TWO TO TANGO

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PROFIT-MAX LEAGUE

Conventional theory of sports leagues [(QFV) Fort and Quirk, 1995; and Vrooman, 1995] begins with simultaneous maximization of twin profit functions in a simplified two-team league:

$$\pi_1 = R_1[m_1, w_1(t_1, t_2)] - ct_1 \qquad \pi_2 = R_2[m_2, w_2(t_2, t_1)] - ct_2 \qquad (1)$$

Revenue R_1 of team 1 is a function of its market size m_1 and its winning percentage w_1, which is determined by a contest function of standard logistic probability form $w_1(t_1, t_2) = t_1/(t_1 + t_2)$.

The zero-sum nature of an n-team league requires $\Sigma w_i = n/2$ and $dw_1/dw_2 = dw_2/dw_1 = -1$. A profit-maximizing owner's objective is to max π_1 with respect to t_1.

At the profit max, team 1 sets its payroll ct_1 by acquiring talent until the marginal revenue product of talent MRP_1 is equal to the marginal cost of talent c, which is assumed to be the same for both teams:

$$MRP_1 = MR_1 MP_1 = (\partial R_1/\partial w_1)(\partial w_1/\partial t_1) = c \qquad (2)$$

Simultaneous profit max (mutual best response) yields:

$$MRP_1 = (\partial R_1/\partial w_1)(\partial w_1/\partial t_1) = c = MRP_2 \qquad (3)$$

The standard logit $w_1 = t_1/(t_1 + t_2)$ yields the marginal product of talent MP_1,

$$MP_1 = \partial w_1/\partial t_1 = (t_2 - t_1\, \partial t_2/\partial t_1)/(t_1 + t_2)^2 \qquad (4)$$

In league equilibrium, the *MRPs* for both teams are equal to their mutual wage rate c:

$$MRP_1 = MR_1 MP_1 = [\partial R_1/\partial w_1][(t_2 - t_1\, \partial t_2/\partial t_1)/T^2] = MRP_2 = c \qquad (5)$$

OPEN & CLOSED CASE

In a *closed league* an inelastic supply of talent $T^* = t_1 + t_2$ is fixed, (similar to N.A. sports leagues) and one team's talent gain is another team's zero-sum talent loss, such that $dt_1/dt_2 = dt_2/dt_1 = -1$. Substitution into (5) yields the *closed league* equilibrium condition:

$$MR_1 = MR_2 = cT^* \qquad (6)$$

By comparison, *open league* teams face an elastic supply of talent at an exogenous wage rate c^* (similar to European football leagues). One team's talent acquisition has zero effect on the talent of the other team, such that $dt_1/dt_2 = dt_1/dt_2 = 0$. Substitution into (5) yields the *open league* solution:

$$MR_1 w_2 = MR_2 w_1 = c^* T \qquad (7)$$

ASYMMETRIC MARKETS

An asymmetric revenue advantage $m_1 > m_2$ for team 1 can be shown generalizing profit-max solutions for a market size parameter $\sigma > 1$. The *Yankee paradox* is the argument that fans prefer close competition. Fan-preference for competitive balance implies strictly concave revenue functions measured by the preference scale parameter $\phi \in [0, 1]$:

$$\pi_1 = \sigma[\phi w_1 + (1-\phi)\, w_1 w_2] - ct_1 \qquad \pi_2 = [\phi w_2 + (1-\phi)\, w_1 w_2] - ct_2 \qquad (8)$$

The *Yankee paradox* suggests $\phi = .5$ and the zero-sum constraint $w_2 = 1 - w_1$ simplifies (8):

$$\pi_1 = \sigma(w_1 - .5w_1^{2}) - ct_1 \qquad \pi_2 = w_2 - .5w_2^{\,2} - ct_2 \qquad (9)$$

In a *closed league* from (6), simultaneous profit max of (9) yields:

$$MR_1 = MR_2 = \sigma w_2 = w_1 = cT^* \qquad (10)$$

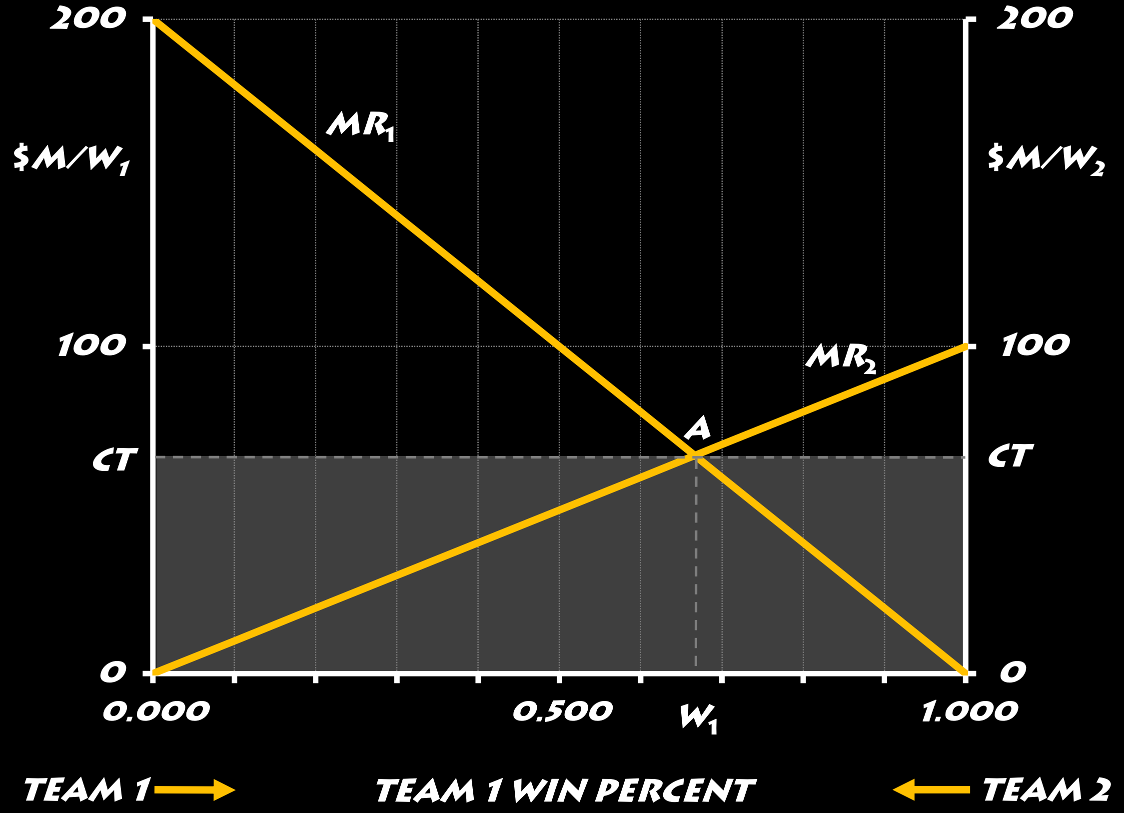
Team 1 dominates a *closed league* by the ratio $w_1/w_2 = \sigma$ with respective win percentages, $w_1 = \sigma/(1+\sigma)$ and $w_2 = 1/(1+\sigma)$. League payroll is: $cT^* = \sigma/(1+\sigma)$ with team payrolls $ct_1 = \sigma/(1+\sigma)^2$ and $ct_2 = 1/(1+\sigma)^2$

By comparison the σ-model *open-league* solution from (7) is:

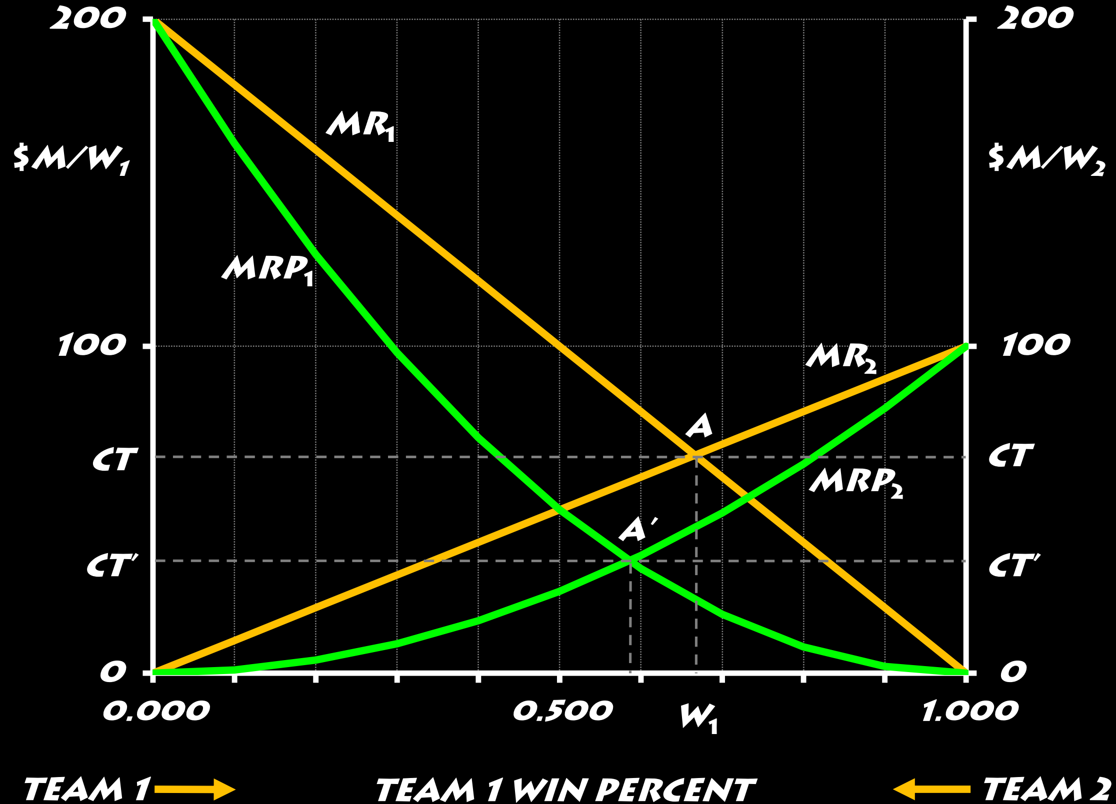
$$MR_1 w_2 = MR_2 w_1 = \sigma w_2^2 = w_1^2 = c^*T \qquad (11)$$

An *open league* has greater competitive balance $w_1/w_2 = \sigma^{1/2}$ for team win percentages $w_1 = \sigma^{1/2}/(1+\sigma^{1/2})$ and $w_2 = 1/(1+\sigma^{1/2})$.

NON-COOPERATIVE LEAGUE EQUILIBRIUM



OPEN & CLOSED PROFIT-MAX LEAGUES



REVENUE-SHARING PARADOX

The *invariance proposition* holds that competitive balance in a sports league will be the same with or without revenue sharing. Revenue sharing serves only to shift monopsony rent from players to owners.

Consider a straight pool-sharing formula $R_1' = \alpha R_1 + (1-\alpha)(R_1+R_2)/2$, where each team blends an α-share of its home revenue with an $(1-\alpha)$ visiting-team share of a league revenue pool, where $\alpha \in [0,1]$.

The zero-sum constraint implies $dw_1/dt_1 = -\,dw_2/dt_1$ and *closed league* α-sharing from (10) yields the σ-solution for $MR_1' = MR_2' = c'T$:

$$\alpha\sigma w_2 + (1-\alpha)(\sigma w_2 - w_1)/2 = \alpha w_1 - (1-\alpha)(\sigma w_2 - w_1)/2 \qquad (12)$$

This yields the same balance $w_1/w_2 = \sigma$ as (10) regardless of α-share. The second term in (12) vanishes at league equilibrium $\sigma w_2 = w_1$, and lower league payroll at $c'T = \alpha\sigma w_2 = \alpha w_1 = \alpha\sigma/(1+\sigma)$ reveals that the rate of exploitation is equal to the degree of revenue sharing $(1-\alpha)$.

By comparison, the *open-league* revenue-sharing solution implies:

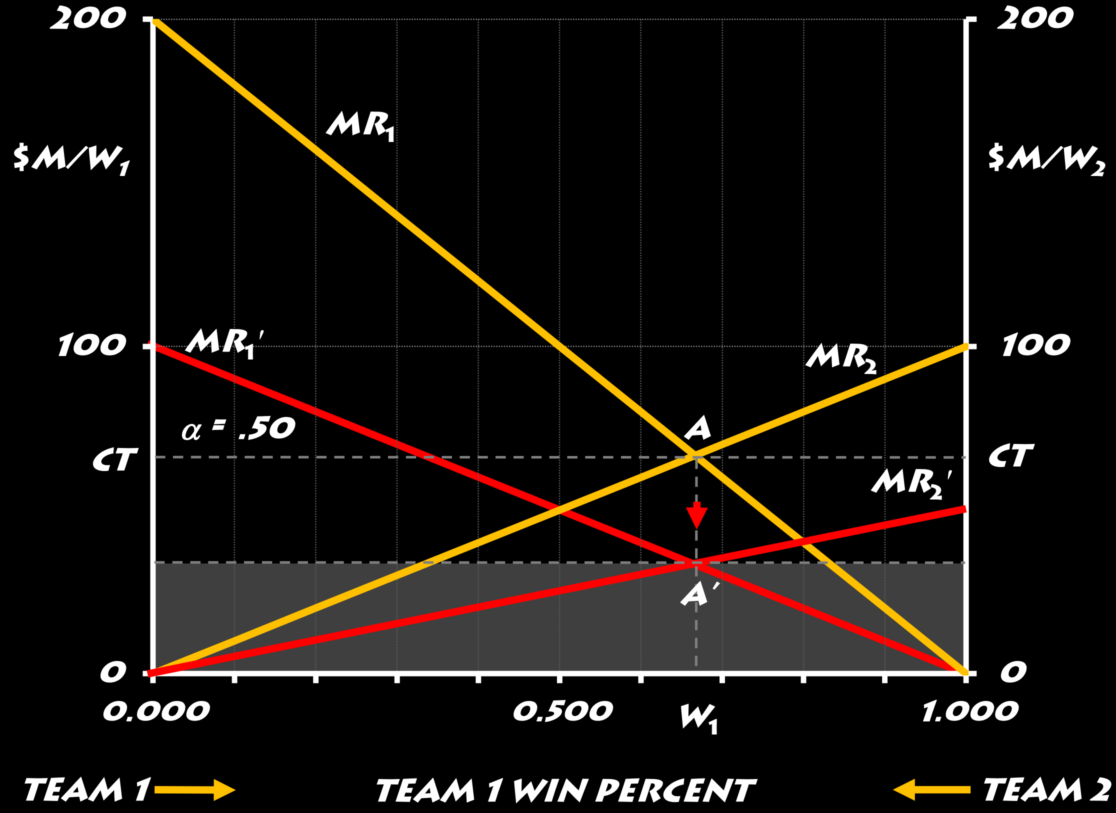
$$2\alpha(\sigma w_2^2 - w_1^2) + (1-\alpha)(\sigma w_2 - w_1)(w_1 + w_2) = 0 \qquad (13)$$

If there is no revenue sharing ($\alpha = 1$) then the second term vanishes and (13) reduces to the *open league* solution $w_1/w_2 = \sigma^{½}$ in (11). As league-sharing approaches a perfect syndicate ($\alpha \to 0$) the first term vanishes and the second term approaches the closed league solution $w_1/w_2 = \sigma$ in (10). At the limit ($\alpha = 0$) open and closed league solutions are identical and cost per unit of talent is reduced to reservation wage.

The invariance proposition still holds in a closed league, but revenue sharing in an open league actually *reduces* competitive balance.

The counter-intuitive conclusion is that revenue sharing does not lead to competitive balance in either closed or open profit-max leagues, but it does create mutual disincentives that lead to the exploitation of talent.

COOPERATIVE CARTEL EQUILIBRIUM



SALARY CAP

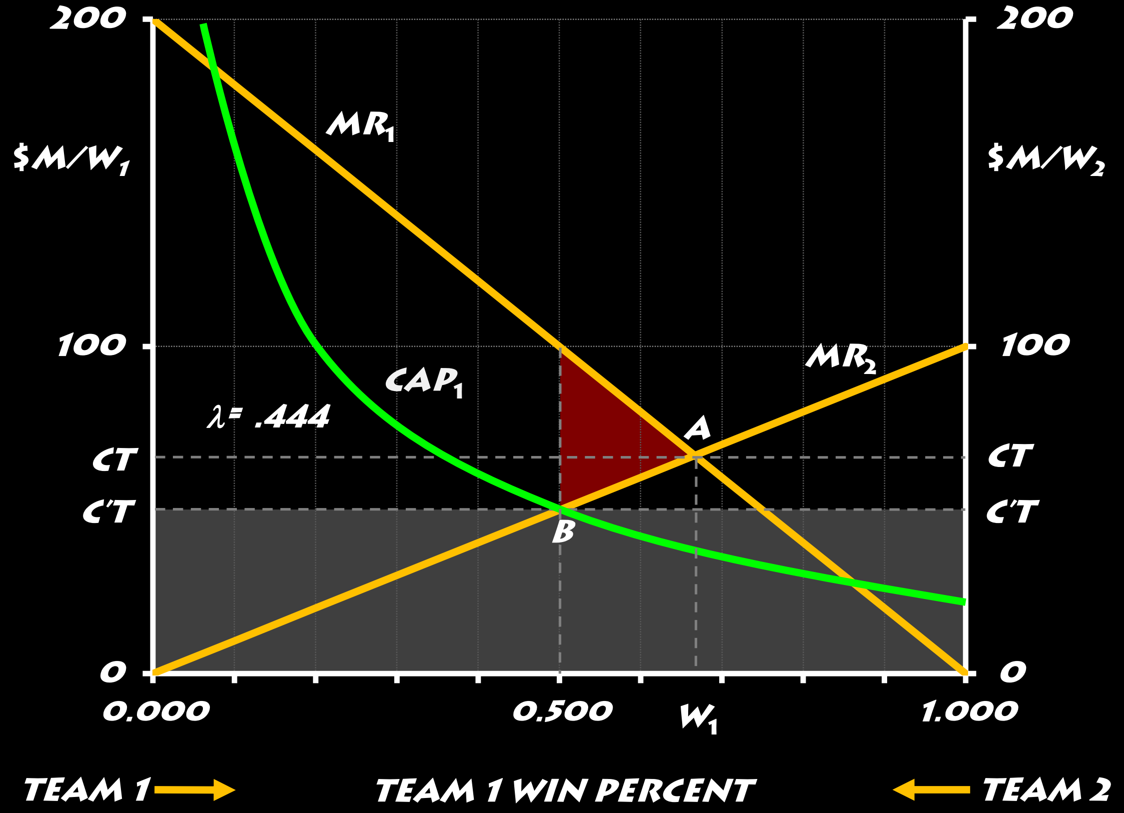
A league-wide payroll cap constrains each team's payroll to a constant λ-share of the average club's revenue such that: $w_1 cT = \lambda(R_1 + R_2)/2$. If CAP_1 is defined as an *iso*-payroll constraint (locus of $\lambda(R_1+R_2)/2$ for all w_1) for team 1, the closed league solution becomes:

$$CAP_1 = MR_2 = \lambda\,(R_1 + R_2)/2w_1 = cT \qquad (14)$$

In order for the cap to constrain team 1, $\lambda \leq 4\sigma^2 / [(1+\sigma)(1+\sigma + \sigma^2)]$. To achieve absolute balance $w_1 = w_2$ the cap should be $\lambda = 1.33/(1+\sigma)$.

The effect of the payroll cap on team 1's profit is ambiguous, because gains from lower payroll are offset by revenue losses from fewer wins. Team 2's improvement is unambiguous because team 2 profits increase from both lower payroll and higher revenue from more wins.

PAYROLL CAP


200
$M/W_1
100
CT
C'T
0
200
$M/W_2
100
CT
C'T
0
MR_1
MR_2
CAP_1
λ= .444
A
B
0.000
0.500
W_1
1.000
TEAM 1
TEAM 1 WIN PERCENT
TEAM 2

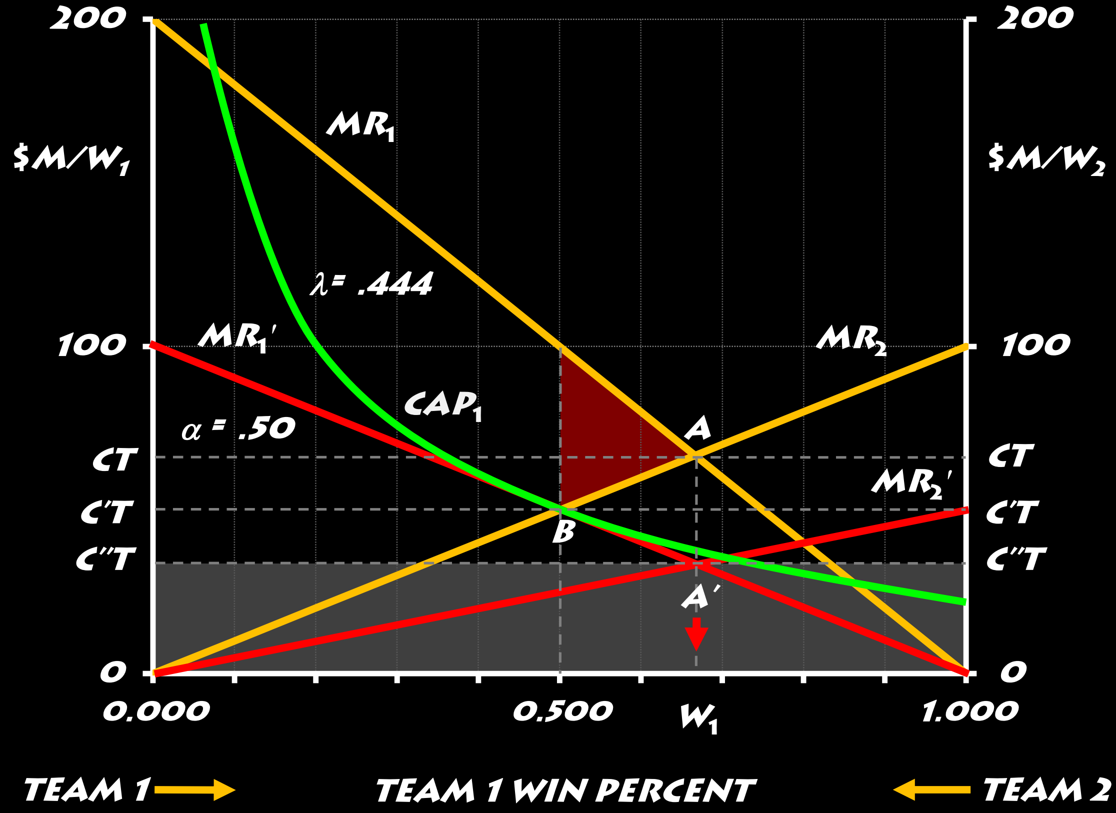
SALARY CAP & REVENUE SHARING

Team 1 has an incentive to circumvent the cap because $MR_1 > MR_2$ at .500. The dead-weight loss (shaded area between MR_1 and MR_2 above .500) suggests mutual gain from a revenue-sharing deal between clubs.

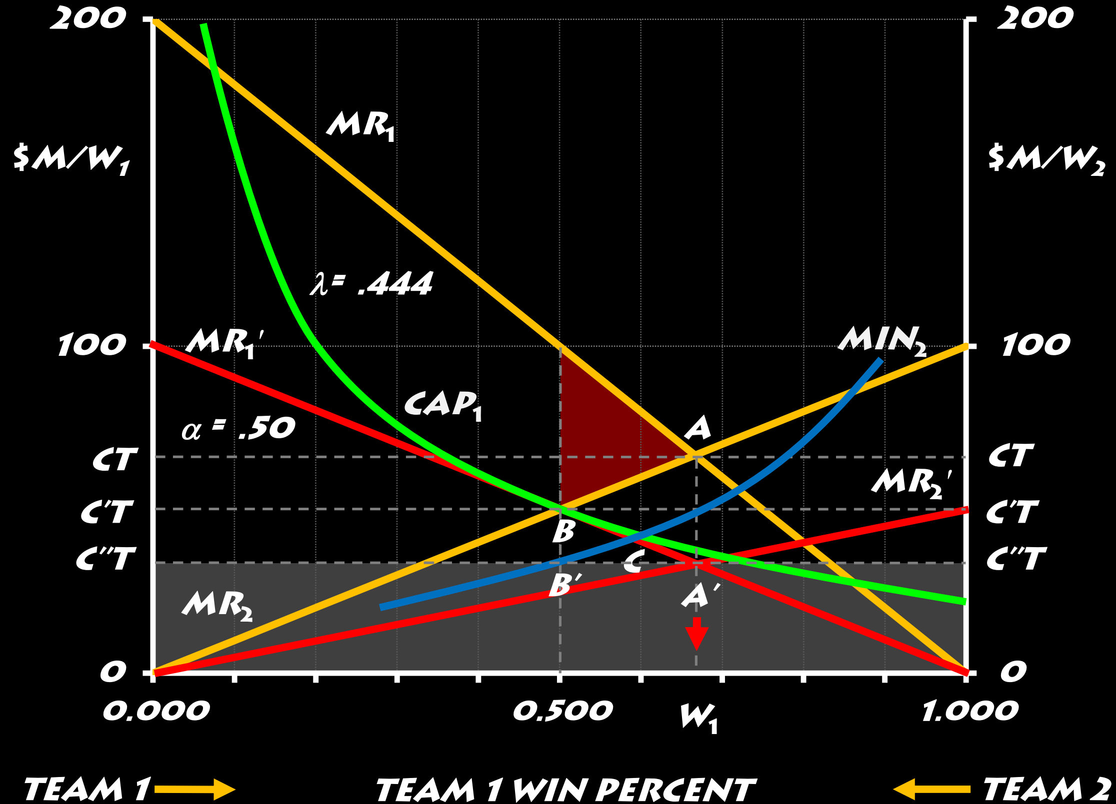
If a payroll cap is combined with revenue sharing then MR_1 and MR_2 are vertically displaced downward and CAP_1 no longer constrains the payroll of team 1. League equilibrium is restored at $MR_1' = MR_2'$ and the original state of imbalance returns to $w_1/w_2 = \sigma$.

This leads to the conclusion that a payroll cap by itself will constrain large market teams in a π-max league and improve competitive balance. But when a payroll cap is combined with revenue sharing the disincentive to win negates the cap and the league returns to its original state of imbalance. *Ironically* a payroll minimum is necessary to create competitive balance in a profit-max league with revenue sharing.

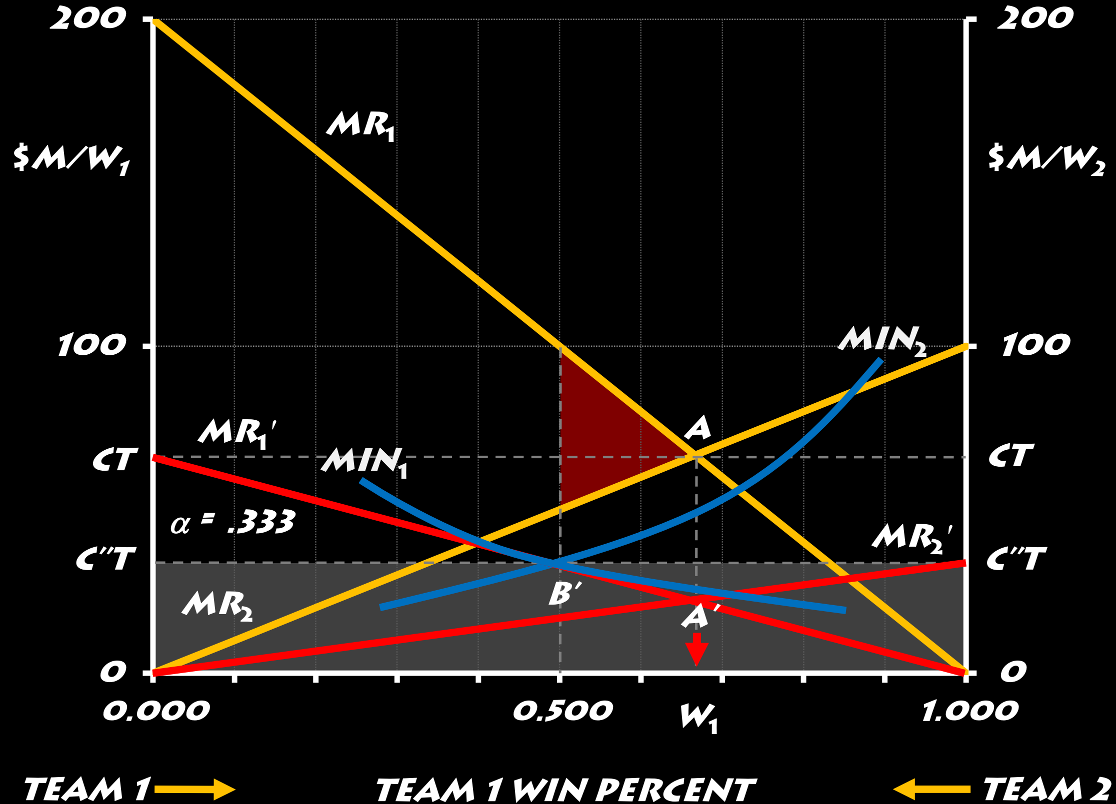
REVENUE SHARING & PAYROLL CAP



REVENUE SHARING & PAYROLL MINIMUM



REVENUE SHARING & PAYROLL MINIMUM



CHAMPION EFFECT

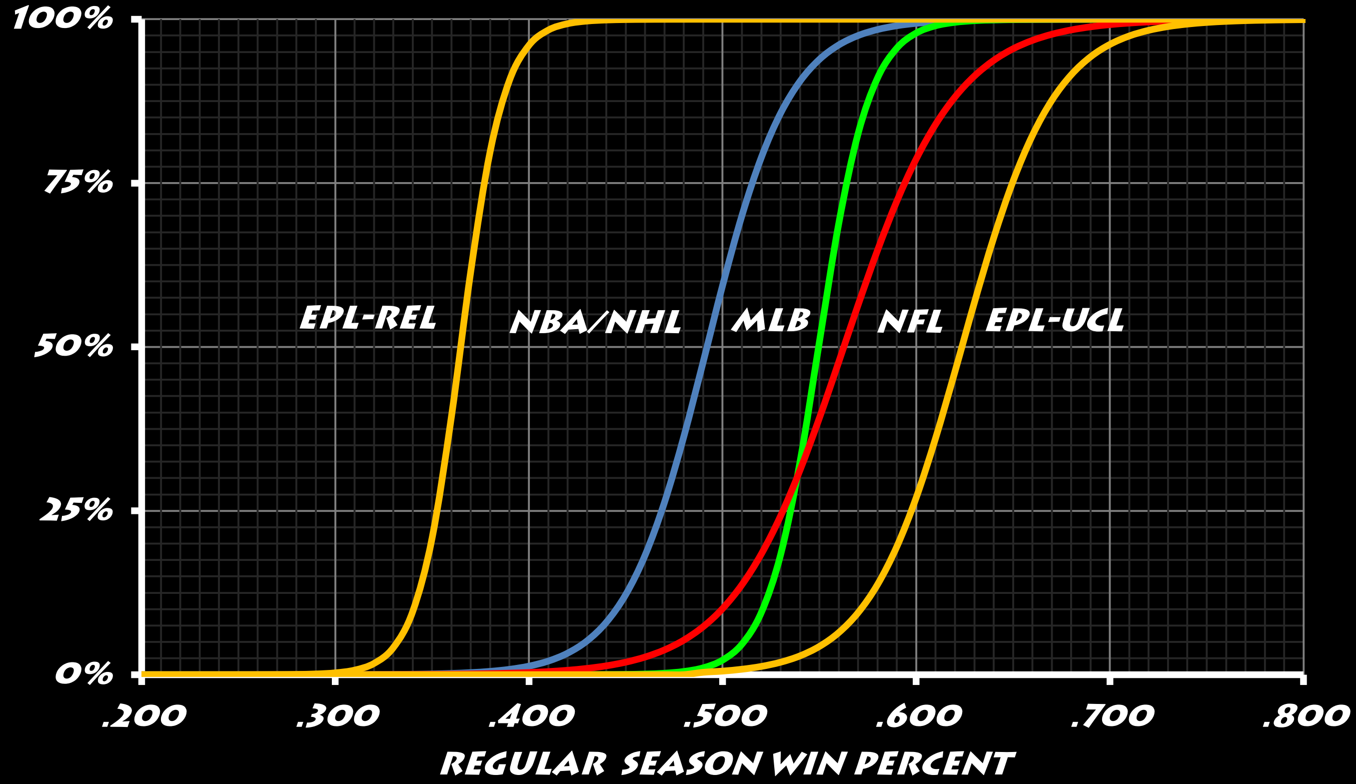
Post-season championship tournaments complicate the convenience of concave revenue functions, because of the redoubled importance of winning. With an added chance for post-season play, each team is built to win the regular season, but also to win the post-season tournament. The *champion effect* is the polarizing feedback that the post-season has on regular season competitive balance.

The degree of revenue convexity caused by the *champion effect* depends on the size and certainty of the champion's prize compared to regular-season revenue. The probability θ_1 of team 1 making the post-season tournament based on its regular-season performance w_1 can be expressed as a logistic cumulative density function (*CDF*):

$$\theta_1 = 1(1 + \exp[(\alpha + \beta w_1)] \qquad (15)$$

where $0 < \theta < 1$; $\alpha < 0$; $\beta > 0$. The mean $\mu = -\alpha/\beta$ is the win-threshold where teams have a 50/50 chance of qualifying for the post-season.

PLAYOFF PROBABILITY

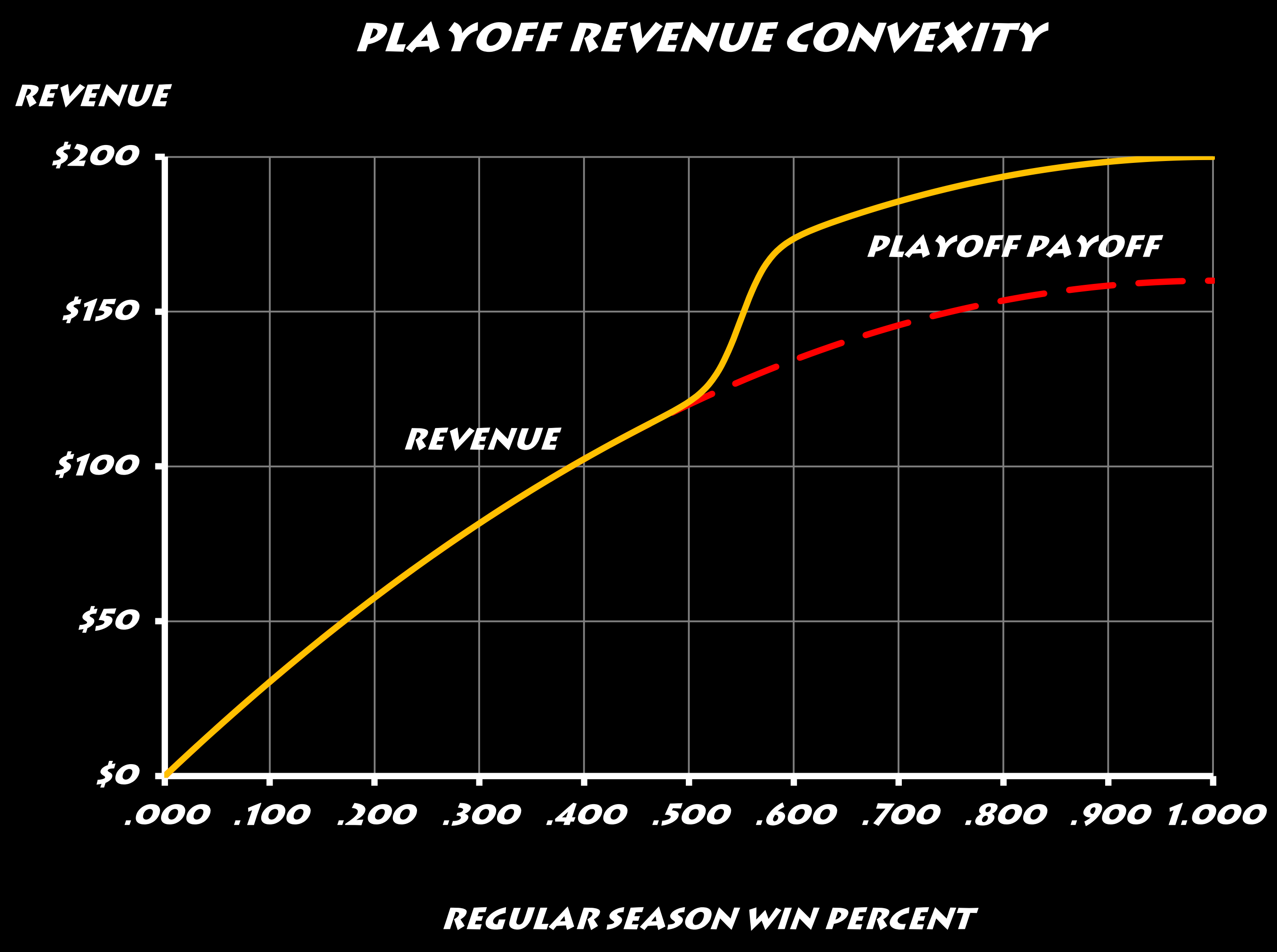


If δ is the ratio of the champion's prize to regular season revenue and $\omega_1 = w_1/(w_1 + \mu)$ is the probability of playoff success against teams with expected win percentage μ, then the combined revenue function R_1* becomes complicated by convexity:

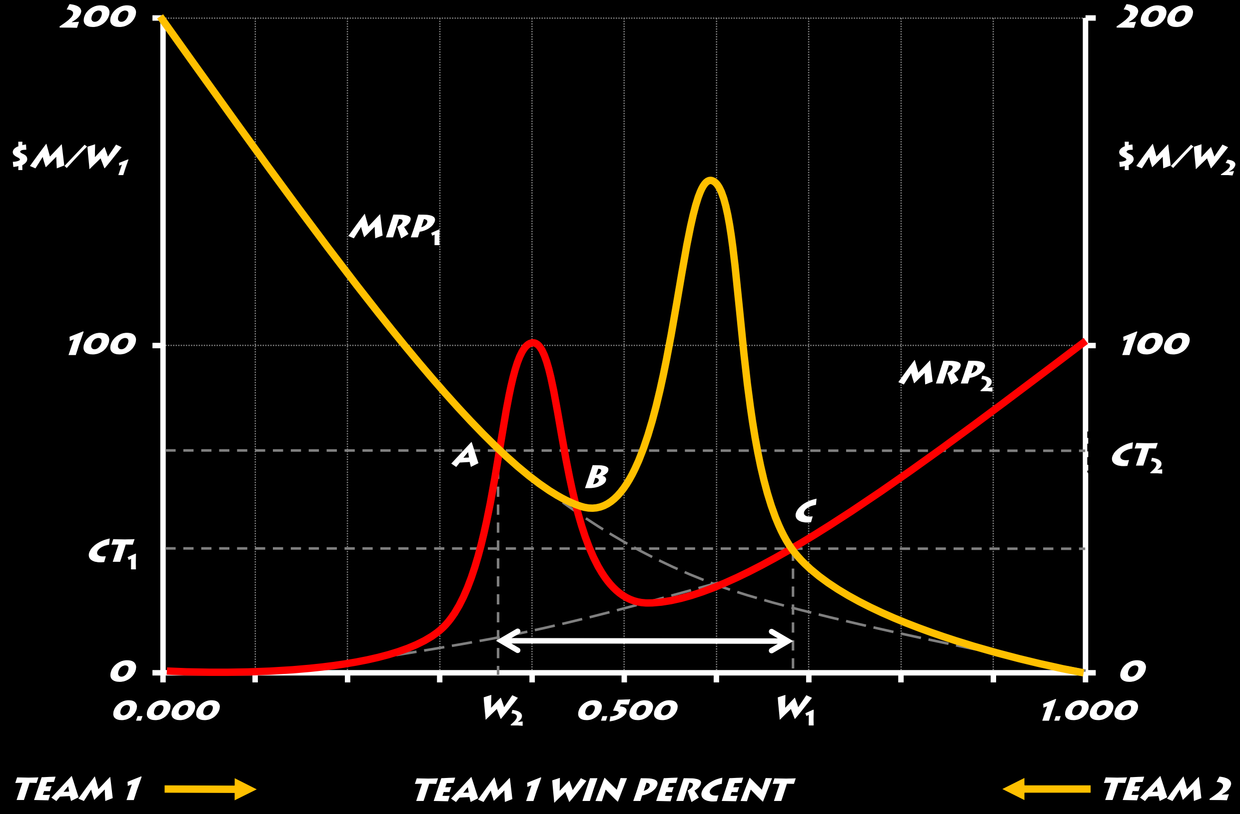
$$R_1^* = \sigma[\, w_1 - .5w_1^2 + \delta\theta(\omega_1 - .5\omega_1^2)] \qquad (16)$$

An important complication of the *champion effect* is that post-season revenue convexity introduces instability and competitive imbalance into the regular-season. The adjusted *MRP*s of both teams reflect probability distribution functions (*PDFs*) as derivatives of the respective *CDFs*.

As either team approaches the playoff threshold μ, the marginal revenue from additional qualifying win explodes and creates an unstable local minimum bracketed by two local maxima. These split equilibria explain polarizing threshold behavior during mid-season transfer windows and trade deadlines for teams on the edge of qualifying for the playoffs.


PLAYOFF REVENUE CONVEXITY
REVENUE
$200
$150
$100
$50
$0
PLAYOFF PAYOFF
REVENUE
.000
.100
.200
.300
.400
.500
.600
.700
.800
.900
1.000
REGULAR SEASON WIN PERCENT

TRADE DEADLINE



SPORTSMAN LEAGUE

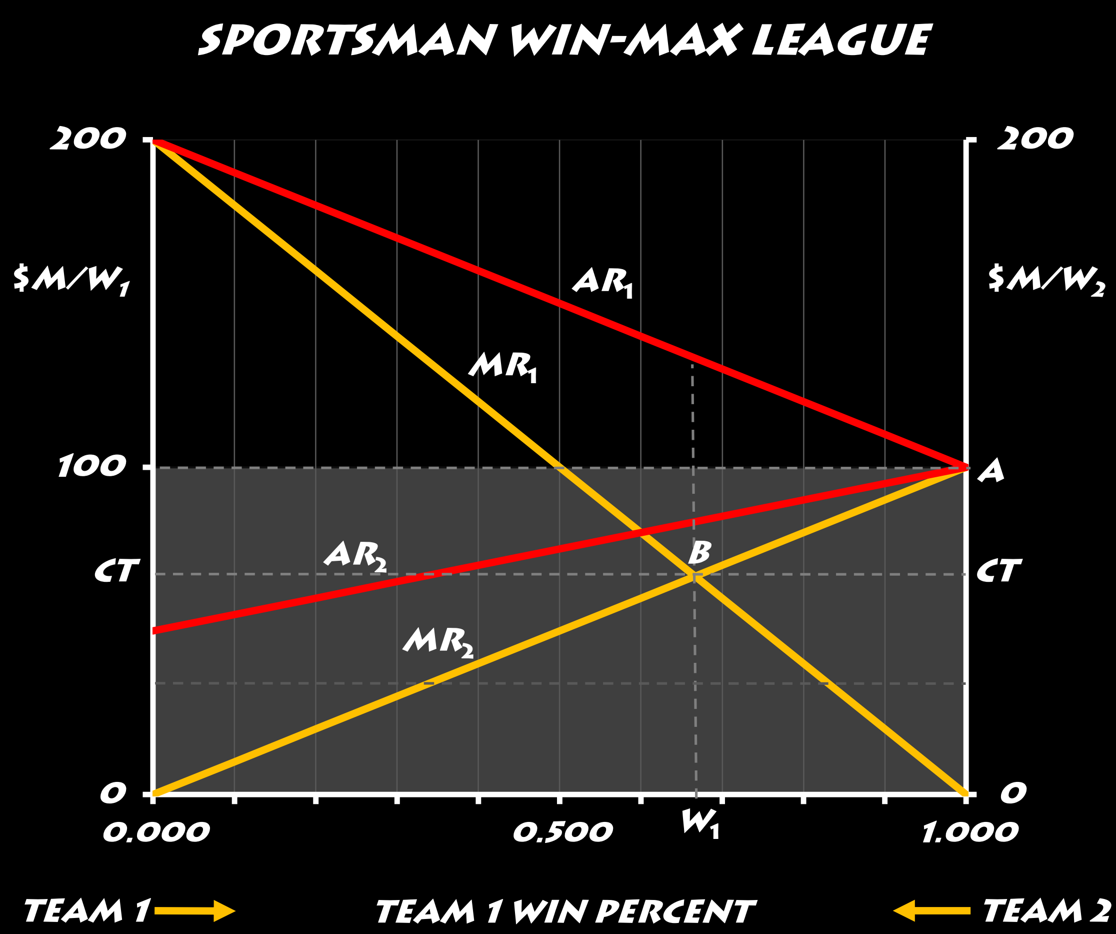
In *sportsman leagues*, team owners sacrifice profit for winning. At the limit, a *pure sportsman* becomes a win maximizer, constrained by zero profit rather than max profit, such that $R_1 = ct_1$ and $R_1/w_1 = ct_1/w_1 = cT$, where $t_1 = w_1T$. The *sportsman league* win-max solution becomes:

$$AR_1 = AR_2 = cT \tag{17}$$

This is true whether talent markets are *open* or *closed*. Substitution of (9) into (17) yields the *pure sportsman* σ-model result:

$$AR_1 = AR_2 = \sigma(1 - .5w_1) = (1 - .5w_2) = cT \tag{18}$$

with less balance than either *open or closed* π-max solution: $w_1/w_2 = (2\sigma - 1)/(2 - \sigma)$ with win percentages $w_1 = (2\sigma - 1)/(1 + \sigma)$ and $w_2 = (2 - \sigma)/(1 + \sigma)$. Existence of the league requires $w_2 > 0$ and therefore constrains $\sigma < 2$ for the assumption that $\phi = .5$ in (9).


SPORTSMAN WIN-MAX LEAGUE
200
200
$M/W1
$M/W2
AR1
MR1
100
A
AR2
B
CT
CT
MR2
0
0
0.000
0.500
W1
1.000
TEAM 1
TEAM 1 WIN PERCENT
TEAM 2

REVENUE SHARING

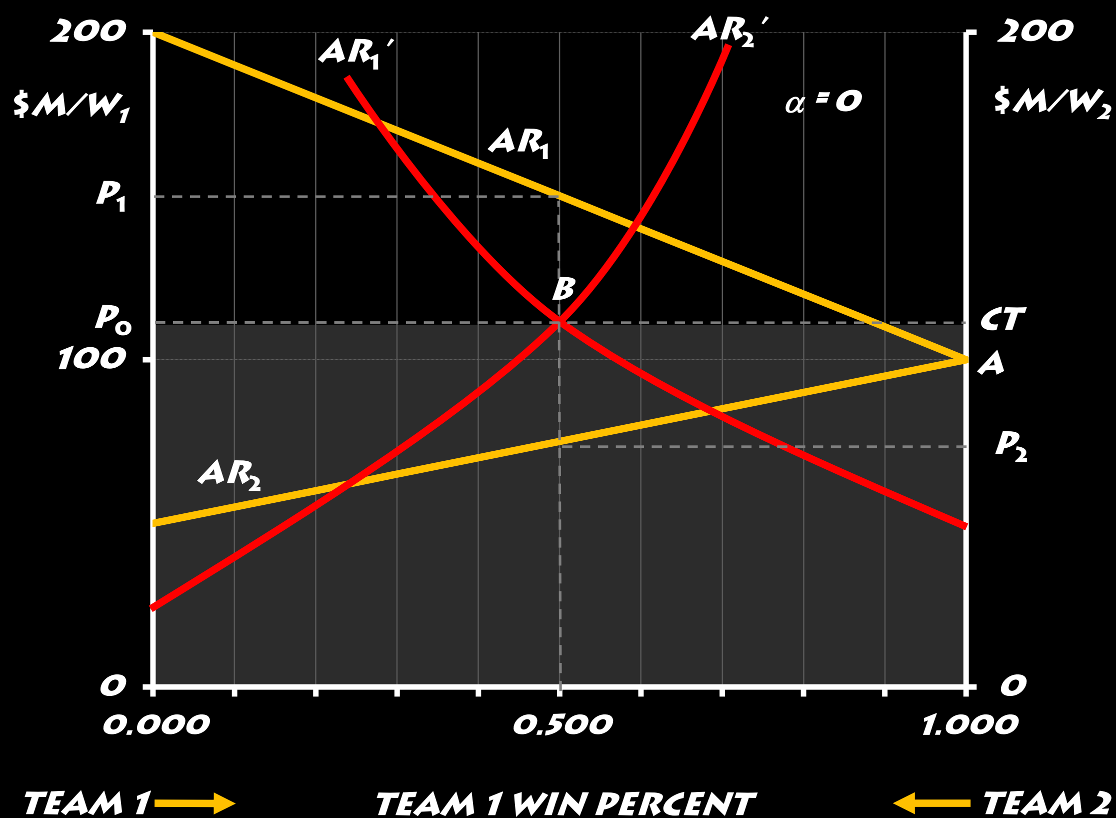
The question whether *strong form* invariance holds in a sportsman win-max league can be answered by modifying the pool-sharing formula in (12) so that $AR_1' = AR_2' = c'T$:

$$\alpha R_1/w_1 + (1-\alpha)(R_1+R_2)/2w_1 = \alpha R_2/w_2 + (1-\alpha)(R_1+R_2)/2w_2 = c'T \qquad (19)$$

If there is no revenue sharing ($\alpha = 1$) then the second term vanishes for each team and $AR_1 = AR_2 = cT$ as in (18). In a pure syndicate ($\alpha = 0$) revenues and payrolls become identical for each team $(R_1+R_2)/2$, which implies that the league is competitively balanced at $w_1 = w_2 = .500$.

In a win-max syndicate league payroll is equal to total revenue, which is divided equally between clubs. Both clubs have zero profits because all revenue is paid to the players to maximize wins. League payroll increases with revenue sharing as competitive balance approaches the total revenue maximum. Maximum league revenue at $\sigma w_2 = w_1$ could be captured by setting $\alpha = [\sigma^4 + \sigma^3 - (\sigma + 1)] / [\sigma^4 + \sigma^3 - (3\sigma + 1)]$. If $\sigma = 2$, for example, then $\alpha = .64$ would yield a league revenue maximum.

WIN-MAX REVENUE SHARING



SALARY CAP

To see the effects of a payroll cap in a win-max league reconsider the cap solution from (13) revised for a *sportsman* league, $CAP_1 = AR_2 = c^*T$:

$$\lambda(R_1 + R_2)/2w_1 = R_2/w_2 = \lambda\,[.5 + \sigma w_1 - .5(\sigma+1)\,w_1^2]\,/2w_1 = (1 - .5w_2) \quad (20)$$

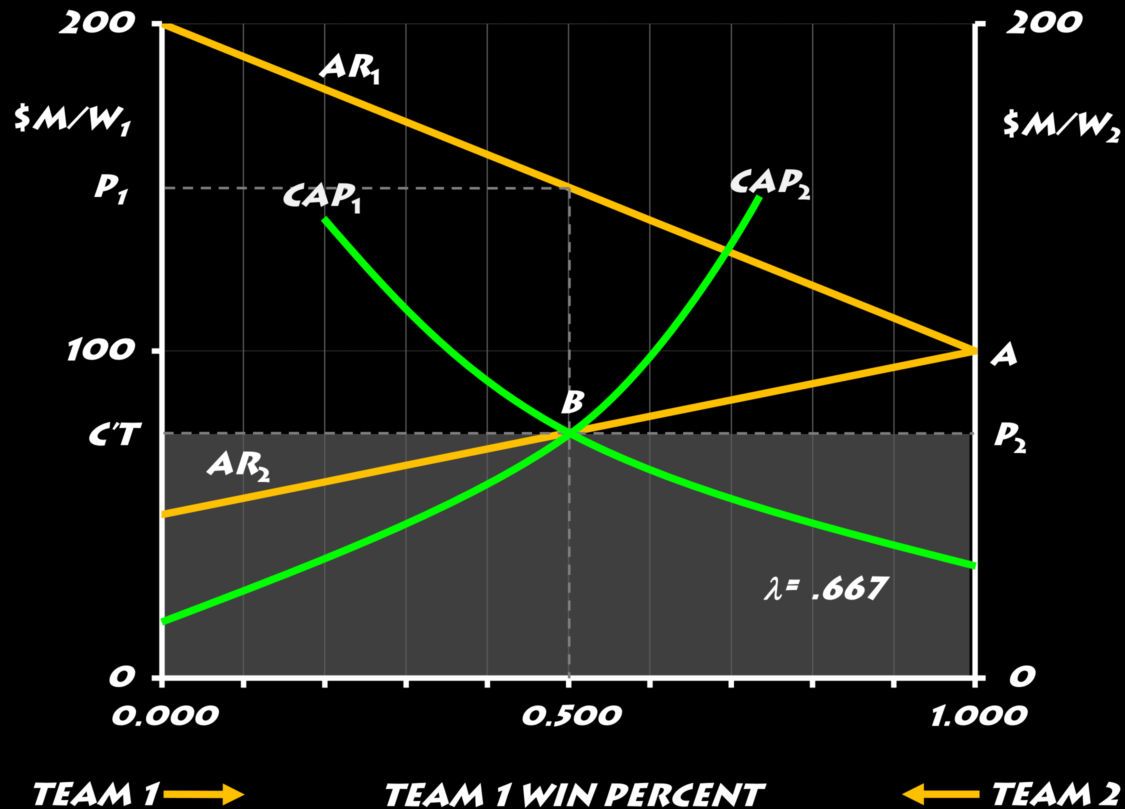
Absolute balance at $w_1 = w_2 = .500$ requires a payroll cap $\lambda = 2/(1+\sigma)$. League revenue max at $\sigma w_2 = w_1$ requires a cap $\lambda = 4\sigma^2/(1+\sigma)(1+\sigma+\sigma^2)$. If $\sigma = 2$, for example, then a payroll cap $\lambda = .76$ yields the revenue max.

Combination payroll cap ($\lambda = .67$) and revenue sharing ($\alpha = 0$) virtually clones equality in team revenues, payrolls and profits.

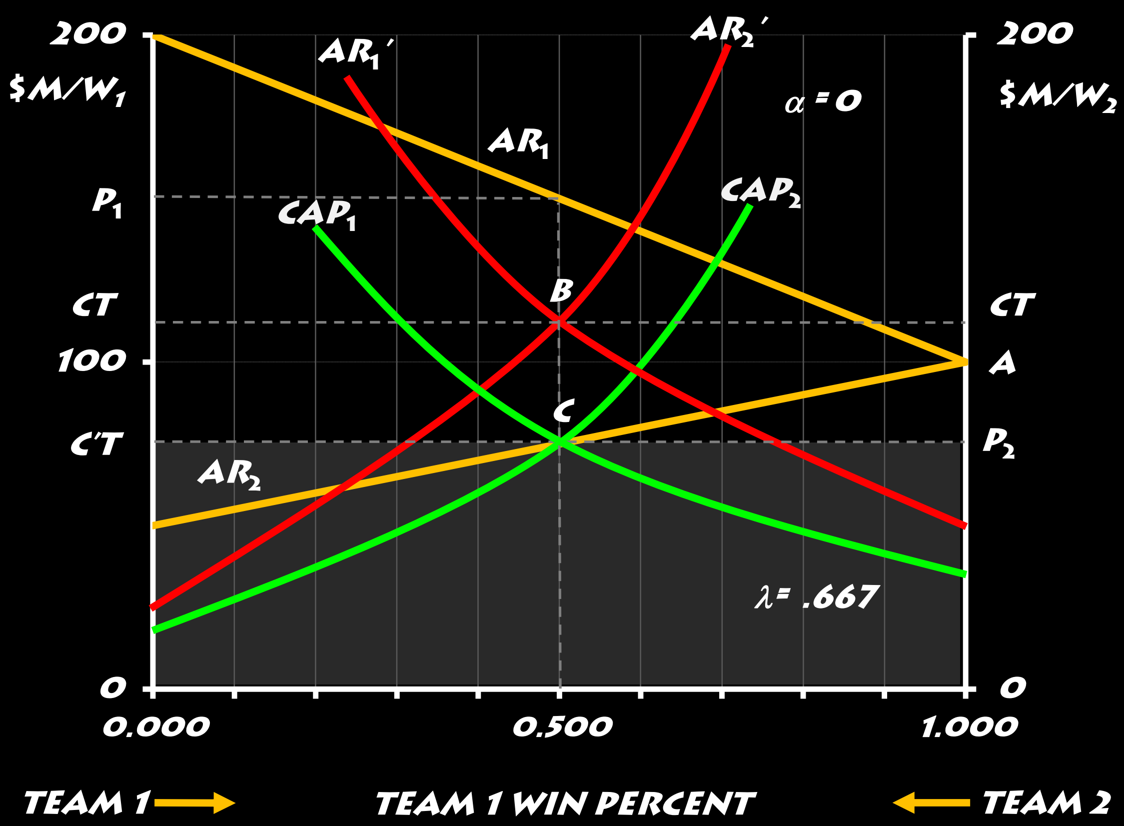
These results lead to opposite conclusions for revenue sharing in π–max and win-max leagues. In π-max leagues revenue sharing increases team profits and talent exploitation but does not increase competitive balance.

Revenue sharing in sportsman leagues increases competitive balance and leads to higher payrolls and revenues toward the maximum $MR_1 = MR_2$.

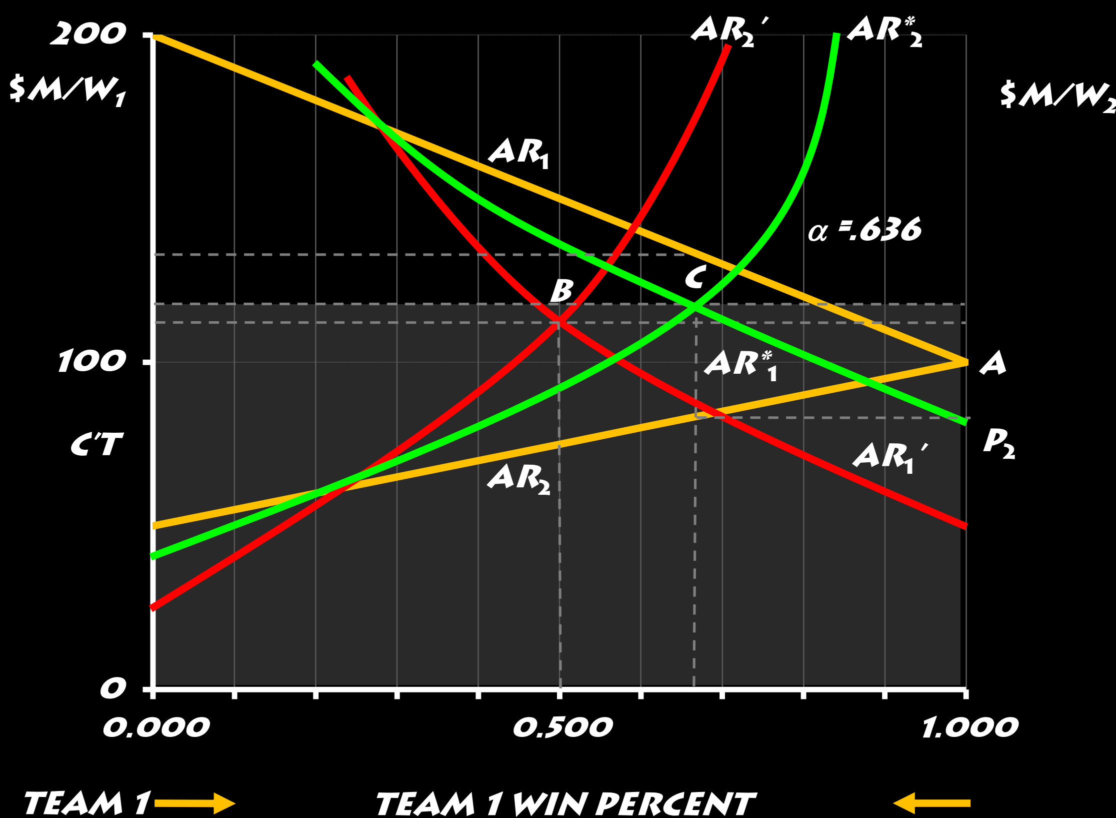
WIN-MAX SALARY CAP



SALARY CAP & REVENUE SHARING



OPTIMUM WIN-MAX REVENUE SHARING



EMPIRICAL EVIDENCE

BETA BALANCE

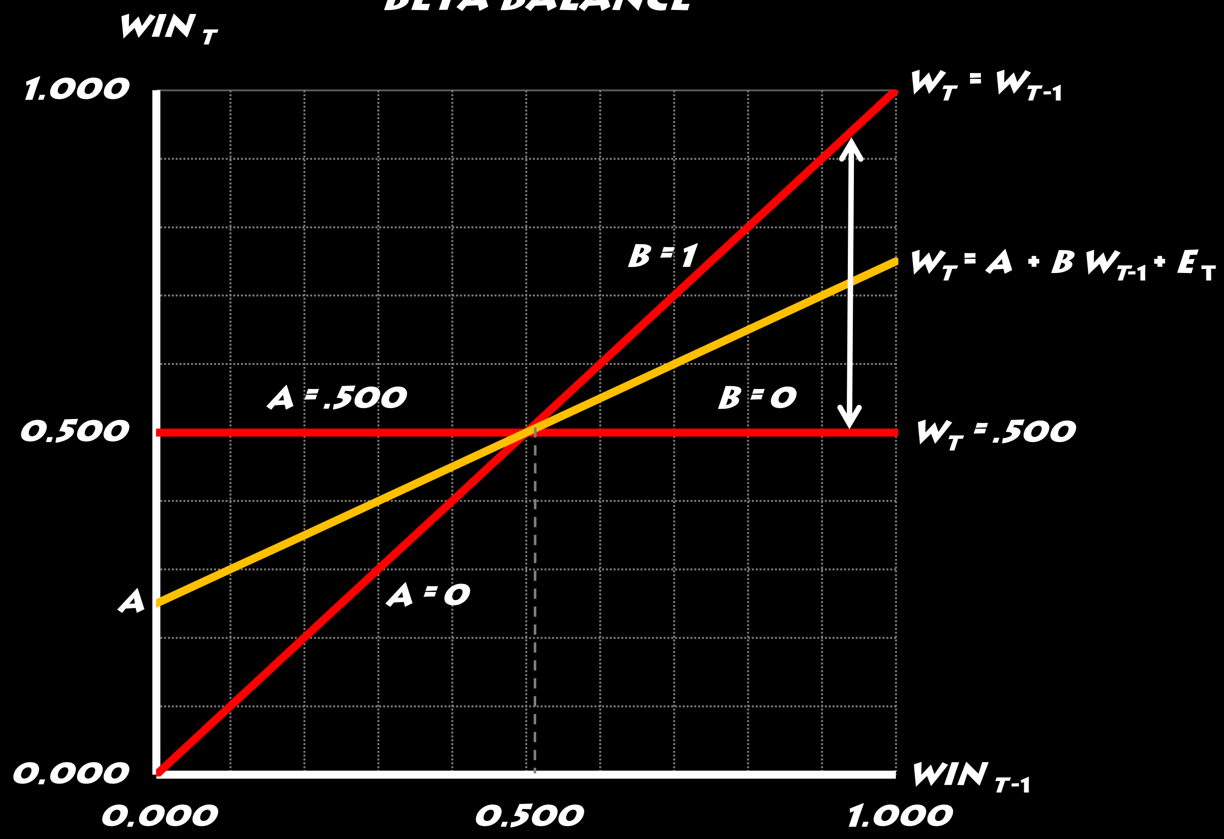
The inter-seasonal dynamics of competitive balance can be captured in an auto-regressive β-estimate of continuity of winning percentages w_{ijt} for team i in league j from season t-1 to season t, where $\beta \in [0,1]$.

$$w_{ijt} = \alpha + \beta w_{ijt-1} + e_{ijt}$$

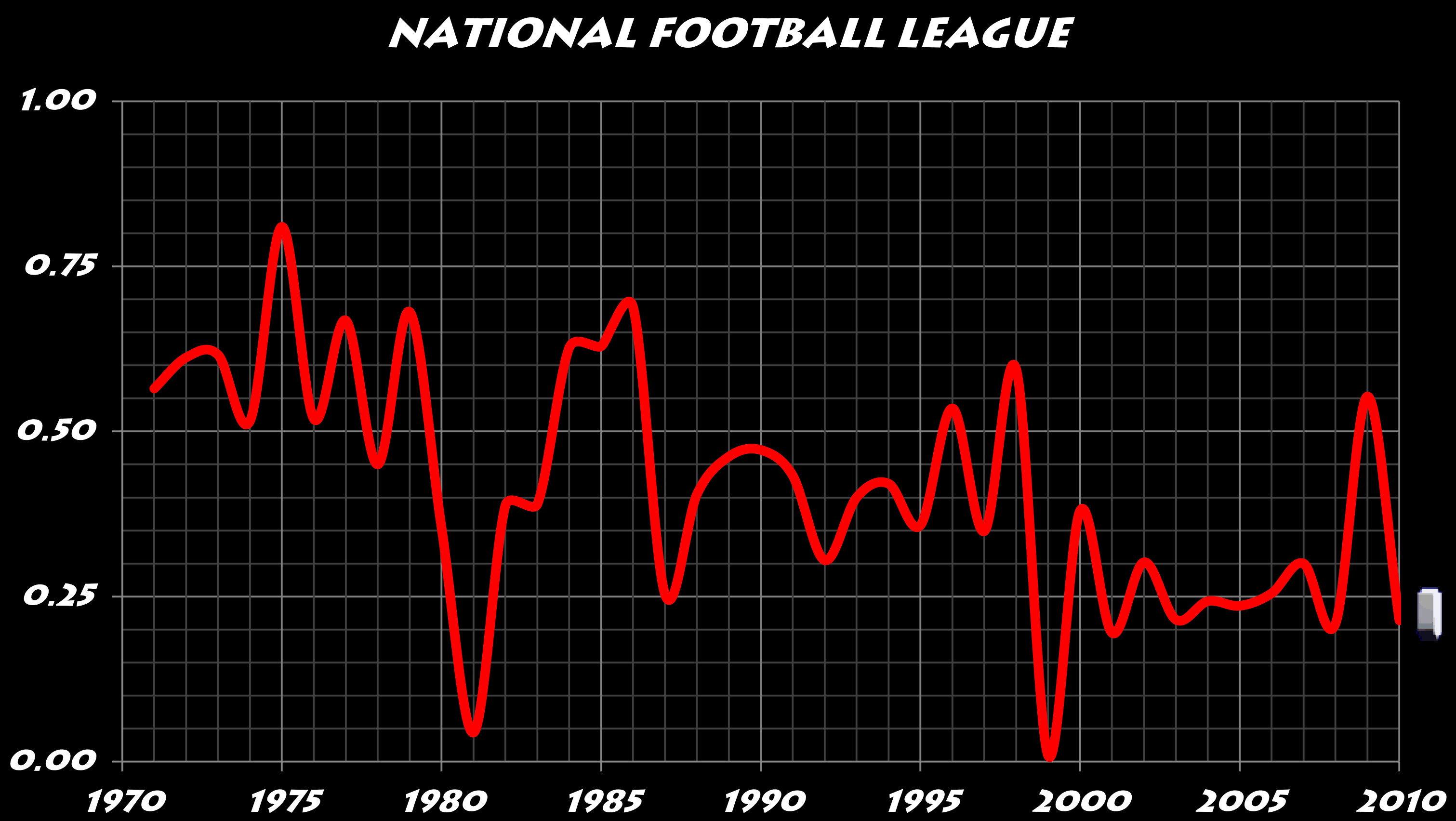
If α =.500 and β = 0 then w_{ijt} = .500 and each season is a random walk and every team has an equal chance to win.

If α=0 and β=1, then $w_{ijt} = w_{ijt-1}$ then outcomes are predetermined.

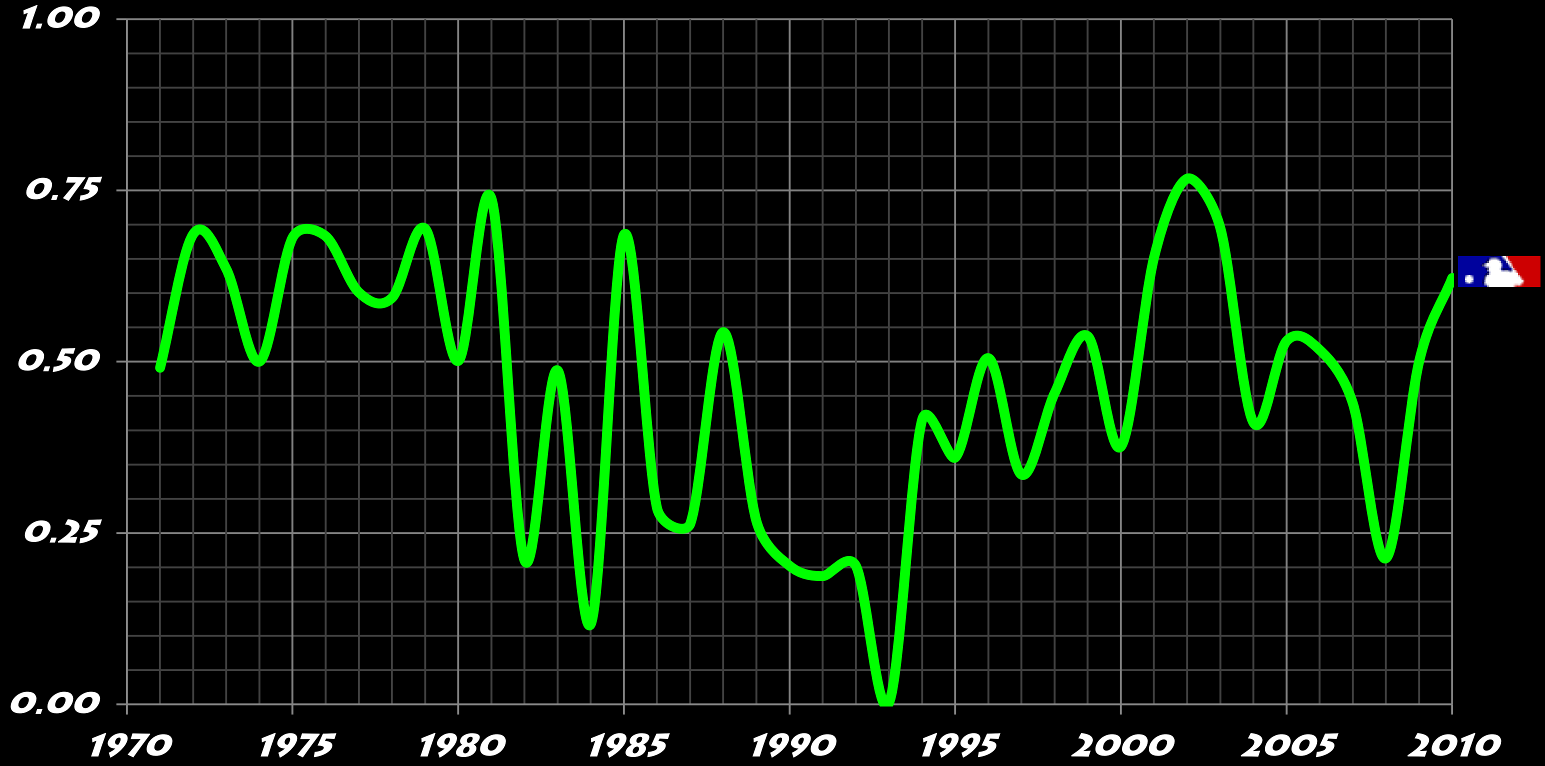
BETA BALANCE



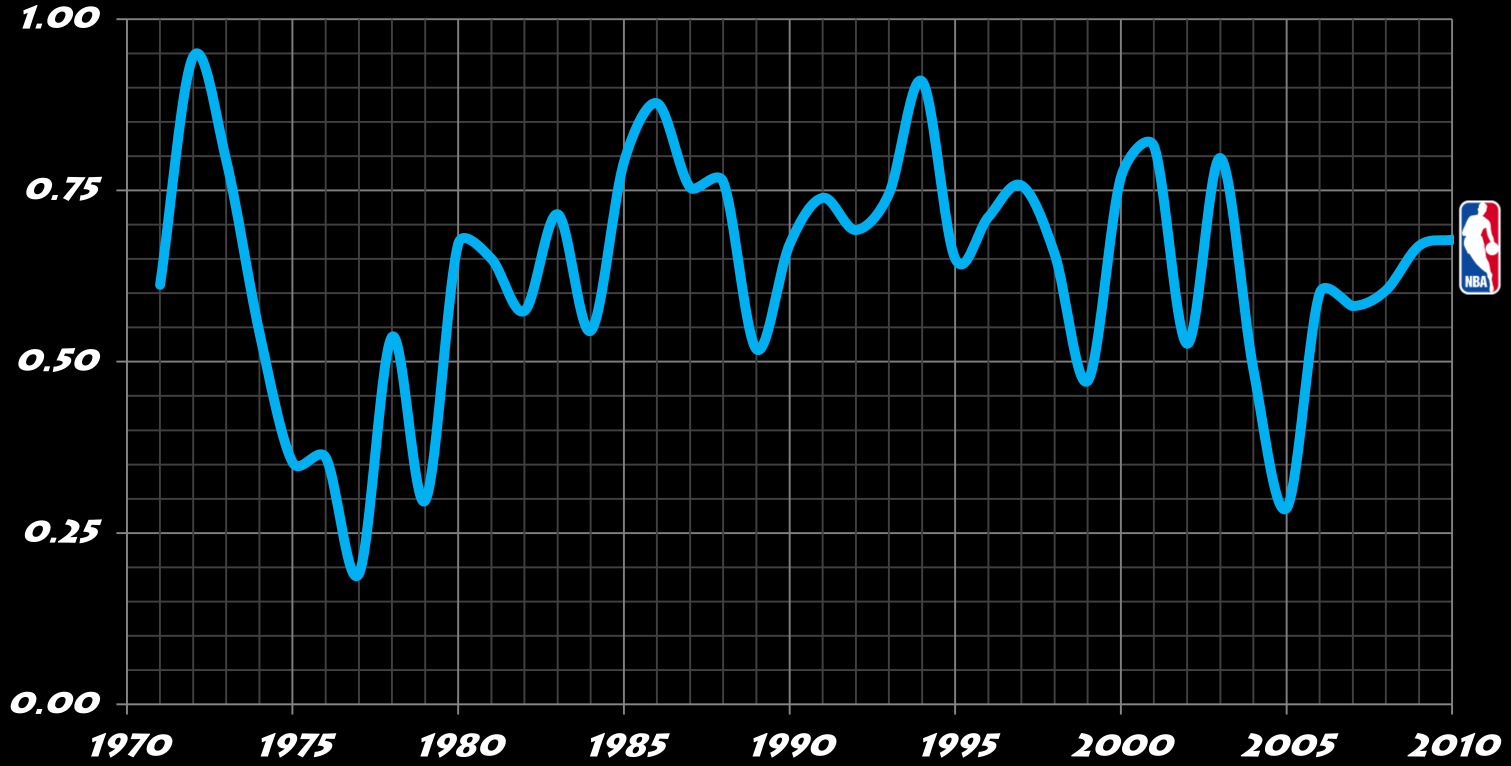
North American Leagues

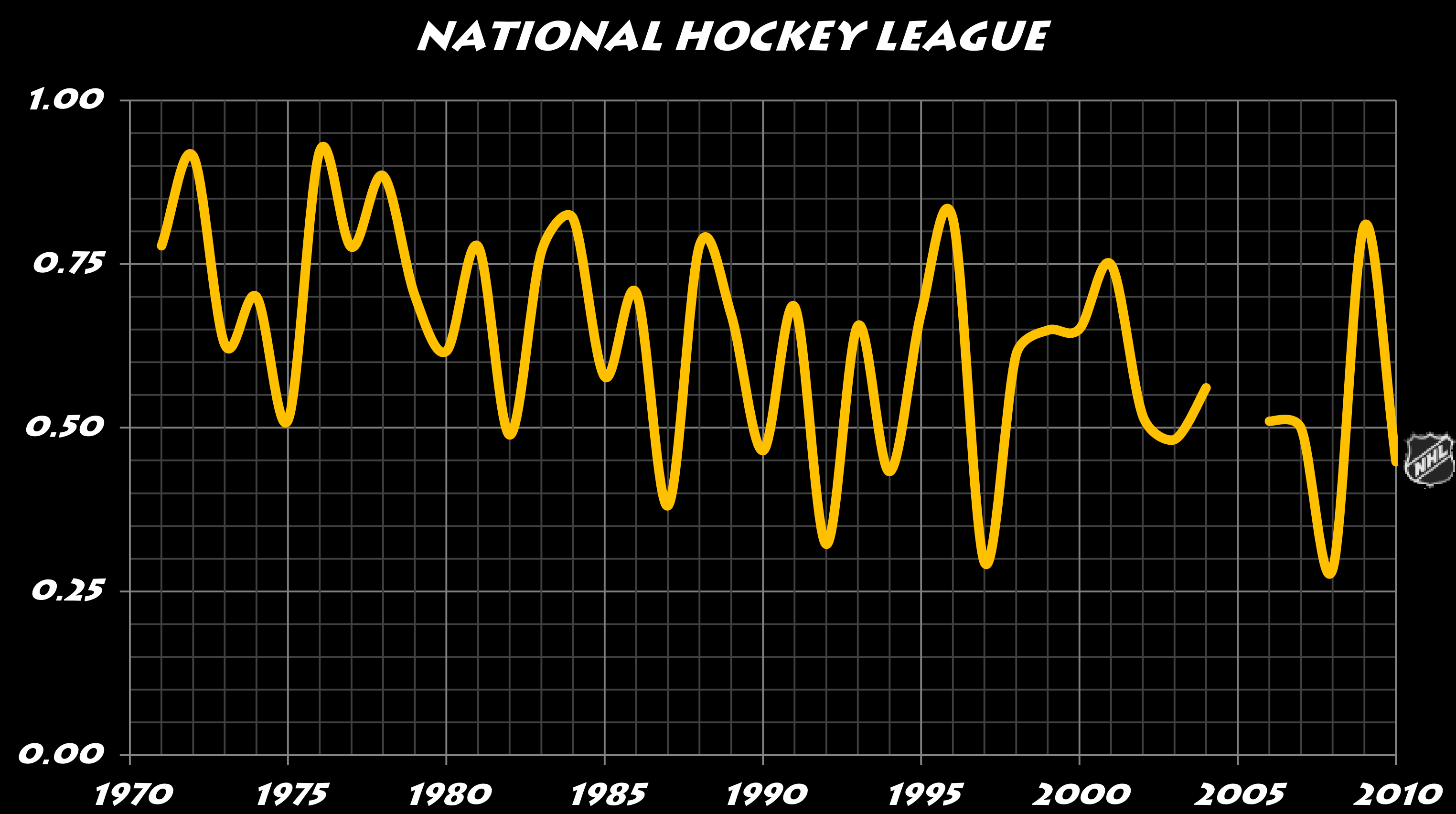

NATIONAL FOOTBALL LEAGUE
1.00
0.75
0.50
0.25
0.00
1970
1975
1980
1985
1990
1995
2000
2005
2010

MAJOR LEAGUE BASEBALL

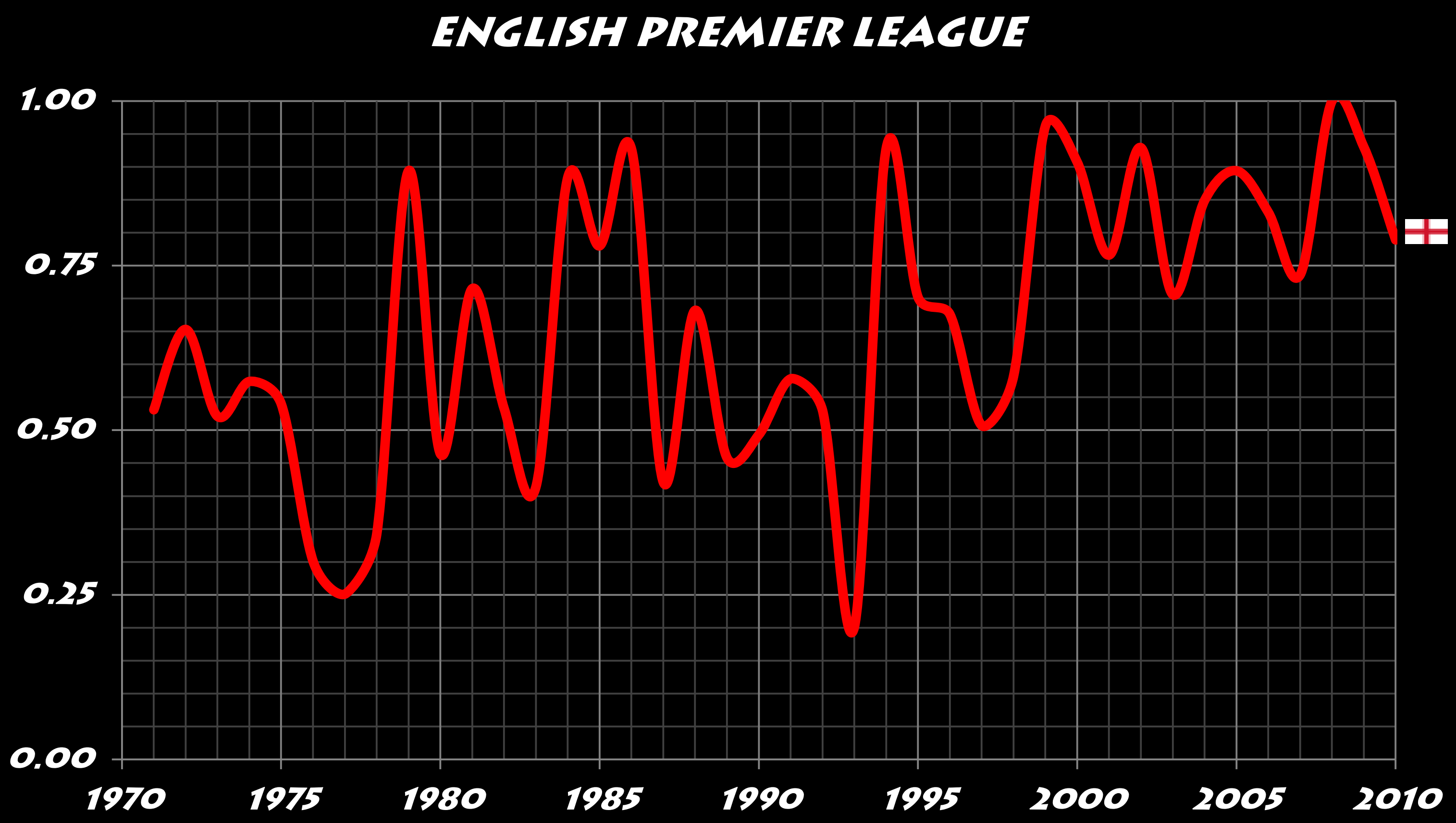


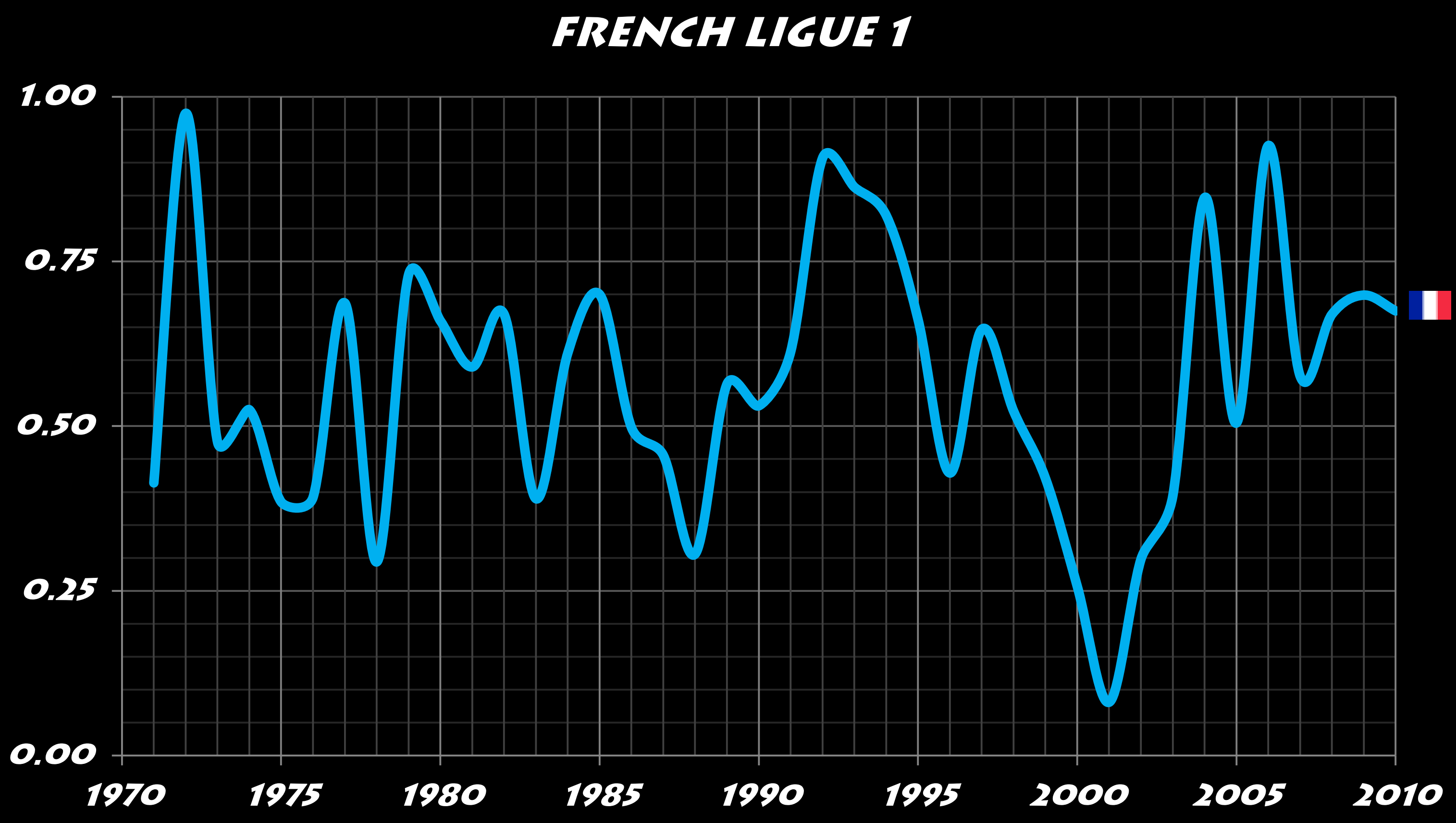
NATIONAL BASKETBALL ASSOCIATION

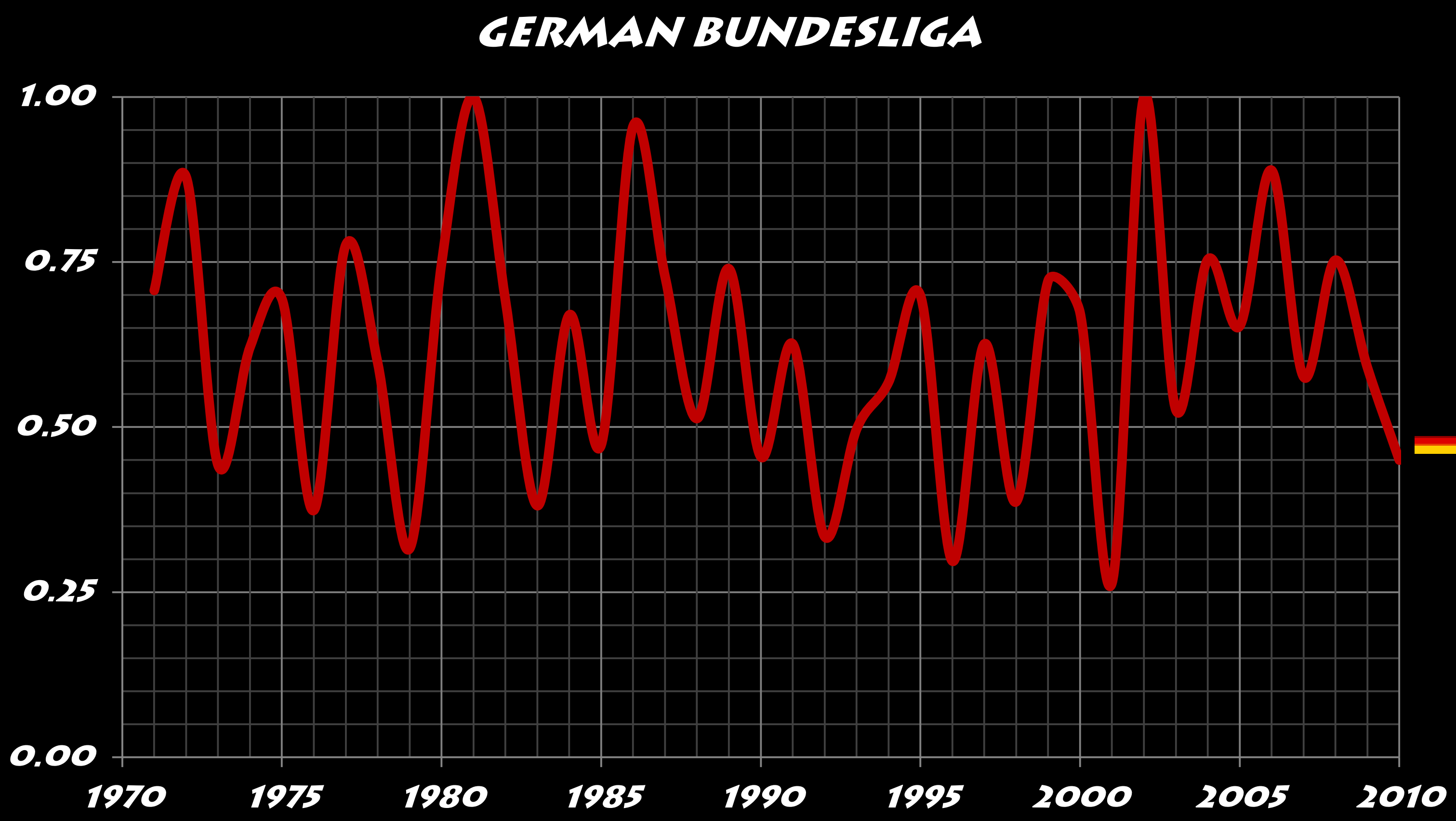


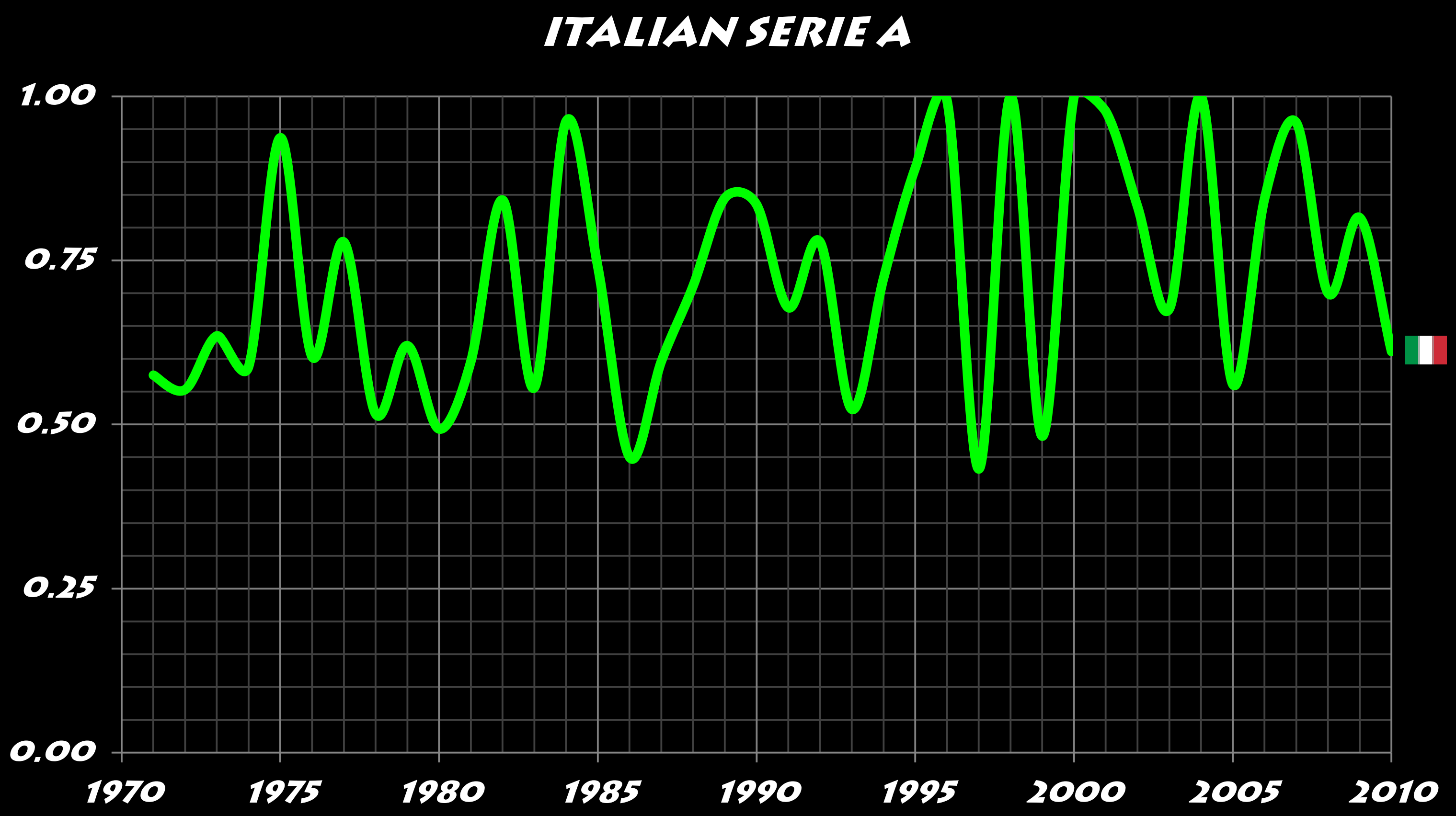

NATIONAL HOCKEY LEAGUE
1.00
0.75
0.50
0.25
0.00
1970
1975
1980
1985
1990
1995
2000
2005
2010
NHL

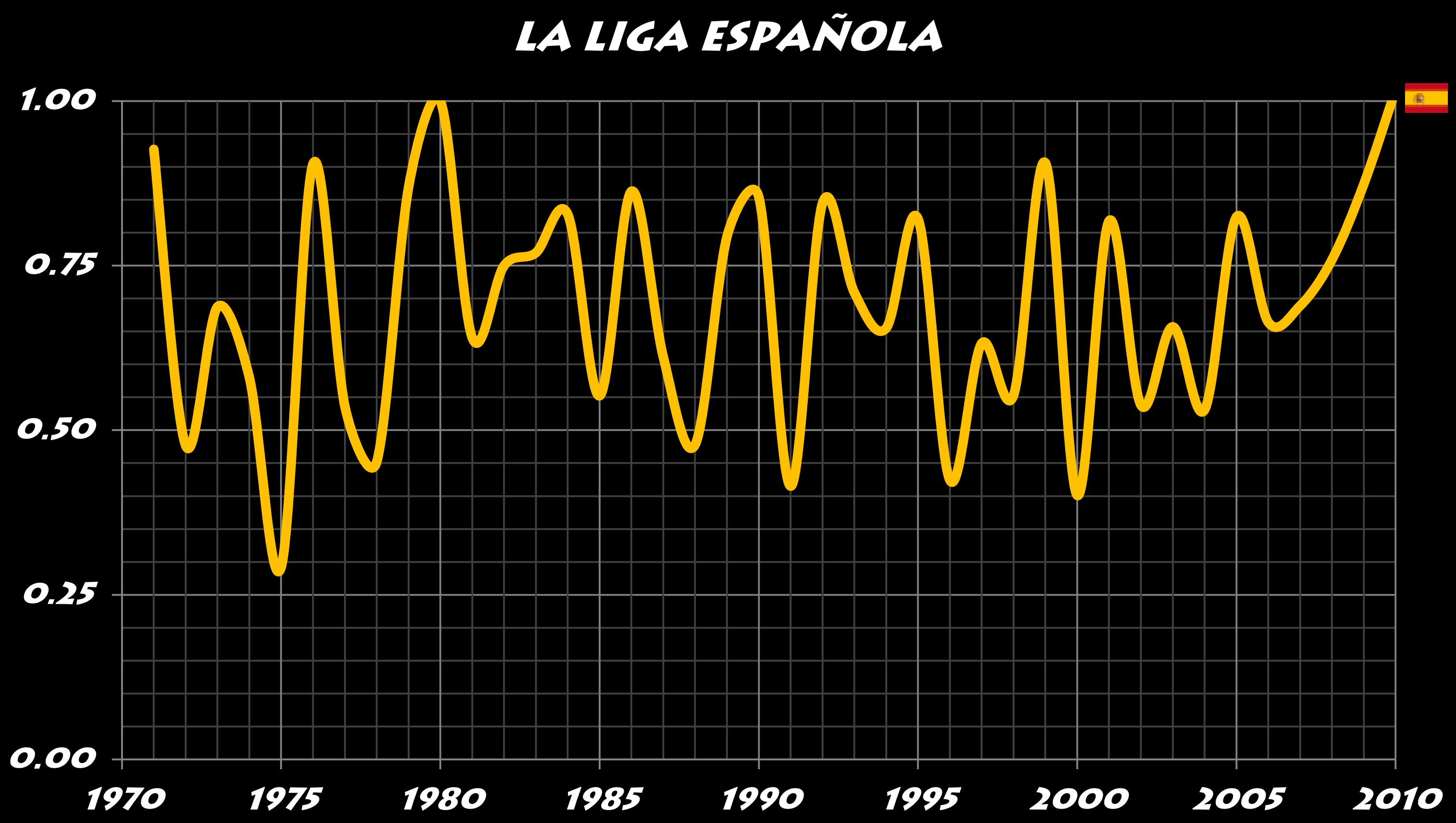
Big 5 European Leagues


ENGLISH PREMIER LEAGUE
1.00
0.75
0.50
0.25
0.00
1970
1975
1980
1985
1990
1995
2000
2005
2010


FRENCH LIGUE 1
1.00
0.75
0.50
0.25
0.00
1970
1975
1980
1985
1990
1995
2000
2005
2010


German Bundesliga
1.00
0.75
0.50
0.25
0.00
1970
1975
1980
1985
1990
1995
2000
2005
2010


ITALIAN SERIE A
1.00
0.75
0.50
0.25
0.00
1970
1975
1980
1985
1990
1995
2000
2005
2010



LA LIGA ESPAÑOLA
1.00
0.75
0.50
0.25
0.00
1970
1975
1980
1985
1990
1995
2000
2005
2010

FIN