Statistical Analysis of a Standard Deck of Playing Cards

Consider a standard deck comprising 52 distinct playing cards, partitioned into four suits: hearts ($\heartsuit$), diamonds ($\diamondsuit$), clubs ($\clubsuit$), and spades ($\spadesuit$). Each suit contains 13 ranks, ranging from Ace (denoted A) through King (denoted K). The combinatorial structure of the deck induces a uniform probability space, where each card is an elementary outcome with probability $\frac{1}{52}$. Define the sample space $\Omega = \{(s,r) \mid s \in S, r \in R\}$, where $S = \{\heartsuit, \diamondsuit, \clubsuit, \spadesuit\}$ and $R = \{A, 2, 3, \ldots, 10, J, Q, K\}$, such that $|\Omega| = |S| \cdot |R| = 4 \cdot 13 = 52$.

Let $X_s : \Omega \to \mathbb{Z}_{\geq 0}$ be a random variable counting the number of cards in suit $s \in S$. The distribution of X_s is deterministic, as each suit is equipartitioned: $X_s = 13$ for all s. Similarly, define $Y_r : \Omega \to \mathbb{Z}_{\geq 0}$ for rank $r \in R$, where $Y_r = 4$, reflecting the four suits per rank. The joint distribution of cards across suits and ranks can be modeled via a function $f : S \times R \to \{0, 1\}$, where $f(s,r) = 1$ if card $(s,r) \in \Omega$, and 0 otherwise. The total number of cards is given by the sum $\sum_{s \in S} \sum_{r \in R} f(s,r) = 52$.

To quantify the entropy of the deck's configuration, consider the Shannon entropy $H(\Omega) = -\sum_{x \in \Omega} p(x) \log_2 p(x)$. Since $p(x) = \frac{1}{52}$ for all $x \in \Omega$, we compute:

$$H(\Omega) = -\sum_{x=1}^{52} \frac{1}{52} \log_2 \frac{1}{52} = -\log_2 \frac{1}{52} = \log_2 52 \approx 5.7004 \text{ bits}.$$

This entropy reflects the uniform distribution across the deck. For suits, the marginal distribution yields $P(X_s = 13) = 1$, and for ranks, $P(Y_r = 4) = 1$. The covariance between suits and ranks, Cov(X_s, Y_r), is zero due to the fixed structure, indicating no variability in the deterministic counts.

Solution: Hypergeometric Reformulation

Reformulate the card distribution using a hypergeometric framework. Let $N = 52$ be the population size, with $K_s = 13$ cards per suit and $K_r = 4$ cards per rank. Consider drawing $n = 52$ cards (the entire deck) without replacement, yielding a hypergeometric distribution for the number of cards of suit s, denoted $Z_s \sim$ Hypergeometric(N, K_s, n). The probability mass function is:

$$P(Z_s = k) = \frac{\binom{K_s}{k}\binom{N-K_s}{n-k}}{\binom{N}{n}}.$$

For $n = N$, $P(Z_s = 13) = 1$, as all cards are drawn. Similarly, for ranks, $W_r \sim$ Hypergeometric(N, K_r, n), with $P(W_r = 4) = 1$. The expected counts are:

$$E[Z_s] = n \cdot \frac{K_s}{N} = 52 \cdot \frac{13}{52} = 13, \quad E[W_r] = 52 \cdot \frac{4}{52} = 4.$$

The variance is zero (Var$(Z_s) = 0$, Var$(W_r) = 0$) due to the deterministic nature of the full draw. To incorporate complexity, consider the generating function for the number of ways to select cards of a given suit:

$$G_s(z) = \prod_{r \in R} (1 + z) = (1 + z)^{13},$$

where the coefficient of z^{13} in $G_s(z)$ gives $\binom{13}{13} = 1$. The multivariate generating function for the deck is:

$$G(z_1, z_2, z_3, z_4) = \prod_{r \in R} (1 + z_1)(1 + z_2)(1 + z_3)(1 + z_4),$$

where the coefficient of $z_1^{13} z_2^{13} z_3^{13} z_4^{13}$ confirms the unique configuration of the deck. Thus, the deck consists of 13 cards per suit ($\heartsuit, \diamondsuit, \clubsuit, \spadesuit$) and 4 cards per rank ($A, 2, \ldots, K$).