Chapter 8 Normal Shock Waves and Related Topics

2019年12月20日 星期五 15:07

# Chapter 8 正激波以及相关概论

## 8.1 引言P550

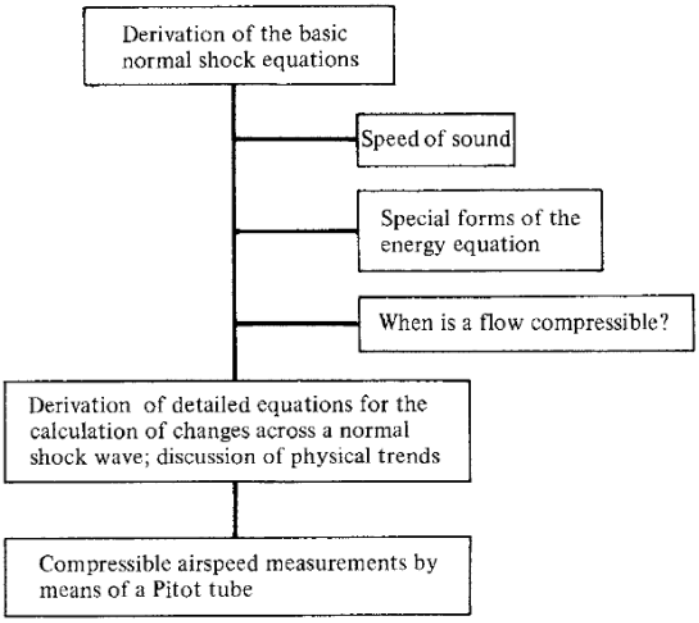


**Figure 8.2** Road map for Chapter 8.

## 8.2 正激波基本方程P551

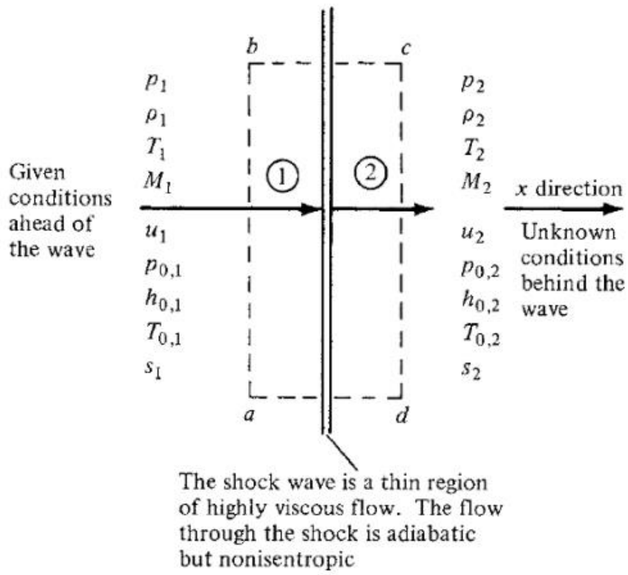


**Figure 8.3** Sketch of a normal wave.

## 8.3 声速P555

### 8.3.1 评注P563

## 8.4 能量方程的特殊形式P564

## 8.5 何时流动是可压缩的？P572

## 8.6 正激波特性的计算P575

### 8.6.1 使用数表求解可压缩流动问题的说明P590

## 8.7 可压缩流中速度的测量P591

### 8.7.1 亚声速可压缩流P591

### 8.7.2 超声速流动P592

## 8.8 总结P596

The speed of sound in a gas is given by

$$a = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_s} \tag{8.18}$$

For a calorically perfect gas,

$$a = \sqrt{\frac{\gamma p}{\rho}} \tag{8.23}$$

or

$$a = \sqrt{\gamma RT} \tag{8.25}$$

The speed of sound depends only on the gas temperature.

For a steady, adiabatic, inviscid flow, the energy equation can be expressed as

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2} \tag{8.29}$$

$$c_p T_1 + \frac{u_1^2}{2} = c_p T_2 + \frac{u_2^2}{2} \tag{8.30}$$

(*continued*)

$$\frac{a_1^2}{\gamma - 1} + \frac{u_1^2}{2} = \frac{a_2^2}{\gamma - 1} + \frac{u_2^2}{2} \tag{8.32}$$

$$\frac{a^2}{\gamma - 1} + \frac{u^2}{2} = \frac{a_0^2}{\gamma - 1} \tag{8.33}$$

$$\frac{a^2}{\gamma - 1} + \frac{u^2}{2} = \frac{\gamma + 1}{2(\gamma - 1)} a^{*2} \tag{8.35}$$

Total conditions in a flow are related to static conditions via

$$c_p T + \frac{u^2}{2} = c_p T_0 \tag{8.38}$$

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2 \tag{8.40}$$

$$\frac{p_0}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma/(\gamma-1)} \tag{8.42}$$

$$\frac{\rho_0}{\rho} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{1/(\gamma-1)} \tag{8.43}$$

Note that the ratios of total to static properties are a function of local Mach number only. These functions are tabulated in Appendix A.

The basic normal shock equations are

*Continuity:* $$\rho_1 u_1 = \rho_2 u_2 \tag{8.2}$$

*Momentum:* $$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2 \tag{8.6}$$

*Energy:* $$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2} \tag{8.10}$$

These equations lead to relations for changes across a normal shock as a function of upstream Mach number $M_1$ only:

$$M_2^2 = \frac{1 + [(\gamma - 1)/2] M_1^2}{\gamma M_1^2 - (\gamma - 1)/2} \tag{8.59}$$

*(continued)*

$$\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} = \frac{(\gamma + 1) M_1^2}{2 + (\gamma - 1) M_1^2} \tag{8.61}$$

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma + 1}(M_1^2 - 1) \tag{8.65}$$

$$\frac{T_2}{T_1} = \frac{h_2}{h_1} = \left[1 + \frac{2\gamma}{\gamma + 1}(M_1^2 - 1)\right] \frac{2 + (\gamma - 1) M_1^2}{(\gamma + 1) M_1^2} \tag{8.67}$$

$$s_2 - s_1 = c_p \ln\left\{\left[1 + \frac{2\gamma}{\gamma + 1}(M_1^2 - 1)\right] \frac{2 + (\gamma - 1) M_1^2}{(\gamma + 1) M_1^2}\right\} - R \ln\left[1 + \frac{2\gamma}{\gamma + 1}(M_1^2 - 1)\right] \tag{8.68}$$

$$\frac{p_{0,2}}{p_{0,1}} = e^{-(s_2 - s_1)/R} \tag{8.73}$$

The normal shock properties are tabulated versus $M_1$ in Appendix B.

For a calorically perfect gas, the total temperature is constant across a normal shock wave:

$$T_{0,2} = T_{0,1}$$

However, there is a loss in total pressure across the wave:

$$p_{0,2} < p_{0,1}$$

For subsonic and supersonic compressible flow, the freestream Mach number is determined by the ratio of Pitot pressure to freestream static pressure. However, the equations are different:

*Subsonic flow:*

$$M_1^2 = \frac{2}{\gamma - 1}\left[\left(\frac{p_{0,1}}{p_1}\right)^{(\gamma-1)/\gamma} - 1\right] \tag{8.74}$$

*Supersonic flow:*

$$\frac{p_{0,2}}{p_1} = \left[\frac{(\gamma+1)^2 M_1^2}{4\gamma M_1^2 - 2(\gamma-1)}\right]^{\gamma/(\gamma-1)} \frac{1 - \gamma + 2\gamma M_1^2}{\gamma + 1} \tag{8.80}$$

## 补充：$M^*$特征马赫数的定义与计算公式

$$\boxed{M^{*2} = \frac{(\gamma+1)M^2}{2 + (\gamma-1)M^2}} \tag{8.48}$$

which gives $M^*$ as a function of $M$. As can be shown by inserting numbers into Equation (8.48) (try some yourself),

$$\begin{aligned} M^* &= 1 && \text{if } M = 1 \\ M^* &< 1 && \text{if } M < 1 \\ M^* &> 1 && \text{if } M > 1 \\ M^* &\to \sqrt{\frac{\gamma+1}{\gamma-1}} && \text{if } M \to \infty \end{aligned}$$

Therefore, $M^*$ acts qualitatively in the same fashion as $M$ except that $M^*$ approaches a finite value when the actual Mach number approaches infinity.

## 8.9 作业题P599