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Solving Scheduling Problems with CP Optimizer (for CPLEX Users)

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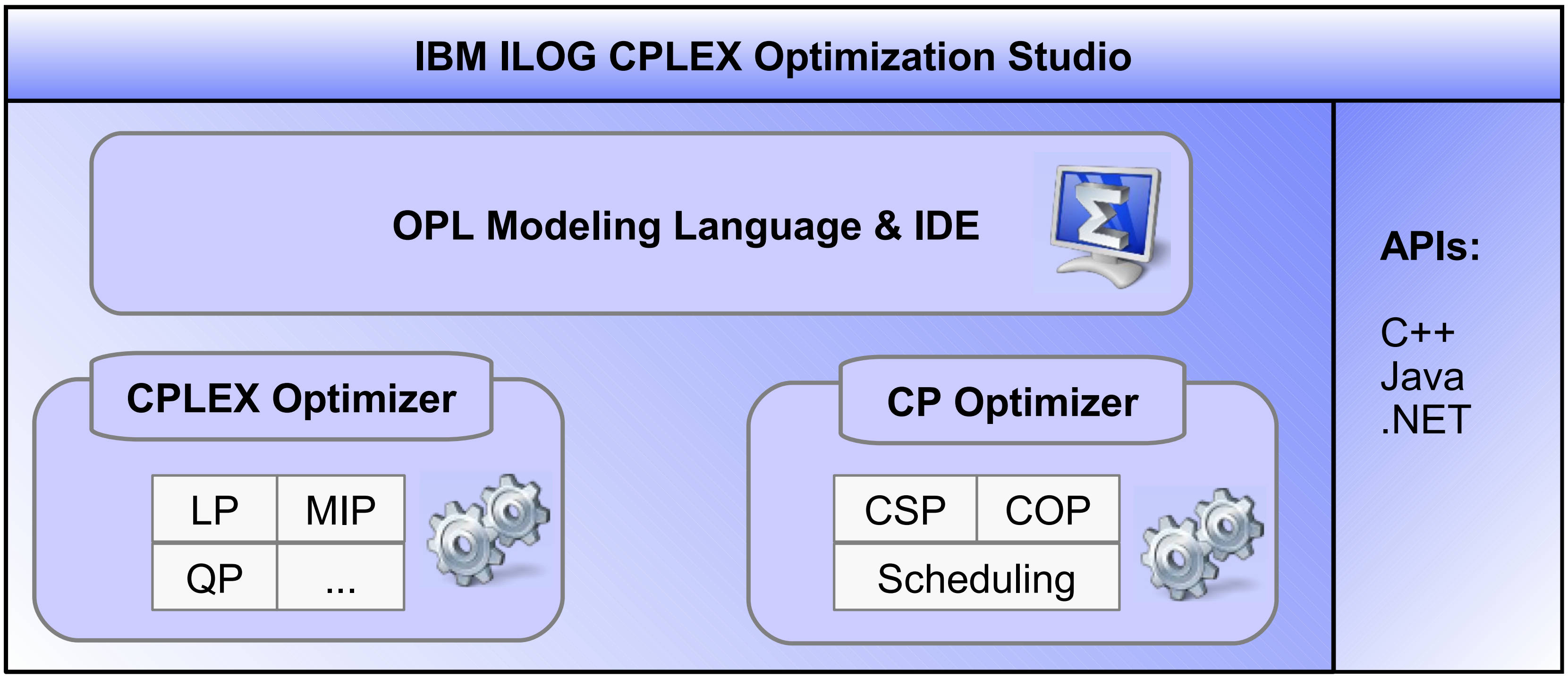
Agenda

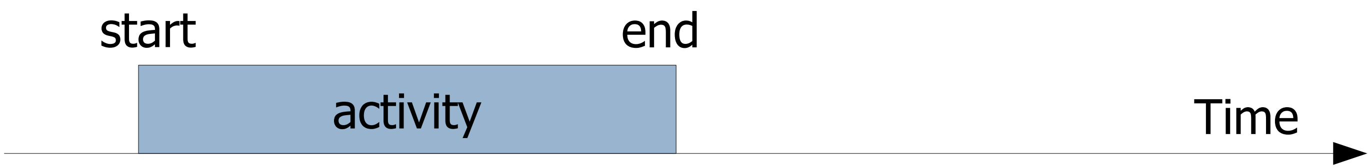
- **Introduction:**
  - **CP Optimizer** a component of CPLEX Optimization Studio
  - **Scheduling**
  - **Example**: Resource Constrained Project Scheduling Problem
- Scheduling **concepts** in CP Optimizer
- **Automatic search** in CP Optimizer
- Some features for **accelerating model development**
- **Q&A**

CP Optimizer, a component of CPLEX Optimization Studio



Scheduling

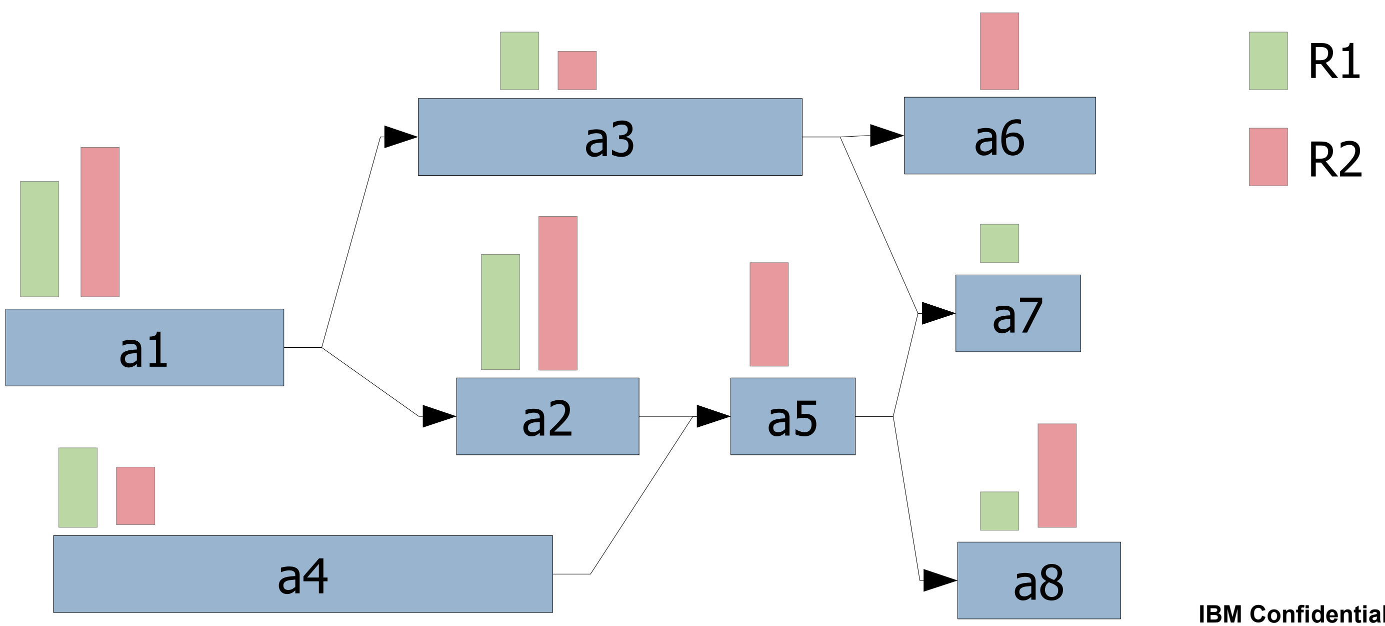
- Scheduling consist of assigning **starting** and **completion times** to a set of activities while satisfying different types of constraints (resource availability, precedence relationships, …) and optimizing some criteria (minimizing tardiness, …)



- Time is considered as a continuous dimension: domain of possible start/completion times for an activity is potentially **very large**
- Beside start and completion times of activities, other types of decision variables are often involved in real industrial scheduling problems (resource **allocation**, **optional** activities …)

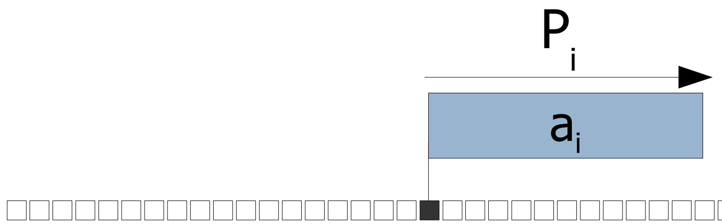
Example: Resource Constrained Project Scheduling Problem

- RCPSP: a very classical academical scheduling problem
  - Tasks a_i with fixed processing time P_i
  - Precedence constraints
  - Discrete resources with limited instantaneous capacity R_k
  - Tasks require some quantity of discrete resources
  - Objective is to minimize the schedule makespan



Example: Resource Constrained Project Scheduling Problem

- RCPSP: Standard time-indexed MIP formulation



$x_{it} \in \{0,1\}$

Standard RCPSP (DT: Discrete Time)

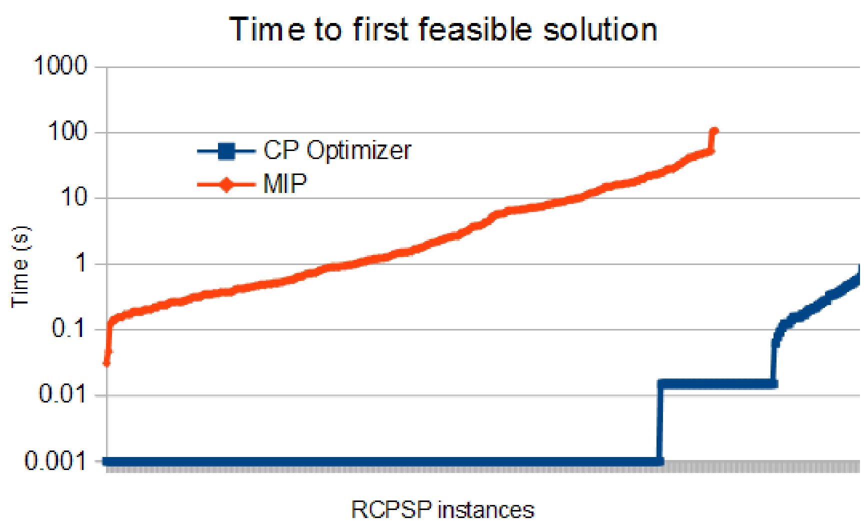
$$
\begin{aligned}
&\text{minimize} \sum_{t \in H} t x_{nt} && \\
&\sum_{t \in H} x_{it} = 1 && \forall i \in \mathcal{A} \\
&\sum_{t \in H} t x_{it} + P_i \leq \sum_{t \in H} t x_{jt} && \forall (i,j) \in \mathcal{P} \\
&\sum_{i \in A, t \leq \tau < t + P_i} Q_{ik} x_{it} \leq R_k && \forall \tau \in H, \forall k \in \mathcal{R} \\
&x_{it} \in \{0,1\} && \forall i \in \mathcal{A}, \forall t \in H
\end{aligned}
$$

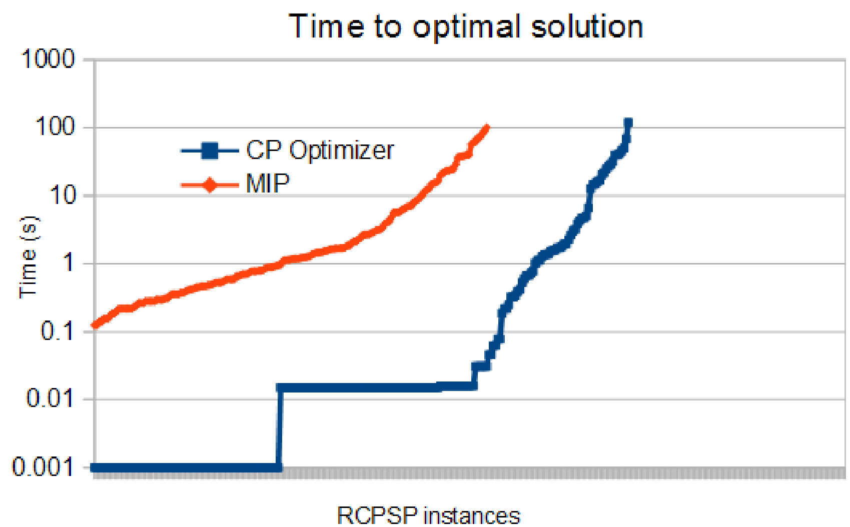
Example: Resource Constrained Project Scheduling Problem

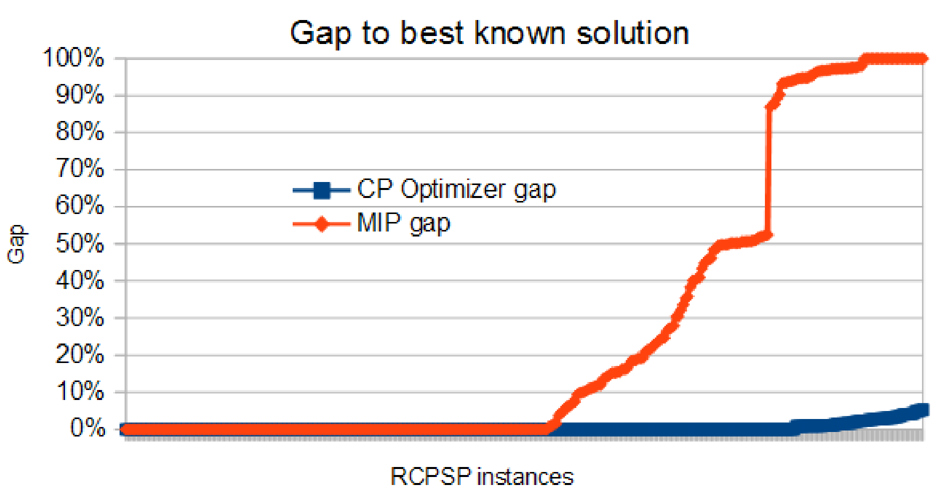
- Comparison of this time-indexed MIP formulation against a simple CP Optimizer model (see later) on a set of:
  - 300 classical **small** RCPSP instances (30-120 tasks) +
  - 40 slightly **more realistic** larger ones (900 tasks)
  - time-limit: 2mn, 4 threads
- Note: industrial scheduling problems are often **larger**, typically several 1.000 tasks (we handled up to 1.000.000 tasks in an RCPSP-like scheduling model)

Example: Resource Constrained Project Scheduling Problem

- Comparison of CP Optimizer and MIP performance on RCPSP







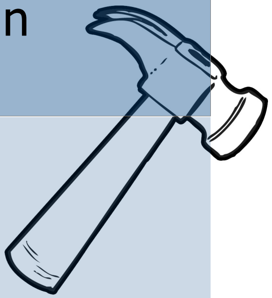
Example: Resource Constrained Project Scheduling Problem

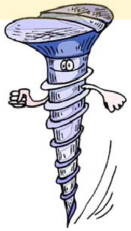
- Some pain points of the MIP approach (also hold for more general scheduling problems) :
  - 1. **Huge number of decision variables**:
    - Time-indexed formulations grow in $O(n.H)$
    - Other formulations (event-based, precedence-based for disjunctive resources) grow in $O(n^2)$
  - 2. The **time dimension is not exploited**. Example: building a first feasible solution by fixing the tasks chronologically is trivial. The MIP engine does not see that.
  - 3. **Global constraints** like resource capacities are split into a number of small individual constraints ($\forall t \in H$) whereas what happens at t+1 is highly correlated with what happens at t.

CP Optimizer for Scheduling

- Modeling and solving scheduling problems

Model & run paradigm	User-implemented solution
MIP (CPLEX)	dispatching rules, heuristics, meta-heuristics





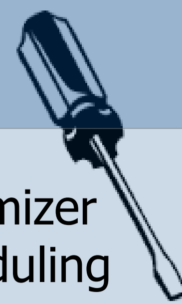
CP Optimizer for Scheduling

- Modeling and solving scheduling problems

Model & run paradigm

MIP (CPLEX)

CP Optimizer for Scheduling



User-implemented solution

dispatching rules, heuristics, meta-heuristics

– Dedicated modeling concepts for scheduling
    - Consistent with Optimization paradigm: new decision variables, expressions and constraints
    - Available in OPL and APIs (C++, Java, .NET)
– Automatic search
    - Complete (provides optimality proofs)
    - Combines tree search, large neighborhood search, meta-heuristics, relaxations, learning, ...

Agenda

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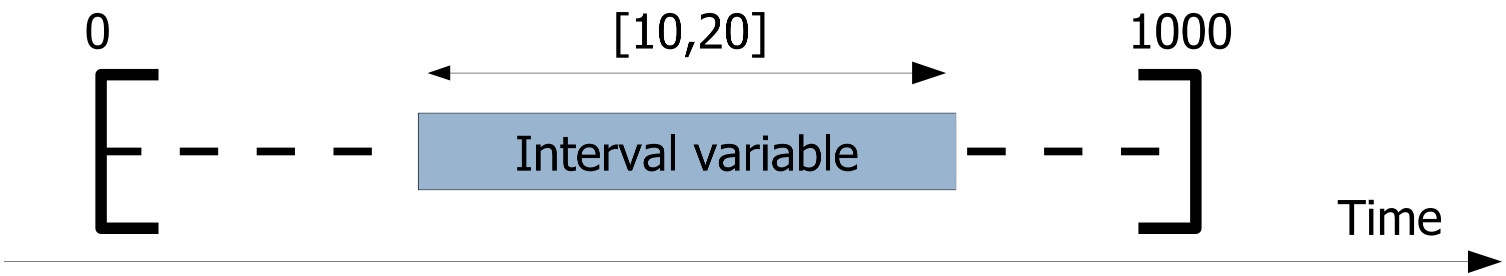
- **Q&A**

Concept: **interval variable**

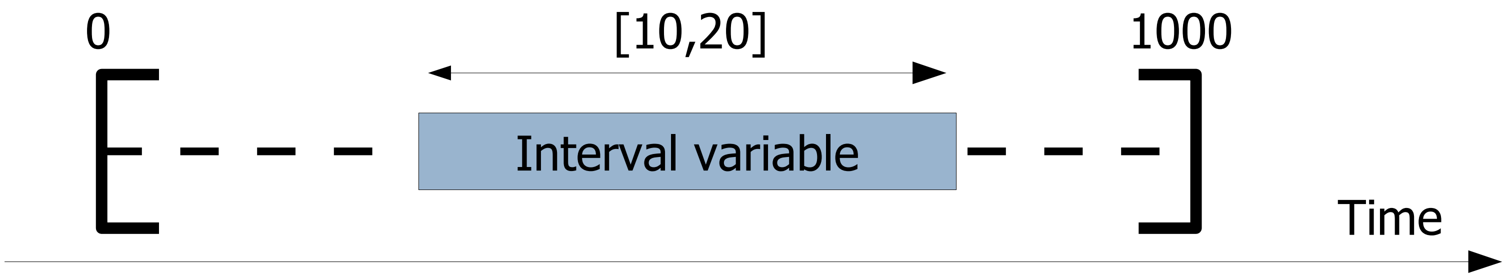
- What for?
  - modeling an interval of time during which a particular property holds (an activity executes, a resource is idle, a tank must be non-empty, …)
- Example:

```
dvar interval x in 0..1000 size in 10..20
```



Concept: **interval variable**

```
dvar interval x in 0..1000 size in 10..20
```



- Properties:
  - The **value** of an interval variable is an integer interval [start,end)
  - **Domain** of possible values: [0,10), [1,11), [2,12),...[990,1000), [0,11),[1,12),...
  - Domain of interval variables is represented **compactly** in CP Optimizer (a few bounds: smin, smax, emin, emax, szmin, szmax)

Concepts: **optional interval variable**

- Interval variables can be defined as being **optional** that is, it is part of the decisions of the problem to decide whether the interval will be **present** or **absent** in the solution
- What for?
  - Modeling optional activities, alternative execution modes for activities, and … most of the discrete decisions in a schedule
- Example:

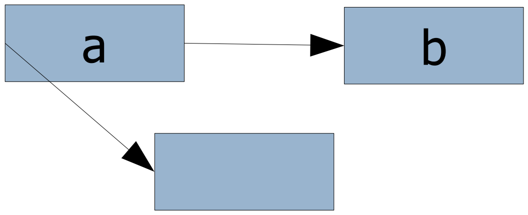
```
dvar interval x optional in 0..1000
                size in 10..20
```

- Properties:
  - An optional interval variable has an additional possible value in its domain (absence value)
  - **Optionality** is a powerful property that you must learn to leverage in your models (more on this later ...)

Concept: **precedence constraint**

- What for ?
  - Modeling temporal constraints between interval variables

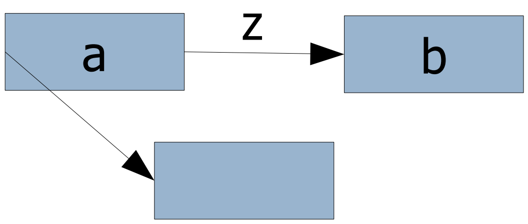
- Example:




```
endBeforeStart(a,b)
startBeforeStart(a,b)
startBeforeEnd(a,b)
endBeforeEnd(a,b)
endAtStart(a,b)
startAtStart(a,b)
startAtEnd(a,b)
endAtEnd(a,b)
```

Concept: **precedence constraint**

- What for ?
  - Modeling temporal constraints between interval variables
  - Modeling constant or variable minimal delays
- Example:




```
endBeforeStart(a,b,z)
startBeforeStart(a,b,z)
startBeforeEnd(a,b,z)
endBeforeEnd(a,b,z)
endAtStart(a,b,z)
startAtStart(a,b,z)
startAtEnd(a,b,z)
endAtEnd(a,b,z)
```

Concept: **precedence constraint**

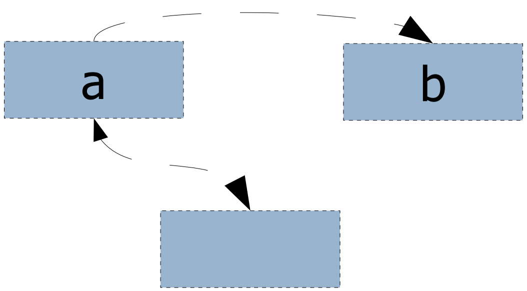
- Properties
  - Semantic of the constraints handles optionality (as for all constraints in CP Optimizer).

    Example of endBeforeStart:

    present(a) AND present(b) $\Rightarrow$ end(a)+z $\leq$ start(b)
  - All precedence constraints are aggregated in a temporal network and handled by dedicated graph algorithms (fast global propagation, negative cycle detection, ...)

Concept: **presence constraint**

- What for:
  - Expressing dependency constraints between execution of optional activities or between allocated resources …
- Examples:




```
 presenceOf(a) =>  presenceOf(b)
 presenceOf(a) ==  presenceOf(b)
 presenceOf(a) => !presenceOf(b)
!presenceOf(a) =>  presenceOf(b)
```

Concept: **presence constraint**

- Properties:
  - All binary constraints between presence status (2-SAT) are aggregated in an implication graph
  - CP Optimizer maintains hypothetical bounds on interval variables (e.g. start min would the interval be present)
  - CP Optimizer exploits the implication graph to perform conditional reasoning between related interval variables

Concept: **expressions on interval variables**

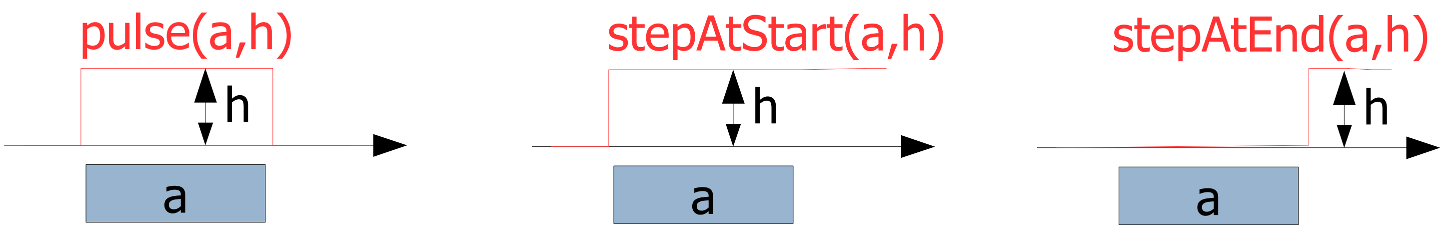
- What for?
  - Evaluating a characteristic of an interval variable (start, end, size) for use in the objective function or in a constraint
- Example:
  - endOf(a, ABSVAL) takes value ABSVAL if interval *a* is absent otherwise it takes value *e* if *a* is present and *a=[s,e)*
  - Typical makespan expression: max(i in 1..n) endOf(a[i])

Concept: **cumul function**

- What for?
  - Modeling and constraining cumulative quantities over time (cumulative use of a discrete resource, level of an inventory with producing and consuming tasks)
  - Restricting the number of intervals that overlap a given date t
  - Forcing a minimal number of intervals to overlap a given date t
  - Complex synchronization constraints between interval variables: e.g. between activities producing/consuming in a tank and the set of time-intervals during which the tank is non-empty

Concept: **cumul function**

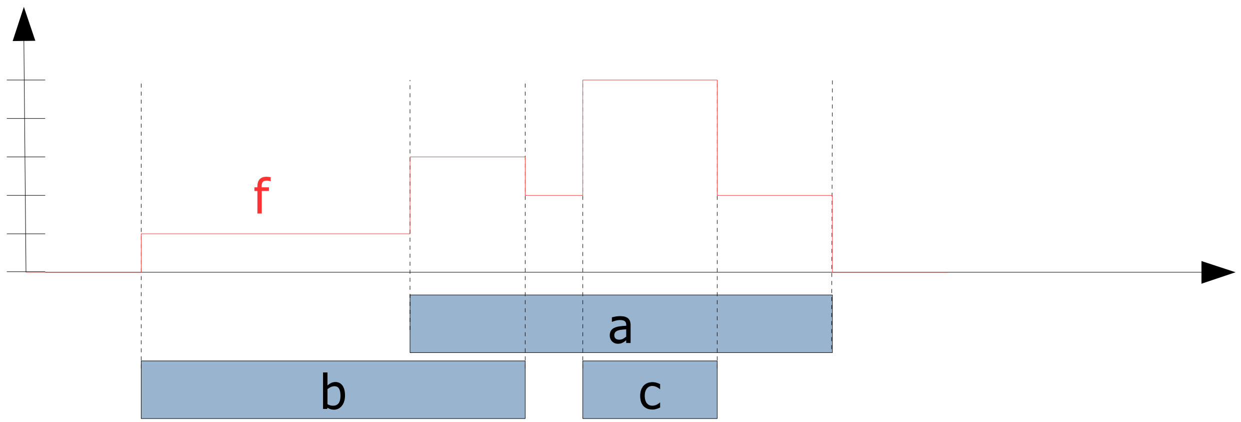
- Cumul functions are built from atomic functions



- A cumul function is the sum of atomic functions or their opposite

Concept: **cumul function**

- Examples:
  - cumulFunction f = pulse(a,2)+pulse(b,1)+pulse(c,3)



  - Constraint f<=C (global capacity limit)
  - Constraint alwaysIn(f,t0,t1,Cmin,Cmax) (capacity profile)
  - Constraint alwaysIn(f,x,Cmin,Cmax) (condition holds over an interval variable x)

Concept: **cumul function**

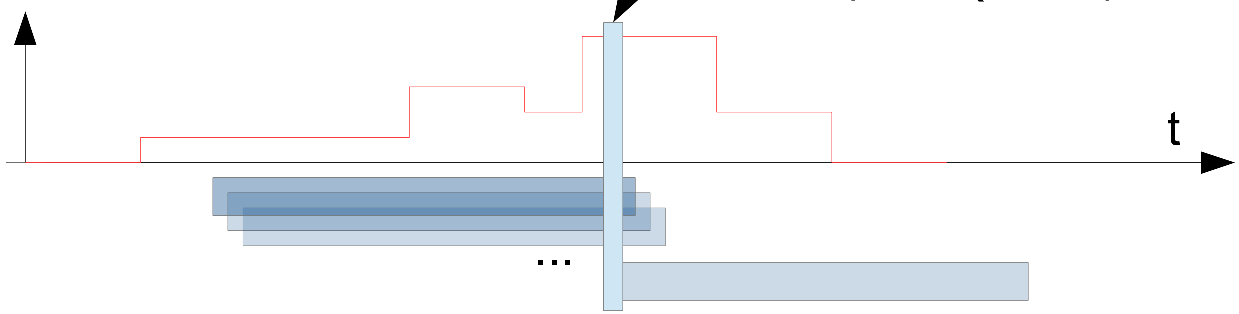
- Properties:
  - Complexity of cumul functions is **independent of the time scale**
  - CP Optimizer is able to reason globally over cumul functions

Concept: **cumul function**

- Properties:
  - Compare:
    - Time-indexed MIP model:

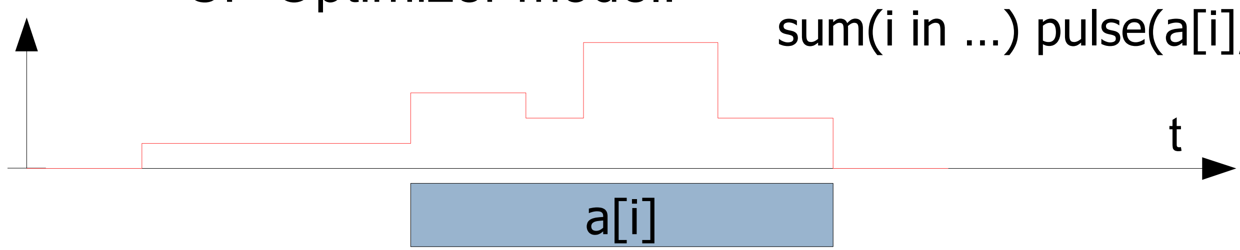
      `x[i][t]∈{0,1}: x[i][t]=1 iff a[i] starts at date t`

      ∀t, sum(i in ..., t' in ...) Q[i]*x[i][t']≤ C



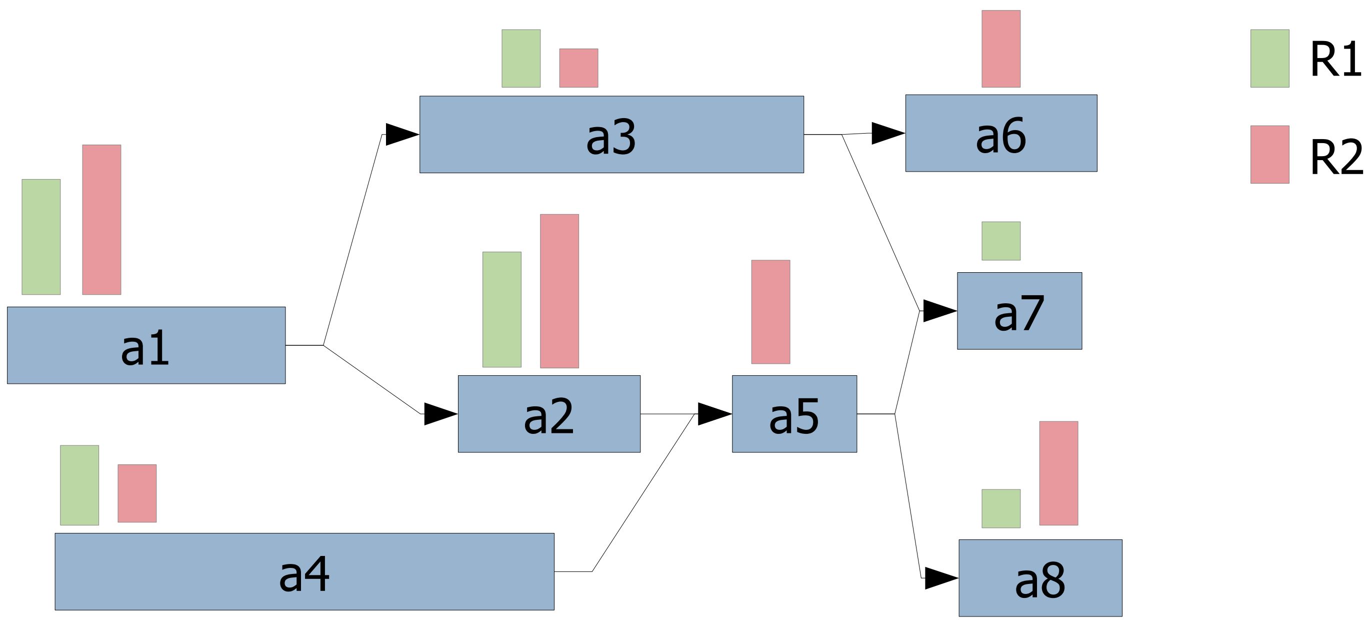
    - CP Optimizer model:

      sum(i in ...) pulse(a[i],Q[i]) ≤ C



Concept: **cumul function**

- Example: Resource-Constrained Project Scheduling Problem (RCPSP)



- Minimization of project makespan

Concept: **cumul function**

- CP Optimizer model for RCPSP:

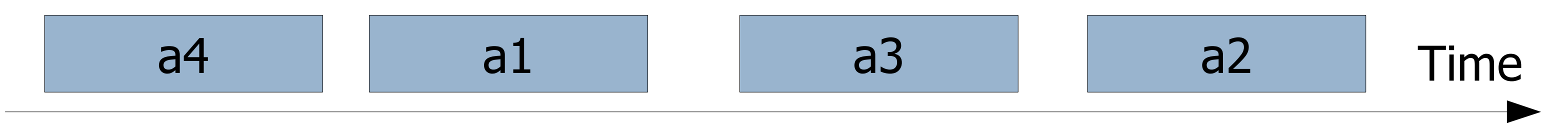
```
dvar interval a[i in Tasks] size i.pt;

cumulFunction usage[r in Resources] =
  sum (i in Tasks: i.qty[r]>0) pulse(a[i], i.qty[r]);

minimize max(i in Tasks) endOf(a[i]);
subject to {
  forall (r in Resources)
    usage[r] <= Capacity[r];
  forall (i in Tasks, j in i.succs)
    endBeforeStart(a[i], a[<j>]);
}
```

Concept: **sequence variable**

- What for?
  - Modeling sequences of events
  - Constraining the transitions between consecutive events (setup times/costs,...)
- Example:

```
dvar sequence s in all(i in Tasks) a[i]
noOverlap(s)
```

- Properties:
  - A value for the sequence variable is a total order on the present interval variables. E.g. a4 → a1 → a3 → a2
  - A constraint noOverlap states that intervals of the sequence do not overlap and follows the total order of the sequence



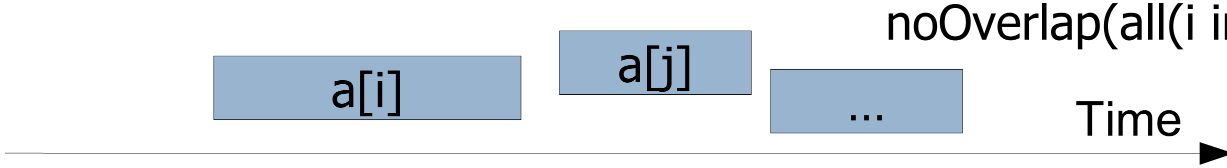
Concept: **sequence variable**

- Properties:
  - Complexity is **independent of the time scale**
  - CP Optimizer is able to reason **globally** over a sequence variable
  - **Avoid quadratic models** over each pair (i,j) of intervals in the sequence
- Compare:
  - Quadratic disjunctive MIP formulation with big-Ms:

```
b[i][j]∈{0,1}: b[i][j]=1 iff a[i] before a[j]
end[i] <= start[j] + M*(1-b[i][j])
end[j] <= start[i] + M*b[i][j]
```

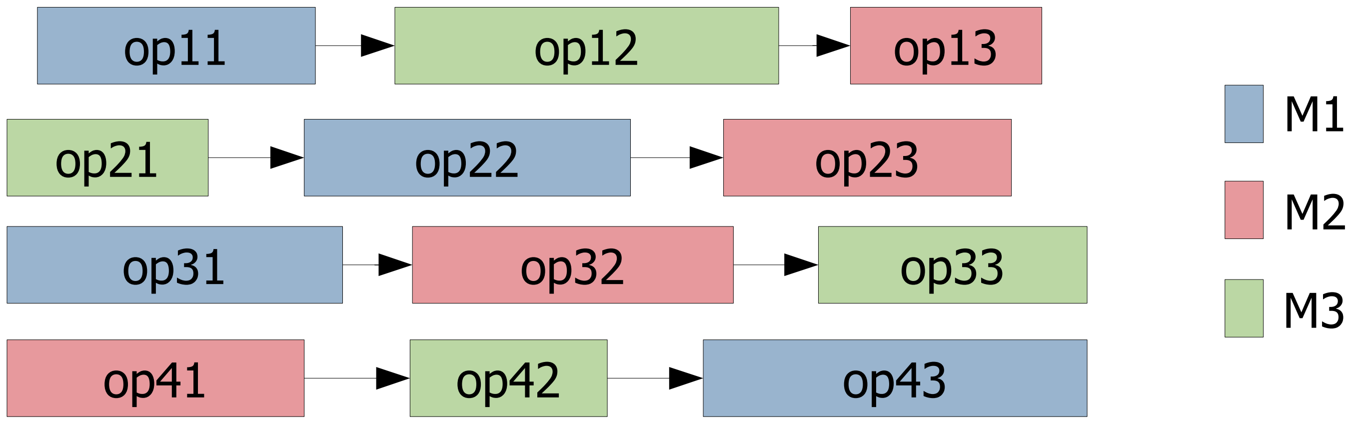
  - CP Optimizer model:

noOverlap(all(i in …) a[i])



Concept: **sequence variable**

- Example: Job-shop Scheduling Problem



- Minimization of makespan

Concept: **sequence variable**

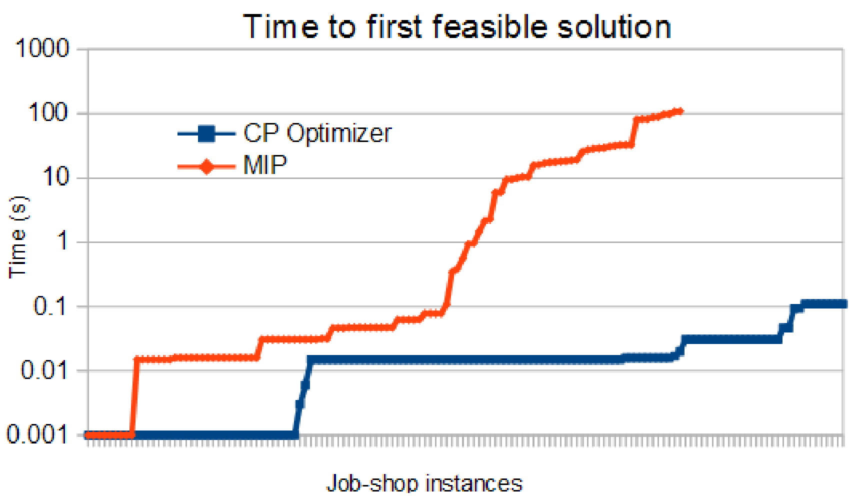
- CP Optimizer model for Job-shop:

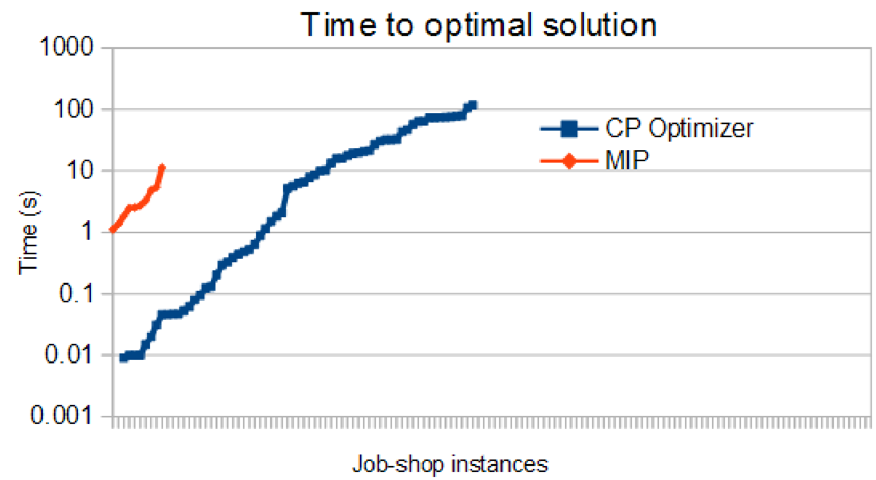
```
dvar interval op[j in Jobs][p in Pos] size Ops[j][p].pt;
dvar sequence mchs[m in Mchs] in
  all(j in Jobs, p in Pos : Ops[j][p].mch == m) op[j][p];

minimize max(j in Jobs) endOf(op[j][nbPos-1]);
subject to {
  forall (m in Mchs)
    noOverlap(mchs[m]);
  forall (j in Jobs, p in 1..nbPos-1)
    endBeforeStart(op[j][p-1], op[j][p]);
}
```

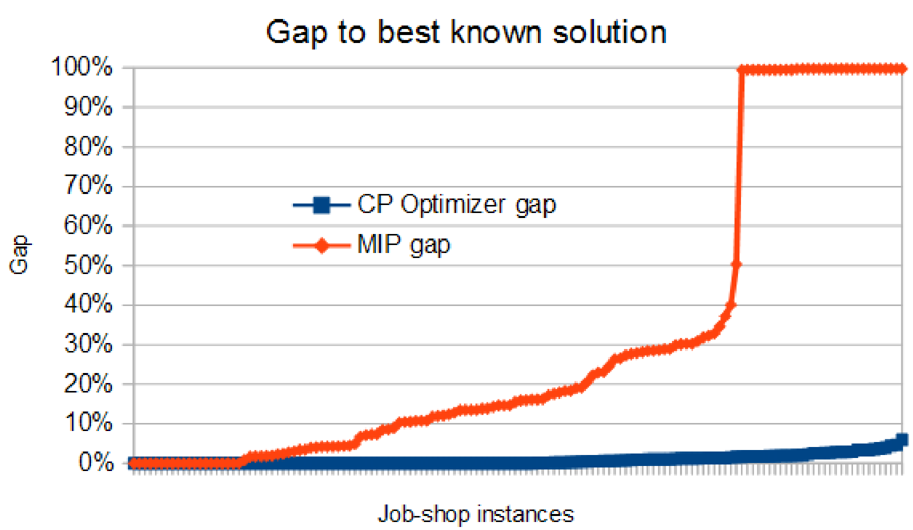
- Comparison of this CP Optimizer model vs a disjunctive MIP formulation on a set of 140 classical Job-shop instances (50-2000 tasks), time-limit: 2mn, 4 threads

Concept: **sequence variable**

- Comparison of CP Optimizer and MIP performance on Job-Shop



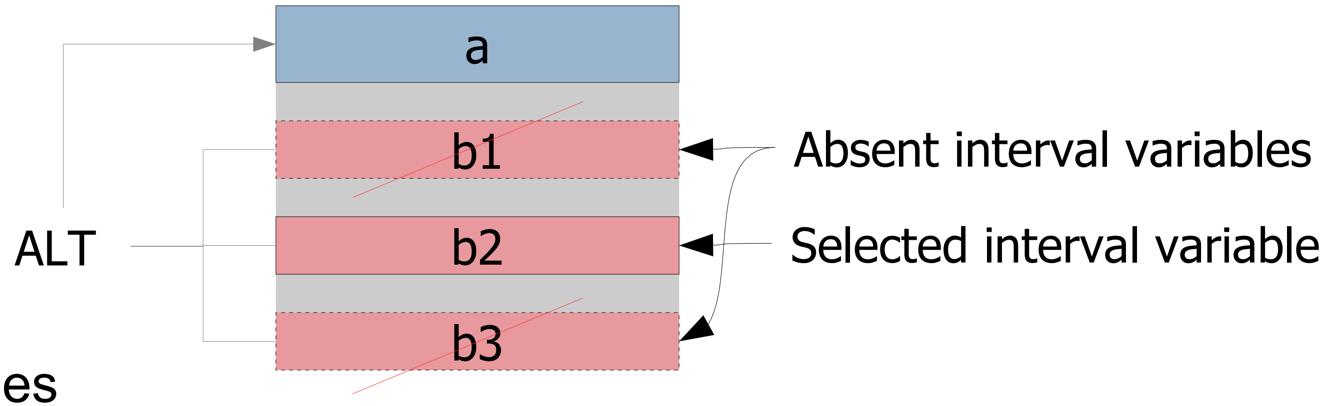




Concept: **alternative constraint**

- What for?
  - Modeling alternative resource/modes/recipes
  - In general modeling a discrete selection in the schedule
- Example

```
alternative(a,[b1,...,bn])
```



- Properties
  - Conditional reasoning allows a strong pruning on the alternative master interval variable *a*
  - Master interval variable *a* can of course be optional

Integer variables and expressions in CP Optimizer

- CP Optimizer is not only about scheduling !
- … and complex scheduling problems may involve other decision variables than interval variables !
- CP Optimizer supports integer decision variables and a large panel of expressions/constraints

<table>
<tr><th>Variables</th><th>Expressions</th><th>Constraints</th></tr>
<tr>
<td>Variables are discrete integer

Domains can be specified as a range [1..50] or as a set of values {1, 3, 5, 7, 9}

dvar int x in 1..50</td>
<td>Expressions can be integer or floating-point, for example 0.37 * y is allowed

Basic arithmetic and more complex operators (min, max, log, pow etc.) are supported

Relational expressions can be treated as 0-1 expressions.
e.g. z = (y < t)

Special expressions:
y == a[x]
t == count(b, 3)
t == cond ? x : y</td>
<td>Rich set of constraints

Standard relational constraints (==, !=, <, >, <=, >=)

Logical combinators (&&, ||, !, =>)

Specialized (global) constraints
allDifferent(b)
allowedAssignments(b, tuples)
forbiddenAssignments(b, tuples)
pack(load, container, size)
lexicographic(b, c)
inverse(b, c)</td>
</tr>
</table>

Integer variables and expressions in CP Optimizer

- Model can mix integer and interval variables

```
dvar int qty[i in 1..n] in …;
dvar interval task[i in 1..n] in 0..H;

maximize sum(i in 1..n)
   qty[i] * pow( H-endOf(task[i]), 1.025 );
```

Agenda

- **Introduction:**
  - **CP Optimizer** a component of CPLEX Optimization Studio
  - **Scheduling**
  - **Example**: Resource Constrained Project Scheduling Problem
- Scheduling **concepts** in CP Optimizer
- **Automatic search** in CP Optimizer
- Some features for **accelerating model development**
- **Q&A**

Automatic Search

- Automatic search is invoked by a **single call** to:
  - IloCP::solve()
- Automatic search is **complete**:
  - If the problem is unfeasible, it will prove unfeasibility
  - If the problem is feasible, it will find an optimal solution and prove optimality
- Note that especially for scheduling problems, **proofs are very challenging** (some scheduling problems with as few as 50 tasks are still open)
- Once a first feasible solution has been found, the search can be interrupted at **any time** to get the best incumbent solution
- Search **parameters** are available to tune and limit the search (e.g. time limit)

Automatic Search

- Core CP techniques used as a building block:
  - Tree search (Depth First)
  - Constraint propagation
- Other techniques used:
  - Model presolve
  - Restarting techniques
  - No-good learning
  - Randomization
  - Impact-based branching
  - Opportunistic probing
  - Dominance rules
  - LP-assisted heuristics
  - Large Neighborhood Search
  - Algorithms portfolios and Machine learning
  - Evolutionary algorithms

Automatic Search

- Performance of the automatic search of CP Optimizer is continuously monitored on a set of more than 90 different scheduling benchmarks (e.g. RCPSP is just one of these benchmarks)

Campaign: PerfWorker1

	COS122	COS123	COS124	COS125	COS1251	COS126
COS123	1.16	-	-	-	-	-
COS124	1.19	1.02	-	-	-	-
COS125	1.48	1.28	1.25	-	-	-
COS1251	1.53	1.32	1.29	1.02	-	-
COS126	3.09	2.65	2.58	2.07	2.07	-
COS12610	3.03	2.57	2.51	1.99	1.99	0.98

- Performance of automatic search has been compared with the problem specific state-of-the-art methods in a number of scheduling benchmarks

Automatic Search

MISTA 2007

- Some results:

Problem type	Benchmark	Problem size	Reference UB	MRD	# Imp. UBs / # Instances
Trolley	[41]	230-460	[19]	−11.8%	15/15
Hybrid flow-shop	[35]	200-1000	[35]	−8.8%	19/20
Job-shop w/ E/T	[3]	30-200	[3]	−5.6%	32/48
Air traffic management	[19]	2000	[19]	−4.0%	1/1
Flow-shop w/ E/T	[27]	30-400	[14]	−3.0%	4/12
Max. quality RCPSP	[33]	30	[33]	−2.3%	NA/3600
Cumulative job-shop	[28]	150-675	[17]	−0.3%	27/86
Single proc. tardiness	[20]	200-500	[20]	0.2%	0/20
Semiconductor testing	[30]	400	[30]	0.2%	7/18
RCPSP w/ E/T	[42]	30-50	[42]	0.4%	15/60
Open-shop	[9, 40, 18]	64-400	[15, 7, 25]	0.9%	0/28
RCPSP	[23]	120	Best PSPLIB	1.6%	0/600[3]
Shop w/ setup times	[10]	50-200	[2]	2.3%	0/15
Parallel machine w/ E/T	[29]	8-200	[4]	2.6%	2/52
Job-shop	[1, 39, 43, 40]	100-500	Best OR-Lib	2.8%	0/33
Air land	[5]	10-50	[5]	3.4%	0/8
Flow-shop w/ buffers	[40]	100-500	[8]	3.6%	12/30
Flow-shop	[40]	100-500	Best OR-Lib	5.9%	0/22
Aircraft assembly	[16]	575	[13]	8.7%	0/1
Single machine w/ E/T	[11, 37, 29]	8-500	[38]	9.8%	1/100
Common due-date	[6]	100-200	[36]	14.7%	0/20

Table 1: Results of SA-LNS on 21 scheduling benchmarks

Automatic Search

CP-AI-OR 2009

Some results:

– Problem #1: Flow-shop with earliness/tardiness cost
– Problem #2: Oversubscribed Satellite Scheduling problem
– Problem #3: Personal tasks scheduling

	OPL Model size	CPO Automatic search (no parameter tuning) vs. state-of-the-art
#1	20 lines	Competitive with state of the art (GA, LNS)
#2	15 lines	Number of unscheduled tasks decreased by 5%
#3	42 lines	Finds solution to more instances Solution quality increased by 12.5%

Automatic Search

CP-AI-OR 2015

Some results:

Benchmark set	Number of instances	Lower bound improvements	Upper bound improvements	Closed instances
JobShop	48	40	3	15
JobShopOperators	222	107	215	208
FlexibleJobShop	107	67	39	74
RCPSP	472	52	1	0
RCPSPMax	58	51	23	1
MultiModeRCPSP (j30)	552	No reference	3	535
MultiModeRCPSPMax	85	84	77	85

Table 1. Results summary

Agenda

Introduction:

- **CP Optimizer** a component of CPLEX Optimization Studio
- **Scheduling**
- **Example**: Resource Constrained Project Scheduling Problem

Scheduling **concepts** in CP Optimizer

Automatic search in CP Optimizer

Some features for **accelerating model development**

Q&A

Some features for accelerating model development

CP Optimizer provides similar features as CPLEX:

- Search log
- Conflict refiner
- Warm start
- Human-readable input/output file format (new in 12.6.1)

```
// --------------------------------------------------------------------------
// IBM ILOG CP Optimizer model export file
// Effective workers: 8
// --------------------------------------------------------------------------
// ------ Interval-related variables: ---------------------------------------
"itvs(0)(5)" = intervalVar(size=2);
"itvs(1)(5)" = intervalVar(size=3);
...
"mchs(0)" = sequenceVar(["itvs(0)(4)", "itvs(1)(1)", "itvs(2)(3)", "itvs(3)(5)", "itvs(4)(4)", "itvs(5)(2)"]);
"mchs(1)" = sequenceVar(["itvs(0)(1)", "itvs(1)(0)", "itvs(2)(2)", "itvs(3)(2)", "itvs(4)(0)", "itvs(5)(4)"]);
...
// ------ Objective: --------------------------------------------------------
minimize(max([endOf("itvs(0)(5)"), endOf("itvs(1)(5)"), ...]));
// ------ Constraints: ------------------------------------------------------
noOverlap("mchs(0)");
noOverlap("mchs(1)");
...
endBeforeStart("itvs(0)(0)", "itvs(0)(1)");
endBeforeStart("itvs(0)(1)", "itvs(0)(2)");
...
```

Some features for accelerating model development

- CPLEX and CP Optimizer engines are available in the same CPLEX Optimization Studio ecosystem and with the same "look&feel"

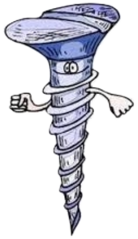
		CPLEX	**CP Optimizer**
Interfaces		OPL, C++, Java, .NET, C, Python	OPL, C++, Java, .NET
Model	Decision variables	int, float	int, interval
	Expressions	linear, quadratic	arithmetic, log, pow, … relational, a[x], count,...
	Constraints	range	relational, logical, specialized, scheduling
Search	Search parameters	✓	✓
	Warm start	✓	✓
	Multi-core //	✓	✓
Tools	Search log	✓	✓
	I/O format	.lp, .mps, ...	.cpo
	Conflict refiner	✓	✓

Some features for accelerating model development

- CPLEX and CP Optimizer engines are available in the same CPLEX Optimization Studio ecosystem and with the same “look&feel”
- A few differences still:
  - In CP, lower bounds are not directly available.
    Rationale: CP uses Depth-First instead of Best-First search, LB becomes known only at the very end of the search space traversal.
  - CP Optimizer allows for the implementation of user-defined constraints and search tree exploration (in C++ only).
    This can be compared to CPLEX user cuts and callbacks.
  - No feas-opt feature available so far for CP Optimizer.
    Rationale: due to the large typology of constraint, the violation degree of a constraint is highly dependent on the context. But optionality of interval variables makes it easy to model oversubscribed problems.

Some features for accelerating model development

- CPLEX and CP Optimizer engines are available in the same CPLEX Optimization Studio ecosystem and with the same "look&feel"
- Real problems are very seldom pure and monolithic



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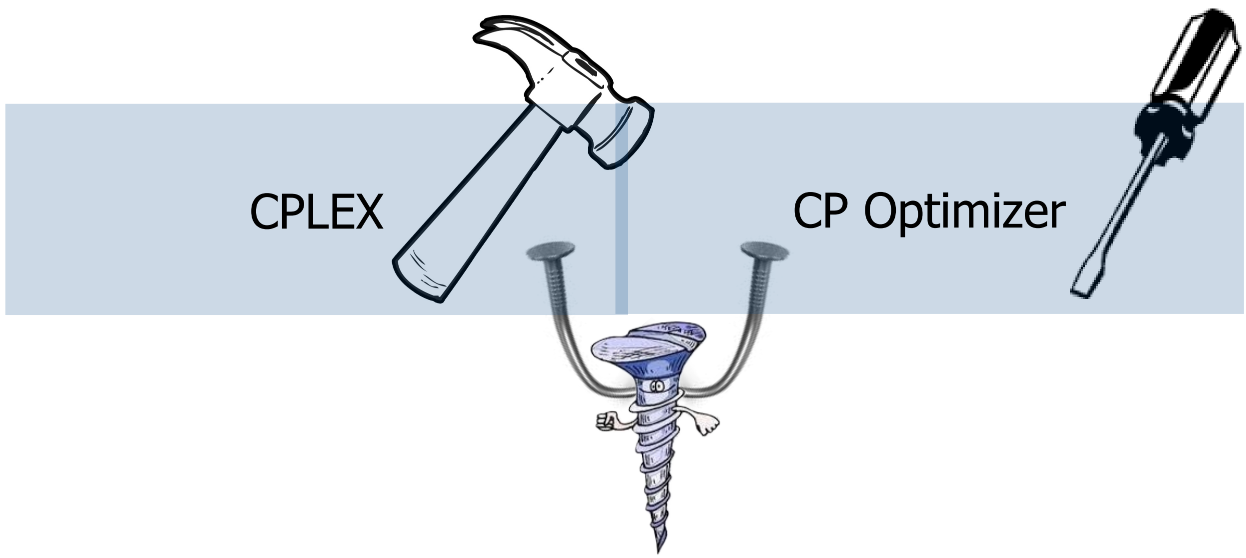

- CPLEX and CP Optimizer are **complementary**:
  - Sequential models (planning → detailed scheduling)
  - Use MIP to compute LB (relaxed model) or guidelines
  - Logic-based Benders decomposition, column generation

Agenda

Introduction:

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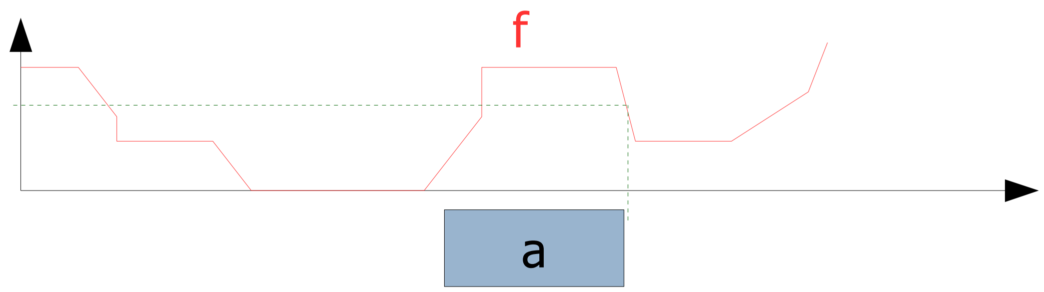
Some features for **accelerating model development**

Q&A

Backup

Concept: **expressions on interval variables**

- What for?
  - Evaluating a piecewise linear function at a particular end-point of an interval variable (earliness-tardiness cost, temporal preference)
- Example:
  - endEval(f,a, ABSVAL) takes value ABSVAL if *a* is absent otherwise it takes value f(*e*) if *a* is present and *a=[s,e)*

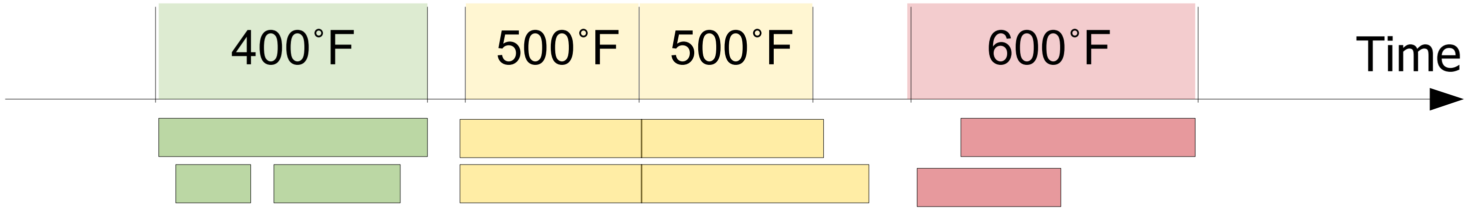


Property:

- f can be any piecewise linear function (non-continuous, non-convex, ...)

Concept: **state function**

- What for?
  - Modeling evolution of a state variable over time
  - Modeling incompatibility relations between activities overlapping a given date t
  - Synchronizing start/end value of compatible tasks (batching)
- Example

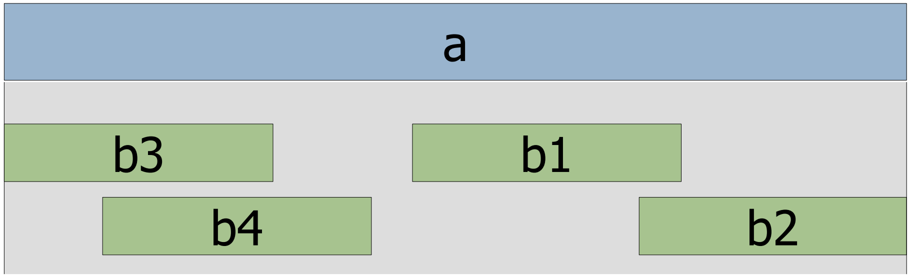


- Properties
  - Complexity is **independent of the time scale**
  - CP Optimizer is able to reason **globally** over a state function
  - **Avoid quadratic models** over each pair (i,j) of incompatible intervals on the state function

Concept: **span constraint**

- What for?
  - Modeling task → sub-task decomposition
  - Modeling immobilization time of a resource
- Example

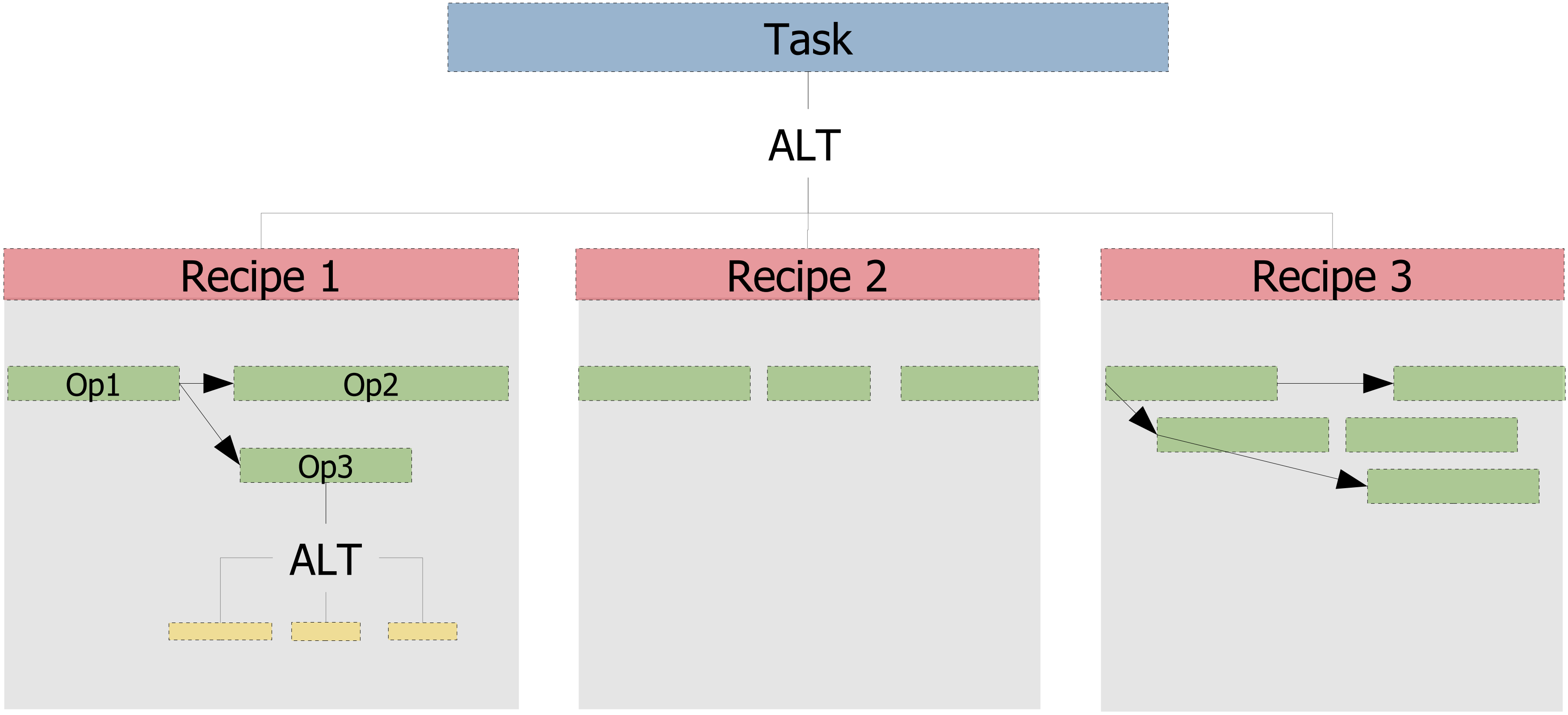
```
span(a,[b1,...,bn])
```



- Properties
  - The constraint of course handles optional interval variables

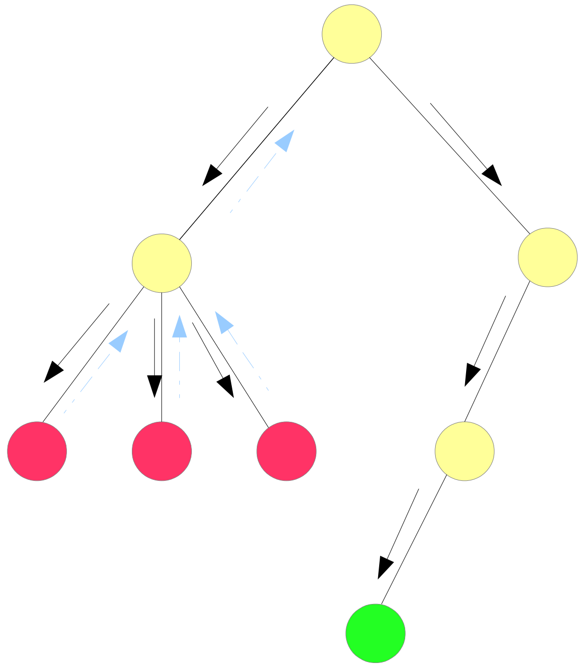
Concept: **span/alternative constraint**

- What for?
  - Modeling Work Breakdown Structure
- Example



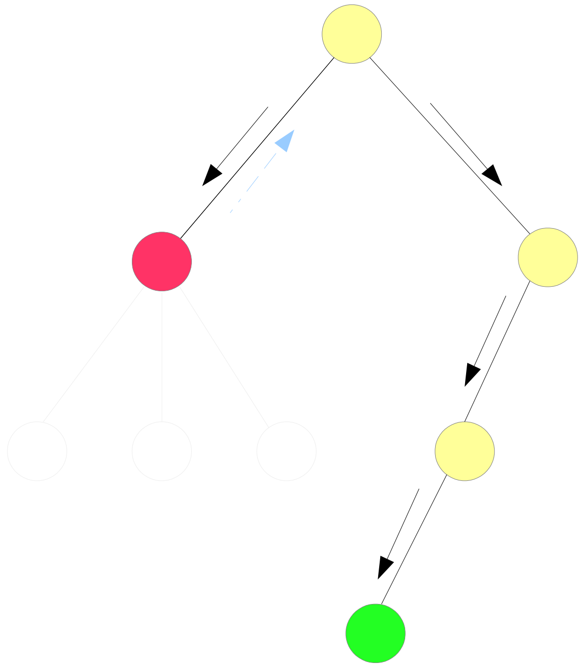
How Constraint Programming Works

- CP is a *constructive* approach
- Values are assigned to variables one at a time to extend a partial solution to a complete solution
- At a point, it may be useless to further extend a partial solution as at least one constraint is already violated by the partial solution
  - The solver *backtracks* and tries a different value for a previously assigned variable
  - All possible assignments of values to variables can be examined in this way



How Constraint Programming Works

- In CP, the basic search behavior is tree search
- Including search space reduction via *domain filtering*
- Domain filtering
  - Before each value-variable assignment, *domain filtering* occurs
  - Each value of a variable which cannot be used in a solution (given the current partial assignment) can be removed
  - Each constraint type has a *specialized* algorithm which filters domains



Domain filtering

All Different

A	B	C	D	E
1	1	1	1	1
2				2
3	3	3		
4		4	4	4
5				5

Domain filtering

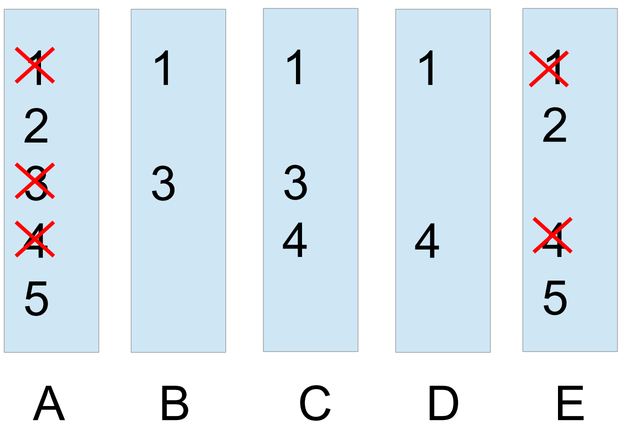
All Different

A	B	C	D	E
~~1~~	1	1	1	~~1~~
2				2
~~3~~	3	3		
~~4~~		4	4	~~4~~
5				5

Uses matching theory

Domain filtering

All Different



Uses matching theory

Other examples in CP Optimizer

- Cardinality (occurrence count)
  - Flow theory
- Packing
  - Subset sum and packing theory
- Table (assignments)
  - Support structures
- Scheduling constraints
  - Energetic reasoning
  - Timetables
  - Edge finding

Model Presolve

- Depending on the input data, models can have redundancies and can contain poorly formulated constraints
- Objective: Automatically improve the model in order to make stronger inferences faster
- Simplifications
  - Constraint compaction
  - Redundancy elimination
  - Constant propagation
  - Common sub-expression factorization
- Aggregations
  - Count expressions & difference constraints
  - Linear constraints over binary {0,1} variables
  - Alternative constraint combined with arithmetic expressions

Examples of Model Presolve

- Elimination of redundant constraints
  - Bound of variables and expressions computed by constraint propagation are used to eliminate redundant constraints before search
- Propagation of constant variables and expressions

$$1 + 2x + 3y + xy - 3x + 7y \leq 8x - 7y + 10$$

$$x == 4 \quad \rightarrow \quad 21y \leq 45$$

- Variable merge

$x==y,\ y==z,\ z==t$ → merge the domains of x, y, z and t and replace y, z, and t by x everywhere

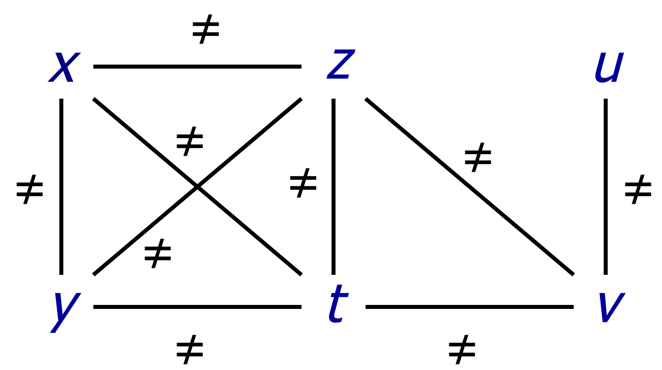
Examples of Model Presolve

- Aggregation of difference constraints
  - Binary ≠ constraints can be lifted to alldiff constraints

A problem with ≠

$$\begin{cases} x \neq y \\ x \neq z \\ x \neq t \\ y \neq z \\ y \neq t \\ z \neq v \\ t \neq z \\ t \neq v \\ u \neq v \end{cases}$$

The associated graph structure



The aggregated model

$$\begin{cases} \texttt{alldiff}(x, y, z, t) \\ \texttt{alldiff}(z, t, v) \\ u \neq v \end{cases}$$

Examples of Model Presolve

- Lifting difference constraints
  1) Pick up a binary constraint $x \neq y$ not already in a lifted alldiff
  2) Find the largest clique containing $x \neq y$ (in practice we use a greedy algorithm – add vertex one by one to the set starting with vertices having the maximum degree)
  3) Add a new lifted alldiff
- Experimentation on graph coloring
  - Color the vertices of a graph such that two adjacent vertices have a different color and the number of colors is minimized
  - The model is stated with a set of binary ≠ constraints
  - Problems are from the DIMACS challenge (COLOR02 set)
    - 9 problems over 60 are solved to optimality without presolve
    - 39 problems over 60 are solved to optimality with presolve

Examples of Model Presolve

Common sub-expression factorization

- Eliminate multiple occurrences of the same expression and replace them by a new variable

  example: $xy \neq z + t$

  $z + xy == a + b$

  $100 \leq z + xy$

- The expressions xy and $z + xy$ appear several times. We introduce two new variables u and v to replace these expressions and add the constraints: $u == xy$, $v == z + u$
- The model becomes $u \neq z + t$

  $v == a + b$

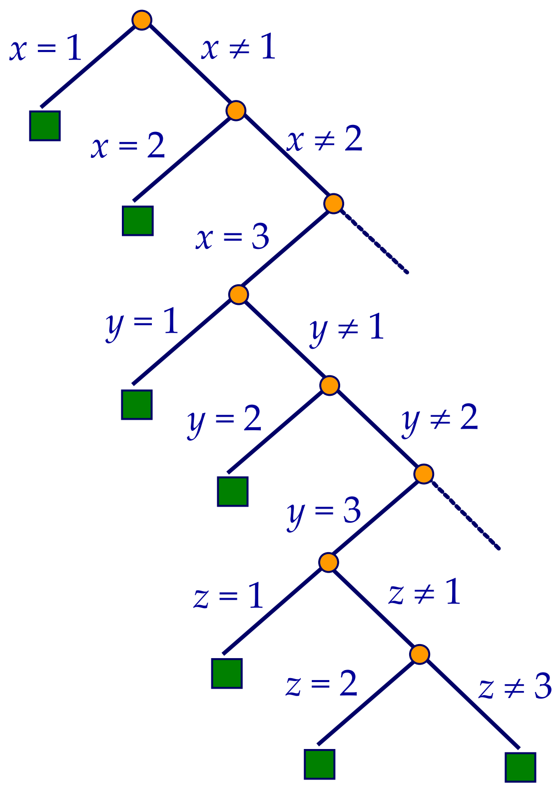
  $100 \leq v$

- Communication of bound reduction on newly introduced variables achieves more domain reduction
- It also reduces the number of expressions and thus involves less computations

Restart and No-goods

- Pure DFS is generally not used
  - Thanks to a parameter, you can select pure DFS if you wish
- Numerous DFS searches of limited effort are favored
  - One element of the CP Optimizer search is the restart
  - Effort limit increases over time to ensure completeness
  - CP Optimizer's restart limit increases geometrically
  - No-good learning helps increase efficiency
  - Benefits well documented in numerous papers:
    - Selman and Gomes, late 90s, theoretical and practical results

Limited DFS Methods - Classic Restarting



Clause Generation

$$\begin{cases} x \neq 1, \\ x \neq 2, \\ x \neq 3 \vee y \neq 1, \\ x \neq 3 \vee y \neq 2, \\ x \neq 3 \vee y \neq 3 \vee z \neq 1, \\ x \neq 3 \vee y \neq 3 \vee z \neq 2, \\ x \neq 3 \vee y \neq 3 \vee z \neq 3, \end{cases}$$

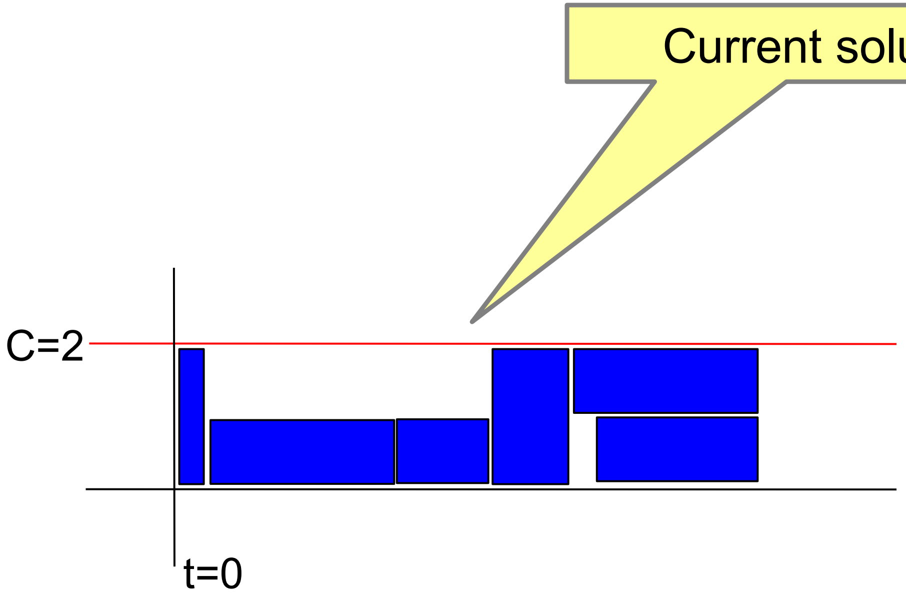
Restart

Increase limit

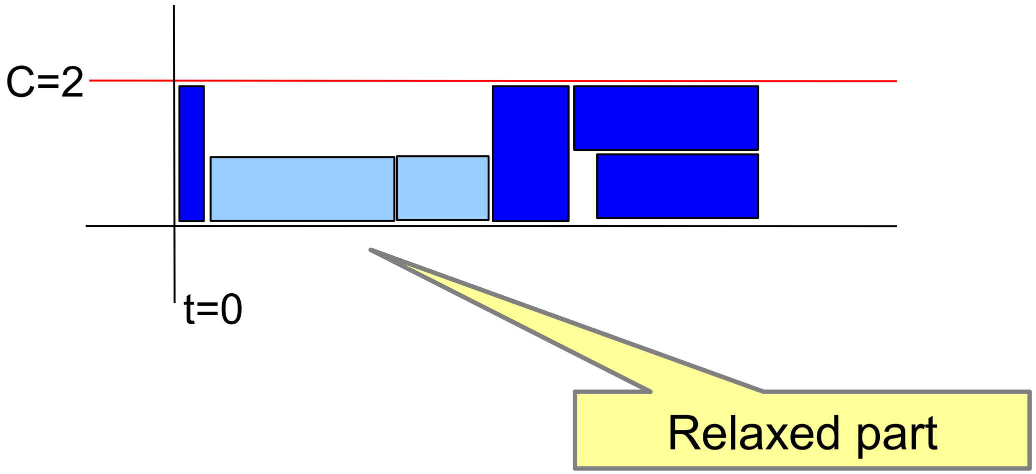
Large Neighborhood Search

- In scheduling, a common way of improving solutions is to use Large Neighborhood Search
  - Shuffling [A&C, 1991] was the first variant
- A number of operations are “relaxed” meaning they can move freely where they want while still obeying problem constraints, such as precedences
- The remaining operations stay “rigid” which means that they can shift in time but stay in much the same order as in the current solution
- The start times of the free and rigid parts are then decided by a limited DFS search using heuristics. The aim is to find a solution of lower objective value

LNS: simple example



LNS: simple example



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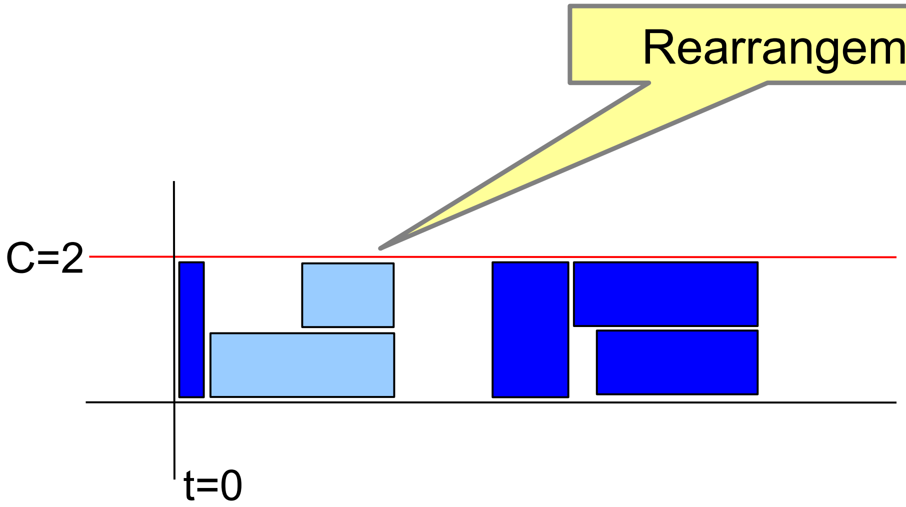
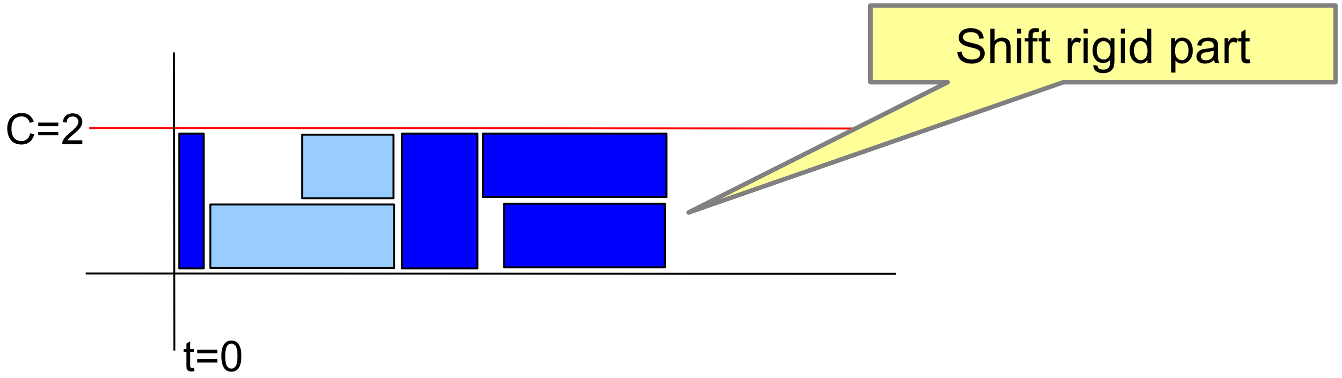


LNS: simple example

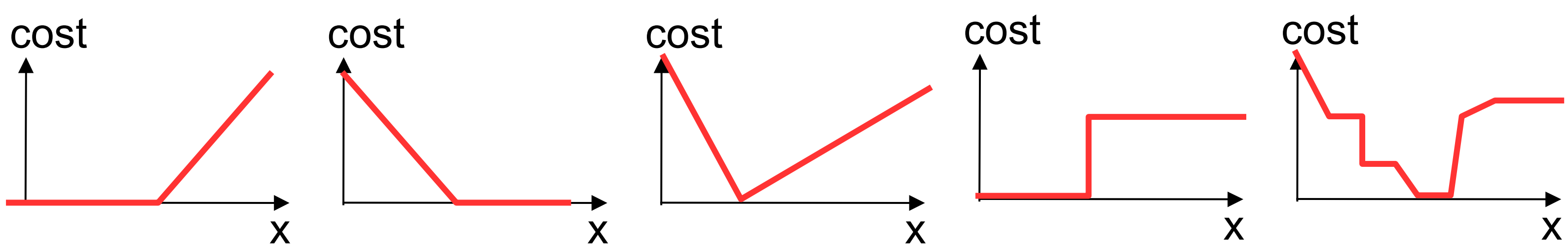


Large Neighborhood Search

- In a classic form of shuffling for job-shop scheduling, the order of operations on one or more resources is fixed and the order of the others is reoptimized
- Problem structure is used to identify the part of the problem to relax and reoptimize. e.g. use resources, alternatives, strongly connected components
- Due to the more general forms of resource supported in CP Optimizer, more subtle forms of "fixing" (creating rigidity in the schedule) are required than simple order
- Use of partial order schedules
  - See [Godard et al., MISTA, 2007], [Pisinger and Ropke, Handbook of Metaheuristics, 2010]

Branching Heuristics – LP-assisted

- Traditionally, early/tardy scheduling problems are challenging for CP solvers as the solvers don't have a good global view of the cost function



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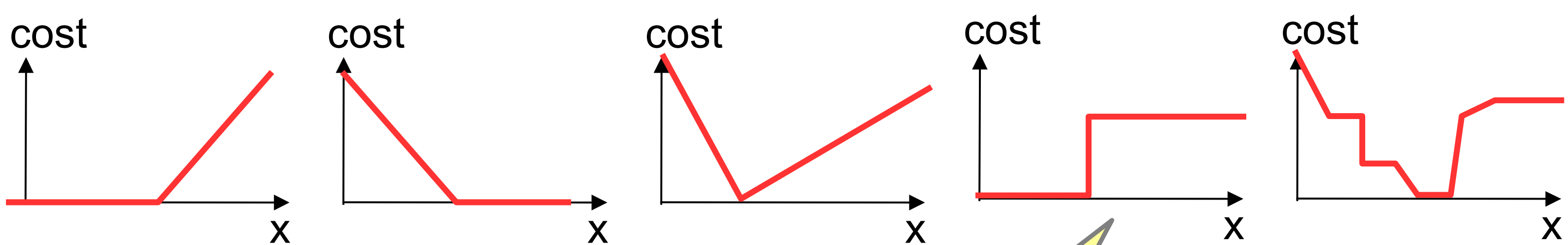

penalty * (startOf(op) > dueDate)

Branching Heuristics – LP-assisted

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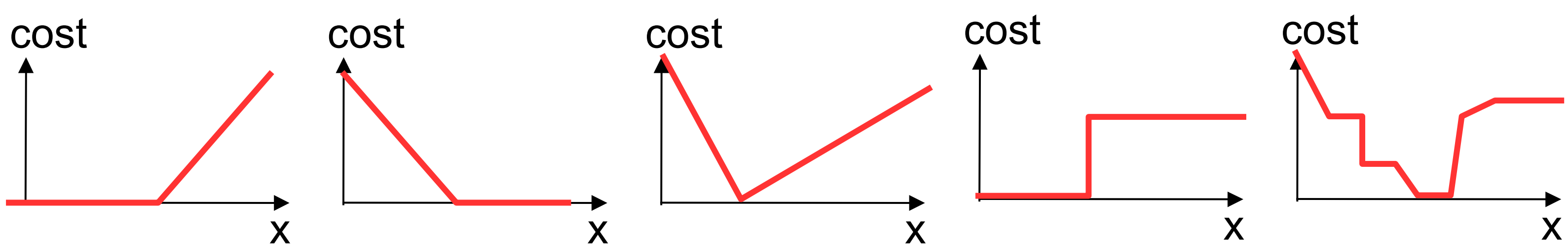
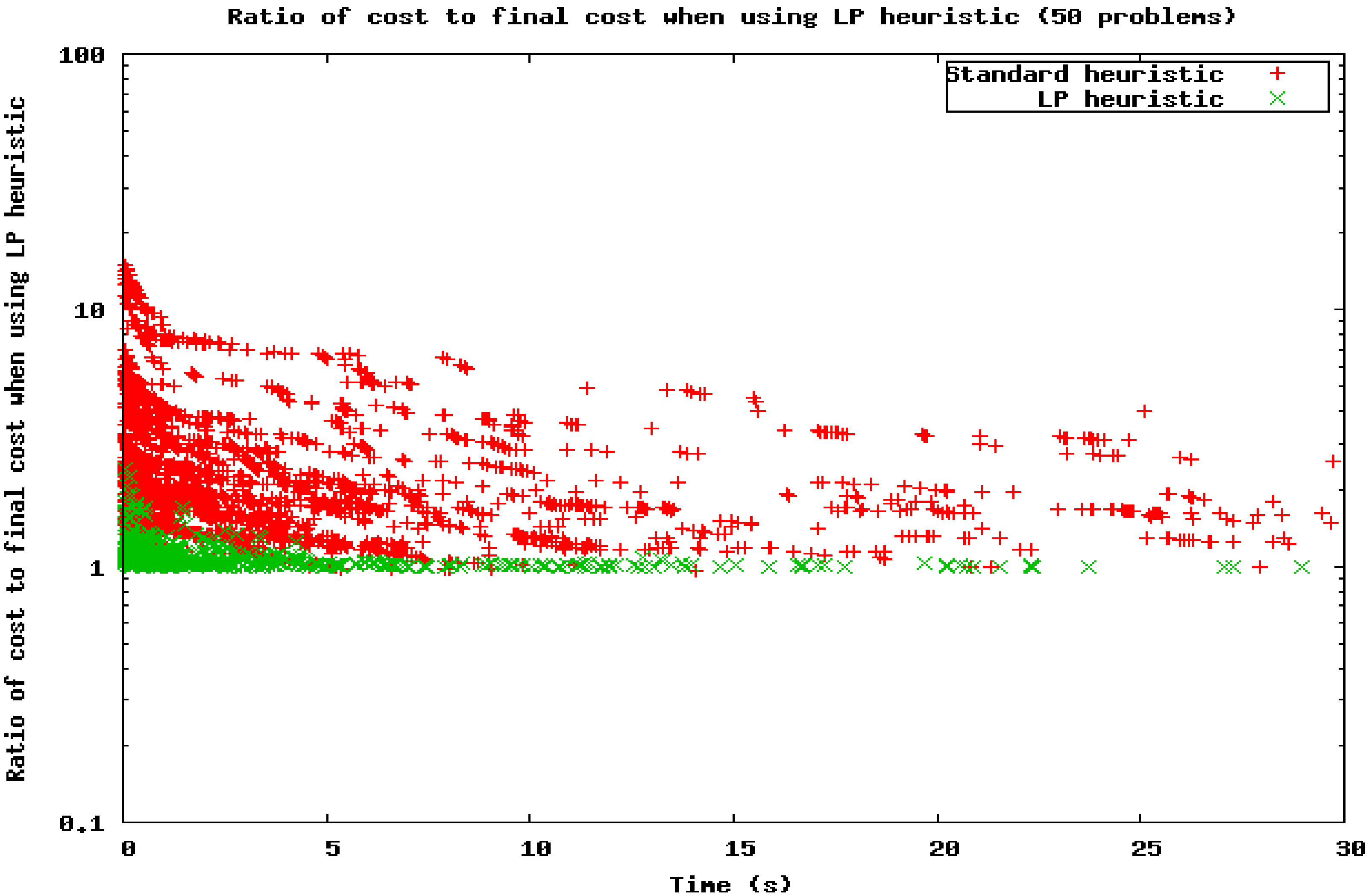


max(0, slope * (startOf(op) - dueDate))

Branching Heuristics – LP-assisted

- Since CPLEX and CP Optimizer are both part of CPLEX Optimization studio, CP Optimizer can take advantage of the availability of CPLEX to help on problems where a good global view of the cost function is important
- CP Optimizer linearizes scheduling constructs such as:
  - Precedences
  - Execution / non-execution costs, alternatives
  - Cost function terms which are functions of start/end times
  - PWL functions can be used to approximate
- CPLEX computes cost lower bound and “best” start times using the CPLEX LP Solver: use the LP solution to guide heuristics. Start an operation as close as possible to the time proposed by CPLEX

Branching Heuristics – LP-assisted



Probing

- Probing is a technique that exploits the fact that in some problems, fixing a variable has a higher than usual probability of causing a failure
  - In this case, at a node other variables as well as the branching variable can be tested by fixing them to a value
  - If the fixing fails, the fixed value can be filtered from the domain, otherwise the fixing is undone
  - Particularly useful for binary and small-domain variables
- In CP Optimizer, a dynamic technique is used to adjust how often probing occurs to avoid paying the probe penalty when probing is ineffective

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