Review of lecture #11/第11次课复习

CHAPTER 9 Oblique Shock and Expansion Waves

第九章 斜激波和膨胀波

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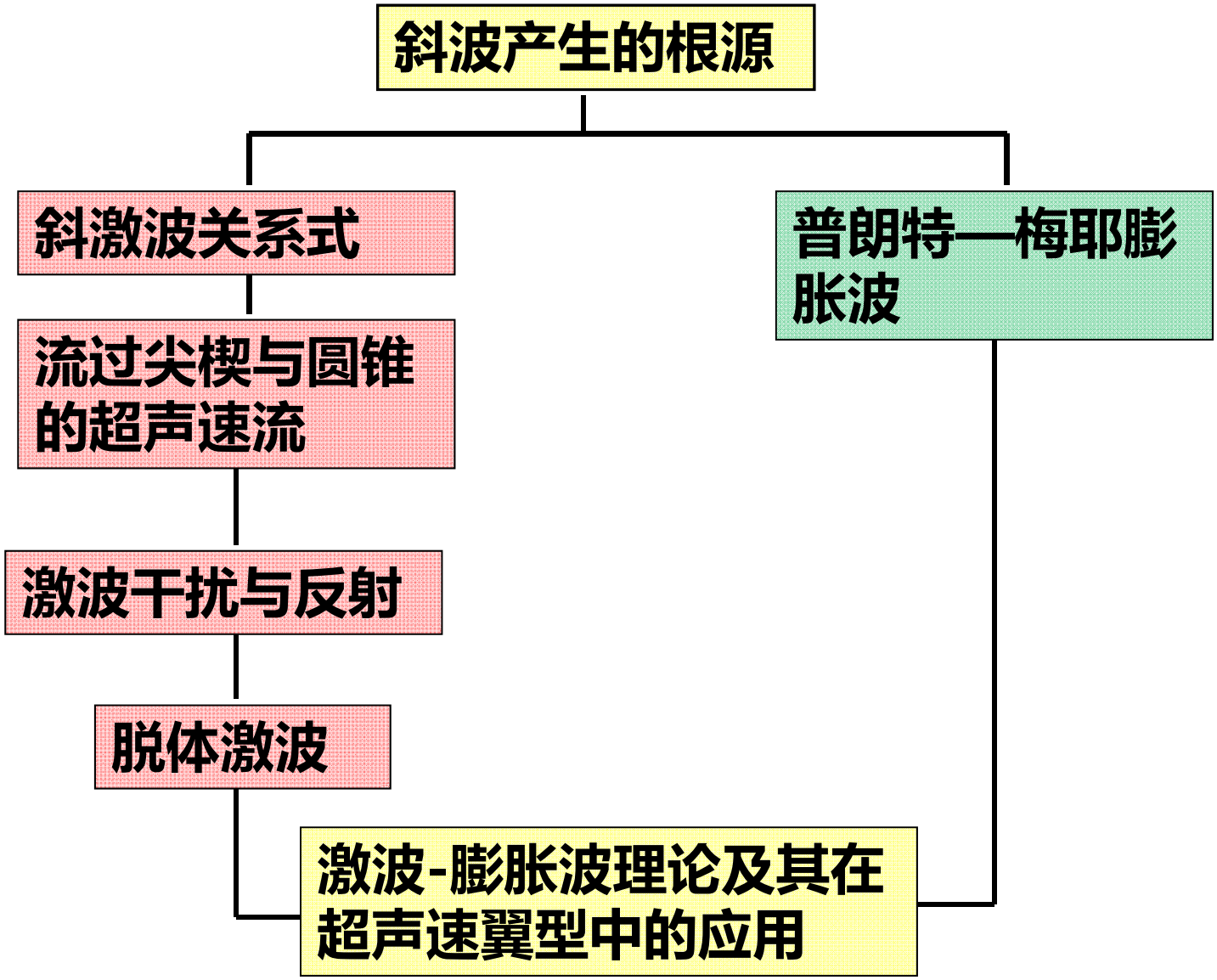
National Key Laboratory of Science and Technology on Aerodynamic Design and Research

Compressible Aerodynamics Course

第九章路线图


斜波产生的根源
斜激波关系式
普朗特—梅耶膨胀波
流过尖楔与圆锥的超声速流
激波干扰与反射
脱体激波
激波-膨胀波理论及其在超声速翼型中的应用

9.5 Detached Shock Wave in Front of a Blunt Body

钝头体前的脱体激波

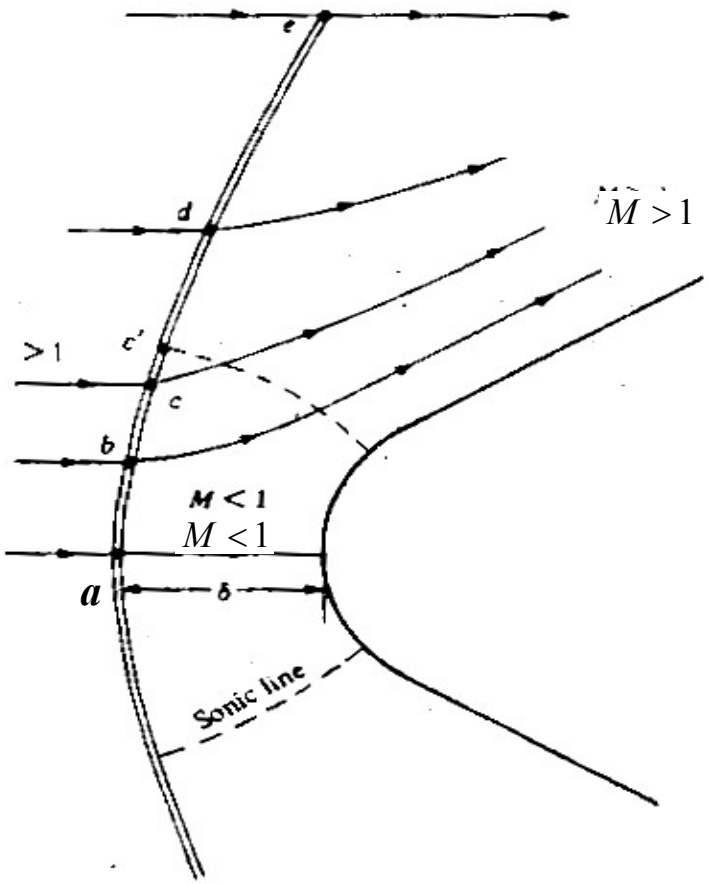


图9.23 绕钝头体的超声速流动

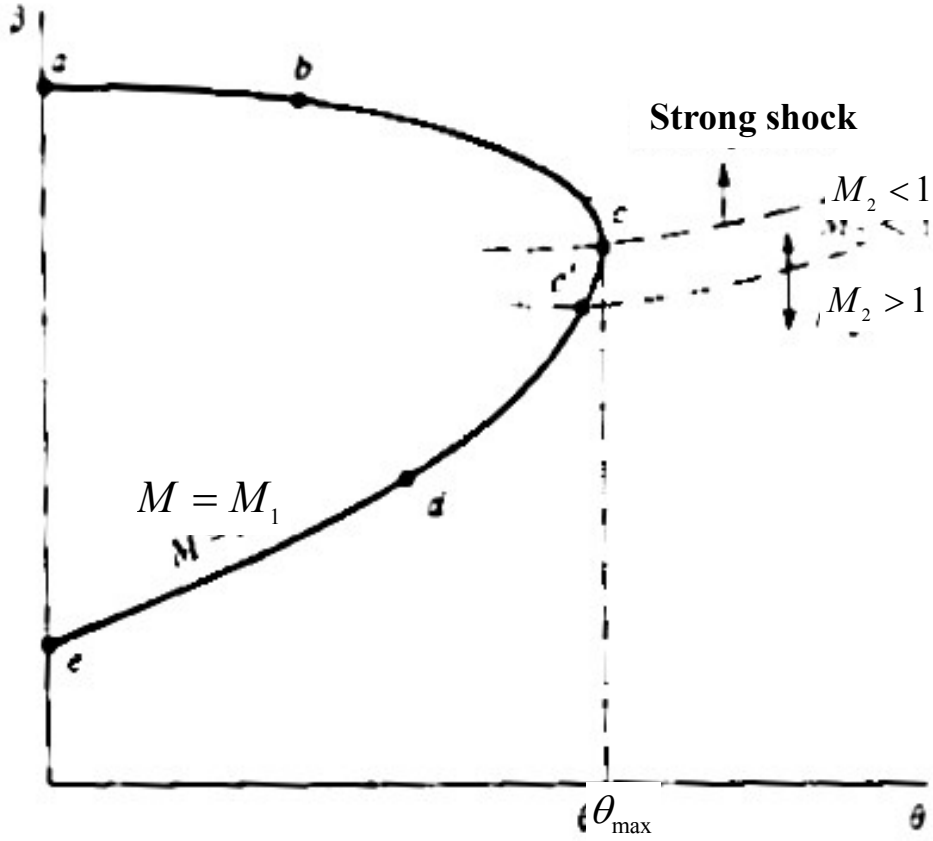


图9.24 与图9.23对应的θ - β – M 曲线

Shock detachment distance ：激波脱体距离；Sonic line:声速线

在图9.23中，点 a 处激波与来流垂直。离开点 a， 激波逐渐变弯变弱，最后在远离物体的地方变为马赫波（图9.23中的e点）。弯曲弓形激波是可让你观察到在同一来流马赫数下*所有*可能的斜激波解的例子之一。对应由a 至 e之间的各点.

a-c: 对应强解；

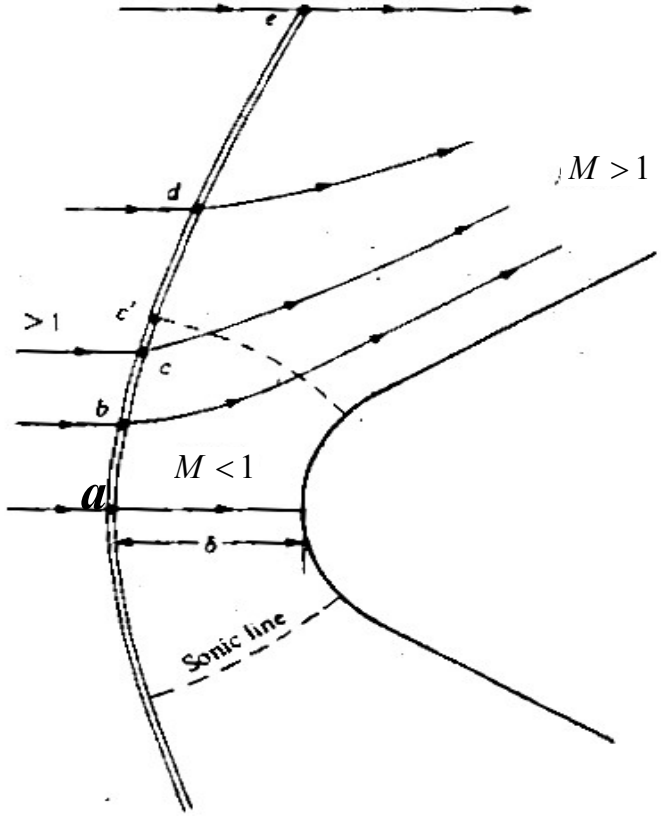
c: 对应最大偏转角θ_{max} 点；

c-c': 对应波后马赫数小于1的弱解；

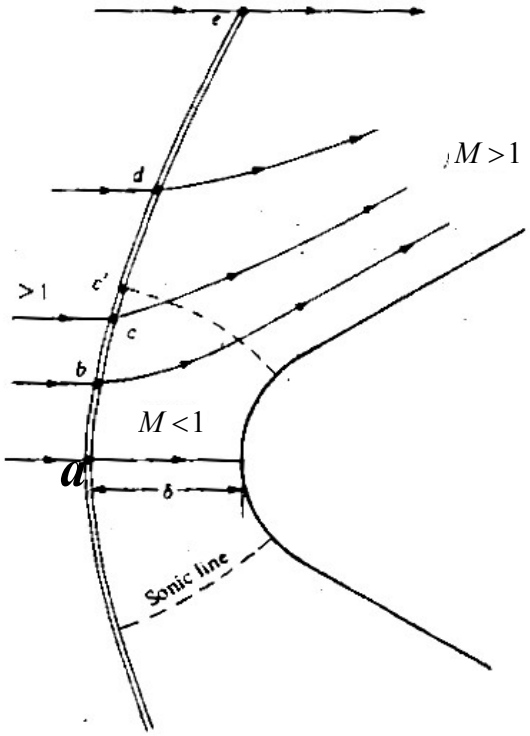
c': 对应激波之后气流速度为声速的点；

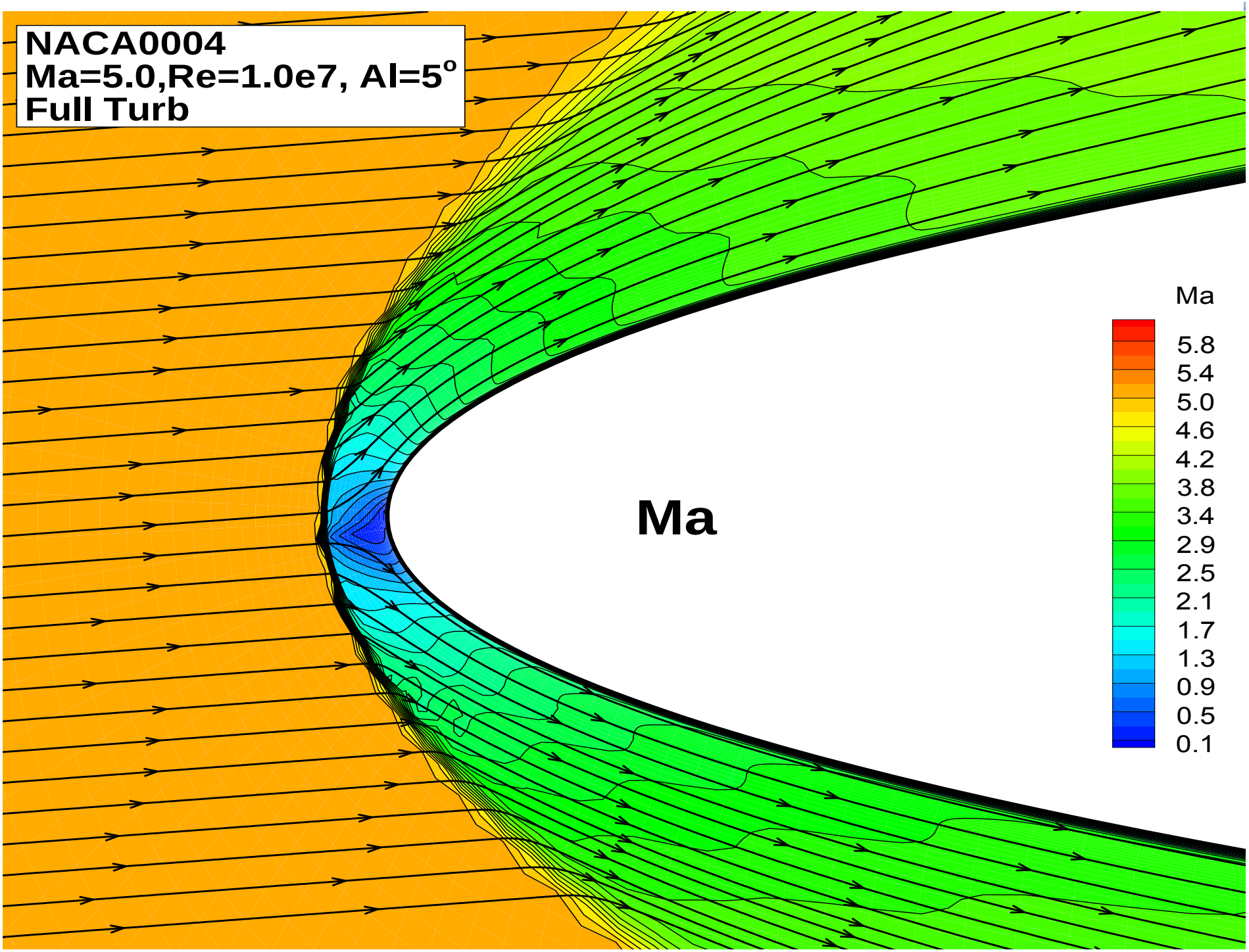
c'-e ：对应激波之后马赫数大于1的弱解；

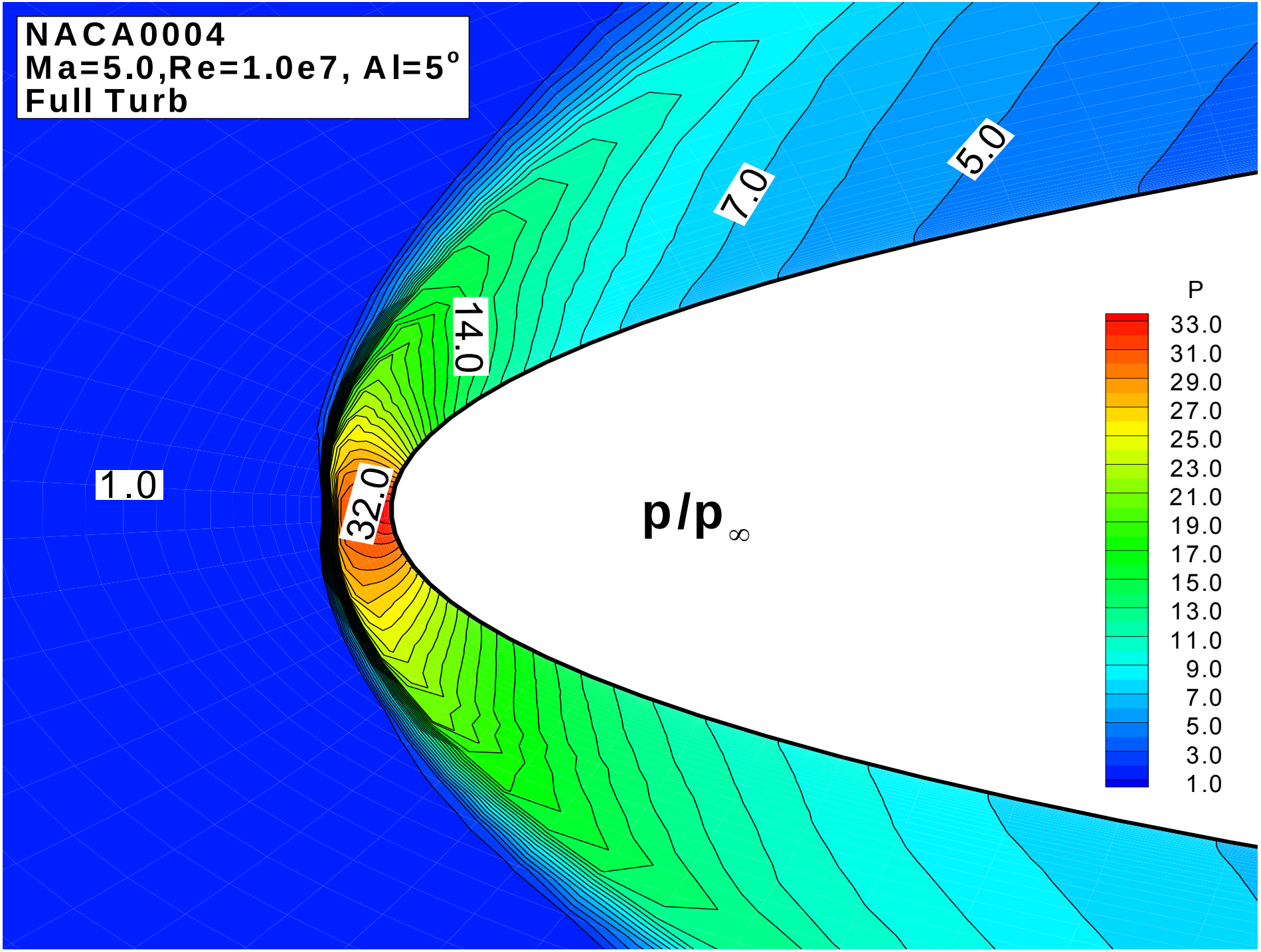
e: 对应马赫波。



- 弓形激波与钝头体之间的流动为超声速流和亚声速流的混合区。亚声速区与超声速区的分界线被称为声速线。
- 脱体激波的形状，激波脱体距离δ，以及整个流场的解由来流马赫数，钝头体的尺寸与形状确定。采用数值求解技术可以得到该流场的解。




NACA0004
Ma=5.0,Re=1.0e7, Al=5°
Full Turb
Ma
Ma
5.8
5.4
5.0
4.6
4.2
3.8
3.4
2.9
2.5
2.1
1.7
1.3
0.9
0.5
0.1


NACA0004
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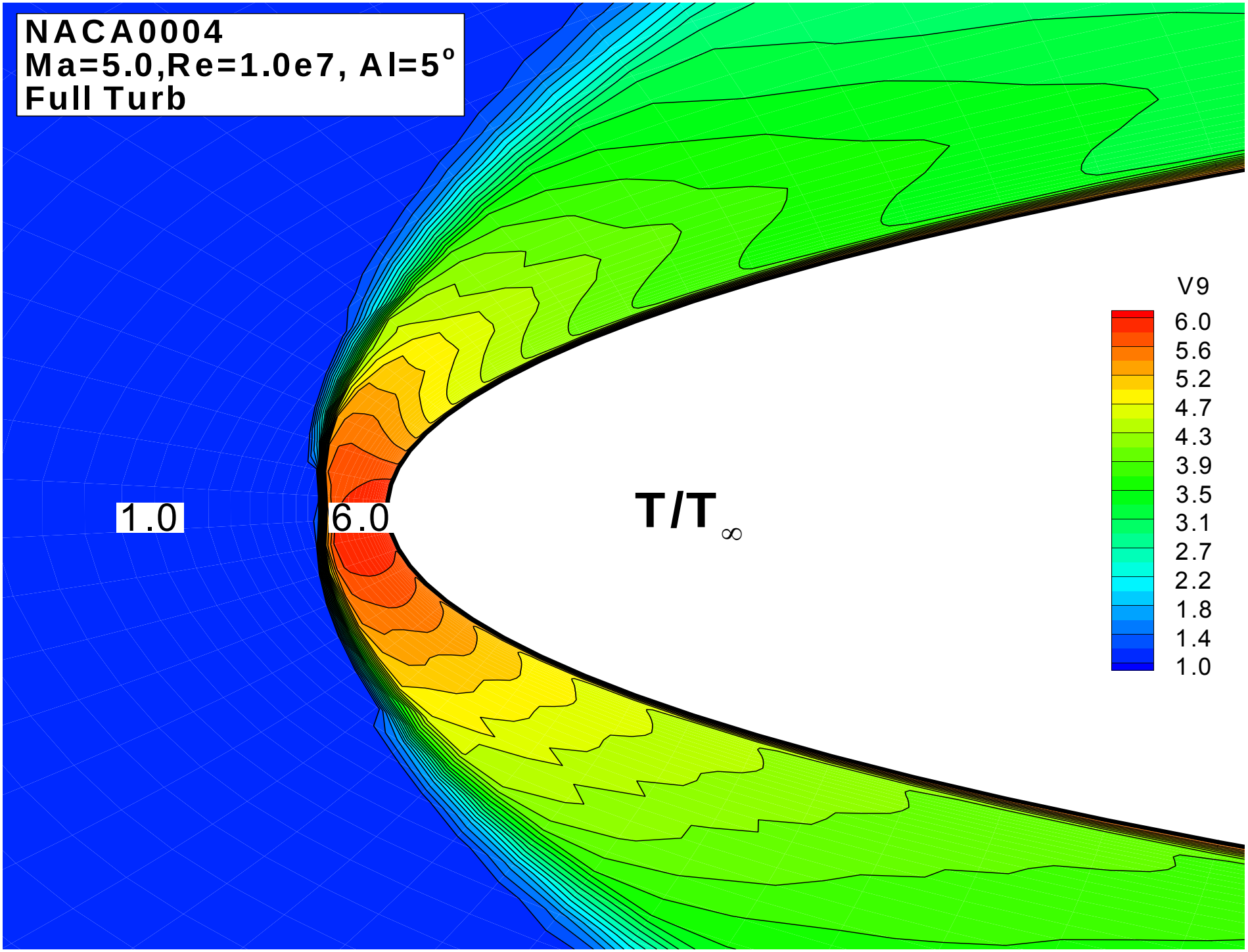
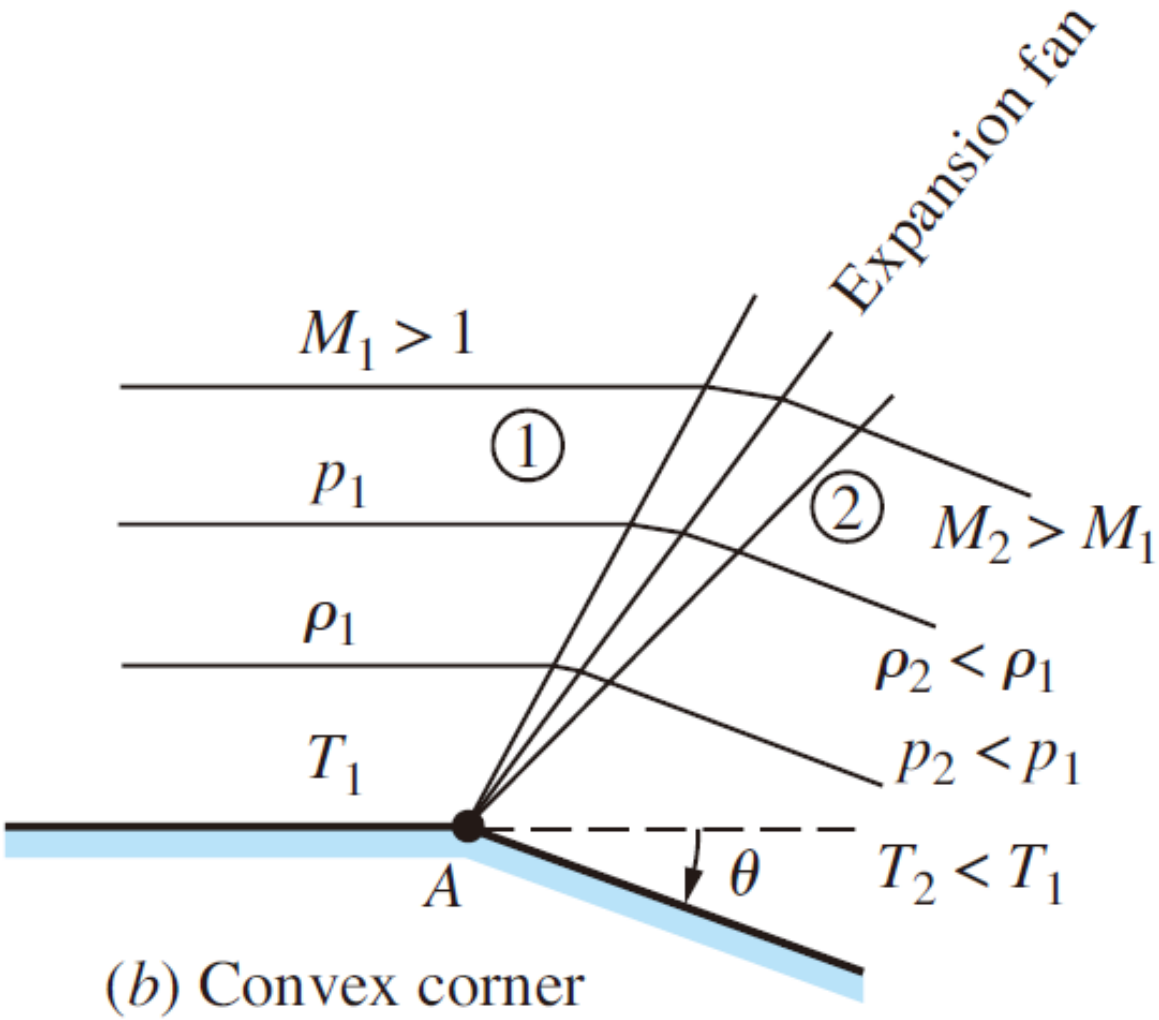
Full Turb
1.0
32.0
14.0
7.0
5.0
p/p∞
P
33.0
31.0
29.0
27.0
25.0
23.0
21.0
19.0
17.0
15.0
13.0
11.0
9.0
7.0
5.0
3.0
1.0

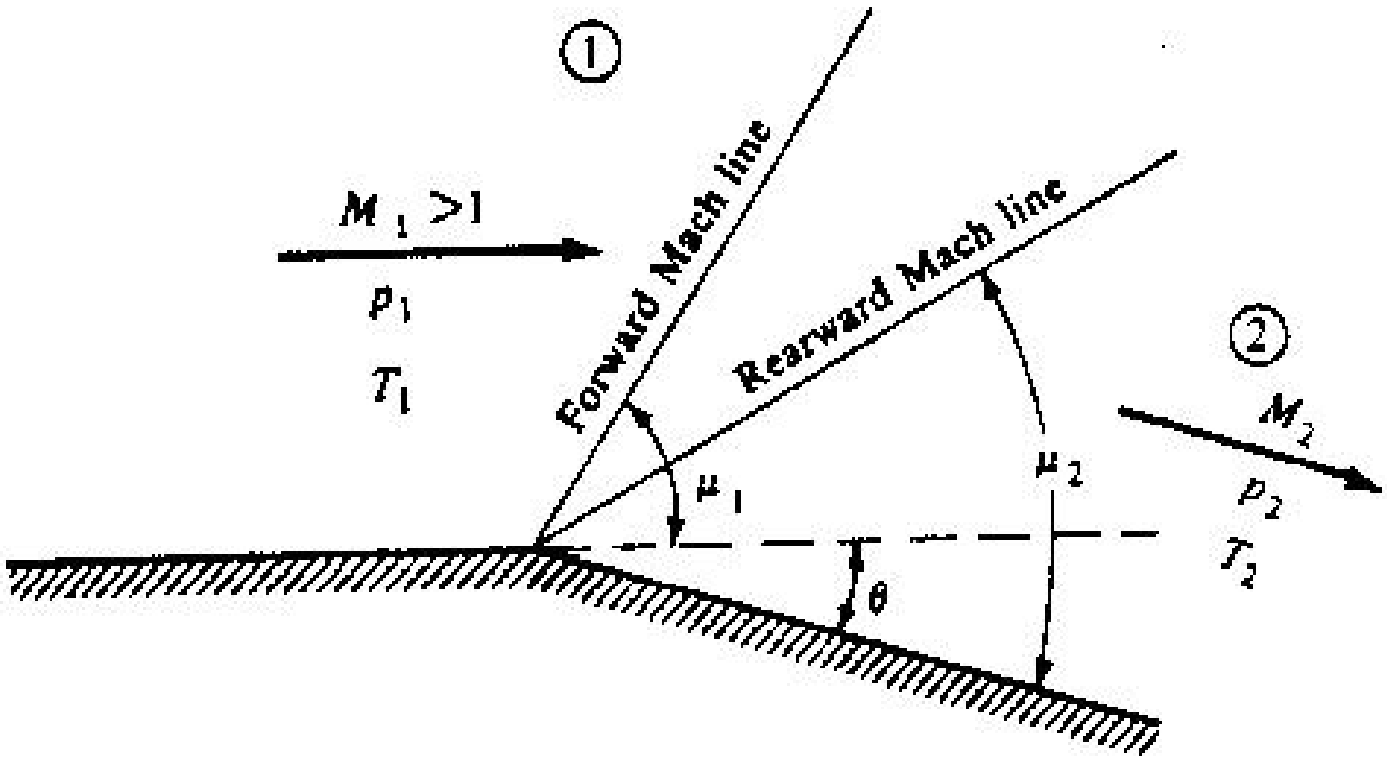

NACA0004
Ma=5.0,Re=1.0e7, Al=5°
Full Turb
1.0
6.0
T/T∞
V9
6.0
5.6
5.2
4.7
4.3
3.9
3.5
3.1
2.7
2.2
1.8
1.4
1.0

Chapter 9.6 Prandle-Meyer Expansion Waves
普朗特–梅耶膨胀波



(*b*) Convex corner

普朗特-梅耶膨胀波



- 特别要注意：膨胀过程是一个等熵过程。
- 要解决的问题是：已知上游马赫数 M_1 及其它流动特性(区 域1），求通过偏转角 θ 膨胀后的下游（区域2）的特性。

普朗特-梅耶膨胀波

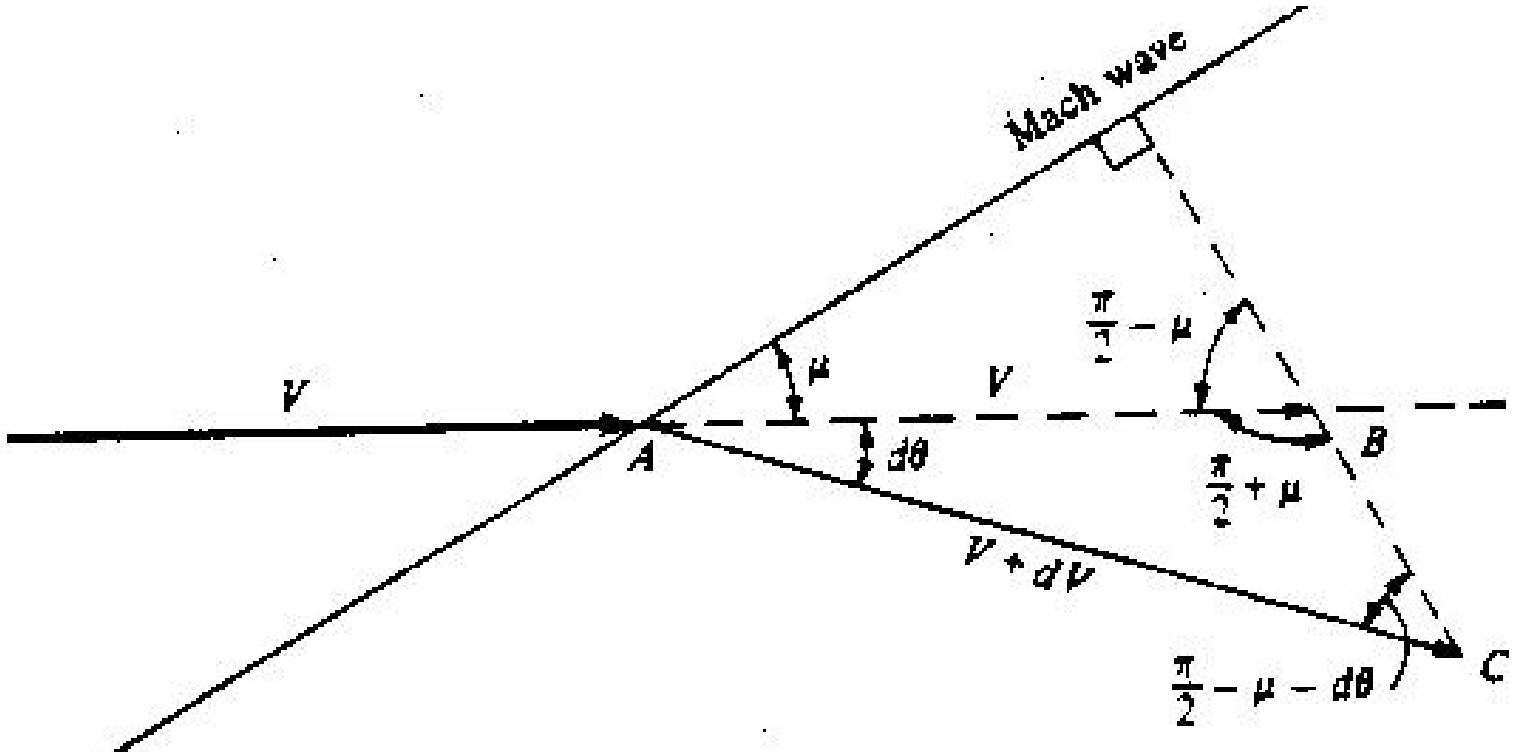
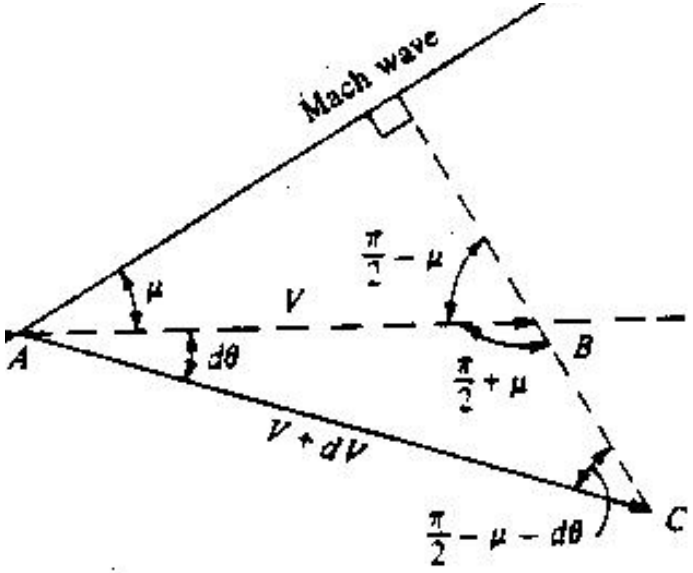


图9.26 通过无限弱波（极限—马赫波）的
无限小变化的几何关系图

注意：波前波后切向速度分量不变保证了CB垂直于马赫波。

对ΔABC应用正弦定理



$$\frac{V+dV}{V}=\frac{\sin(\pi/2+\mu)}{\sin(\pi/2-\mu-d\theta)} \qquad (9.24)$$

$$\sin(\frac{\pi}{2}+\mu)=\sin(\frac{\pi}{2}-\mu)=\cos\mu \qquad (9.25)$$

$$\sin(\frac{\pi}{2}-\mu-d\theta)=\cos(\mu+d\theta)=\cos\mu\cos d\theta-\sin\mu\sin d\theta \qquad (9.26)$$

$$1+\frac{dV}{V}=\frac{\cos\mu}{\cos\mu\cos d\theta-\sin\mu\sin d\theta} \qquad (9.27)$$

对于无限小的 $d\theta$, Sin $d\theta \approx d\theta$, Cos $d\theta \approx 1$,所以有：

$$1+\frac{dV}{V}=\frac{\cos\mu}{\cos\mu-d\theta\sin\mu}=\frac{1}{1-d\theta\tan\mu} \qquad (9.28)$$

泰勒展开后的结果

因为：
$$\frac{1}{1-x}=1+x+x^2+x^3+\cdots$$

所以：
$$1+\frac{dV}{V}=1+d\theta\tan\mu+\cdots \tag{9.29}$$

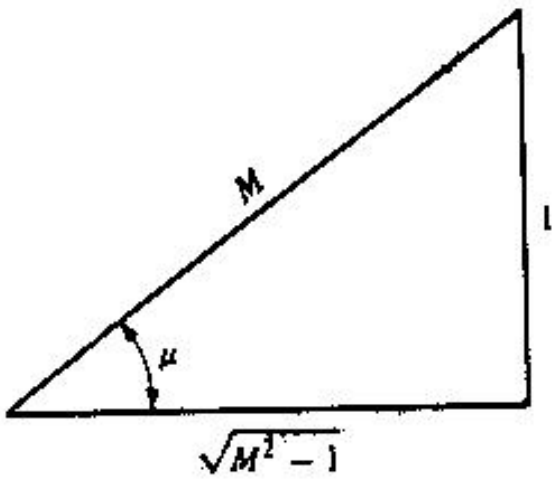


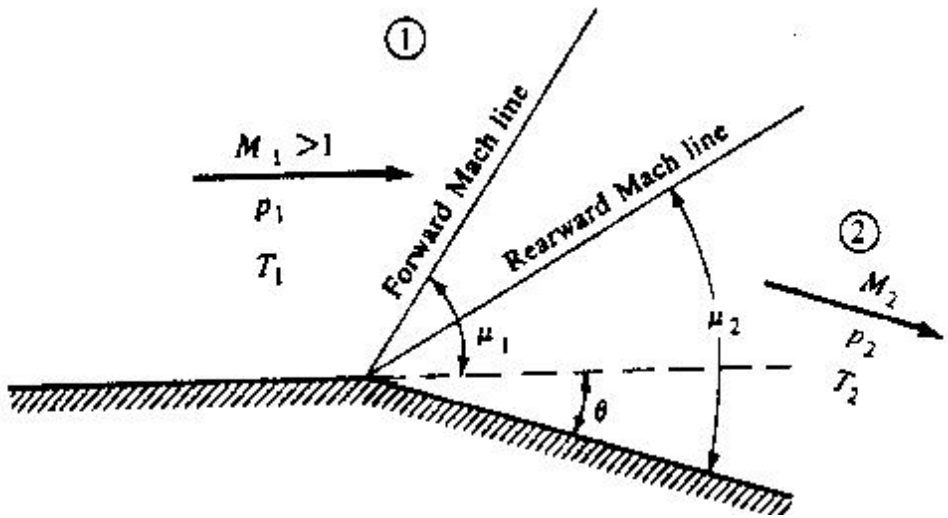
图9.27 与马赫角相联系的直角三角形

$$d\theta=\frac{dV/V}{\tan\mu} \tag{9.30}$$

$$\tan\mu=\frac{1}{\sqrt{M^2-1}} \tag{9.31}$$

$d\theta$ 与 dV 的关系式

$$d\theta = \sqrt{M^2 - 1}\frac{dV}{V} \qquad (9.32)$$



(9. 32)式是一个精确描述膨胀波内部变化的微分方程。

将(9.32)式从偏角为零，马赫数为M_1的区域1，积分到偏角为θ，马赫数为M_2的区域2

$$\int_0^{\theta} d\theta = \theta = \int_{M_1}^{M_2} \sqrt{M^2 - 1}\frac{dV}{V} \qquad (9.33)$$

将 dV/V 用马赫数 M 表示

因为： $V = M \cdot a$

所以：

$$\ln V = \ln M + \ln a \tag{9.34}$$

$$\frac{dV}{V} = \frac{dM}{M} + \frac{da}{a} \tag{9.35}$$

因为：

$$\left(\frac{a_0}{a}\right)^2 = \frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2 \tag{9.36}$$

所以：

$$a = a_0 (1 + \frac{\gamma - 1}{2} M^2)^{-\frac{1}{2}} \tag{9.37}$$

$$\frac{da}{a} = -\left(\frac{\gamma - 1}{2}\right) M (1 + \frac{\gamma - 1}{2} M^2)^{-1} dM \tag{9.38}$$

$$\frac{dV}{V} = \frac{1}{1 + [(\gamma - 1)/2] M^2} \frac{dM}{M} \tag{9.39}$$

普朗特-梅耶函数

$$\theta = \int_{M_1}^{M_2} \sqrt{M^2 - 1}\frac{dV}{V} \qquad (9.33)$$

将 (9.39) 式代入到（9.33)式，积分得：

$$\boxed{\theta = \int_{M_1}^{M_2} \frac{\sqrt{M^2 - 1}}{1 + [(\gamma - 1)/2]M^2}\frac{dM}{M}} \qquad (9.40)$$

$$\boxed{\nu(M) = \int \frac{\sqrt{M^2 - 1}}{1 + [(\gamma - 1)/2]M^2}\frac{dM}{M}} \qquad (9.41)$$

$\nu(M)$ 被称为 Prandtl – Meyer 函数，其具体表达式如下：

$$\boxed{\nu(M) = \sqrt{\frac{\gamma + 1}{\gamma - 1}}\tan^{-1}\sqrt{\frac{\gamma - 1}{\gamma + 1}(M^2 - 1)} - \tan^{-1}\sqrt{M^2 - 1}} \qquad (9.42)$$

因此，(9.40)的积分可以表示为：

$$\theta = \nu(M_2) - \nu(M_1) \qquad (9.43)$$

普朗特–梅耶函数（续）

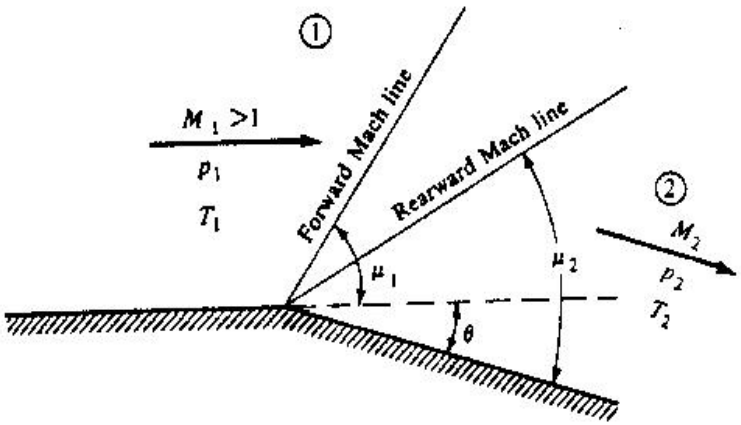
$$\boxed{\theta = \nu(M_2) - \nu(M_1)} \qquad (9.43)$$

对于量热完全气体, $\nu(M)$ 由 (9.42) 式给定。Prandtl-Meyer 函数 $\nu(M)$ 非常重要，它是计算通过膨胀波气体特性变化的关键

$$\boxed{\nu(M) = \theta + \nu(1) = \theta}$$

Prandtl-Meyer 函数物理意义:
从 马赫数 1 的流动膨胀加速到 *M* 时，需要的偏转角

普朗特-梅耶膨胀波求解的步骤



1. 对于给定M_1，由附录C查得 $\nu(M_1)$ 。
2. 由 $\nu(M_2)=\nu(M_1)+\theta$ 计算 $\nu(M_2)$ 。
3. 根据第 2 步计算出的$\nu(M_2)$ ，查附录 C 得到 M_2。
4. 因为膨胀波是等熵的，因此 p_0 和 T_0 通过膨胀波保持不变。即 $T_{0,1}=T_{0,2}$, $p_{0,1}=p_{0,2}$。由(8.40)式， (8.42) 式，我们有

$$\frac{T_2}{T_1}=\frac{T_2/T_{0,2}}{T_1/T_{0,1}}=\frac{1+[(\gamma-1)/2]M_1^{\ 2}}{1+[(\gamma-1)/2]M_2^{\ 2}} \qquad (9.44)$$

$$\frac{p_2}{p_1}=\frac{p_2/p_{0,2}}{p_1/p_{0,1}}=\left(\frac{1+[(\gamma-1)/2]M_1^{\ 2}}{1+[(\gamma-1)/2]M_2^{\ 2}}\right)^{\gamma/(\gamma-1)} \qquad (9.45)$$

DESIGN BOX

高超声速飞行器验证机和概念机

以上高超声速飞行器的动力系统均为超燃发动机

DESIGN BOX

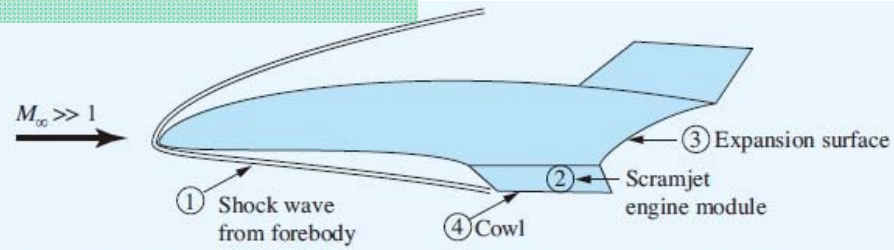


Figure 9.30 Sketch of a generic hypersonic vehicle powered by a SCRAMjet engine.

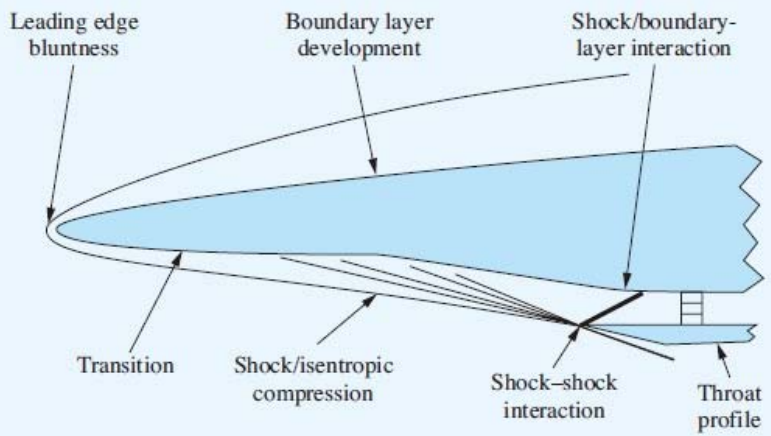


Figure 9.31 Sketch of some of the flow features on the forebody of a SCRAMjet-powered hypersonic vehicle.

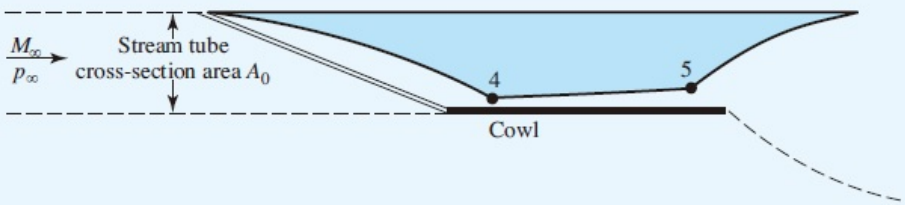


Figure 9.32 Flow path through a SCRAMjet engine.

高超声速飞行器的整个底部是吸气式超燃发动机的集成部分

① **前体激波**——头部开始的激波压缩气流；

② **超燃冲压发动机舱** ——

通过进气道内反射激波系进一步压缩气流并与燃料混合，从发动机尾部膨胀喷出；

③ **飞行器后部膨胀面** ——

使喷出气流进一步膨胀

④ **发动机整流罩** —— 在设计飞行状态要求前体激波接到整流罩前缘

DESIGN BOX

如图9.31所示：前体激波之后还可以通过等熵压缩在气流进入进气道之前进一步压缩。

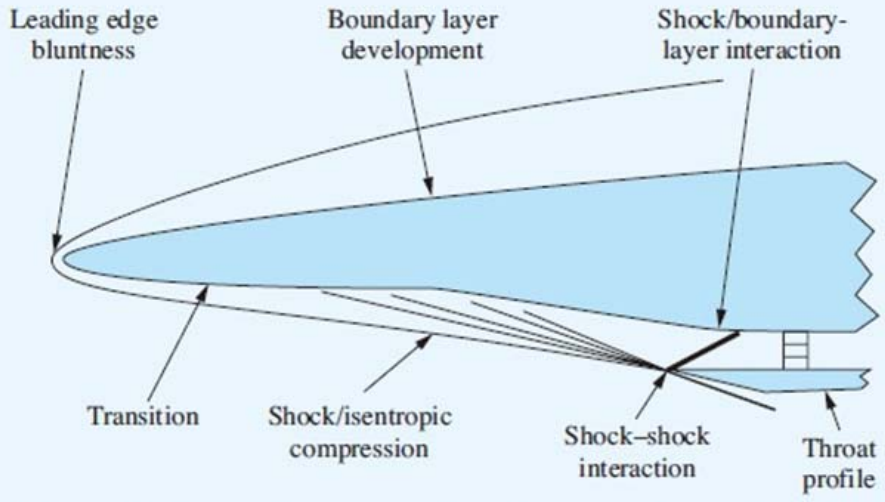


Figure 9.31 Sketch of some of the flow features on the forebody of a SCRAMjet-powered hypersonic vehicle.

影响超燃冲压发动机性能和气动特性的因素与物理问题有：

前缘钝头——降低头部气动加热

粘性边界层及激波边界层干扰 —— 产生阻力和气动加热；引起气流分离与再附

层流向湍流转捩 —— 影响阻力和气动热

激波间干扰 —— 引起发动机整流罩严重的气动加热问题

超燃冲压发动机的原理草图：

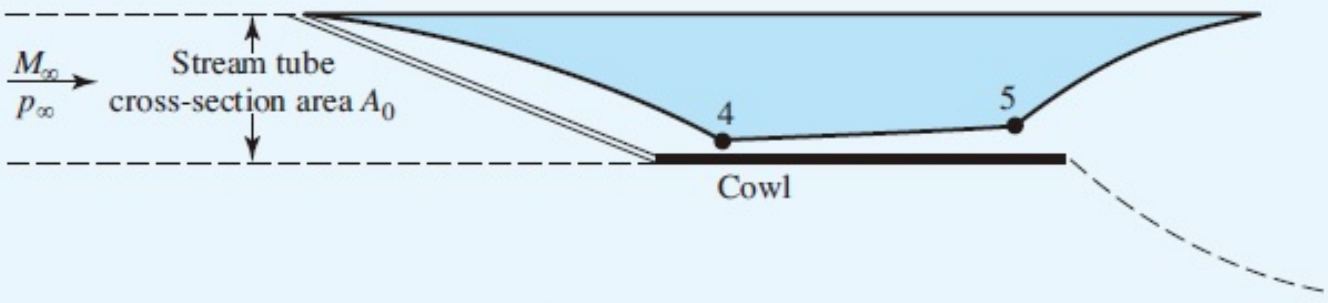


Figure 9.32 Flow path through a SCRAMjet engine.

4-5：燃烧室（Combustor)

4：入口　5：出口

M_∞	高度(m)	M_4	A_0/A_4	P_4/p_∞	$P_4(N/m^2)$	$T_4(K)$	$V_4(m/s)$
7	24407	3.143	10.85	47	911.1	806	1755
10	29108	4.143	16.49	89.6	851.3	1087	2665
15	34823	5.502	25.23	185.9	763.2	1600	4239
20	41989	6.650	33.11	313.6	479.8	2263	5989

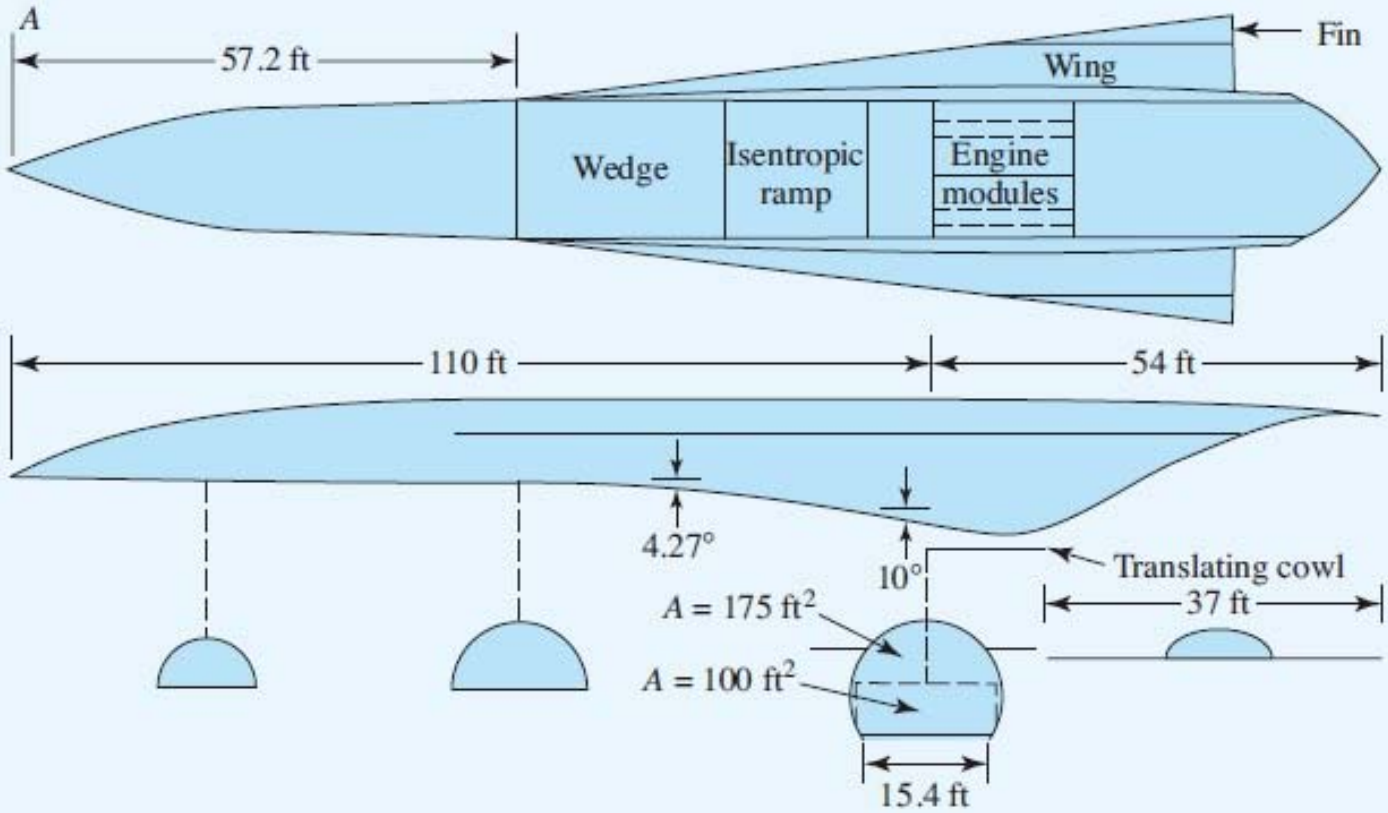
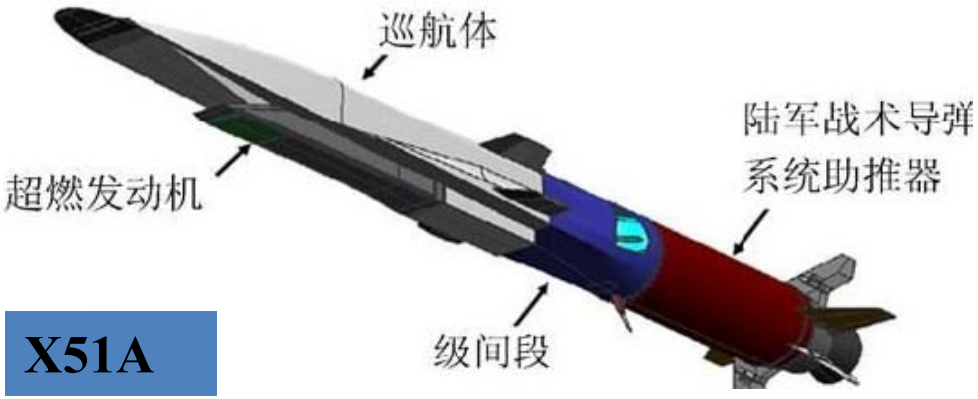


Figure 9.33 Two-sided view of a conceptual vehicle design. (*Source:* Billig, Frederick S.: "Design and Development of Single-Stage-to-Orbit Vehicles," *Johns Hopkins Applied Physics Laboratory Technical Digest*, vol. 11, nos. 3 and 4, July–December 1990, pp. 336–352).

Isentropic ramp
Translating cowl

X-43A挂载在改装过的B-52轰炸机飞至高空后，点燃连结在X-43A上的飞马座火箭将X-43A推进到超声速的状态后，再点燃自身的超声速冲压发动机进行高超声速飞行。 **2004年成功首飞，第一次接近马赫7，第二次接近马赫10。**

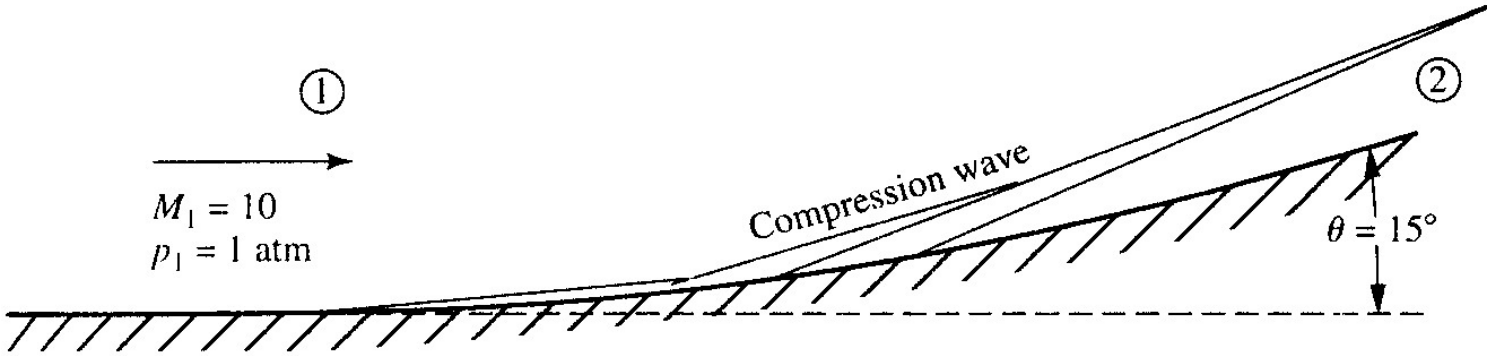
X51



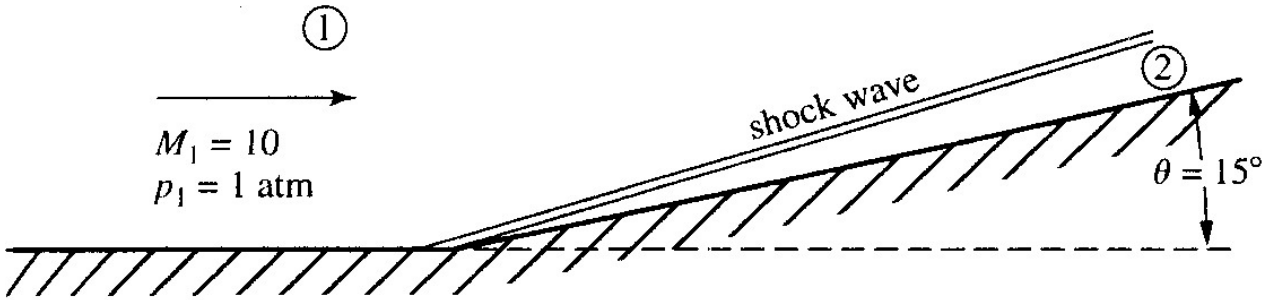
X51A

X-51A飞行器长4.3米，采用超燃冲压发动机。最近一次试飞在2013年5月，6分钟内达到了最高5.1马赫的飞行速度。

Example 9.10在前面的对超燃冲压发动机的讨论中，提到了等熵压缩波作为可能的压缩机制之一。考虑如图9.35a所示的等熵压缩。压缩波上游的马赫数和静压分别为：M_1=10，p_1=1atm。气流总的偏转角为15度。计算压缩波后区域2的马赫数和压强。



(*a*) Isentropic compression corner



(*b*) Shock compression corner

Fig 9.35 Figures for (a)Example 9.10 and (b) Example 9.11

由附录C，M_1=10 时， $\nu_1 = 102.3^0$

在区域2， $\nu_2 = \nu_1 - \theta = 102.3^0 - 15^0 = 87.3^0$

由附录C，根据 ν_2 的值，可得： $M_2 = 6.4$

由附录A，得到 $p_{0,1}/p_1$，$p_{0,2}/p_2$ 的值，即可得：

$$p_2 = \frac{p_2}{p_{0,2}} \frac{p_{0,2}}{p_{0,1}} \frac{p_{0,1}}{p_1} p_1$$

$$= \left(\frac{1}{0.2355 \times 10^4}\right)(1)(0.4244 \times 10^5)$$

$$= 18.02\text{atm}$$

$$p_{0,2} = p_{0,1} = 0.4244 \times 10^5 \text{atm} = 42440\text{atm}$$

Example 9.11 考虑如图9.34b所示的压缩凹角。上游的马赫数和静压与例9.10一样分别为：M_1 = 10，p_1 = 1atm。气流通过一道斜激波偏转角15度。计算激波后区域2的马赫数、静压和总压。与例9.10相比较并讨论比较的意义。

解: 由图9.9可得：$M_1 = 10$，$\theta = 15$度，对应

$$\beta = 20^0$$

$$M_{n,1} = M_1 \sin\beta = 10\sin 20^0 = 3.42$$

$M_{n,1} = 3.42$，查表B得

$$p_2/p_1 = 13.32, \quad p_{0,2}/p_{0,1} = 0.2322, \quad M_{n,2} = 0.4552$$

因此：

$$M_2 = \frac{M_{n,2}}{\sin(\beta-\theta)} = \frac{0.4552}{\sin(20^0 - 15^0)} = 5.22$$

$$p_2 = (\frac{p_2}{p_1}) p_1 = 13.32(\text{atm})$$

$$p_{0,2} = (\frac{p_{0,2}}{p_{0,1}})(\frac{p_{0,1}}{p_1}) p_1 = 0.2322 \times 0.4244 \times 10^5 \times 1 = 9.85 \times 10^3 (\text{atm})$$

例9.9： $p_2 = 18.02\text{atm}$

$$p_{0,2} = p_{0,1} = 0.4244 \times 10^5 \text{atm} = 42440 \text{ atm}$$

比较例9.9和9.10，我们可以明显地看出，等熵压缩是一个更高效的压缩，下游马赫数和压强都比斜激波下游对应的马赫数和压强高。等熵压缩没有总压损失，而斜激波总压损失高达77%。

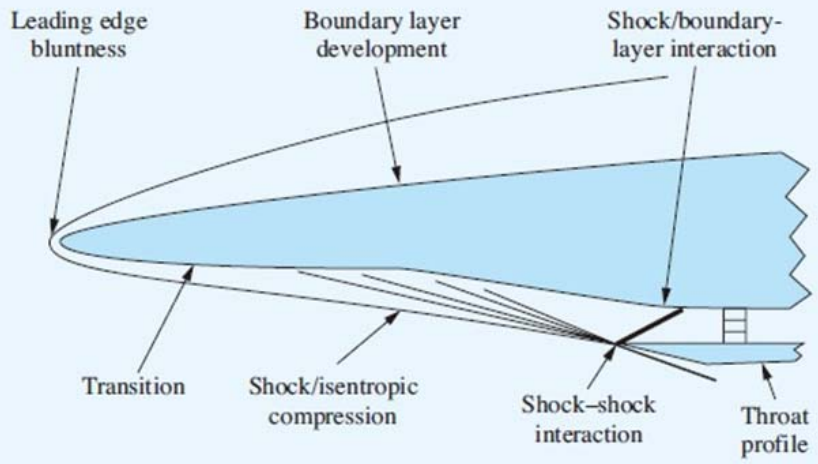


Figure 9.31 Sketch of some of the flow features on the forebody of a SCRAMjet-powered hypersonic vehicle.

这是超声速/高超声速进气道设计者为什么希望通过等熵压缩波压缩气流的原因。但是，就像我们针对超燃发动机的讨论提到的那样，等熵压缩在现实中是极其难以实现的，压缩表面的外形必须十分精准，是一个针对一个确定上游马赫数的点设计。在非设计马赫数下，最佳的压缩型面也会出现激波。

投票 最多可选1项

设置

前一次课的掌握情况投票

A 完全掌握了这部分知识内容

B 掌握了大部分

C 掌握了一小部分

D 完全不懂

提交

Review of Lecture # 11 Ended !

Lecture #12/第12次课

CHAPTER 9 Oblique Shock and Expansion Waves

第九章 斜激波和膨胀波

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2019年11月13日 Wednesday

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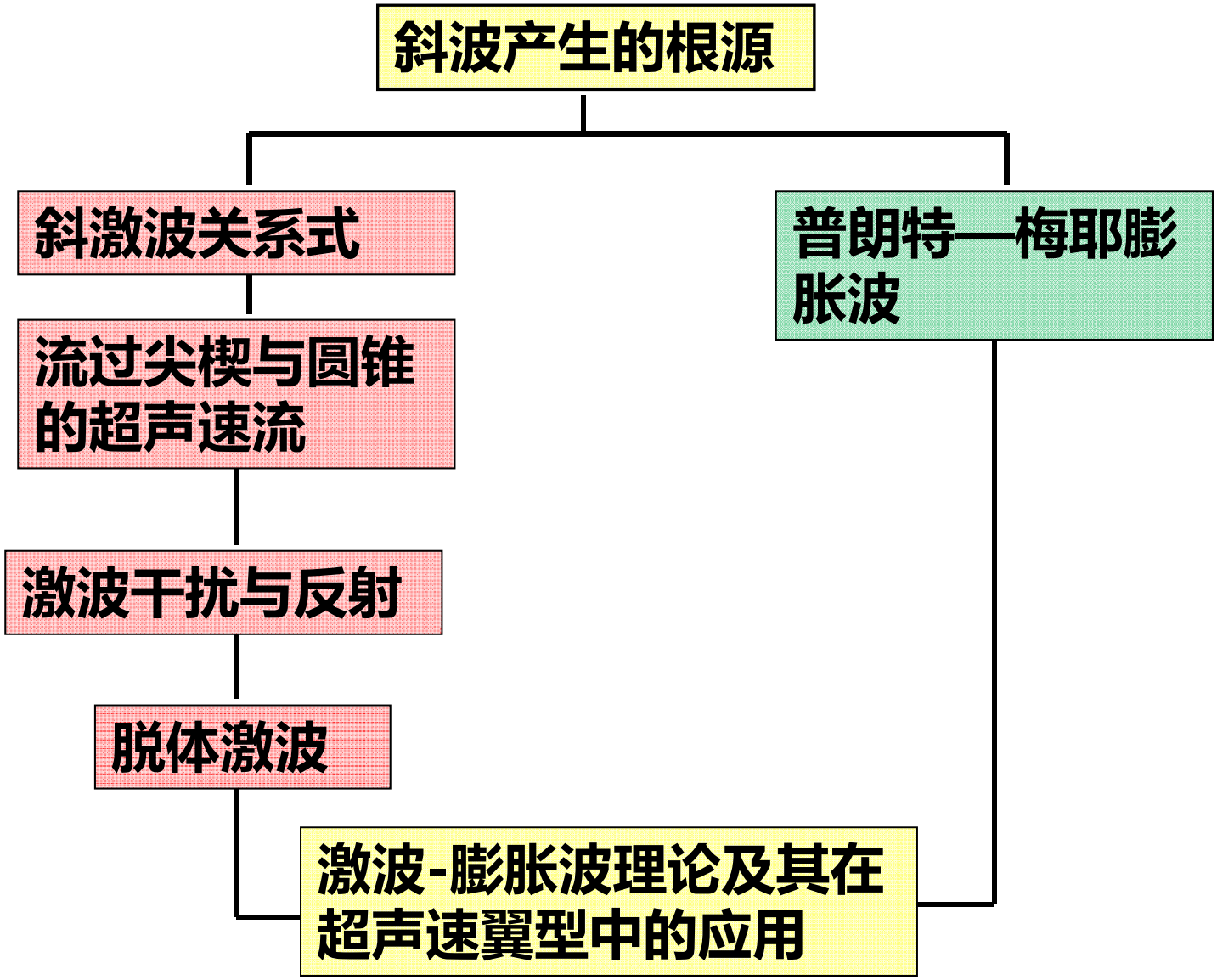
National Key Laboratory of Science and Technology on Aerodynamic Design and Research

Compressible Aerodynamics Course

第九章路线图


斜波产生的根源
斜激波关系式
流过尖楔与圆锥的超声速流
激波干扰与反射
脱体激波
普朗特—梅耶膨胀波
激波-膨胀波理论及其在超声速翼型中的应用

Chapter 9.7 Shock-Expansion Theory: Application to Supersonic Airfoils/激波-膨胀波理论及其对超声速翼型的应用

例1 平板翼型：

$$R' = (p_3 - p_2)c \quad (9.46)$$

$$L' = (p_3 - p_2)c \cdot \cos\alpha \quad (9.47)$$

$$D' = (p_3 - p_2)c \cdot \sin\alpha \quad (9.48)$$

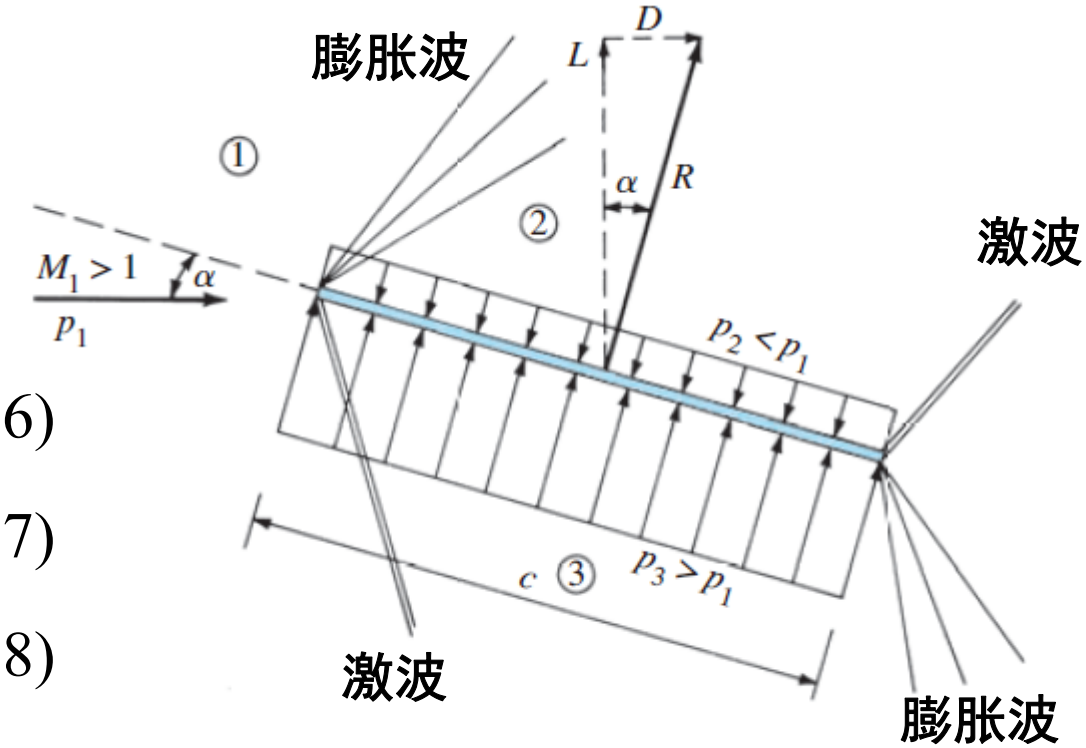


Figure 9.36 Flat plate at an angle of attack in a supersonic flow.

象这样由激波-膨胀波理论(*shock-expansion waves theory*)计算得到的解是精确解。

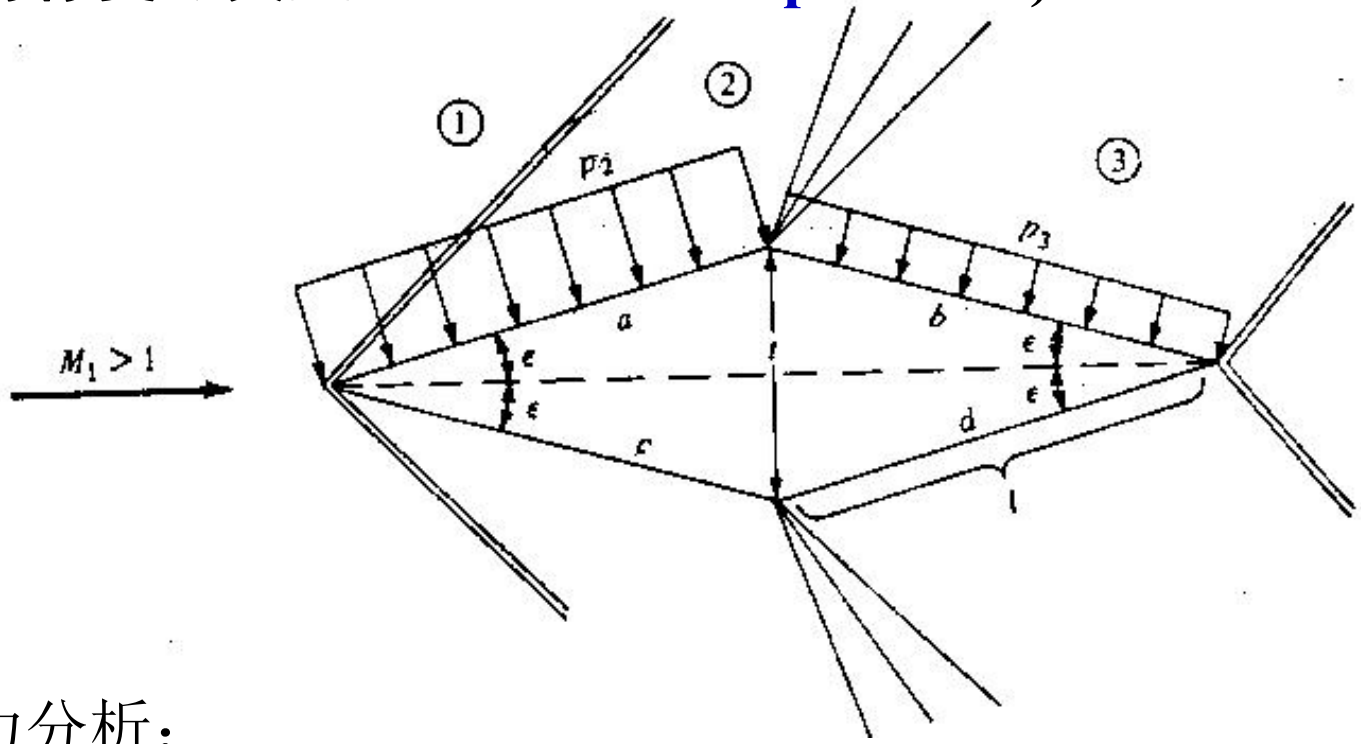
问题：什么情况下可以利用激波-膨胀波理论来求解翼型的气动特性？

Whenever we have a body made up of straight-line segments and the deflection angles are small enough so that no detached shock waves occur, the flow over the body goes through a series of distinct oblique shock and expansion waves, and the pressure distribution on the surface (hence the lift and drag) can be obtained *exactly* from both the shock- and expansion wave theories discussed in this chapter.

on P649 of the 5th edition

- 翼型是由直线段组成的，
- 流动偏转角足够小能保证没有脱体激波
- 绕翼型的超声速流动是由一系列斜激波、膨胀波组成的
- 因此，我们可以应用激波-膨胀波理论精确地求解翼型表面的压力分布进而翼型的升力和阻力。

例2：对称菱形翼型（Diamond-shape airfoil)



受力分析：

- a、c面压强均匀相等，用 p_2 表示，为压缩偏转角为 ε 的斜激波后的压强；
- b、d面压强均匀相等，用 p_3 表示,为膨胀偏转角为2ε 的膨胀波后的压强。

因为流动是上下 对称的，所以 L' =0；而由于 $p_2 > p_3$, 所以会有阻力分量 D' 。

$$D' = 2(p_2 l \sin\varepsilon - p_3 l \sin\varepsilon) = 2(p_2 - p_3)\frac{t}{2}$$

即：

$$D' = (p_2 - p_3)t \qquad (9.49)$$

(9.49)式中，p_2 由斜激波特性计算而得，p_3 由膨胀波特性计算而得。而且这些压强是超声速无粘流绕菱形翼型的精确值。

讨论：

- 无粘、超声速流动的一个非常重要的特征：二维翼型在超声速流中将受到一定的阻力。
- 即，在超声速流中，二维物体要受到阻力的作用，这一阻力被称为波阻。
  - 降低波阻是超声速翼型设计中的一个重要考虑因素。
  - 波阻的存在在本质上与翼型产生的激波有关，即与通过激波的熵增和总压损失有关。
- 在同样来流马赫数下，翼型厚度越大，其零升波阻越大。

单选题 1分

设置

超声速飞机的翼型是尖而薄的，如此设计翼型的主要目的是：

A 降低波阻

B 增大升力

C 降低气动热

D 增大绕焦点的俯仰力矩

提交

Example 9.11 计算来流马赫数为 3，迎角为 5°的平板翼型的升力系数和阻力系数。

解：

第一步, 首先计算上表面的 p_2/p_1. 由 M_1 =3, 查附表C，

得 $\nu_1 = 49.76^0$。由 $\nu_2 = \nu_1 + \theta$ 及 $\theta = \alpha$，得 $\nu_2 = 54.76^0$；

查附表C得M_2 = 3.27。

所以：
$$\frac{p_2}{p_1} = \frac{p_{0,1}}{p_1} \Big/ \frac{p_{0,2}}{p_2} = \frac{36.73}{55} = 0.668$$

其中：$p_{0,1}/p_1$ 与 $p_{0,2}/p_2$均由附表A查得。

第二步，计算下表面的p_3/p_1。由图9.9可知，对于

$M_1 = 3$， $\theta = \alpha = 5^0$ ，$\beta = 23.1^0$ ，因此

$$M_{n,1} = M_1 \sin\beta = 3\sin 23.1^0 = 1.177$$

查附表B，对于$M_{n,1} = 1.177$, $p_3/p_1 = 1.458$。

$$L' = (p_3 - p_2)c \cdot \cos\alpha$$

$$c_l = \frac{L'}{q_1 S} = \frac{L'}{\frac{\gamma}{2} p_1 M_1^{\ 2} c} = \frac{2}{\gamma M_1^{\ 2}}\left(\frac{p_3}{p_1} - \frac{p_2}{p_1}\right)\cos\alpha$$

$$= \frac{2}{(1.4)\cdot 3^2}(1.458 - 0.668)\cos 5^0 = 0.125$$

$$D' = (p_3 - p_2)c \cdot \sin\alpha$$

$$c_d = \frac{D'}{q_1 S} = \frac{D'}{\frac{\gamma}{2} p_1 M_1^{\ 2} c} = \frac{2}{\gamma M_1^{\ 2}}\left(\frac{p_3}{p_1} - \frac{p_2}{p_1}\right)\sin\alpha$$

$$= \frac{2}{(1.4)\cdot 3^2}(1.458 - 0.668)\sin 5^0 = 0.011$$

本例的阻力系数还可利用下面关系简便求解：

$$\frac{c_d}{c_l} = \tan\alpha$$

因此： $c_d = c_l \tan\alpha = 0.125 \tan 5^0 = 0.011$

例9.13

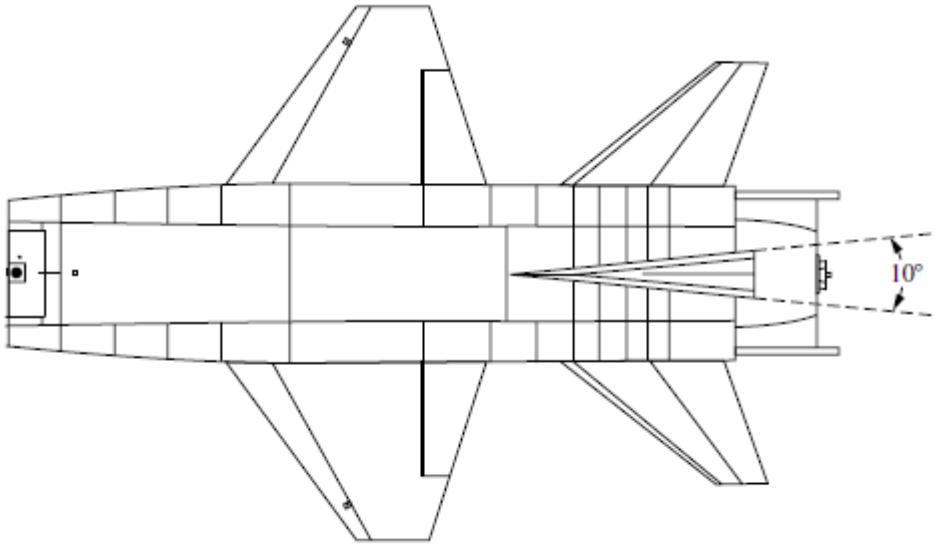


Figure 9.38 Illustration of the wedge-shaped tail on the X-15.

X15的垂尾（vertical tail）采用10度尖楔， NACA工程师C.H.McLellan 通过理论计算发现10度尖楔比超声速薄翼型效率更高。为什么？我们通过以下例子来回答这一问题。

例9.13 如图9.39*a*所示的平板和9.39b所示的10度顶角尖楔，均处于来流马赫数7，10度迎角的条件下。(a)计算平板的升力系数；(b)计算尖楔的升力系数。

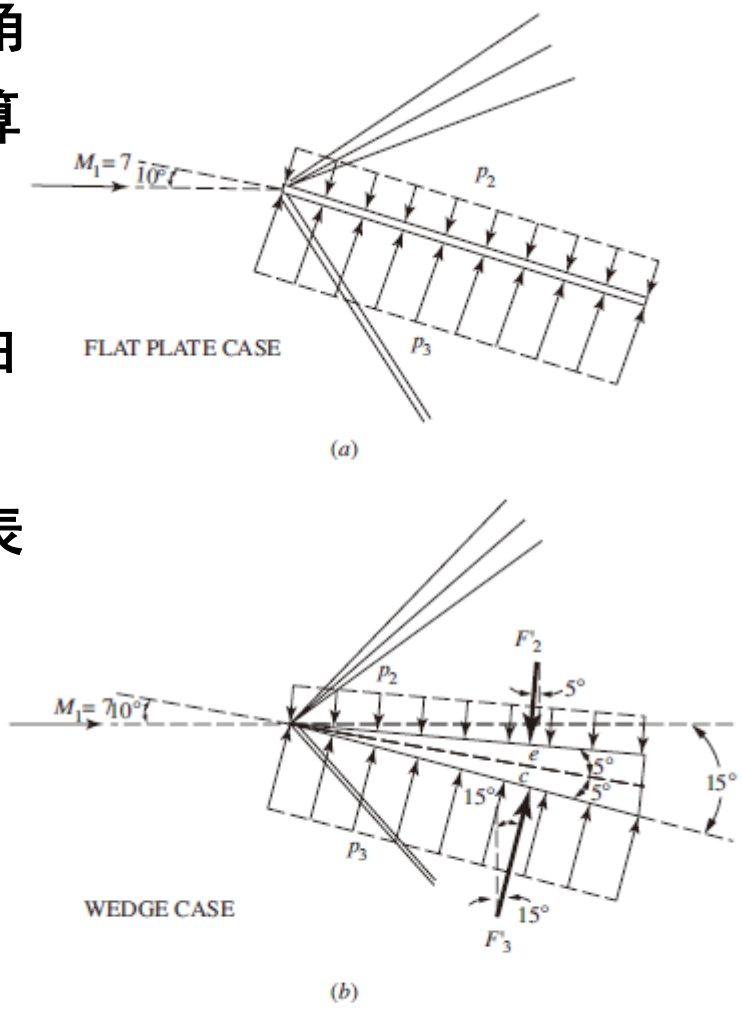


Figure 9.39 Schematic of hypersonic flow over (*a*) a flat plate, and (*b*) a wedge, both at a 10° angle of attack. Not to scale.

解：(*a*)首先计算上表面膨胀波后的 p_2/p_1. 由 $M_1=7$, 查附表C，得 $\nu_1 = 90.97^0$ 。由及 $\theta = \alpha$ ，得 $\nu_2 = \nu_1 + \theta$ ；查附表C得： $\nu_2 = 100.97^0$

$$M_2 = 9 + \frac{100.97 - 99.32}{102.3 - 99.32}(1) = 9.56$$

由公式： $\frac{p_0}{p} = (1 + \frac{\gamma - 1}{2} M^2)^{\frac{\gamma}{\gamma-1}}$

$$\frac{p_2}{p_1} = \frac{p_{0,1}}{p_1} \Big/ \frac{p_{0,2}}{p_2} = \frac{4140}{31461} = 0.1316$$

第二步，计算下表面的p_3/p_1。由图9.9可知，对于$M_1=7$，$\theta=\alpha=10^0$，$\beta=16.5^0$，因此

$$M_{n,1}=M_1\sin\beta=7\sin 16.5^0=1.99$$

由公式 $\frac{p_3}{p_1}=1+\frac{2\gamma}{\gamma+1}(M_{n,1}{}^2-1)$ 得： $\frac{p_3}{p_1}=4.4534$

$$L'=(p_3-p_2)c\cdot\cos\alpha$$

$$c_l=\frac{2}{\gamma M_1{}^2}\left(\frac{p_3}{p_1}-\frac{p_2}{p_1}\right)\cos\alpha$$

$$=\frac{2}{(1.4)\cdot 7^2}(4.4534-0.1316)\cos 10^0=0.124$$

(*b*)首先计算上表面膨胀波后的 p_2/p_1. 由 膨胀波后气流偏转角 $\theta = 5^o$ ，查附表C，

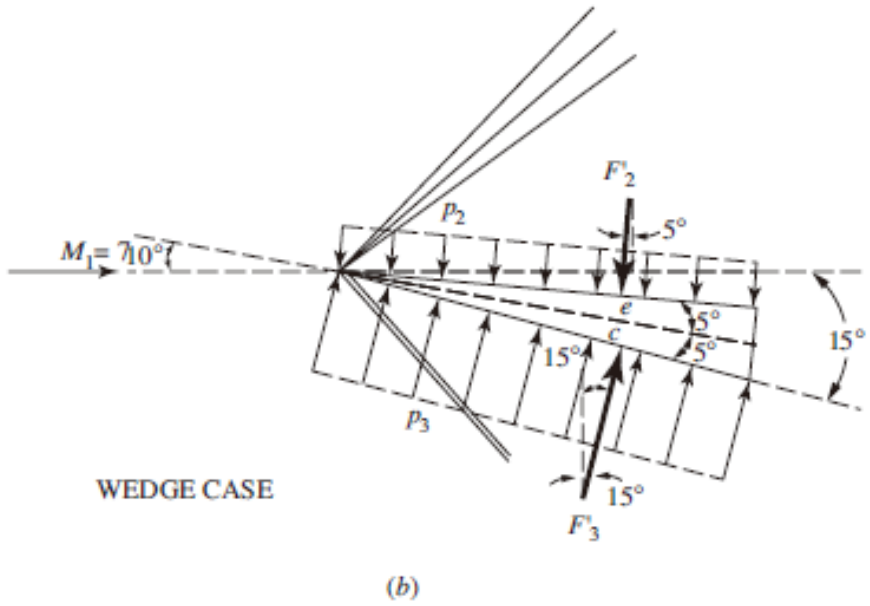
得 $\nu_2 = \nu_1 + \theta = 90.97^o + 5^o = 95.97^o$ 。

查附录C并插值得:

$$M_2 = 8 + \frac{95.97 - 96.62}{99.32 - 95.62}(1) = 8.1$$

由 $$\frac{p_0}{p} = (1 + \frac{\gamma - 1}{2} M^2)^{\frac{\gamma}{\gamma - 1}}$$

$$\frac{p_2}{p_1} = \frac{p_{0,1}}{p_1} \Big/ \frac{p_{0,2}}{p_2} = \frac{4140}{10583} = 0.3912$$



Note: Angles not to scale

Figure 9.39 Schematic of hypersonic flow over (*a*) a flat plate, and (*b*) a wedge, both at a 10° angle of attack. Not to scale.

作用于上表面的垂直物面的力为： $$F_2' = p_2 l = (\frac{p_2}{p_1}) p_1 l = 0.3912 p_1 l$$

计算下表面斜激波后的p_3/p_1。由图9.9可知，对于M_1=7，$\theta = \alpha + 5^o = 15^0$ ，

因此 $\beta = 23.5^0$ ，

$$M_{n,1} = M_1 \sin\beta = 7\sin 23.5^0 = 2.79$$

由公式 $\dfrac{p_3}{p_1} = 1 + \dfrac{2\gamma}{\gamma+1}(M_{n,1}{}^2 - 1)$ 得：$\dfrac{p_3}{p_1} = 8.9148$

作用于尖楔下表面的垂直物面的力为： $F_3' = p_3 l = (\dfrac{p_3}{p_1}) p_1 l = 8.9148 p_1 l$

升力为F_2'，F_3'在垂直于来流方向的合力：

$$\begin{aligned} L' &= F_3'\cos 15^o - F_2'\cos 5^o = 0.9659\,F_3' - 0.9962\,F_2' \\ &= \left[0.9659\,(8.9148) - 0.9962\,(0.3912)\right] p_1 l \\ &= 8.2211\,p_1 l \end{aligned}$$

$$l = \frac{c}{\cos 5^o} = 1.0038\,c$$

$$L' = 8.2211\,p_1 l = 8.2424\,p_1 c$$

$$c_l = \frac{L'}{q_1 c} = \frac{2L'}{\gamma M_1^{\ 2} p_1 c} = \frac{2(8.2424)\,p_1 c}{1.4(7^2)\,p_1 c} = 0.2403$$

对于垂尾而言，升力就是侧向力，很明显，尖楔提供的控制偏航的力矩比平板代表的薄翼型更大。

单选题 1分

设置

我们在第一章量纲分析中学过，物体的升力系数和阻力系数由物体的形状、迎角、马赫数、雷诺数确定。如果忽略粘性，物体的升力系数和阻力系数由什么确定？

A 物体形状、马赫数

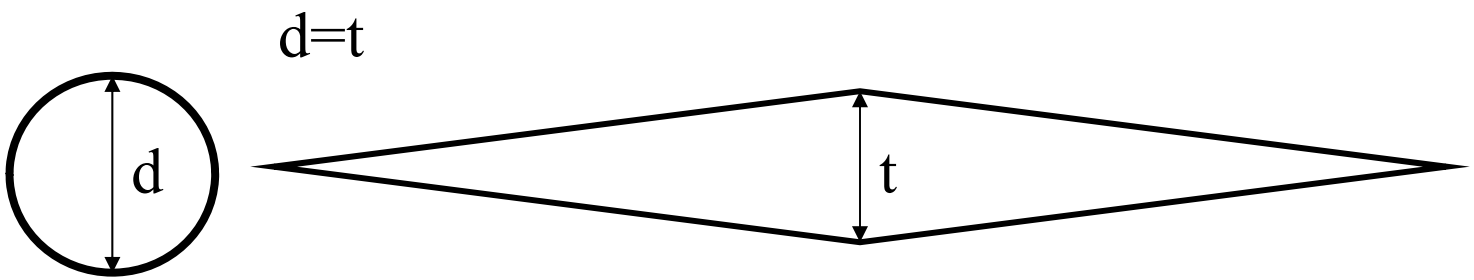
B 物体形状、迎角、马赫数

C 物体形状、迎角

提交

补充例题：Problem 9.16

习题9.16 考虑一个体轴与来流垂直的圆柱体和一个迎角为零、半顶角为5°的对称菱形翼型。菱形翼型厚度与圆柱的直径相同。来流马赫数为5时，圆柱的阻力系数为4/3（基于迎风投影面积），计算在相同来流马赫数5时圆柱阻力与对称菱形翼型的阻力的比。由本题计算结果（比较超声速流中的钝头体和尖头细长体的气动性能），可以得出什么结论？



解：

对于基于迎风投影面积的阻力系数为C_d的圆柱体，其阻力为：

$$\left(D'\right)_{cyl} = q_\infty S C_d = q_\infty d\left(l\right)\left(4/3\right) = \frac{4}{3}\left(d\right)q_\infty$$

对于对称菱形翼型：

$$\left(D'\right)_w = \left(p_2 - p_3\right)t$$

$$\frac{\left(D'\right)_{cyl}}{\left(D'\right)_w} = \frac{\frac{4}{3}\left(d\right)q_\infty}{\left(p_2 - p_3\right)t}$$

$$\because\ d = t, \quad q_\infty = \frac{\gamma}{2} p_1 M_1^2$$

$$\therefore \quad \frac{(D')_{cyl}}{(D')_w} = \frac{\frac{4}{3}\left(\frac{\gamma}{2}\right) M_1^2}{\left(\frac{p_2}{p_1} - \frac{p_3}{p_1}\right)} = \frac{\frac{2}{3}\gamma M_1^2}{\left[\frac{p_2}{p_1} - \frac{p_3}{p_1}\right]}$$

To calculate p_2/p_1, we have, for M_1 =5 and $\theta = 5^\circ \ \beta = 15.1^\circ$

$$M_{n,1} = M_1 \sin\beta = 5\sin(15.1^\circ) = 1.303$$

From Appendix B,for $M_{n,1} = 1.303$, $\dfrac{p_2}{p_1} = 1.805$

Also, $$M_2 = \frac{M_{n,2}}{\sin(\beta - \theta)} = \frac{0.786}{\sin(15.1 - 5)} = 4.48$$

To calculate $\dfrac{p_3}{p_1}$ **, the flow is expanded through an angle of** 10°

From Table C,for $M_2 = 4.48 \rightarrow \nu_2 = 71.83$

$$\nu_3 = \nu_2 + \theta = 71.83 + 10 = 81.83^\circ$$

Hence, $M_3 = 5.6$ **(nearest entry)**

From Appendix A: **for** $M_1 = 5:$ $\dfrac{p_{01}}{p_1} = 529.1$

for $M_3 = 5.6:$ $\dfrac{p_{03}}{p_3} = 1037$

From Appendix B: For $M_{n,1} = 1.303:$ $\dfrac{p_{02}}{p_{01}} = 0.9794$

Thus,

$$\frac{p_3}{p_1} = \frac{p_3}{p_{03}}\frac{p_{03}}{p_{02}}\frac{p_{02}}{p_{01}}\frac{p_{01}}{p_1}$$

$$= \left[\frac{1}{1037}\right](1)(0.9794)(529.1) = 0.5$$

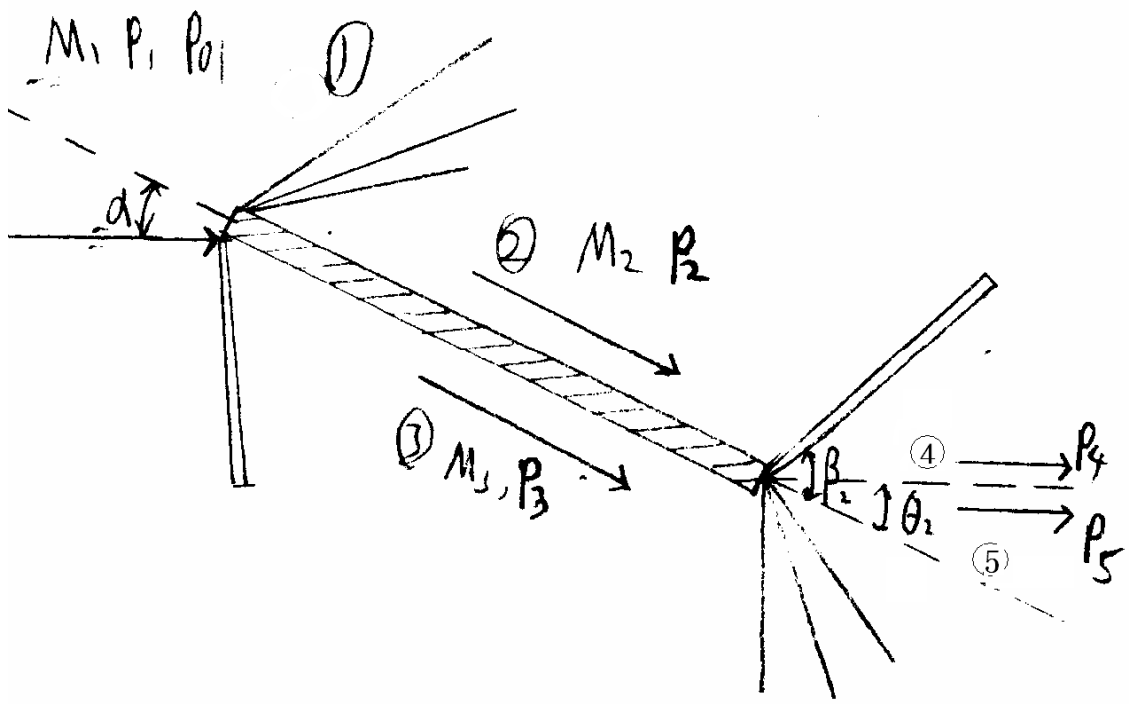
因此

$$\frac{\left(D'\right)_{cyl}}{\left(D'\right)_{w}}=\frac{\frac{2}{3}\gamma M_1^2}{\left[\frac{p_2}{p_1}-\frac{p_3}{p_1}\right]}=\frac{\frac{2}{3}(1.4)(5)^2}{(1.805-0.5)}=17.9$$

提示：钝头体的阻力要大得多，这就是我们为什么在超声速飞行器中避免使用钝头前缘的原因。（而在高超声速条件下，钝头前缘可以减小气动热）

补充例题：problem 9.17

考虑如图9.36所示的超声速绕平板流动，在平板翼型后缘的气流速度方向与来流方向近似（不是精确地）一致，为什么？给出确定后缘处激波、膨胀波后流动特性以及流动方向的方法。



原因：通过膨胀波和激波的压强变化不是线性的，上表面膨胀波、激波强度和下表面膨胀波、激波强度是不同的，因此如果气流回到原方向不能保证$p_4=p_5$的条件。

反证：假设气流回到原方向

$$M_1 = 2.6 \quad , \quad \alpha = 15^\circ$$

上表面：

从①区—②区，经过膨胀波， $p_{01} = p_{02}$

由 $M_1 = 2.6$ **查附表C得：** $\nu_1 = 41.41^0$

$\nu_2 = \nu_1 + \theta_1 = \nu_1 + \alpha = 56.41^0$ **查附表C得：** $M_2 = 3.37$

由 $M_1 = 2.6, M_1 = 3.37$ 查表A得: $\dfrac{p_{0,1}}{p_1} = 19.95, \dfrac{p_{0,2}}{p_2} = 63.33$

$$p_2 = \frac{p_2}{p_{0,2}} \times \frac{p_{0,2}}{p_{0,1}} \times \frac{p_{0,1}}{p_1} \times p_1 = 0.315 p_1$$

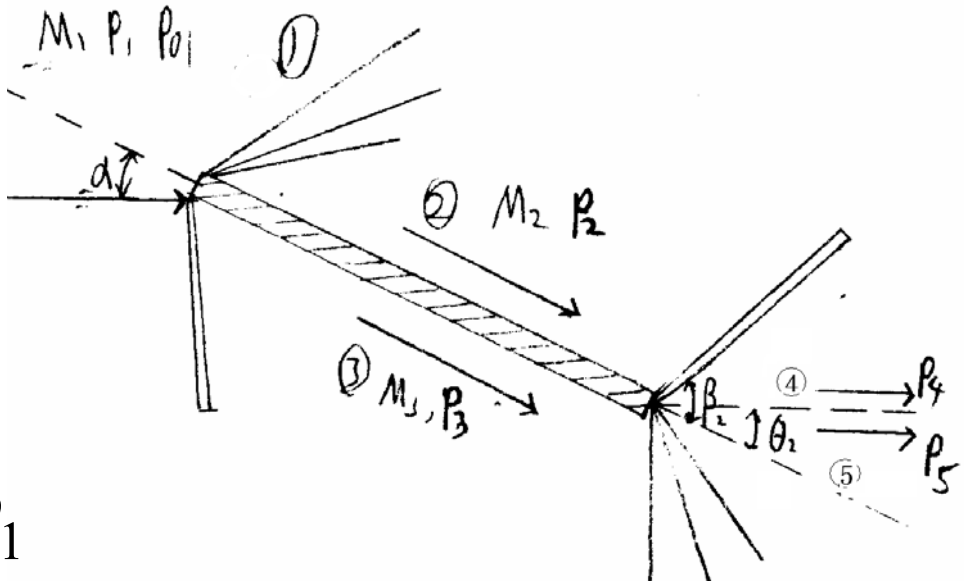
从②区—④区，经过斜激波，如果假设气流回到来流方向，则

$$\theta_2 = \alpha = 15^o \quad M_2 = 3.37$$

查 $\theta - \beta - M$ 图可知：$\beta_2 = 30^\circ$

$$M_{n2} = M_2 \times \sin 30^\circ = 1.685$$

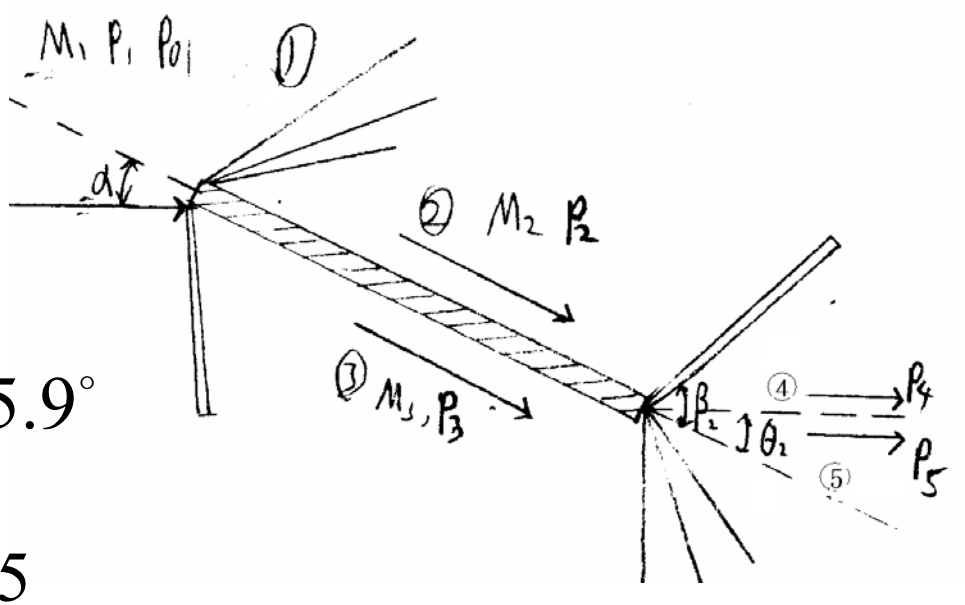
由附表B得：



$$\frac{p_4}{p_2} = 3.146$$

$$p_4 = \frac{p_{4'}}{p_2} \times p_2 = 0.99 p_1$$

下表面：



从①区—③区，经过斜激波，

$$\theta_1 = \alpha = 15^o$$

查 $\theta - \beta - M$ 图可知：$\beta_1 = 35.9^\circ$

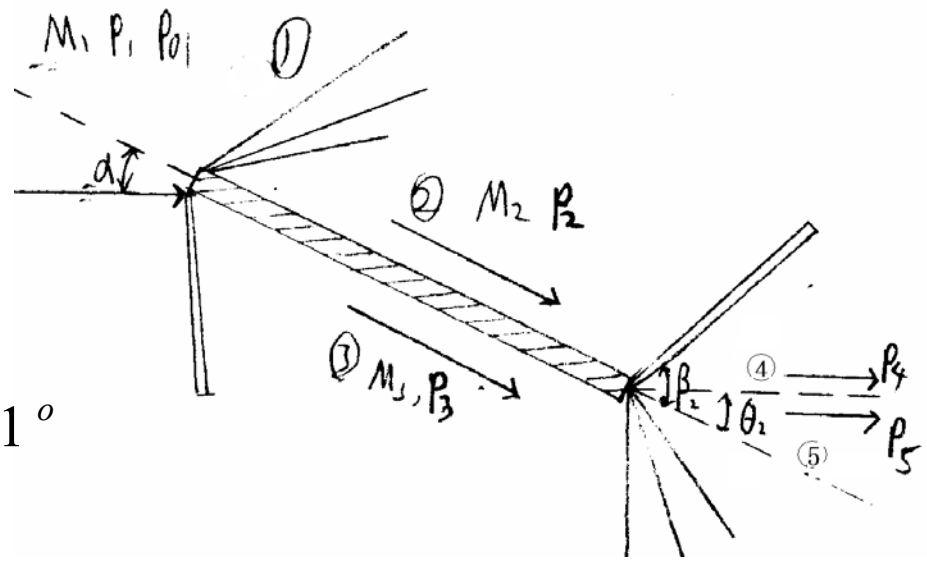
$$M_{n1} = 2.6 \times \sin 35.9^\circ = 1.525$$

由附表B得： $\dfrac{p_3}{p_1} = 2.547,\quad M_{n3} = 0.6941$

$$M_3 = \frac{M_{n3}}{\sin(\beta_1 - \theta_1)} = 1.94 \qquad p_3 = 2.547\, p_1$$

从③区—⑤区，经过膨胀波，

$$p_{0,3} = p_{0,5}$$



由 $M_3 = 1.94$ 查附表C得:

$$\nu(M_3) = 24.71^o$$

$$\nu(M_5) = \nu(M_3) + \theta_2 = 39.71^o$$

查附表C得： $M_5 = 2.53$

$M_3 = 1.94$ $M_5 = 2.53$ 查表A得: $\dfrac{p_{0,3}}{p_3} = 7.128, \dfrac{p_{0,5}}{p_5} = 17.78$

$$p_5 = \frac{p_5}{p_{0,5}} \times \frac{p_{0,5}}{p_{0,3}} \times \frac{p_{0,3}}{p_3} \times p_3 = 1.02 p_1$$

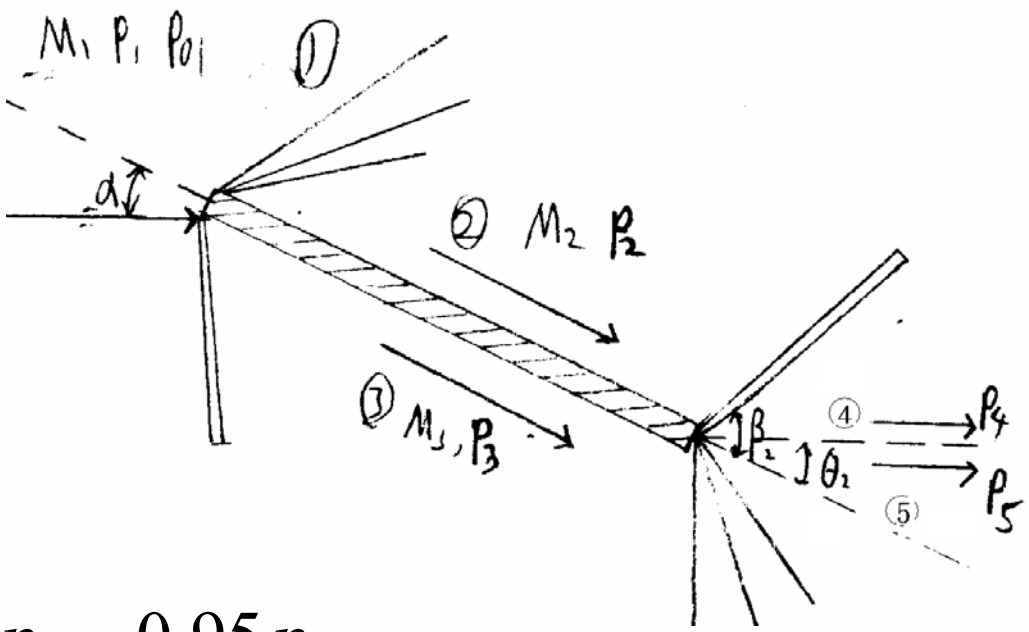
$$p_4 = 0.99 p_1 < p_5 = 1.02 p_1$$

而滑移线两侧压力应相等，所以 $\theta_2 \neq 15^o$

即:经平板后缘后，气流方向与来流方向并不精确一致。

若假设： $\theta_2 = 16^o$

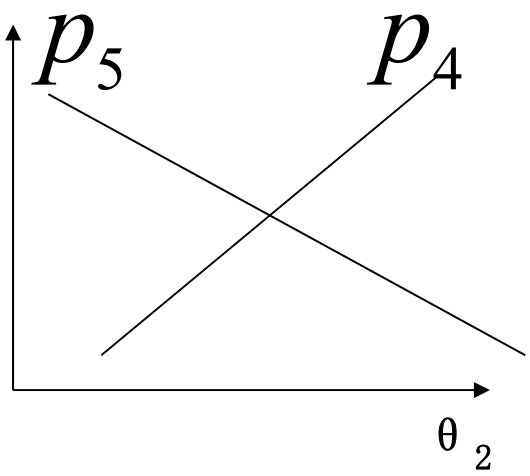
重复上面过程，则得：



$$p_4 = \frac{p_4}{p_2} \times p_2 = 1.03 p_1$$

$$p_5 = \frac{p_5}{p_{0,5}} \times \frac{p_{0,5}}{p_{0,3}} \times \frac{p_{0,3}}{p_3} \times p_3 = 0.95 p_1$$

$$p_4 > p_5$$



θ_2 一定在15^0和16^0之间，可根据插值得到。

9.9 粘性流:激波/边界层干扰

无粘假设下，图9.19的B点为一逆压梯度为无穷的奇点。如果考虑气体粘性，壁面存在附面层，会出现什么现象？

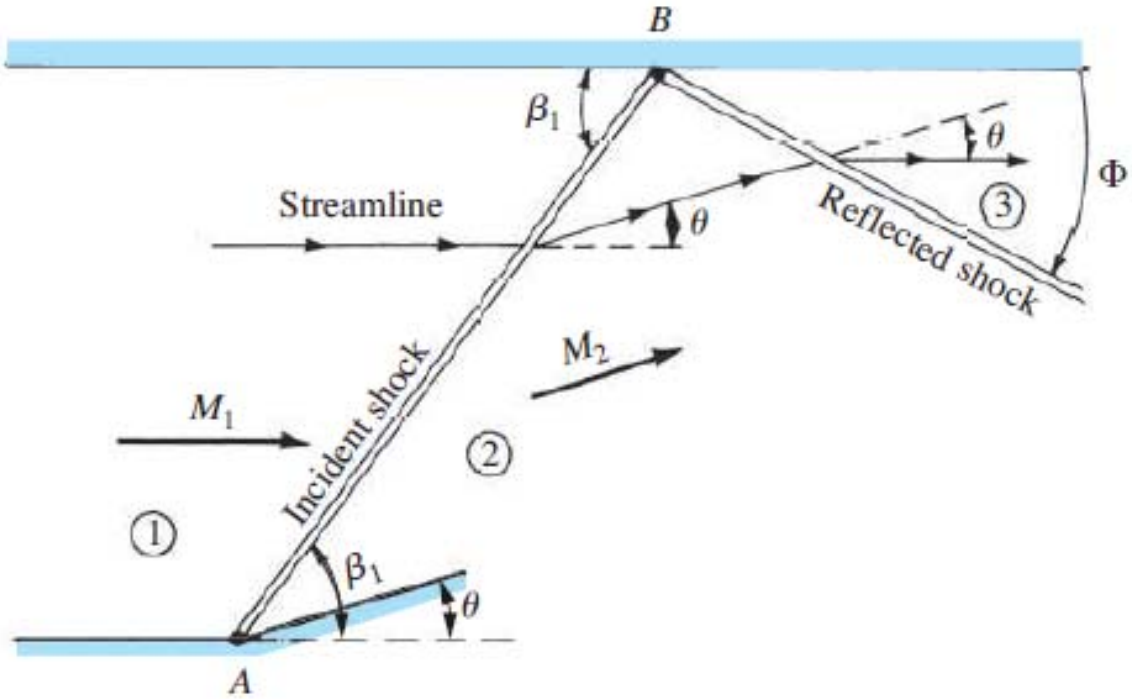


Figure 9.19 Regular reflection of a shock wave from a solid boundary.

粘性流:激波/边界层干扰

附面层在大的逆压梯度下会分离!

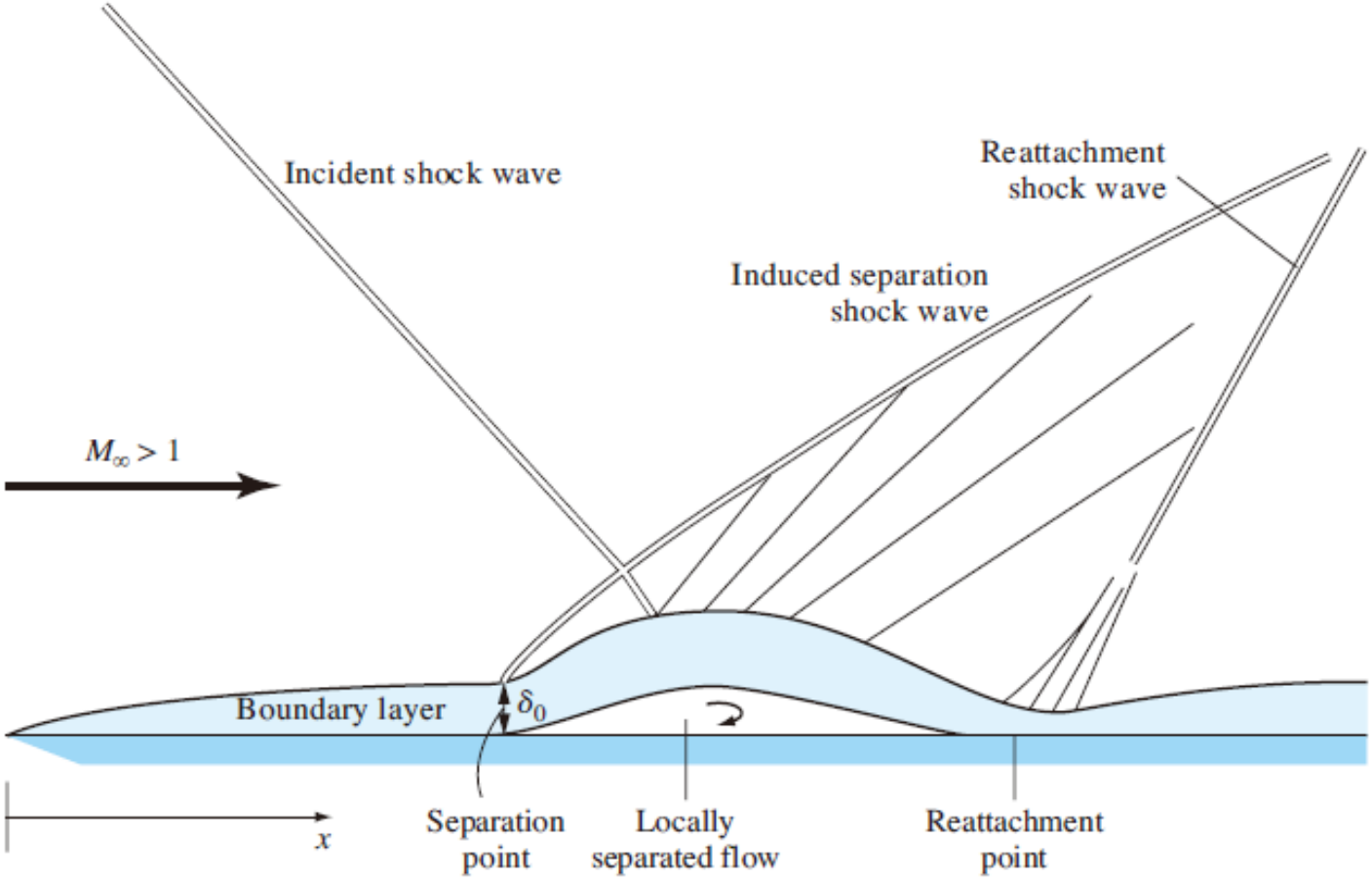


Figure 9.40 Schematic of the shock-wave/boundary-layer interaction.

粘性流：激波/边界层干扰

- 边界层分离引起外流向来流方向偏转（压缩），引起第二个激波的出现——分离诱导激波（Induced separation shock wave）
- 分离的边界层向平板折回，再附于分离点下游的某点。（Reattachment point）
- 再附引起边界层外的超声速流动又一次向来流方向偏转，形成第三个激波——再附激波（Reattachment shock wave）
- 在分离激波和再附激波之间，超声速气流向远离自身方向偏转，产生如图9.40所示膨胀波

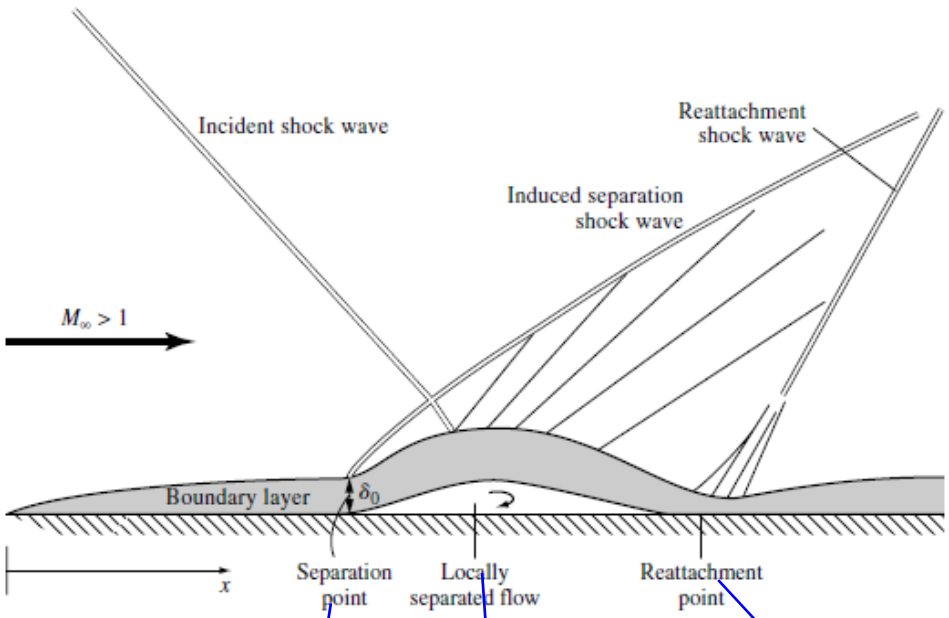


Figure 9.40 Schematic of the shock-wave/boundary-layer interaction.

再附点

分离点

当地分离流

- 图9.40所示的激波/边界层干扰的规模和严重程度取决于流动流态是层流还是湍流。由于层流与湍流相比更容易分离，因此干扰更严重。但层流激波/边界层干扰和湍流激波/边界层干扰的定性特征是一致的。

- 激波/边界层干扰对沿壁面的压力分布、剪切应力分布和热传导率分布都有重要影响。

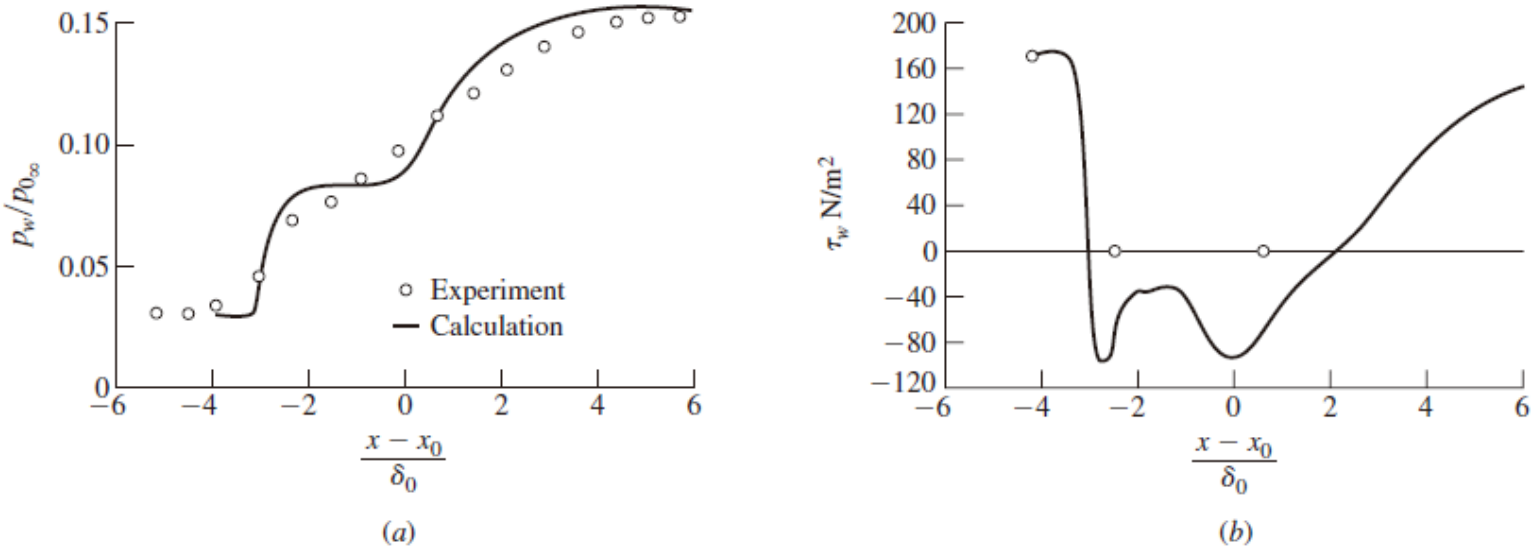


Figure 9.41 Effects of shock-wave/boundary-layer interaction on (*a*) pressure distribution, and (*b*) shear stress for Mach 3 turbulent flow over a flat plate.

图9.41中x_0为无粘流假设下理论入射激波与壁面的交点

Chapter 9.12 Summary/小结

超声速多维流动中的无限微弱扰动产生与来流夹角为马赫角 μ 的马赫波。马赫角的定义如下：

$$\mu = \sin^{-1}\frac{1}{M} \qquad (9.1)$$

Chapter 9.12 Summary/小结（续）

通过斜激波流动特性的变化由斜激波前的法向速度分量决定。对于量热完全气体，上游法向马赫数是决定性参数。通过斜激波的流动参数变化可利用第8章中的正激波关系式对应波前法向马赫数$M_{n,1}$求得。

$$M_{n,1} = M_1 \sin\beta \qquad (9.13)$$

通过斜激波的气体特性变化取决于两个参数，M_1, β 或 M_1, θ。图9.9给出了M_1, β, θ曲线，必须仔细地研究它。图9.9对应的θ-β-M关系式为：

$$\boxed{\tan\theta = 2\cot\beta \frac{M_1^2 \sin^2\beta - 1}{M_1^2(\gamma + \cos 2\beta) + 2}} \qquad (9.23)$$

Chapter 9.12 Summary/小结（续）

斜激波入射到固壁表面上将会从表面反射，反射波以保证物面处流动相切条件的形式出现。不同斜激波会相互干扰，其干扰结果取决于激波的具体形式。

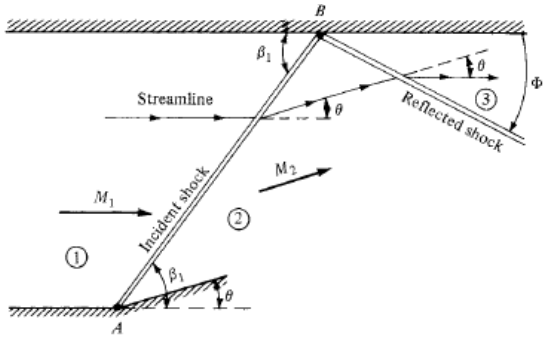


Figure 9.19 Regular reflection of a shock wave from a solid boundary.

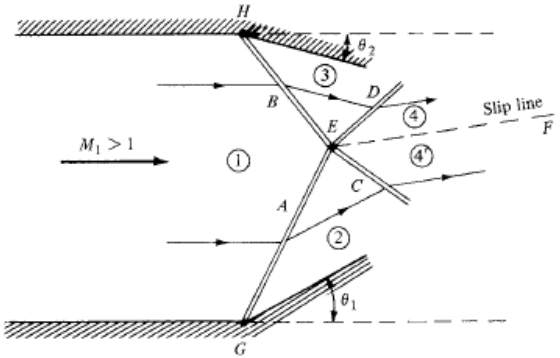
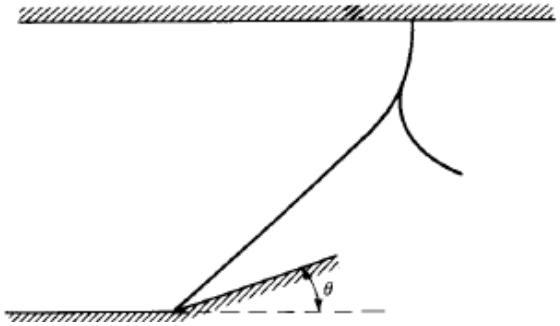


Figure 9.21 Intersection of right- and left-running shock waves.



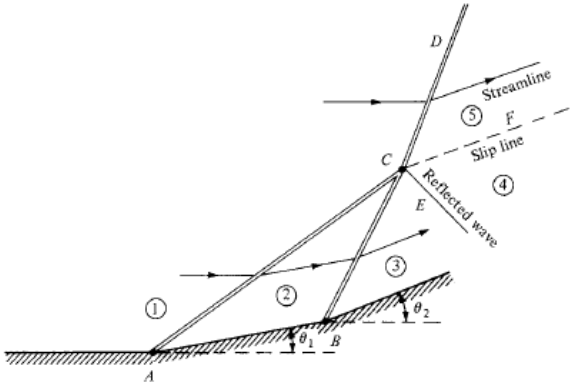


Figure 9.22 Intersection of two left-running shock waves.

Chapter 9.12 Summary/小结（续）

精确描述膨胀波内部变化的微分方程：

$$\boxed{d\theta = \sqrt{M^2 - 1}\,\frac{dV}{V}} \qquad (9.32)$$

普朗特 - 梅耶函数 $\nu(M)$：

$$\boxed{\nu(M) = \sqrt{\frac{\gamma+1}{\gamma-1}}\tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}(M^2-1)} - \tan^{-1}\sqrt{M^2-1}} \qquad (9.42)$$

决定中心膨胀波的参数是普朗特 - 梅耶函数 $\nu(M)$。联系下游马赫数M_2、上游马赫数M_1及偏转角θ的重要方程是：

$$\theta = \nu(M_2) - \nu(M_1) \qquad (9.43)$$

Chapter 9.12 Summary/小结（续）

由直线段组成的超声速翼型的压强分布可以用斜激波、膨胀波理论精确地计算出来。

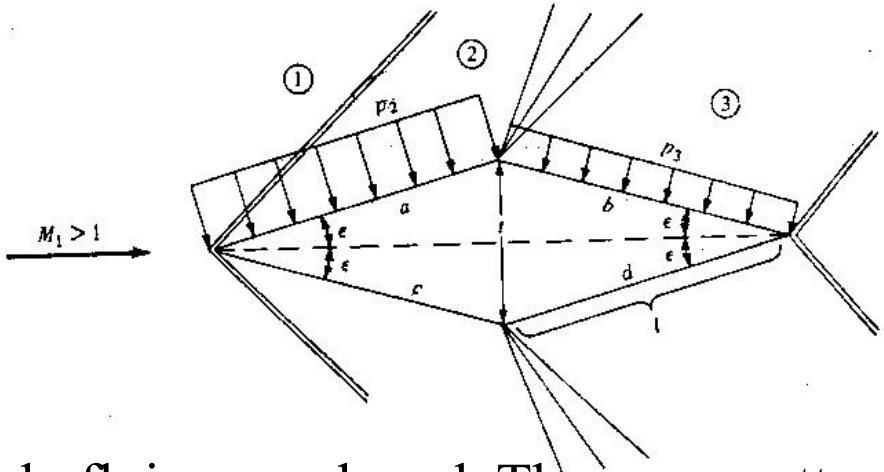
以上论述需满足的条件：

（1）流动偏转角足够小能保证没有脱体激波

（2）绕翼型的超声速流动是由一系列斜激波、膨胀波组成的

补充：声爆的概念

当飞机做超声速飞行时，空气受到扰动而产生激波。激波传播到地面，地面观察者会听到类似爆炸的声音，被称为“声爆”。



Imagine this body flying overheard. The wave pattern from the bottom of the airfoil propagates toward the ground, and when it sweeps past you standing on the ground, your eardrums pick up the pressure changes across the waves, creating a booming sound. What you hear is the “sonic boom.” The body is dragging this wave pattern with it as it flies through the atmosphere, and hence the “sonic boom” is sweeping over the ground at the same speed as the body is flying through the air.

N波——声压在某一瞬间的空间分布

Two shock waves are produced, one at the nose of the body, and one at the tail of the body. These two shocks propagate downward with the expansion wave contained between them. Also shown in Figure 9.43 is the change in pressure, Δp through the wave pattern. At large distances below the body, the variation in Δp shows a jump increase across the bow wave, an almost linear decrease between the bow and tail waves, and another jump increase across the tail wave. The variation of Δp through the wave pattern resembles the **capital letter N**, and for this reason the sonic boom pressure ave is called **an "N-wave."**

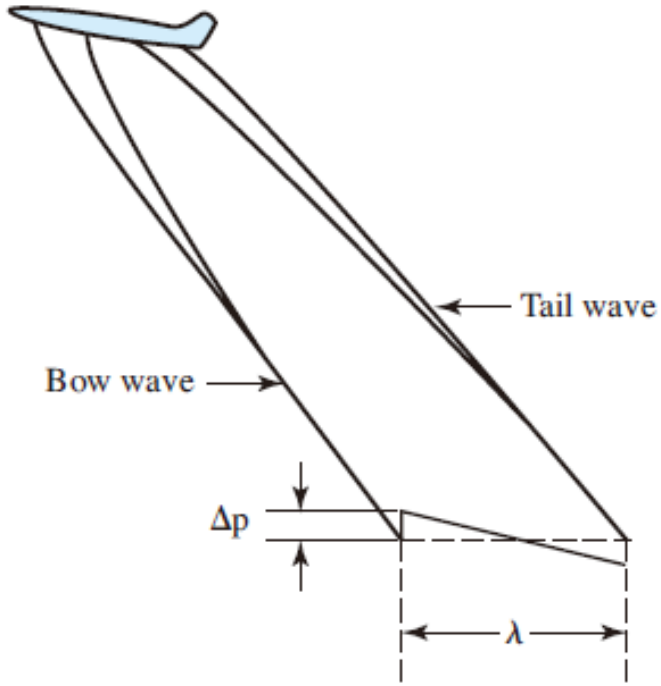
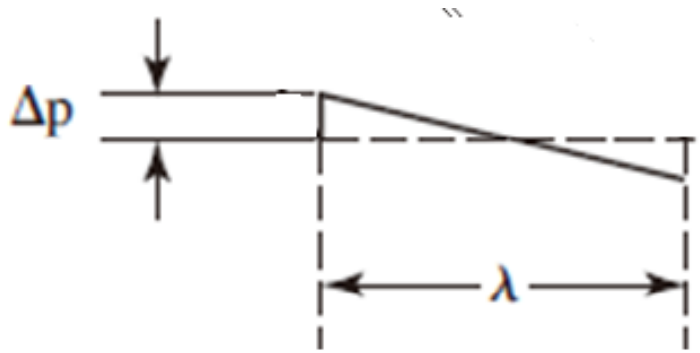


Figure 9.43 Classic N-wave sonic boom generation.

Note that when this N-wave sweeps past you on the ground, you hear two booms separated in time by $t = \lambda/V_\infty$, where λ is the distance along the ground between the bow and tail waves, and V_∞ is the flight velocity of the supersonic body. In this case, what you hear as the "sonic boom" is in reality two booms very closely spaced in time, the so-called classic double-boom signature. For example, the Concorde supersonic transport cruised at a Mach number of 2 at 50,000 ft, where the standard atmospheric temperature is 390◦R. The speed of sound at this temperature is a =968 ft/s, hence $V\infty = (968)(2) = 1936$ ft/s. Assume the distance across the N-wave at ground level is 200 ft. Then the time interval between the two booms would be $t = \lambda/V\infty = 200/1936 = 0.103$ s. This is equivalent to the response time sensed by the human ear, and therefore two distinct booms will be heard, the double-boom. However, if t is shorter, the average ear cannot distinguish between the two pulses shown in Figure 9.43, and the sonic boom will be heard as only one boom.

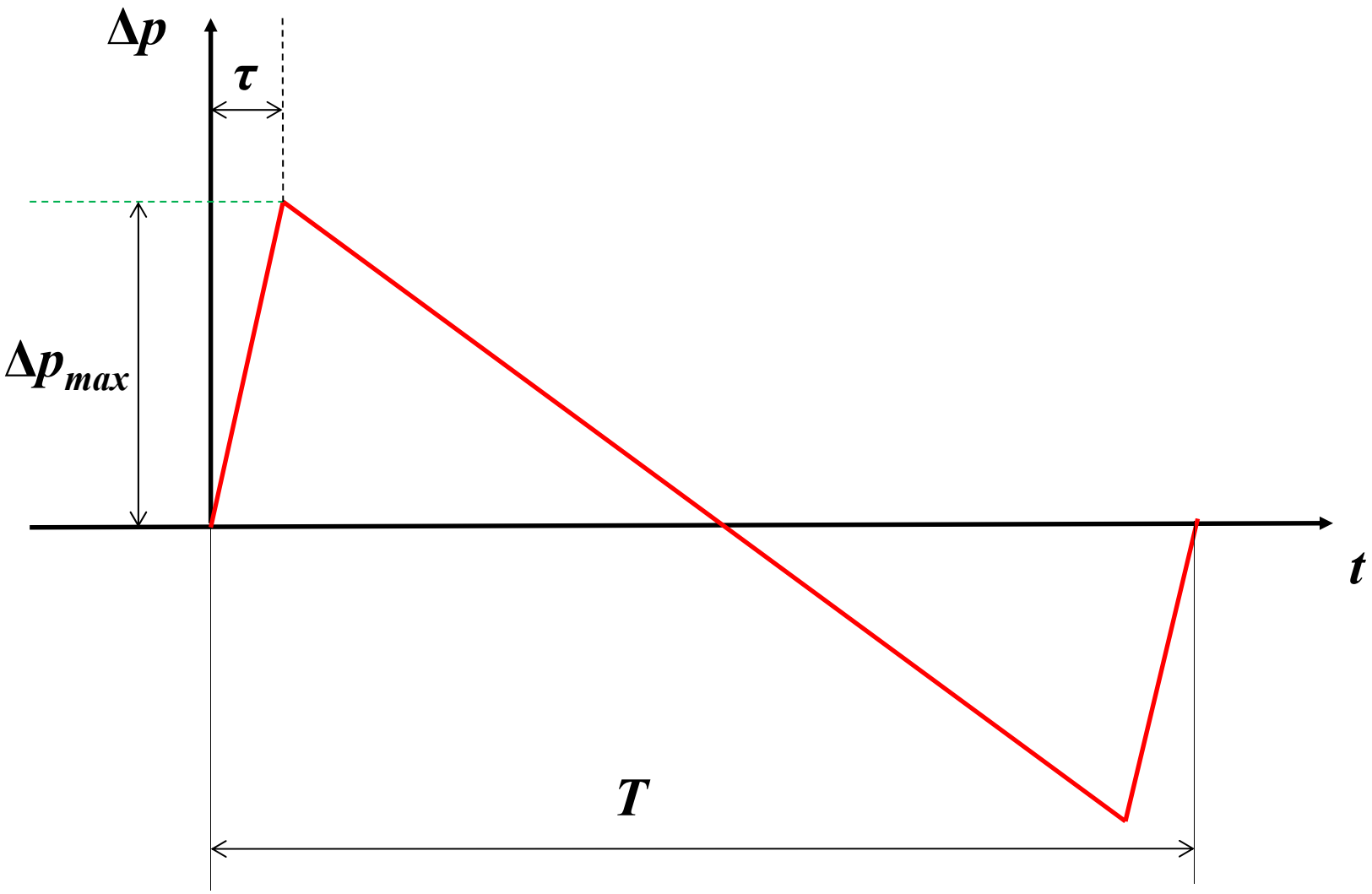
a =968 ft/s=295m/s

$V\infty = (295)(2) = 590$m/s



The most critical aspect of the sonic boom is the environmental impact caused by the magnitude of Δp felt on the ground. In the early days of supersonic flight, the sonic booms generated by airplanes sometimes exceeded an increase in pressure greater than $\Delta p = 2$ lb/ft^2 （95.76Pa/m^2), greatly affecting human health and causing structural damage (broken windows, crackedwalls, etc.). At the time of the design of the Concorde SST during the 1960s, a maximum value of $\Delta p = 2$ lb/ft2 was considered acceptable. This was an error in judgment because soon after the Concorde went into service with British Airways and Air France, nations all over the world (including the United States) began to ban supersonic flight of the Concorde over land, severely hurting the economic value of the airplane. At the time of writing, such bans on overland supersonic flight are still in effect.

N波——声压的时间分布



T：N波持续时间；τ：过压上升时间

降低声爆的措施：

1. 通过外形优化降低Δp

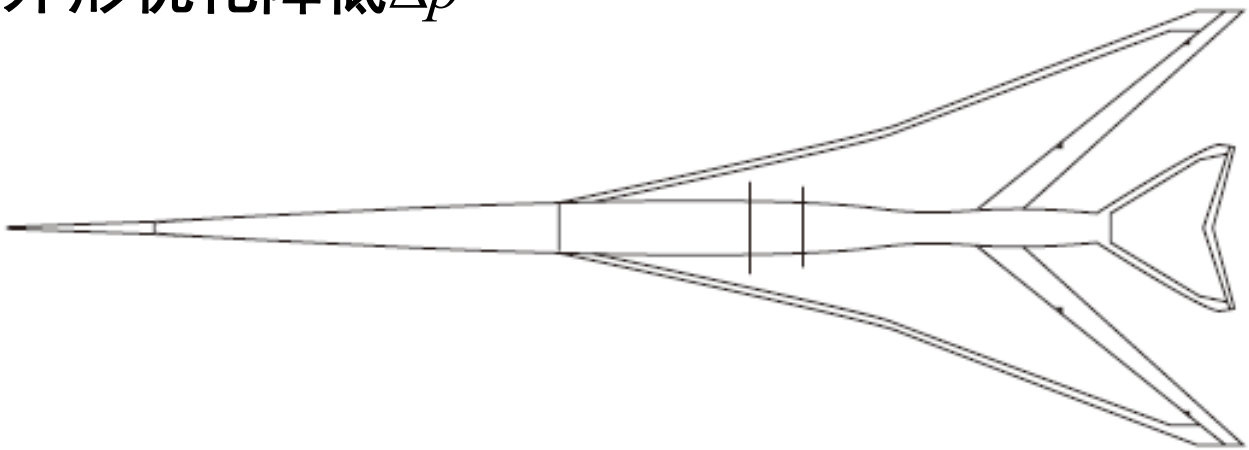


Figure 9.44 Planform of a generic low-boom fuselage, wing, and tail supersonic airplane.

- Sharp-nosed slender bodies will generate weaker shock waves than blunter-nosed thicker bodies
- Expansion waves generated over portions of the body can interact with, and weaken, shock waves

降低声爆的措施：

2. 增大上升到Δp_{max}的上升时间τ

过压上升时间τ对人的主观感觉很重要，最大过压为35N/m^2, 上升时间为1毫秒与过压125N/m^2, 上升时间为10毫秒的情况，人们所感受到的响度（声爆）是一样的。

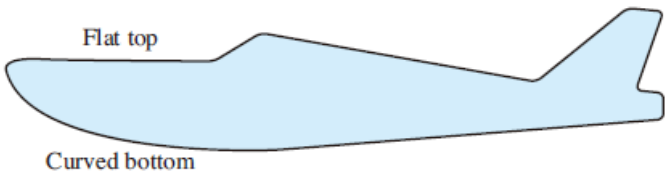


Figure 9.45 Schematic of a supersonic vehicle with a nose shape to produce a shaped sonic boom.

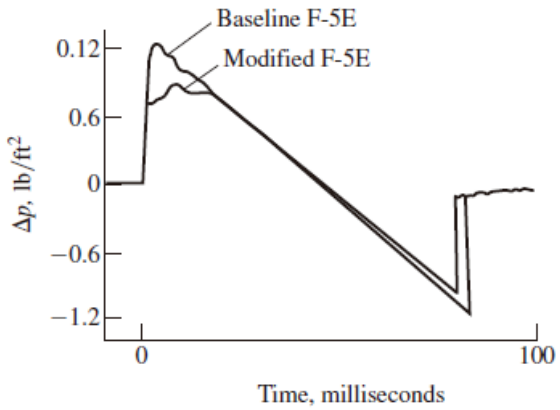


Figure 9.46 Comparison of a classic sonic boom signature (generated by a baseline F-5E aircraft) with a shaped boom signature (generated by a modified F-5E with a curved-bottom nose).

图9.46

- a flat top
- a curved bottom

The expansion wave from the curved bottom surface interacts with the bow shock wave in a fashion that smooths the sharp increase in Δp

Decrease Δp from 1.2 lb/ft^2 to 0.8 lb/ft^2, increase **τ to** 20 milliseconds

NASA X-59 QueSST

作业：9.13-9.15，9.18-9.20，9.21选做（9.21关键是看懂题）

Lecture # 12 Ended！

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