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A Binary Differential Evolutionary Algorithm for Airline Revenue Management: A Case Study

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Abstract

In the current highly competitive airline market, many companies have failed due to their low revenue rates. For this reason, many of them have to develop strategies to increase their revenue. In this study, we develop a revenue management (RM) strategy for the Iranian airline industry. More specifically, we present a mathematical model that considers some conditions not studied in previous researches. A binary differential evolutionary (BDE) algorithm is used to solve the model due to the stochastic data and the NP-hard nature of the model. In order to achieve maximum revenue among the six types of airplanes that fly among the four main cities, the studied airline is recommended to implement only 21 flights to these cities and cancel the rest of the flights.

Keywords: Revenue management; Airline industry; Optimization; Binary differential evolutionary algorithm; Booking; Overloading; Cancelation.

1. Introduction

The airline industry, as a part of the transportation industry, plays a fundamental role in the economy of every country. Due to the popularity of air transports, the number of users of this service is increasing dramatically. As a result, many flight agencies join the airline industry. Besides the increasing demand for air transportation, the economic performance of airlines has become one of the major concerns.

RM is a powerful tool for service industries, especially in the airline industry (Talluri & Van Ryzin, 2006). More particularly, RM facilitates effective demand management and pricing decisions without damaging the firms' revenues. Moreover, airline companies must attempt to attract passengers with low costs and superior services. Literature review shows that many researchers have devoted their attention to the RM applications in service industries (e.g. Wang & Brennan, 2014; Guadix et al., 2011; Denizci Guillet & Mohammed, 2015; Abrate & Viglia, 2016; Heo, 2017).

Many studies have considered various options to solve the mathematical models in order to optimize airline firms' revenues (e.g. Lin et al. 2017; Grauberger & Kimms, 2016a; Grauberger & Kimms, 2016b; Aslani et al. 2014; Aydın, Birbil, Frenk & Noyan, 2012; Zhang et al. 2010; Gosavi et al. 2007; Mukhopadhyay et al. 2007; Zhang & Cooper, 2005). Due to its geopolitical location, Iran is a highly competitive airline transportation hub in the Middle East. Fifteen airlines currently operate in Iran for the transportation of passengers both in Iran and abroad.

Nowadays, there are many airline companies in Iran, which compete with each other. Although Iran Air airline is the oldest and biggest one, today some new airlines are emerging. The planes of Iran Air are very old fashioned and their probability of crashing is very high. Therefore, people are afraid to travel with them. This causes that this company is on the edge of bankruptcy. Instead, new emerging companies employ new planes for traveling and therefore their market share increases dramatically. Hence, Iran Air Company requires planning and managing effectively its planes in order to optimize its revenue.

In this study, we address the problem by implementing the RM in the Iran airline market for a two-year period during 2015-16 years. We propose a mathematical model for revenue optimization in Iran Air Company using the BDE algorithm (Storn & Price, 1997). The primary contribution of this study is to provide different options and methods to solve the above problems. In previous studies, scholars have considered the availability of seats, overloading, price optimization, cancellation and booking using linear programming, (Gosavi et al., 2007; Zhang & Cooper, 2009; Oancea, 2016), system dynamics (Sierag et al., 2015), and network programming (Grauberger & Kimms, 2016a).

We consider three options for cancellation, booking, and overloading and their relevant penalties simultaneously, which have not been considered in previous researches. We use stochastic data for these options to make our presented model more realistic. The stochastic data are in the form of intervals containing lower and upper bounds. Then, we model the investigated revenue optimization problem by formulating it as an integer programming model.

Notably, in solving NP-hard problems, DE is proven as an efficient evolutionary algorithm with obtained acceptable results (Karaboga & Cetinkaya, 2004; Mayer et al. 2005). Therefore, we utilize the BDE algorithm for solving the integer programming model due to the existence of binary variables. In addition, the BDE algorithm is an efficient stochastic optimization algorithm,

whose usage for solving an NP-hard model with stochastic data is reasonable (see Wang et al., 2012). It is noteworthy that differential evolutionary algorithm has been previously applied for different problems under various applications (e.g. Varadarajan & Swarup, 2008; Qin et al. 2009; Wang et al., 2012; Tang, Zhao & Liu, 2014; Wang et al. 2017).

This paper includes seven sections. Section 1 is the introduction. Section 2 reviews the RM and its application to the airline industry. Section 3 is about the proposed model. The case study is described in Section 4. Section 5 focuses on the results of the paper. Section 6 describes our solution method. The final section is the concluding remarks.

2. Related works

During the past three decades, most companies have realized the importance of revenue or yield management. The airline industry is one of the industries in which implementing RM has led to some advantages (Vardi et al., 2016). It is evident from the literature that there are many studies that have studied revenue optimization in the airline industries (such as Parker, 2003; Yu et al., 2004; Oliveira, 2003; Graf & Kimms, 2013; Wittman & Belobaba, 2016; Grauberger & Kimms, 2016a; Grauberger & Kimms, 2016b; Chao & Li, 2017).

Reviewing previous studies demonstrates that researchers have proposed airline RM models considering several relevant options. The seat inventory control is one of the options that has been extensively considered in the earlier studies. Several researchers such as Williamson, (1992), Bertsimas & De Boer (2005), Zhang & Cooper (2005), Klophaus & Pölt, (2007) and have considered flight ticket booking as an option the RM models. In addition to booking limit, thicket flight cancelation is another important option that is considered in the previous RM models. Some studies such as GOSAVII et al., (2002), El-Haber & El-Taha, (2004), Gosavi et al. (2007), Aydın et al., (2012), Sierag et al. (2015) and Vardi et al., (2016) have simultaneously considered booking and cancelation limits in the airline business.

Beside the mentioned seat inventory control options, some studies have devoted their attention to competition options among airlines and strategic airline companies' alliance. In this regard, some game theory models like a noncooperative game model (Hu et al., 2013) and duopoly game mode (Li et al., 2016) and Markov-game model (Wright et al., 2010) have been applied to find the equilibrium strategy for airline companies under competition. Ticket pricing is also one

of the other factors in which the airline companies compete with rivals in the market. Some researchers have focused on the pricing option like Zhang & Cooper (2009), Birbil et al. (2013), Li & Peng, (2007), Aslani et al. (2014) and Otero & Akhavan-Tabatabaei, (2015) to find the optimal pricing strategies. Furthermore, few other options in the literature which have been considered in the previous researches contain buy-back policy (Lin, Lee, & Yang, 2017) and commitment (Aydın et al., 2016). Nonetheless, these options are less used in the airline RM models.

Loading is also an option that is seldom studied in the previous airline RM researches and refers to the allowed weighting of cargo for each passenger in a specific flight. Recently, Chao & Li (2017) have investigated the optimal loading option and have studied the optimal loading density for a flight. However, they have studied air cargo companies, not airline ones. Considering the fact that the investigating the overloading option is neglected in the airline RM literature, in this study we consider three options of overbooking, cancelation, and overloading to optimize airline company revenue. To the best of our knowledge, this is the first study that simultaneously considers the three mentioned options in the airline RM context. Table 1 highlights the mentioned research gap this study aims to fill.

Insert Table 1 Here

Focusing on the solution methods in airline RM models indicates that the most applied method for solving the presented models are simulation-based optimization models (e.g. Bertsimas & De Boer, 2005; Zhang & Cooper, 2005; Gosavi et al., 2007; Graf & Kimms, 2013; Doreswamy et al., 2015). It should be noted that a metaheuristic approach which well matches the stochastic environment of RM has not been utilized until now. This motivated us to use a BDE algorithm to solve our presented model.

3. Proposed RM model

In this section, we introduce the airline revenue optimization model considering three options for booking, cancelation, and loading. The mathematical model is explained as follows:

Notations:

I Set of origins

i Number of origins $i = 1,2,\dots,I$

J	Set of destinations
j	Number of destination $j = 1,2,\dots,J$
V	Set of planes
v	Kind of planes $v = 1,2,\dots,V$
B	Indicator of booking by passengers
b	Number of seats that booking by passenger $b = 1,2,\dots,B$
C	Indicators of cancellation by passengers
c	Number of cancellation seats by passengers $c = 1,2,\dots,C$
O	Indicators of overloading of passengers
o	Number of passengers with overloading $o = 1,2,\dots,O$

Parameters:

P_{jvb}	Profit by booking
P_{jvc}	Profit by cancelation
P_{jvo}	Profit by overloading
$P_{x_{ij}}$	Price of flight from origin i to destination j
$N_{x_{ij}}$	Number of passengers in a plane from i to j
G_{ijvc}	Total expected profit from ticket cancellation
D_{ijvb}	Total expected profit from ticket booking
H_{ijvo}	Total expected profit from overloading

$\Pr(c) =$ Probability of canceling ticket (between 0 and 0.2)

$\Pr(o) =$ Probability of overloading (between 0 and 0.03)

$\Pr(b) =$ Probability of ticket booking (between 0.8 and 1)

Decision variable:

$$x_{ijv} = \begin{cases} 1 & \text{if the flight goes to destination} \\ 0 & \text{if the flight does not go to destination} \end{cases}$$

Suppose that there are V airplanes with different capacities belonging to the set $V \in \{v_1, v_2, \dots, v_V\}$. These planes go from the origins identified by i where $i \in \{i_1, i_2, \dots, i_I\}$ to the destinations shown by j where $j \in \{j_1, j_2, \dots, j_J\}$. Based on the kinds of planes and the number of destinations we can determine the decision variables. The first decision variable is x_{ijv} that is related to going or not going of a specific plane v from origin i to destination j. According to the objective of this study, simultaneously considering three relevant options including booking, cancellation and overloading, we have shown these options by b, c and o, respectively. Each considered option includes a penalty for the passenger.

The presented model in this study consists of the objective function (introduced by z) which calculates the amount of revenue that an airline company can receive from the flights between cities with various kinds of airplanes. P_{jvb} refers to the profit from buying a ticket in the situation of booking, P_{jvc} represents the penalty of canceling a ticket by a passenger and also P_{jvo} denotes the penalty of overloading more than allowed amount per kilogram by a passenger. These parameters of the model are probabilistic where the ticket cancelation, booking and overloading limits are taken into consideration and we show their probabilities by $\Pr(o)$, $\Pr(c)$ and $\Pr(b)$. Eventually, $N_{x_{ij}}$ introduces the number of passengers in each plane, which is a parameter, and $P_{x_{ij}}$ refers to the price of a ticket for a specific flight from. The proposed mathematical model is presented as bellow:

$$Max\ z : \sum_{j=1}^{J} \sum_{i=1}^{I} \sum_{c=1}^{C} \sum_{b=1}^{B} \sum_{o=1}^{O} \sum_{v=1}^{V} x_{ijv}\, N_{x_{ij}} (P_{jvc} + P_{jvb} + P_{jvo}) \tag{1}$$

s.t:

$$x_{ijv} \times \Pr(c) \times P_{jvc} \times N_{x_{ij}} \le G_{ijvc} \quad \forall v, c, i, \mathrm{j} \tag{2}$$

$$x_{ijv} \times \Pr(b) \times P_{jvb} \times N_{x_{ij}} \times P_{x_{ij}} \le D_{ijvb} \qquad \forall b, v, i, j \tag{3}$$

$$x_{ijv} \times \Pr(o) \times P_{jvo} \times N_{x_{ij}} \le H_{ijvo} \qquad \forall v, o, i, j \tag{4}$$

$$x_{ijv} \in \{0,1\} \tag{5}$$

Equations 1 to 5 demonstrate the mathematical formulation for revenue optimization. Eq. (1) which represents the objective function, seeks for the maximum revenue of the airline company considering booking and penalties of overloading and cancellation options. The first constraint, Eq. 2, is related to the penalty of cancellation as well as its probability. Eq. 3 refers to the probability of booking the seats of a plane. The probability of overloading is indicted by Eq. 4 as the third constraint.

4. A case study of the Iranian Airline Industry

4.1. Problem description

Iran Air Company is the oldest and most popular airline in Iran. This company transfers passengers to most cities in Iran as well as cities outside of its borders. However, after some years as a monopolist, *Iran Air* begins to face challenges from other airline companies. Hence, RM is vital for this company to survive in this competitive market. This company has the plane types and their capacities as seen in Table 2.

Insert Table 2 Here

This airline has many routes for transferring passengers from cities. However, most of these routes use the same planes. In this study, we focused our planning on megacities with a high travel demand; the cities were Tabriz, Shiraz, Mashhad, and Isfahan.

Insert Figure 1 Here

Table 3 describes the notation for each plane that flies to these cities and table 4 shows the prices of tickets from each evaluated city as follow:

Insert Tables 3 & 4 Here

Following the objective of the study, the proposed mathematical model related to the Iran Air Company revenue optimization is explained in details in the next section.

4.2. Assumptions of the model

In the real world, there are too many restrictions to fully solve the problem. If we want to solve this problem, these restrictions must be taken into account. Hence, the assumptions used in our model are as follows:

- All of the passengers are adults: Since the price of children thicket is less than that of adults, calculations and estimations considering how many children travel by plane and finding the

function distribution of such a situation are very difficult. Therefore, we assumed all of the passengers to be adults.

- The price of tickets is stable: Some days the price of the tickets will be increased dramatically. Although the revenue of airlines increases dramatically on such days, such an occurrence is based on chance. Therefore, we assumed the price to be stable.

- If the number of passengers is below 80% of the plane capacity, then that flight will be canceled. Airlines have computed that under 80% of seats occupied, then the flight is not worthwhile and the income that would be earned is not enough to cover maintenance, fuel, and other concerns.

- The average overloading weight is 8 kg: Since the weight of some passengers' freight is within the normal range and that of others is higher, the calculation of how many passengers are over and by how much is very difficult. Therefore, we considered 8 kg to be the average for overloading based on the *Iran Air* airline.

- Only one season (summer) is considered for calculations: In each season, the demands for airlines change. Since all of the schools in Iran are shut down during the summer and the demand for trips increases dramatically during this season, we considered only the summer season for this research.

- Only one airline is considered in this study: As mentioned before, there are many airlines in Iran, but we only focused on one, which is the oldest and most popular airline in Iran.

- Flights are two-way and the planes return to parking for maintenance checks after returning to the original location. In some plane trips, the journey is not as short. Some planes go from the original location to the first destination, then to the next destination and then, after reaching the third destination, they return back to the original location. Since the other destinations will not be megacities, we assumed that the plane would travel to only one location and back again.

4.3. Mathematical model
4.3.1. Objective function. First, we sought to maximize the revenue. We decide whether a certain type of plane should be allocated to a flight and develop a variable that indicates the allocation of a plane.

4.2.2. Booking option for seats

This airline has 6 kinds of airplanes with diverse capacities. Moreover, the price of tickets is different based on the demand and also on the length of the flights. Hence, the revenue from buying, booking, and canceling seats can be described as follows. In this section, these planes followed a simple rule. If more than 80% of seats were occupied, then they would travel, and if less than 80% of seats were occupied, then the flight would be canceled. Therefore, the rate of occupation was from 80% to 100%, and this rate was multiplied by the number of seats on each plane. Moreover, the ticket to each city has a unique price.

4.2.3. Penalty for cancellation

There are many reasons that some passengers have to cancel their flights, and most airlines have the same rules regarding this. The penalties for cancellation depend on the time of cancellation. If this time is close to the time of the flight, then this rate increases dramatically. In this study, we assume that this penalty rate is the average of the lowest and highest penalty rates. This rate is between 0 and 20%.

4.2.4. Overloading

Another kind of resource that profits airlines is overloading. Most passengers usually buy many souvenirs when they travel and, therefore, the weight of the return flight may be more than the normal weight. In this study, we assume that the average weight of overloading was 8 kilograms. Then, this average was multiplied by the number of passengers, the probability of overloading, and the penalty cost for 31,000 Rials per kilogram. The following formula shows this computation. Table 3 shows a description of the formula noted above.

Insert Table 5 Here

It is noteworthy that Iran Air has 54 airplanes which have more than 300 flights daily. Furthermore, the number of seats in each airplane varies from 104 to 239 seats. Considering the large size of the understudied problem and the stochastic parameters of the presented model, we dealt with an integer programming model and due to the computational complexity, exact solutions were unable to efficiently solve the problem. As a result, we resort to BDE as a viable heuristic approach.

5. Solution methodology
5.1. BDE algorithm

Like many exiting meta-heuristic algorithms, DE is an approach inspired by the environment. The simplicity of usage, accuracy, and solution time are all benefits of this method. Hence, many papers have been published about this method (Das, Abraham, & Konar, 2008; Huang, Qin, & Suganthan, 2006; Onwubolu & Davendra, 2006; Qin, Huang, & Suganthan, 2009; Qin & Suganthan, 2005; Storn & Price, 1997; Tang et al. 2014; Yildiz, 2013). Since the model used in this paper has several constraints, we used the following procedure to solve this problem. During the first stage, the constraints of the problem are added to the objective function and, hence, we used the BDE approach while considering the penalty function. Therefore, the violation was used as follows:

Min f(x)

s.t: 　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　(6)

$$g_i(x) \geq 0 \qquad i = 1,2,\dots,n$$

So we have

Min f(x)-$\sum_{i=1}^{n} \lambda_i * g_1$ 　　　　　　　　　　　　　　　　　　　　　(7)

If the constraint to satisfy is $g(x) \leq g_0$ then the violation will be

$$V\{g(x) \geq g_0\} = \begin{cases} 0 & g(x) \leq g_0 \\ 1 - \frac{g(x)}{g_0} & g(x) < g_0 \end{cases}$$

If the constraint to satisfy is $g(x) \geq g_0$ then the violation will be

$$V\{g(x) \leq g_0\} = \begin{cases} 0 & g(x) \geq g_0 \\ \frac{g(x)}{g_0} - 1 & g(x) > g_0 \end{cases}$$

We have three types of penalty function as below:

1. Additive penalty function

$$\hat{f}(x) = f(x) + \alpha * P(v)$$

s.t:

$v \geq 0$ 　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　(8)

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$\hat{f}(x)$ in Eq.8 indicates the changes of f(x) where $f(x)$ is the penalty function. α is the coefficient of penalty, P(v) is the increment function of v where v represents the violation amount of the model.

2. Multiplicative penalty function

$$\hat{f}(x) = f(x)(1 + \beta*P(v)) \qquad \forall\, f(x) \geq 0$$

s.t: $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ (9)

$v \geq 0$

3. The hybrid additive-multiplicative penalty function

$$\widehat{f}_1(x) = (f(x) + \alpha*P(v))\,(1 + \beta*Q(v))$$

$$\widehat{f}_2(x) = (f(x)\,(1 + \beta*Q(v)) + \alpha * P(v) \qquad\qquad (10)$$

s.t:

$v \geq 0$

In this study, the multiplicative penalty function was used to solve the problem.

5.2. Solution procedure and findings

The procedure for solving this problem is as follows:

```
1: Start
2: Create the objective function
3: Create the constraints
4: Set the number of variables
5: Set the violation coefficient
6: Identify the population size (nPoP)?
7: Identify the maximum iteration
8: Set the crossover rate
9: Find the results
10: If the solution has violation, return to line 5 and set the
coefficient again and repeat all later steps
11: If the result does not have violation, it is the best
solution
12: End
```

After running this model with MATLAB version 2015(b) with a PC CPU core (i) 3 and 5 GB RAM, the results were computed. The total revenue earned from the flights was 6.2592 million Iran Rials. The computation time was 29 seconds. nPop in this study was 1,000, the maximum iteration was 100, and the crossover rate of the problem was 0.3. Out of approximately 24 flights among these cities, our results showed that only 7 planes would not fly in order to obtain the best revenue. This means that 17 planes can transfer passengers between these cities. Table 5 shows the results of our analysis.

Insert Table 6 Here

Insert Figure 2 Here

6. Analysis of the results

The obtained findings of the study indicate that among the 4 mega-cities, Shiraz should cease 3 return flights from Tehran to Shiraz. McDonnell Douglas MD-82, Fokker 100, and Airbus A320-200 airplanes should not be used in these cities. The main reason for this is that these planes have low capacities in comparison to that of the other types of planes. The number of passengers that want to travel by plane from Tehran to Shiraz and then return back again is not very large, so the types of planes noted above and big planes with high capacities are sufficient for transferring passengers on these routes. The next highest number of flight cancellations was in Tabriz with two flights cancellation. The two kinds of planes to be allocated for this city are the Airbus A300B4-200 and Fokker 100. Therefore, *Iran Air* should not use the planes that have the lowest capacity or the highest capacity. This result indicates that this city always has the same number of passengers and the distribution function of these passengers is uniform.

The rest of cities, Mashhad and Isfahan, had the same number of plane cancellations at one flight, but the kinds of the plane are different. In Mashhad, the McDonnell Douglas MD-82 plane should be canceled and the Airbus A300-600R should not fly into Isfahan in order to achieve the best revenue. These differences indicate that the demand for flights from Tehran to Mashhad and then back again is very high and planes with high capacities need to be allocated here more than other planes with lower capacities. However, the result was opposite for Isfahan, where people have less need to use planes for many reasons, such as the availability of buses, which serve as

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suitable methods of transportation with a lower price than an airplane. Hence, big planes should be allocated to other cities in order to increase the revenue of *Iran Air*.

Analysis of the kinds of planes showed that the McDonnell Douglas MD-82 is the type of plane that brings the lowest revenue to *Iran Air*, and it is also the most frequently canceled type of plane, with a cancellation frequency of 4 times for planes with low capacities versus 2 times for larger planes. The results of this study revealed that the demand for *Iran Air* planes in megacities is very high and, therefore, if this airline does not focus on this matter, it will lose customers and its revenue will decrease.

7. Concluding remarks

RM is an essential and effective tool that allows managers to make appropriate decisions, which maximize the revenue. Following this objective, managers require to consider all aspects of the problem, where some of the data are deterministic while other data are stochastic. For the competitive markets in the airline business, the tool developed in this study would be able to help airline companies to better compete in the market.

In this study, we aimed to help the managers of Iran air to optimize profit. We proposed a mathematical model considering the passenger overloading carriages and cancellation. DE algorithm was used to solve this problem. *Iran Air Company*, which is the oldest airline in Iran, was analyzed to show how an airline company can allocate planes to megacities in order to obtain superior revenue management. In addition, various options that help to increase revenue were also considered in this study. These options were penalties due to ticket cancellation and overloading. Our data was interval, the size of the problem was large, and this model was run for 1,000 flights for each city with a different type of plane. This model cannot be developed into a closed form solution. Therefore, an NP-hard problem in the form DE, an evolutionary algorithm, is used to mitigate the drawbacks of previous methods. After running the model and obtaining our results, we provided some suggestions and insights for the management of the studied company. Adding more options as well as novel options to future RM models can be an interesting idea for the further studies.

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FIGURES
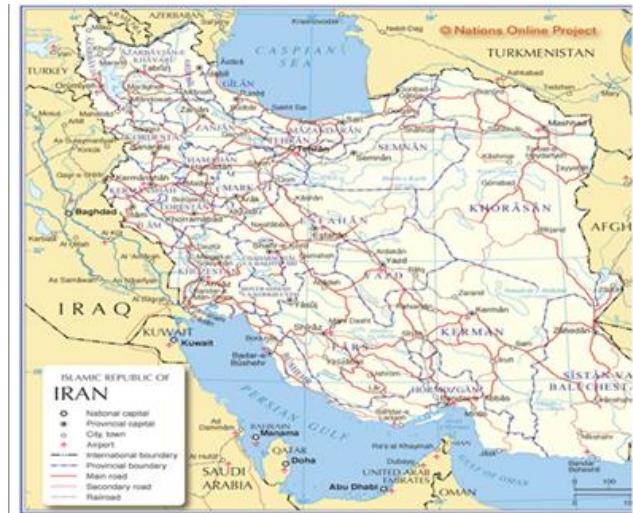
Figure 1. Map of Iran
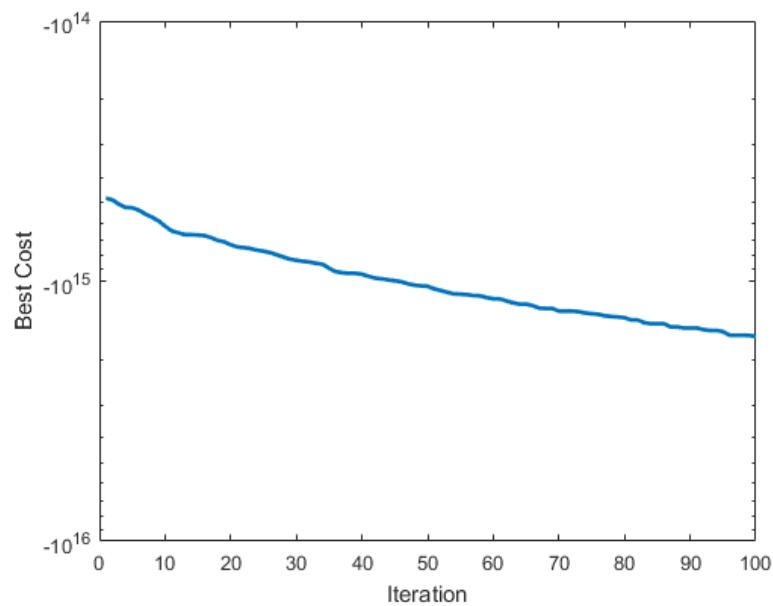
Figure 2. Obtained results

TABLES

Table 1. Considered options of airline RM model in the recent literature

Author(s)	Considered options					
	Booking	Cancelation	Overloading	Pricing	Competition	Buy-back policy
Gosavi et al. (2007)	√	√				
Zhang & Cooper (2009)				√		
Aydın et al. (2012)	√	√				
Birbil et al. (2013)	√			√		
Aslani et al. (2014)				√		
Grauberger, & Kimms (2016a)	√				√	
Grauberger, & Kimms (2016b)	√			√	√	
Oancea, O. (2016)	√			√		
Li et al. (2016)	√				√	
Chao & Li (2017)			√	√		
Lin et al. (2017)	√					√
Yoon et al., (2017)	√	√		√		
The current study	√	√	√			

Table 2. Types of planes

Type of plane	Number of passengers
McDonnell Douglas MD-82	140
Fokker 100	104
Airbus A320-200	144
Airbus A310-300	198
Airbus A300-600R	239
Airbus A300B4-200	236

Table 3. Notation of planes and cities

Type of plane	Booking	Cancelation	Overloading	City
McDonnell Douglas MD-82	P_{IbMD}	P_{IcMD}	P_{IoMD}	Isfahan
Fokker 100	P_{Ibf100}	P_{Icf100}	P_{Iof100}	
Airbus A320-200	P_{IbA200}	P_{IcA200}	P_{IoA200}	
Airbus A310-300	P_{IbA300}	P_{IcA300}	P_{IoA300}	
Airbus A300-600R	P_{IbA600}	P_{IcA600}	P_{IoA600}	
Airbus A300B4-200	P_{IbB200}	P_{IcB200}	P_{IoB200}	
McDonnell Douglas MD-82	P_{TbMD}	P_{TcMD}	P_{ToMD}	Tabriz
Fokker 100	P_{Tbf100}	P_{Tcf100}	P_{Tof100}	
Airbus A320-200	P_{TbA200}	P_{TcA200}	P_{ToA200}	
Airbus A310-300	P_{TbA300}	P_{TcA300}	P_{ToA300}	
Airbus A300-600R	P_{TbA600}	P_{TcA600}	P_{ToA600}	
Airbus A300B4-200	P_{TbB200}	P_{TcB200}	P_{ToB200}	
McDonnell Douglas MD-82	P_{MbMD}	P_{McMD}	P_{MoMD}	Mashhad
Fokker 100	P_{Mbf100}	P_{Mcf100}	P_{Mof100}	
Airbus A320-200	P_{MbA200}	P_{McA200}	P_{MoA200}	
Airbus A310-300	P_{MbA300}	P_{McA300}	P_{MoA300}	
Airbus A300-600R	P_{MbA600}	P_{McA600}	P_{MoA600}	
Airbus A300B4-200	P_{MbB200}	P_{McB200}	P_{MoB200}	
McDonnell Douglas MD-82	P_{SHbMD}	P_{SHcMD}	P_{SHoMD}	Shiraz
Fokker 100	$P_{SHbf100}$	$P_{SHcf100}$	$P_{SHof100}$	
Airbus A320-200	$P_{SHbA200}$	$P_{SHcA200}$	$P_{SHoA200}$	
Airbus A310-300	$P_{SHbA300}$	$P_{SHcA300}$	$P_{SHoA300}$	
Airbus A300-600R	$P_{SHbA600}$	$P_{SHcA600}$	$P_{SHoA600}$	
Airbus A300B4-200	$P_{SHbB200}$	$P_{SHcB200}$	$P_{SHoB200}$	

Table 4. The prices of flights

City	Price (Based on Iranian currency, Rial)
Isfahan	1660000
Shiraz	2000000
Tabriz	1554000
Mashhad	2508000

Table 5. Formulas for calculating each option, type, and the city of flights

Type of Plane	Booking	Cancelation	Overloading	City
McDonnell Douglas MD-82	$P_{IbMD}(r_i \geq 0.8)$ $= \int_{0.8}^{1} P_{IbMD} \, dr_i$	$P_{IcMD}(r_i \geq 0)$ $= \int_{0}^{0.2} P_{IcMD} \, dr_i$	$P_{IoMD}(r_i \geq 0)$ $= \int_{0}^{0.03} P_{IoMD} \, dr_i$	
Fokker 100	$P_{Ibf100}(r_i \geq 0.8)$ $= \int_{0.8}^{1} P_{Ibf100} \, dr_i$	$P_{Icf100}(r_i \geq 0)$ $= \int_{0}^{0.2} P_{Icf100} \, dr_i$	$P_{Iof100}(r_i \geq 0)$ $= \int_{0}^{0.03} P_{Iof100} \, dr_i$	
Airbus A320-200	$P_{IbA200}(r_i \geq 0.8)$ $= \int_{0.8}^{1} P_{IbA200} \, dr_i$	$P_{IcA200}(r_i \geq 0)$ $= \int_{0}^{0.2} P_{IcA200} \, dr_i$	$P_{IoA200}(r_i \geq 0)$ $= \int_{0}^{0.03} P_{IoA200} \, dr_i$	
Airbus A310-300	$P_{IbA300}(r_i \geq 0.8)$ $= \int_{0.8}^{1} P_{IbA300} dr_i$	$P_{IcA300}(r_i \geq 0)$ $= \int_{0}^{0.2} P_{IcA300} \, dr_i$	$P_{IoA300}(r_i \geq 0)$ $= \int_{0}^{0.03} P_{IoA300} \, dr_i$	**Isfahan**
Airbus A300-600R	$P_{IbA600}(r_i \geq 0.8)$ $= \int_{0.8}^{1} P_{IbA600} \, dr_i$	$P_{IcA600}(r_i \geq 0)$ $= \int_{0}^{0.2} P_{IcA600} \, dr_i$	$P_{IoA600}(r_i \geq 0)$ $= \int_{0}^{0.03} P_{IoA600} \, dr_i$	
Airbus A300B4-200	$P_{IbB200}(r_i \geq 0.8)$ $= \int_{0.8}^{1} P_{IbB200} \, dr_i$	$P_{IcB200}(r_i \geq 0)$ $= \int_{0}^{0.2} P_{IcB200} \, dr_i$	$P_{IoB200}(r_i \geq 0)$ $= \int_{0}^{0.03} P_{IoB200} \, dr_i$	
McDonnell Douglas MD-82	$P_{TbMD}(r_i \geq 0.8)$ $= \int_{0.8}^{1} P_{TbMD} \, dr_i$	$P_{TcMD}(r_i \geq 0)$ $= \int_{0}^{0.2} P_{TcMD} \, dr_i$	$P_{ToMD}(r_i \geq 0)$ $= \int_{0}^{0.03} P_{ToMD} \, dr_i$	**Tabriz**
Fokker 100	$P_{Tbf100}(r_i \geq 0.8)$ $= \int_{0.8}^{1} P_{Tbf100} \, dr_i$	$P_{Tcf100}(r_i \geq 0)$ $= \int_{0}^{0.2} P_{Tcf100} \, dr_i$	$P_{Tof100}(r_i \geq 0)$ $= \int_{0}^{0.03} P_{Tof100} \, dr_i$	

Airbus A320-200	$P_{TbA200}(r_i \geq 0.8)$ $= \int_{0.8}^{1} P_{TbA200}\, dr_i$	$P_{TcA200}(r_i \geq 0)$ $= \int_{0}^{0.2} P_{TcA200}\, dr_i$	$P_{ToA200}(r_i \geq 0)$ $= \int_{0.8}^{1} P_{ToA200}\, dr_i$	
Airbus A310-300	$P_{TbA300}(r_i \geq 0.8)$ $= \int_{0.8}^{1} P_{TbA300}\, dr_i$	$P_{TcA300}(r_i \geq 0.$ $= \int_{0}^{0.2} P_{TcA300}\, dr_i$	$P_{ToA300}(r_i \geq 0)$ $= \int_{0}^{0.03} P_{ToA300}\, dr_i$	
Airbus A300-600R	$P_{TbA600}(r_i \geq 0.8)$ $= \int_{0.8}^{1} P_{TbA600}\, dr_i$	$P_{TcA600}(r_i \geq 0)$ $= \int_{0}^{0.2} P_{TcA600}\, dr_i$	$P_{ToA600}(r_i \geq 0)$ $= \int_{0}^{0.03} P_{ToA600}\, dr_i$	
Airbus A300B4-200	$P_{TbB200}(r_i \geq 0.8)$ $= \int_{0.8}^{1} P_{TbB200}\, dr_i$	$P_{TcB200}(r_i \geq 0)$ $= \int_{0}^{0.2} P_{TcB200}\, dr_i$	$P_{ToB200}(r_i \geq 0)$ $= \int_{0}^{0.03} P_{ToB200}$	
McDonnell Douglas MD-82	$P_{MbMD}(r_i \geq 0.8)$ $= \int_{0.8}^{1} P_{MbMD}\, dr_i$	$P_{McMD}(r_i \geq 0)$ $= \int_{0}^{0.2} P_{McMD}\, dr_i$	$P_{MoMD}(r_i \geq 0)$ $= \int_{0}^{0.03} P_{MoMD}\, dr_i$	
Fokker 100	$P_{Mbf100}(r_i \geq 0.8)$ $= \int_{0.8}^{1} P_{Mbf100}\, dr_i$	$P_{Mcf100}(r_i \geq 0)$ $= \int_{0}^{0.2} P_{Mcf100}\, dr_i$	$P_{Mof100}(r_i \geq 0)$ $= \int_{0}^{0.03} P_{Mof100}\, dr_i$	**Mashhad**
Airbus A320-200	$P_{MbA200}(r_i \geq 0.8)$ $= \int_{0.8}^{1} P_{MbA200}\, dr_i$	$P_{McA200}(r_i \geq 0)$ $= \int_{0}^{0.2} P_{McA200}\, dr_i$	$P_{MoA200}(r_i \geq 0)$ $= \int_{0}^{0.03} P_{MoA200}\, dr_i$	
Airbus A310-300	$P_{MbA300}(r_i \geq 0.8)$ $= \int_{0.8}^{1} P_{MbA300}\, dr_i$	$P_{McA300}(r_i \geq 0)$ $= \int_{0}^{0.2} P_{McA300}\, dr_i$	$P_{MoA300}(r_i \geq 0)$ $= \int_{0}^{0.03} P_{MoA300}\, dr_i$	

Airbus A300-600R	$P_{MbA600}(r_i \geq 0.8)$ $= \int_{0.8}^{1} P_{MbA600} \, dr_i$	$P_{McA600}(r_i \geq 0)$ $= \int_{0}^{0.2} P_{McA600} \, dr_i$	$P_{MoA600}(r_i \geq 0)$ $= \int_{0}^{0.03} P_{MoA600} \, dr_i$	
Airbus A300B4-200	$P_{MbB200}(r_i \geq 0.8)$ $= \int_{0.8}^{1} P_{MbB200} \, dr_i$	$P_{McB200}(r_i \geq 0)$ $= \int_{0}^{0.2} P_{McB200} \, dr_i$	$P_{MoB200}(r_i \geq 0)$ $= \int_{0}^{0.03} P_{MoB200} \, dr_i$	
McDonnell Douglas MD-82	$P_{SHbMD}(r_i \geq 0.8)$ $= \int_{0.8}^{1} P_{SHbMD} \, dr_i$	$P_{SHcMD}(r_i \geq 0)$ $= \int_{0}^{0.2} P_{SHcMD} \, dr_i$	$P_{SHoMD}(r_i \geq 0)$ $= \int_{0}^{0.03} P_{SHoMD} \, dr_i$	
Fokker 100	$P_{SHbf100}(r_i \geq 0.8)$ $= \int_{0.8}^{1} P_{SHbf100} \, dr_i$	$P_{SHcf100}(r_i \geq 0)$ $= \int_{0}^{0.2} P_{SHcf100} \, dr_i$	$P_{SHof100}(r_i \geq 0)$ $= \int_{0}^{0.03} P_{SHof100} \, dr_i$	
Airbus A320-200	$P_{SHbA200}(r_i \geq 0.8)$ $= \int_{0.8}^{1} P_{SHbA200} \, dr_i$	$P_{SHcA200}(r_i \geq 0)$ $= \int_{0}^{0.2} P_{SHcA200} \, dr_i$	$P_{SHoA200}(r_i \geq 0)$ $= \int_{0}^{0.03} P_{SHoA200} \, dr_i$	**Shiraz**
Airbus A310-300	$P_{SHbA300}(r_i \geq 0.8)$ $= \int_{0.8}^{1} P_{SHbA300} \, dr_i$	$P_{SHcA300}(r_i \geq 0)$ $= \int_{0}^{0.2} P_{SHcA300} \, dr_i$	$P_{SHoA300}(r_i \geq 0)$ $= \int_{0}^{0.03} P_{SHoA300} \, dr_i$	
Airbus A300-600R	$P_{SHbA600}(r_i \geq 0.8)$ $= \int_{0.8}^{1} P_{SHbA600} \, dr_i$	$P_{SHcA600}(r_i \geq 0)$ $= \int_{0}^{0.2} P_{SHcA600} \, dr_i$	$P_{SHoA600}(r_i \geq 0)$ $= \int_{0}^{0.03} P_{SHoA600} \, dr_i$	
Airbus A300B4-200	$P_{SHbB200}(r_i \geq 0.8)$ $= \int_{0.8}^{1} P_{SHbB200} \, dr_i$	$P_{SHcB200}(r_i \geq 0)$ $= \int_{0}^{0.2} P_{SHcB200} \, dr_i$	$P_{SHoB200}(r_i \geq 0)$ $= \int_{0}^{0.03} P_{SHoB200} \, dr_i$	

Continue of table 5

Table 6. Final results

Type of Plane	Accept/Reject	City
McDonnell Douglas MD-82	*Accept*	Isfahan
Fokker 100	*Accept*	
Airbus A320-200	*Accept*	
Airbus A310-300	*Accept*	
Airbus A300-600R	*Reject*	
Airbus A300B4-200	*Accept*	
McDonnell Douglas MD-82	*Reject*	Tabriz
Fokker 100	*Accept*	
Airbus A320-200	*Accept*	
Airbus A310-300	*Accept*	
Airbus A300-600R	*Accept*	
Airbus A300B4-200	*Reject*	
McDonnell Douglas MD-82	*Reject*	Mashhad
Fokker 100	*Accept*	
Airbus A320-200	*Accept*	
Airbus A310-300	*Accept*	
Airbus A300-600R	*Accept*	
Airbus A300B4-200	*Accept*	
McDonnell Douglas MD-82	*Reject*	Shiraz
Fokker 100	*Reject*	
Airbus A320-200	*Reject*	
Airbus A310-300	*Accept*	
Airbus A300-600R	*Accept*	
Airbus A300B4-200	*Accept*	