Side Channel Analysis Using a Model Counting Constraint Solver and Symbolic Execution

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Joint work with:
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Verification Laboratory (VLab)
University of California, Santa Barbara (UCSB)

- VLab: Research on automated verification, program analysis, formal methods, software engineering, computer security
- Recent research: String analysis, Model counting constraint solvers, Side channel analysis, Data model verification, Web application verification and security
- Always looking for talented and hard working graduate students!

Publications most closely related to this talk

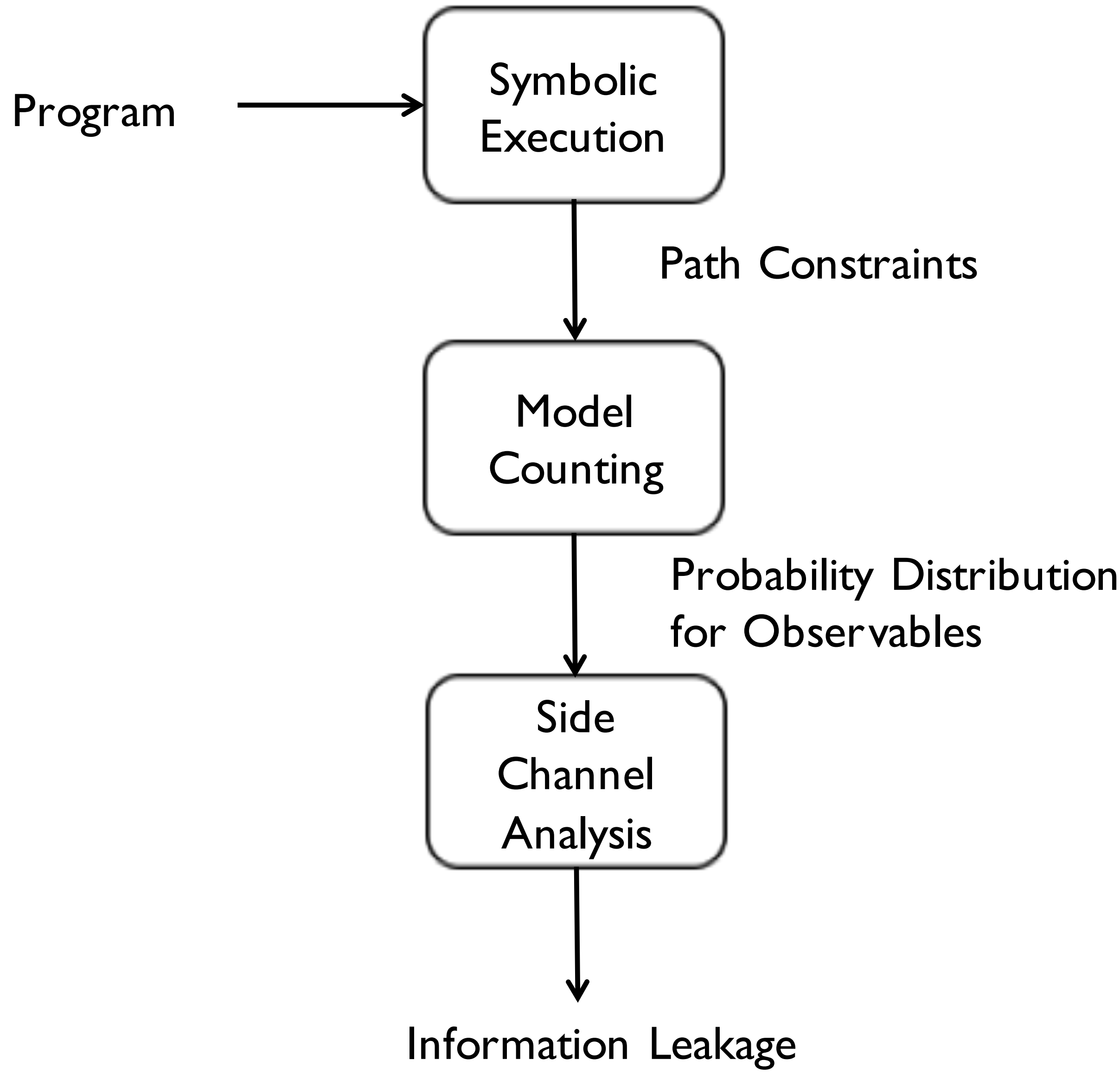
- "String Analysis for Side Channels with Segmented Oracles." Lucas Bang , Abdulbaki Aydin , Quoc-Sang Phan, Corina S. Pasareanu , Tevfik Bultan, FSE'16.
- "Automata-based Model Counting for String Constraints." Abdulbaki Aydin, Lucas Bang, Tevfik Bultan, CAV'15.

Quantitative Information Flow Problem

Given a program and a secret that the program accesses:

Figure out how much information is leaked about the secret by observing the behavior of the program

Overview



Program
Symbolic Execution
Path Constraints
Model Counting
Probability Distribution for Observables
Side Channel Analysis
Information Leakage

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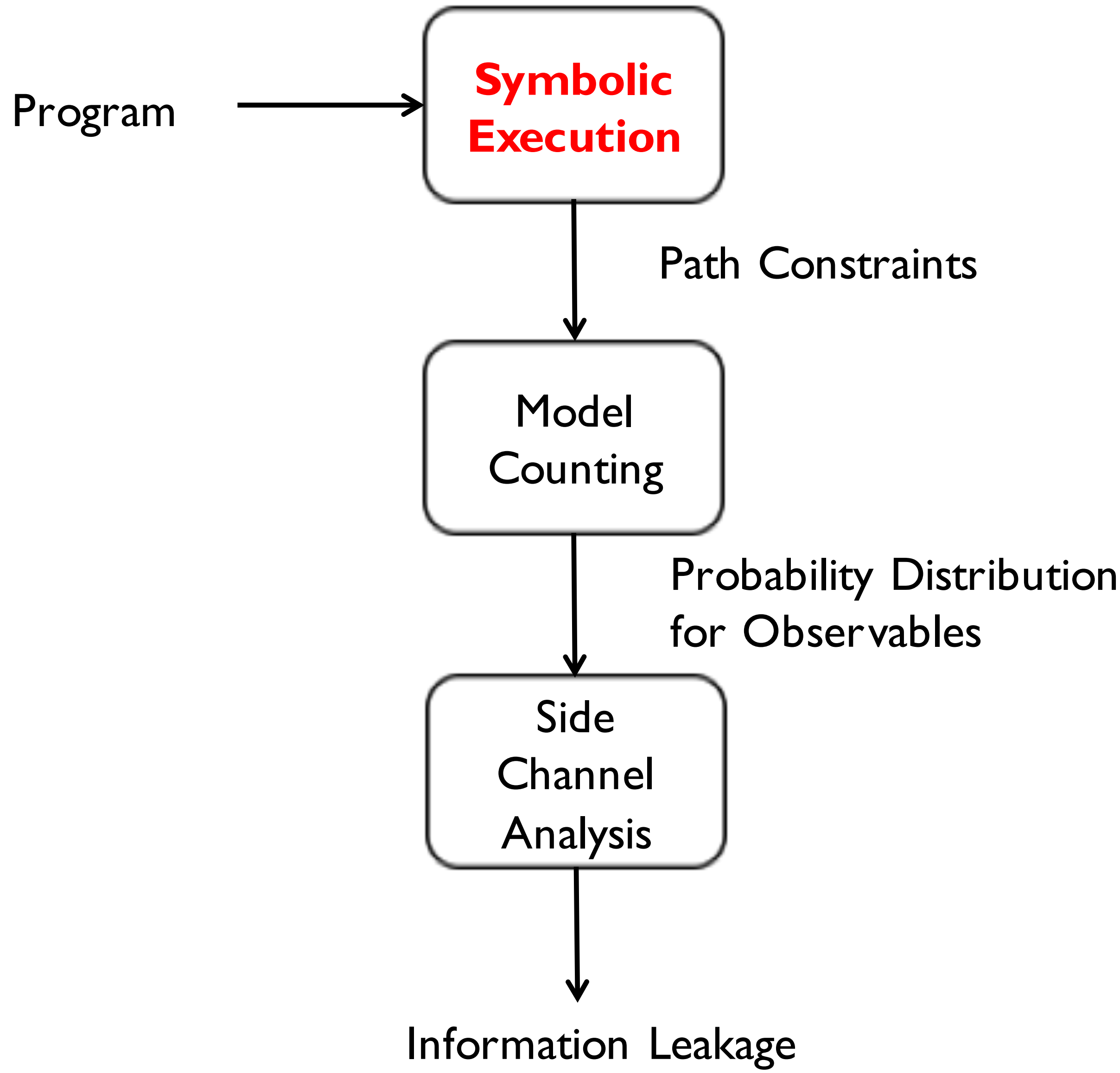
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A 4-digit PIN Checker

```
bool checkPIN(guess[])
for(i = 0; i < 4; i++)
 if(guess[i] != PIN[i])
  return false
return true
```

P: PIN, *G*: guess

Symbolic Execution of PIN Checker

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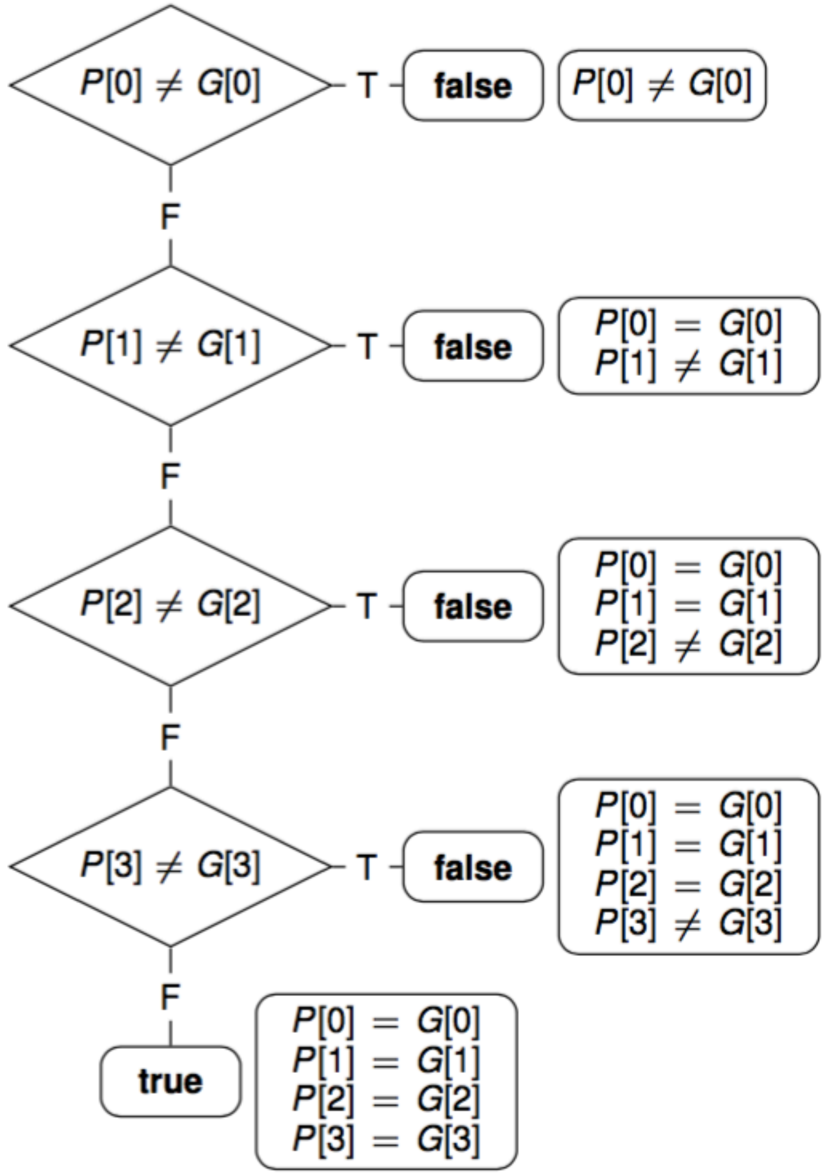

Probabilistic Symbolic Execution

Can we determine the probability of executing a program path?

- Let PC_i denote the path constraint for a program path
- Let $|PC_i|$ denote the number of possible solutions for PC_i
- Let $|D|$ denote the size of the input domain
- Assume uniform distribution over the input domain
- Then the probability of executing that program path is:

$$p(PC_i) = |PC_i| \,/\, |D|$$

Probabilistic Symbolic Execution of PIN Checker

- Assume binary 4 digit PIN, P and G each have 4 bits
- $|D| = 2^8 = 256$

i	0	1	2	3	4
PC_i	$P[0] \neq G[0]$	$P[0] = G[0]$ $P[1] \neq G[1]$	$P[0] = G[0]$ $P[1] = G[1]$ $P[2] \neq G[2]$	$P[0] = G[0]$ $P[1] = G[1]$ $P[2] = G[2]$ $P[3] \neq G[3]$	$P[0] = G[0]$ $P[1] = G[1]$ $P[2] = G[2]$ $P[3] = G[3]$
$\lvert PC_i \rvert$					
p_i					

- $p(PC_i) = |PC_i| / |D|$

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$\lvert PC_i \rvert$	128				
p_i	1/2				

- $p(PC_i) = |PC_i| / |D|$

Probabilistic Symbolic Execution of PIN Checker

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$\|PC_i\|$	128	64			
p_i	1/2	1/4			

- $p(PC_i) = |PC_i| / |D|$

Probabilistic Symbolic Execution of PIN Checker

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- $|D| = 2^8 = 256$

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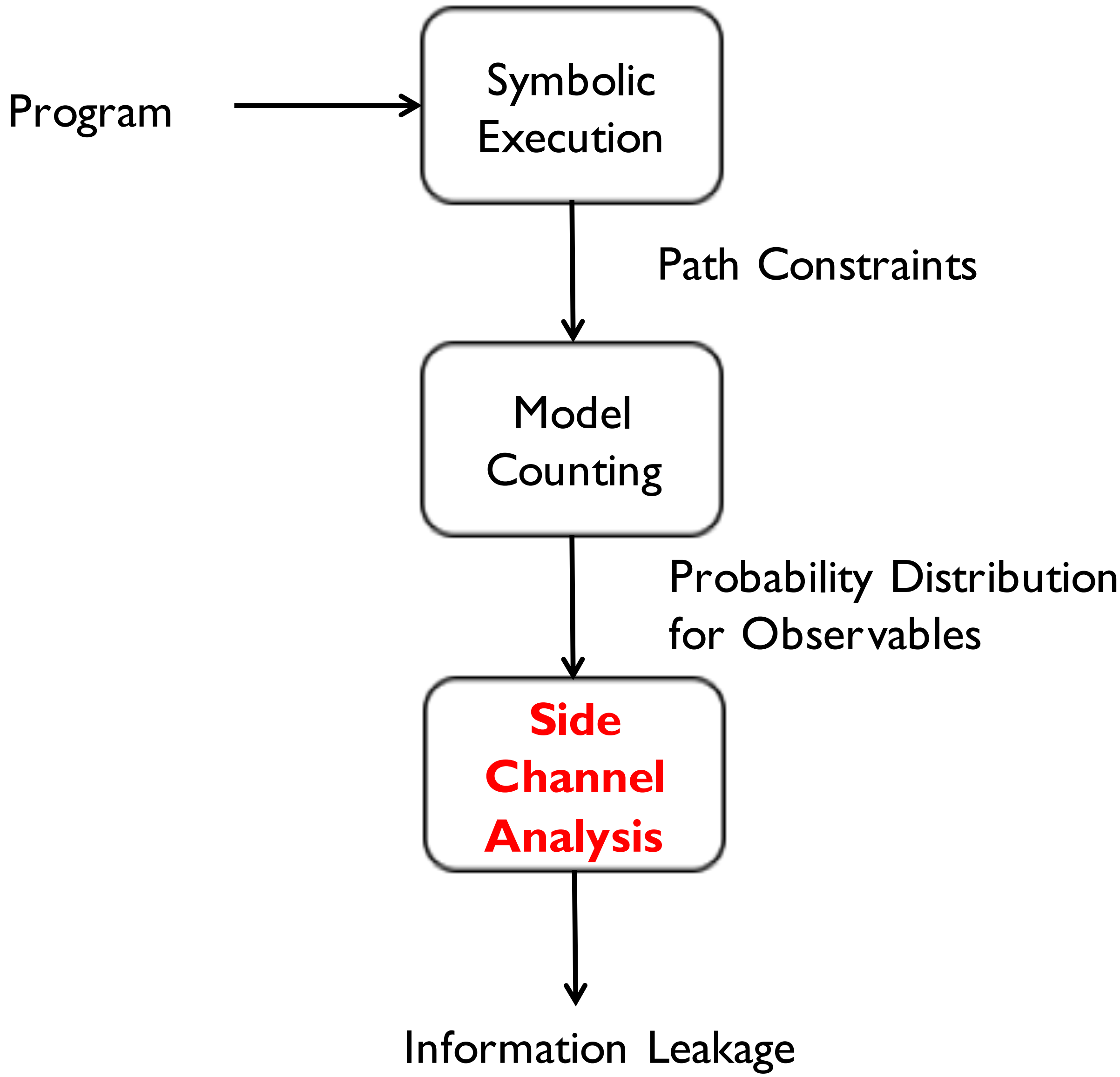
$\lvert PC_i \rvert$	128	64	32	16	16
p_i	1/2	1/4	1/8	1/16	1/16

- Probability that an adversary can guess a prefix of length i in one guess is given by p_i

Overview



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Information Leakage

- Note that any PIN checker leaks information about the secret (secret is the pin value P)
- When an adversary tries a guess G there are two scenarios:
  - If G matches P then adversary learns the PIN
  - If G does not match P, then the adversary learns that the PIN value is not G
- This is due to the public output of the PIN checker
  - This is called the **main channel**
- However, there may be other observations one can make about the PIN checker that reveals more information about P

Information Leakage

- An adversary may observe more than just the public output of a program, such as
  - execution time
  - memory usage
  - file size
  - network package size

- There may be information leakage about the secret from these observable values

- These are called **side channels**

Entropy: Quantifying Information Leakage

- How can we quantify information leakage?
- Shannon Entropy

$$H = \sum p_i \log \frac{1}{p_i} = E\left[\log \frac{1}{p_i}\right]$$

- Intuition:
- The ***expected*** amount of ***information gain*** (i.e., the expected amount of surprise) expressed in terms of ***bits***

Entropy: Quantifying Information Leakage

$$H = \sum p_i \log \frac{1}{p_i} = E\left[\log \frac{1}{p_i}\right]$$

- Entropy example:
- Seattle weather in December: Always raining
- $p_{rain} = 1$, $p_{sun} = 0$
- Entropy: H = 0

- San Francisco weather in December: Coin flip
- $p_{rain} = ½$, $p_{sun} = ½$
- Entropy: H = 1

- Santa Barbara weather in December: Almost always beautiful:
- $p_{rain} = 1/10$, $p_{sun} = 9/10$
- Entropy: H = 0.496

Information Leakage via Side Channels

- Side channels produce a set of observables that partition the secret: $\mathcal{O} = \{o_1, o_2, ...o_m\}$

- By computing the probability of observable values we can compute the entropy:

$$\mathcal{H}(P) = -\sum_{i=1,m} p(o_i)\log_2(p(o_i))$$

- We can compute the probability of observable values using model counting:

the probability of observing o_i is:

$$p(o_i) = \frac{\sum_{cost(\pi_j)=o_i} \sharp(PC_j(h,l))}{\sharp D}$$

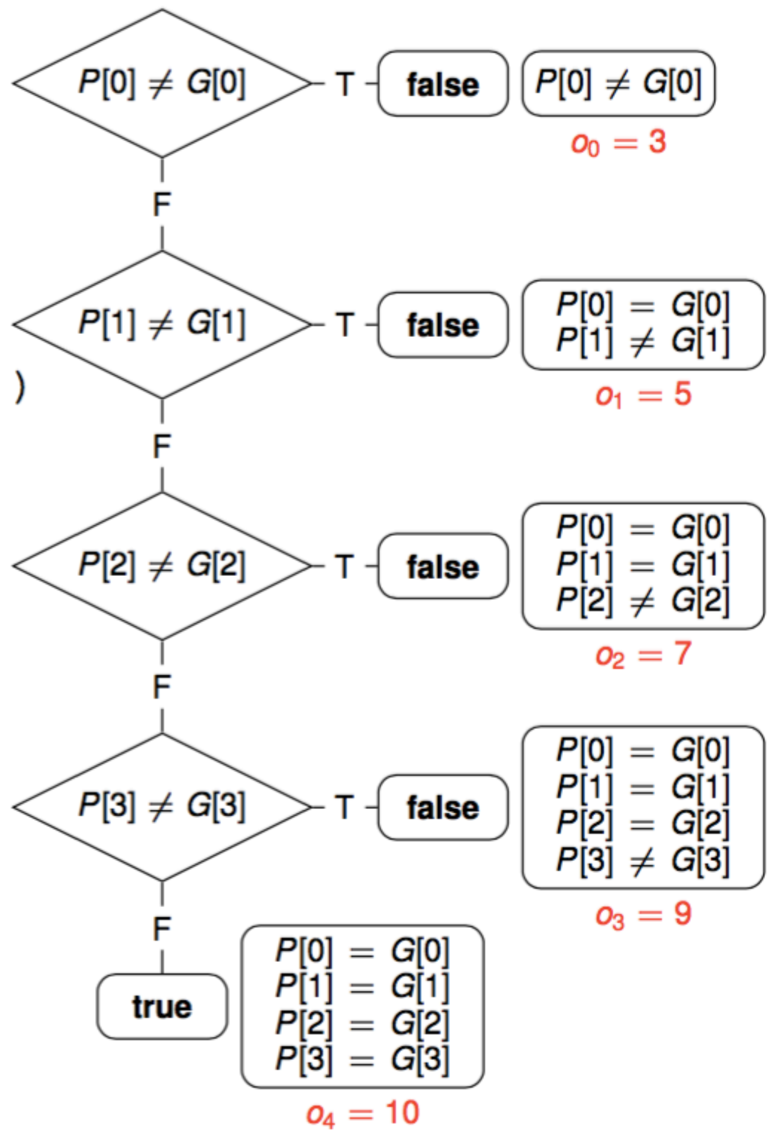
Bang et al., String Analysis for Side Channels with Segmented Oracles (FSE'16)

Symbolic Execution of PIN Checker

```
bool checkPIN(guess[])
for(i = 0; i < 4; i++)
 if(guess[i] != PIN[i])
  return false
return true
```

P: PIN, *G*: guess

o_i = lines of code



Probabilistic Symbolic Execution of PIN Checker

- Assume binary 4 digit PIN, P and G each have 4 bits
- $|D| = 2^8 = 256$

i	0	1	2	3	4
PC_i	$P[0] \neq G[0]$	$P[0] = G[0]$ $P[1] \neq G[1]$	$P[0] = G[0]$ $P[1] = G[1]$ $P[2] \neq G[2]$	$P[0] = G[0]$ $P[1] = G[1]$ $P[2] = G[2]$ $P[3] \neq G[3]$	$P[0] = G[0]$ $P[1] = G[1]$ $P[2] = G[2]$ $P[3] = G[3]$
return	false	false	false	false	true
$\|PC_i\|$	128	64	32	16	16
p_i	1/2	1/4	1/8	1/16	1/16
o_i	3	5	7	9	10

Information Leakage

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return	false	false	false	false	true
$\|PC_i\|$	128	64	32	16	16
p_i	1/2	1/4	1/8	1/16	1/16
o_i	3	5	7	9	10

$$H = \sum p_i \log \frac{1}{p_i} = 1.8750$$

- H: The expected amount of information gain by the adversary

A secure PIN checker

```
public verifyPassword (guess[])
  matched = true
  for (int i = 0; i < 4; i++)
    if (guess[i] != PIN[i])
          matched = false
    else
          matched = matched
  return matched
```

- Only two observables (just the main channel, no side channel): o_0: does not match, o_1: full match
- $p(o_0) = 15/16$, $p(o_1) = 1/16$
- $H_{secure} = 0.33729$

Secure vs. insecure PIN checker

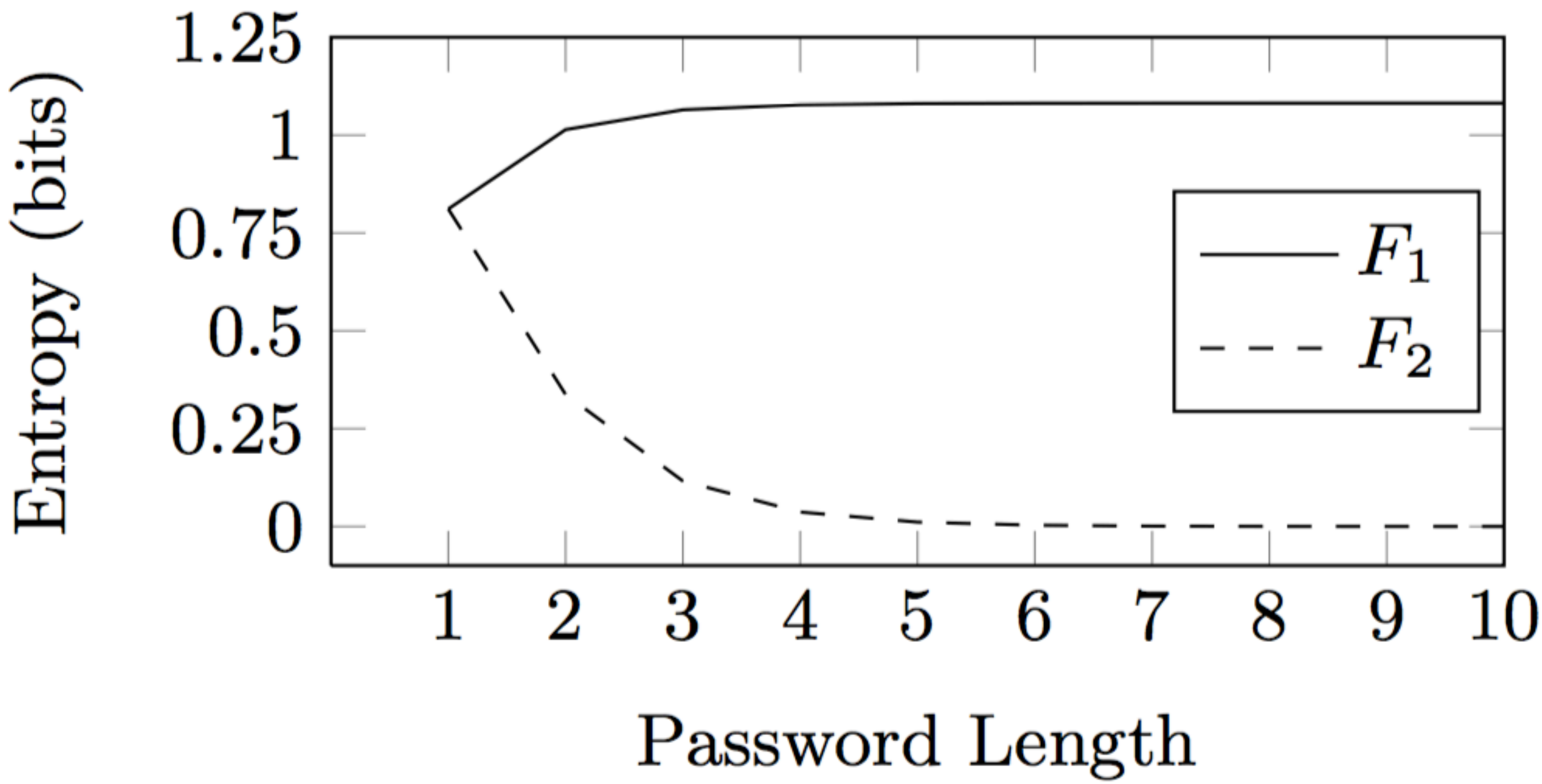
- Given a PIN of length L where each PIN digit has K values
- Secure PIN checker
  - K^L guesses in the worst case
  - Example: 16 digit password where each digit is ASCII

128^{16} tries in the worst case, which would take a lot of years

- Insecure PIN checker
  - A **prefix attack** that determines each digit one by one starting with the leftmost digit
  - Example: 16 digit password where each digit is ASCII

128×16 tries in the worst case, which would not take too much time

Secure vs. insecure PIN checker



Not just a toy example

Vulnerabilities that are similar to the simple PIN example happen in real software systems

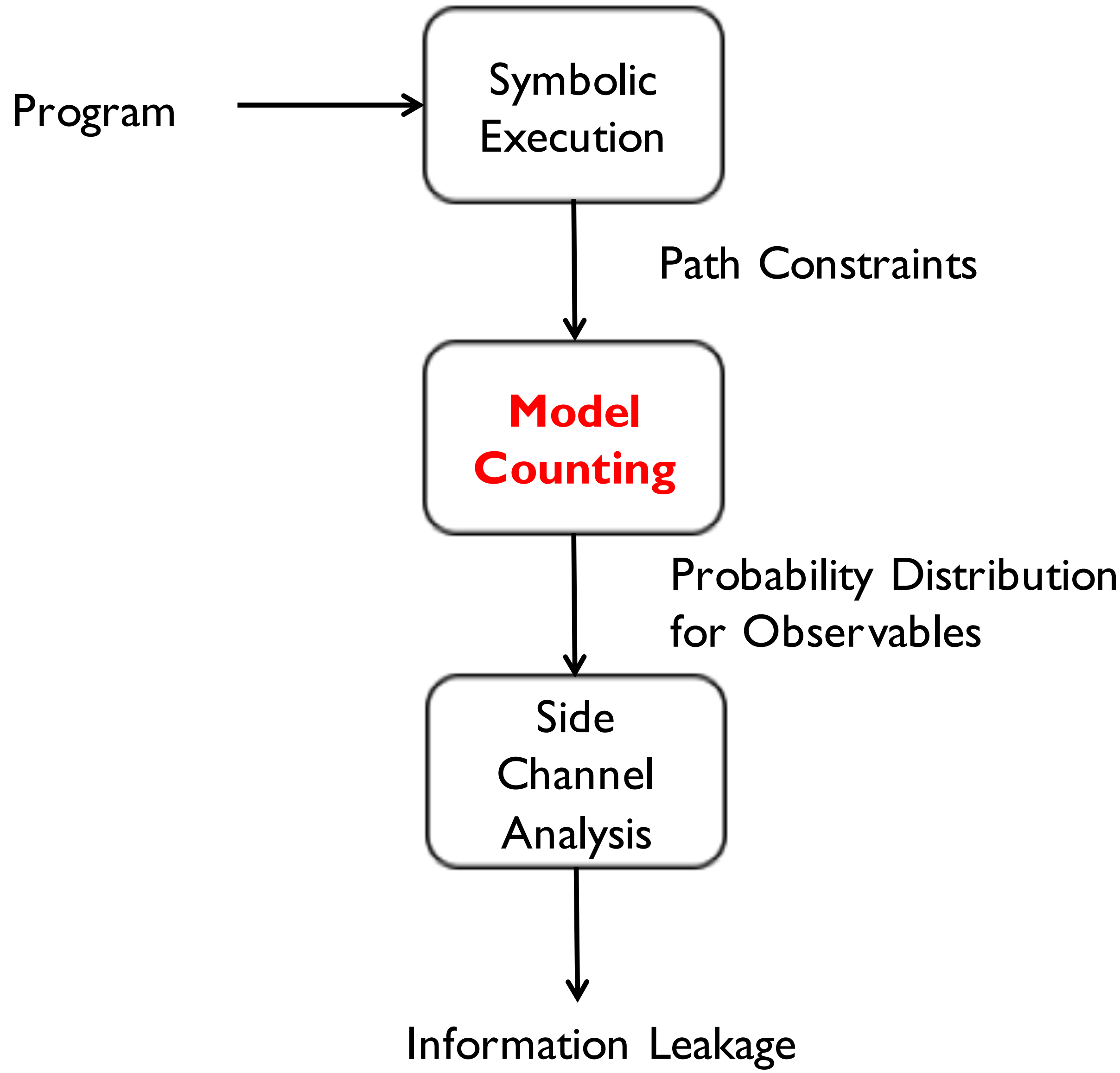
Timing Side Channels

- HMAC keys: Google Keyczar Library, Xbox 360
- Authorization Frameworks: OAuth, OpenID
- Java's Array.equals, String.equals
- C's memcmp

Network Packet Size Side Channel

- Compression Ratio Infoleak Made Easy (CRIME)

Overview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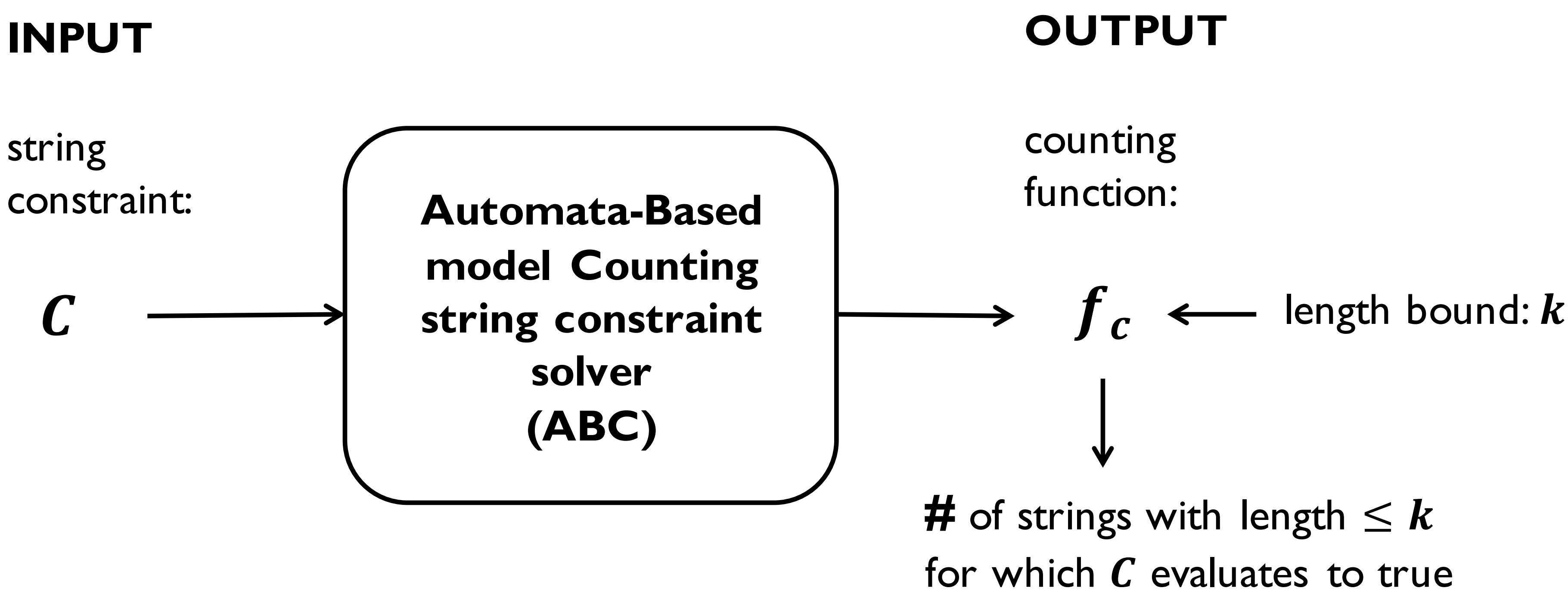
Model Counting String Constraint Solver



Automata Based Counter (ABC)
A Model Counting String Constraint Solver

INPUT

string constraint:

C

Automata-Based model Counting string constraint solver (ABC)

OUTPUT

counting function:

f_C

length bound: k

\# of strings with length $\leq k$ for which C evaluates to true

Aydin et al., Automata-based Model Counting for String Constraints. (CAV'15)

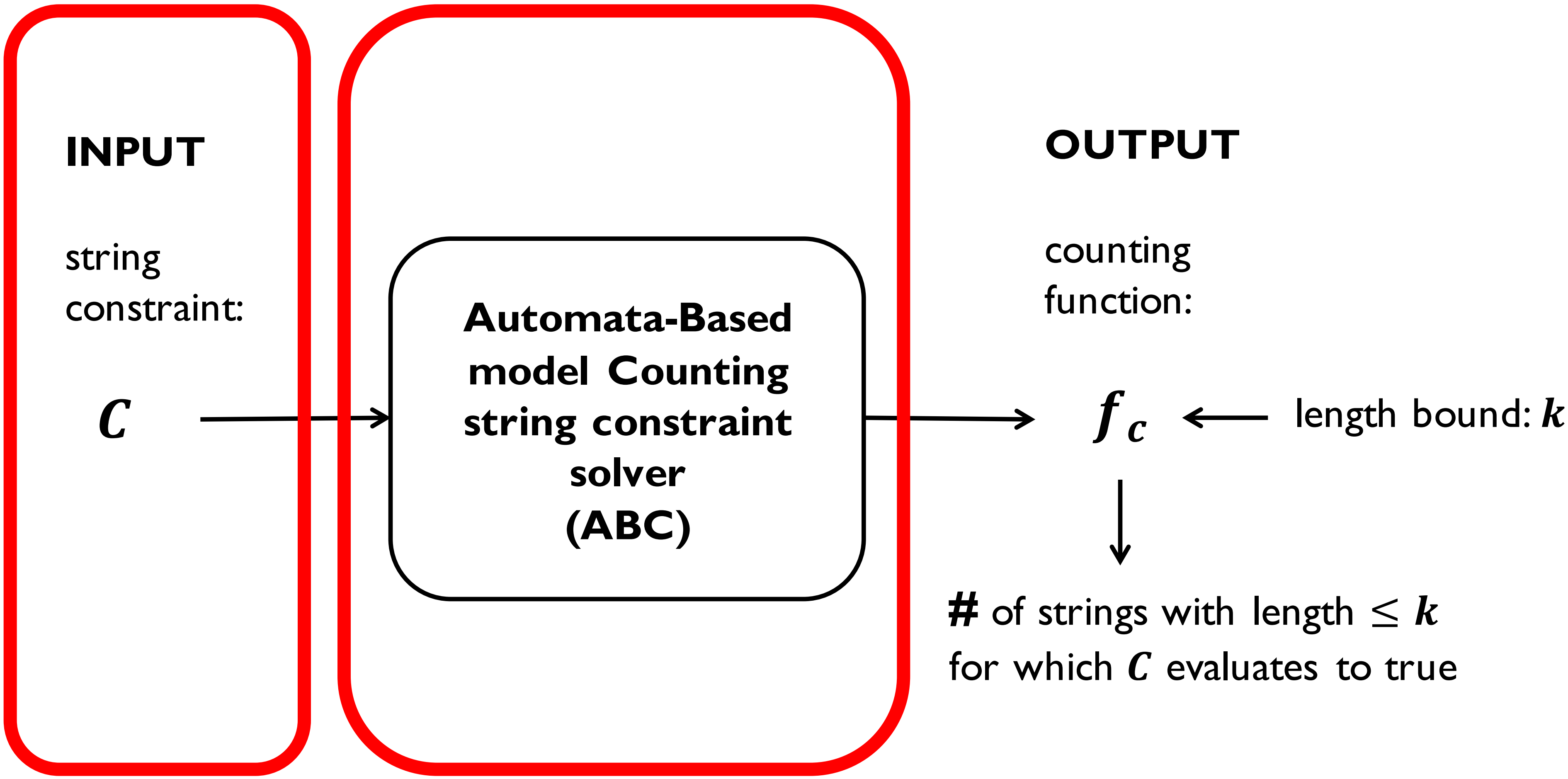
String Constraint Language

$$
\begin{array}{lcl}
C & \longrightarrow & bterm \\
\\
bterm & \longrightarrow & \mathtt{v} \mid \mathtt{true} \mid \mathtt{false} \\
 & \mid & \neg bterm \mid bterm \wedge bterm \mid bterm \vee bterm \mid (bterm) \\
 & \mid & sterm = sterm \\
 & \mid & \mathtt{match}(sterm, sterm) \\
 & \mid & \mathtt{contains}(sterm, sterm) \\
 & \mid & \mathtt{begins}(sterm, sterm) \\
 & \mid & \mathtt{ends}(sterm, sterm) \\
 & \mid & iterm = iterm \mid iterm < iterm \mid iterm > iterm \\
\\
iterm & \longrightarrow & \mathtt{v} \mid \mathtt{n} \\
 & \mid & iterm + iterm \mid iterm - iterm \mid iterm \times \mathtt{n} \mid (iterm) \\
 & \mid & \mathtt{length}(sterm) \mid \mathtt{toint}(sterm) \\
 & \mid & \mathtt{indexof}(sterm, sterm) \\
 & \mid & \mathtt{lastindexof}(sterm, sterm) \\
\\
sterm & \longrightarrow & \mathtt{v} \mid \varepsilon \mid \mathtt{s} \\
 & \mid & sterm.sterm \mid sterm|sterm \mid sterm^* \mid (sterm) \\
 & \mid & \mathtt{charat}(sterm, iterm) \mid \mathtt{tostring}(iterm) \\
 & \mid & \mathtt{toupper}(sterm) \mid \mathtt{tolower}(sterm) \\
 & \mid & \mathtt{substring}(sterm, iterm, iterm) \\
 & \mid & \mathtt{replacefirst}(sterm, sterm, sterm) \\
 & \mid & \mathtt{replacelast}(sterm, sterm, sterm) \\
 & \mid & \mathtt{replaceall}(sterm, sterm, sterm)
\end{array}
$$

Example String Expressions

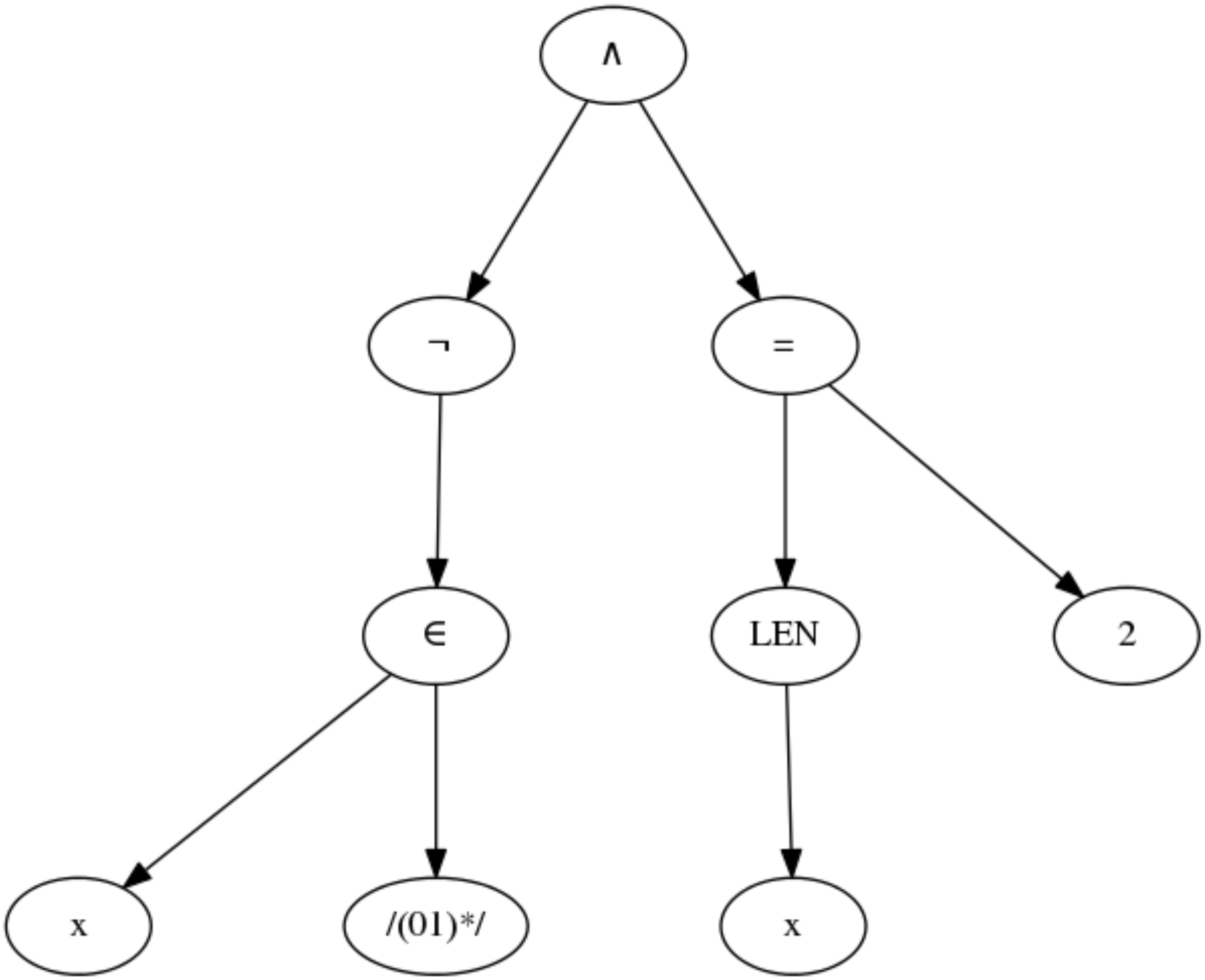
<table>
<tr><th></th><th>String Expression</th><th>Constraint Language</th></tr>
<tr><td rowspan="5">Java</td><td>s.length()</td><td>length(s)</td></tr>
<tr><td>s.isEmpty()</td><td>length(s) == 0</td></tr>
<tr><td>s.startsWith(t,n)</td><td>0 ≤ n ∧ n ≤ |s| ∧
begins(substring(s,n,|s|),t)</td></tr>
<tr><td>s.indexOf(t,n)</td><td>indexof(substring(s,n,|s|),t)</td></tr>
<tr><td>s.replaceAll(p,r)</td><td>replaceall(s,p,r)</td></tr>
<tr><td rowspan="4">PHP</td><td>strrpos(s, t)</td><td>lastindexof(s,t)</td></tr>
<tr><td>substr_replace(s,
t,i,j)</td><td>substring(s,0,i).t.substring(s,j,|s|)</td></tr>
<tr><td>strip_tags(s)</td><td>replaceall(s,("<a>"|"<p>"|...),"")</td></tr>
<tr><td>mysql_real_escape
_string(s)</td><td>...replaceall(s
,replaceall(s,"\\","\\\\")
,"'", "\'")...</td></tr>
</table>

Model Counting String Constraint Solver



String Automata Construction

$$C \equiv \neg(x \in (01)^*) \land LEN(x) = 2$$



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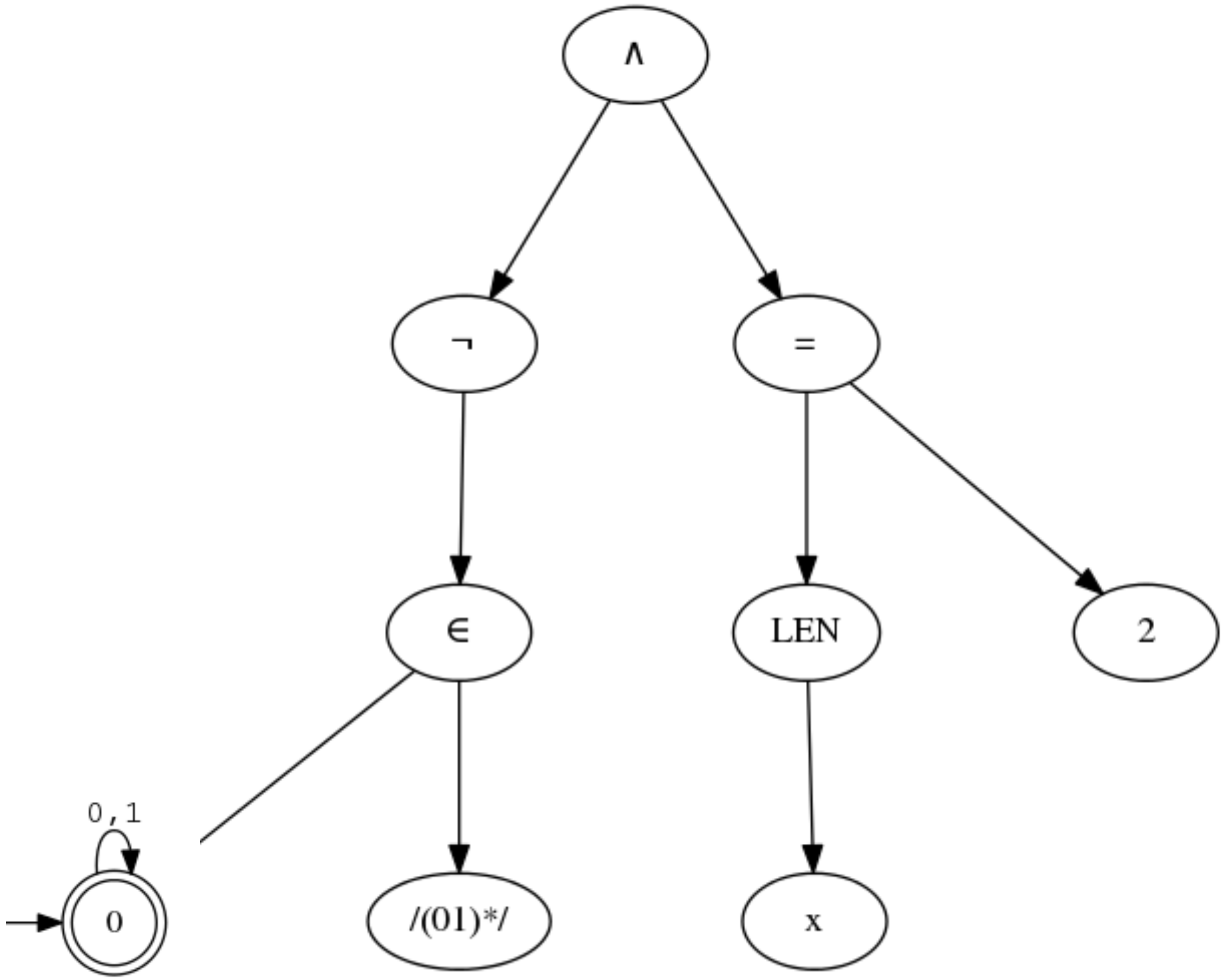

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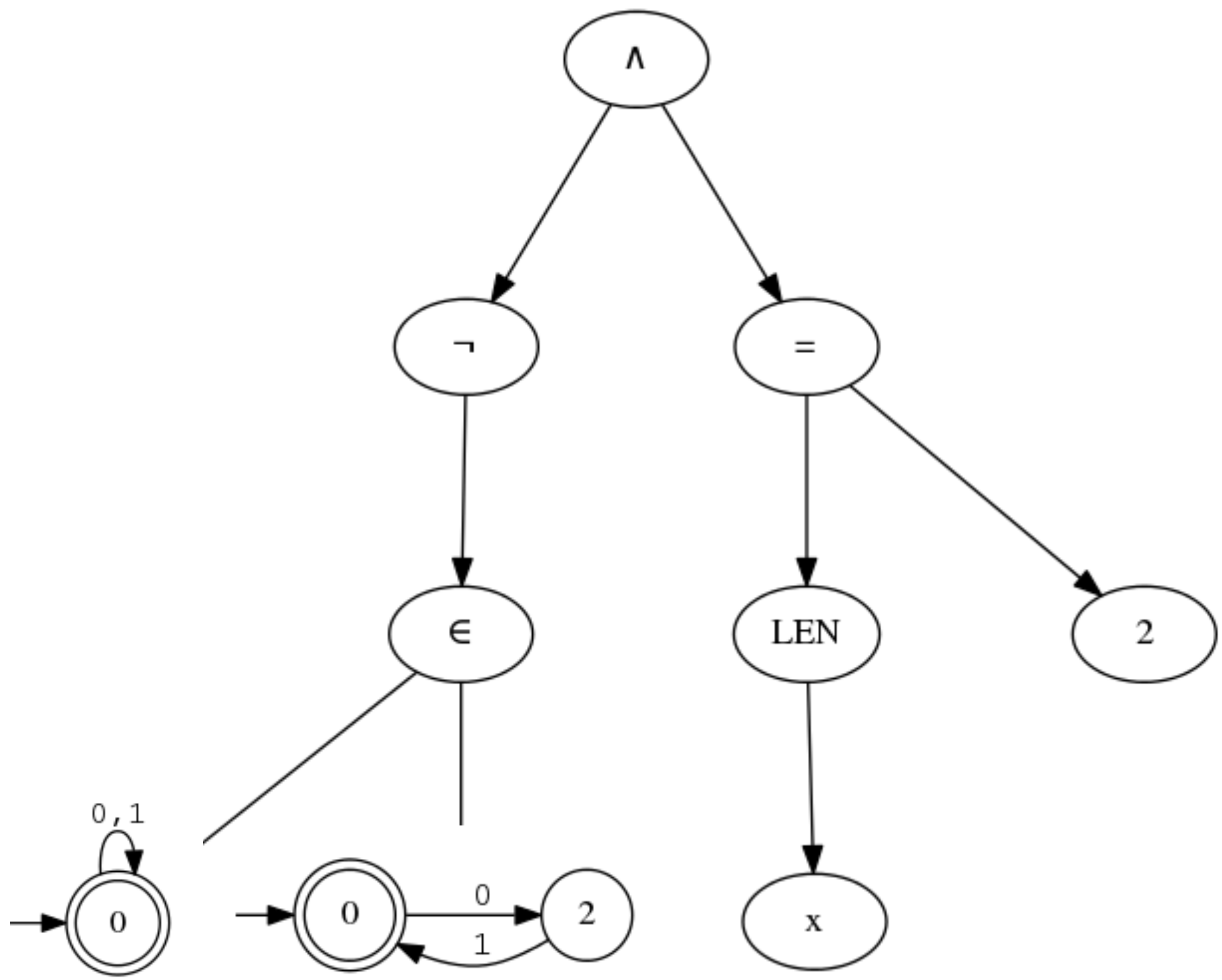

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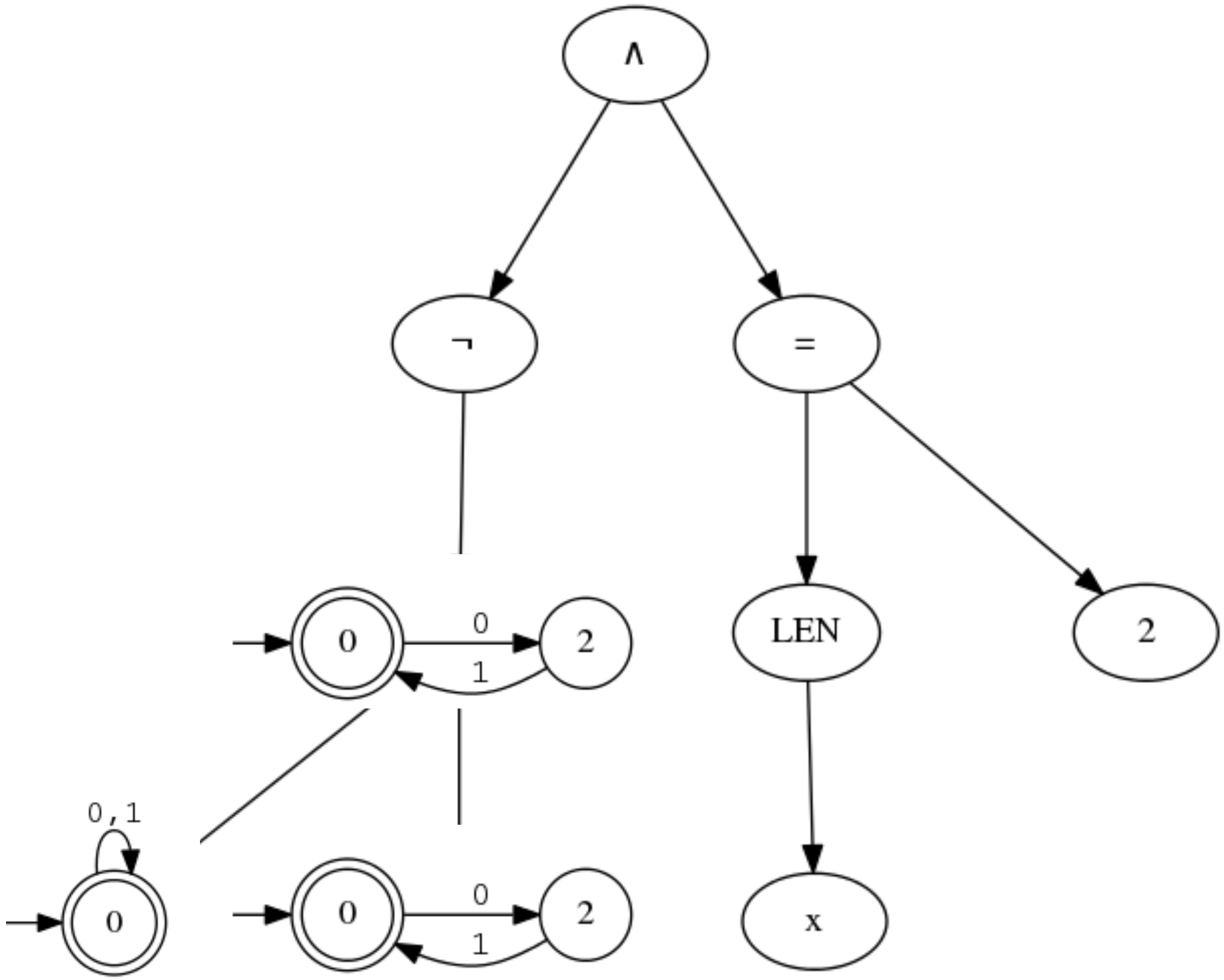

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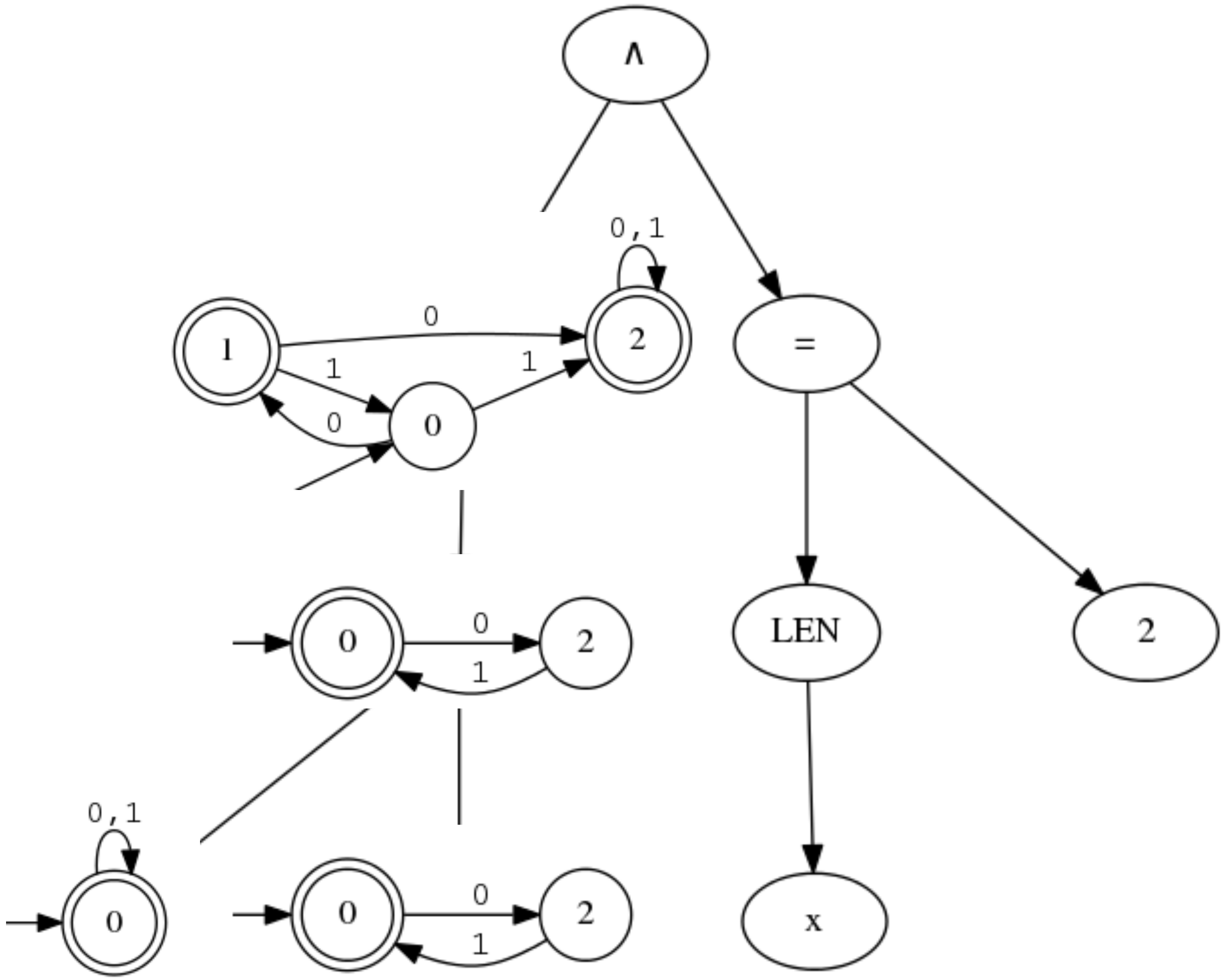

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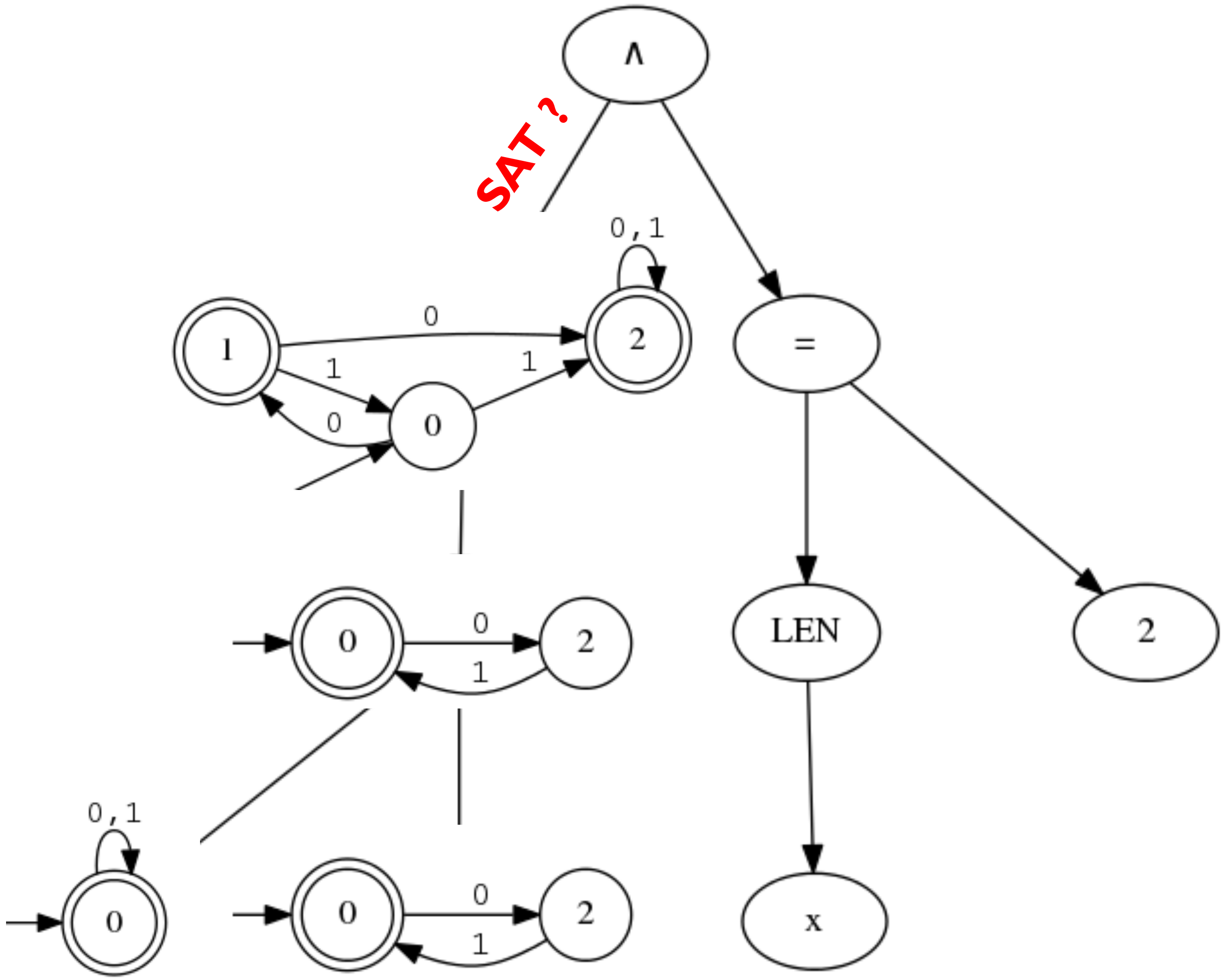

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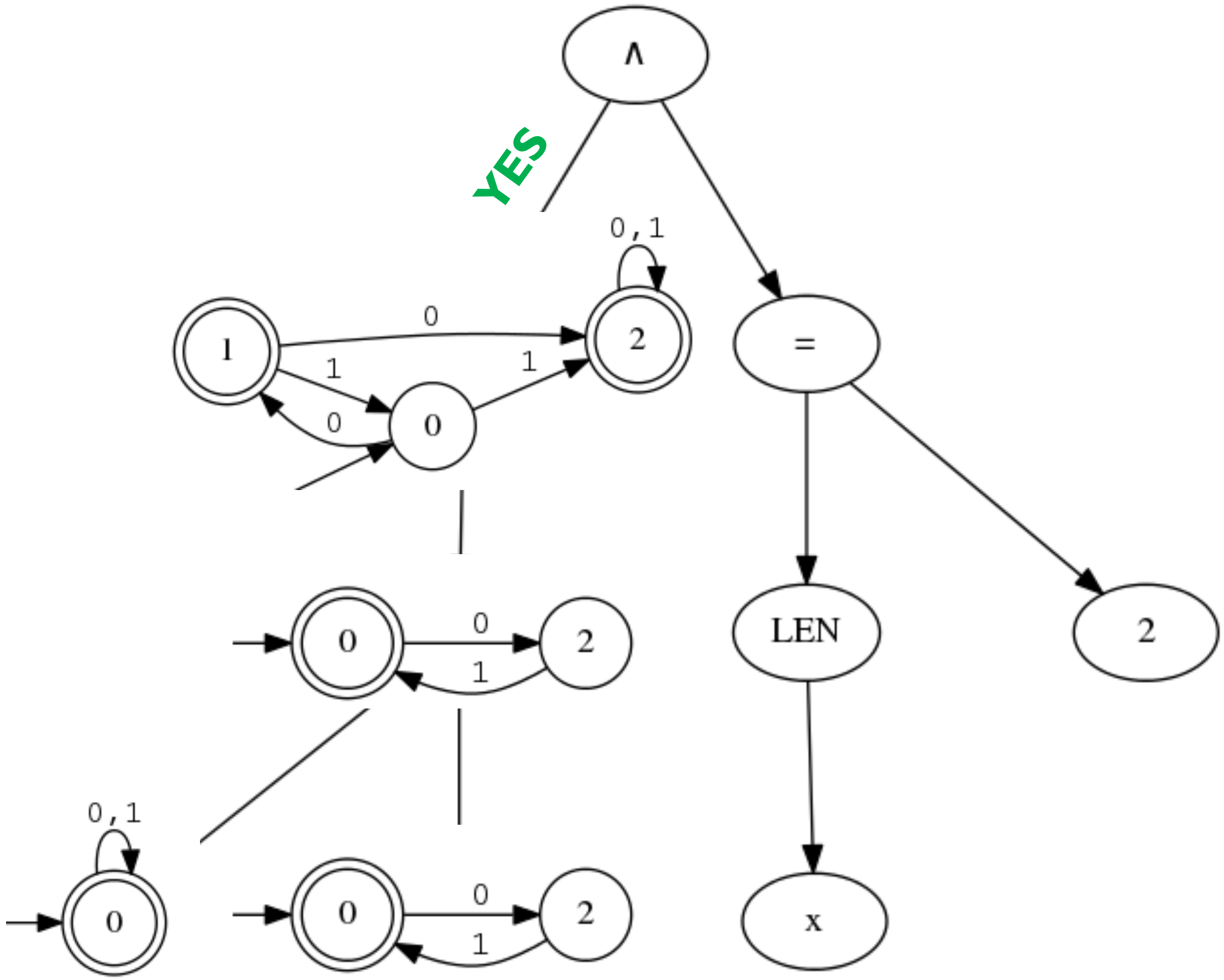

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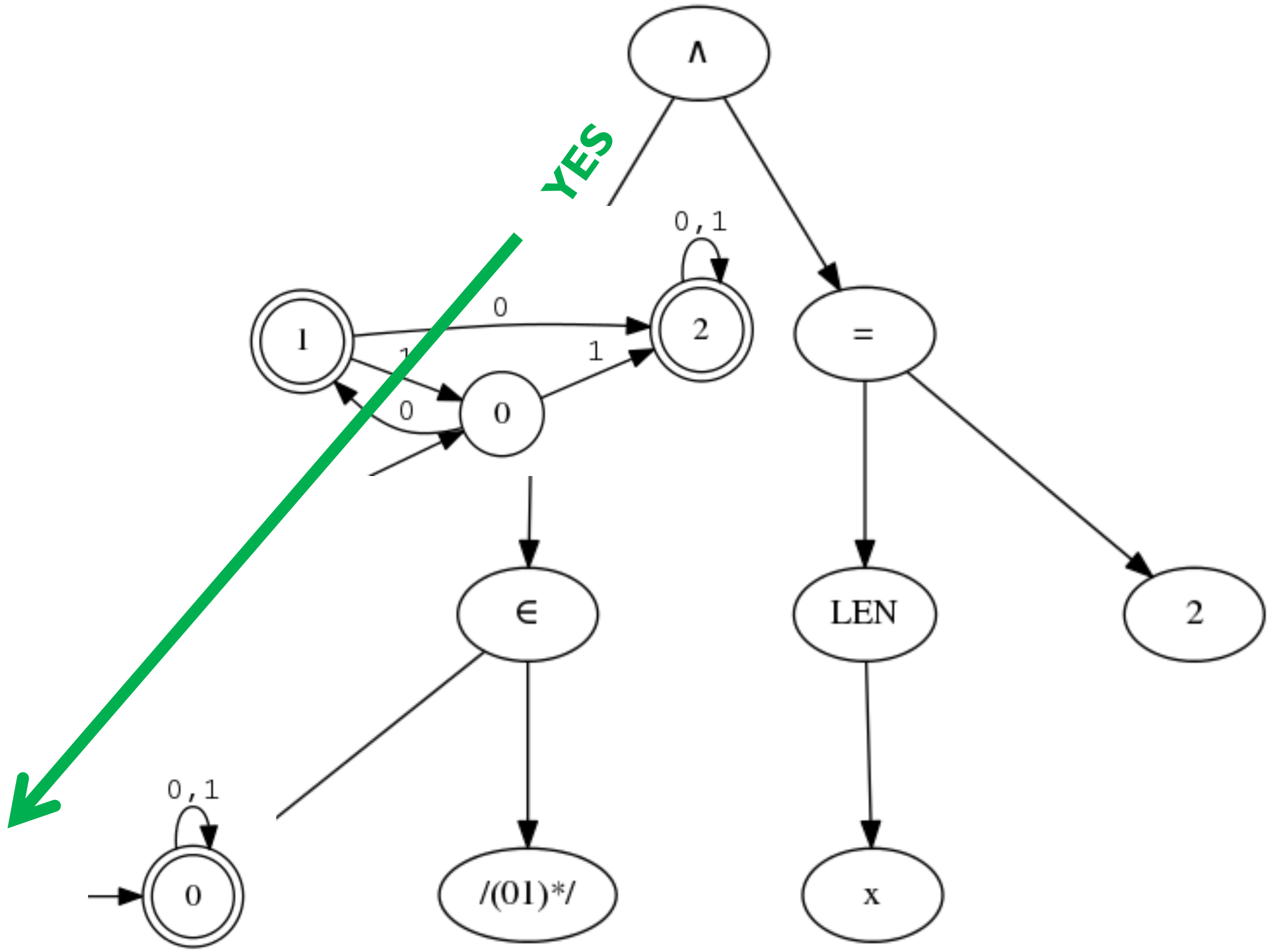

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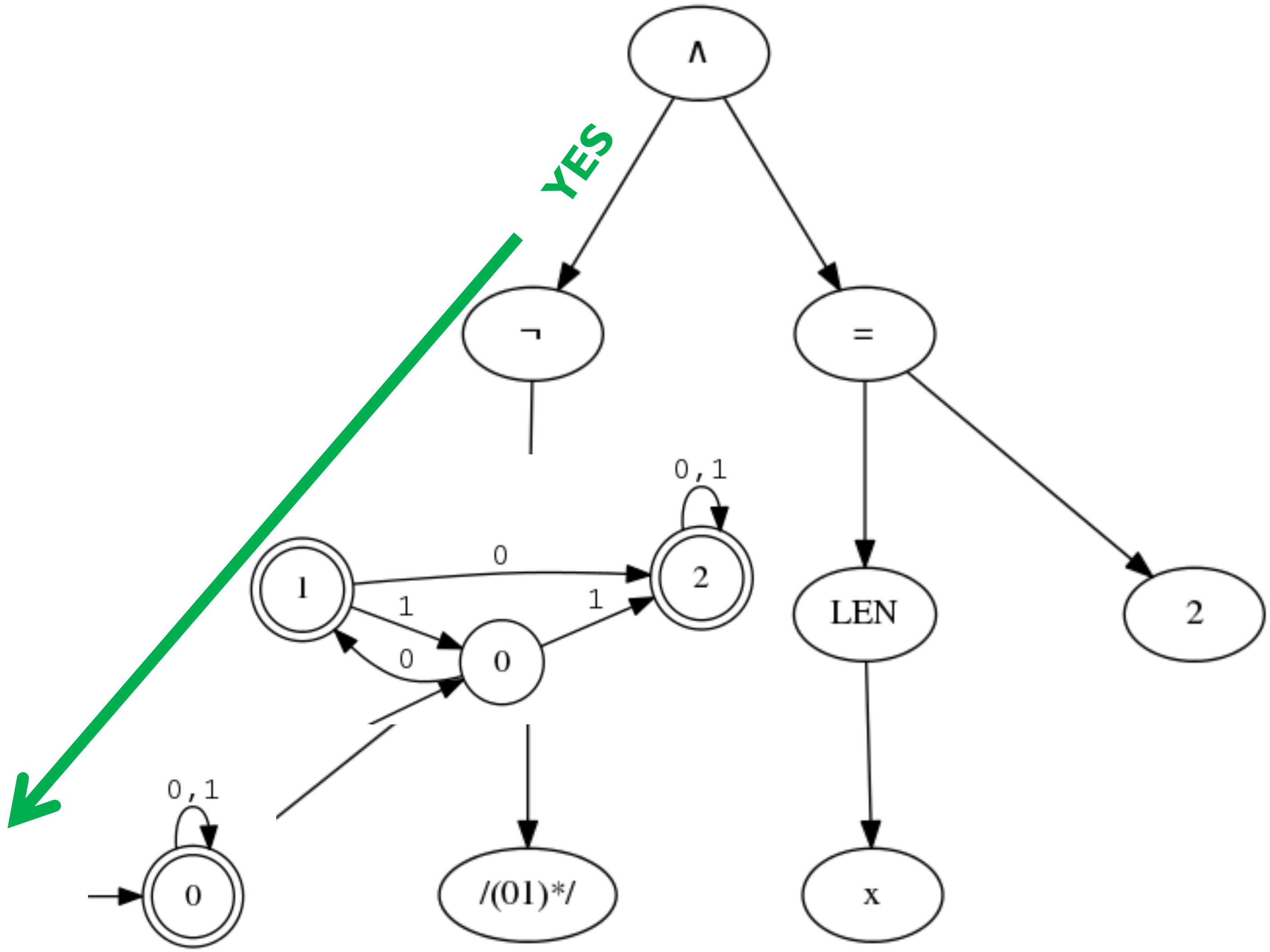

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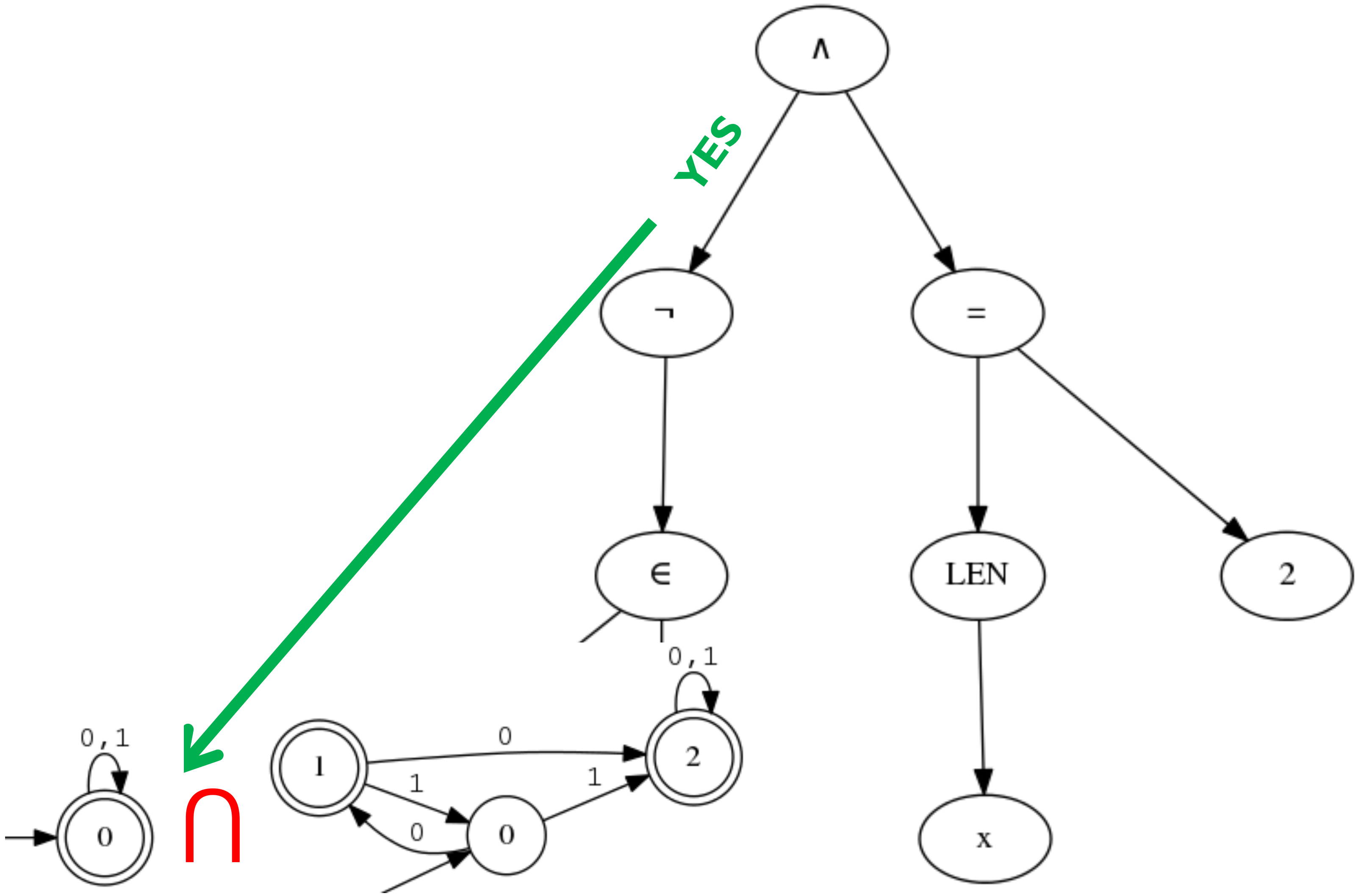

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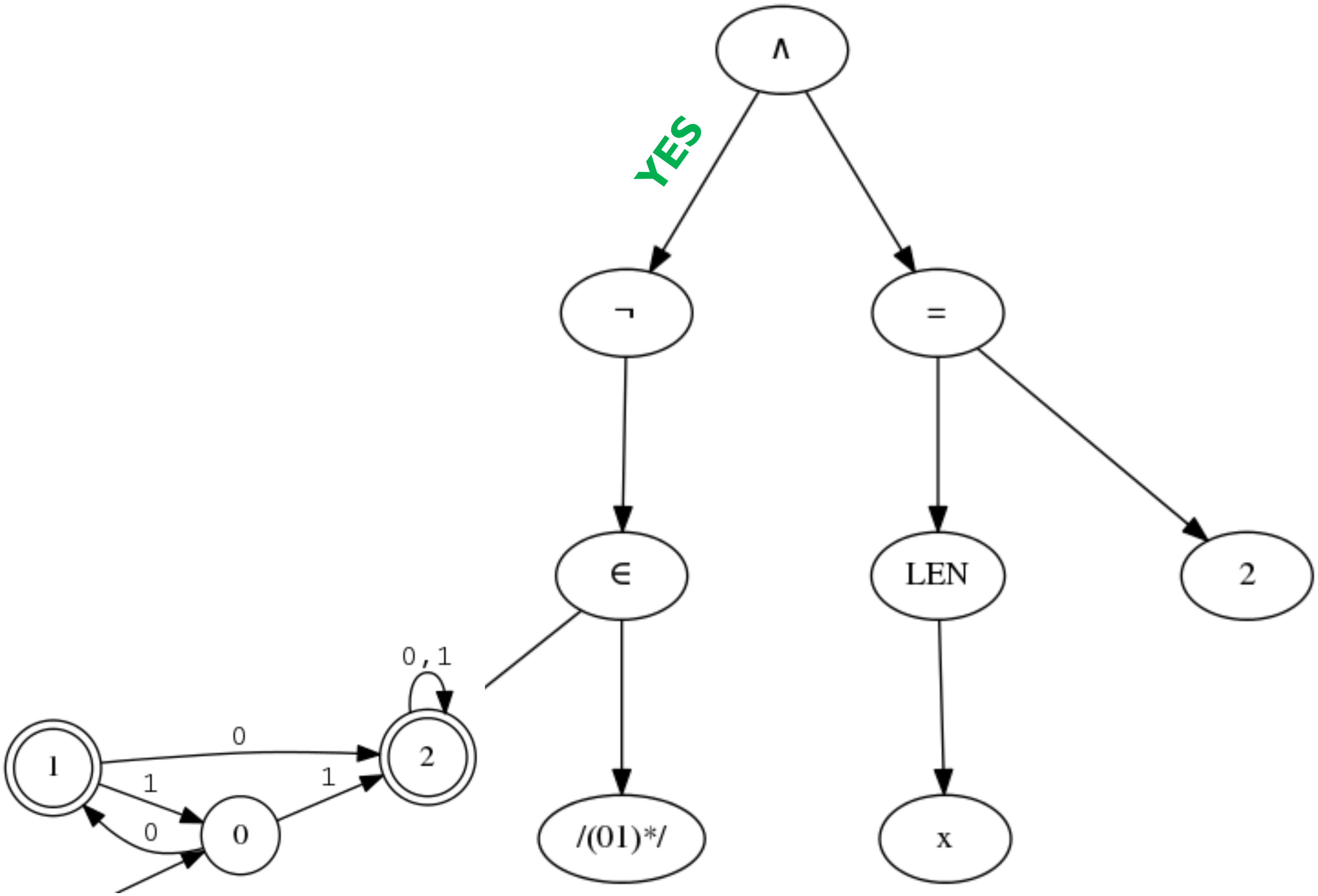

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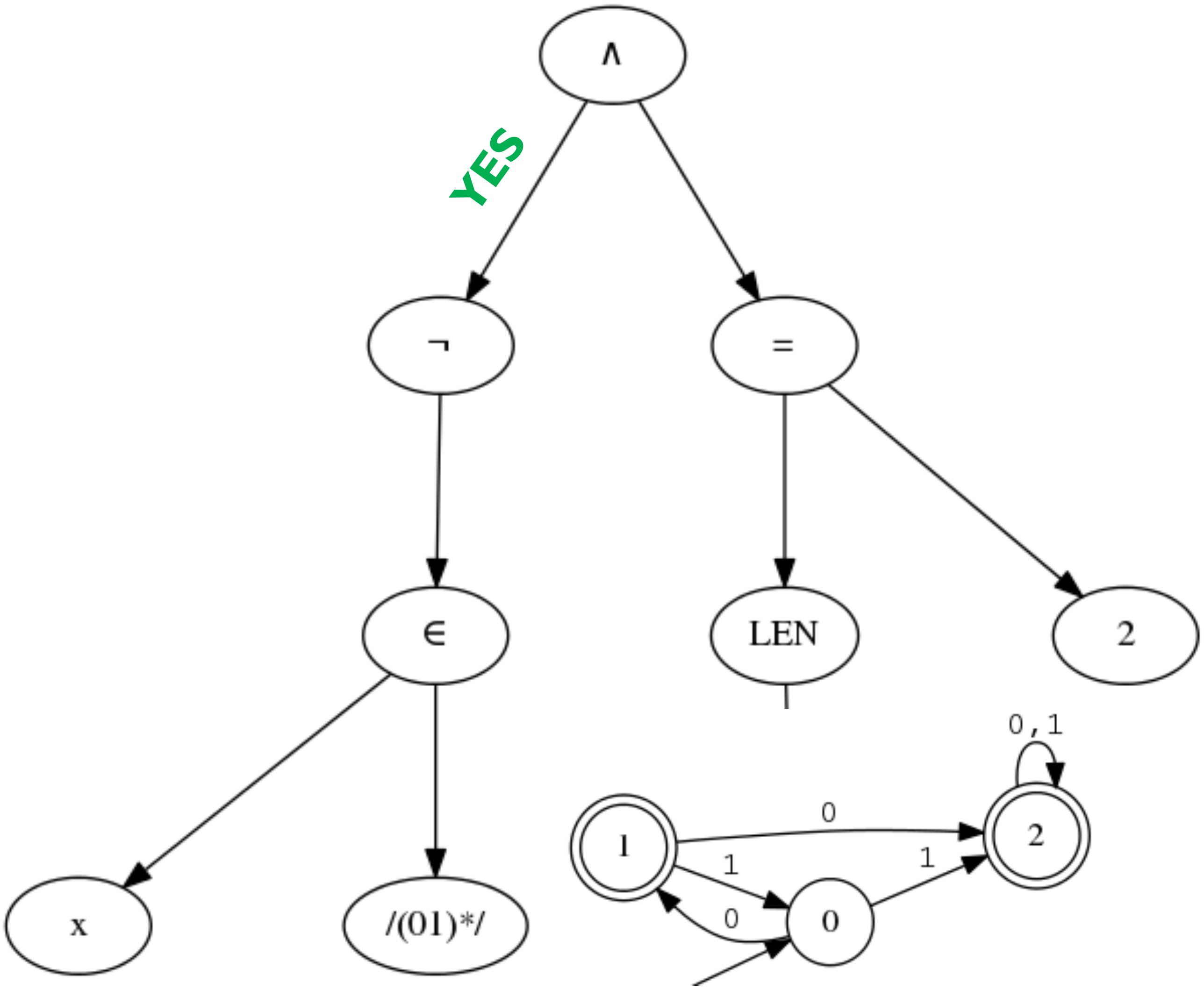

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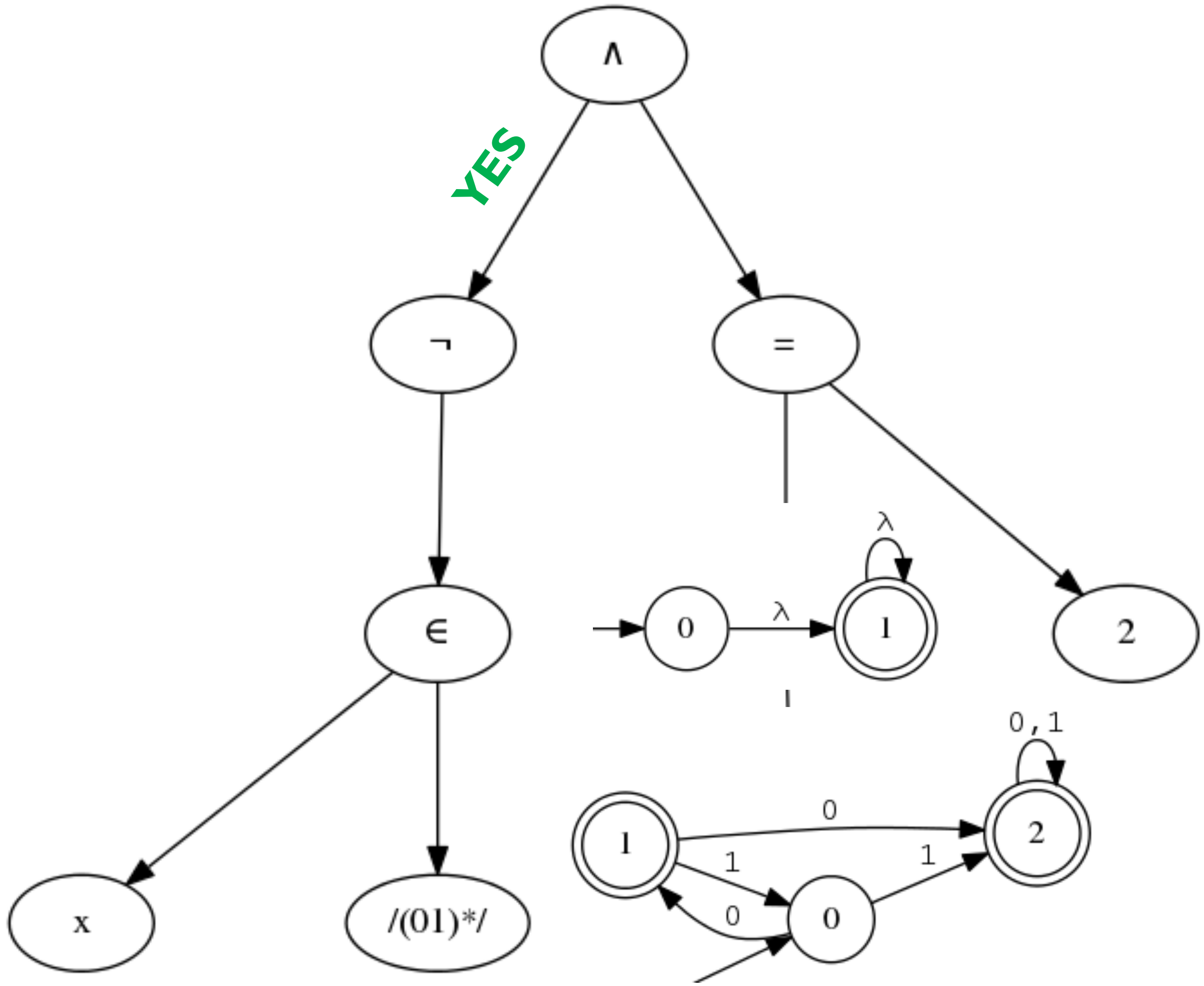

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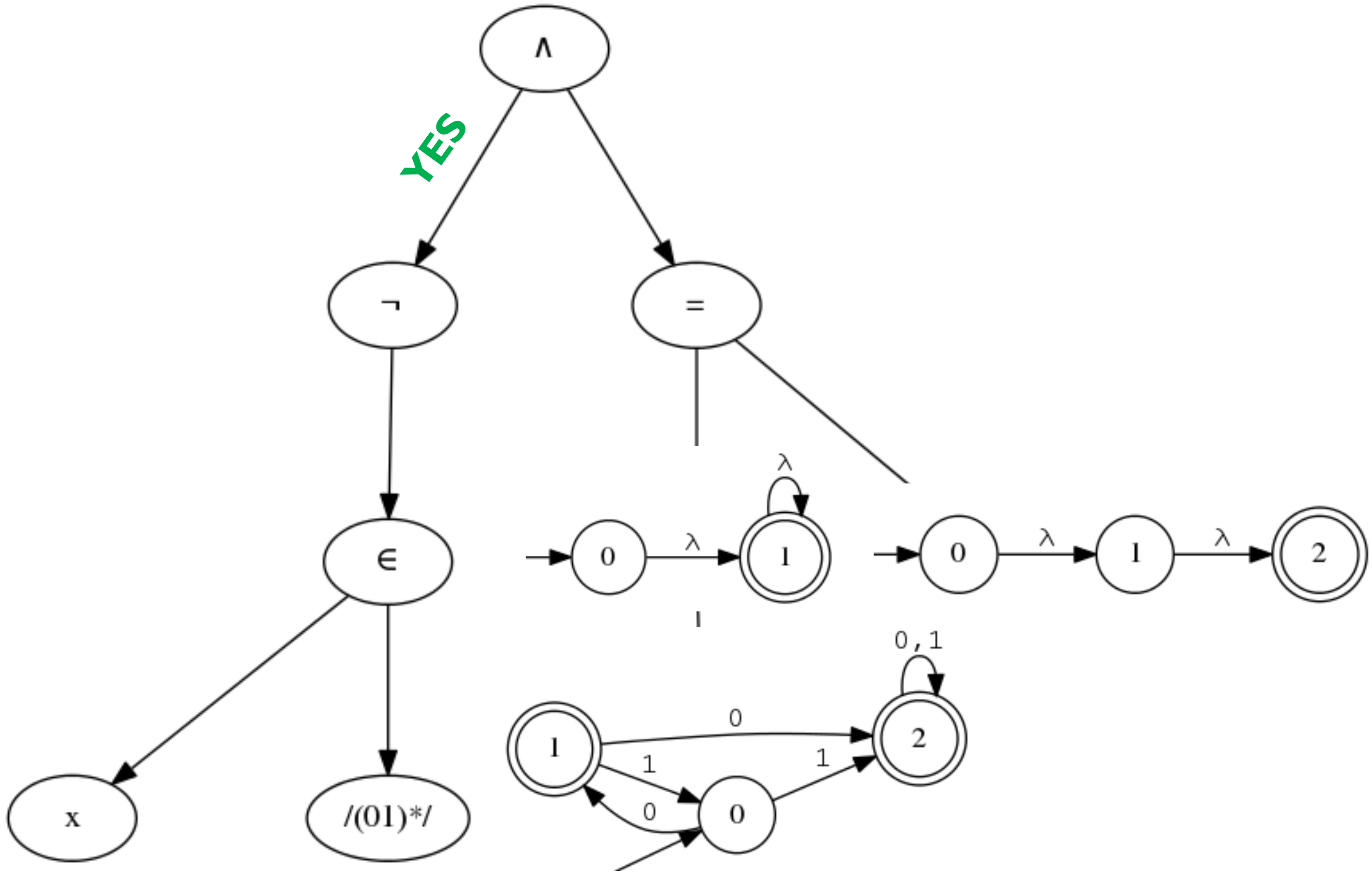

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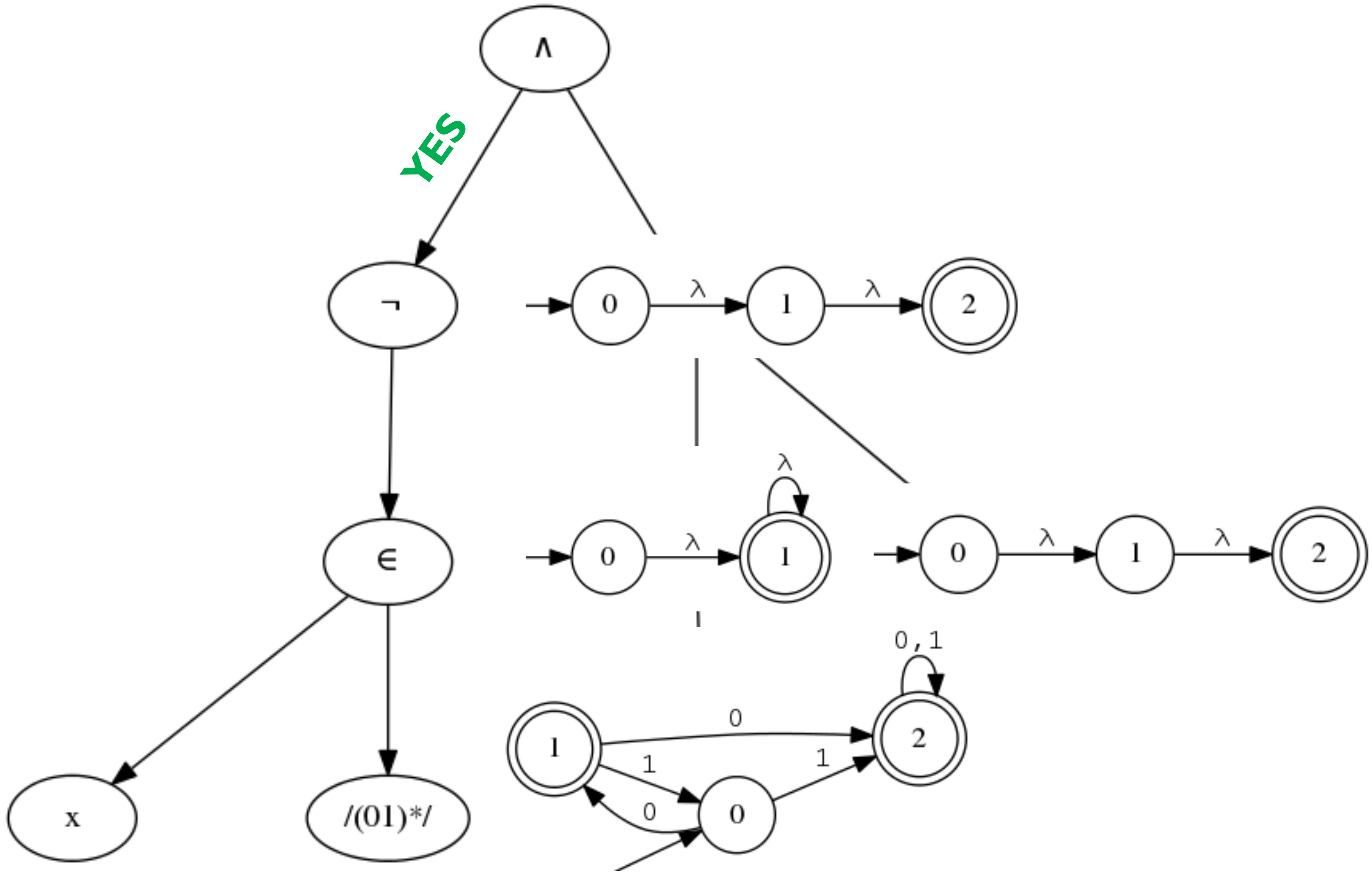

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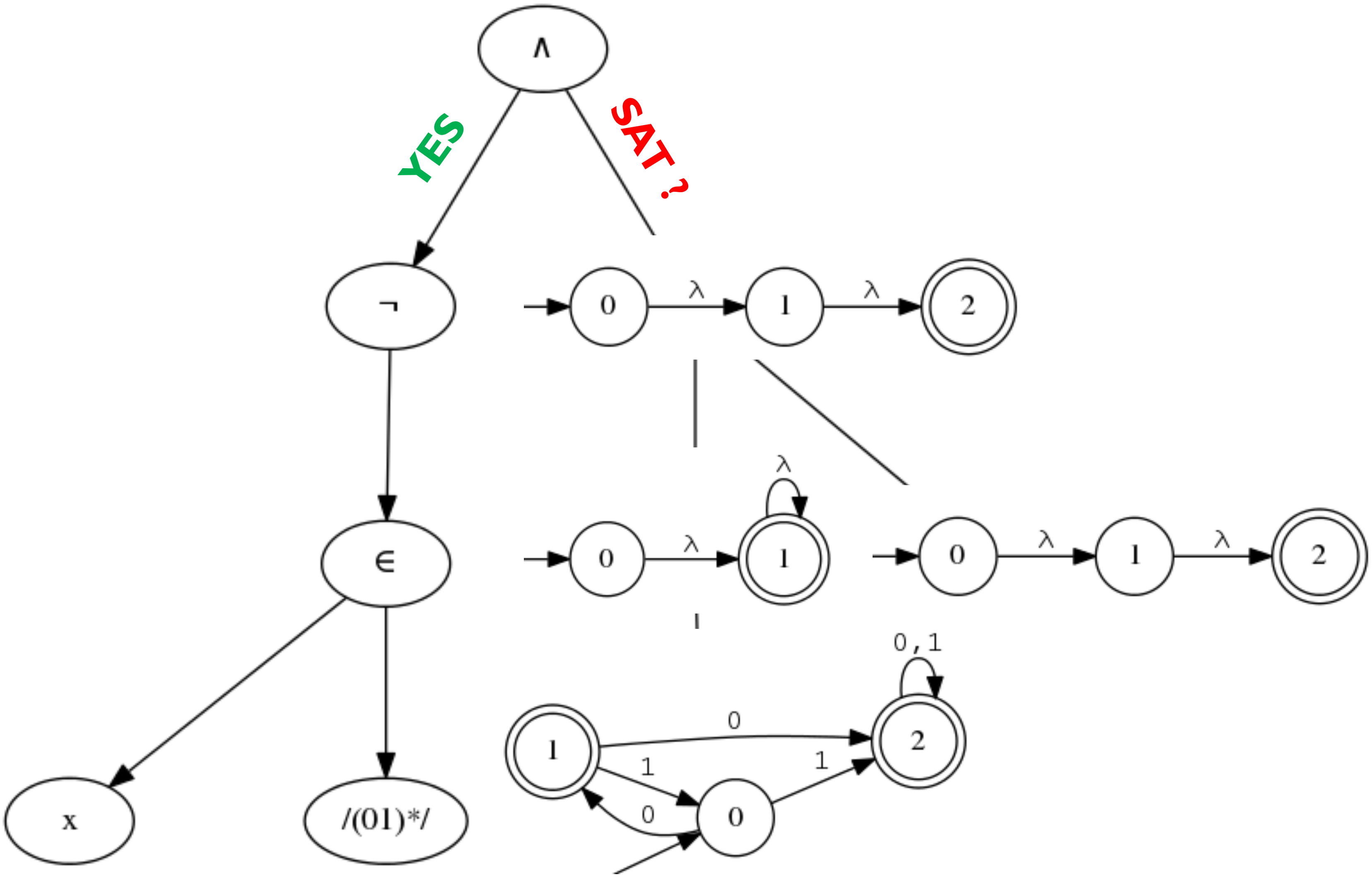

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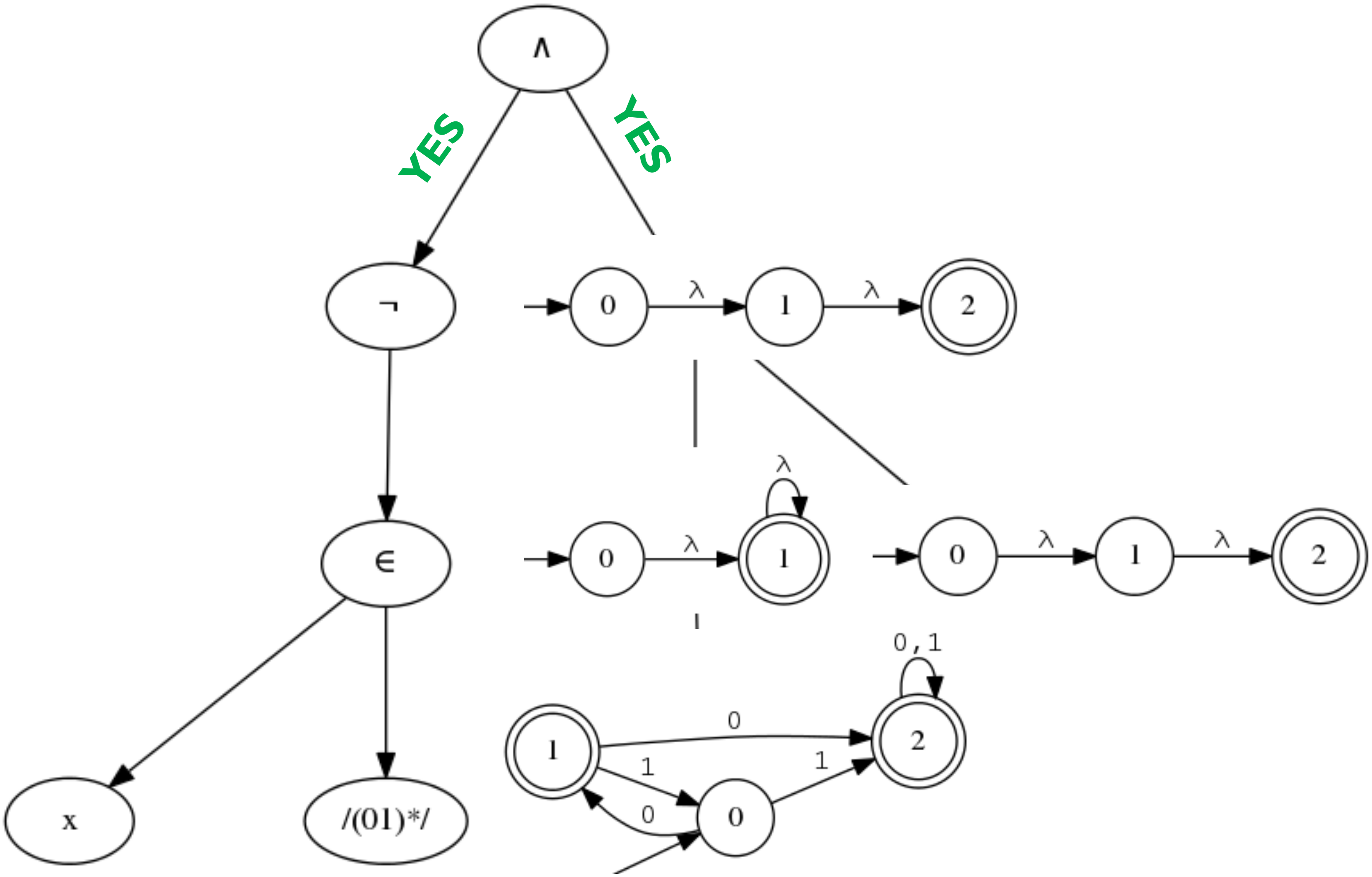

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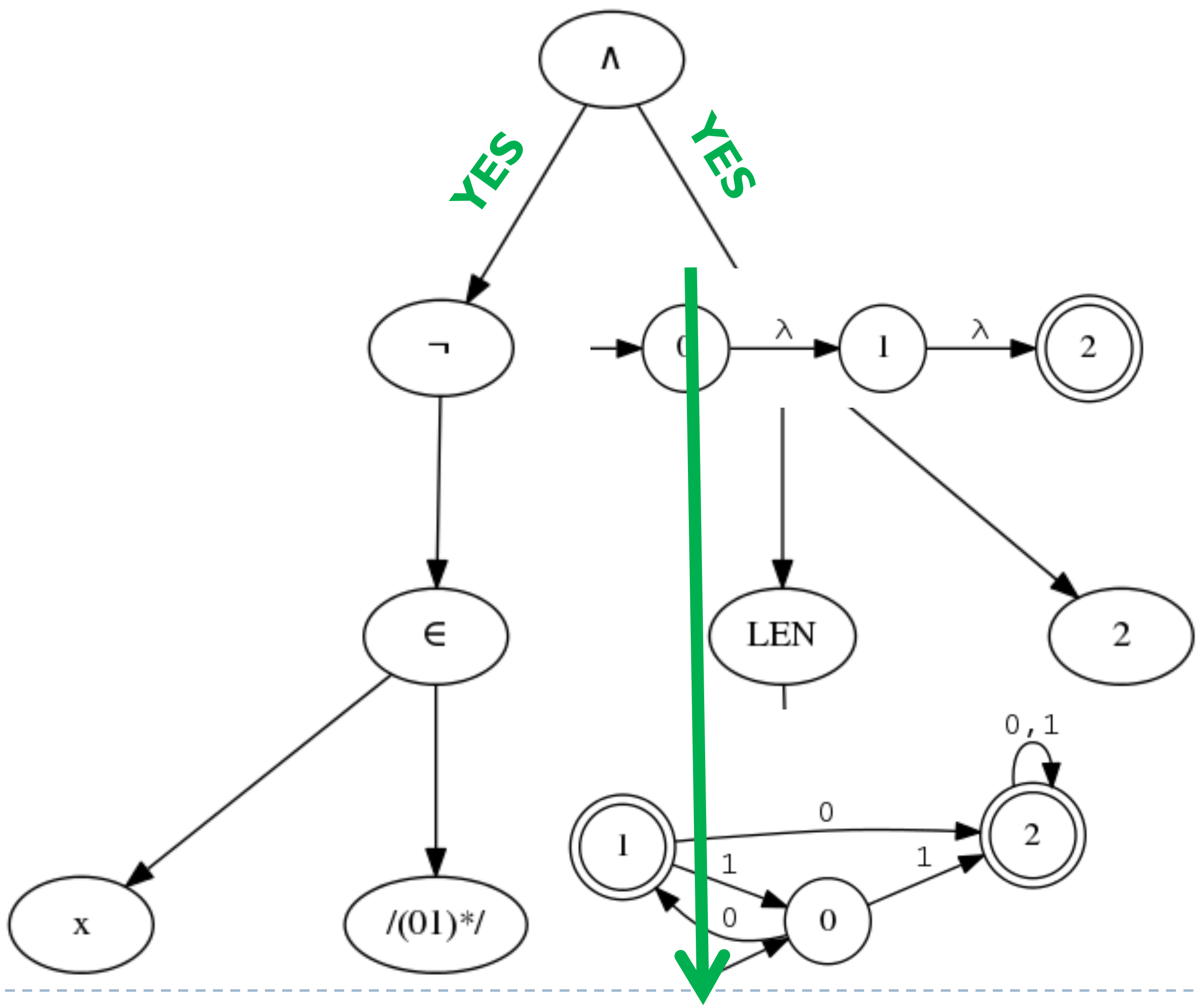

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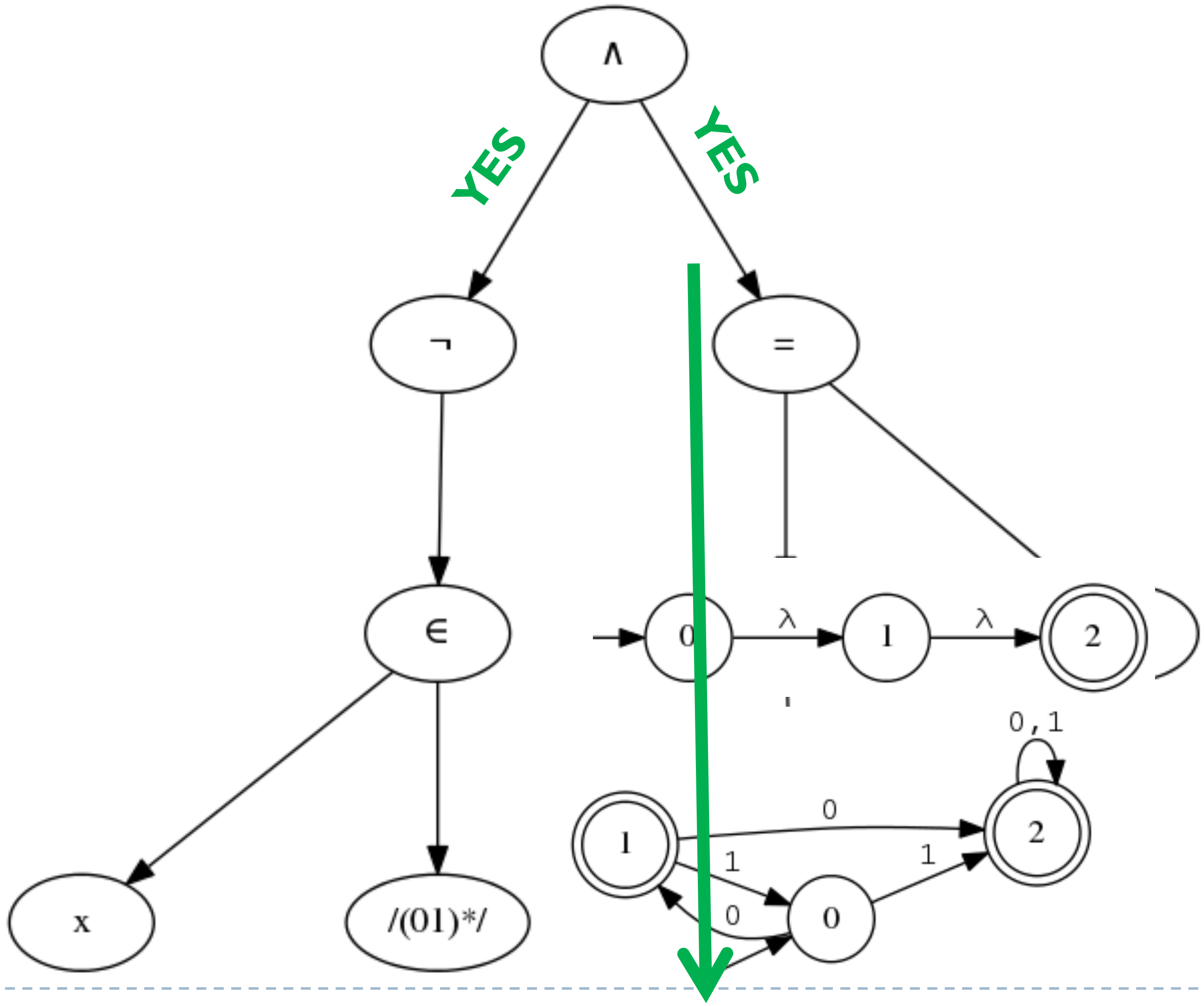

String Automata Construction

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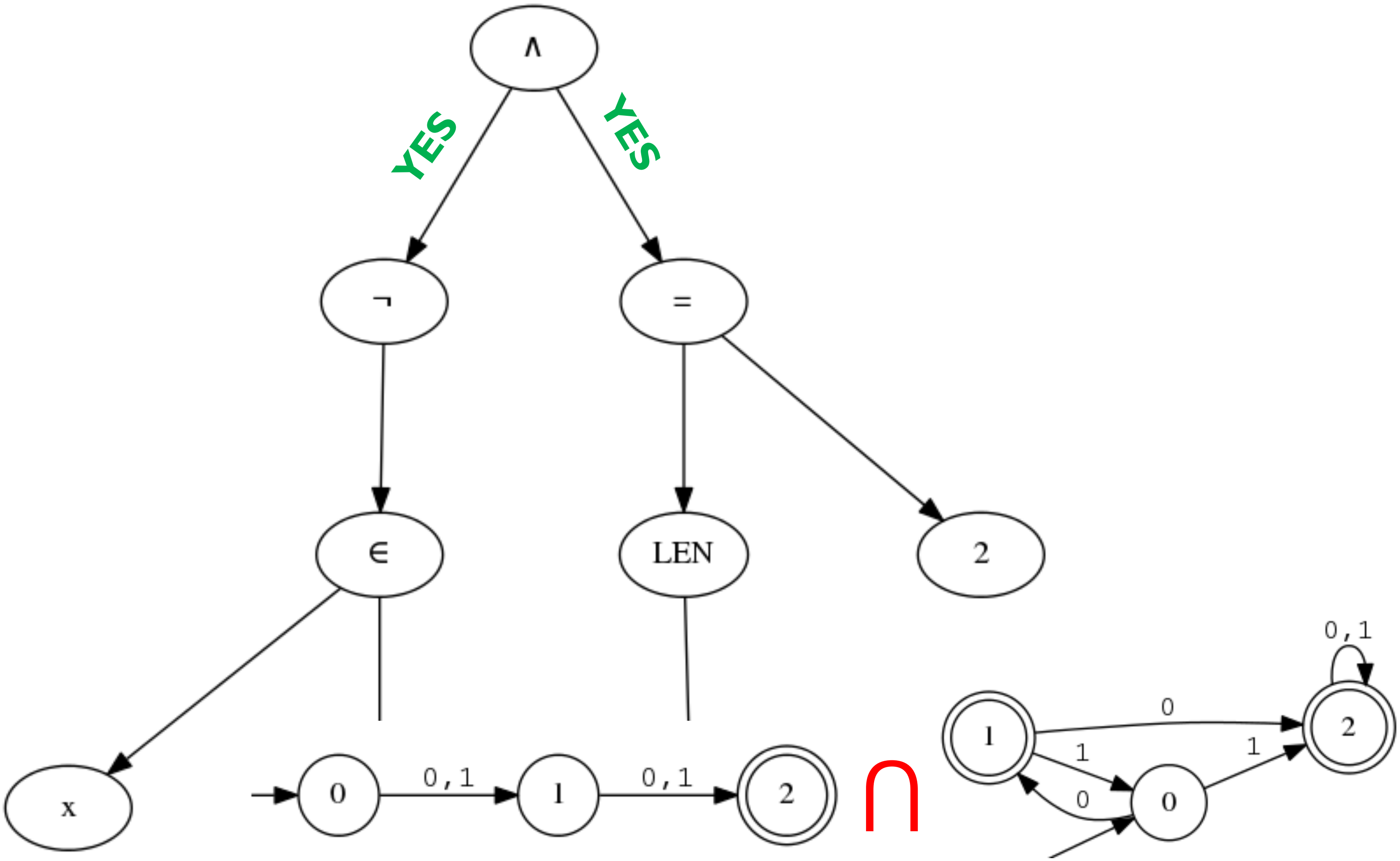

String Automata Construction

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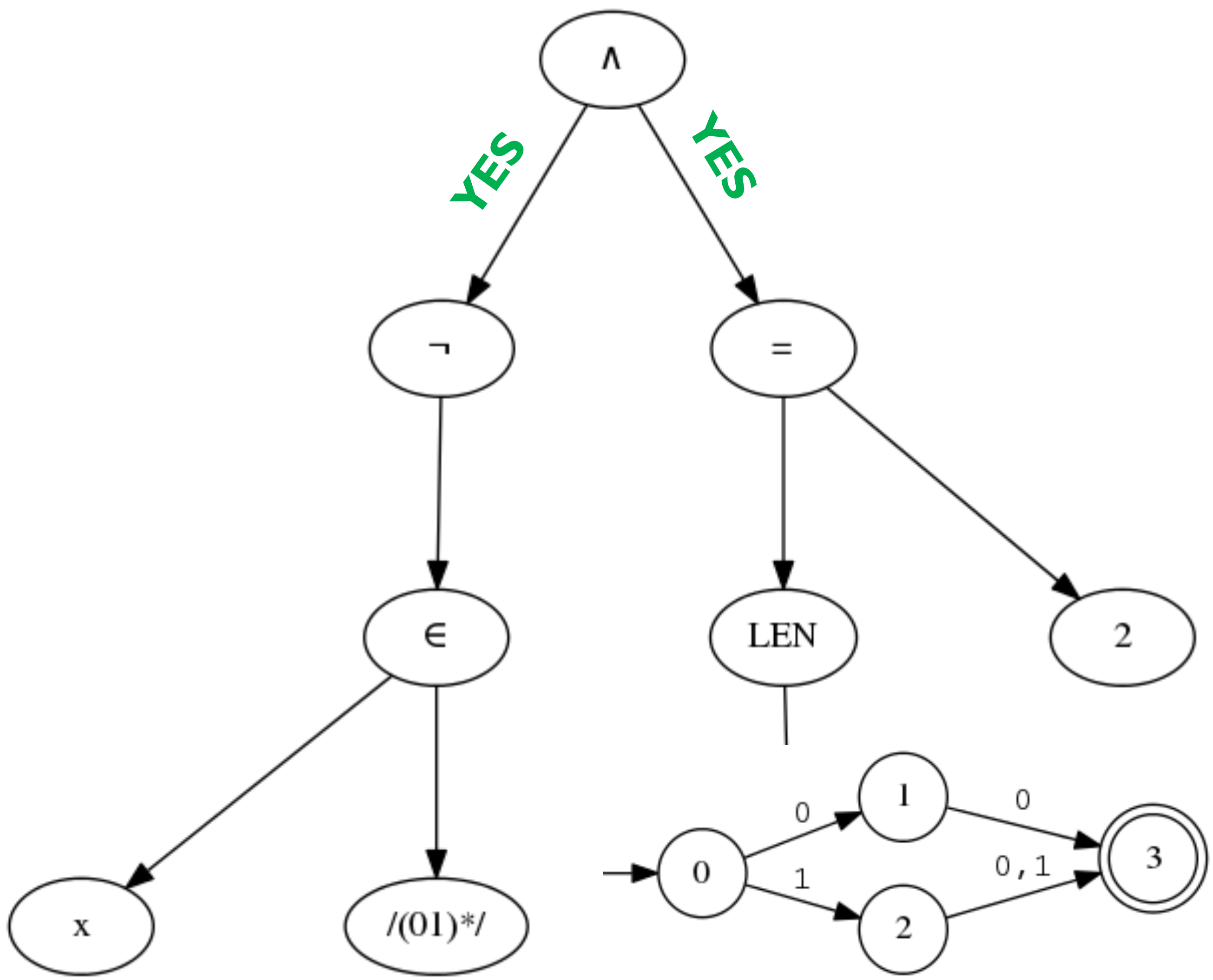

String Automata Construction

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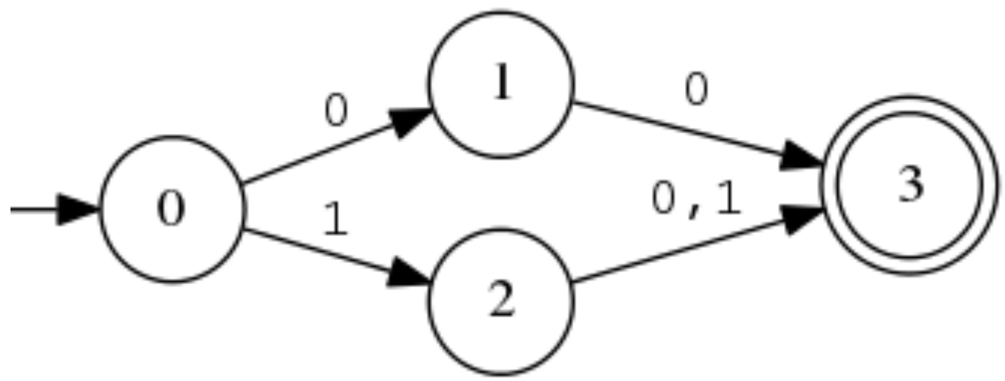


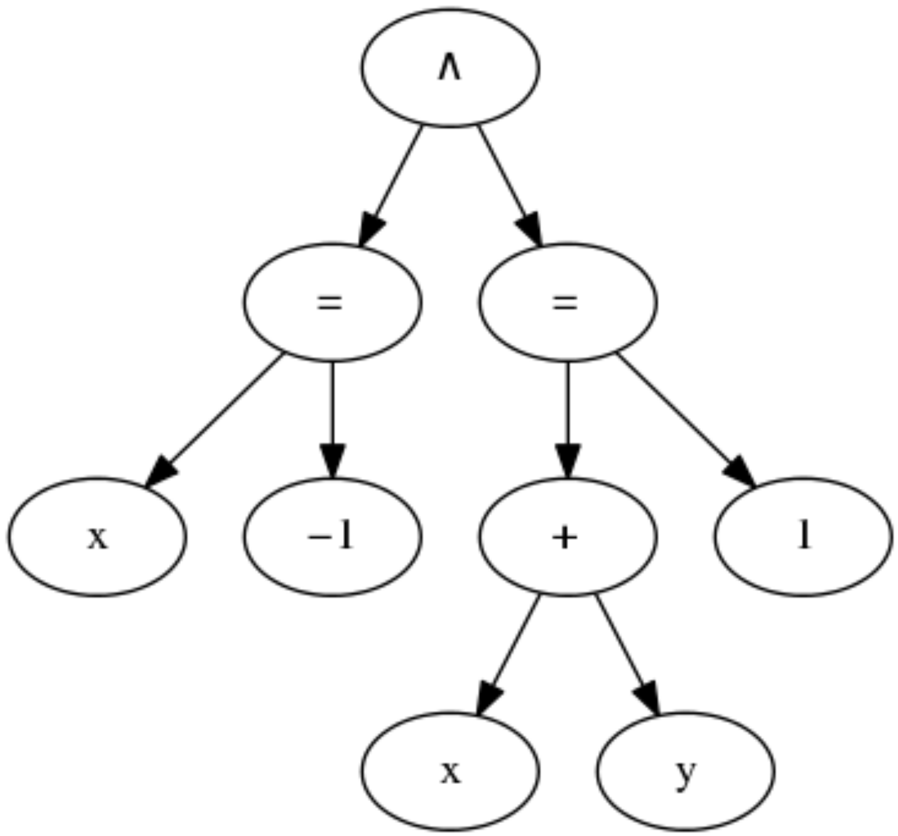
00, 10, 11

Integer Constraints

$$
\begin{array}{lcl}
C & \longrightarrow & bterm \\
\\
bterm & \longrightarrow & \mathtt{v} \mid \mathtt{true} \mid \mathtt{false} \\
& \mid & \neg bterm \mid bterm \wedge bterm \mid bterm \vee bterm \mid (bterm) \\
& \mid & sterm = sterm \\
& \mid & \mathtt{match}(sterm, sterm) \\
& \mid & \mathtt{contains}(sterm, sterm) \\
& \mid & \mathtt{begins}(sterm, sterm) \\
& \mid & \mathtt{ends}(sterm, sterm) \\
& \mid & iterm = iterm \mid iterm < iterm \mid iterm > iterm \\
\\
iterm & \longrightarrow & \mathtt{v} \mid \mathtt{n} \\
& \mid & iterm + iterm \mid iterm - iterm \mid iterm \times \mathtt{n} \mid (iterm) \\
& \mid & \mathtt{length}(sterm) \mid \mathtt{toint}(sterm) \\
& \mid & \mathtt{indexof}(sterm, sterm) \\
& \mid & \mathtt{lastindexof}(sterm, sterm) \\
\\
sterm & \longrightarrow & \mathtt{v} \mid \varepsilon \mid \mathtt{s} \\
& \mid & sterm.sterm \mid sterm|sterm \mid sterm^{*} \mid (sterm) \\
& \mid & \mathtt{charat}(sterm, iterm) \mid \mathtt{tostring}(iterm) \\
& \mid & \mathtt{toupper}(sterm) \mid \mathtt{tolower}(sterm) \\
& \mid & \mathtt{substring}(sterm, iterm, iterm) \\
& \mid & \mathtt{replacefirst}(sterm, sterm, sterm) \\
& \mid & \mathtt{replacelast}(sterm, sterm, sterm) \\
& \mid & \mathtt{replaceall}(sterm, sterm, sterm)
\end{array}
$$

Integer Automata Construction

$$C \equiv x = -1 \wedge \mathrm{x} + \mathrm{y} = 1$$

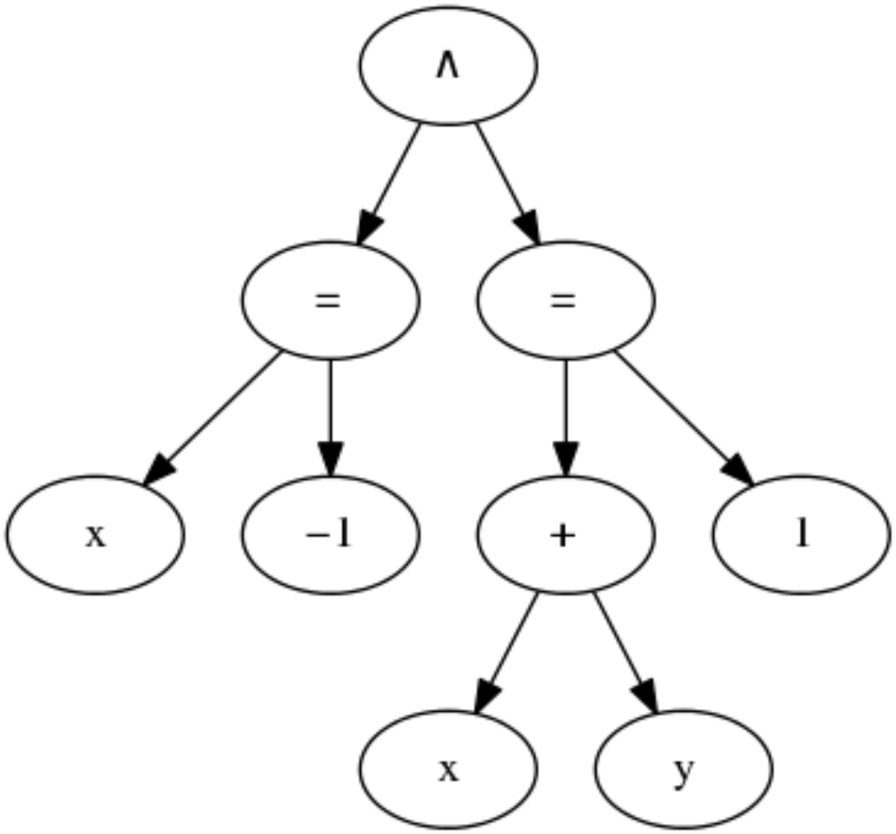


Integer Automata Construction

$$C \equiv x = -1 \wedge \mathrm{x} + \mathrm{y} = 1$$

$$C_1 \equiv x + 0 * y + 1 = 0 \Rightarrow [1\ 0\ 1]$$

$$C_2 \equiv x + y\ - 1 = 0 \Rightarrow [1\ 1 - 1]$$



Integer Automata Construction

$$C \equiv x = -1 \wedge x + y = 1$$

$$C_1 \equiv x + 0 * y + 1 = 0 \Rightarrow [1\ 0\ 1]$$

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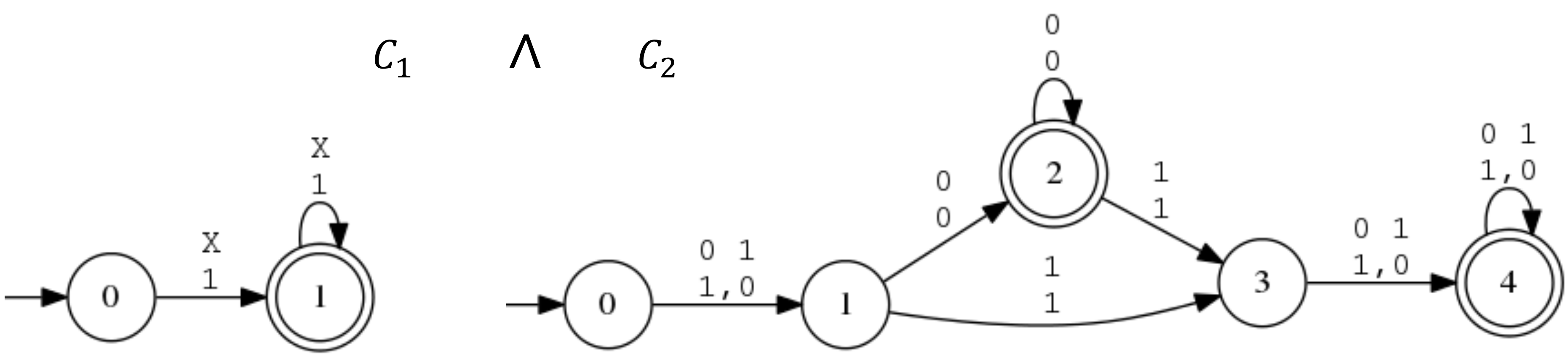
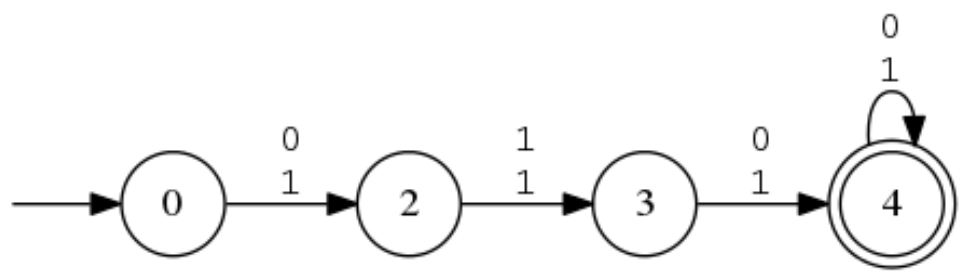


- Using automata construction techniques described in:

C. Bartzis and Tevfik Bultan. Efficient symbolic representations for arithmetic constraints in verification. *Int. J. Found. Comput. Sci.*, 2003

Integer Automata Construction

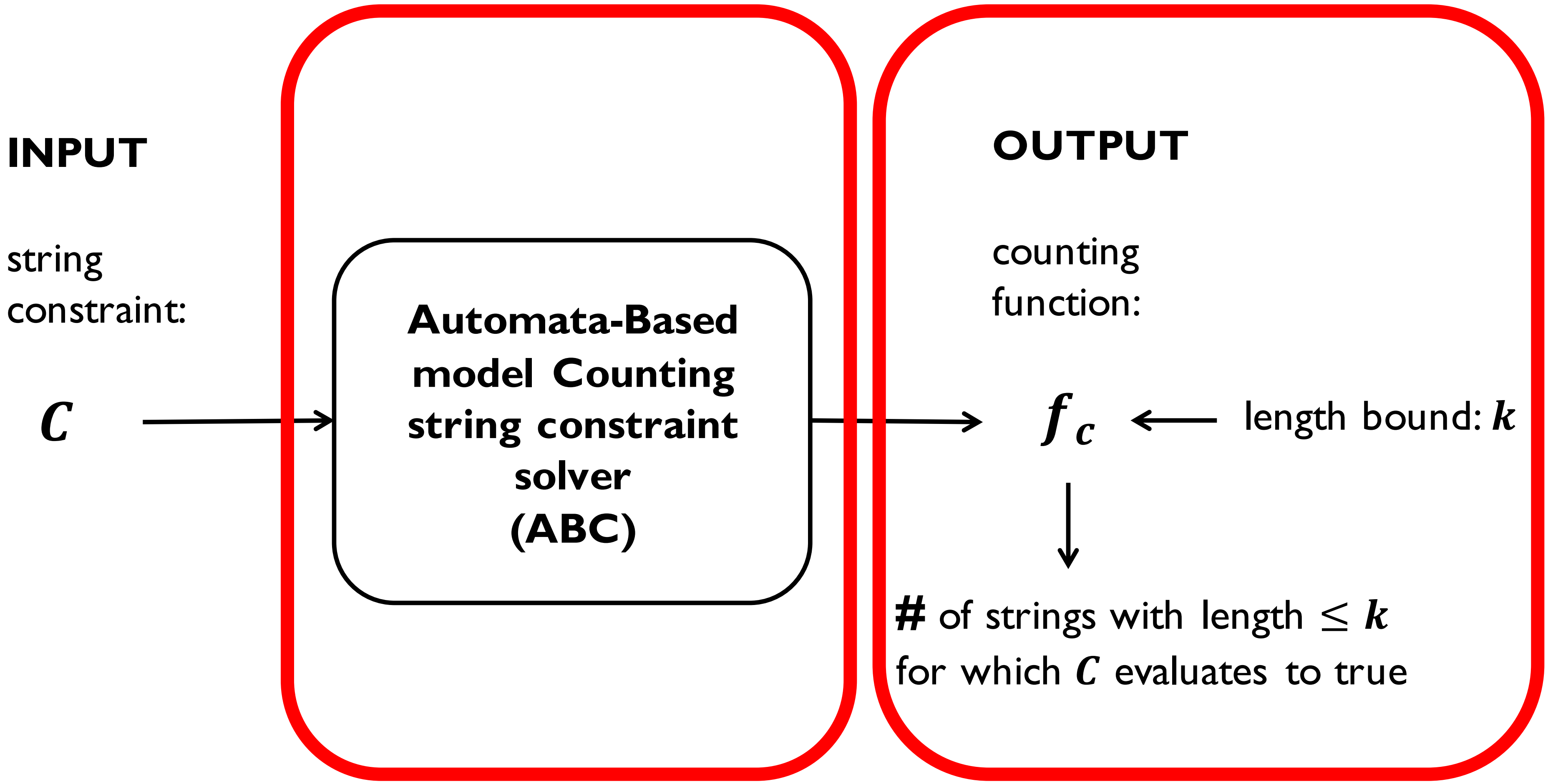
$$C \equiv x = -1 \land \mathrm{x} + \mathrm{y} = 1$$



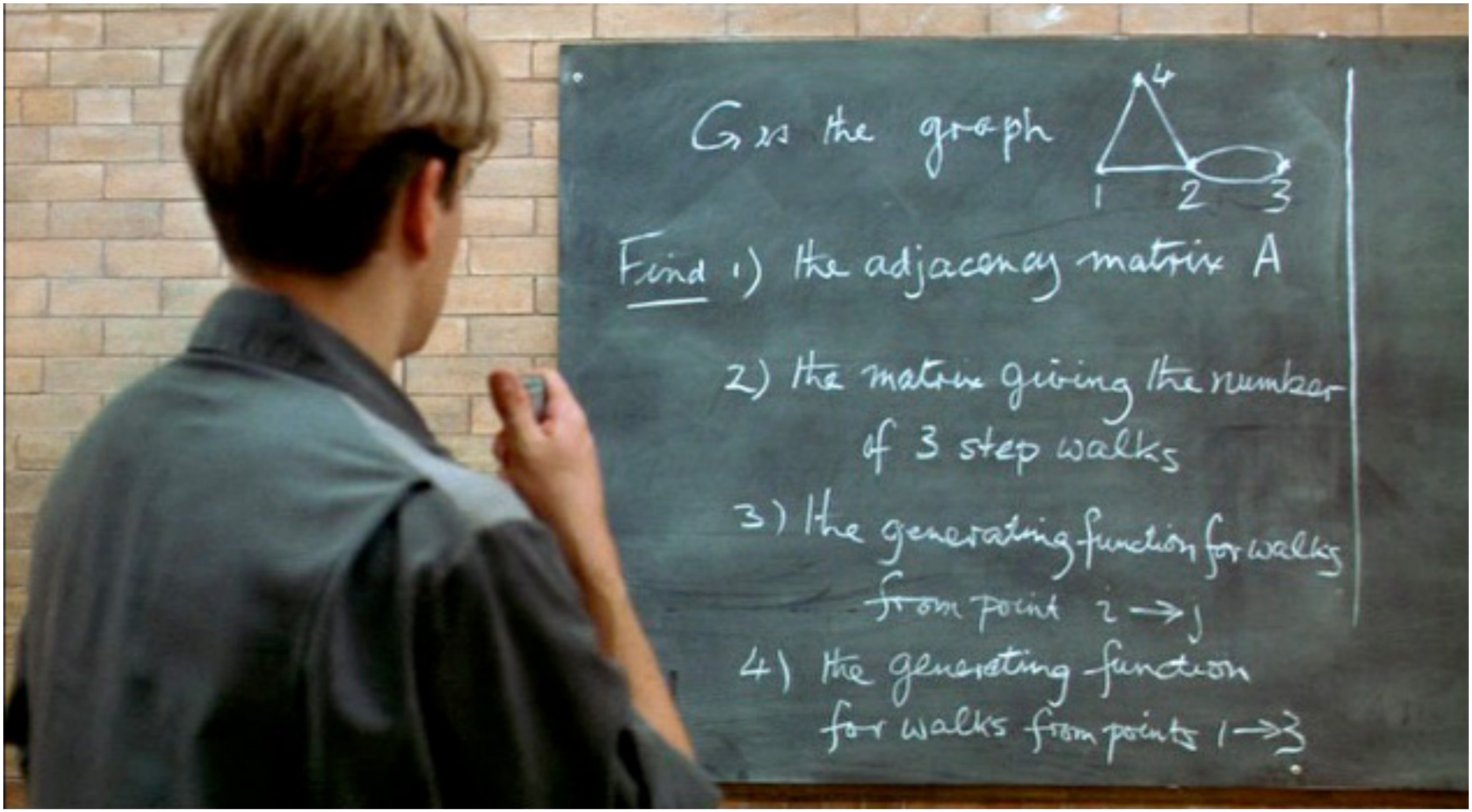
(111, 010) = (−1, 2)

- Conjunction and disjunction is handled by automata product, negation is handled by automata complement

Model Counting String Constraints Solver



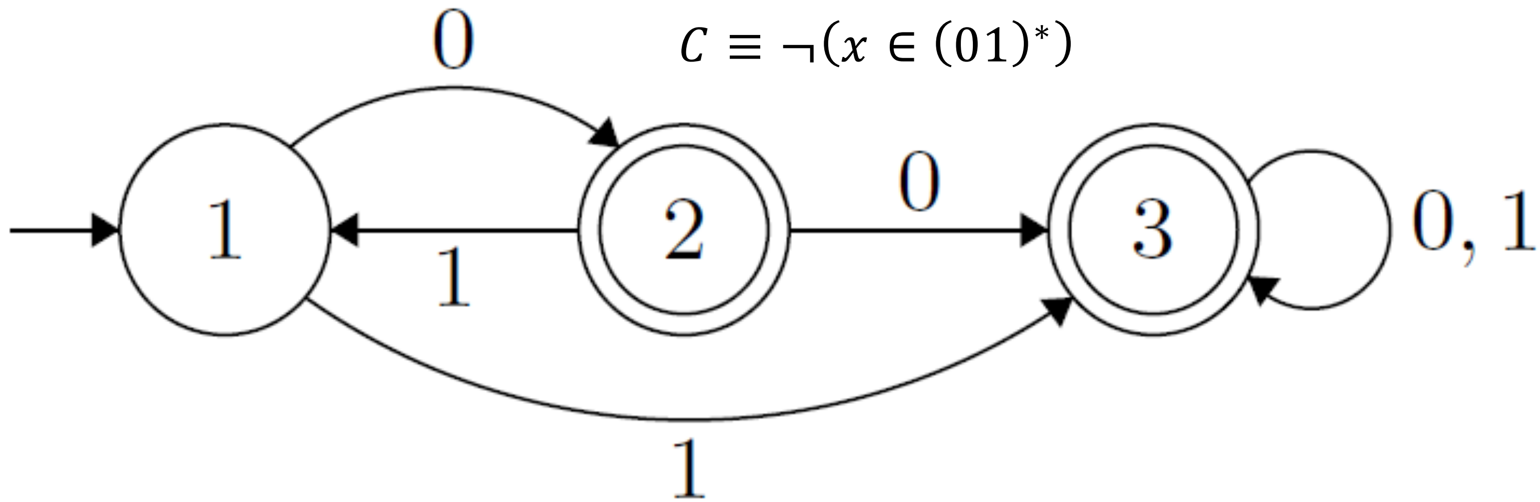
Can you solve it Will Hunting?



Automata-based Model Counting

- Converting constraints to automata reduces the model counting problem to path counting problem in graphs

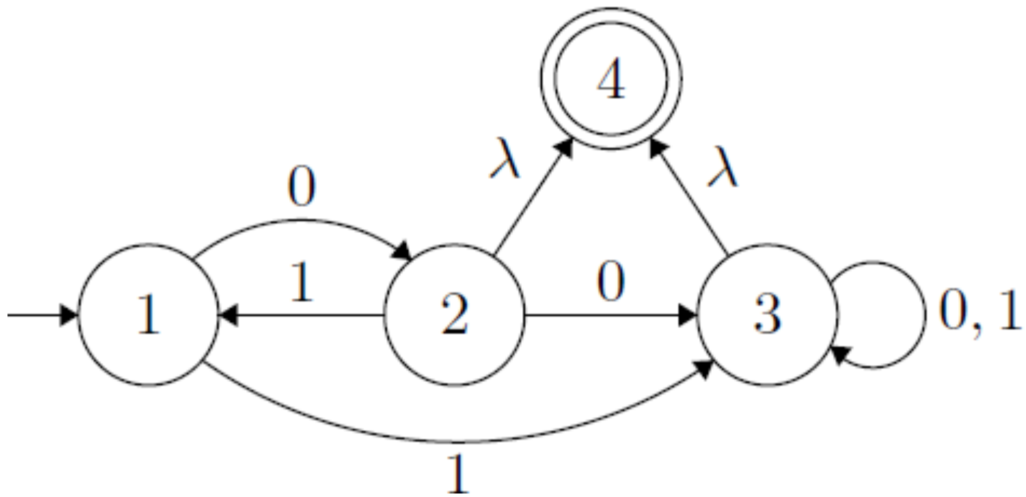
$C \equiv \neg(x \in (01)^*)$



- We will generate a function $f(k)$
  - Given length bound k, it will count the number of paths with length k.
  - $f(0) = 0,\ \{\}$
  - $f(1) = 2, \{0,1\}$
  - $f(2) = 3, \{00,10,11\}$

Path Counting via Matrix Exponentiation

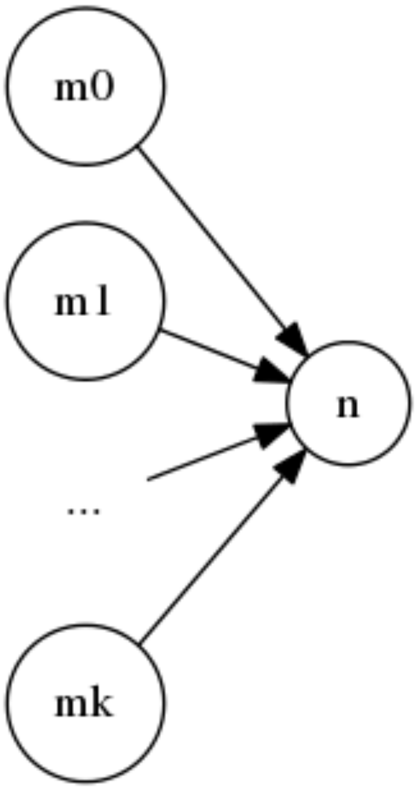
$$C = \neg(x \in (01)^*)$$



$$T = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, T^2 = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 4 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}, T^3 = \begin{bmatrix} 0 & 1 & 7 & 3 \\ 1 & 0 & 7 & 4 \\ 0 & 0 & 8 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}, T^4 = \begin{bmatrix} 0 & 1 & 15 & 8 \\ 1 & 0 & 15 & 7 \\ 0 & 0 & 16 & 8 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$f(0) = 0$ $\quad$ $f(1) = 2$ $\quad$ $f(2) = 3$ $\quad$ $f(3) = 8$

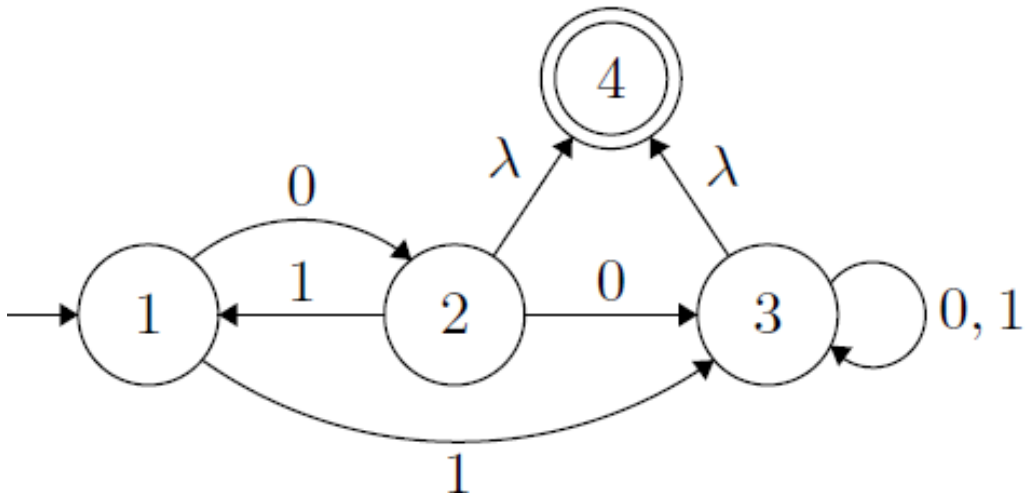
Path Counting via Recurrence Relation



$$f(n,k) = \sum_{(m,n)\in E} f(m,k-1)$$

$$f(0,0) = 1$$
$$f(1,0) = 0$$
$$f(2,0) = 0$$
...
$$f(i,0) = 0$$

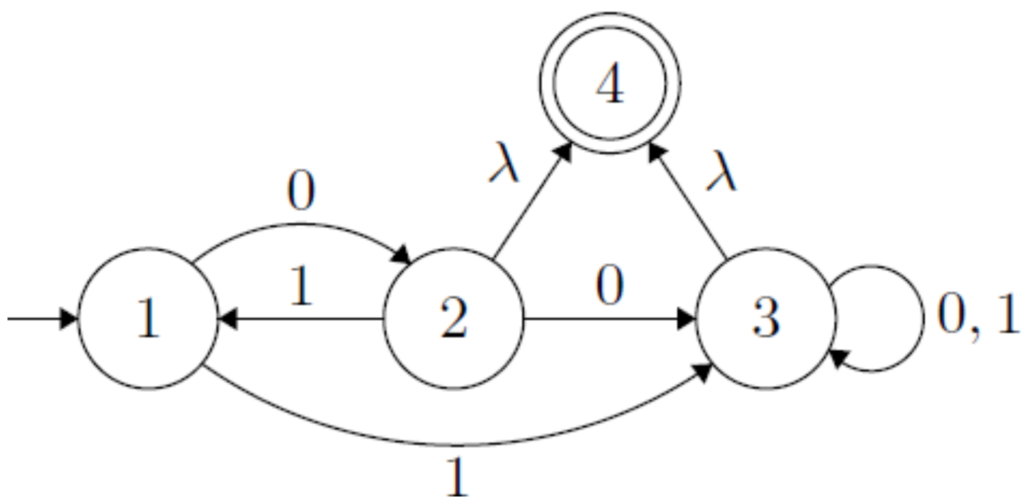
Path Counting via Recurrence Relation



$$f(4,k) = f(2,k-1) + f(3,k-1)$$
$$f(3,k) = f(1,k-1) + f(2,k-1) + f(3,k-1)$$
$$f(2,k) = f(1,k-1)$$
$$f(1,k) = f(2,k-1)$$
$$f(1,0) = 1, f(2,0) = 0, f(3,0) = 0, f(4,0) = 0$$

Path Counting via Recurrence Relation

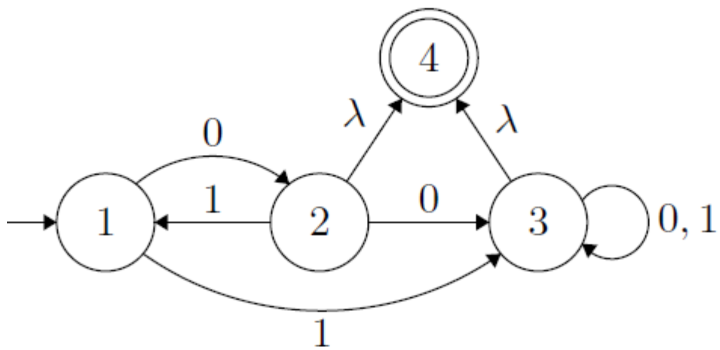
- We can solve system of recurrence relations for final node



$$f(0) = 0, f(1) = 2, f(2) = 3$$
$$f(k) = 2f(k-1) + f(k-2) - 2f(k-3)$$

Counting Paths via Generating Functions

- We can compute a generating function, $g(z)$, for a DFA from the associated matrix



$$T = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$g(z) = (-1)^n \frac{\det(I - zT{:}\, n+1,1)}{z \times \det(I - zT)} = \frac{2z - z^2}{1 - 2z - z^2 + 2z^3}$$

Counting Paths via Generating Functions

$$g(z) = \frac{2z - z^2}{1 - 2z - z^2 + 2z^3}$$

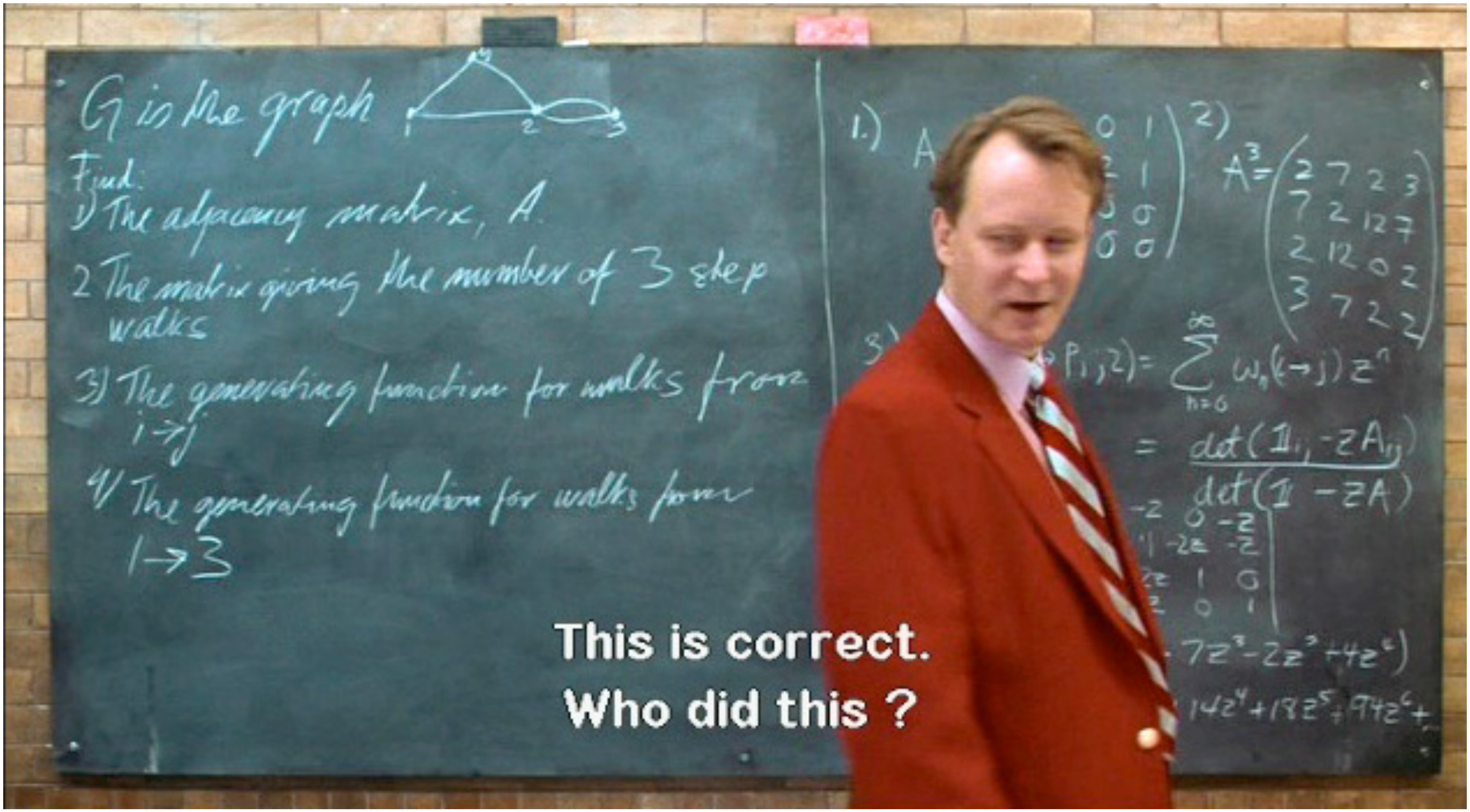
- Each $f(i)$ can be computed by Taylor expansion of $g(z)$

$$g(z) = \frac{g(0)}{0!}z^0 + \frac{g^{(1)}(0)}{1!}z^1 + \frac{g^{(2)}(0)}{2!}z^2 + \cdots + \frac{g^{(n)}(0)}{n!}z^n + \cdots$$

$$g(z) = 0z^0 + 2z^1 + 3z^2 + 8z^3 + 15z^4 + \cdots$$

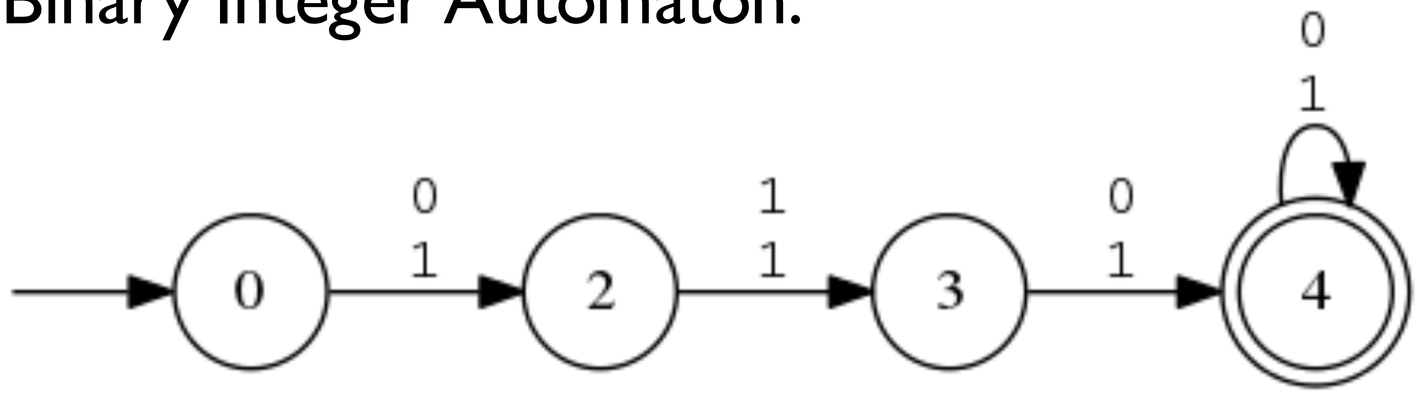
$$g(z) = f(0)z^0 + f(1)z^1 + f(2)z^2 + f(3)z^3 + f(4)z^4 + \cdots$$

Good job Will Hunting!

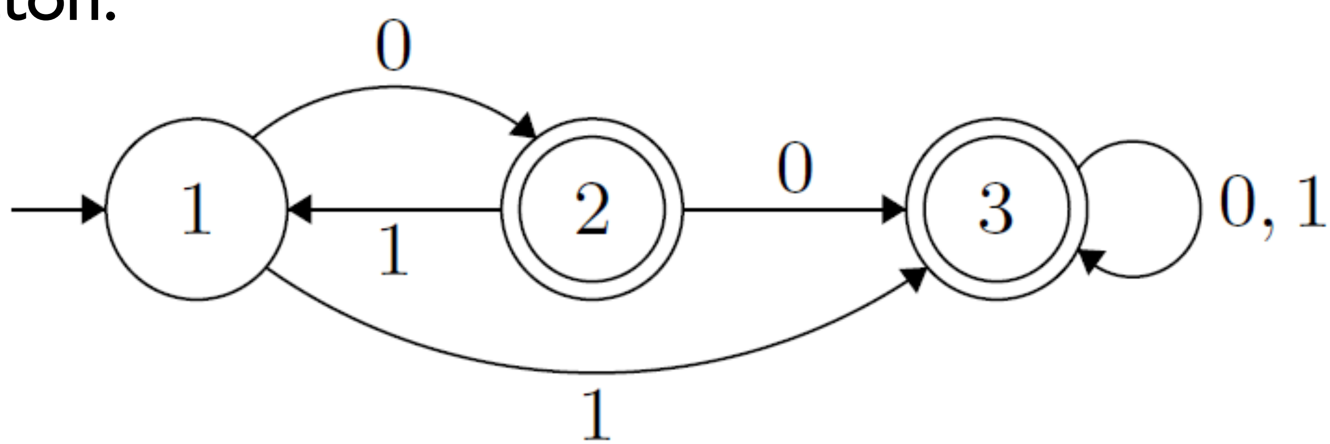


Applicable to Both Automata

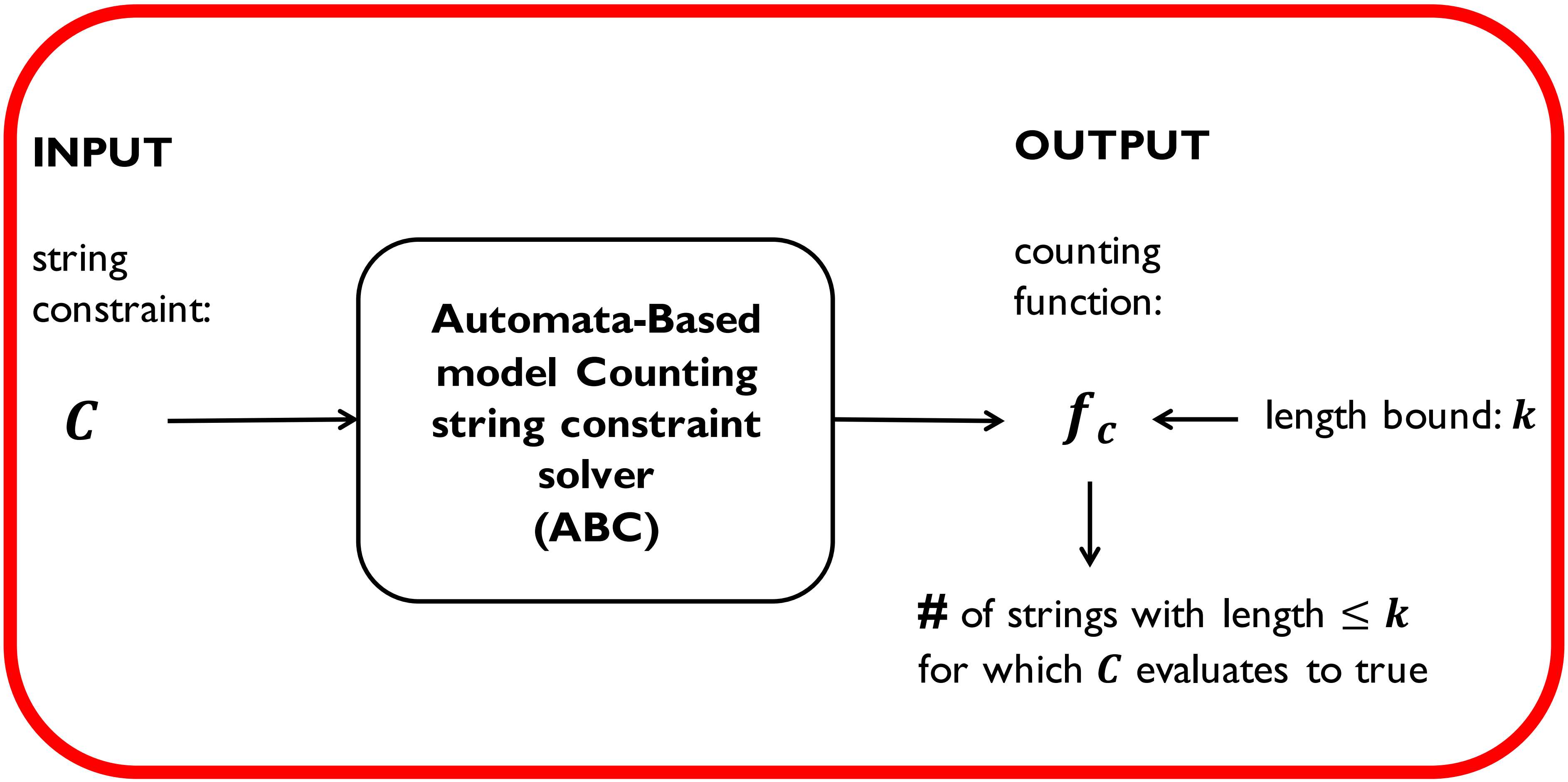
- Multi-track Binary Integer Automaton:



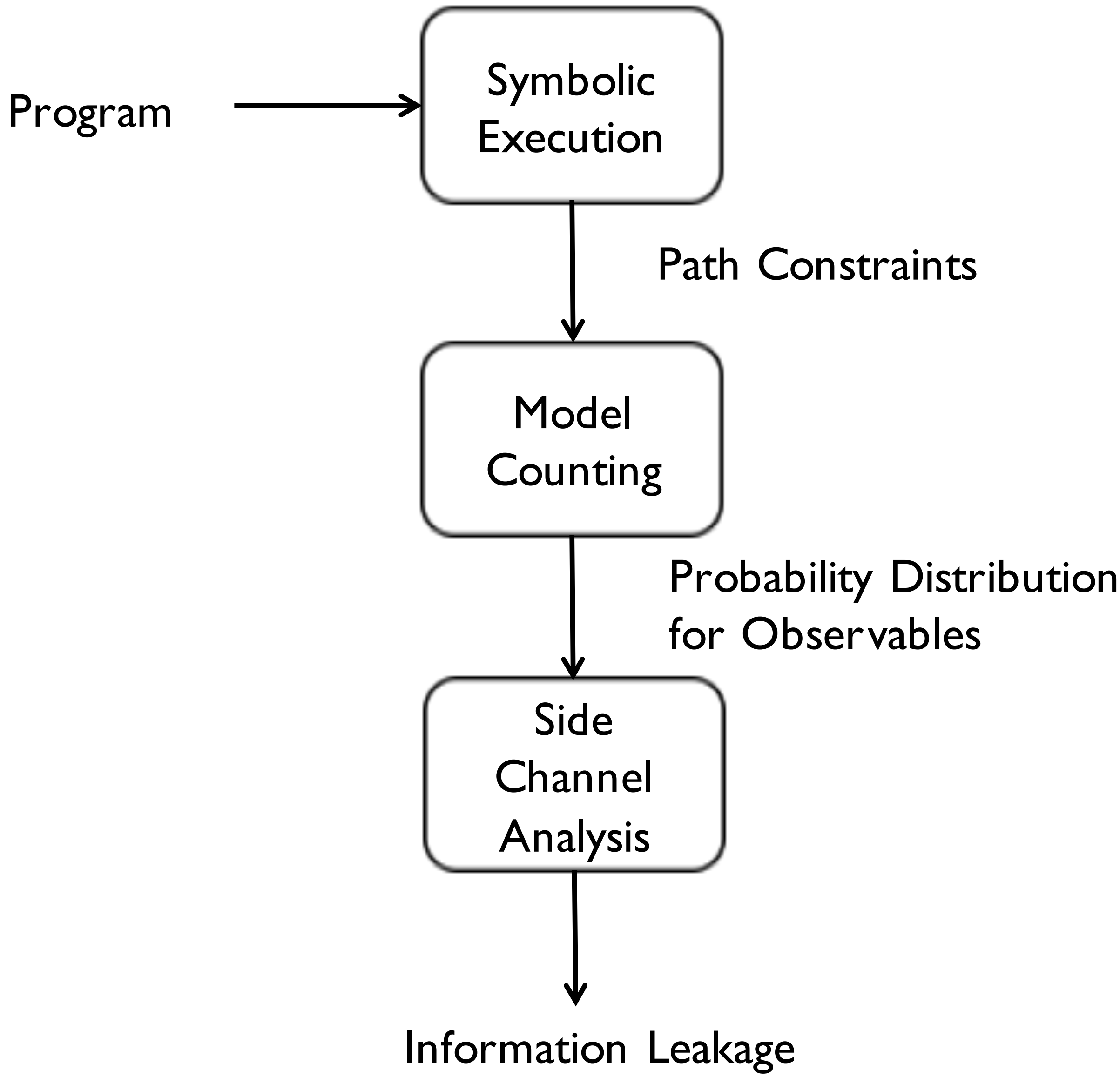
- String Automaton:



Model Counting String Constraints Solver



Overview


Program
Symbolic Execution
Path Constraints
Model Counting
Probability Distribution for Observables
Side Channel Analysis
Information Leakage

A case study

- A web service with a database that contains restricted & unrestricted employee IDs
- Supports SEARCH & INSERT queries

- Question: Is there a side channel in time that a third party can determine the value of a single restricted ID in the database

Code Inspection

- Using code inspection we identified that the SEARCH and INSERT operations are implemented in:

```
class UDPServerHandler

method channelRead0

switch case 1: INSERT

switch case 8: SEARCH
```

SPF Driver

```
public class Driver {
       public static void main(String[] args){
        BTree tree = new BTree(10);
        CheckRestrictedID checker = new CheckRestrictedID();
        // create two concrete unrestricted ids
        int id1 = 64, id2 = 85;
        tree.add(id1, null, false);
        tree.add(id2, null, false);
        // create one symbolic restricted id
        int h = Debug.makeSymbolicInteger("h");
        Debug.assume(h!=id1 && h!=id2);
        tree.add(h, null, false);
        checker.add(h);
        UDPServerHandler handler = new UDPServerHandler(tree,checker);
        int key = Debug.makeSymbolicInteger("key");
        handler.channelRead0(8,key);  // send a search query with
       }                             // with search range 50 to 100
}
```

SPF Output

```
>>>>> There are 5 path conditions and 5 observables
cost: 9059
(assert (<= h 100))
(assert (> h 85))
(assert (> h 64))
(assert (not (= h 85)))
(assert (not (= h 64)))
Count = 15
-----------------------
cost: 8713
(assert (<= h 85))
(assert (> h 64))
(assert (not (= h 85)))
(assert (not (= h 64)))
Count = 20
-----------------------
cost: 7916
(assert (> h 100))
(assert (> h 85))
(assert (> h 64))
(assert (not (= h 85)))
(assert (not (= h 64)))
Count = 923
-----------------------
cost: 8701
(assert (>= h 50))
(assert (<= h 64))
(assert (not (= h 85)))
(assert (not (= h 64)))
Count = 14
-----------------------
cost: 7951
(assert (< h 50))
(assert (<= h 64))
(assert (not (= h 85)))
(assert (not (= h 64)))
Count = 50
-----------------------
*************************************************************
PC equivalance class model counting results.
*************************************************************
Cost: 9059    Count:     15  Probability:  0.014677
Cost: 8713    Count:     20  Probability:  0.019569
Cost: 7916    Count:    923  Probability:  0.903131
Cost: 8701    Count:     14  Probability:  0.013699
Cost: 7951    Count:     50  Probability:  0.048924

Domain Size: 1022
Single Run Leakage:  0.6309758112933285
```

Observation & Proposed Attack

- SEARCH operation:

takes longer when the secret is within the search range (9059, 8713, 8701 byte code instructions)

as opposed to the case when the secret is out of the search range (7916, 7951 byte code instructions)

- Proposed attack:

Measure the time it takes for the search operation to figure out if there is a secret within the search range.

Attack

- Binary search on the ranges of the IDs
- Send two search queries at a time and compare their execution time.
- Refine the search range based on the result.

```
min= 0; max=MAX_ID        //assume MAX_ID is a power of 2
while ( min < max )
{
  half = (max-min-1)/2;
  if (time(search(min.. min+half-1) > time(search(min+half .. max)))
     max = min+half-1;
  else
     min = min+half;
}
```

Attack Output

Running [0, 40000000] at 0.
Comparing 467821 vs 612252...
Running [20000000,40000000] at 2.
Comparing 400377 vs 333665...
Running [20000000,30000000] at 4.
Comparing 200603 vs 237025...
Running [25000000,30000000] at 6.
Comparing 163564 vs 115072...
Running [25000000,27500000] at 8.
Comparing 95736 vs 37388...
Running [25000000,26250000] at 10.
Comparing 85305 vs 30118...
Running [25000000,25625000] at 12.
Comparing 22765 vs 72958...
Running [25312500,25625000] at 14.
Comparing 2147483647 vs 19353...
Running [25312500,25468750] at 16.
Comparing 517 vs 2147483647...
Running [25390625,25468750] at 18.
Comparing 317 vs 2147483647...
Running [25429687,25468750] at 20.
Comparing 2147483647 vs 302...
Running [25429687,25449218] at 22.
Comparing 2147483647 vs 287...
Running [25429687,25439452] at 24.
Comparing 336 vs 2147483647...
Running [25434569,25439452] at 26.
Comparing 300 vs 2147483647...
Running [25437010,25439452] at 28.
Comparing 2147483647 vs 265...
Running [25437010,25438231] at 30.
Comparing 2147483647 vs 328...
Running [25437010,25437620] at 32.
Comparing 280 vs 2147483647...
Running [25437315,25437620] at 34.
Comparing 293 vs 2147483647...
Running [25437467,25437620] at 36.
Comparing 2147483647 vs 281...
Running [25437467,25437543] at 38.
Comparing 2147483647 vs 613...
Running [25437467,25437505] at 40.
Comparing 2147483647 vs 258...
Running [25437467,25437486] at 42.
Comparing 2147483647 vs 291...
Running [25437467,25437476] at 44.
Comparing 362 vs 2147483647...
Running [25437471,25437476] at 46.
Comparing 311 vs 2147483647...
Running [25437473,25437476] at 48.
Comparing 2147483647 vs 2147483647...
Checking oracle for: 25437474...true
Checking oracle for: 25437475...false

Multi-Run Analysis

- The side channel analysis I discussed so far is for analyzing a single execution of a program
- Can we do model multi-run analysis?
- Adversary runs the program on multiple inputs one after another
- Can we determine the amount of information leakage in such a scenario?

Multi-Run Analysis

- For multi-run analysis we need an adversary model
  - Adversary behavior influences the analysis

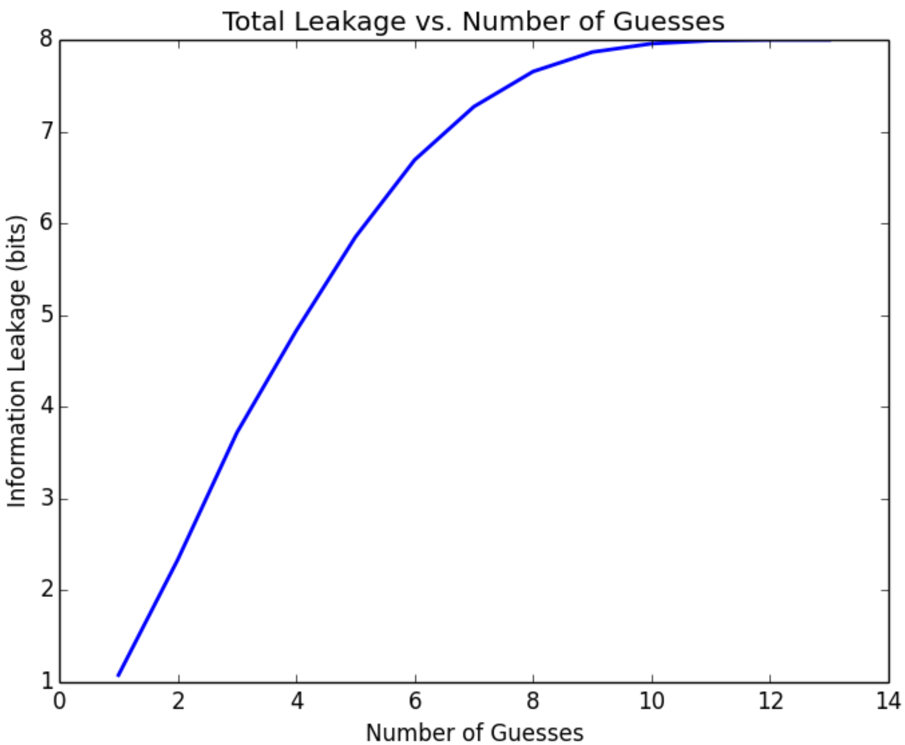
- It would make sense to calculate the leakage for the best adversary

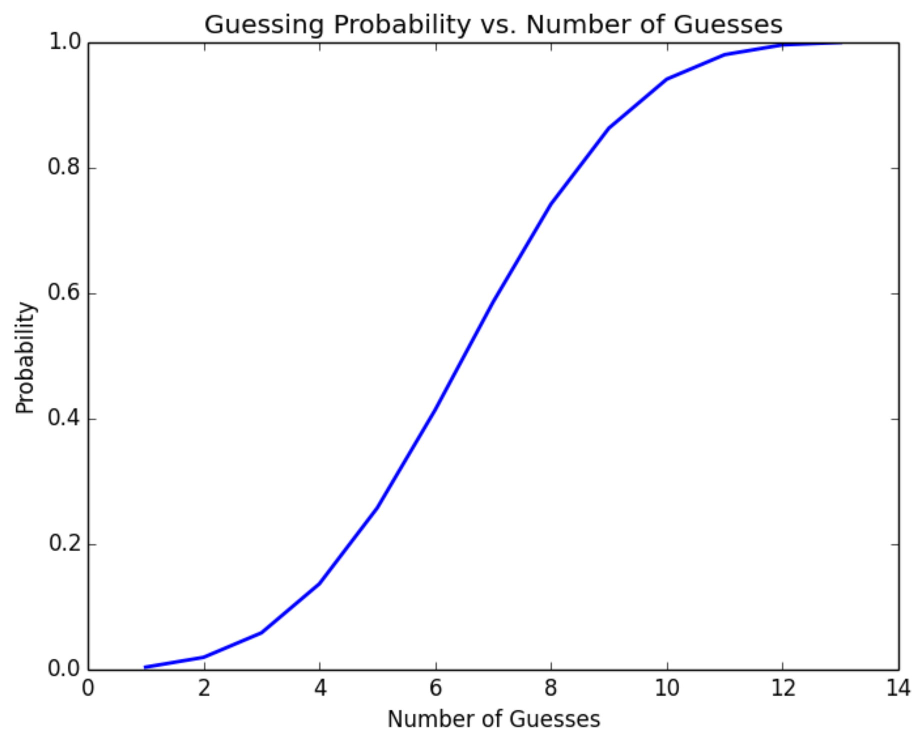
- For a class of side channels called "segmented oracles" we can use symbolic execution and entropy calculation from a single run to compute the change in the entropy for multiple runs

- This can be used to automatically compute how many tries it will take to reveal the secret.

Results for Password Check

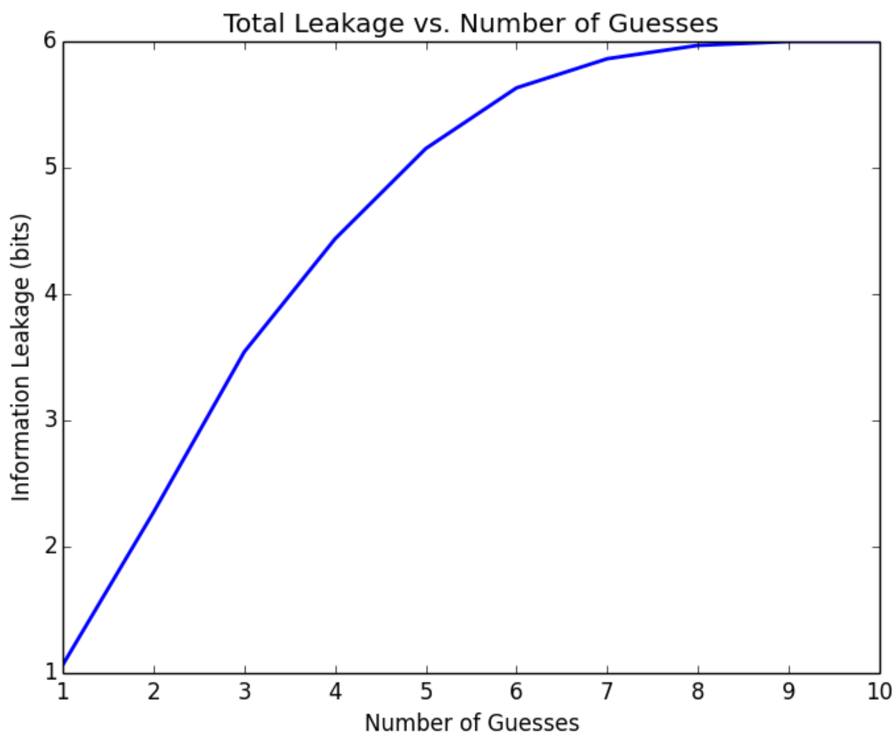
Results for 4 segments with 4 values (8 bits of information)

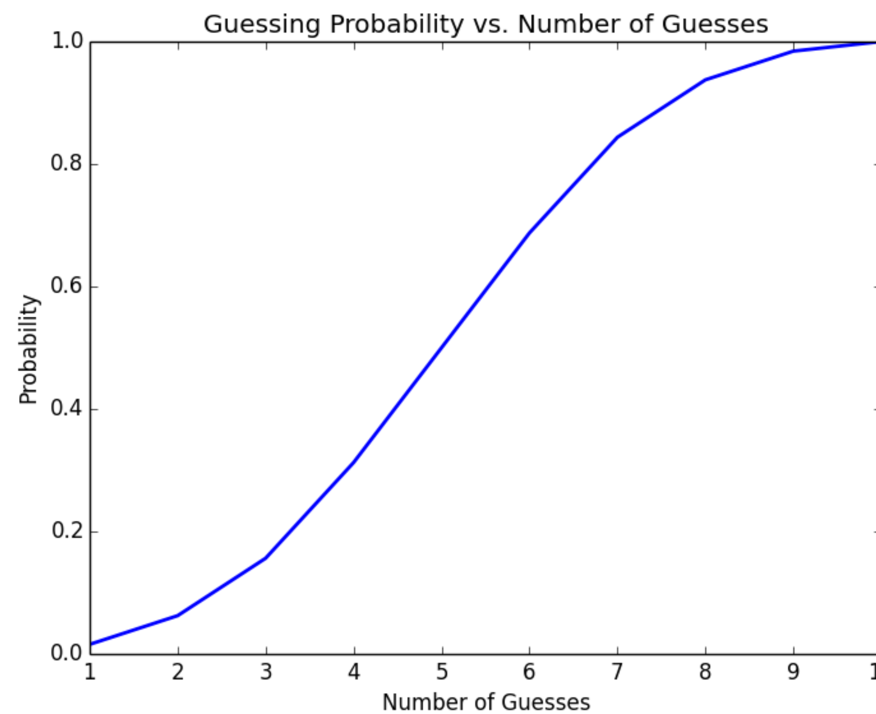




Results for CRIME

Results for 3 segments with 4 values (6 bits of information)



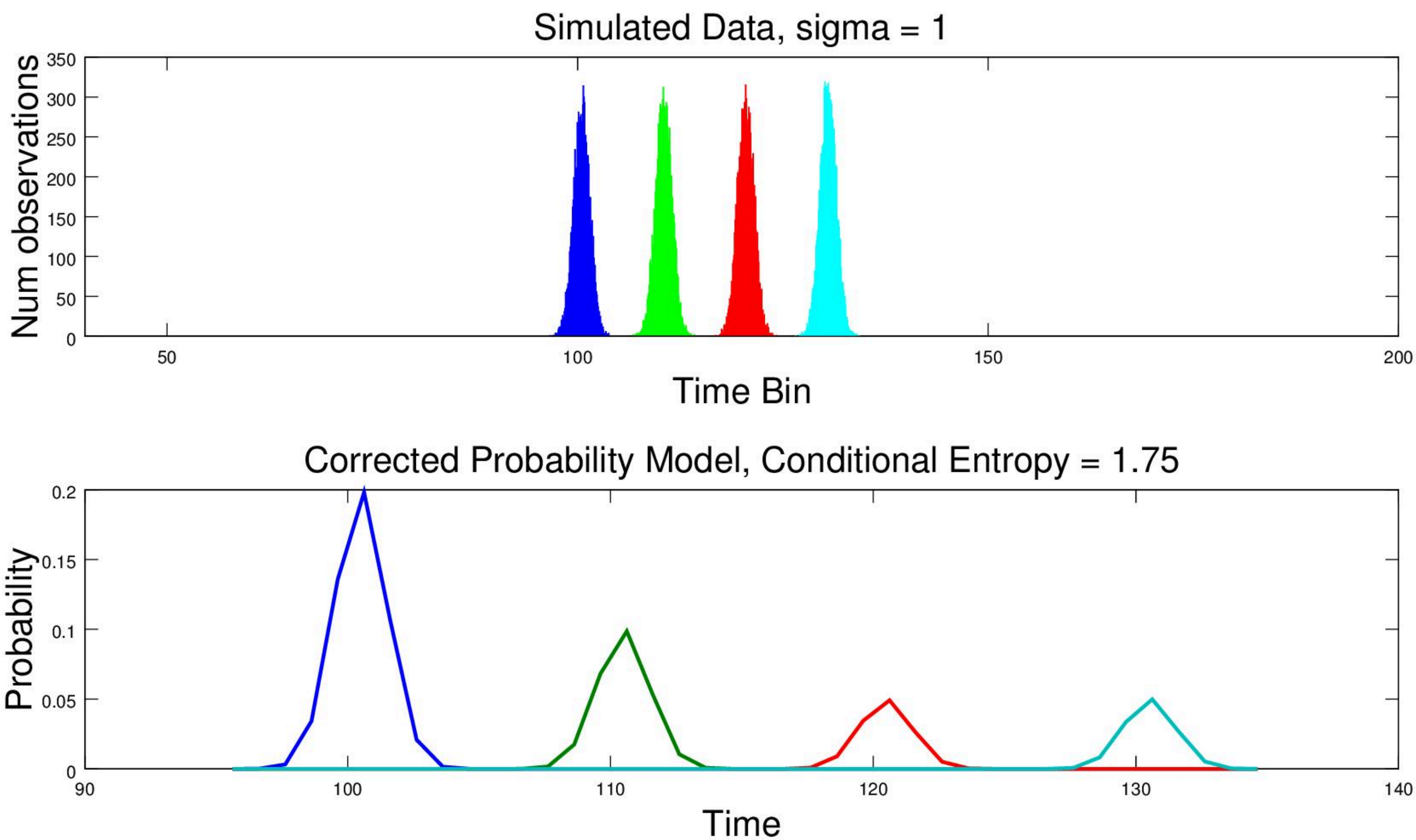


Noisy Observations

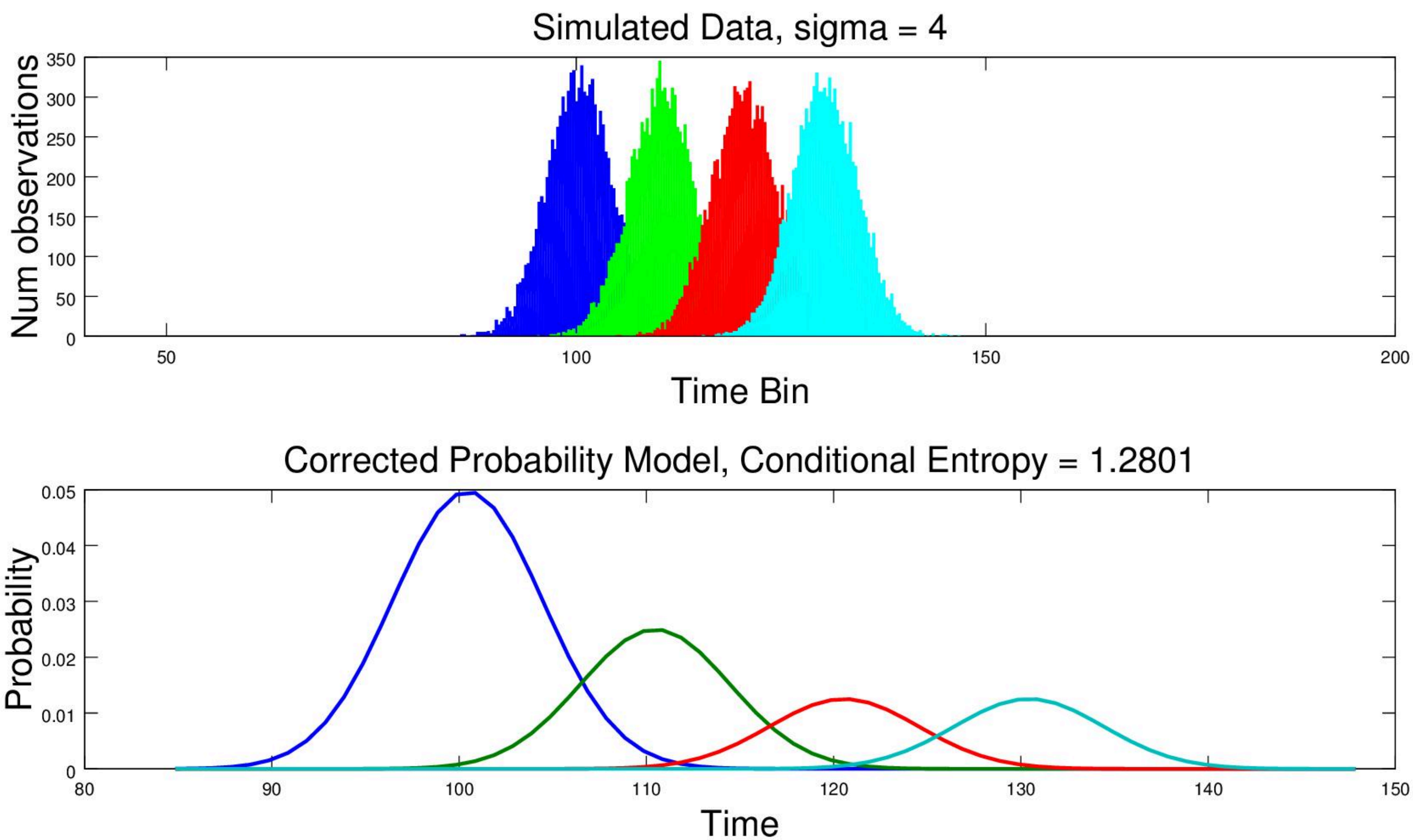
- Entropy computations we have shown so far do not take observation noise into account

- One approach we are investigating to handle noise:
- Assume a noise distribution (for example normal distribution)
- Run fuzzing to observe parameters of the distribution (mean and standard deviation)
- Update entropy calculations using the noise model

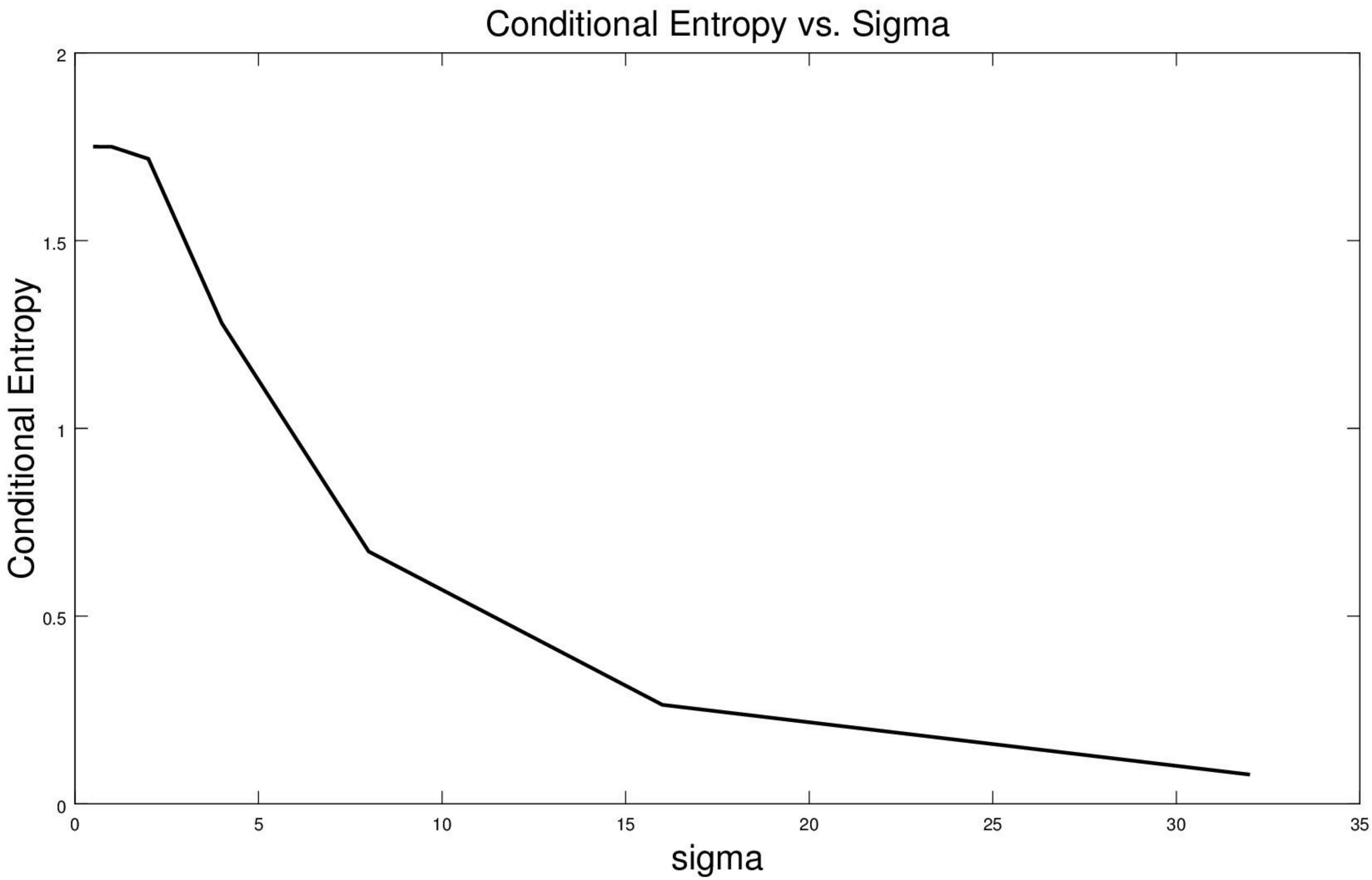
Noisy Observation Simulation



Noisy Observation Simulation



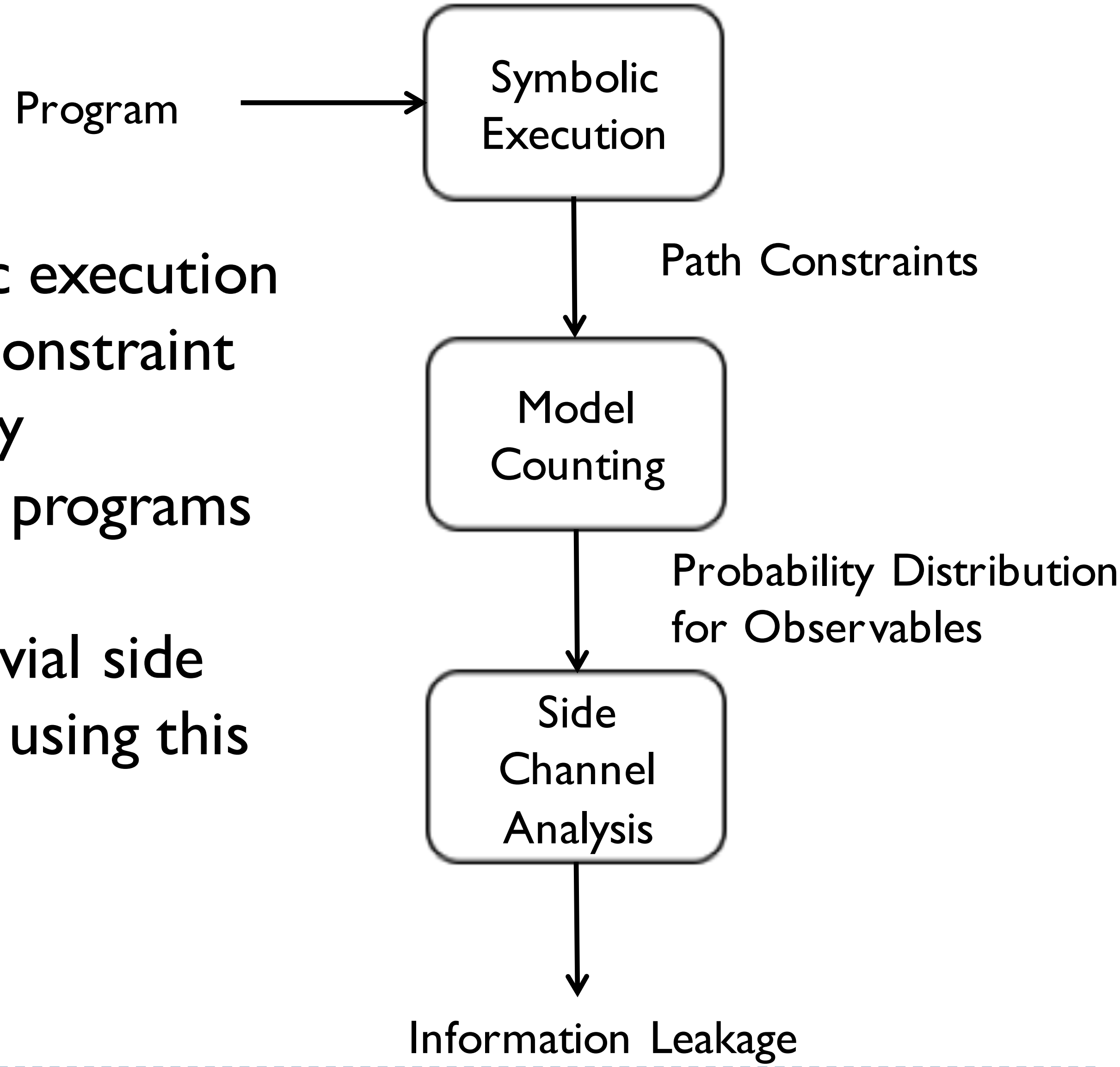
Entropy vs. Noise



Conclusions

- By combining symbolic execution with model counting constraint solvers we can quantify information leakage in programs

- We can detect non-trivial side channel vulnerabilities using this approach



Current & Future Work

- More efficient model counting
- More expressive model counting
- Handling noise in observations
- Attack synthesis

Related work: Quantitative Information Flow

- Geoffrey Smith. "On the Foundations of Quantitative Information Flow." FOSSACS 2009:288-302
- Pasquale Malacaria. "Assessing security threats of looping constructs." POPL 2007:225-235
- David Clark, Sebastian Hunt, Pasquale Malacaria. "A static analysis for quantifying information flow in a simple imperative language." Journal of Computer Security 15(3): 321-371 (2007)
- Jonathan Heusser, Pasquale Malacaria. "Quantifying information leaks in software." ACSAC 2010:261-269
- Quoc-Sang Phan, Pasquale Malacaria, Oksana Tkachuk, Corina S. Pasareanu. "Symbolic quantitative information flow." ACM SIGSOFT Software Engineering Notes 37(6):1-5 (2012)
- Quoc-Sang Phan, Pasquale Malacaria, Corina S. Pasareanu, Marcelo d'Amorim. "Quantifying information leaks using reliability analysis." SPIN 2014: 105-108
- Stephen McCamant, Michael D. Ernst. "Quantitative information flow as network flow capacity." PLDI 2008: 193-205
- Michael Backes, Boris Köpf, Andrey Rybalchenko. "Automatic Discovery and Quantification of Information Leaks." IEEE Symposium on Security and Privacy 2009:141-153
- Shuo Chen, Rui Wang, XiaoFeng Wang, Kehuan Zhang. "Side-Channel Leaks in Web Applications: A Reality Today, a Challenge Tomorrow." IEEE Symposium on Security and Privacy 2010: 191-206
- Goran Doychev, Dominik Feld, Boris Köpf, Laurent Mauborgne, Jan Reineke. "CacheAudit: A Tool for the Static Analysis of Cache Side Channels." USENIX Security 2013:431-446

Related work: Model Counting

- SMC: “A Model Counter For Constraints Over Unbounded Strings.” Loi Luu, Shweta Shinde, Prateek Saxena.
- Latte, Barvinok:“A Polynomial Time Algorithm for Counting Integral Points in Polyhedra When the Dimension Is Fixed.” Alexander I. Barvinok
- “Effective lattice point counting in rational convex polytopes.” Jesús A. De Loerab, Raymond Hemmeckeb, Jeremiah Tauzera, Ruriko Yoshidab.
- “From Weighted to Unweighted Model Counting.” Supratik Chakraborty, Dror Fried, Kuldeep S. Meel, Moshe Y. Vardi.
- “Algorithmic Improvements in Approximate Counting for Probabilistic Inference.” From Linear to Logarithmic SAT Calls Supratik Chakraborty, Kuldeep S. Meel, Moshe Y. Vardi.
- “Approximate Probabilistic Inference via Word-Level Counting.” Supratik Chakraborty, Kuldeep S. Meel, Rakesh Mistry, Moshe Y. Vardi