Investigate the exponential distribution

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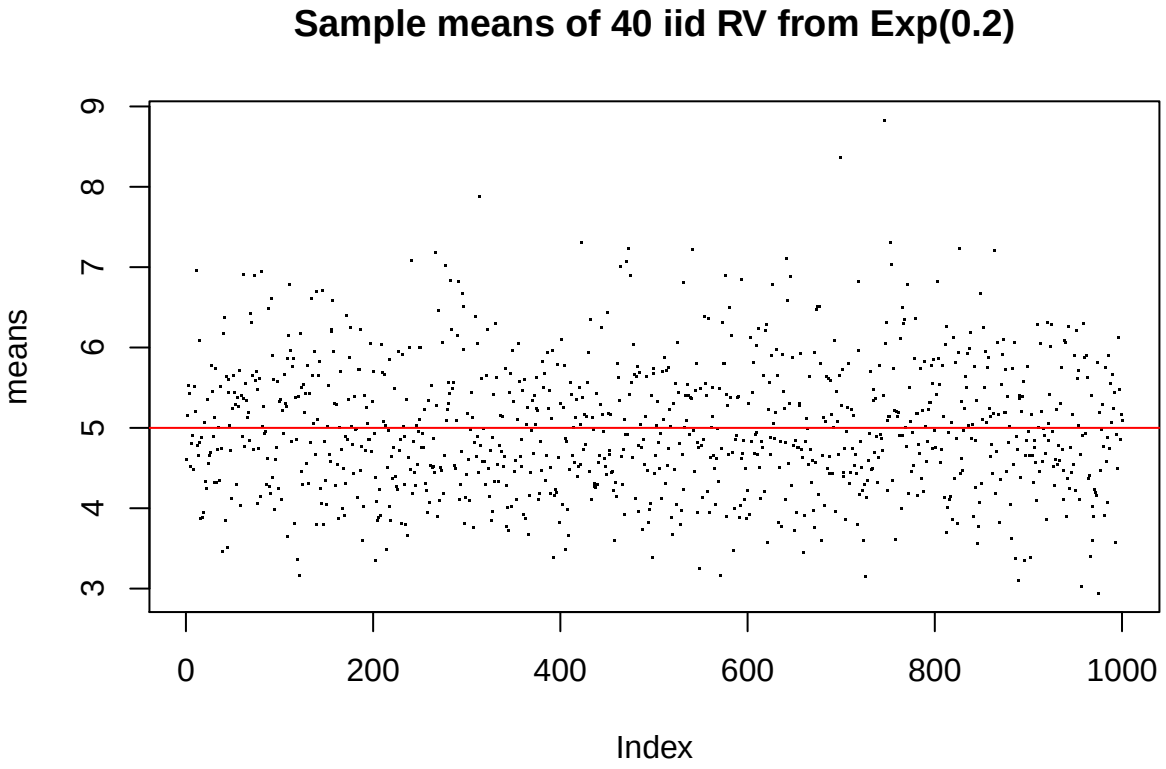
Overview

We simulated 1000 sample means $\bar{X}_i$ of 40 iid random variables from exponential distribution. Their mean, variance, and distribution are compared to the normal distribution to verify Central limit theorem.

Simulation

We sample 1000x40 random variables(RV) from exponential distribution with parameter $\lambda = 0.2$. They are divided into 1000 sets of 40 numbers. The 1000 means of the sets are the sample we are going to research into.

```
options(digits=3)
lambda = 0.2
M = 40
N = 1000
samples = matrix(rexp(N*M, lambda), N, M)
means = rowMeans(samples)
plot(means, pch='.', main='Sample means of 40 iid RV from Exp(0.2)')
abline(h=5, col='red')
```



Sample Parameters versus Theoretical Parameters

Central Limit Theorem stats that: as $n \to \infty$

$$\bar{X} \overset{d}{\sim} N(\mu, \sigma^2/n)$$

In our case, $\mu = \sigma = 1/\lambda$.

```
theory_m = 1/lambda
theory_var = (1/lambda)^2 / M

sample_m = mean(means)
sample_var = var(means)
```

Sample mean is 5.005, compare to theoretical mean of 5. Sample vairance is 0.672, compare to theoretical variance of 0.625. They are very close.

Distribution

The distribution of sample means are summarized in the following figure. The dashed purple line is a normal distribution given by Central Limit Therom, which is a close fit for the sample distribution line.