Review of Lecture #18/第18课复习

Chapter 11 Subsonic Compressible Flow Over Airfoils: Linear Theory

(绕翼型的可压缩亚音速流:线化理论)

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十一章路线图

速度势方程

↓

线化速度势方程 → 普朗特 – 葛劳特压缩性修正；改进的压缩性修正

↓

临界马赫数

↓

阻力发散马赫数：声障

↓

跨声速面积律

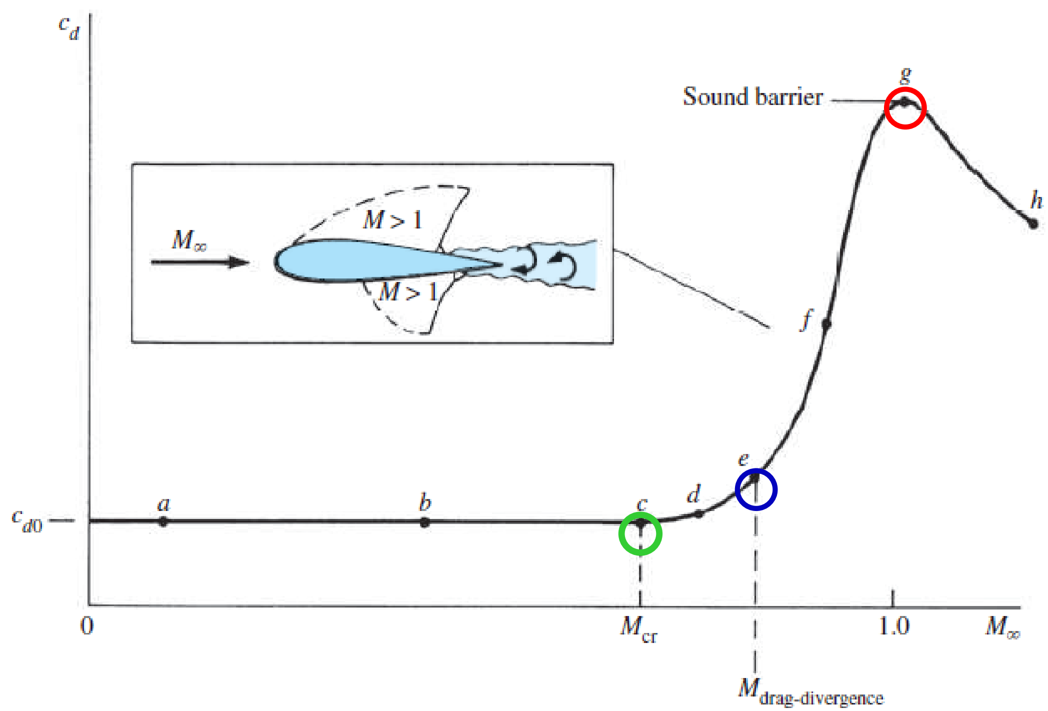
↓

超临界翼型

11.7 DRAG-DIVERGENCE MACH NUMBER: THE SOUND BARRIER（阻力发散马赫数：声障）

阻力发散马赫数的定义:

阻力系数和马赫数的关系



阻力开始急剧增大时所对应的自由来流马赫数，称为阻力发散马赫数。

通常用M_{dd}表示。

$$M_{drag\text{-}divergence}$$

阻力发散马赫数的确定

道格拉斯定义(常用):

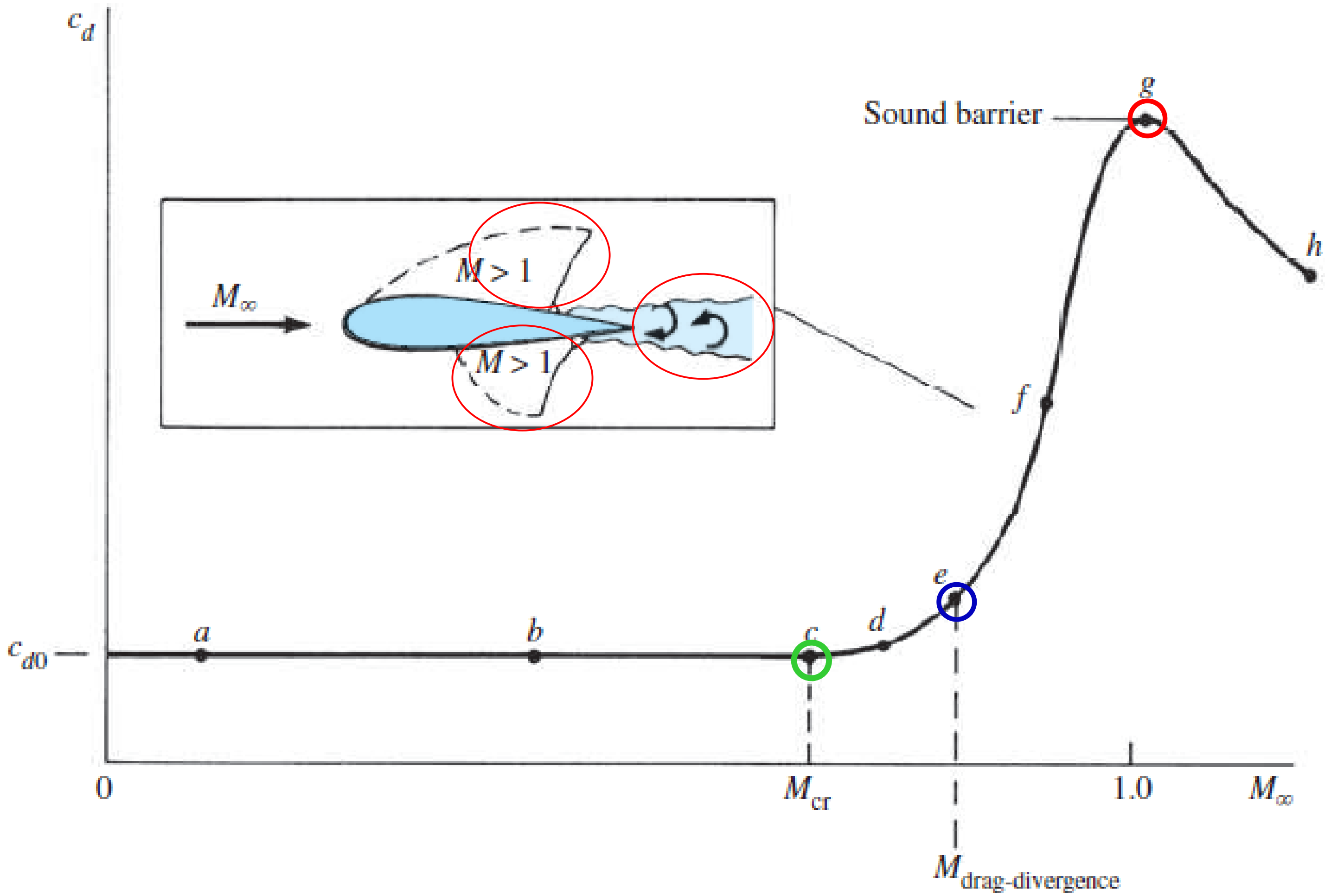
$$\left.\frac{\partial c_d}{\partial M_\infty}\right|_{Mdd} = 0.1$$

波音定义(较少使用):

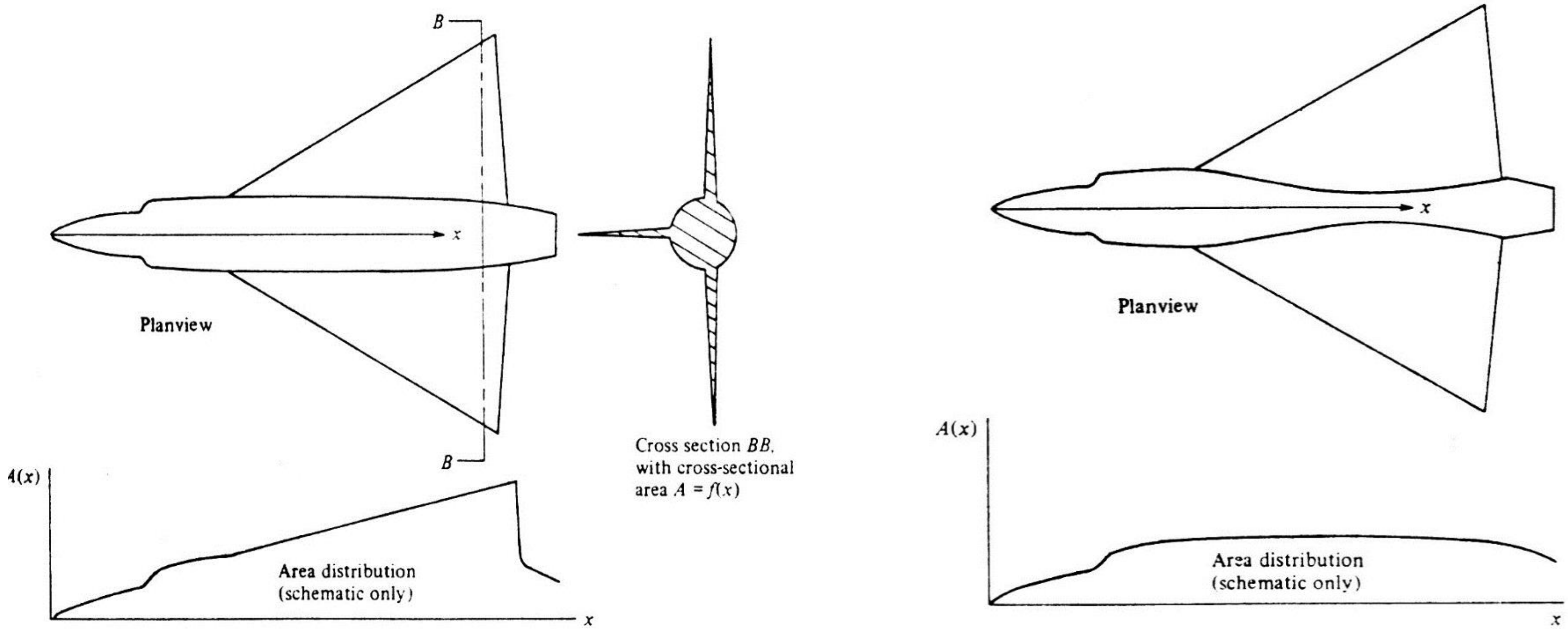
$$\Delta c_d = c_d(M_{dd}) - c_{d,0} = 0.002$$

引起阻力急剧增大的原因:

出现了大的超声速区和强激波，以及出现激波诱导的分离。



11.8 The Area Rule（面积律）



跨音速面积律：

沿机身轴线，一个飞机包括机身、机翼和尾翼的横截面积分布应该是光滑连续的，这样其跨声速阻力可以得到有效减小。

应用面积律的减阻效果示意图和实例

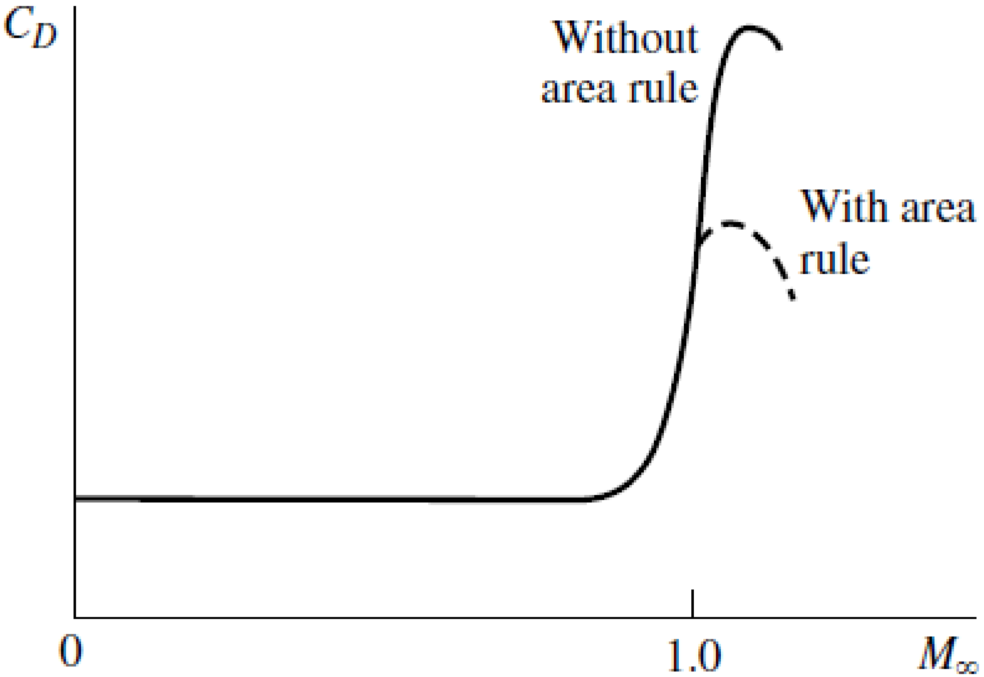


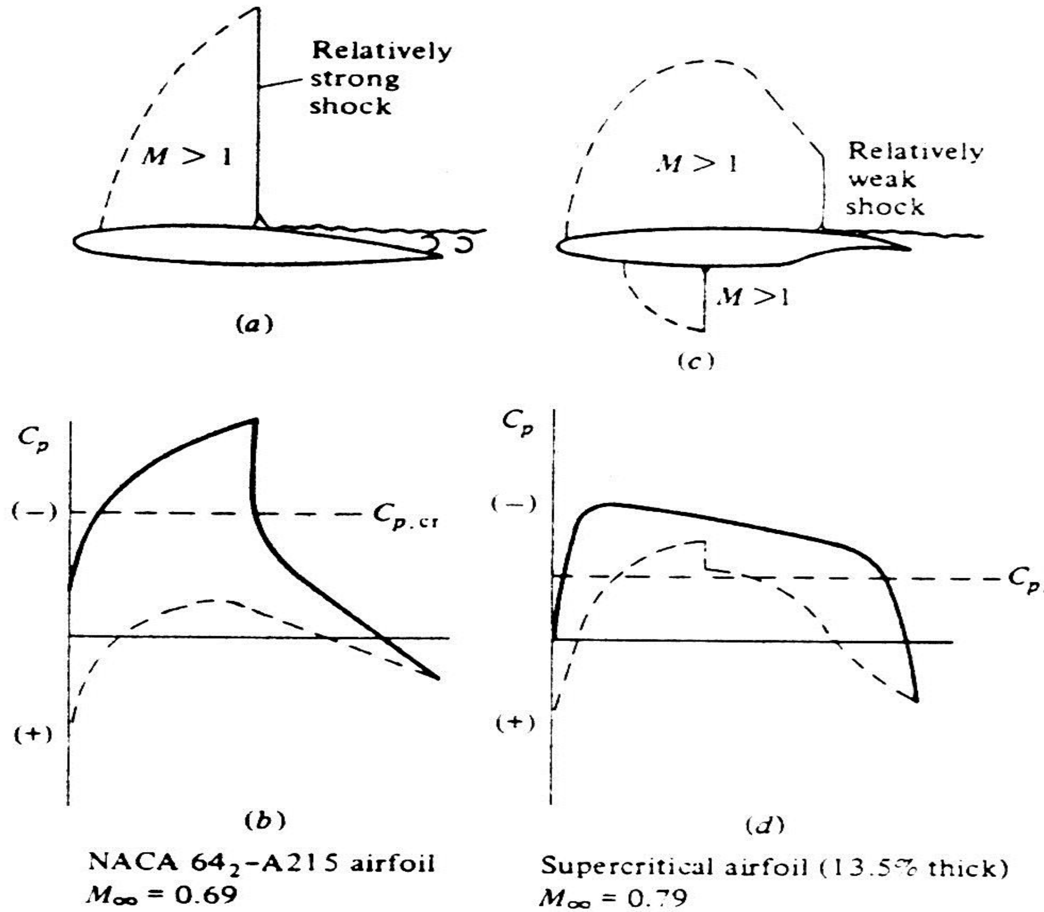
Figure 11.18 The drag-rise properties of area-ruled and non-area-ruled aircraft (schematic only).

歼10飞机的“蜂腰型”机身

11.9 THE SUPERCRITICAL AIRFOIL（超临界翼型）

The purpose of a supercritical airfoil is to increase the value of drag-divergence Mach number.

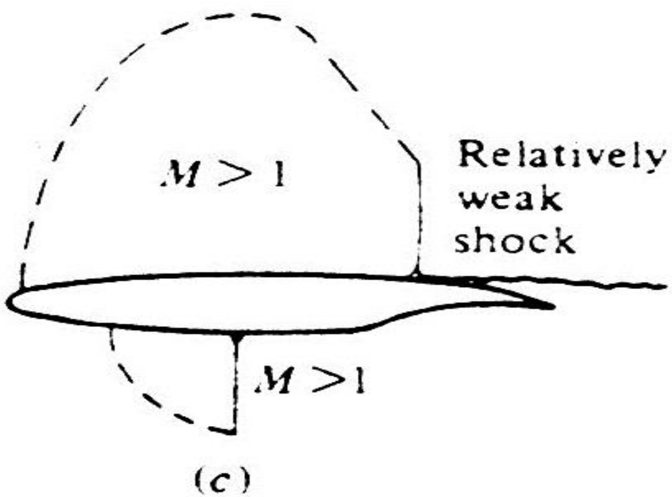
超临界翼型是经过特殊设计的、以增加阻力发散马赫数为目的翼型，使来流马赫数超过临界马赫数后无激波或仅有弱激波的翼型

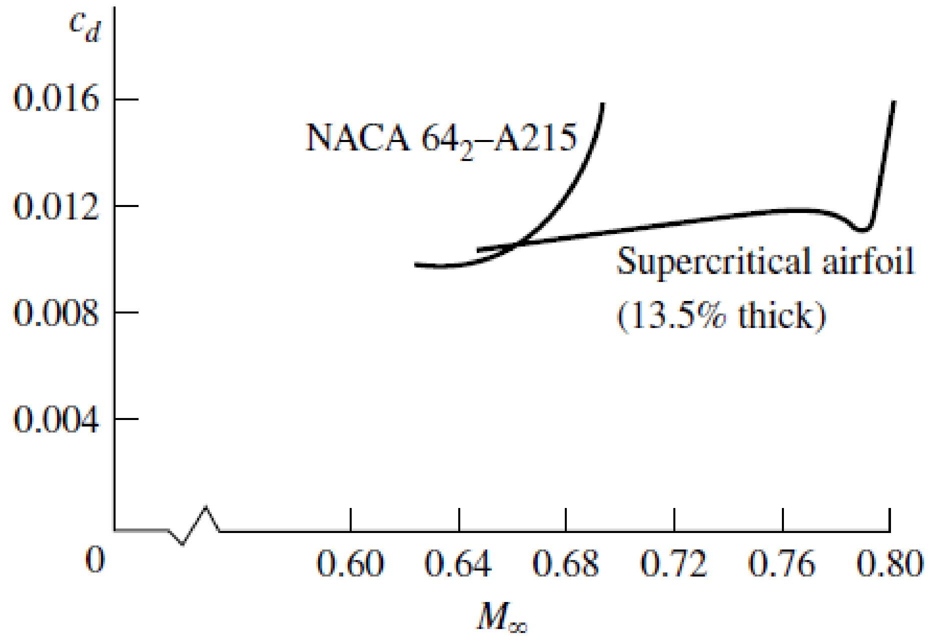


超临界翼型的特点：

具有相对平缓的上表面，因此激波较弱，产生的波阻较小。

由于超临界翼型上表面比较平，翼型在前60%的部分具有负弯度，所以使升力降低，为弥补这一不足，超临界翼型的后30%一般具有较大的正弯度，称为“后加载”。





11.1 应用空气动力学：翼身融合体

(1988 年） **Bushnell, the chief scientist of the NASA Langley Research Center**, asked if new, innovative thinking could result in a commercial jet transport that would provide a quantum leap in efficiency and performance in comparison to the standard tube-fuselage, swept wing airplane with jet engines pod-mounted under the wings, such as pioneered by the historic Boeing 707.

Boeing 707

Responding to this challenge， a small group of aerodynamicists at McDonnell Douglas led by **Dr. Robert Liebeck** conceived the blended wing body.

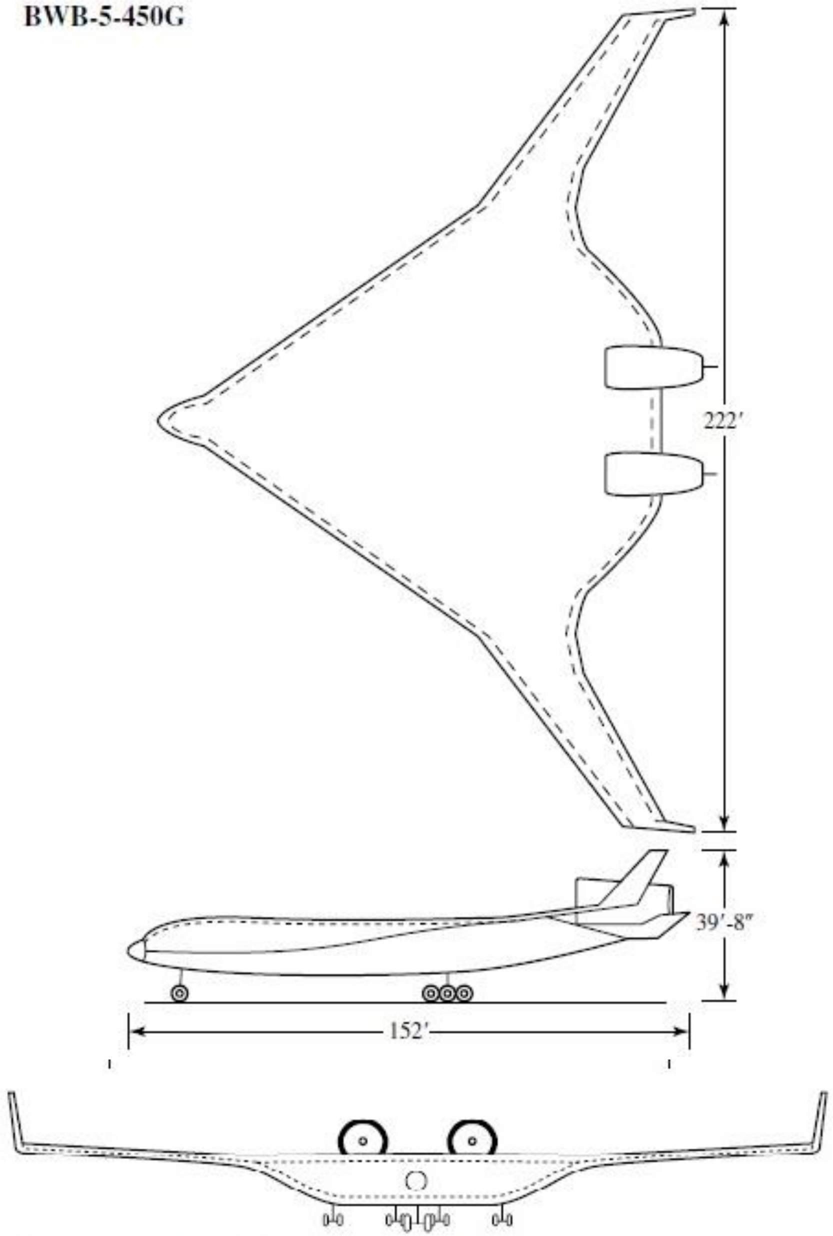


Figure 11.24 Three-view of a blended wing body.

The aerodynamics of the blended wing body is a graphic illustration of the application of many of the fundamentals highlighted in this book. For this reason, the blended wing body is chosen for attention in this applied aerodynamics section. We will see that the BWB is an advanced futuristic flight vehicle applying the very fundamental aerodynamics that is the subject of this book, underscoring the fact that such fundamentals are **timeless(永恒的）**.

- Center body that itself is an efficient lifting surface
- The center body must be large enough and thick enough to accommodate the passenger and cargo load, which drives the increase of both the airfoil chord length and t/c for the center body section
- Spanwise lift distribution from one wing tip to the other is closer to the ideal elliptical distribution

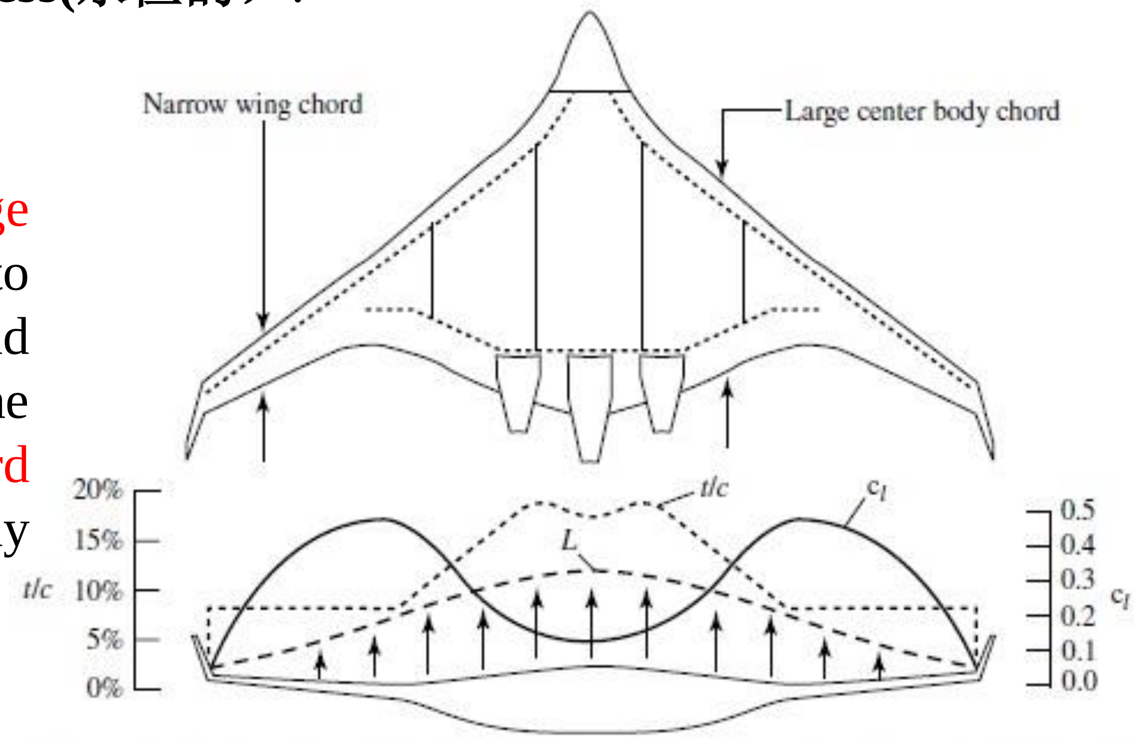


Figure 11.25 Typical spanwise distribution of lift L, lift coefficient c_l, and thickness-to-chord ratio for the blended wing body.

The BWB is a high-speed subsonic airplane intended to fly at the lower end of the transonic flight regime. Hence, major efforts are made to obtain as high a drag divergence Mach number as possible. Toward that end, the BWB incorporates two design features, both of which deal with aerodynamic fundamentals discussed in this chapter.

1. *Supercritical airfoils.*

2. *Area rule*

1. *Supercritical airfoils. The function of a supercritical airfoil is discussed in* Section 11.9. The outer portions of the BWB wing incorporate a modern supercritical airfoil section with aft camber, similar to that shown in Figure 11.19*c*. *The center body profile is also an airfoil section. In the first* generation of the BWB development, the airfoil shape chosen was a Liebeck LW102A airfoil (Ref. 93) point designed for *cl = 0.25 at a Mach* number of 0.7. A side view of the resulting center body profile is shown in Figure 11.26*a*. *The new-generation BWB uses an advanced customized* transonic airfoil design for the center body profile. Taking into account the constraints in cross-sectional area required to effectively hold passengers, baggage, and cargo, the new transonic airfoil design dealt with a careful three-dimensional contouring of the center body smoothly blending into the outer wing panels.

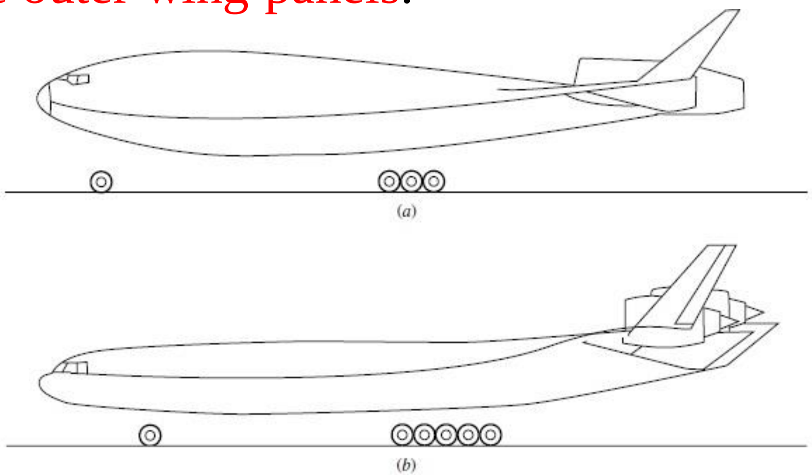


Figure 11.26 Center body profiles: (*a*) first generation; (*b*) recent generation.

→ Liebeck LW102A airfoil
（亚声速高升力翼型）

→ *An advanced customized* transonic airfoil
（专门设计先进跨声速翼型）

2. *Area rule. The notion of the area rule is discussed in Section 11.8. The* blended wing body, with its smooth contours and smoothly varying cross section, is almost naturally area-ruled. Figure 11.27 compares the cross-sectional area distributions as a function of longitudinal coordinate for the BWB (solid curve) and a conventional subsonic transport, the MD-11 (dashed curve). Clearly, the BWB area distribution is much smoother than that of the MD-11, thus exhibiting good area-rule qualities. Liebeck (Ref. 94) states that for the BWB "there appears to be no explicit boundary for increasing the cruise Mach number beyond 0.88." Indeed, a set of blended wing bodies have been designed for Mach numbers of 0.85,0.9., 0.93, and 0.95.

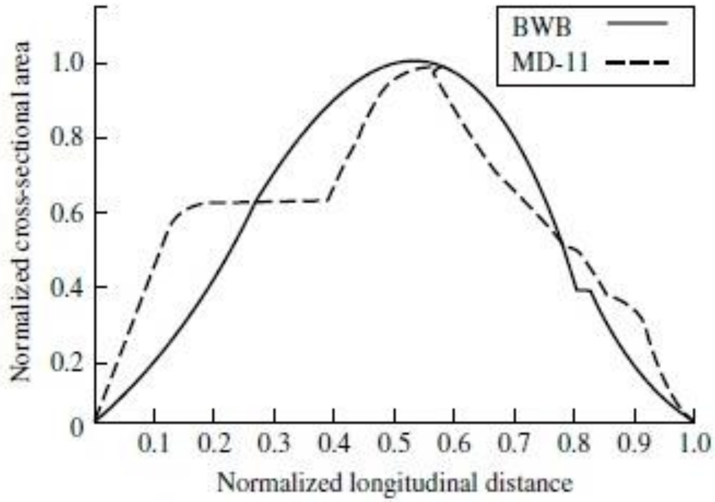


Figure 11.27 Longitudinal distributions of cross-sectional area comparing the blended wing body with a conventional wide-body civil transport, the McDonnell-Douglas MD-11.

Static pressure contours over the top surface of the BWB obtained with a Navier-Stokes CFD solution

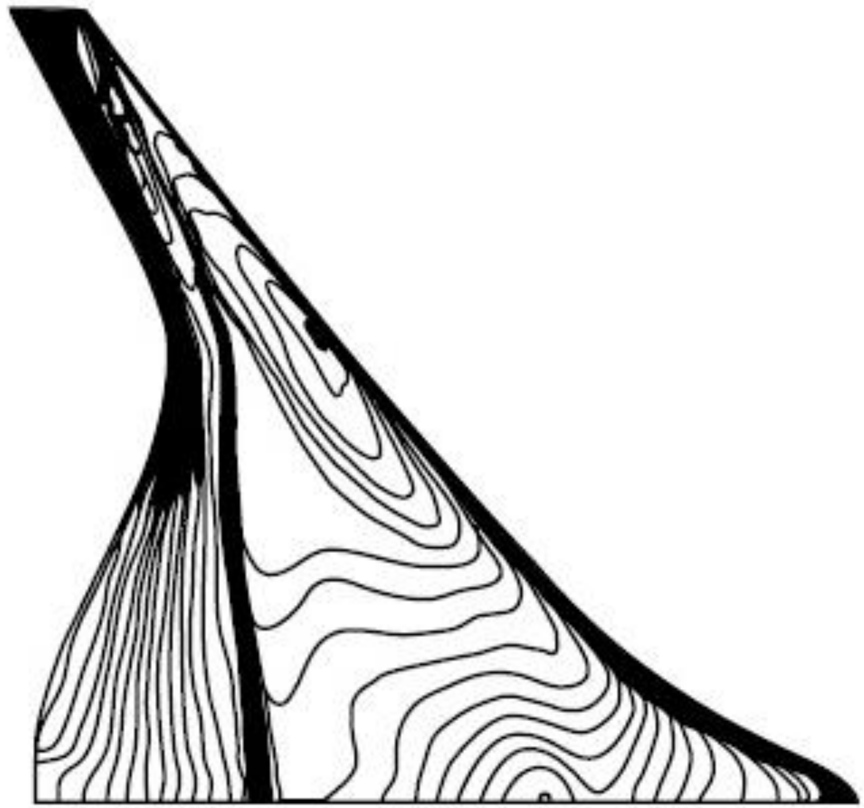


Figure 11.28 Representative CFD calculations of the static pressure contours over the top surface of the BWB at midcruise condition.

CFD 计算与试验的对比：

Experimental data of a blended wing body model is tested in the National Transonic Facility (NFT) at the NASA Langley Research Laboratory at almost full-scale Reynolds number.

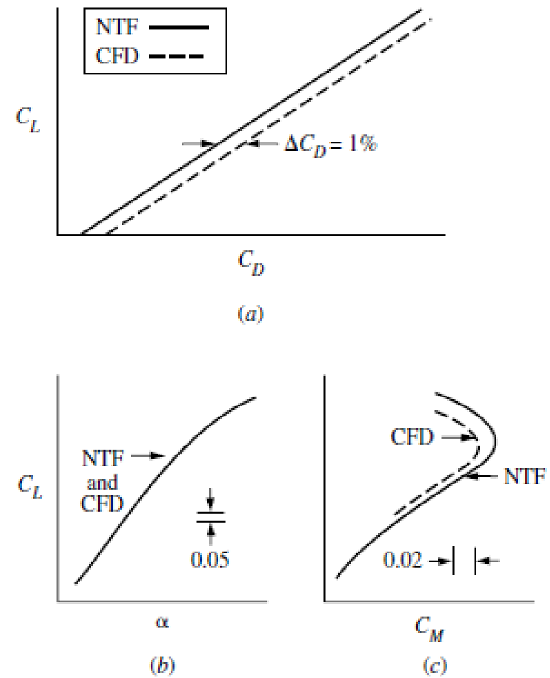


Figure 11.29 Comparison between CFD calculations and experimental data for the blended wing body. The CFD calculations are made with the CFL3D code, and the experimental data are from the National Transonic Facility at the NASA Langley Research Laboratoty. (*a*) C_L versus CD (drag polar); (*b*) C_L versus angle of attack; (*c*) C_L versus moment coefficient.

11.12 小结

对于二维、无旋、等熵、定常的可压缩流， 精确的速度势方程为：

$$\left[1-\frac{1}{a^2}(\frac{\partial\phi}{\partial x})^2\right]\frac{\partial^2\phi}{\partial x^2}+\left[1-\frac{1}{a^2}(\frac{\partial\phi}{\partial y})^2\right]\frac{\partial^2\phi}{\partial y^2}-\frac{2}{a^2}(\frac{\partial\phi}{\partial x})(\frac{\partial\phi}{\partial y})\frac{\partial^2\phi}{\partial x\partial y}=0$$

（11.12)

其中

$$a^2=a_0{}^2-\frac{\gamma-1}{2}\left[(\frac{\partial\phi}{\partial x})^2+(\frac{\partial\phi}{\partial y})^2\right] \qquad (11.13)$$

这一方程是精确的，但它是非线性的，因此很难求解。在目前，还找不到该方程的解析解。

对于小扰动情况（细长体、小迎角），精确速度势方程可以近似为：

$$\boxed{(1-M_\infty^2)\frac{\partial^2\hat{\phi}}{\partial x^2}+\frac{\partial^2\hat{\phi}}{\partial y^2}=0} \qquad (11.18)$$

以上小扰动速度势方程是近似的，但它是线性的，因此求解容易得多。这一方程在亚音速下（ $0\le M_\infty\le 0.8$ ）和超音速下（ $M_\infty\ge 1.2$ ）成立。在跨音速（ $0.8\le M_\infty\le 1.2$ ）和高超音速（ $M_\infty>5$ ）不成立。

线性化压强系数表示为：

$$\boxed{C_p=-\frac{2\hat{u}}{V_\infty}} \qquad (11.32)$$

近似物面边界条件为:

$$\boxed{\frac{\partial\hat{\phi}}{\partial y}=V_\infty\tan\theta} \qquad (11.34)$$

Prandtl-Glauert 相似律是一个压缩性修正公式，可将不可压流动的结果经过修改来考虑压缩性的影响（注意其适用范围）

$$\boxed{C_p = \frac{C_{p,0}}{\sqrt{1-M_\infty^2}}} \tag{11.51}$$

$$\boxed{c_l = \frac{c_{l,0}}{\sqrt{1-M_\infty^2}}} \tag{11.52}$$

$$\boxed{c_m = \frac{c_{m,0}}{\sqrt{1-M_\infty^2}}} \tag{11.53}$$

临界马赫数是指物体表面流速度最快点为声速时所对应的自由来流马赫数。对于薄翼型，可由图11.9估算出其临界马赫数。

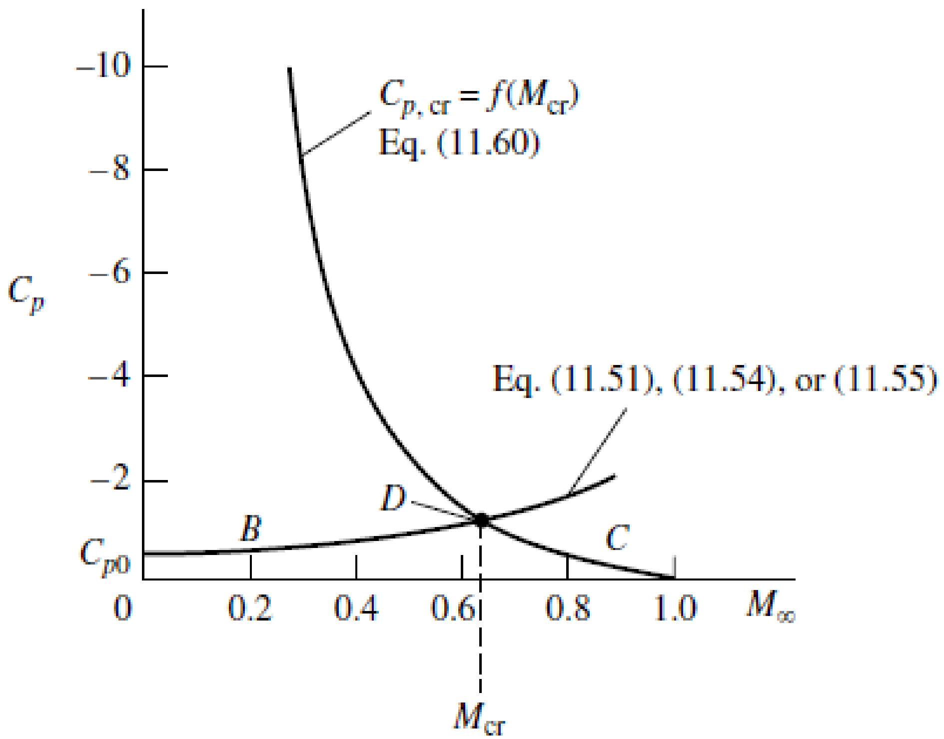
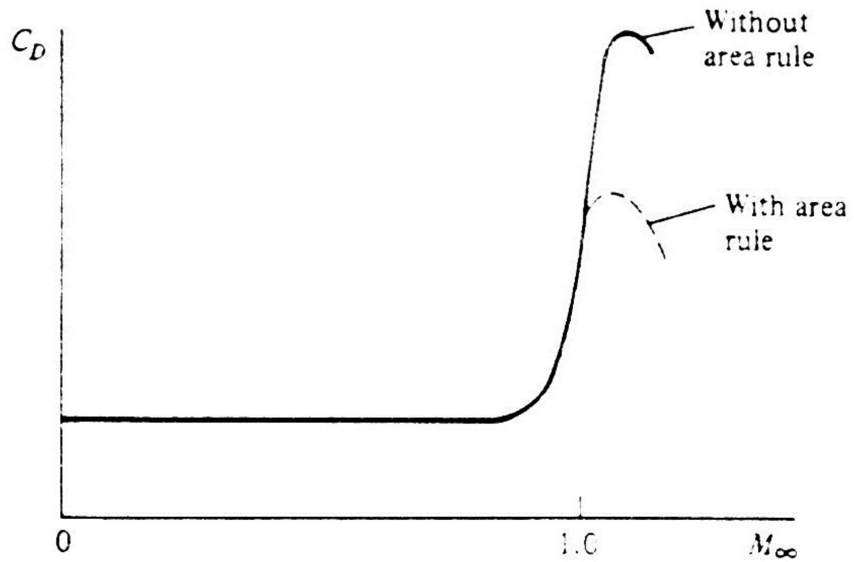


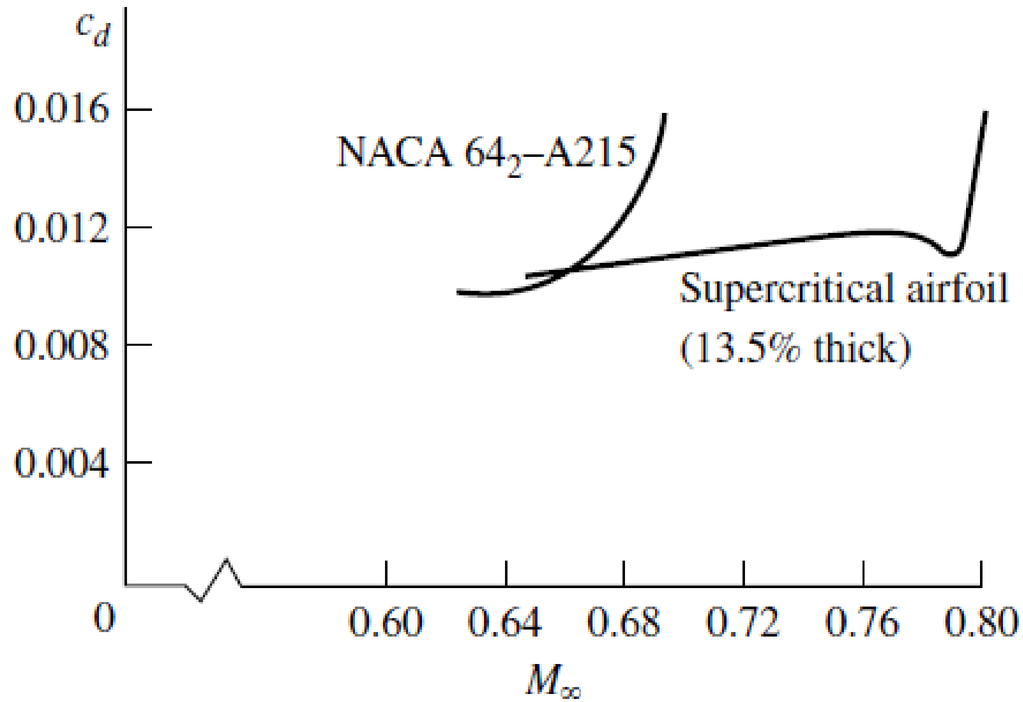
Figure 11.6 Estimation of critical Mach number.

The area rule for transonic flow states that the cross-sectional area distribution of an airplane, including fuselage, wing, and tail, should have a smooth distribution along the axis of the airplane. 跨声速面积律指出，沿飞机轴线其包括机身，机翼，尾翼的横截面积分布应该是光滑连续的，这样其跨声速阻力可以得到有效减小。



Supercritical airfoils are specially designed profiles to increase the drag-divergence Mach number.

超临界翼型是经过特殊设计的翼型,其目的是增大翼型的阻力发散马赫数.



投票 最多可选1项

设置

请对上一次课内容的掌握情况进行投票

- A 完全掌握了这部分知识内容
- B 掌握了大部分
- C 掌握了一小部分
- D 完全不懂

提交

Review of Lecture #18 ended!

Lecture #19/第19次课

CHAPTER 12 LINERIZED SUPERSONIC FLOW
线化超声速流动

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12.1 前言

线化小扰动速度势方程

$$(1-M_\infty^2)\frac{\partial^2\hat{\phi}}{\partial x^2}+\frac{\partial^2\hat{\phi}}{\partial y^2}=0 \qquad (11.18)$$

$1-M_\infty^2 > 0$ 　亚声速，椭圆型偏微分方程

(*elliptic* partial differential equation)

$1-M_\infty^2<0$ 　超声速，双曲型偏微分方程

(hyperbolic partial differential equation)。

线化小扰动速势方程

方程(11.18)中第一项系数的符号改变使方程的解发生了很大变化，其超声速解和亚声速完全不同。

本章的目的就是找到方程(11.18)的超声速解，进而计算超声速翼型的气动特性。

12.2 线化压强系数计算公式的推导
----- 超声速线化小扰动方程的通解

对于超声速流，定义 $\lambda=\sqrt{M_\infty^2-1}$ ，我们可以将线化速度势方程（11.18）重写为如下形式：

$$\lambda^2\frac{\partial^2\hat{\phi}}{\partial x^2}-\frac{\partial^2\hat{\phi}}{\partial y^2}=0 \qquad (12.1)$$

这一方程有个一般解:

$$\hat{\phi}=f(x-\lambda y)+g(x+\lambda y) \qquad (12.2a)$$

书上:

$$\hat{\phi}=f(x-\lambda y) \qquad (12.2)$$

将（12.2)代入（12.1)，即可证明其正解性（见书上证明）。

超声速线化小扰动方程的特征线

仔细观察（12.2)，f 可以是 x-λy 的任意函数，g 可以是 x+λy 的任意函数，因此这是一个一般解，而不是一个确定的解。

沿 $x-\lambda y=const.,\ f=const.$

沿 $x+\lambda y=const.,\ g=const.$

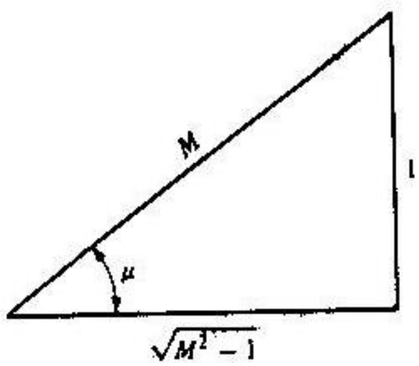
$x-\lambda y=const.$ 和 $x+\lambda y=const.$

称为方程（12.1)的特征线。

超声速线化小扰动方程的特征线

$$x-\lambda y=const. \qquad \frac{dy}{dx}=\frac{1}{\lambda}=\frac{1}{\sqrt{M_\infty{}^2-1}} \qquad (12.7)$$

$$x+\lambda y=const. \qquad \frac{dy}{dx}=-\frac{1}{\lambda}=-\frac{1}{\sqrt{M_\infty{}^2-1}}$$



$$\tan\mu=\frac{1}{\sqrt{M_\infty{}^2-1}} \qquad (12.8)$$

φ沿马赫线为常数

结论: 方程的特征线是自由来流马赫数对应的马赫线。

超声速线化小扰动解的特点

对于超声速流过图12.1所示的物体:

$$g = 0$$

$$\hat{\phi} = f(x - \lambda y) \qquad (12.2)$$

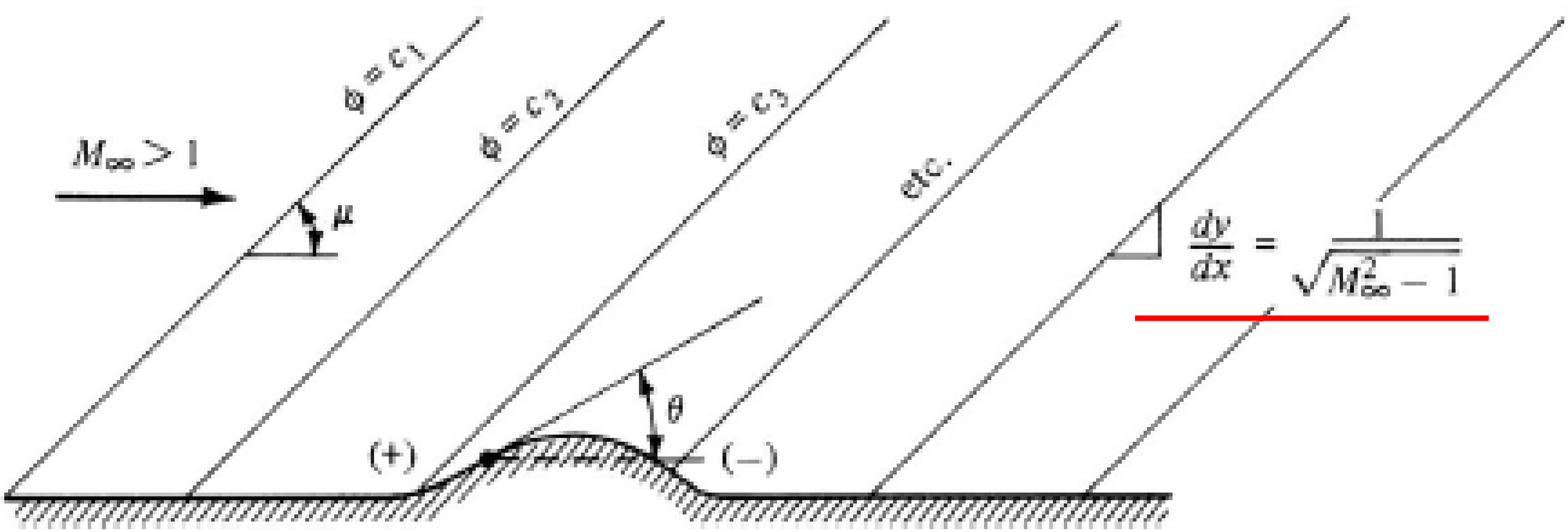
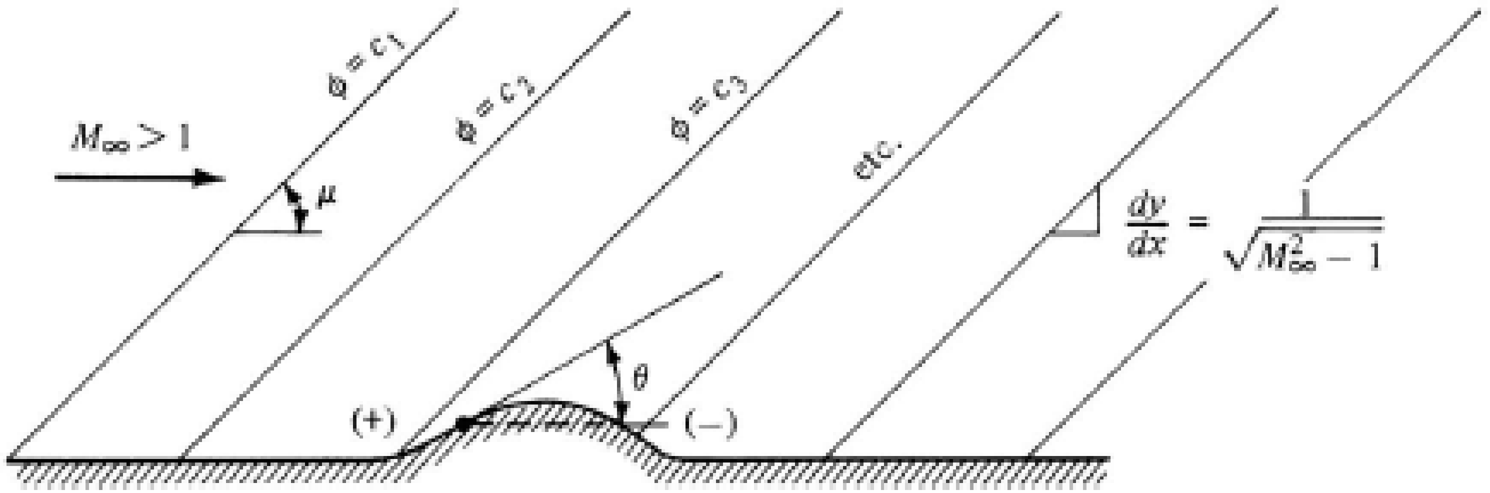


Figure 12.1 Linearized supersonic flow.

超声速线化小扰动解的特点（续）

由图(12.1)可以看出，马赫波向下游方向倾斜，因此，壁面发出的任何扰动不能向上游传播（*any disturbance at the wall cannot propagate upstream*）； 这和亚声速情况完全不同， 在亚声速流中， 扰动向四周传播（disturbances propagate everywhere throughout a subsonic flow）.

Read P780-788



超声速线化小扰动解的特点（续）

$$\hat{\phi} = f(x - \lambda y) \qquad (12.2)$$

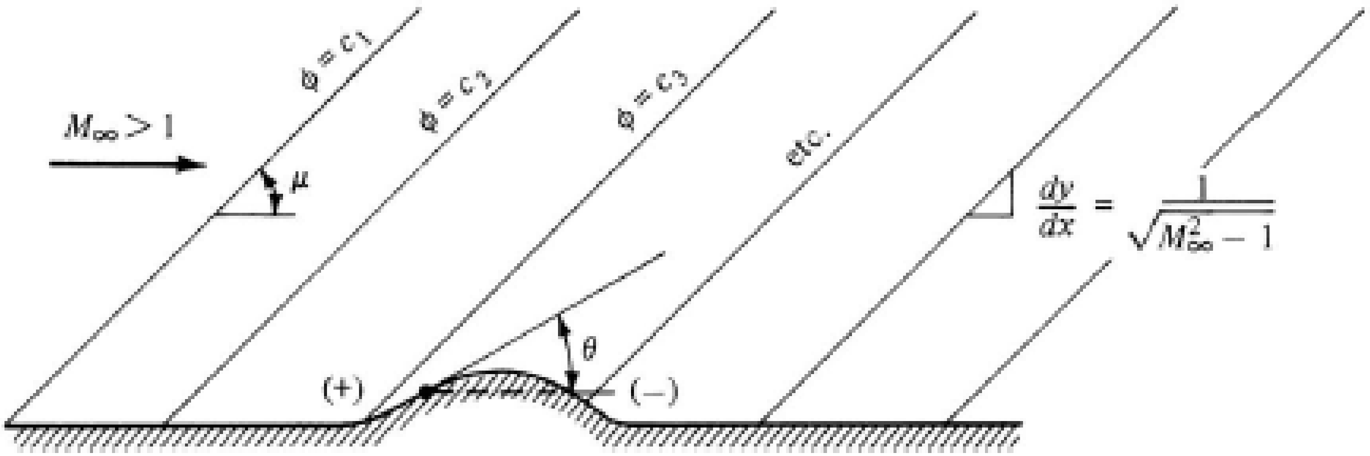


Figure 12.1 Linearized supersonic flow.

结论：超声速线化理论是将超声速流场中的压缩波、膨胀波全部简化为对应自由来流马赫数的马赫波。

物面压强系数表达式的推导：

$$\hat{u} = \frac{\partial \hat{\phi}}{\partial x} = f' \qquad (12.9)$$

$$\hat{v} = \frac{\partial \hat{\phi}}{\partial y} = -\lambda f' \qquad (12.10)$$

$$\hat{u} = -\frac{\hat{v}}{\lambda} \qquad (12.11)$$

物面压强系数表达式的推导：

小扰动条件下的物面边界条件为

$$\hat{v}=\frac{\partial\hat{\phi}}{\partial y}=V_\infty\tan\theta \qquad (12.12) \text{ 即}(11.34)$$

当 θ 为小量时， $\tan\theta\approx\theta$

所以有 $\hat{v}=V_\infty\theta$ (12.13)

将(12.13) 代入(12.11) $\hat{u}=-\hat{v}/\lambda$，得

$$\hat{u}=-\frac{V_\infty\theta}{\lambda} \qquad (12.14)$$

物面压强系数表达式

因为:

$$C_p = -2\hat{u} / V_\infty \tag{11.32}$$

$$\hat{u} = -V_\infty \theta / \lambda \tag{12.14}$$

所以有:

$$\boxed{C_p = \frac{2\theta}{\sqrt{M_\infty^2 - 1}}} \tag{12.15}$$

Conclusion: The linearized supersonic pressure coefficient is directly proportional to the local surface inclination with respect to the free stream. **Read 783**

结论: 线化假设下超声速压强系数正比于当地物面切线方向与自由来流的夹角。

物面压强系数公式中θ的正、负号讨论（见图12.1）

$$\boxed{C_p = \frac{2\theta}{\sqrt{M_\infty^2 - 1}}} \qquad (12.15)$$

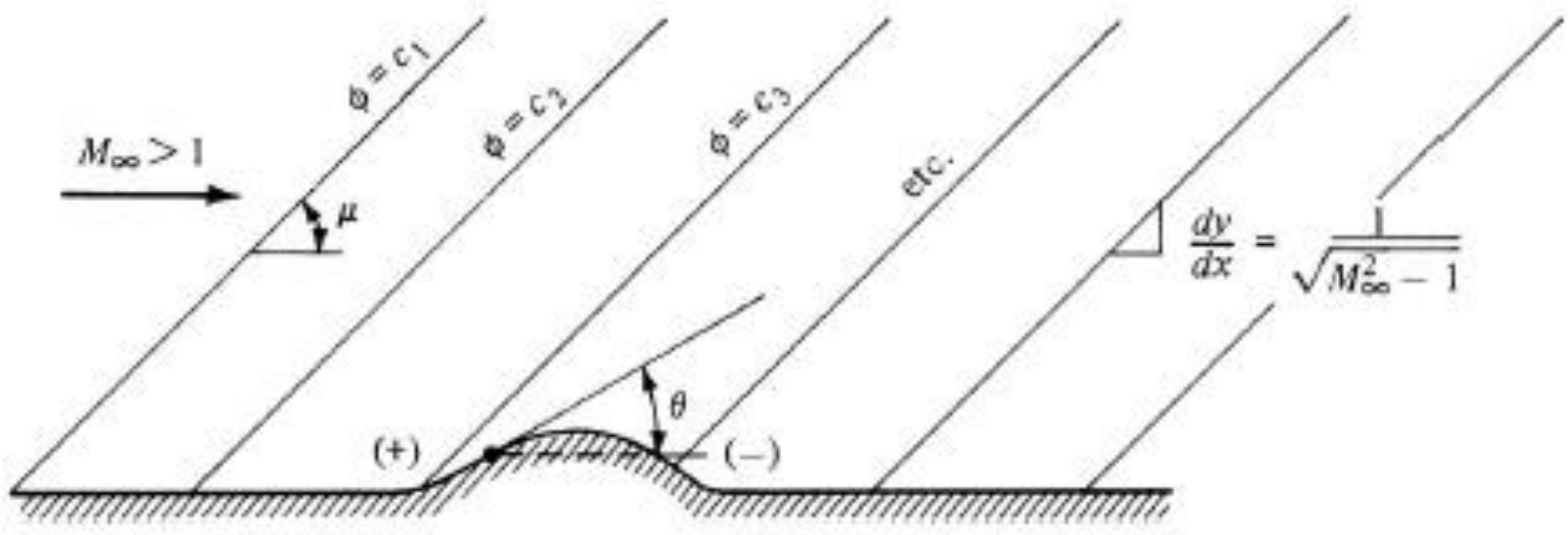
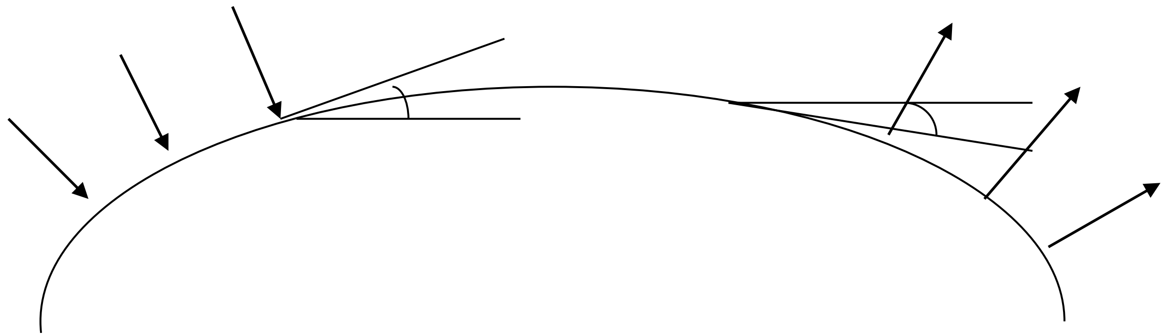


Figure 12.1 Linearized supersonic flow.

波阻的产生原理

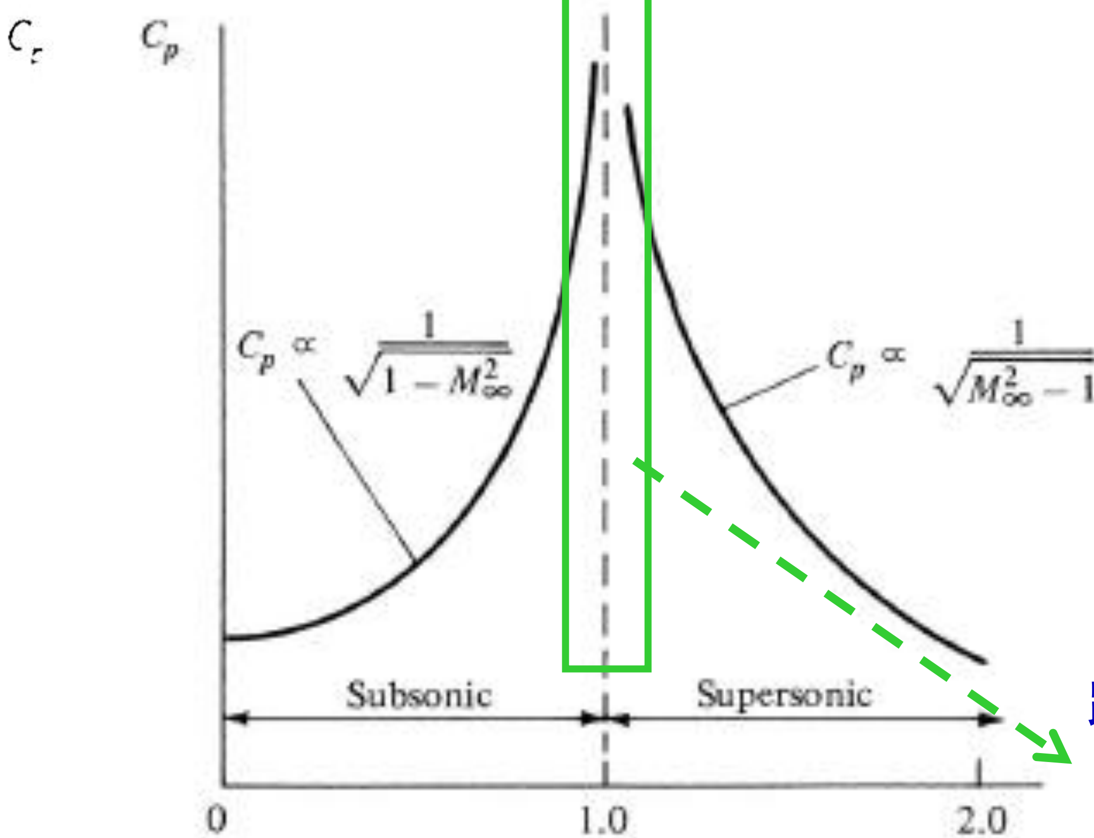
The pressure is higher on the front section of the hump, and lower on the rear section. So there is a drag force exerting on the hump. This drag is called ***wave drag.***

前面的压强高，后面压强低，便产生了波阻



线化压强系数随马赫数变化

$$\boxed{C_p = \frac{C_{p,0}}{\sqrt{1-M_\infty^2}}}$$



$$\boxed{C_p = \frac{2\theta}{\sqrt{M_\infty^2-1}}}$$

跨声速区
(12.15)、
(11.51)不成立

Figure 12.2 Variation of the linearized pressure coefficient with Mach number (schematic).

单选题 2分

设置

下列结论不正确的是：

A 线化假设下超声速流的压强系数正比于当地物面切线方向与自由来流的夹角

B 小扰动速度势方程 $(1-M_\infty^2)\frac{\partial^2\hat{\phi}}{\partial x^2}+\frac{\partial^2\hat{\phi}}{\partial y^2}=0$ 在超声速时的一般解是：$\hat{\phi}=f(x-\lambda y)+g(x+\lambda y)$ 其中 $\lambda=\sqrt{M_\infty{}^2-1}$

C 超声速流动中扰动可以向上游传播

D 超声速线化理论是将超声速流场中的压缩波、膨胀波全部简化为对应自由来流马赫数的马赫波。

提交

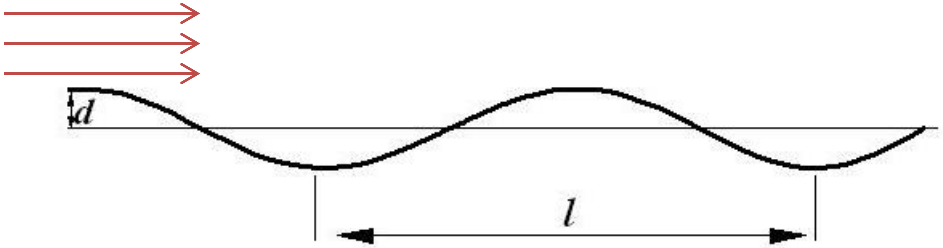
补充：超声速小扰动流绕波形壁的解

$$\lambda^2 \frac{\partial^2 \hat{\phi}}{\partial x^2} - \frac{\partial^2 \hat{\phi}}{\partial y^2} = 0 \qquad \lambda = \sqrt{M_\infty{}^2 - 1}$$

物面边界条件：

$$\left(\frac{\partial \hat{\phi}}{\partial y}\right)_{y=0} = V_\infty \frac{dy}{dx}$$

波形壁表面方程： $M_\infty > 1$



$$y_s = d\cos(2\pi x/l)$$

$$\hat{\phi} = f(x - \lambda y)$$

$$\left(\frac{\partial \hat{\phi}}{\partial y}\right)_{y=0} = -\lambda f'(x) = V_\infty \frac{dy_s}{dx} = -\frac{2\pi d V_\infty}{l} sin\left(\frac{2\pi x}{l}\right)$$

$$f'(x) = \frac{2\pi d V_\infty}{\lambda l} sin\left(\frac{2\pi x}{l}\right)$$

$$f(x) = -\frac{V_\infty d}{\lambda} cos\left(\frac{2\pi x}{l}\right)$$

$$f(x-\lambda y)=-\frac{V_\infty d}{\lambda}\cos[\frac{2\pi(x-\lambda y)}{l}]$$

$$\hat{\phi}=-\frac{V_\infty d}{\lambda}\cos[\frac{2\pi(x-\lambda y)}{l}]$$

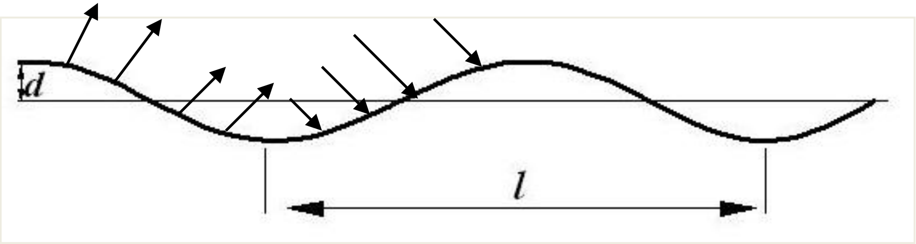
$$\hat{u}=\frac{\partial\hat{\phi}}{\partial x}=\frac{2\pi V_\infty d}{\lambda l}\sin\frac{2\pi(x-\lambda y)}{l}$$

$$\hat{v}=\frac{\partial\hat{\phi}}{\partial y}=-\frac{2\pi V_\infty d}{l}\sin\frac{2\pi(x-\lambda y)}{l}$$

$$C_p = -\frac{2}{V_\infty}\frac{\partial \hat{\phi}}{\partial x} = -\frac{4\pi d}{\lambda l}\sin\frac{2\pi(x-\lambda y)}{l}$$

$$(C_p)_{y=0} = -\frac{2}{V_\infty}\frac{\partial \hat{\phi}}{\partial x} = -\frac{4\pi d}{\lambda l}\sin\frac{2\pi x}{l}$$

阻力系数Cd>0



对比： 亚声速解 超声速解

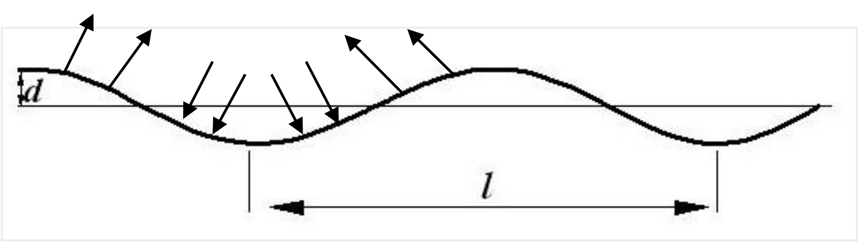
$$\hat{\phi} = (V_\infty d/\beta) e^{-\frac{2\pi\beta y}{l}} \sin(2\pi x/l)$$

$$\hat{u} = (2\pi V_\infty d/\beta l) e^{-\frac{2\pi\beta y}{l}} \cos(2\pi x/l)$$

$$\hat{v} = -(2\pi V_\infty d/l) e^{-\frac{2\pi\beta y}{l}} \cos(2\pi x/l)$$

$$C_p = -(4\pi d/\beta l) e^{-\frac{2\pi\beta y}{l}} \cos(2\pi x/l)$$

$$(C_p)_{y=0} = -(4\pi d/\beta l)\cos(2\pi x/l)$$



阻力系数Cd=0

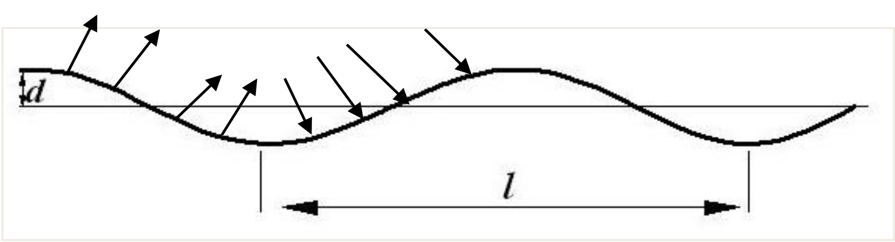
$$\hat{\phi} = -\frac{V_\infty d}{\lambda}\cos[\frac{2\pi(x-\lambda y)}{l}]$$

$$\hat{u} = \frac{2\pi V_\infty d}{\lambda l}\sin\frac{2\pi(x-\lambda y)}{l}$$

$$\hat{v} = -\frac{2\pi V_\infty d}{l}\sin\frac{2\pi(x-\lambda y)}{l}$$

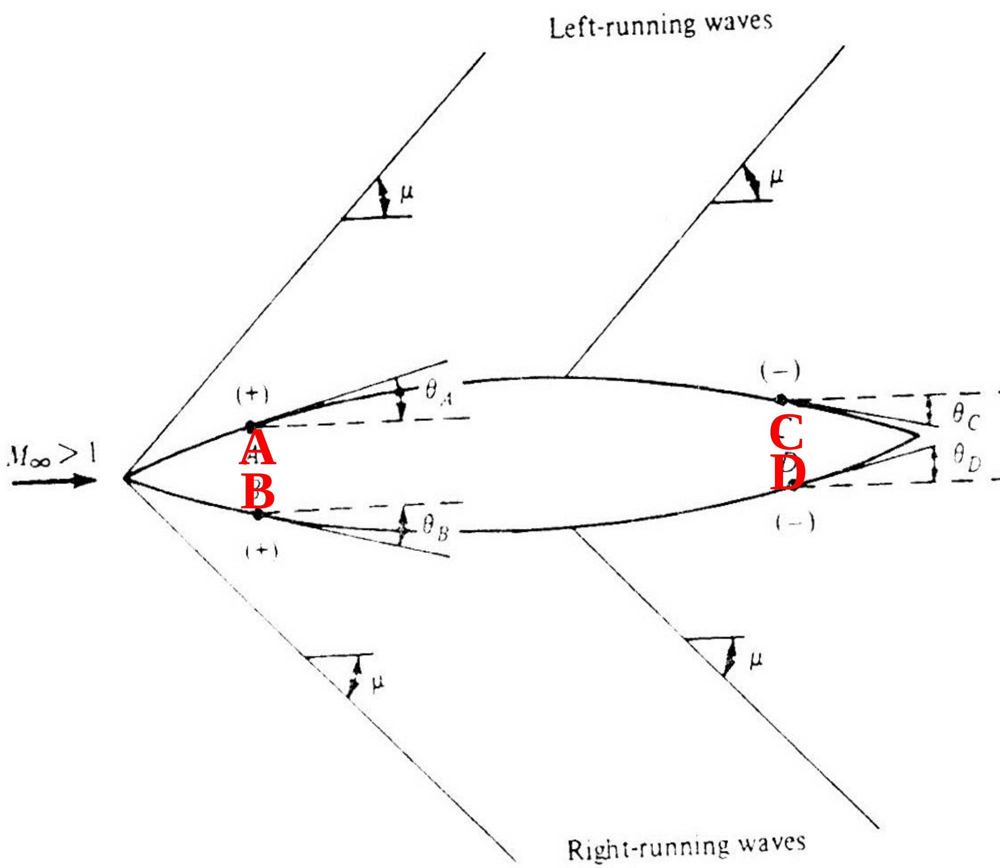
$$C_p = -\frac{4\pi d}{\lambda l}\sin\frac{2\pi(x-\lambda y)}{l}$$

$$(C_p)_{y=0} = -\frac{4\pi d}{\lambda l}\sin\frac{2\pi x}{l}$$



阻力系数Cd>0

12.3 APPLICATION TO SUPERSONIC AIRFOILS 在超声速翼型上的应用



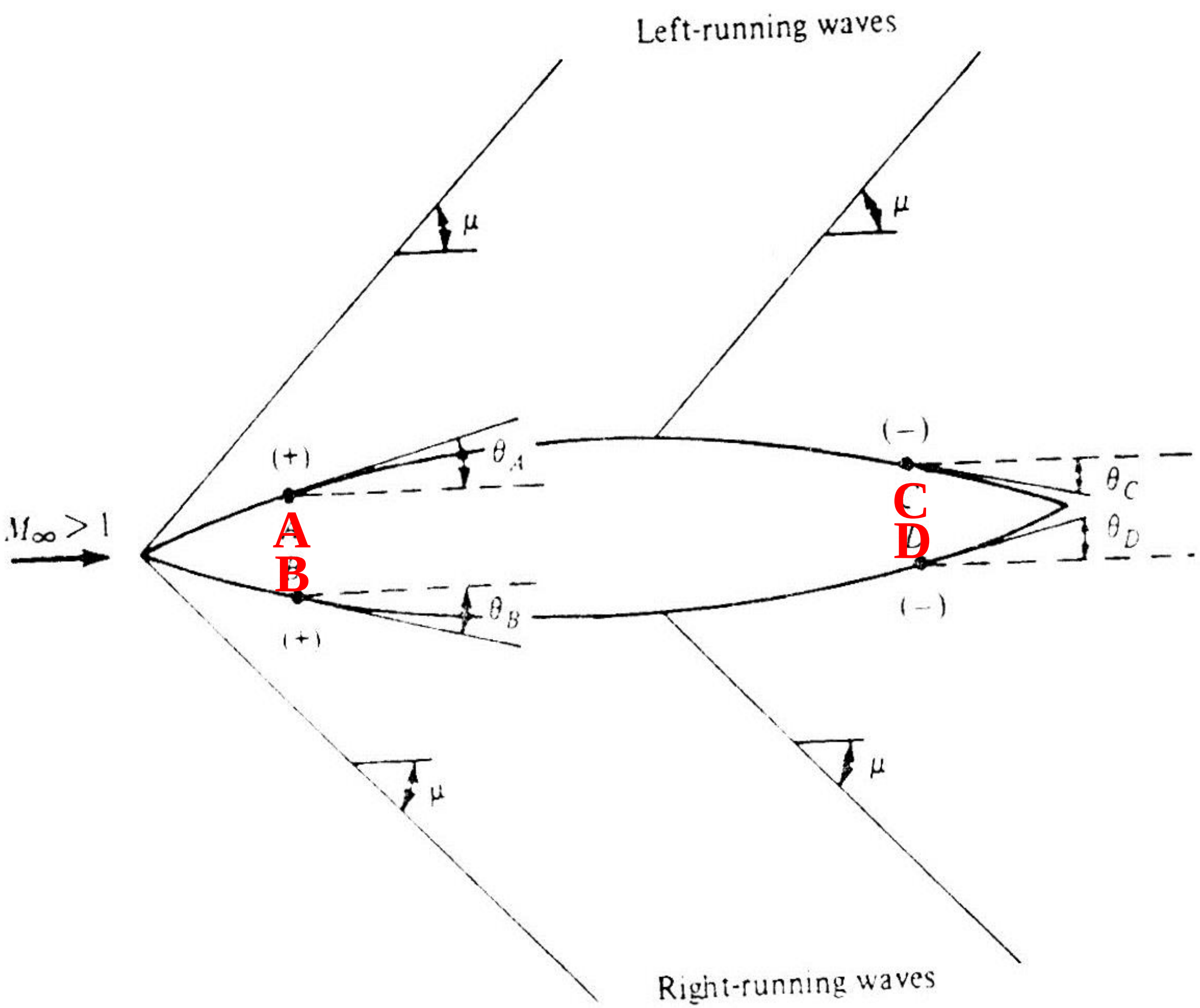
The definition of sign of θ :

θ 的正负号定义（更普适，见图12.3）

When the surface is inclined into the freestream direction, θ is taken positive,

When the surface is inclined away from the freestream direction, θ is taken negative.

θ 也可以始终规定为正



$$C_{p,A} = \frac{2\theta_A}{\sqrt{M_\infty^2 - 1}}$$

$$C_{p,B} = \frac{2\theta_B}{\sqrt{M_\infty^2 - 1}}$$

$$C_{p,C} = -\frac{2\theta_C}{\sqrt{M_\infty^2 - 1}}$$

$$C_{p,D} = -\frac{2\theta_D}{\sqrt{M_\infty^2 - 1}}$$

超声速平板流动

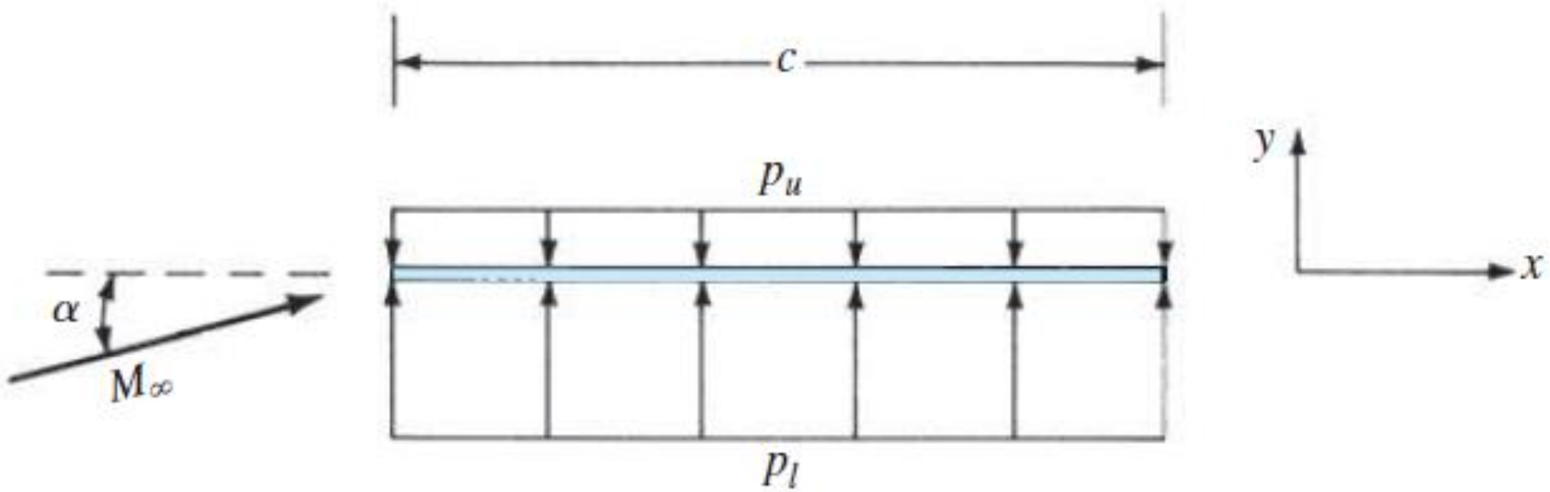


Figure 12.4 A flat plate at angle of attack in a supersonic flow.

$$C_{p,l} = \frac{2\alpha}{\sqrt{M_\infty^2 - 1}} \qquad (12.16)$$

$$C_{p,u} = -\frac{2\alpha}{\sqrt{M_\infty^2 - 1}} \qquad (12.17)$$

小扰动线化理论下超声速平板的气动力系数

$$c_n = \frac{1}{c}\int_0^c (C_{p,l} - C_{p,u})dx \qquad (12.18)$$

法向力系数 $$c_n = \frac{4\alpha}{\sqrt{M_\infty{}^2 - 1}} \frac{1}{c}\int_0^c dx \qquad (12.19)$$

$$c_a = \frac{1}{c}\int_0^c (C_{p,l} - C_{p,u})dy \qquad (12.20)$$

轴向力系数 $c_a = 0$

小扰动线化理论下超声速平板的气动力系数 （续）

升力和阻力为

$$c_l = c_n \cos\alpha - c_a \sin\alpha \qquad (1.18)$$

$$c_d = c_n \sin\alpha + c_a \cos\alpha \qquad (1.19)$$

对于小迎角:

$$c_l = c_n - c_a \alpha \qquad (12.21)$$

$$c_d = c_n \alpha + c_a \qquad (12.22)$$

小扰动线化理论下超声速平板的气动力系数 （续）

$$c_l = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}} \tag{12.23}$$

$$c_d = \frac{4\alpha^2}{\sqrt{M_\infty^2 - 1}} \tag{12.24}$$

$$c_{m,LE} = -\frac{2\alpha}{\sqrt{M_\infty^2 - 1}} = -\frac{c_l}{2}$$

结论：　绕前缘力矩系数公式说明与亚声速流动相比，超声速时翼型压力中心后移，焦点后移。

任意外形的薄翼型超声速小攻角流动

$$c_l = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}}$$

$$c_d = \frac{4}{\sqrt{M_\infty^2 - 1}}(\alpha^2 + g_c{}^2 + g_t{}^2)$$

$$c_{m,LE} = -\frac{2\alpha}{\sqrt{M_\infty^2 - 1}} + \frac{4K_1}{\sqrt{M_\infty^2 - 1}}$$

$$c_{mac} = \frac{4K_1}{\sqrt{M_\infty^2 - 1}}$$

g_c和g_t分别是翼型弯度和厚度的函数。

参考文献[25]、[26]

结论：根据超声速小扰动线化理论，翼型升力系数只与攻角有关，与厚度和弯度无关；这与低速或亚声速翼型不同。

补充：任意外形的薄翼型超声速小攻角流动的气动力系数计算公式

对于薄翼型，如果：

$$\begin{cases} y_u = f_u(x) \\ y_l = f_l(x) \end{cases}$$

注意：x 轴取来流方向，小攻角的影响也可反映在外形上。

$$C_{p,u} = \frac{2}{\sqrt{M_\infty^2 - 1}}\left(\frac{dy_u}{dx}\right)$$

$$C_{p,l} = \frac{2}{\sqrt{M_\infty^2 - 1}}\left(-\frac{dy_l}{dx}\right)$$

任意外形的薄翼型超声速小攻角流动

$$c_l = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}}$$

$$c_d = \frac{2}{\sqrt{M_\infty^2 - 1}} \frac{1}{c} \int_0^c \left[\left(\frac{dy_u}{dx}\right)^2 + \left(\frac{dy_l}{dx}\right)^2 \right] dx$$

体现厚度、弯度、迎角的影响

$$c_{mLE} = \frac{2}{\sqrt{M_\infty^2 - 1}} \frac{1}{c^2} \int_0^c \left[\left(\frac{dy_u}{dx}\right) + \left(\frac{dy_l}{dx}\right) \right] x dx$$

举例：平板翼型

$$y_u = -\alpha x$$

$$y_l = -\alpha x$$

$$C_{p,u} = \frac{2}{\sqrt{M_\infty^2 - 1}}\left(\frac{dy_u}{dx}\right) = -\frac{2\alpha}{\sqrt{M_\infty^2 - 1}}$$

$$C_{p,l} = \frac{2}{\sqrt{M_\infty^2 - 1}}\left(-\frac{dy_l}{dx}\right) = \frac{2\alpha}{\sqrt{M_\infty^2 - 1}}$$

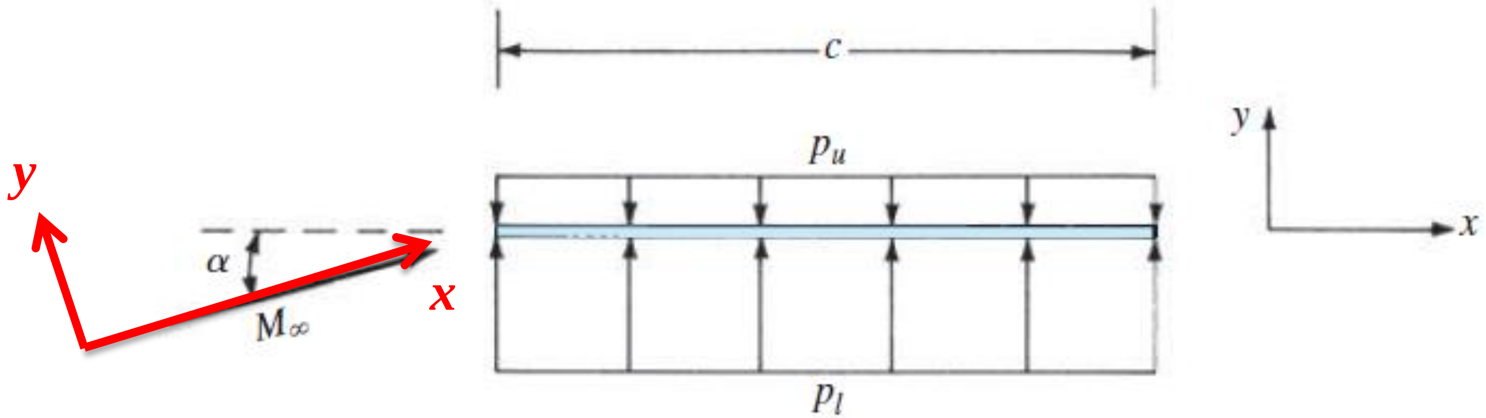


Figure 12.4 A flat plate at angle of attack in a supersonic flow.

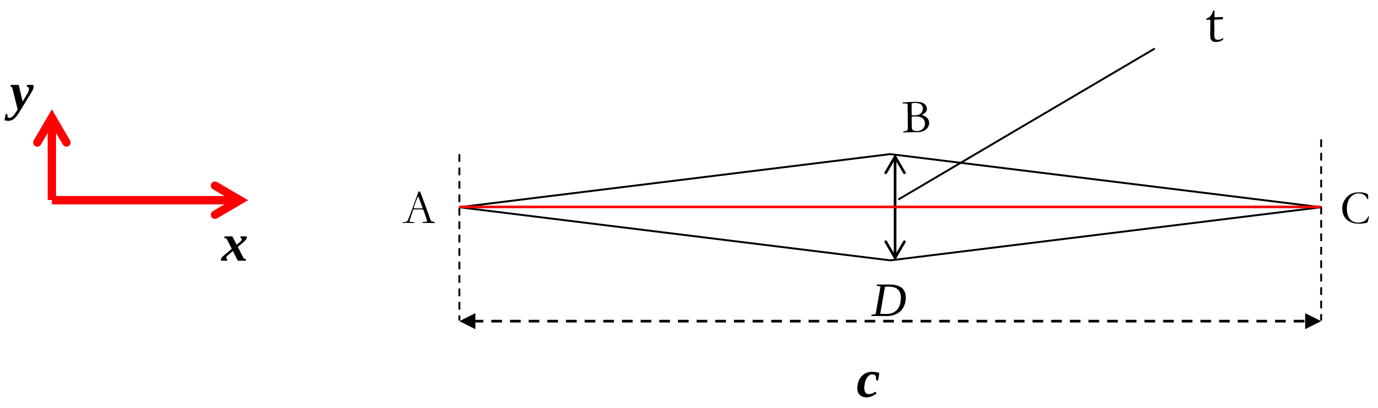
$$c_l = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}}$$

$$c_d = \frac{2}{\sqrt{M_\infty^2 - 1}} \frac{1}{c} \int_0^c \left[\left(\frac{dy_u}{dx}\right)^2 + \left(\frac{dy_l}{dx}\right)^2 \right] dx = \frac{4\alpha^2}{\sqrt{M_\infty^2 - 1}}$$

$$c_{mLE} = \frac{2}{\sqrt{M_\infty^2 - 1}} \frac{1}{c^2} \int_0^c \left[\left(\frac{dy_u}{dx}\right) + \left(\frac{dy_l}{dx}\right) \right] x dx = -\frac{2\alpha}{\sqrt{M_\infty^2 - 1}}$$

对称菱形翼型零升波阻:

书上公式:



$$C_{p,AB} = C_{p,AD} = \frac{2}{\sqrt{M_\infty^2 - 1}}\left(\frac{t/2}{c/2}\right) = \frac{2}{\sqrt{M_\infty^2 - 1}}\left(\frac{t}{c}\right)$$

$$C_{p,BC} = C_{p,DC} = \frac{2}{\sqrt{M_\infty^2 - 1}}\left(\frac{-t/2}{c - c/2}\right) = \frac{2}{\sqrt{M_\infty^2 - 1}}\left(-\frac{t}{c}\right)$$

对称菱形翼型零升波阻:

$$c_d = \frac{1}{c}\int_0^c \left(C_{p,u}\frac{dy_u}{dx} - C_{p.l}\frac{dy_l}{dx}\right)dx \qquad (1.16)$$

$$c_d = \frac{1}{c}\int_0^{c/2}[C_{p,AB}(\frac{t}{c}) - C_{p,AD}(-\frac{t}{c})dx]$$
$$+\int_{c/2}^{c}[C_{p,BC}(-\frac{t}{c}) - C_{p,DC}(\frac{t}{c})dx]$$

$$\boxed{c_d = \frac{4}{\sqrt{M_\infty^2 - 1}}\left(\frac{t}{c}\right)^2}$$

小扰动假设下，超声速翼型的相对厚度要尽量小以降低波阻

对称菱形翼型零升波阻（续）

补充公式：

$$c_d=\frac{2}{\sqrt{M_\infty^2-1}}\frac{1}{c}\int_0^c\left[(\frac{dy_u}{dx})^2+(\frac{dy_l}{dx})^2\right]dx$$

$$=\frac{2}{\sqrt{M_\infty^2-1}}\frac{1}{c}\left\{\int_0^{\frac{c}{2}}\left[(\frac{t}{c})^2+(-\frac{t}{c})^2\right]dx+\int_{\frac{c}{2}}^{c}\left[(-\frac{t}{c})^2+(\frac{t}{c})^2\right]dx\right\}$$

$$=\frac{4}{\sqrt{M_\infty^2-1}}(\frac{t}{c})^2$$

$$\boxed{c_d=\frac{4}{\sqrt{M_\infty^2-1}}(\frac{t}{c})^2}$$

可以证明：在小扰动线化假设下，对称菱形翼型是最小零升波阻翼型（相同厚度）

（参考文献26的作业题）

多选题 2分

设置

下列说法正确的是：

A 超声速线化小扰动假设下，平板翼型的压力中心在1/2弦长处,焦点也在1/2弦长处

B 超声速线化小扰动假设下，翼型的升力系数只和翼型的迎角有关，和厚度弯度无关

C 在小扰动线化假设下，对称菱形翼型是最小零升波阻翼型

D 超声速线化小扰动假设下，翼型的阻力系数和力矩系数只和翼型的迎角有关，和厚度弯度无关

提交

补充：线性化小扰动速度势方程的进一步讨论

可叠加性：方程可叠加、边界条件可叠加

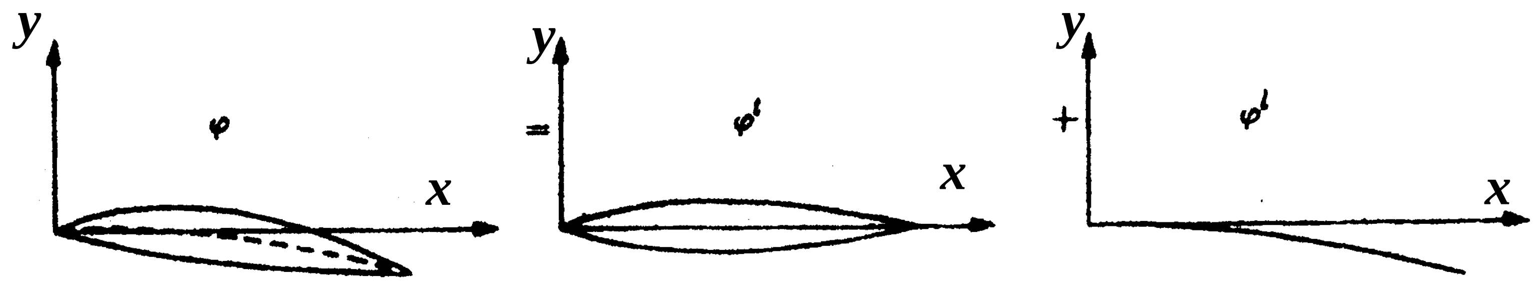
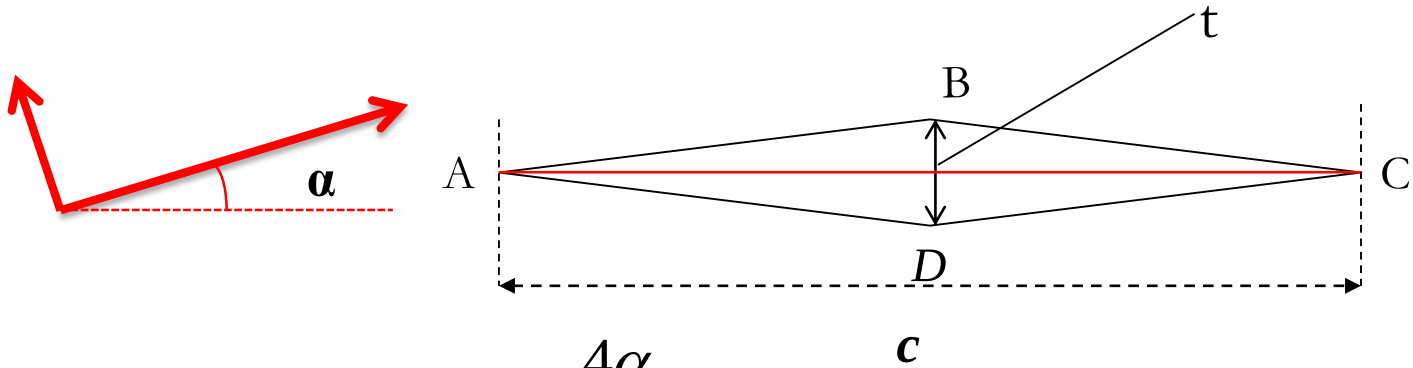


Fig. 5–3. Separation of thickness and lift problems.



$$c_l = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}}$$

$$c_d = \frac{4\alpha^2}{\sqrt{M_\infty^2 - 1}} + \frac{4}{\sqrt{M_\infty^2 - 1}}\left(\frac{t}{c}\right)^2$$

Example 12.1 $M_\infty = 3$，$\alpha = 5^\circ$，计算平板翼型的升力系数和阻力系数，并与例9.11比较。

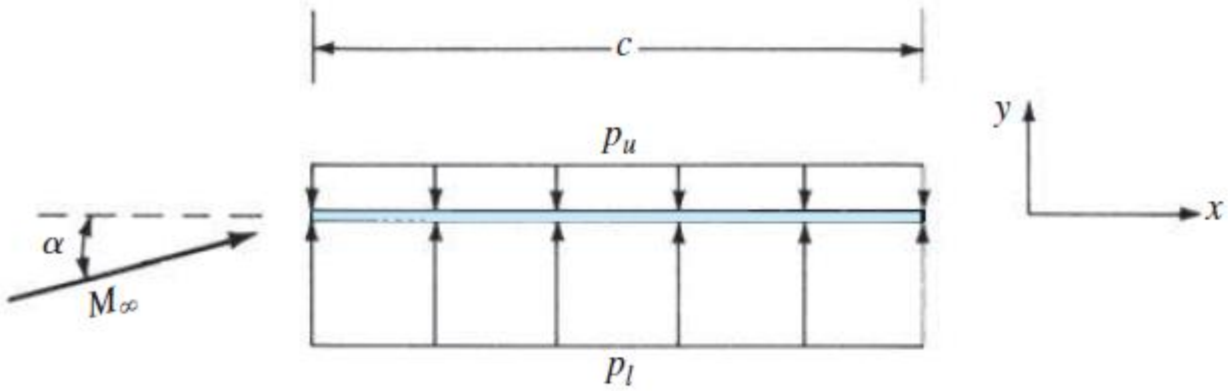


Figure 12.4 A flat plate at angle of attack in a supersonic flow.

$$c_l = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}} = \frac{4 \times 5 \times \pi / 180}{\sqrt{3^2 - 1}} = 0.123$$

$$c_d = \frac{4\alpha^2}{\sqrt{M_\infty^2 - 1}} = 0.123 \times 5 \times \pi / 180 = 0.011$$

例9.11

$$c_l = 0.125$$

$$c_d = 0.011$$

Example 12.2 $M_\infty = 2$，H =11km，$S = 18.21m^2$，W = 9400kgf. 假设升力完全由机翼产生，且机翼的升力系数可由二维翼型的升力系数近似，计算飞机的飞行迎角。

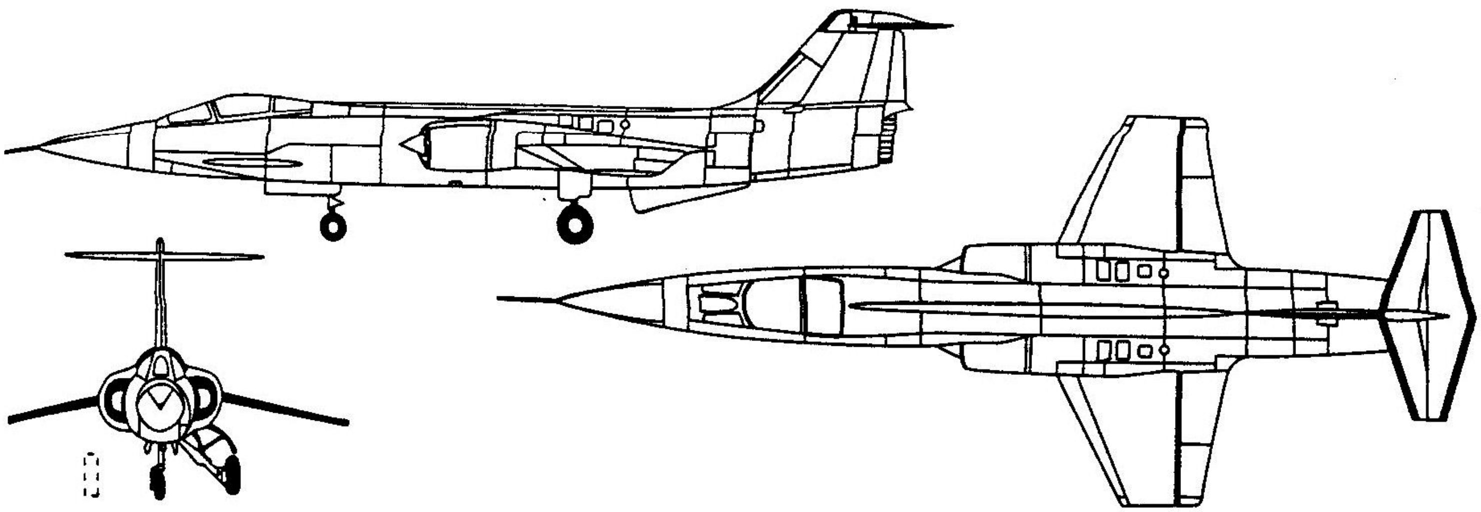


Figure 12.5 Lockheed F-104.

11km高空，查附表D，可知

$$T_\infty = 216.78K \qquad \rho_\infty = 0.3648\,kg/m^3$$

$$a_\infty = \sqrt{\gamma R T_\infty} = \sqrt{(1.4)(287)(216.78)} = 295(m/s)$$

$$V_\infty = M_\infty a_\infty = (2)295 = 590(m/s)$$

$$C_L = \frac{W}{q_\infty S} = \frac{9400\times 9.8}{0.5\times 0.3648\times(2\times 295)^2\times 18.21} = 0.08$$

$$C_L = c_l = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}}$$

$$\alpha = \frac{0.08\times\sqrt{2^2-1}}{4} = 0.035\,rad = 1.98^o$$

DESIGN BOX

举例：F16 – 遵循跨声速面积律和超声速面积律

百度图片（F-16)

F-16沿机身轴线的的横截面积分布

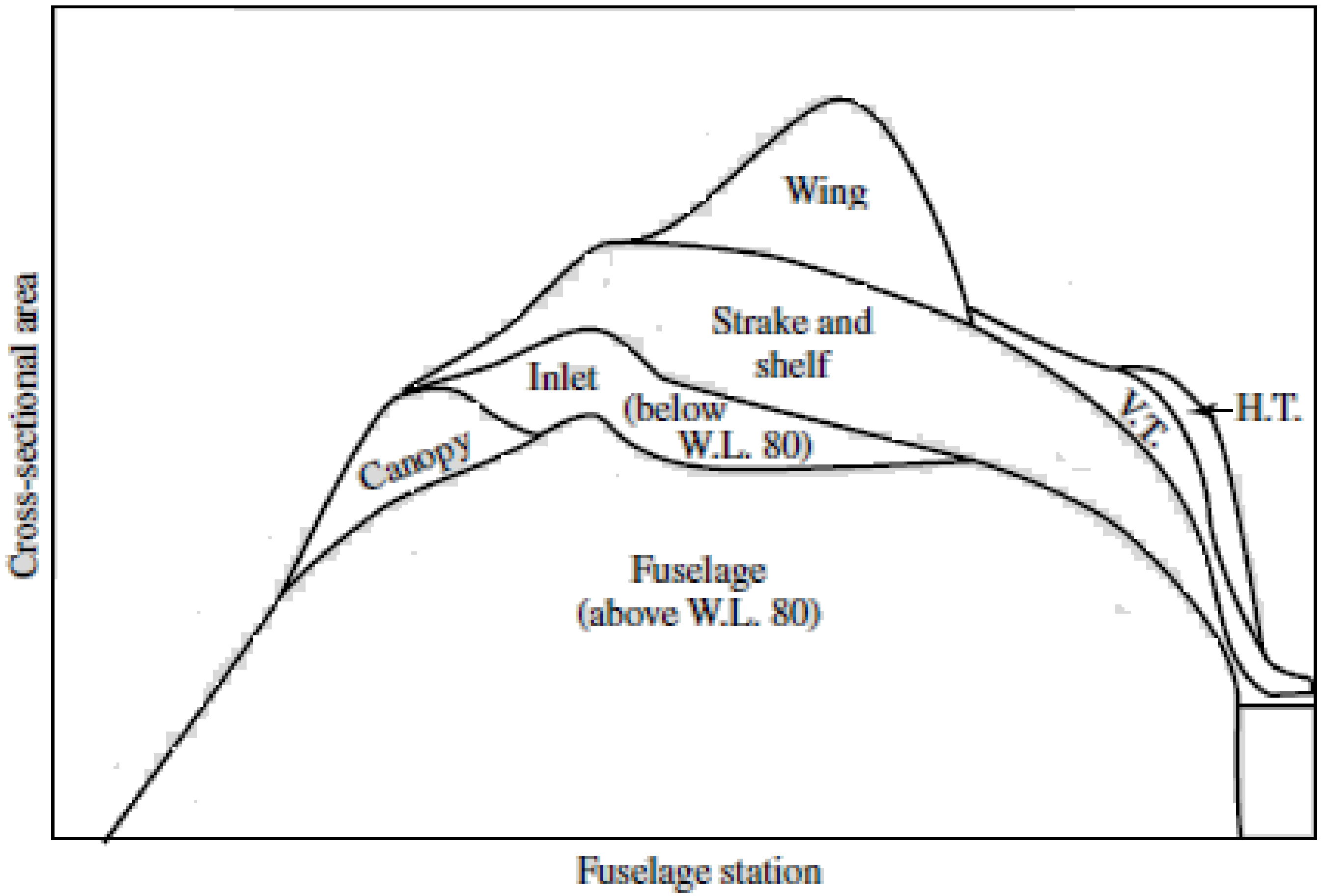


Figure 12.6 Transonic area ruling for the F-16. Variation of normal cross-sectional area as a function of location along the fuselage axis.

Wing-机翼

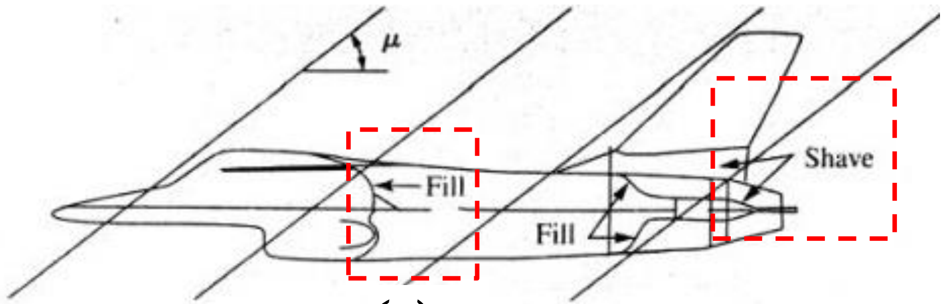
Inlet-进气道

Canopy-座舱罩

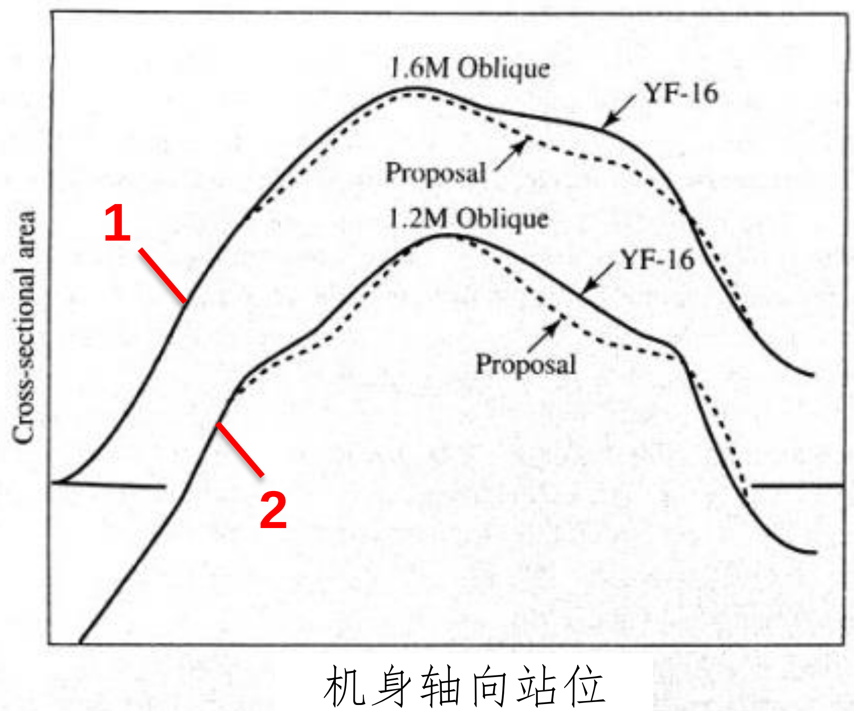
Strake and shelf -挂板和托（弹）架

H.T.（Horizontal Tail）-水平尾翼

V.T.(Vertical Tail)-垂直尾翼

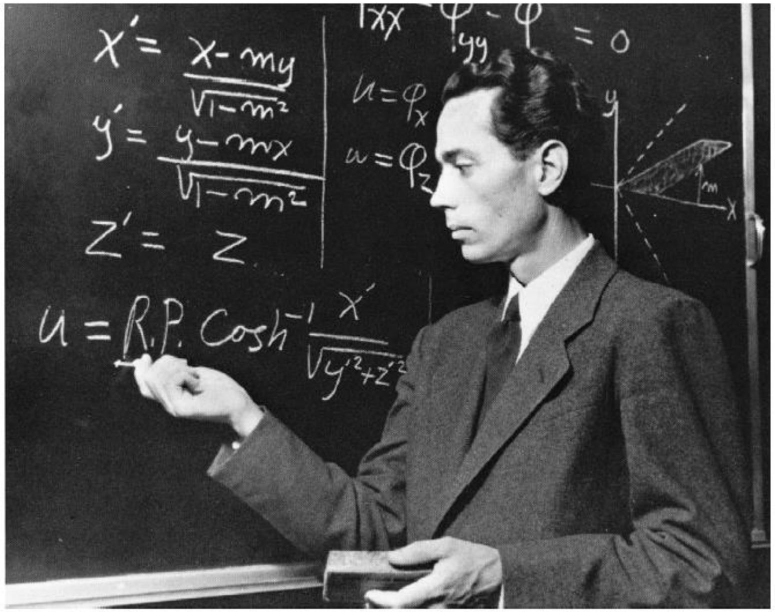


(a)



(b)

图12.7 F16的超声速面积律（M=1.6和M=1.2时斜切平面的面积沿机身轴线坐标的变化，实线为实际面积分布，虚线为早期设计的面积分布）。



Robert T. Jones
（1910-1999）
A leading aerodynamicist at the NACA Langley Memorial Laboratory

超声速面积律最初是由著名的NACA和NASA空气动力学家R.T. Jones提出的，他在这方面的工作首次发表在NACA报告中，题目为“Theory of Wing-Body Drag at Supersonic Speeds”，报告号为NACA TR 1284，1953年7月8日（印刷于NACA 1956年年度报告）。感兴趣的同学可以课后参阅这份报告，从中获取应用超声速面积律的更详细知识。。

12.4 Viscous Flow： Supersonic Airfoil Drag 超声速翼型粘性阻力

不可压层流平板摩阻

$$C_f = \frac{1.328}{\sqrt{Re_c}}$$

可压缩层流平板摩阻

$$C_f = \frac{F(M_e, Pr, T_w/T_e)}{\sqrt{Re_c}}$$

不可压湍流流平板摩阻

$$C_f = \frac{0.074}{Re_c^{1/5}}$$

可压缩湍流平板摩阻

$$C_f = \frac{G(M_e, Pr, T_w/T_e)}{Re_c^{1/5}}$$

Pr为反映粘性耗散能量和热传导能量比值的相似参数，被称为普朗特数

$$Pr = \frac{\mu c_p}{k}$$

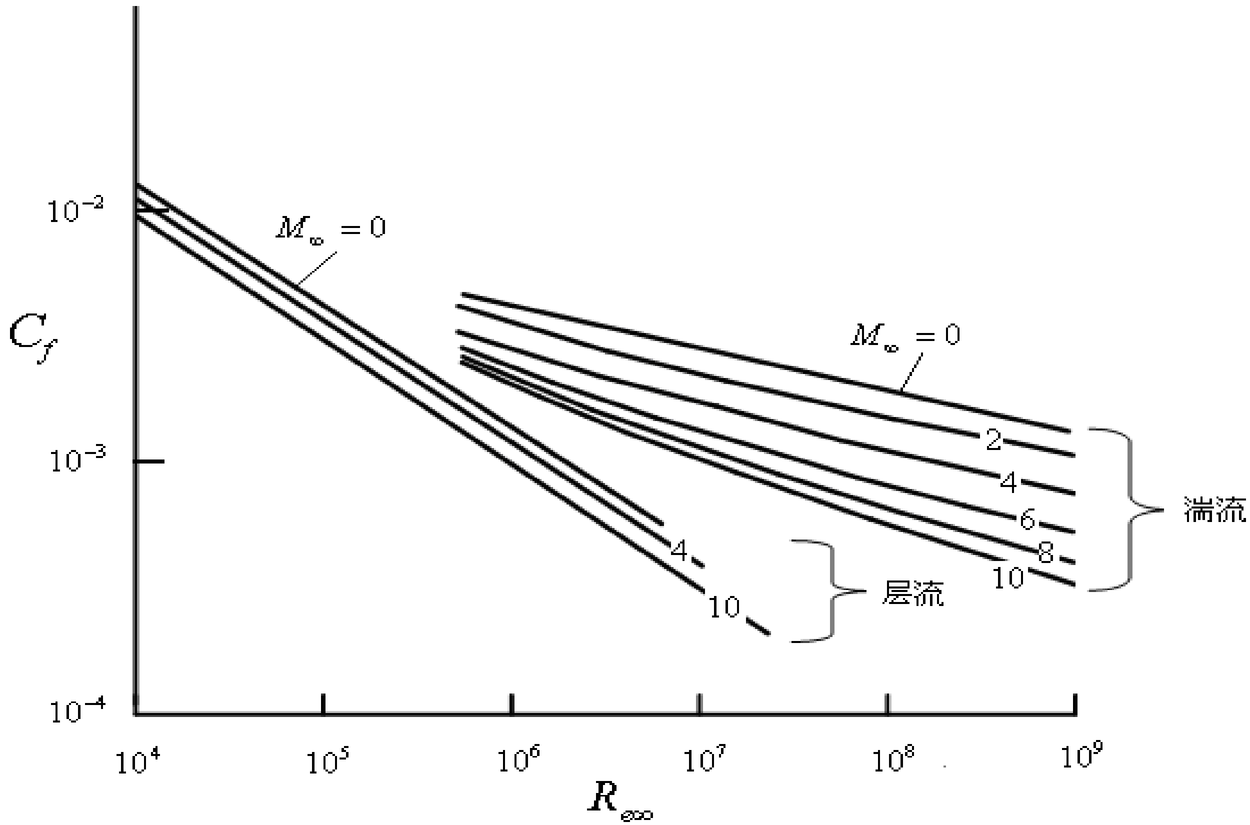


图19.1　平板摩擦阻力系数随雷诺数、马赫数的变化曲线（绝热壁，Pr=0.75）

The most important phenomena to observe in this figure is that C_f decreases as M_∞ increases.

Example 12.3 F-104

M_∞=2 H=11km $\alpha = 0.035 rad = 1.98^o$ **， c=2.2m**

假设翼型表面为湍流，计算

（a）翼型的摩擦阻力系数，

（b）翼型的波阻系数。并进行两种阻力的比较。

解:

(a) 摩阻计算

$$a_\infty = \sqrt{\gamma R T_\infty} = \sqrt{(1.4)(287)(216.78)} = 295 (m/s)$$

$$V_\infty = M_\infty a_\infty = (2)295 = 590 (m/s)$$

$$\frac{\mu_\infty}{\mu_0} = \left(\frac{T_\infty}{T_0}\right)^{3/2} \frac{T_0 + 110}{T_\infty + 110}$$

$$\mu_\infty = 1.4226 \times 10^{-5} \ \ \boldsymbol{kg/(m \cdot s)}$$

$$Re = \frac{\rho_\infty V_\infty c}{\mu_\infty} = 3.33 \times 10^{7}$$

查图19.1 得： $C_f = 2.15 \times 10^{-3}$

考虑上下表面： $C_f = 2(2.15 \times 10^{-3}) = 4.3 \times 10^{-3}$

(b) 波阻系数计算：

$$c_d = \frac{4\alpha^2}{\sqrt{M_\infty^2 - 1}} = \frac{4(0.035)^2}{\sqrt{2^2 - 1}} = 2.83\times10^{-3}$$

总阻力系数计算：

$$c_{d,total} = 2.83\times10^{-3} + 4.3\times10^{-3} = 7.13\times10^{-3}$$

升阻比对比：

无粘	有粘
$\frac{L}{D} = 28.3$	$\frac{L}{D} = 11.2$

12.5 SUMMARY

在线化超声速流中，信息沿马赫线传播。这些马赫线的来流夹角为基于M∞的马赫角，$\mu=\sin^{-1}(1/M_\infty)$ 。物面引起的扰动信息沿这些直的、平行的马赫线向下游传播。因此，在定常超声速流中，扰动不能向上游传播。

基于线化理论，当物面切线方向与来流成一小角***θ***时，物面的压强系数表达式为：

$$C_p=\frac{2\theta}{\sqrt{M_\infty^2-1}} \tag{12.15}$$

如果物面向来流倾斜，C_p 为正；物面向远离来流倾斜，C_p 负。

基于线化超声速理论，小迎角下平板翼型的升力系数和波阻系数分别是时：

$$c_l = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}} \qquad （12.23)$$

$$c_d = \frac{4\alpha^2}{\sqrt{M_\infty^2 - 1}} \qquad (12.24)$$

升力系数计算公式**(12.23)**适用于任意形状薄翼型。而波阻系数还依赖于翼型的中弧线形状和厚度。

Problems：12.1-12.4

Lecture #19 ended！

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