飞行性能课后习题答案（仅供参考）

2017 年 11 月 1 日

习题 2.1

证明滑翔机的最大航时为 $t_{\max}=\dfrac{3E_m}{2\sqrt[4]{3}V_R}(h_i-h_f)$

解：

根据运动学方程：

$$\dot{h}=V\gamma \tag{1}$$

变形得：

$$\mathrm{d}t=\frac{\mathrm{d}h}{V\gamma} \tag{2}$$

假设法向力平衡，可得到：

$$V=\sqrt{\frac{2W}{\rho SC_L}} \tag{3}$$

由动力学方程：

$$Q+W\sin\gamma=0 \tag{4}$$

假设 γ 比较小时，$\sin\gamma=\gamma$，则：

$$\gamma=-\frac{D}{W}=-\frac{L}{W}=-E \tag{5}$$

将 γ 及 V 代入式 2得：

$$t=-\int_{h_i}^{h_f}\sqrt{\frac{\rho S}{2W}}\left(\frac{C_L^{\frac{3}{2}}}{C_D}\right)\mathrm{d}h=\sqrt{\frac{\rho S}{2W}}\left(\frac{C_L^{\frac{3}{2}}}{C_D}\right)(h_i-h_f) \tag{6}$$

上式表示当 $\left(\dfrac{C_L^{\frac{3}{2}}}{C_D}\right)$ 最大，即 $\left(\dfrac{C_D}{C_L^{\frac{3}{2}}}\right)$ 最小时，航时最大。

$$\frac{C_D}{C_L^{\frac{3}{2}}}=\frac{C_{D0}+kC_L^2}{C_L^{\frac{3}{2}}}=C_{D0}C_L^{-\frac{3}{2}}+kC_L^{\frac{1}{2}} \tag{7}$$

$$\frac{\mathrm{d}}{\mathrm{d}C_L}\left(\frac{C_D}{C_L^{\frac{3}{2}}}\right)=-\frac{3}{2}C_{D0}C_L^{-\frac{5}{2}}+\frac{1}{2}kC_L^{-\frac{1}{2}}=0 \tag{8}$$

解得：

$$\frac{3}{2}C_{D0}C_L^{-\frac{5}{2}}=\frac{1}{2}kC_L^{-\frac{1}{2}} \tag{9}$$

代入 6得：

$$t_{\max}=\sqrt{\frac{\rho S}{2W}}\sqrt[4]{\frac{27}{k^3C_{D0}}}\left(\frac{h_i-h_f}{4}\right) \tag{10}$$

有：

$$V_R = \sqrt{\frac{2W}{\rho S}} \sqrt[4]{\frac{k}{C_{D0}}} \tag{11}$$

$$E_m = \frac{1}{2\sqrt{kC_{D0}}} \tag{12}$$

则：

$$t_{\max} = \sqrt{\frac{\rho S}{2W}} \sqrt[4]{\frac{27}{k^3 C_{D0}}} \left(\frac{h_i - h_f}{4}\right) \tag{13}$$

$$= \frac{1}{V_R} \sqrt[4]{\frac{27}{k^2 C_{D0}^2}} \left(\frac{h_i - h_f}{4}\right) \tag{14}$$

$$= \frac{E_m}{V_R} \frac{\sqrt[4]{27}}{2} (h_i - h_f) \tag{15}$$

$$= \frac{3E_m}{2\sqrt[4]{3} V_R} (h_i - h_f) \tag{16}$$

证毕。

习题 2.2

滑翔机重 $4500N$，翼载为 $600N/m$，其阻力极曲线由 $C_D = 0.01 + 0.022C_L^2$ 给出。在 $400m$ 高空的平静大气中将其发射，求

（a）相对地面的最大滑翔距离，[参考答案：$13.484km$]

（b）空中停留的最长时间，[参考答案：$6.72min$]

（c）$10m/s$ 的顺风对以上两项性能的影响。[参考答案：$\Delta R = 3.45km$, $\Delta t = 0$]

解：

(a)

$$C_{D0} = 0.01 \tag{17}$$

$$k = 0.022 \tag{18}$$

因此：

$$E_m = \frac{1}{2\sqrt{kC_{D0}}} = \frac{1}{2\sqrt{0.022 \times 0.01}} = 33.7 \tag{19}$$

$$R_{\max} = E_m \Delta h = 33.7 \times 400 = 13.48km \tag{20}$$

(b)

$$t_{\max} = \sqrt{\frac{\rho S}{2W}} \sqrt[4]{\frac{27}{k^3 C_{D0}}} \left(\frac{\Delta h}{4}\right) = \sqrt{\frac{1.225}{2 \times 600}} \sqrt[4]{\frac{27}{0.022^3 \times 0.01}} \left(\frac{400}{4}\right) \tag{21}$$

$$= 403.2sec = 6.72min \tag{22}$$

(c) 为了满足 E_m 下飞行，滑翔速度应为 V_R:

$$V_R = \sqrt{\frac{2W}{\rho S}} \sqrt[4]{\frac{k}{C_{D0}}} = \sqrt{\frac{2 \times 600}{1.225}} \sqrt[4]{\frac{0.022}{0.01}} = 38.1m/s \tag{23}$$

顺风时空速为 V_R，对应的地速为 $V = V_R + V_w$，实际的滑翔距离为：

$$\Delta R = R_{\max} \frac{V_w}{V_R} = 13.48 \times \frac{10}{38.1} = 3.54km \tag{24}$$

习题 2.3

某飞机重 $44440N$，翼载为 $1433.55N/m^2$，阻力极曲线由 $C_D = 0.02 + 0.04C_L^2$ 给出，$C_{L,\max} = 1.2$。从 $600m$ 高空做无动力滑翔，求

（a）最大滑翔距离，[参考答案：$10.6066km$]

（b）空中停留的最长时间。[参考答案：$210.125s$]

解：

(a)

$$C_{D0} = 0.02 \tag{25}$$

$$k = 0.04 \tag{26}$$

因此：

$$E_m = \frac{1}{2\sqrt{kC_{D0}}} = \frac{1}{2\sqrt{0.04 \times 0.02}} = 17.7 \tag{27}$$

$$R_{\max} = E_m \Delta h = 17.7 \times 600 = 10.62km \tag{28}$$

(b)

$$t_{\max} = \sqrt{\frac{\rho S}{2W}} \sqrt[4]{\frac{27}{k^3 C_{D0}}} \left(\frac{\Delta h}{4}\right) = \sqrt{\frac{1.225}{2 \times 1433.55}} \sqrt[4]{\frac{27}{0.04^3 \times 0.02}} \left(\frac{600}{4}\right) \tag{29}$$

$$= 210.1sec = 3.50min \tag{30}$$

习题 2.5

一喷气式飞机重 $30000N$，翼载为 $1000N/m^2$，海平面处产生的推力为 4000N，推力随高度变化关系为 $T = T_0\sigma^{0.8}$。假设 $C_D = 0.015 + 0.024C_L^2$，$C_{L,\max} = 1.4$，求

（a）海平面处的平飞最大和最小速度，[参考答案：$119.2132m/s$ 和 $34.1494m/s$]

（b）飞机的静升限。[参考答案：$13.30km$]

解：

(a)

$$E_m = \frac{1}{2\sqrt{kC_{D0}}} = \frac{1}{2\sqrt{0.024 \times 0.015}} = 26.35 \tag{31}$$

$$z = \frac{TE_m}{W} = \frac{4000 \times 26.35}{30000} = 3.513 \tag{32}$$

$$V_R = \sqrt{\frac{2W}{\rho S}} \sqrt[4]{\frac{k}{C_{D0}}} = \sqrt{\frac{2 \times 1000}{1.225}} \sqrt[4]{\frac{0.024}{0.015}} = 45.4m/s \tag{33}$$

$$u_{\max} = \sqrt{z + \sqrt{z^2 - 1}} = \sqrt{3.513 + \sqrt{3.513^2 - 1}} = 2.623 \tag{34}$$

$$u_{\min} = \sqrt{z - \sqrt{z^2 - 1}} = \sqrt{3.513 - \sqrt{3.513^2 - 1}} = 0.381 \tag{35}$$

$$V_{\max} = u_{\max} V_R = 2.623 \times 45.4 = 119.1m/s \tag{36}$$

$$V_{\min} = u_{\min} V_R = 0.381 \times 45.4 = 17.3m/s \tag{37}$$

$$V_{stall} = \sqrt{\frac{2W}{\rho S C_{L,\max}}} = \sqrt{\frac{2 \times 1000}{1.225 \times 1.4}} = 34.1m/s \tag{38}$$

由于 $V_{stall} > V_{\min}$，所以最小飞行速度为 $V_{stall} = 34.1m/s$。

(b) 升限处 $z = 1$。

$$T = \frac{zW}{E_m} = \frac{1 \times 30000}{26.35} = 1138.5N \tag{39}$$

$$\sigma = \left(\frac{T}{T_0}\right)^{\frac{1}{0.8}} = \left(\frac{1138.5}{4000}\right)^{1.25} = 0.2079 \tag{40}$$

$$\rho = \sigma\rho_0 = 0.2079 \times 1.225 = 0.2547kg/m^3 \tag{41}$$

查表，得升限为 13.3km。

习题 2.6

喷气式飞机重 $50000N$，翼载为 $1800N/m^2$，海平面处最大平飞速度为 $241.83m/s$，求对应的需用推力。假设 $C_D = 0.02 + 0.04C_L^2$，$C_{L,\max} = 1.5$。[参考答案：$20000N$]

解：

$$S = \frac{W}{W/S} = \frac{50000}{1800} = 27.78m^2 \tag{42}$$

$$qS = \frac{1}{2}\rho V^2 S = \frac{1}{2} \times 1.225 \times 241.83^2 \times 27.78 = 9.95 \times 10^5 \tag{43}$$

$$C_L = \frac{W}{qS} = \frac{50000}{9.95 \times 10^5} = 0.0503 \tag{44}$$

$$C_D = 0.02 + 0.04C_L^2 = 0.02 + 0.04 \times 0.0503^2 = 0.0201 \tag{45}$$

$$T_R = D = qSC_D = 9.95 \times 10^5 \times 0.0201 = 20000N \tag{46}$$

习题 2.8

螺旋桨飞机的翼载为 $1750N/m^2$，机翼面积 $30m^2$，$C_D = 0.02 + 0.04C_L^2$，$C_{L,\max} = 1.5$，推进效率为 0.85。为了达到海平面处的最大爬升率 $12m/s$，求活塞式发动机需要产生多大的功率。

解：

$$V_R = \sqrt{\frac{2W}{\rho S}}\sqrt[4]{\frac{k}{C_{D0}}} = \sqrt{\frac{2 \times 1750}{1.225}}\sqrt[4]{\frac{0.04}{0.02}} = 63.6m/s \tag{47}$$

$$V_{R/C,\max} = \frac{1}{\sqrt[4]{3}}V_R = \frac{1}{\sqrt[4]{3}}63.6 = 48.3m/s \tag{48}$$

$$E_m = \frac{1}{2\sqrt{kC_{D0}}} = \frac{1}{2\sqrt{0.04 \times 0.02}} = 17.7 \tag{49}$$

$$(R/C)_{\max} = \frac{k'\eta_p P(kW)}{W} - \frac{V_{mp}}{0.866E_m} \tag{50}$$

$$12 = \frac{1000 \times 0.85P(kW)}{1750 \times 30} - \frac{48.3}{0.866 \times 17.7} \tag{51}$$

$$P(kW) = 935.8 \tag{52}$$

习题 2.9

喷气式飞机的阻力极曲线由下式给出

$$C_D = C_{D0} + \frac{C_L^2}{\pi Ae} \tag{53}$$

假设推力与飞行速度无关，证明爬升率最大时的动压为

$$q = \frac{T}{6SC_{D0}} + \sqrt{\left(\frac{T}{6SC_{D0}}\right)^2 + \frac{W^2}{3\pi AeS^2C_{D0}}} \tag{54}$$

解：

$$q = \frac{1}{2}\rho V^2 \tag{55}$$

$$V = \sqrt{\frac{2q}{\rho}} \tag{56}$$

$$W = qSC_L \tag{57}$$

$$C_L = \frac{W}{qS} \tag{58}$$

$$R/C = \frac{V(T-D)}{W} \tag{59}$$

$$= \frac{1}{W}\sqrt{\frac{2q}{\rho}}(T - qS(C_{D0} + \frac{\left(\frac{W}{qS}\right)^2}{\pi Ae})) \tag{60}$$

$$\frac{\mathrm{d}(R/C)}{\mathrm{d}q} = 0 \tag{61}$$

$$\frac{\mathrm{d}\left(q^{\frac{1}{2}}T - q^{\frac{3}{2}}SC_{D0} - \frac{W^2}{\pi AeS}q^{-\frac{1}{2}}\right)}{\mathrm{d}q} = 0 \tag{62}$$

$$\frac{1}{2}q^{-\frac{1}{2}}T - \frac{3}{2}q^{\frac{1}{2}}SC_{D0} + \frac{1}{2}\frac{W^2}{\pi AeS}q^{-\frac{3}{2}} = 0 \tag{63}$$

$$3SC_{D0}q^2 - Tq - \frac{W^2}{\pi AeS} = 0 \tag{64}$$

$$q = \frac{T \pm \sqrt{T^2 + \frac{12C_{D0}W^2}{\pi Ae}}}{6SC_{D0}} \tag{65}$$

舍去负根，得：

$$q = \frac{T}{6SC_{D0}} + \sqrt{\left(\frac{T}{6SC_{D0}}\right)^2 + \frac{W^2}{3\pi AeS^2C_{D0}}} \tag{66}$$

习题 2.10

喷气式飞机在海平面处的最大速度为 $790km/h$，飞机总重为 $160000N$，机翼面积 $50m^2$，$C_D = 0.02 + 0.04C_L^2$。求

（a）发动机提供的推力，[参考答案：30190.35N]

（b）以最大气动效率的 75% 飞行时的爬升角和爬升率，[参考答案：6.5036deg 和 14.4951m/s]

（c）最大爬升率和对应速度。[参考答案：14.5195m/s 和 132.1841m/s]

解：

(a)

$$V_R = \sqrt{\frac{2W}{\rho S}} \sqrt[4]{\frac{k}{C_{D0}}} = \sqrt{\frac{2 \times 160000}{1.225 \times 50}} \sqrt[4]{\frac{0.04}{0.02}} = 86 m/s \tag{67}$$

$$u_{\max} = \frac{V_{\max}}{V_R} = \frac{790/3.6}{86} = 2.55 \tag{68}$$

$$E_m = \frac{1}{2\sqrt{k C_{D0}}} = \frac{1}{2\sqrt{0.02 \times 0.04}} = 17.7 \tag{69}$$

$$\tag{70}$$

由 $u^4 - 2zu^2 + 1 = 0$ 得

$$z = \frac{u_{\max}^4 + 1}{2u_{\max}^2} = \frac{2.55^4 + 1}{2 \times 2.55^2} = 3.33 \tag{71}$$

$$T = \frac{zW}{E_m} = \frac{3.33 \times 160000}{17.7} = 30102 N \tag{72}$$

(b)

$$E = C_L/C_D = 0.75 E_m = 0.75 \times 17.7 = 13.3 \tag{73}$$

$$C_D = 0.02 + 0.04 C_L^2 \tag{74}$$

此处有两个解，题目中没有更多条件（例如 $C_{L\max}$），因此无法去掉一个：

$$C_L = 1.559, 0.321 \tag{75}$$

$$C_D = 0.117, 0.024 \tag{76}$$

求速度

$$V = \sqrt{\frac{W}{\frac{1}{2}\rho S C_L}} = \sqrt{\frac{160000}{0.5 \times 1.225 \times 50 \times C_L}} = 57.9, 127.6 \tag{77}$$

$$R/C = \frac{(T-D)V}{W} = \frac{(30102 - 0.5 \times 1.225 \times V^2 \times 50 \times C_D) \times V}{160000} \tag{78}$$

$$= 6.5, 14.5 m/s \tag{79}$$

$$\gamma = \frac{(T-D)}{W} = \frac{30102 - 0.5 \times 1.225 \times V^2 \times 50 \times C_D}{160000} \tag{80}$$

$$= 0.113, 0.113 rad = 6.5, 6.5 deg \tag{81}$$

(c)

$$u_m = \sqrt{\frac{z + \sqrt{z^2+3}}{3}} = \sqrt{\frac{3.33 + \sqrt{3.33^2+3}}{3}} = 1.54 \tag{82}$$

$$V_m = u_m V_R = 1.54 \times 86 = 132.4 m/s \tag{83}$$

$$(R/C)_{\max} = \frac{V_R}{2E_m}\left[2zu_m - \left(u_m^3 + \frac{1}{u_m}\right)\right] \tag{84}$$

$$= \frac{86}{2 \times 17.7}\left[2 \times 3.33 \times 1.54 - \left(1.54^3 + \frac{1}{1.54}\right)\right] \tag{85}$$

$$= 14.5 m/s \tag{86}$$

习题 2.11

喷气式飞机海平面处速度为 $200m/s$，可以达到 $20m/s$ 的爬升率，若飞行员选择等高度加速而非爬升，求海平面处可以达到的最大速度。假设 $W = 200000N$，$S = 60m^2$，$C_D = 0.021 + 0.042C_L^2$ 。[参考答案：$257.8538m/s$]

解：

$$V_R = \sqrt{\frac{2W}{\rho S}}\sqrt[4]{\frac{k}{C_{D0}}} = \sqrt{\frac{2 \times 200000}{1.225 \times 60}}\sqrt[4]{\frac{0.042}{0.021}} = 87.7m/s \tag{87}$$

$$E_m = \frac{1}{2\sqrt{kC_{D0}}} = \frac{1}{2\sqrt{0.021 \times 0.042}} = 16.8 \tag{88}$$

$$u = \frac{V}{V_R} = \frac{200}{87.7} = 2.28 \tag{89}$$

$$\sin\gamma = \frac{1}{2E_m}\left[2z - \left(u^2 + \frac{1}{u^2}\right)\right] \tag{90}$$

$$\frac{20}{200} = \frac{1}{2 \times 16.8}\left[2z - \left(2.28^2 + \frac{1}{2.28^2}\right)\right] \tag{91}$$

$$z = 4.38 \tag{92}$$

$$u_{\max} = \sqrt{z + \sqrt{z^2 - 1}} = \sqrt{4.38 + \sqrt{4.38^2 - 1}} = 2.94 \tag{93}$$

$$V_{\max} = u_{\max}V_R = 2.94 \times 87.7 = 257.8m/s \tag{94}$$

习题 2.12

喷气式飞机重 $160000N$，零升阻力系数为 0.008，机翼面积 $42m^2$ 。在海平面处飞行速度为 $100m/s$ 时，爬升率为 $11.5m/s$，发动机推力为 $27000N$。求海平面处最大爬升率和对应的飞行速度。[] 参考答案：$21.6013m/s$，$214.5577m/s$]

解：首先求解 k

$$\sin\gamma = \frac{R/C}{V} = \frac{11.5}{100} = 0.115 \tag{95}$$

$$\sin\gamma = \frac{T - D}{W} \tag{96}$$

$$qS = \frac{1}{2}\rho V^2 S = 0.5 \times 1.225 \times 100^2 \times 42 = 257250N \tag{97}$$

$$L = W\cos\gamma = 160000 \times \sqrt{1 - 0.115^2} = 158938N \tag{98}$$

$$C_L = \frac{L}{qS} = \frac{158938}{257250} = 0.618 \tag{99}$$

$$D = T - W\sin\gamma = 27000 - 160000 \times 0.115 = 8600N \tag{100}$$

$$= qS(C_{D0} + kC_L^2) = 257250(0.008 + k \times 0.618^2) \tag{101}$$

$$k = 0.0666 \tag{102}$$

$$\tag{103}$$

$$E_m = \frac{1}{2\sqrt{kC_{D0}}} = \frac{1}{2\sqrt{0.0666 \times 0.008}} = 21.66 \tag{104}$$

$$z = \frac{TE_m}{W} = \frac{27000 \times 21.66}{160000} = 3.66 \tag{105}$$

$$u_m = \sqrt{\frac{z + \sqrt{z^2 + 3}}{3}} = \sqrt{\frac{z + \sqrt{z^2 + 3}}{3}} = 1.6 \tag{106}$$

$$V_R = \sqrt{\frac{2W}{\rho S}} \sqrt[4]{\frac{k}{C_{D0}}} = \sqrt{\frac{2 \times 160000}{1.225 \times 42}} \sqrt[4]{\frac{0.0666}{0.008}} = 134 m/s \tag{107}$$

$$(R/C)_{\max} = \frac{V_R}{2E_m}\left[2zu_m - \left(u_m^3 + \frac{1}{u_m}\right)\right] = \frac{134}{2 \times 21.66}\left[2 \times 3.66 \times 1.6 - \left(1.6^3 + \frac{1}{1.6}\right)\right] \tag{108}$$

$$= 21.6 m/s \tag{109}$$

$$V = V_R u_m = 134 \times 1.6 = 214.4 m/s \tag{110}$$

习题 2.13

喷气式飞机重 150000N，机翼面积 30m^2，阻力极曲线由 $C_D = 0.015 + 0.025C_L^2$ 给出，海平面处推力 T_0 为 23200N，推力与高度关系为 $T = T_0\sigma$，海平面处耗油率 $c_0 = 1.2N/Nh$，与高度关系为 $c = c_0\sigma$。求对应于 Breguet 航程 2500km 时的

（a）最经济巡航高度，[参考答案：12.31km]

（b）燃油量和巡航速度。[参考答案：5796.96N 和 205.166m/s]

解：

(a) 教材上的答案是按照最大升阻比计算的

$$E_m = \frac{1}{2\sqrt{kC_{D0}}} = \frac{1}{2\sqrt{0.025 \times 0.015}} = 25.8 \tag{111}$$

$$\sigma = \left(\frac{W_0}{ET_0}\right)^{\frac{1}{\beta}} = \frac{150000}{25.8 \times 23200} = 0.2506 \tag{112}$$

$$h = 44.3(1 - \sigma^{0.235}) = 44.3(1 - 0.2506^{0.235}) = 12.3km \tag{113}$$

最经济巡航高度约为 12.3km。

实际上无论等高巡航和等速巡航的升力系数都应该为 $\sqrt{\frac{C_{D0}}{3k}}$

$$C_L = \sqrt{\frac{C_{D0}}{3k}} = \sqrt{\frac{0.015}{3 \times 0.025}} = 0.447 \tag{114}$$

$$C_D = 0.015 + 0.025C_L^2 = 0.015 + 0.025 \times 0.447^2 = 0.02 \tag{115}$$

$$E = C_L/C_D = 0.447/0.02 = 22.35 \tag{116}$$

$$\sigma = \left(\frac{W_0}{ET_0}\right)^{\frac{1}{\beta}} = \frac{150000}{22.35 \times 23200} = 0.289 \tag{117}$$

$$h = 44.3(1 - \sigma^{0.235}) = 44.3(1 - 0.289^{0.235}) = 11.2km \tag{118}$$

最经济巡航高度约为 11.2km。

(b) 教材上的答案是按照最大升阻比及 V_R 计算的

$$V_R = \sqrt{\frac{2W_0}{\rho S}}\sqrt[4]{\frac{k}{C_{D0}}} = \sqrt{\frac{2\times 150000}{0.307\times 30}}\sqrt[4]{\frac{0.025}{0.015}} = 205.1m/s \tag{119}$$

$$c = 1.2\times 0.2506N/Nh = 8.3533\times 10^{-5} \tag{120}$$

$$R = \frac{V}{c}E\ln\frac{W_0}{W_1} \tag{121}$$

$$2500\times 1000 = \frac{205.1\times 25.8}{8.3533\times 10^{-5}}\ln\frac{150000}{W_1} \tag{122}$$

$$W_1 = 144186N \tag{123}$$

$$W_f = W_0 - W_1 = 5814N \tag{124}$$

实际上应该按照 $\sqrt[4]{3}V_R$ 计算

$$V = \sqrt[4]{3}V_R = \sqrt[4]{3}\times 205.1 = 269.9m/s \tag{125}$$

$$\rho = \rho_0\sigma = 1.225\times 0.2506 = 0.307kg/m^3 \tag{126}$$

$$C_L = \frac{W}{\frac{1}{2}\rho V^2 S} = \frac{150000}{0.5\times 0.307\times 269.9^2\times 30} = 0.447 \tag{127}$$

$$E = \frac{C_L}{0.015 + 0.025C_L^2} = 22.4 \tag{128}$$

$$R = \frac{V}{c}E\ln\frac{W_0}{W_1} \tag{129}$$

$$2500\times 1000 = \frac{269.9\times 22.4}{8.3533\times 10^{-5}}\ln\frac{150000}{W_1} \tag{130}$$

$$W_1 = 144907N \tag{131}$$

$$W_f = W_0 - W_1 = 5093N \tag{132}$$

习题 2.14

如果习题 2.13 中的飞机在巡航中遇到 35km/h 持续的顺风，可以节省多少燃油。[参考答案：263.88N]

解：

$$R = \frac{V + V_w}{c}E\ln\frac{W_0}{W_1} \tag{133}$$

$$2500\times 1000 = \frac{(205.1 + 35/3.6)\times 25.8}{8.3533\times 10^{-5}}\ln\frac{150000}{W_1} \tag{134}$$

$$W_1 = 144453N \tag{135}$$

$$W_f = 5547N \tag{136}$$

$$\Delta W_f = 5814 - 5547 = 267N \tag{137}$$

考虑到 2.13 中计算方式的不同，此处结果也不同。

习题 2.16

喷气式飞机各项数据如下：$W = 50000N$，$C_D = 0.02 + 0.04C_L^2$，$C_{L,\max} = 1.2$，$S = 30m^2$，$T = 14500\sigma^{0.65}N$，$c = 1.5\sigma^{0.25}N/Nh$，每 $25N$ 的燃油量时会有 $1N$ 的燃油损耗，$W_f = 20000N$。求
（a）$5000m(\sigma = 0.6)$ 处无风时的航程和航时，[参考答案：2075.05km]
（b）如果在巡航时有 $20m/s$ 持续的逆风会减小多少航程。[参考答案：448.49km]

解：根据题目，为等高巡航。

$$C_L = \sqrt{\frac{C_{D0}}{3k}} = \sqrt{\frac{0.02}{3 \times 0.04}} = 0.4082 \tag{138}$$

$$C_D = 0.02 + 0.04C_L^2 = 0.02 + 0.04 \times 0.4082^2 = 0.02666 \tag{139}$$

$$E_m = \frac{1}{2\sqrt{kC_{D0}}} = \frac{1}{2 \times \sqrt{0.04 \times 0.02}} = 17.68 \tag{140}$$

由于燃油损失产生的实际耗油率（转换为 N/Ns）：

$$c = 1.5 \times 0.6^{0.25}/3600 \times \frac{25}{24} \tag{141}$$

$$= 0.000382N/Ns \tag{142}$$

按照等高巡航的航程计算：

$$R = \frac{\sqrt{C_L}}{C_D}\frac{2}{c}\sqrt{\frac{2}{\rho S}}\left(\sqrt{W_0} - \sqrt{W_1}\right) \tag{143}$$

$$= \frac{\sqrt{0.4082}}{0.02666}\frac{2}{0.000382}\sqrt{\frac{2}{1.225 \times 0.6 \times 30}}\left(\sqrt{50000} - \sqrt{30000}\right) \tag{144}$$

$$= 1904 \times 10^3 m = 1904km \tag{145}$$

最大航时：

$$t = \frac{E_m}{c}\ln\frac{W_0}{W_1} = \frac{17.68}{0.000382}\ln\frac{50000}{30000} = 23642s = 6.6h \tag{146}$$

此题关于“每 $25N$ 的燃油量时会有 $1N$ 的燃油损耗”这句话的理解有争议，以上算法与参考答案不同。

习题 2.17

喷气式飞机各项数据如下：$W = 50000N$，$C_D = 0.025 + 0.03C_L^2$，$C_L = 0.08\alpha$，$S = 30m^2$，$T = 15000\sigma^{0.6}N$，$c = 1.0\sigma^{0.2}N/Nh$。如果飞机以 $5°$ 的迎角巡航，求当 $11km(\sigma = 0.293)$ 高度航程为 $1500km$ 时，燃油消耗的变化量与能达到的最小燃油消耗量的百分比。假设飞行员始终调整速度以避免高度变化。[参考答案：2.6102]

解：按照最大航程等高巡航

$$C_L = \sqrt{\frac{C_{D0}}{3k}} = \sqrt{\frac{0.025}{3 \times 0.03}} = 0.5270 \tag{147}$$

$$C_D = 0.025 + 0.03C_L^2 = 0.025 + 0.03 \times 0.5270^2 = 0.03333 \tag{148}$$

$$R = \frac{\sqrt{C_L}}{C_D}\frac{2}{c}\sqrt{\frac{2}{\rho S}}\left(\sqrt{W_0} - \sqrt{W_1}\right) \tag{149}$$

$$= \frac{\sqrt{0.5270}}{0.03333}\frac{2}{1.0 \times 0.293^{0.2}/3600}\sqrt{\frac{2}{1.225 \times 0.293 \times 30}}\left(\sqrt{50000} - \sqrt{W_1}\right) \tag{150}$$

$$= 86393 \times \left(\sqrt{50000} - \sqrt{W_1}\right) = 1500000m \tag{151}$$

$$W_1 = \left(\sqrt{50000} - \frac{1500000}{86393}\right)^2 = 42537 \tag{152}$$

$$W_f = 50000 - 42537 = 7463N \tag{153}$$

按照等迎角巡航

$$C_L = 0.08\alpha = 0.08 \times 5 = 0.4 \tag{154}$$

$$C_D = 0.025 + 0.03C_L^2 = 0.025 + 0.03 \times 0.4^2 = 0.0298 \tag{155}$$

$$R = \frac{\sqrt{C_L}}{C_D}\frac{2}{c}\sqrt{\frac{2}{\rho S}}\left(\sqrt{W_0} - \sqrt{W_1}\right) \tag{156}$$

$$= \frac{\sqrt{0.4}}{0.0298}\frac{2}{1.0 \times 0.293^{0.2}/3600}\sqrt{\frac{2}{1.225 \times 0.293 \times 30}}\left(\sqrt{50000} - \sqrt{W_1}\right) \tag{157}$$

$$= 84183 \times \left(\sqrt{50000} - \sqrt{W_1}\right) = 1500000m \tag{158}$$

$$W_1 = \left(\sqrt{50000} - \frac{1500000}{84183}\right)^2 = 42348N \tag{159}$$

$$W_f = 50000 - 42348 = 7652N \tag{160}$$

$$(7652 - 7463)/7463 = 0.0253 = 2.53\% \tag{161}$$

习题 2.18

螺旋桨飞机重 $60000N$，翼载 $2000N/m^2$，机翼展弦比为 6，零升阻力系数 0.021，Oswald 效率因子为 0.920，发动机功率为 $750kW$，推进效率为 0.82，耗油率为 $3.5N/kW/h$。求下列情况下飞机应携带的燃油量：

（a）无风时的航程为 $1500km$，[参考答案：6985.3523N]

（b）空中持续飞行时间为 $8h$。假设飞机的飞行高度为 2.4km($\sigma = 0.7892$)。[参考答案：9156.86N]

解：按照最大航程等高巡航

$$k = \frac{1}{\pi eA} = \frac{1}{\pi \times 0.92 \times 6} = 0.05766 \tag{162}$$

$$E_m = \frac{1}{2\sqrt{kC_{D0}}} = \frac{1}{2\sqrt{0.05766 \times 0.021}} = 14.37 \tag{163}$$

$$R = \frac{\eta_p}{c}E_m \ln\left(\frac{W_0}{W_1}\right) \tag{164}$$

$$= \frac{0.82}{3.5/3600/1000} \times 14.37 \times \ln\left(\frac{60000}{W_1}\right) \tag{165}$$

$$= 12120000 \times \ln\left(\frac{60000}{W_1}\right) \tag{166}$$

$$= 1500000m \tag{167}$$

$$W_1 = \frac{60000}{2.718^{\frac{1500000}{12120000}}} = 53015N \tag{168}$$

$$W_f = W_1 - W_0 = 60000 - 53015 = 6985N \tag{169}$$

按照最大航时等高巡航

$$S = \frac{60000N}{2000N} = 30m^2 \tag{170}$$

$$C_L = \sqrt{\frac{3C_{D0}}{k}} = \sqrt{\frac{3 \times 0.021}{0.05766}} = 1.045 \tag{171}$$

$$C_D = 0.021 + 0.05766C_L^2 = 0.021 + 0.05766 \times 1.045^2 = 0.08397 \tag{172}$$

$$t = \frac{2\eta_p}{c}\frac{C_L^{\frac{3}{2}}}{C_D}\sqrt{\frac{\rho S}{2}}\left(\sqrt{\frac{1}{W_1}} - \sqrt{\frac{1}{W_0}}\right) \tag{173}$$

$$= \frac{2 \times 0.82}{3.5/3600/1000}\frac{1.045^{1.5}}{0.08397}\sqrt{\frac{1.225 \times 0.7892 \times 30}{2}}\left(\sqrt{\frac{1}{W_1}} - \sqrt{\frac{1}{60000}}\right) \tag{174}$$

$$= 8.172 \times 10^7 \times \left(\sqrt{\frac{1}{W_1}} - 0.004082\right) \tag{175}$$

$$= 8 \times 3600 \tag{176}$$

$$W_1 = \frac{1}{\left(\frac{8 \times 3600}{8.172 \times 10^7} + 0.004082\right)^2} = 50854N \tag{177}$$

$$W_f = W_1 - W_0 = 60000 - 50854 = 9146N \tag{178}$$

习题 2.19

（a）对于水平面内的协调等速盘旋，推导喷气式飞机在极限过载情况下的盘旋速率和盘旋半径的表达式。
（b）证明对于 MSTR 和 SST，升阻比分别为
1）$E = \dfrac{2nE_m}{1+n^2}$
2）$E = \dfrac{2znE_m}{1+n^2z^2}$

解：

(a) 根据静态性能方程 $n^2 = 2zu^2 - u^4$，此方程表示协调盘旋时过载随速度的变化，当 $\mathrm{d}n/\mathrm{d}u$ 为 0 时，盘旋过载最大，上式对速度 u 求导:

$$2n\frac{\mathrm{d}n}{\mathrm{d}u} = 4u(z - u^2) = 0 \tag{179}$$

$$u = \sqrt{z} \tag{180}$$

$$n_{\max} = 2zu^2 - u^4 = z \tag{181}$$

$$\omega = \frac{g\sqrt{n^2-1}}{V} = \frac{g\sqrt{z^2-1}}{\sqrt{z}V_R} = \frac{g}{V_R}\sqrt{\frac{z^2-1}{z}} \tag{182}$$

$$R = \frac{V^2}{g\sqrt{n^2-1}} = \frac{zV_R^2}{g\sqrt{z^2-1}} \tag{183}$$

(b.1) 对于任何盘旋：

$$L = nW \tag{184}$$

$$D = T = \frac{zW}{E_m} \tag{185}$$

$$E = \frac{L}{D} = \frac{nW}{\frac{zW}{E_m}} = \frac{nE_m}{z} \tag{186}$$

根据 MSTR，$n=\sqrt{2z-1}$

$$z=\frac{n^2+1}{2} \tag{187}$$

$$E=\frac{nE_m}{z}=\frac{2nE_m}{1+n^2} \tag{188}$$

(b.2) 根据 SST：

$$n=\frac{\sqrt{2z^2-1}}{z} \tag{189}$$

$$2z^2=n^2z^2+1 \tag{190}$$

$$z=\frac{n^2z^2+1}{2z} \tag{191}$$

$$E=\frac{nE_m}{z}=\frac{2znE_m}{1+n^2z^2} \tag{192}$$

习题 2.21

喷气式飞机重 $80000N$，最大推力 $30000N$，机翼升力线斜率为 $5.0/rad$，零升迎角为 $-2°$，最大升力系数 1.5，机翼面积 $25m^2$，阻力极曲线由 $C_D=0.018+0.08C_L^2$ 给出，结构极限过载为 6.0。当飞机在 $1500m(\rho=1.058kg/m^3)$ 高度的水平面内做倾斜盘旋时，求

（a）最快盘旋速率，[参考答案：$\omega=13.9748deg/s$]

（b）对应的倾斜角，[参考答案：$\mu=70.3948deg$]

（c）迎角，[参考答案：$\alpha=14.2deg$]

（d）升阻比。[参考答案：$E=7.9477$]

$$V_R=\sqrt{\frac{2W}{\rho S}}\sqrt[4]{\frac{k}{C_{D0}}}=\sqrt{\frac{2\times80000}{1.058\times25}}\sqrt[4]{\frac{0.08}{0.018}}=112.9m/s \tag{193}$$

$$E_m=\frac{1}{2\sqrt{kC_{D0}}}=\frac{1}{2\sqrt{0.08\times0.018}}=13.18 \tag{194}$$

$$z=\frac{TE_m}{W}=\frac{30000\times13.18}{80000}=4.94 \tag{195}$$

$$\omega=\frac{g\sqrt{2z-2}}{V_R}=\frac{9.81\times\sqrt{2\times4.94-2}}{112.9}=0.244rad/s=13.98deg/s \tag{196}$$

$$n=\sqrt{2z-1}=\sqrt{2\times4.94-1}=2.98 \tag{197}$$

$$\mu=\cos^{-1}(1/n)=\cos^{-1}(1/2.98)=70.39deg \tag{198}$$

$$C_L=\frac{nW}{\frac{1}{2}\rho V^2S}=\frac{2.98\times80000}{0.5\times1.058\times112.9^2\times25}=1.414 \tag{199}$$

$$C_L=C_{L\alpha}(\alpha-\alpha_{0L}) \tag{200}$$

$$\alpha=\frac{C_L}{C_{L\alpha}}+\alpha_{0L}=\frac{1.414}{5}\times57.3+(-2)=14.2deg \tag{201}$$

$$E=C_L/C_D=1.414/(0.018+0.08\times1.414^2)=7.95 \tag{202}$$

习题 2.22

喷气式飞机各项数据如下：$W = 45000N$，推重比 0.49，$C_{L\alpha} = 4.6/rad$，$\alpha_{0L} = -2.2deg$，$S = 25m^2$。求

（a）在 2250m($\sigma = 0.8$) 高度处以 $\alpha = 8deg$ 和 $n = 4$ 做适当倾斜的水平盘旋时的盘旋半径；[参考答案：472.18m]

（b）在 $C_L = 1.5$ 和 $E = 6.0$ 条件下做 $5g$ 盘旋所需的附加推力；[参考答案：15450N]

（c）以 25% 附加推力和 $20deg$ 侧滑角盘旋时，盘旋半径的减少量。[参考答案：24.23m]

解：a

$$C_L = C_{L\alpha}(\alpha - \alpha_{0L}) = 4.6 \times (8 + 2.2)/57.3 = 0.819 \tag{203}$$

$$V = \sqrt{\frac{2nW}{\rho S C_L}} = \sqrt{\frac{2 \times 4 \times 45000}{1.225 \times 0.8 \times 25 \times 0.819}} = 133.9m/s \tag{204}$$

$$R = \frac{V^2}{g\sqrt{n^2 - 1}} = \frac{133.9^2}{9.81\sqrt{4^2 - 1}} = 471.9m \tag{205}$$

b

$$V = \sqrt{\frac{2nW}{\rho S C_L}} = \sqrt{\frac{2 \times 5 \times 45000}{1.225 \times 0.8 \times 25 \times 1.5}} = 110.7m/s \tag{206}$$

$$C_D = C_L/E = 1.5/6 = 0.25 \tag{207}$$

$$T = D = \frac{1}{2}\rho V^2 S C_D = 0.5 \times 1.225 \times 0.8 \times 110.7^2 \times 25 \times 0.25 = 37529N \tag{208}$$

$$\Delta T = 3529 - 45000 \times 0.49 = 15479N \tag{209}$$

习题 2.23

喷气式战斗机重 $70000N$，机翼面积 $25m^2$，$C_D = 0.015 + 0.06C_L^2$，$C_{L\max} = 1.4$，结构极限过载为 8.0。求飞机在海平面处 $15s$ 以 $300km/h$ 的速度完成 180° 盘旋的

（a）倾斜角，[参考答案：$\mu = 60.656$]

（b）升力系数，[参考答案：$C_L = 1.3433$]

（c）升阻比，[参考答案：$E = 10.8975$]

（d）盘旋半径，[参考答案：$R = 397.9615m$]

（e）过载，[参考答案：$n = 2.0406$]

（f）需用推力。[参考答案：$T = 13111.32N$]

解：

$$V = 300km/h = 83.33m/s \tag{210}$$

$$qS = \frac{1}{2}\rho V^2 S = 0.5 \times 1.225 \times 83.33^2 \times 25 = 106328 \tag{211}$$

$$\omega = 180/15 = 12deg/s = 0.2094rad/s \tag{212}$$

$$\mu = \tan^{-1}\frac{V\omega}{g} = \tan^{-1}\frac{83.33 \times 0.2094}{9.81} = 60.656deg \tag{213}$$

$$n = \frac{1}{\cos\mu} = \frac{1}{\cos(60.656)} = 2.04 \tag{214}$$

$$C_L = \frac{nW}{qS} = \frac{2.04 \times 70000}{106328} = 1.343 \tag{215}$$

$$C_D = 0.015 + 0.06C_L^2 = 0.015 + 0.06 \times 1.343^2 = 0.1232 \tag{216}$$

$$E = \frac{C_D}{C_D} = \frac{1.343}{0.1232} = 10.9 \tag{217}$$

$$T = D = qSC_D = 106328 \times 0.1232 = 13100N \tag{218}$$

$$R = \frac{V^2}{g\sqrt{n^2-1}} = \frac{83.33^2}{9.81\sqrt{2.04^2-1}} = 398m \tag{219}$$

习题 2.24

喷气式飞机重 $50000N$，海平面处最大推力 $6000N$，翼载 $1500N/m^2$，$C_D = 0.02+0.08C_L^2$，$C_{L\max} = 1.8$，最大允许过载为 2.5。求飞机以全推力在海平面处以 $1500m$ 半径做协调盘旋时 (该问题有两个解):

（a）倾斜角，[参考答案：$\mu = 37.5726deg,\ 7.4005deg$]

（b）升力系数，[参考答案：$C_L = 0.2729,\ 1.2945$]

（c）升阻比，[参考答案：$E = 10.4962,\ 8.4004$]

（d）盘旋速率，[参考答案：$\omega = 4.0644deg/s,\ 1.6685deg/s$]

（e）过载。[参考答案：$n = 1.2617,\ 1.0084$]

解：

$$S = 50000/1500 \tag{220}$$

$$= 33.33m^2 \tag{221}$$

根据一致条件，可以列出以下三个方程

$$R = \frac{V^2}{g\sqrt{n^2-1}} \tag{222}$$

$$T = \frac{1}{2}\rho V^2 S(0.02 + 0.08C_L^2) \tag{223}$$

$$nW = \frac{1}{2}\rho V^2 S C_L \tag{224}$$

带入已知数值:

$$1500 = \frac{V^2}{9.81\sqrt{n^2-1}} \tag{225}$$

$$6000 = 20.42 \times V^2(0.02 + 0.08C_L^2) \tag{226}$$

$$n \times 50000 = 20.42V^2 C_L \tag{227}$$

整理得

$$V^4 = 14715^2(n^2-1) \tag{228}$$

$$293.8 = V^2(0.02 + 0.08C_L^2) \tag{229}$$

$$n = V^2 C_L/2449 \tag{230}$$

$$V^4 = 14715^2(V^4 C_L^2/2449^2 - 1) \tag{231}$$

$$C_L^2 = 3672.5/V^2 - 0.25 \tag{232}$$

$$V^4 = 36.1V^4 \times (3672.5/V^2 - 0.25) - 14715^2 \tag{233}$$

$$0 = 10V^4 - 132577V^2 + 14715^2 \tag{234}$$

$$V^2 = 11350, 1908 \tag{235}$$

$$V = 106.5, 43.7m/s \tag{236}$$

$$C_L = \sqrt{3672.5/V^2 - 0.25} = 0.271, 1.294 \tag{237}$$

$$n = V^2 C_L/2449 = 1.26, 1.008 \tag{238}$$

$$\omega = \frac{g\sqrt{n^2-1}}{V} = 0.0706, 0.0285rad/s = 4.05, 1.63deg/s \tag{239}$$

$$\mu = \cos^{-1}(1/n) = 34.5, 7.2deg \tag{240}$$

$$E = \frac{C_L}{0.02 + 0.08C_L^2} = 10.5, 8.4 \tag{241}$$

习题 2.25

战斗机重 $75000N$，机翼面积 $27m^2$，采用增升装置时的最大升力系数为 1.8，结构极限过载为 6.0，当以 $250km/h$ 速度飞行时，飞机在海平面处 $8s$ 完成等高度的 $90°$ 盘旋，此时迎角对应的升阻比为 8.0。求

（a）倾斜角，[参考答案：$\mu = 54.26deg$]

（b）过载，[参考答案：$n = 1.7120$]

（c）盘旋半径，[参考答案：$R = 353.7665$]

（d）需用推力。[参考答案：$T = 16050N$]

解：

$$\omega = 90/8 = 11.25deg/s = 0.196rad/s \tag{242}$$

$$V = 250km/h = 69.4m/s \tag{243}$$

$$n = \sqrt{(\frac{\omega V}{g})^2 + 1} = \sqrt{(\frac{0.196 \times 69.4}{9.81})^2 + 1} = 1.71 \tag{244}$$

$$\mu = \cos^{-1}(1/n) = \cos^{-1}(1/1.71) = 54.2deg \tag{245}$$

$$R = \frac{V^2}{g\sqrt{n^2-1}} = \frac{69.4^2}{9.81 \times \sqrt{1.71^2 - 1}} = 353.9m \tag{246}$$

$$T = D = nW/E = 1.71 \times 75000/8 = 16031N \tag{247}$$

习题 2.26

喷气式飞机重 $85000N$，机翼面积 $32m^2$，$C_{L\max} = 1.7$，$C_D = 0.04 + 0.0833C_L^2$，$n_{lim} = 6.0$，当飞机在海平面处 $6s$ 完成 $90°$ 的协调盘旋时，求其需用推力。[参考答案：31681.9N]

缺少条件。

习题 2.27

喷气式飞机重 $68000N$，推重比 0.6，假设 $S=24m^2$，$C_D=0.025+0.07C_L^2$，$C_{L\max}=1.2$，$n_{lim}=8.0$，求下列情况下飞机在 $2250m(\sigma=0.8)$ 高度做 180° 盘旋的最短时间：
（a）保持高度不变，[参考答案：9.4366s]
（b）允许高度损失，[参考答案：7.6886s]
（c）求（b）情况下的高度损失。[参考答案：360.3337m]

解：

a 如果按照 MSTR 计算

$$V_R=\sqrt{\frac{2W}{\rho S}}\sqrt[4]{\frac{k}{C_{D0}}}=\sqrt{\frac{2\times 68000}{1.225\times 0.8\times 24}}\sqrt[4]{\frac{0.07}{0.025}}=98.4m/s \tag{248}$$

$$E_m=\frac{1}{2\sqrt{kC_{D0}}}=\frac{1}{2\sqrt{0.025\times 0.07}}=11.95 \tag{249}$$

$$z=\frac{TE_m}{W}=0.6\times 11.95=7.17 \tag{250}$$

$$n=\sqrt{2z-1}=\sqrt{2\times 7.17-1}=3.65 \tag{251}$$

$$C_L=\frac{nW}{\frac{1}{2}\rho V^2 S}=\frac{3.65\times 68000}{0.5\times 1.225\times 0.8\times 98.4^2\times 24}=2.18>1.2 \tag{252}$$

由于升力系数大于最大升力系数，因此无法满足 MSTR 盘旋的条件。此时按照最大升力系数计算阻力系数，跟据定常关系（推力等于阻力）反过来确定速度。

$$C_L=1.2 \tag{253}$$

$$C_D=0.025+0.07C_L^2=0.025+0.07\times 1.2^2=0.1258 \tag{254}$$

$$V=\sqrt{\frac{T}{\frac{1}{2}\rho S C_D}}=\sqrt{\frac{68000\times 0.6}{0.5\times 1.225\times 0.8\times 24\times 0.1258}}=166m/s \tag{255}$$

$$n=\frac{\frac{1}{2}\rho V^2 S C_L}{W}=\frac{0.5\times 1.225\times 0.8\times 166^2\times 24\times 1.2}{68000}=5.72 \tag{256}$$

$$\omega=\frac{g\sqrt{n^2-1}}{V}=\frac{9.81\times\sqrt{5.72^2-1}}{166}=0.333rad/s=19.1deg/s \tag{257}$$

$$t_{180}=180/19.1=9.42s \tag{258}$$

b 根据角点速度计算盘旋

$$V_{corner}=\sqrt{\frac{2n_{\lim}W}{\rho S C_{L,\max}}}=\sqrt{\frac{2\times 8\times 68000}{1.225\times 0.8\times 24\times 1.2}}=196.3m/s \tag{259}$$

$$C_D=0.025+0.07C_L^2=0.025+0.07\times 1.2^2=0.1258 \tag{260}$$

$$D=\frac{1}{2}\rho V^2 S C_D=0.5\times 1.225\times 0.8\times 196.3^2\times 24\times 0.1258=57007N \tag{261}$$

$$\gamma=\sin^{-1}\frac{T-D}{W}=\sin^{-1}\frac{68000*0.6-57007}{68000}=13.79deg \tag{262}$$

$$\omega=\frac{g\sqrt{n^2-\cos^2\gamma}}{V\cos\gamma}=\frac{9.81\times\sqrt{8^2-\cos^2(13.79)}}{196.3\cos 13.79}=23.4deg/s \tag{263}$$

$$t_{180}=180/23.4=7.69s \tag{264}$$

c(此处爬升率不考虑方向，只考虑大小)

$$R/C=V_{corner}\sin\gamma=196.3\times\sin 13.79=46.79m/s \tag{265}$$

$$\Delta h=46.79\times 7.69=360m \tag{266}$$

习题 2.28

证明逆风时飞机起飞地面滑跑距离的减少量近似为 $\Delta s_1 = (2V_w/V_1)s_1$。

解：

无风滑跑距离

$$s_1 = \frac{W}{2g}\frac{V_1^2}{F_0 - F_1}\ln\frac{F_0}{F_1} \tag{267}$$

$$\tag{268}$$

有风滑跑距离

$$s_1' = \frac{W}{2g}\frac{(V_1 - V_w)^2}{F_0 - F_1}\ln\frac{F_0}{F_1} \tag{269}$$

$$\tag{270}$$

则

$$\frac{\Delta s_1}{s_1} = \frac{V_1^2 - (V_1 - V_w)^2}{V_1^2} \tag{271}$$

$$= \frac{2V_w}{V_1} + \frac{V_w^2}{V_1^2} \tag{272}$$

$$\tag{273}$$

由于 $\frac{V_w}{V_1}$ 是小量，则 $\frac{V_w^2}{V_1^2}$ 可以忽略：

$$\Delta s_1 = \frac{2V_w}{V_1}s_1 \tag{274}$$

习题 2.29

飞机重 $294300N$，机翼面积 $100m^2$，推力 $73575N$，阻力极曲线由 $C_D = 0.02 + 0.05C_L^2$ 给出，襟翼偏转时的最大升力系数为 1.80。假设飞机轮胎与水泥地面之间的摩擦系数为 0.05，求海平面处最短滑跑距离和对应时间。[参考答案：$s_1 = 925.1527m$]

解：

$$V_{stall} = \sqrt{\frac{2W}{\rho S C_{L,\max}}} = \sqrt{\frac{2\times 294300}{1.225\times 100\times 1.8}} = 51.67m/s \tag{275}$$

$$V_1 = 1.2V_{stall} = 1.2\times 51.67 = 62m/s \tag{276}$$

$$C_L^* = \frac{\mu}{2k} = \frac{0.05}{2\times 0.05} = 0.5 \tag{277}$$

$$D = \frac{1}{2}\rho V_1^2 S(C_{D0} + kC_L^2) = \frac{1}{2}\times 1.225\times 62^2\times 100\times(0.02 + 0.05\times 0.5^2) = 7652N \tag{278}$$

$$F_0 = T - \mu W = 73575 - 0.05\times 294300 = 58860N \tag{279}$$

$$F_1 = T - D = 73575 - 7652 = 65923N \tag{280}$$

$$s_1 = \frac{W}{2g}\frac{V_1^2}{F_0 - F_1}\ln\frac{F_0}{F_1} = \frac{294300}{2\times 9.80665}\times\frac{62^2}{58860 - 65923}\ln\frac{58860}{65923} \tag{281}$$

$$= 920m \tag{282}$$

$$a = F_0 = 58860 \tag{283}$$

$$b = \frac{F_1 - F_0}{V_1^2} = \frac{65923 - 58860}{62^2} = 1.837 \tag{284}$$

$$t_1 = \frac{W}{g\sqrt{ab}} \tan^{-1} \sqrt{\frac{b}{a}} V_1 = \frac{294300}{9.81 \times \sqrt{58860 \times 1.837}} \tan^{-1}(\sqrt{\frac{1.837}{58860}}) \times 62 \tag{285}$$

$$= 31.6s \tag{286}$$

习题 2.30

假设障碍高度为 15m，求习题 2.29 中飞机总的起飞距离和时间。[参考答案：990.4157m 和 31.50s]

解：

$$\gamma = \sin^{-1} \frac{T - D}{W} = \sin^{-1} \frac{73575 - 7652}{294300} \tag{287}$$

$$= 0.2259rad = 12.94deg \tag{288}$$

$$s_2 = \frac{h_{obst}}{\tan \gamma} = \frac{15}{\tan 0.2259} = 65m \tag{289}$$

$$t_2 = \frac{s_2}{V_1 \cos \gamma} = \frac{65}{62 \cos 0.2259} = 1s \tag{290}$$

$$s = s_1 + s_2 = 920 + 65 = 985m \tag{291}$$

$$t = t_1 + t_2 = 31.6 + 1 = 32.6s \tag{292}$$

习题 2.31

战斗机重 $78480N$，机翼面积 $25m^2$，升力线斜率为 $0.06/deg$，$C_{L\max} = 0.95$，$C_D = 0.0254 + 0.178C_L^2$，该飞机需要降落在位于 $1000m(\sigma = 0.9074)$ 高度处的跑道上，假设轮胎与跑道之间的摩擦系数为 0.02，进场下滑角为 $3.5deg$，求

（a）空中飞行距离 (包括改平段)[参考答案：290m]

（b）地面滑跑距离。假设障碍高度为 15m。假设接地时襟翼偏转使 $C_{L\max}$ 增加了 0.45，C_D 增加了 0.05。进一步假设使用刹车使摩擦系数增加了 0.4。[参考答案：1038m]

解：

a

$$s_1 = \frac{h_{obst}}{\tan \gamma} = \frac{15}{\tan(3.5/57.3)} = 245.3m \tag{293}$$

$$V_A = 1.3V_{stall} = 1.3\sqrt{\frac{2W}{\rho S C_{L\max}}} \tag{294}$$

$$= 1.3\sqrt{\frac{2 \times 78480}{1.25 \times 0.9074 \times 25 \times 0.95}} \tag{295}$$

$$= 99.23m/s \tag{296}$$

$$R = \frac{V_A^2}{0.69g} = \frac{99.23^2}{0.69 \times 9.81} = 1455m \tag{297}$$

$$s_2 = \frac{1}{2} R\gamma = \frac{1}{2} \times 1455 \times \frac{3.5}{57.3} = 44.4m \tag{298}$$

$$s_1 + s_2 = 289.7m \tag{299}$$

b

$$V_A = 1.3V_{stall} = 1.3\sqrt{\frac{2W}{\rho S C_{L\max}}} \tag{300}$$

$$= 1.3\sqrt{\frac{2 \times 78480}{1.25 \times 0.9074 \times 25 \times (0.95 + 0.45)}} \tag{301}$$

$$= 81.74m/s \tag{302}$$

$$F_1 = T_R + D \tag{303}$$

$$= 0 + \frac{1}{2} \times 1.225 \times 0.9074 \times 81.74^2 \times 25 \times (0.0254 + 0.178 \times (0.95 + 0.45)^2 + 0.05) \tag{304}$$

$$= 39388N \tag{305}$$

$$F_0 = T_R + \mu W = 0 + (0.02 + 0.4) \times 78480 = 32962N \tag{306}$$

$$s_3 = \frac{W}{2g}\frac{V_1^2}{F_1 - F_0}\ln\frac{F_1}{F_0} = \frac{78480}{2 \times 9.81}\frac{81.74^2}{39388 - 32962}\ln\frac{39388}{32962} \tag{307}$$

$$= 740.7m \tag{308}$$