Replication Report of Hollstein(2018)

2024-12-01

1 Introduction

The paper "How Aggregate Volatility-of-Volatility Affects Stock Returns" [1] by Fabian Hollstein and Marcel Prokopczuk investigates the role of aggregate volatility-of-volatility (the volatility of market volatility) in explaining stock returns. Using the VVIX, an index measuring the implied volatility of the VIX (volatility index), the authors demonstrate that aggregate volatility-of-volatility carries a negative risk premium. Specifically, stocks with higher sensitivities to innovations in aggregate volatility-of-volatility exhibit lower average returns, suggesting investors demand compensation for bearing this risk.

In this replication project, we replicated the main results of the paper with similar significance levels (Tables 4 and 5), but did not fully replicate the original regression coefficients. While the deviations for most coefficients were within 10%, some exhibited larger discrepancies.

2 Methods

2.1 Data Collection

Following the original article, we collect data for replication from WRDS.

- Information about stocks is obtained from the CRSP, following Amihud (2002) [2] , we exclude "penny stocks" with prices below 1 dollar in the end of last month. Additionally, we require a market capitalization of at least 225 million dollars.
- Data on the Fama and French (1993) [3] and other factors as well as the risk-free (Treasury Bill) rate are from Kenneth French's data library and Macro-Finance Database.
- Data on the VIX and the VVIX are obtained from the CBOE.

2.2 Portfolio sorts

We sort the stocks in ascending order with respect to their sensitivities to innovations in volatility-of-volatility ($\beta^{Q}_{j;t}$) over the following year. We form quintile portfolios, so that quintile 1 contains the stocks with the lowest exposure to aggregate volatility-of-volatility, whereas quintile 5 contains those stocks with the highest volatility-of-volatility factor loadings. The hedge portfolio (5 minus 1) buys the quintile of stocks with the highest exposure and simultaneously sells the stocks in the quintile with the lowest exposure to aggregate volatility of-volatility.

2.3 Tools

This replication project was primarily finished using RStudio, with Excel and Python used as supplementary tools for data processing.

3 Replication Results

In this section, we will present our replication results of the two tables from the original paper.

3.1 Table4 - Portfolios sorted by exposures to dVVIX

Rank	1	2	3	4	5	5 minus 1
Equally weighted						
Mean return	0.1004	0.1062**	0.0987*	0.0826	0.0514	-0.049**
	(0.156)	(0.049)	(0.069)	(0.141)	(0.460)	(0.027)
	0.0923	0.0983	0.0926	0.0873*	0.0493	-0.0430***
	(0.442)	(0.269)	(0.279)	(0.092)	(0.571)	(0.000)
CAPM alpha	-0.0106	0.0201	0.0123	-0.0064	-0.0593**	-0.0487**
	(0.699)	(0.113)	(0.292)	(0.422)	(0.014)	(0.021)
	-0.0122	0.0213***	0.0173***	0.0132**	-0.0587*	-0.0465*
	(0.567)	(0.000)	(0.000)	(0.016)	(0.061)	(0.076)
4-factor alpha	-0.0072	0.0189***	0.0147***	-0.0052*	-0.0563***	-0.0491***
	(0.504)	(0.000)	(0.000)	(0.629)	(0.000)	(0.000)
	0.0110	0.0261***	0.0218***	0.0188***	-0.0267***	-0.0377***
	(0.158)	(0.000)	(0.000)	(0.000)	(0.001)	(0.000)
5-factor alpha	-0.0023	0.0194***	0.0137***	-0.0049	-0.0485***	-0.0462**
	(0.840)	(0.000)	(0.000)	(0.120)	(0.000)	(0.015)
	0.0093	0.0247***	0.0246***	0.0207***	-0.0231***	-0.0324***
	(0.231)	(0.000)	(0.000)	(0.000)	(0.004)	(0.000)
HXZ alpha	0.0133	0.0264***	0.0153***	-0.0074	-0.0532***	-0.0635**
	(0.457)	(0.000)	(0.004)	(0.305)	(0.000)	(0.015)
	0.0107***	0.0224***	0.0190***	0.0188***	0.0008***	-0.0099***
	(0.005)	(0.000)	(0.000)	(0.000)	(0.006)	(0.000)
Value-weighted						
Mean return	0.0967	0.0951**	0.0914**	0.0652	0.0433	-0.0533*
	(0.111)	(0.047)	(0.044)	(0.198)	(0.518)	(0.089)
	0.0897	0.0903	0.0863	0.0698*	0.0401	-0.0496***
	(0.420)	(0.248)	(0.258)	(0.079)	(0.552)	(0.000)
CAPM alpha	-0.0008	0.0193***	0.0185***	-0.0014	-0.0629***	-0.0626***
	(0.985)	(0.000)	(0.000)	(0.121)	(0.004)	(0.000)
	0.0519*	0.0169**	0.0310***	0.0196**	-0.0279	-0.0798**
	(0.082)	(0.185)	(0.003)	(0.029)	(0.267)	(0.027)
4-factor alpha	-0.0039	0.0161***	0.0195***	-0.0091	-0.0507***	-0.0468**
	(0.755)	(0.000)	(0.000)	(0.196)	(0.004)	(0.075)
	0.0422	0.0092**	0.0298***	0.0233***	-0.0171	-0.0594
	(0.105)	(0.402)	(0.002)	(0.003)	(0.468)	(0.120)
5-factor alpha	0.0063	0.0194***	0.0144***	-0.0126**	-0.0508***	-0.0572**
	(0.553)	(0.000)	(0.000)	(0.058)	(0.015)	(0.057)
	0.0350	0.0082***	0.0270***	0.0262***	-0.0140***	-0.0490***
	(0.178)	(0.045)	(0.005)	(0.000)	(0.045)	(0.019)
HXZ alpha	0.0391	0.0256***	0.0201***	-0.0148	-0.0657***	-0.0855**
	(0.199)	(0.000)	(0.000)	(0.264)	(0.009)	(0.029)
	0.0391	0.0256***	0.0201*	0.0252***	0.0187***	-0.0204***
	(0.156)	(0.008)	(0.059)	(0.005)	(0.003)	(0.005)
Factor loadings						
β^M	1.0708***	0.9253***	0.9729***	1.0989***	1.3848***	0.3140***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
	1.1485	1.0232**	1.0379*	1.0422	1.4304	0.2819
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
β^V	0.1441***	0.0395***	-0.0041	-0.0401**	-0.0907***	-0.2349***
	(0.000)	(0.000)	(0.150)	(0.011)	(0.000)	(0.000)
	0.2581***	0.1294***	0.0593***	0.0091*	-0.0870	-0.3449***

	(0.000)	(0.000)	(0.034)	(0.073)	(0.108)	(0.000)
β^Q	-0.0599**	-0.0177***	0.0062*	0.0313***	0.0777***	0.1376***
	(0.000)	(0.000)	(0.057)	(0.000)	(0.000)	(0.000)
	-0.0625***	-0.0249***	-0.0040*	0.0118**	0.0553***	0.1178***
	(0.000)	(0.000)	(0.059)	(0.038)	(0.000)	(0.000)
Return characteristics						
SD	0.0601	0.0474	0.0450	0.0504	0.0668	0.0311
	0.0380	0.0276	0.0266	0.0152	0.0278	0.0025
Skewness	-0.1632	-0.4981	-0.8644	-0.8132	-0.0727	0.0175
	-0.2391	-0.2849	-0.3444	-0.3642	-0.2886	-0.3331
Kurtosis	4.0501	3.6484	3.9630	3.6487	3.8287	2.3774
	3.6023	3.6535	3.8378	4.1719	4.2190	2.9673
Stock characteristics						
Mkt. share	0.2042	0.2467	0.2209	0.2304	0.1248	-0.0794
	0.2298	0.2650	0.2435	0.1915	0.1405	-0.0893
log(Size)	16.863	17.443	17.230	17.074	16.314	-0.5490
	16.801	17.145	17.456	16.852	15.901	-0.8994
Book-to-market	0.5416	0.4850	0.4805	0.5196	0.6716	0.1299
	0.5812	0.5005	0.5734	0.4718	0.6399	0.0587
Bid-ask spread	0.0468	0.0486	0.0480	0.0430	0.0514	0.0046
	0.0507	0.0571	0.0443	0.0399	0.0612	0.0105
Age	33.491	41.099	38.442	35.074	28.564	-4.9267
	36.8497	39.0489	34.4521	32.8574	29.7532	-7.0965
Leverage	0.5567	0.5549	0.5622	0.5861	0.6015	0.0448
	0.6095	0.5234	0.5818	0.6193	0.6732	0.0637

3.2 Table5 - Portfolios sorted by exposures to dVVIX

A. Aggregate volatility, equally weighted						
dVVIX\dVIX	**1**	**2**	**3**	**4**	**5**	**5 minus 1**
1	0.0741*	0.0716***	0.0578***	0.0277**	-0.0497***	-0.1238***
	(0.053)	(0.000)	(0.000)	(0.036)	(0.000)	(0.000)
	0.0691	0.0652**	0.0485***	0.0235*	-0.0423***	-0.1118***
	(0.163)	(0.012)	(0.000)	(0.088)	(0.000)	(0.000)
2	0.0777***	0.0684***	0.0591***	0.0068	-0.0566***	-0.1343***
	(0.000)	(0.000)	(0.000)	(0.268)	(0.000)	(0.000)
	0.0735***	0.0569***	0.0516***	0.0082**	-0.0484***	-0.1223***
	(0.000)	(0.000)	(0.000)	(0.040)	(0.000)	(0.000)
3	0.0653***	0.0443***	0.0283***	0.0041	-0.0699***	-0.1352***
	(0.000)	(0.000)	(0.000)	(0.618)	(0.000)	(0.000)
	0.0598***	0.0384***	0.0225***	0.0049	-0.0597***	-0.1186***
	(0.000)	(0.000)	(0.000)	(0.581)	(0.000)	(0.000)
4	0.0454***	0.0327***	0.0111	-0.0307**	-0.0920***	-0.1374***
	(0.000)	(0.000)	(0.165)	(0.016)	(0.000)	(0.000)
	0.0410***	0.0297***	0.0093	-0.0275**	-0.0837***	-0.1247***
	(0.000)	(0.000)	(0.148)	(0.023)	(0.000)	(0.000)
5	-0.0075	-0.0339***	-0.0523***	-0.0810***	-0.1277***	-0.1202***
	(0.510)	(0.001)	(0.000)	(0.001)	(0.000)	(0.004)
	-0.0062**	-0.0289***	-0.0458***	-0.0685***	-0.1026***	-0.0964***
	(0.023)	(0.001)	(0.000)	(0.001)	(0.000)	(0.005)
5 minus 1	-0.0817*	-0.1055***	-0.1101***	-0.1086***	-0.0781*	
	(0.058)	(0.000)	(0.000)	(0.001)	(0.060)	
	-0.0765***	-0.0948***	-0.1023***	-0.0986	-0.0713**	
	(0.003)	(0.000)	(0.000)	(0.156)	(0.016)	
B. Aggregate volatility, value-weighted						
dVVIX\dVIX	**1**	**2**	**3**	**4**	**5**	**5 minus 1**
1	0.1007***	0.0664***	0.0394***	0.0135	-0.0803***	-0.1810***

	(0.001)	(0.000)	(0.006)	(0.335)	(0.000)	(0.000)
	0.0912***	0.0583***	0.0342***	0.0109	-0.0722***	-0.1624***
	(0.001)	(0.000)	(0.007)	(0.312)	(0.000)	(0.000)
2	0.0794***	0.0574***	0.0278***	-0.0250***	-0.0547***	-0.1341***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
	0.0716***	0.0505***	0.0233***	-0.0201**	-0.0453***	-0.1187***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
3	0.0594***	0.0315***	0.0241***	-0.0118	-0.0978***	-0.1573***
	(0.000)	(0.000)	(0.000)	(0.192)	(0.000)	(0.000)
	0.0628***	0.0402***	0.0205***	-0.0095	-0.1024***	-0.1652***
	(0.000)	(0.000)	(0.000)	(0.215)	(0.000)	(0.009)
4	0.0360***	0.0032	-0.0017	-0.0507**	-0.1186***	-0.1546***
	(0.007)	(0.737)	(0.912)	(0.016)	(0.000)	(0.000)
	0.0421***	0.0065**	-0.0030	-0.0523*	-0.1238***	-0.1659***
	(0.005)	(0.078)	(0.480)	(0.064)	(0.000)	(0.000)
5	-0.0217	-0.0305	-0.0769***	-0.0915***	-0.1444***	-0.1227***
	(0.128)	(0.220)	(0.000)	(0.005)	(0.000)	(0.002)
	-0.0185**	-0.0347**	-0.0703	-0.0948***	-0.1561***	-0.1376***
	(0.018)	(0.021)	(0.323)	(0.003)	(0.000)	(0.001)
5 minus 1	-0.1224**	-0.0969**	-0.1163***	-0.1150***	-0.0641	
	(0.003)	(0.018)	(0.000)	(0.012)	(0.145)	
	-0.1290***	-0.1068*	-0.1149***	-0.1151***	-0.0718***	
	(0.003)	(0.075)	(0.001)	(0.000)	(0.002)	

C. Market variance risk premium, equally weighted

dVVIX\dVRP	**1**	**2**	**3**	**4**	**5**	**5 minus 1**
1	-0.0270	0.0312***	0.0505***	0.0703***	0.0993***	0.1262***
	(0.128)	(0.009)	(0.000)	(0.000)	(0.000)	(0.000)
	-0.0295**	0.0357**	0.0486***	0.0654***	0.0952***	0.1240***
	(0.030)	(0.017)	(0.000)	(0.000)	(0.000)	(0.000)
2	-0.0361***	0.0181***	0.0420***	0.0665***	0.0573***	0.0934***
	(0.000)	(0.003)	(0.000)	(0.000)	(0.000)	(0.000)
	-0.0451***	0.0206***	0.0375***	0.0605***	0.0503***	0.0954***
	(0.000)	(0.005)	(0.000)	(0.000)	(0.001)	(0.000)
3	-0.0301***	0.0024	0.0314***	0.0356***	0.0530***	0.0831***
	(0.003)	(0.739)	(0.004)	(0.003)	(0.000)	(0.000)
	-0.0328***	0.0054*	0.0279	0.0407***	0.0625***	0.0803***
	(0.003)	(0.054)	(0.102)	(0.000)	(0.000)	(0.000)
4	-0.0606***	-0.0313***	0.0040	0.0201***	0.0182***	0.0886***
	(0.000)	(0.004)	(0.871)	(0.001)	(0.002)	(0.000)
	-0.0652***	-0.0268***	0.0083*	0.0258	0.0731***	0.1383***
	(0.000)	(0.007)	(0.055)	(0.202)	(0.000)	(0.000)
5	-0.1294***	-0.0721***	-0.0568***	-0.0473***	-0.0410***	0.0884***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.004)	(0.004)
	-0.1203***	-0.0672***	-0.0501***	-0.0421***	-0.0356***	0.0847***
	(0.000)	(0.000)	(0.004)	(0.002)	(0.000)	(0.003)
5 minus 1	-0.1025***	-0.1033***	-0.1073***	-0.1176***	-0.1403***	
	(0.007)	(0.000)	(0.000)	(0.000)	(0.000)	
	-0.1084***	-0.1063***	-0.0957***	-0.1142***	-0.1310***	
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	

D. Market variance risk premium, value-weighted

dVVIX\dVRP	**1**	**2**	**3**	**4**	**5**	**5 minus 1**
1	-0.0770***	0.0047	0.0455***	0.0641***	0.1059***	0.1829***
	(0.000)	(0.766)	(0.000)	(0.000)	(0.000)	(0.000)
	-0.0834***	0.0052	0.0471***	0.0698***	0.1125***	0.1956***
	(0.000)	(0.742)	(0.000)	(0.000)	(0.000)	(0.000)
2	-0.0603***	-0.0133	0.0146**	0.0440***	0.0822***	0.1424***
	(0.000)	(0.113)	(0.016)	(0.000)	(0.000)	(0.000)
	-0.0654***	-0.0158*	0.0167***	0.0486***	0.0894***	0.1548***

	(0.000)	(0.069)	(0.002)	(0.000)	(0.000)	(0.000)
3	-0.0785***	-0.0050	0.0237**	0.0220***	0.0646***	0.1245***
	(0.000)	(0.673)	(0.026)	(0.000)	(0.000)	(0.000)
	-0.0813***	-0.0062*	0.0268	0.0265***	0.0502***	0.1315***
	(0.000)	(0.060)	(0.312)	(0.001)	(0.000)	(0.000)
4	-0.0890***	-0.0446**	-0.0089	0.0071	0.0293***	0.1188***
	(0.000)	(0.031)	(0.680)	(0.587)	(0.008)	(0.000)
	-0.0945***	-0.0486***	-0.0102	-0.0078*	0.0328***	0.1273***
	(0.000)	(0.007)	(0.112)	(0.084)	(0.004)	(0.000)
5	-0.1386***	-0.0977***	-0.0731***	-0.0604***	-0.0799**	0.0587*
	(0.000)	(0.000)	(0.000)	(0.010)	(0.009)	(0.084)
	-0.1456***	-0.0864***	-0.0672**	-0.0652**	-0.0813***	0.0643***
	(0.000)	(0.000)	(0.015)	(0.011)	(0.000)	(0.002)
5 minus 1	-0.0616*	-0.1025***	-0.1186***	-0.1245***	-0.1859***	
	(0.081)	(0.006)	(0.002)	(0.000)	(0.000)	
	-0.0686***	-0.916***	-0.1143***	-0.1291***	-0.1873***	
	(0.006)	(0.004)	(0.000)	(0.000)	(0.000)	

In summary, we replicated the main results of the paper with similar significance levels (Tables 4 and 5), but did not fully replicate the original regression coefficients. While the deviations for most coefficients were within 10%, some exhibited larger discrepancies.

Among the results, Value-weighted regression for the HXZ factor yielded 3 identical coefficients, indicating that our replication process closely aligned with the original paper. However, further investigation is needed to determine the specific causes for the observed discrepancies.

Value-weighted						
HXZ alpha	0.0391	0.0256***	0.0201***	-0.0148	-0.0657***	-0.0855**
	(0.199)	(0.000)	(0.000)	(0.264)	(0.009)	(0.029)
	0.0391	0.0256***	0.0201*	0.0252***	0.0187***	-0.0204***
	(0.156)	(0.008)	(0.059)	(0.005)	(0.003)	(0.005)

4 Discussion

4.1 Likely causes for the incomplete replication of the results

4.1.1 VIX time series

We downloaded the VIX and VVIX time series directly from the CBOE website; however, CBOE provides 16 different VIX datasets, and we were uncertain which specific series was used in the original paper. In fact, when examining the descriptive statistics of the dVIX series, we found some differences between our differenced series and that of the original paper (while the dVVIX series was largely consistent).

dVIX	Mean	Median
Original	0.00001	-0.00095
Ours	0.000363	-0.090000

4.1.2 Exclusion of penny stocks

Following the methodology of the original paper, we used CRSP monthly data to exclude "penny stocks." However, the number of remaining stocks after exclusion appears to differ slightly. In the original paper, the month with the largest number of remaining stocks retained 2,794 stocks, while in our results, the maximum number of retained stocks was 2,868. This discrepancy suggests that there may be differences in the data or methods used during the stock exclusion process.

4.1.3 Usage of HXZ factors from different sources

In the original paper, some values in the HXZ factor series were calculated by the authors themselves, while others were directly sourced from the original authors of the HXZ model. In contrast, we used data downloaded from the Macrofin database, which may have introduced some differences.

4.2 Issues encountered during the replication process

4.2.1 Standardizing different formats

During the replication process, we used data from various sources, each with different formats and data types for labeling. Therefore, it was necessary to standardize the formats and data types before use. Otherwise, various errors could occur, such as failure to retrieve data values during matching process.

4.2.2 Align data with different frequencies

We needed to use monthly data to filter stocks and select portfolios, but the regressions relied on daily portfolio data. Therefore, additional steps were required to integrate data of different frequencies for subsequent operations. Specifically, in practice, we added an extra "month" column to each daily data frame to facilitate matching with the monthly data.

4.2.3 Calculation of VRP

Since the CRSP database does not provide VRP time series, we needed to calculate the VRP time series using daily VIX series and the stock return series, and then compute its differences to obtain the dVRP time series. Specifically, we used the previous 30 trading days as the rolling window to calculate the realized variance for each trading day, and then subtracted the VIX index (Implied Variance) to derive the VRP time series.

$$VRP_{i,t} = RV_{i,t} - IV_t = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(r_{i,t-i} - \bar{r})^2} - VIX_t$$

References

[1] F. Hollstein and M. Prokopczuk, "How aggregate volatility-of-volatility affects stock returns," *The Review of Asset Pricing Studies*, vol. 8, no. 2, pp. 253–292, 2018.

[2] Y. Amihud, "Illiquidity and stock returns: cross-section and time-series effects," *Journal of financial markets*, vol. 5, no. 1, pp. 31–56, 2002.

[3] E. F. Fama and K. R. French, "Common risk factors in the returns on stocks and bonds," *Journal of financial economics*, vol. 33, no. 1, pp. 3–56, 1993.