Chapter 7 Compressible Flow: Some Preliminary Aspects

2019年12月20日 星期五 15:06

Chapter 7 可压缩流动：一些预备知识

7.1 引言P516

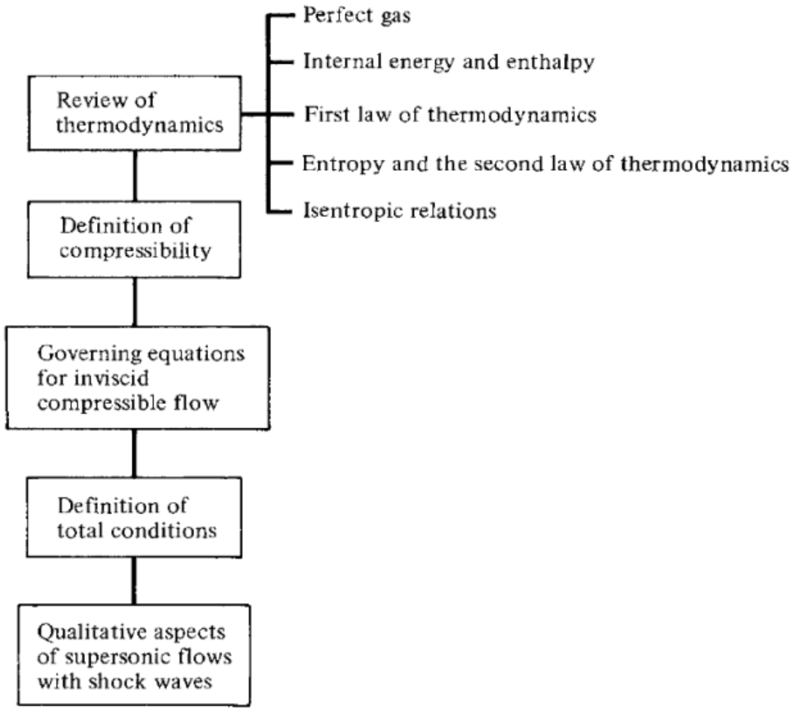

Figure 7.1 Road map for Chapter 7.

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7.2.1 完全气体P518

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7.4 无粘可压缩流动的控制方程P531

7.5 总（滞止）状态的定义P533

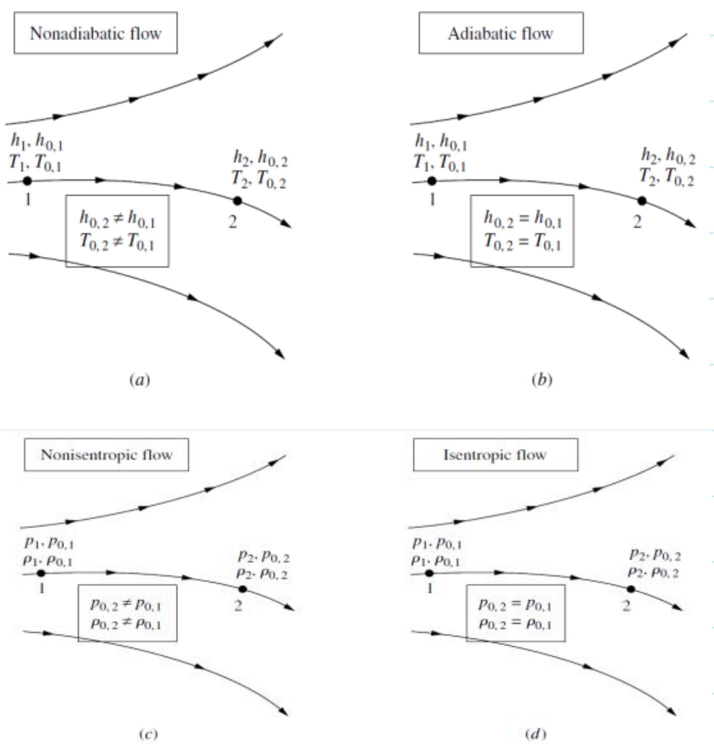

Figure 7.4 Comparisons between (*a*) nonadiabatic, (*b*) adiabatic, (*c*) nonisentropic, and (*d*) isentropic flows.

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7.7 总结P544

Thermodynamic relations:

Equation of state: $$p = \rho R T \tag{7.1}$$

For a calorically perfect gas,

$$e = c_v T \quad \text{and} \quad h = c_p T \tag{7.6a and b}$$

$$c_p = \frac{\gamma R}{\gamma - 1} \tag{7.9}$$

$$c_v = \frac{R}{\gamma - 1} \tag{7.10}$$

Forms of the first law:

$$\delta q + \delta w = de \tag{7.11}$$

$$T\,ds = de + p\,dv \tag{7.18}$$

$$T\,ds = dh - v\,dp \tag{7.20}$$

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Definition of entropy:

$$ds = \frac{\delta q_{\text{rev}}}{T} \tag{7.13}$$

Also

$$ds = \frac{\delta q}{T} + ds_{\text{irrev}} \tag{7.14}$$

The second law:

$$ds \geq \frac{\delta q}{T} \tag{7.16}$$

or, for an adiabatic process,

$$ds \geq 0 \tag{7.17}$$

Entropy changes can be calculated from (for a calorically perfect gas)

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} \tag{7.25}$$

and

$$s_2 - s_1 = c_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} \tag{7.26}$$

For an isentropic flow,

$$\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} = \left(\frac{T_2}{T_1}\right)^{\gamma/(\gamma-1)} \tag{7.32}$$

General definition of compressibility:

$$\tau = -\frac{1}{v}\frac{dv}{dp} \tag{7.33}$$

For an isothermal process,

$$\tau_T = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T \tag{7.34}$$

For an isentropic process,

$$\tau_s = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_s \tag{7.35}$$

The governing equations for inviscid, compressible flow are

Continuity:

$$\frac{\partial}{\partial t}\iiint_{\mathcal{V}} \rho\, d\mathcal{V} + \oiint_{S} \rho \mathbf{V} \cdot \mathbf{dS} = 0 \tag{7.39}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{V} = 0 \tag{7.40}$$

Momentum:

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$$\frac{\partial}{\partial t}\oiiint_{\mathcal{V}} \rho \mathbf{V}\, d\mathcal{V} + \oiint_{S} (\rho \mathbf{V}\cdot \mathbf{dS})\mathbf{V} = -\oiint_{S} p\, \mathbf{dS} + \oiiint_{\mathcal{V}} \rho \mathbf{f}\, d\mathcal{V} \quad (7.41)$$

$$\rho\frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \rho f_x \quad (7.42a)$$

$$\rho\frac{Dv}{Dt} = -\frac{\partial p}{\partial y} + \rho f_y \quad (7.42b)$$

$$\rho\frac{Dw}{Dt} = -\frac{\partial p}{\partial z} + \rho f_z \quad (7.42c)$$

Energy:

$$\frac{\partial}{\partial t}\oiiint_{\mathcal{V}} \rho\left(e + \frac{V^2}{2}\right) d\mathcal{V} + \oiint_{S} \rho\left(e + \frac{V^2}{2}\right)\mathbf{V}\cdot \mathbf{dS}$$

(*continued*)

$$= \oiiint_{\mathcal{V}} \dot{q}\rho\, d\mathcal{V} - \oiint_{S} p\mathbf{V}\cdot \mathbf{dS} + \oiiint_{\mathcal{V}} \rho(\mathbf{f}\cdot \mathbf{V})\, d\mathcal{V} \quad (7.43)$$

$$\rho\frac{D(e + V^2/2)}{Dt} = \rho\dot{q} - \nabla\cdot p\mathbf{V} + \rho(\mathbf{f}\cdot \mathbf{V}) \quad (7.44)$$

If the flow is steady and adiabatic, Equations (7.43) and (7.44) can be replaced by

$$h_0 = h + \frac{V^2}{2} = \text{const}$$

Equation of state (perfect gas):

$$p = \rho RT \quad (7.1)$$

Internal energy (calorically perfect gas):

$$e = c_v T \quad (7.6a)$$

Total temperature T_0 and total enthalpy h_0 are defined as the properties that would exist if (in our imagination) we slowed the fluid element at a point in the flow to zero velocity adiabatically. Similarly, total pressure p_0 and total density ρ_0 are defined as the properties that would exist if (in our imagination) we slowed the fluid element at a point in the flow to zero velocity isentropically. If a general flow field is adiabatic, h_0 is constant throughout the flow; in contrast, if the flow field is nonadiabatic, h_0 varies from one point to another. Similarly, if a general flow field is isentropic, p_0 and ρ_0 are constant throughout the flow; in contrast, if the flow field is nonsentropic, p_0 and ρ_0 vary from one point to another.

Shock waves are very thin regions in a supersonic flow across which the pressure, density, temperature, and entropy increase; the Mach number, flow velocity, and total pressure decrease; and the total enthalpy stays the same.

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