Labor Supply

Econ 3470 Lecture 4

Labor Economics

2016-2017 Term 1

Outline

Basic Static Model of Labor Supply

Assumptions:

- Agents maximize own utility of the simple form $U = U(C, L)$, where C is consumption, L is leisure.
- All workers live one period.
- Workers earn a wage W per unit of time.
- Each worker is endowed with one unit of time.
- Each worker has unearned income V even if the worker does not work.
- Price of consumption goods is P.

Basic Static Model of Labor Supply

Objective function of the worker

$$\max_{C,L} U(C, L) \quad s.t. \quad V + W(1 - L) = PC$$

$$\mathfrak{L} = U(C, L) + \lambda[V + W(1 - L) - PC]$$

Basic Static Model of Labor Supply

Assuming interior solution, the first order conditions are:

$$\frac{\partial \mathfrak{L}}{\partial C} = 0 \Rightarrow U_C - \lambda P = 0$$
$$\frac{\partial \mathfrak{L}}{\partial L} = 0 \Rightarrow U_L - \lambda W = 0$$
$$\frac{\partial \mathfrak{L}}{\partial \lambda} = 0 \Rightarrow V + W(1 - L) - PC = 0$$

Basic Static Model of Labor Supply

$$\frac{U_L}{U_C} = \frac{W}{P}$$

- $\frac{U_L}{U_C}$ - MRS: amount of C willing to give up for 1 unit of L.
- $\frac{W}{P}$ - price ratio: number of units of C obtained from giving up 1 unit of L.

when U is fixed

$$U(C, L) = \bar{U}$$
$$U_L dL + U_C dC = 0$$
$$\frac{dC}{dL} = -\frac{U_L}{U_C}$$

Basic Static Model of Labor Supply

Total differentiate FOC:

$$U_{CC}dC + U_{CL}dL - Pd\lambda = \lambda dP$$
$$U_{LC}dC + U_{LL}dL - Wd\lambda = \lambda dW$$
$$-PdC - WdL = -(1-L)dW - dV + CdP$$

Basic Static Model of Labor Supply

Slutsky equation: effect of a wage increase on the consumption of leisure

$$\frac{dL}{dW} = (1 - L)\frac{dL}{dV} - \frac{U_C^2}{\lambda \Delta}$$

where $\Delta = \begin{vmatrix} U_{CC} & U_{CL} & -P \\ U_{LC} & U_{LL} & -W \\ -P & -W & 0 \end{vmatrix} > 0$

Total effect = income effect + substitution effect

Basic Static Model of Labor Supply

The substitution effect is

$$\frac{dL}{dW}|_U = -\frac{U_C^2}{\lambda \Delta} < 0$$

The income effect is $(1 - L)\frac{dL}{dV}$

> 0 if L is normal

< 0 if L is inferior

Basic Static Model of Labor Supply

Hours of work $H = 1 - L$, is a function of P, V and W.

- If L is inferior, then both the income and substitution effects are negative.
  Increasing W will reduce consumption of leisure and increase the hours of work.
- If L is normal, the two effects are offsetting, and the net result is ambiguous.

Basic Static Model of Labor Supply

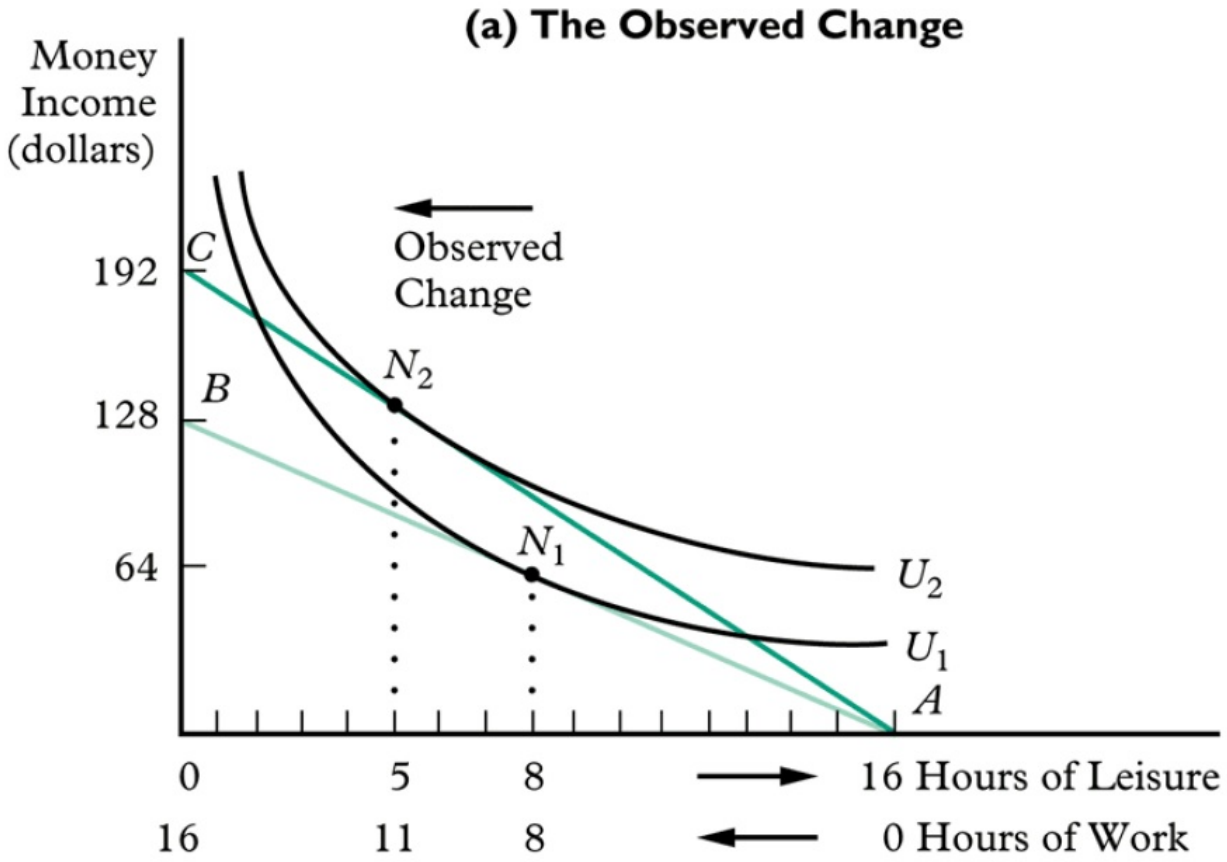


Figure 1: Isolating Income and Substitution Effects

Basic Static Model of Labor Supply

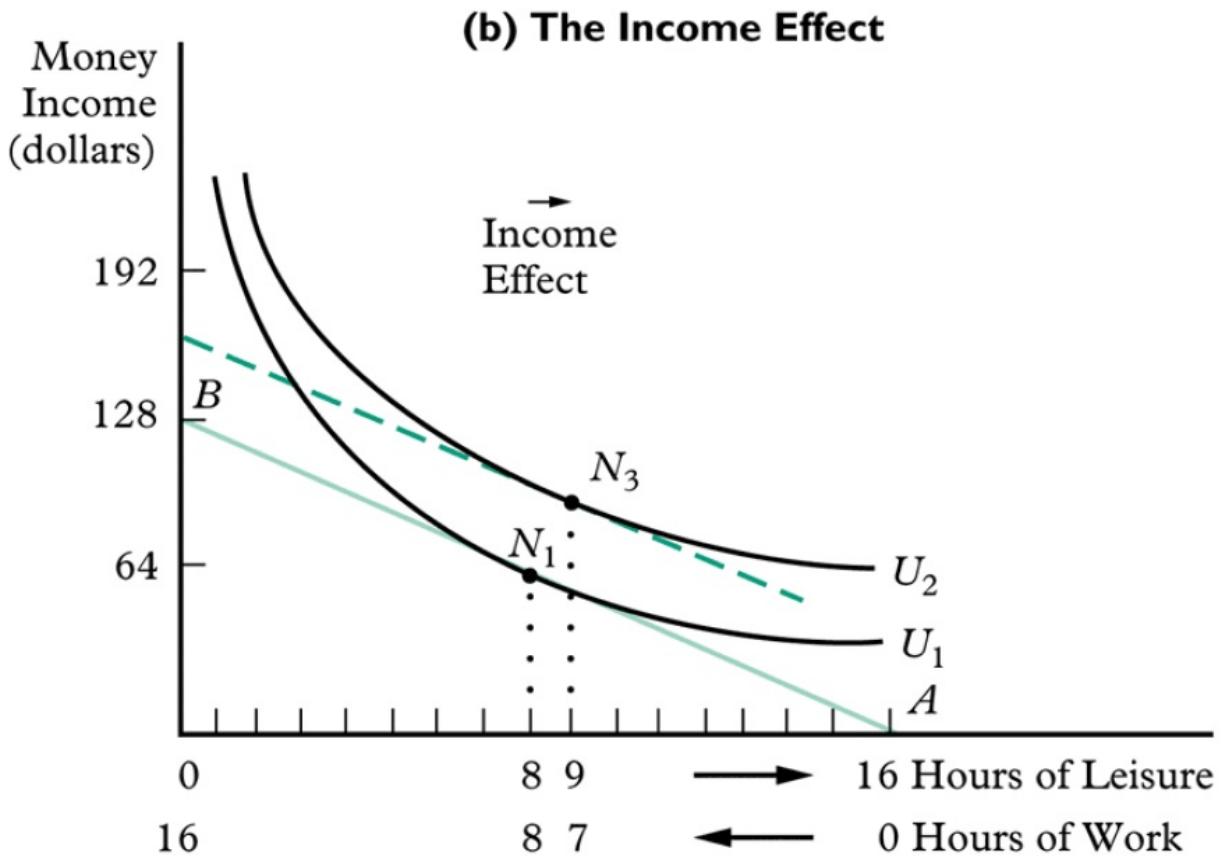


Figure 2: Isolating Income and Substitution Effects

Basic Static Model of Labor Supply

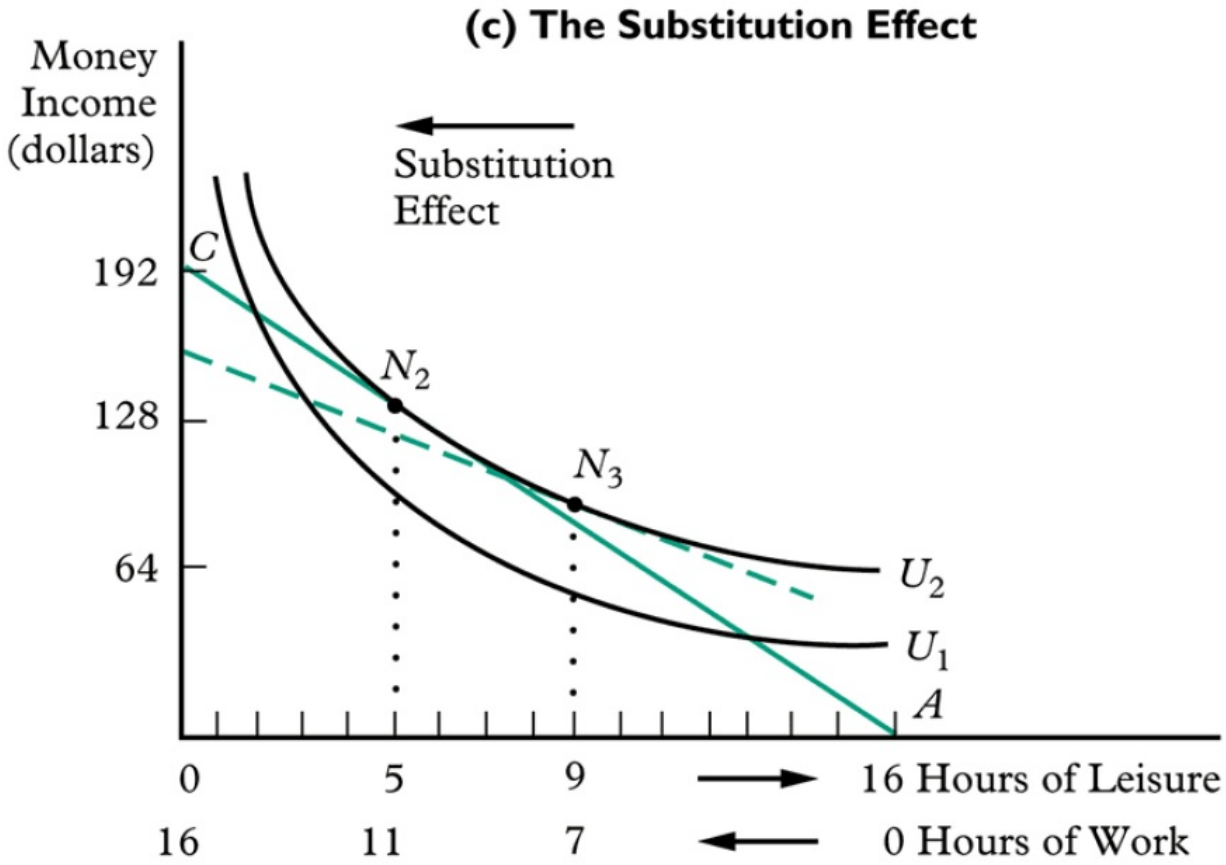


Figure 3: Isolating Income and Substitution Effects

Basic Static Model of Labor Supply

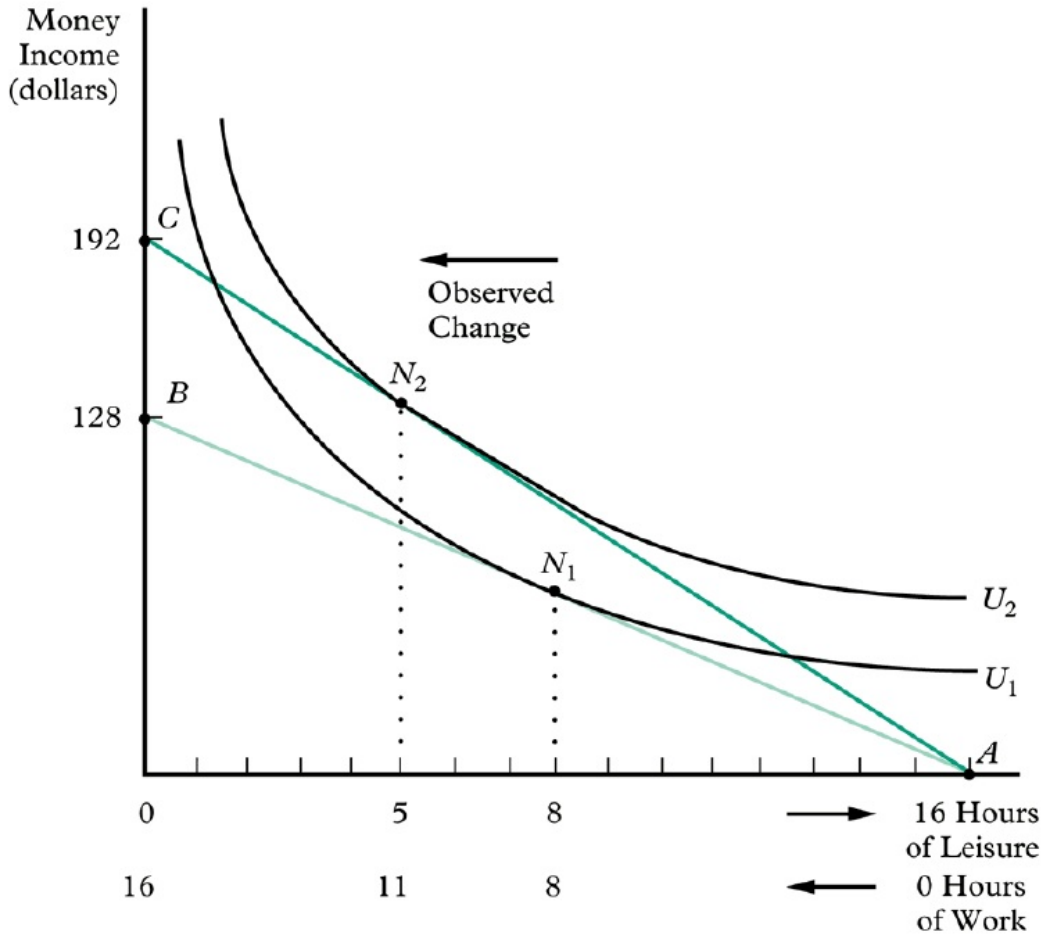


Figure 4: Wage Increase with Substitution Effect Dominating

Basic Static Model of Labor Supply

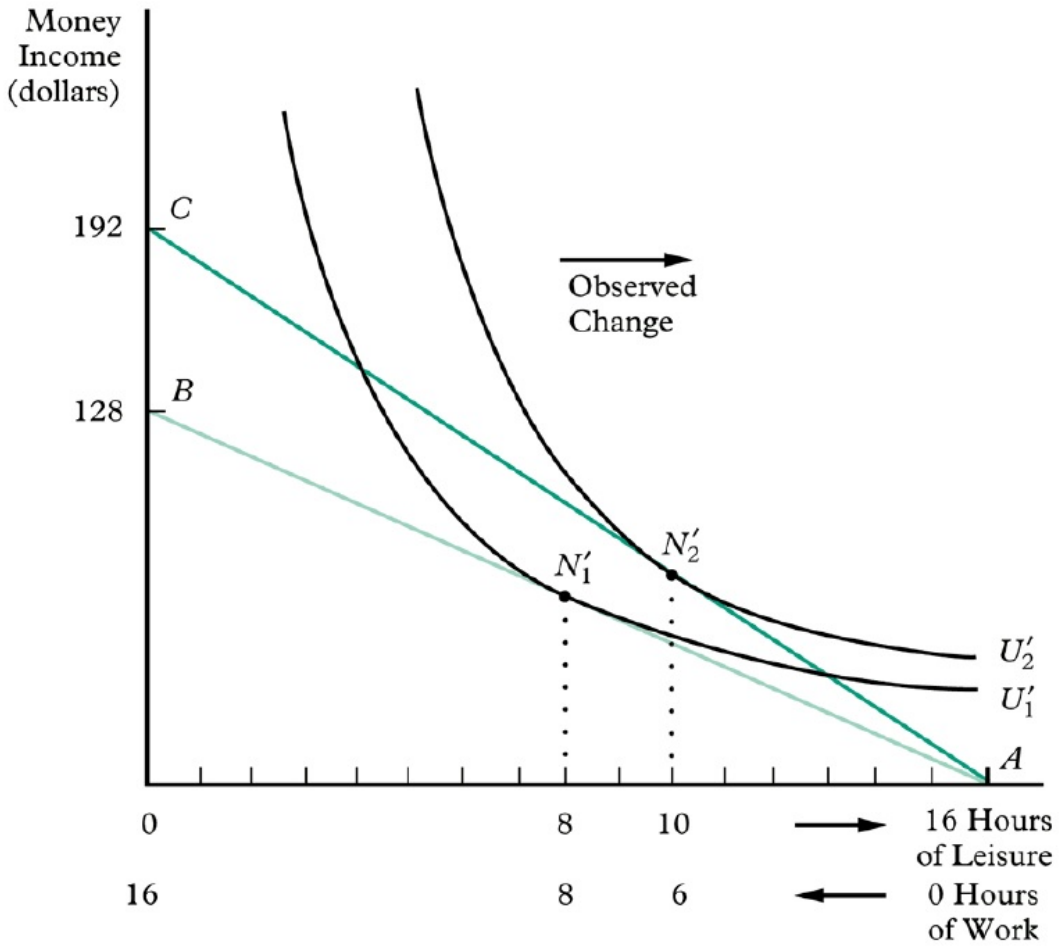


Figure 5: Wage Increase with Income Effect Dominating

Basic Static Model of Labor Supply

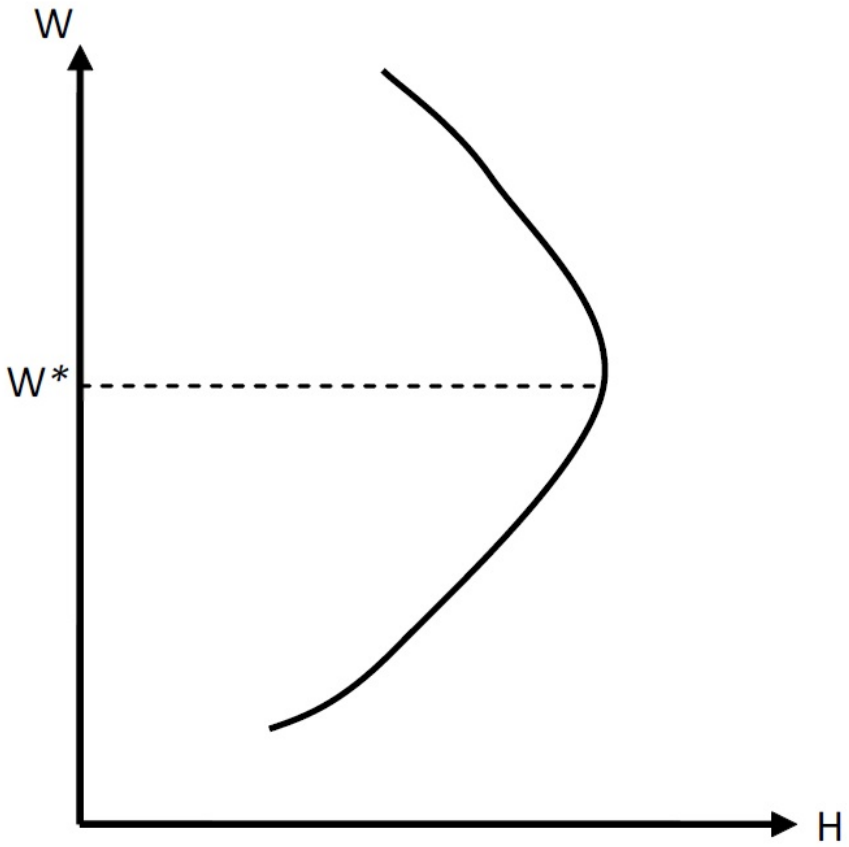


Figure 6: An Individual Backward Bending Labor Supply Curve

Basic Static Model of Labor Supply

- Initial hours of work is low, income effect is small and dominated by substitution effect $\Rightarrow$ labor supply curve is positively sloped
- As H increases, income effect may dominate $\Rightarrow$ the well known backward-bending labor supply curve

When W increases, 2 effects

- Substitution effect: $W \uparrow$, L more expensive $\rightarrow L \downarrow$
- Income effect: $W \uparrow$, real income $\uparrow \rightarrow L \uparrow$ if L is normal, $L \downarrow$ if L is inferior

Basic Static Model of Labor Supply

If L is normal

- total effect ambiguous, but at low W
- low H, income effect small $\Rightarrow$ substitution effect dominates
- as $W \uparrow \Rightarrow L \downarrow$ and $H \uparrow$
- at hight H, income effect gets stronger

If L is inferior, $W \uparrow \Rightarrow L \downarrow$.

Basic Static Model of Labor Supply

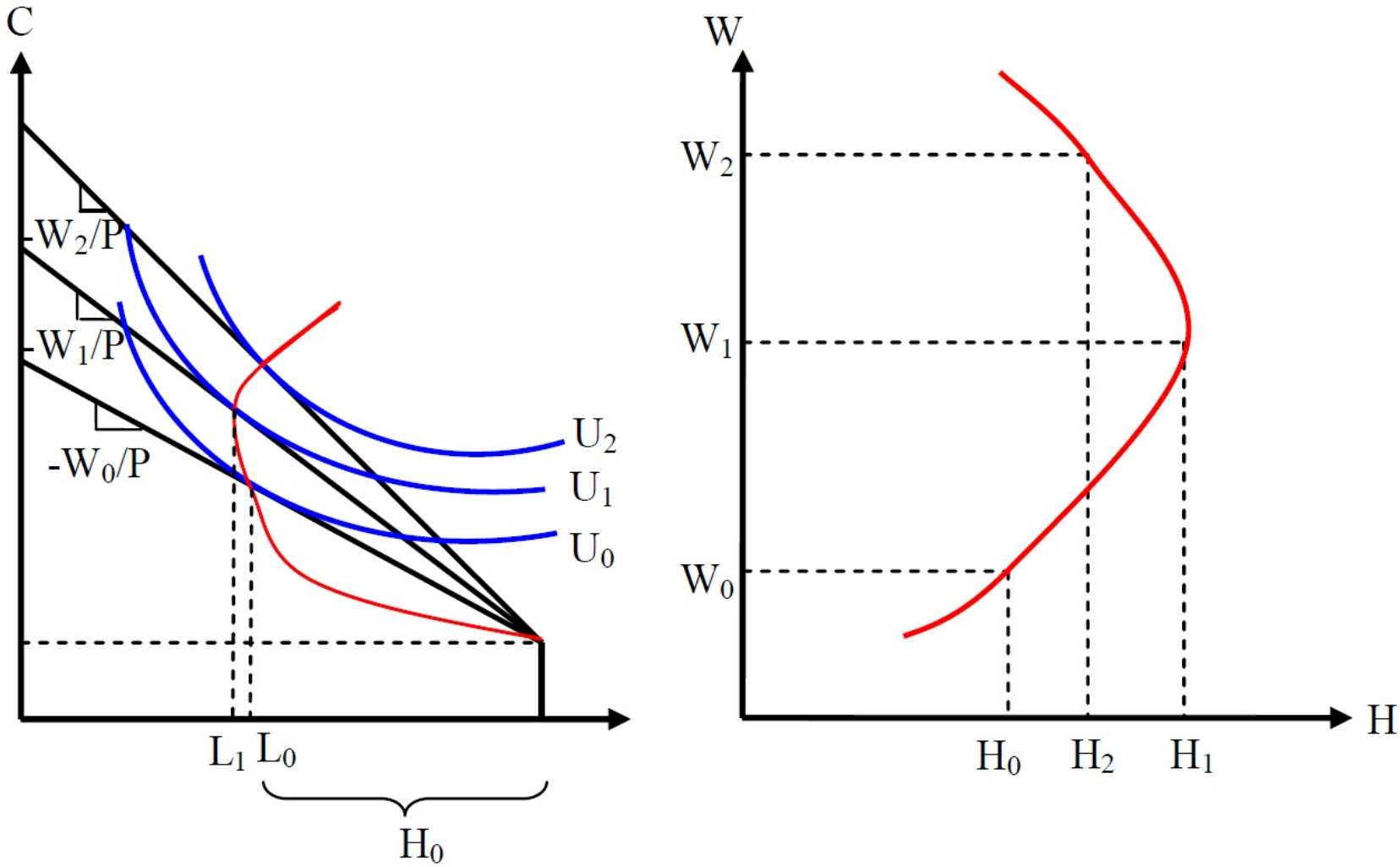
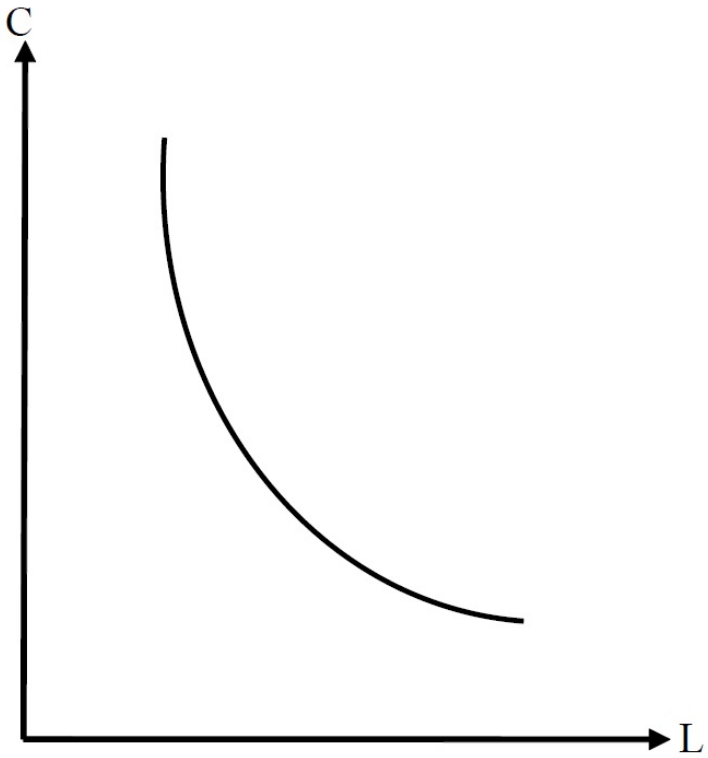
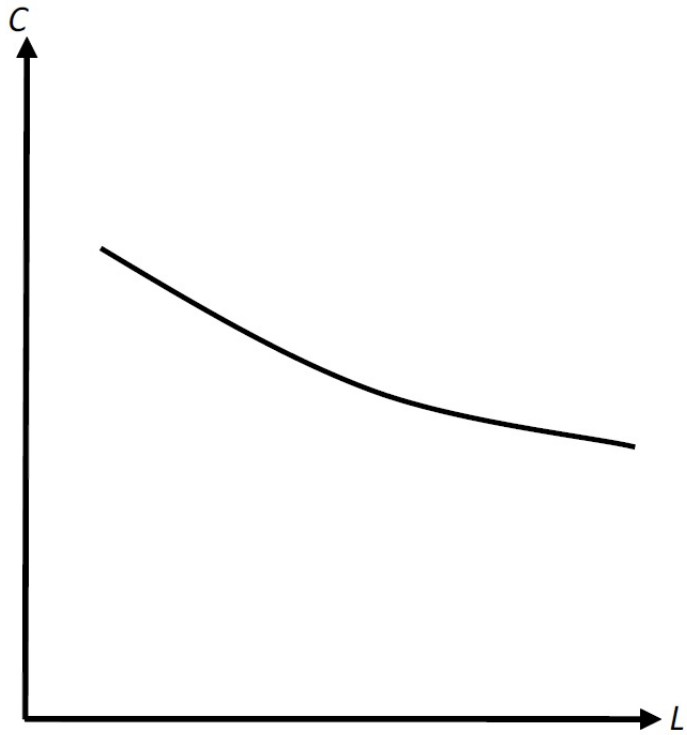


Figure 7: An Individual Backward Bending Labor Supply Curve

Slope of Utility Function



(a) High Value of Leisure



(b) Low Value of Leisure

Difference between Male and Female Labor Supply

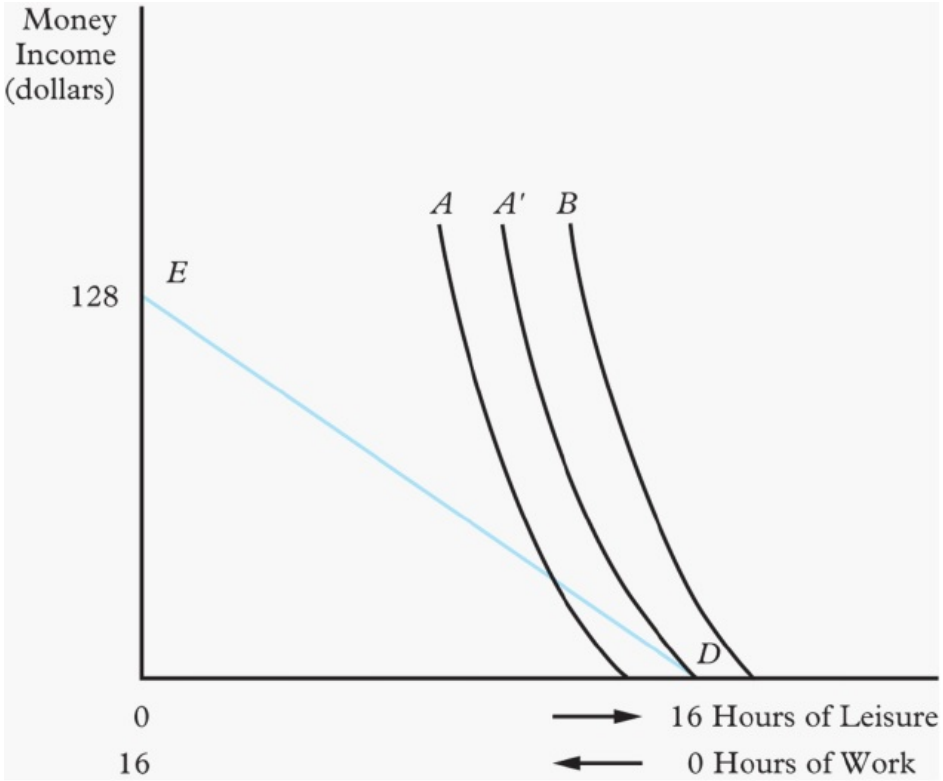
- Men: both income and substitution effects are small
- Women: larger response to wage change; substitution effect $>$ income effect

Recent evidence

- Men: LS slightly backward bending. Effect is small and could be even zero.
- Women: LS strongly upward slopping $\Rightarrow$ strong substitution effect

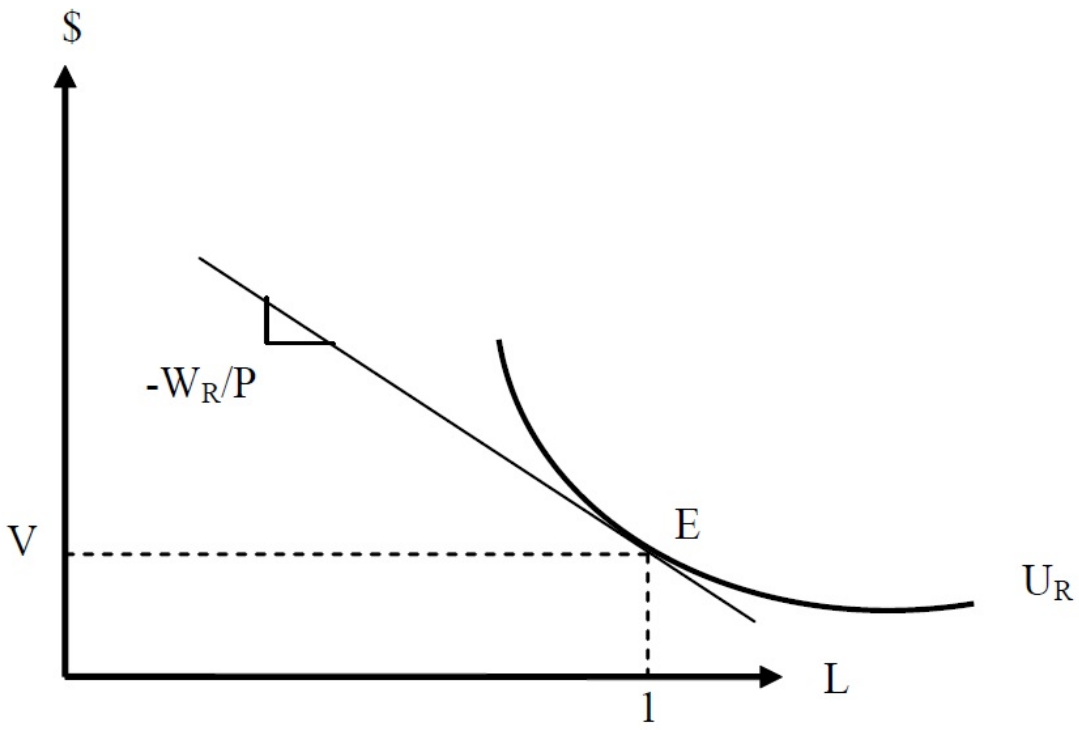
Corner Solution

So far we have only looked at interior solution: $1 - L > 0$.
Solution need not be interior.
In a no-work solution, $U_L > \lambda W$ for $L \in [0, 1]$, so that $L = 1$ and $H = 0$.



Corner Solution

Given V, the slope of the indifference curve passing through the allocation $(1, V)$ represents the individual's reservation wage.



Corner Solution

By definition

$$\frac{U_L(V,1)}{U_C(V,1)} = \frac{W_R}{P}$$

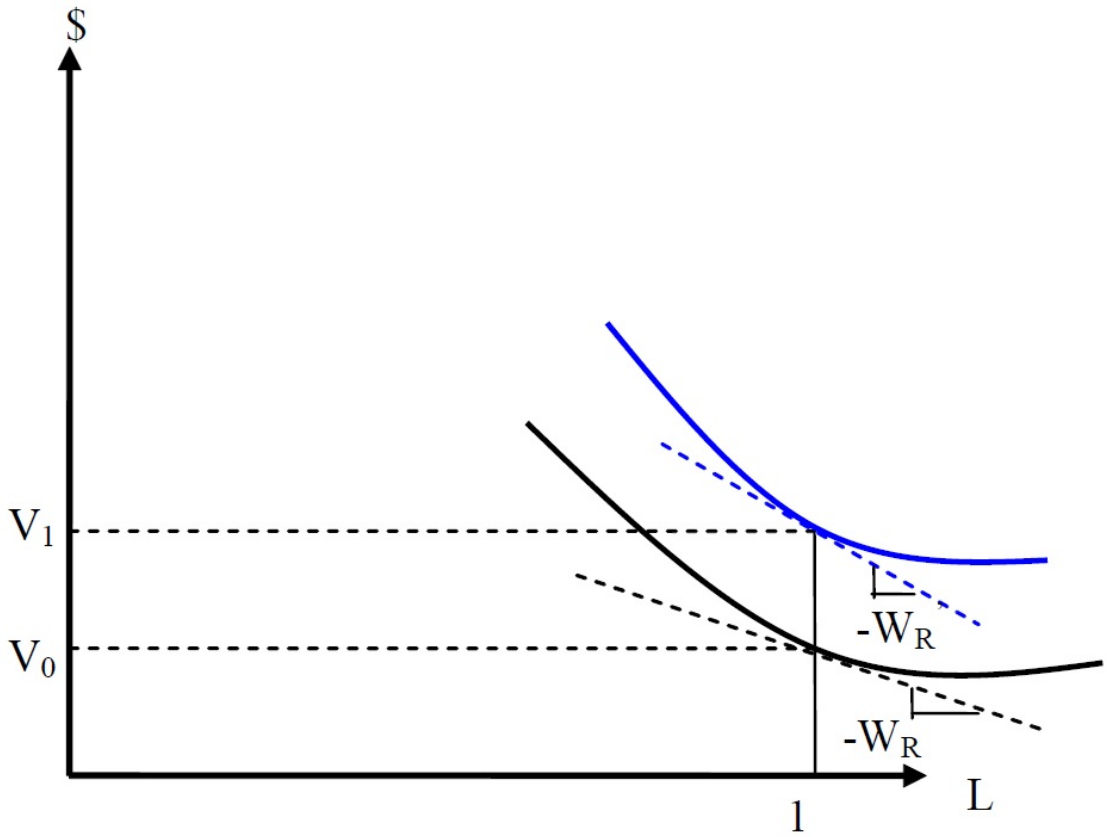
or

$$W_R = \frac{PU_L(V,1)}{U_C(V,1)}$$

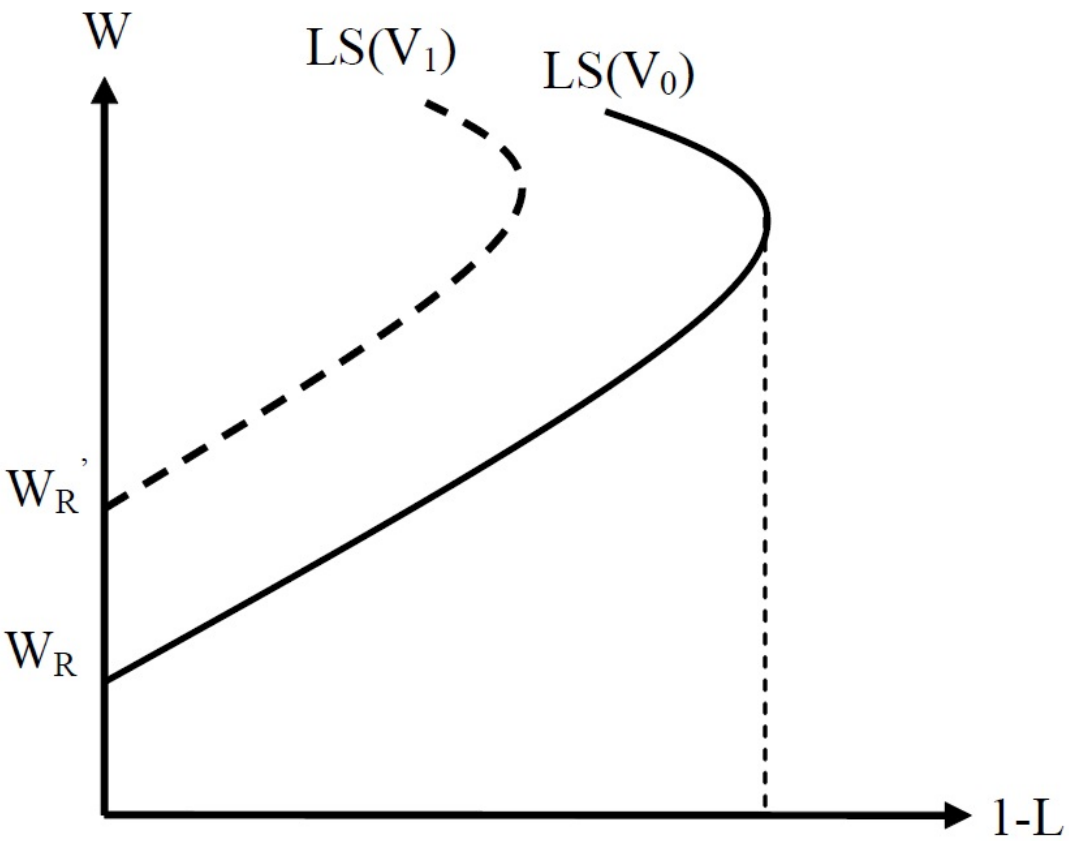
For individuals to be willing to work $W > W_R$.

Corner Solution

W_R depends on the level of V. If leisure is normal, then W_R increases with V.



Corner Solution



Aggregate LS Curve

Aggregate LS curve is simply the horizontal summation of the individual LS curves.

W increases

- $\uparrow$ or $\downarrow$ hours of work of existing workers
- must increase the number of workers in the labor force, i.e. more are willing to work.

Hence, even though individual labor supply may be backward bending, the aggregate curve is usually assumed to be positively sloped.

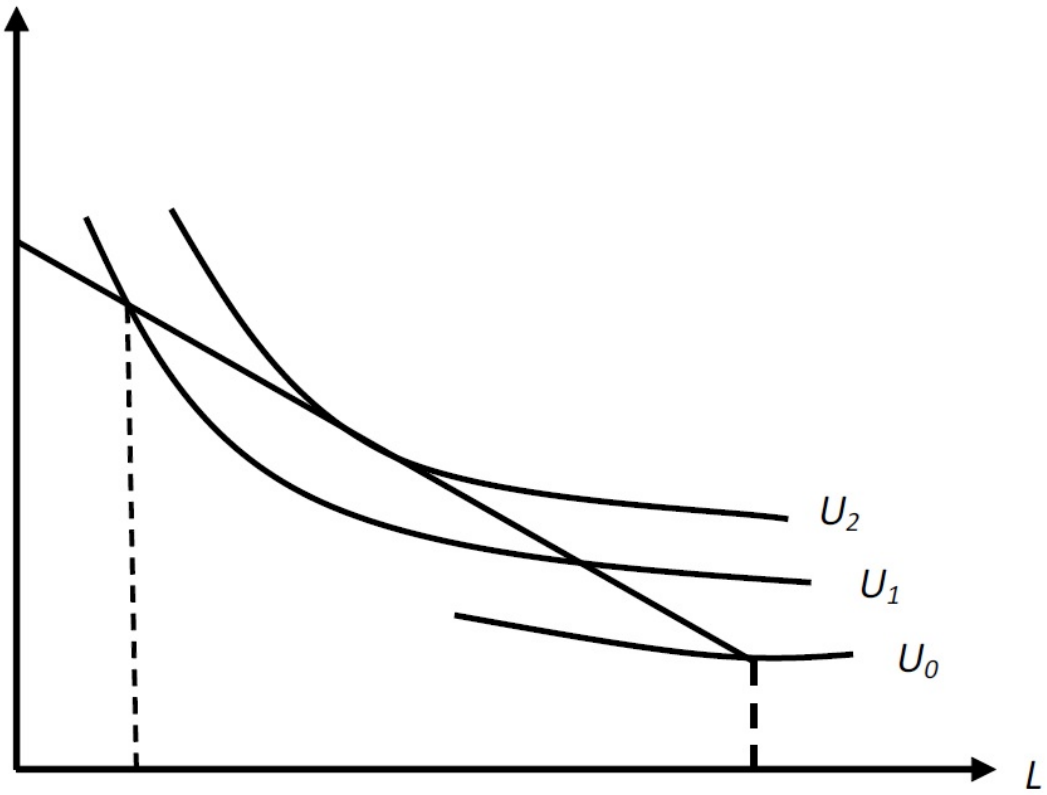
Discontinuous Budget Constraint

Assumption that there is free choice of working hours at a constant wage rate is often considered as unrealistic.
Often argued that hours of work is not subject to choice in most cases.

Why?

Constraint on working hours may be due to nature of the work such as some work involves team production (e.g. movie-making, performing arts) or supervision. In that case work schedule has to be more regulated.
With constraint on wage and hours $\Rightarrow$ discontinuous budget constraint exists in employment contracts, implicit or explicit.

Discontinuous Budget Constraint



Discontinuous Budget Constraint

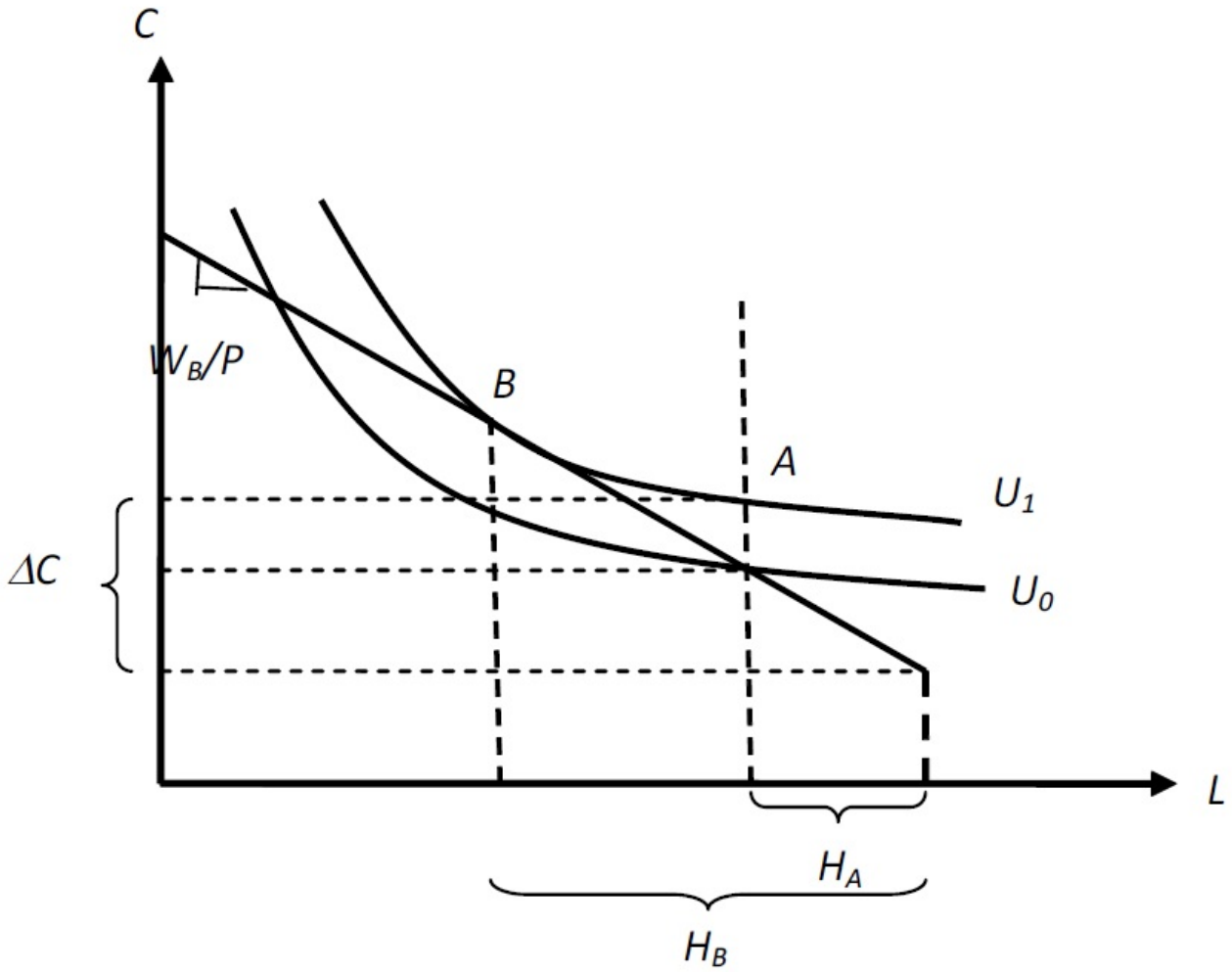
Either work or not work $\Rightarrow$ both are not efficient.
However, if choose to work, it is not necessary inefficient.

In the real world, monitoring to reduce shirking is more efficient when workers work the same hours.
Acceptance of a take-it or leave-it package of wage and hours by worker would not necessarily represent an interior solution.

Given large enough range of jobs, each offering different working hours at the same wage corresponding to a worker's marginal product, then the budget constraint would be linear and continuous even though each point or segment on the line corresponds to a different job.

Discontinuous Budget Constraint

When hours of work is not flexible, may need to pay a compensating wage.



Discontinuous Budget Constraint

At W_B utility is maximized at U_1 and the individual want to work H_B hours.
The firm is only willing to allow him to work for H_A hours, which for the individual will be on a lower utility at the given wage.
In order to induce that person to work for H_A hours, the firm have to offer wage W_A that is higher than W_B, which is called the compensating wage.

$$W_A = \frac{\Delta C}{H_A} > W_B$$

Fixed Costs of Working

Costs that any individual who wants to work must pay every time he works. Be careful not to confuse with the fixed hiring or training costs incurred by the firm or the worker at the beginning of employment.

These costs can be classified into two different types

- Fixed money costs
- Fixed time costs

Fixed Money Costs

Assume an individual need to pay α per working day, i.e. commuting cost, socializing, baby sitting, etc.
The budget will be

$$U = \begin{cases} U(V, 1) & \text{if } L = 1 \\ U(V + WH - \alpha, 1 - H) & \text{if } L < 1 \end{cases}$$

If no fixed money cost ($\alpha = 0$)

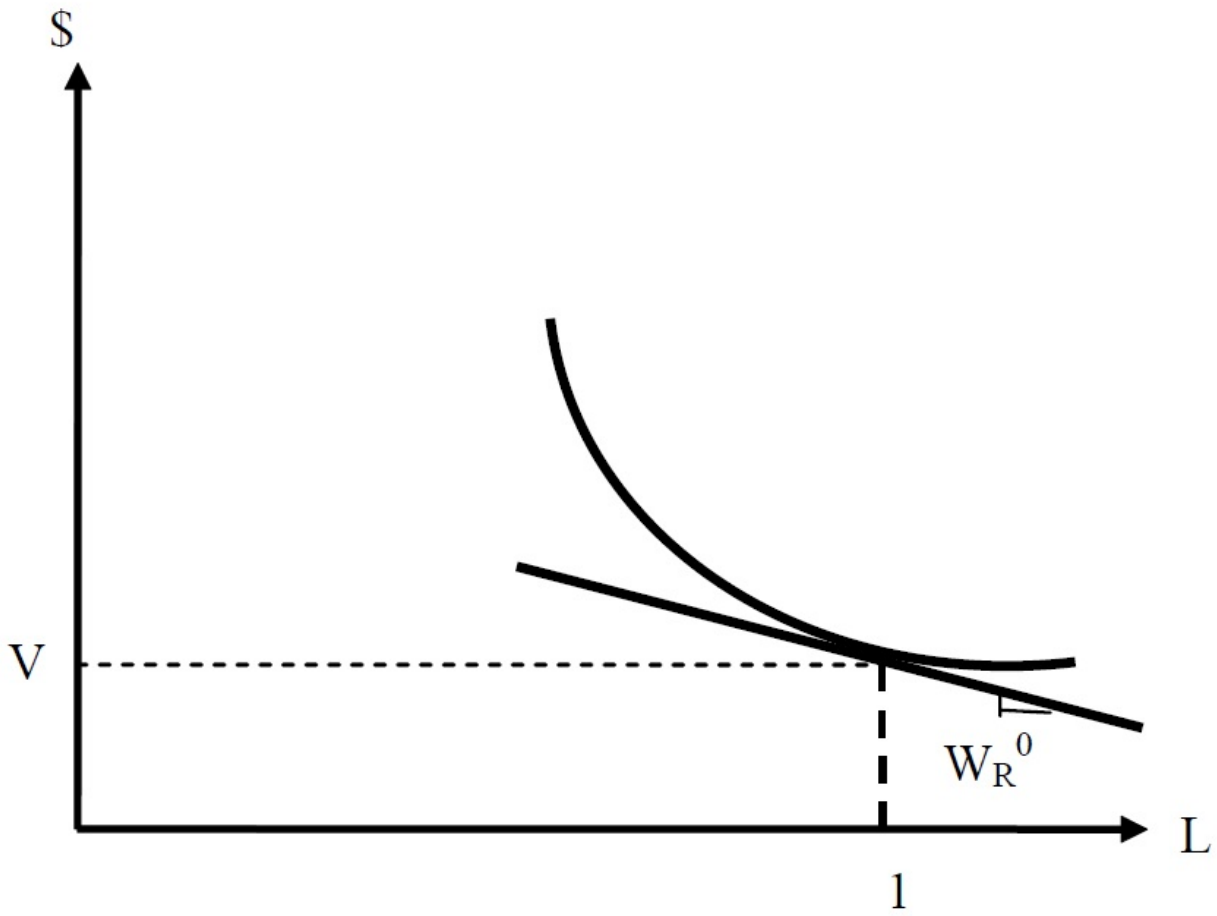
$$W_R = W_R^0; H_R = 0$$

If positive fixed money cost ($\alpha > 0$)

$$W_R = W_R^1; H_R > 0, \text{ where } W_R^1 > W_R^0$$

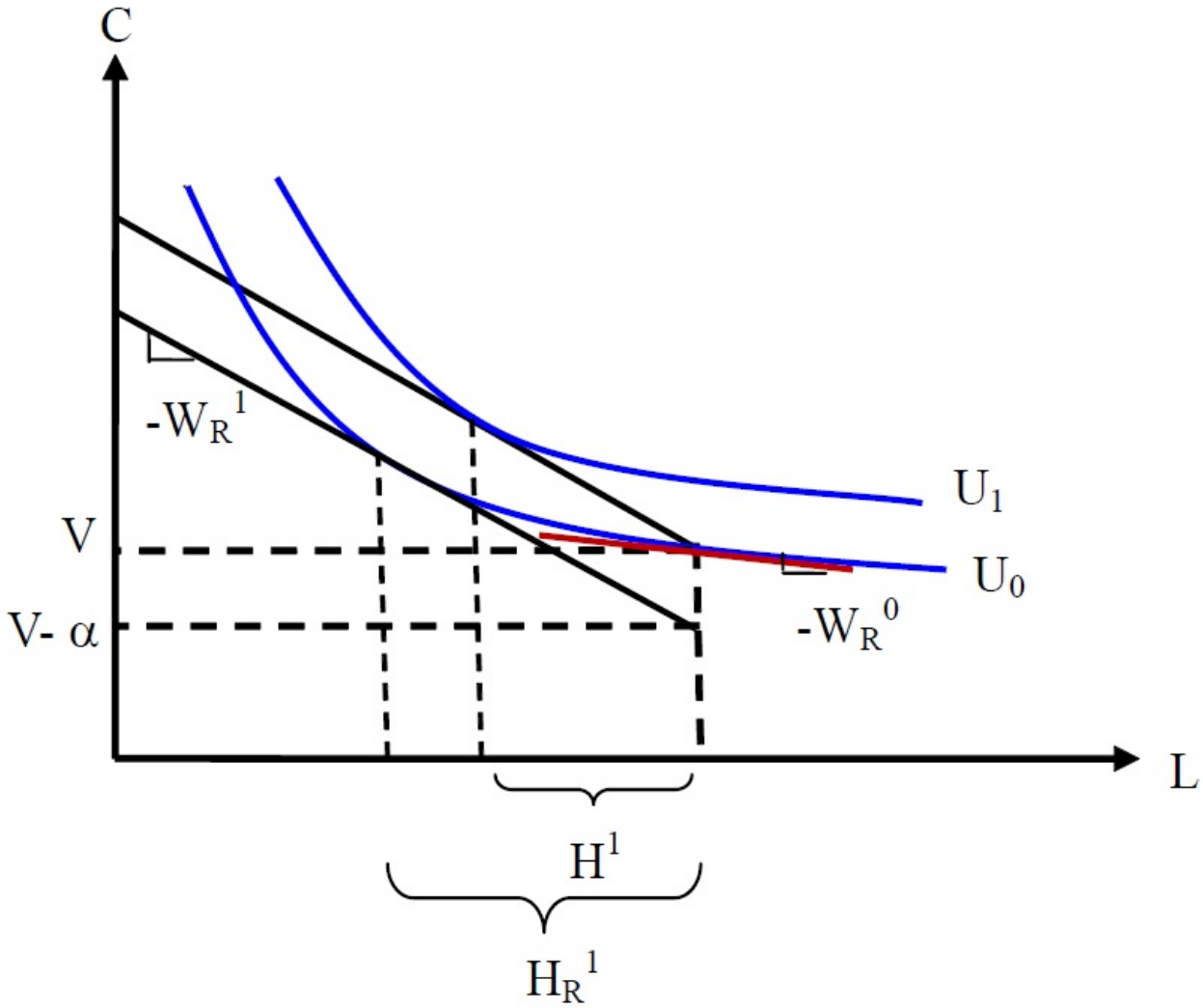
Fixed Money Costs

If no fixed money cost



Fixed Money Costs

If positive fixed money cost



Fixed Money Costs

Note that:

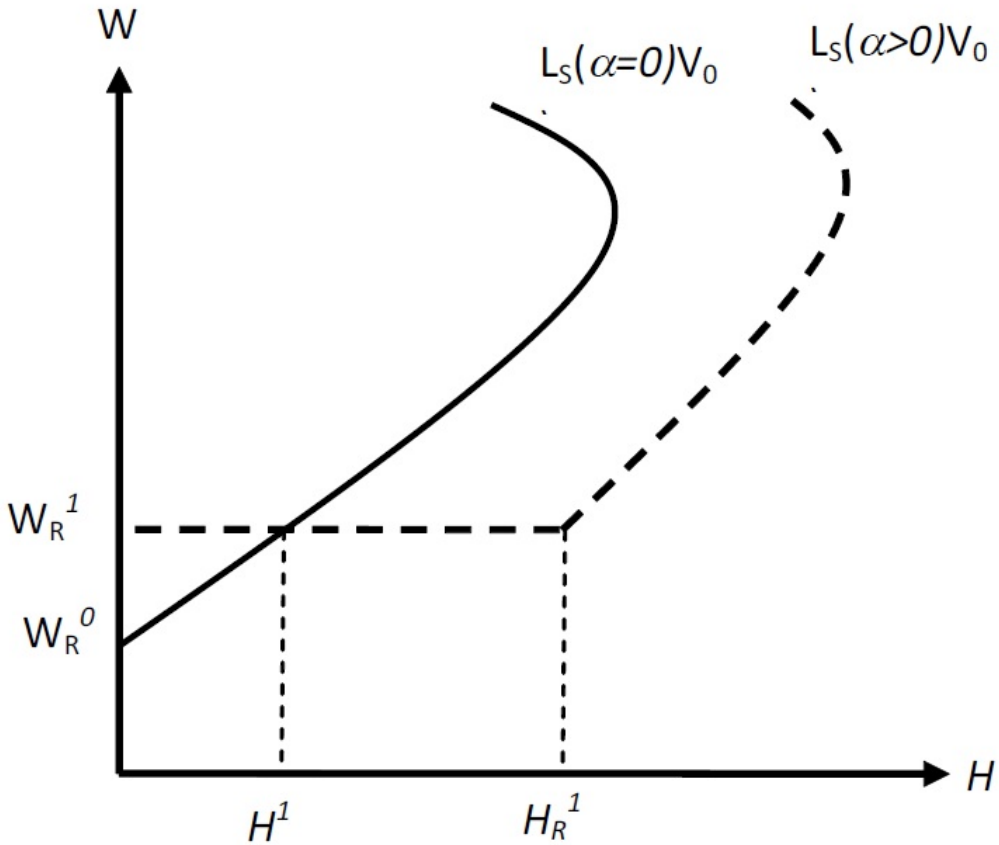
- W_R is higher with fixed cost.
  With $\alpha > 0$, labor supply discontinuous; $H = 0$ for wage $< W_R^1$.
  Reservation wage, $W_R^1 > W_R^0$, will be higher, so that the probability of a corner solution is higher.
- H will not be $< H_R$.
  Minimum hours of work is H_R.
  Because it is not worthwhile to work just a few hours in the face of fixed cost.

Fixed Money Costs

- If L is normal, the LS curve shift to the right as a result of fixed cost. For any given wage $> W_R^1$, the hours of work will be higher than if there is no fixed cost and L is normal, because the fixed cost reduces his income, so that the income effect implies lower consumption of leisure, or more hours of work.

Fixed Money Costs

Labor Supply



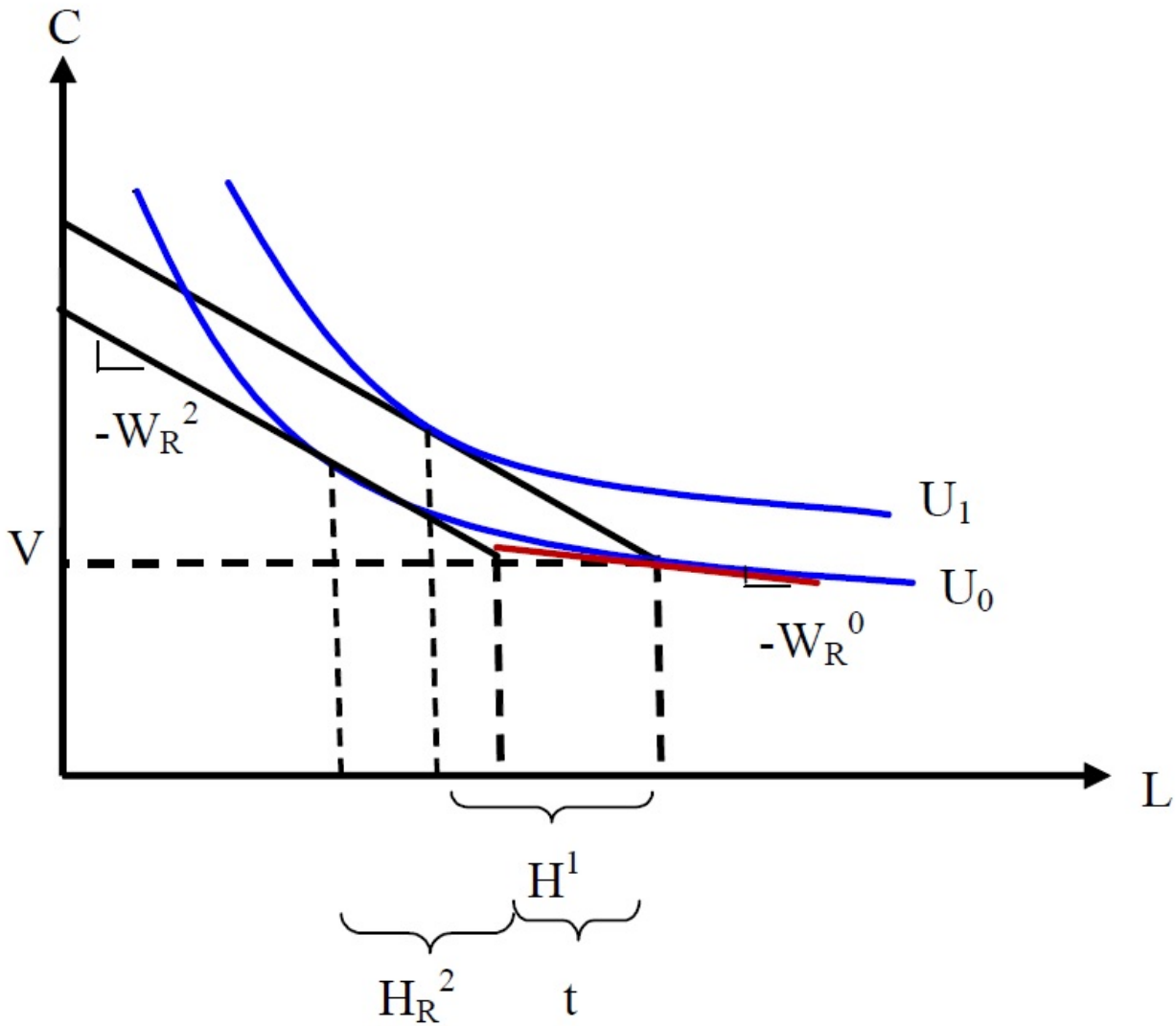
Fixed Time Costs

Instead of money costs, sometimes working involves time cost (t), e.g. commuting time each working day.
Such time costs do not lower the worker's income, but reduce the number of hours available to him. Again, this will result in a budget set as in

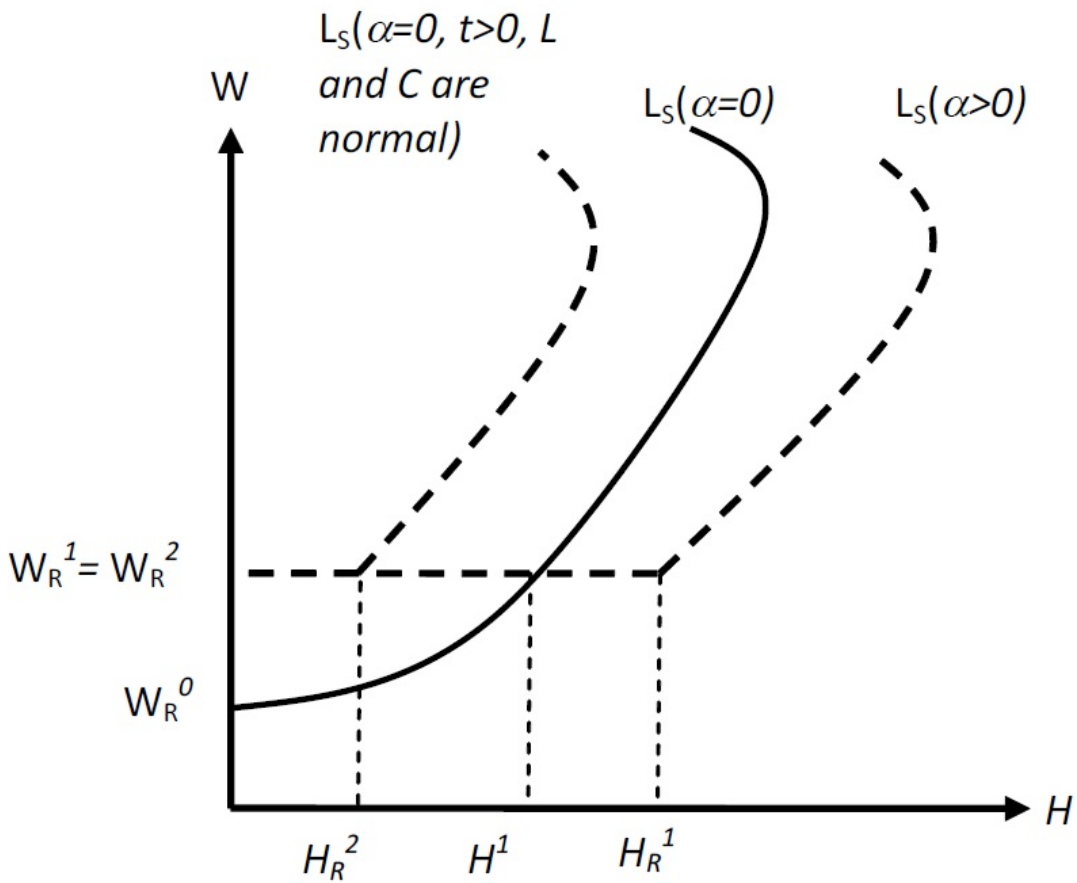
$$U = \begin{cases} U(V, 1) & \text{if } L = 1 \\ U(V + WH, 1 - H - t) & \text{if } L < 1 \end{cases}$$

The reservation wage will again increase, making work less likely.
Fixed time cost $t > 0 \Rightarrow W_R^2 \uparrow, H_R^2 > 0$.
$H_R^2 < H^1$ work less hours than the case without t.

Fixed Time Costs



Fixed Time Costs



Income Replacement Program

Governments often run programs which compensate individuals for the loss of income, usually due to circumstances out of the individuals' control. Examples are unemployment insurance and disability compensation. The problem with full income replace is that there is a very large disincentive effect.

Income Replacement Program

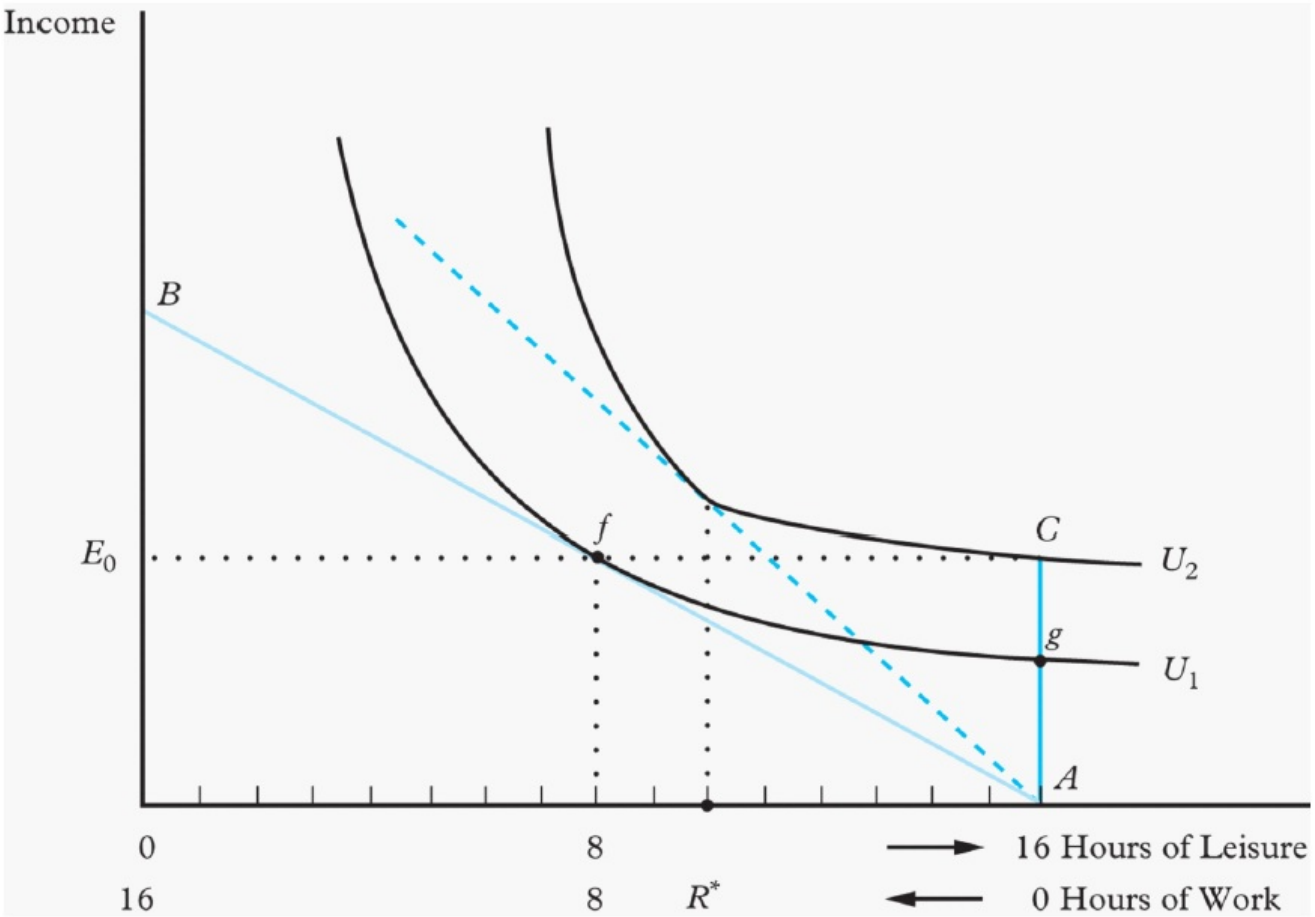


Figure 8: Budget constraint with a spike

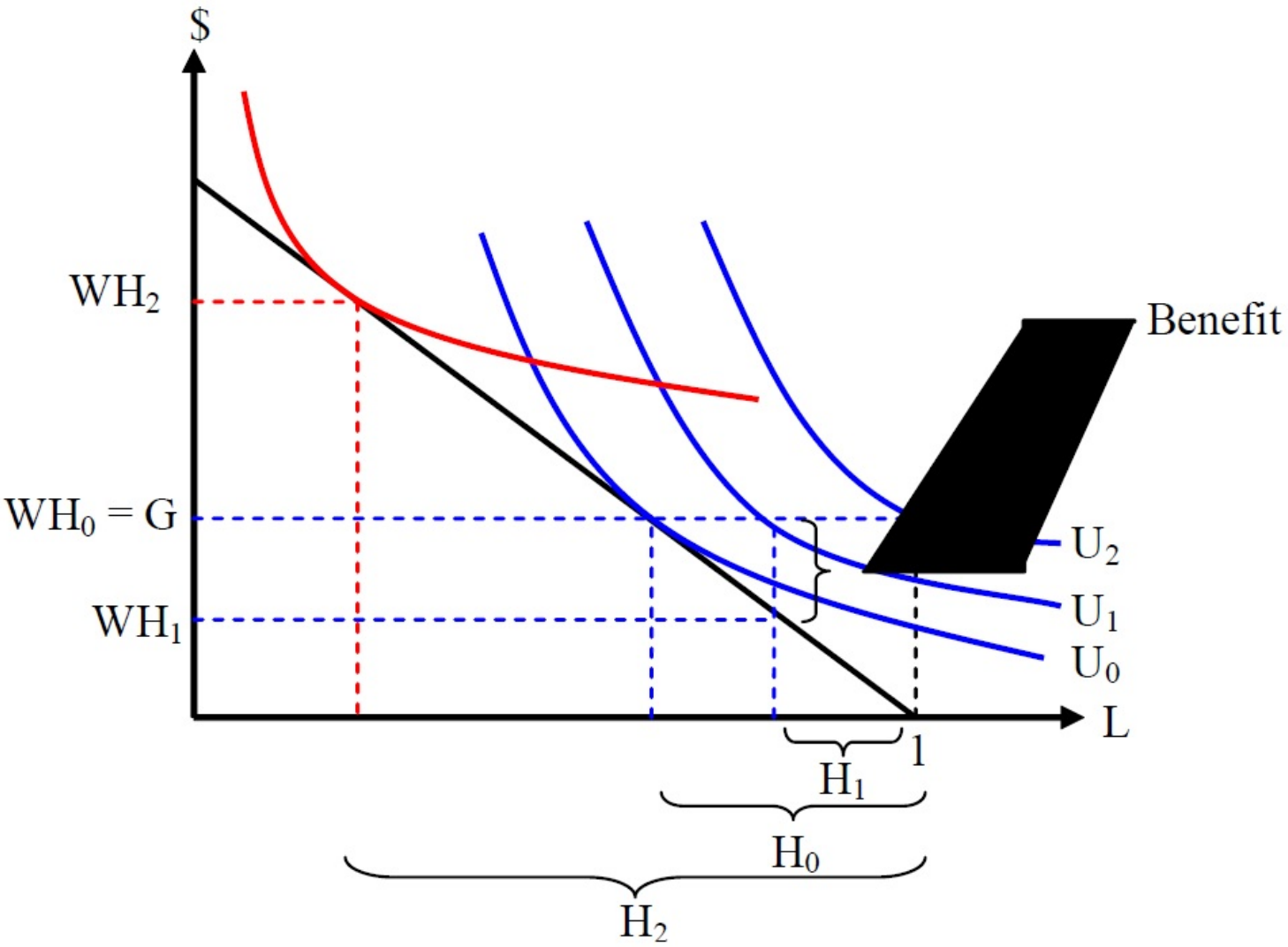
Income Maintenance Program

Another type of transfer program protects the income level of individuals or households.
One way of maintaining the level of income is by guaranteeing a minimum.

If G is the protected income level, then

$$\text{Benefit} = \begin{cases} G - WH & \text{if } WH < G \\ 0 & \text{if } WH \geq G \end{cases}$$

Income Maintenance Program



Income Maintenance Program

Originally choose to work H_0 hours of work and earn income WH_0.
Out of work due to reasons not controlled by worker, the government provides same income, i.e. $WH_0 = G$.
Utility increases from U_0 to U_2.

If workers work less than before, say H_1 instead of H_0.
He will get $WH_1 + (G - WH_1) = G$, utility is U_1 which is lower than not working at all. Therefore large disincentive to work.

If utility at H_2 hours of work is higher than U_2, then out of welfare program with higher utility.

Income Maintenance Program

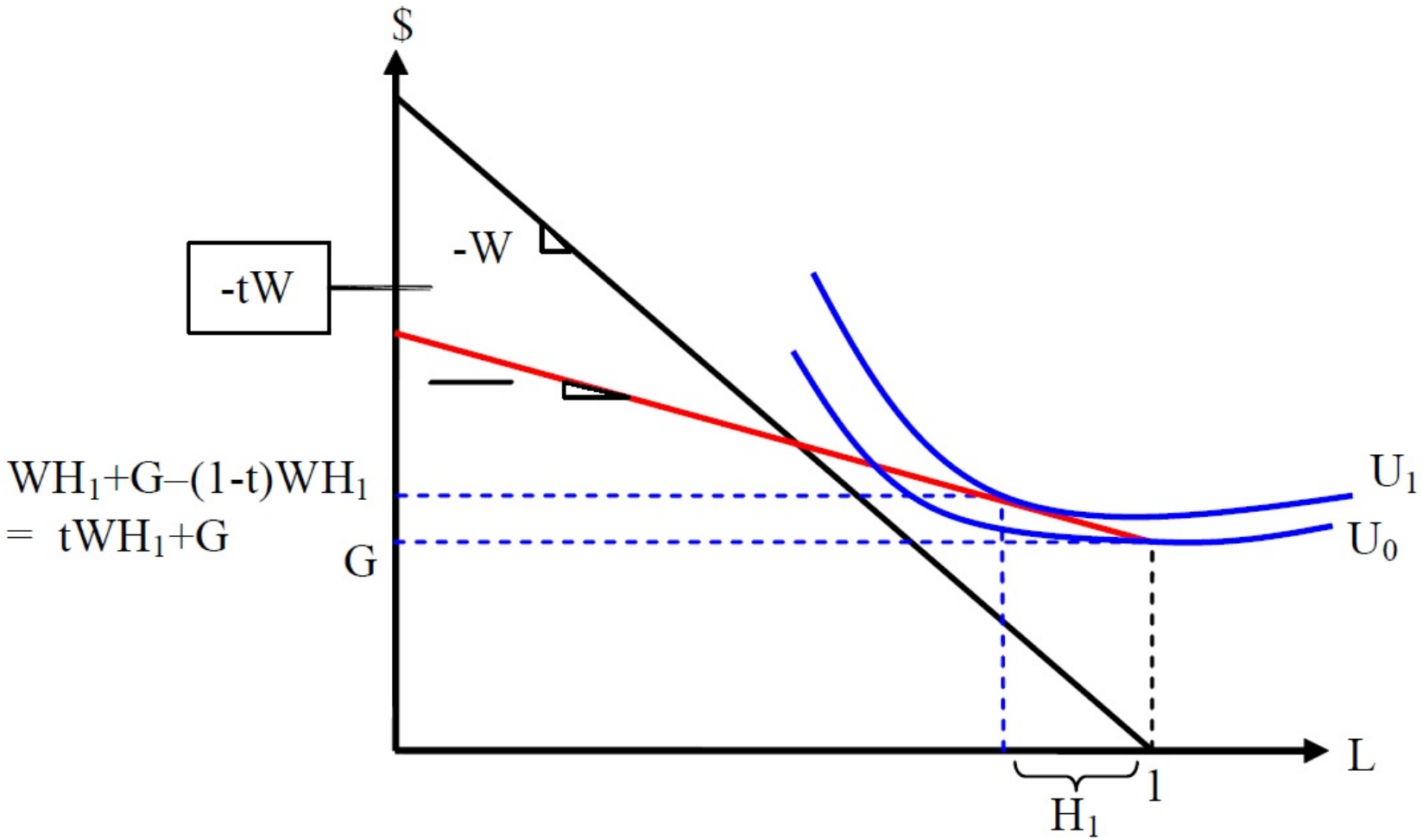
In order to provide less disincentive to work and make people out of welfare program, government can impose a less than 100% tax rate.

$$\text{Benefit} = \begin{cases} G - (1-t)WH & \text{if } (1-t)WH < G \\ 0 & \text{if } (1-t)WH \geq G \end{cases}$$

$$H^* = \frac{G}{W(1-t)}$$

If $H > H^*$, do not get benefit.

Income Maintenance Program



Income Maintenance Program

With welfare and no work at U_0.

Under the new scheme, work H_1 hours and earn income $tWH_1 + G$ and achieve utility U_1.
It's better for the worker to work.

$\uparrow$ welfare $\rightarrow$ work less but better than 100% income replacement.

Income Maintenance Program

Example
Wage $= \$10/\text{hr}$
Welfare if no work $= \$500$
Tax is 50% $\Rightarrow$ new wage $= \$5/\text{hr}$
Now choose point R and work 10 hours $\Rightarrow$ work less

Income Maintenance Program

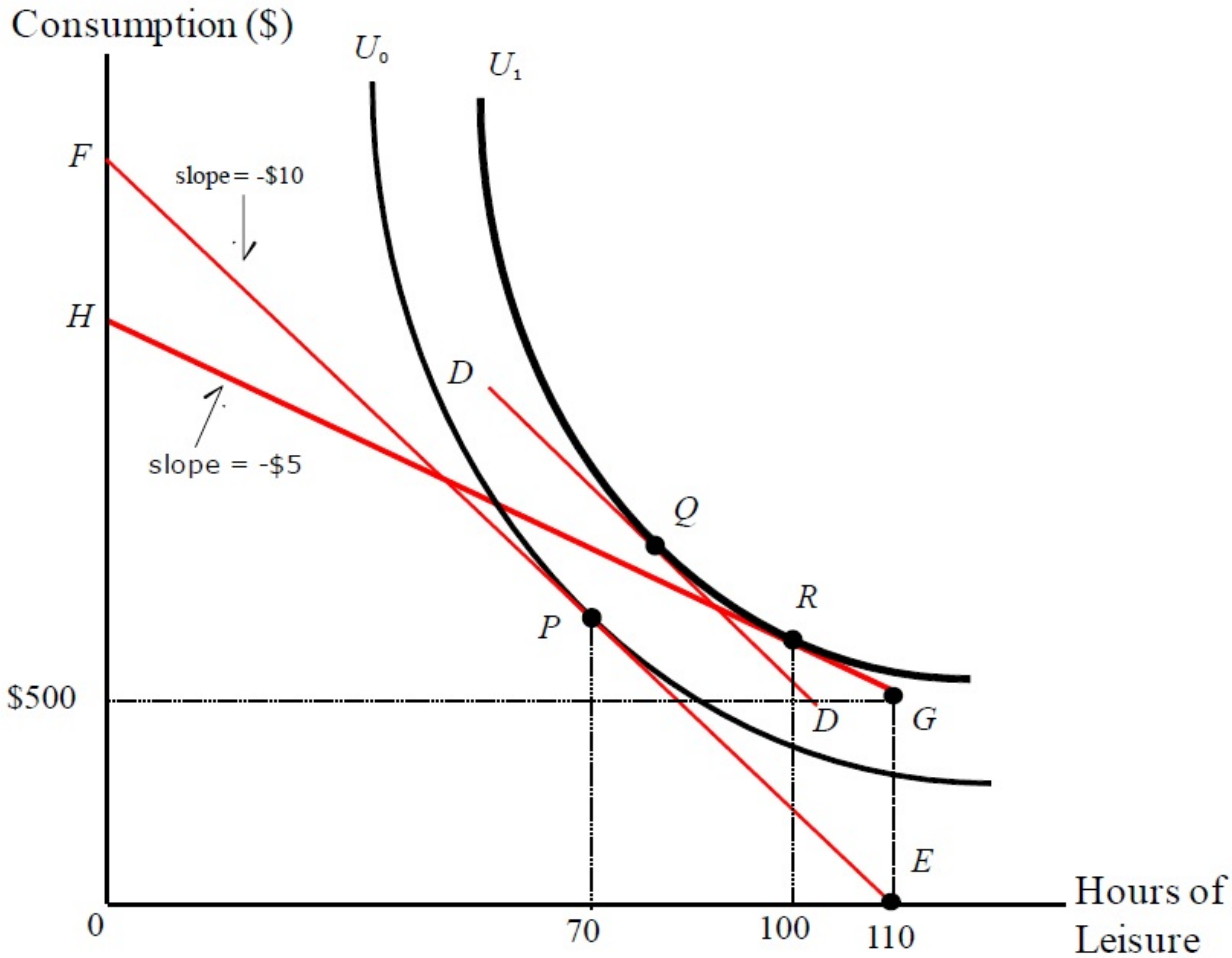


Figure 9: Effect of a welfare program on hours of work

Model of Household Production (Becker 1965)

Previous assumption - utility is a function of consumption goods and leisure
Leisure - represent non-market time

In reality, consumption does not simply represent instantaneous derivation of utility from consumption goods purchases in the market. Rather, consumption goods and leisure are no more than inputs into a production process that transform these ingredients into other commodities which are then consumed for utility.

不是买了就有 Utility
还要 time

Model of Household Production

Often we combine inputs of market goods, X, and household time, T, to produce commodities, which ultimately yield satisfaction or utility, U, to the household.
Some examples:

$$\text{eating} = \text{food} + \text{time}$$

$$\text{travelling} = \text{money} + \text{time}$$

$$U = f(Z_1, Z_2, .., Z_n) = F(Z) = h(X, T)$$

where $Z_i = g(x_i, t_i) \quad Z = g(X, T)$.

Model of Household Production

Slope of the isoquant = "wage rate" or the implicit valuation of its time since it shows how many market goods, X, can be saved by using an additional unit of household time, T.

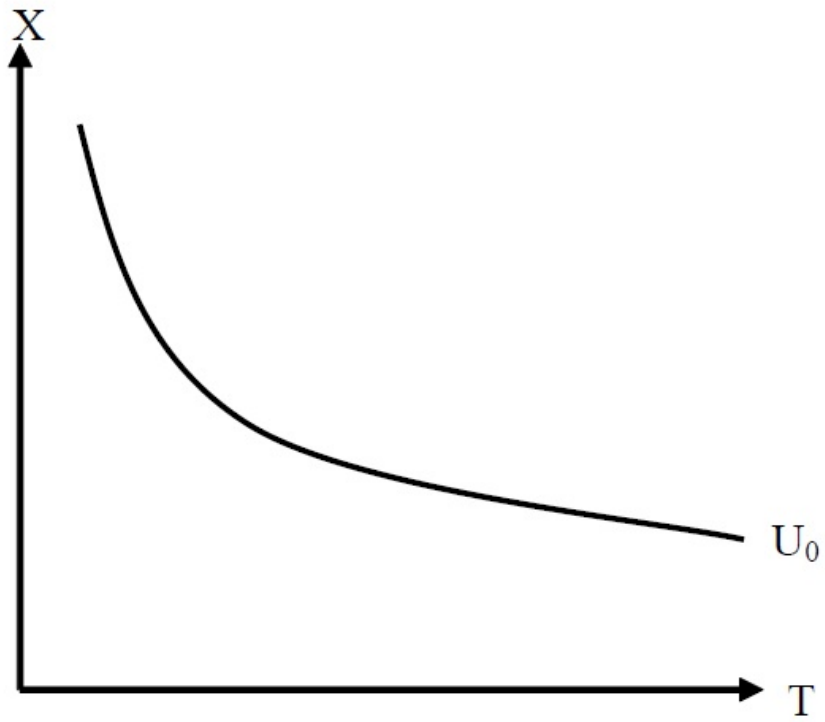
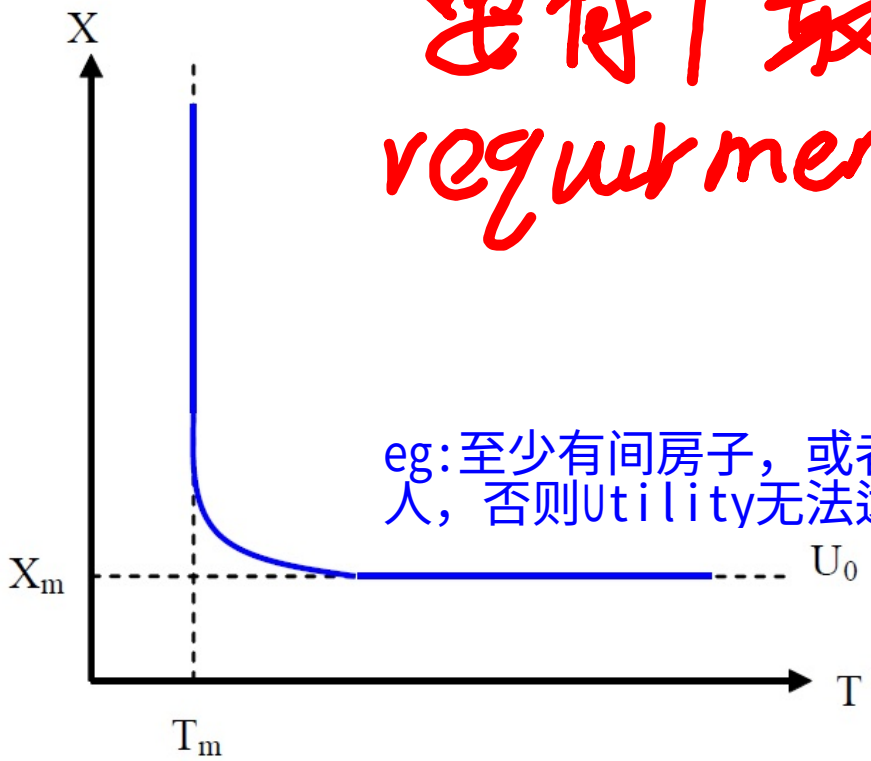


Figure 10: Utility Isoquants

Model of Household Production

Substitutability in household production

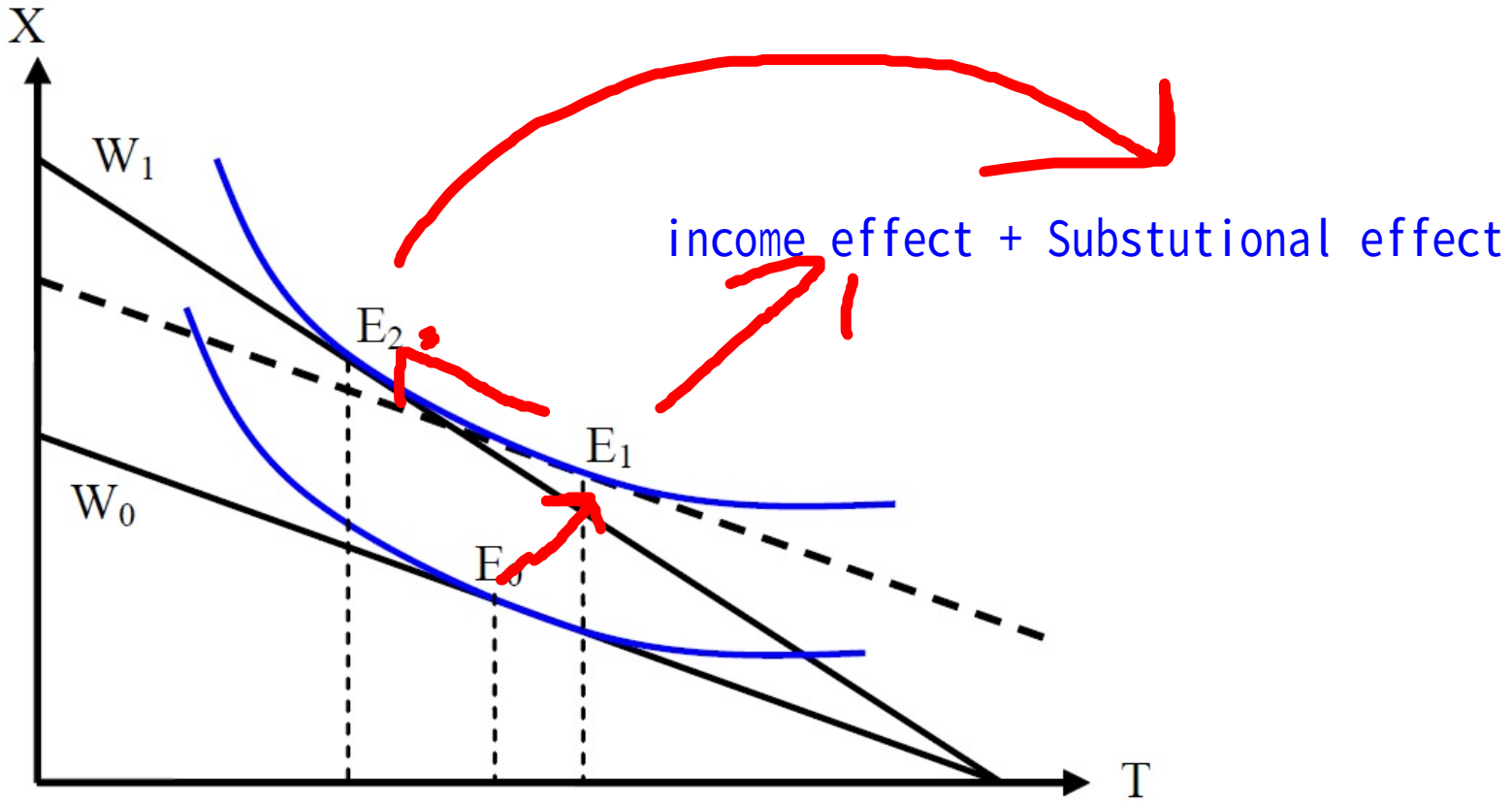
Threshold levels: no substitutability if either one is below the level.



要有个最小的 requirment

eg:至少有间房子，或者至少用多长时间陪家人，否则Utility无法达到

Model of Household Production



Model of Household Production

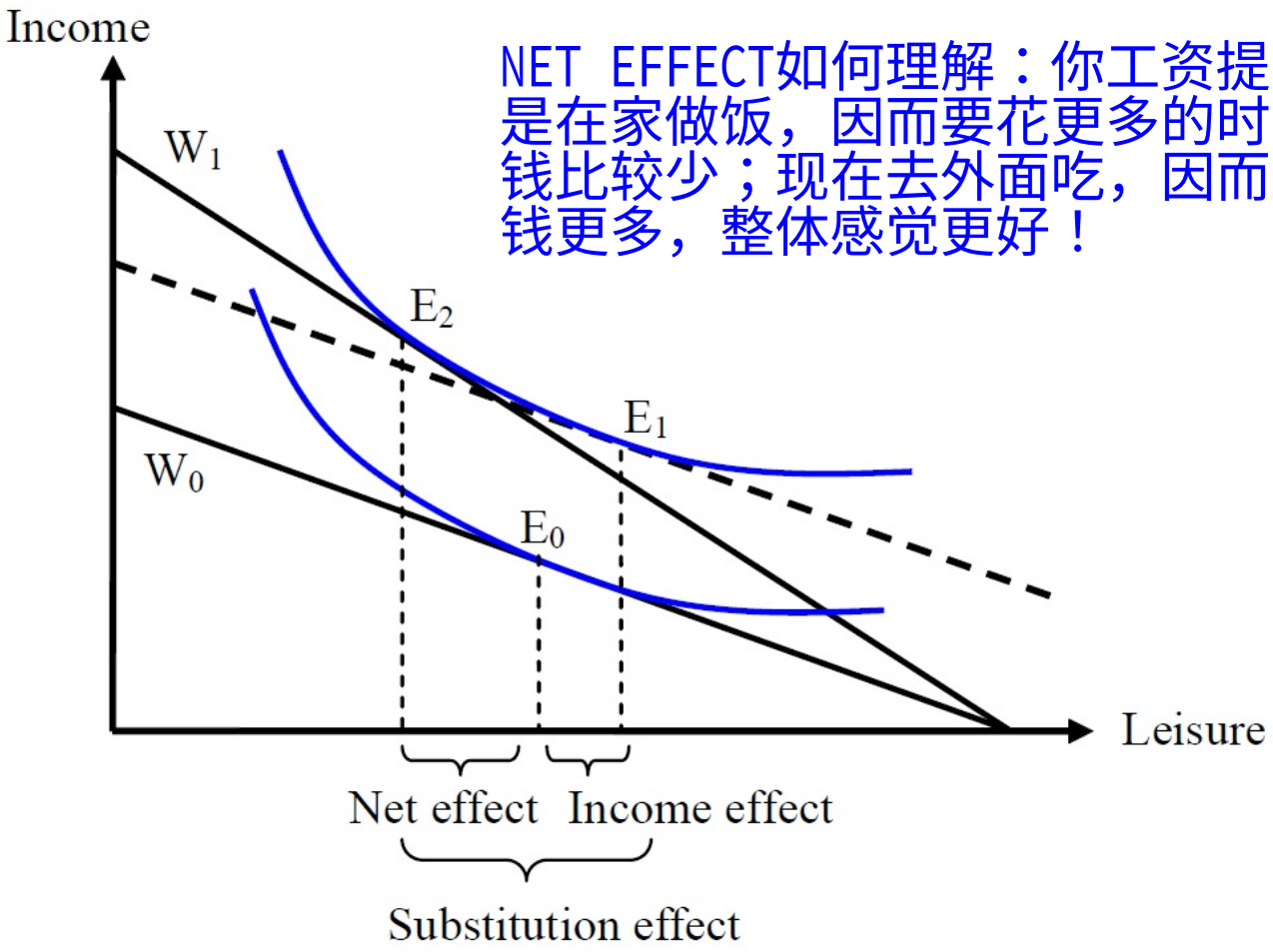
Wage increase

- Income effect and substitution effect increase for X
- Income effect and substitution effect for T works in opposite direction

$W_0 \uparrow\rightarrow W_1$

- Observed equilibrium $E_0 \rightarrow E_2$ 这里先看income effect再看substitute effect
- Income effect $E_0 \rightarrow E_1$, consume more of both goods, X and T intensive activities
- Substitution effect $E_1 \rightarrow E_2$, utilize more good-intensive and less time-intensive activities (substitute away from T into X)

Model of Household Production



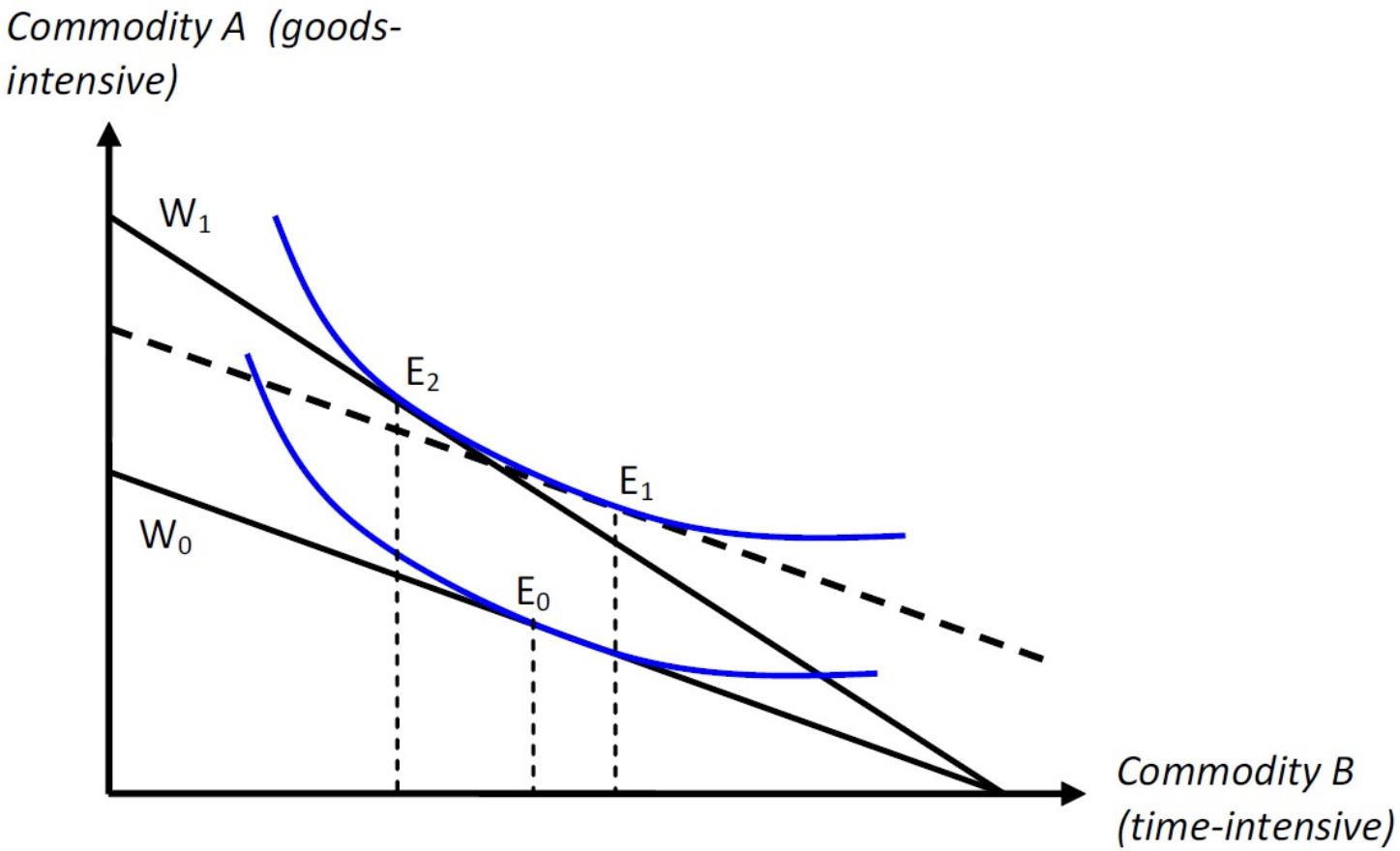
NET EFFECT如何理解：你工资提高以后，原来是在家做饭，因而要花更多的时间，但是花的钱比较少；现在去外面吃，因而时间更少，花钱更多，整体感觉更好！

Model of Household Production

Two major differences, between conventional consumer demand theory and the household production approach:

- Household production approach emphasizes that commodities consumed require both goods and time inputs. Focus on the extent to which particular commodities are good intensive or time intensive. i.e. wage increase $W_0 \to W_1$; equilibrium $E_0 \to E_2$
  - Commodity A (good intensive): income and substitution effects work in the same direction $\to$ increase go to the restaurant
  - Commodity B (time intensive): income effect positive, substitution effect negative cook at home

Model of Household Production



Model of Household Production

- Substitution effect in the household production approach has two reinforcing components.
  i.e. wage increase
  - Use less time inputs and use more goods inputs (in production)
  - Consume less time-intensive commodities and more goods-intensive commodities (in consumption) 在produce和consumption两个过程中都有体现！

  $\Rightarrow$ substitution occurs in household production as well as in household consumption.
  Household is a producer as well as consumer.

Model of Household Production

Some implications:

- Wage $\uparrow \Rightarrow$ goods-intensive commodities $\uparrow$, e.g. fast-food outlets, prepared foods, day-care, etc.
- Even sleep (a very time-intensive activity) time is affected by wage: Biddle and Hamermesh (1990, JPE) find that people of higher potential earnings power sleep less because it costs them more (in terms of forgone income) to sleep longer.
- Market wages, non-labor income, household production efficiency: not only the first two matter, the third one is also important.

Model of Household Production

Some implications:

- Comparative advantage (market vs. household) and specialization, economies of scale, co-operative daycare arrangement
- Time reallocation between household production and labor market (in response to wage increase) will be small if few substitutes exist for household labor.
- Life cycle time allocation and nature of activities in different stages of life cycle: work more during peak earning years, travel before and after working ages

Life Cycle Aspect of Labor Supply

A further refinement: triple choices
time = market work, household work, leisure

So a wage increase has two substitution effects

- between market work and household work
- between market work and leisure

Life Cycle Aspect of Labor Supply

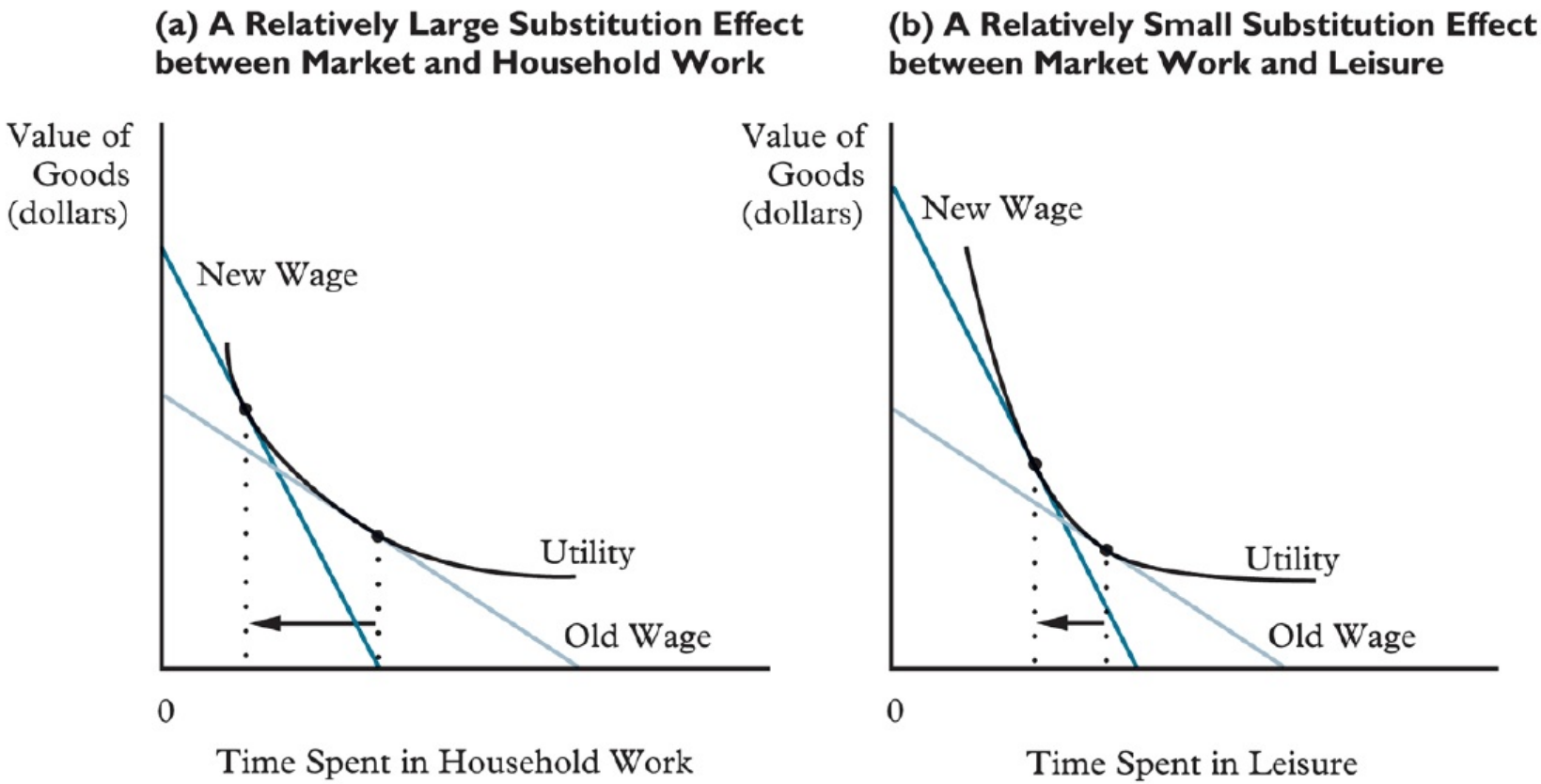


Figure 11: Large versus small substitution effects

Life Cycle Aspect of Labor Supply

- Left figure: shows the tradeoffs between goods and household work time that keep utility constant.
  - Gradual bend implies reduction in hours of household work can be easily compensated for by purchasing more goods.
  - Explain why female's substitution effect of wage increase is large.
- Right figure: shows the goods/leisure trade-off
  - Steeper bend implies greater difficulty in substituting goods for leisure time without loss of utility.

Life Cycle Aspect of Labor Supply

Weekly Hours Spent in Household Work, Paid Work, and Leisure Activities by Men and Women over Age 18, 2008

	Households with Children < 6		Households with Children 6–17		Households with No Children < 18	
	Women	Men	Women	Men	Women	Men
Paid Work[a]	20	42	26	39	21	30
Household Work[b]	41	23	32	18	24	16
Leisure[c]	32	32	36	38	45	46
Personal Care[d]	74	70	73	72	76	74

Figure 12: Weekly hours spent in household work, paid work, and leisure activities by men and women over age 18, 2008

Life Cycle Aspect of Labor Supply

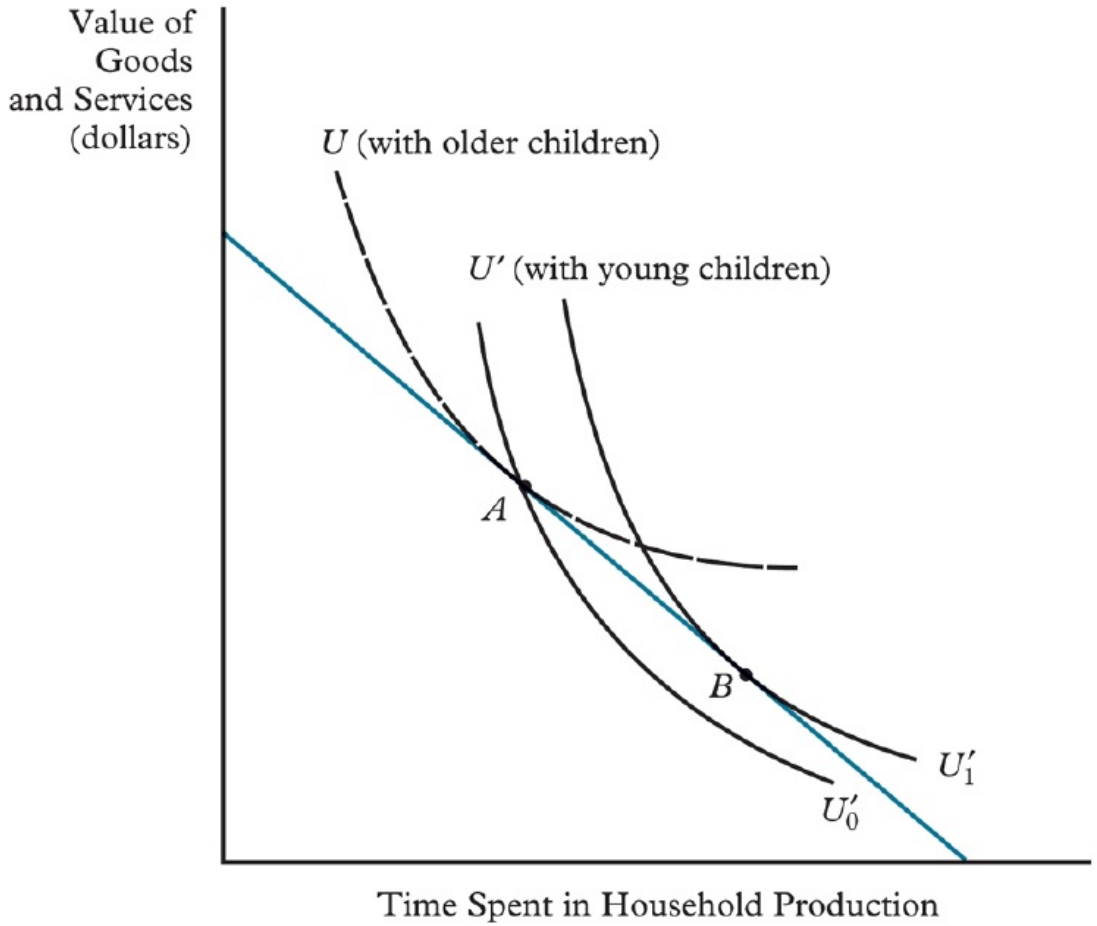


Figure 13: Household productivity can change over the life cycle

Life Cycle Aspect of Labor Supply

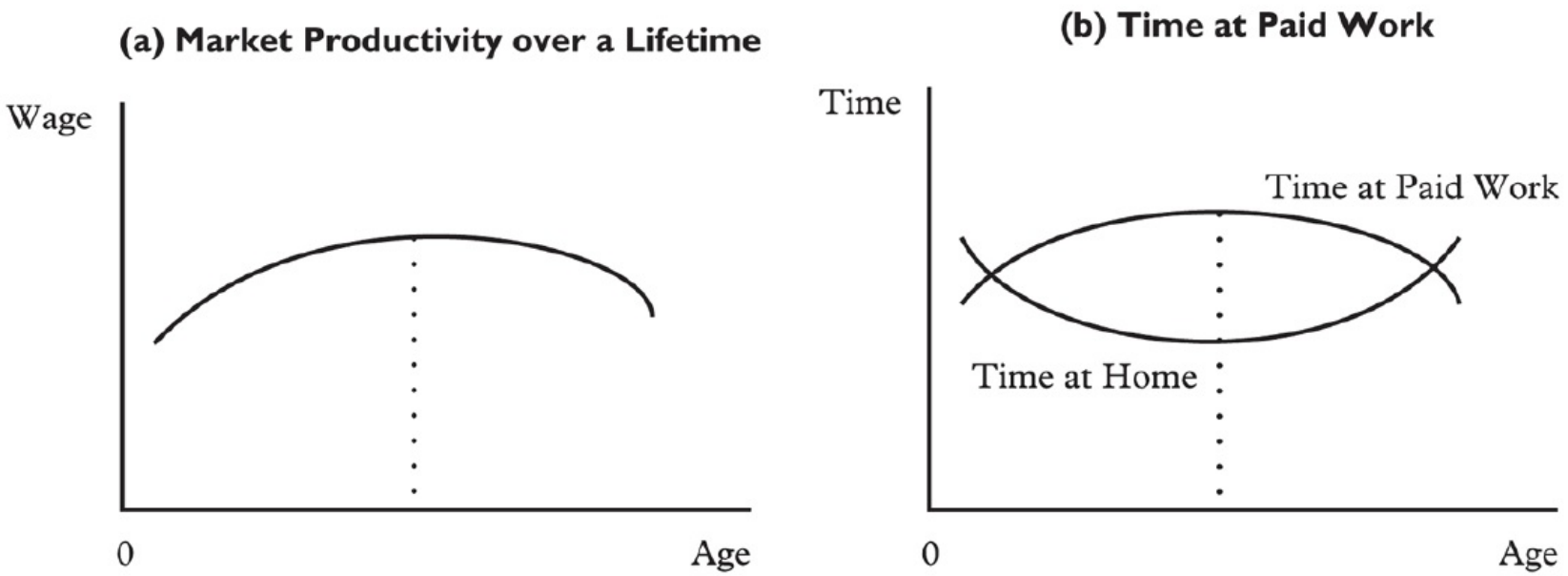


Figure 14: Life-cycle allocation of time

Life Cycle Aspect of Labor Supply

Other considerations:

- Specialization of function. Do both work for pay?
- Labor supply in recession: "discouraged" vs. "additional" worker effect

Example

Jack and Jill both have 10 hours of time
Jack can either produce $ 10 worth of household goods/hr or $20/hr market wage.
Jill can either produce $25 worth of household goods/hr or $15/hr market wage.

Individual's Budget

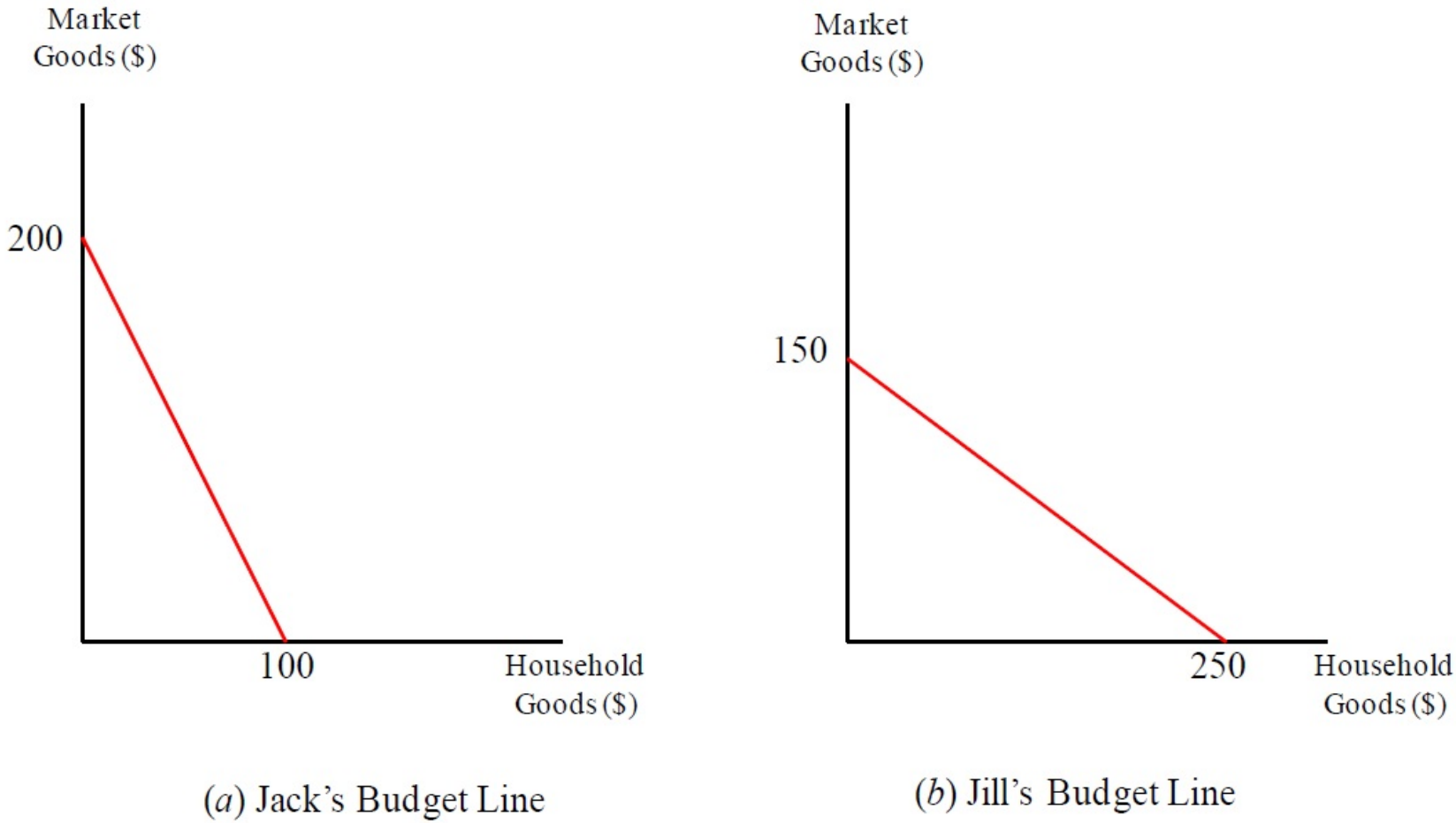


Figure 15: Budget lines and opportunity frontier of married couple

Joint Household Budget

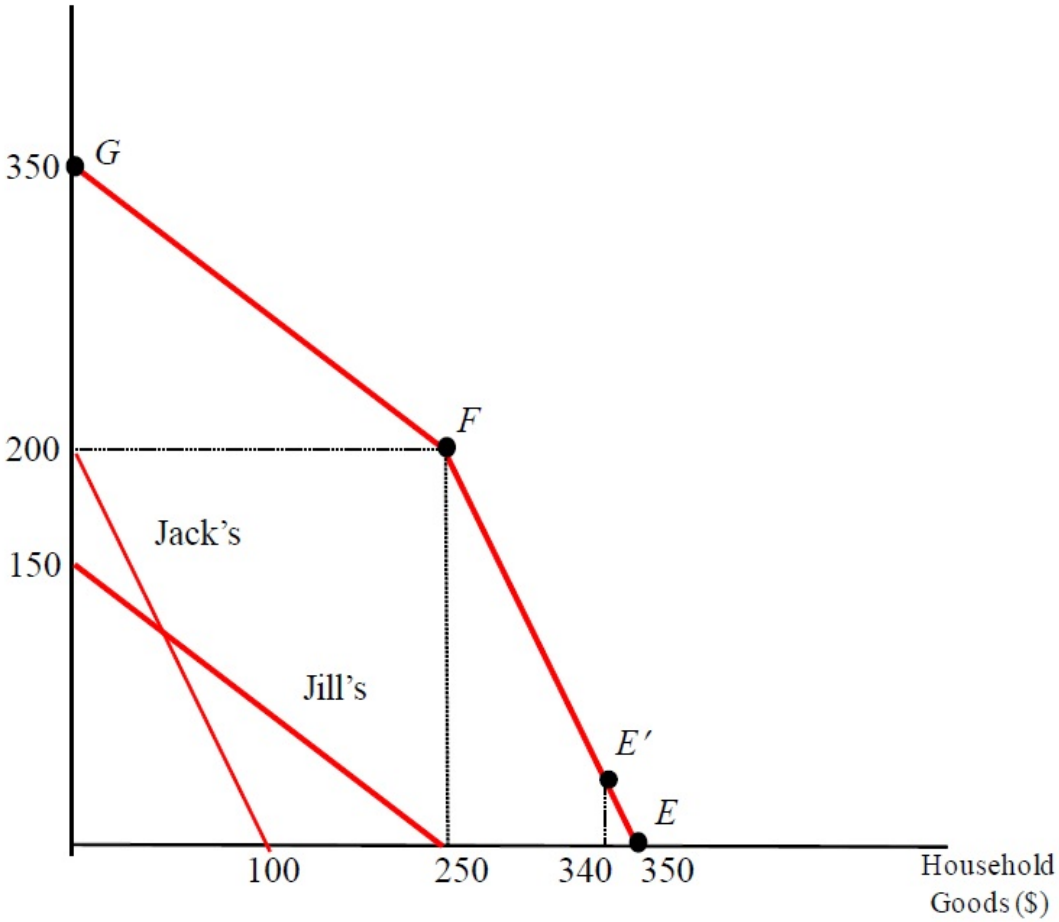


Figure 16: Budget lines and opportunity frontier of married couple

Who Work's Where?

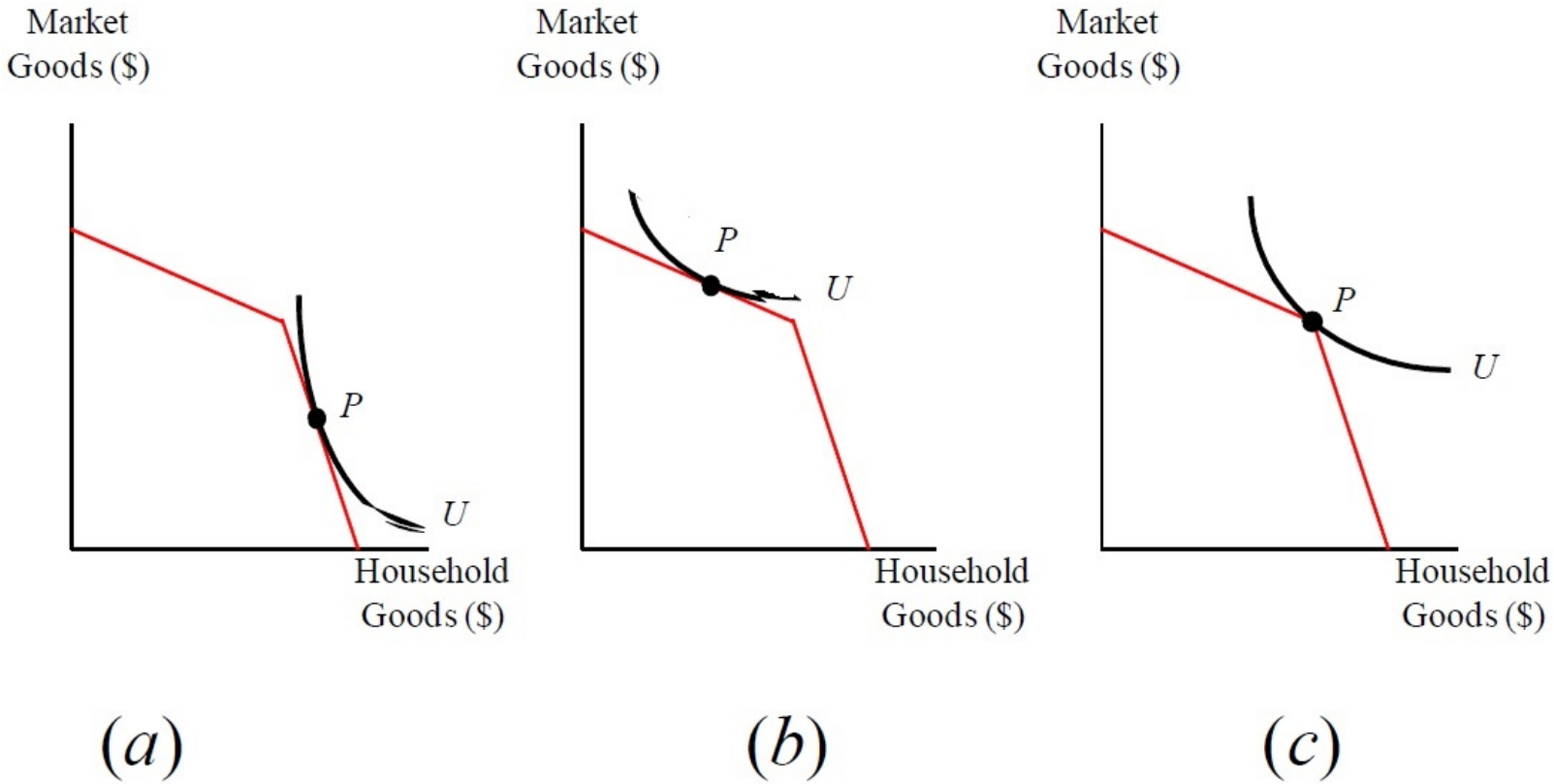


Figure 17: The division of labor in the household

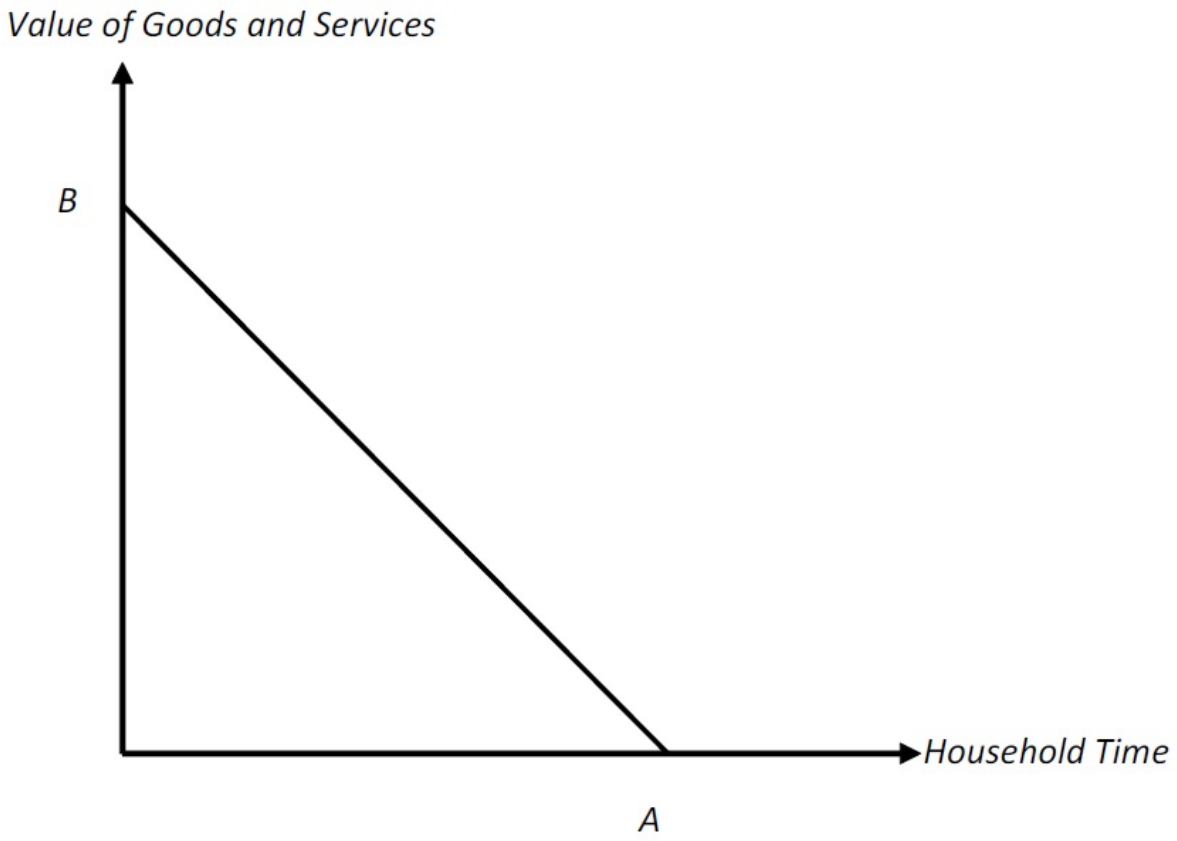
Who Work's Where?

At point P:

- Panel a: Jill specializes in the household sector and Jack divides his time between the labor market and the household.
- Panel b: Jack specializes in the labor market and Jill divides her time between the labor market and the household.
- Panel c: Jack specializes in the labor market and Jill specializes in the household sector.

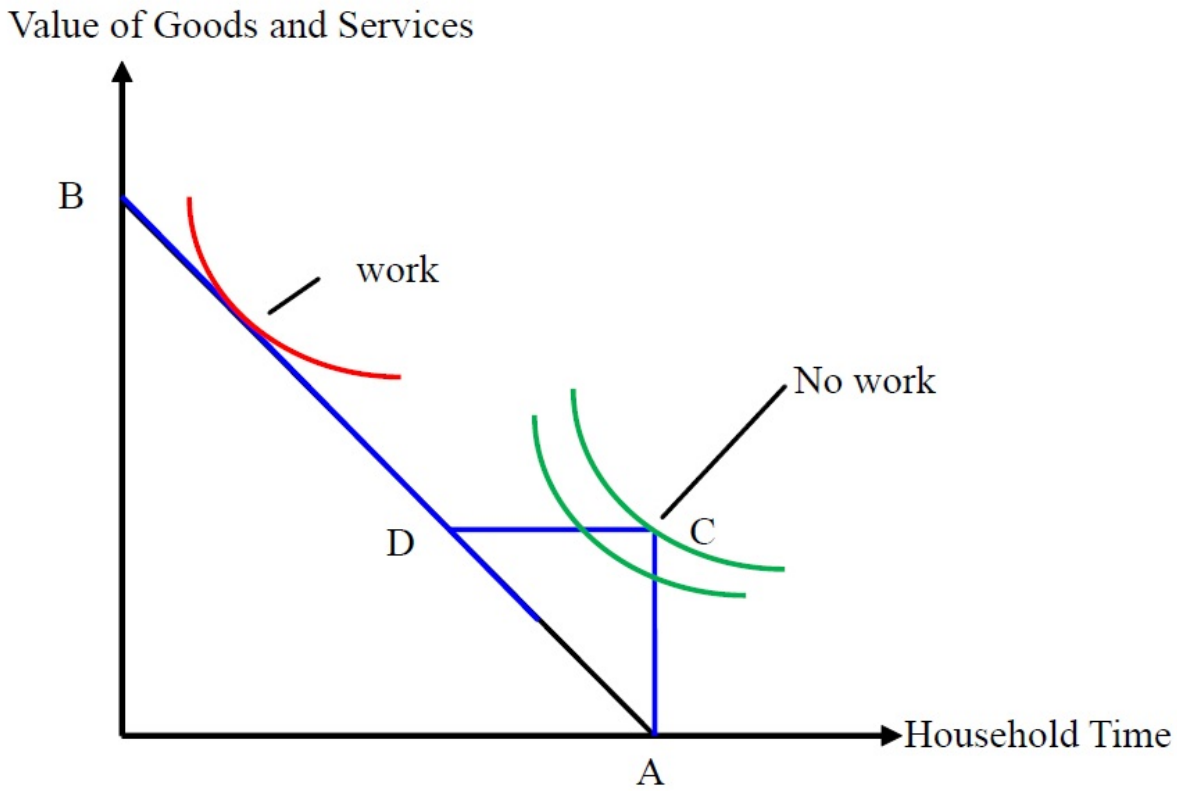
Policy Implication: Child Care and Labor Supply

Market wage is along budget constraint AB
Single parent with dependent child
Without program, do not work and no income



Policy Implication: Child Care and Labor Supply

If guaranteed welfare such that income is at least AC (for earnings $\leq AC$, \$1 earning $\uparrow \Rightarrow$ \$1 welfare $\downarrow$)

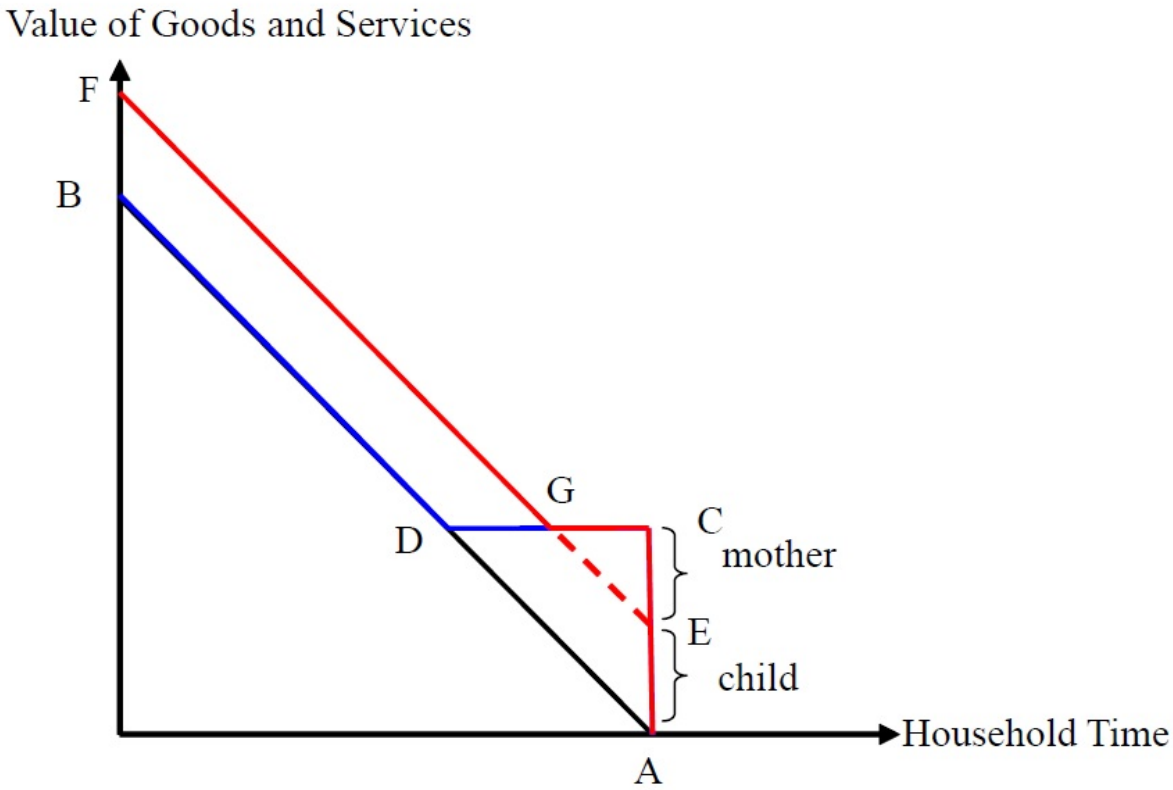


Policy Implication: Child Care and Labor Supply

Modify the welfare program such that it is a child support assurance program

Welfare for both mother and child: *ACGF*

If mother work *AEF*



Policy Implication: Child Care and Labor Supply

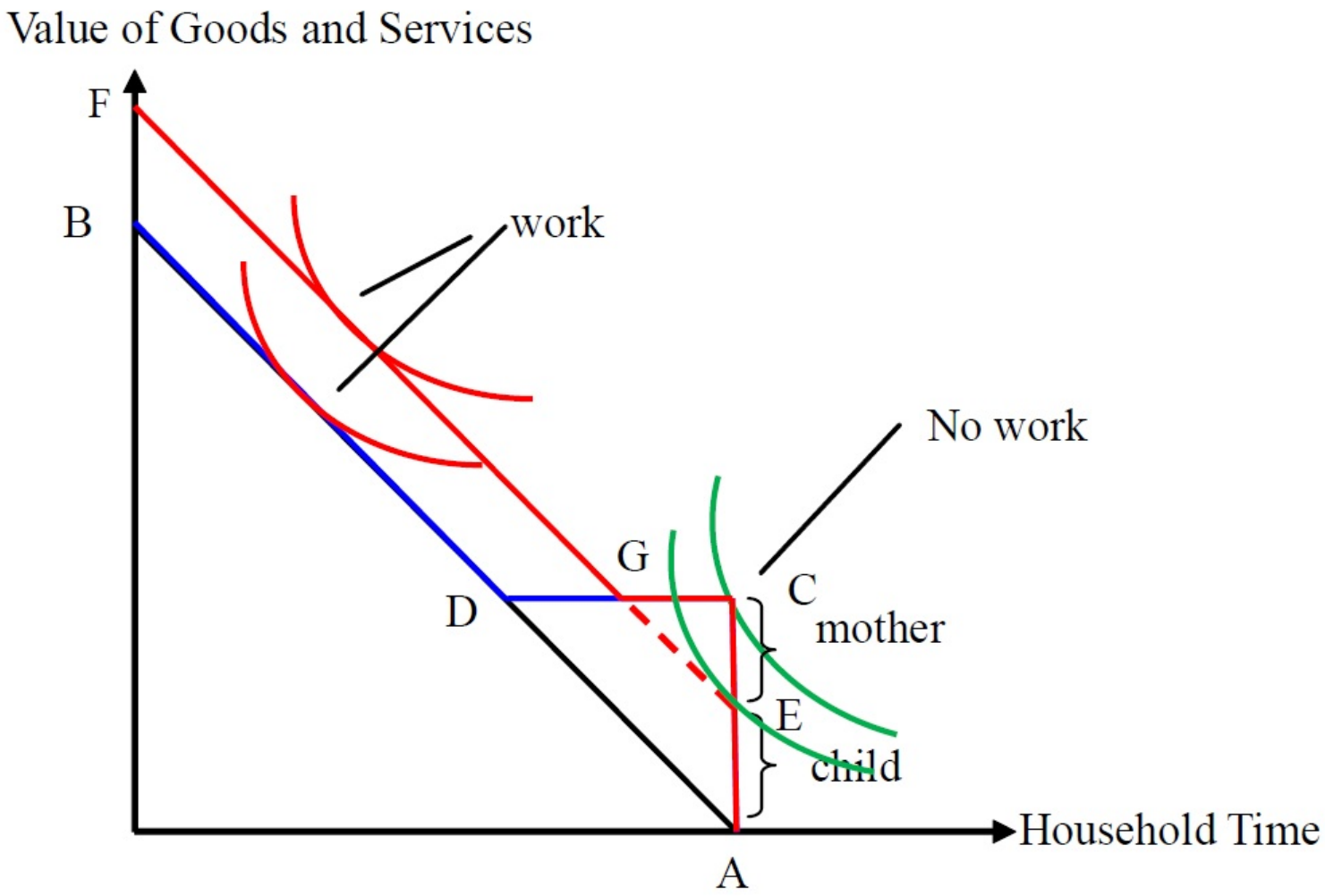
For working hours $\leq GC \Rightarrow AE$ (for the child program) $+EC$ (from the welfare)
For working hours $\geq GC \Rightarrow$ receive only $AE+$ earnings

Three different responses from three kinds of people

- remain at C (U isoquant very steep)
- reduce working hours if they already work before the child support program
- join labor force, move from C to H

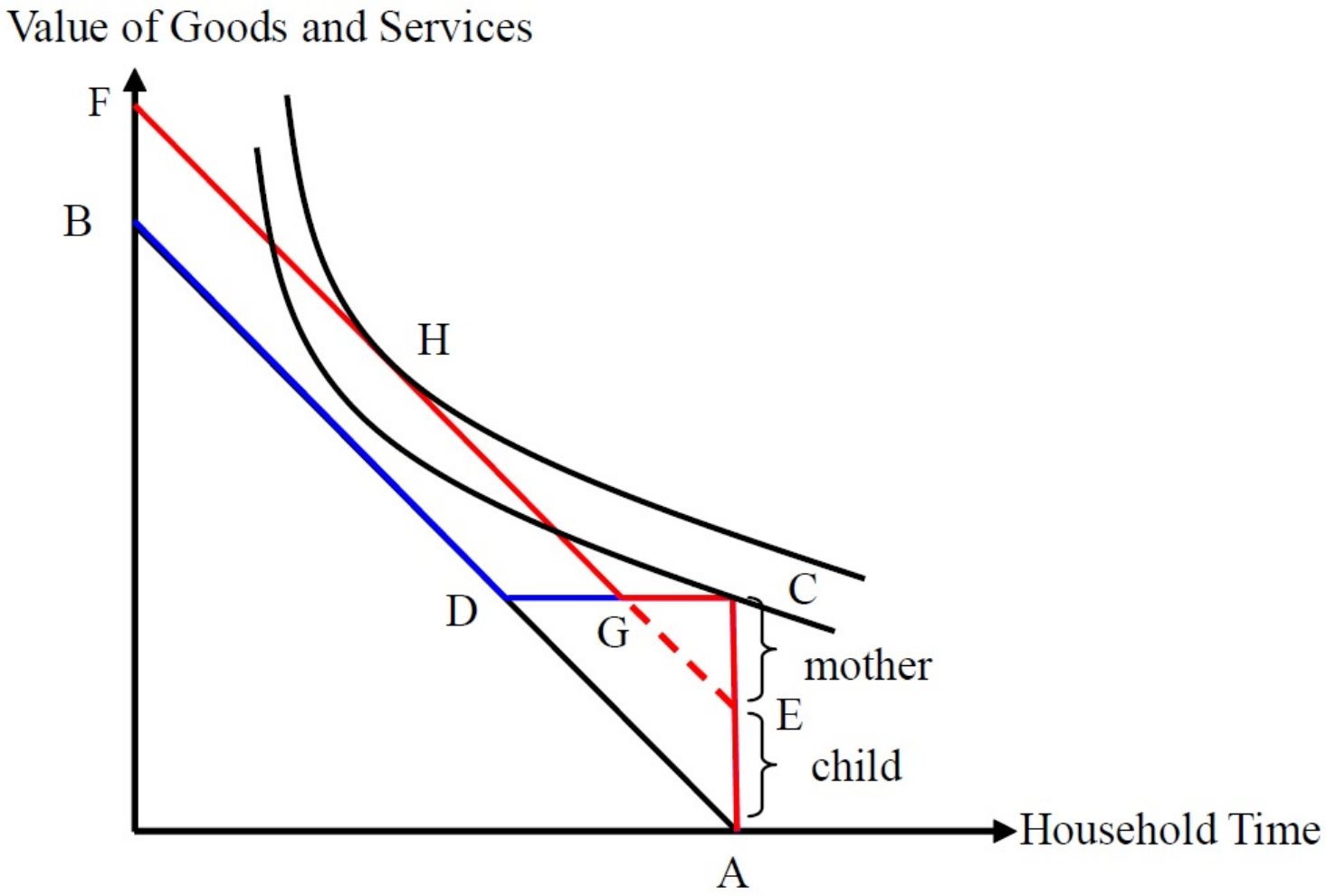
Policy Implication: Child Care and Labor Supply

The response from the first 2 kinds of people



Policy Implication: Child Care and Labor Supply

The response from the third kind of people



Reference

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- Ehrenberg and Smith (2015) *Modern Labor Economics: Theory and Public Policy, Chapter 6 and 7*