习题7.13答案更正

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2019年10月28日 Monday

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7.13 伯努利方程(3.13)、(3.14)或(3.15)，是由第三章的牛顿第二定律推导出来的。它遵循的基本定律是

力=质量×加速度，即F=ma

然而，伯努利方程的每一项具有单位体积能量的量纲，请自行证明。提示：如果伯努利方程是不可压流动的能量方程，这一方程应该可以从本章推导出的无粘，绝热，可压缩能量方程（7.53）式中推导出来，试做出适当的假设，由（7.53）式推导伯努利方程。

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2 \qquad (3.13)$$

$$p + \frac{1}{2}\rho V^2 = const \quad \text{along a streamline} \qquad (3.14)$$

$$p + \frac{1}{2}\rho V^2 = const \quad \text{througout the flow} \qquad (3.15)$$

$$p+\frac{1}{2}\rho V^{2}=const \qquad （3.15）$$

伯努利方程的每一项的量纲是单位体积的能量：

压强单位:$N/m^2=N\cdot m/m^3=J/m^3$

$\frac{1}{2}\rho V^{2}$: $Kg/m^3\cdot(m/s)^2=Kg\cdot m/s^2\cdot m/m^3=N\cdot m/m^3=J/m^3$

$$h+\frac{1}{2}V^{2}=const \qquad （7.53）$$

错误做法：

$$h=\frac{\gamma RT}{\gamma-1}=\frac{\gamma}{\gamma-1}\frac{p}{\rho}$$

$$\gamma\to\infty,\frac{\gamma}{\gamma-1}\frac{p}{\rho}=\frac{p}{\rho}$$

比热比 γ 是空气的物理属性，在不可压假设下不可能趋于无穷。

γ =1.4是常数。

h的组成是两个不同物理属性的能量，内能e和压力做功能力pv。

正确的推导应该是:

$$h+\frac{1}{2}V^2=e+pv+\frac{1}{2}V^2=c_vT+pv+\frac{1}{2}V^2=const \quad (7.53)$$

在不可压假设下，温度是常数，见7.4节531-532页*第一段*。

$$\because c_vT=const, \therefore pv+\frac{1}{2}V^2=const$$

在不可压假设下，密度是常数

$$v=\frac{1}{\rho}=const \Longrightarrow p+\frac{1}{2}\rho V^2=const$$

Review of 6th course/第六次课复习

CHAPTER 8 NORMAL SHOCK WAVES AND RELATED TOPICS
第八章 正激波及有关问题

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第八章 路线图

理论部分

8.2 正激波基本控制方程推导

8.3 声速

8.4 能量方程特殊形式

8.5 什么情况下流动可压？

计算方法

8.6 通过正激波气体特性变化的方程的详细推导; 物理特性变化趋势讨论

实际应用

8.7 用皮托管测量可压缩流的流动速度

8.5 WHEN IS A FLOW COMPRESSIBLE?
什么条件下流动是可压缩的？

We have stated several times in the preceding that a flow can be reasonably assumed to be incompressible when M<0.3, whereas it should be considered compressible when M>0.3. Why?

$$\frac{\rho_0}{\rho} = (1+\frac{\gamma-1}{2}M^2)^{1/(\gamma-1)} \quad \text{即} \quad \frac{\rho}{\rho_0} = (1+\frac{\gamma-1}{2}M^2)^{-1/(\gamma-1)}$$

8.5 WHEN IS A FLOW COMPRESSIBLE?
什么条件下流动是可压缩的？

ρ_0 可以看做是静止气体的密度

根据：

$$\frac{\rho}{\rho_0} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-1/(\gamma-1)}$$

可得：

$$If \quad M < 0.32, \quad \frac{\Delta\rho}{\rho_0} < 5\%$$

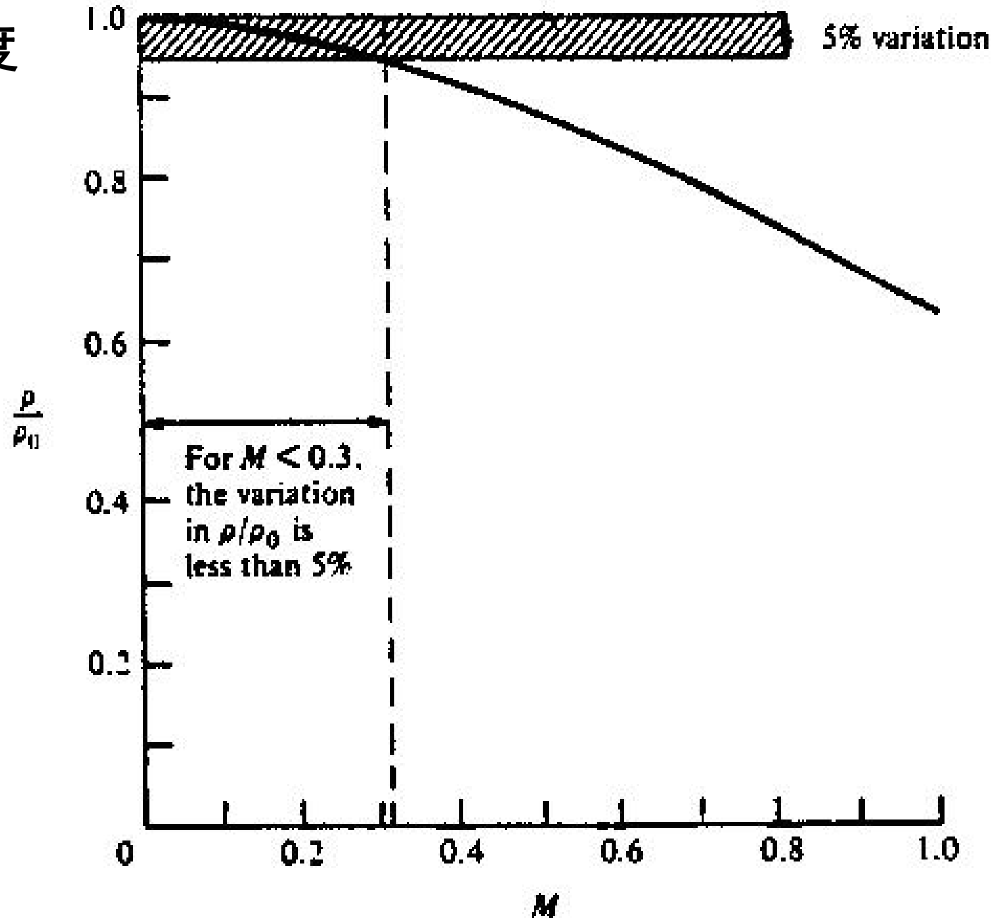


FIGURE 8.5
Isentropic variation of density with Mach number.

8.6 Calculation of Normal Shock Wave Properties
正激波性质的计算

- 本节的要点
  - 计算通过正激波后流动参数的变化

$$M_2 = ? \quad \frac{p_2}{p_1} = ? \quad \frac{\rho_2}{\rho_1} = ? \quad \frac{u_2}{u_1} = ? \quad \frac{T_2}{T_1} = ? \quad s_2 - s_1 = ? \quad \frac{T_{0,2}}{T_{0,1}} = ? \quad \frac{p_{0,2}}{p_{0,1}} = ?$$

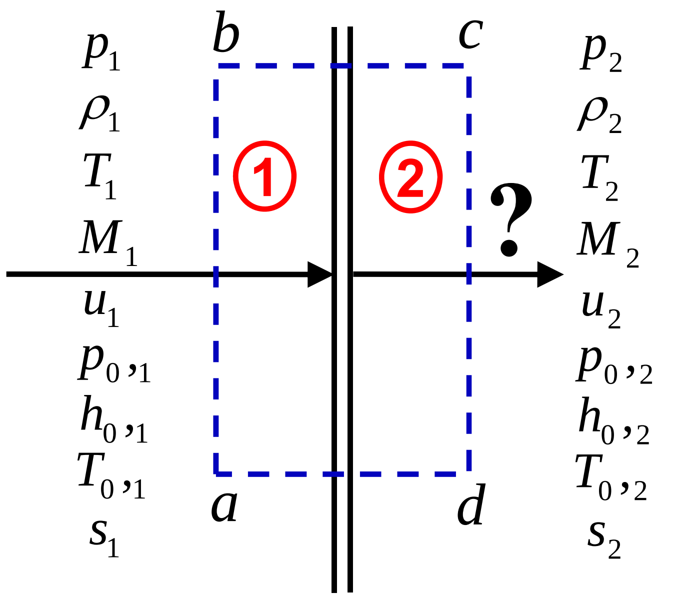


图8.3　正激波示意图

问题：　已知激波前区域1的条件，计算激波后区域2的条件。

正激波关系式推导

- 连续方程 $\rho_1 u_1 = \rho_2 u_2$ **(8.2)**

- 动量方程 $p_1 + \rho_1 u_1^{\,2} = p_2 + \rho_2 u_2^{\,2}$ **(8.6)**

- 能量方程 $\dfrac{a_1^{\,2}}{\gamma - 1} + \dfrac{u_1^{\,2}}{2} = \dfrac{a_2^{\,2}}{\gamma - 1} + \dfrac{u_2^{\,2}}{2} = \dfrac{(\gamma+1)a^{*2}}{2(\gamma-1)} = const.$ **(8.36)**

$$\Rightarrow \boxed{a^{*2} = u_1 u_2}$$ **(8.55)**

正激波关系式归纳（这些关系式在附录B中以列表形式给出）

$$\boxed{M_2^2 = \frac{1+[(\gamma-1)/2]M_1^2}{\gamma M_1^2-(\gamma-1)/2}} \qquad (8.59)$$

$$\boxed{\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} = \frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}} \qquad (8.61)$$

$$\boxed{\frac{p_2}{p_1} = 1+\frac{2\gamma}{\gamma+1}(M_1^2-1)} \qquad (8.65)$$

$$\boxed{\frac{T_2}{T_1} = \frac{h_2}{h_1} = \left[1+\frac{2\gamma}{\gamma+1}(M_1^2-1)\right]\frac{2+(\gamma-1)M_1^2}{(\gamma+1)M_1^2}} \qquad (8.67)$$

应用我们在第七章推导出的熵增公式，我们可以得到通过正激波的熵增计算公式：

$$s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1} \qquad (7.25)$$

$$\boxed{\begin{aligned} s_2 - s_1 = c_p \ln\left\{\left[1+\frac{2\gamma}{\gamma+1}(M_1{}^2-1)\right]\frac{2+(\gamma-1)M_1{}^2}{(\gamma+1)M_1{}^2}\right\} \\ -R\ln\left[1+\frac{2\gamma}{(\gamma+1)}(M_1{}^2-1)\right] \end{aligned}} \qquad (8.68)$$

From Eq. (8.68), we see that the entropy change s_2-s_1 across the shock is a function of M_1 only. 由方程(8.68)可以看出，通过正激波的熵增s_2-s_1 只是波前马赫数M_1的函数。

The second law dictates that

$$s_2-s_1 \geq 0$$

In Eq. (8.68), if $M_1=1$, $s_2=s_1$, and if $M_1>1$, then $s_2-s_1>0$, both of which obey the second law. However, if $M_1<1$, then Eq.(8.68) gives $s_2-s_1<0$, which is not allowed by the second law. Consequently, in nature, only cases involving $M_1 \geq 1$ are valid, i.e., normal shock waves can occur only in supersonic flow.

译文:热力学第二定律指出:

$$s_2-s_1 \geq 0$$

由方程(8.68)可以看出，如果$M_1=1$, $s_2=s_1$ ；如果 $M_1>1$, then $s_2-s_1>0$， 两种情况都符合热力学第二定律。然而，如果$M_1<1$，则.(8.68) 式的结果为 $s_2-s_1<0$，其不符合热力学第二定律。因此，只有$M_1 \geq 1$的情况会发生，即，正激波只能在超音速流动中发生。

总参数T_0与p_0如何变化？

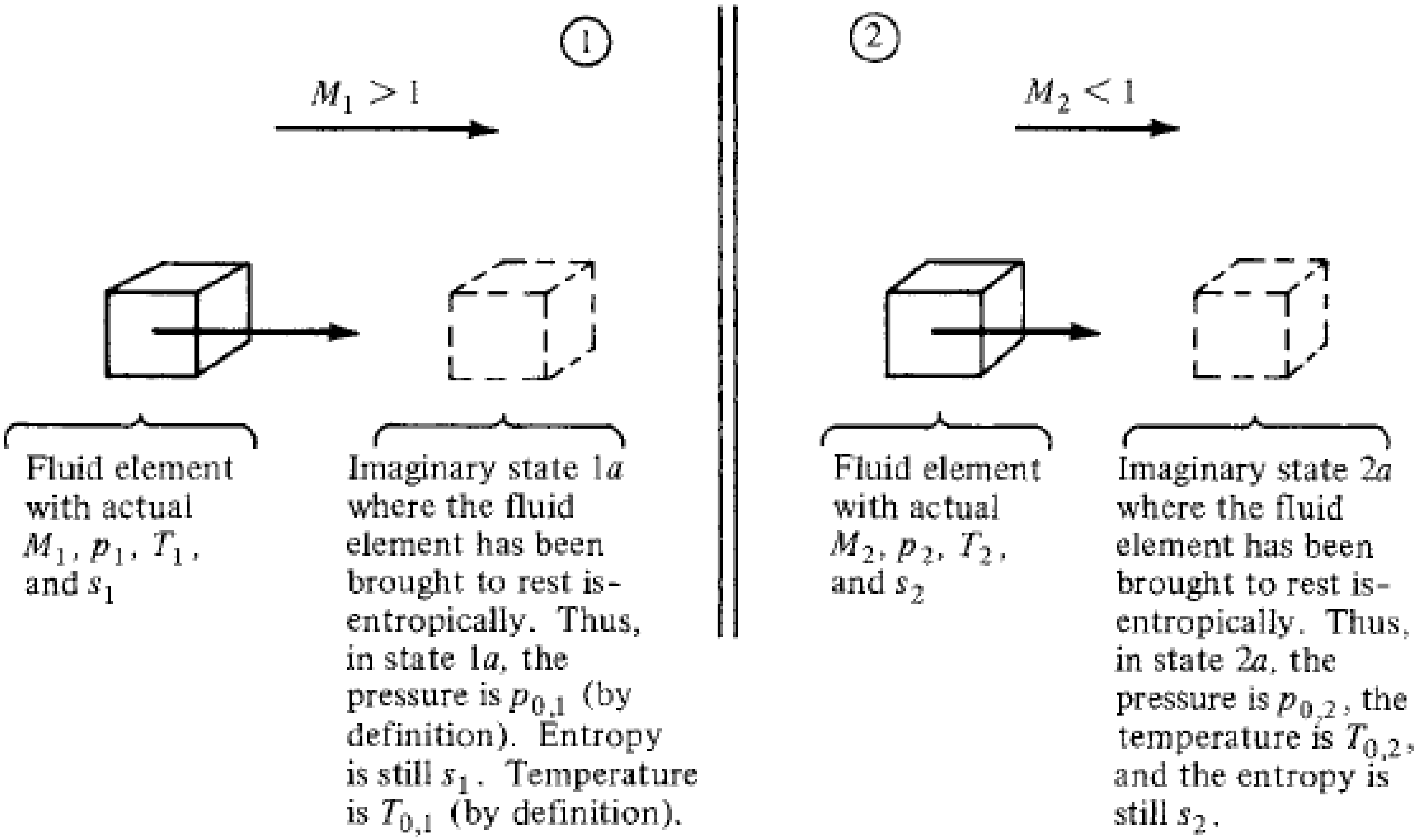


Figure 8.7 Total conditions ahead of and behind a normal shock wave.

首先回答T_0如何变化？

能量方程：

$$\boxed{c_p T_1 + \frac{u_1^2}{2} = c_p T_2 + \frac{u_2^2}{2}} \qquad (8.30)$$

总温定义：

$$\boxed{c_p T_0 = c_p T + \frac{u^2}{2}} \qquad (8.38)$$

$$\boxed{c_p T_{0,1} = c_p T_{0,2}}$$

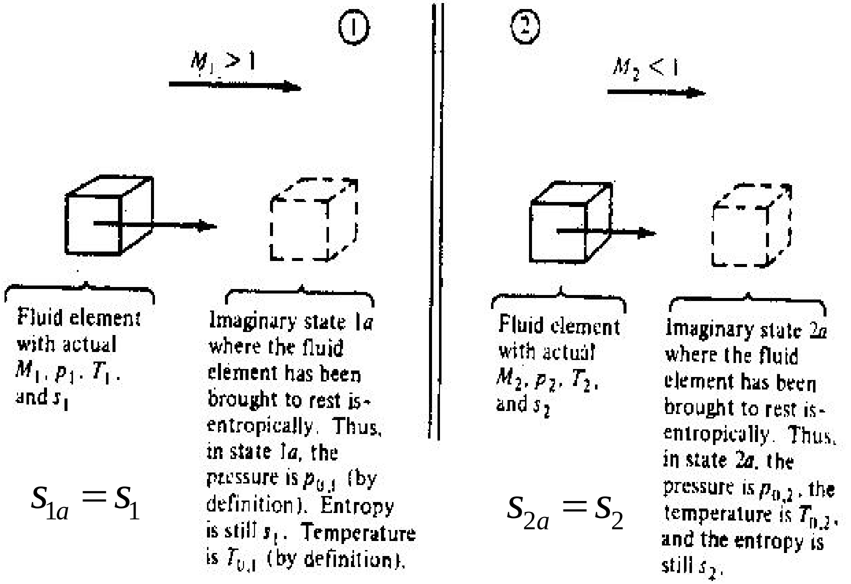
$$\boxed{T_{0,1} = T_{0,2}} \qquad (8.39)$$

- **Equation (8.39) states that *total temperature is constant across a stationary normal shock wave.*** 方程（8. 39）表明：***通过静止正激波总温不变。***

总压如何变化？可借助熵增计算公式求出：

$$s_{2a}-s_{1a}=c_p\ln\frac{T_{2a}}{T_{1a}}-R\ln\frac{p_{2a}}{p_{1a}}$$

$$s_2-s_1=s_{2a}-s_{1a}=c_p\ln\frac{T_{0,2}}{T_{0,1}}-R\ln\frac{p_{0,2}}{p_{0,1}}$$



$$\boxed{s_2-s_1=-R\ln\frac{p_{0,2}}{p_{0,1}}} \qquad \textbf{(8.72)}$$

$$\boxed{\frac{p_{0,2}}{p_{0,1}}=e^{-(s_2-s_1)/R}} \qquad \textbf{(8.73)}$$

From Eq. (8.68), we know that $s_2-s_1>0$ **for a normal shock wave. Hence, Eq. (8.73) states that :** $p_{0,2}<p_{0,1}$**. The total pressure decreases across a shock wave. The total pressure ratio** $p_{0,2}/p_{0,1}$ **across a normal shock wave is a function of** M_1 **only.**

由公式（8.68），我们知道对于正激波$s_2-s_1>0$，因此，式（8.73)表明： $p_{0,2}<p_{0,1}$。 ***通过正激波总压降低，且通过正激波总压比*** $p_{0,2}/p_{0,1}$ ***只是波前马赫数*** M_1 ***的函数。***

至此，我们已经全部回答了本节开始提出的问题：

$$M_2 = ? \quad \frac{p_2}{p_1} = ? \quad \frac{\rho_2}{\rho_1} = ? \quad \frac{u_2}{u_1} = ? \quad \frac{T_2}{T_1} = ? \quad s_2 - s_1 = ? \quad \frac{T_{0,2}}{T_{0,1}} = ? \quad \frac{p_{0,2}}{p_{0,1}} = ?$$

$$\boxed{M_2^{\ 2} = \frac{1+[(\gamma-1)/2]M_1^{\ 2}}{\gamma M_1^{\ 2}-(\gamma-1)/2}}$$

$$\boxed{\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} = \frac{(\gamma+1)M_1^{\ 2}}{2+(\gamma-1)M_1^{\ 2}}}$$

$$\boxed{\frac{p_2}{p_1} = 1+\frac{2\gamma}{\gamma+1}(M_1^{\ 2}-1)}$$

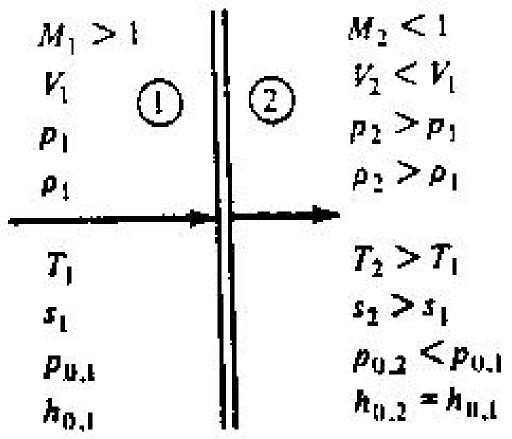
$$\boxed{\frac{T_2}{T_1} = \frac{h_2}{h_1} = \left[1+\frac{2\gamma}{\gamma+1}(M_1^{\ 2}-1)\right]\frac{2+(\gamma-1)M_1^{\ 2}}{(\gamma+1)M_1^{\ 2}}}$$

$$\boxed{s_2 - s_1 = c_p \ln\left\{\left[1+\frac{2\gamma}{\gamma+1}(M_1^{\ 2}-1)\right]\frac{2+(\gamma-1)M_1^{\ 2}}{(\gamma+1)M_1^{\ 2}}\right\} - R\ln\left[1+\frac{2\gamma}{(\gamma+1)}(M_1^{\ 2}-1)\right]}$$

$$\boxed{T_{0,1} = T_{0,2}} \qquad \boxed{\frac{p_{0,2}}{p_{0,1}} = e^{-(s_2-s_1)/R}}$$

这些关系式在附录B中以列表形式给出。

- In summary, we have now verified the qualitative changes across a normal shock wave as sketched in Fig.7.5b and as originally discussed in Sec. 7.6.



(b) Normal shock wave

图7.5b

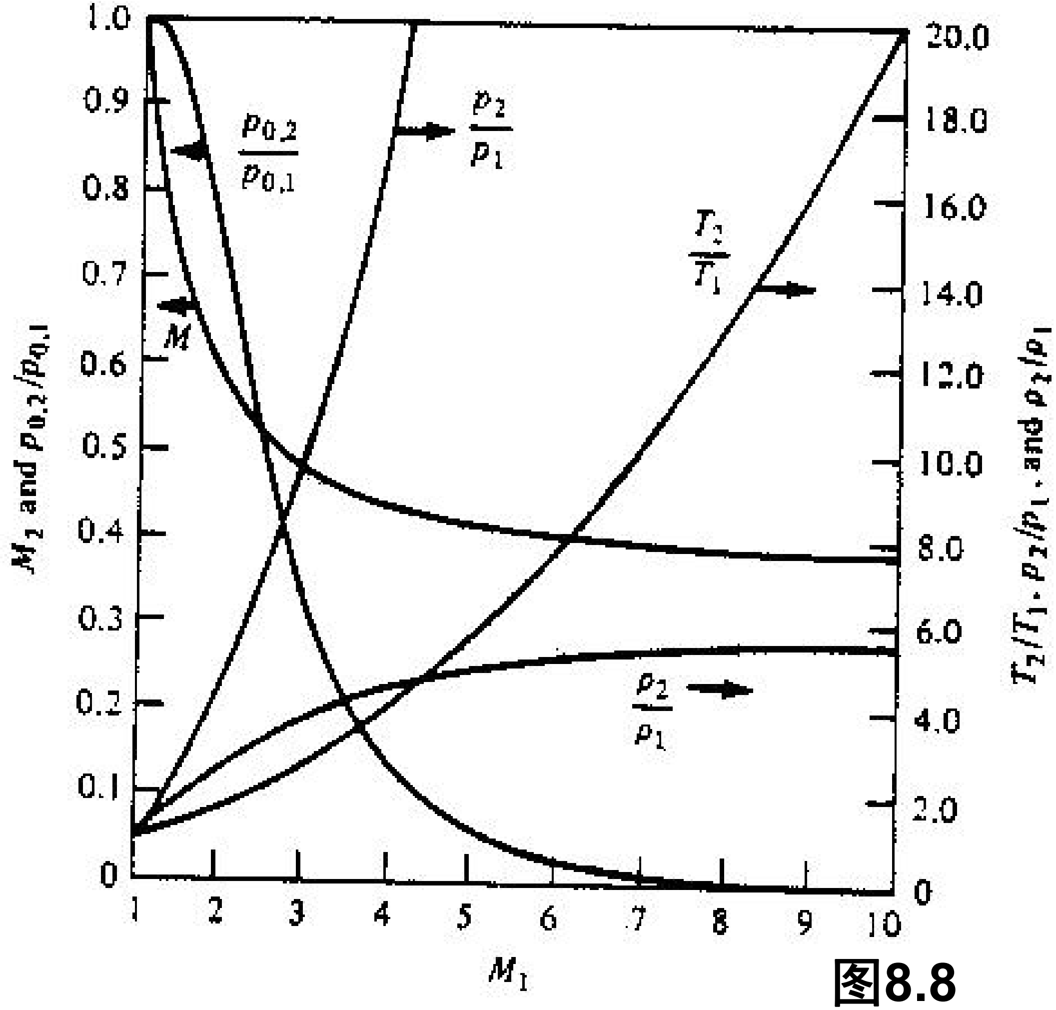


图8.8

投票 最多可选1项　　设置

前一次课的掌握情况投票

- A 完全掌握了这部分知识内容
- B 掌握了大部分
- C 掌握了一小部分
- D 完全不懂

提交

Review Lecture #6 ended!

Lecture #7

CHAPTER 8 NORMAL SHOCK WAVES AND RELATED TOPICS
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Road Map

8.2 Derivation of the basic normal shock equations

8.3 Speed of sound

8.4 Special form of the energy equation

8.5 When is a flow compressible

8.6 Derivation of detailed equations for the calculation of changes across a normal shock wave: discussion of physical trends

8.7 Compressible airspeed measurements by means of a Pitot tube

例 8.13 如图8.9所示冲压发动机，其进气道入口前有一脱体激波。点1之前的激波为正激波。气流通过激波后由点1至点2的流动是等熵的。冲压发动机的飞行高度是10km，飞行马赫数为2，大气压强和温度分别为$2.65\times10^4 N/m^2$和223.3K。当点2处的马赫数为0.2时，计算该点处气体的温度和压强。

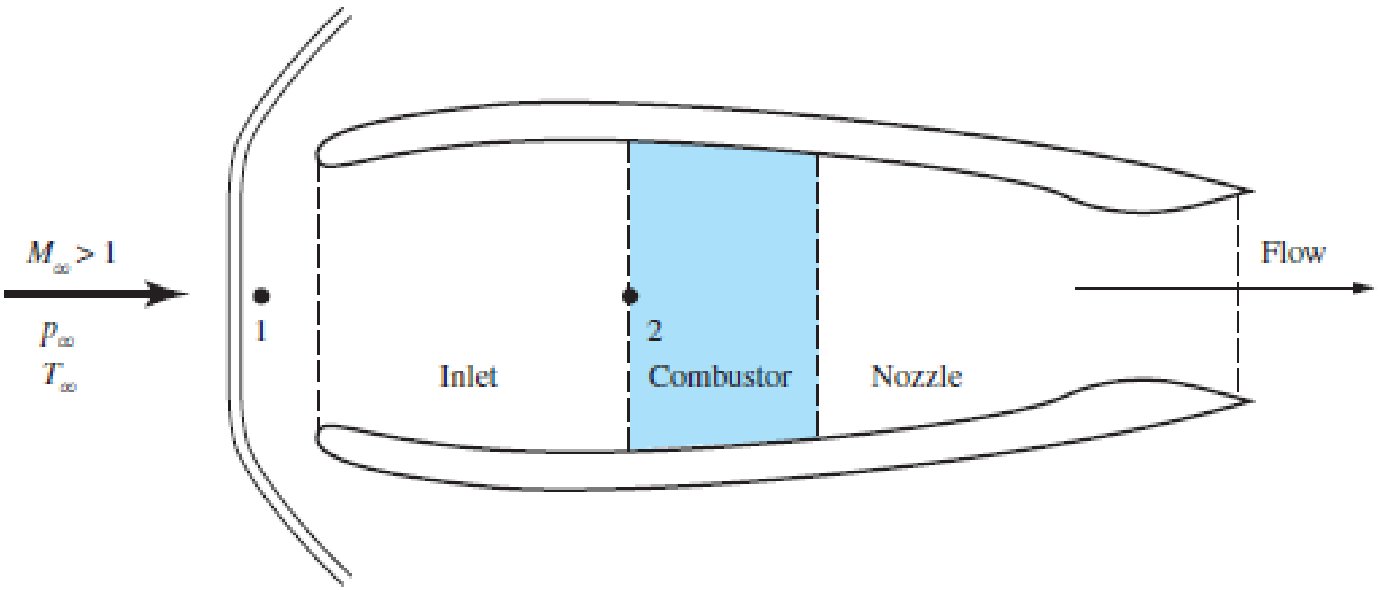


Figure 8.9 Schematic of a conventional subsonic-combustion ramjet engine.

解：由附录A，可查表得M_∞ = 2时的$p_{0,\infty}/p_\infty$，$T_{0,\infty}/T_\infty$

$$p_{0,\infty} = \frac{p_{0,\infty}}{p_\infty} p_\infty = 7.824 \times (2.64 \times 10^4) = 2.07 \times 10^5 (N/m^2)$$

$$T_{0,\infty} = \frac{T_{0,\infty}}{T_\infty} T_\infty = 1.8 \times (223.3) = 401.9K$$

正激波后1点的总压，可查附表B得到，M_∞ = 2时

$$p_{0,1}/p_{0,\infty} = 0.7209$$

$$p_{0,1} = \frac{p_{0,1}}{p_{0,\infty}} p_{0,\infty} = 0.7209 \times (2.07 \times 10^5) = 1.49 \times 10^5 (N/m^2)$$

$$T_{0,1} = T_{0,\infty} = 401.9K$$

由点1至点2为等熵流动，因此总压，总温不变。在点2 M = 0.2，查附表A，可得$p_{0,2}/p_2$， $T_{0,2}/T_2$

$$p_2 = \frac{p_2}{p_{0,2}} p_{0,2} = \frac{1.49\times10^5}{1.028} = 1.45\times10^5 (N/m^2) = 1.42(atm)$$

$$T_2 = \frac{T_2}{T_{0,2}} T_{0,2} = \frac{401.9K}{1.008} = 399K$$

例8.14 飞行马赫数为10，其它条件和**例8.13**相同。当点2处的马赫数为0.2时，计算该点处气体的温度和压强。

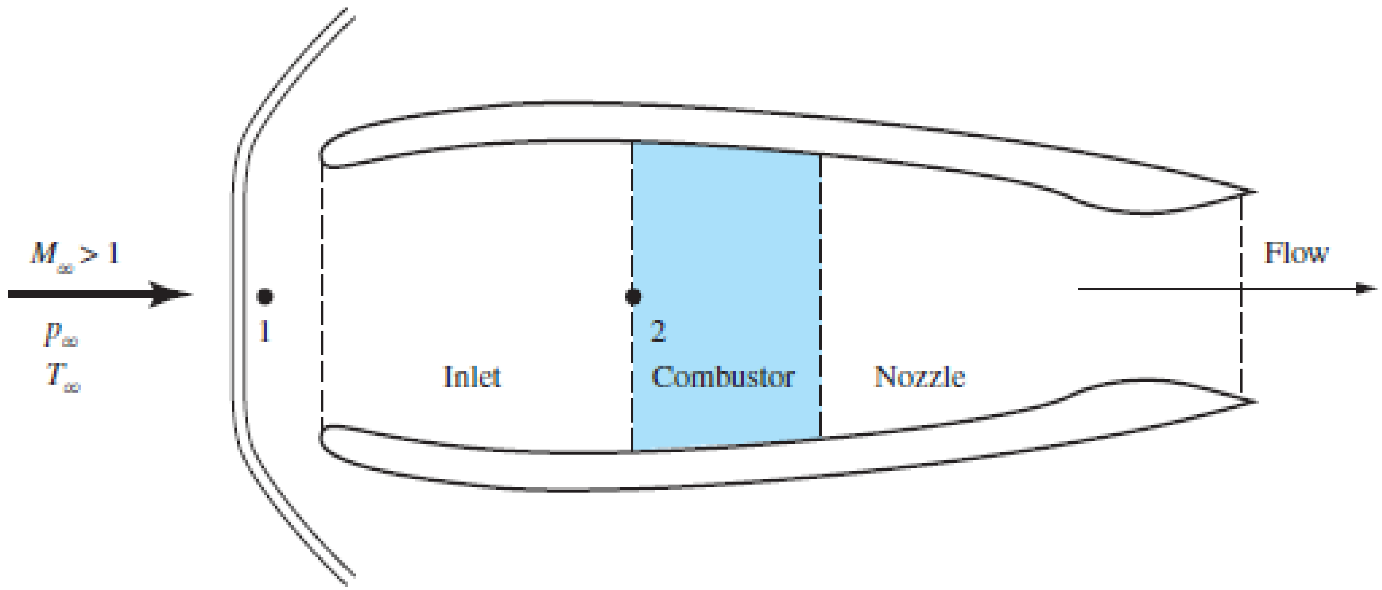


Figure 8.9 Schematic of a conventional subsonic-combustion ramjet engine.

解：由附录A，可查表得M_∞=10时的$p_{0,\infty}/p_\infty$，$T_{0,\infty}/T_\infty$

$$p_{0,\infty}=\frac{p_{0,\infty}}{p_\infty}p_\infty=(0.4244\times10^5)(2.64\times10^4)=1.125\times10^9(N/m^2)$$

$$T_{0,\infty}=\frac{T_{0,\infty}}{T_\infty}T_\infty=21\times(223.3)=4690K$$

正激波后1点的总压，可查附表B得到，M_∞=10时 $p_{0,1}/p_{0,\infty}=0.3045\times10^{-2}$

$$p_{0,1}=\frac{p_{0,1}}{p_{0,\infty}}p_{0,\infty}=0.3045\times10^{-2}\times(1.125\times10^9)=3.43\times10^6(N/m^2)$$

$$T_{0,1}=T_{0,\infty}=4690K$$

由点1至点2为等熵流动，因此总压，总温不变。在点2 M=0.2，查附表A，可得$p_{0,2}/p_2$， $T_{0,2}/T_2$

$$p_2 = \frac{p_2}{p_{0,2}} p_{0,2} = \frac{3.43\times10^6}{1.028} = 3.34\times10^6 (N/m^2) = 32.7(atm)$$

$$T =_2 \frac{T_2}{T_{0,2}} T_{0,2} = \frac{4690K}{1.008} = 4653K$$

Note：本例中气流以极高压强和极高温度进入燃烧室，燃料在如此高温下不能燃烧，而是分解了，因此不会产生推力。进一步，即使有可以抵挡如此高温的材料，在如此大的压力作用下，受极其严苛的结构设计要求限制，燃烧室会非常重。

简而言之，在常规冲压发动机中，气流在进入燃烧室前被减速到低亚声速，这样的常规冲压发动机是不能在高马赫数下，既高超声速下工作的。

解决途径：

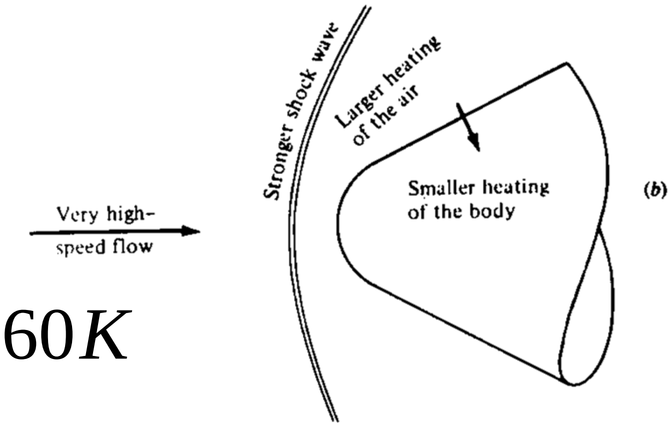
超燃冲压发动机：前沿研究热点

（SCRAMjet-Supersonic combustion ramjet）

X43验证机：2005年11月

Problem 8.17 阿波罗号指挥舱从月球返回，以马赫数36再入地球大气层。用本章的公式（比热比γ=1.4）预计阿波罗号指挥舱在飞行马赫数为36，自由来流温度为300K的高度时的驻点温度。评价这个结果的合理性。

解：通过正激波流动为绝热流，因此自由来流的总温即为驻点温度。



查表A：　对于M_1=36，　T_1=300K, 有：

$$T_{0,1}=\frac{T_{0,1}}{T_1}\cdot T_1=260.2\times300K=78060K$$

$$T_{0,2}=T_{0,1}=78060K$$

结论：假设比热比为γ=1.4的常数，高估了驻点温度。（太阳表面温度约为6000°C）

Problem 8.18 阿波罗号指挥舱进入大气的驻点温度为11000K，和8.17题中从量热完全气体假设（比热比γ=1.4）出发得出的结果相差很大。造成这种差别的原因是高温条件下空气发生了化学反应——分解和电离，比热比为常数1.4的量热完全气体假设对于这样的化学反应流不再成立。然而，在工程近似中，为模拟高温化学反应流，有时可以采用较小的比热比值，被称为所谓的“有效γ”，仍然利用量热完全气体假设来估算驻点温度。对于本题条件，计算对应驻点温度为11000K的有效γ的值。假设自由来流温度为300K。

解：

因为：

$$\frac{T_{0,1}}{T_1} = 1 + \frac{\gamma - 1}{2} M_1{}^2$$

所以：

$$\gamma = 1 + \frac{2}{M_1{}^2}\left(\frac{T_{0,1}}{T_1} - 1\right) = 1.055$$

问题：下图黄圈内是什么飞机的部件？

波音777的L型空速管

飞行参数的主要测量系统（大气数据传感器）：总静压探头（皮托管或空速管）、静压孔（测气压高度）、总温传感器、攻角传感器、加速度传感器等

8.7 Measurement of Velocity in a Compressible Flow/ 可压缩流动的速度测量

复习：低速飞行的速度测量

伯努利方程 $$\boxed{p_1+\frac{1}{2}\rho V^2=p_0}$$

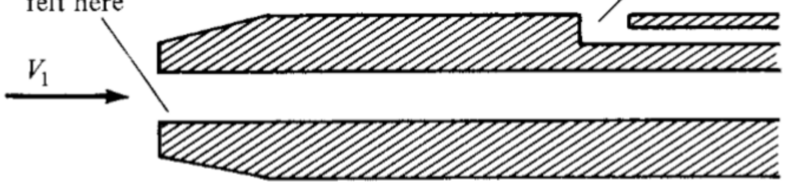


Figure 3.12 Pitot-static probe.

$$\boxed{V=\sqrt{\frac{2(p_{0,1}-p_1)}{\rho}}} \quad (3.34)$$

Q：高速飞行时速度如何测量呢？

提示： $$\frac{p_0}{p_1}=(1+\frac{\gamma-1}{2}M_1{}^2)^{\gamma/(\gamma-1)}$$

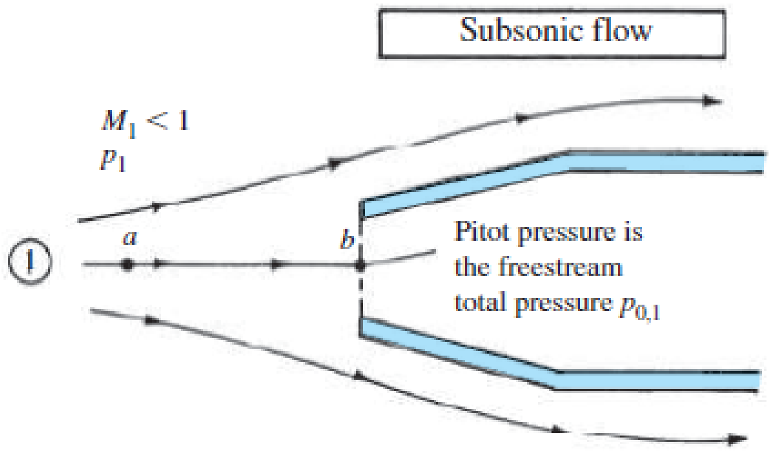
8.7 Measurement of Velocity in a Compressible Flow/ 可压缩流动的速度测量

8.7.1 Subsonic Compressible Flow /亚声速可压缩流

$$\frac{p_{0,1}}{p_1} = \left(1+\frac{\gamma-1}{2}M_1^{\ 2}\right)^{\gamma/(\gamma-1)} \qquad (8.42)$$

$$\boxed{M_1^{\ 2} = \frac{2}{\gamma-1}\left[\left(\frac{p_{0,1}}{p_1}\right)^{(\gamma-1)/\gamma} - 1\right]} \qquad (8.74)$$

$$u_1^{\ 2} = \frac{2a_1^{\ 2}}{\gamma-1}\left[\left(\frac{p_{0,1}}{p_1}\right)^{(\gamma-1)/\gamma} - 1\right] \qquad (8.75)$$



(a)

$$u_1^{\ 2} = \frac{2a_1^{\ 2}}{\gamma - 1}\left[\left(\frac{p_{0,1}}{p_1}\right)^{(\gamma-1)/\gamma} - 1 \right] \qquad (8.75)$$

- **From Eq. (8.75) , we see that , unlike incompressible flow, a knowledge of $p_{0,1}$ and p_1 is not sufficient to obtain u_1; we also need the freestream speed of sound, a_1.**

- 从（8. 75）式可以看到：与不可压缩流不同，只知道 $p_{0,1}$ 和 p_1 还不足以得到速度u_1；我们还需要知道自由流的声速： a_1。

8.7.2 Supersonic Flow/超声速流

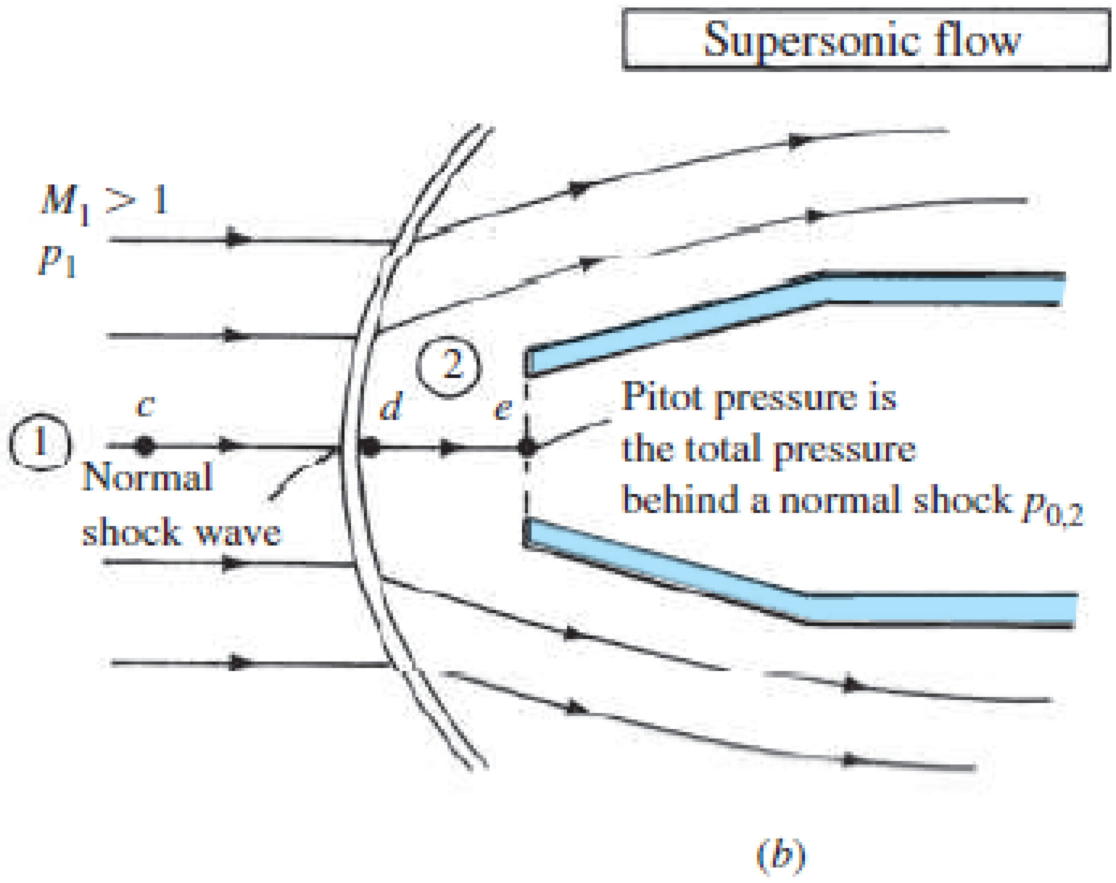
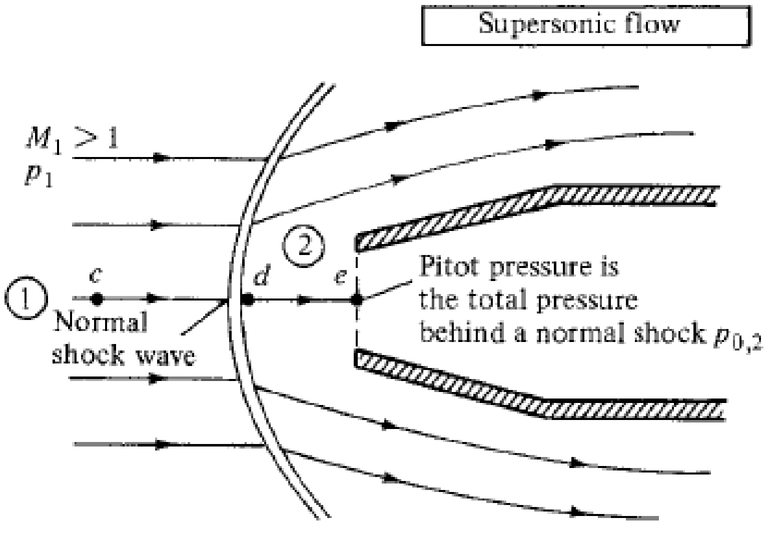


Fig. 8.10 A Pitot tube in supersonic flow

- A fluid element moving along streamline *cde* will first decelerated *nonisentropically* to a subsonic velocity at point *b* just behind the shock . Then it is isentropically compressed to zero velocity at point *e*. As a result, the pressure at point *e* is not the total pressure of the freestream but rather the total pressure *behind a normal shock wave*, $p_{0,2}$. This is the Pitot pressure read at the end of the tube.

- 沿流线 *cde* 的流体微团首先非等熵地在 *d*点减速为亚声速，然后被等熵地在*e*点压缩为驻点速度零。因此， *e*点的压强不是自由流的总压而是正激波后的总压 $p_{0,2}$。这是皮托管测得的总压(称为皮托管压强)。



(b)

$$\frac{p_{0,2}}{p_1} = \frac{p_{0,2}}{p_2}\frac{p_2}{p_1} \tag{8.76}$$

$$\frac{p_{0,2}}{p_2} = (1 + \frac{\gamma - 1}{2} M_2^{\ 2})^{\gamma/(\gamma-1)} \tag{8.77}$$

$$M_2^{\ 2} = \frac{1 + [(\gamma - 1)/2] M_1^{\ 2}}{\gamma M_1^{\ 2} - (\gamma - 1)/2} \tag{8.78}$$

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma + 1}(M_1^{\ 2} - 1) \tag{8.79}$$

$$\boxed{\frac{p_{0,2}}{p_1} = \left(\frac{(\gamma+1)^2 M_1^{\ 2}}{4\gamma M_1^{\ 2} - 2(\gamma - 1)}\right)^{\gamma/(\gamma-1)} \frac{1 - \gamma + 2\gamma M_1^{\ 2}}{\gamma + 1}} \tag{8.80}$$

$$\boxed{\frac{p_{0,2}}{p_1}=\left(\frac{(\gamma+1)^2 M_1^{\ 2}}{4\gamma M_1^{\ 2}-2(\gamma-1)}\right)^{\gamma/(\gamma-1)}\frac{1-\gamma+2\gamma M_1^{\ 2}}{\gamma+1}} \qquad \textbf{(8.80)}$$

Equation (8.80) is called the *Rayleigh Pitot tube formula*. It relates the Pitot pressure $p_{0,2}$ and the freestream static pressure p_1 to the freestream Mach number M_1. Equation (8.80) gives M_1 as an implicit function of $p_{0,2}/p_1$ and allows the calculation of M_1 from known $p_{0,2}/p_1$. For convenience in making calculations, the ratio $p_{0,2}/p_1$ is tabulated versus M_1 in App.B.

(8.80)式被称为雷利皮托管公式。它将皮托管测得的总压$p_{0,2}$和自由来流静压p_1与自由来流马赫数M_1 联系起来了。(8.80)式中M_1为$p_{0,2}/p_1$ 的隐式函数，可以由$p_{0,2}/p_1$ 的值计算出M_1 。为方便应用， 附录B给出了$p_{0,2}/p_1$ 随M_1的变化表。

例8.22 A pitot tube is inserted into an airflow where the static pressure is 1atm. Calculate the flow Mach Number when the Pitot tube measures (a) 1.276atm; (b) 2.714atm; (c) 12.06atm.

解： 首先我们必须确定流动是亚声速的还是超声速的。当马赫数为1时，皮托管测出的总压为$p_0=p/0.528=1.893\text{atm}$.

因此，皮托管测出的总压$p_0<1.893\text{atm}$时，来流是亚声速的；

测出的总压$p_0>1.893\text{atm}$时，来流是超声速的。

(a) p_0=1.276atm, 来流是亚声速的。皮托管测得的总压就是来流的总压，查表A得，p_0/p=1.276时， M=0.6

(b) p_0=2.714atm, 来流是超声速的。皮托管测得的总压是正激波后的总压。查表B得，$p_{0,2}/p$=2.714时，M=1.3

(c) p_0=12.06atm, 来流是超声速的。皮托管测得的总压是正激波后的总压。查表B得，$p_{0,2}/p$=12.06时， M=3.0

例8.23 Consider a hypersonic missile flying at Mach 8 at an altitude of 20,000ft, where the pressure is 973.3lb/ft². The nose of missile is Figure 8.1. Calculate the pressure at the stagnation point on the nose.

- 解法1： $p_\infty = 973.3 lb/ft^2$

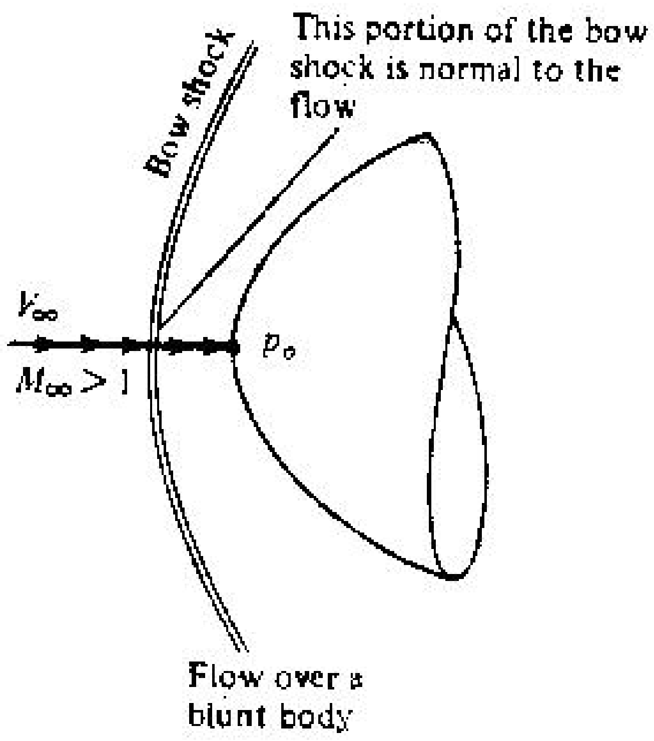
$$= 973.3 \times 47.88 N/m^2$$

$$= 4.660 \times 10^4 N/m^2$$

查表A, $M_\infty = 8$, 有 $p_{0,1}/p_\infty = 9763$

查表B, $M_1 = 8$, 有

$$p_{0,2}/p_{0,1} = 0.8488 \times 10^{-2}$$



所以: $$p_{0,2}=\frac{p_{0,2}}{p_{0,1}}\frac{p_{0,1}}{p_{\infty}}p_{\infty}$$

$$=0.8488\times10^{-2}\times9763\times4.6602\times10^{4}N/m^{2}$$

$$=3.8618\times10^{6}N/m^{2}\approx38.1atm$$

- 解法2:

直接查表B, $M_1=8$, $p_{02}/p_1=82.87$

所以:

$$p_{0,2}=\frac{p_{0,2}}{p_{\infty}}p_{\infty}=82.87\times4.6602\times10^{4}N/m^{2}$$

$$=3.8619\times10^{6}N/m^{2}\approx38.1atm$$

例8.24 Consider the Lockheed Blackbird shown in Figure 8.11 flying at a standard attitude of 25km. The pressure measured by a Pitot tube on this plane is 3.88×10^4 N/m^2. Calculate the velocity of the plane.

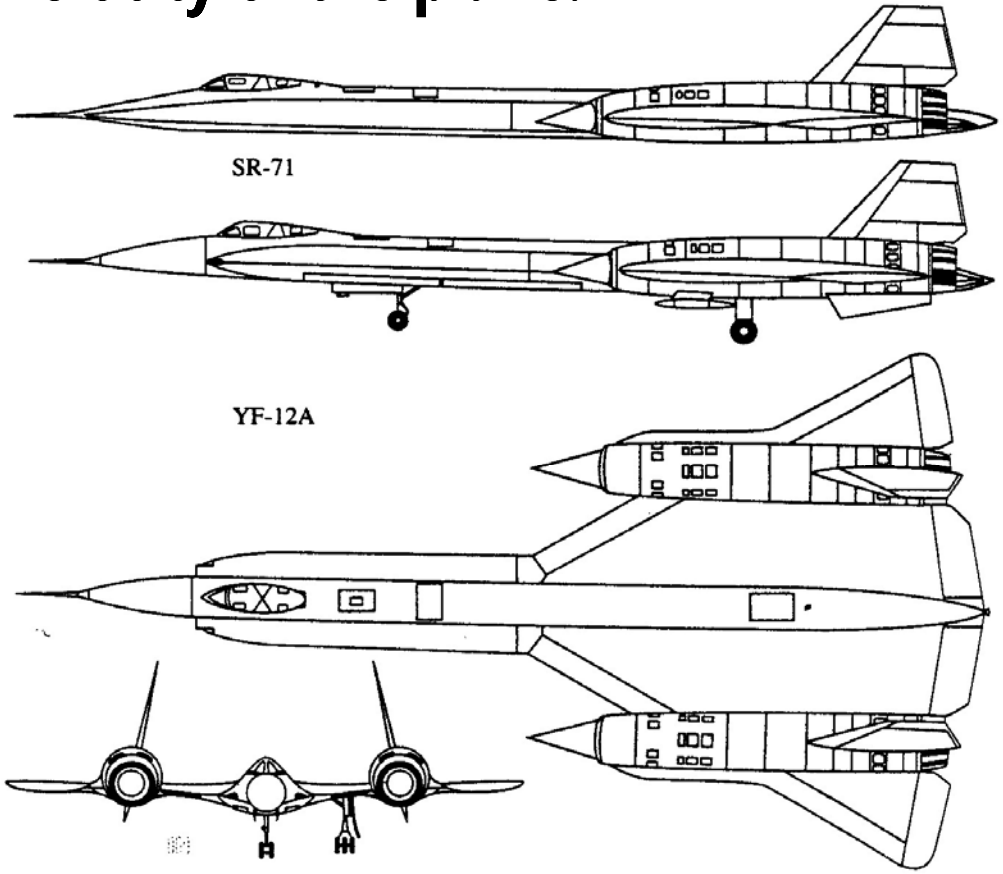


Figure 8.11 The Lockheed SR-71/YF-12A Blackbird.

堪萨斯宇宙与太空中心博物馆入门大厅的黑鸟

例8.24 如图所示黑鸟战斗机，飞行高度25km。飞机上的皮托管测得的压强为 $3.88\times10^4 N/m^2$ 。计算飞机的飞行速度。

解： 查标准大气表, 得

$$h=25km\text{时}，p_\infty=2.5273\times10^3 N/m^2,\ T_\infty=216.66K$$

$$\frac{p_{0,2}}{p_\infty}=\frac{3.88\times10^4}{2.5273\times10^3}=15.35$$

查表B，p_{02}/p_1=15.35时，M_∞=3.4

$$a_\infty = \sqrt{\gamma R T_\infty} = \sqrt{1.4 \times 287 \times 216.66} = 295(m/s)$$

所以，飞机的速度为

$$V_\infty = 3.4 \times 295 = 1003(m/s)$$

8.8 Summary

气体的声速由下式给出:

$$a = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_s} \tag{8.18}$$

对于量热完全气体

$$a = \sqrt{\frac{\gamma p}{\rho}} \tag{8.23}$$

或

$$a = \sqrt{\gamma RT} \tag{8.25}$$

声速只依赖于气体的温度。

对于定常、绝热、无粘流动，能量方程可以表示为：

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2} \qquad (8.29)$$

$$\boxed{c_p T_1 + \frac{u_1^2}{2} = c_p T_2 + \frac{u_2^2}{2}} \qquad (8.30)$$

$$\boxed{\frac{a_1^2}{\gamma - 1} + \frac{u_1^2}{2} = \frac{a_2^2}{\gamma - 1} + \frac{u_2^2}{2}} \qquad (8.32)$$

$$\frac{a_1^2}{\gamma - 1} + \frac{u_1^2}{2} = \frac{a_2^2}{\gamma - 1} + \frac{u_2^2}{2} = \frac{a_0^2}{\gamma - 1} \qquad (8.34)$$

$$\frac{a_1^2}{\gamma - 1} + \frac{u_1^2}{2} = \frac{a_2^2}{\gamma - 1} + \frac{u_2^2}{2} = \frac{(\gamma + 1) a^{*2}}{2(\gamma - 1)} = const. \qquad (8.36)$$

滞止声速和临界声速的定义：

$$\boxed{\frac{{a_0}^2}{\gamma-1}=\frac{a^2}{\gamma-1}+\frac{u^2}{2}} \qquad (8.33)$$

$$\boxed{\frac{(\gamma+1)}{2(\gamma-1)}a^{*2}=\frac{a^2}{\gamma-1}+\frac{u^2}{2}} \qquad (8.35)$$

总参数与静参数通过下式联系起来:

$$\boxed{\frac{T_0}{T}=1+\frac{\gamma-1}{2}M^2} \qquad (8.40)$$

$$\frac{p_0}{p}=\left(1+\frac{\gamma-1}{2}M^2\right)^{\gamma/(\gamma-1)} \qquad (8.42)$$

$$\frac{\rho_0}{\rho}=\left(1+\frac{\gamma-1}{2}M^2\right)^{1/(\gamma-1)} \qquad (8.43)$$

注意总参数与静参数的比只是当地马赫数的函数。由附录A以列表形式提供。

特征马赫数与当地马赫数关系

$$\boxed{M^{*2}=\frac{(\gamma+1)M^2}{2+(\gamma-1)M^2}} \qquad (8.48)$$

正激波基本方程：

连续方程： $\rho_1 u_1 = \rho_2 u_2$ (8.2)

动量方程： $p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$ (8.6)

能量方程： $h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}$ (8.10)

由这些方程导出通过正激波的气体特性变化由波前马赫数唯一确定。

$$\boxed{M_2^2 = \frac{1 + [(\gamma - 1)/2] M_1^2}{\gamma M_1^2 - (\gamma - 1)/2}} \quad (8.59)$$

$$\boxed{\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} = \frac{(\gamma + 1) M_1^2}{2 + (\gamma - 1) M_1^2}} \quad (8.61)$$

接前框：

$$\frac{p_2}{p_1}=1+\frac{2\gamma}{\gamma+1}(M_1{}^2-1) \tag{8.65}$$

$$\frac{T_2}{T_1}=\frac{h_2}{h_1}=\left[1+\frac{2\gamma}{\gamma+1}(M_1{}^2-1)\right]\frac{2+(\gamma-1)M_1{}^2}{(\gamma+1)M_1{}^2} \tag{8.67}$$

$$s_2-s_1=c_p\ln\left\{\left[1+\frac{2\gamma}{\gamma+1}(M_1{}^2-1)\right]\frac{2+(\gamma-1)M_1{}^2}{(\gamma+1)M_1{}^2}\right\} -R\ln\left[1+\frac{2\gamma}{(\gamma+1)}(M_1{}^2-1)\right] \tag{8.68}$$

$$\frac{p_{0,2}}{p_{0,1}}=e^{-(s_2-s_1)/R} \tag{8.73}$$

附录B给出了随M_1变化的正激波性质。

对于量热完全气体，通过正激波总温不变：

$$\boxed{T_{0,2} = T_{0,1}}$$

然而，通过正激波总压有损失：

$$\boxed{p_{0,2} < p_{0,1}}$$

对于亚声速与超声速可压缩流动，自由来流马赫数确定了皮托管的总压与自流来流静压比。但是亚声速与超声速情况对应的方程不同：

亚声速流：

$$\boxed{M_1^{\ 2} = \frac{2}{\gamma - 1}\left[\left(\frac{p_{0,1}}{p_1}\right)^{(\gamma-1)/\gamma} - 1\right]} \qquad \textbf{(8.74)}$$

超声速流：

$$\boxed{\frac{p_{0,2}}{p_1} = \left(\frac{(\gamma+1)^2 M_1^{\ 2}}{4\gamma M_1^{\ 2} - 2(\gamma - 1)}\right)^{\gamma/(\gamma-1)} \frac{1 - \gamma + 2\gamma M_1^{\ 2}}{\gamma + 1}} \qquad \textbf{(8.80)}$$

主观题 2分

设置

在压强为1atm，温度为288K的气流中放入皮托管，皮托管测得的压强是1.524atm。问气流的马赫数是多少，速度是多少？

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作答

主观题 2分

设置

通过正激波的$p_2/p_1=7.125$,波前马赫数$M_1=$？

波后马赫数$M_2=$？

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作答

Problem 8.11r, 8.12, 8.13, 8.14r, 8.15, 8.16r，选作：8.17.8.18.8.19

8.11r Consider a flow with a pressure and temperature of 1 atm and 288 K. A Pitot tube is inserted into this flow and measures a pressure of **1.655** atm. What is the velocity of the flow?

8.12 Consider a flow with a pressure and temperature of 2116 lb/ft^2 and 519°R, respectively. A Pitot tube is inserted into this flow and measures a pressure of 7712.8 lb/ft^2. What is the velocity of this flow? (换算至公制单位）

8.13r Repeat Problems 8.11r and 8.12 using (incorrectly) Bernoulli's equation for incompressible flow. Calculate the percent error induced by using Bernoulli's equation.

8.14 Derive the Rayleigh Pitot tube formula, Equation (8.80).

8.15 On March 16, 1990, an Air Force SR-71 set a new continental speed record, averaging a velocity of 2112 mi/h at an altitude of 80,000 ft. Calculate the temperature (in degrees Fahrenheit) at a stagnation point on the vehicle.(换算至公制单位）

8.16r In the test section of a supersonic wind tunnel, a Pitot tube in the flow reads a pressure of **1.366** atm. A static pressure measurement (from a pressure tap on the sidewall of the test section) yields 0.1 atm. Calculate the Mach number of the flow in the test section.

Lecture #7 ended!

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