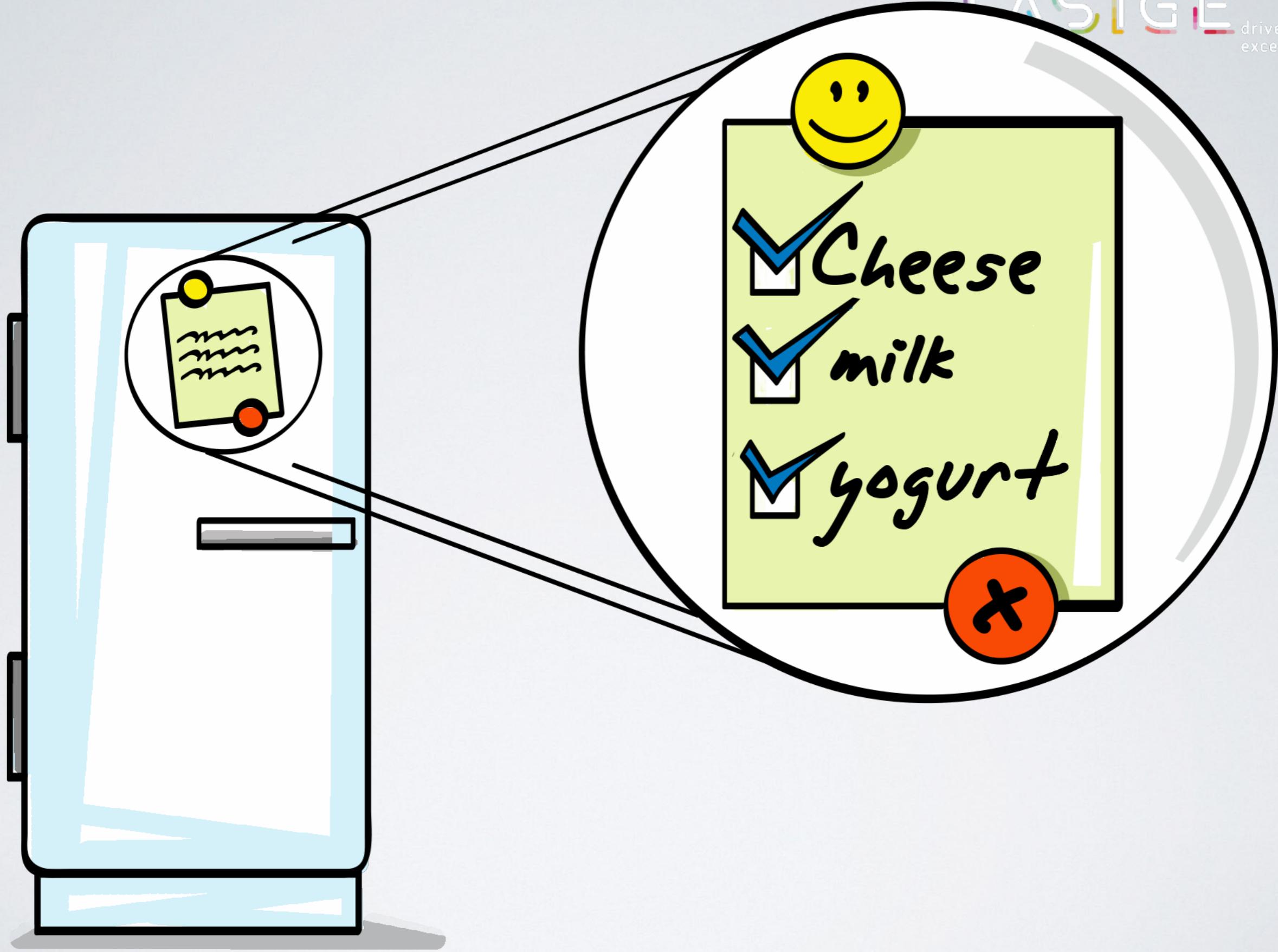
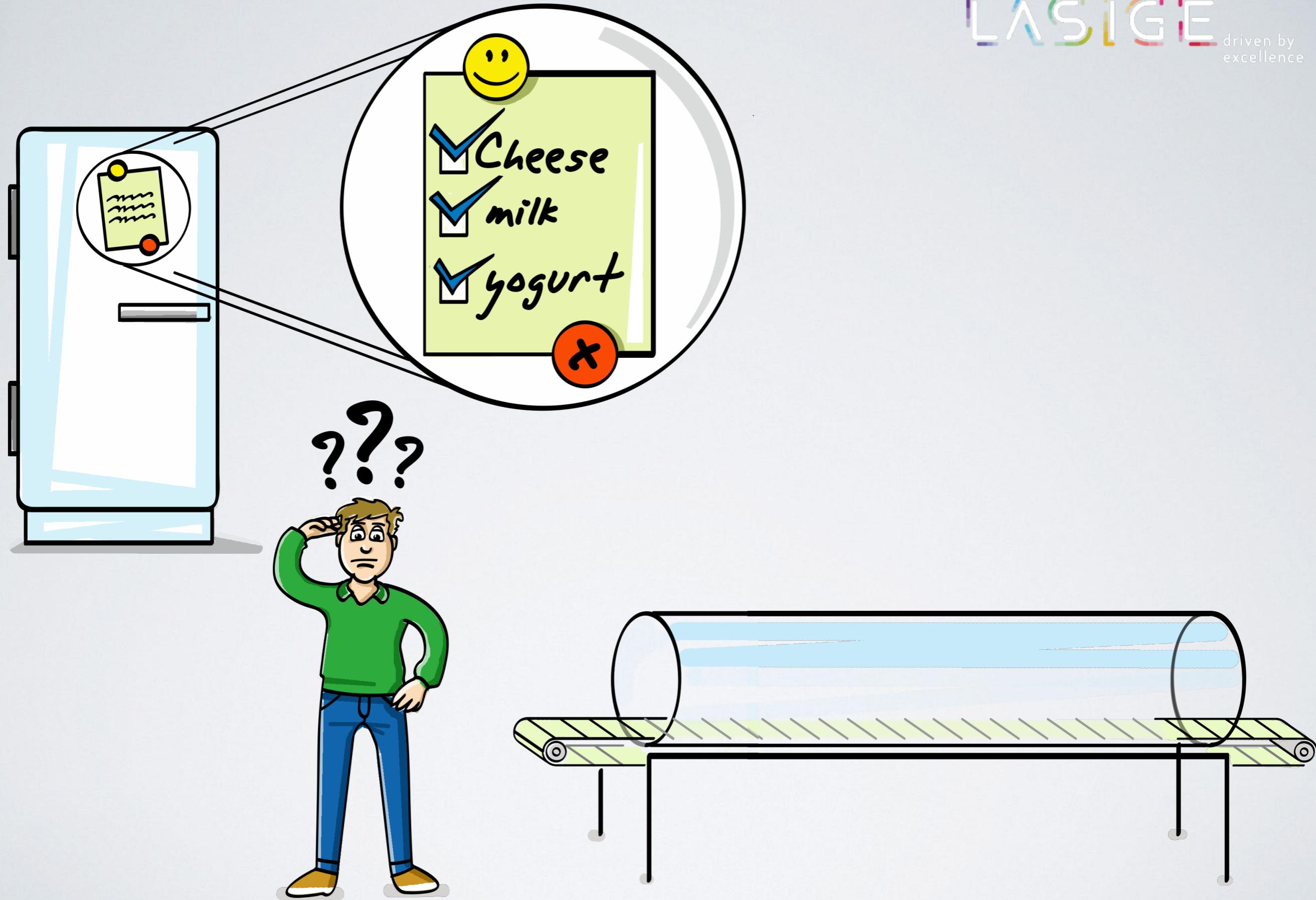


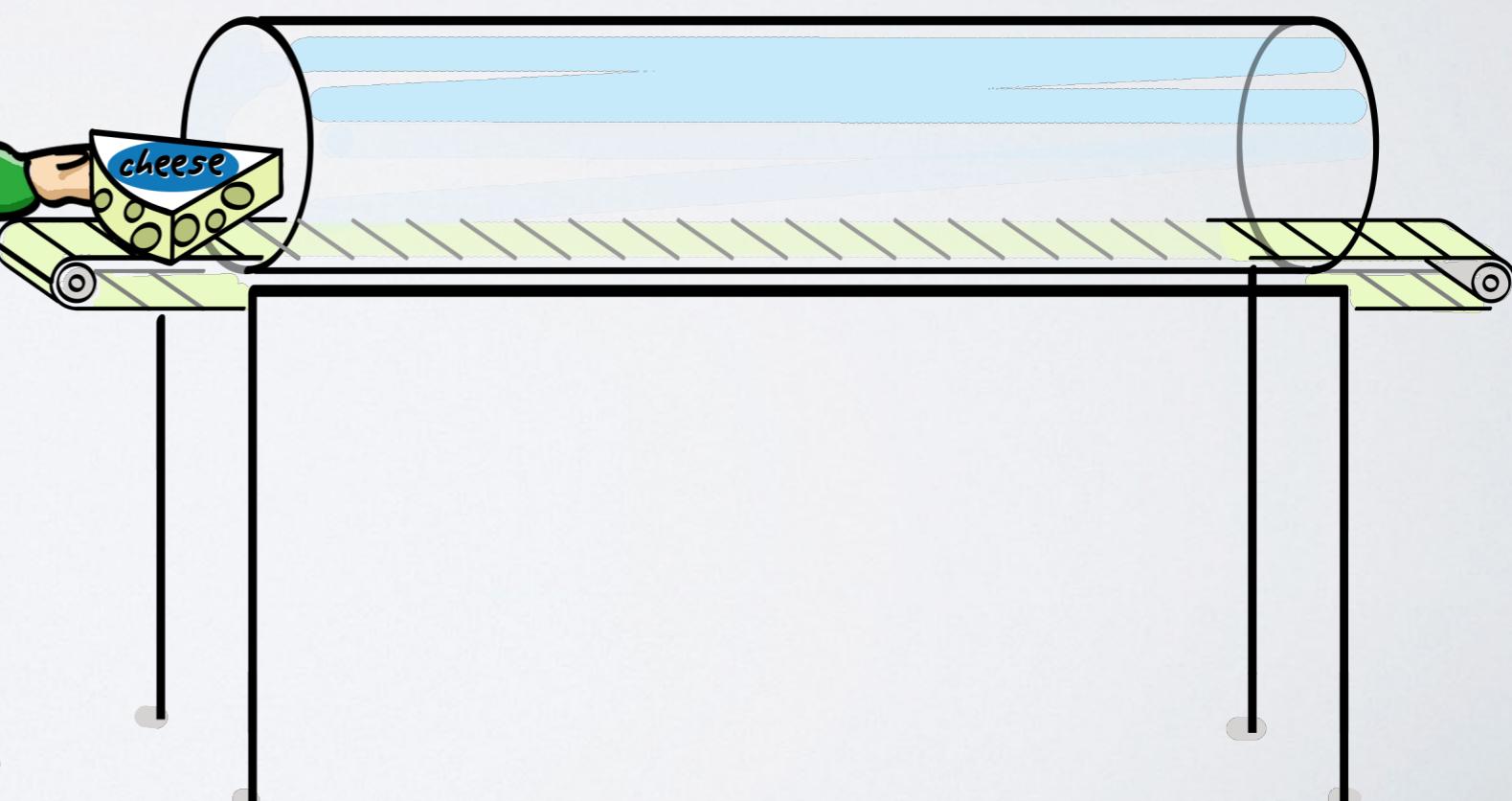
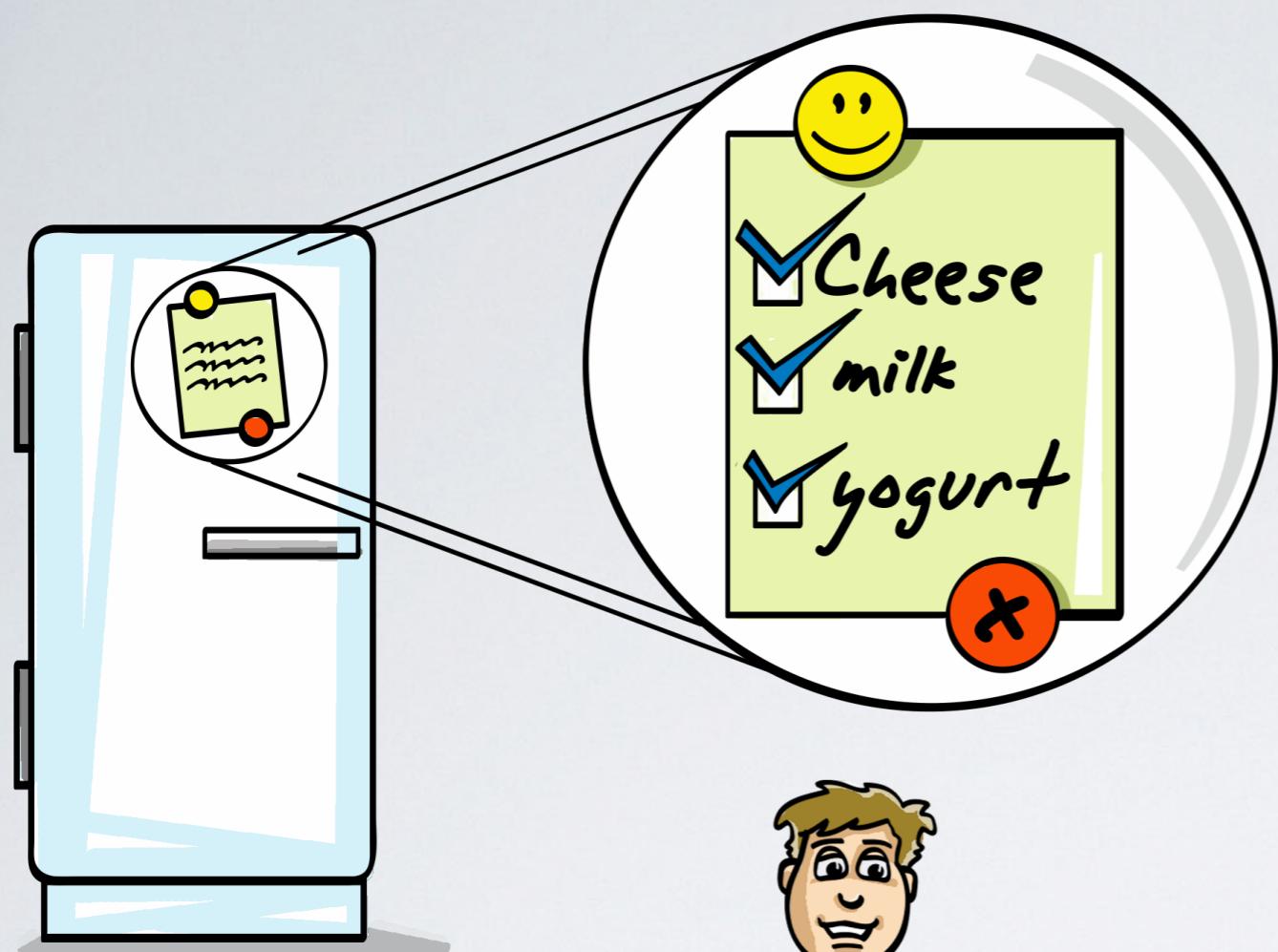
FREEST _ CONTEXT-FREE SESSION TYPES IN CONCURRENT FUNCTIONAL LANGUAGE

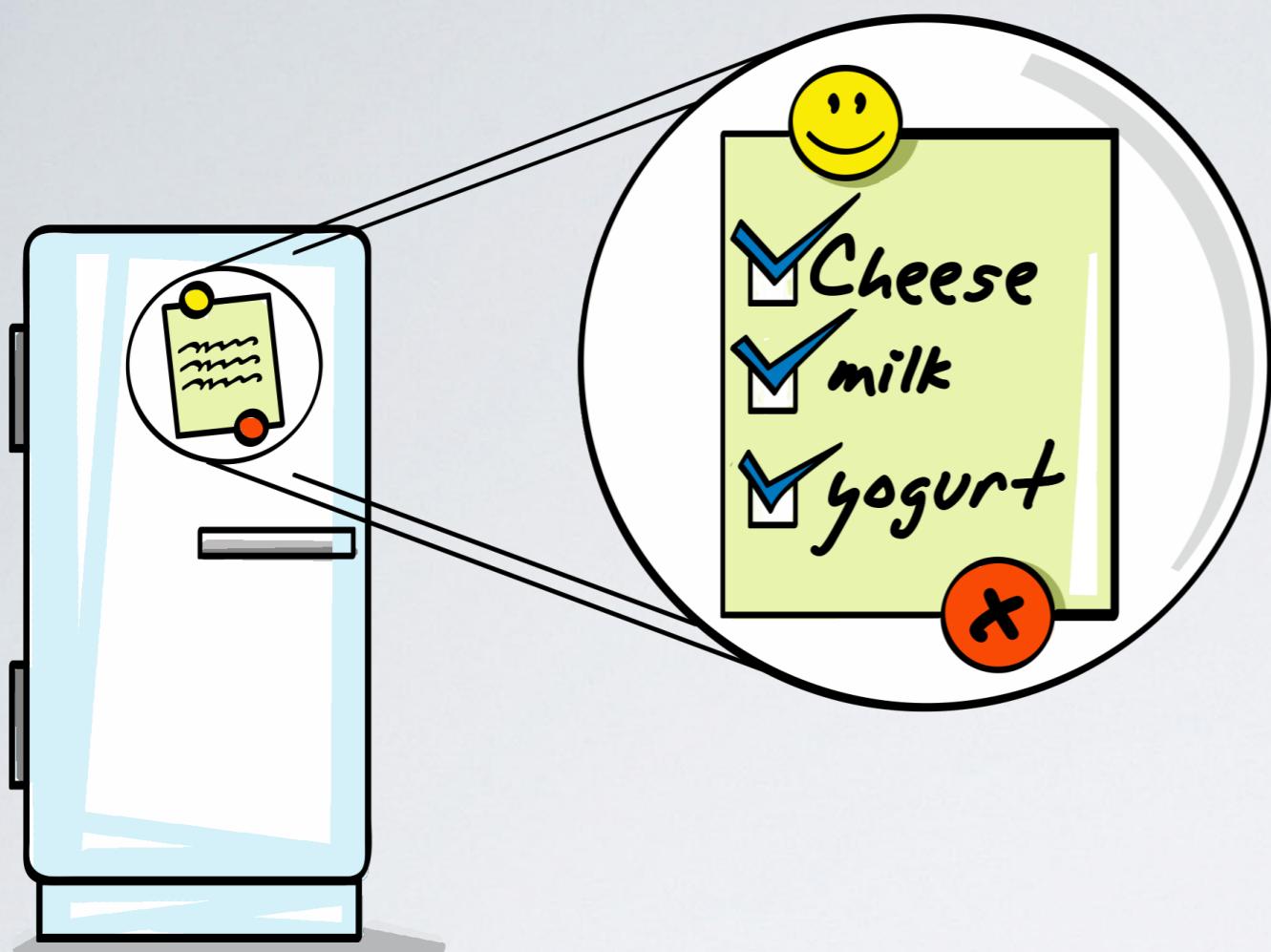
Vasco T. Vasconcelos, Bernardo Almeida, Andreia Mordido
University of Lisbon

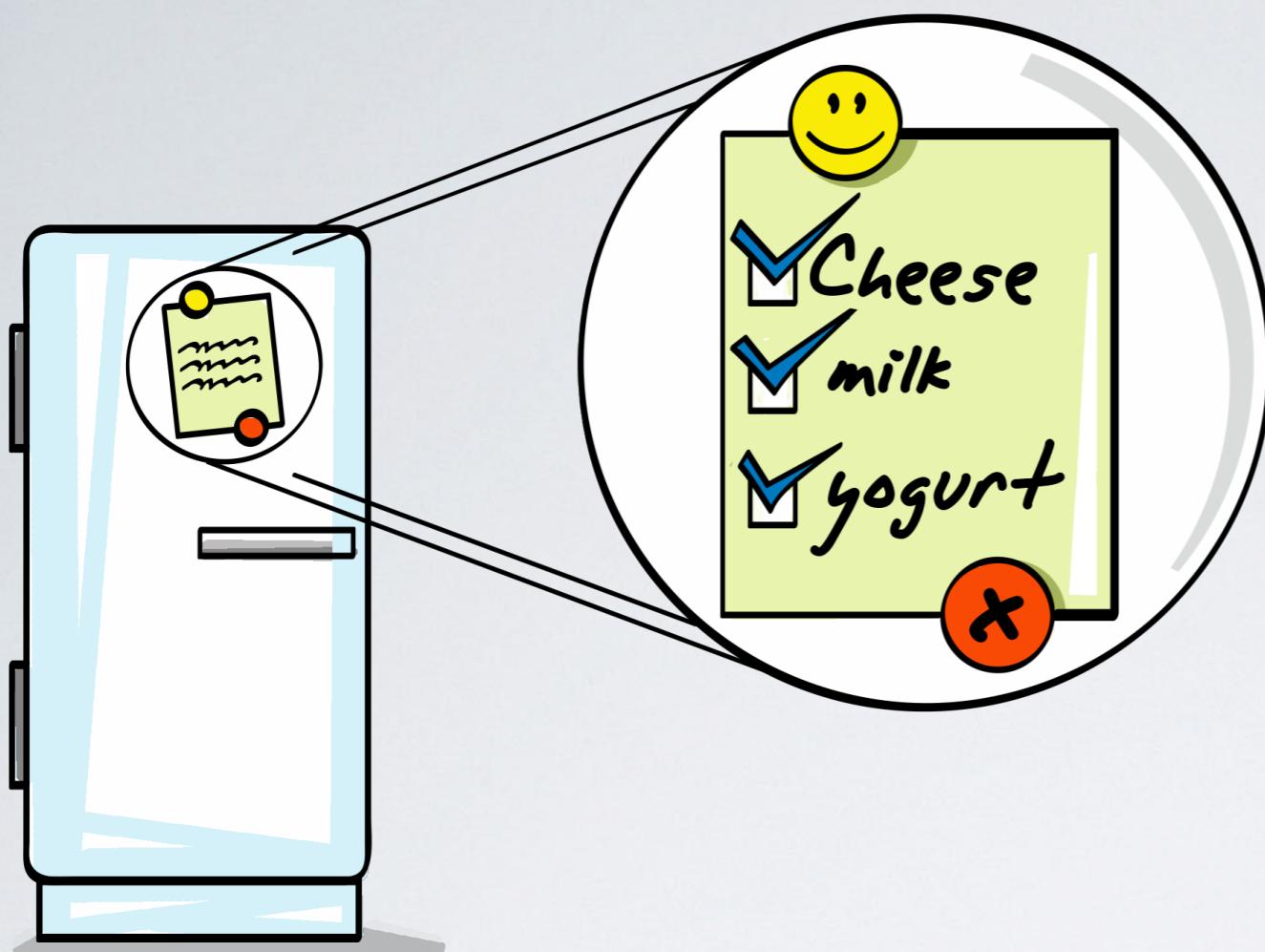
Programming Languages for Distributed Systems
Shonan, May 2019

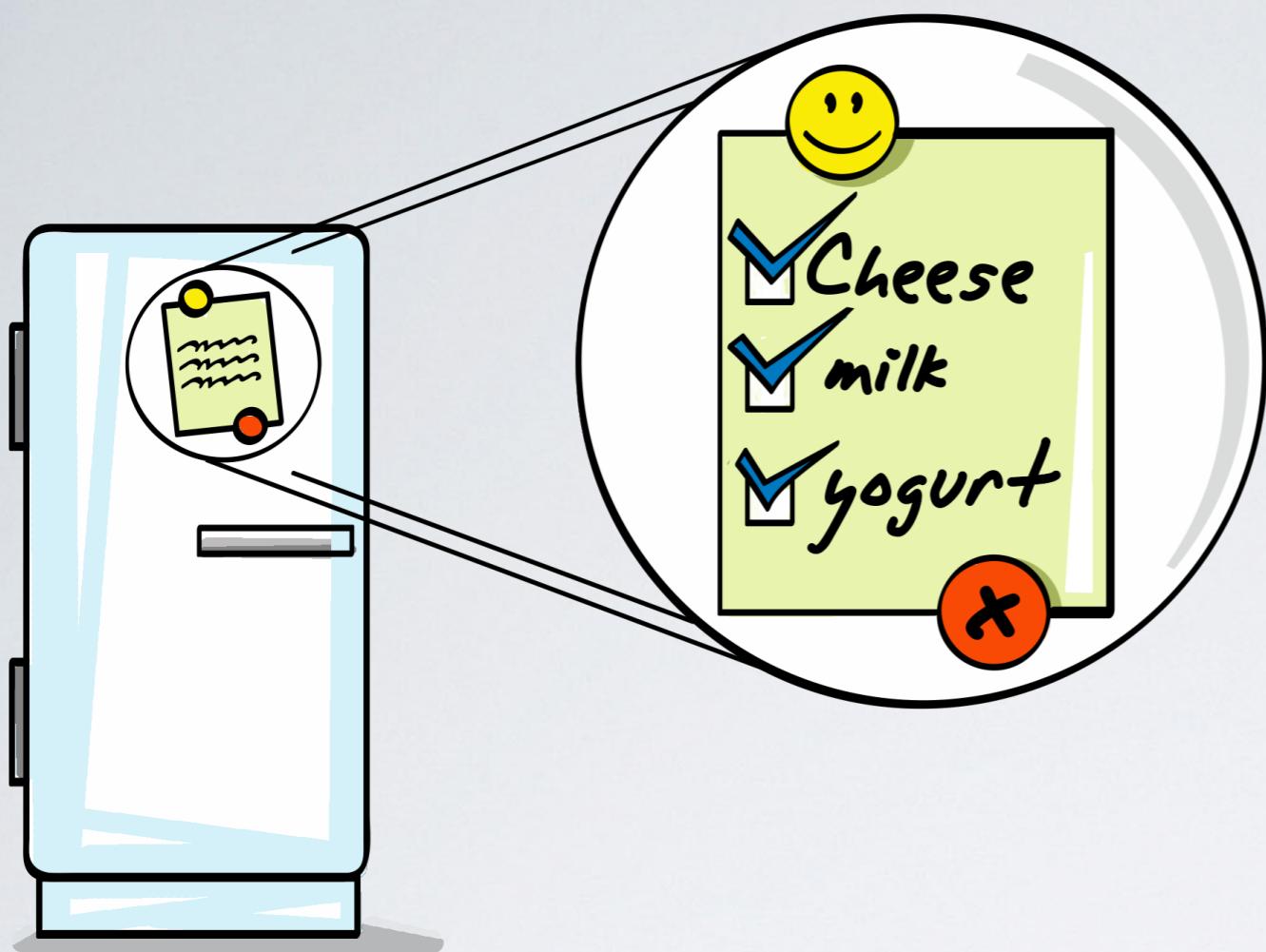














REGULAR SESSION-TYPES

The type of the server

Choice

type ListServer = &{

Choice label

Nil : end,

Termination

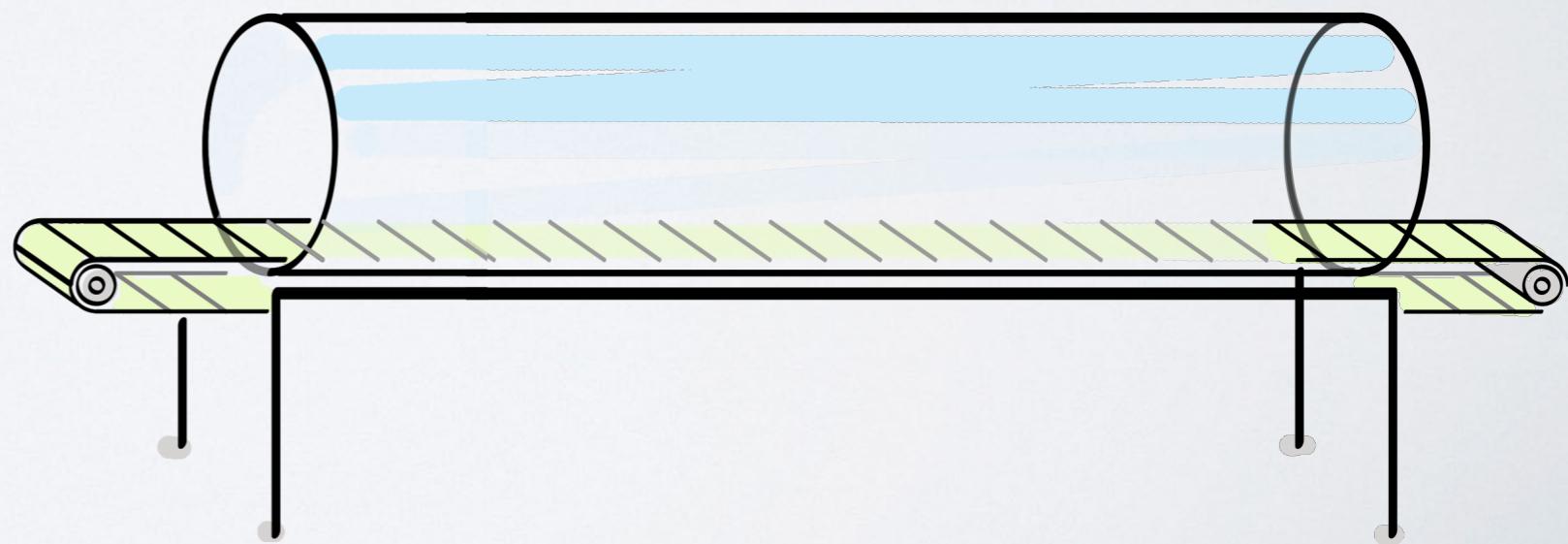
Cons: ?Int . ListServer

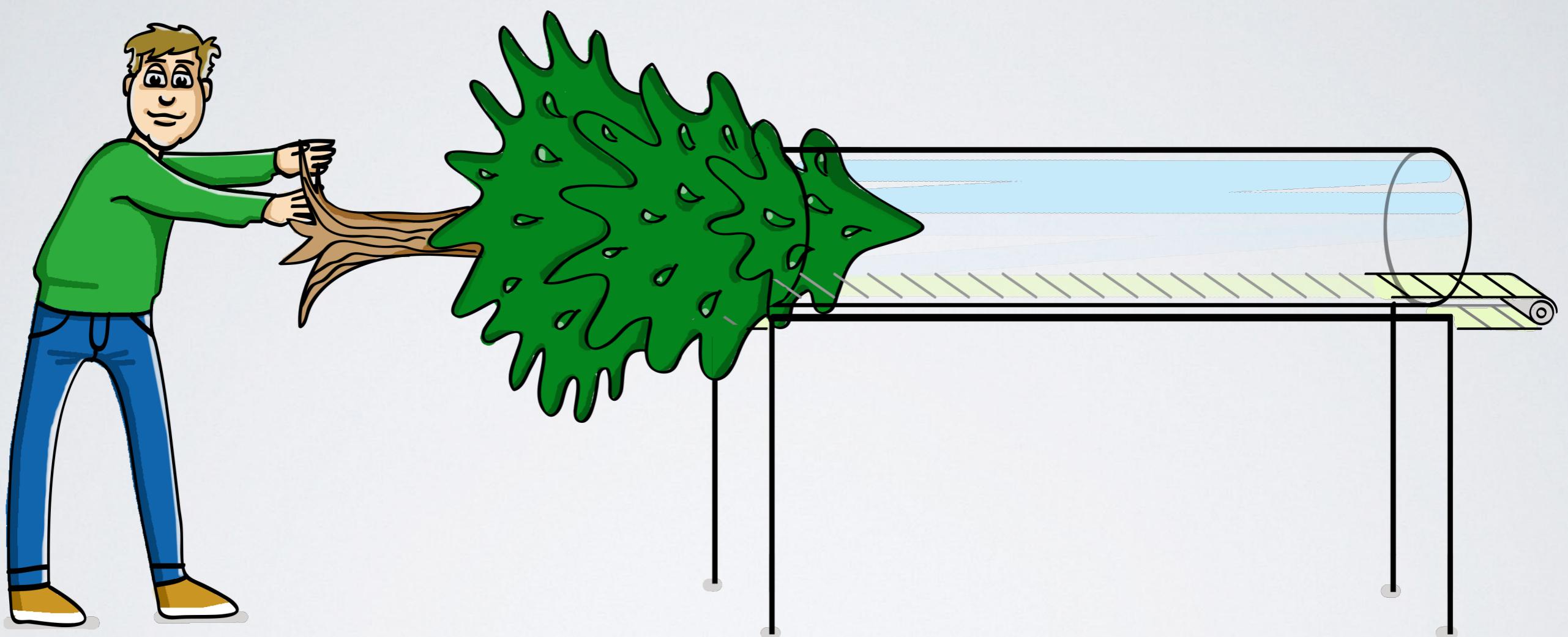
Receive

Recursion

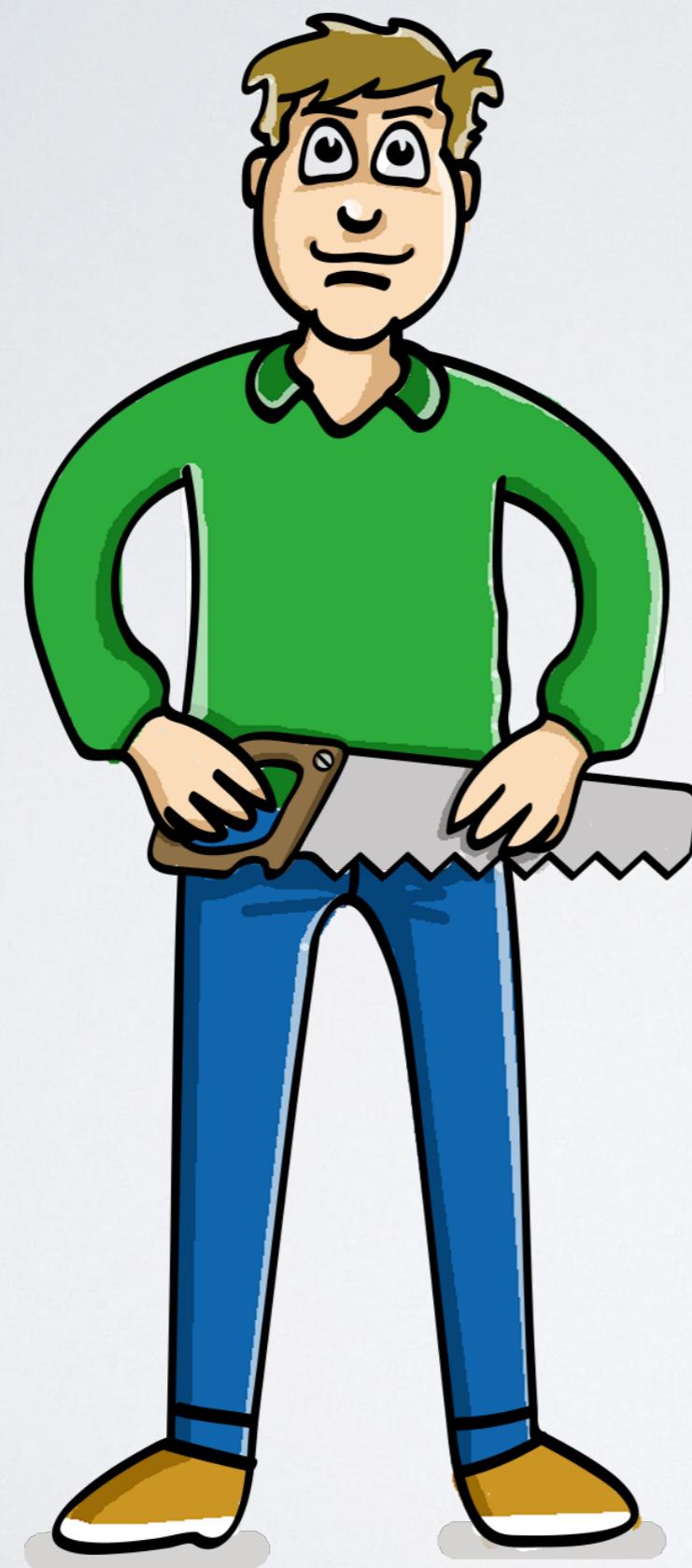


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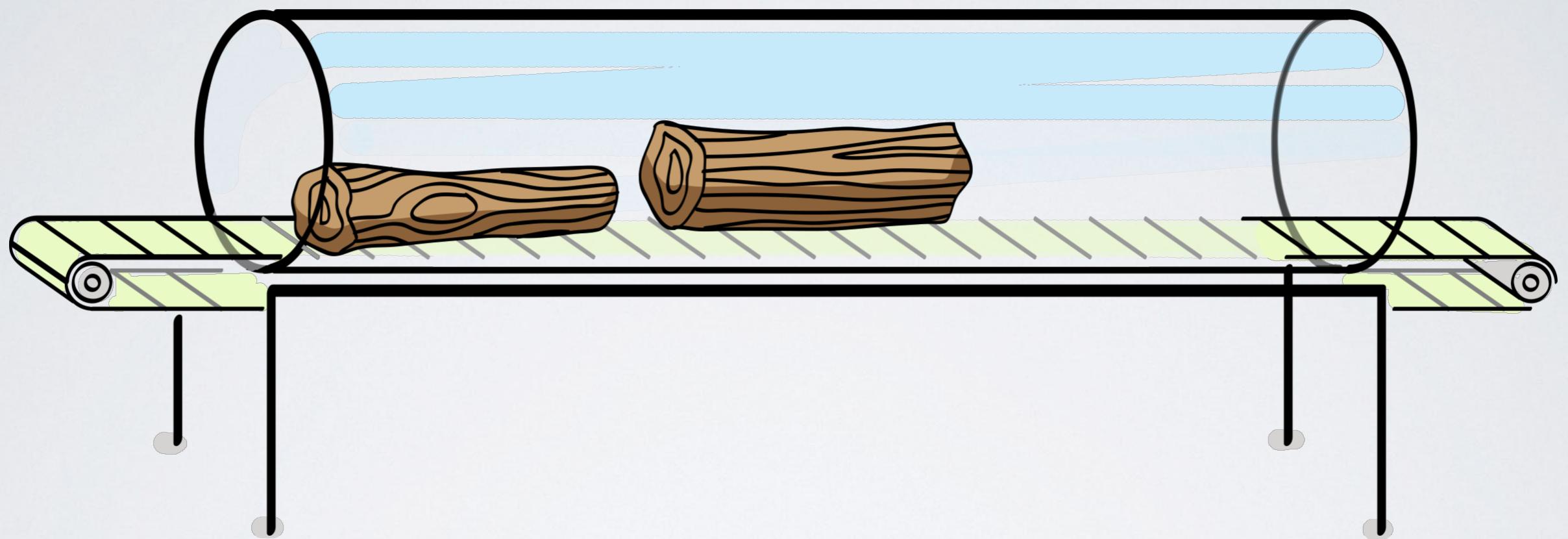


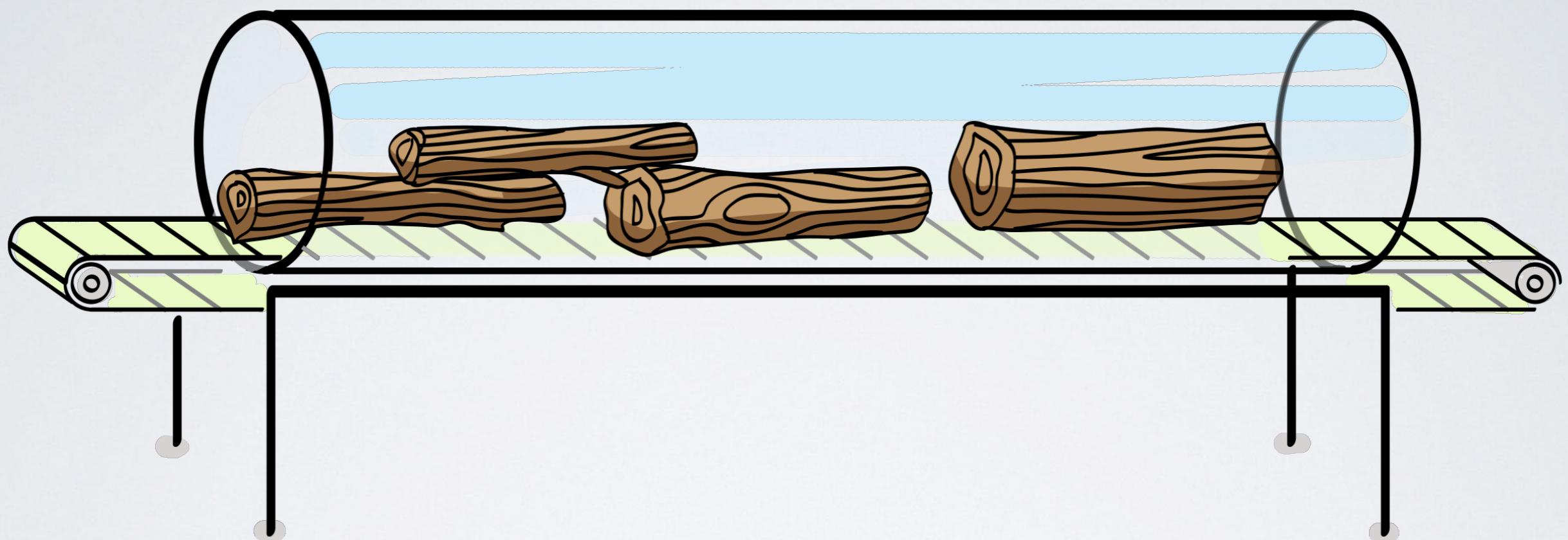


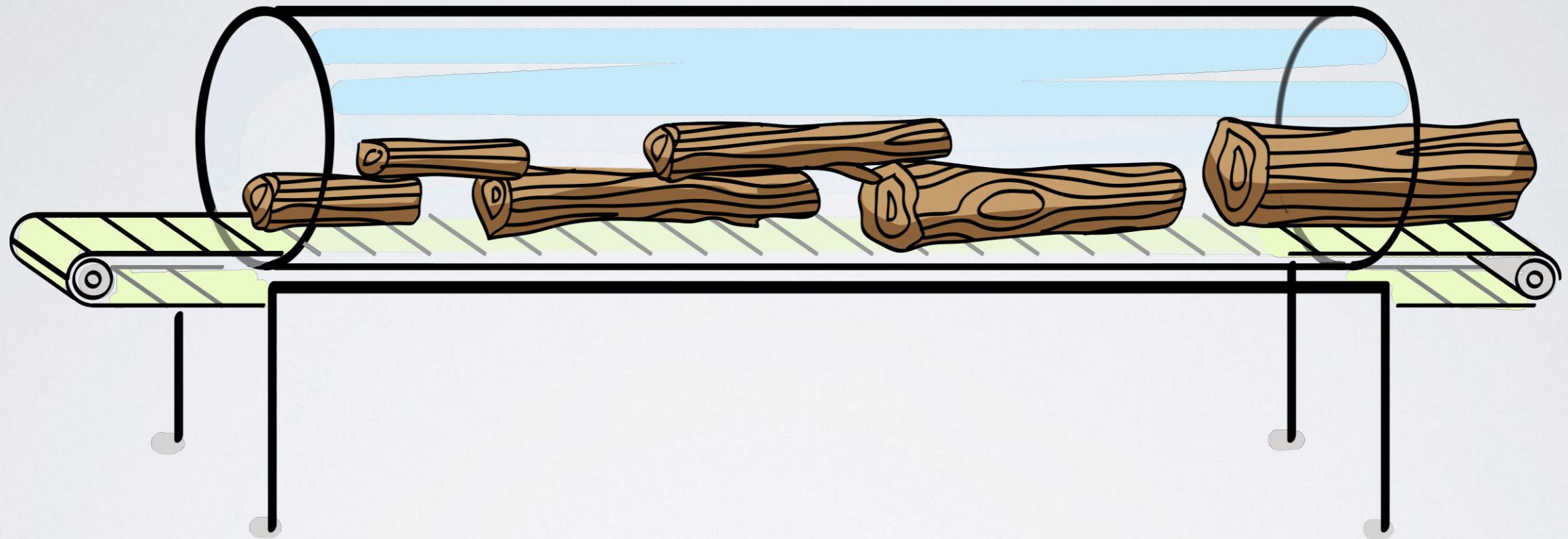


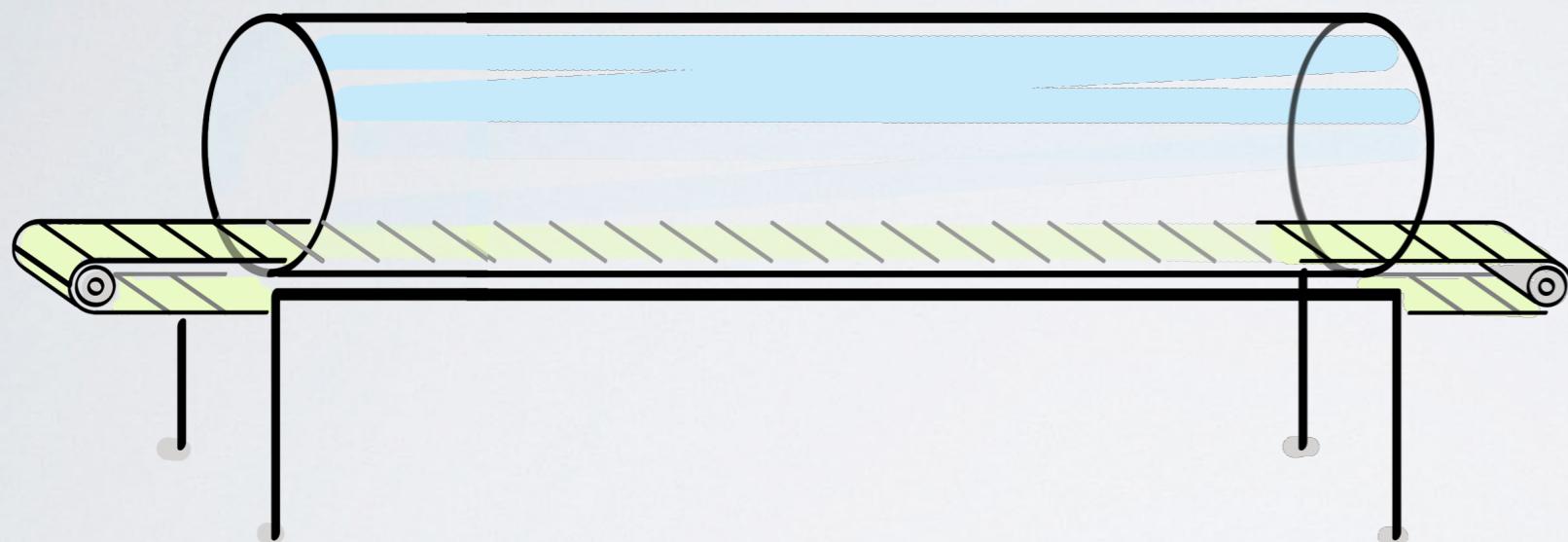












SESSION TYPES FOR LIST AND TREE SERVERS

```
type ListServer = &{  
    Nil: End,  
    Cons: ?Int .  
        ListServer  
}
```

```
type TreeServer = &{  
    LeafS: Skip,  
    NodeS: TreeServer ;  
        ?Int ;  
    TreeServer  
}
```

TYPE EQUIVALENCE _ SOME LAWS

- Recursion is silently unfolded (eqirecursive)

- Identity laws

$$T ; \text{skip} \sim T ; \text{skip} \sim T$$

- Associativity

$$(T ; U) ; V \sim T ; (U ; V)$$

- Distributivity

$$(\&\{l:T, m:U\}); V \sim \&\{l:T; V, m:U; V\}$$

FREEST

CONTEXT-FREE SESSION TYPES IN A CONCURRENT FUNCTIONAL LANGUAGE

DEMO AVAILABLE

```

data Tree = Leaf | Node Int Tree Tree

type TreeC |= +{LeafC: Skip, NodeC: !Int;TreeC;TreeC;?Int}

transform : forall a => Tree -> TreeC;a -> (Tree, a)
transform tree c =
  case tree of {
    Leaf ->
      (Leaf, select LeafC c);
    Node x l r ->
      let c = select NodeC c in
      let c = send c x in
      let l, c = transform[TreeC;?Int;a] l c in
      let r, c = transform[?Int;a] r c in
      let y, c = receive c in
      (Node y l r, c)
  }

```

```

treeSum : forall a => dualof TreeC;a -> (Int, a)
treeSum c =
  match c with {
    LeafC c -> (0, c);
    NodeC c ->
      let x, c = receive c in
      let l, c = treeSum[dualof TreeC;!Int;a] c in
      let r, c = treeSum[!Int;a] c in
      (x + l + r, send c (x + l + r))
  }

main : Tree
main =
  let w, r = new TreeC in
  let _ = fork (treeSum[Skip] r) in
  fst (transform[Skip] aTree w)

```

RAISING A TREE

```
data Tree =  
    Leaf |  
    Node Tree Int Tree
```

```
raiseTree c =  
  case c of  
    LeafS c0 → (Leaf, c0)  
    NodeS c0 →  
      let (t1, c1) = raiseTree c0  
          (v, c2) = receive c1  
          (t2, c3) = raiseTree c2  
      in (Node t1 v t2, c3)
```

choice
label

AIM: LOW-LEVEL SOLUTION

- Communication restricted to base types (no structures, no closures)
- First-order session types (no channel passing)

RAISING A LIST

```
type ListServer = &{
  Nil : end,
  Cons: ?Int . ListServer
}
```

:: ListServer

```
raiseList c =
  case c of
    Nil   → λc0. ([] , c0)
    Cons → λc0.
      let (x, c1) = receive c0
          (xs, c2) = raiseList c1
      in (x:xs, c2)
```

branch
on a choice
type

read from a
channel

WHAT TYPING SHOULD GUARANTEE

- No ill-formed streams

Node 5 Leaf Leaf



Tree Int Tree

- Stream does not end too soon

Node Leaf 5



missing right tree

- No extra elements after the end of the stream

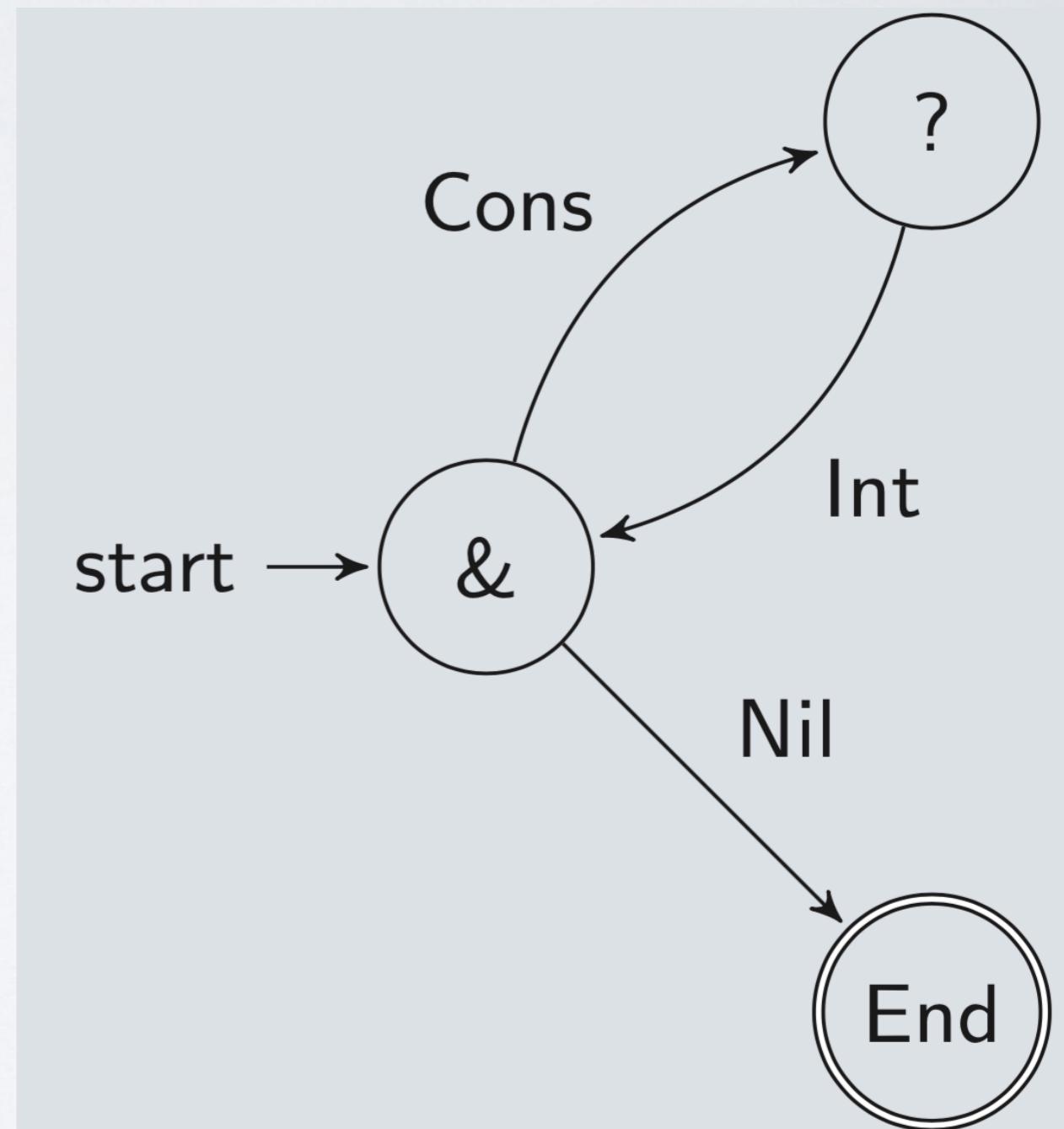
Leaf 5

RECALL THE LIST SERVER

```
type ListServer = &{
    Nil : end,
    Cons: ?Int . ListServer
}
```

- Regular trace language

$(\&\text{Cons} . ?\text{Int})^* . \&\text{Nil}$



THE TRACE LANGUAGE OF A SESSION TYPE

- The trace language of any (first-order) session type is:
 - recognised by a finite automaton
 - a (ω -) regular language
- The trace language of raiseTree is a context-free language:

$$T \rightarrow \&\text{LeafS}$$
$$T \rightarrow \&\text{NodeS} T ? \text{Int} T$$

THEREFORE...

- There is no standard session type for `raiseTree`. It cannot be typed in (e.g.) GV

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Linear type theory for asynchronous session types

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ENTER CONTEXT-FREE SESSION TYPES

REGULAR & CONTEXT-FREE SESSION TYPES

Regular	Context free
$!B.S$	$!B$
$?B.S$	$?B$
end	$S;S$ skip
Hardwired sequencing of communication	Explicit sequencing
Tail recursive	Unrestricted recursion

ARITHMETIC EXPRESSION SERVER

```
type TermChan = &{Const: !Int,  
                    Add: TermChan;TermChan,  
                    Mult: TermChan;TermChan}
```

```
computeService :: TermChan;!Int →o skip
```

```
computeService c =
```

```
  let n,c1 = receiveEval c in send n c1
```

```
receiveEval :: ∀a.TermChan;a →o Int ⊗ a
```

```
receiveEval c =
```

```
  case c of {
```

```
    Const c1 → receive c1
```

```
    Add c1 → let n1,c2 = receiveEval 1  
              n2,c3 = receiveEval c2  
              in (n1+n2,c3)
```

```
    Mult c1 → let n1,c2 = receiveEval c ...
```

```
}
```

REQUIREMENTS FOR THE META THEORY

TYPING RAISETREE

- At the top level:

```
type TreeServer = &{  
    LeafS : skip,  
    NodeS : TreeServer ;  
    ?int ;  
    TreeServer}
```

Linear
function

Linear pair

raiseTree :: TreeServer →o (Tree ⊗ skip)

TYPING RAISETREE

- At the first recursive call

```
raiseTree c = case c of
```

```
  Node → λc0. let (t1 , c1) = raiseTree c0
                (v, c2) = receive c1
                (t2 , c3) = raiseTree c2
```

```
raiseTree :: (TreeServer;?Int;TreeServer ;...) →o
          Tree ⊗ (?Int; TreeServer;...)
```

```
type TreeServer = &{
  LeafS : skip,
  NodeS : TreeServer ;
  ?int ;
  TreeServer}
```

TYPING RAISETREE

- At the top level:

`raiseTree :: TreeServer → (Tree ⊗ skip)`

- At the first recursive call

`raiseTree :: (TreeServer; ?Int; TreeServer ;...) →`
`Tree ⊗ (?Int; TreeServer;...)`

- A polymorphic type

`raiseTree :: ∀a. TreeServer;a → (Tree ⊗ a)`

INSTANTIATING A POLYMORPHIC TYPE

`raiseTree :: ∀a. TreeServer;a →o (Tree ⊗ a)`

- Replace α with `skip`

`raiseTree :: TreeServer; skip →o (Tree ⊗ skip)`

- We need

`TreeServer; skip ~ TreeServer`

TYPE EQUIVALENCE IS A BISIMULATION

- ... over a suitable labelled transition system
- How do we decide type equivalence?
- For regular types there is a simple (polynomial) algorithm
- But context-free types is a different story...

TYPE EQUIVALENCE IS DECIDABLE

- Via translation to BPA, Basic Process Algebra [Bergstra, Klop 1988]
- Bisimulation for BPA is decidable [Christiansen, Hüttel, Stirling 1995]
- Unfortunately, the decidability result does not yield an obvious algorithm

CONCLUSION

- Programming with context free session types is fun
- Compiler ready soon
- Implementing type equivalence challenging