Review of 3rd course/第三课复习

Chapter 7 Compressible Flow: Some Preliminary Aspects
可压缩流动基础

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第七章路线图

- 热力学复习
  - 完全气体
  - 内能和焓
  - 热力学第一定律
  - 熵及热力学第二定律
  - 等熵关系式
- 压缩性定义
- 无粘可压缩流动的控制方程
- 总参数定义
- 有激波可压缩流动的定性了解

7.3 Definition of Compressibility (压缩性定义)

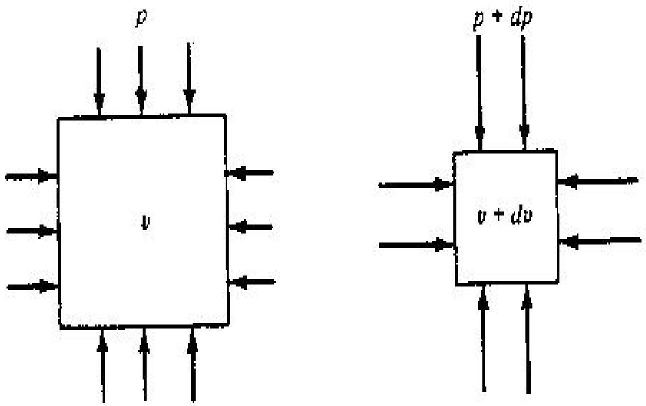


FIGURE 7.3
Definition of compressibility.

$$\tau = -\frac{1}{v}\frac{dv}{dp}$$

$$v = \frac{1}{\rho} \qquad \tau = \frac{1}{\rho}\frac{d\rho}{dp}$$

Physically, the compressibility is a fractional change in volume of the fluid element per unit change in pressure.

(从物理上讲，压缩性就是每单位压强变化引起的流体微元单位体积内的体积变化)

或：压缩性就是每单位压强变化引起的密度的相对变化

等温压缩性和等熵压缩性

等温压缩性

$$\tau_T = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T \tag{7.34}$$

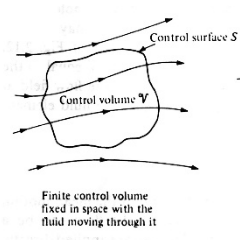
等熵压缩性

$$\tau_s = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_s \tag{7.35}$$

7.4 Governing Equations for inviscid Compressible Flow (无粘、可压缩流控制方程)

1. Continuity equation (连续方程）　　质量守恒

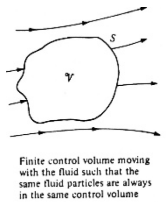
①



$$\boxed{\frac{\partial}{\partial t}\iiint_{v}\rho\, dv+\oiint_{S}\rho\vec{V}\cdot \mathrm{dS}=0}$$

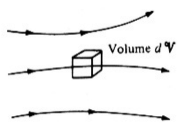
(7. 39) 即 (2. 48)

②



$$\frac{D}{Dt}\iiint_{V}\rho dV=0$$

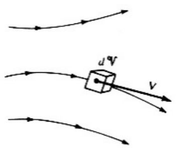
③



$$\frac{\partial\rho}{\partial t}+\nabla\cdot\left(\rho\vec{V}\right)=0$$

(7. 40) 即 (2. 52)

④

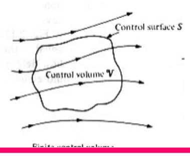


$$\frac{D\rho}{Dt}+\rho\nabla\cdot\vec{V}=0$$

2. Momentum（动量方程）

牛顿第二定律：力与动量变化率的关系 $\vec{F}=\frac{d}{dt}\left(m\vec{V}\right)$

积分形式



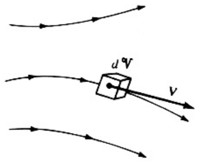
$$\boxed{\frac{\partial}{\partial t}\oiiint_{v}\rho\vec{V}dv+\oiint_{s}\left(\rho\vec{V}\cdot\mathrm{dS}\right)\vec{V}=-\oiint_{s}p\,\mathrm{dS}+\oiiint_{v}\rho\vec{f}\,dv}$$

（7. 41）

即（2. 64）$\vec{F}_{viscous}=0$

其中 $\vec{f}$ 为体积力，如重力和电磁力

微分形式



$$\vec{F}=m\vec{a}$$

$$\boxed{\rho\frac{D\vec{V}}{Dt}=-\nabla p+\rho\vec{f}}\;\rightarrow\;
\begin{cases}
\rho\dfrac{Du}{Dt}=-\dfrac{\partial p}{\partial x}+\rho f_x\\[2mm]
\rho\dfrac{Dv}{Dt}=-\dfrac{\partial p}{\partial y}+\rho f_y\\[2mm]
\rho\dfrac{Dw}{Dt}=-\dfrac{\partial p}{\partial z}+\rho f_z
\end{cases}$$

$$\xrightarrow{\frac{D}{Dt}\equiv\frac{\partial}{\partial t}+(\vec{V}\bullet\nabla)}$$

$$\rho\frac{\partial u}{\partial t}+\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}+\rho w\frac{\partial u}{\partial z}=-\frac{\partial p}{\partial x}+\rho f_x$$

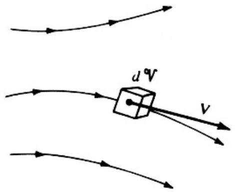
$$\rho\frac{\partial v}{\partial t}+\rho u\frac{\partial v}{\partial x}+\rho v\frac{\partial v}{\partial y}+\rho w\frac{\partial v}{\partial z}=-\frac{\partial p}{\partial y}+\rho f_y$$

$$\rho\frac{\partial w}{\partial t}+\rho u\frac{\partial w}{\partial x}+\rho v\frac{\partial w}{\partial y}+\rho w\frac{\partial w}{\partial z}=-\frac{\partial p}{\partial z}+\rho f_z$$

3. Energy equation (能量方程)

能量守恒

微分形式



$$\boxed{\frac{\partial}{\partial t}\oiiint_{v}\rho\left(e+\frac{V^2}{2}\right)dv+\oiint_{s}\rho\left(e+\frac{V^2}{2}\right)\vec{V}\cdot d\mathrm{S} =\oiiint_{v}\dot{q}\,\rho\,dv-\oiint_{s}p\vec{V}\cdot d\mathrm{S}+\oiiint_{v}\rho\left(\vec{f}\cdot\vec{V}\right)dv}$$

(7. 43)
(2. 95)

$$\dot{Q}_{viscous}=0,\quad \dot{W}_{viscous}=0$$

$$\rho\frac{D\left(e+V^2/2\right)}{Dt}=\rho\dot{q}-\nabla\cdot p\vec{V}+\rho\left(\vec{f}\cdot\vec{V}\right)$$

(7. 44)
(2. 96)

$$\dot{q}=0\qquad \vec{f}=0\qquad \boxed{\rho\frac{D\left(e+V^2/2\right)}{Dt}=-\nabla\cdot p\vec{V}}$$

4. Equation of state for a perfect gas/完全气体状态方程

$$\boxed{p = \rho R T}$$

5. Internal energy for a calorically perfect gas/量热完全气体内能方程:

$$\boxed{e = c_v T}$$

We have now *5 equations* for *5 unknowns* / 五个方程，五个未知数 / 上述方程联立可求解出所有流动变量！

7.5 Definition of Total (Stagnation) Conditions (总参数的定义）(滞止参数的定义：*流体力学中的综合测评）*

- **总温的定义：**

假想流体微团被*绝热地*减速为静止所对应的温度，定义此时流体微团对应的温度为总温。用 T_0 表示。

- **总焓的定义：**

$$h_0 = c_p T_0$$

- **总焓和总温的物理意义：**

单位质量流体具有的总能量，包括内能、压力势能和宏观动能；总温大小也就标征了总能量大小。

由能量方程计算总焓和总温

$$\rho\frac{D\left(e+V^2/2\right)}{Dt}=\rho\dot{q}-\nabla\cdot p\vec{V}+\rho\left(\vec{f}\cdot\vec{V}\right) \tag{7.44}$$

$$\Rightarrow\quad \rho\frac{D\left(e+V^2/2\right)}{Dt}=-\nabla\cdot p\vec{V} \tag{7.45}$$

前提条件：无粘、绝热、忽略体积力

用 h 表示

$$\Rightarrow\quad \rho\frac{D\left(h+V^2/2\right)}{Dt}=\frac{\partial p}{\partial t}$$

定常流

$$\Rightarrow\quad \rho\frac{D\left(h+V^2/2\right)}{Dt}=0$$

沿流线成立的能量方程

$$\Rightarrow\quad h+\frac{V^2}{2}=const$$

总焓的定义

$$\Rightarrow\quad \boxed{h+\frac{V^2}{2}=h_0}$$

$$T_0=h_0/c_p$$

由总焓表示的能量方程

有了总焓的定义，能量方程可以用总焓来表示：对于定常、绝热、无粘流动，方程（7.52）可以写成：

$$\rho\frac{Dh_0}{Dt}=0 \qquad \text{or} \qquad \boxed{h_0=const} \qquad (7.55)$$

i.e. the total enthalpy is constant along a streamline.

即总焓沿流线为常数。

由总温表示的能量方程

对于量热完全气体，$h_0 = c_p T_0$。因此，上面的结果也表明了对于定常、无粘、绝热的量热完全气体，总温保持不变，即

$$\boxed{T_0 = const} \qquad (7.56)$$

容易混淆的概念

- 对于一个*非绝热*流动，比如具有热传导的粘性附面层内的流动，　以下能量方程(7. 51), (7. 52), (7. 53)不成立：

$$\boxed{\rho\frac{D\left(h+V^2/2\right)}{Dt}=\frac{\partial p}{\partial t}}\quad \boxed{\rho\frac{D\left(h+V^2/2\right)}{Dt}=0}\quad \boxed{h+\frac{V^2}{2}=const}$$

$$\boxed{h_0=const}\qquad \boxed{T_0=const}$$

- 但是在非绝热流动中的每一点，(7. 54)式成立！

$$\boxed{h+\frac{V^2}{2}=h_0} \qquad (7.54)$$

容易混淆的概念（续）

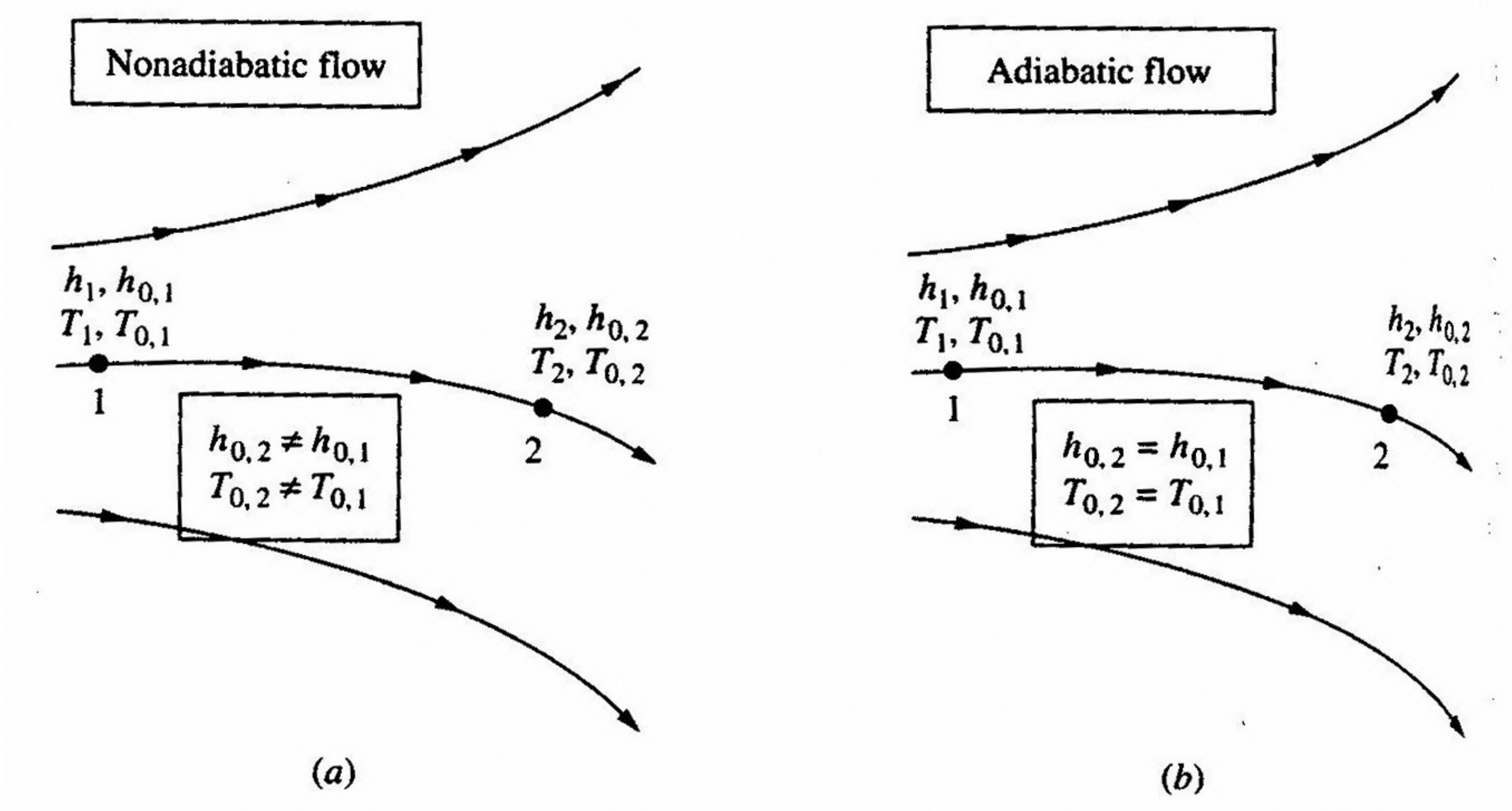


图7.4　非绝热流与绝热流的比较

投票(匿名) 最多可选1项

设置

请对上一次课内容的掌握情况进行投票

A 完全掌握了这部分知识内容

B 掌握了大部分

C 掌握了一小部分

D 完全不懂

提交

Review of Lecture #3
The End！

Lecture #4(1)

Chapter 7 Compressible Flow: Some Preliminary Aspects
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2019年10月16日 Wednesday

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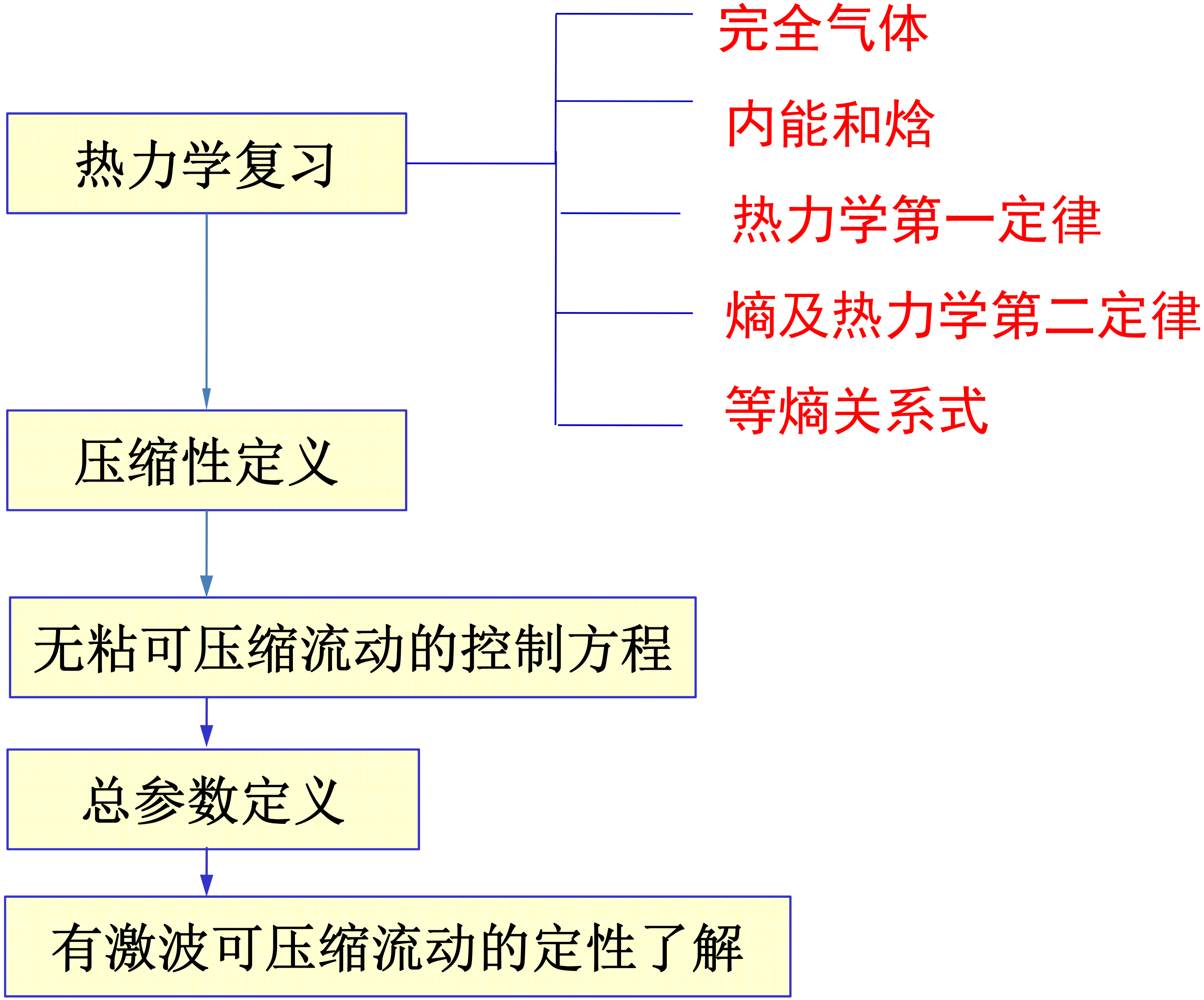
第七章路线图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总参数定义
有激波可压缩流动的定性了解

总压&总密度

Total Pressure & Total Density

回到本节的开头，我们考虑流体微团通过一个给定点，对应的当地压强、温度、密度、马赫数、速度分别为

$$p, T, \rho, M, \vec{V}$$

Once again, imagine that you grab hold of the fluid element and slow it down to zero velocity, but this time, let us slow it down both *adiabatically* and *reversibly*. That is, let us slow the fluid element down to zero velocity *isentropically.*

总压与总密度定义
Definition of Total Pressure & Total Density

定义： 当流体微元被*等熵地*减速至静止时对应的压强和密度被定义为其总压 p_0 和总密度 ρ_0 。

Definition: When the fluid element is brought to rest *isentropically*, the resulting pressure and density are defined as the total pressure p_0 and total density ρ_0

As before, keep in mind that we do not have to actually bring the flow to rest in real life in order to talk about total pressure and total density; rather, they are *defined* quantities that would exist at a point in a flow *if* (in our imagination) the fluid element passing through that point were brought to rest *isentropically*. Therefore, at a given point in a flow, where the static pressure and static density are p and ρ, respectively, we can also assign a value of total pressure p_0 , and total density ρ_0 defined as above.

注意：

- 总压和总密度只是定义量，是假想将流体速度等熵地降为零来定义的，而并不是真的这样做。
- 对于流场中任意一点，我们都可以定义总压和总密度；
- 总压和总密度的概念适用于任意等熵流或非等熵流。

因为等熵过程是绝热的，因此对于等熵流，总温不变。

Since an *isentropic* process is also *adiabatic*, total temperature remains unchanged.

- **总压的物理意义：**

流体微元作有用功的能力

物理解释

思考：流体微元等熵地减速到零速度时流体微元的总能量由什么组成？

$$h_0 = c_v T + pv + \frac{V^2}{2}(\text{等熵减为速度零}) = c_v T_0 + p_0 / \rho_0$$

$\frac{V^2}{2}\downarrow$ 在能量转换中的贡献？

总压定义的关注点：

$\dfrac{V^2}{2}$等熵变为零使内能（$e = c_v T_0$）增加：　　**不关注**

$\dfrac{V^2}{2}$等熵变为零使pv增加，即做有用功能力增加　　**关注！！**

总结：总压是流体微元做有用功能力的度量。

问题：为什么等熵流沿流线总压和总密度不变？

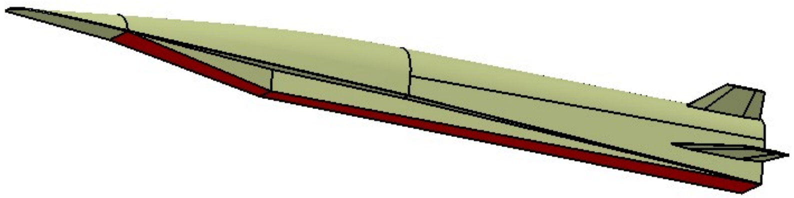
Q：The total pressure and total density are constants along a streamline in an isentropic flow. Why?

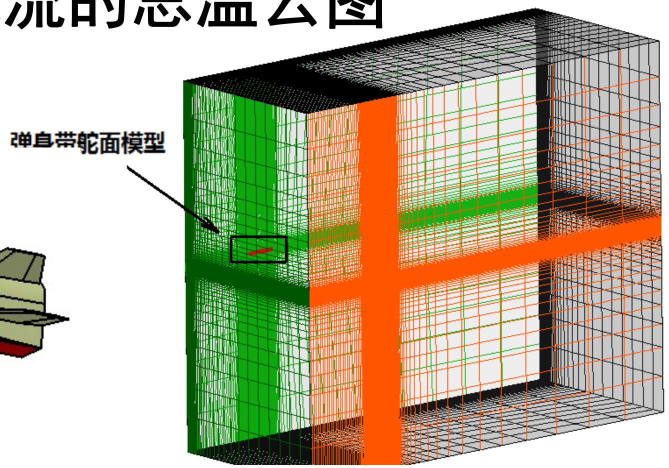
$$\boxed{\frac{p_2}{p_1}=\left(\frac{\rho_2}{\rho_1}\right)^{\gamma}=\left(\frac{T_2}{T_1}\right)^{\gamma/(\gamma-1)}} \Rightarrow \frac{p}{T^{\gamma/(\gamma-1)}}=\text{contant}$$

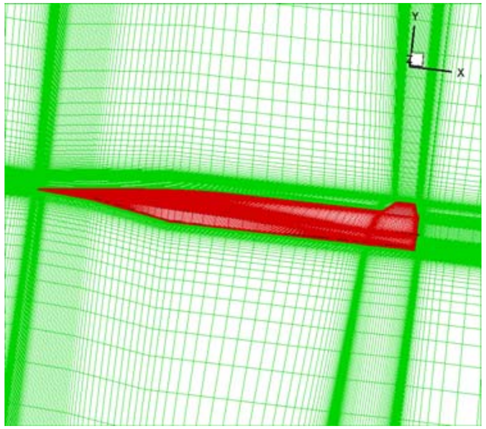
$$\Rightarrow \frac{p_0}{T_0^{\gamma/(\gamma-1)}}=\text{contant} \quad \xrightarrow{T_0=\text{contant}} \quad p_0=\text{contant}$$

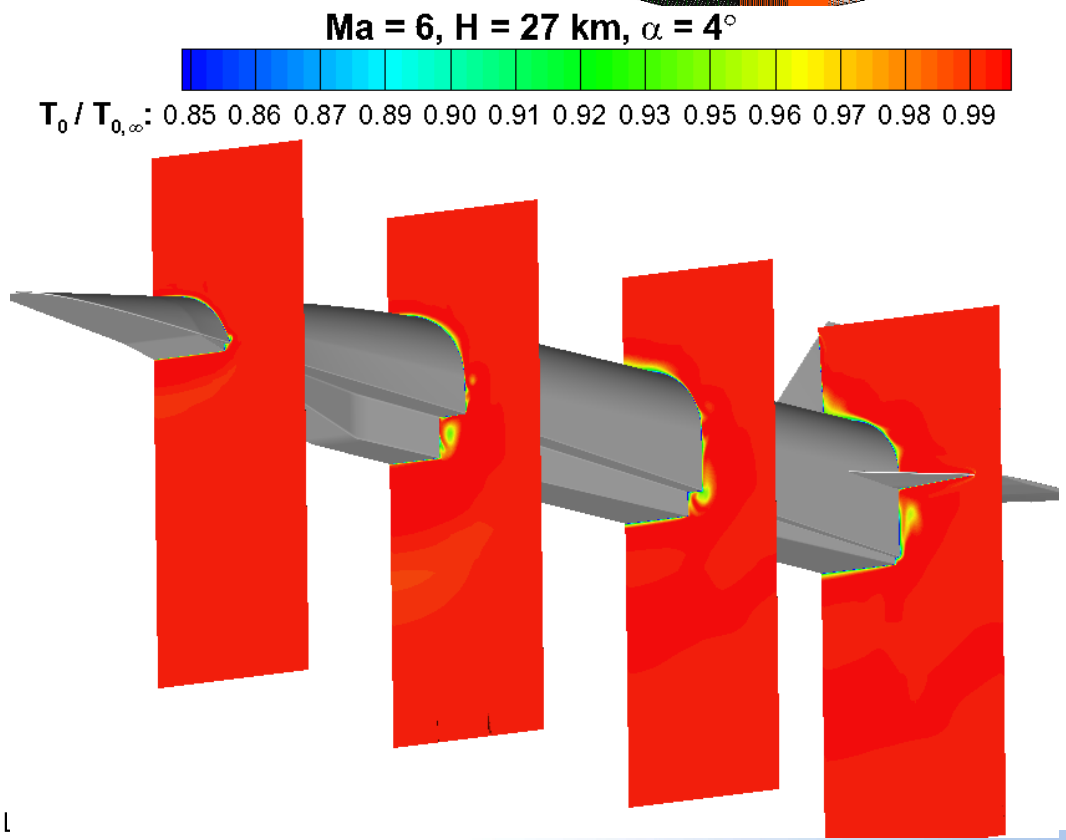
$$\Rightarrow \rho_0=\text{contant}$$

举例：高超声速乘波体绕流的总温云图

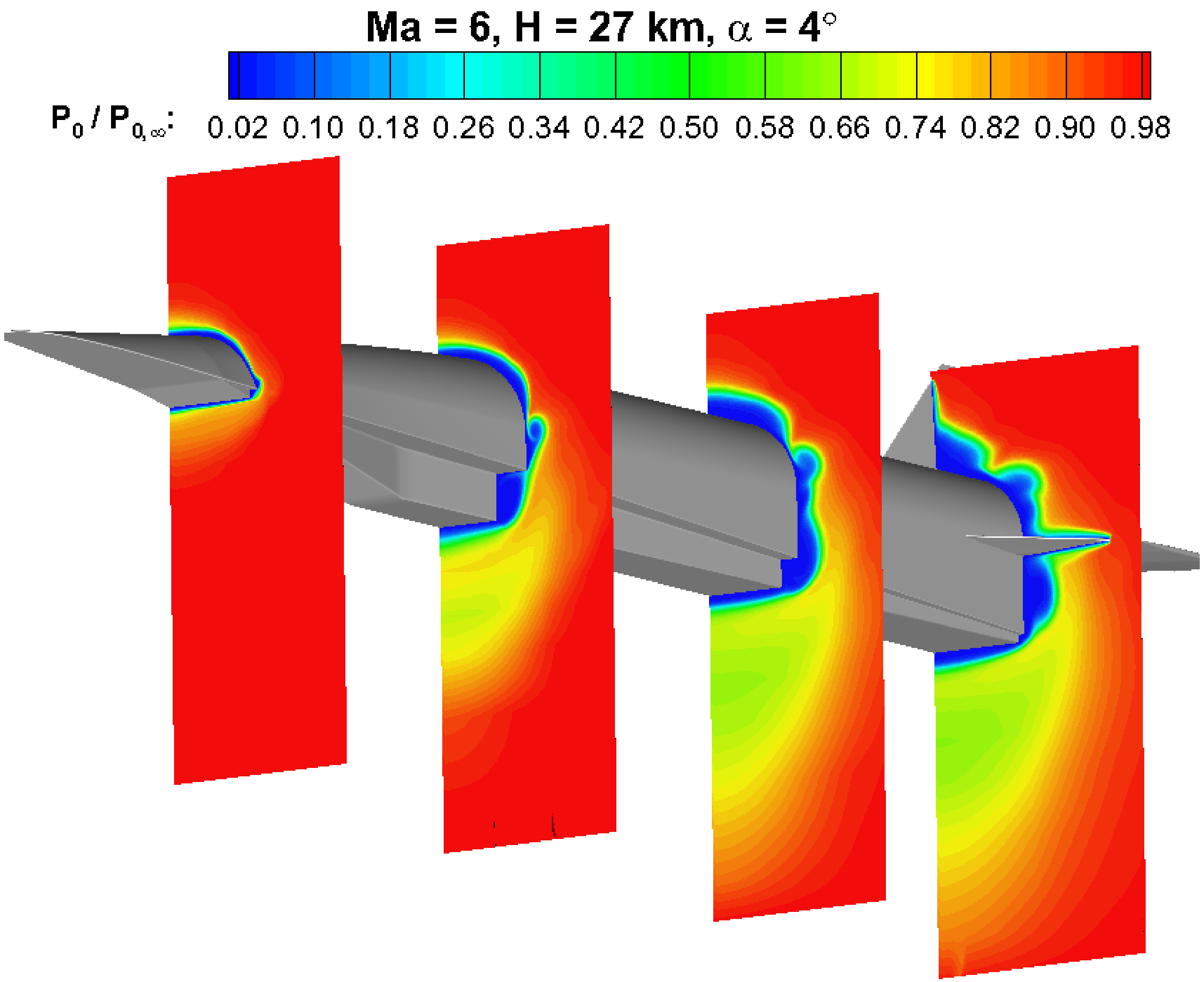








举例：高超声速类乘波体绕流的总压云图



有关临界温度的定义——总温定义的推论（corollary）

T*的定义:

假想把具有当地静温T的流体微元*绝热地*由亚声速加速到声速,或绝热地由超声速减速到声速，流体微元对应的温度，被定义为T*。T*同样是对于一给定点的定义量。

Summary

- Total temperature T_0 and total enthalpy h_0 are defined as the properties that would exist if the flow were slowed to *zero velocity adiabatically*.
- Total pressure p_0 and total density ρ_0 are defined as the properties that would exist if the flow were slowed to *zero velocity isentropically*.
- If the general flow field is adiabatic, h_0 is constant throughout the flow.
- If the general flow field is *isentropic*, p_0 and ρ_0 are constant throughout the flow.

单选题 1分

设置

流体微元总压的正确定义是

A 流体微元速度为零时具有的压强

B 流体微元被绝热地减速至静止时对应的压强被定义为流体微元的总压

C 流体微元被等熵地减速至静止时对应的压强被定义为流体微元的总压

提交

Example 7.6 气流中一点处的压强、温度和速度分别为1atm, 320K,1000m/s。计算这一点的总温和总压。

$$h+\frac{V^2}{2}=h_0$$

$$\because \quad h=c_pT, \quad c_p=\frac{\gamma R}{\gamma-1}$$

$$\therefore \quad c_pT+\frac{V^2}{2}=c_pT_0$$

$$\therefore \quad T_0=T+\frac{V^2}{2c_p}=T+(\frac{\gamma-1}{2\gamma R})V^2$$

$$\therefore \quad T_0=320+\frac{0.4\times1000^2}{2\times1.4\times287}=817.8(K)$$

$$\frac{p_0}{p}=\left(\frac{T_0}{T}\right)^{\gamma/(\gamma-1)}$$

$$p_0=p(\frac{T_0}{T})^{\frac{\gamma}{\gamma-1}}=1(\frac{817.8}{320})^{\frac{1.4}{0.4}}$$

$$=26.7(atm)$$

Example 7.7 飞机10000ft海平面高度飞行。机头皮托管测得压强为2220lb/ft²。飞机以大于300mph的高亚声速飞行速度飞行。由3.1节讨论的那样，流动需要按可压缩流处理。计算飞机的速度。

解：皮托管测得的是总压，但总压不能按3.4节中不可压伯努利方程计算总压。

10000ft=3048m，$T_\infty = 268.67K$ ， $p_\infty = 70121 N/m^2$

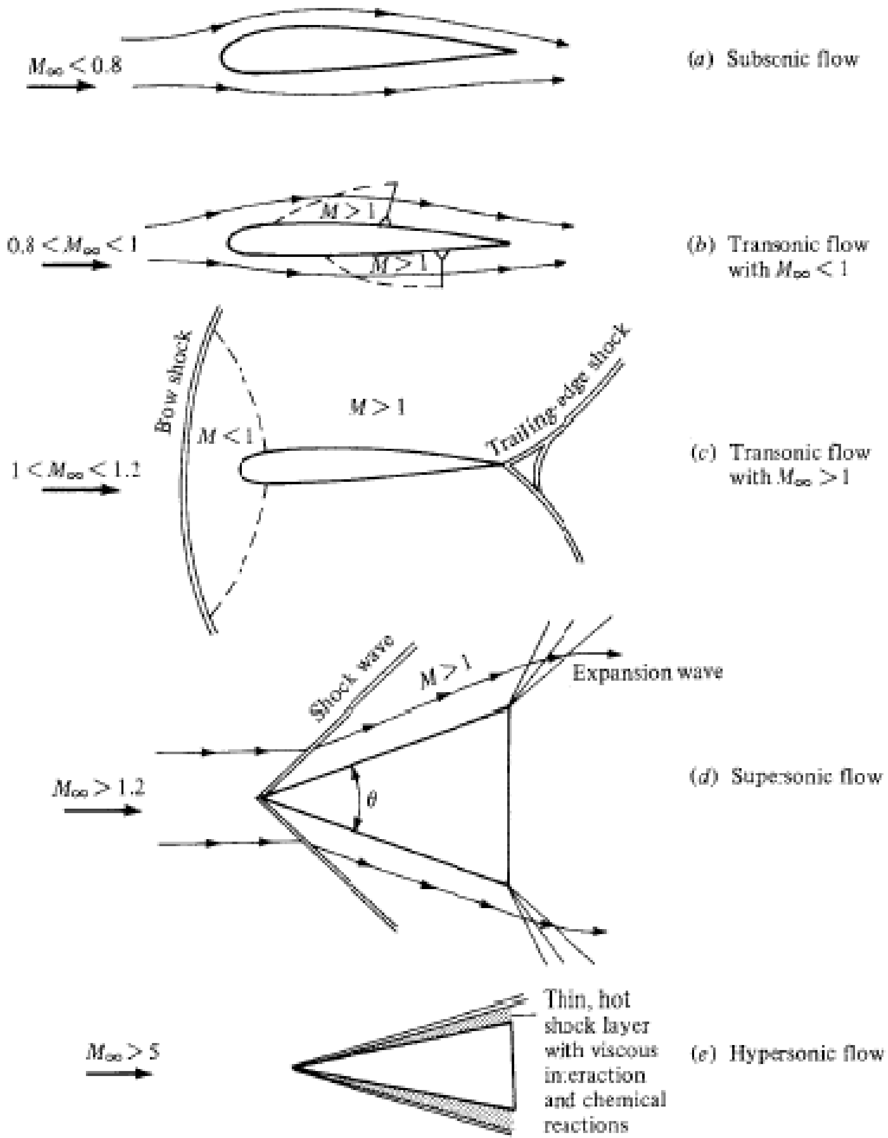
已知：$p0 = 2220 lb/ft^2 = 2220*47.88 = 106293.6 (N/m^2)$

$$T_0 = T_\infty \left(\frac{p_0}{p_\infty}\right)^{\frac{\gamma-1}{\gamma}} = 303.11K$$

$$\frac{1}{2}V_\infty^{\ 2} = c_p(T_0 - T_\infty) \qquad V_\infty = \sqrt{2c_p(T_0 - T_\infty)} = 263.04(m/s)$$

7.6 Some Aspects of Supersonic Flow: Shock Waves

超音速流的一些特征：激波



66页图1.44

An essential ingredient of a supersonic flow is the calculation of the shape and strength of shock waves. This is the main thrust of chaps. 8 and 9.

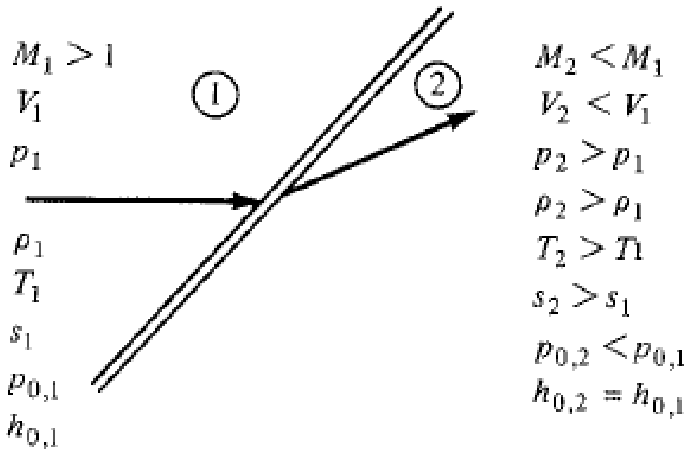
超声速流动研究的一个重要内容就是计算激波的形状和强度。这是第8章和第9章的主题。

什么是激波：对激波的初步认识

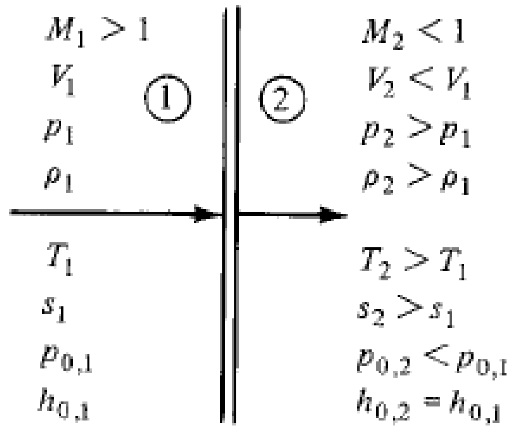
A shock wave is an extremely thin region, typically on the order of 10^{-5} cm, across which the flow properties can change drastically.

激波是一个极其薄的区域，厚度大约只有10^{-5}cm的量级，通过激波流动特性发生剧烈变化。

激波是一个大振幅压缩波。



(a) Oblique shock wave



(b) Normal shock wave

Figure 7.5 Qualitative pictures of flow through oblique and normal shock waves.

图7.5

7.7 Summary(小结)

1、热力学关系式:

- 状态方程:

$$p = \rho RT \qquad (7.1)$$

- 对于量热完全气体:

$$e = c_v T \qquad (7.6a)$$

$$h = c_p T \qquad (7.6b)$$

$$c_p = \frac{\gamma R}{\gamma - 1} \qquad (7.9)$$

$$c_v = \frac{R}{\gamma - 1} \qquad (7.10)$$

- 热力学第一定律的各种表达形式：

$$\delta q + \delta w = de \qquad (7.11)$$

- 可逆过程：

$$\begin{cases} Tds = de + pdv & (7.18) \\ Tds = dh - vdp & (7.20) \end{cases}$$

- 熵的定义：

$$ds = \frac{\delta q_{rev}}{T} \qquad (7.13)$$

$$\text{及} \quad ds = \frac{\delta q}{T} + ds_{irrev} \qquad (7.14)$$

- 热力学第二定律：

$$ds \geq \frac{\delta q}{T} \qquad (7.16)$$

- 或对于绝热过程：

$$ds \geq 0 \qquad (7.17)$$

- 量热完全气体的熵增计算公式：

$$s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1} \qquad (7.25)$$

$$s_2 - s_1 = c_v \ln\frac{T_2}{T_1} + R\ln\frac{v_2}{v_1} \qquad (7.26)$$

- 等熵流动的等熵关系式：

$$\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} = \left(\frac{T_2}{T_1}\right)^{\gamma/(\gamma-1)} \qquad (7.32)$$

2、压缩性

- 压缩性的一般定义：

$$\tau = -\frac{1}{v}\frac{dv}{dp} \qquad (7.33)$$

- 对于等温过程：

$$\tau_T = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T \qquad (7.34)$$

- 对于等熵过程：

$$\tau_s = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_s \qquad (7.35)$$

3、无粘、可压缩流的控制方程：

- 连续方程：

$$\frac{\partial}{\partial t}\iiint_v \rho\, dv + \oiint_S \rho \vec{V}\cdot d\vec{S} = 0 \qquad (7.39)$$

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{V}) = 0 \qquad (7.40)$$

- 动量方程：

$$\frac{\partial}{\partial t}\iiint_v \rho\vec{V} dv + \oiint_s (\rho\vec{V}\cdot d\vec{S})\vec{V} = -\oiint_s p\, d\vec{S} + \iiint_v \rho\vec{f}\, dv \qquad (7.41)$$

$$\rho\frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \rho f_x \qquad (7.41a)$$

$$\rho\frac{Dv}{Dt} = -\frac{\partial p}{\partial y} + \rho f_y \qquad (7.41b)$$

$$\rho\frac{Dw}{Dt} = -\frac{\partial p}{\partial z} + \rho f_z \qquad (7.41c)$$

- **能量方程：**

$$\frac{\partial}{\partial t}\oiiint_{v}\rho\left(e+\frac{V^2}{2}\right)dv+\oiint_{s}\rho\left(e+\frac{V^2}{2}\right)\vec{V}\cdot ds$$
$$=\oiiint_{v}\dot{q}\,\rho\,dv-\oiint_{s}p\vec{V}\cdot dS+\oiiint_{v}\rho\left(f\cdot\vec{V}\right)dv \qquad (7.43)$$

$$\rho\frac{D\left(e+V^2/2\right)}{Dt}=\rho\dot{q}-\nabla\cdot p\vec{V}+\rho\left(\vec{f}\cdot\vec{V}\right) \qquad (7.44)$$

- **对于定常、绝热、无粘流，（7.44)和(7.43)可以写成：**

$$h+\frac{V^2}{2}=const \quad 或 \quad h_0=const \qquad (7.53),(7.55)$$

- **完全气体状态方程：** $p=\rho RT$ (7.1)
- **量热完全气体内能：** $e=c_v T$ (7.6a)

4、总温、总焓、总压、总密度的定义及概念：

- “总温” T_0 和“总焓” h_0 定义为把流体微元（在我们的想象中）*绝热地* 减速为静止时流体微元所对应的温度和焓值。
- 类似地，“总压” p_0 和“总密度” ρ_0 定义为把流体微元（在我们的想象中）*等熵地* 减速为静止时流体微元所对应的压强和密度。
- 在均匀自由来流的绝热流场中，总焓 h_0 在全流场中为常数，相反，在非绝热流场中，h_0 随流场点的不同而不同。
- 类似地，在等熵流场中，总压和总密度在整个流场中为常数，相反，在非等熵流场中，总压和总密度随流场点的不同而不同。

5、激波

- 激波为超音速流中很薄的一层
- 通过激波
  - 压强、密度、温度和熵增加；
  - 马赫数、流动速度、总压降低；
  - 总焓、总温不变。

Problem 7.11,7.12，7.13

下面课讲正激波

→ 第八章

菱形翼型的激波观察纹影试验

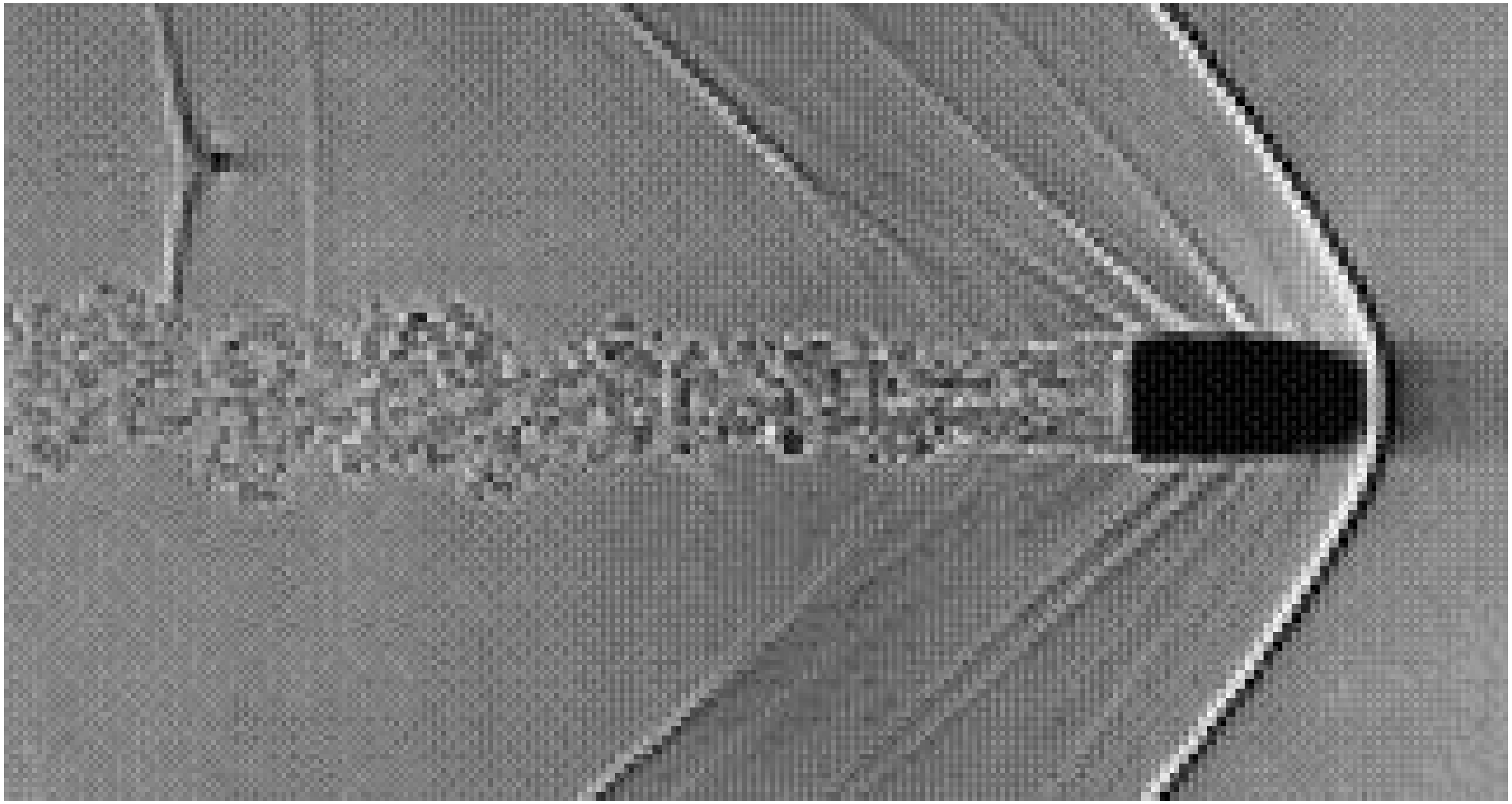
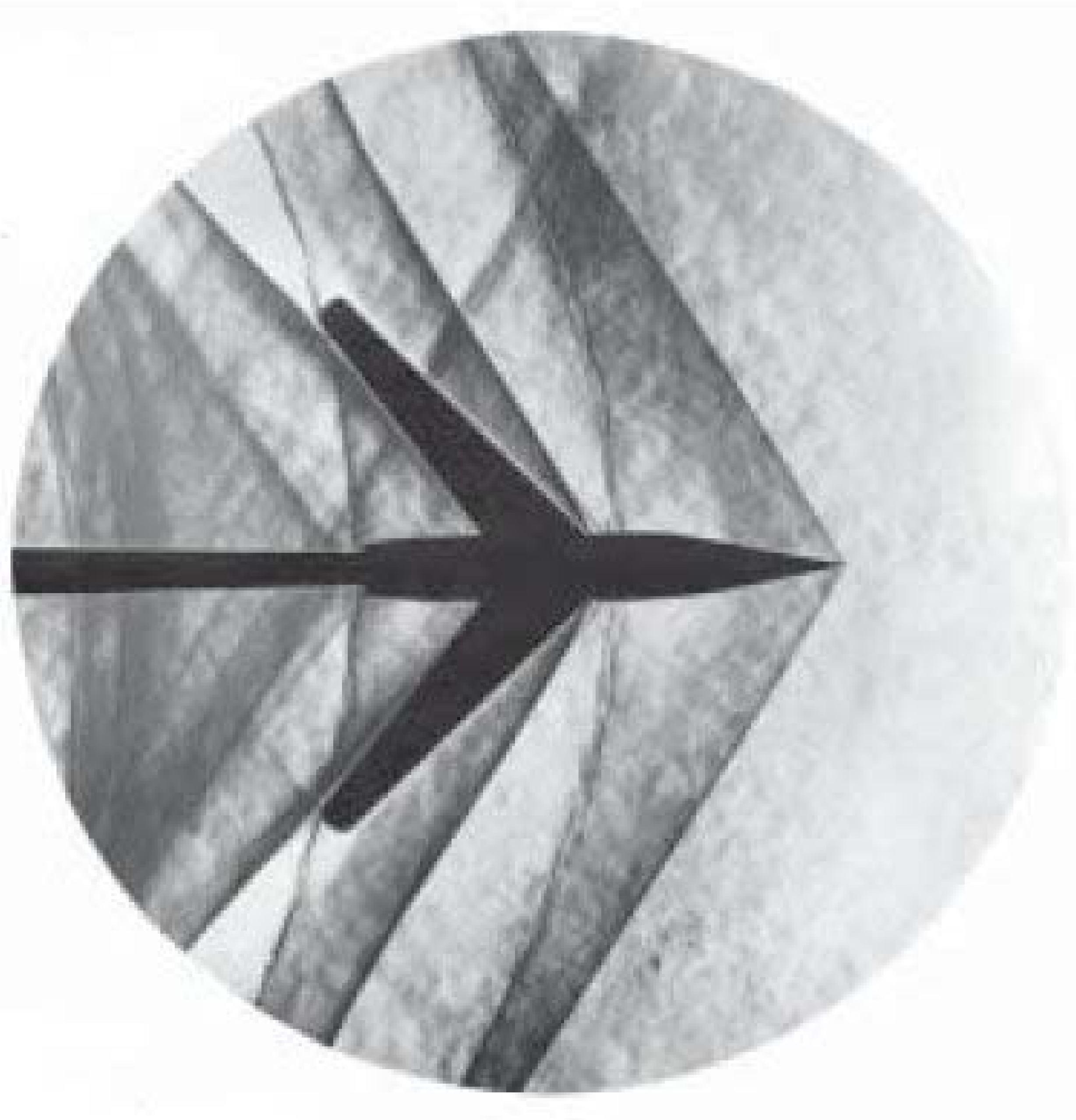
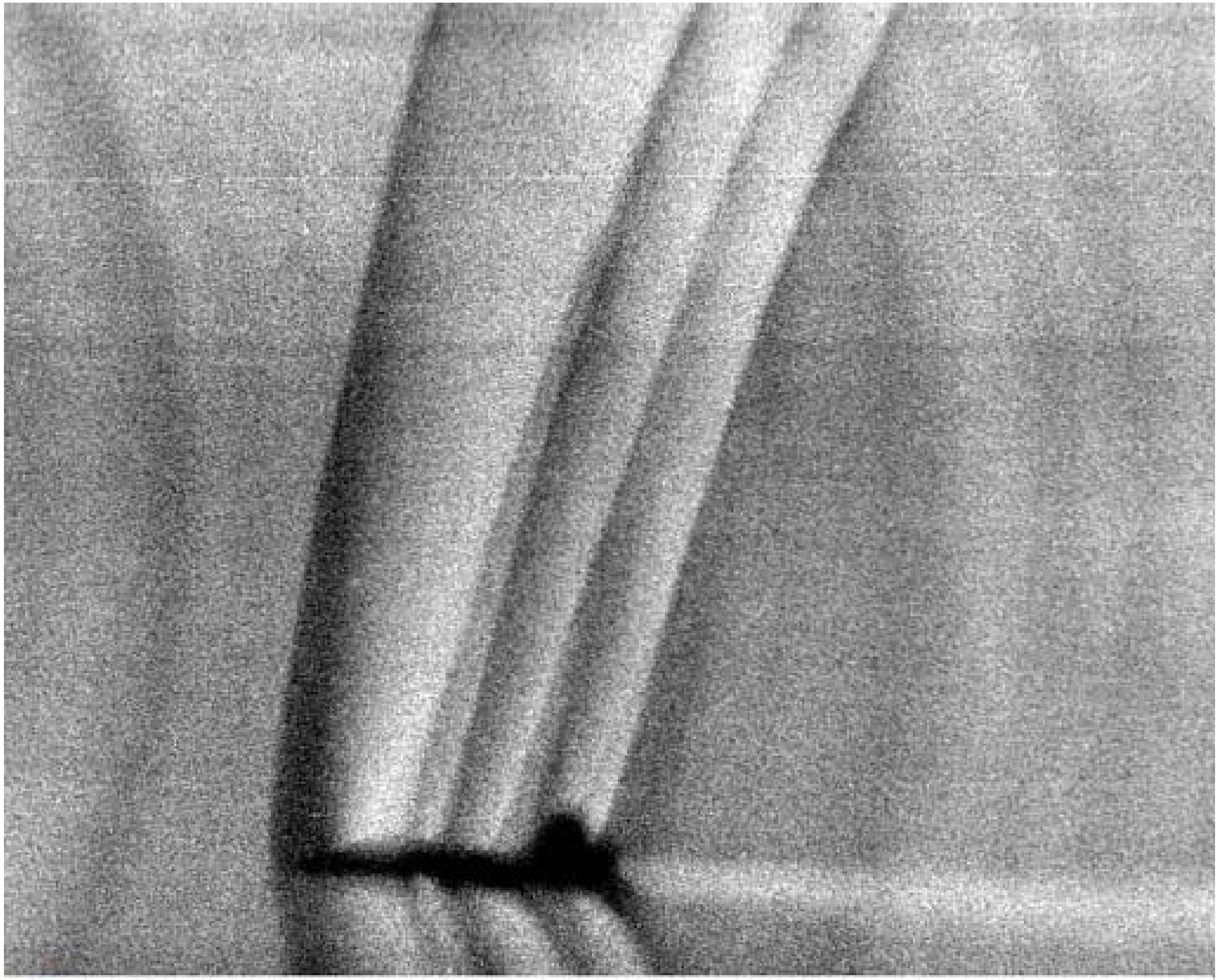


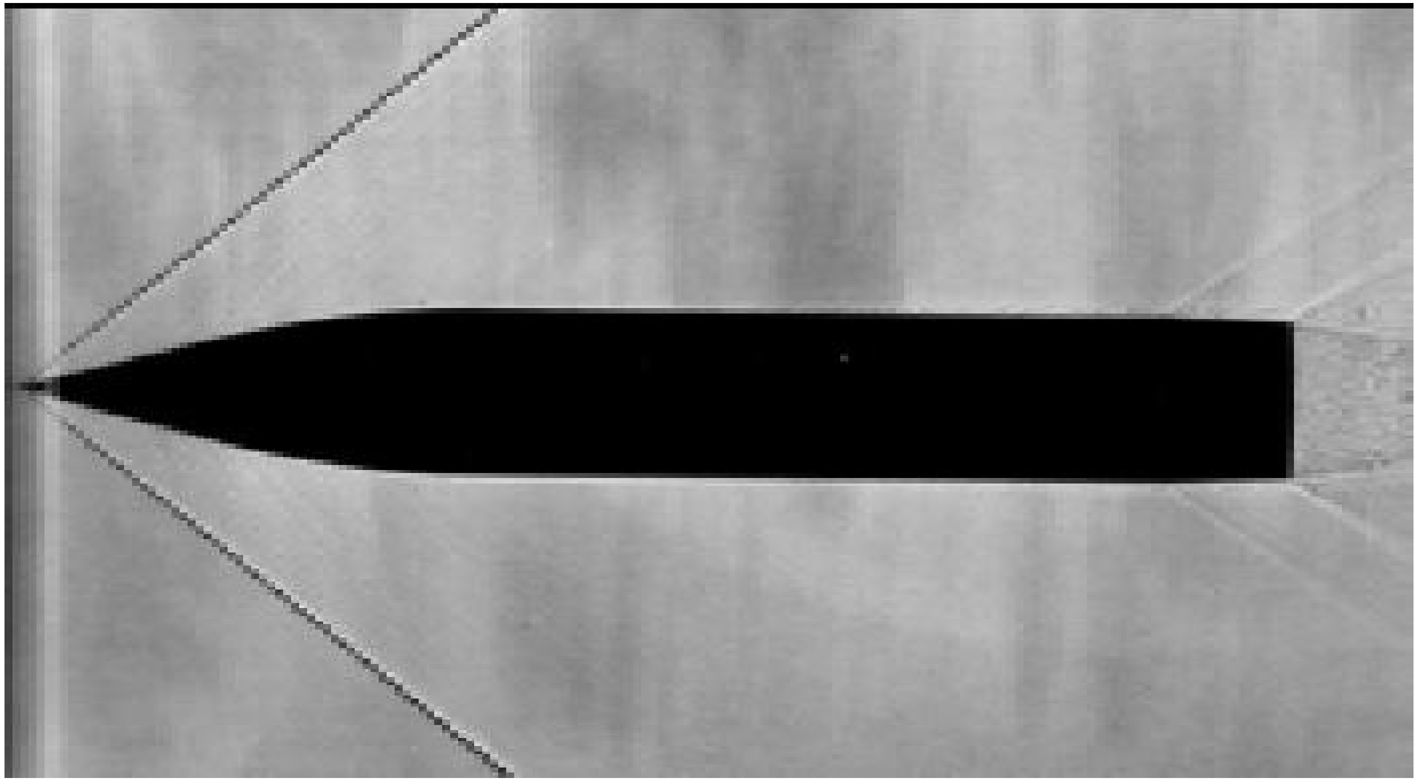
Fig. 14 Schlieren photograph taken during Hyper-X stage separation testing in AEDC Tunnel B with the drop jaw at 60 deg, $x_{sep} = -4$, $A_{sep} = 0$ deg.





NASA Dryden Flight Research Center Photo Collection
http://www.dfrc.nasa.gov/gallery/photo/index.html
NASA Photo: EC94-42528-1 Date: December 13, 1993 Photo by: Dr. Leonard Weinstein

Schlieren photograph of T-38 shock waves at Mach 1.1, 13,000 feet



返回

Problem 7.11,7.12,7.13

The End！

Lecture #4(2)

CHAPTER 8 NORMAL SHOCK WAVES AND RELATED TOPICS
第八章 正激波及有关问题

Presented by Wenping Song
E-mail: wpsong@nwpu.edu.cn

2019年10月16日 Wednesday

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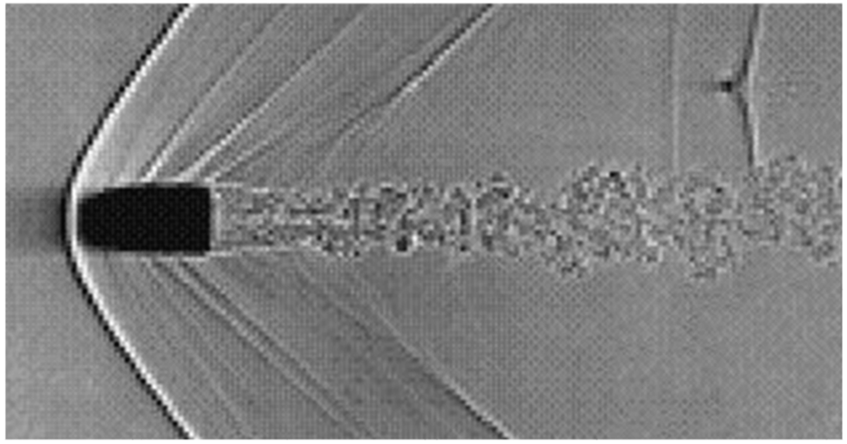
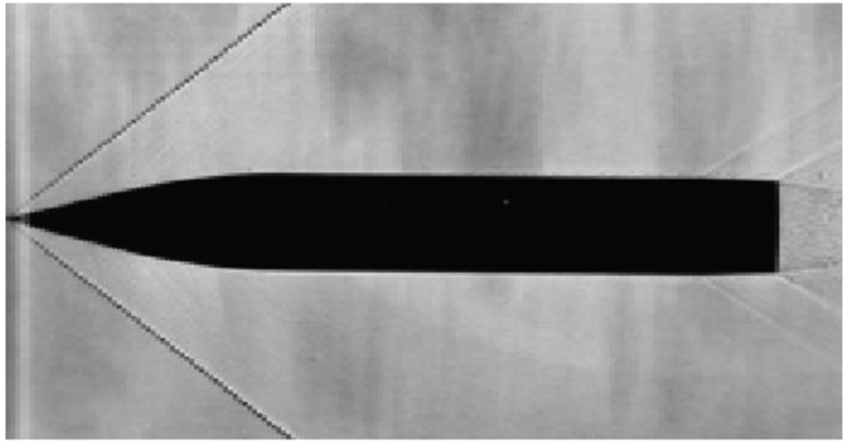
What is shock wave? 什么是激波？

- Shock wave: A large-amplitude compression wave, such as that produced by an explosion, caused by supersonic motion of a body in a medium.
- 激波是一个大振幅波，如由爆炸产生的波或物体在介质中超声速运动而引起的波。





8.1 Introduction前言

- The purpose of this chapter and Chap.9 is to develop shock-wave theory, thus giving us the means to calculate the changes in the flow properties across a wave
- 本章和第九章的目的是推导激波理论,因而得出计算通过激波的流动特性变化量的公式

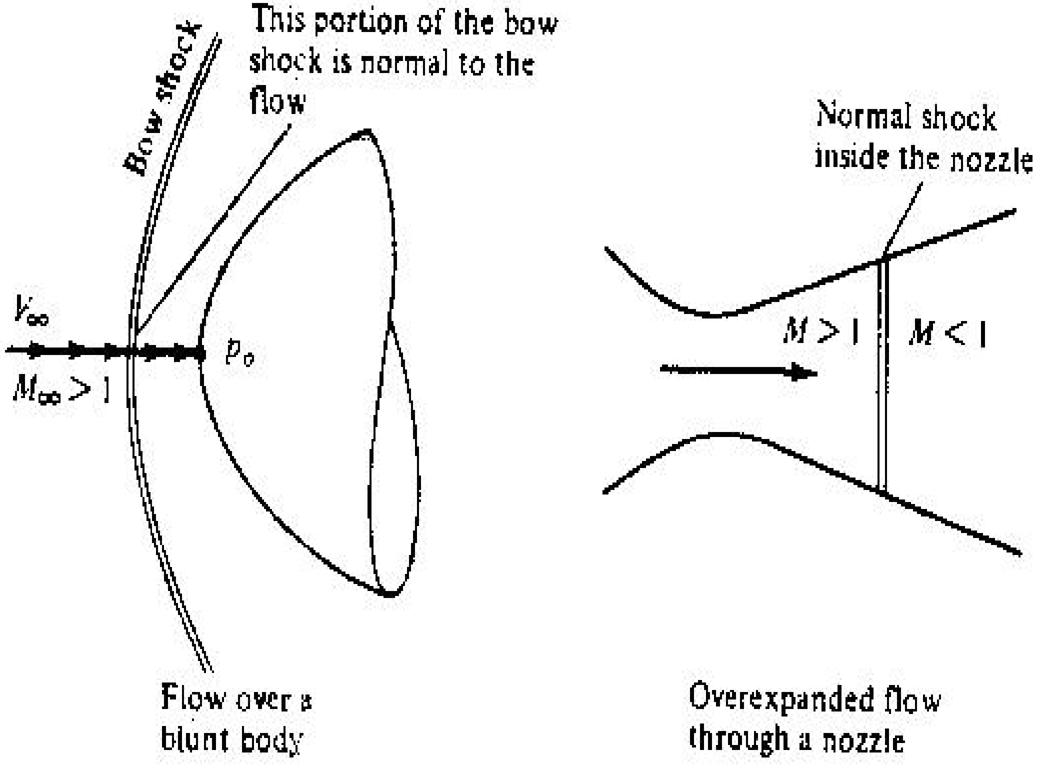


FIGURE 8.1
Two examples where normal shock waves are of interest.

路线图

8.2 正激波基本控制方程推导

8.3 声速

8.4 能量方程特殊形式

8.5 什么情况下流动可压?

8.6 通过正激波气体特性变化的方程的详细推导; 物理特性变化趋势讨论

8.7 用皮托管测量可压缩流的流动速度

8. 2 Basic Normal Shock Equations
正激波基本控制方程

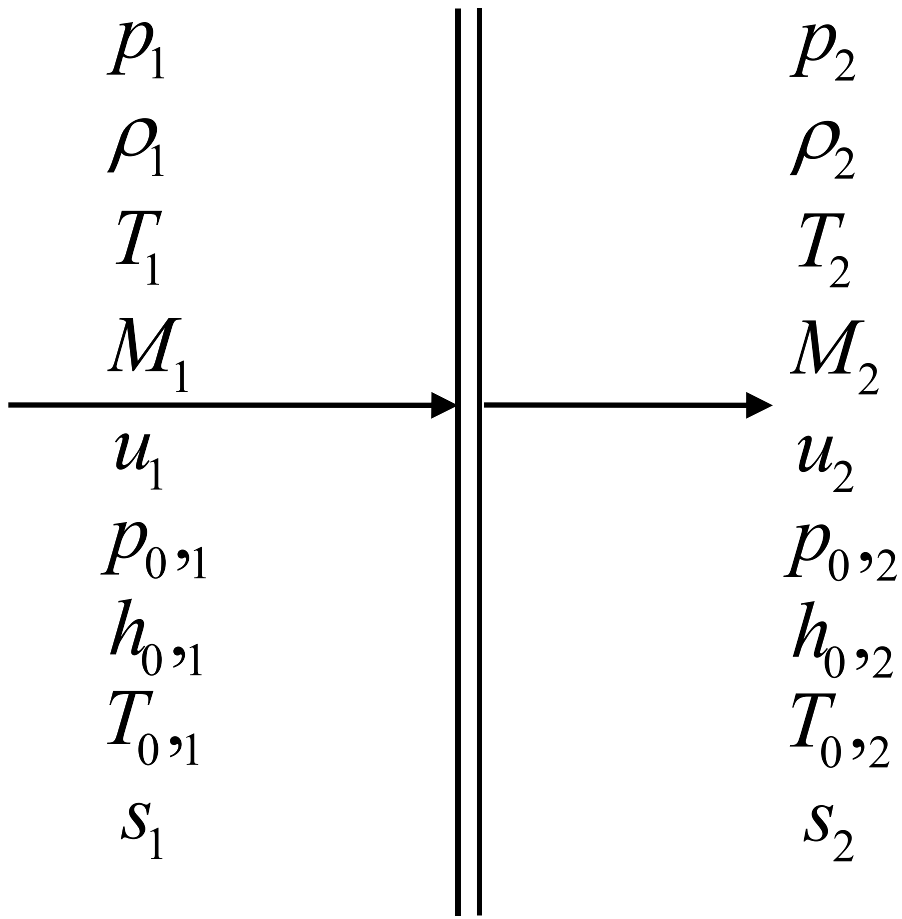


图8.3 正激波示意图

- The shock wave is a thin region of highly viscous flow. The flow through the shock is adiabatic but nonisentropic
- 激波是很薄的、具有强粘性的区域。通过激波流动是绝热、但不等熵的

8. 2 Basic Normal Shock Equations
正激波基本控制方程

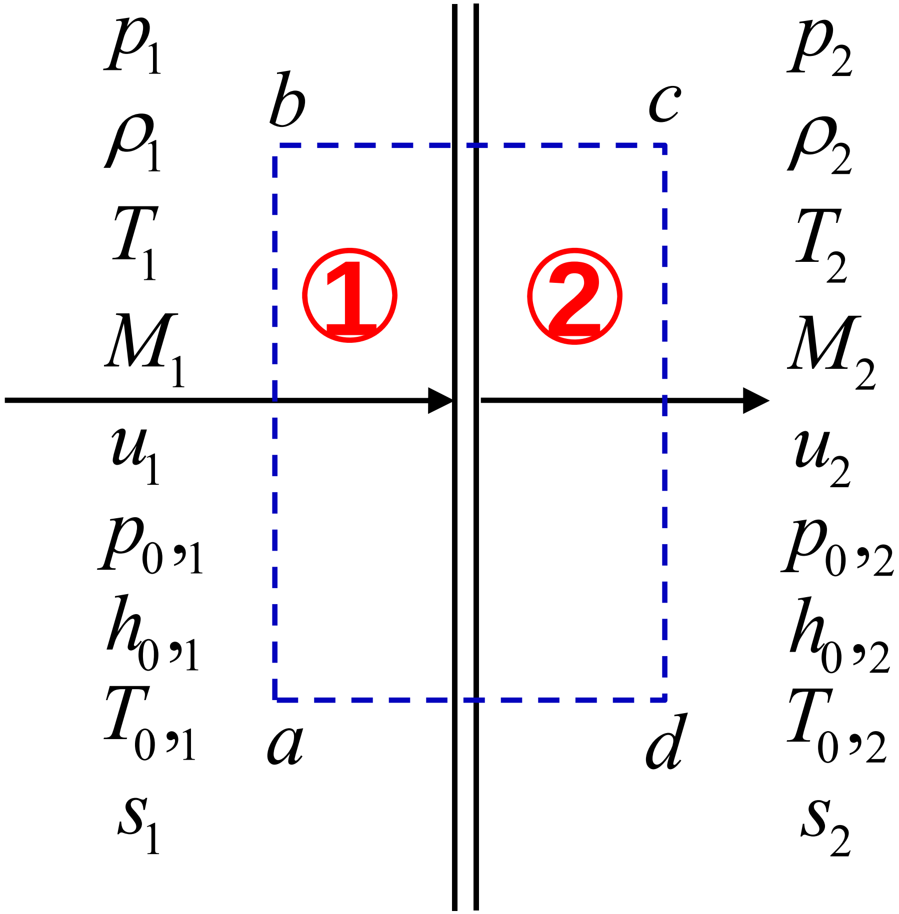


图8.3 正激波示意图

- Consider the rectangular control volume *abcd* given by the dashed line in Fig.8.3. The shock wave is inside the control volume
- 考虑矩形控制体*abcd*如图8.3虚线所示，激波在控制体内

8.2 Basic Normal Shock Equations 正激波基本控制方程

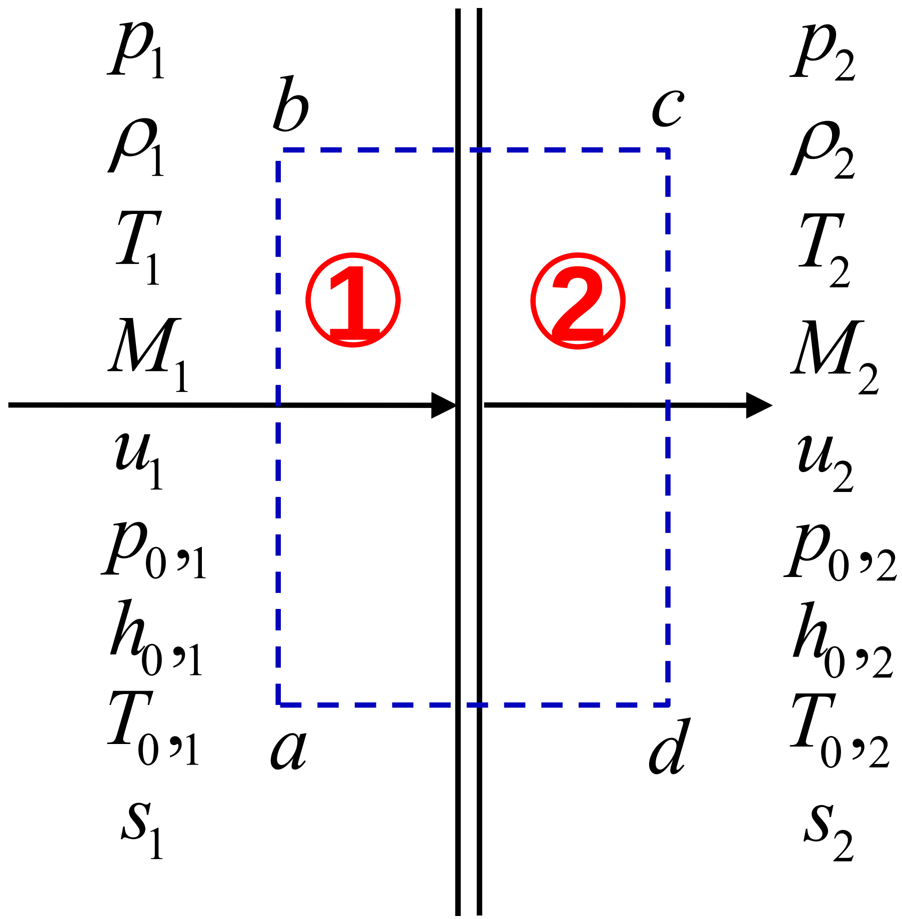


图8.3 正激波示意图

图8.3 所示控制体内流动具有如下特点

- The flow is *steady*, i.e. $\partial/\partial t = 0$. 流动是定常的
- flow is *adiabatic*: no heat is added or taken away from the control volume. 流动是绝热的，没有加入和带出控制体的热量.
- There are *no viscous effects* on the sides of the control volume.控制体的边界上没有粘性的作用
- There are no body forces; f=0。没有体积力

8. 2 Basic Normal Shock Equations 正激波基本控制方程

- Continuity equation/ 连续方程

$$\oiint_S \rho \vec{V} \cdot d\vec{S} = 0 \qquad (8.1)$$

$$\longrightarrow \boxed{\rho_1 u_1 = \rho_2 u_2} \qquad (8.2)$$

- Momentum equations/ 动量方程

$$\oiint_S \left(\rho \vec{V} \cdot d\vec{S}\right) \vec{V} = -\oiint_S p\, d\vec{S} \qquad (8.3)$$

  - Component in x-direction/ X方向分量

$$\oiint_S \left(\rho \vec{V} \cdot d\vec{S}\right) u = -\oiint_S \left(p\, d\vec{S}\right)_x \qquad (8.4)$$

$$\longrightarrow \boxed{p_1 + \rho_1 u_1^{\ 2} = p_2 + \rho_2 u_2^{\ 2}} \qquad (8.6)$$

8. 2 Basic Normal Shock Equations
正激波基本控制方程

- Energy equation/ 能量方程

$$\oiint_{S} \rho\left(e+\frac{V^2}{2}\right)\vec{V}\cdot d\vec{S} = -\oiint_{S} p\vec{V}\cdot \overrightarrow{dS} \qquad (8.7)$$

$$\longrightarrow \boxed{h_1+\frac{u_1^{\,2}}{2}=h_2+\frac{u_2^{\,2}}{2}} \qquad (8.10)$$

- **Repeating the above results for clarity, the basic normal shock equations are/为了清晰起见，我们重复写出正激波基本方程：**

  - Continuity equation/ 连续方程 $$\boxed{\rho_1 u_1 = \rho_2 u_2} \qquad (8.2$$
  - Momentum equations/ 动量方程 $$\boxed{p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2} \qquad (8.6)$$
  - Energy equation/ 能量方程 $$\boxed{h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}} \qquad (8.10)$$
  - State equation/ 完全气体状态方程 $$\boxed{p_2 = \rho_2 R T_2}$$
  - Internal energy equation/ 内能方程 $$\boxed{h_2 = C_p T_2} \qquad C_p = \frac{\gamma R}{\gamma - 1}$$

激波内部强粘性区的进一步理解

$$\rho \frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \rho f_x + (\mathcal{F}_x)_{\text{viscous}} \qquad \textbf{(2.113a)}$$

$$(\mathcal{F}_x)_{\text{viscous}} = ?$$

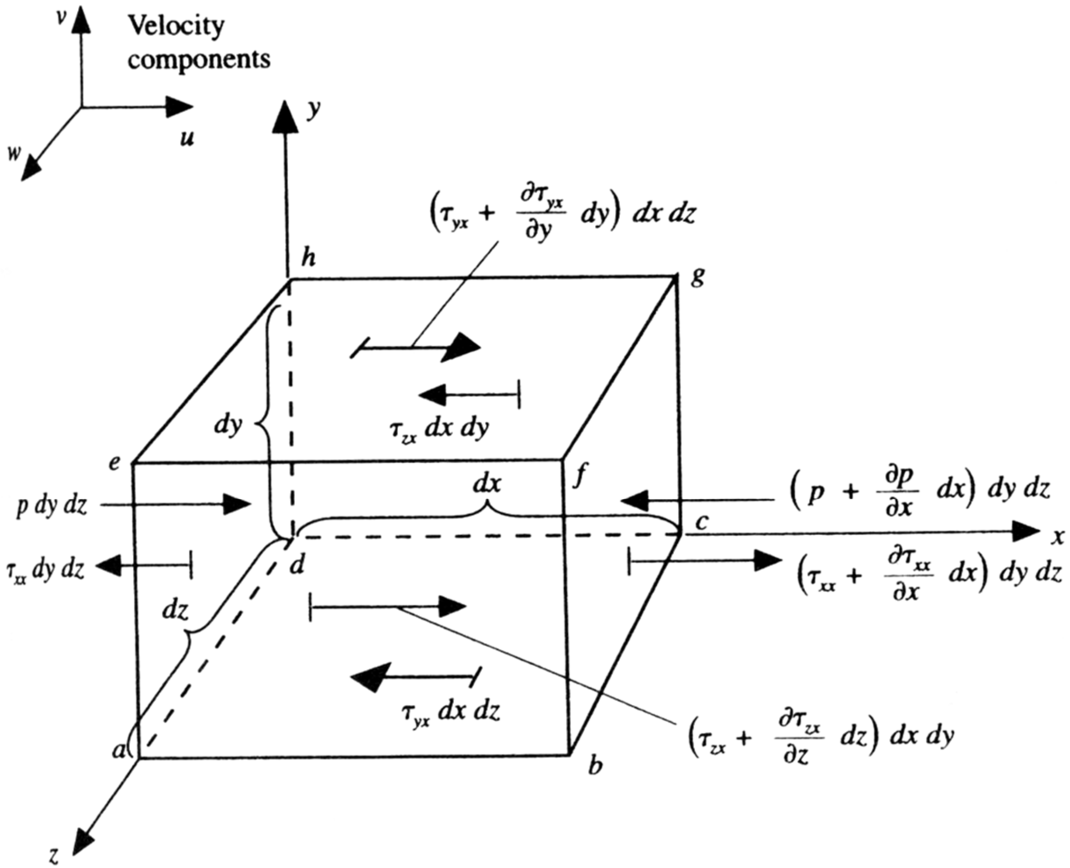


FIG. 2.8
Infinitesimally small, moving fluid element. Only the forces in the x direction are shown. Model used for the derivation of the x component of the momentum equation.

$$\rho \frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + \rho f_x$$

Stokes 在1845年得出(本构关系-constitutive equation)：

$$\tau_{xx} = \lambda(\nabla \cdot \vec{V}) + 2\mu \frac{\partial u}{\partial x} \qquad (2.57a)$$

$$\tau_{yy} = \lambda(\nabla \cdot \vec{V}) + 2\mu \frac{\partial v}{\partial y} \qquad (2.57b)$$

$$\tau_{zz} = \lambda(\nabla \cdot \vec{V}) + 2\mu \frac{\partial w}{\partial z} \qquad (2.57c)$$

$$\tau_{xy} = \tau_{yx} = \mu\left[\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right] \qquad (2.57e)$$

$$\tau_{xz} = \tau_{zx} = \mu\left[\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right] \qquad (2.57f)$$

$$\tau_{yz} = \tau_{zy} = \mu\left[\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right] \qquad (2.57g)$$

The shock wave is a thin region of highly viscous flow. The flow through the shock is adiabatic but nonisentropic

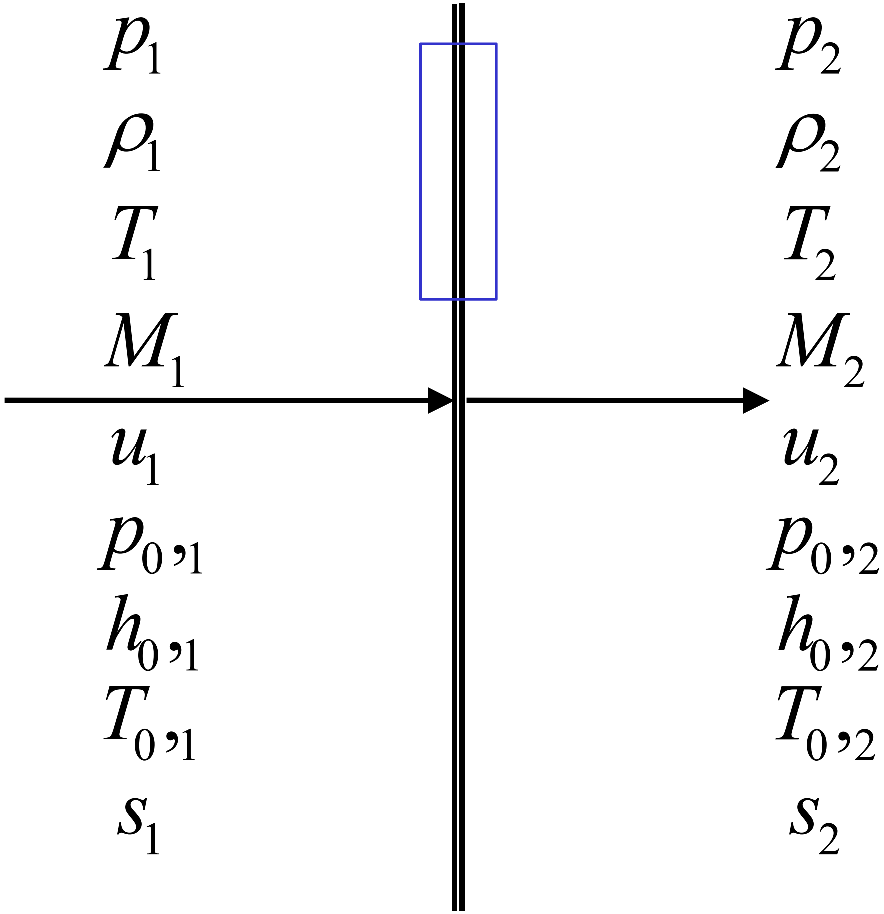
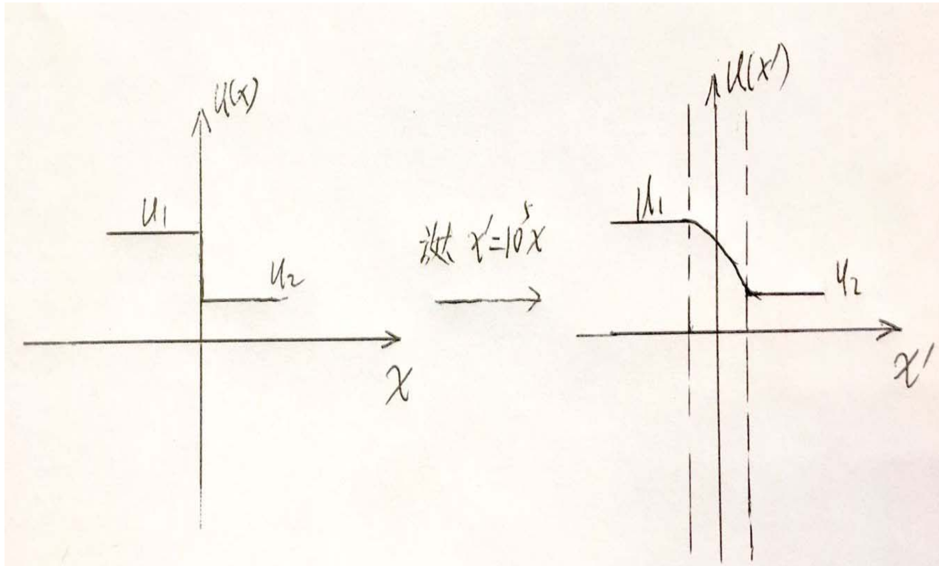


图8.3 正激波示意图



激波内部粘性正应力：

$$\tau_{xx} = \lambda(\nabla \cdot \vec{V}) + 2\mu \frac{\partial u}{\partial x}$$

多选题 1分

设置

通过激波流动参数如何变化？

A 压强、密度、温度、熵增大

B 马赫数、速度减小

C 总温、总焓不变

D 总压减小

提交

Have a rest!