COURSEWORK 1

G. Hilbert

Deadline: 4/4/2018

INSTRUCTIONS

- Write your name and student ID clearly on the papers;
- Submit your coursework to the assistant before the deadline;
- Copy is forbidden, otherwise the score for the coursework would be 0 this time;
- Typewriting with LaTeX is encouraged that 1 more score will be awarded this time.

Warm-up

Exercise 1.1. *The same thing is* $f(x) = g(x) = \mathbf{S} = \mathrm{Ox}$.

Exercise 1.2. *The same thing is*

$$\prod_{x=y=0}^{6,9} f(x,y) = g(x,y) = 2x^2 + y + 2018.$$

Medium

Definition 1.3. Suppose M is a non-empty set and $P \subseteq M$.

(1) X is definable over $(M, \in)$ from parameters in P if there is an $\mathscr{L}_\in$ formula $\varphi(x, \vec{y})$ and some $\vec{b}$ from P such that

$$X = \{a \in M \mid (M, \in) \vDash \varphi(x, \vec{y})[a, \vec{b}]\}.$$

(2) $D(M) = \{X \subseteq M \mid X \text{ is definable over } (M, \in) \text{ from parameters in } M\}$.

(3) In particular, $D(\varnothing) = \{\varnothing\}$.

Definition 1.4 (ZF−P, Gödel)**.** By transfinite recursion on $\alpha \in \mathsf{ON}$, define L_α by

$$\begin{array}{rcll} L_\alpha & = & \varnothing, & \alpha = 0, \\ L_\alpha & = & D(L_\delta), & \alpha = \delta + 1, \\ L_\alpha & = & \bigcup_{\delta<\alpha} L_\delta, & \alpha \text{ is a limit.} \end{array}$$

And set $L = \bigcup_{\alpha\in\mathsf{ON}} L_\alpha$, and we call L the constructible universe.

Advanced

Definition 1.5. X is transitive if any element of X is a subset (or subclass) of X.

Lemma 1.6. *For any ordinals ζ, ξ,*

(1) $L_\zeta \subseteq V_\zeta$.

(2) L_ζ is transitive.

(3) if $\zeta < \xi$, then $L_\zeta \subseteq L_\xi$.

(4) if $\zeta < \xi$, then $L_\zeta \in L_\xi$.

(5) $L_\zeta \cap \mathsf{ON} = \zeta$.

(6) L is transitive.

(7) L is a proper class.

A, B, C, D, S, O, P, u, v, w, x, y, z

EA, the theory of so-called elementary arithmetic, which is what you get by taking Q and adding Δ_0-induction plus the axiom which says that 2^n is a total function.