APPENDIX

1976 US Standard Atmosphere Model

B

B.1 The atmospheric environment

The chemical composition of the sensible atmosphere is essentially constant and by mole fraction it is comprised of 78% nitrogen, 21% oxygen, and 1% other gases. (Argon 0.93%, CO_2 0.03%, and neon, helium, krypton, hydrogen, xenon, and ozone in increasingly smaller amounts.) For most thermo-chemical purposes the atmosphere is considered a binary mixture of 79% nitrogen and 21% oxygen. This fixed composition approximation makes the temperature distribution a reliable means for dividing up the various important regions in the atmosphere. The pressure and density in the regions of specified temperature behavior may be determined from the equation of state and the conditions of hydrostatic equilibrium.

B.2 Equation of state and hydrostatic equilibrium

If one considers the atmosphere to behave as a perfect gas, then

$$p = \rho RT = \rho \frac{R_u}{W_m} T$$

The molecular weight of the mixture of atmospheric gases is essentially constant up to 100 km and is given by $W_m = 28.96$, and the atmospheric gas constant $R = 0.287\,\text{kJ/kg-K}$ (or $1716\,\text{ft}^2/\text{s}^2$-R). The hydrostatic equation for the atmosphere is

$$dp = -\rho g dz$$

Since the gravitational acceleration depends on altitude a new altitude function may be defined. This is the *geopotential* altitude, h, and it is related to the geometric altitude by the relation

$$g_E dh = g dz$$

Here $g_E = 9.087\,\text{m/s}^2$ (or $32.15\,\text{ft/s}^2$) is the gravitational acceleration at the surface of the Earth, $z = 0$. The gravitational acceleration which varies with altitude according to Newton's law of gravitation may be written as

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$$g = \frac{g_E R_E^2}{(R_E + z)^2} = g_E \frac{1}{\left(1 + \frac{z}{R_E}\right)^2}$$

The radius of the Earth is taken to be $R_E = 6357$ km (3950 miles or 3430 nm) so that for altitudes typical of commercial aircraft (z < 15 km or 49,000 ft) the ratio $z/R_E \sim 0.0024$, and therefore the difference between h and z is relatively small. Integrating the relation between h and z yields the geopotential altitude

$$h = z \frac{1}{1 + \frac{z}{R_E}}$$

Thus the difference between the geometric and geopotential altitudes for commercial aircraft is less than 1%. Using the equation of state in the hydrostatic equilibrium equation yields

$$\frac{dp}{p} = -g \frac{dz}{RT} = -g_E \frac{dh}{RT}$$

With an appropriate relation for temperature in terms of geopotential altitude, $T = T(h)$, one may integrate the hydrostatic equation and find $p = p(h)$ and $\rho = \rho(h)$. The 1976 standard atmosphere defines atmospheric layers, each with $T = T_i + \lambda_i (h - h_i)$, where T_i is the temperature of the start of layer i, h_i is the altitude at the start of layer i, and λ_i is the lapse rate, i.e., dT/dh, in that layer. Integration of the hydrostatic equation for non-zero λ yields

$$p = p_i \left[\frac{T_i}{T_i + \lambda_i (h - h_i)} \right]^{\frac{g_0}{R \lambda_i}}$$

In isothermal layers where $\lambda = 0$, the temperature $T = T_i =$ constant, and the pressure is instead given by

$$p = p_i \exp\left[-\frac{g_E}{RT_i} (h - h_i) \right]$$

The temperature at the Earth's surface is taken as $T = 15\,°\mathrm{C} = 288.15$ K (518.7R) and $g_E/R = 34.17$ K/km (0.01874R/ft). For general reference the properties of this atmospheric model in the various layers are given in Table B.1.

The distribution of pressure, in kPa, in the various layers is then given by the following:

Layer 1 (0–11 km):

$$p = 101.3 \left(\frac{288.15}{288.15 - 6.5h} \right)^{\frac{34.17}{-6.5}}$$

Layer 2 (11–20 km):

Table B.1 Definition of the Layers in the 1976 Model Atmosphere[a]

Layer	Geopotential Altitude, *h* (km)	Geopotential Altitude, *h* (kft)	Lapse Rate, λ_i (K/km)	Thermal Type
1	0	0	−6.5	Neutral
2	11	36.1	0	Isothermal
3	20	65.6	+1.0	Inversion
4	32	105	+2.8	Inversion
5	47	154	0	Isothermal
6	51	167	−2.8	Neutral
7	71	233	−2.0	Neutral
8	84.85	278	+1.65	Inversion
9	100			

[a] *The eighth layer lapse rate is based on the 1962 Standard Atmosphere. The difference between the 1976 and 1962 Standard Atmospheres is small for altitudes h < 150 km.*

$$p = 22.62 \exp\left(\frac{-34.17\,[h - 11]}{216.65}\right)$$

Layer 3 (20–32 km):

$$p = 5.47 \left(\frac{216.65}{216.65 + [h - 20]}\right)^{\frac{34.17}{1}}$$

Layer 4 (32–47 km):

$$p = 0.8669 \left(\frac{228.65}{228 + 2.8\,[h - 32]}\right)^{\frac{34.17}{2.8}}$$

Layer 5 (47–51 km):

$$p = 0.1107 \exp\left(\frac{-34.17\,[h - 47]}{270.65}\right)$$

Layer 6 (51–71 km):

$$p = 0.06681 \left(\frac{270.65}{270.65 - 2.8\,[h - 51]}\right)^{\frac{34.17}{-2.8}}$$

Layer 7 (71–84.85 km):

$$p = 0.003946 \left(\frac{214.65}{214.65 - 2\,[h - 71]}\right)^{\frac{34.17}{-2}}$$

Layer 8 (84.85–100 km)

$$p = 0.0003724\left(\frac{186.95}{186.95 + 1.65\,[h - 84.85]}\right)^{\frac{34.17}{1.65}}$$

The density may be found from the equation of state. Note that in Layer 2 the temperature is constant at about 216 K or −57 °C (390R or −70 °F) in the altitude range of 10–20 km. This region is called the stratosphere and is the domain of high-speed manned flight, from jet airliners to military aircraft up to the Mach 3 SR-71 Blackbird. Because the speed of sound is proportional to the square root of the temperature, and the temperature through the stratosphere has little variation, the speed of sound is relatively constant. It is common to assume a constant value for preliminary design purposes and this can be used with minimal error. A table of useful pressure, density, temperature, sound speed, and kinematic viscosity data for this atmosphere model is given in Tables B.2 and B.3 for English units and in Tables B.4 and B.5 for

Table B.2 Properties of the 1976 US Standard Atmosphere for Altitudes between Sea Level and 50,000 ft in 2000 ft Increments

z (ft)	δ	θ	σ
0	1.0000	1.0000	1.0000
2000	0.9298	0.9862	0.9428
4000	0.8636	0.9725	0.8881
6000	0.8013	0.9587	0.8358
10,000	0.6877	0.9312	0.7384
12,000	0.6359	0.9175	0.6931
14,000	0.5874	0.9037	0.6500
16,000	0.5419	0.8900	0.6089
18,000	0.4993	0.8762	0.5698
20,000	0.4595	0.8625	0.5327
22,000	0.4222	0.8487	0.4975
24,000	0.3875	0.8350	0.4641
26,000	0.3551	0.8212	0.4324
28,000	0.3249	0.8075	0.4024
30,000	0.2969	0.7937	0.3740
32,000	0.2708	0.7800	0.3472
34,000	0.2467	0.7662	0.3219
36,000	0.2243	0.7525	0.2980
38,000	0.2005	0.7519	0.2666
40,000	0.1821	0.7519	0.2422
42,000	0.1654	0.7519	0.2200
44,000	0.1502	0.7519	0.1998
46,000	0.1365	0.7519	0.1815
48,000	0.1239	0.7519	0.1649
50,000	0.1126	0.7519	0.1497

Table B.3 Properties of the 1976 US Standard Atmosphere for Altitudes Between Sea Level and 50,000 ft in 2000 ft Increments (Concluded)

z (ft)	p (lb/ft^2)	T (R)	ρ(slug/ft^3)	a (ft/s)	a (kts)	ν (ft^2/s)
0	2116	518.7	2.377E-03	1116	660.9	1.573E-04
2000	1968	511.5	2.241E-03	1109	656.4	1.650E-04
4000	1828	504.4	2.111E-03	1101	651.8	1.733E-04
6000	1696	497.3	1.987E-03	1093	647.2	1.821E-04
10,000	1455	483.0	1.755E-03	1077	637.8	2.014E-04
12,000	1346	475.9	1.648E-03	1069	633.1	2.121E-04
14,000	1243	468.7	1.545E-03	1061	628.3	2.234E-04
16,000	1147	461.6	1.447E-03	1053	623.5	2.356E-04
18,000	1057	454.5	1.354E-03	1045	618.7	2.486E-04
20,000	972.3	447.3	1.266E-03	1037	613.8	2.626E-04
22,000	893.6	440.2	1.183E-03	1028	608.9	2.775E-04
24,000	820.0	433.1	1.103E-03	1020	604.0	2.936E-04
26,000	751.5	426.0	1.028E-03	1012	599.0	3.109E-04
28,000	687.7	418.8	9.565E-04	1003	593.9	3.294E-04
30,000	628.3	411.7	8.891E-04	994.5	588.8	3.495E-04
32,000	573.1	404.6	8.253E-04	985.9	583.7	3.710E-04
34,000	522.0	397.4	7.652E-04	977.1	578.6	3.944E-04
36,000	474.6	390.3	7.084E-04	968.3	573.3	4.196E-04
38,000	424.2	390.0	6.337E-04	967.9	573.1	4.687E-04
40,000	385.3	390.0	5.756E-04	967.9	573.1	5.160E-04
42,000	350.0	390.0	5.229E-04	967.9	573.1	5.680E-04
44,000	317.9	390.0	4.749E-04	967.9	573.1	6.254E-04
46,000	288.8	390.0	4.314E-04	967.9	573.1	6.885E-04
48,000	262.3	390.0	3.919E-04	967.9	573.1	7.580E-04
50,000	238.3	390.0	3.559E-04	967.9	573.1	8.344E-04

SI units. The dynamic viscosity, calculated using Sutherland's law for air, is given below for English and SI units, respectively:

$$\mu = 2.270 \times 10^{-7}\left[\frac{T^{3/2}}{T+198.6}\right]\frac{\text{lb}-\text{s}}{\text{ft}^2}$$

$$\mu = 1.461 \times 10^{-6}\left[\frac{T^{3/2}}{T+111}\right]\frac{\text{N}-\text{s}}{\text{m}^2}$$

An important parameter for commercial aircraft is the equivalent velocity $V_E = \sqrt{\sigma}V = \sqrt{\sigma}aM$. The variation of V_E/M with altitude is shown in Figure B.1, along with the variation of the local free stream static pressure with altitude. A

Table B.4 Properties of the 1976 US Standard Atmosphere for Altitudes Between Sea Level and 20 km in 1 km Increments

z (km)	δ	θ	σ
0	1.0000	1.0000	1.0000
1	0.8870	0.9774	0.9075
2	0.7846	0.9549	0.8216
3	0.6919	0.9323	0.7421
4	0.6083	0.9098	0.6687
5	0.5331	0.8872	0.6009
6	0.4656	0.8647	0.5385
7	0.4052	0.8421	0.4812
8	0.3513	0.8195	0.4287
9	0.3034	0.7970	0.3807
10	0.2609	0.7744	0.3369
11	0.2234	0.7519	0.2971
12	0.1908	0.7519	0.2538
13	0.1630	0.7519	0.2168
14	0.1392	0.7519	0.1851
15	0.1189	0.7519	0.1581
16	0.1015	0.7519	0.1351
17	0.08673	0.7519	0.1154
18	0.07408	0.7519	0.0985
20	0.05404	0.7519	0.0719

useful simple approximation for the atmospheric pressure ratio is $p/p_{sl} = \exp(-z/H)$ where H = 24,000 ft (7.32km). A similar approximation for the density ratio uses H = 29,000 ft (8.84 km). Three aircraft and their typical cruise speeds are shown at their typical flight altitudes. The equivalent velocity for each is indicated and we note that the dynamic pressure $q = \frac{1}{2}\rho_{sl}V_E^2$. In Figure B.1 it is seen that a conventional jet transport (B747) flies at an equivalent airspeed about 50% higher than does a conventional turboprop regional airliner (ATR) and therefore experiences more than twice the dynamic pressure. Similarly, a supersonic airliner like the Concorde flies at an equivalent airspeed about 50% higher than does a conventional jet transport and therefore it encounters almost five times the dynamic pressure than does the regional turboprop.

Another important parameter for commercial airliners is the Reynolds number

$$\mathrm{Re} = \frac{\rho V l}{\mu} = \frac{aMl}{\nu}$$

The ratio Re/Ml depends upon altitude alone and its variation is shown in Figure B.2. Three representative aircraft are again shown on that plot at their usual flight altitude and Mach number. The corresponding Reynolds number for each, based on fuselage length l, is given in Figure B.2. Note that the Reynolds numbers are all

Table B.5 Properties of the 1976 US Standard Atmosphere for Altitudes Between Sea Level and 20 km in 1 km Increments (Concluded)

z (km)	*p* (kPa)	*T* (K)	ρ (kg/m^3)	*a* (m/s)	*a* (kts)	ν (m^2/s)
0	101.3	288.2	1.225	340.3	660.9	1.461E-05
1	89.85	281.7	1.112	336.4	653.5	1.582E-05
2	79.48	275.2	1.006	332.5	645.9	1.716E-05
3	70.09	268.7	0.9091	328.6	638.2	1.864E-05
4	61.63	262.2	0.8191	324.6	630.5	2.029E-05
5	54.01	255.7	0.7361	320.5	622.6	2.212E-05
6	47.17	249.2	0.6597	316.4	614.6	2.418E-05
7	41.05	242.7	0.5895	312.3	606.6	2.649E-05
8	35.59	236.2	0.5252	308.0	598.4	2.908E-05
9	30.74	229.7	0.4663	303.8	590.1	3.200E-05
10	26.43	223.2	0.4127	299.4	581.7	3.531E-05
11	22.63	216.7	0.3639	295.0	573.2	3.907E-05
12	19.33	216.7	0.3109	295.0	573.2	4.573E-05
13	16.51	216.7	0.2655	295.0	573.2	5.355E-05
14	14.10	216.7	0.2268	295.0	573.2	6.269E-05
15	12.04	216.7	0.1937	295.0	573.2	7.340E-05
16	10.29	216.7	0.1654	295.0	573.2	8.594E-05
17	8.786	216.7	0.1413	295.0	573.2	1.006E-04
18	7.504	216.7	0.1207	295.0	573.2	1.178E-04
20	5.47	216.7	0.0880	295.0	573.2	1.615E-04

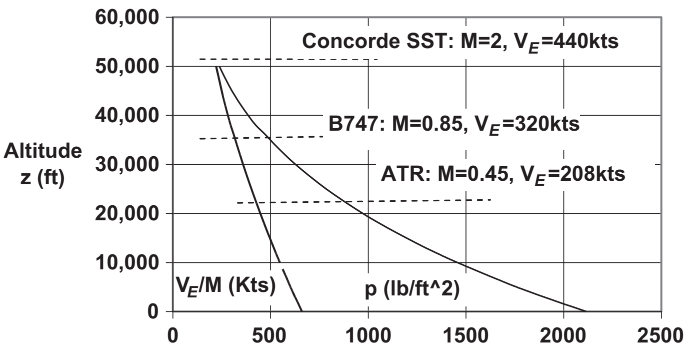

FIGURE B.1

The variation of pressure and V_E/M is shown as a function of altitude for the standard atmosphere. Three representative aircraft are depicted at their flight altitude and Mach number and the corresponding V_E is indicated.

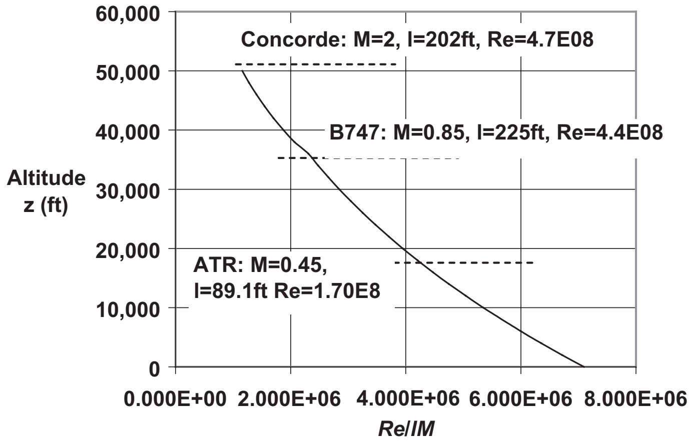

FIGURE B.2

The ratio *Re*/*lM* is shown as a function of altitude. Three representative aircraft are depicted at their flight altitude and Mach number. Their Reynolds numbers based on fuselage length *l* are indicated.

in excess of 100 million. Because the mean aerodynamic chord is on the order of 10–20% of the fuselage length, we see that the Reynolds numbers on the wings of these aircraft are on the order of 10 million. The flow over the wings and fuselage of these aircraft are then going to be turbulent over most of the surface giving rise to frictional drag much higher than would be the case if the flow were laminar. The search for means to maintain larger areas of laminar flow on aircraft components is driven by the potentially large drag reductions that might be realized.

Note that the model atmosphere does not account for day-to-day variations in the characteristics of the atmosphere. More detailed models for narrower geographical and seasonal data are usually given as mid-latitude winter, mid-latitude summer, subarctic winter, subarctic summer, and tropical and will show deviations from the US standard atmosphere. A description of the various models in use is provided by ANSI/AIAA (2004).

B.3 Nomenclature

a	speed of sound
h	geopotential altitude
g	acceleration of gravity
g_E	acceleration of gravity at the Earth's surface $z=0$
l	length
M	Mach number V/a

P	pressure
q	dynamic pressure
R	gas constant
R_E	radius of the Earth
Re	Reynolds number
R_u	universal gas constant
T	temperature
V	velocity
V_E	equivalent velocity $\sqrt{\sigma}V$
W_m	molecular weight
z	geometric altitude
δ	pressure ratio p/p_{sl}
λ	lapse rate dT/dz
μ	dynamic viscosity
ν	kinematic viscosity μ/ρ
p	density
σ	density ratio ρ/ρ_{sl}
θ	temperature ratio T/T_{sl}

B.3.1 Subscripts

i	denotes a specific layer in the atmosphere
sl	denotes sea lecel conditions

Reference

ANSI/AIAA, 2004. Guide to Reference and Standard Atmosphere Models, G-003B-2004. American Institute of Aeronautics and Astronautics, Reston, VA.