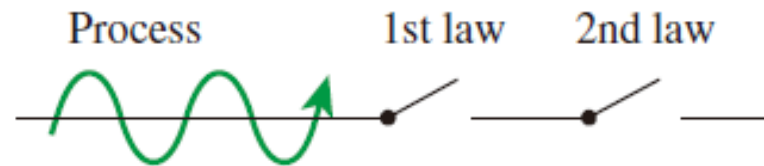
答：热力学第二定律



- 热力学**第一定律**解决了能量在一个过程中的守恒问题。
- 热力学**第二定律**则要解决过程会向哪个方向进行的问题，表明能量不仅有“量”（quantity）的大小，还有“质”（quality）的高低。
- 一个过程只能向同时满足热力学**第一定律**和**第二定律**的方向进行 (A process cannot take place unless it satisfies both the first and second law.)

**FIGURE 6-4**

Processes occur in a certain direction, and not in the reverse direction.

**FIGURE 6-5**

A process must satisfy both the first and second laws of thermodynamics to proceed.



## 熵的定义

- The second law leads to the definition of a new property called entropy. (热力学第二定律引入了一个新性质, 熵)
- Let us define a new **state variable**, the *entropy*  $s$ , as follows (熵的定义如下):

$$ds = \frac{\delta q_{rev}}{T} \quad (7.13)$$

where  $\delta q_{rev}$  is an incremental amount of heat added reversibly to the system and  $T$  is the system temperature. (  $\delta q_{rev}$  为向系统可逆地加入的热量,  $T$  为系统的温度。)



## 熵的定义（续）

- The quantity  $\delta q_{rev}$  is just an artifice(人为的量)
- **can always be assigned to relate the initial and end points of an irreversible process, where the *actual amount of heat added is*  $\delta q$  (实际的加热量), 因此, 另一个熵定义的表达式为:**

$$ds = \frac{\delta q}{T} + ds_{irrev}$$

$ds_{irrev}$  is the generation of entropy due to the irreversible, dissipative phenomena(由于耗散现象导致的熵增).

**注意：熵是状态量，可用于任何可逆或不可逆过程**



- These dissipative phenomena always increase the entropy  $s$ :

$$ds_{irrev} \geq 0 \quad (7.15)$$

- In (7.15), the equals sign denotes a reversible process (等号表示可逆过程). Therefore:

$$ds \geq \frac{\delta q}{T} \quad (7.16)$$

- If the process is adiabatic, then  $\delta q = 0$

$$ds \geq 0 \quad (7.17)$$





## 热力学第二定律

公式表达方式:  $ds_{irrev} \geq 0$   $ds \geq \frac{\delta q}{T}$  或 (绝热):  $ds \geq 0$

- This is the second law of thermodynamics that tells in what direction a process will take place. (热力学第二定律指明过程进行的方向)
- The entropy of the system and its surroundings always increases or, at best, stays the same. (热力学第二定律的语言表达: 系统和其环境的熵总是增加的或不变的)
- In summary, the concept of entropy in combination with the second law allows us to predict the direction that nature takes. (总之, 熵与热力学第二定律相结合, 使我们能预计过程自发进行的方向)



## 热力学第二定律的各种表述方法

- 系统和其环境（孤立系统）的熵总是增加的或不变的
- 不可能将热从低温物体传至高温物体而不引起其它变化——克劳斯表述（1850年）
- 不可能从单一热源取热，并使之完全变为有用功而不引起其它变化——开尔文表述



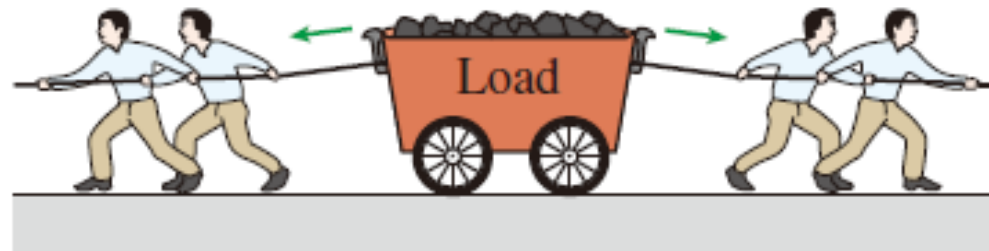
## 热力学第二定律的进一步理解

- 热力学第二定律不仅指明了过程发展的方向，而且说明了能量不仅有“量”的大小，还有“质”的高低。
- 一个单位的“高质量”能量比三个单位的“低质量”能量更有价值。例如，对于一个热电厂的工程师来说，一定量（quantity）的高温热能就比大量的低温热能更有吸引力。



## 熵的概念的进一步理解

- 熵是重要的热力学性质，是表示热力学第二定律的有用工具。
- 熵可以看作是分子混乱度（disorder）的度量。混乱度增加，熵增加。

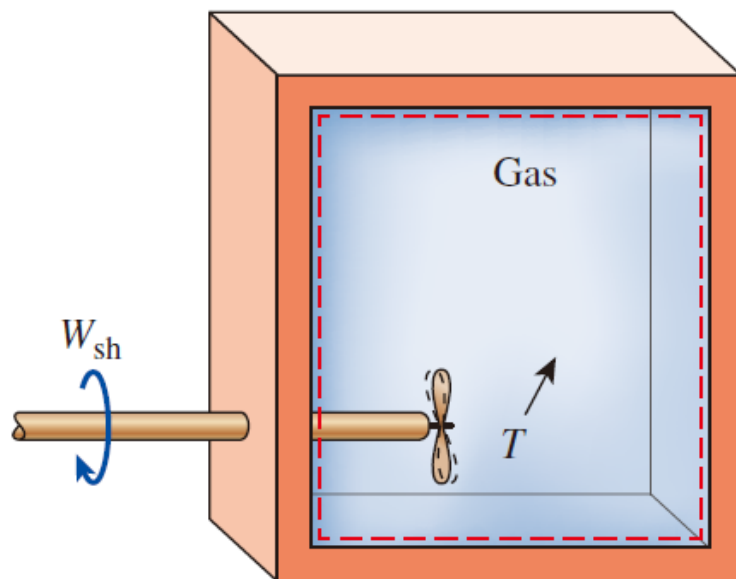


**FIGURE 7-22**

Disorganized energy does not create much useful effect, no matter how large it is.

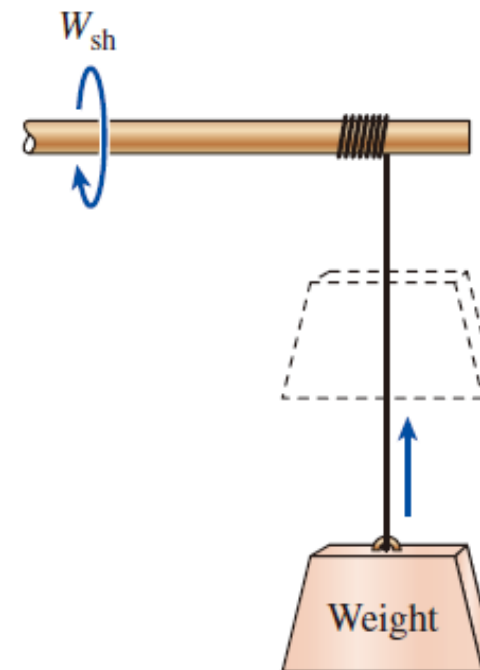


## 熵的概念的进一步理解（续）



**FIGURE 7-24**

The paddle-wheel work done on a gas increases the level of disorder (entropy) of the gas, and thus energy is degraded during this process.



**FIGURE 7-23**

In the absence of friction, raising a weight by a rotating shaft does not create any disorder (entropy), and thus energy is not degraded during this process.



关于熵及热力学第二定律的描述不正确的是：

- ☐ A 系统和其环境（孤立系统）的熵总是增加的或不变的
- ☐ B 熵是一个状态量
- ☐ C 热力学第二定律确定了一个过程的自发方向
- ☒ D 熵是一个和过程有关的量



## Practical calculation of entropy(熵的实际计算)

Assuming that heat is added *reversibly*, then:

$$\delta q = Tds$$

and

$$Tds = de + pdv \quad (7.18)$$

From the definition of enthalpy

$$h = e + pv$$

$$\longrightarrow dh = de + pdv + vdp \quad (7.19)$$



## Practical calculation of entropy(熵的实际计算) (续)

and, by combining these equations:

$$Tds = dh - vdp \quad (7.20)$$

For a perfect gas,

$$de = c_v dT$$

and

$$dh = c_p dT$$

Substituting into (7.18) and (7.20), respectively, one obtain



$$ds = c_v \frac{dT}{T} + \frac{p dv}{T} \quad (7.21)$$

$$ds = c_p \frac{dT}{T} - \frac{v dp}{T} \quad (7.22)$$





## Practical calculation of entropy(熵的实际计算) (续)

With the equation of state

$$p\nu = RT \quad \text{or} \quad \frac{p}{T} = \frac{R}{\nu} \quad \text{or} \quad \frac{\nu}{T} = \frac{R}{p}$$

From (7.21)、(7.22), we get

$$\begin{aligned} \longrightarrow \quad ds &= c_v \frac{dT}{T} + \frac{R d\nu}{\nu} \\ ds &= c_p \frac{dT}{T} - R \frac{dp}{p} \end{aligned} \quad (7.23)$$



## Practical calculation of entropy(熵的实际计算) (续)

Consider a thermodynamic process with initial state “1” and end state “2”, the integration leads to:

$$s_2 - s_1 = \int_{T_1}^{T_2} c_p \frac{dT}{T} - \int_{p_1}^{p_2} R \frac{dp}{p}$$

For a calorically perfect gas, both  $R$  and  $c_p$  are constant; hence:

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} \quad (7.25)$$

In a similar fashion, we obtain:

$$s_2 - s_1 = c_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} \quad (7.26)$$



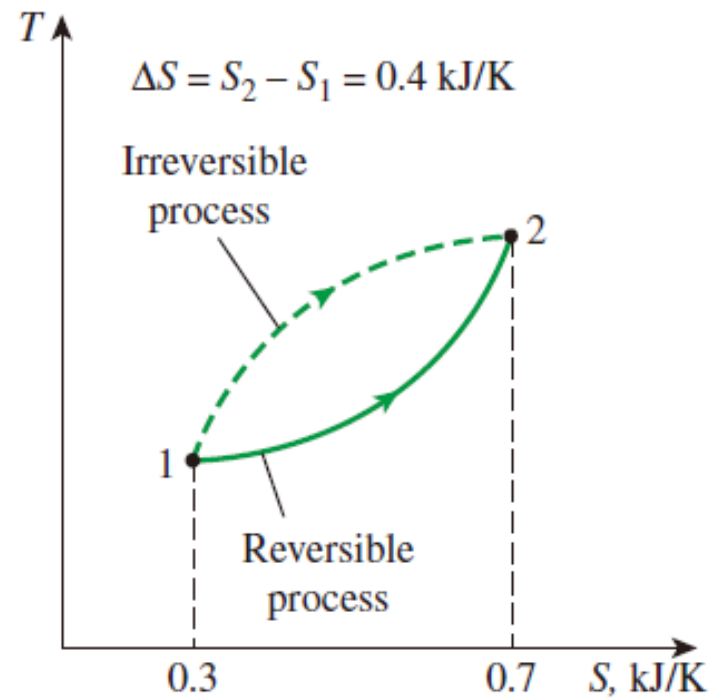
Note that “s” is a function of two thermodynamic state variables;  
e.g.  $s = s(p, T)$  or  $s = s(v, T)$

问题：上述熵增计算公式是从可逆过程假设下推导出来的，是否适用于不可逆过程？

Q: Is the aforementioned formula derived under the assumption of reversible process suited for irreversible process?

适用！更有意义！反映过程不可逆程度的大小！





**FIGURE 7–3**

The entropy change between two specified states is the same whether the process is reversible or irreversible.



## 7.2.5 Isentropic relations (等熵关系式)

➤ For an isentropic process,  $ds = 0$

or

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} = 0$$

Then we get :

$$\ln \frac{p_2}{p_1} = \frac{c_p}{R} \ln \frac{T_2}{T_1}$$

or

$$\frac{p_2}{p_1} = \left( \frac{T_2}{T_1} \right)^{c_p / R} \quad (7.27)$$

With  $\frac{c_p}{R} = \frac{\gamma}{\gamma - 1} \rightarrow$

$$\frac{p_2}{p_1} = \left( \frac{T_2}{T_1} \right)^{\gamma / (\gamma - 1)} \quad (7.28)$$



## 7.2.5 Isentropic relations (等熵关系式) (续)

➤ In a similar fashion , from (7.26)

$$s_2 - s_1 = c_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} = 0 \quad \rightarrow \quad \ln \frac{T_2}{T_1} = -\frac{R}{c_v} \ln \frac{v_2}{v_1}$$

$$\rightarrow \quad \frac{v_2}{v_1} = \left( \frac{T_2}{T_1} \right)^{-c_v/R} \quad (7.29)$$

With

$$\boxed{\frac{c_v}{R} = \frac{1}{\gamma - 1}}$$

$$\rightarrow \quad \frac{v_2}{v_1} = \left( \frac{T_2}{T_1} \right)^{-1/(\gamma-1)} \quad (7.30) \quad \text{and}$$

$$\boxed{\frac{\rho_2}{\rho_1} = \left( \frac{T_2}{T_1} \right)^{1/(\gamma-1)}} \quad (7.31)$$



## 等熵关系式总结 (Summary of isentropic relations)

$$\frac{p_2}{p_1} = \left( \frac{\rho_2}{\rho_1} \right)^\gamma = \left( \frac{T_2}{T_1} \right)^{\gamma/(\gamma-1)} \quad (7.32)$$

Equation (7.32) is very important ; it relates pressure, density , and temperature for an isentropic process.

方程 (7.32) 非常重要，它反映了等熵过程中的压强、密度温度的关系。



$$\frac{p_2}{p_1} = \left( \frac{\rho_2}{\rho_1} \right)^\gamma = \left( \frac{T_2}{T_1} \right)^{\gamma/(\gamma-1)} \quad (7.32)$$

Also, keep in mind the source of Equation (7.32); it stems from the first law and the definition of entropy. Therefore, Equation (7.32) is basically an energy relation for an isentropic process.

另外，要牢记公式（7.32）的起源；它是由热力学第一定律和熵的定义相结合得到的。因此，公式（7.32）实际上就是一个等熵过程的能量关系式。





**Question: Why is the equation (7.32) so important?**

**An isentropic process seems so restrictive, requiring both adiabatic and reversible conditions?**

**Answer:** because a large number of practical compressible flows can be assumed to be isentropic; equation (7.32) is a powerful relation valid for the flows outside the boundary layer, which is small compared to the entire flow field

(因为绝大多数实际流动问题可以被假设为等熵的, 7.32式适用于与整个流场区域相比很小的附面层之外的绝大多数流动区域)。



## 例7.4

波音747飞机在36000  $ft$  高空飞行。机翼表面某点的压强为400  $lb/ft^2$ 。假设流过机翼的流动为等熵流动，计算该点的温度。

国际单位制：

解：  $h=36000\text{ }ft=36000\times 0.3048\text{m}=10972.8\text{m}$

$$p_{\infty} = 2.261 \times 10^4 \text{ Pa} \quad T_{\infty} = 216.5 \text{ K}$$

$$p = 400 \times 47.88 = 19152 (\text{Pa}) = 1.9152 \times 10^4 (\text{Pa})$$

$$T = T_{\infty} \left( \frac{p}{p_{\infty}} \right)^{\frac{\gamma-1}{\gamma}} = 217 \left( \frac{19152}{22610} \right)^{\frac{0.4}{1.4}} = 206.9 \text{ K}$$



## 单选题 1分



高压气罐中静止大气的压强为 $20\text{atm}$ ，温度为 $300\text{K}$ 。  
气罐内的气体膨胀进入风洞的管道,某一截面处气流压强为1大气压。（a）气流等熵膨胀。  
（b）气流以熵增 $320\text{J}/(\text{Kg}\cdot\text{R})$ 非等熵膨胀。  
问题：能否用所学知识计算出这一截面的温度。

- ☐ A （a）能，（b）不能
- ☒ B （a）能,(b)能
- ☐ C （a）不能,(b)能
- ☐ D （a）不能,(b)能



Example 7.5 Consider the gas in the reservoir of the supersonic wind tunnel discussed in Examples 7.2 and 7.3. The pressure and temperature of the air in the reservoir are 20 atm and 300 K respectively. The air in the reservoir expands through the wind tunnel duct. At a certain location in the duct, the pressure is 1 atm. Calculate the air temperature at this location if: (a) the expansion is isentropic and (b) the expansion is nonisentropic with an entropy increase through the duct to this location of 320 J/(kg · K).

■ **Solution**

(a) From Equation (7.32),

$$\frac{p_2}{p_1} = \left( \frac{T_2}{T_1} \right)^{\gamma/(\gamma-1)}$$

or,

$$\begin{aligned} T_2 &= T_1 \left( \frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} = 300 \left( \frac{1}{20} \right)^{\frac{0.4}{1.4}} = 300 (0.05)^{0.2857} \\ &= 300 (0.4249) = \boxed{127.5 \text{ K}} \end{aligned}$$



(b) From Equation (7.25),

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$$

Using Equation (7.9) to obtain the value of  $c_p$ ,

$$c_p = \frac{\gamma R}{\gamma - 1} = \frac{(1.4)(287)}{0.4} = 1004.5 \frac{\text{J}}{\text{kg} \cdot \text{K}}$$

we have from Equation (7.25),

$$\begin{aligned} 320 &= 1004.5 \ln \left( \frac{T_2}{300} \right) - (287) \ln \left( \frac{1}{20} \right) \\ &= 1004.5 \ln \left( \frac{T_2}{300} \right) - (-859.78) \end{aligned}$$

Thus,

$$\ln \left( \frac{T_2}{300} \right) = \frac{320 - 859.78}{1004.5} = -0.5374$$

$$\frac{T_2}{300} = e^{-0.5374} = 0.5843$$

$$T_2 = (0.5843)(300) = \boxed{175.3 \text{ K}}$$



*Comment:* Comparing the results from parts (a) and (b), note that the entropy increase results in a higher temperature at the point in the expansion where  $p = 1\text{ atm}$  compared to that for the isentropic expansion. This makes sense. From Equation (7.25) we see that entropy is a function of both temperature and pressure, increasing with an increase in temperature and decreasing with an increase in pressure. In this example, the final pressure for both cases (a) and (b) is the same, but the entropy for case (b) is higher.

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} \quad (7.25)$$



Thus, from Equation (7.25), we see that the final temperature for case (b) must be higher than that for case (a). On a more qualitative basis, the physical mechanisms that could produce the change in entropy would be viscous dissipation (friction), the presence of shock waves in the duct, or heat addition from the surroundings through the walls of the duct. Intuitively, all these irreversible mechanisms would result in a higher gas temperature than the isentropic expansion that, by definition, assumes an adiabatic and reversible (no friction) expansion.

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} \quad (7.25)$$



## 补充: Concept of barotropic :/正压流的概念

- 满足密度是压力的唯一函数的流动, 称为正压流, 即  $\rho = \rho(p)$
- 根据定义, 不可压流是正压流, 等熵流也是正压流。

在3.2节中, 我们推导出无粘、忽略体积力的微分形式Euler方程:

$$dp = -\rho V dV \quad (3.12)$$

不可压假设和等熵假设应用于上述方程?





## 补充: Concept of barotropic :/正压流的概念 (续)

1. 不可压流:  $\rho = \text{const.}$  我们得到伯努利方程:

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$$

- 对于无粘、不可压缩有旋流动

$$p + \frac{1}{2}\rho V^2 = \text{const} \quad \textit{along a streamline} \quad (3.14)$$

- 对于无粘、不可压缩无旋流动

$$p + \frac{1}{2}\rho V^2 = \text{const} \quad \textit{throughout the flow} \quad (3.15)$$

结论:不可压假设下, 可用伯努利方程代替能量方程.



## 补充: Concept of barotropic :/正压流的概念 (续)

2. 等熵流:  $\frac{p}{\rho^\gamma} = \text{const.}$  我们由:  $\int_1^2 \frac{dp}{\rho} = -\int_1^2 V dV$

$$\begin{aligned} \int_1^2 \frac{dp}{\rho} &= \int_1^2 \frac{\text{const.} \cdot \gamma \rho^{\gamma-1} d\rho}{\rho} = \text{const.} \cdot \frac{\gamma}{\gamma-1} \rho^{\gamma-1} \Big|_1^2 \\ &= \frac{\gamma}{\gamma-1} \text{const.} (\rho_2^{\gamma-1} - \rho_1^{\gamma-1}) = \frac{\gamma}{\gamma-1} \left( \frac{p_2}{\rho_2} - \frac{p_1}{\rho_1} \right) \end{aligned}$$

$$\frac{\gamma}{\gamma-1} \frac{p_1}{\rho_1} + \frac{V_1^2}{2} = \frac{\gamma}{\gamma-1} \frac{p_2}{\rho_2} + \frac{V_2^2}{2} \quad \text{又可表示为} \quad h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

结论: 等熵假设可以使控制方程得到简化, 可用等熵关系式代替能量方程。



Problem 7.3, 7.4, 7.5

**The End !**

