# Chapter 3

## Logic Machines

### Introduction

The popular conception of the computer is one of a giant calculator, a machine that can carry out millions of arithmetic operations at lightning-fast speeds. But if this were all that computers are, they would be unable to do most of the tasks they are commonly assigned. They could not sort or organize data, as they do each time we do word processing or use a database; they could not even carry out complex computations, because these involve making nonarithmetic decisions, e.g., deciding when to stop one arithmetic process and begin another. Computers are powerful because they are able to carry out long and complex sequences of logical as well as arithmetical operations and modify these sequences according to information presented to them, without any direct human intervention. Without the ability to make logical decisions, computers would have nothing more than an uncontrolled, raw arithmetic power, which would make them only slightly more useful than simple adding machines.

The computer was not the first calculating technology able to make logical decisions. Many punched-card systems, relay calculators, and electronic calculators of the 1930s and early 1940s (all of which are described in later chapters) had rudimentary logical capabilities. But there is an even earlier stream of development,

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The author greatly appreciates the suggestions of Michael S. Mahoney and Linda M. Strauss in the preparation of this essay.

beginning around 1800, having as its central purpose the construction of machines capable of making logical decisions. This chapter traces the history of these machines built to solve problems of Aristotelian and symbolic logic, and shows how their development fits into a much older tradition of automata–devices and machines built to mimic mental and physical aspects of human behavior. This chapter also traces the growing understanding prior to the Second World War of the relationship between logic and the theory of computing, which is the foundation for computer science today.

## The Automata Tradition

The automata tradition extends back into antiquity. In the Hellenistic period complex mechanisms were constructed to give the appearance of human animation. For example, around 200 B.C., Heron of Alexandria constructed a theater in which the god Dionysius would emerge and spray wine from his staff while the Bacchants danced in his honor. These Hellenistic mechanisms were powered in many different ways: by falling water, sand, or mustard seeds; heat; atmospheric pressure; and in one case by a primitive steam engine. The great civic clocks constructed in major European cities, beginning in the thirteenth century, also are part of this tradition. Human and other figures ornamenting the clocks became animated at the tolling of certain hours. For example, from the clock at Strasbourg the three Magi emerged and a cock crowed each day at dawn. Over time, in the late Middle Ages and the Renaissance, these clockwork automata became more elaborate and were built separately from the civic clocks.

Following the rediscovery and translation of Heron's writings, the great formal gardens of sixteenth- and seventeenth-century Europe were adorned with hydraulic automata. Elaborate nymphs, shepherds, and musicians were empowered by falling water. In eighteenth- and nineteenth-century France, miniaturized automata powered by spring mechanisms were produced in quantity and sold to the upper classes. Some of these works involved great craftsmanship: a girl able to sign her name, a flying bird with three hundred moving parts in its wing, a figure able to play the dulcimer.

Most of these automata modeled physical rather than mental processes. Of the latter variety were several attempts to construct

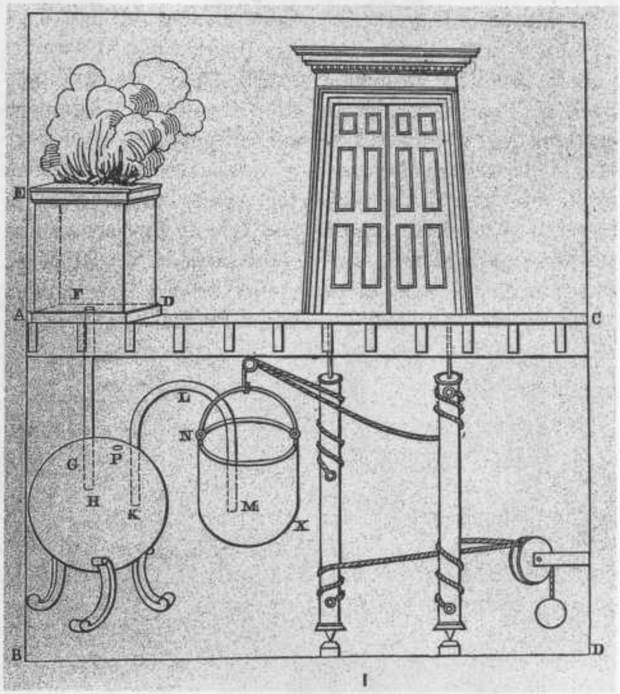
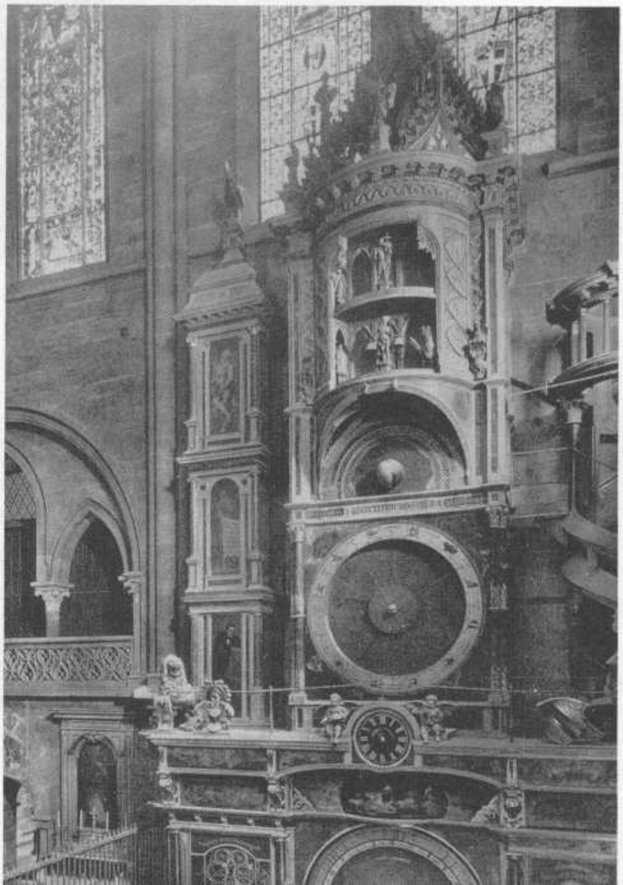



Figure 3.1. A pneumatic mechanism to open and close a door, designed by Heron of Alexandria.

Figure 3.2. The astronomical clock of Strasbourg, with its mechanical cock.



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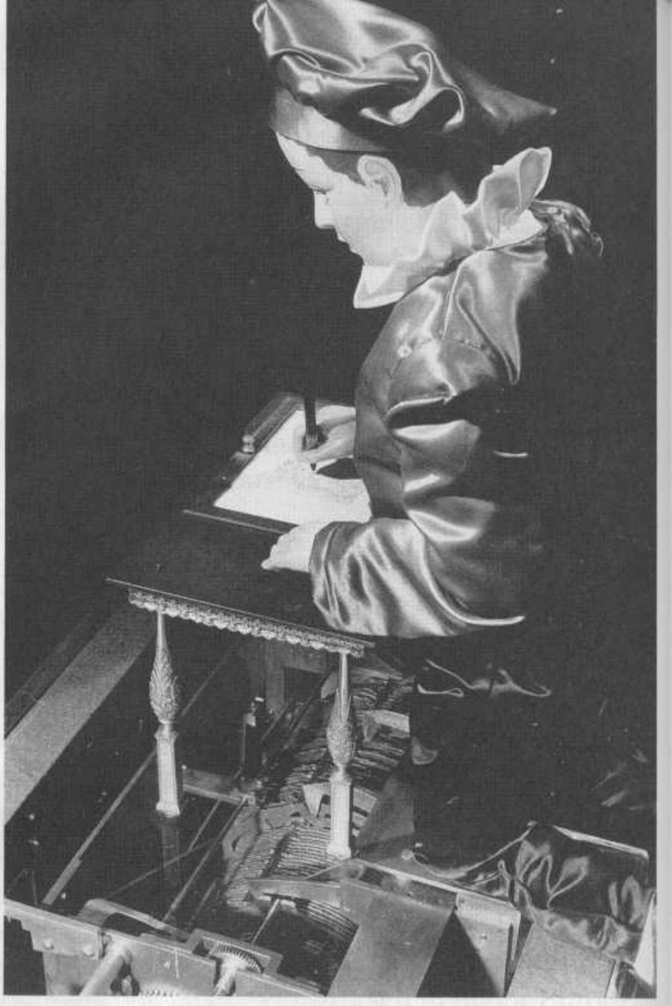

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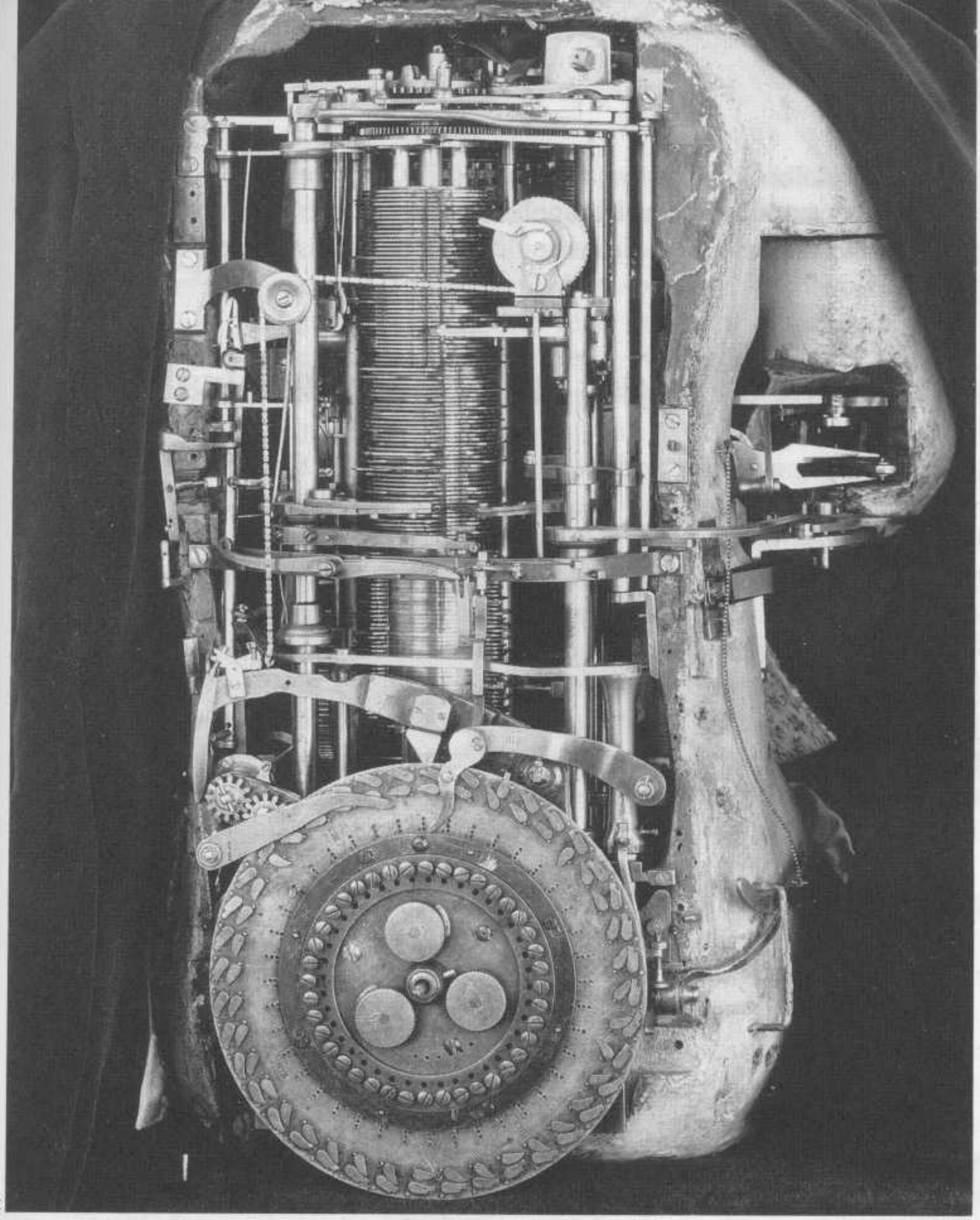
Figure 3.3. (a) Henry Maillardet's eighteenth-century autamaton that draws and writes in French and English. Courtesy Franklin Institute. (b) The Jaquet-Droz Writer of 1774. Courtesy Neuchatel Museum of Art and History. (c) The mechanism of the Jaquet-Droz Writer. Courtesy Neuchatel Museum of Art and History.

talking automata and perhaps more importantly van Kempelen's 1769 chess player, which though fraudulent (hiding a man inside the player) engendered a seventy-year debate over the possibility of mechanizing human thought processes. But the number of automata of this type on the Continent were few, especially in comparison to the number developed in England, where craftsmanship was not nearly so advanced. There are probably many reasons to explain why this is so, but one may have been philosophical rather than

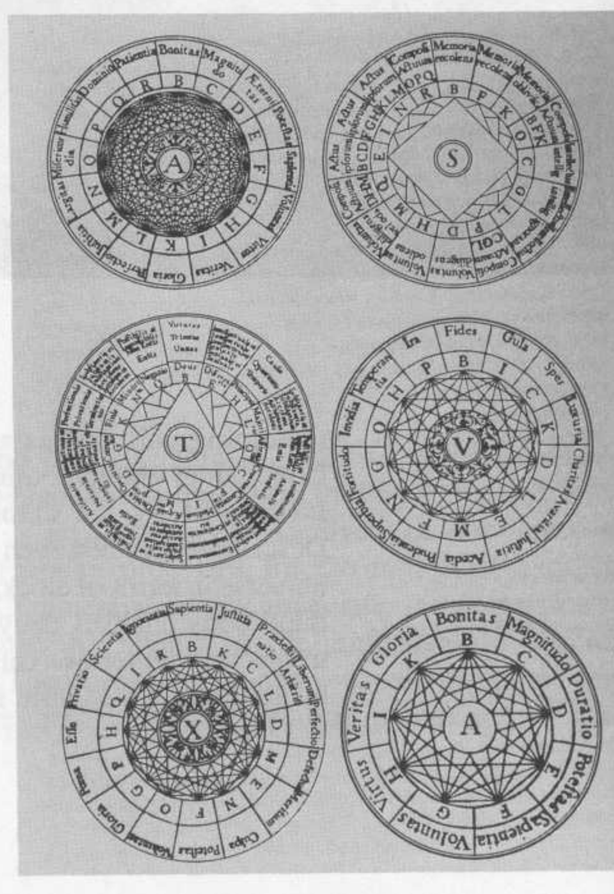


technological. Cartesian philosophy colored every aspect of Continental thought throughout the eighteenth century. Perhaps influenced by the elaborate clockwork automata of his time, Descartes explained even the most complex physical processes of the universe in terms of clocklike mechanisms. But he maintained a strict mind-body dualism, denying that mental processes can be explained in mechanical terms. This rationalist dualism was questioned, e.g., by Julien de La Mettrie in *Man the Machine* (1748) and by Baron d'Holbach in *System of Nature* (1770) as well as by the discussions surrounding van Kempelen's automaton, but the influence of Descartes' world view should not be underestimated.

## The Development of Logic and Its Mechanization

Another line of development, sometimes closely intertwined with the automata tradition, was the effort to mechanize logic, historically regarded as the most central of the rational processes. In his *Ars Magna* the Spanish theologian Raymond Lull (1235–1315) used geometrical diagrams and primitive logical devices to try to demonstrate the truths of Christianity (Figure 3.4). He believed that each domain of knowledge involves a finite number of basic principles, so that by enumerating the permutations of these basic principles in pairs, triples, and larger combinations a list of the basic building blocks for theological discourse could be assembled.

Figure 3.4. The logical diagrams of Ramon Lull. Courtesy Martin Gardner.



Lull mechanized the process of forming these permutations by constructing devices with two or more concentric circles, each listing the basic principles around the circumference. The permutations could then be formed by spinning the dials so as to line up different permutations. One such device was used for studying the divine attributes. Each of two circles contained the fourteen accepted attributes (goodness, greatness, eternity. . .), and the device would give you the 196 (i.e., 14 x 14 = 196) permutations, e.g., "God is good and God is eternal," "God is eternal and God is great," etc. Similar devices were constructed for study of the soul and the seven deadly sins. Although these devices did not really offer labor savings or additional logical power, Lull's "great art" was admired by many Renaissance clerics and commented on by such noted scholars as Nicholas of Cusa, Athanasius Kircher (who is notable for his interest in automata, e.g., his plans for building a talking head), and Wilhelm Gottfried Leibniz.

Leibniz (1646-1716) was enamored with the power that algebraic symbolism and method had added to geometry during the previous century. In his *De Arte Combinatoria* (1666) and in later fragmentary works he described an "algebraico-logical synthesis" by which one could reason mechanically in all fields as one could reason in algebra. The first step was to devise a universal language, his "universal characteristic," for expressing thoughts in an unambiguous, symbolic way. Leibniz experimented with various linguistic schemes, e.g., representing primitive ideas by prime numbers and complex ideas by the product of these numbers. He also moved towards an algebra of logic by implicitly giving logical interpretations to the algebraic operators and relations +, x, –, =. But he never achieved substantial results, and this work became widely known only in the twentieth century when his fragmentary writings were first published.

The algebrization of logic, primarily the work of Augustus de Morgan (1806-1871) and George Boole (1815-1864), was important to the transformation of Aristotelian logic into modern logic and to the introduction of logic machines in the automation of logical reasoning. In his *Formal Logic* (1847) the British mathematician de Morgan began the algebrization process. He introduced quantification into logic. By using algebraic variables to represent the numbers of members of classes mentioned in a syllogism, e.g., there are *a* A's and *b* B's, he could strengthen a conclusion like "Some A's are B's" to "At least *k* A's are B's," where *k* is an algebraic expression involving *a*, *b*, and other variables that appeared in the premises.

In his *Mathematical Analysis of Logic* (1847) and *An Investigation of the Laws of Thought* (1854) the Irish professor of mathematics Boole rigorized logic by introducing algebraic symbolism and method. He let $x$, $y$, $z$ represent classes, $X$, $Y$, $Z$ individual members, 1 the universal class, 0 the null (empty) class, $xy$ the intersections of classes $x$ and $y$, x + y the union of (disjoint) classes $x$ and $y$, and $1 - x$ the complement of class $x$. He then presented in symbolic form, as the axioms of his logic, what he considered to be the basic "laws of thought." His axioms include, for example:

$$x(1 - x) = 0$$

(The intersection of a set and its complement is null.)

$$x(y + z) = xy + xz$$

(De Morgan's law on the distribution of intersection over union).

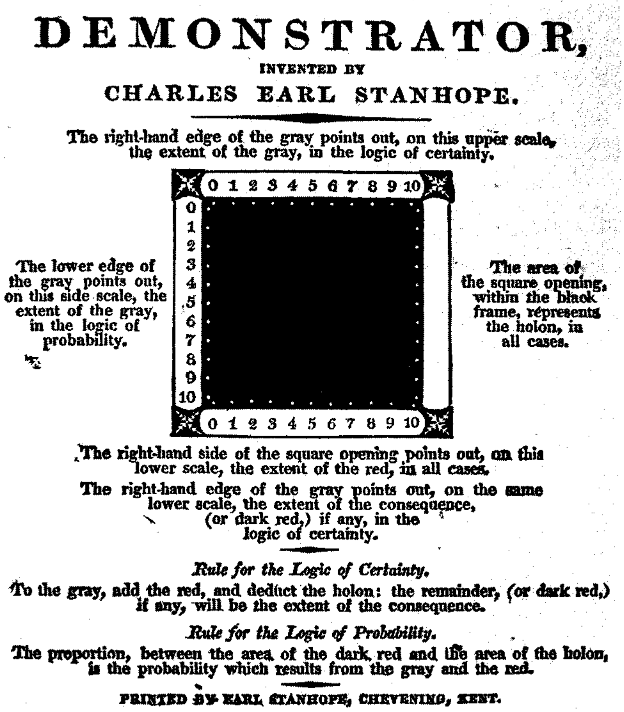
Boole could then formally deduce more complex "laws of thought" through algebraic manipulation.

These first efforts to reform Aristotelian logic were continued in the late nineteenth and early twentieth centuries by Charles Saunders Peirce, Gottlob Frege, Guiseppe Peano, Bertrand Russell, Alfred North Whitehead, and others. Their efforts further stimulated the mechanization of logic because machines could conduct or abet the algebraic manipulation that now represented logical reasoning.

## Logic Machines

The first logic machine, the Stanhope Demonstrator, appeared prior to the algebrization of logic. Charles, third Earl of Stanhope, (1753-1816) was a politician and inventor of independent means. His scientific abilities were recognized early, leading to his induction into the Royal Society of London at the age of nineteen. Stanhope invented a microscopic lens, a hand printing press, a tuner for musical instruments, an improved system of canal locks, and an arithmetical calculating machine, as well as a theory of electricity. Stanhope's Demonstrator (Figure 3.5), refined over a thirty-year span, is a device able to solve mechanically traditional syllogisms, numerical syllogisms, and elementary probability problems. It consists of a 4" × 4.5" × 0.75" mahogany block with a brass top, having carved out of it a window 1" × 1" × 0.5". Slots were grooved in three sides of the block to allow transparent red and gray slides to enter and cover a portion of the window. On the brass face, along three sides of the window, integer calibrations from zero to ten were marked.

Figure 3.5. The face of Lord Stanhope's Logical Demonstrator.




To solve a numerical syllogism, for example:

Eight of ten $A$'s are $B$'s;
Four of ten $A$'s are $C$'s;
Therefore, at least two $B$'s are $C$'s.

Stanhope would push the red slide (representing $B$) eight units across the window (representing $A$) and the gray slide (representing $C$) four units from the opposite direction. The two units that the slides overlapped represented the minimum number of *B*'s that were also *C*'s. To solve a probability problem like:

Prob ($A$) = 1/2;
Prob ($B$) = 1/5;
Therefore, Prob ($A$ and $B$ ) = 1/10.

Stanhope would push the red slide (representing $A$) from the north side five units (representing five tenths) and the gray slide from the east two units (representing two tenths). The portion of the window ($5/10 \times 2/10 = 1/10$) over which the two slides overlapped represents the probability of $A$ and $B$.

In a similar way the Demonstrator could be used to solve a traditional syllogism like:

No $M$ is $A$.
All $B$ is $M$.
Therefore, No $B$ is $A$.

The Demonstrator had obvious limitations. It could not be extended to syllogisms involving more than two premises or to probability problems with more than two events (always assumed to be independent of one another). Any of the problems it could handle were solved easily and quickly without the aid of the machine. Nonetheless, Stanhope believed he had made a fundamental invention. The few friends and relatives who received his privately distributed account of the Demonstrator, *The Science of Reasoning Clearly Explained Upon New Principles* (1800), were advised to remain silent lest "some bastard imitation" precede his intended publication on the subject. This publication never appeared and the Demonstrator remained unknown until the Reverend Robert Harley described it in the *Philosophical Transactions* in 1879. The Demonstrator was important mainly because it demonstrated to others, most notably to William Stanley Jevons, that problems of logic could be solved by mechanical means.

The second major figure was Alfred Smee (1818-1877), senior surgeon to the Royal General Dispensary and to the Central London Opthalmic Hospital. Also a Fellow of the Royal Society, he published a series of books on a field he called "electro-biology," the relation of electricity to the vital functions of the human body. Stimulated by the lectures of Herbert Mayo on the physiology of the brain, his laboratory work under John Frederic Daniell (inventor of the Daniell battery), and the prevailing theory of Luigi Galvani on the effect of electrical stimulation on nerves and muscles, Smee determined to study how the functions of the brain are related to the electrical stimulation of the nervous system.

In 1851, Smee published his most important book, *Process of Thought Adapted to Words and Language*, which, he stated, "is a deduction from the general system of Electro-biology." He planned to produce an artificial system of reasoning based upon natural principles, one that processes ideas in the same way that the human nervous system processes them. Little was known about the brain in 1850, and there were no good tools for its study. Smee had to rely on speculation rather than experimentation to gain his understanding of human thinking. The outcome of these speculations was to be demonstrated in his electro-biological machine.

According to his theory, each idea is determined by the presence or absence of certain properties (redness, roundness, etc.), and each property is represented in the brain by the electrical stimulation of a nerve fiber. Thus, for Smee, an idea consists of a collection of electrically stimulated nerve fibers. One might envision Smee building an elaborate electromechanical machine with artificial nerve fibers and cortex. But consistent with the technology of 1850, the machines Smee conceived were entirely mechanical. His Relational Machine, so called because it represented the relationship between the various properties that comprise an idea, was intended to represent one thought, idea, or mental image at a time. One version of it was constructed from a large piece of sheet metal, repeatedly divided into halves by metal hinges. Half of the metal would represent the presence, the other the absence, of a property. The metal flaps, representing absent properties, would be folded out of sight until all that remained was a piece of metal representing the collection of properties that formed the idea.

Smee designed a second machine to compare ideas. This Differential Machine consisted of two Relational Machines linked together by an interface able to compare the properties represented by each Relational Machine and then to judge whether the ideas agree, probably agree, possibly agree, or disagree. Representation of ideas and judgments about them, the tasks his machines were designed to do, comprised the entire rational thinking faculty for Smee.

Smee was confident his machines could model human thought. He was concerned, however, about the feasibility of constructing his machines because of the elaborate mechanical engineering involved and the problem of scale. He wrote in *Process of Thought* that

> when the vast extent of a machine sufficiently large to include all words and sequences is considered, we at once observe the absolute impossibility of forming one for practical purposes, inasmuch as it would cover an area exceeding probably all London, and the very attempt to move its respective parts upon each other, would inevitably cause its own destruction.

Although Smee may have built small scale models of his machine (even this is doubtful), he realized that his hope for a machine that could represent the natural processes of thought and judgment was beyond his reach. Nevertheless, his books were popular in mid-nineteenth-century Britain and spread his conviction of the possibility of mechanized thought.

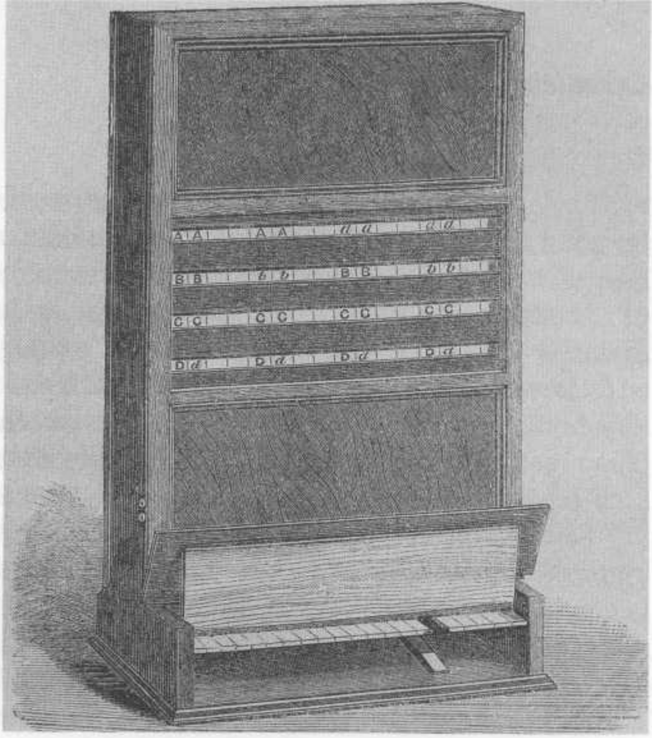
Stanhope's work inspired William Stanley Jevons to construct his "logic piano," the best known logic machine of the nineteenth century. Jevons (1835-1882) was professor of logic and political economy at Owens College, Manchester, and later at University College, London. His scientific interests were broad, and while working as an assayer in Australia early in his career, he made important contributions to anthropology, natural history, meteorology, and chemistry. His research in logic was encouraged by his teacher, Augustus de Morgan. Today, Jevons is perhaps best known for his unfortunate theory of the correlation between sunspots and economic cycles.

In his 1869 logic textbook, *Substitution of Similars*, Jevons announced the construction of the logic piano (Figure 3.6). It was the culmination of a long series of inventions and aids to the calculation of syllogisms: logical alphabet, logical slate, logical stamp, and logical abacus–all tools to write quickly the lines of a truth table in a logical argument.

The logic piano was a box approximately three feet high. A faceplate above the keyboard displayed the entries of the truth table. Like a piano, the keyboard had black-and-white keys, but here they were used for entering premises. As the keys were struck, rods would mechanically remove from the face of the piano the truth-table entries inconsistent with the premises entered on the keys.

A truth-table for $n$ proposition requires $2^n$ entries. The table for

Figure 3.6. The logic piano designed by William Stanley Jevons.



$n = 4$ is as follows, if we represent the truth of a proposition by an upper case letter, and its falsity by the same letter in lower case:

Table 3.1. Truth-table for n = 4

| | | | |
|---|---|---|---|
| *PQRS* | *PQRs* | *PQrs* | *PQrs* |
| *PqRS* | *PqRs* | *Pqrs* | *Pqrs* |
| *pQRS* | *pQRs* | *pQrs* | *pQrs* |
| *pqRS* | *pqRs* | *pqrS* | *pqrs* |

The proposition "if *P*, then *Q*," is true just in case *P* is false or *Q* is true. If this proposition were entered on the keyboard of the logic piano, the face would show:

Table 3.2. Truth-table for the proposition "if P, then Q" is true in case P is false or Q is true, when n = 4

| | | | |
|---|---|---|---|
| *PQRs* | *PQRs* | *PQrs* | *PQrs* |
| | | (second line removed) | |
| *pQRS* | *pQRs* | *pQrs* | *pQrs* |
| *pqRs* | *pqRs* | *pqrs* | *pqrs* |

As propositions were entered on the keyboard, representing additional premises that must be satisfied simultaneously, other inconsistent entries would disappear from the face.

The machine was limited to solving problems involving four or fewer propositions, although these could easily be handled manually. Jevons once planned a ten-term machine, but abandoned the project because the proposed machine would have occupied an entire side of his study. As the philosopher Francis Bradley pointed out, the action of the logic piano did not result in a conclusion stated in the form of a proposition, but only in the truth table entries consistent with the conclusion. Jevons worked unsuccessfully to resolve this problem, which he termed the "inverse problem" and which he somewhat misleadingly associated with the process of mathematical induction. And, as his adversary John Venn noted, the logic piano has no practical purpose, for there are no circumstances in which difficult syllogisms arise or in which syllogisms must be resolved repeatedly enough to justify mechanization of the process. Jevons countered that it was a convenience to his personal work and useful in his logic classes.

The Reverend John Venn (1834-1923) was lecturer in moral science and fellow of Gonville and Caius College, Cambridge. He published on moral science, history, probability, and logic. His *Symbolic Logic* (1894) was the most widely used logic textbook of its day. In it he presented his famous technique for diagramming logical arguments, described a logical diagramming machine, and discussed the general purposes and possibilities of logic machines.

Diagramming of logical arguments has a long history. In the Middle Ages diagrams were devised for remembering various forms of the Aristotelian syllogism. In the seventeenth and eighteenth centuries, the mathematicians Gottfried W. Leibniz, Leonhard Euler, and J. H. Lambert all had developed systems for diagramming logic. The first practical system of diagramming was announced by Venn in an 1880 article in *Philosophical Magazine*. It described his method of Venn diagrams, which is only a slight variation on the method of intersecting circles still taught in schools today.

Venn also designed a diagramming machine for logical arguments involving four propositions. (Venn diagrams treat at most three.) This is somewhat surprising because of Venn's belief that logic machines are both useless and unworthy of the name "logical." Like Jevons, Venn first developed other laborsaving devices: a rubber stamp of his intersecting circles and a puzzle board in which each piece of the intersecting circles could be removed separately. Then he developed the machine, with four intersecting ellipses hung on pegs by strings such that each section, attached by a separate peg, represented one of the sixteen possible logical combinations. To exclude a combination, the appropriate peg would be released, allowing the section it held to fall below its normal level. The keyboard consisted simply of the sixteen pegs to be individually manipulated. No device was added by which a number of pegs could be removed at once. Thus, it is more properly categorized as a diagram than a machine.

The last major figure in the development of nineteenth-century logic machines was Allen Marquand (1853-1924). After studying at Johns Hopkins University with C. S. Pierce, who probably taught him about logic machines, Marquand was appointed tutor of logic at the College of New Jersey, as Princeton University was then called. Marquand soon abandoned logic to become professor of art and archeology. Besides important work on classical Greek art and archeology, he contributed to the algebra of logic and built several logic machines.

Marquand improved upon Jevon's logic piano. He constructed a crude version in 1881, and a Princeton colleague, Charles Rockwood, followed the next year with a more elaborate version. It measured $12" \times 8" \times 6"$ and used a mechanical action, with rods and levers connected by pins and catgut strings (Figure 3.7). Marquand

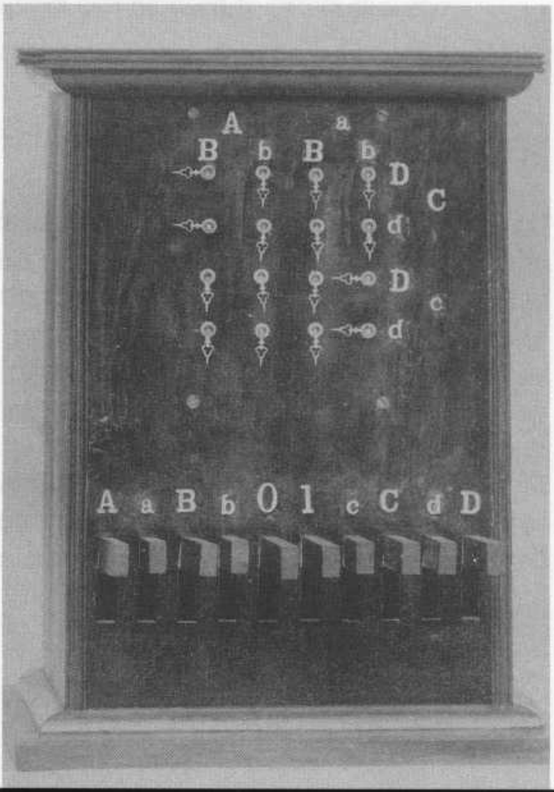



Figure 3.7. Allen Marquand's logic machine.

proposed a third version that would have changed the action of the machine from mechanical to electromechanical, but difficulties with the new electrical technology prevented him from advancing beyond building a prototype from a hotel annunciator.

Marquand's machine was designed for syllogisms involving four propositions. The front of the machine displayed pointers representing the sixteen possible logical combinations. The pointers would turn to indicate the consistency or inconsistency of the logical combinations with the premises. Marquand improved upon Jevons' keyboard for entering premises, opening the possibility of constructing a machine capable of handling many more propositions. However, both machines were limited in the complexity of argument they could handle, and both produced only logical combinations consistent with the concluding proposition rather than the proposition itself.

In 1936 Benjamin Burack, a psychologist at Roosevelt College in Chicago, constructed the first electrical logic machine (Figure 3.8). It was packaged in a small suitcase and powered by batteries. The bottom of the case contained wooden blocks representing propositions. These blocks held metal contacts, and when the blocks were moved to certain positions, circuits would be activated showing whether a syllogism was valid or which of seven categories of fallacies occurred. Burack's machine offered little advantage over manual checking and was generally unknown until it was described in the literature in 1947.

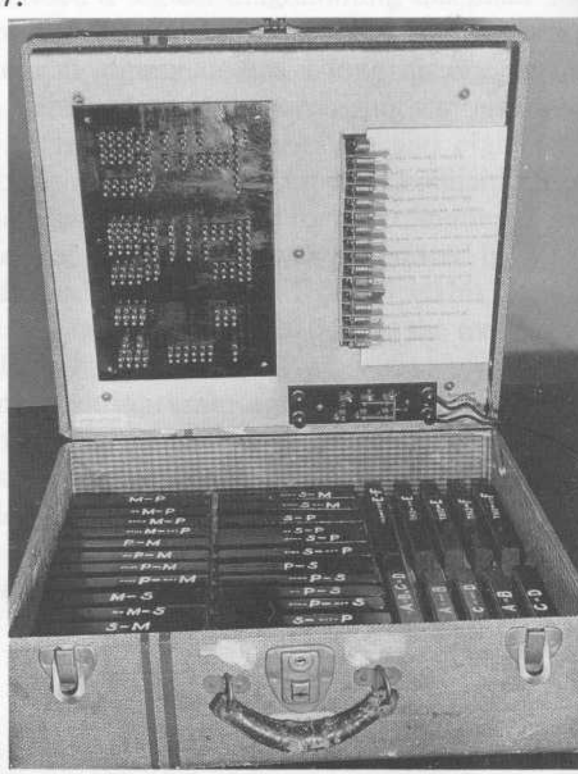


Figure 3.8. Benjamin Burack's portable electical logic machine. Courtesy Martin Gardner.

Better known was a logic machine built in 1947 by Harvard University undergraduates William Burkhardt and Theodore Kalin. Their machine was essentially an electrical version of Jevon's logic piano, capable of handling syllogisms with as many as twelve terms. Logical premises were entered by setting switches that established an electrical circuit logically isomorphic to the premises. Lights indicated the lines of the truth-table consistent with the premises. Use of the Burkhardt-Kalin machine was much faster than checking the syllogisms manually. And, unexpectedly, their machine could establish the well-known indeterminacy of truth value of the logical paradoxes by lights that alternated true and false. (An example of a logical paradox is "this statement is false." It is easy to establish that the statement in the quotation marks is true if and only if it is false.)

After the Second World War, it became apparent that general-purpose stored-program computers could achieve the same results as any of these special-purpose logic machines. Subsequently, all major logic machines have been programmed on computers. The first such effort was made by Hao Wang in 1960. He programmed an IBM 704 computer to test the first 220 theorems of the propositional calculus as presented in Bertrand Russell and Alfred North Whitehead's *Principia Mathematica*. The process was completed in less than three minutes, at least a thousand times faster than could be done manually. Since 1950 a number of computers have been programmed to act as logic machines. They have been used either to try to discover new logical results or to investigate the general principles by which computers can be used to prove theorems.

## Logic and Computing

The logic machines described here did not have any practical significance. They did not provide meaningful control of the daily information flow in the factory or business office, nor did they enable scientists to solve problems they could not otherwise easily solve by hand. Although logic machines were occasionally used as didactic aids, their chief importance was theoretical. They demonstrated that logical processes could be mechanized. Thus, it should come as no surprise that their principal role in modern computing is also theoretical. The existence of logic machines reinforced the relationship between logic and computing, and helped

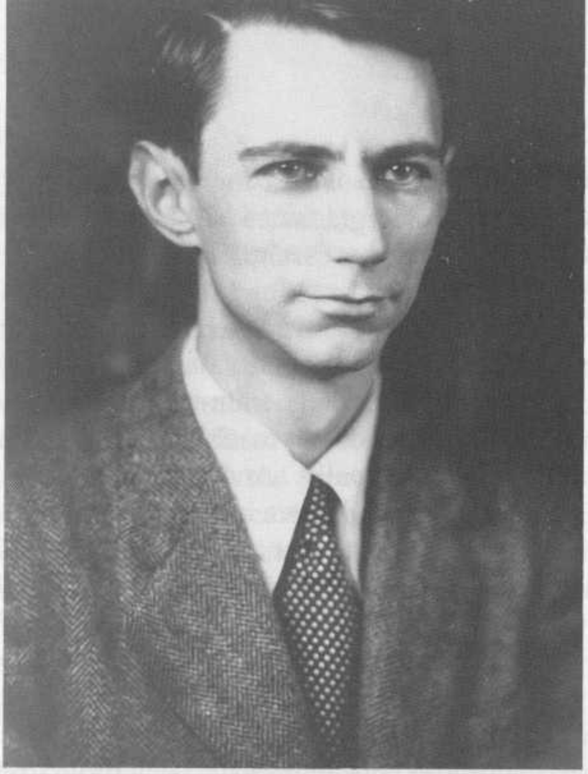


Figure 3.9. Claude Shannon, who discovered the isomorphism between switching circuits and the propositional calculus. Courtesy AT&T Archives.

to set the context in which two theoretical papers of the 1930s were written, papers that provided the underpinning for the modern theory of computing.

In a 1938 paper based upon his master's thesis at MIT, Claude Shannon demonstrated how relay and switching circuits could be expressed in the logical symbolism of the propositional calculus, and vice versa. Some examples of the correspondence he discovered are:

| logic | circuit |
|---|---|
| true | closed |
| false | open |
| and | serial |
| or (inclusive) | parallel |

Similar circuit interpretations can be given for the logical connectives not, nand (not both), exclusive or, and equivalence. This isomorphism between propositional calculus and relay and switching circuits became a powerful new design tool. Inspired by Shannon's paper, Burkhardt and Kalin employed it in the design of their special-purpose electrical logic machine. A more important application was to electrical circuit design for computers. Complex circuits could be more readily simplified by simplifying the corresponding Boolean expression; and in many cases it was easier for a circuit designer to express his design in a logical expression and only later translate that into a circuit design. Hundreds of papers followed Shannon's, building this fundamental isomorphism between logic and computing into a theory of switching circuits and a practical design methodology.

Shannon was not the first to suggest this isomorphism. The idea had been suggested in the Russian literature in 1910 by Paul Ehrenfest and followed up in 1934 by V. I. S. Sestakov. It also appeared in a 1936 Japanese publication by Akira Nakasima and Masao Hanzawa. However, none of these received the wide attention of Shannon's paper, mainly because his paper was published in English and presented a detailed account of the isomorphism in a way that highlighted its value to circuit design theory.

The other important theoretical paper of the 1930s was Alan Turing's "On Computable Numbers" (1937). Turing characterized which functions (or, as he equivalently considered, which numbers) in mathematics are effectively computable. By this he understood functions that can be computed in a mechanical fashion ~~by a~~ well-defined algorithm that requires no human intervention during the course of the computation. Turing's paper was one of the original contributions to the area known as recursive function theory, a subject in vogue then because of the interest in the methods used in Kurt Godel's famous incompleteness results, concerns about the constructivist foundations of mathematics, and other independent research in logic.

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Turing phrased his characterization in terms of theoretical machines, known today as Turing machines. He defined a mathematical function to be effectively computable just in case it could be calculated by one of his machines and demonstrated that one of his machines, the Universal Turing Machine, was able to simulate any of his other machines. Thus, by Turing's criteria, a mathematical function is effectively computable if and only if it can be computed by the Universal Turing Machine.

Figure 3.10. Alan Turing, whose characterization of effectively computable functions gave the first theoretical description of the stored-program computer.

A Turing machine consists of an infinite tape, broken into cells, and a mechanical device capable of scanning the tape and performing a few basic read and write operations. At any moment, depending on the internal state of the machine and the symbol in the cell being scanned, the machine may move the tape one square left or right, or print or erase a symbol in the scanned cell. Function arguments are entered as a coded sequence of 0s and 1s on consecutive cells. Function values are read off as another coded sequence of 0s and 1s when the machine completes its activity. If the activity never ceases, the function is not effectively computable for that argument. The universal machine represents essentially a function of two variables, one being the number of a particular Turing machine it is to simulate and the other being the function argument.

The importance of the Universal Turing Machine to computer science becomes clear once it is recognized that it is a theoretical model of a digital, stored-program computer. Instructions programming the operation of the machine, as well as data, are entered on the tape. The tape serves the dual function of input-output medium and memory—similar to magnetic tape in computers (which is used, however, only as a secondary storage medium). Information is stored, processed, and transferred digitally. Central processing takes place at the read-write mechanism, which is able to carry out logical and arithmetic operations on the scanned cell and those adjacent to it—whether they represent instructions, input data, or intermediate results. Many programming features, like conditional and unconditional branching and recursive loops, have their Turing machine equivalents.

Just as Shannon's paper served as the starting point for the theory of switching and relay circuits, Turing's paper opened the field of automata theory—the theoretical study of the computing capabilities of well-defined information processing automata—whether they be natural, physical artifact, or theoretical. This provided an abstract model and formal description for what was occurring in computer design.

Turing's methods, and the methods of recursive function theory more generally, were also employed in another area of theoretical computer science, the theory of complexity. This field considers the complexity of information-processing problems in terms of the amount of time, cost, storage space, or other computational resources that are required to compute a solution to the problem. Turing had demonstrated the existence of a class of problems too complex for solution by his machines. The most important of these was the halting problem: given the number describing to the universal machine a particular Turing machine and a given input, decide whether the machine will ever halt its computation. Turing demonstrated that the halting problem is computationally undecidable, that no Turing machine can make this decision. This placed a theoretical limit on what is mechanically computable and on our practical abilities to predict computation lengths and systematically diagnose programming errors. Working within the bounds set by Turing, many other researchers have developed finer meshes for ascertaining computational complexities of problems.

It has been a long and sometimes tenuous line of development from the logic machines of Stanhope and Jevons to modern computer science theory. But today logic is the foundation for automata theory, switching theory, and other theoretical areas of computer study; and the computer is a tool much more capable of logical processing than any of the special-purpose machines of the past.

# Further Reading

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