Uncertainty-Aware Linear Schedule Optimization: A Space-Time Constraint-Satisfaction Approach

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Abstract: Schedules and physical workspaces are two key elements of linear construction projects that are extremely interdependent. Any negligence in incorporating spatial and temporal constraints in developing and improving schedules of linear projects results in inevitable delays and workspace congestions and can substantially hinder the performance of the activity resources. This study augments the current linear scheduling methods by presenting an uncertainty-aware optimization framework to optimize the duration of linear projects while minimizing their potential congestions. The methodology is built upon the new concept of space-time float for explicit consideration of spatio-temporal constraints of activities and their inherent uncertainty. A constraint satisfaction approach was used for the two-tier optimization of duration and congestion. A fuzzy inference system was also incorporated to assess the inherent uncertainty in the schedule. Two case examples from literature are analyzed. The results demonstrate the effectiveness of the proposed method in planning and control of the unforeseen variations from planned schedules of linear projects. **DOI: [10.1061/(ASCE)CO.1943-7862.0001276](https://doi.org/10.1061/(ASCE)CO.1943-7862.0001276).** © *2016 American Society of Civil Engineers.*

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Introduction

Space is a dynamic and strictly limited resource on construction job sites. Construction workspaces constantly change to accomplish different activities throughout the preconstruction and construction phases of a project. As such, site space planning has been known to be one of the key factors of efficient management of construction projects (Zouein and Tommelein 2001). Site space planning in construction has been studied in two different domains: (1) the site layout problem, which focuses on the location of temporary facilities of various kinds (Karan and Ardeshir 2008; Zhou et al. 2009; Andayesh and Sadeghpour 2014; Pradhananga and Teizer 2014; Hammad et al. 2016); and (2) the space scheduling problem, which focuses on planning task execution spaces (Thabet and Beliveau 1997; Dawood and Mallasi 2006; Koo et al. 2013; Choi et al. 2014). The main purpose of integrating spatial information into schedules is to prevent available and/or potential congestions, i.e., interferences between resources of activities in space and time, on job sites. When congestion occurs on construction job sites, it not only causes work disruptions and reduces resource productivity but also impacts safety and may lengthen project durations (Thabet and Beliveau 1994; Su and Cai 2013).

Linear projects, such as highways, bridges, pipelines, and railways, are not an exception to this rule. This type of project is characterized by a series of repetitive activities whose resources share the same space either in sequential or parallel manner.

The frequent movement of linear activities' resources over limited shared space needs to be well-planned to avoid potential issues during execution of linear projects. As a result, schedules developed for these projects need not only to take into account all the logical, project-dependent and precedence constraints of activities (Moselhi and Hassanein 2003; Polat et al. 2009; Song et al. 2009) but also should incorporate the space and time constraints that coexist for the movement of their resources (Roofigari-Esfahan et al. 2015).

The current linear project scheduling methods schedule activities by planning the spatial progress of activities over time, i.e., their productivity rate. This, in fact, is the main difference between the linear scheduling methods and the duration-based methods, such as critical path method (CPM). In other words, the development and accuracy of linear schedules is highly dependent upon the productivity achieved from their activity resources (Duffy et al. 2011). However, the conventional linear scheduling methods do not account for the potential congestions that are likely to happen due to deviations from the planned productivity rate of activity resources. As such, integrating spatio-temporal constraints and flexibilities of movement of resources will improve the efficacy of the linear schedules in dealing with not-as-planned situations during construction.

As a continuation of the previous work, this study proposes and develops an uncertainty-aware schedule optimization framework for linear projects. In authors' previous work (Roofigari-Esfahan et al. 2015) conventional linear scheduling method (LSM) is augmented through the introduction of space-time floats. Adding space-time floats to the schedules of linear projects enables the linear schedules to adapt to variable production rates at each time interval during their construction (in essence exchanging time for space and vice versa). Due consideration to space and time flexibilities of linear activities achieved through space-time floats not only helps in generation of more realistic and flexible schedules but also enables identification and forecasting of potential space-time conflicts/congestions. Subsequently, this paper takes previous work to the next level to optimize the generated schedules and to minimize their potential congestions while taking into account the

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inherent uncertainties of the linear activities. As such, this paper uses and adds to the essential benefits of the space-time floats.

To this end, the framework uses constraint satisfaction approach to capture the optimum duration of linear projects that is associated with minimized potential congestions. For this purpose, the method uses the whole range of possible productivity rates that are available to each activity's resources at each point in time and space in form of space-time floats (Roofigari Esfahan et al. 2013, 2015). This available range of productivity rates relates to different project-dependent and management-dependent factors. Examples of these factors include effects of the site condition on the productivity of the resources and managerial decisions to use alternative equipment type or size to perform the same activity. By considering the whole range of possible productivity rates in the scheduling phase, the spatio-temporal constraints and flexibilities of the activities' resources are better understood and can accordingly be used to identify and avoid potential congestions. The congestion in this research specifically means the workspace interference when two activities have both spatial and temporal overlaps. This congestion can potentially happen as a result of deviations from planned productivities.

A new type of buffer, uncertainty-aware productivity buffer, is also introduced and used to account for the uncertainties inherent in project activities. This buffer is defined as allowable deviations from planned productivity rates of linear activities that need to be predicted and incorporated into the activity in the planning phase to prevent its unforeseen occurrence during construction. In other words, the uncertainty buffer identifies the amount of uncertainty in productivity rates of linear activities that the schedule can manage and cause no negative impact. In contrast to the conventional duration buffers, the uncertainty-aware productivity buffer presents the buffer available for the productivity rates of linear activities. Consequently, this buffer is realized both in terms of time and space that is more suitable to the nature of linear schedules. The framework makes use of fuzzy inference systems to calculate and determine the uncertainty-aware productivity buffer of linear activities.

The rest of the paper is organized as follows: first, the state of the art in scheduling and control of linear projects, as well as the literature of space and time congestion/conflict reduction, is reviewed. The three phases of the proposed method including (1) constraint-based optimization of linear schedules; (2) uncertainty-aware productivity buffer estimation using fuzzy inference system; and (3) constraint-based congestion minimization of linear achedules are then discussed. The different phases of the proposed framework are then evaluated in sequential steps using two case studies from literature (Mattila and Abraham 1998; Georgy 2008; Tang et al. 2014a, b). The first case study is presented to evaluate the efficacy of the first phase in minimizing the duration of linear schedules. The second case study is then used to examine the ultimate framework. These case studies were selected as they best presented highway project activities as a representative of linear projects and were frequently used by researchers for education and model verification purposes. The case studies are used here to present the added benefits of the proposed framework for optimizing linear schedules with due consideration to all their space-time constraints.

Relevant Literature

The literature on scheduling and planning linear projects is rich. A number of methods including linear scheduling method, repetitive scheduling method (RSM), and line of balance (LOB) are presented in the literature to plan, schedule, and control linear projects

(O'Brien 1975; Johnston 1981; Stradal and Cacha 1982; Chrzanowski Jr and Johnston 1986; Harmelink and Rowings 1998; Harris and Ioannou 1998; Cosma 2003). Much research has also been performed to predict the productivity rate of linear projects based on simulation, probability, or regression analysis (Yamin 2001; Kuo 2004; O'Connor and Huh 2005; Jiang and Wu 2007; Watkins et al. 2009; Duffy et al. 2011; Woldesenbet et al. 2012). Other methods consider variable productivity rates for linear activities (Lucko 2008; Duffy 2009; Lucko et al. 2014). There are also methods that attempted to visualize linear schedules in four-dimensional (4D) computer-aided design (CAD) (Staub-French et al. 2008), Excel spreadsheet (Lluch 2009) or by using the slip charts from the aerospace industry (Lucko et al. 2015).

The methods presented in literature for optimizing schedules of linear projects can be divided into three main categories: (1) the methods whose purpose is to maintain work continuity in terms of minimized resource fluctuations (Mattila and Abraham 1998; Shu-Shun and Chang-Jung 2007; Georgy 2008; Tang et al. 2014a, b) or minimized resource idle times (Vanhoucke 2006; Gonzalez et al. 2013; Ioannou and Yang 2016); (2) the methods that optimize linear project schedules considering minimization of project cost as objective (Handa and Barcia 1986; Senouci and Eldin 1996; Hegazy and Wassef 2001; Moselhi and Hassanein 2003; Ipsilandis 2007; Ezeldin and Soliman 2009; Menesi et al. 2013); and (3) the methods that tend to minimize the duration of linear projects (Russell and Caselton 1988; Fan and Lin 2007; Bakry et al. 2013; Cho et al. 2013; Bakry et al. 2014).

As stated earlier, simultaneous consideration of space and time constraints, flexibilities, and requirements is a major factor in scheduling linear construction projects. Currently available network techniques and linear scheduling methods, however, mainly consider technological constraints and resource requirements when generating schedules for linear projects. Such methods have often overlooked the fact that construction sequences are often constrained by the subsequent occupation of workspaces (Zouein and Tommelein 2001). The importance of space as a construction resource has been recognized through a number of studies and has subsequently been incorporated as an integral part of planning construction projects (Thabet and Beliveau 1994; Winch and North 2006; Hildum and Smith 2007).

Accordingly, the issue of space-time conflicts in the integrated schedules was raised through a number of studies. Guo (2002) analyzed spatial conflicts and temporal conflicts separately and accordingly introduced two independent interference indicators, the interference space percentage and the interference duration percentage. Graphical methods have also been used to present potential congestions in collided areas of construction sites and to detect interferences among trades (Chua et al. 2010; Koo et al. 2013). Many construction practitioners and researchers developed four-dimensional models by linking the three-dimensional (3D) components of buildings with the network activities of a project schedule (Mallasi 2006; Moon et al. 2014; Wang et al. 2014).

A limited number of methods similar to the one discussed in this paper have been proposed for linear projects with different purposes. Duffy et al. (2011) divided linear project schedules into areas of time and location for which unique production variables, called working windows, can be assigned. Working windows display locations and times when production variables of activities may change along the linear project's timeline. In a recent attempt Shah (2014) reported the information needed for identification of time–space conflicts in earthwork operations of road and railway construction projects. This paper aims to add to the current body of knowledge by proposing an effective schedule optimization framework that integrates workspaces of linear activities' resources as
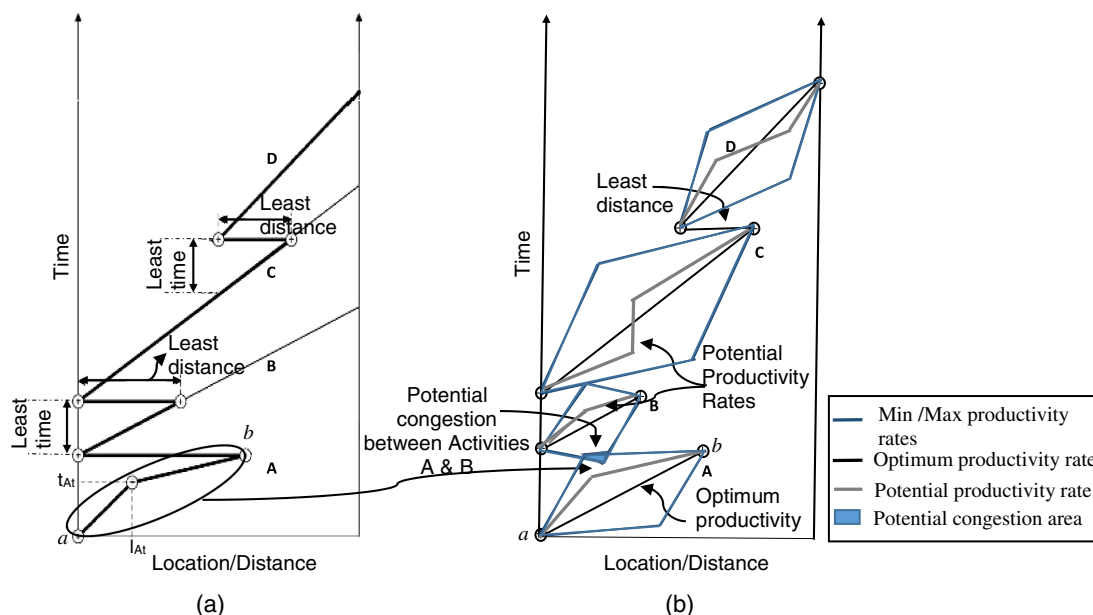
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Fig. 1. Comparison of (a) conventional LSM with (b) the space-time float LSM

well as activities' inherent uncertainties into their schedule in the planning stage. The framework is able to depict the optimum duration for linear projects that is associated with the least potential congestions.

The schedule optimization framework proposed in this paper uses different tools to achieve optimum duration for linear projects that is associated with the least potential congestions. These tools include: (1) space-time float; (2) constraint-satisfaction optimization; and (3) fuzzy inference systems. A brief description of the tools followed by the three-step schedule optimization framework is subsequently presented.

Space-Time Float

The space-time float (Roofigari-Esfahan et al. 2015) is used in this study as a means to incorporate space and time constraints and flexibilities of the movement of linear activities' resources into linear schedules. Fig. 1 illustrates the transformation of the linear scheduling method through the use of space-time floats. In most other methods presented for linear schedules, each linear activity is presented by its start and end coordinates in the form of [location(x), time (t)], the productivity rate planned for the whole activity or a set of productivity rates planned for different time intervals of the activity as shown in Fig. 1(a). As it is demonstrated in Fig. 1(a), Activity A is scheduled using its start and end points, shown as a and b, respectively, and two productivity rates alongside its duration. Other activities (B, C, and D) are planned to have one productivity rate throughout. The least distance and time required between succeeding activities also are taken into account when planning activities using LSM as illustrated in Fig. 1(a).

When using space-time floats, a range of potential productivity rates for each activity is also taken into account when scheduling linear projects. This range is bounded by the minimum and maximum possible productivity rates that are available to the resources of activity. These boundaries form the space-time float polygon of activities as shown in Fig. 1(b). In practice, the variability in the potential productivity rates of each activity can be caused by different factors. Project environment and uncertainties in work

quantities, variations in performance of the resources in response to site condition causing slow-down/speed-up alterations as well as alternative managerial plans to use different size and/or type of the resource for an activity are a few of the factors leading to variations in activities' planned productivities.

As such, the space-time float is an envelope for all potential movement paths that resources of activity can take considering the time and space constraints of that activity. The variation of productivity rates planned for Activity A using LSM, presented in Fig. 1(a), still falls inside the space-time float of that activity as shown in Fig. 1(b). Vertical paths, as shown for Activity C in Fig. 1(b), demonstrate idle times of activity when no productive work is actually executed. Space-time floats also consist of a set of space-time coordinates representing the activity start/end time and locations as well as other related constraints and milestones [shown as a and b in Figs. 1(b) and 2(a)].

Simultaneous consideration of space and time floats makes it possible to trade off one for the other, which consequently allows planning for potential delays before the actual execution commences. Furthermore, generation of space-time float polygons helps in the detection of potential congestions in linear schedules. These potential congestions are caused as a result of deviations from planned productivity rates of activities and are depicted through overlaps between space-time float polygons of activities as shown in Fig. 2(b). As such, the congestions are predicted before the construction phase starts, which consequently helps in generating alternative plans to prevent such potential congestion from happening.

Constraint Programming

Constraint programming (CP) is presented as a programming paradigm for solving constraint satisfaction problems (CSPs) through using a combination of mathematics, artificial intelligence, and operations research techniques (Chan and Hu 2002; Liu and Wang 2012; Tang et al. 2014b). Constraint satisfaction problems are defined by a set of variables, $X1; X2; \ldots; Xn$, and a set of constraints, $C1; C2; \ldots; Cm$. Each variable Xi has a nonempty domain Di of
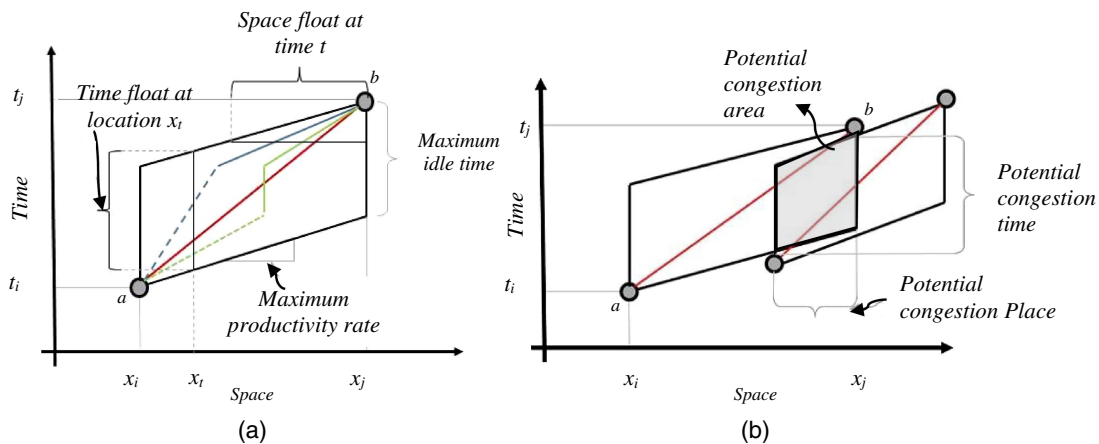
Fig. 2. Realization of (a) space-time float prism; (b) potential congestion areas

possible values. Each constraint Ci involves some subset of the variables and specifies the allowable combinations of values for that subset. A state of the problem is defined by an assignment of values to some or all of the variables. An assignment that does not violate any constraints is called a consistent assignment. A complete assignment is one in which every variable is mentioned, and a solution to a CSP is a complete assignment that satisfies all the constraints.

The objective function in a CSP optimization problem is treated as a constraint. This additional constraint forces the new feasible solution to have a better value than the current solution. When a better objective function value is found, the upper or lower bounds of the constraint are replaced. The propagation mechanism reduces the size of the search space by narrowing down the domains of decision variables. The search terminates when no feasible solution is found (Pindeo 2002; Liu and Wang 2008).

Constraint programming has particular advantages in solving scheduling problems (Heipcke 1999; Chan and Hu 2002; Menesi et al. 2013). Some of the advantages include (1) efficient solution search mechanism as explained above; (2) flexibility to consider a variety of constraint types; and (3) convenience of model formulation. As such, constraint programming is suitable for modeling and solution finding of the project scheduling optimization of linear projects and accordingly has been selected to be used for this study.

Fuzzy Inference System

A fuzzy inference system (FIS) provides a means for using fuzzy logic to map an input space to an output space. The goal of FIS is to formalize the reasoning process of human language by means of fuzzy logic through building fuzzy IF-THEN rules. Using membership functions, fuzzy if-then rules also called as fuzzy conditional statements capture the spirit of a "rule of thumb" used by humans. In other words, fuzzy if-then rules are expressions of the form IF A THEN B, where A and B are characterized by appropriate membership functions (Zadeh 1965). To this end, fuzzy inference systems are basically composed of five main functional blocks (Jang 1993) "(1) a rule base containing a number of fuzzy if-then rules; (2) a database which defines the membership functions of the fuzzy sets used in the fuzzy rules; (3) a decision-making unit which performs the inference operations on the rules; (4) a fuzzification interface which transforms the crisp inputs into degrees of match with linguistic values; and (5) a defuzzification interface which transforms the fuzzy results of the inference into a crisp output." The concise form of the fuzzy inference system's if-then rules enables capturing

of the imprecise modes of reasoning and making decisions in an environment of uncertainty and imprecision (Jang 1993). As such, FIS has been selected in the method presented here to present and compute the uncertainty associated with the productivity of the linear activities.

Research Methodology

The three essential phases of the proposed uncertainty-aware schedule optimization framework developed in the present study are illustrated in Fig. 3. As it is schematically illustrated in Fig. 3, the first phase of the proposed framework makes use of space-time floats presented in authors' previous work as a tool. The framework then builds upon it to minimize the duration of the linear projects taking into account the spatio-temporal constraints and flexibilities provided through application of space-time floats. Constraint satisfaction approach is used in the first phase to achieve this objective. Having generated the optimum schedule in the first phase, the method then presents and quantifies uncertainty-aware productivity buffers of the linear activities using fuzzy inference system. The method in this phase also detects and quantifies potential congestions between space-time floats of linear activities in the schedule generated in the first phase. The project schedule is then optimized in the third phase to minimize the potential congestions detected in the second phase taking into account project constraints as well as the uncertainty-aware productivity buffers defined in the second phase. The details of each phase of the proposed framework are presented in the following sections.

Schedule Optimization Framework for Linear Projects

As discussed earlier, the three phases of the developed method in this paper include (1) constraint-based optimization of linear schedules; (2) uncertainty-aware productivity buffer estimation using fuzzy inference system; and (3) constraint-based congestion minimization of linear schedules. The details of each phase follow.

Phase 1. Constraint-Based Optimization of Linear Schedules

The first phase of the proposed schedule optimization framework is a constraint satisfaction problem (CSP) based optimization model.
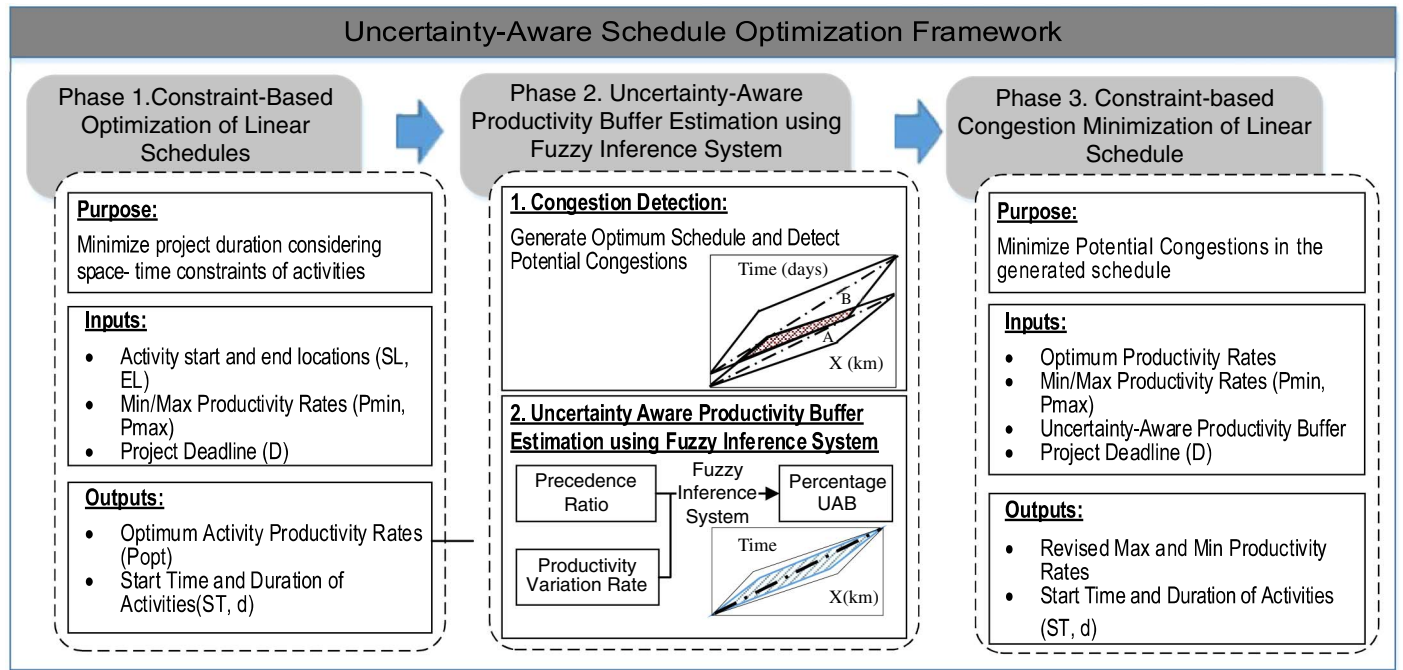
Fig. 3. Uncertainty-aware schedule optimization framework

This phase establishes a minimum duration schedule with due consideration to all spatio-temporal constraints and flexibilities of linear activities. In order to facilitate the use of CP algorithms in scheduling problems, a powerful optimization package termed *IBM ILOG CPLEX* Optimization Studio (Beck et al. 2011) was developed. A CP Optimizer engine is incorporated into the package to solve scheduling problems. *IBM ILOG CPLEX* Optimization Studio was used, and the ILOG OPL language was adopted in this study as the model formulation language.

In the problem specification stage of the optimization, the objective and the variables are determined. The objective of this phase is to minimize the duration, and the decision variables include the start time (ST) and optimum productivity rates (Popt) of all linear activities. The search engine then explores these options and finds the optimum productivity rate for each activity that minimizes the overall project duration. The following constants, variables, constraints, and objectives were adopted in generation of the first phase of the CSP-based model.

Constants
The values of constants will not change during problem solving of the CSP. These constants include
SL_i = start location of Activity i;
El_i = end location of Activity i;
$P_{\min,i}$, = minimum productivity rate of Activity i;
$P_{\max,i}$, = maximum productivity rate of Activity i;
$L_{t\min i,j}$ = minimum time required between Activity i and Activity j;
$L_{t\max i,j}$ = maximum time required between Activity i and Activity j; and
D = project deadline; where $i, j \in \{0, \ldots, n\}$, n: number of project activities.

Decision Variables
P_{opti} = optimum resource productivity rate of Activity i;

$$P_{opti} \in \{P_{\min,i}, P_{\max,i}\}$$

ST_i = start time of Activity i;

$$ST_i \in [0, D];$$

where $P_{opt,i}$ = optimum productivity rate of Activity i; ST_i = start time of Activity i; $P_{\min,i}$ = minimum productivity rate of activity i; and $P_{\max,i}$ = maximum productivity rate of Activity i.

Constraints
The precedence relationships between activities are considered as the main constraints applied to the optimization model. In this study, all types of precedence relations are taken into account, namely: finish to start (FT), start to finish (SF), finish to finish (FF), and start to start (SS). In addition, the model is not limited to the constraints between succeeding activities. Accordingly, any type of constraint between any two activities is taken into account. The minimum and maximum times required between subsequent activities are also accounted for when considering the precedence relationships between succeeding activities. The following fixed constraints are also applied to the optimization model.

Fixed constraints $ST_i \geq 0 \quad ET_i \leq D \quad ET_{la} \leq D \quad ST_{fa}=0$

where ET_i = end time of Activity i; ET_{la} = last activity of the project; and ST_{fa} = first activity of the project.

Objective Function

$$\text{Minimize}[\text{Max}(ET_i)], \qquad \text{For all } i \in \{1, \ldots, n\} \qquad (1)$$

$$ET_i = ST_i + \frac{EL_i - SL_i}{P_{opti}} \qquad (2)$$

The output of this phase is the start time and the optimum productivity rates for all project activities that lead to the minimum project duration. Activity durations are then calculated using Eq. (3)
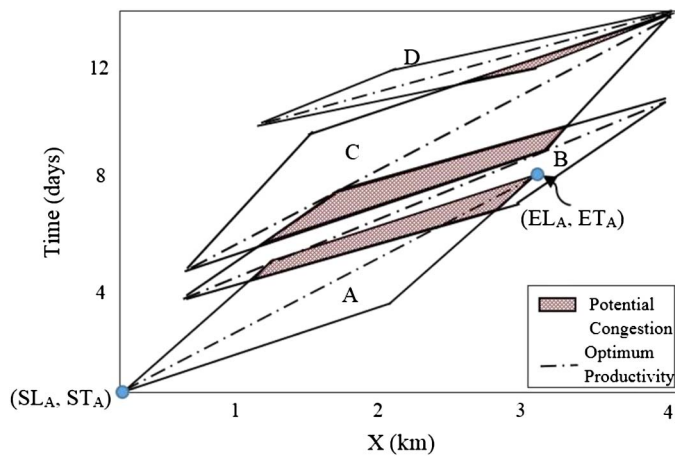
Fig. 4. Detection of potential congestions between activities

Fig. 5. Uncertainty-aware productivity buffer

$$D_i = ET_i - ST_i \tag{3}$$

The start and end coordinates, i.e., start and end location and times, of all activities along with their maximum and minimum productivity rates are then used in the second phase to generate the schedule in the form of space-time float polygons and to calculate uncertainty-aware productivity buffers.

Phase 2. Uncertainty-Aware Productivity Buffer Estimation Using Fuzzy Inference System

After the schedule is optimized in the first phase and optimum project duration along with the optimum productivity rates for each activity are identified, the schedule is generated in this phase. The second phase is modeled in a *MATLAB* environment. To transfer the data generated in the previous phase, a spreadsheet interface is used between the *CPLEX* and *MATLAB* environments. The data transferred includes the number of project activities, activity IDs, and the information required to generate space-time float polygons for each activity as shown in Fig. 4. This information includes the start and end coordinates of all activities in the form of (SL, ST) and (EL, ET), respectively, their optimum productivity rates (P_{opt}) as well as minimum and maximum productivity rates (polygon boundaries). The potential congestions in the generated schedule are then detected as follows.

Space-Time Congestions

As it is shown in Fig. 4, the generated schedule may contain overlaps between the space-time float polygons of succeeding activities. Such overlaps accordingly demonstrate the potential congestion between resources of the overlapping activities caused by deviations from planned productivities. Deviations from planned productivity rates of linear activities cause their respective resources to be placed in the same space over the same period of time, resulting in congestions on the job site. Such deviations mainly occur due to uncertainties in weather, design, labor efficiency, equipment efficiency, and site conditions. These unforeseen uncertainties need to be anticipated and accounted for upfront, i.e., in the planning stage, to be able to efficiently address any changes occurring during execution. Identification of places and times where congestions are probable to happen provides a potential to minimize delays caused by such congestions. To this end, the method presented here introduces a novel approach to account for uncertainties associated with planned productivities of linear projects in activity level. For this purpose, a new type of buffer named uncertainty-aware productivity
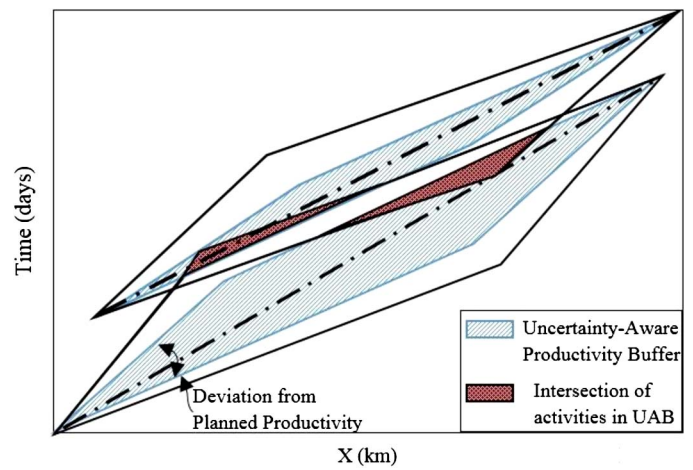
buffer is introduced and estimated using the fuzzy inference system as described in the next section.

Uncertainty-Aware Productivity Buffer

The uncertainty-aware productivity buffer (UAB) presented in this study is defined as the maximum tolerable (versus potential) deviation (uncertainty) from the planned productivity rate whose impact on the schedule needs to be minimized (Fig. 5). In other words, a certain percentage deviation from planned productivity is accepted to occur when planning activities. Any congestions within the accepted buffer, as presented by darker-shaded areas in Fig. 5, need to be prevented from occurring through improvement of the schedule. In contrary to the other methods that model uncertainty buffer associated with activity durations, the UAB considers the uncertainty associated with planned productivity rates of the linear activities. This buffer also enables due consideration to both time and space uncertainties. It should be mentioned that uncertainty-aware productivity buffer is considered here in activity level, and is different from one activity to another. A fuzzy inference system is proposed for calculation of the buffer for individual project activities.

Various project-based and activity-based factors can affect the uncertainty buffer that needs to be considered for each linear activity. In the proposed framework, contextual information about precedence relationships and productivity variation of the activities form the input variables of the fuzzy inference system, and the activities' uncertainty-aware buffer is the output. Although theses contextual variables have an effect on the amount of uncertainty buffer, their boundaries, as well as a level of contribution to the buffer are fuzzy as they are highly dependent on the activity itself and also on the project's particular conditions. These boundaries need to be estimated by experts through judgmental statements that are vague and imprecise. As such, fuzzy inference system was selected as a means for formulating the mapping from these input variables to the output using fuzzy logic. Consequently, uncertainty is captured by the notion of belonging to a range. The design parameters of the fuzzy system, in addition to their definitions, and the designed inference rules are subsequently explained.

Fuzzy Input Variables

To better account for the inherent uncertainties and to more accurately define the buffer size, various distinctive project attributes need to be considered. To calculate the UAB size for individual activities, first, the factors causing uncertainties need to be identified. In this study, precedence ratio (PR) and productivity variation
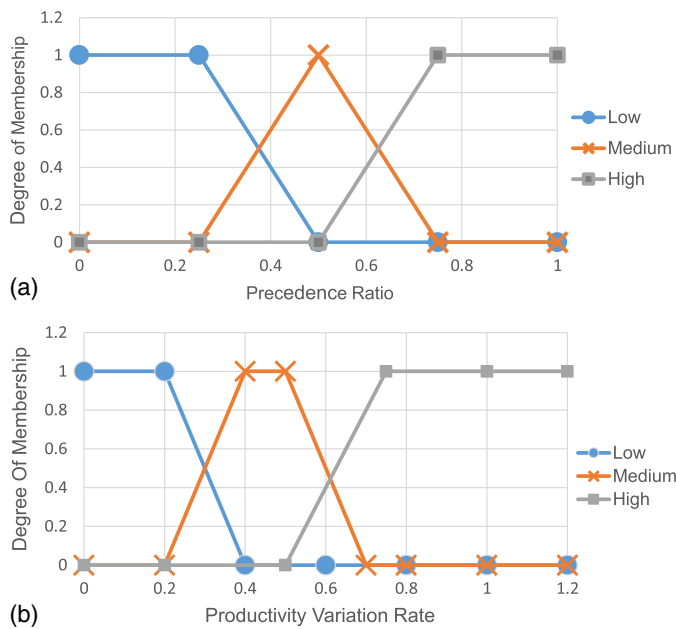
(a)

(b)

Fig. 6. Fuzzy input variable membership function: (a) PR membership function; (b) PVR membership function

rate (PVR) are considered as the two factors that can define the tolerability in deviation from planned productivity. These variables are arbitrarily used on the basis of availability and management judgment. The definition of the variables follows.

Precedence Ratio
The contextual variable precedence ratio implies the relevance of a particular activity with other activities in the project network. When an activity has a greater number of successor activities in the project, its uncertainties, and resulting delays and distortions, are more likely to influence its succeeding activities and cause delays in the project. Thus, a smaller uncertainty-aware productivity buffer is desirable for this activity. The PR can be measured using Eq. (4)

$$PR_i = N_{Succ,i}/N_{A,i} \qquad (4)$$

where $N_{succ,i}$ = number of the successors of Activity i; and $N_{A,i}$, = number of activities in the path(s) that contains Activity i. The PR fuzzy variable can have values of high, medium, and low, depending on the ratio. Fig. 6(a) shows the membership function for this input variable.

Productivity Variation Rate
The contextual variable productivity variation rate (PVR) is directly related to the space-time floats of linear activities and represents the flexibility of a particular activity. The flexibility of a linear activity here is defined as its productivity boundaries from minimum to maximum productivity rates, used to generate a space-time float polygon for that activity. In other words, linear activity flexibilities are presented by the possible variation from the optimum productivity rate of that activity within its productivity boundaries. It has been found through previous studies that activities with less flexibility are less likely to be completed on time (Yang et al. 2009), and therefore, larger buffers are required for these activities. The value of the PVR of Activity i can be determined as shown by Eq. (5)

$$PVR_i = \frac{P_{\max,i} - P_{\min,i}}{P_{\mathrm{opt},i}} \qquad (5)$$
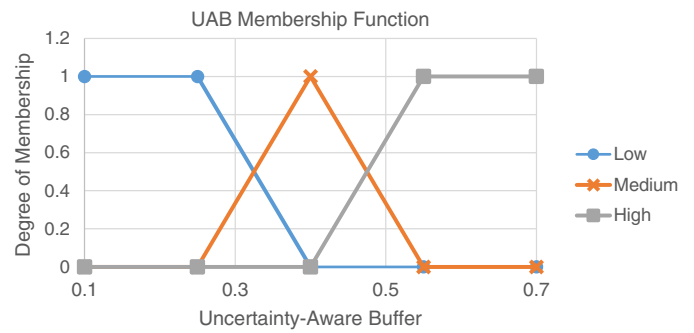

Fig. 7. Fuzzy output variable membership function

where PVR_i = productivity variation rate for Activity i; $P_{\max,i}$, $P_{\min,i}$, and $P_{\mathrm{opt},i}$ = maximum, minimum, and optimum productivity rates of Activity i, respectively. Consequently, when Activity i has low flexibility and thus small space-time float, it imposes a higher risk on the on-time completion of succeeding activities and accordingly requests for a higher buffer. In contrast, when Activity i has high flexibility and thus large space-time float, it is less likely to delay succeeding activities, and accordingly, a smaller amount of buffer needs to be assigned. In this fuzzy inference system, PVR takes values of high, medium, and low depending on the flexibility of the activity. Fig. 6(b) presents the membership function for this input variable.

Fuzzy Output Variable
The output variable is the uncertainty-aware buffer size that is presented in terms of percentage variation from optimum productivity rate and is calculated based on the contextual data as described above. Similar to the input variables, fuzzy values of low, medium, and high can be assigned to the UAB. Fig. 7 presents the membership function for the fuzzy output variable and its three fuzzy values.

Fuzzy Inference Rules
Fuzzy inference rules map input into output variables. As discussed earlier, both factors in this study have an inverse relationship with the final result. In other words, the more successors an activity has, the less buffer should be assigned to that activity. Likewise, the more flexible an activity is, the less uncertainty buffer is required for that activity. A sample rule from the fuzzy inference rules engine is "If (PR is Low) or (PVR is Low) then (UAB is High)." The final output of the ruling process is then defuzzified, and an absolute number in the buffer range for the activity is achieved. The surface generated from fuzzy inference system rules is illustrated in Fig. 8.

Defuzzification
To convert a fuzzy value to an absolute number, defuzzification method is used. The buffer size (in terms of percentage) is then used in the third phase for minimizing potential congestions in the generated schedule. There are three methods of defuzzification: center of gravity, center of sums, or mean of maxima. Because of the simplicity of calculation, the simple center of gravity or centroid method was chosen for use in this study.

The output of the second phase identifies the uncertainty-aware productivity buffer size for each activity. After the UABs are calculated for all activities in this phase, this data is transferred to the third phase to be used as one of the constraints applied to the congestion minimization model. The schedule is then revised and improved to minimize the detected congestions.
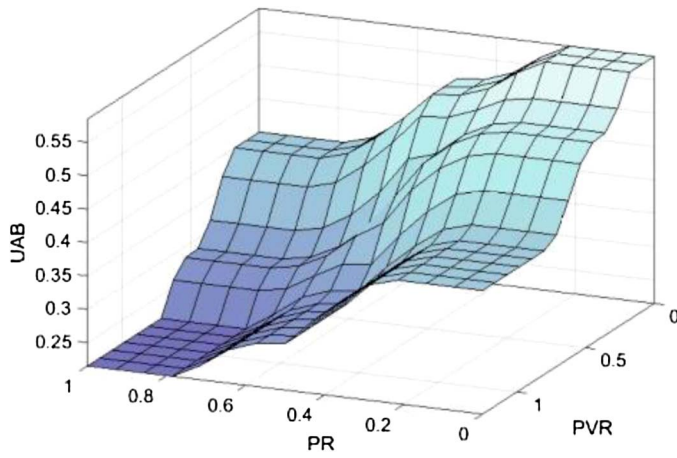
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Fig. 8. Fuzzy inference system output surface

Phase 3. Constraint-Based Congestion Minimization of Linear Schedules

The purpose of the third phase is to minimize the summation of potential congestion areas between all linear activities. For this purpose, the optimization results of the first phase, i.e., optimum productivity rates of activities and their respective time interval, as well as the uncertainty-aware productivity buffers, calculated in the second phase are transferred and used as known information. This information is used to reoptimize the schedule generated in Phase 1 to minimize the detected congestions between activities in the second phase and avoid congestions in the uncertainty-aware productivity buffer areas.

As discussed earlier and shown in Fig. 4, the overlaps of space-time float polygons, i.e., potential congestion between activities, are detected in the *MATLAB* model. To detect and quantify the potential congestion between activities, a matrix is generated in which the overlap areas are calculated. In that matrix, if the element aij is nonzero, it means there exist potential congestion between activities i and j. Essentially, the element aij will be equal to zero if no potential congestion exists between the two activities i and j.

In this optimization phase, decision variables include polygon boundaries, i.e., minimum and maximum productivity rates of activities, as well as the start time of activities. To minimize the potential congestion (overlap areas) between activities, the space-time float polygon for either of the overlapping activities (or both) becomes narrower on the congested side, or the start time of the activities is moved. Changing the boundaries of the polygon only on the congested side provides the advantage of not reducing the overall space-time float available to activities where it is not necessary. Therefore, the floats are only compensated where their existence leads to the occurrence of congestion between activities. The optimization inputs and outputs are listed below.

Constants

SL_i = start location of Activity i;

EL_i = end location of Activity i;

$P_{opt i}$ = optimum resource production rate of Activity i (achieved from Phase 1);

$L_{t \min i,j}$ = minimum time required between Activities i and j;

$L_{t \max i,j}$ = maximum time required between Activities i and j;

UAB_i = uncertainty-aware productivity buffer for Activity i (in percentage);

D = project deadline

Decision Variables

$P1_{min,i}$ = minimum productivity rate of Activity i, which minimizes congestions

$$P1_{min,i} \in [P_{min,i}, P_{opt,i}]$$

$P1_{max,i}$ = maximum productivity rate of Activity i, which minimizes congestions

$$P1_{max,i} \in [P_{opt,i}, P_{max,i}]$$

ST_i = start time of Activity i

$$ST_i \in [0, D]$$

Constraints

The precedence relationships between activities are again considered as the main constraints applied to the optimization model. The uncertainty-aware productivity buffers are also the hard constraints added to the optimization model. For this purpose, first the area of the UAB for each activity is computed, and accordingly, no overlap is allowed to occur within the UAB area. To do so, $A_{UABi,j}$ is calculated as the summation of uncertainty buffer overlap areas between Activities i and j. The total uncertainty buffer congestion areas in the schedule (A_{UAB}), calculated using Eq. (6), is set to zero to prevent these congestions from happening

$$A_{UAB} = \sum_{j=1}^{n} \sum_{i=1}^{n} A_{UABi,j} = 0, i,j \in \{1, \ldots, n\} \quad (6)$$

Objective Function

After all the constraints are defined, the total congestion area in the schedule is calculated and minimized using Eq. (7)

$$min \left(A_{cong} = \sum_{j=1}^{n} \sum_{i=1}^{n} A_{cong i,j} \right); \qquad i,j \in \{1, \ldots, n\} \quad (7)$$

where $A_{cong,i,j}$ = congestion area between space-time float polygons of Activities i and j; and A_{cong} = total congestion area available in the generated optimum schedule.

The congestion areas used in Eqs. (6) and (7) are calculated as the area of a polygon formed by the overlap between activity space-time float polygons or uncertainty-aware productivity buffer areas. As such, the algebraic area calculation formula for closed polygons with known coordinates of vertices is used as shown in Eq. (8)

$$A = \left| \frac{(x_1 y_2 - y_1 x_2) + (x_2 y_3 - y_2 x_3) + \cdots + (x_n y_1 - y_n x_1)}{2} \right| \quad (8)$$

where $x1$ = x coordinate of Vertex 1; and yn = y coordinates of the nth vertex. The vertices are numbered in order, going either clockwise or counterclockwise, starting at any vertex. Examples of triangular and rectangular intersections are depicted in Fig. 9. The coordinate of the intersection vertices is presented in (xi, ti) format. However, to use in the optimization model, the vertices need to be formulated using decision variables, i.e., $P1_{min,i}$, $P1_{max,i}$, and ST_i. This is done through generating a parametric system of linear equations for the intersecting sides of the activity polygons as shown in Eqs. (9) and (10)

$$x_i - SL_i = P1_{min,i}(t_i - ST_i) \quad (9)$$

$$x_i - SL_j = P1_{max,j}(t_i - ST_j) \quad (10)$$

After solving the parametric system of linear equations, the intersection points are identified in terms of start time (ST) and
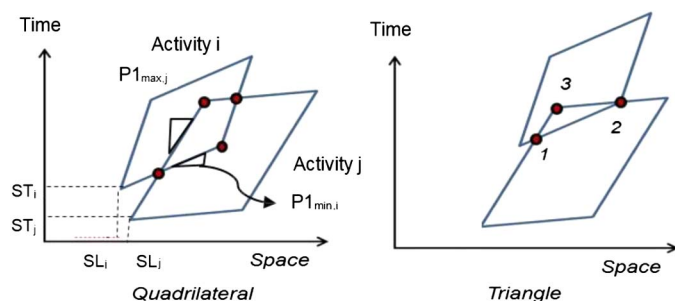
Fig. 9. Different congestion areas between polygons

Table 1. Input Data to the Optimization Process

Task name	Task	SuccsId	SL	EL	P_{max}	P_{max}
Ditch excavation	1	2,3	0	50	3.3	10
Culvert installation	2	—	0	50	1	5
Concrete pavement removal	3	4,5	0	50	1.67	5.83
Peat excavation and swamp backfill	4	5	30	50	8	12
Embankment	5	6	0	50	2.5	8.75
Utility work	6	7	30	50	10	15
Sub-base	7	8	0	50	2.56	6.41
Gravel	8	9	0	50	5	12.5
Paving	9	—	0	50	8.33	20.83

Note: Number of tasks 9, deadline 38.

Table 2. Output of the Optimization

Activities	Start	End	Duration	OptPro
Ditch excavation	0	12	12	4.17
Culvert installation	0	4	4	2.00
Concrete pavement removal	3	24	21	2.38
Peat excavation and swamp backfill	5	7	2	6.00
Embankment	7	26	19	2.63
Utility work	26	28	2	10.00
Sub-base	12	32	20	2.50
Gravel	24	34	10	5.00
Paving	30	36	6	8.33

minimum and maximum productivity rates ($P1_{min}$ and $P1_{max}$) of the intersecting polygons. The coordinates of the intersection points are then used in Eq. (8) to calculate the congestions in the schedule. Similarly, the congestion areas within the identified uncertainty-aware buffers of activities is computed and formulated using Eqs. (8)–(10). This area is then set to zero to prevent overlaps from occurring within the uncertainty buffers of activities. The optimization process then optimizes the schedule by minimizing the total congestion in the project network.

The output of this phase, i.e., the revised minimum and maximum productivity rates of activities (revised polygon boundaries) and start and end time of activities, is then used to generate the revised schedule. The step-by-step application of the proposed framework to two numerical examples drawn from literature follows.

Case Study

The optimization framework presented here was applied to two case studies from the literature to verify the proposed method. The first case study examines the efficacy of the first stage of the optimization framework in generating optimum duration for linear projects with due consideration to their spatio-temporal constraints and flexibilities. The second case study examines the three-phase optimization framework in practically optimizing schedules of linear projects.

Case Study 1

The first example is previously presented in the literature (Mattila and Abraham 1998; Georgy 2008; Tang et al. 2014a, b). This highway construction project was used by various researchers for verifying the resource leveling, scheduling, and optimization capabilities of their proposed models. The same project is used here to verify the capabilities of the first phase of the proposed framework in generating an optimum duration project schedule that takes into account space and time constraints in addition to logical constraints. Although no congestion was detected in the optimum schedule generated, the first phase of the optimization achieved 36 days duration for the project, which is two days shorter than other methods. As such, the results of the first phase are presented and compared with other methods here. The minimum and maximum productivities are calculated from the minimum and maximum resources available in the original example. The project network of this example consists of nine activities. The project included widening of a segment of U.S. Route 41, located in northern Michigan. The main activities include removal of existing concrete paving, ditch excavation, embankment, sub-base, gravel, and bituminous paving. The duration presented in the literature for this

example is 38 days. The description of each activity, as well as inputs and outputs of the first optimization phase, are listed in Table 1.

As it is shown in Table 1, the input data of the optimization process includes activity IDs, start and end locations, minimum and maximum achievable productivity rates, and successors of all activities. This information is used in the first phase to look for the lowest duration of the project. If some activities are required to start or finish on a certain day, this information is also included as a constraint. The output of this process identifies optimum productivity rates as well as start and end times of all activities. The optimum duration for each activity is then calculated from the optimum productivities attained in the optimization process. The achieved optimum productivity rates are listed in Table 2. The computed optimum duration for the project is 36 days, which is two days shorter than the other methods shown in Table 3 (Mattila and Abraham 1998; Georgy 2008; Tang et al. 2014a).

Table 3 also presents the comparison of durations achieved previously for this example versus the results of this study. By considering the whole range of possible productivity rates for each activity, the current method was able to relax some activities (Activities 1, 5, 7, and 8) causing less required resources per day. If nonrepetitive activities also exist in the project, the start time is calculated based on the precedence relationships with their predecessors and successors.
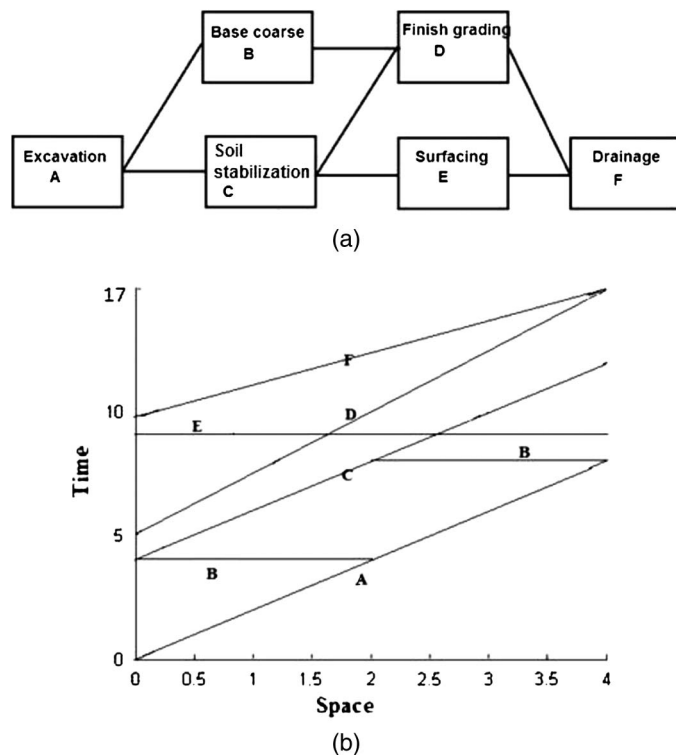
This simple example demonstrates the ability of the proposed method to derive alternative plans to meet project deadlines. The generated optimized schedule shows that generating schedules with due consideration of the space-time float for each activity enables better scheduling and control of linear projects.

Case Study 2

The second case study is adapted from the literature (Georgy 2008) and is modified to verify the capabilities of the ultimate framework

Table 3. Comparison of the Results

Method	Total duration (days)
Study by Mattila and Abraham (1998)	38
Study by Georgy (2008)	38
Study by Tang et al. (2014a)	38
Current study	36

(a)

(b)

Fig. 10. (a) CPM network; (b) original LSM schedule

in optimizing duration of linear schedules and minimize their potential congestions. In this example, the minimum and maximum productivity rates are hypothetical. However, in the real-world examples, the information about activity productivity ranges as well as specific project and activity constraints can be obtained from the project team. The project includes construction of a 4-km altering access road. Four activities (A, C, D, and F) are assumed to be linear activities while the other two (B and E) were considered to be the delivery of the required material. Activities B and E are therefore assumed nonrepetitive and, as such, are represented by horizontal lines. Activity B is the delivery of base coarse aggregates and is planned to be delivered in two steps, at days four and eight, and is illustrated with two horizontal lines. The CPM network of the project in addition to the original linear schedule

are presented in Fig. 10. The description of each activity along with inputs and outputs of the optimization process are included in Table 4.

The project was initially scheduled to be completed in 17 days. Using the proposed optimization framework, an optimum duration equal to 15 days was achieved from the first phase of optimization. The results of the first phase, i.e., optimum productivity rates and durations, were used to generate a schedule using space-time float polygons and to calculate uncertainty-aware buffers for the activities in the second phase.

As it is illustrated in Fig. 11(a), there exist overlaps between space-time float polygons of subsequent activities, which accordingly represent potential congestions in the generated schedule. As seen in Fig. 11(a), these potential congestions exist between Activities C and D, as well as between Activities D and F. This calls for minimizing these congestions in the third optimization phase. To this end, first, the uncertainty-aware productivity buffer sizes are calculated for all linear activities using the developed fuzzy inference system. The inputs and output of the fuzzy inference system are listed in Table 5. Fig. 12 also shows example fuzzy rules for Activity D. As shown in Fig. 11(a), there exists an overlap between the uncertainty-aware buffer of Activities C and D that should be eliminated.

The output of this phase is then used in the third phase to minimize existing potential congestions in the schedule. This input data includes activity IDs, their start and end locations, successors, minimum and maximum productivity rates, as well as optimum productivity rates from the first phase and uncertainty buffers from the second phase. This information is used to search for the optimum duration of the project that is associated with the least potential congestion in the schedule. The project deadline (17 days) and uncertainty-aware buffers are inputted as hard constraints that cannot be violated. The output of this process includes optimum minimum and maximum productivity rates for all repetitive activities as well as start and end times of all the activities. The optimum duration achieved in this process might be longer than the previously determined duration for the project but still satisfies the deadline constraint. Fig. 11(a) shows the schedule generated in the second phase, using optimization output from the first phase and activity productivity boundaries. The optimum schedule achieved in the third phase is also shown in Fig. 11(b). As seen in Fig. 11(b), all the potential congestions detected in the second phase are eliminated in the optimum schedule achieved in the third phase.

Table 5 shows the results of the schedule after the proposed three-phase optimization process. It can be inferred from Table 5 that the scheme of the schedule generated is significantly different from that of the initial schedule in which congestion was not considered. The final schedule finishes at 16 days, which still satisfies the 17-days deadline constraint. Although the achieved duration is one day longer than the previously achieved minimum duration,

Table 4. Input of the Optimization Process (Initial Schedule)

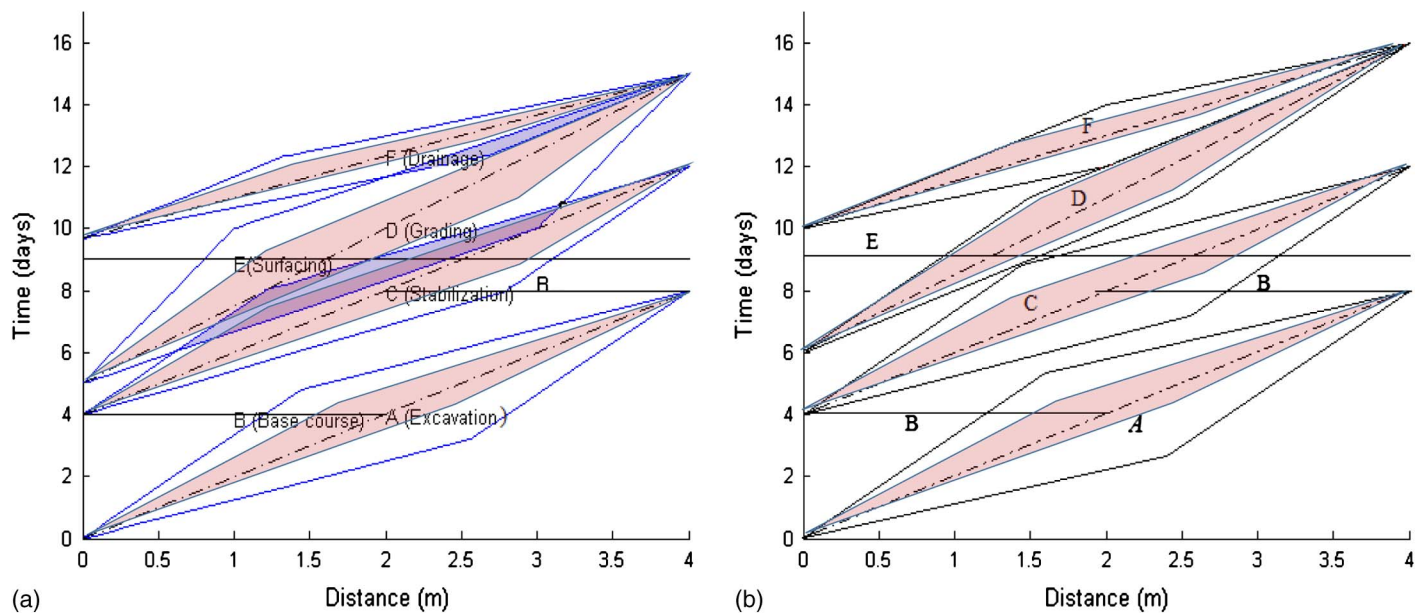| Task	SL	EL	P_{max}	P_{max}	SuccsId	Duration	Productivity
A	0	4	0.3	0.9	{2,3}	8	0.5
B	0	4	Nonrepetitive	Nonrepetitive	{4}	Nonrepetitive	Nonrepetitive
C	0	4	0.3	0.8	{4,5}	8	0.5
D	0	4	0.2	0.7	{6}	10	0.4
E	0	4	Nonrepetitive	Nonrepetitive	{6}	Nonrepetitive	Nonrepetitive
F	0	4	0.5	1	—	6	0.75

Note: Number tasks 6, deadline 17.

Fig. 11. (a) Initial schedule visualized in the second phase versus (b) the schedule with minimized congestion

Table 5. Uncertainty-Aware Productivity Buffers

Task	PR	PVR	UAB (%)
A	0.83	1.2	21.5
B	—	—	—
C	0.6	1	27.6
D	0.2	1.25	40
E	—	—	—
F	0	0.67	43

the total area of potential congestions is reduced to zero in the optimized schedule. For this purpose, as can be seen in Table 6 and Fig. 11(b), Activities D and F are moved by one day, i.e., from day five to six and nine to 10, respectively. Also, the available

productivity interval for Activity D (its minimum and maximum productivity rates) is reduced from [0.2, 0.7] to [0.3, 0.5], which practically means less float will be available to the resources of this activity. The result attained through this example ensures the efficacy of the method in optimizing schedules of linear projects. Although the main contribution of the presented framework is in eliminating potential congestions in the schedule while still satisfying the deadline constraint, the one to two day reduction in duration achieved for this case study is still significant compared to the original duration (17 days). For longer projects, however, due consideration to space and time flexibilities and constraints in addition to other project constraints might potentially lead to more reductions, although it is not within the premises of the present method.
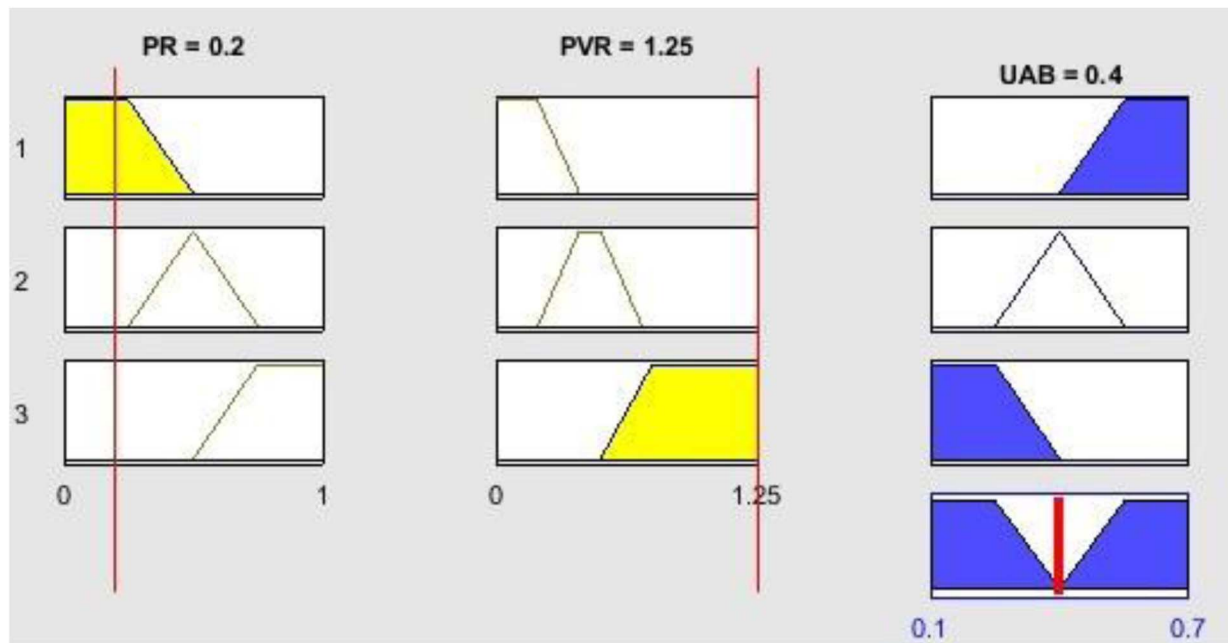

Fig. 12. Example fuzzy rules

Table 6. Optimization Output

Activity	$P_{\max(opt)}$	$P_{\max(opt)}$	ST	ET	Duration
A	0.3	0.9	0	8	8
C	0.3	0.8	4	12	8
D	0.3	0.5	6	16	10
F	0.5	1	10	16	6

Summary and Concluding Remarks

The developments presented in this study, aimed to add to the current scheduling methods for linear construction projects while keeping their essential benefits. The framework presented here contributed to the body of knowledge of scheduling linear projects and augmented the conventional methods in the following areas: (1) taking into account potential congestions when optimizing the duration of linear projects. This, in fact, helps in generation of more realistic schedules for linear projects that are associated with less risk of delays resulted from congestions; (2) incorporating uncertainties inherent to linear activities. This accordingly provides a means to account for unforeseen deviations from plans and preventing their negative effect on linear schedules; and (3) presenting the new concept of an uncertainty-aware productivity buffer to linear activities. Instead of modeling the uncertainty buffer associated with activity duration only, the uncertainty-aware productivity buffer is realized both in terms of time and space that is more suitable to the nature of linear schedules. As such, a significant advantage of the proposed framework is that it can comprehensively convey a detailed work schedule that can be used by construction managers in making projectwide decisions.

The method in its current format only addresses the duration and congestion objectives using space and time factors. Although the cost aspect is not directly integrated into this model, the output still helps the management teams of linear projects to prevent cost overruns through efficiently planning their activities and decreasing potential delays. As an improvement, the cost factor can be added directly into the model in the future to improve the decisions made. Two numerical examples were analyzed to demonstrate the added benefits of the proposed method. The results obtained promise applicability of the proposed method in improving scheduling of linear construction projects. Although the main objective of the proposed framework is to guarantee the generation of a project schedule with minimum potential congestion within the project timeline, as the results illustrate, the optimized schedules not only satisfy the objective but also achieve shorter duration compared to other scheduling methods. As such, reducing the duration of the project, as was achieved for both case studies, is an additional advantage on top of the expected added benefits of the proposed method. As a result, the method provides the project management team with the ability to better understand the unplanned changes in the project and their impact on productivity, time, space, and cost. As such, the proposed framework helps project planners in making alternative plans to prevent/treat the identified issues before execution starts.

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