**Review of 2nd course/第二课复习**

# Chapter 7 Compressible Flow: Some Preliminary Aspects
# 可压缩流动基础

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# 第七章路线图

- 热力学复习
  - 完全气体
  - 内能和焓
  - 热力学第一定律
  - 熵及热力学第二定律
  - 等熵关系式

↓

压缩性定义

↓

无粘可压缩流动的控制方程

↓

总参数定义

↓

有激波可压缩流动的定性了解

# 定容比热与内能的关系的推导：

$$e = e(T, v)$$

$$de = \left(\frac{\partial e}{\partial T}\right)_v dT + \left(\frac{\partial e}{\partial v}\right)_T dv$$

$$\delta q = de + p dv = \left(\frac{\partial e}{\partial T}\right)_v dT + \left[\left(\frac{\partial e}{\partial v}\right)_T + p\right] dv$$

$$\because dv = 0 \qquad \therefore c_v = \frac{(\delta q)_v}{dT} = \left(\frac{\partial e}{\partial T}\right)_v$$

# 定压比热与焓的关系的推导：

$$h = h(T, p)$$

$$dh = \left(\frac{\partial h}{\partial T}\right)_p dT + \left(\frac{\partial h}{\partial p}\right)_T dp$$

$$\delta q = dh - vdp = \left(\frac{\partial h}{\partial T}\right)_p dT + \left[\left(\frac{\partial h}{\partial p}\right)_T - v\right] dp$$

$$\because dp = 0 \quad \therefore c_p = \frac{(\delta q)_p}{dT} = \left(\frac{\partial h}{\partial T}\right)_p$$

# 7.2.4 熵及热力学第二定律 (Entropy and the Second Law of Thermodynamics)

- 热力学第一定律解决了能量在一个过程中的守恒问题。
- 热力学第二定律则要解决过程会向哪个方向进行的问题，表明能量不仅有“量”（quantity)的大小, 还有“质”（quality)的高低 。
- 一个过程只能向同时满足热力学第一定律和第二定律的方向进行 (A process cannot take place unless it satisfies both the first and second law.)


**FIGURE 6–4**
Processes occur in a certain direction, and not in the reverse direction.

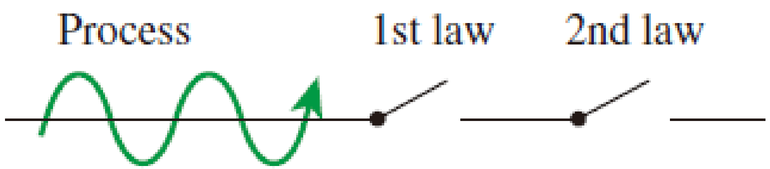



**FIGURE 6–5**
A process must satisfy both the first and second laws of thermodynamics to proceed.

# 熵的定义

- The second law leads to the definition of a new property called entropy. （热力学第二定律引入了一个新性质，熵）
- Let us define a new state variable, the *entropy s*, as follows (熵的定义如下):

$$\boxed{ds = \frac{\delta q_{rev}}{T}} \qquad (7.13)$$

$$\boxed{ds = \frac{\delta q}{T} + ds_{irrev}} \qquad (7.14)$$

注意：熵是状态量，可用于任何可逆或不可逆过程

# 热力学第二定律的表达方式：

$$\boxed{ds_{irrev} \geq 0}$$

$$\boxed{ds \geq \frac{\delta q}{T}, \text{or } ds \geq 0}$$

- This is the second law of thermodynamics that tells in what direction a process will take place:（热力学第二定律指明过程进行的方向）
- The entropy of the system and its surroundings always increases or, at best, stays the same.（热力学第二定律:系统和其环境的熵总是增加的或不变的）
- In summary, the concept of entropy in combination with the second law allows us to predict the direction that nature takes.（总之，熵与热力学第二定律相结合，使我们能预计过程进行的方向）

# 热力学第二定律的其他表达方式：

**1850**年克劳修斯从能量传递方向性的角度表述：

不可能将热从低温物体传至高温物体而不引起其他变化

**1851**年开尔文从热功转换的角度表述

不可能从单一热源取热，并使之完全变为有用功而不引起其他变化

---

- 违反热力学第二定律的热机被称为第二类永动机。
- 热力学第二定律揭示热过程的方向性、条件与限度。
- 熵的引入使热力学第二定律得以量化，熵的变化表征了热交换的方向和大小。

# 热力学第二定律的进一步理解

- 节约能源的概念是从热力学第二定律的角度考虑的

- 提高效率的概念也是从热力学第二定律的角度考虑的

- 孤立系统的熵增的大小表示能量贬值或功耗散（作功能力下降）的程度

# 熵的物理意义的理解（level of disorder)

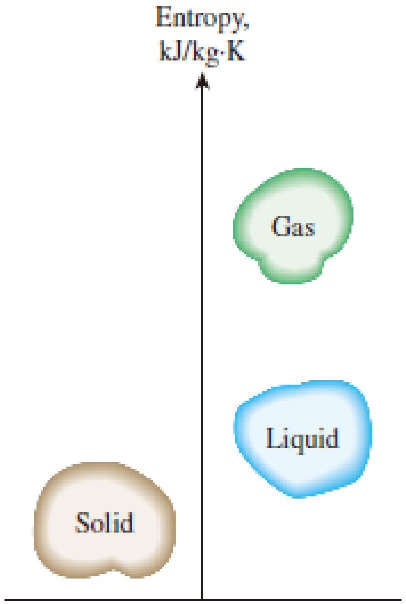



**FIGURE 7–20**
The level of molecular disorder (entropy) of a substance increases as it melts or evaporates.

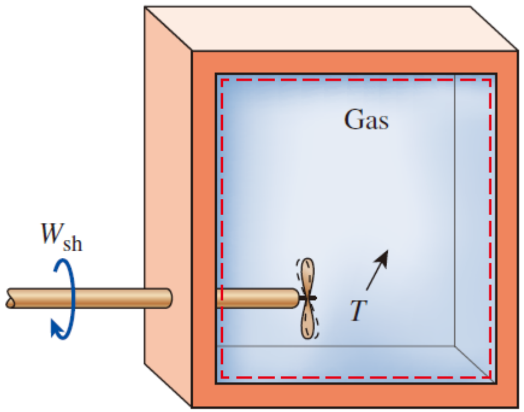



**FIGURE 7–24**
The paddle-wheel work done on a gas increases the level of disorder (entropy) of the gas, and thus energy is degraded during this process.

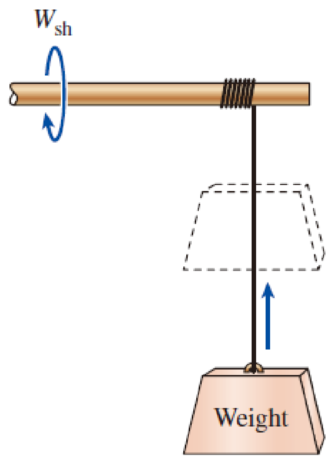



**FIGURE 7–23**
In the absence of friction, raising a weight by a rotating shaft does not create any disorder (entropy), and thus energy is not degraded during this process.

The *quantity* of energy is always preserved during an actual process(the first law), but the *quality* is bound to decrease (the second law).

## 生活中有关熵的概念和熵增的例子(Entropy and Entropy Generation in Daily Life)：

Efficient people lead low-entropy (highly organized) lives. They have a place for everything (minimum uncertainty), and it takes minimum energy for them to locate something. Inefficient people, on the other hand, are disorganized and lead high-entropy lives. It takes them minutes (if not hours) to find something they need, and they are likely to create a bigger disorder as they are searching since they will probably conduct the search in a disorganized manner. People leading high-entropy lifestyles are always on the run, and never seem to catch up.

# 生活中有关熵的概念和熵增的例子(Entropy and Entropy Generation in Daily Life)：

We know that mechanical friction is always accompanied by entropy generation, and thus reduced performance. We can generalize this to daily life: ***friction in the workplace*** with fellow workers is bound to generate entropy, and thus adversely affect performance (Fig. 7–26). It results in reduced productivity.

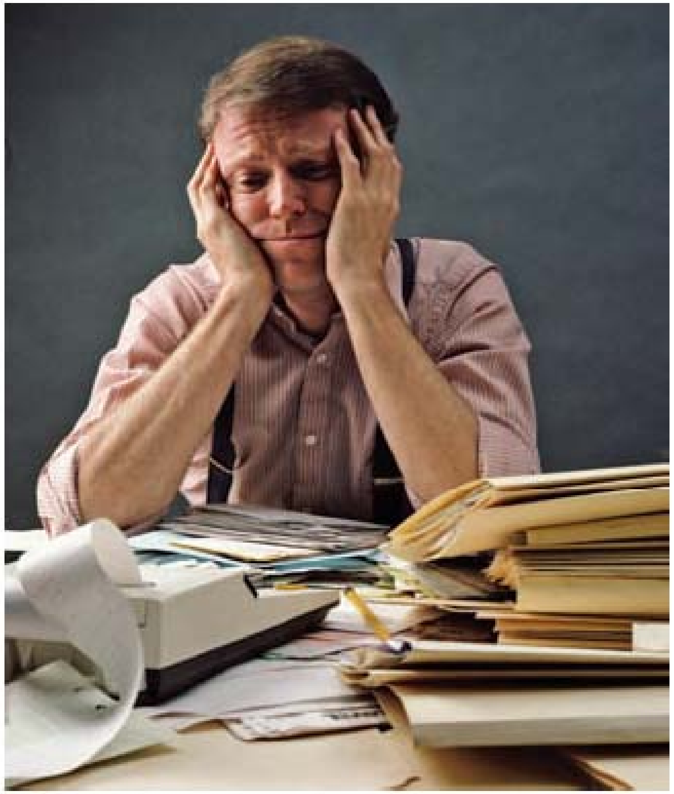


**FIGURE 7–26**
As in mechanical systems, friction in the workplace is bound to generate entropy and reduce performance.
*© PhotoLink/Getty Images RF*


**FIGURE 7–26**
The use of entropy (disorganization, uncertainty) is not limited to thermodynamics.
*BLONDIE © KING FEATURES SYNDICATE.*

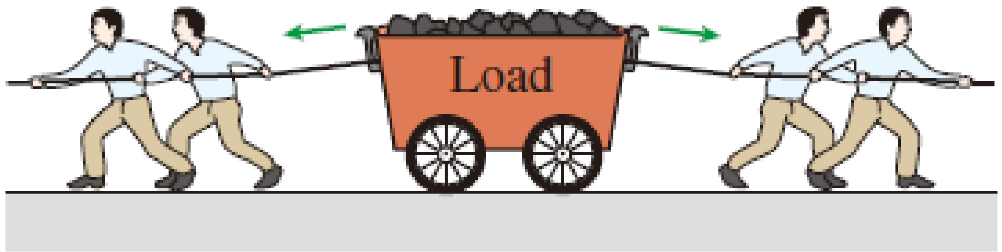



**FIGURE 7–22**
Disorganized energy does not create much useful effect, no matter how large it is.

# Practical calculation of entropy(熵的实际计算）

从状态“1”到状态“2”，根据熵的定义计算熵增 $S_2 - S_1$

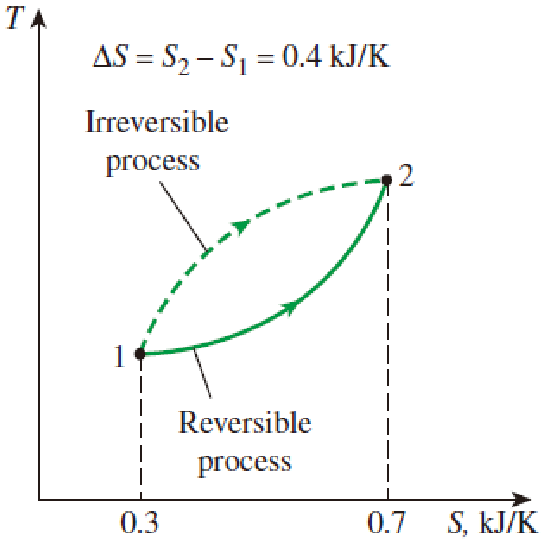



**FIGURE 7–3**
The entropy change between two specified states is the same whether the process is reversible or irreversible.

$$\boxed{s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}} \quad (7.25)$$

$$\boxed{s_2 - s_1 = c_v \ln\frac{T_2}{T_1} + R\ln\frac{v_2}{v_1}} \quad (7.26)$$

Note that “s” is a function of two thermodynamic state variables;
e.g. $s = s(p,T)$ or $s = s(v,T)$

# 等熵关系式（isentropic relations）

$$\boxed{\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} = \left(\frac{T_2}{T_1}\right)^{\gamma/(\gamma-1)}} \qquad (7.32)$$

Equation (7.32) is very important ; it relates pressure, density , and temperature for an isentropic process.

方程（7. 32)非常重要，它将等熵过程中的压强、密度、温度联系起来。

$$\frac{p}{\rho^{\gamma}} = const.$$

# 补充：Concept of barotropic :/正压流的概念 （续）

$$dp = -\rho V dV \qquad (3.12)$$

**1. 不可压流：** $\rho = const.$ 我们得到伯努利方程：

$$p_1 + \frac{1}{2}\rho V_1^{\ 2} = p_2 + \frac{1}{2}\rho V_2^{\ 2}$$

- 对于无粘、不可压缩有旋流动

$$p + \frac{1}{2}\rho V^2 = const \qquad \textbf{\textit{along a streameline}} \qquad (3.14)$$

- 对于无粘、不可压缩无旋流动

$$p + \frac{1}{2}\rho V^2 = const \qquad \textbf{\textit{throughout the flow}} \qquad (3.15)$$

结论:不可压假设下，可用伯努利方程代替能量方程.

# 补充：Concept of barotropic :/正压流的概念 （续）

2. 等熵流： $\dfrac{p}{\rho^{\gamma}}=const.$ 我们由： $\displaystyle\int_1^2\frac{dp}{\rho}=-\int_1^2 VdV$

$$\int_1^2\frac{dp}{\rho}=\int_1^2\frac{const.\cdot\gamma\rho^{\gamma-1}d\rho}{\rho}=const.\frac{\gamma}{\gamma-1}\rho^{\gamma-1}\Big|_1^2$$

$$=\frac{\gamma}{\gamma-1}const.(\rho_2{}^{\gamma-1}-\rho_1{}^{\gamma-1})=\frac{\gamma}{\gamma-1}\left(\frac{p_2}{\rho_2}-\frac{p_1}{\rho_1}\right)$$

$$\frac{\gamma}{\gamma-1}\frac{p_1}{\rho_1}+\frac{V_1{}^2}{2}=\frac{\gamma}{\gamma-1}\frac{p_2}{\rho_2}+\frac{V_2{}^2}{2}$$ 又可表示为 $$h_1+\frac{V_1{}^2}{2}=h_2+\frac{V_2{}^2}{2}$$

结论:等熵假设可以使控制方程得到简化, 可用等熵关系式代替能量方程。

## 投票(匿名) 最多可选1项

设置

# 前一次课的掌握情况投票

- A 完全掌握了这部分知识内容
- B 掌握了大部分
- C 掌握了一小部分
- D 完全不懂

提交

# 学习空气动力学到底有什么用？


A380


A350

B747

B787

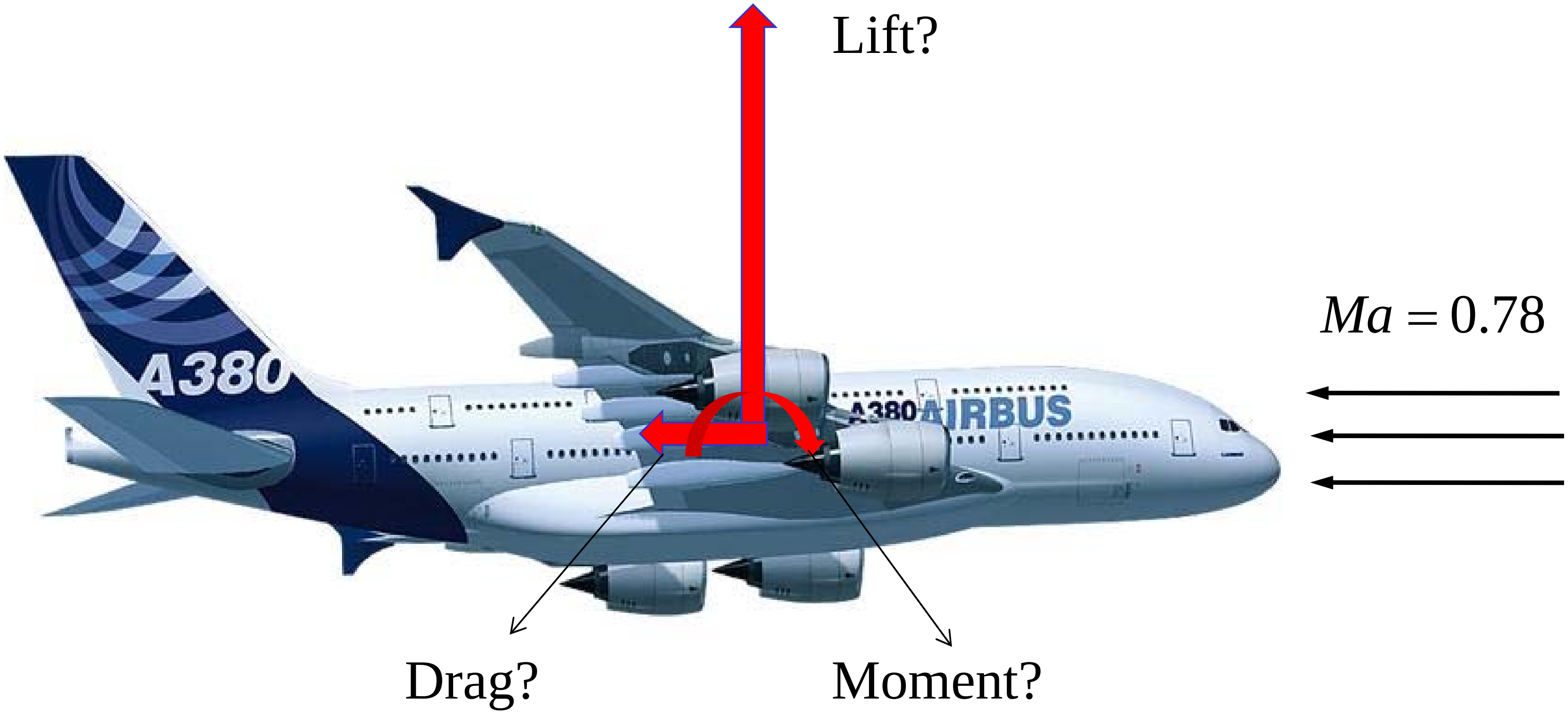
科学的作用在于“认识世界”和“改造世界”

1. 认识与空气流动相关的流动现象及其规律

2. 设计更好或新的飞行器、汽车、风力机等与空气具有相对运动的运载工具或器件

提高1%的最大升力系数，意味着多载22个旅客或2000kg的货物；
升阻比提高1%可以转化为相当于多载18个旅客或1270kg的货物

# 飞机设计的一个基本问题：给定外形，如何计算其气动力？



## Lecture #3

# Chapter 7 Compressible Flow: Some Preliminary Aspects
# 可压缩流动基础

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2019年10月15日 Tuesday

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# 第七章路线图

- 热力学复习
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  - 热力学第一定律
  - 熵及热力学第二定律
  - 等熵关系式
- 压缩性定义
- 无粘可压缩流动的控制方程
- 总参数定义
- 有激波可压缩流动的定性了解

# 7.3 Definition of Compressibility (压缩性定义)

All real substances are ***compressible*** to some greater or lesser extend. When you squeeze or press on them, their density will change. This is particularly true of ***gases***.

(所有的真实物质都是可压缩的,当我们压挤它们时,它们的密度会发生变化,对于气体尤其是这样。)

The amount by which a substance can be compressed is given by a specific property of the substance called the compressibility.

(物质可被压缩的大小程度称为物质的压缩性，是物质的特定物理属性)


2019/10/15 Tuesday

# 7.3 Definition of Compressibility (压缩性定义) (续)

Consider a small element of fluid of volume $v$ . The pressure exerted on the sides of the element is $p$. If the pressure is increased by an infinitesimal amount $dp$, the volume will change by a negative amount $dv$ .

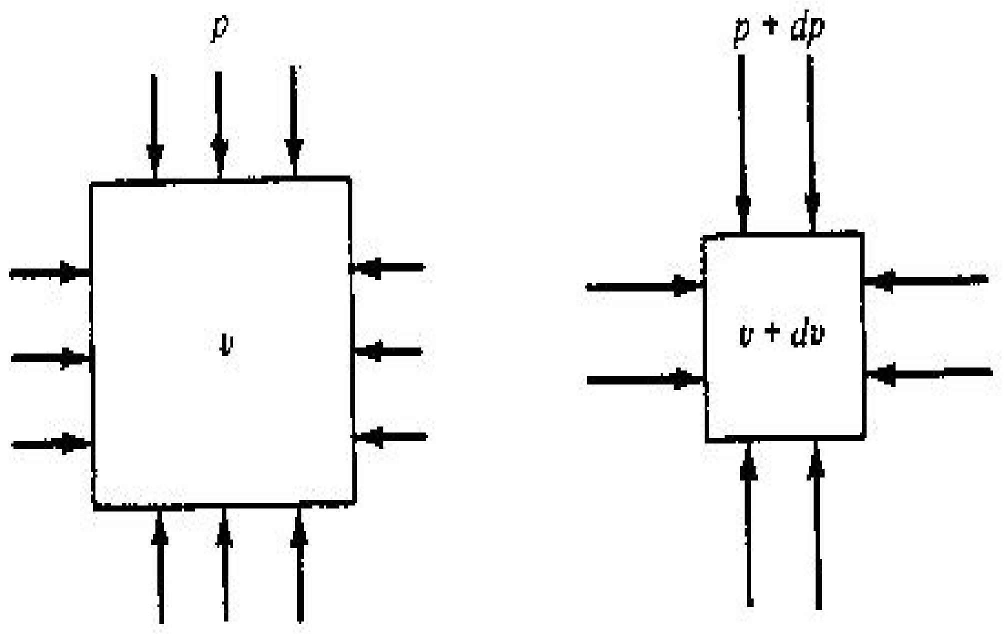


**FIGURE 7.3**
**Definition of compressibility.**


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# 压缩性定义

By definition, the compressibility is given by:

$$\tau = -\frac{1}{v}\frac{dv}{dp} \quad (7.33)$$

as

$$v = \frac{1}{\rho}$$

$$\tau = \frac{1}{\rho}\frac{d\rho}{dp} \quad (7.36)$$

Physically, the compressibility is a fractional change in volume of the fluid element per unit change in pressure.

(从物理上讲，压缩性就是每单位压强变化引起的流体微元单位体积内的体积变化)

# 等温压缩性和等熵压缩性

If the temperature of the fluid element is held constant, then $\tau$ is identified as the isothermal compressibility (等温压缩性)

$$\tau_T = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T \qquad (7.34)$$

If the process takes place isentropically, then

$\tau$ is identified as the isentropic compressibility (等熵压缩性).

$$\tau_s = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_s \qquad (7.35)$$


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# 如何判定流动是否可压？

$$d\rho = \rho \tau \, dp \qquad (7.37)$$

- If the fluid is liquid. $\tau$ is very small, and $d\rho$ is small . So, a liquid flow is incompressible
- If the fluid is a gas, where compressibility $\tau$ is large, then for a given pressure change $dp$ from one point to another in the flow , Eq.(7.37) states that $d\rho$ can be large. Thus a flow of gas is compressible.
- For low speed flow of a gas, $\tau$ is large but $dp$ is small. Therefore $d\rho$ is small. It can be regarded as incompressible flow

注意区分流体的压缩性与流动是否为可压缩流动！


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# 判定空气流动是否是可压缩流动的更常用的判据是什么？

$$M = \frac{V}{a}$$

8.3节引入声速的概念后，可证明 $a = \sqrt{\dfrac{1}{\rho \tau_s}}$

$M \le 0.3$ 时，可假设流动是不可压的。

$M > 0.3$ 时，必须按可压缩流动处理。


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## 单选题 1分

设置

**等熵压缩性的正确定义式是:**

A $$\tau_s = -\left(\frac{\partial v}{\partial p}\right)_s$$

B $$\tau_s = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_s$$

C $$\tau_T = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T$$

D $$\tau_T = -\left(\frac{\partial v}{\partial p}\right)_T$$

提交

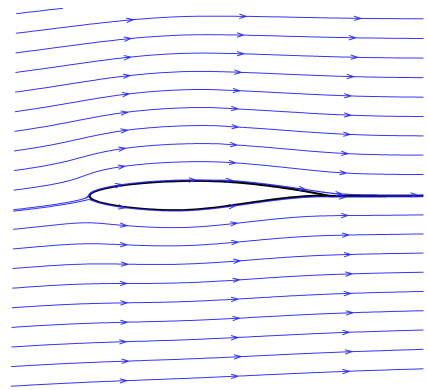
# 7.4 Governing Equations for inviscid Compressible Flow (无粘、可压缩流控制方程)

复习：对于无粘、不可压缩流动，基本变量是压强 $p$ 和速度 $\vec{V}$ 。因此我们只需要两个基本方程，即连续方程和动量方程。

Review：for inviscid, *incompressible* flow, the primary dependent variables are the pressure *p* and the velocity $\vec{V}$ . Hence, we need only *two basic equations*, namely the *continuity* and the *momentum equations.*

# 复习：求解无粘、不可压缩流的两个基本方程？

**Review: Basic flow equations for solving an inviscid and incompressible flow?**



- 结合连续方程和动量方程
  - 拉普拉斯方程 (Laplace’s equation) $\frac{\partial \Phi^2}{\partial x^2}+\frac{\partial \Phi^2}{\partial y^2}+\frac{\partial \Phi^2}{\partial z^2}=0$
  - 伯努利方程 (Bernoulli’s equation) $p+\frac{1}{2}\rho V^2=\text{const}$

•不可压缩流动我们假定“密度”和“温度”保持不变
•不可压流动遵循的主要是机械能守恒定律，不需要引入热力学。

# 相比之下，求解无粘、可压缩流涉及哪些量，应该引入哪些方程？

**How about solving a compressible flow ?**

对于可压缩流，与不可压流相反的是，$\rho$ 是一个变量，并且是一个未知数。 因此，我们需要一个附加方程 — *能量方程* — 进而引入未知数内能 $e$ 。

In contrast, for compressible flow, $\rho$ is variable and becomes an unknown. Hence we need an additional equation – the energy equation – which in turn introduces internal energy $e$ as an unknown.

# 无粘、可压缩流的基本变量和控制方程

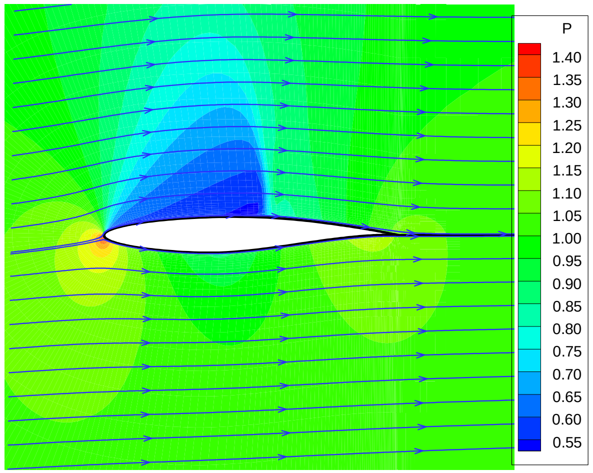
## Primary variables & Governing equations

Internal energy *e* is related to temperature, then *T* also becomes an important variable. Therefore, the 5 primary dependent variables are:

$$\boxed{p, \vec{V}, \rho, e, and\ T}$$

To solve for these five variables, we need five governing equations:

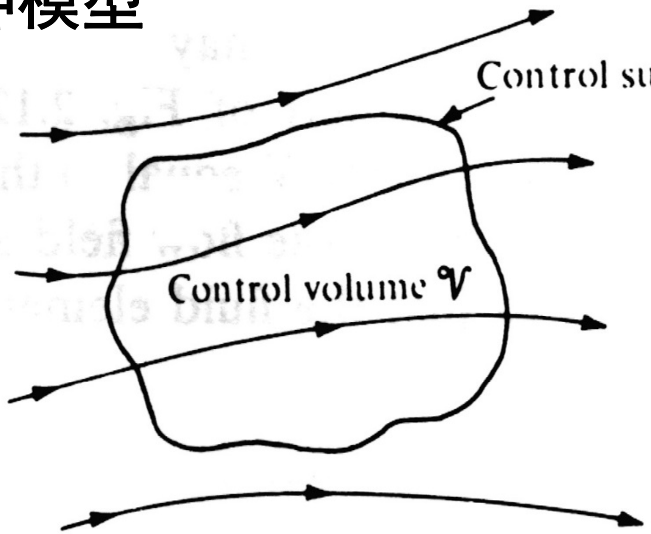
1.Continuity (连续方程）
2.Momentum (动量方程）
3.Energy（能量方程）
4.Equation of state (状态方程）
5.Internal energy (内能方程）



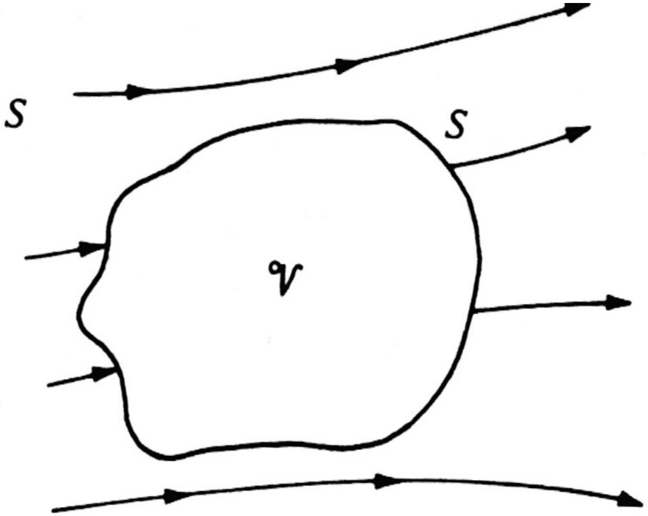
# 复习：连续方程、动量方程和能量方程的推导

## Review: derivation of governing equations

４种模型




Finite control volume
fixed in space with the
fluid moving through it

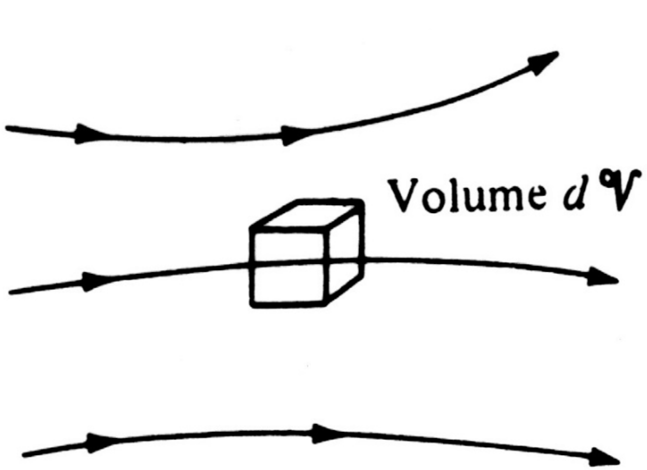



Finite control volume moving
with the fluid such that the
same fluid particles are always
in the same control volume

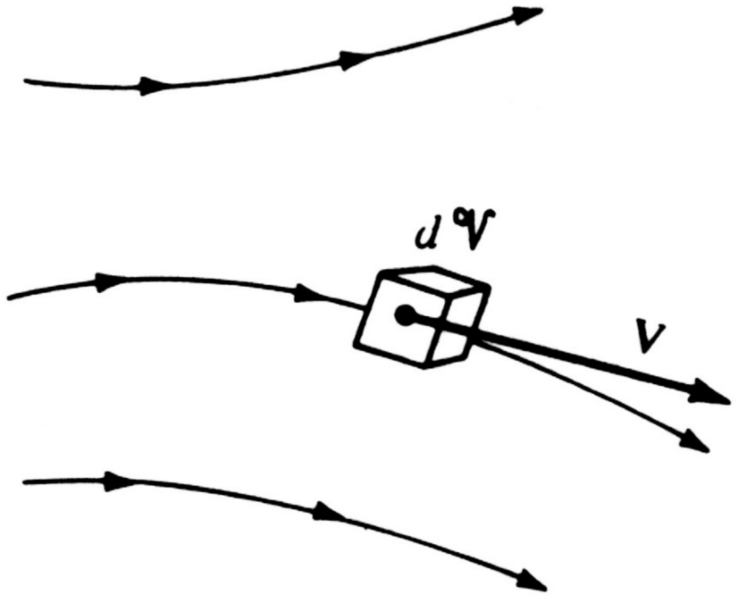
① Fixed finite control volume

② Moving finite control volume

# 4 种模型 (续)




③ Fixed infinitesimally small element




④ Moving infinitesimally small element

**Different models of flow lead to different forms of governing equations**

# 1. Continuity equation (连续方程）

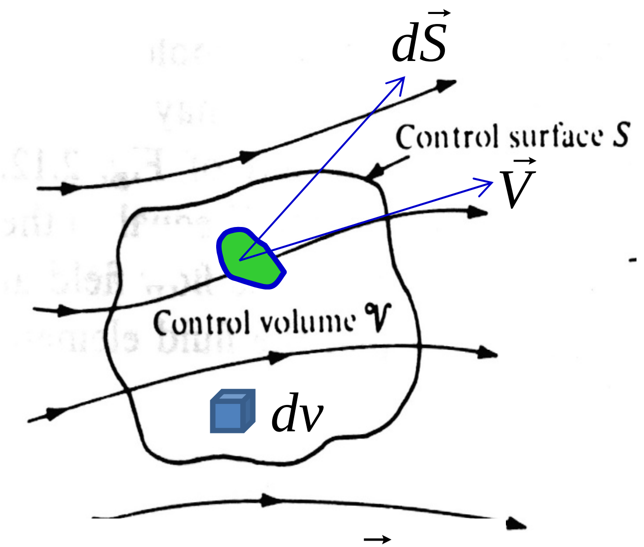
**Physical principle: mass is conserved**质量守恒

| **Net mass flow out of control volume through surface *S*** | = | **time rate of decrease of mass inside control volume *V*** |
|---|---|---|
| 通过控制体表面S流出控制体的净质量流量 | ＝ | 控制体内的质量减少率 |

$$\boxed{\frac{\partial}{\partial t}\iiint\limits_{v}\rho\, dv + \oiint\limits_{S}\rho\vec{V}\cdot d\mathrm{S} = 0}$$

(7. 39)
即 (2. 48)

①




$$\rho V_n dS = \rho\vec{V}\cdot d\mathrm{S}$$

# 连续方程的其它形式

- Model of the Finite Control Volume Moving with the Fluid (Assignment)

$$\frac{D}{Dt}\iiint_V \rho dV = 0 \qquad ②$$

- Model of an infinitesimally Small Element Fixed in Space (Assignment)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho \vec{V}\right) = 0 \qquad ③$$

- Model of an infinitesimally Small Element Moving with the Fluid (Assignment)

$$\frac{D\rho}{Dt} + \rho \nabla \cdot \vec{V} = 0 \qquad ④$$

# 2. Momentum（动量方程）

Physical principle: Newton's second law, $\vec{F} = \frac{d}{dt}\left(m\vec{V}\right)$

①

$$\boxed{\frac{\partial}{\partial t}\iiint_v \rho\vec{V}dv + \oiint_s \left(\rho\vec{V}\cdot \mathrm{d}\mathrm{S}\right)\vec{V} = -\oiint_s p\,\mathrm{d}\mathrm{S} + \iiint_v \rho\,\vec{f}\,dv}$$

（7.41）
即（2.64）

$$\vec{F}_{viscous} = 0$$

where $\vec{f}$ are the body forces (彻体力), such as gravity, or electromagnetic forces

③

$$\frac{\partial(\rho u)}{\partial t} + \nabla\cdot(\rho u\vec{V}) = -\frac{\partial p}{\partial x} \qquad (2.70a)$$

$$\frac{\partial(\rho v)}{\partial t} + \nabla\cdot(\rho v\vec{V}) = -\frac{\partial p}{\partial y} \qquad (2.70b)$$

$$\frac{\partial(\rho w)}{\partial t} + \nabla\cdot(\rho w\vec{V}) = -\frac{\partial p}{\partial z} \qquad (2.70c)$$

$$\vec{F}_{viscous} = 0 \qquad \vec{f} = 0$$

# 2. Momentum（动量方程）(续)

In terms of substantial derivative: $\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + (\vec{V} \bullet \nabla)$

④

$$\rho \frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \rho f_x \qquad (7.42a)(2.113a)$$

$$\rho \frac{Dv}{Dt} = -\frac{\partial p}{\partial y} + \rho f_y \qquad (7.42b)(2.113b)$$

$$\rho \frac{Dw}{Dt} = -\frac{\partial p}{\partial z} + \rho f_z \qquad (7.42c)\ (2.113c)$$

写成矢量形式：

$$\boxed{\rho \frac{D\vec{V}}{Dt} = -\nabla p + \rho \vec{f}}$$

# 3. Energy equation (能量方程)

Physical principle: energy is conserved/能量守恒

Energy can be neither created nor destroyed; it can only change in form

①

$$\boxed{\frac{\partial}{\partial t}\iiint_v \rho\left(e+\frac{V^2}{2}\right)dv+\oiint_s \rho\left(e+\frac{V^2}{2}\right)\vec{V}\cdot d\mathbf{S} = \iiint_v \dot{q}\,\rho\, dv-\oiint_s p\vec{V}\cdot d\mathbf{S}+\iiint_v \rho\left(\vec{f}\cdot\vec{V}\right)dv}$$

(7. 43)
(2. 95)

$\dot{Q}_{viscous}=0,\quad \dot{W}_{viscous}=0$

④

$$\rho\frac{D\left(e+V^2/2\right)}{Dt}=\rho\dot{q}-\nabla\cdot p\vec{V}+\rho\left(\vec{f}\cdot\vec{V}\right)$$

(7. 44)
(2. 114)

$\dot{q}=0 \qquad \vec{f}=0 \qquad \rho\frac{D\left(e+V^2/2\right)}{Dt}=-\nabla\cdot p\vec{V}$ (7. 45)

**4. Equation of state for a perfect gas/完全气体状态方程**

$$\boxed{p = \rho RT}$$

**5. Internal energy for a calorically perfect gas/量热完全气体内能方程:**

$$\boxed{e = c_v T}$$

**We have now *5 equations* for *5 unknowns* /** 五个方程，五个未知数 / 上述方程联立可求解出所有流动变量！

# 问题1：伯努利方程是否适用于可压缩流动？

## Q1　：Is Bernoulli's equation valid for compressible flow?

A：　No.

复习:

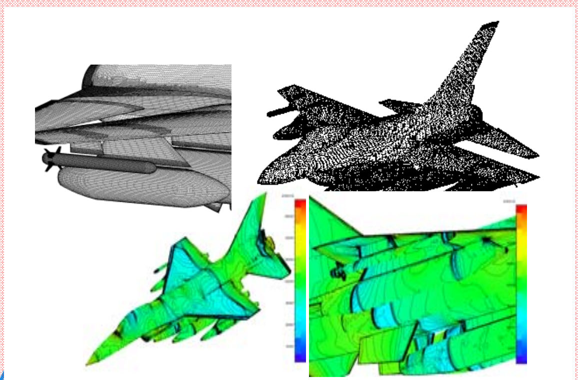
$$p+\frac{1}{2}\rho V^2=\text{const}$$

不可压、无粘定常流动：
(1)无旋时全流场成立；
(2)有旋时沿流线成立。

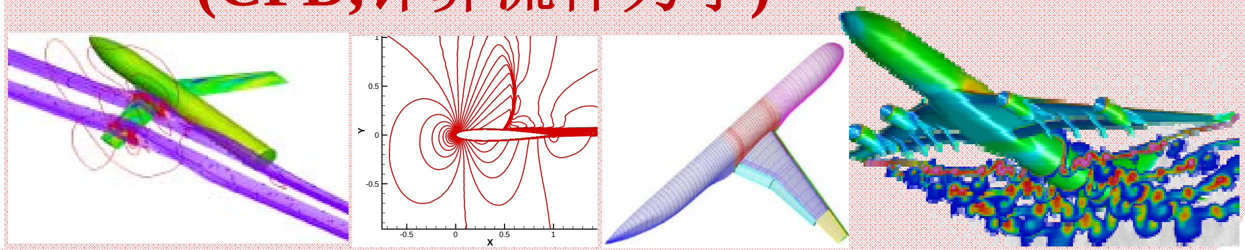
# 问题2：如何求解这些方程？

## Q2：How to solve these equations?

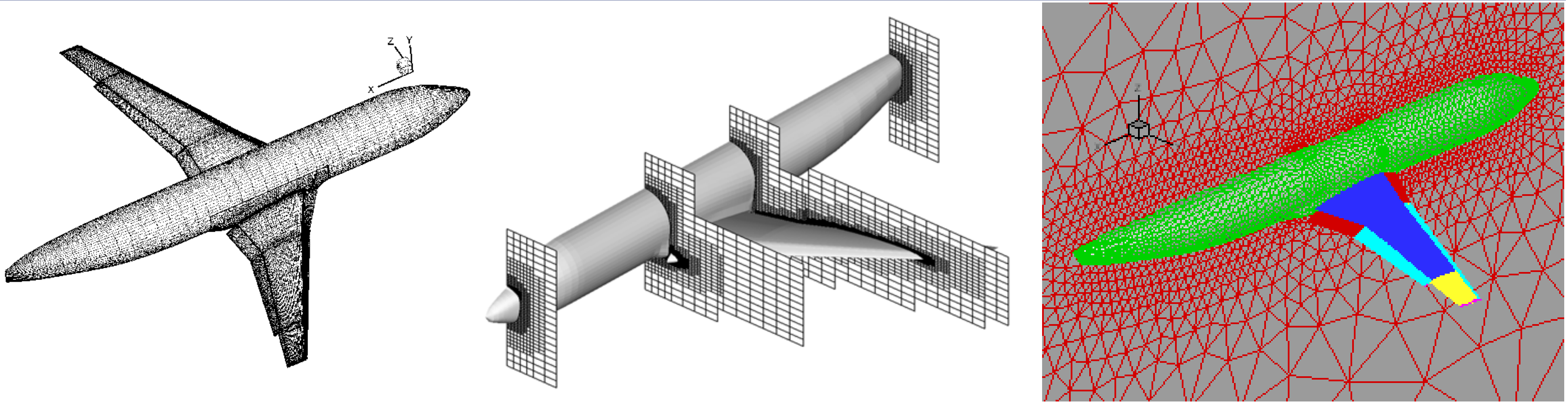
- **解析求解**（第8、9、10章）：相对简单的问题
- **小扰动线化求解**(第11、12章)，亚、超声速；薄物体、小攻角
- **数值求解（CFD）**(第13章)：任意外形、任意流动状态、可考虑层流、湍流、转捩、分离等复杂流动现象。



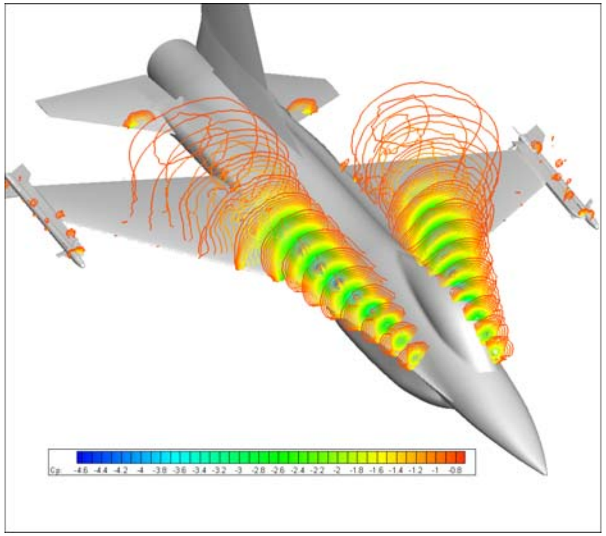
**computational fluid dynamics**
**(CFD,计算流体力学)**

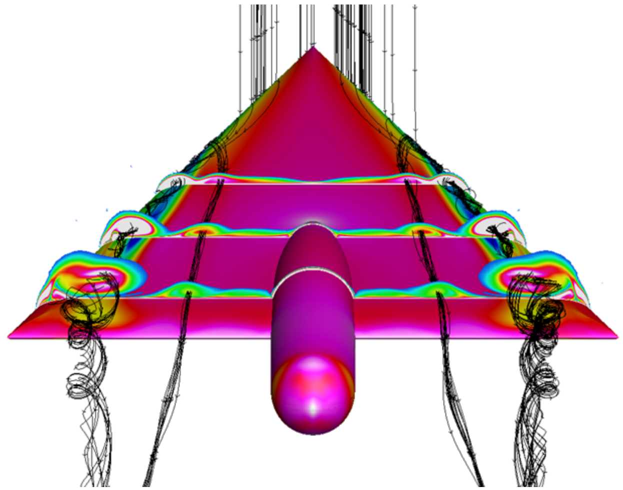


# Numerical method for solving compressible flows



**Grid generation methods**





**Euler results** **RANS results** **DES results**

# 7.5 Definition of Total (Stagnation) Conditions (总参数的定义）(滞止参数的定义：*流体力学中的综合测评）*

Consider a fluid element passing through a given point in a flow where the local pressure, density, temperature, Mach number, and velocity (local conditions) are $p,\ \rho, T, M,$ and $\vec{V}$, respectively.

假设流体微团通过一个给定点，对应的当地压强、温度、密度、马赫数、速度分别为

$$p,\ \rho, T, M, \vec{V}$$

思考：

单位质量的动能是？ $\frac{V^2}{2}$

单位质量的内能是？ $c_v T$

单位质量流体的体积为比容 $v$，静压为$p$， $pv$是？

$pv$ 代表流动能，即流体进入或离开控制体 $v$ 拥有的能量，也被称作压力势能。

$$pv=\frac{p}{\rho}$$

总结： $p, \rho, T, M, \vec{V}$ 都是和流体微元能量有关的变量。

问题：如何衡量流体微元拥有总能量的大小？

# 总温 / Total temperature

Here, $p, T, and\ \rho$ are static quantities, i.e., ***static pressure, static temperature, static density***, respectively.

这里，$p, T$，$\rho$分别是静参数，即静压、静温、静密度。

Now imagine that you grab hold of the fluid element and ***adiabatically*** slow it down to zero velocity. Clearly, you would expect (correctly) that the values of $p, T, and\ \rho$ would change as the element is brought to rest.

In particular, the value of the temperature of the fluid element after it has been brought to rest ***adiabatically*** is defined as the ***total temperature***, denoted by $T_0$ .

特别地，假想流体微团被*绝热地*减速为静止时所对应的温度，定义为流体微团的总温 $T_0$ 。

# 总焓／Total Enthalpy

The enthalpy corresponding to $T_0$ is defined as *total enthalpy* $h_0$,

where $h_0 = c_p T_0$ for a calorically perfect gas.

*total enthalpy* $h_0$：总焓

Keep in mind that we do not *actually* have to bring the flow to rest in real life in order to talk about the total temperature or total enthalpy; rather, the are *defined quantities* that would exist at a point in a flow *if* (in our imagination) the fluid element passing through that point were to brought to rest adiabatically.

注意：总参数只是假想流体微元绝热地减速到静止的定义量。

流动中任意一点都可以计算其总参数

# 能量方程

The energy equation, Eq. (7.44), provides some important information about total enthalpy and hence total temperature.

由能量方程（7.44)可以得到总焓、因而总温的重要信息。

$$\rho \frac{D\left(e+V^2/2\right)}{Dt} = \rho \dot{q} - \nabla \cdot p\vec{V} + \rho\left(\vec{f} \cdot \vec{V}\right) \qquad (7.44)$$

Assume that the flow is *adiabatic* and that *body forces are negligible*, then the equation of energy can be written as:

$$\rho \frac{D\left(e+V^2/2\right)}{Dt} = -\nabla \cdot p\vec{V} \qquad (7.45)$$

注意：(7. 45)式的前提条件：无粘、绝热、忽略体积力.

# 能量方程

如果将（7.45）式表示成用焓 $h$ 表示的能量方程，我们可以得到总焓的信息。因为：

$$h = e + \frac{p}{\rho}$$

所以，我们求出 $\rho \frac{D(p/\rho)}{Dt}$ 与（7.45）式相加

就可得到关于焓$h$的能量方程。

又因为：

$$\rho \frac{D(p/\rho)}{Dt} = \rho \frac{\rho \frac{Dp}{Dt} - p \frac{D\rho}{Dt}}{\rho^2} = \frac{Dp}{Dt} - \frac{p}{\rho}\frac{D\rho}{Dt} \qquad \textbf{(7.47)}$$

# 能量方程

将连续方程 $\frac{D\rho}{Dt}+\rho\nabla\cdot\vec{V}=0$ （2.108） 代入上式得：

$$\rho\frac{D(p/\rho)}{Dt}=\frac{Dp}{Dt}-\frac{p}{\rho}\frac{D\rho}{Dt}=\frac{Dp}{Dt}+p\nabla\cdot\vec{V}$$

$$=\frac{\partial p}{\partial t}+\vec{V}\cdot\nabla p+p\nabla\cdot\vec{V} \qquad (7.48)$$

$$=\frac{\partial p}{\partial t}+\nabla\cdot p\vec{V}$$

$$\nabla\cdot p\vec{V}\equiv p\nabla\cdot\vec{V}+\vec{V}\cdot\nabla p \qquad (7.46)$$

$$\rho\frac{D(e+V^2/2)}{Dt}=-\nabla\cdot p\vec{V} \qquad h=e+\frac{p}{\rho}$$

$$+$$

$$\rho\frac{D(p/\rho)}{Dt}=\frac{\partial p}{\partial t}+\nabla\cdot p\vec{V}$$

$$\Rightarrow \quad \rho\frac{D(h+V^2/2)}{Dt}=\frac{\partial p}{\partial t}$$

# 能量方程

If the flow is steady, (如果流动是定常的） $\frac{\partial p}{\partial t}=0$

$$\rho\frac{D\left(h+V^2/2\right)}{Dt}=0 \qquad \textbf{(7.52)}$$

The time rate of change of $h+V^2/2$ following a moving fluid element is zero， that is:

$$\boxed{h+\frac{V^2}{2}=const} \qquad \textbf{(7.53)}$$

Recall that the assumptions which led to Eq. (7.53) are that the flow is steady, adiabatic, and inviscid.

（7. 53）式成立的假设是定常、绝热、无粘。

# 总焓

Since $h_0$ is defined as that enthalpy which would exist at a point if *the fluid element were brought to rest adiabatically*, where $V = 0$ and hence $h = h_0$, then the value of the constant is $h_0$.

因为我们定义总焓 $h_0$ 为流体微元被绝热地减速为静止时对应的焓值，因此由能量方程我们可以得到总焓的值，即为上式(7.53)中的常数。

因此有:

$$\boxed{h + \frac{V^2}{2} = h_0} \qquad (7.54)$$

# 总焓

$$\boxed{h + \frac{V^2}{2} = h_0} \tag{7.54}$$

Equation (7.54) is important; it states that at any point in a flow, the total enthalpy is given by the sum of the static enthalpy plus the kinetic energy , all per unit of mass.

方程(7.54)很重要，它表明在流动中任一点，总焓由每单位质量的静焓和动能之和组成。

# 能量方程

有了总焓的定义，能量方程可以用总焓来表示：对于定常、绝热、无粘、忽略体积力流动，方程（7.52）可以写成:

$$\rho \frac{Dh_0}{Dt} = 0 \quad \text{or} \quad h_0 = const \qquad (7.55)$$

i.e. the total enthalpy is constant along a streamline.

即总焓沿流线为常数。

# 能量方程

If all the streamlines of the flow originate from a common uniform freestream (as the usually the case), then the $h_0$ is the same for each line.
如果像通常的情况那样，所有的流线都来自均匀自由来流，那么$h_0$在不同流线也是相等的。

$h_0$ *=const*, throughout the *entire* flow, and $h_0$ is equal to its freestream value.
总焓在整个流场中为常数，等于自由来流对应的总焓。

# 总温&能量方程

For a calorically perfect gas, $h_0 = c_p T_0$ . Thus, the above results also state that the total temperature is constant throughout the *steady, inviscid, adiabatic flow of a calorically perfect gas; i.e.*

对于量热完全气体，$h_0 = c_p T_0$。因此，上面的结果也表明了对于定常、无粘、绝热的量热完全气体，总温保持不变，即

$$\boxed{T_0 = const} \qquad (7.56)$$

# 容易混淆的概念

Keep in mind that the above discussion marbled two trains of thought:

- On the one hand, we dealt with the general concept of an adiabatic flow field [which led to Eqs. (7.51) to (7.53)],
- and on the other hand, we dealt with the definition of total enthalpy [which led to Eq. (7.54)].

要牢记在心的是：上面的讨论是沿着两条思路进行的，

- 一方面，我们讨论了绝热流场的一般概念,导出了能量方程 [(7.51) 至(7.53)]；
- 另一方面，我们讨论了总焓的定义 [给出了（7.54）式]。

$$\boxed{\rho\frac{D\left(h+V^2/2\right)}{Dt}=\frac{\partial p}{\partial t}} \quad (7.51)$$

$$\boxed{\rho\frac{D\left(h+V^2/2\right)}{Dt}=0} \quad (7.52)$$

$$\boxed{h+\frac{V^2}{2}=const} \quad (7.53)$$

$$\boxed{h+\frac{V^2}{2}=h_0} \quad (7.54)$$

# 容易混淆的概念（续）

- 对于一个非绝热流动，比如具有热传导的粘性附面层内的流动， 以下能量方程(7.51),(7.52),(7.53)不成立：

$$\rho\frac{D\left(h+V^2/2\right)}{Dt}=\frac{\partial p}{\partial t}$$

$$\rho\frac{D\left(h+V^2/2\right)}{Dt}=0$$

$$h+\frac{V^2}{2}=const$$

$$h_0=const$$

$$T_0=const$$

- 但是在非绝热流动中的每一点，(7.54)式成立！

$$h+\frac{V^2}{2}=h_0 \qquad (7.54)$$

# 容易混淆的概念（续）

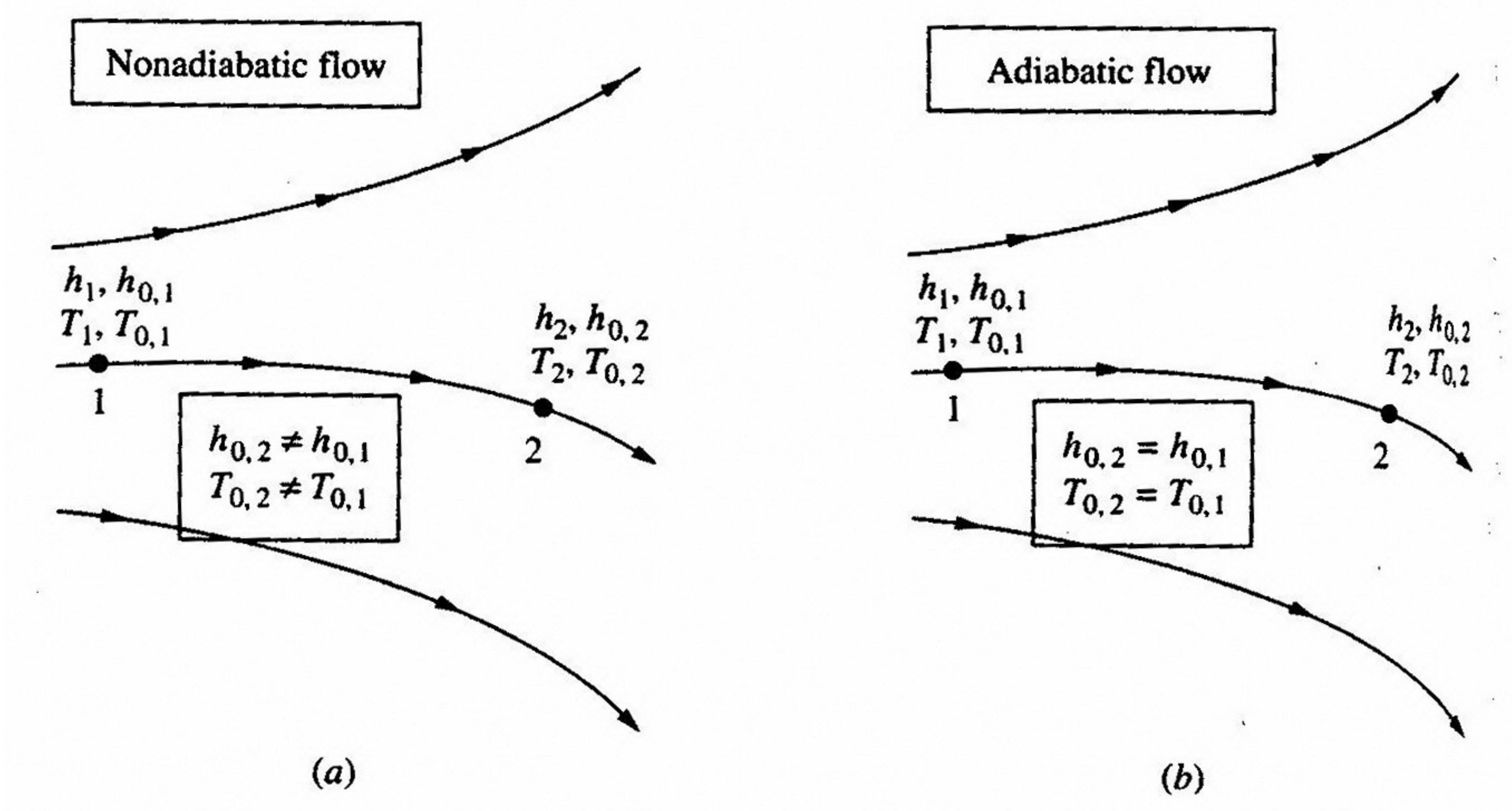



图7.4　非绝热流与绝热流的比较

## 思考：总温/总焓的物理意义？

$$h_0 = h + \frac{V^2}{2} = e + pv + \frac{V^2}{2}$$

内能　压力势能　宏观动能

总温/总焓反映流体微元所具有的总能量的大小

## 单选题 1分

设置

**总温的正确定义是：**

A 假想流体微团被绝热地减速到静止时所对应的温度，称为流体微元的总温

B 流体微团减速为静止时所对应的温度，称为流体微元的总温

C 假想流体微团被等熵地减速为静止时所对应的温度，称为流体微元的总温

提交

# 下次课讲总压和总密度的定义

Problem 7.6, 7.7, 7.8, 7.9, 7.10

# The End！

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