Review of Lecture #16/第16次课复习

Chapter 11 Subsonic Compressible Flow Over Airfoils: Linear Theory
(绕翼型的可压缩亚音速流:线化理论)

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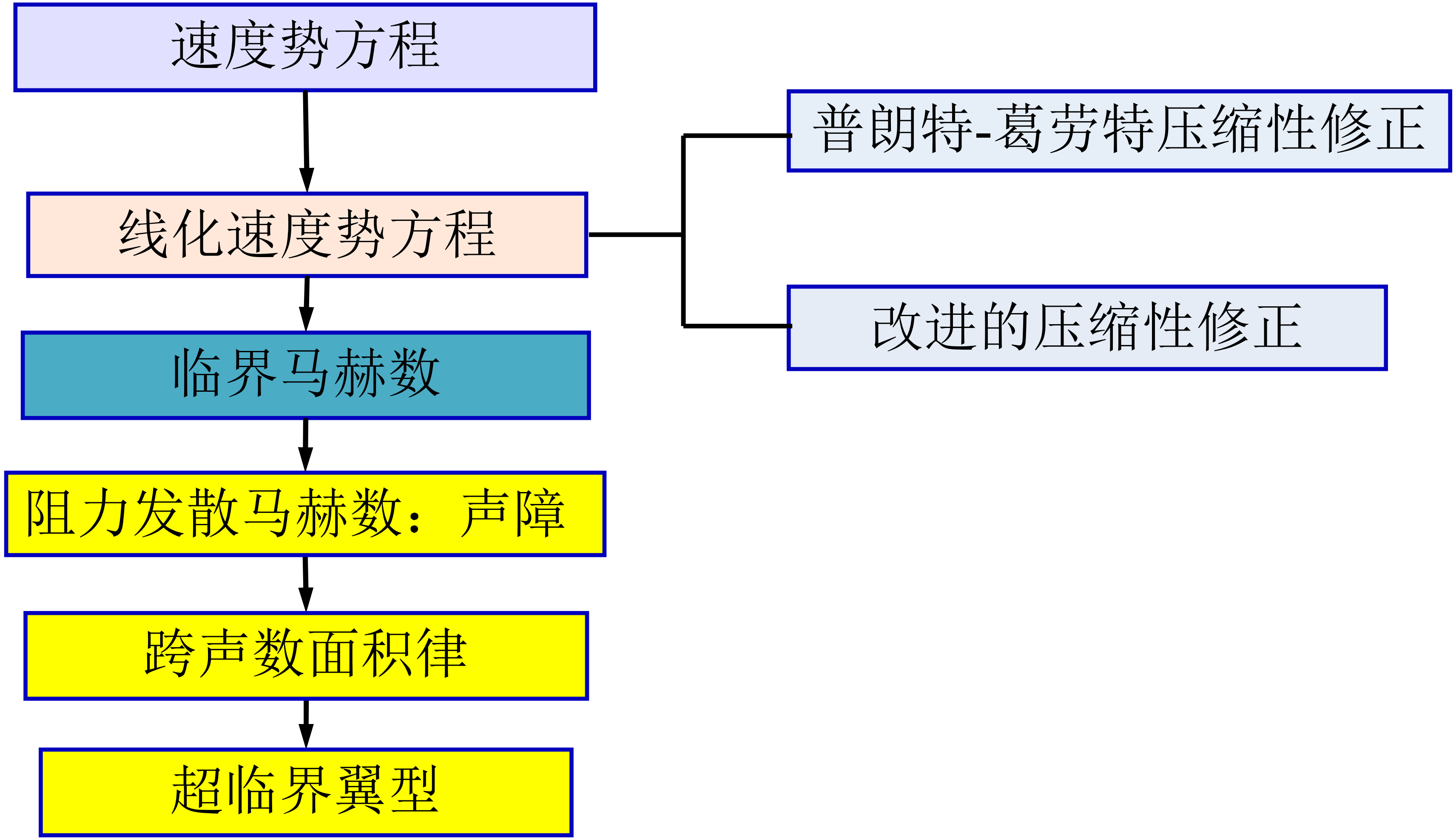
National Key Laboratory of Science and Technology on Aerodynamic Design and Research

十一章路线图


速度势方程
线化速度势方程
临界马赫数
阻力发散马赫数：声障
跨声数面积律
超临界翼型
普朗特-葛劳特压缩性修正
改进的压缩性修正

11.2 The Velocity Potential Equation (速度势方程)

For two-dimensional , steady, **irrotational** , isentropic flow, a velocity potential $\phi = \phi(x, y)$ can be defined such that :

$$V = \nabla \phi \qquad (11.1)$$

or in terms of the Cartesian velocity components:

$$u = \frac{\partial \phi}{\partial x} \qquad (11.2a)$$

$$v = \frac{\partial \phi}{\partial y} \qquad (11.2b)$$

速度势方程推导

The continuity equation for steady, two-dimensional flow is :

$$\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}=0 \tag{11.3}$$

or

$$\rho\frac{\partial u}{\partial x}+u\frac{\partial \rho}{\partial x}+v\frac{\partial \rho}{\partial y}+\rho\frac{\partial v}{\partial y}=0 \tag{11.4}$$

Substituting $u=\frac{\partial \phi}{\partial x}, v=\frac{\partial \phi}{\partial y}$ into it, we get

$$\rho(\frac{\partial^2 \phi}{\partial x^2}+\frac{\partial^2 \phi}{\partial y^2})+\frac{\partial \phi}{\partial x}\frac{\partial \rho}{\partial x}+\frac{\partial \phi}{\partial y}\frac{\partial \rho}{\partial y}=0 \tag{11.5}$$

速度势方程推导 （续）

To eliminate ρ from above equation, we consider the momentum equation :

$$dp = -\rho V dV \tag{3.12}$$

$$dp = -\rho V dV = -\frac{\rho}{2} dV^2 = -\frac{\rho}{2} d(u^2 + v^2)$$

$$dp = -\frac{\rho}{2} d\left[(\frac{\partial \phi}{\partial x})^2 + (\frac{\partial \phi}{\partial y})^2 \right] \tag{11.6}$$

Since the flow we are considering is isentropic, therefore

$$\frac{dp}{d\rho} = \left(\frac{\partial p}{\partial \rho} \right)_s = a^2 \tag{11.7}$$

速度势方程推导 （续）

Hence
$$d\rho = -\frac{\rho}{2a^2} d\left[(\frac{\partial\phi}{\partial x})^2 + (\frac{\partial\phi}{\partial y})^2 \right] \qquad (11.9)$$

$$\frac{\partial\rho}{\partial x} = -\frac{\rho}{2a^2}\frac{\partial}{\partial x}\left[(\frac{\partial\phi}{\partial x})^2 + (\frac{\partial\phi}{\partial y})^2 \right] \qquad (11.10)$$

$$\frac{\partial\rho}{\partial y} = -\frac{\rho}{2a^2}\frac{\partial}{\partial y}\left[(\frac{\partial\phi}{\partial x})^2 + (\frac{\partial\phi}{\partial y})^2 \right] \qquad (11.11)$$

Substitute them into

$$\rho(\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2}) + \frac{\partial\phi}{\partial x}\frac{\partial\rho}{\partial x} + \frac{\partial\phi}{\partial y}\frac{\partial\rho}{\partial y} = 0 \qquad (11.5)$$

速度势方程推导 （续）

We get the *velocity potential equation*:

$$\boxed{\left[1-\frac{1}{a^2}(\frac{\partial\phi}{\partial x})^2\right]\frac{\partial^2\phi}{\partial x^2}+\left[1-\frac{1}{a^2}(\frac{\partial\phi}{\partial y})^2\right]\frac{\partial^2\phi}{\partial y^2} -\frac{2}{a^2}(\frac{\partial\phi}{\partial x})(\frac{\partial\phi}{\partial y})\frac{\partial^2\phi}{\partial x\partial y}=0} \tag{11.12}$$

Here, the speed of sound is also the function of ϕ :

声速形式
能量方程
(8.33)

$$a^2=a_0^2-\frac{\gamma-1}{2}\Big[\underbrace{(\frac{\partial\phi}{\partial x})^2+(\frac{\partial\phi}{\partial y})^2}_{V^2}\Big] \tag{11.13}$$

$$a_0^2=a_\infty^2+\frac{\gamma-1}{2}V_\infty^2=const.$$

边界条件

- **The infinite boundary condition (远场边界条件) is**

$$u = \frac{\partial \phi}{\partial x} = V_\infty$$

Neumann边界条件

$$v = \frac{\partial \phi}{\partial y} = 0$$

- **The wall boundary condition (物面边界条件) is**

$$\frac{\partial \phi}{\partial n} = 0$$

Neumann边界条件

求解速度势方程后如何计算流场参数?

1. Calculate u and v : $u=\dfrac{\partial\phi}{\partial x}$ and $v=\dfrac{\partial\phi}{\partial y}$

2. Calculate a : $$a=\sqrt{a_0^{\;2}-\frac{\gamma-1}{2}\left[\left(\frac{\partial\phi}{\partial x}\right)^2+\left(\frac{\partial\phi}{\partial y}\right)^2\right]}$$

3. Calculate M : $$M=\frac{V}{a}=\frac{\sqrt{u^2+v^2}}{a}$$

4. Calculate T, p, ρ :

$$T=T_0\left(1+\frac{\gamma-1}{2}M^2\right)^{-1}$$

$$p=p_0\left(1+\frac{\gamma-1}{2}M^2\right)^{-\frac{\gamma}{\gamma-1}}$$

$$\rho=\rho_0\left(1+\frac{\gamma-1}{2}M^2\right)^{-\frac{1}{\gamma-1}}$$

11.3 The Linearized Velocity Potential Equation
线化速度势方程

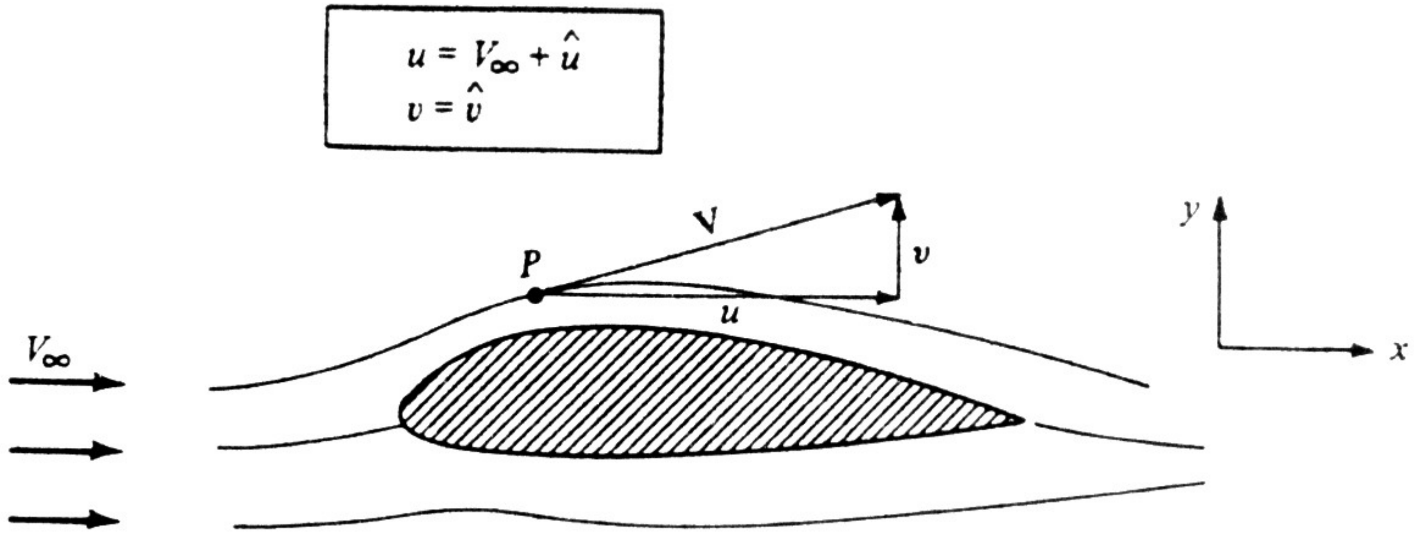


FIGURE 11.2
Uniform flow and perturbed flow.

If we define $u = V_\infty + \hat{u}$, $v = \hat{v}$, we call $\hat{u}$ and $\hat{v}$ the perturbation velocities (扰动速度).

首先，将全速势方程整理为扰动速度形式

$$
\begin{aligned}
(1-M_\infty^2)\frac{\partial\hat{u}}{\partial x}+\frac{\partial\hat{v}}{\partial y} &= M_\infty^2\left[(\gamma+1)\frac{\hat{u}}{V_\infty}+\frac{\gamma+1}{2}\frac{\hat{u}^2}{V_\infty^2}+\frac{\gamma-1}{2}\frac{\hat{v}^2}{V_\infty^2}\right]\frac{\partial\hat{u}}{\partial x} \\
&+M_\infty^2\left[(\gamma-1)\frac{\hat{u}}{V_\infty}+\frac{\gamma+1}{2}\frac{\hat{v}^2}{V_\infty^2}+\frac{\gamma-1}{2}\frac{\hat{u}^2}{V_\infty^2}\right]\frac{\partial\hat{v}}{\partial y} \\
&+M_\infty^2\left[\frac{\hat{v}}{V_\infty}\left(1+\frac{\hat{u}}{V_\infty}\right)\left(\frac{\partial\hat{u}}{\partial y}+\frac{\partial\hat{v}}{\partial x}\right)\right]
\end{aligned}
\tag{11.16}
$$

该方程对无粘、无旋、等熵流动，无论扰动大或小，仍精确成立

其次，推导小扰动线化(扰动)速度势方程

引入小扰动假设后

$$(1-M_\infty^2)\frac{\partial \hat{u}}{\partial x}+\frac{\partial \hat{v}}{\partial y}=0 \qquad (11.17)$$

线化小扰动速势方程

$$\boxed{(1-M_\infty^2)\frac{\partial^2 \hat{\phi}}{\partial x^2}+\frac{\partial^2 \hat{\phi}}{\partial y^2}=0} \qquad (11.18)$$

近似的

小扰动速势方程的适用条件

1、*Small* perturbation, i.e., thin bodies at small angles of attack (小扰动，即小攻角下的薄物体）

2、*Subsonic* and *supersonic* Mach numbers (亚音速或超音速）

$0 \le M_\infty \le 0.8 \quad 1.2 \le M_\infty < 5$

讨论问题：绕翼型流动是否处处满足小扰动假设?

驻点附近不满足（钝头翼型前缘，有后缘角的翼型后缘）

压强系数 （Pressure coefficient）

压强系数的精确表达式为/Exact expression for pressure coefficient

$$C_p = \frac{2}{\gamma M_\infty^2}\left(\frac{p}{p_\infty} - 1\right) \qquad (11.22)$$

引入小扰动线化后的压强系数/Linearized pressure coefficient

（忽略高阶小量）

$$\boxed{C_p = -\frac{2\hat{u}}{V_\infty}} \qquad (11.32)$$

线化方程的边界条件/Boundary conditions for linearized equation

远场边界条件

$$\hat{\phi} = \text{constant}; \quad or \ \hat{u} = \hat{v} = 0$$

物面边界条件

$$\tan\theta = \frac{v}{u} = \frac{\hat{v}}{V_\infty + \hat{u}} \tag{11.33}$$

公式(11.33)为精确的流动与物面相切条件。在小扰动假设下，它近似为

$$\tan\theta = \frac{\hat{v}}{V_\infty} \qquad \boxed{\frac{\partial\hat{\phi}}{\partial y} = V_\infty \tan\theta} \tag{11.34}$$

投票 最多可选1项

设置

请对上一次课内容的掌握情况进行投票

- A 完全掌握了这部分知识内容
- B 掌握了大部分
- C 掌握了一小部分
- D 完全不懂

提交

Review of Lecture #16 ended !

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十一章路线图

速度势方程

↓

线化速度势方程 → 普朗特-葛劳特压缩性修正；改进的压缩性修正

↓

临界马赫数

↓

阻力发散马赫数：声障

↓

跨声速面积律

↓

超临界翼型

11.4 PRANDTL-GLAUERT COMPRESSIBILITY CORRECTION (普朗特-葛劳特压缩性修正)

Compressibility correction（压缩性修正）

The methods that allow simple corrections to existing incompressible flow results which would approximately take into account of the effects of compressibility

通过修正不可压缩流的结果来近似考虑压缩性影响的方法。

Compressibility correction（压缩性修正）

要解决的问题：绕翼型的亚声速可压缩流（小扰动线化条件）

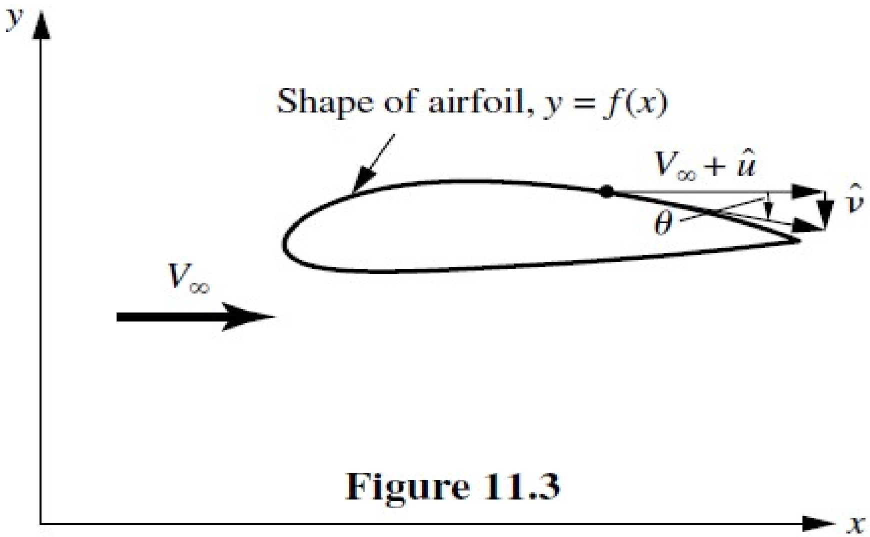


Figure 11.3

满足的控制方程：

$$\boxed{(1-M_\infty^2)\frac{\partial^2\hat{\phi}}{\partial x^2}+\frac{\partial^2\hat{\phi}}{\partial y^2}=0} \qquad (11.18)$$

满足的物面边界条件：

$$\frac{\partial\hat{\phi}}{\partial y}=V_\infty \tan\theta \quad \Rightarrow \quad \frac{\partial\hat{\phi}}{\partial y}=V_\infty\frac{df}{dx}$$

普朗特-葛劳特压缩性修正

对于亚声速流动，定义 $\beta^2 \equiv 1 - M_\infty^2$

线化速度势方程可写为 $$\beta^2 \frac{\partial^2 \hat{\phi}}{\partial x^2} + \frac{\partial^2 \hat{\phi}}{\partial y^2} = 0 \qquad (11.35)$$

将物理坐标系x,y变换到新空间(可称为转换坐标系，或ξ,η坐标系)

$$\xi = x \qquad (11.36a)$$

$$\eta = \beta y \qquad (11.36b)$$

引入新的扰动速度势

$$\overline{\phi}(\xi, \eta) = \beta \hat{\phi}(x, y) \qquad (11.36c)$$

普朗特-葛劳特压缩性修正（续）

一阶导数的变换

In（11.35):

$$\frac{\partial \hat{\phi}}{\partial x} = \frac{\partial \hat{\phi}}{\partial \xi}\frac{\partial \xi}{\partial x} + \frac{\partial \hat{\phi}}{\partial \eta}\frac{\partial \eta}{\partial x} \qquad (11.37)$$

$$\frac{\partial \hat{\phi}}{\partial y} = \frac{\partial \hat{\phi}}{\partial \xi}\frac{\partial \xi}{\partial y} + \frac{\partial \hat{\phi}}{\partial \eta}\frac{\partial \eta}{\partial y} \qquad (11.38)$$

由坐标变换公式可知 $\frac{\partial \xi}{\partial x} = 1 \quad \frac{\partial \xi}{\partial y} = 0 \quad \frac{\partial \eta}{\partial x} = 0 \quad \frac{\partial \eta}{\partial y} = \beta$

因此

$$\frac{\partial \hat{\phi}}{\partial x} = \frac{\partial \hat{\phi}}{\partial \xi} \qquad (11.39)$$

$$\frac{\partial \hat{\phi}}{\partial y} = \beta \frac{\partial \hat{\phi}}{\partial \eta} \qquad (11.40)$$

普朗特-葛劳特压缩性修正 （续）

一阶导数的变换

Since $$\bar{\phi}(\xi,\eta)=\beta\hat{\phi}(x,y) \qquad (11.36c)$$

we have $$\frac{\partial\hat{\phi}}{\partial x}=\frac{1}{\beta}\frac{\partial\bar{\phi}}{\partial\xi} \qquad (11.41)$$

$$\frac{\partial\hat{\phi}}{\partial y}=\frac{\partial\bar{\phi}}{\partial\eta} \qquad (11.42)$$

二阶导数的变换

$$\frac{\partial^2\hat{\phi}}{\partial x^2}=\frac{\partial}{\partial\xi}\left(\frac{1}{\beta}\frac{\partial\bar{\phi}}{\partial\xi}\right)\frac{\partial\xi}{\partial x}=\frac{1}{\beta}\frac{\partial^2\bar{\phi}}{\partial\xi^2} \qquad (11.43)$$

$$\frac{\partial^2\hat{\phi}}{\partial y^2}=\frac{\partial}{\partial\eta}\left(\frac{\partial\bar{\phi}}{\partial\eta}\right)\frac{\partial\eta}{\partial y}=\beta\frac{\partial^2\bar{\phi}}{\partial\eta^2} \qquad (11.44)$$

普朗特-葛劳特压缩性修正（续）

$\hat{\phi},\ x,\ y\ \rightarrow\ \overline{\phi},\ \xi,\ \eta$

原有的线化扰动速度势方程最后变换为

新速度势在转换坐标系中的拉普拉斯方程

$$\frac{\partial^2 \overline{\phi}}{\partial \xi^2} + \frac{\partial^2 \overline{\phi}}{\partial \eta^2} = 0 \qquad (11.45)$$

Laplace's equation is the governing equation for inviscid , incompressible flow. This means that we can relate the compressible flow in (x,y) space to an incompressible flow in (ξ,η) space.

Laplace方程是无粘、不可压流的控制方程。这就意味着我们将(x,y)空间内的可压缩流动与(ξ,η)空间的不可压缩流动联系起来了。

Boundary Condition in Physical Space 物理空间边界条件		Boundary Condition in Transformed Space 转换空间的边界条件
物面外形	$y=f(x)$	$\eta=q(\xi)$
精确	$\dfrac{\hat{v}}{V_\infty+\hat{u}}=\dfrac{df}{dx}$	$\dfrac{\bar{v}}{V_\infty+\bar{u}}=\dfrac{dq}{d\xi}$
小扰动近似	$\dfrac{\partial\hat{\phi}}{\partial y}=V_\infty\dfrac{df}{dx}$	$\dfrac{\partial\bar{\phi}}{\partial\eta}=V_\infty\dfrac{dq}{d\xi}$ (11.47)

$$\frac{\partial\hat{\phi}}{\partial y}=\frac{\partial\bar{\phi}}{\partial\eta} \tag{11.42}$$

$$\frac{dq}{d\xi}=\frac{df}{dx} \tag{11.48}$$

普朗特-葛劳特压缩性修正（续）

$$\frac{dq}{d\xi} = \frac{df}{dx} \tag{11.48}$$

结论：

- 物理空间和变换空间内的翼型形状相同
- 以上变换将“物理空间中绕翼型的可压缩流动”与“变换空间中绕相同翼型的不可压流动”联系起来

线化压强系数/Linearized pressure coefficient

$$\boxed{C_p = -\frac{2\hat{u}}{V_\infty}} = -\frac{2}{V_\infty}\underline{\frac{\partial\hat{\phi}}{\partial x}} = -\underline{\frac{2}{V_\infty}\frac{1}{\beta}\frac{\partial\overline{\phi}}{\partial x}} = \frac{1}{\beta}\left(-\frac{2}{V_\infty}\underline{\frac{\partial\overline{\phi}}{\partial\xi}}\right) \qquad (11.49)$$

（11.32）

转换空间内不可压缩流动的ξ方向扰动速度 $\overline{u}$

因此

$$C_p = \frac{1}{\beta}\left(-\frac{2\overline{u}}{V_\infty}\right) \qquad (11.50)$$

$$\left(-\frac{2\overline{u}}{V_\infty}\right) = C_{p,0}$$

转换空间中不可压流动的线化压强系数

普朗特–葛劳特准则

$$\boxed{C_p = \frac{C_{p,0}}{\beta} = \frac{C_{p,0}}{\sqrt{1 - M_\infty^2}}} \qquad (11.51)$$

It is called Prandtl-Glauert rule; it states that , if we know the incompressible pressure distribution over an airfoil, then the compressible flow over the same airfoil can be obtained from (11.51).

该式称为“普朗特–葛劳特准则”。它表明：如果已知翼型在不可压流动下的压力分布，就可通过该准则获得相同翼型在可压缩条件下的压力分布。

普朗特-葛劳特准则

$$\boxed{C_p = \frac{C_{p,0}}{\sqrt{1-M_\infty^2}}} \tag{11.51}$$

$$c_l = \frac{c_{l,0}}{\sqrt{1-M_\infty^2}} \tag{11.52}$$

$$c_m = \frac{c_{m,0}}{\sqrt{1-M_\infty^2}} \tag{11.53}$$

适用于薄物体、小迎角、亚声速（高亚声速时偏差大）

- **1922**年普朗特在哥廷根的一个讲座中给出了这个结果
- **1928**年英国空气动力学家葛劳特正式发表了其推导过程

Example 11.1 在翼型表面一给定点，已知在绕流速度极低时的压强系数为-0.3。如果自由来流马赫数为0.6，计算这一点的压强系数。

$$C_p = \frac{C_{p,0}}{\sqrt{1 - {M_\infty}^2}} = \frac{-0.3}{\sqrt{1 - (0.6)^2}} = -0.375$$

Example 11.2　由第四章，我们得出绕对称、薄翼型的不可压流动的理论升力系数为 $c_l = 2\pi\alpha$ 。计算自由来流马赫数为 0.7 时的升力系数。

$$c_l = \frac{c_{l,0}}{\sqrt{1-M_\infty{}^2}} = \frac{2\pi\alpha}{\sqrt{1-(0.7)^2}} = 8.8\alpha$$

单选题 1分

设置

普朗特-葛劳特压缩性修正公式适用于下列哪类问题？

A 小迎角条件下，绕薄翼型的超声速流动问题

B 小迎角条件下，绕薄翼型的亚声速流动问题

C 大迎角条件下，绕薄翼型的亚声速流动问题

D 大迎角条件下，绕薄翼型的超声速流动问题

提交

11.5 IMPROVED COMPRESSIBILITYCORRECTIONS 改进的压缩性修正

卡门-钱公式:

$$C_p = \frac{C_{p,0}}{\sqrt{1-M_\infty^2} + [M_\infty^2/(1+\sqrt{1-M_\infty^2})]C_{p,0}/2} \tag{11.54}$$

Laitone 公式:

$$C_p = \frac{C_{p,0}}{\sqrt{1-M_\infty^2} + [M_\infty^2(1+\frac{\gamma-1}{2}M_\infty^2)/(2\sqrt{1-M_\infty^2})]C_{p,0}} \tag{11.55}$$

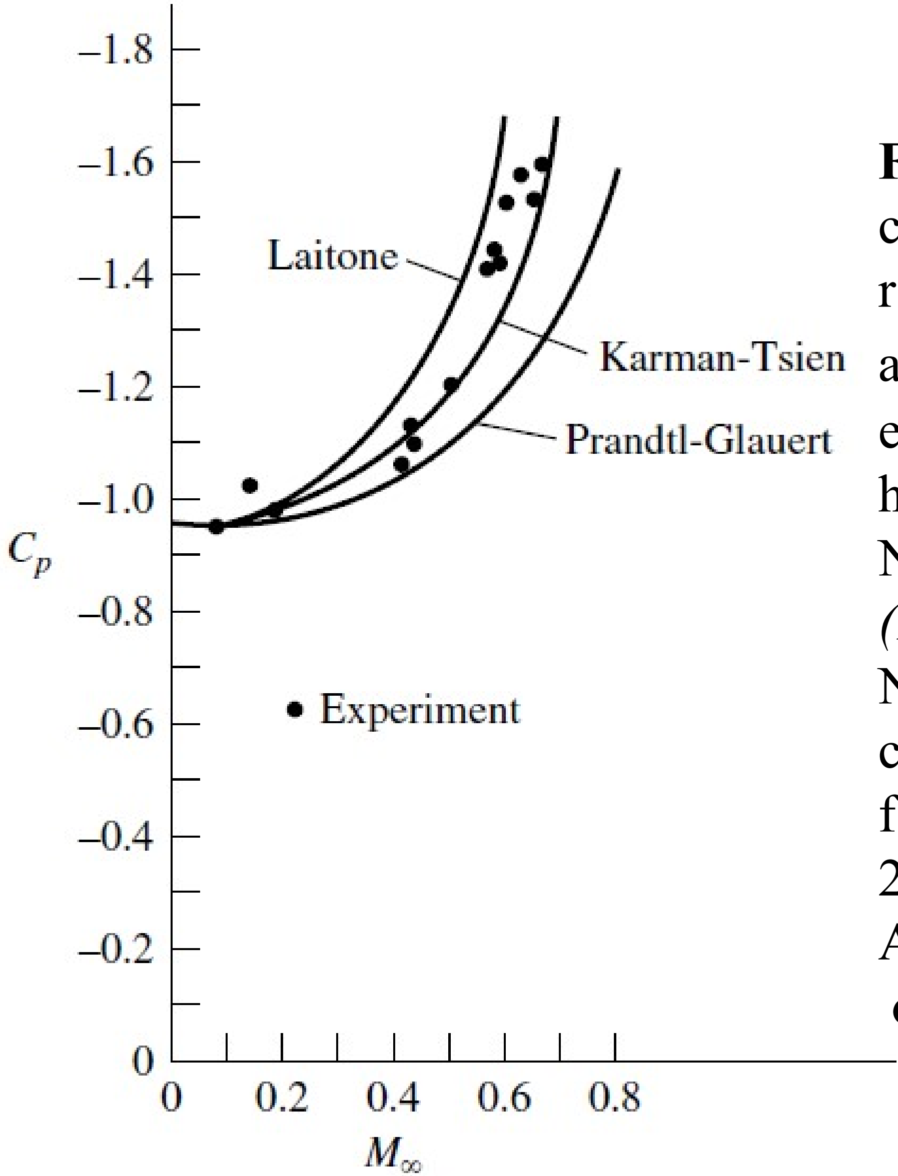


Figure 11.4 Several compressibility corrections compared with experimental results for an NACA 4412 airfoil at an angle of attack $\alpha = 1^\circ\ 53'$. The experimental data are chosen for their historical significance; they are from NACA report no. 646, published in 1938 *(Reference 30)*. This was the first major NACA publication to address the compressibility problem in a systematic fashion; it covered work performed in the 2-ft high-speed tunnel at the Langley Aeronautical Laboratory and was carried out during 1935–1936.

Comments for compressibility corrections

- Prandtl-Glauert rule：基于线性理论，因此适用于薄物体、小迎角、亚声速、不适合高亚声速
- Karmen-Tisen和Laitone公式: 都试图反映高亚声速时流动的非线性特征，高亚声速也适用

钱学森

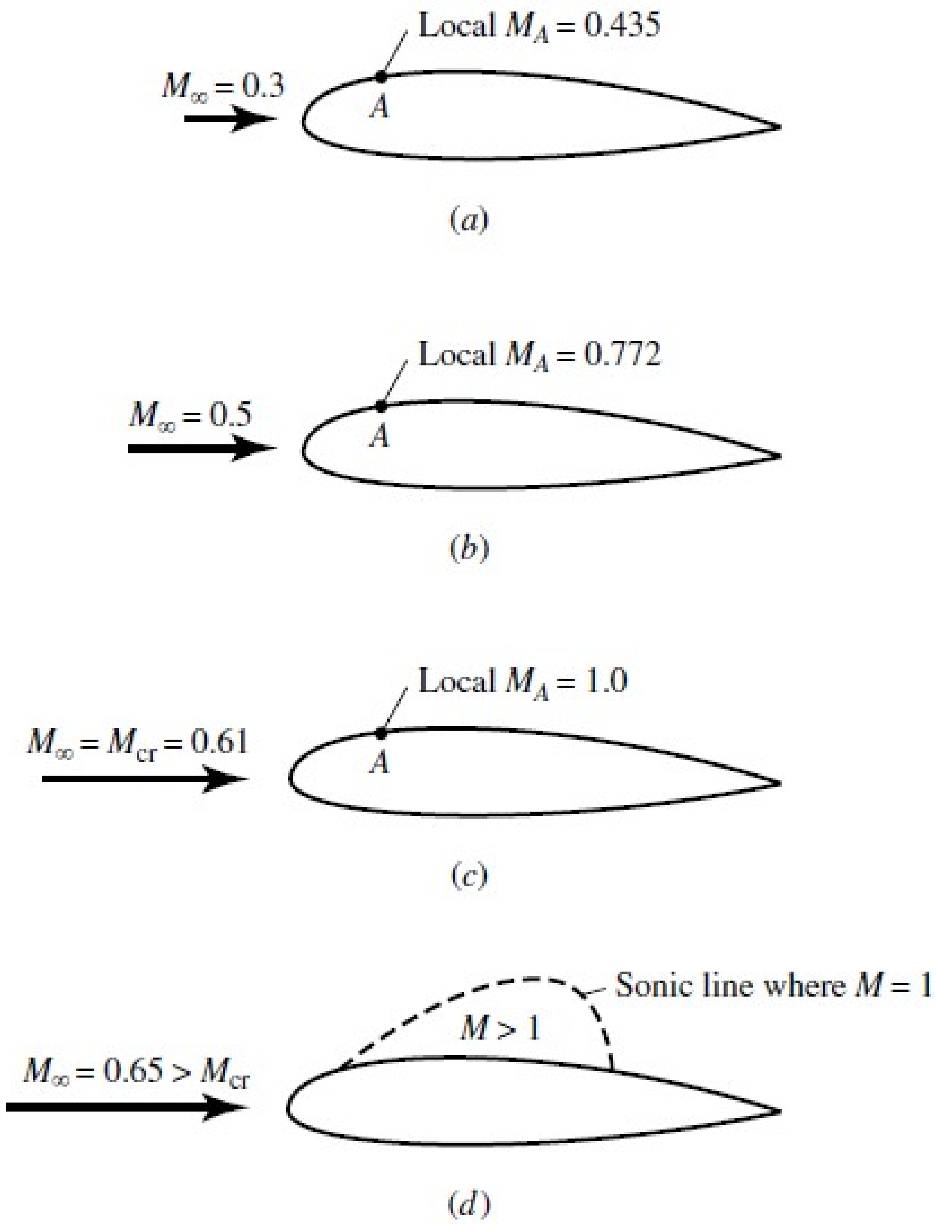
冯.卡门

普朗特

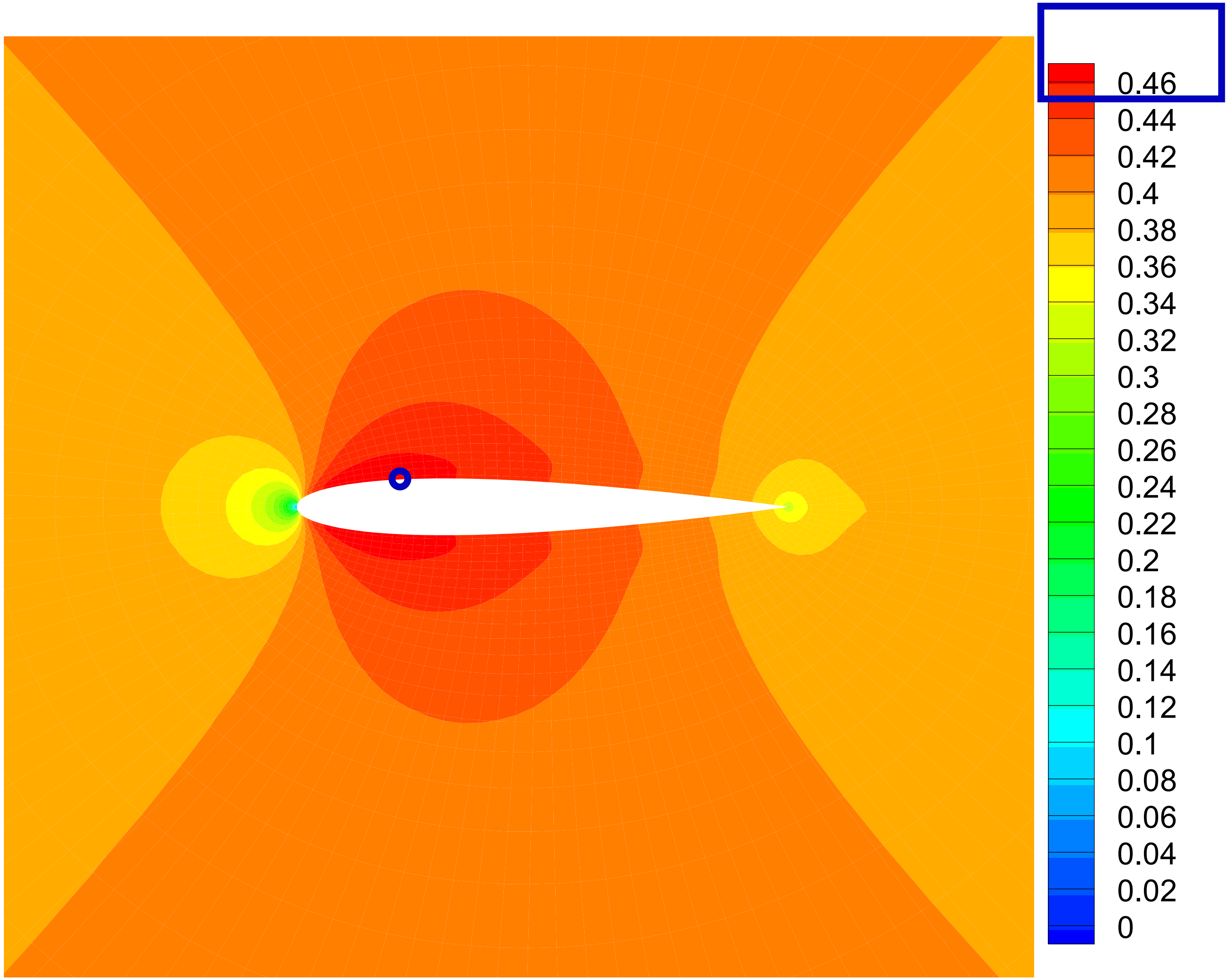
11.6 CRITICAL MACH NUMBER（临界马赫数）

We deal with several aspects of transonic flow from a qualitative point of view.

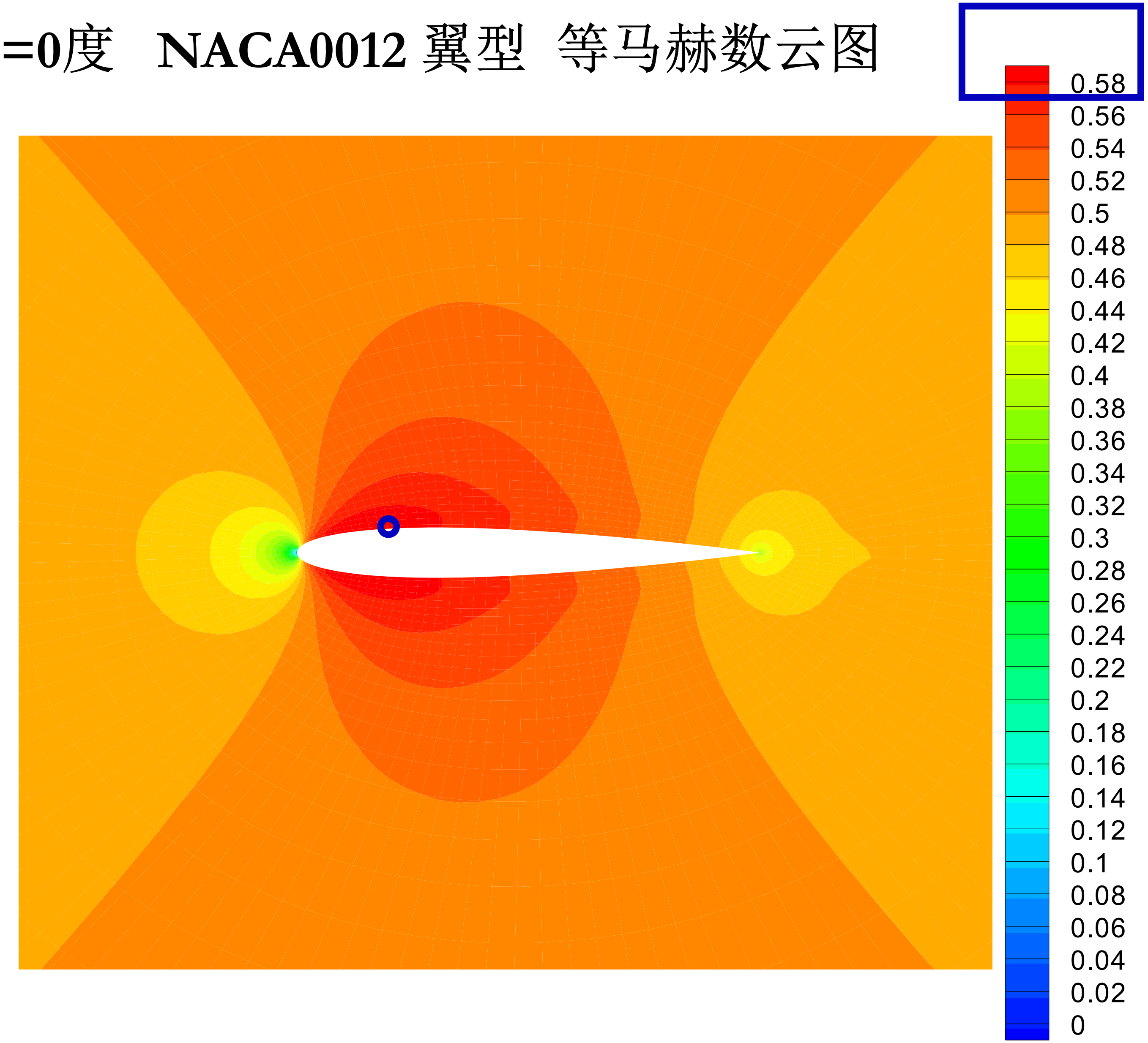
本节主要定性地讨论跨声速流动的特征。



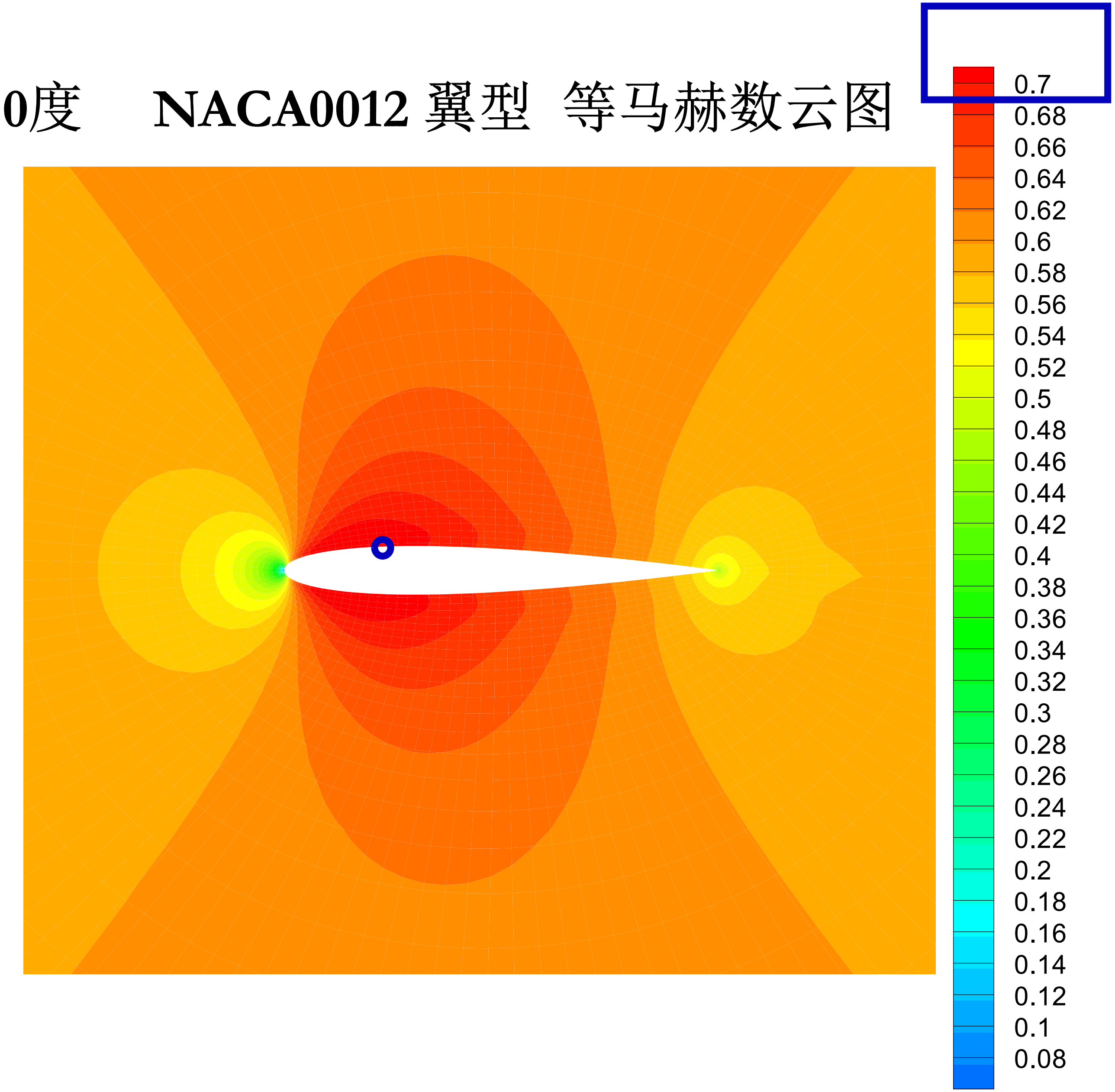
$M_\infty=0.4$ $\alpha=0$度，NACA0012 翼型 等马赫数云图



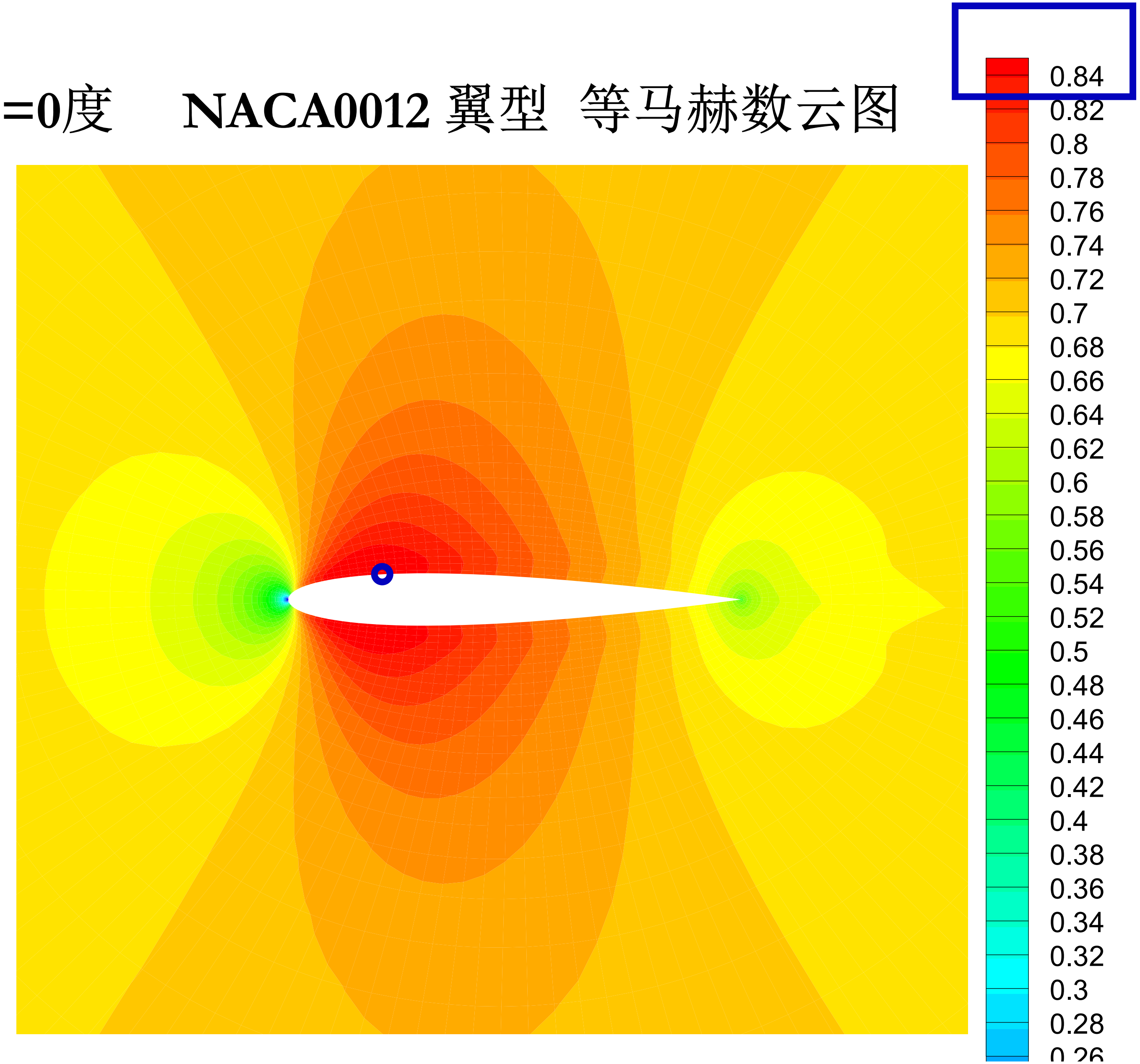
M_∞=0.5 α=0度 NACA0012 翼型 等马赫数云图



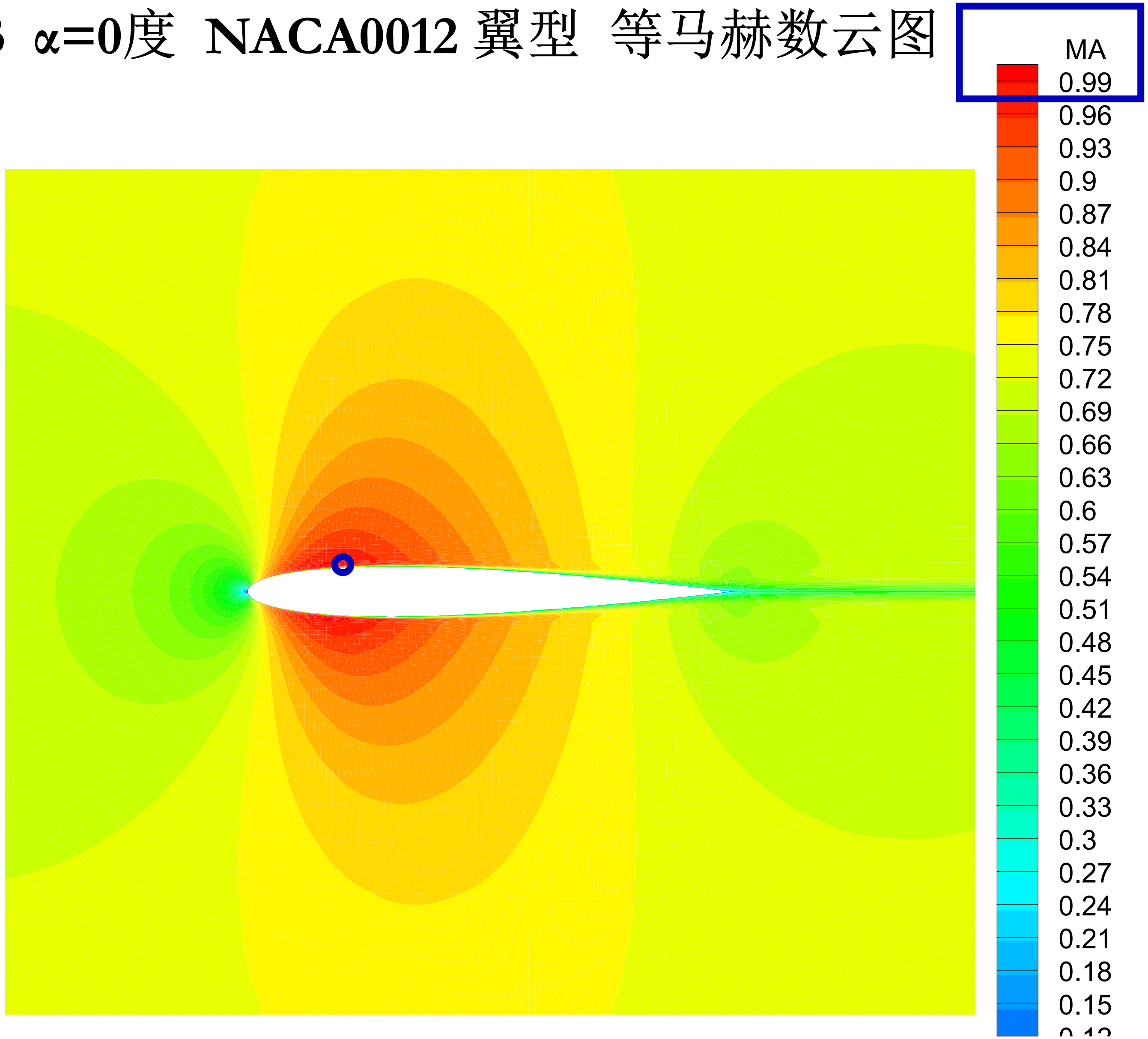
M_∞=0.6 α=0度   NACA0012 翼型 等马赫数云图



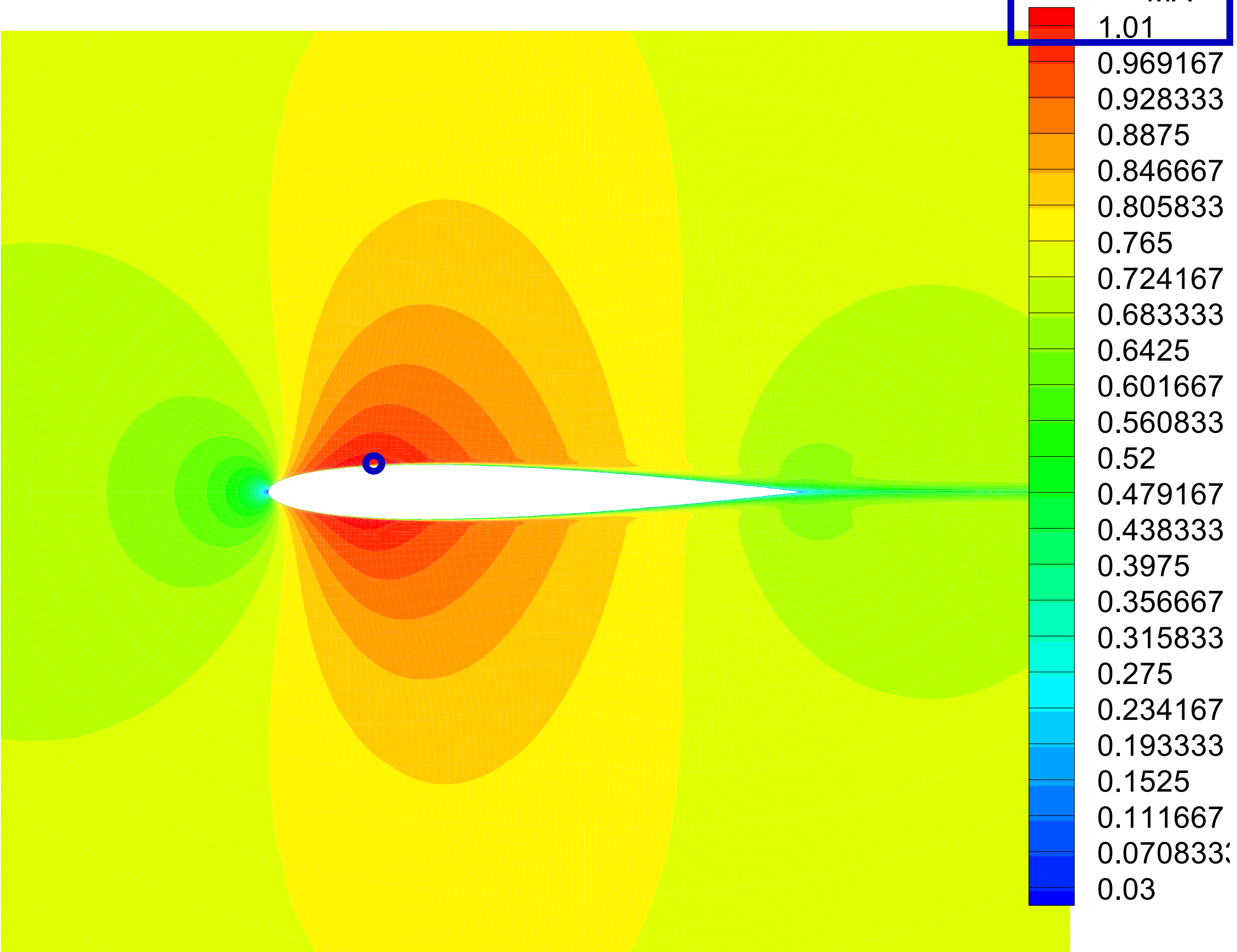
M_∞=0.7 α=0度 NACA0012 翼型 等马赫数云图



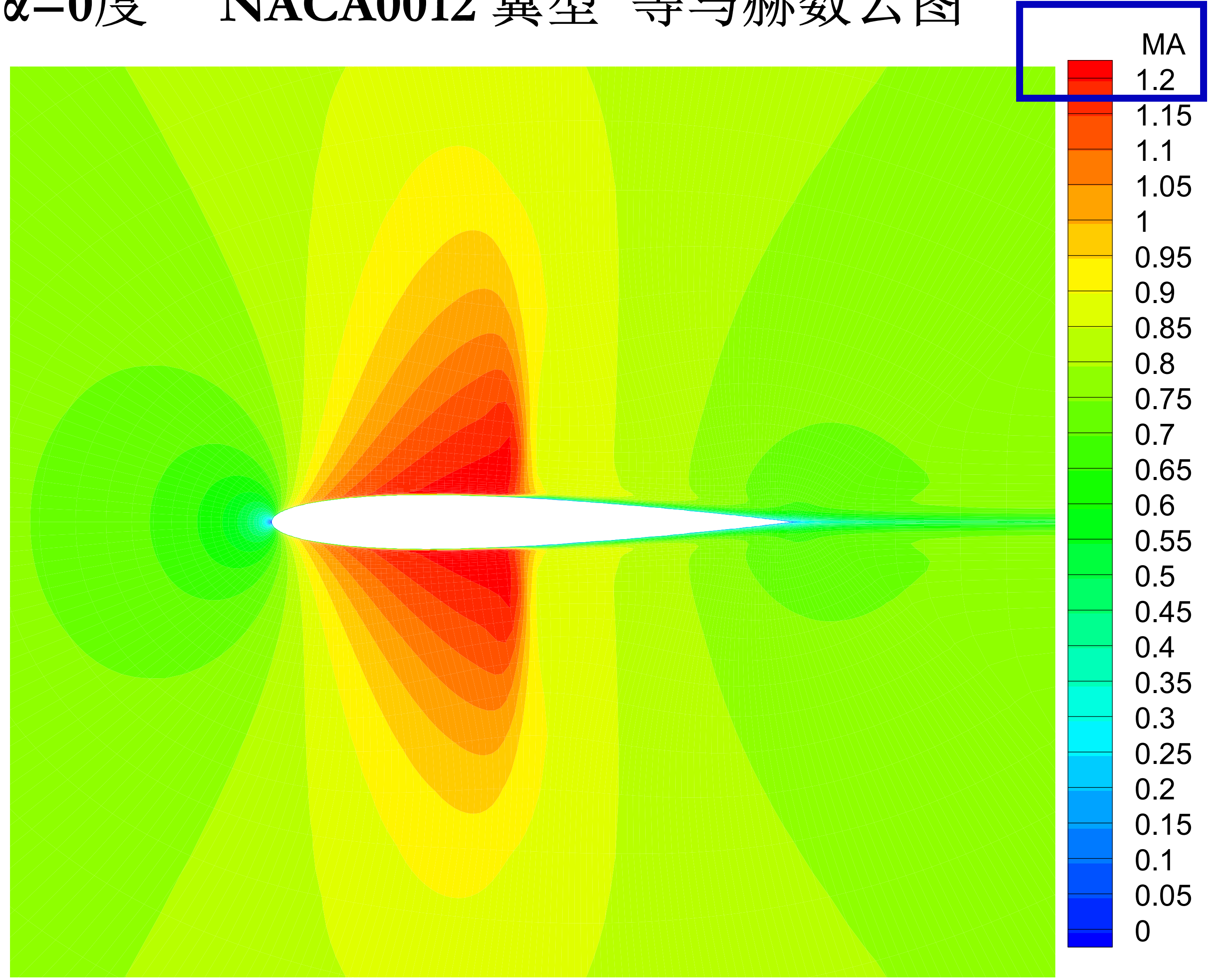
M_∞=0.73 α=0度 NACA0012 翼型 等马赫数云图



$M_\infty=0.74$ α=0度 NACA0012 翼型 等马赫数云图



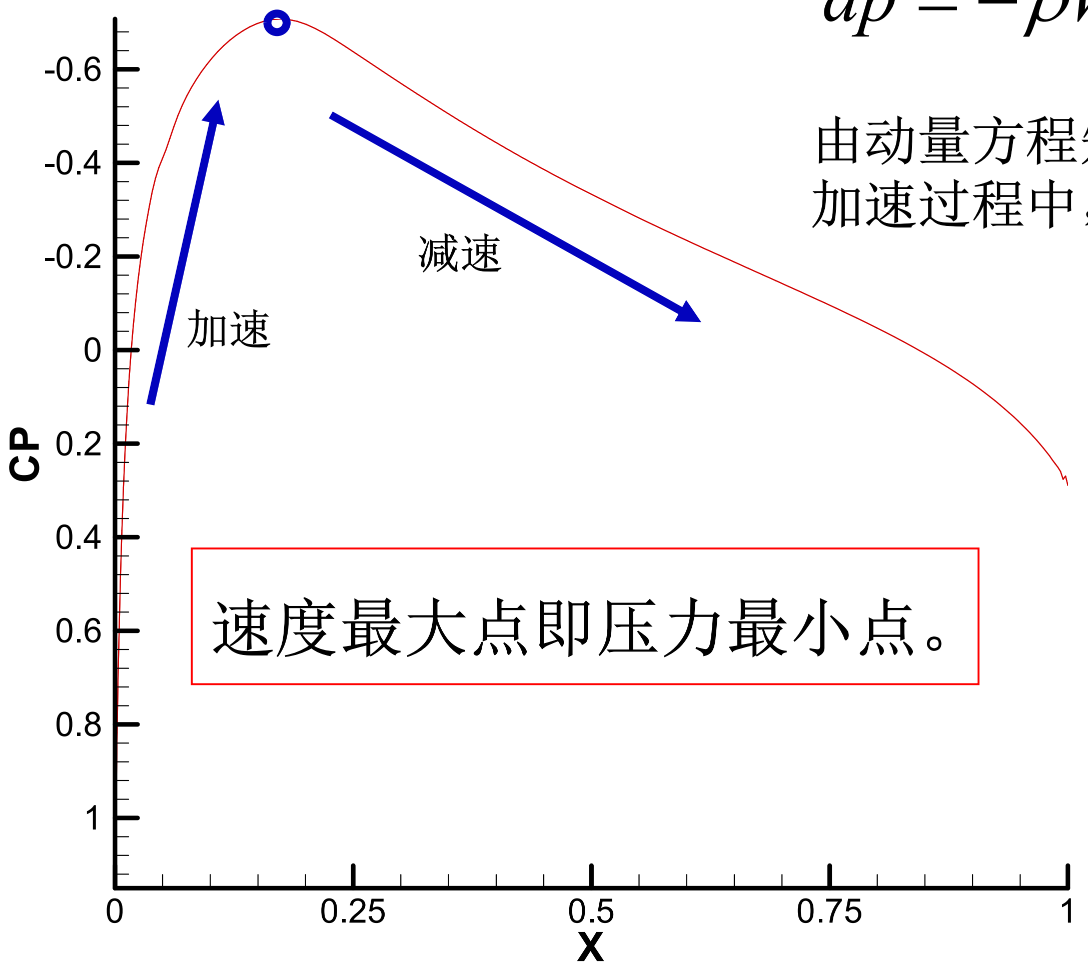
M_∞=0.8 α=0度　NACA0012 翼型 等马赫数云图



M_∞=0.74 α=0度　NACA0012 翼型表面压力系数分布

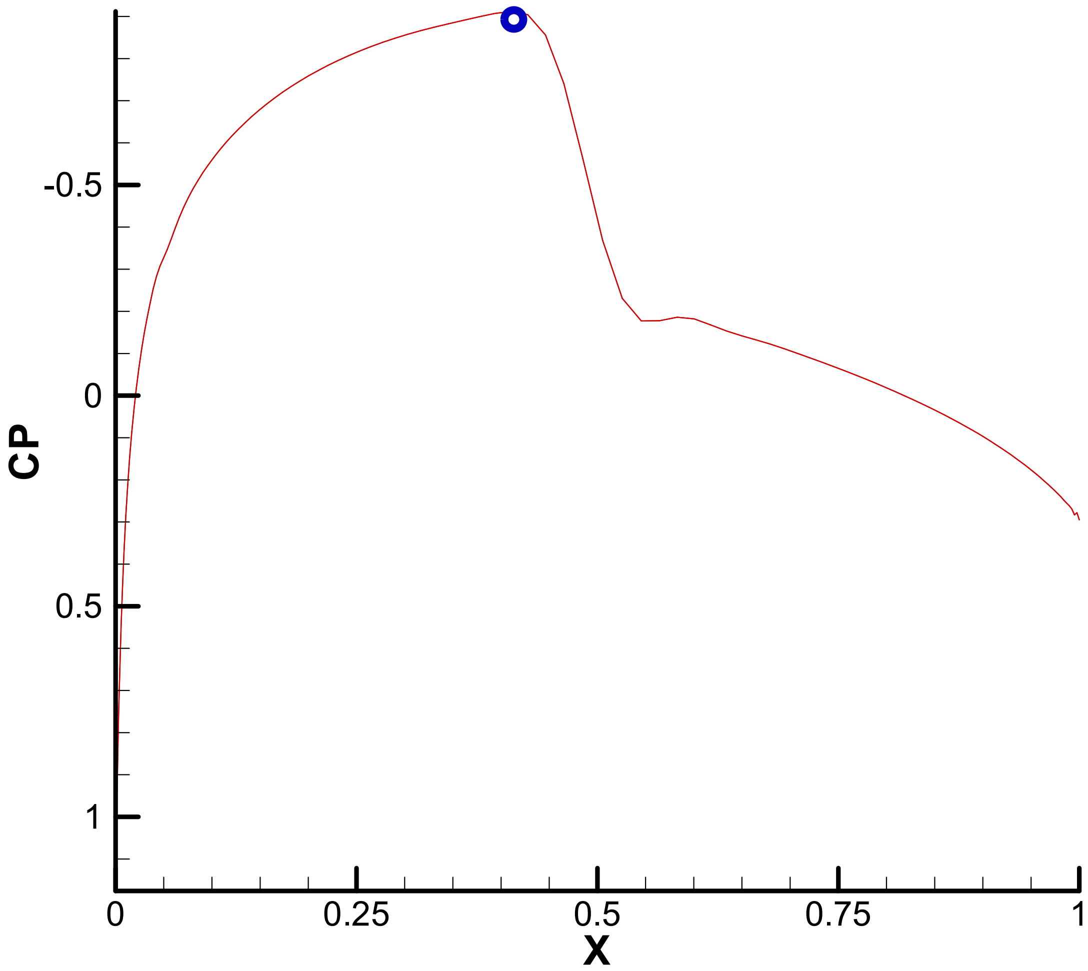
$$dp = -\rho V dV \qquad (3.12)$$

由动量方程知，气流在翼型表面加速过程中，压强会不断下降



速度最大点即压力最小点。

M_∞=0.80 α=0度 NACA0012 翼型表面压力系数分布



临界马赫数的定义？

The critical Mach number is that freestream Mach number at which sonic flow is first achieved on the airfoil surface/翼型表面速度最大点刚好达到声速时对应的*自由来流马赫数*，称为翼型的临界马赫数。

Derivation of *critical pressure coefficient* (临界压强系数的推导)

临界压强系数的定义：当地马赫数为1时对应的压强系数。

流场中任意一点A的压强系数可以通过如下推导得到：

$$\boxed{\frac{p_0}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}}}$$

$$\frac{p_A}{p_\infty} = \frac{p_A/p_0}{p_\infty/p_0} = \left(\frac{1+[(\gamma-1)/2]M_\infty^2}{1+[(\gamma-1)/2]M_A^2}\right)^{\gamma/(\gamma-1)} \quad (8.42)\rightarrow(11.56)$$

$$C_{p,A} = \frac{2}{\gamma M_\infty^2}\left[\frac{p_A}{p_\infty} - 1\right] \quad (11.22)\rightarrow(11.57)$$

将（11.56)代入(11.57)得：

$$C_{p,A} = \frac{2}{\gamma M_\infty^2}\left[\left(\frac{1+[(\gamma-1)/2]M_\infty^2}{1+[(\gamma-1)/2]M_A^2}\right)^{\gamma/(\gamma-1)} - 1\right] \quad (11.58)$$

Derivation of *critical pressure coefficient* (临界压强系数的推导)

根据临界压强系数的定义 $M_A = 1$

$$C_{p,cr} = \frac{2}{\gamma M_\infty^2}\left[\left(\frac{1+[(\gamma-1)/2]M_\infty^2}{1+(\gamma-1)/2}\right)^{\gamma/(\gamma-1)} - 1\right] \qquad (11.59)$$

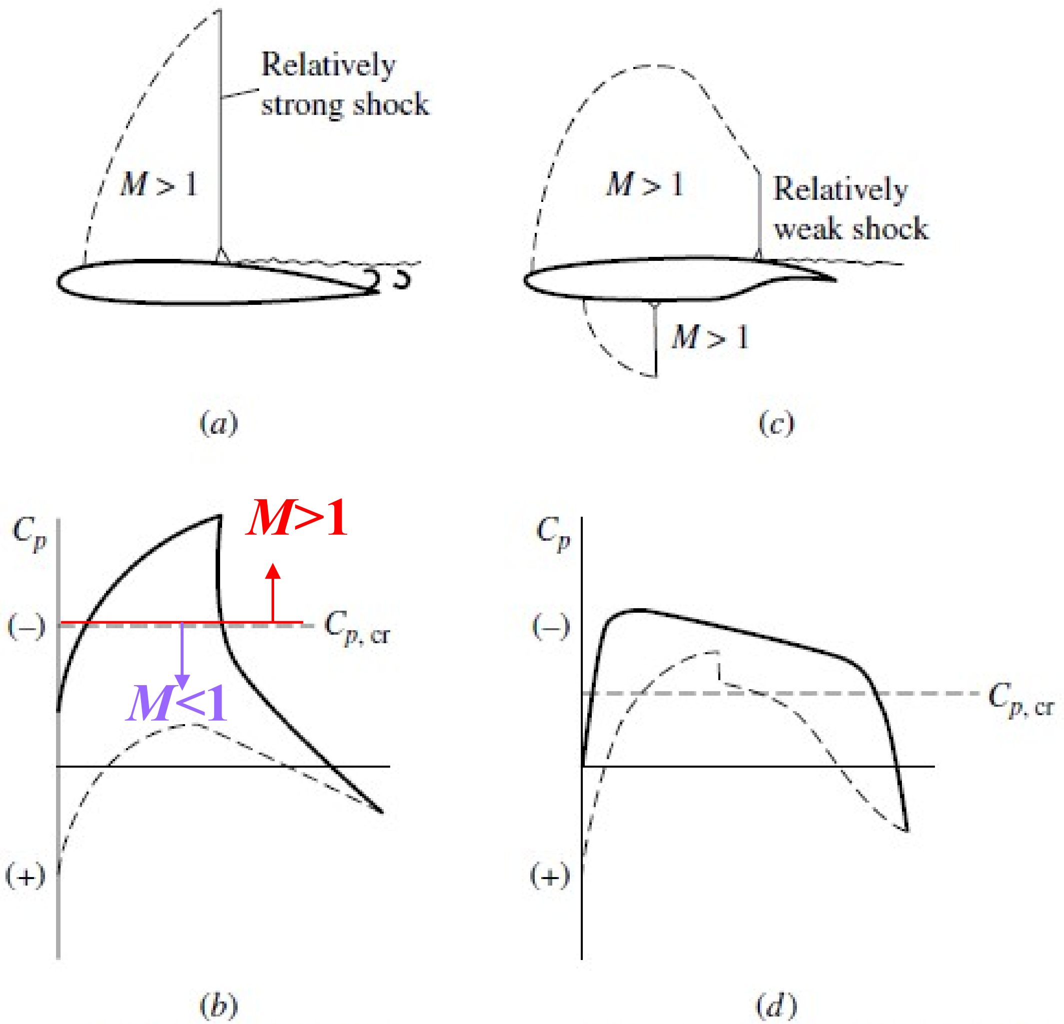
如果 $M_\infty = M_{cr}$，流场中刚好只有一点的马赫数达到1，

则有：

$$\boxed{C_{p,cr} = \frac{2}{\gamma M_{cr}^2}\left[\left(\frac{1+[(\gamma-1)/2]M_{cr}^2}{1+(\gamma-1)/2}\right)^{\gamma/(\gamma-1)} - 1\right]} \qquad (11.60)$$

方程(11.60)表明：临界压强系数是临界马赫数的唯一函数

临界压强系数的意义



NACA 64_2–A215 airfoil
$M_\infty = 0.69$

Supercritical airfoil (13.5% thick)
$M_\infty = 0.79$

临界马赫数的估算

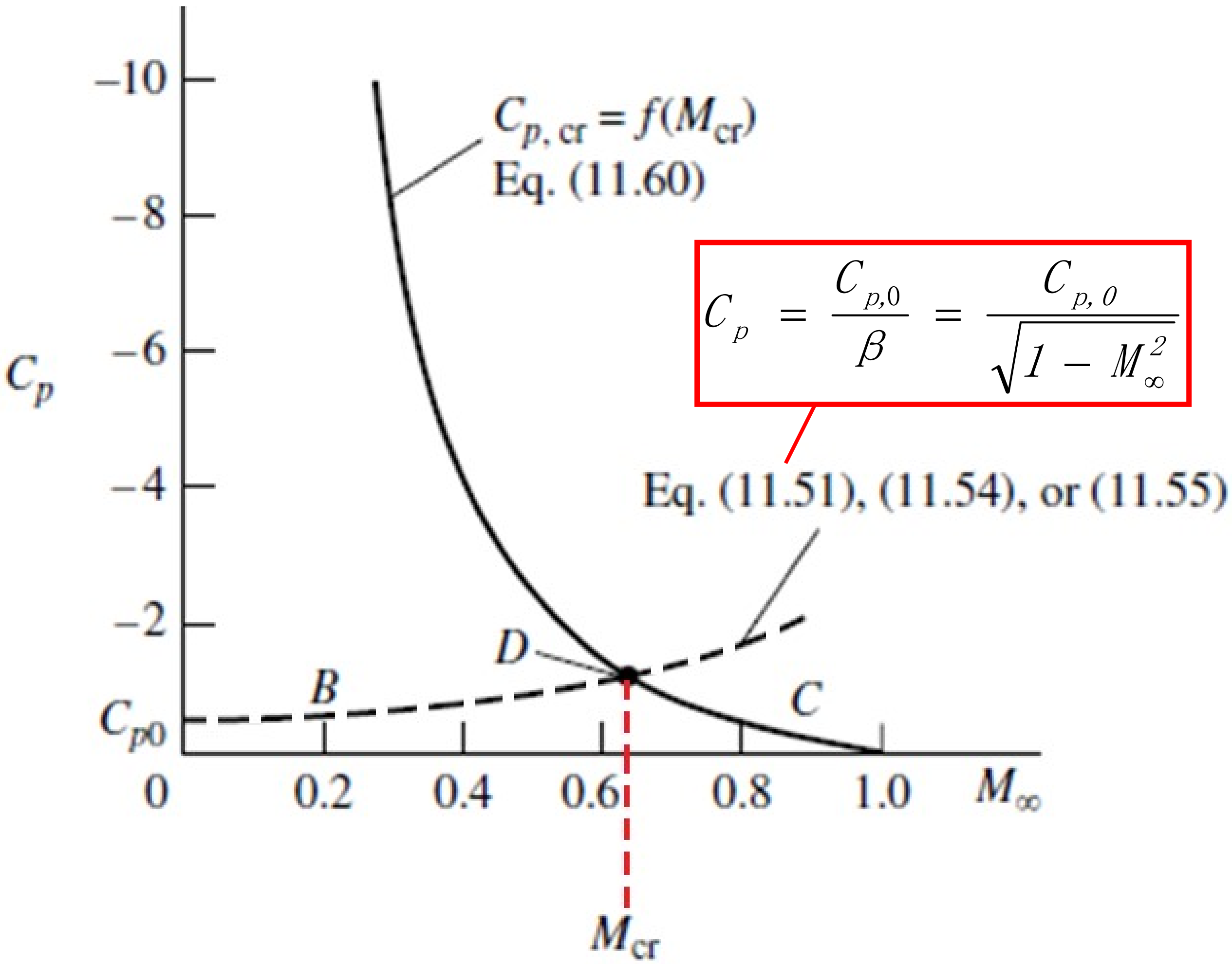


Figure 11.6 Estimation of critical Mach number.

临界马赫数的估算

结合压缩性修正公式和临界压强系数计算公式，我们可以估算出一个翼型的临界马赫数：

1、通过实验或理论方法，得到翼型在低速不可压绕流下的表面最小压力点的压强系数 C_p 。

2、用压缩性修正公式得到最小压强系数C_p随自由来流马赫数 M_∞ 的变化曲线 B。

3、求出曲线 B 与方程（11.60）代表的曲线 C 的交点，对应的横坐标位置就是 M_{cr} 的估算值。

翼型厚度对临界马赫数的影响

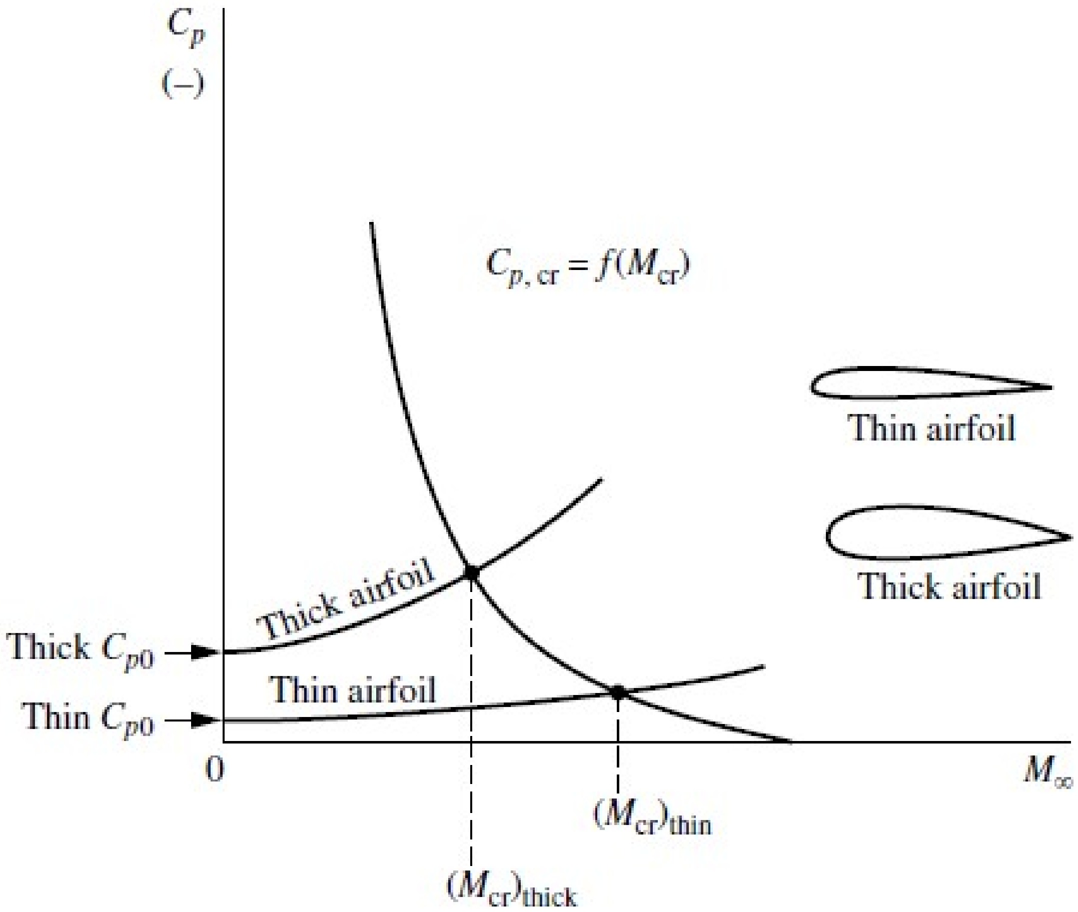


Figure 11.7 Effect of airfoil thickness on critical Mach number.

结论：翼型越厚，临界马赫数越小。

多选题 1分

设置

下列说法正确的是：

A NACA0012翼型的临界马赫数小于NACA0010翼型的临界马赫数

B 翼型表面速度最大点刚好达到声速时对应的自由来流马赫数，称为翼型的临界马赫数

C 翼型表面速度最大点刚好达到声速时对应的马赫数，称为翼型的临界马赫数

D NACA0012翼型的临界马赫数大于NACA0010翼型的临界马赫数

提交

本节小结：

一、临界马赫数的定义：临界压强系数的定义及计算公式

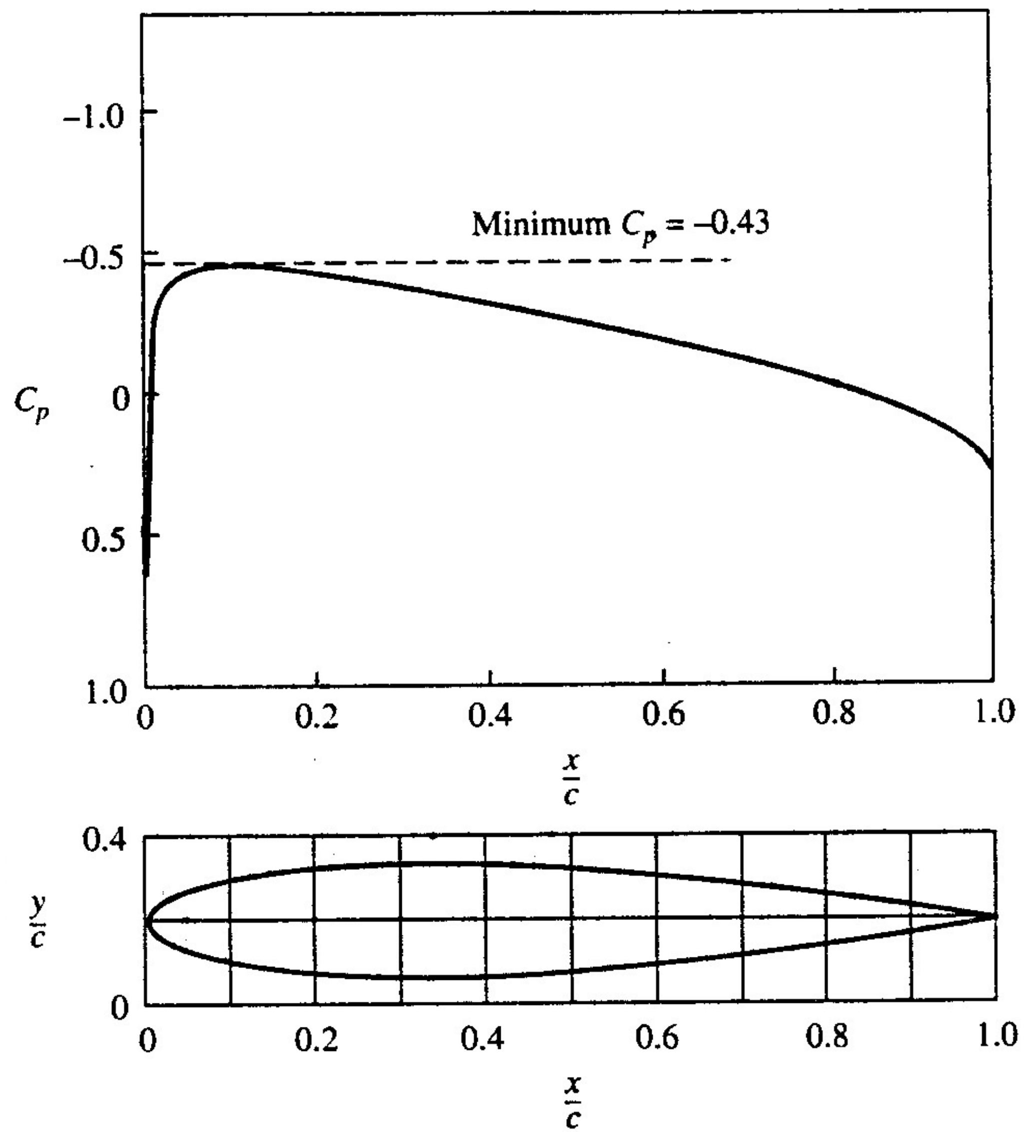
临界马赫数是一个特殊的来流值

二、临界马赫数的估算方法

三、翼型厚度对临界马赫数的影响

例11.3

（a） 用作图法求NACA0012翼型的临界马赫数；



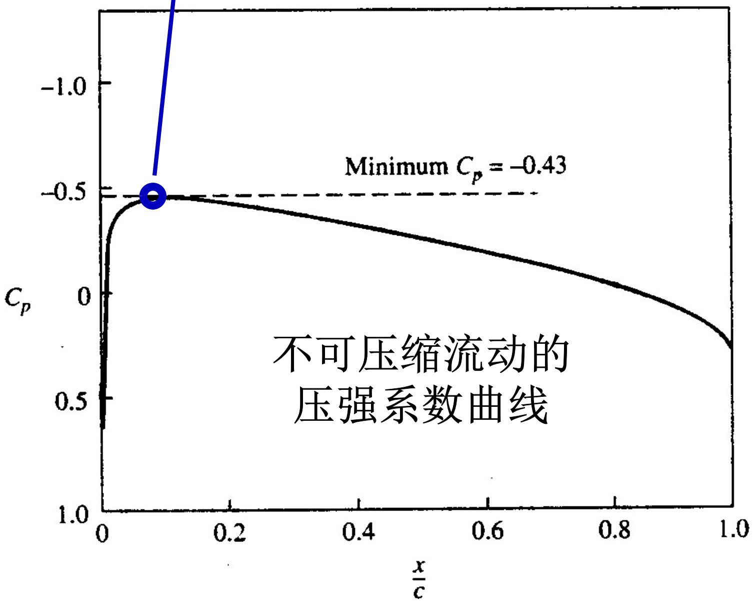
绘制曲线C, (11,60)

M_∞	0.4	0.5	0.6	0.7	0.8	0.9
$C_{p,cr}$	-3.66	-2.13	-1.29	-0.779	-0.435	-0.188

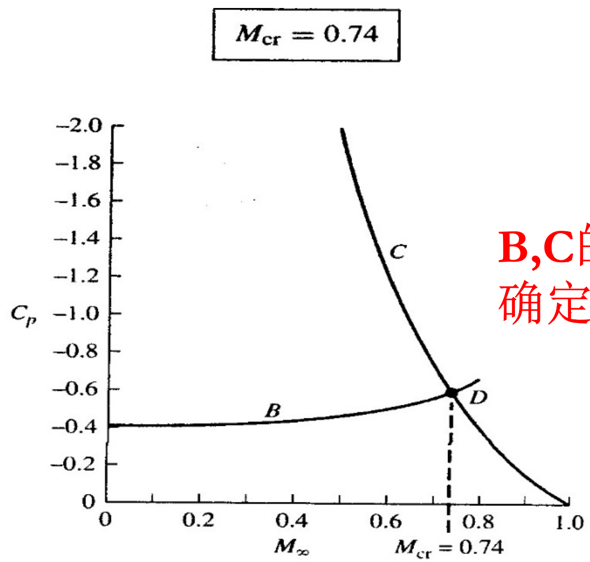
$$(C_p)_{\min} = \frac{\boxed{(C_{p,0})_{\min}}}{\sqrt{1-M_\infty^{\ 2}}}$$

绘制曲线B, (11.61)

M_∞	0	0.2	0.4	0.6	0.8
$(C_p)_{min}$	**-0.43**	**-0.439**	**-0.469**	**-0.538**	**-0.717**



不可压缩流动的
压强系数曲线



B,C的交点
确定M_{cr}

Figure 11.9 Graphical solution for the critical Mach number.

例11.3

(b) 用解析法求NACA0012翼型的临界马赫数。

(实质上是用解析法求曲线B，C的交点)

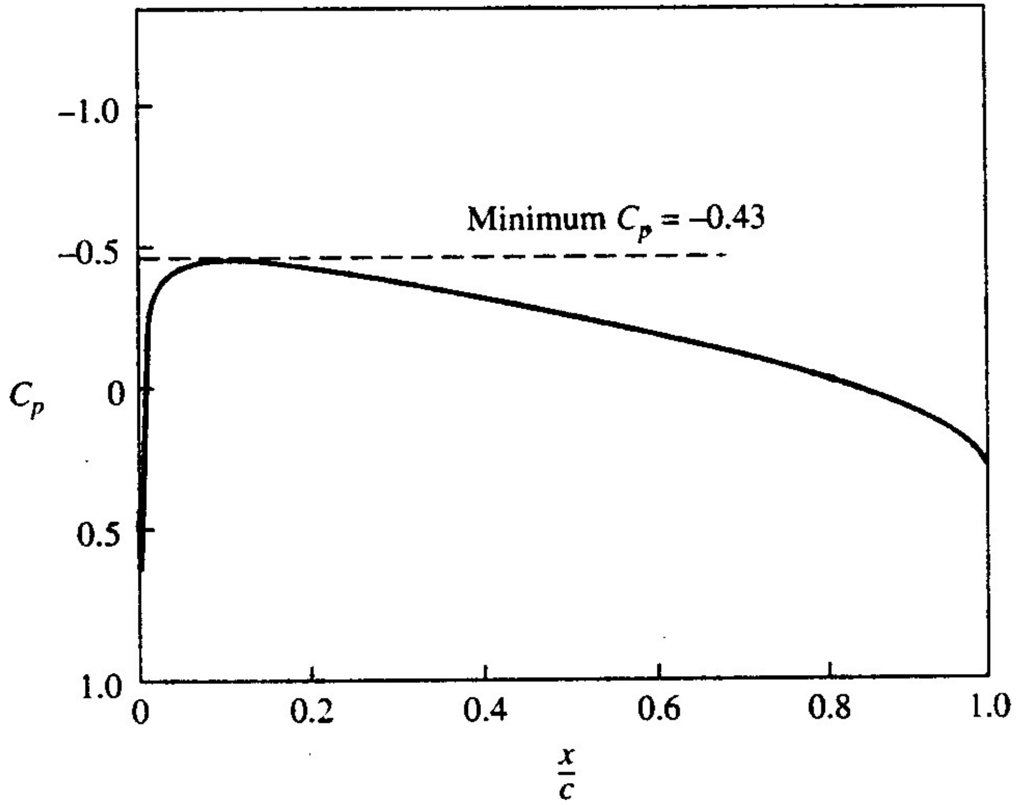


Figure 11.8 Low-speed pressure coefficient distribution over the surface of an NACA 0012 airfoil at zero angle of attack. Re $= 3.65 \times 10^6$. (*Source:* R. J. Freuler and G. M. Gregorek, "An Evaluation of Four Single Element Airfoil Analytical Methods," in *Advanced Technology Airfoil Research*, NASA CP 2045, 1978, pp. 133–162.)

$$\frac{-0.43}{\sqrt{1-M_{cr}^{2}}}=\frac{2}{\gamma M_{cr}^{2}}\left[\left(\frac{1+[(\gamma-1)/2]M_{cr}^{2}}{1+(\gamma-1)/2}\right)^{\gamma/(\gamma-1)}-1\right]$$

M_{cr}	$\frac{-0.43}{\sqrt{1-M_{cr}^{2}}}$	$\frac{2}{\gamma M_{cr}^{2}}\left[\left(\frac{1+[(\gamma-1)/2]M_{cr}^{2}}{1+(\gamma-1)/2}\right)^{\gamma/(\gamma-1)}-1\right]$
0.72	-0.6196	-0.6996
0.73	-0.6292	-0.6621
0.74	-0.6393	-0.6260
0.738	-0.6372	-0.6331
0.737	-0.6362	-0.6367
0.7371	-0.6363	-0.6363

Question: How accurate is the estimate of the critical Mach number in this example? Error of 1 -2 percent.

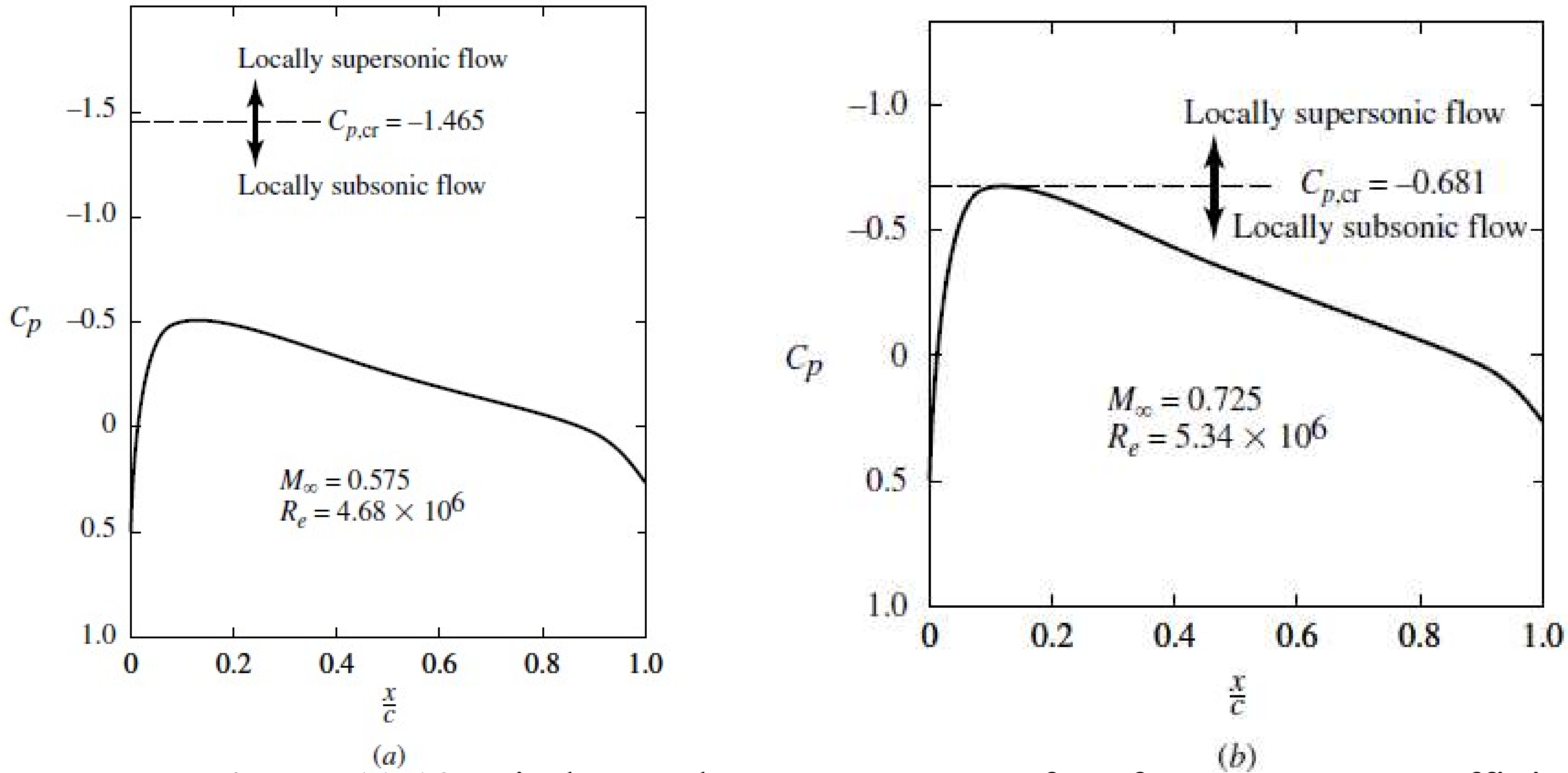
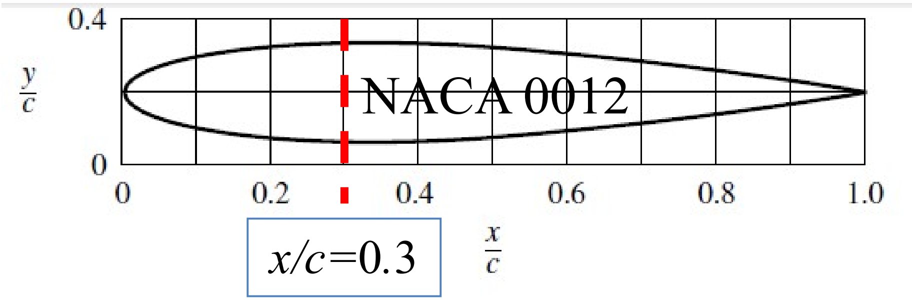


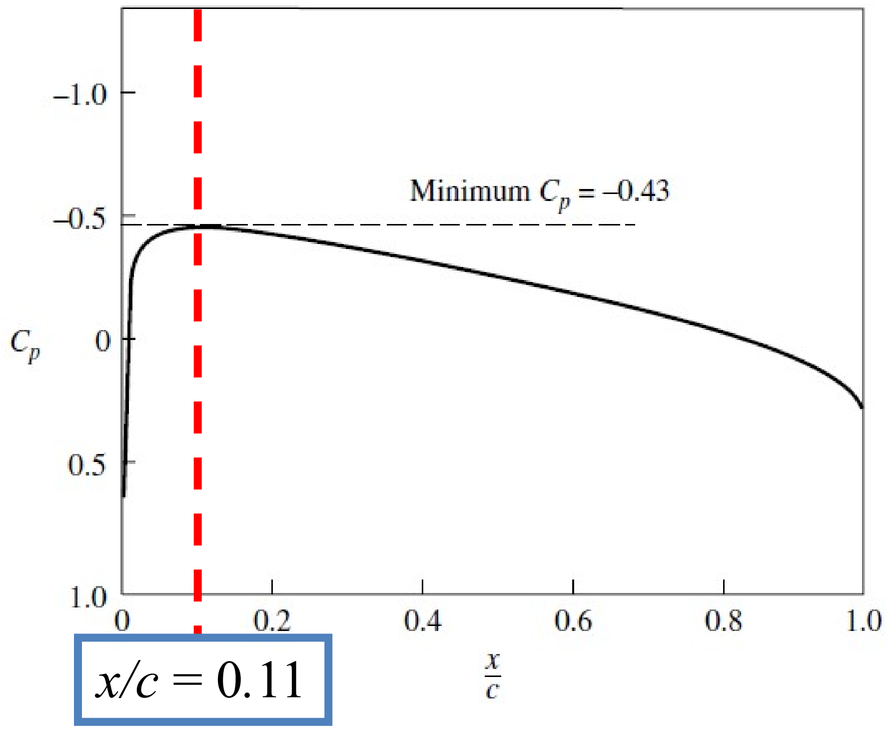
Figure 11.10 Wind tunnel measurements of surface pressure coefficient distribution for the NACA 0012 airfoil at zero angle of attack. Experimental data of Frueler and Gregorek, NASA CP 2045. (*a*) $M_\infty = 0.575$, (*b*) $M_\infty = 0.725$.

11.6.1 A Comment on the Location of Minimum Pressure (Maximum Velocity)/最小压力（最大速度）位置的讨论

右图说明，最小压力点（最大速度点）出现的位置并不和翼型的最大厚度位置相对应。

Nature places the maximum velocity at a point which satisfies the physics of the *whole flow field*, not just what is happening in a local region of the flow. The point of maximum velocity is dictated by *the complete* shape of the airfoil, not just by the shape in a local region.





作业：**11.2r-11.5r**

11.2 Using the Prandtl-Glauert rule, calculate the lift coefficient for an NACA 2412 airfoil at 4.5° angle of attack in a Mach 0.58 freestream. (Refer to Figure 4.5 for the original airfoil data.)

11.3 Under low-speed incompressible flow conditions, the pressure coefficient at a given point on an airfoil is −0.51. Calculate *Cp* at this point when the freestream Mach number is 0.60, using

a. The Prandtl-Glauert rule

b. The Karman-Tsien rule

c. Laitone's rule

11.4 In low-speed incompressible flow, the peak pressure coefficient (at the minimum pressure point) on an airfoil is −0.39. Estimate the critical Mach number for this airfoil, using the Prandtl-Glauert rule.

11.5 For a given airfoil, the critical Mach number is 0.78. Calculate the value of p/p_∞ at the minimum pressure point when $M_\infty = 0.78$.

Lecture #17 ended！

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