Chapter 12 Linearized Supersonic Flow

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Chapter 12 线化超声速流

12.1 引言P780

The linearized perturbation velocity potential equation derived in Chapter 11, Equation (11.18), is

$$(1 - M_\infty^2)\frac{\partial^2\hat{\phi}}{\partial x^2} + \frac{\partial^2\hat{\phi}}{\partial y^2} = 0 \tag{11.18}$$

and holds for both subsonic and supersonic flow. In Chapter 11, we treated the case of subsonic flow, where $1 - M_\infty^2 > 0$ in Equation (11.18). However, for supersonic flow, $1 - M_\infty^2 < 0$. This seemingly innocent change in sign on the first term of Equation (11.18) is, in reality, a very dramatic change. Mathematically, when $1 - M_\infty^2 > 0$ for subsonic flow, Equation (11.18) is an *elliptic* partial differential equation, whereas when $1 - M_\infty^2 < 0$ for supersonic flow, Equation (11.18) becomes a *hyperbolic* differential equation. The details of this mathematical difference are beyond the scope of this book; however, the important point is that there *is* a difference. Moreover, this portends a fundamental difference in the physical aspects of subsonic and supersonic flow—something we have already demonstrated in previous chapters.

The purpose of this chapter is to obtain a solution of Equation (11.18) for supersonic flow and to apply this solution to the calculation of supersonic airfoil properties. Since our purpose is straightforward, and since this chapter is relatively short, there is no need for a chapter road map to provide guidance on the flow of our ideas.

12.2 线化超声速流压力系数计算公式的推导P780

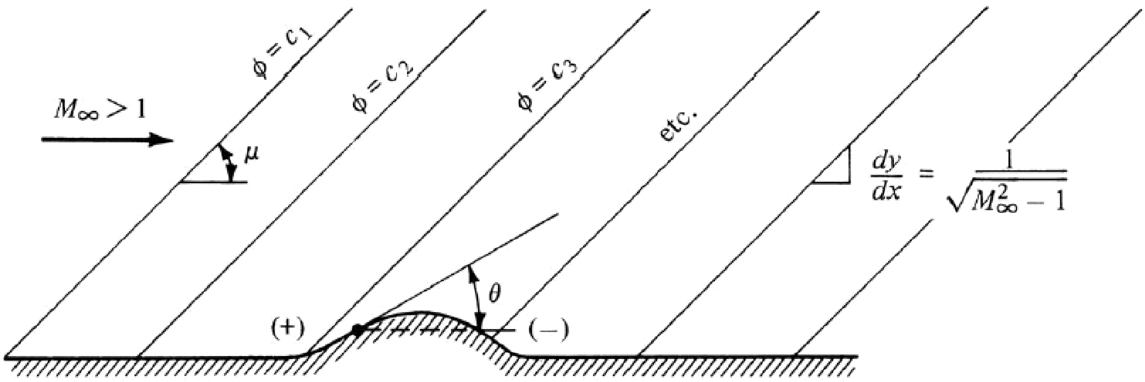

Figure 12.1 Linearized supersonic flow.

$$\boxed{C_p = \frac{2\theta}{\sqrt{M_\infty^2 - 1}}} \tag{12.15}$$

12.3 （线化理论对）超声速翼型的应用P784

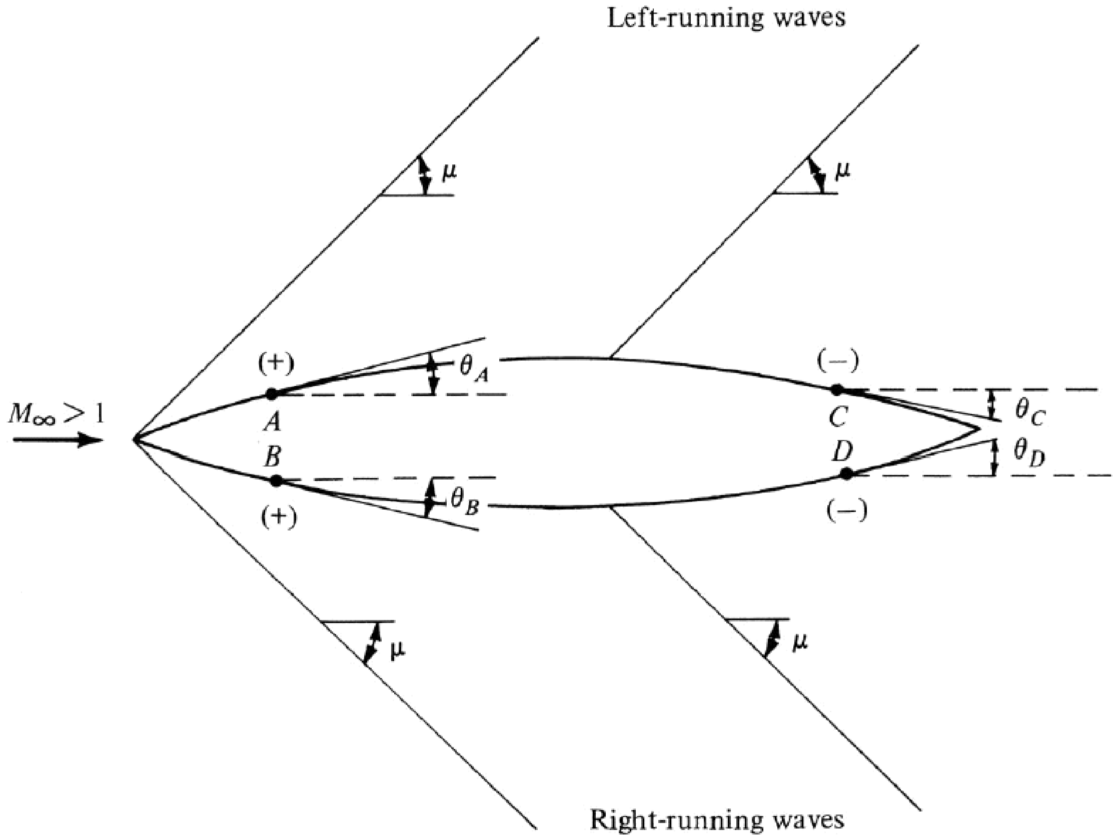

Figure 12.3 Linearized supersonic flow over an airfoil.

$$\boxed{c_l = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}}} \tag{12.23}$$

$$\boxed{c_d = \frac{4\alpha^2}{\sqrt{M_\infty^2 - 1}}} \tag{12.24}$$

12.4 粘性流动：超声速翼型阻力特性P790

12.5 总结P793

In linearized supersonic flow, information is propagated along Mach lines where the Mach angle $\mu = \sin^{-1}(1/M_\infty)$. Since these Mach lines are all based on M_∞, they are straight, parallel lines which propagate away from and downstream of a body. For this reason, disturbances cannot propagate upstream in a steady supersonic flow.

The pressure coefficient, based on linearized theory, on a surface inclined at a small angle θ to the freestream is

$$C_p = \frac{2\theta}{\sqrt{M_\infty^2 - 1}} \tag{12.15}$$

If the surface is inclined into the freestream, C_p is positive; if the surface is inclined away from the freestream, C_p is negative.

Based on linearized supersonic theory, the lift and wave-drag coefficients for a flat plate at an angle of attack are

$$c_l = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}} \tag{12.23}$$

and

$$c_d = \frac{4\alpha^2}{\sqrt{M_\infty^2 - 1}} \tag{12.24}$$

Equation (12.23) also holds for a thin airfoil of arbitrary shape. However, for such an airfoil, the wave-drag coefficient depends on both the shape of the mean camber line and the airfoil thickness.

12.6 作业题P794