No.5 Homework

Note: The standard sea level value of viscosity coefficient for air is
$\mu = 1.7894\times10^{-5}\ kg/(m\cdot s) = 3.7373\times10^{-7}\ slug/(ft\cdot s)$.

19.1 The wing on a Piper Cherokee general aviation aircraft is rectangular, with a span of 9.75m and a chord of 1.6m. The aircraft is flying at cruising speed (141 mi/h) at sea level. Assume that the skin friction drag on the wing can be approximated by the drag on a flat plate of the same dimensions. Calculate the skin friction drag:

a. If the flow were completely laminar (which is not the case in real life);

b. If the flow were completely turbulent (which is more realistic);

Compare the two results.

解19.1: ① 首先计算飞行的巡航速度与需要变量。

由理想气体状态方程 $p=\rho RT$，可得到自由来流密度为:

$$\rho_\infty = \frac{p_\infty}{RT_\infty} = \frac{1.01\times10^5}{(287)(288)} = 1.2219\ kg/m^3$$

由声速公式可得自由来流声速为:

$$a_\infty = \sqrt{\gamma RT_\infty} = \sqrt{(1.4)(287)(288)} = 340.1741\ m/s$$

又由巡航来流速度为 $V_\infty = 141\ mi/h = 63.0326\ m/s$

则巡航马赫数(海平面状态下)为 $M_\infty = \frac{V_\infty}{a_\infty} = \frac{63.0326}{340.1741} = 0.1853 < 0.3$;

故马赫数足够低使得飞行时来流可假设为不可压缩的(Incompressible flow);

② 分两种情况计算摩擦阻力如下:

a. 假设气流完全是层流状态(实际生活中不可能)

由

$$Re_c = \frac{\rho_\infty V_\infty c}{\mu_\infty} = \frac{(1.2219)(63.0326)(1.6)}{1.7894\times10^{-5}} = 6.8867\times10^6$$

故层流状态下

$$C_f = \frac{1.328}{\sqrt{Re_c}} = \frac{1.328}{\sqrt{6.8867\times10^6}} = 5.0605\times10^{-4}$$

将机翼所受摩擦阻力近似认为是相同尺寸平板所受摩擦阻力，则有

$$D_f = \frac{1}{2}\rho_\infty V_\infty^2 S C_f = \frac{1}{2}(1.2219)(63.0326)^2(9.75)(1.6)(5.0605\times10^{-4}) = 19.1625\ N$$

由实际机翼两面均受摩擦阻力，则有层流来流，总表面摩擦阻力为

$$D_{laminar} = 2D_f = 2(19.1625) = 38.3250\ N$$

b. 假设来流完全为湍流状态.（更贴近生活实际）

则湍流状态下

$$C_f = \frac{0.074}{(Re_c)^{1/5}} = \frac{0.074}{(6.8867\times10^6)^{1/5}} = 3.1742\times10^{-3}$$

$$D_f = \frac{1}{2}\rho_\infty V_\infty^2 S C_f = \frac{1}{2}(1.2219)(63.0326)^2(9.75)(1.6)(3.1742\times10^{-3}) = 120.1959\ N$$

$$D_{turbulent} = 2D_f = 2(120.1959) = 240.3918\ N$$

③ 比较来流完全为层流和完全为湍流两种情况下机翼所受摩擦阻力，有

$$\frac{D_{turbulent}}{D_{laminar}} = \frac{(240.3918)}{(38.3250)} = 6.2725$$

可见平板湍流边界层带来的壁面摩擦阻力远大于层流状况，因此，尽量保证飞机飞行时来流与绕流边界层为层流，可有效降低机翼表面摩擦阻力。

19.2 For the case in Problem 19.1, calculate the boundary-layer thickness at the trailing edge for

a. Completely laminar flow;

b. Completely turbulent flow.

解19.2 ① 仍考虑完全层流与完全湍流两种极端情况下机翼尾部的边界层厚度。

a. 假设来流完全是层流状态.

由机翼弦长 $c = 1.6m$，且 Problem 19.1 已求得 $Re_c = 6.8867\times10^6$，故机翼后缘处边界层厚度为

$$\delta_{x=c,laminar} = \frac{5.0c}{\sqrt{Re_c}} = \frac{(5.0)(1.6)}{\sqrt{6.8867\times10^6}} = 3.048\times10^{-3}\ m = 3.048\ mm$$

b. 假设来流完全是湍流状态、

由机翼弦长 $c = 1.6m$，且 Problem 19.1 已求得 $Re_c = 6.8867\times10^6$，
故机翼后缘处边界层厚度为

$$\delta_{x=c,\,turbulent} = \frac{0.37c}{Re_c^{1/5}} = \frac{(0.37)(1.6)}{(6.8867\times10^6)^{1/5}} = 0.02539\ m = 25.393\ mm$$

② 比较来流完全为湍流与完全为层流两种情况下机翼后缘处边界层厚度，有

$$\frac{\delta_{x=c,\,turbulent}}{\delta_{x=c,\,laminar}} = \frac{0.02539}{3.048\times10^{-3}} = 8.3301$$

可见平板湍流边界层在相同发展段（机翼后缘）的边界层厚度远大于层流状态。

19.3 For the case in Problem 19.1, calculate the skin friction drag accounting for transition. Assume the transition Reynolds number $= 5\times10^5$.

解 19.3: 对于 Problem 19.1 所给情况，考虑一种简单的边界层转捩情况，转捩雷诺数为 $Re_t = 5\times10^5$，当 $Re_x \le Re_t$ 时，平板边界层近似为层流状态；当 $Re_x > Re_t$，平板边界层由层流状态转捩为湍流状态。

首先计算发生转捩点（位置），由 $Re_x = \frac{\rho_\infty V_\infty x}{\mu_\infty}$
令

$$Re_x = \frac{\rho_\infty V_\infty x}{\mu_\infty} = \frac{(1.2219)(63.0326)x}{1.7894\times10^{-5}} = 5\times10^5$$

则可解得：$x = x_t = 0.1162\ m$，此位置为层流边界层与湍流边界层分界处。

a. 考虑机翼近似平板从前缘 0 位置至 $x_t = 0.1162m$ 这一段为层流边界层，$Re_t = 5\times10^5$；
故层流边界层段总摩擦阻力系数为

$$C_{f,\,laminar} = \frac{1}{x_t}\int_0^{x_t} c_f\,dx = \frac{1}{x_t}\int_0^{x_t}\frac{0.664}{\sqrt{Re_x}}\,dx$$

$$= \frac{1}{x_t}(0.664)\sqrt{\frac{\mu_\infty}{\rho_\infty V_\infty}}\int_0^{x_t} x^{-\frac{1}{2}}dx = \frac{1.328}{x_t}\sqrt{\frac{\mu_\infty x_t}{\rho_\infty V_\infty}}$$

$$= \frac{1.328}{\sqrt{Re_t}} = \frac{1.328}{\sqrt{5\times10^5}} = 1.8781\times10^{-3}$$

则层流边界层段壁面所受总摩擦阻力为

$$D_{f,\,laminar} = 2\times\frac{1}{2}\rho_\infty V_\infty^2 b x_t C_{f,\,laminar} = (1.2219)(63.0326)^2(9.75)(0.1162)(1.8781\times10^{-3})$$

$$= 10.3299\ N$$

b. 考虑机翼近似平板从 $x_t = 0.1162m$ 至后缘 $x_c = 1.6m$ 这一段为湍流边界层，$Re_t = 5\times10^5$，$Re_c = 6.8867\times10^6$；

若近似有湍流边界层中摩擦阻力系数不随 Re 改变，

故湍流边界层段总摩擦阻力系数为

$$C_{f,\,turbulent} = \frac{0.074}{Re_c^{1/5}} = \frac{0.074}{(6.8867\times10^6)^{1/5}} = 3.1742\times10^{-3}$$

则湍流边界层壁面所受总摩擦阻力为

$$D_{f,\,turbulent} = 2\times\frac{1}{2}\rho_\infty V_\infty^2\, b(x_c - x_t)\, C_{f,\,turbulent}$$

$$= (1.2219)(63.0326)^2(9.75)(1.6-0.1162)(3.1742\times10^{-3}) = 222.9361\ N$$

∴ 综上 a. b. 可得

考虑简单边界层转捩情况下，机翼壁面所受总摩擦阻力为

$$D_{f,\,tot} = D_{f,\,laminar} + D_{f,\,turbulent} = 10.3299 + 222.9361 = 233.2660\ N$$

与完全层流或完全湍流流动相比更贴近生活工程实际。

19.4 Consider Mach 4 flow at standard sea level conditions over a flat plate of chord 5 in. Assuming all laminar flow and adiabatic wall conditions, calculate the skin friction drag on the plate per unit span.

解19.4 由于来流马赫数 $M_\infty = 4 \gg 0.3$，故流动为可压缩流动。

① 首先计算来流速度。

由理想气体状态方程 $p = \rho RT$，可得来流密度（标准海平面状态）为

$$\rho_\infty = \frac{p_\infty}{RT_\infty} = \frac{1.01\times10^5}{(287)(288)} = 1.2219\ kg/m^3$$

由声速公式可得自由来流声速为

$$a_\infty = \sqrt{\gamma RT_\infty} = \sqrt{(1.4)(287)(288)} = 340.1741\ m/s$$

则来流速度为

$$V_\infty = M_\infty a_\infty = 4(340.1741) = 1360.6964\ m/s$$

② 下面采用 The Meador-Smart Reference Temperature Method 求解平板单截层长摩擦阻力。

在标准海平面状况下，Prandtl Number 为 $P_r = 0.71 = P_r^*$

故恢复系数 $r = \dfrac{T_{aw}-T_e}{T_0-T_e} = \sqrt{P_r} = \sqrt{0.71} = 0.8426$

查阅 Appendix A，当 $M_e = M_\infty = 4$ 时，$\frac{T_0}{T_e} = 4.20$

则有

$$\frac{T_{aw}}{T_e} = 1 + r\left(\frac{T_0}{T_e} - 1\right) = 1 + 0.8426(4.20-1) = 3.6963 = \frac{T_w}{T_e}$$

$$\frac{T^*}{T_e} = 0.45 + 0.55\left(\frac{T_w}{T_e}\right) + 0.16r\left(\frac{\gamma-1}{2}\right)M_e^2$$

$$= 0.45 + (0.55)(3.6963) + (0.16)(0.8426)\left(\frac{1.4-1}{2}\right)(4)^2 = 2.9144$$

故 $T^* = 2.9144 T_e = (2.9144)(288) = 839.3435\ K$

根据参考温度 T^* 计算参考密度 ρ^* 和参考粘性系数 μ^* 如下。

$$\rho^* = \frac{p^*}{RT^*} = \frac{1.01\times10^5}{(287)(839.3435)} = 0.4193\ kg/m^3$$

由 Sutherland 公式可得：

$$\frac{\mu^*}{\mu_0} = \left(\frac{T^*}{T_0}\right)^{\frac{3}{2}} \frac{T_0+110}{T^*+110} = \left(\frac{839.3435}{288}\right)^{\frac{3}{2}} \frac{288+110}{839.3435+110} = 2.0858$$

又 $\mu_0 = \mu_\infty = 1.7894\times10^{-5}\ kg/(m.s)$，

则 $\mu^* = 2.0858\mu_0 = (2.0858)(1.7894\times10^{-5}) = 3.7324\times10^{-5}\ kg/(m.s)$

有 $Re^* = \frac{\rho^* V_\infty c}{\mu^*} = \frac{(0.4193)(1360.6964)(5\times0.0254)}{3.7324\times10^{-5}} = 1.9413\times10^6$

故考虑来流流动均为层流情况下，有

$$C_f^* = \frac{1.328}{\sqrt{Re_c^*}} = \frac{1.328}{\sqrt{1.9413\times10^6}} = 9.5312\times10^{-4}$$

∴ 因此，考虑单位展长平板壁面的两侧总摩擦阻力为

$$D_{f,tot} = 2\times\frac{1}{2}\rho^* V_\infty^2 c\, C_f^*$$

$$= (0.4193)(1360.6964)^2(5\times0.0254)(9.5312\times10^{-4})$$

$$= 93.9719\ N$$