Labor Demand

Econ 3470 Lecture 2

Labor Economics

2015-2016 Term 1

Outline

1. Short-Run Labor Demand
   - Firm Labor Demand
     - Competitive Market
     - Monopolist Product Market
     - Monopsony Factor Market
   - Industry Labor Demand

2. Long-Run Labor Demand
   - Firm Labor Demand (Competitive Case)
   - Industry Labor Demand

Factors affecting Demand for Labor

1. Technology of production
2. Supply of other factors of production
3. Demand for the final product

- Firm vs. industry
- Short-run vs. long-run

Assumptions

1. Production function: $q = f(K, L)$.
2. All factors homogeneous
3. All firms are profit maximizing

Marginal Product

- Marginal product of labor MP_L

$$MP_L = \Delta Q / \Delta L \text{ (holding capital constant)}$$

- Marginal product of capital MP_K

$$MP_K = \Delta Q / \Delta K \text{ (holding labor constant)}$$

- Marginal revenue
  - Competitive product market: $MR = P$ (product price)
  - Differentiated product market (some monopoly power): $MR < P$ (extra units can be sold only if product price is reduced)

Marginal Product

- Value of Marginal product of labor

$$VMP_L = MP_L \cdot MR$$

- Marginal expense
  - Competitive labor market: $ME_L = W$ (market wage, flat labor supply curve)
  - Monopsony labor market: upward sloping labor supply curves

Short-Run Demand (Competitive Market)

Competitive - take all prices (P, W, r) as given, i.e. price takers - faces a horizontal product demand curve and a horizontal supply factor curve.

Firm maximize

$$\max_{L} \pi = Pf(\bar{K}, L) - r\bar{K} - WL$$

Only 1 degree of freedom, i.e. only able to change L to $\max \pi$.

Short-Run Demand (Competitive Market)

FOC

$$\frac{\partial \pi}{\partial L} = 0$$
$$= Pf_L - W$$

where $Pf_L = VMP_L$

$$Pf_L(\bar{K}, L) = W$$
$$VMP_L = W$$
$$f_L(\bar{K}, L) = \frac{W}{P}$$

Short-Run Demand (Competitive Market)

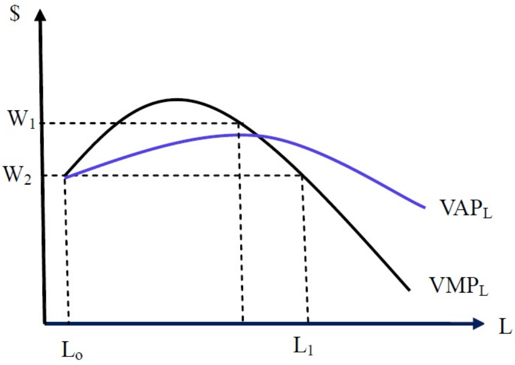
SOC

$$\frac{\partial^2 \pi}{\partial L^2} = Pf_{LL} < 0 \Rightarrow VMP_L \text{ negatively slope}$$

Value of average product of labor is $VAP_L = P \cdot AP_L$
Value of marginal product of labor is $VMP_L = P \cdot MP_L$

$\Rightarrow$ The competitive firm's labor demand curve is simply that part of the value of marginal product curve which is below the average product curve.

Short-Run Demand (Competitive Market)



Short-Run Demand (Competitive Market)

Proof.

$$TVC \leq TR$$
$$WL \leq Pf$$
$$W \leq \frac{Pf}{L} = VAP_L$$

FOC implies that $Pf_L(\bar{K}, L) = W$. □

Short-Run Demand (Competitive Market)

Take total derivative of FOC

$$Pf_{LL}dL + Pf_{LK}dK + f_L dP = dW$$

Effect of an increase in wage

$$\frac{\partial L}{\partial W} = \frac{1}{Pf_{LL}} < 0$$

Effect of an increase in demand (P)

$$\frac{\partial L}{\partial P} = -\frac{f_L}{Pf_{LL}} > 0$$

P increase, VMP and VAP curves shift up. L increases.

Short-Run Demand (Competitive Market)

$$Pf_{LL}dL + Pf_{LK}dK + f_L dP = dW$$

Since P and W will not change we get

$$Pf_{LL}dL = -Pf_{LK}dK$$
$$\frac{\partial L}{\partial K} = -\frac{f_{LK}}{f_{LL}}$$

where $f_{LL} < 0$ by SOC.

$f_{LK} > 0 \Rightarrow$ complements. i.e. $\uparrow K \rightarrow \uparrow L$.
$f_{LK} < 0 \Rightarrow$ substitutes. i.e. $\uparrow K \rightarrow \downarrow L$.

Short-Run Demand (Product Market Monopolist)

Monopolist - only seller in the market, i.e. price setter
P is also a function of $q = f(K, L)$.

$$\begin{aligned}\max_L \pi &= P(q)f(\bar{K}, L) - r\bar{K} - WL \\ &= P(f(\bar{K}, L))f(\bar{K}, L) - r\bar{K} - WL\end{aligned}$$

Still 1 degree of freedom, price depends on output

Short-Run Demand (Product Market Monopolist)

FOC

$$Pf_L + f\frac{\partial P}{\partial f}f_L - W = 0$$
$$P(1+\frac{1}{\eta})f_L = W$$

where $\eta = \frac{P}{f}\frac{\partial f}{\partial P} < 0$ is price elasticity of product demand.
$MR = P(1+\frac{1}{\eta}) < P$.

FOC indicates that at any given level of P and W, f_L is greater and L is smaller under monopolistic condition as compared to perfect competition in the output market.

Short-Run Demand (Labor Market Monopsony)

Monopsony - a market with only one buyer.
In the labor market we are talking about there is only one employer in the labor market.
The employer faces the market supply curve.
Marginal expense of labor, ME_L - the extra expenditures associated with hiring one additional unit of labor

$$\max_L \pi = Pf(\bar{K}, L) - r\bar{K} - W(L)L$$

Short-Run Demand (Labor Market Monopsony)

FOC

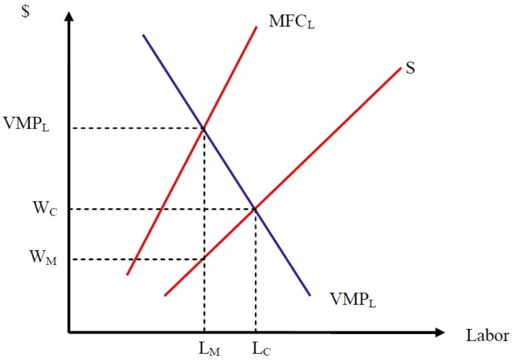
$$Pf_L(\bar{K}, L) - [W(L) + W'(L)L] = 0$$

$$Pf_L(\bar{K}, L) = W(1 + \frac{1}{\epsilon})$$

$$VMP_L = ME_L$$

where $\epsilon = \frac{dL}{dW}\frac{W}{L}$ is the price elasticity of labor supply

Short-Run Demand (Labor Market Monopsony)



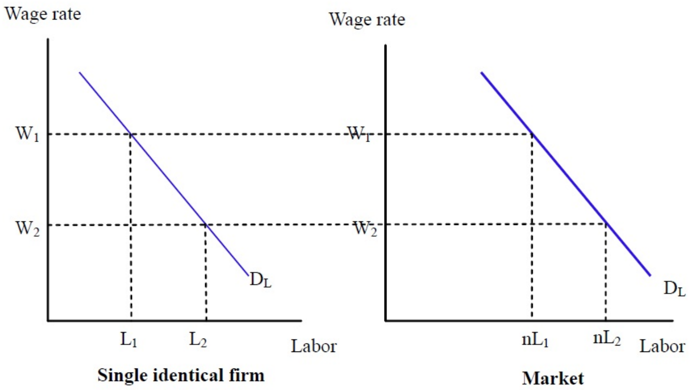
Short-Run Industry Demand (Competitive Market)

Assume that

- there are n identical firms in the industry;
- there are no technological externalities in production in the industry.

Short-Run Industry Demand (Competitive Market)

If product price is constant (i.e. the product demand is perfectly elastic) $\Rightarrow$ the industry labor demand curve will simply be the horizontal summation of the labor demand curves of all n firms.

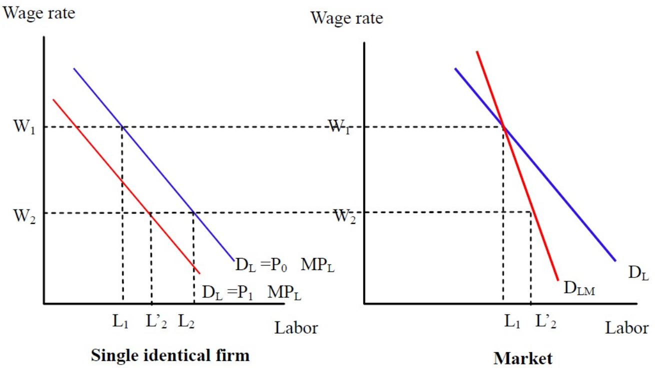


Short-Run Industry Demand (Competitive Market)

If product demand is not perfectly elastic.
When firms expand output
$\rightarrow\downarrow$ product price
$\rightarrow\downarrow$ VMP_L
$\rightarrow\downarrow$ demand for labor and output

Therefore, the industry factor demand curve is less elastic than the horizontal aggregation of the VMP_L curves of the individual firms.

Short-Run Industry Demand (Competitive Market)



Long-Run Demand (Competitive Market)

$$\max_{K,L} \pi = Pf(K,L) - WL - rK$$

FOC

$$\frac{\partial \pi}{\partial L} = 0 \Rightarrow Pf_L - W = 0$$
$$\frac{\partial \pi}{\partial K} = 0 \Rightarrow Pf_K - r = 0$$

Long-Run Demand (Competitive Market)

FOC for π imply

$$\frac{W}{f_L} = \frac{r}{f_K} = P$$

$\frac{1}{f_L}$ is the number of units of labor required to produced an additional unit of output

$\Rightarrow \frac{W}{f_L}$ is the cost of producing an additional unit of output when only labor input is increased.

Long-Run Demand (Competitive Market)

- Isoquant: possible combinations of K and L that produce the same level of output
  - Isoquants must be downward sloping.
  - Isoquants do not intersect.
  - Higher isoquants are associated with higher levels of output.
  - Isoquants are convex to the origin.
- Isocost: all combinations of K and L that are equally costly

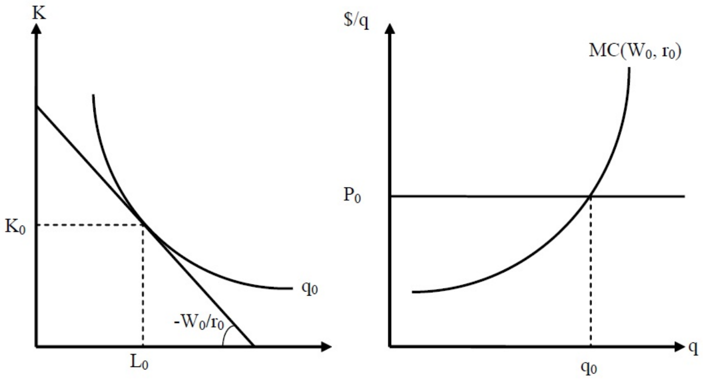
  $$C = WL + rK$$

  The isocost line is

  $$K = \frac{C}{r} - \frac{W}{r}L$$

  with intercept C/r and slope $-W/r$.

Long-Run Demand (Competitive Market)



Slope of isocost line $= -\frac{W}{r} = -\frac{f_L}{f_K} =$ slope of isoquant

Marginal rate of substitution: $\frac{dK}{dL} = -\frac{MP_L}{MP_K}$

Long-Run Demand (Competitive Market)

FOC

$$Pf_L = W$$
$$Pf_K = r$$

SOC

$$Pf_{LL}dL + Pf_{LK}dK = dW$$
$$Pf_{KL}dL + Pf_{KK}dK = dr$$

Long-Run Demand (Competitive Market)

Assume $dr = 0$
Define $\Delta = P(f_{LL} f_{KK} - f_{LK}^2)$

$$\frac{dL}{dW} = \frac{f_{KK}}{\Delta} < 0$$
$$\frac{dK}{dW} = -\frac{f_{KL}}{\Delta} \gtrless 0$$

where $\Delta > 0$ by the SOC.

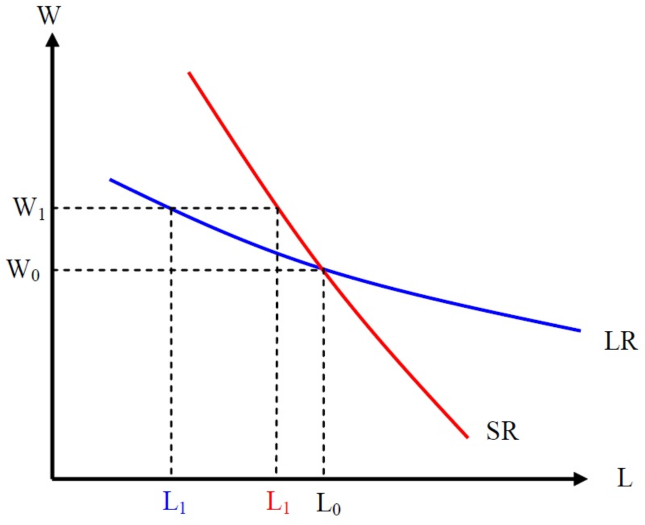
Long-Run Demand (Competitive Market)

Compare with SR where K is fixed, i.e. $\frac{\partial L}{\partial W} = \frac{1}{Pf_{LL}}$

$$\frac{\partial L}{\partial W}\Big|_K = \frac{1}{Pf_{LL}} > \frac{f_{KK}}{P(f_{LL}f_{KK} - f_{LK}^2)} = \frac{1}{Pf_{LL} - \frac{Pf_{LK}^2}{f_{KK}}} = \frac{\partial L}{\partial W}$$

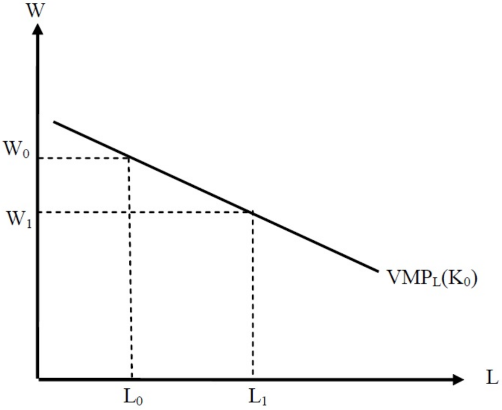
Thus, labor demand is more elastic in the long run than in the short run (i.e. LR is flatter than SR).

Long-Run Demand (Competitive Market)



Long-Run Demand (Competitive Market)

Intuitively, in the SR when there is a change in market wage, labor input will adjust along the $VMP_L(\bar{K})$.



Long-Run Demand (Competitive Market)

Given time, capital input can adjust.
$\Rightarrow$ shift the VMP_L curve (even if $\bar{P}$ is constant for a competitive firm)
Effect of wage on capital in the LR, depends on the sign of f_{KL}

$$\frac{\partial K}{\partial W} = -\frac{f_{KL}}{\Delta} \begin{cases} < 0 \text{ if } f_{KL} > 0 \text{ (complements)} \\ > 0 \text{ if } f_{KL} < 0 \text{ (substitutes)} \end{cases}$$

Long-Run Demand (Competitive Market)

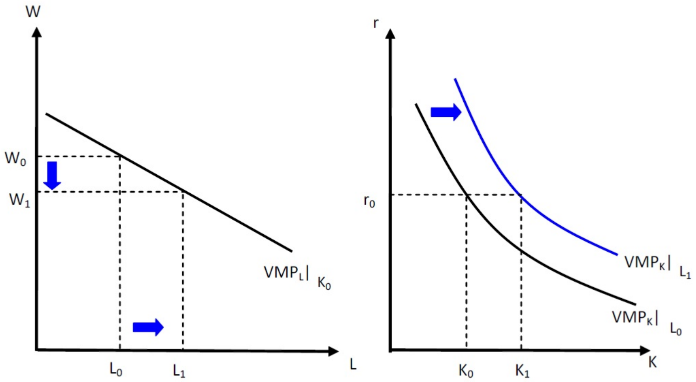
When K and L are complements

$$f_{KL} > 0$$
$$\frac{\partial K}{\partial W} < 0$$

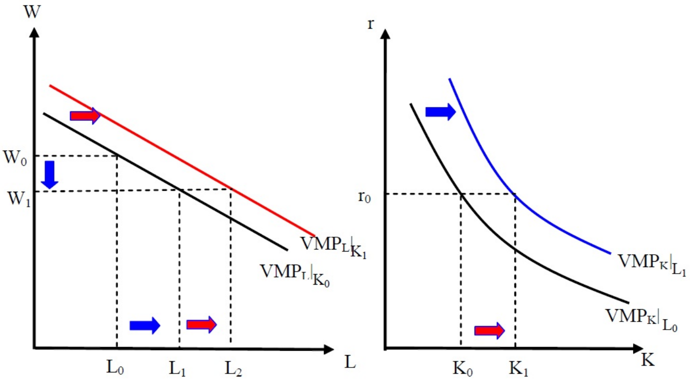
As wage decrease

- labor increase
- shift VMP_K curve to right, same r, K increase
- shift VMP_L curve to right, L increase further
- both curves will shift until it converges to new equilibrium

Long-Run Demand (Competitive Market)



Long-Run Demand (Competitive Market)



LR response in this case is stronger than the SR case.

Long-Run Demand (Competitive Market)

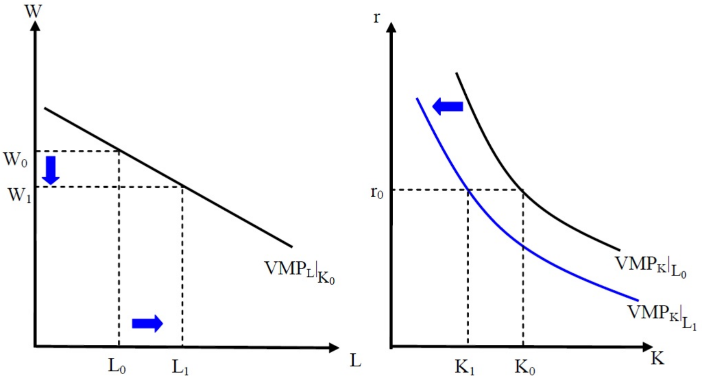
When K and L are substitutes

$$f_{KL} < 0$$
$$\frac{\partial K}{\partial W} > 0$$

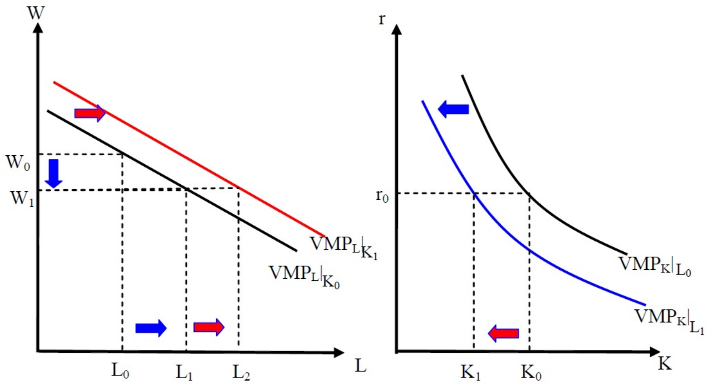
As wage decrease

- labor increase
- shift VMP_K curve to left, same r, K decrease

Will SR labor demand shift to the left and LR labor demand be steeper?



No!
$f_{KL} < 0, MP_L$ increase, labor demand shift out at a lower K



Long-Run Demand

Labor demand function have a negative slope implies: wage increase, quantity demand for labor must decrease.
This observation can be decomposed into 2 effects:

- Substitution effect
- Scale effect

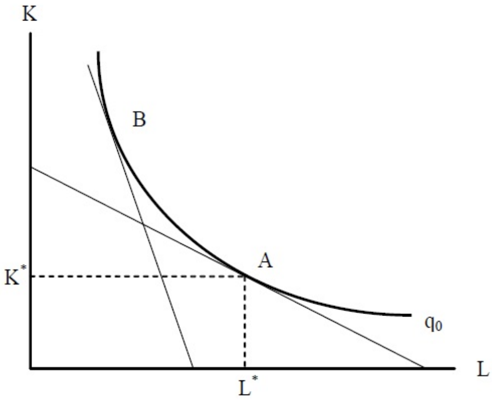
$$\frac{\partial L}{\partial W} = \frac{\partial L}{\partial W}\Big|_{\bar{q}} + \frac{\partial L}{\partial W}_{(q \text{ variable})}$$

Substitution Effect

$$\frac{\partial L}{\partial W}|_{\bar{q}} < 0$$

Effect of a factor price change where output is held constant must be negative.
In other words, at constant output (along a given isoquant), an increase in wage must reduce demand for labor.

Substitution Effect



Scale Effect

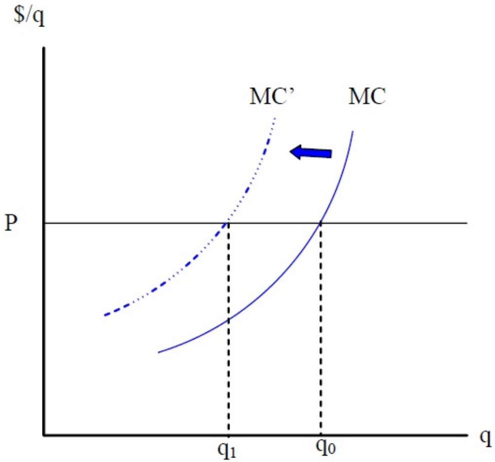
$$\frac{\partial L}{\partial W}_{(q \text{ variable})}$$

When W changes, it will affect the scale of production.

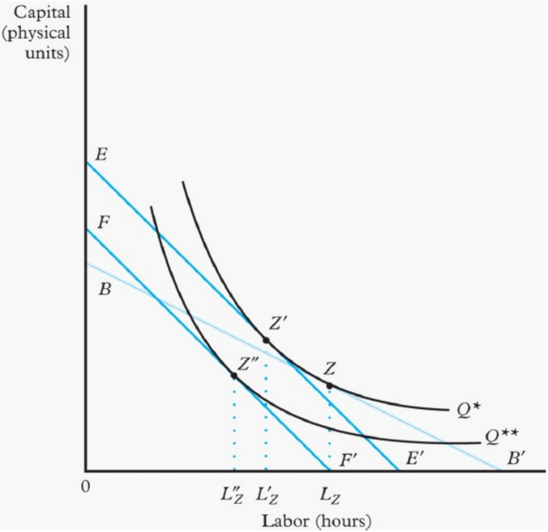
$$\begin{aligned} W \uparrow \rightarrow & \quad \uparrow MC \text{ shift supply curve upwards/left} \\ \rightarrow & \quad \downarrow Q \text{ (i.e. } \downarrow \text{ in scale)} \\ \rightarrow & \quad \downarrow L \end{aligned}$$

Same direction as substitution effect.

Scale Effect



Substitution and Scale Effects of a Wage Increase



Substitution and Scale Effects of a Wage Increase

Overall response L_z to L''_Z

- Substitution effect: L_Z to L'_Z
- Scale effect: L'_Z to L''_Z

Both are negative effect $\Rightarrow$ LR labor demand curve slopes downward

Substitution and Scale Effects

Components of the Own-Wage Elasticity of Demand for Labor: Empirical Estimates Using Plant-Level Data

	Estimated Elasticity
Short-Run Scale Effect	
British manufacturing firms, 1974–1982	−0.53
Substitution Effect	
32 studies using plant or narrowly defined industry data	Average: −0.45 (typical range: −0.15 to −0.75)
Overall Labor Demand Elasticity	
British plants, 1984	−0.93
British coal mines, 1950–1980	−1.0 to −1.4

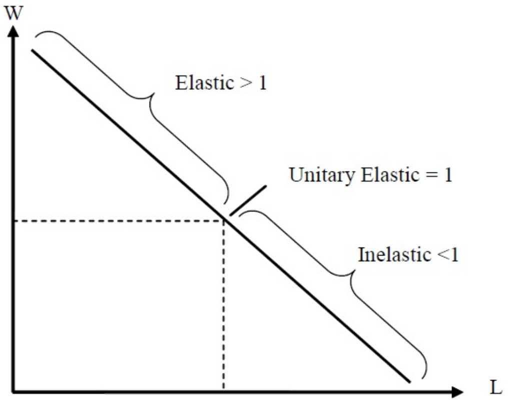
Long-Run Industry Labor Demand

Industry demand for labor $\neq$ horizontal summation of firm's demand, because

- Pecuniary externalities (the price of the product will adjust as each firm changes its output in response to an exogenous change in wage)
- Technological externalities
- Exit and entry $\Rightarrow$ zero economic profit

So these factors must be incorporated into our analysis when deriving the LR industry labor demand.

Labor Demand Elasticity



Straight-line demand curve: a unit change in wages induces the same response in units of employment at each point along the curve.

Same responses in unit changes along the demand curve do not imply equal percentage changes.

Own Wage Elasticity

$$\epsilon_{ii} = \frac{\%\Delta L_i}{\%\Delta W_i}$$

$$\epsilon_{ii} \begin{cases} > 1 \text{ elastic} \\ = 1 \text{ unitary elastic} \\ < 1 \text{ inelastic} \end{cases}$$

Cross Wage Elasticity

$$\epsilon_{jk} = \frac{\%\Delta L_j}{\%\Delta W_k}$$

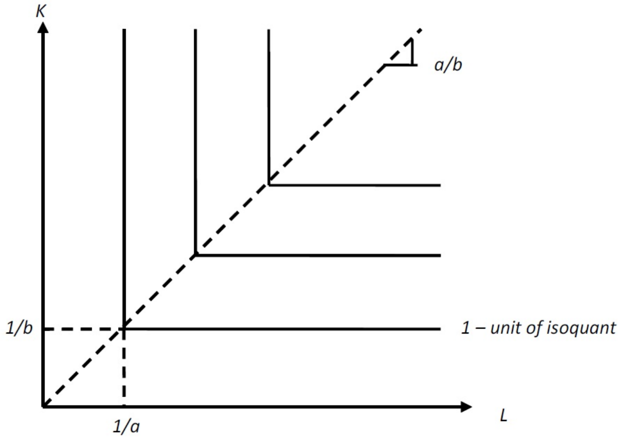
$$\epsilon_{jk} \begin{cases} > 0 \text{ substitutes} \\ < 0 \text{ complements} \end{cases}$$

Industry Long-Run Demand

Assume

- No technology externalities
- Constant returns to scale

Example: $q = \min(aL, bK)$
Leontief - no substitution effect



Scale Effect

Each firm

$$L = \frac{q}{a}$$
$$K = \frac{q}{b}$$

Industry (assume there are n identical firms)

$$L = \frac{Q}{a} = \frac{nq}{a}$$
$$K = \frac{Q}{b} = \frac{nq}{b}$$

Scale Effect

Cost to individual firm

$$\begin{aligned} c &= WL + rK \\ &= W\frac{q}{a} + r\frac{q}{b} \end{aligned}$$

Average cost and marginal cost

$$\begin{aligned} AC = MC &= \frac{c}{q} \\ &= \frac{W}{a} + \frac{r}{b} = P \end{aligned}$$

Product demand is $g()$

$$Q = g(P) = g(\frac{W}{a} + \frac{r}{b})$$

$$L = \frac{Q}{a} = \frac{g(\frac{W}{a} + \frac{r}{b})}{a}$$

Scale Effect

Take ln on both side

$$\ln L = \ln g(\frac{W}{a} + \frac{r}{b}) - \ln a$$

then differentiate

$$\begin{aligned} d\ln L &= d\ln g - d\ln a \\ &= \frac{\partial \ln g}{\partial \ln P} d\ln P - d\ln a \\ &= \eta_Q^D d\ln P - d\ln a \end{aligned}$$

where η_Q^D is elasticity of product demand.

Scale Effect

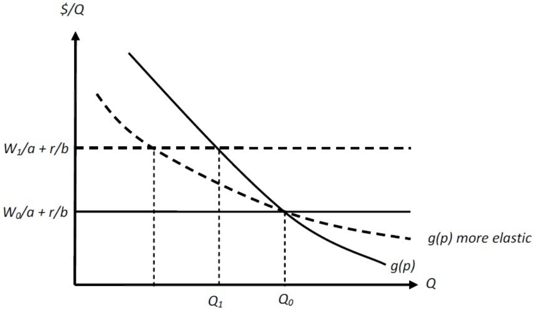
$$
\begin{aligned}
d\ln P &= d\ln\left(\frac{W}{a}+\frac{r}{b}\right) \\
&= \frac{\frac{W}{a}}{\left(\frac{W}{a}+\frac{r}{b}\right)} d\ln\frac{W}{a} + \frac{\frac{r}{a}}{\left(\frac{W}{a}+\frac{r}{b}\right)} d\ln\frac{r}{b}
\end{aligned}
$$

Let $v_L = \frac{\frac{W}{a}}{\left(\frac{W}{a}+\frac{r}{b}\right)}$, where v_L is the factor share of labor

$$
\begin{aligned}
d\ln P =& v_L(d\ln W - d\ln a) + (1-v_L)(d\ln r - d\ln b) \\
d\ln L =& \eta_Q^D[v_L(d\ln W - d\ln a) + (1-v_L)(d\ln r - d\ln b)] - d\ln a \\
=& \eta_Q^D v_L d\ln W + (1-v_L)\eta_Q^D d\ln r - (1+v_L\eta_Q^D) d\ln a \\
& - (1-v_L)\eta_Q^D d\ln b
\end{aligned}
$$

Elasticity of Labor Demand

$$\epsilon_{LL}^{\text{scale}} = \frac{d \ln L}{d \ln W} = \eta_Q^D v_L < 0$$



$$W \uparrow, MC \uparrow, P \uparrow \quad \Rightarrow Q \downarrow \quad \Rightarrow L \downarrow$$

Example

Given $d \ln W = 0.1$

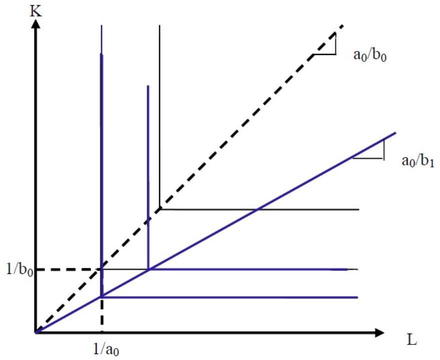
$$\begin{aligned} \text{capital intensive } v_L &= 0.1 \\ \%\Delta AC &= 0.01 \end{aligned}$$

$$\begin{aligned} \text{labor intensive } v_L &= 0.8 \\ \%\Delta AC &= 0.08 \end{aligned}$$

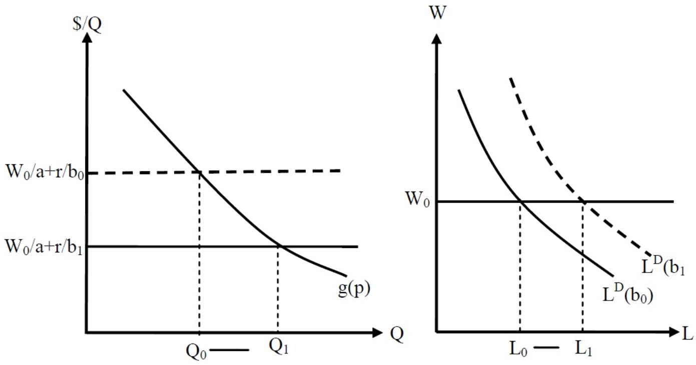
Technology Progress

$\uparrow b$

$$\frac{d \ln L}{d \ln b} = -(1 - v_L)\eta_Q^D > 0$$



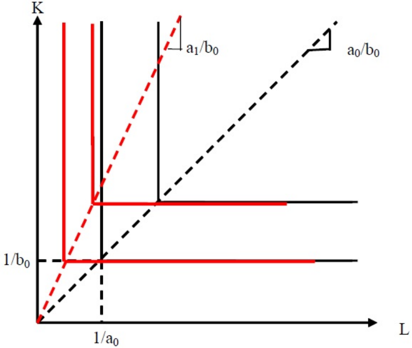
Technology Progress



$$b\uparrow, MC\downarrow, P\downarrow \;\; \Rightarrow Q\uparrow \;\; \Rightarrow L\uparrow$$

Technology Progress

$\uparrow a$



$$\frac{d\ln L}{d\ln a} = -(1 + v_L \eta_Q^D) \gtrless 0$$

$$\uparrow MP_L \left\{ \begin{array}{l} \Rightarrow \downarrow L \\ \Rightarrow \uparrow Q \Rightarrow \uparrow L \end{array} \right\} \text{uncertain}$$

Cross Wage Elasticity

$\uparrow r$. Assume that capital supply is elastic.

$$\frac{d \ln L}{d \ln r} = (1 - v_L)\eta_Q^D < 0$$

$$r \uparrow, MC \uparrow, P \uparrow \quad \Rightarrow Q \downarrow \quad \Rightarrow L \downarrow$$

Substitution Effect

$$
\begin{aligned}
q_0 &= f(K, L) \\
dq_0 &= 0 \\
&= f_K \frac{dK}{dW} + f_L \frac{dL}{dW} \\
&= f_K \left(\frac{dK}{dW}\Big|_{q_0} + \frac{f_L}{f_K} \frac{dL}{dW}\Big|_{q_0} \right)
\end{aligned}
$$

Substitution Effect

$$
\begin{aligned}
\frac{f_L}{f_K}\frac{dL}{dW}|_{q_0} + \frac{dK}{dW}|_{q_0} &= 0 \\
\frac{W}{r}\frac{dL}{dW}|_{q_0} + \frac{dK}{dW}|_{q_0} &= 0 \\
\frac{WL}{c}\frac{W}{L}\frac{dL}{dW}|_{q_0} + \frac{rK}{c}\frac{W}{K}\frac{dK}{dW}|_{q_0} &= 0 \\
v_L\frac{d\ln L}{d\ln W} + (1-v_L)\frac{d\ln K}{d\ln W} &= 0
\end{aligned}
$$

Substitution Effect

$$v_L \frac{d \ln L}{d \ln W} = -(1 - v_L) \frac{d \ln K}{d \ln W}$$

Add $(1 - v_L) \frac{d \ln L}{d \ln W}$ to both sides

$$\begin{aligned} \epsilon^c_{LL} \equiv \frac{d \ln L}{d \ln W} &= -(1 - v_L)\left(\frac{d \ln K}{d \ln W}|_{q_0} - \frac{d \ln L}{d \ln W}|_{q_0}\right) \\ &= -(1 - v_L) \frac{d \ln(K/L)}{d \ln W}|_{q_0} \\ &= -(1 - v_L) \frac{d \ln(K/L)}{d \ln(W/r)}|_{q_0} \end{aligned}$$

Since $d \ln r = 0$ implies $d \ln W = d \ln W - d \ln r = d \ln(W/r)$.

Substitution Effect

Let S_{LK} be the direct elasticity of substitution

$$S_{LK} = \frac{d\ln(\frac{K}{L})}{d\ln(\frac{f_L}{f_K})} = \frac{d\ln(\frac{K}{L})}{d\ln(\frac{W}{r})}$$

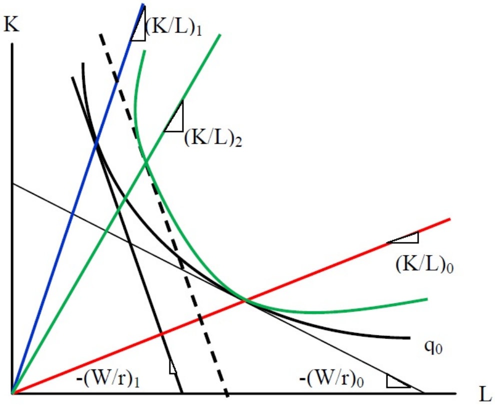
then

$$\epsilon^C_{LL} = -(1 - v_L)S_{LK}$$

properties of S_{LK}

- > 0 in 2-factor case
- $S_{LK} = S_{KL}$, symmetric.

Substitution Effect



$\downarrow S_{LK} \Rightarrow$ substitutability between factors $\downarrow \Rightarrow$ curvature of isoquants $\uparrow$

Total Effect

If r is constant (supply of K is perfectly elastic), then total effect = substitution effect + scale effect.

$$\begin{aligned}\epsilon_{LL} &= \epsilon^{c}_{LL} + \epsilon^{\text{scale}}_{LL} \\ &= -(1 - v_L)S_{LK} + v_L\eta^D_Q \\ |\epsilon_{LL}| &= (1 - v_L)|S_{LK}| + v_L|\eta^D_Q|\end{aligned}$$

weighted average of S_{LK} and η^D_Q

Total Effect

LR elasticity of labor demand depends on

- factor share (v_L)
- product demand elasticity (η_Q^D)
- elasticity of substitution (S_{LK})

Total Effect

Other things equal, when there is more than one factor, then the own-wage elasticity of demand for an input is high if $|\epsilon_{LL}| \uparrow$

- $|\eta_Q^D|$ is large - product elasticity is high
- S_{LK} is large - inputs can be easily substituted
- v_L is large - factor share of labor is high

First Law

$|\eta_Q^D|$ is large - product elasticity is high (elastic).
Mainly pertains to scale effect

$$\begin{aligned} \uparrow W \rightarrow \uparrow MC & \\ & \downarrow\downarrow Q \text{ (large scale effect)} \\ & \downarrow\downarrow L \end{aligned}$$

Second Law

S_{LK} is large - inputs can be easily substituted.
Related to substitution effect

$$\begin{aligned} \uparrow W \rightarrow & \downarrow L \\ & \uparrow K \\ & \downarrow\downarrow L \text{ (if highly substitutable)} \end{aligned}$$

Third Law

v_L is large - factor share of labor is high.
Factor share affects scale effect

$$\uparrow W(50\%) \rightarrow$$
$$v_L = 10\% \rightarrow \text{ total cost increase by } 5\% \quad \downarrow Q$$
$$v_L = 80\% \rightarrow \text{ total cost increase by } 40\% \quad \downarrow\downarrow Q$$

Reference

- Borjas, George (2013) *Labor Economics, Chapter 3 and 4*
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