A Solution Plan and CSP Relation

In this appendix, we demonstrate a way to build an assignment to the CSP variables such that the encoded plan matches a given solution plan. This is needed as a preliminary to the completeness proof.

For a given solution plan Π, the variables in the Constraint Satisfaction Problem (CSP) are:

$$\begin{aligned}\mathcal{V} = &\left\{ p_a^i \mid a \in \mathbb{A}, p_a^i \in P_a \right\} \\ &\cup \left\{ s_a \mid a \in \mathbb{A} \right\} \cup \left\{ e_a \mid a \in \mathbb{A} \right\} \\ &\cup \left\{ \mathit{pres}(a) \mid a \in \mathbb{A} \right\} \\ &\cup \left\{ v_\kappa \mid \kappa \in T_R \right\} \\ &\cup \left\{ m_\varepsilon \mid \varepsilon \in T_E^A \right\}\end{aligned}$$

There are three groups of variables:

- Variables related to actions: $\left\{ p_a^i \mid a \in \mathbb{A}, p_a^i \in P_a \right\} \cup \left\{ s_a \mid a \in \mathbb{A} \right\} \cup \left\{ e_a \mid a \in \mathbb{A} \right\} \cup \left\{ \mathit{pres}(a) \mid a \in \mathbb{A} \right\}$
- Variables related to readings: $\left\{ v_\kappa \mid \kappa \in T_R \right\}$
- Variables related to assignments: $\left\{ m_\varepsilon \mid \varepsilon \in T_E^A \right\}$

CSP Action Variables

For each action instance $a \in \mathbb{A}$, if the action is present in the solution plan, then the presence variable $\mathit{pres}(a)$ is set to true ($\top$). Moreover, the parameters p_a^i, the start time s_a, and the end time e_a of the action are set to the corresponding values in the solution plan.

However, if the action is not present in the solution plan, then the presence variable $\mathit{pres}(a)$ is set to false ($\bot$) and the other variables are set to arbitrary values since they are not used in the CSP thanks to implies constraints.

This is graphically summarized in the Figure 1.

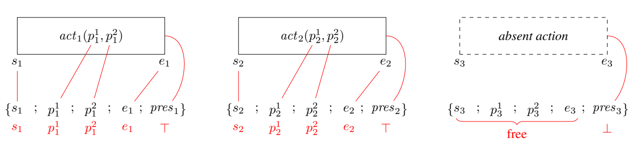

Figure 1: Relation between the solution plan actions and the CSP action variables

CSP Reading Variables

For each reading $\kappa \in T_R$, the variable v_κ is set to the required value in the solution plan.

For a condition $[t]sv(x) = V$ in an action $a \in \mathbb{A}$, a read token $\langle\, \mathit{pres}(a) : [t]sv(x) = v \,\rangle \in T_R$ is created. If the action is present in the solution plan ($\mathit{pres}(a) = \top$), then the variable v is set to V. Otherwise, the variable v is set to an arbitrary value since it is not used in the CSP thanks to implies constraints.

For a condition $[t]sv(x) \bowtie V$, where $\bowtie \in \{ <, \leq, >, \geq \}$, in an action $a \in \mathbb{A}$, a read token $\langle\, \mathit{pres}(a) : [t]sv(x) = v \,\rangle \in T_R$ is created. If the action is present in the solution plan ($\mathit{pres}(a) = \top$), then the variable v is set to $V + k$, with $k \in \mathbb{Z}$ such that the condition is respected. Otherwise, the variable v is set to an arbitrary value since it is not used in the CSP thanks to implies constraints.

This is graphically summarized in the Figure 2.

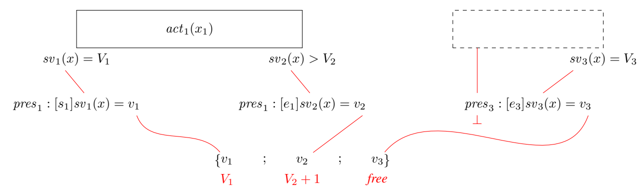

Figure 2: Relation between the solution plan conditions and the CSP reading variables

CSP Mutex Variables

For each assign effect $\varepsilon \in T_E^A$, the variable m_ε is set to the value of the next assign effect execution time-point affecting the same state variable in the solution plan decreased by ε. However, in the case where the assign effect is the last one affecting the state variable, the variable m_ε is set just after the horizon, *i.e.*, $m_\varepsilon = H + \varepsilon$.

This is graphically summarized in the Figure 3.

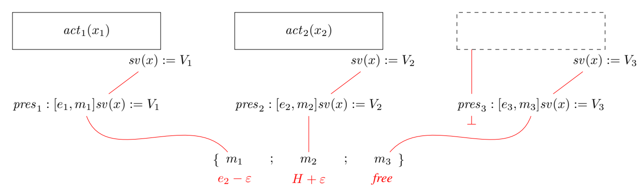

Figure 3: Relation between the solution plan assign effects and the CSP mutex variables

Solution Plan from the CSP

The previous relations allow to build an assignment of the CSP variables from a solution plan.

In order to create a solution plan from the CSP variable values, only the relations between the action variables and the solution plan actions are needed. If the presence variable $\mathit{pres}(a)$ is set to $\top$, then the action is present in the solution plan and the parameters p_a^i, the start time s_a, and the end time e_a are set to the corresponding values in the CSP. However, if the presence variable $\mathit{pres}(a)$ is set to $\bot$, then the action is not present in the solution plan and the other variables are ignored.

B Completeness Proof

In this appendix, we prove the Theorem 1 stating that the proposed encoding is complete, *i.e.*, if the planning problem has a solution, then the generated CSP has a solution.

Solution Plan Definition

Let's consider a solution plan Π to the planning problem. Then, according to the Definition 11 of a solution plan, the following holds:

1. Π is coherent, *i.e.*, for any $t \in \mathcal{T}$ and $sv(x) \in \mathcal{SV}$, $E_{\Pi}^{[t]sv(x)}$ is either:
   (a) $E_{\Pi}^{[t]sv(x)} = \emptyset$, empty
   (b) $E_{\Pi}^{[t]sv(x)} = \{ := v \}$, with a single assign
   (c) $E_{\Pi}^{[t]sv(x)} = \{ += N_i \mid 1 \leq i \leq k \}$, with k increases
2. Numerical state variables are always within their bounds: $\forall sv(x) \in \mathcal{SV}_{\mathcal{N}}, \forall t \in \mathcal{T}, lb(sv) \leq [t]sv(x) \leq ub(sv)$.
3. All actions $a \in \Pi$ are applicable, meaning all constraints and conditions in C_a are entailed by the trace of Π.
4. All goals $g \in G$ are satisfied at the associated time-point.
5. All effects in the plan are executed before the horizon.

Associated CSP Definition

Let's consider k wide enough for $\mathbb{A}_k$ to contain enough action instances to represent the solution plan Π. Then, the associated CSP is the tuple variables / constraints $(\mathcal{V}, \mathcal{X})$, where:

$$
\begin{aligned}
\mathcal{V} = &\left\{ p_a^i \mid a \in \mathbb{A}, p_a^i \in P_a \right\} \\
&\cup \left\{ s_a \mid a \in \mathbb{A} \right\} \cup \left\{ e_a \mid a \in \mathbb{A} \right\} \\
&\cup \left\{ \mathit{pres}(a) \mid a \in \mathbb{A} \right\} \\
&\cup \left\{ v_\kappa \mid \kappa \in T_R \right\} \\
&\cup \left\{ m_\varepsilon \mid \varepsilon \in T_E^A \right\} \\
\mathcal{X} = &\left\{ \mathit{coherent}\left(\varepsilon_1, \varepsilon_2\right) \mid \varepsilon_1, \varepsilon_2 \in T_E^A, \varepsilon_1 \neq \varepsilon_2 \right\} \\
&\cup \left\{ \mathit{supported}\left(\kappa\right) \mid \kappa \in T_R, sv(x_\kappa) \in \mathcal{SV} \setminus \mathcal{SV}_{\mathcal{N}} \right\} \\
&\cup \left\{ \mathit{num\text{-}supported}\left(\kappa\right) \mid \kappa \in T_R, sv(x_\kappa) \in \mathcal{SV}_{\mathcal{N}} \right\} \\
&\cup \left\{ \mathit{consistent}\left(\beta\right) \mid \beta \in T_C \right\} \\
&\cup \left\{ \mathit{structure\text{-}action}\left(a\right) \mid a \in \mathbb{A} \right\} \\
&\cup \left\{ \mathit{structure\text{-}assign}\left(\varepsilon\right) \mid \varepsilon \in T_E^A \right\} \\
&\cup \left\{ \mathit{structure\text{-}increase}\left(\varphi\right) \mid \varphi \in T_E^I \right\}
\end{aligned}
$$

Let's consider the variable assignments of the CSP described in the Appendix A to prove by absurdity that the CSP has a solution. We therefore assume that the CSP has no solution, *i.e.*, a constraint of $\mathcal{X}$ is not verified. The following of the proof is organized by the type of constraint that will be first supposed to be violated, and then the contradiction will be shown.

Coherence Constraints

Let's assume that a coherence constraint is not verified, *i.e.*, there exists a pair of assign effects $\varepsilon_1, \varepsilon_2 \in T_E^A$ such that $\varepsilon_1 \neq \varepsilon_2$ and *coherent* $(\varepsilon_1, \varepsilon_2)$ is not verified:

$$
\begin{aligned}
&\neg \left(p_1 \wedge p_2 \implies \left(m_1 \leq t_2 \vee m_2 \leq t_1 \vee x_1 \neq x_2\right)\right) \\
\Leftrightarrow\ & p_1 \wedge p_2 \wedge m_1 > t_2 \wedge m_2 > t_1 \wedge x_1 = x_2
\end{aligned}
$$

$p_1 \wedge p_2 \wedge x_1 = x_2$ means that both assign effects are present in the plan and applied to the same state variable.

Moreover, since $t_1 < m_1$ and $t_2 < m_2$, then $m_1 > t_2 \wedge m_2 > t_1$ means that the two mutex periods overlap. This is a **contradiction with the point 1.(b)** of a solution plan, as $E_{\Pi}^{[t]sv(x)}$ can only contain at most one assign effect for each state variable at each time-point.

Therefore, all **coherence constraints are verified**.

Support Constraints

Let's assume that a support constraint is not verified, *i.e.*, there exists a read token value $\kappa \in T_R$ with a state variable $sv(x_\kappa) \in \mathcal{SV} \setminus \mathcal{SV}_{\mathcal{N}}$ such that *supported* (κ) is not verified:

$$
\begin{aligned}
&\neg \left(p_\kappa \implies \bigvee_{\varepsilon \in T_E^A} \mathit{supported\text{-}by}\left(\kappa, \varepsilon\right)\right) \\
\Leftrightarrow\ & p_\kappa \wedge \bigwedge_{\varepsilon \in T_E^A} \neg \mathit{supported\text{-}by}\left(\kappa, \varepsilon\right) \\
\Leftrightarrow\ & p_\kappa \wedge \bigwedge_{\varepsilon \in T_E^A} \left(\begin{array}{l} \neg p_\varepsilon \vee t_\varepsilon \geq t_\kappa \vee t_\kappa > m_\varepsilon \\ \vee x_\kappa \neq x_\varepsilon \vee v_\kappa \neq v_\varepsilon \end{array} \right)
\end{aligned}
$$

This means that the read token value is present in the plan, and each assign effect is either:

- $\neg p_\varepsilon$: not present in the plan
- $t_\varepsilon \geq t_\kappa$: executed after the reading
- $t_\kappa > m_\varepsilon$: ended before the reading
- $x_\kappa \neq x_\varepsilon$: applied to a different state variable
- $v_\kappa \neq v_\varepsilon$: assigning a different value

Therefore, there is no assign effect that assigns the required value to the state variable at the time-point of the read token value. This is a **contradiction with the points 3 and 4** of a solution plan, as all actions are applicable and all goals are satisfied.

Finally, all **support constraints are verified**.

Numerical Support Constraints

Let's assume that a numerical support constraint is not verified, *i.e.*, there exists a read token value $\kappa \in T_R$ with a state variable $sv(x_\kappa) \in \mathcal{SV}_{\mathcal{N}}$ such that *num-supported* (κ) is not verified:

$$
\begin{aligned}
&\neg \left(p_\kappa \implies \bigvee_{\varepsilon \in T_E^A} \mathit{num\text{-}supported\text{-}by}\left(\kappa, \varepsilon, T_E^I\right)\right) \\
\Leftrightarrow\ & p_\kappa \wedge \bigwedge_{\varepsilon \in T_E^A} \neg \mathit{num\text{-}supported\text{-}by}\left(\kappa, \varepsilon, T_E^I\right) \\
\Leftrightarrow\ & p_\kappa \wedge \bigwedge_{\varepsilon \in T_E^A} \left(\begin{array}{l} \neg \chi_\varepsilon\left(\kappa, \varepsilon\right) \vee lb(sv) > v_\kappa \vee v_\kappa > ub(sv) \\ \vee \left(v_\kappa \neq v_\varepsilon + \sum_{i=1}^{\left| T_E^I \right|} \left[\chi_\varphi\left(\kappa, \varepsilon, \varphi_i\right)\right] \cdot v_\varphi^i \right) \end{array} \right)
\end{aligned}
$$

This means that the read token value is present in the plan, and for each assign effect:

- $\neg \chi_\varepsilon\left(\kappa, \varepsilon\right)$: the assign effect token does not assign a value to the state variable or does not maintains its mutex period until the end of the evaluation time. The reasoning is the same as for the support constraints, and it is a **contradiction with the points 3 and 4** of a solution plan.
- $lb(sv) > v_\kappa \vee v_\kappa > ub(sv)$: the read value is not within the bounds of the state variable. This is a **contradiction with the point 2** of a solution plan, as all numerical state variables are within their bounds.

- $v_\kappa \neq v_\varepsilon + \sum_{i=1}^{\left| T_E^I \right|} \left[\chi_\varphi \left(\kappa, \varepsilon, \varphi_i \right) \right] \cdot v_\varphi^i$: the assign and the increase effects do not assign the required value to the state variable. This is **impossible** based on the construction of the assignment, see Appendix A.

Therefore, all **numerical support constraints are verified**.

Consistency Constraints

Let's assume that a consistency constraint is not verified, *i.e.*, there exists a condition token $\beta \in T_C$ such that *consistent* (β) is not verified:

$$\begin{aligned} & \neg \left(p_\beta \implies B_\beta \right) \\ \Leftrightarrow \quad & p_\beta \wedge \neg B_\beta \end{aligned}$$

This means that the condition token is present in the plan, but the associated boolean expression is not satisfied. This is a **contradiction with the point 3 and 4** of a solution plan, as all actions are applicable and all goals are satisfied.

Therefore, all **consistency constraints are verified**.

Action Structure Constraints

Let's assume that a constraint on an action structure is not verified, *i.e.*, there exists an action instance $a \in \mathbb{A}$ such that *structure-action* (a) is not verified:

$$\begin{aligned} & \neg \left(pres(a) \implies s_a \leq e_a \wedge e_a \leq H \right) \\ \Leftrightarrow \quad & pres(a) \wedge \left(s_a > e_a \vee e_a > H \right) \end{aligned}$$

This means that the action instance is present in the plan, but the start time is after the end time, which is a **contradiction with the definition of an action**, or the end time is after the horizon, which is a **contradiction with the point 5** of a solution plan.

Therefore, all **constraints on action structure are verified**.

Assign Effect Structure Constraints

Let's assume that a constraint on an assign effect structure is not verified, *i.e.*, there exists an assign effect token $\varepsilon \in T_E^A$ such that *structure-assign* (ε) is not verified:

$$\begin{aligned} & \neg \left(p_\varepsilon \implies t_\varepsilon < H \wedge t_\varepsilon < m_\varepsilon \right) \\ \Leftrightarrow \quad & p_\varepsilon \wedge \left(t_\varepsilon \geq H \vee t_\varepsilon \geq m_\varepsilon \right) \end{aligned}$$

This means that the assign effect token is present in the plan, but the execution time is after the horizon or the mutex time-point, which is a **contradiction with the point 5** of a solution plan or with the definition of an assign effect.

Therefore, all **constraints on assign effect structure are verified**.

Increase Effect Structure Constraints

Let's assume that a constraint on an increase effect structure is not verified, *i.e.*, there exists an increase effect token $\varphi \in T_E^I$ such that *structure-increase* (φ) is not verified:

$$\begin{aligned} & \neg \left(p_\varphi \implies t_\varphi < H \right) \\ \Leftrightarrow \quad & p_\varphi \wedge t_\varphi \geq H \end{aligned}$$

This means that the increase effect token is present in the plan, but the execution time is after the horizon, which is a **contradiction with the point 5** of a solution plan.

Therefore, all **constraints on increase effect structure are verified**.

Conclusion

We have shown by absurdity that all constraints of the CSP are verified when the planning problem has a solution. Therefore, **the proposed encoding is complete**, *i.e.*, if the planning problem has a solution, then the generated CSP has a solution.

$\square$

C Soundness Proof

In this appendix, we prove the Theorem 2 stating that the proposed encoding is sound, *i.e.*, if the generated CSP has a solution, then the original planning problem has a solution as well.

CSP Solution Definition

Let's consider a solution to our CSP encoding, *i.e.*, a set of values for the variables in $\mathcal{V}$ that satisfies all the constraints in $\mathcal{X}$:

$$\begin{aligned}
\mathcal{V} = &\left\{ p_a^i \mid a \in \mathbb{A}, p_a^i \in P_a \right\} \\
&\cup \left\{ s_a \mid a \in \mathbb{A} \right\} \cup \left\{ e_a \mid a \in \mathbb{A} \right\} \\
&\cup \left\{ \mathit{pres}(a) \mid a \in \mathbb{A} \right\} \\
&\cup \left\{ v_\kappa \mid \kappa \in T_R \right\} \\
&\cup \left\{ m_\varepsilon \mid \varepsilon \in T_E^A \right\} \\
\mathcal{X} = &\left\{ \mathit{coherent}\left(\varepsilon_1, \varepsilon_2\right) \mid \varepsilon_1, \varepsilon_2 \in T_E^A, \varepsilon_1 \neq \varepsilon_2 \right\} \\
&\cup \left\{ \mathit{supported}\left(\kappa\right) \mid \kappa \in T_R, sv(x_\kappa) \in \mathcal{SV} \setminus \mathcal{SV}_\mathcal{N} \right\} \\
&\cup \left\{ \mathit{num\text{-}supported}\left(\kappa\right) \mid \kappa \in T_R, sv(x_\kappa) \in \mathcal{SV}_\mathcal{N} \right\} \\
&\cup \left\{ \mathit{consistent}\left(\beta\right) \mid \beta \in T_C \right\} \\
&\cup \left\{ \mathit{structure\text{-}action}\left(a\right) \mid a \in \mathbb{A} \right\} \\
&\cup \left\{ \mathit{structure\text{-}assign}\left(\varepsilon\right) \mid \varepsilon \in T_E^A \right\} \\
&\cup \left\{ \mathit{structure\text{-}increase}\left(\varphi\right) \mid \varphi \in T_E^I \right\}
\end{aligned}$$

Associated Plan Definition

Let's consider a plan Π of the planning problem built from the solution to the CSP encoding as described in the Appendix A.

Let's prove that the plan Π is a solution to the original planning problem, *i.e.*, the following holds:

1. Π is coherent, *i.e.*, for any $t \in \mathcal{T}$ and $sv(x) \in \mathcal{SV}$, $E_\Pi^{[t]sv(x)}$ is either:
   (a) $E_\Pi^{[t]sv(x)} = \emptyset$, empty
   (b) $E_\Pi^{[t]sv(x)} = \left\{ := v \right\}$, with a single assign
   (c) $E_\Pi^{[t]sv(x)} = \left\{ += N_i \mid 1 \leq i \leq k \right\}$, with k increases
2. Numerical state variables are always within their bounds: $\forall sv(x) \in \mathcal{SV}_\mathcal{N}, \forall t \in \mathcal{T}, \mathit{lb}(sv) \leq [t]sv(x) \leq \mathit{ub}(sv)$.
3. All actions $a \in \Pi$ are applicable, meaning all constraints and conditions in C_a are entailed by the trace of Π.
4. All goals $g \in G$ are satisfied at the associated time-point.
5. All effects in the plan are executed before the horizon.

Coherence of the Plan

For all pair of assign effect tokens $\varepsilon_1, \varepsilon_2 \in T_E^A$, the constraint *coherent* $(\varepsilon_1, \varepsilon_2)$ is satisfied:

$$p_1 \wedge p_2 \implies \left(m_1 \leq t_2 \vee m_2 \leq t_1 \vee x_1 \neq x_2\right)$$

If at least one of the assign effects is not present in the plan, the constraint is trivially satisfied. Otherwise, let's consider two assign effects $\varepsilon_1, \varepsilon_2 \in T_E^A$ present in the plan. If $m_1 \leq t_2 \vee m_2 \leq t_1$ holds, then one of the assign effects is executed before the other. If $x_1 \neq x_2$ holds, then the assign effects are on different state variables. Therefore, two assign effects on the same state variable cannot be executed at the same time.

Moreover, for all read token $\kappa \in T_R$ related to a numerical state variable, *i.e.*, $sv(x_\kappa) \in \mathcal{SV}_\mathcal{N}$, the constraint *num-supported* (κ) is satisfied:

$$p_\kappa \implies \bigvee_{\varepsilon \in T_E^A} \begin{pmatrix} \chi_\varepsilon\left(\kappa, \varepsilon\right) \wedge \mathit{lb}(sv) \leq v_\kappa \wedge v_\kappa \leq \mathit{ub}(sv) \\ \wedge \left(v_\kappa = v_\varepsilon + \sum_{i=1}^n \left[\chi_\varphi\left(\kappa, \varepsilon, \varphi_i\right)\right] \cdot v_\varphi^i\right) \end{pmatrix}$$

This implies that for each increase effect token $\varphi_i \in T_E^I$ present in the plan, there is a read token $\kappa \in T_R$ and an assign effect token $\varepsilon \in T_E^A$ such that $\chi_\varphi\left(\kappa, \varepsilon, \varphi_i\right)$ holds:

$$p_\varphi^i \wedge t_\varepsilon < t_\varphi^i \wedge t_\varphi^i \leq t_\kappa \wedge x_\kappa = x_\varphi^i$$

- p_φ^i: The increase effect token is present in the plan.
- $t_\varepsilon < t_\varphi^i$: The assign effect token is executed strictly before the increase effect token.
- $t_\varphi^i \leq t_\kappa$: The read token is executed at the same time or after the increase effect token.
- $x_\kappa = x_\varphi^i$: The increase effect token is related to the same state variable as the assign effect token because $\chi_\varepsilon\left(\kappa, \varepsilon\right)$ holds so $x_\varphi^i = x_\kappa = x_\varepsilon$.

Finally, for all assign and increase effects related to the same state variable, it is impossible to have two assign effects or an increase effect with an assign effect at the same time. Therefore, the plan Π is coherent and **the first point of the definition is satisfied**.

Numerical State Variable Bounds

For all read token $\kappa \in T_R$ related to a numerical state variable, *i.e.*, $sv(x_\kappa) \in \mathcal{SV}_\mathcal{N}$, the constraint *num-supported* (κ) is satisfied:

$$p_\kappa \implies \bigvee_{\varepsilon \in T_E^A} \begin{pmatrix} \chi_\varepsilon\left(\kappa, \varepsilon\right) \wedge \mathit{lb}(sv) \leq v_\kappa \wedge v_\kappa \leq \mathit{ub}(sv) \\ \wedge \left(v_\kappa = v_\varepsilon + \sum_{i=1}^n \left[\chi_\varphi\left(\kappa, \varepsilon, \varphi_i\right)\right] \cdot v_\varphi^i\right) \end{pmatrix}$$

In particular, we have $\mathit{lb}(sv) \leq v_\kappa \wedge v_\kappa \leq \mathit{ub}(sv)$ iff the reading is present in the plan. Therefore, it is impossible for a numerical state variable to be out of bounds and **the second point of the definition is satisfied**.

Applicability of the Actions

For all condition token $\beta \in T_C$, the constraint *consistent* (β) is satisfied:

$$p_\beta \implies B_\beta$$

Therefore, the condition or the constraint is satisfied or absent in the plan. Moreover, for all state variable initially present in the boolean expression B_β, the constraints related *supported* $(.)$ and *num-supported* $(.)$ are satisfied.

Remark. For simplicity, only the *supported* $(.)$ constraint will be considered for the rest of this subsection, but the reasoning is the same for the *num-supported* $(.)$ constraint.

Therefore, for all read token $\kappa \in T_R$ related to a state variable present in the boolean expression B_β, the constraint *supported* (κ) is satisfied:

$$p_\kappa \implies \bigvee_{\varepsilon \in T_E^A} \begin{pmatrix} p_\varepsilon \wedge t_\varepsilon < t_\kappa \wedge t_\kappa \leq m_\varepsilon \\ \wedge x_\kappa = x_\varepsilon \wedge v_\kappa = v_\varepsilon \end{pmatrix}$$

This means that there is an assign effect token $\varepsilon \in T_E^A$ such that:

- p_ε: The assign effect token is present in the plan.
- $t_\varepsilon < t_\kappa \wedge t_\kappa \leq m_\varepsilon$: The read token is contained in the interval of the assign effect token.
- $x_\kappa = x_\varepsilon$: The read token is related to the same state variable as the assign effect token.
- $v_\kappa = v_\varepsilon$: The value read is the value assigned by the assign effect token.

Therefore, there exist assign effects for all read tokens in the boolean expression B_β giving the expected value to the boolean expression. Finally, all actions in the plan are applicable and **the third point of the definition is satisfied**.

Goal Satisfaction

Because all goals are encoded as conditions, the previous reasoning applies to the goals as well. Therefore, all goals are satisfied at the associated time-point and **the fourth point of the definition is satisfied**.

Effects Before Horizon

For every assign effect token $\varepsilon \in T_E^A$, the constraint *structure-assign* (ε) is satisfied:

$$p_\varepsilon \implies t_\varepsilon < H \wedge t_\varepsilon < m_\varepsilon$$

Therefore, all present assign effects are executed before the horizon. Moreover, for all increase effect token $\varphi \in T_E^I$, the constraint *structure-increase* (φ) is satisfied:

$$p_\varphi \implies t_\varphi < H$$

Therefore, all present assign and increase effects are executed before the horizon and **the fifth point of the definition is satisfied**.

Plan Structure Check

For every assign effect token $\varepsilon \in T_E^A$, the constraint *structure-assign* (ε) is satisfied:

$$p_\varepsilon \implies t_\varepsilon < H \wedge t_\varepsilon < m_\varepsilon$$

Therefore, all present assign effects have a mutex period $m_\varepsilon - t_\varepsilon$ which is positive. Moreover, for all action $a \in \mathbb{A}$, the constraint *structure-action* (a) is satisfied:

$$pres(a) \implies s_a \leq e_a \wedge e_a \leq H$$

Therefore, all present actions have a positive duration $e_a - s_a$ and are executed before the horizon. Finally, the actions and the assign effects are valid based on their definition.

Conclusion

We have shown that the structure of the plan Π build from the solution of the CSP is valid and satisfies all the constraints to be a solution to the original planning problem. Therefore, **the proposed encoding is sound**, *i.e.*, if the generated CSP has a solution, then the original planning problem has a solution as well.

$\square$

D Size of the Encoding

In this appendix, we analyze the size of the proposed CSP encoding of the planning problem. The generated CSP is built from the set of variables $\mathcal{V}$ and constraints $\mathcal{X}$:

$$\begin{aligned}
\mathcal{V} = & \left\{ p_a^i \mid a \in \mathbb{A}, p_a^i \in P_a \right\} \\
\cup & \left\{ s_a \mid a \in \mathbb{A} \right\} \cup \left\{ e_a \mid a \in \mathbb{A} \right\} \\
\cup & \left\{ pres(a) \mid a \in \mathbb{A} \right\} \\
\cup & \left\{ v_\kappa \mid \kappa \in T_R \right\} \\
\cup & \left\{ m_\varepsilon \mid \varepsilon \in T_E^A \right\} \\
\mathcal{X} = & \left\{ \textit{coherent}\,(\varepsilon_1, \varepsilon_2) \mid \varepsilon_1, \varepsilon_2 \in T_E^A, \varepsilon_1 \neq \varepsilon_2 \right\} \\
\cup & \left\{ \textit{supported}\,(\kappa) \mid \kappa \in T_R, sv(x_\kappa) \in \mathcal{SV} \setminus \mathcal{SV}_\mathcal{N} \right\} \\
\cup & \left\{ \textit{num-supported}\,(\kappa) \mid \kappa \in T_R, sv(x_\kappa) \in \mathcal{SV}_\mathcal{N} \right\} \\
\cup & \left\{ \textit{consistent}\,(\beta) \mid \beta \in T_C \right\} \\
\cup & \left\{ \textit{structure-action}\,(a) \mid a \in \mathbb{A} \right\} \\
\cup & \left\{ \textit{structure-assign}\,(\varepsilon) \mid \varepsilon \in T_E^A \right\} \\
\cup & \left\{ \textit{structure-increase}\,(\varphi) \mid \varphi \in T_E^I \right\}
\end{aligned}$$

Number of Variables

The number of parameters in $\left\{ p_a^i \mid a \in \mathbb{A}, p_a^i \in P_a \right\}$ is:

$$\sum_{a \in \mathbb{A}} \left| P_a \right| = \mathcal{O}\left(\left| \mathbb{A} \right| \cdot \max_{a \in \mathbb{A}} \left| P_a \right| \right)$$

The other variables related to actions are s_a, e_a, and $pres(a)$ for each action instance $a \in \mathbb{A}$, *i.e.*, $3 \cdot \left| \mathbb{A} \right|$ in total. Moreover, the number of read token values is $\left| T_R \right|$, and the number of assign effect token mutexes is $\left| T_E^A \right|$.

Therefore, the total number of variables is:

$$\begin{aligned}
\left| \mathcal{V} \right| &= \mathcal{O}\left((3 + \max_{a \in \mathbb{A}} \left| P_a \right|) \cdot \left| \mathbb{A} \right| + \left| T_R \right| + \left| T_E^A \right| \right) \\
&= \mathcal{O}\left(\left| \mathbb{A} \right| + \left| T_R \right| + \left| T_E^A \right| \right)
\end{aligned}$$

Number of Constraints

The size of each group of constraints is as follows:

- Coherence: $\mathcal{O}\left(\left| T_E^A \right|^2 \right)$
- Support: $\left| T_R \right| \cdot \left| \left\{ T_E^A \mid sv \notin \mathcal{SV}_\mathcal{N} \right\} \right|$
- Num Support: $\left| T_R \right| \cdot \left| \left\{ T_E^A \mid sv \in \mathcal{SV}_\mathcal{N} \right\} \right| \cdot \left| T_E^I \right|$
- Consistency: $\left| T_C \right|$
- Structure: $\left| \mathbb{A} \right| + \left| T_E^A \right| + \left| T_E^I \right|$

Therefore, the total number of support constraints (including numerical support) is:

$$\begin{aligned}
& \left| T_R \right| \cdot \left| \left\{ T_E^A \mid sv \notin \mathcal{SV}_\mathcal{N} \right\} \right| \\
+ \quad & \left| T_R \right| \cdot \left| \left\{ T_E^A \mid sv \in \mathcal{SV}_\mathcal{N} \right\} \right| \cdot \left| T_E^I \right| \\
= \quad & \mathcal{O}\left(\left| T_R \right| \cdot \left| T_E^A \right| \cdot \max\left(1, \left| T_E^I \right| \right) \right)
\end{aligned}$$

Finally, the total number of constraints is:

$$\left| \mathcal{X} \right| = \mathcal{O}\left(\begin{array}{cl} & \left| T_E^A \right|^2 + \left| T_C \right| \\ + & \left| \mathbb{A} \right| + \left| T_E^A \right| + \left| T_E^I \right| \\ + & \left| T_R \right| \cdot \left| T_E^A \right| \cdot \max\left(1, \left| T_E^I \right| \right) \end{array} \right)$$