

Temporal Reasoning Based on Semi-Intervals

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Abstract

A generalization of Allen's interval-based approach to temporal reasoning is presented. The scope of reasoning capabilities can be considerably extended by using relations between semi-intervals rather than intervals as the basic units of knowledge. Semi-intervals correspond to beginnings or endings of temporal events. We develop a representational framework in which relations between semi-intervals appear as coarse knowledge in comparison with relations between intervals. We demonstrate the advantages of reasoning on the basis of semi-intervals: 1) coarse knowledge can be processed directly; computational effort is saved; 2) incomplete knowledge about temporal intervals can be fully exploited; 3) incomplete inferences made on the basis of complete knowledge can be used directly for further inference steps; 4) there is no trade-off in computational strength for the added flexibility and efficiency; 5) semi-intervals correspond to natural entities both from a cognitive and from a computational point of view. The presented scheme supports reasoning on the basis of fine-grained or complete knowledge, on the basis of coarse or incomplete knowledge, and on combinations of both kinds of knowledge. The notion of 'conceptual neighborhood' is central to the presented approach. Besides enhancing the reasoning capabilities in several directions, this notion allows for a drastic compaction of the knowledge base underlying Allen's inference scheme. A connection to fuzzy reasoning on the basis of 'conceptual neighborhood' is drawn. It is suggested that reasoning based on the simplified knowledge base may be particularly suited for the implementation of parallel inference engines.

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TIME IS A MASK WORN BY SPACE
Robert Fulton [1977]

1 Introduction

In his paper on maintaining knowledge about temporal intervals Allen [1983] discusses the formal problem that arises when representing the everyday concept of a ‘point in time’ by a point in the mathematical sense. This problem is due to the fact that logical inconsistencies arise when events are allowed to have zero duration. Besides the arguments Allen provides against the representation of events of zero duration, namely physical and logical arguments, mathematical time points are not appropriate from a cognitive point of view either. We know, for example, that events have to have a certain extent, both in time and in space, in order to be perceivable. Thus, when we talk about a ‘point in time’ colloquially, we do not refer to a mathematical (bottom-up) point, but rather to a conceptual (top-down) point, an entity which merely is not resolved further, in the given situation.

Thus, we agree with Allen that events must not be represented by points on the real axis. We also agree that qualitative knowledge about temporal affairs can be based on events. However, we do not agree with Allen’s conclusion that intervals should be used as the representational primitives for reasoning about events.

We must carefully distinguish between the representation of temporal relations and the representation of *knowledge about* temporal relations. If we know everything about all relations, the distinction is a philosophical one; but if we want to deal with incomplete knowledge, the distinction becomes a practical matter as well. Even if we start out with complete knowledge about the relation between certain temporal intervals, after only one inference step we may not have complete knowledge about relations between intervals which we want to use in future reasoning steps. If we use intervals as the primitives for our representation, incomplete knowledge leads to a cognitively awkward situation: the less we know, the more complex the representation of what we know becomes. What is *not* known is represented in terms of disjunctions of what *could be* the case.

From a cognitive point of view, we prefer to represent what is known more directly and in such a way that less knowledge corresponds to a simpler representation than more knowledge does. For this reason and for reasons stated in the following sections we will use

'beginnings' and 'endings' of events as representational primitives. We may only know the beginning or the ending of a temporal event. For example, we may only know the birthdate of a person or the date of his death, but not both; or we may know that a certain event Y did not start before a given event X, but we do not know if X and Y started simultaneously or if Y started after X. In many cases useful inferences can be drawn from such incomplete knowledge, in some cases even without any loss of information.

2 Temporal knowledge about the physical world

An event is something that happens. Beginnings of events always take place before their endings. If we let two event boundaries (beginnings or endings of events) have three possible qualitative relations: $<$, $=$, $>$, then two events which start in a beginning and terminate in an ending have thirteen possible qualitative relations. These correspond to the relations that two ordered pairs of real numbers (the boundaries of real-valued intervals) can have under the relations $<$, $=$, $>$. Note that we do not imply that beginnings and endings of events correspond to the end points of real-valued intervals. They do not. Rather, they can be viewed as (recursively) corresponding to events themselves, when considered at a higher resolution.

Allen denotes the thirteen relations between two events with *before* ($<$), *after* ($>$), *during* (d), *contains* (di), *overlaps* (o), *overlapped-by* (oi), *meets* (m), *met-by* (mi), *starts* (s), *started-by* (si), *finishes* (f), *finished-by* (fi), and *equals* ($=$). Figure 1 shows how the thirteen relations may be distinguished by considering only a subset of relations between the beginnings α , A and the endings ω , Ω of the two events.

The reason that such incomplete information about events suffices for fully characterizing their qualitative relations is due to two domain-inherent conditions: 1) the beginnings of events take place before their endings ($\alpha < \omega$, $A < \Omega$) and 2) the relations $<$, $=$, $>$ are transitive. Without these conditions, $3^4 = 81$ relations between the four defining boundaries of two events would be possible.

Allen's approach to temporal reasoning uses the thirteen possible relations between two events as a basis for a theoretical framework for temporal reasoning. In addition to Allen's theory we will take into account considerations about how cognitive systems can establish relevant relations from observing the real world. In observing the real world, there will be situations in which only incomplete knowledge about the domain is available and uncertainty exists as to which of the mutually exclusive abstract relations holds.

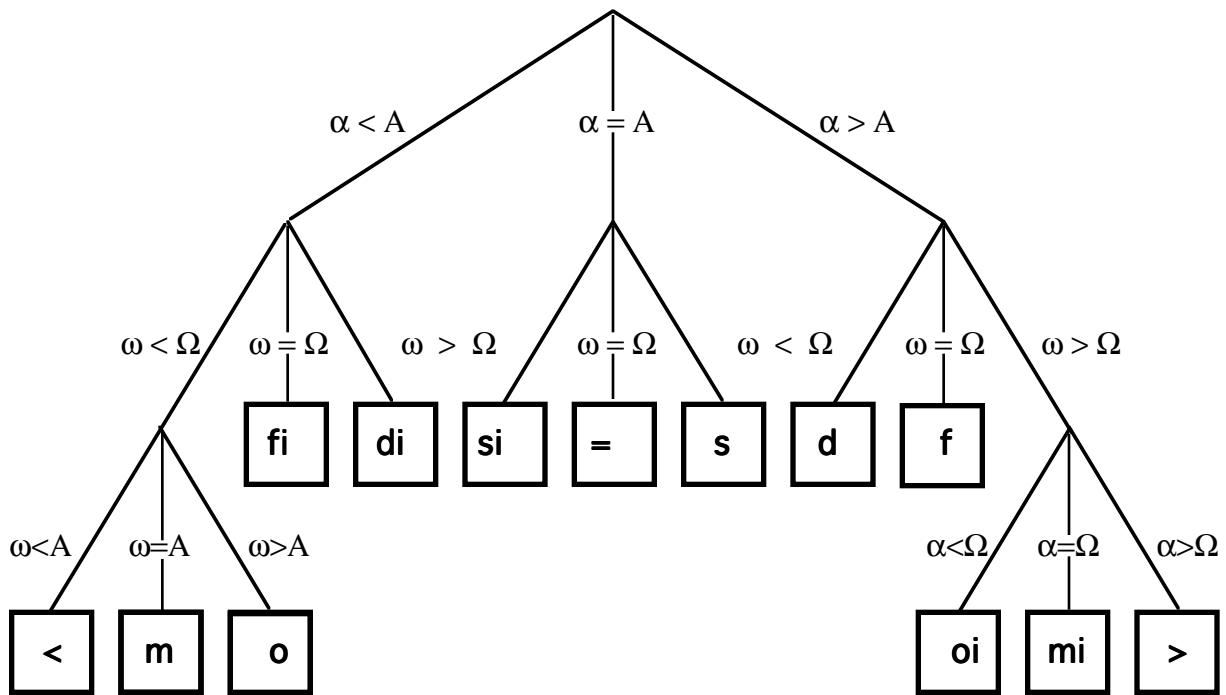


Figure 1: The thirteen qualitative relations between two events characterized by relations between their beginnings α , A and their endings ω , Ω .

2.1 Incomplete knowledge about events

In many temporal reasoning situations we do not have to know everything about the involved events in order to infer what we want to know. For example, in order to determine that Newton lived *before* Einstein it is sufficient to know that Newton's death was before Einstein's birth; it does not help if we additionally know when Newton was born or when Einstein died. Actually, we can derive complete qualitative knowledge about the relations between the birth and death dates involved, due to the domain-inherent constraints mentioned in the previous section.

Of course we may encounter situations in which the available knowledge is insufficient for determining the complete answer to a query; however, a partial answer may be better than no answer at all. For example, we may want to know if two artists may have been influenced by each others work. All we know is that X was born before Y's death and that X died after Y. We do not know who was born first. From this information we can conclude that Y lived *during* X' lifetime or he *started* X' lifetime or his life *overlapped* with X' life. Although we

cannot infer who was the older artist or which was the period when they both lived, at least we know that there was a common period.

With Allen's representation, it is possible to express the situation of the example given above as follows: "X was born before Y's death" can be expressed as "X lived *before* Y or X' life *meets* Y's life or X' life *overlaps* Y's life or X' life is *finished by* Y's life or X' life *contains* Y's life or X' life is *started by* Y's life or X' life *equals* Y's life or X' life *starts* Y's life or X lived *during* Y's life or X *finishes* Y's life or X' life is *overlapped by* Y's life", and "X died after Y" can be expressed as "X' life *contains* Y's life or X' life is *started by* Y's life or X' life is *overlapped by* Y's life or X' life is *started by* Y's life or X lived *after* Y". The inference step then consists of forming the conjunction of the two sets of disjunctions: "X' life *contains* Y's life or X' life is *started by* Y's life or X' life is *overlapped by* Y's life" which is equivalent to the conclusion derived above.

2.2 Neighborhoods vs. disjunctions

As suggested in the introduction, it does not appear cognitively adequate to represent coarse knowledge in terms of disjunctions of finely grained alternative propositions, although this representation may be logically correct. Coarse knowledge is a special form of incomplete knowledge. The missing knowledge corresponds to fine distinctions which are not made. The alternatives all lie in the same ballpark, they are 'conceptual neighbors'. For use in future parts in this paper, we make the following definitions:

Definition 1: Two relations are 'conceptual neighbors' (short: 'neighbors'), if a direct transition from one relation to the other can occur upon an arbitrarily small change in the referenced domain.

Definition 2: A set of relations form a 'conceptual neighborhood' (short: 'neighborhood') when they are connected through conceptual neighbor-relations.

Definition 3: Incomplete knowledge is called 'coarse knowledge' if the referred entities belong to the same conceptual neighborhood on the basis of complete knowledge.

If temporal relations are perceived incompletely, the resulting knowledge is typically coarse knowledge. A perception channel will not generate the set of alternatives $X \sqcap Y$ or

$X \circ Y$ or $X \circ i Y$ (compare Figure 3 in Güsgen [1989]), for example. The reason for this is that the last two alternatives of this disjunction have drastically different perceptual appearances – they vary in several aspects. If the system cannot distinguish alternatives differing in several aspects, then it is reasonable to expect that it cannot distinguish alternatives differing only in a subset of these aspects; thus, it should consider the neighboring intermediate alternatives as well.

Incomplete knowledge consisting of non-neighboring alternatives may be available, however, from more abstract knowledge sources, for example from story understanding systems. Here, abstract information about X and Y may have been lost (for example about their identity) such that two perceptually distant alternatives cannot be distinguished while perceptually closer alternatives can. In this case, two distinct (mental) images would correspond to the two alternatives, rather than a single coarse image. In such situations, it appears ‘cognitively justified’ to use the abstract concept of a disjunction for the representation of alternatives.

Thus, we make a distinction between knowledge incompleteness which does not permit a fine *resolution* of closely related variants within a neighborhood (lack of knowledge about details) and incompleteness which does not permit the *selection* of the appropriate alternative (lack of knowledge about essentials). In the former situation, we want to express knowledge directly on the granularity level on which it is available, i.e., we want to represent neighborhoods. As a side-effect, we will have to represent only knowledge which is positively available; we do not have to carry along the burden of the possibilities that remain open due to lack of more detailed knowledge. In the latter situation, knowledge representation in terms of disjunctions may be appropriate. However, by restructuring the knowledge, it may be possible to find representations in terms of neighborhoods for such situations as well.

3 Semi-intervals and conceptual neighborhoods

According to the considerations presented in the previous sections, we will represent knowledge about time in terms of relationships between beginnings and endings of events. We will call beginnings and endings of events ‘semi-intervals’. In order to support the discussion mnemonically, we will introduce special labels for the relationships between semi-intervals. These labels will be used in addition to the labels introduced by Allen.

We will say X is *older* (*ol*) than Y when the beginning of X is less than the beginning of Y. X is *head to head* (*hh*) with Y when their beginnings are equal and X is *younger* (*yo*) than Y when the beginning of X is greater than the beginning of Y. Accordingly, we will say, X is *survived by* (*sb*) Y, X is *tail to tail* (*tt*) with Y, or X *survives* (*sv*) Y when the ending of X is less than, equal, or greater than the ending of Y, respectively. We will say X *precedes* (*pr*) Y, when the ending of X is not greater than the beginning of Y, we will say, X *succeeds* (*sd*) Y, when the beginning of X is not less than the ending of Y, otherwise (i.e., when the intersection of X and Y is not empty) X is a *contemporary* (*ct*) of Y. If X does not *precede* Y, we will say X is *born before death of* (*bd*) Y and if X does not *succeed* Y, we will say X *died after birth of* (*db*) Y. These relations are shown in Figure 2 (compare Figure 2 in Allen [1983]).

Relation	Label	Inverse	Illustration
X is <i>older</i> than Y Y is <i>younger</i> than X	<i>ol</i> <i>yo</i>		XXX???? YY
X is <i>head to head</i> with Y	<i>hh</i>	<i>hh</i>	XXX?? YYYY
X <i>survives</i> Y Y is <i>survived by</i> X	<i>sv</i> <i>sb</i>		???XXX YY
X is <i>tail to tail</i> with Y	<i>tt</i>	<i>tt</i>	??XXX YYYY
X <i>precedes</i> Y Y <i>succeeds</i> X	<i>pr</i> <i>sd</i>		XXX? YYY
X is a <i>contemporary</i> of Y	<i>ct</i>	<i>ct</i>	?XXX??? ???YYY?
X is <i>born before death of</i> Y Y <i>died after birth of</i> X	<i>bd</i> <i>db</i>		XXX????? ????YYY

Figure 2: Eleven relationships between semi-intervals. Question marks (?) in the pictorial illustration stand for either the symbol denoting the event depicted in the same line (X or Y) or for a blank. The number of question marks reflects the number of qualitatively alternative implementations of the given relation.

Combining constraints from above we obtain the relations *older & survived by* (ob), *younger & survives* (ys), *older contemporary* (oc), *surviving contemporary* (sc), *survived-by contemporary* (bc), and *younger contemporary* (yc).

3.1 Uncertainty about temporal relations

Allen's transitivity table establishes the set of theoretically possible relations between two intervals which both have a known qualitative relation to a third interval. The table does not represent knowledge about the effects of small variations or degradations in the input knowledge, specifically, lack of knowledge about certain details. Such variations may be present in the knowledge about the real world due to perceptual uncertainty and/or the dynamics of the domain. For example, we may not know if event X takes place *before* event Y, if X *meets* Y, or if X *overlaps* Y, but we can distinguish these three options from the remaining ten alternatives.

Uncertainty as to which temporal relation holds between two events does not typically mean that any of the thirteen relations are considered possible by a perceiving cognitive system – otherwise the system is not perceiving. Rather, uncertainty may exist between few options. Furthermore, perceptual uncertainties usually do not cause large jumps in the conclusions; rather, conceptually related options are obtained. In order to model such lateral knowledge dependencies, we structure the temporal relations between events according to a conceptual neighborhood relation. This neighborhood relation is determined by our understanding as to which uncertainties in perception are cognitively plausible.

3.2 Conceptual neighborhoods among temporal relations

natura non facit saltus
Linnaeus

According to our definition of conceptual neighborhood in section 2.2, we arrange the thirteen mutually exclusive relations between events in such a way that conceptually neighboring relations become neighbors in our depiction. Figure 3 shows this arrangement. The two events are depicted by a rectangle and a dumbbell-shaped line, respectively; time is assumed to proceed from left to right. We obtain two dimensions of neighborhood: the vertical dimension corresponds to the relative time at which the events take place; the horizontal dimension corresponds to the relative duration of the events.

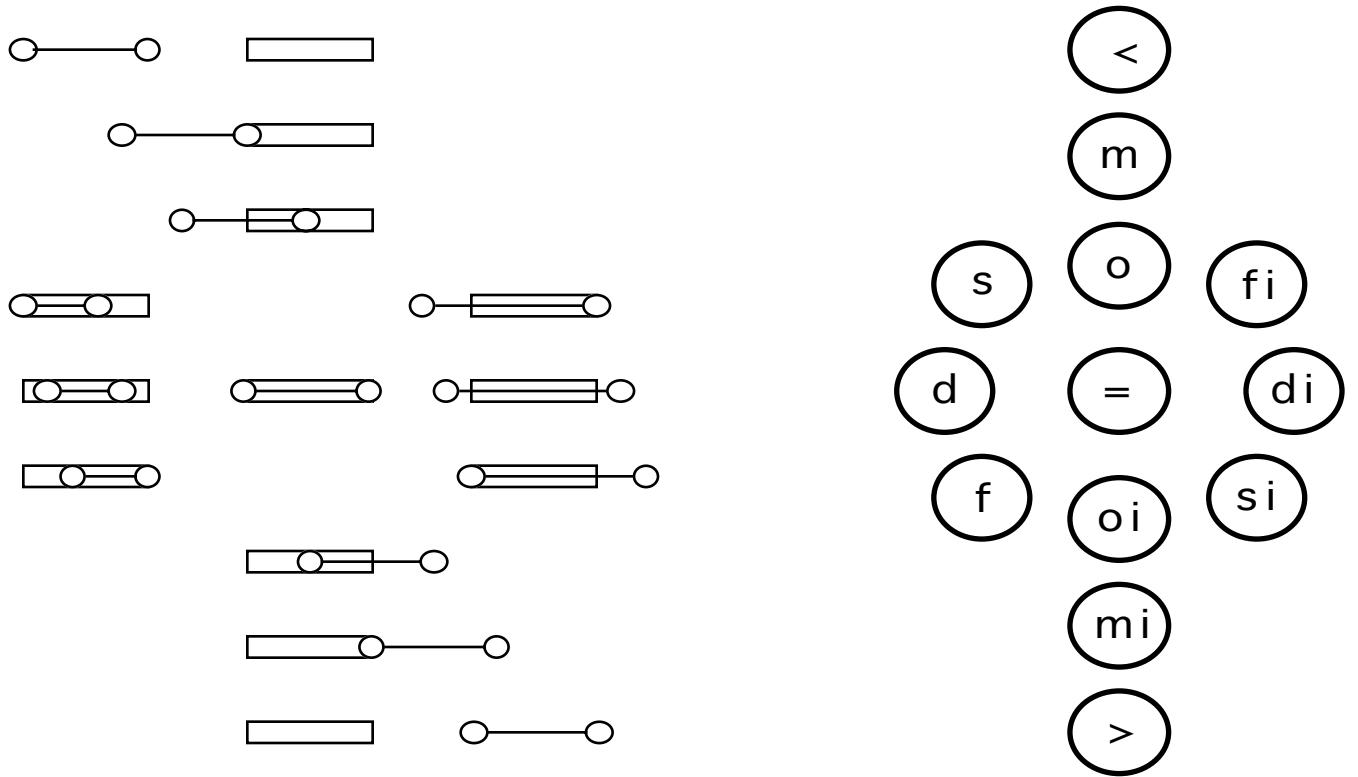


Figure 3: Left: Temporal relations between two events arranged according to their conceptual neighborhood. Right: The corresponding labels arranged accordingly.

For easy visual reference to the thirteen relations between two intervals and their neighborhood relations as depicted in Figure 3, we will use icons symbolizing the neighborhood structure as shown in Figure 4. The black dots indicate which of the thirteen relations within the structure is being referred to. Below the icons are the corresponding labels.

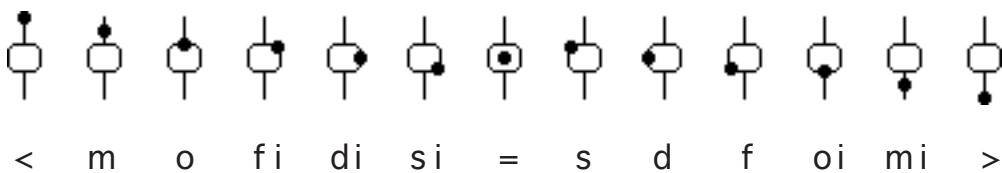


Figure 4: The thirteen qualitative relations between intervals depicted by icons and their corresponding labels.

Thought is imaginative
George Lakoff [1987]

3.3 The aquarium metaphor

The conceptual neighborhood relation between events can be easily imagined by means of the ‘aquarium metaphor’ (compare Freksa [1981]). In this metaphor, temporal events are transformed into spatial objects, namely fishes. Heads and tails of the fishes correspond to beginnings and endings of events. When one fish moves relative to the other, we may observe the sequence of relations $<, m, o, fi, di, si, oi, mi, >$ (or $<, m, o, =, oi, >$, if the fishes appear to be of the same size when passing one another). Any sequence of neighboring relations is conceivable (provided the appearance of the relative sizes of the fishes can vary); other sequences are not possible.

The conceptual neighborhoods between the presented relations are not only due to the physics of the movement of the fishes, but also due to the geometry of the optics in the smooth aquarium world. When the observer moves in front of a static aquarium, the sequences of possible relationships between the fishes will be constrained by the same neighborhood relations.

I am putting forward the thesis here that if a cognitive system is uncertain as to which relation holds between two events, uncertainty can be expected particularly between neighboring concepts. The introduction of movement into our static domain of relations is not intended to make a complex situation even more difficult; rather I suggest that we easily can discover neighboring concepts by imagining gradual changes in the represented world and observing the corresponding state transitions in the conceptual world. This will turn out to be very helpful for adequately representing static situations, particularly for representing uncertainty.

4 Neighborhood-based reasoning

In order to utilize the conceptual neighborhood relations, we present Allen's transitivity table (Allen [1983], Figure 4) in an arrangement which preserves some of the neighborhood relations: the entries along the rows and the columns are arranged in such a way that neighboring entries always correspond to neighboring relations in the sense defined above. The two neighborhood dimensions span a 4-dimensional (2×2) transitivity structure. Since it is not easy to depict a 4-dimensional structure on paper,[¥] we depict a linearized version of the structure: in Figure 5 we arrange the entries according to the sequence suggested in Figure 1: $<, m, o, fi, di, si, =, s, d, f, oi, mi, >$. (This is one of two possible ways of listing the relations in such a way that each is listed exactly once and that all neighbors in the list correspond to neighbors in Figure 3).

In addition, we use icons instead of mnemonic labels for the entries of the transitivity structure in order to simplify its visual inspection. Disjunctions of several relations from Allen's table are depicted by superposition of the corresponding icons from Figure 4. For example,

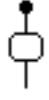
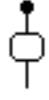
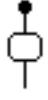
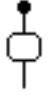
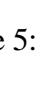
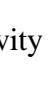
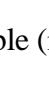
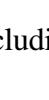
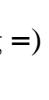
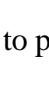
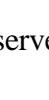
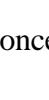
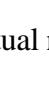
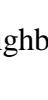
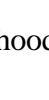


corresponds to the disjunction of the relations $<, m, o, d, f$ and



corresponds to the disjunction of all thirteen relations which means that no constraint on the relationships between the events is given.

[¥] For the discovery of the regularities and the development of the reasoning system presented, a "thinktool" was implemented in HyperCard in order to represent and manipulate the 4-dimensional neighborhood structure.

														
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Looking at the transitivity structure depicted in Figure 5, we can make a number of observations:

- 1) only a fraction of the inferences that can be drawn from the transitivity table are given in terms of unique relations between intervals; most of the conclusions appear in terms of disjunctions of alternative relations;
- 2) the sets of alternative relations in the entries of Allen's transitivity table always form connected neighborhoods;
- 3) in many cases, the transition to neighboring entries leads to sub-neighborhoods or super-neighborhoods rather than to completely different relations;
- 4) the transition to neighboring entries never causes to a jump to non-neighboring relations;
- 5) only a small fraction of combinatorially possible neighborhoods actually appears in the table;
- 6) there is a lot of symmetry which may be exploited for temporal reasoning;
- 7) the observations are valid not only with respect to the presented linearized neighborhood table; they hold for the complete 4-dimensional structure.

Observations 1) and 2) should not be completely surprising, since the structure exhibits gradual transitions from one qualitative state to another; only one (micro-) feature is changed at a time; these microfeature transitions correspond to the neighborhood relation in the structure. Observation 3) is very useful since it allows for reasoning under uncertainty. Observations 4) and 5) provoke the question of the cognitive significance of the neighborhoods. If the neighborhoods appear to correspond to cognitive relevant concepts, may be the reasoning should be done based on these neighborhoods which correspond to classes of relations rather than on the individual relations themselves.

Note that the neighborhoods that are found in the table either are contained in our list of concepts derived from relating semi-intervals (Figure 2) or are obtained by conjoining such concepts (except for the non-informative entry ? corresponding to the disjunction of all thirteen relations between two intervals). Figure 6 associates the icons and their corresponding neighborhoods with their mnemonics, their associated labels, the corresponding list of Allen-relations and the corresponding constraints between beginnings and endings of the respective events. With help from Figure 6 we can read the transitivity table as follows: If X *meets* Y and Y is *after* Z then X *survives* Z; or: If X *overlaps* Y and Y is *overlapped by* Z then X is a *contemporary* of Z.

ICON LABEL MNEMONIC

ALLEN

CONSTRAINTS

	?		< m o fi di si = s d f oi mi >	none
	ol	<i>older</i>	< m o fi di	$\alpha < A$
	hh	<i>head to head with</i>	si = s	$\alpha = A$
	yo	<i>younger</i>	d f oi mi >	$\alpha > A$
	sb	<i>survived by</i>	< m o s d	$\omega < \Omega$
	tt	<i>tail to tail with</i>	fi = f	$\omega = \Omega$
	sv	<i>survives</i>	di si oi mi >	$\omega > \Omega$
	pr	<i>precedes</i>	< m	$\omega \leq A$
	bd	<i>born before death of</i>	< m o fi di si = s d f oi	$\alpha < \Omega$
	ct	<i>contemporary of</i>	o fi di si = s d f oi	$\alpha < \Omega, \omega > A$
	db	<i>died after birth of</i>	o fi di si = s d f oi mi >	$\omega > A$
	sd	<i>succeeds</i>	mi >	$\alpha \geq \Omega$
	ob	<i>older & survived by</i>	< m o	$\alpha < A, \omega < \Omega$
	oc	<i>older contemporary of</i>	o fi di	$\alpha < A, \omega > A$
	sc	<i>surviving contemporary of</i>	di si oi	$\alpha < \Omega, \omega > \Omega$
	bc	<i>survived by contemporary of</i>	o s d	$\omega > A, \omega < \Omega$
	yc	<i>younger contemporary of</i>	d f oi	$\alpha > A, \alpha < \Omega$
	ys	<i>younger & survives</i>	oi mi >	$\alpha > A, \omega > \Omega$

Figure 6: Neighborhoods, their icons, labels, mnemonics, correspondences, and constraints.

4.1 Coarse reasoning based on neighborhoods

We have seen that the inferences that can be drawn from the transitivity table may be coarser than the initial conditions. We may have several reasons for stating the initial conditions in coarser terms, as well:

- 1) uncertainty may exist as to which initial condition in terms of the thirteen mutually exclusive relations holds;
- 2) the initial conditions may be stated in coarser terms corresponding to knowledge relating beginnings and/or endings of events (compare examples in section 2.1);
- 3) we want to use the conclusions from one inference step as initial conditions for further inference steps.

Coarser knowledge can always be expressed in terms of disjunctions of finer knowledge, inferences can be drawn on the basis of the finer knowledge, and conclusions can be derived by forming conjunctions of these inferences on a case by case basis. However, this approach is comparable to deriving general algebraic relationships from specific numerical instances. Rather than solving problems on the finest level of resolution we would like to solve them on the coarsest possible level.

In order to do this, we use as initial conditions disjunctions of relations corresponding to neighborhoods which can be found as conclusions in the transitivity table. We select the neighborhoods in such a way that finer initial conditions can be expressed in terms of conjunctions of coarser initial conditions, if necessary. This step corresponds to aggregating neighboring Allen-relations. Initially, we do not aggregate $<$, m , mi , $>$, since these relations do not have enough neighbors to form conjunctions for refinement. The following ten relations and neighborhoods were selected. An interesting question is, how much knowledge about the corresponding temporal domain can be recovered from the coarse representation (compare ‘coarse coding’ in distributed representations, e.g. Hinton et al. [1986]).

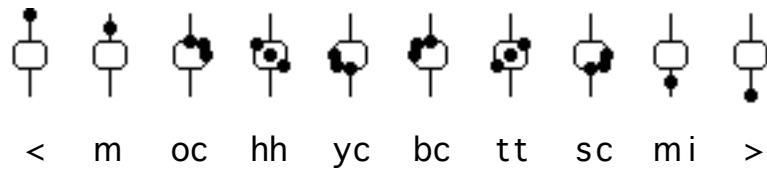


Figure 7: 10 relations and neighborhoods used as initial conditions for coarse reasoning.

We now form a transitivity table whose cell values consist of the disjunctions of all the combinations of transitivities of the constituent relations of the initial neighborhoods (Figure 8). Not surprisingly, we obtain similar patterns as in the case of fine reasoning. Due to the fact that we did not have any abrupt jumps between neighboring patterns, we only find connected neighborhoods again; due to the fact that in many cases neighboring patterns were sub-neighborhoods or super-neighborhoods of each other, we do not get many new patterns.

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Figure 8: Transitivity table condensed for coarse reasoning based on neighborhoods.

In fact, the patterns **ob** and **ys** disappeared and two new patterns corresponding to the disjunctions **ct ob** and **ct ys** appeared. These new neighborhoods will be called *born before death of* (**bd**) and *died after birth of* (**db**), respectively (compare Figure 6).

The aggregated table shown in Figure 8 permits us to do two things: coarse reasoning and fine reasoning. For coarse reasoning, we simply look up the neighborhood of possible relations in the table. For example, if we know that X is an *older contemporary of* (**oc**) Y and Y is a *younger contemporary of* (**yc**) Z, we can infer that X is a *contemporary of* (**ct**) Z or if instead Y is *head to head with* (**hh**) Z then X is also an *older contemporary of* (**oc**) Z (see Figure 9). So, the conclusions do not necessarily become coarser when the initial conditions become coarser.

The computational pay-off of coarse reasoning is best when the granularity of the processed knowledge agrees with the sizes of the involved neighborhoods, since this minimizes the number of cases that have to be combined by disjunctions and/or conjunctions.

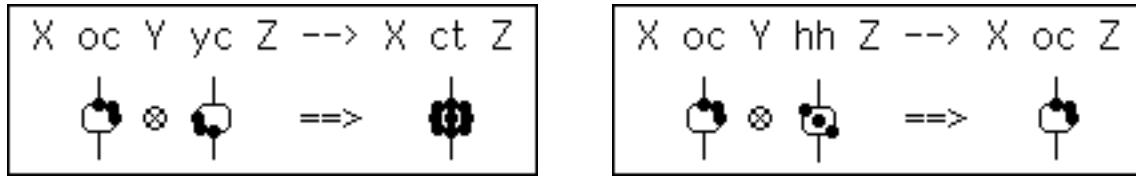


Figure 9: Two instances of the coarse transitivity relation (denoted by \otimes).

4.2 Fine reasoning based on neighborhoods

For fine reasoning, we form the conjunctions of the inferences we can draw by coarse reasoning. By algebraic considerations we obtain at least all fine relations which are obtained by fine reasoning. For example (see Figure 10), the fine relation $X \text{ f } Y$ corresponds to the two coarse relations $X \text{ yc } Y$ and $X \text{ tt } Y$; the fine relation $Y \text{ o } Z$ corresponds to the two coarse relations $Y \text{ oc } Z$ and $Y \text{ bc } Z$. So, if $X \text{ f } Y$ and $Y \text{ o } Z$ hold, then the corresponding coarse relations also hold and so do the conclusions which we can draw from the interactions of the coarse relations. These interactions yield the neighborhoods ?, **bd**, **db**, **bc**; the intersection of these neighborhoods is **bc**.

The result is identical to the result we get by fine reasoning. In fact, we obtain the correct optimal result in all cases. This is not due to the algebraic properties of the operations performed but due to the independence of constraints between the neighborhoods that are being combined. For example, **yc** corresponds to the constraints $\alpha > A$, $\alpha < \Omega$ and **tt** corresponds

to the independent constraint $\omega = \Omega$ which combined yield the constraints $\alpha < \Omega$ and $\omega = \Omega$, which are the conditions for the relation f . (The constraint $\alpha < \Omega$ follows from $\omega = \Omega$ and the domain-inherent constraint $\alpha < \omega$.)

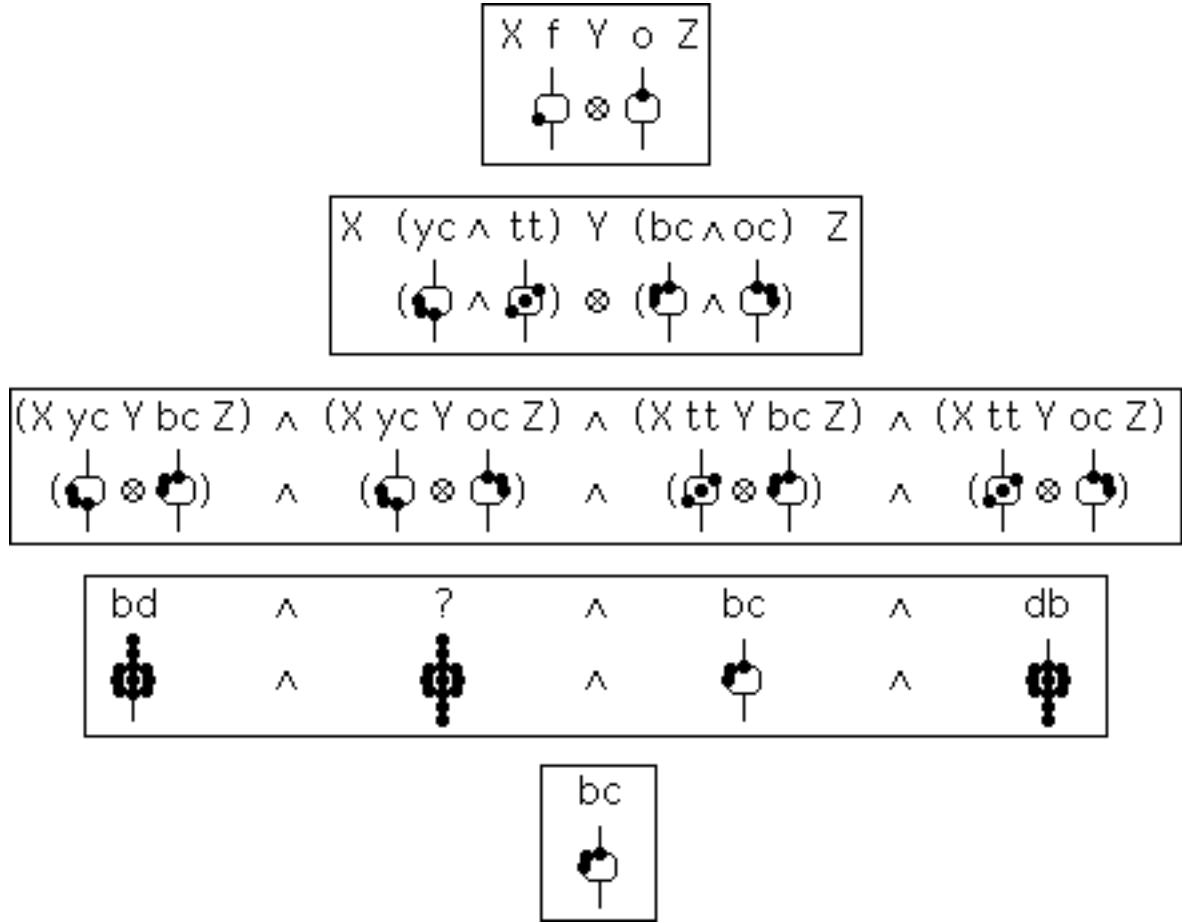


Figure 10: Elaborate fine reasoning by intersecting results from coarse reasoning.

In the same manner it is possible to combine fine knowledge (i.e., complete knowledge about the relation between two events) and coarse knowledge (here: knowledge about the relation between semi-intervals), in this inference scheme.

5 Compacting the knowledge base

Computationally viewed, the main advantage of coarse reasoning is the fact that we can reason with coarse knowledge by direct lookup – in Allen’s scheme we would have had to perform a disjunction for every missing fine relation; on the other hand, of course, we have to perform a few operations in order to do fine reasoning. Thus, it depends on the granularity of the processed knowledge which granularity should be chosen for reasoning. Since knowledge tends to become coarser beyond the first level of reasoning, there is a bias towards coarse reasoning for multi-level reasoning tasks.

By aggregating relations for neighborhood-based reasoning, the transitivity table shrank from $13 \times 13 = 169$ to $10 \times 10 = 100$ entries. We may be able to further simplify the knowledge base underlying the reasoning scheme. In the example given in section 4.2 we showed how fine knowledge can be obtained by combining intersecting pieces of coarse knowledge. The final conclusion we obtained in the inference process was already present in full detail in one of the four inferences we combined, namely in the inference drawn from $X \text{ tt } Y$ and $Y \text{ bc } Z$. Do we always have to form the conjunction of all possible inferences from the intersecting initial neighborhoods or can we systematically simplify the procedure?

Inspection of the condensed transitivity table (Figure 8) shows that in no case more than two inferences contribute towards the solution. In fact, only 54 of the 100 entries of the table yield useful constraints for the reasoning procedure. These entries are depicted in Figure 11. The remaining entries provide no additional information.

A												
<												
m												
oc												
hh												
yc												
bc												
tt												
sc												
mi												
>												

Figure 11: Condensed transitivity table without non-contributing entries.

5.1 Symmetry and redundancy

There is much symmetry in the neighborhood-oriented transitivity table which can be exploited for matrix simplification. Most obvious is the symmetry between the top and bottom halves of the table: if for both initial conditions A and B (compare Figure 11) the icons are

flipped vertically, the table entries are flipped vertically. This corresponds to the symmetry between ' $<$ ' and ' $>$ ' when comparing semi-intervals.

After compaction of the table due to this symmetry, the columns corresponding to the neighborhoods bc , tt , sc are not needed and can be eliminated. In addition, the first two columns can be merged. This corresponds to forming a neighborhood of the relations $<$ and m . Figure 12 shows the table compacted to 25 entries. On the right hand and bottom sides of the table the initial conditions for the entries to be vertically flipped are shown.

Figure 12: Transitivity table compacted to 25 entries.

Further symmetries and other regularities allow the elimination of all but seven table entries: The entries in the upper right hand corner of the table shown in Figure 12 can be mapped into the lower left half of the table by exchanging the x- and y-axes of the initial conditions and by flipping the corresponding entries horizontally. This transformation yields layers II - ii of initial conditions (Figure 13).

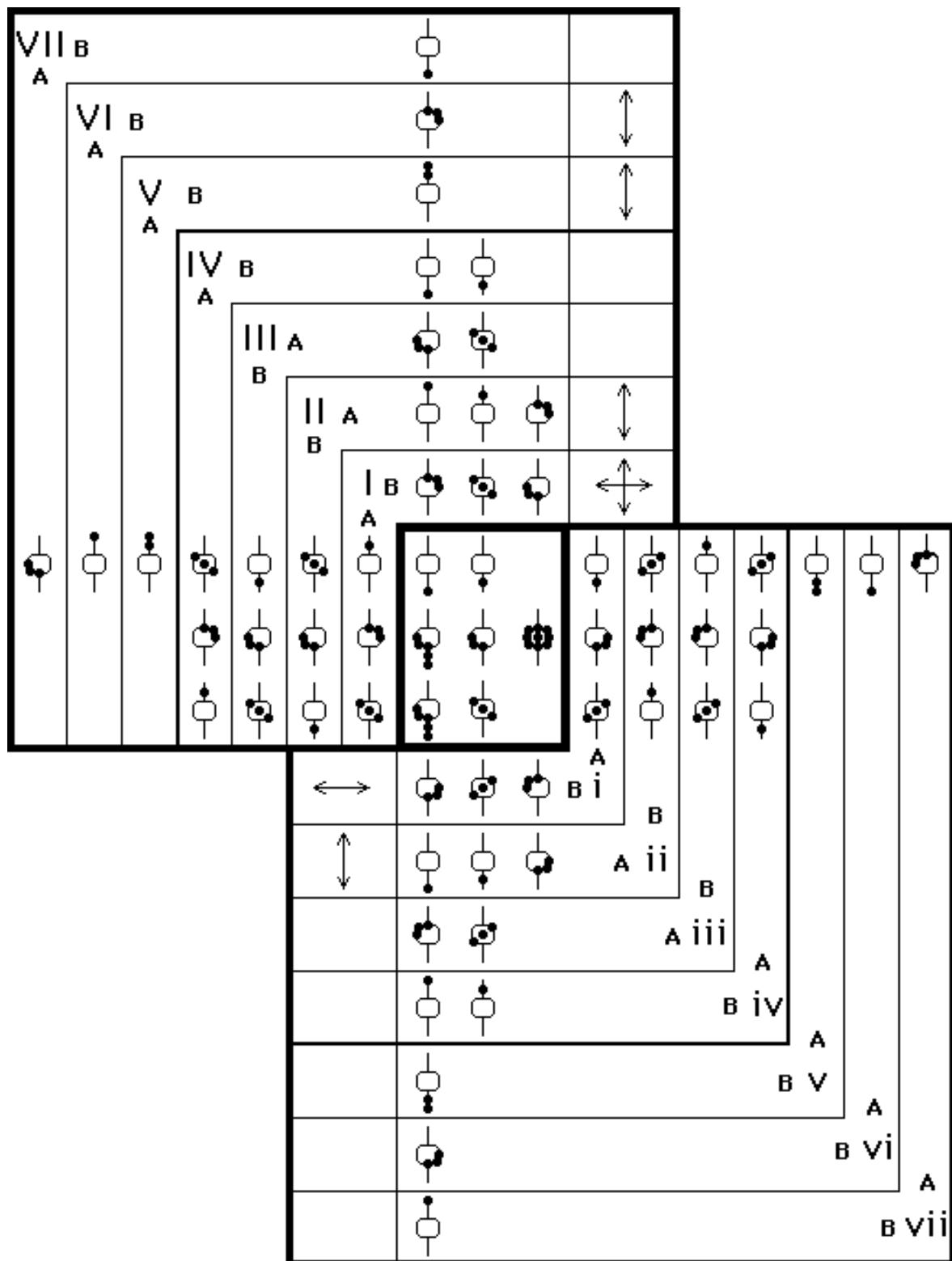


Figure 13: Transitivity table compressed to 7 entries.

The entries in the lower right hand corner of the table in Figure 12 can be mapped into the upper left half of the table by inverting the x- and y-axes of the initial conditions and flipping the table entries both vertically and horizontally (or equivalently, by rotating them by 180 degrees). This transformation yields layers III - iii and IV - iv.

Furthermore, we find four identical entries in the upper left hand corner of the table. They can be mapped into a single entry by adding layers V - v, VI - vi, and VII - vii, each containing one singleton of initial conditions.

Finally, the table can be simplified in order to minimize transformations on the table entries; this is done by rotating the remaining entries by 180 degrees.

Figure 13 shows the compressed table consisting of seven entries which are accessible by 2×7 layers of initial conditions. Two of the entry patterns are identical (yo). The initial conditions in each layer are mutually exclusive. Neighboring initial conditions always correspond to neighboring neighborhoods, in each layer. Only pairs of conditions belonging to the same layer (I-VII or i-vii) have to be used for accessing the entries. Entries derived from layers marked with a vertical double arrow have to be flipped vertically, entries derived from layers marked with a horizontal double arrow have to be flipped horizontally, entries derived from layers marked with both arrows have to be flipped in both directions.

5.2 Reasoning based on the compressed transitivity table

The compressed transitivity table represents very general regularities corresponding to the symmetries involved in the relations between neighborhoods. Only the first 2×4 layers of initial conditions (I - IV and i - iv) actually define the structure of the table; the other 2×3 layers (V - VII and v - vii) all refer to the same single entry in the table.

In order to use this table for temporal reasoning, neighborhoods of event relations and/or individual event relations are matched with the corresponding initial conditions for the table as in the previous transitivity tables. Then the conjunction of the corresponding entries is formed. The entries corresponding to the initial conditions marked with arrows must be flipped as suggested by the arrow before the conjunction is formed.

Flipping the entry patterns corresponds to a very simple re-labeling of relations. Specifically, horizontal flipping corresponds to exchanging the labels f_i and s_i , d_i and m_i , s_i and f_i ; vertical flipping corresponds to exchanging the labels $<$ and $>$, m and mi , o and oi , f and s .

and si , s and f ; flipping both dimensions corresponds to exchanging the labels $<$ and $>$, m and mi , o and oi , fi and f , di and d , si and s (compare Figure 14).

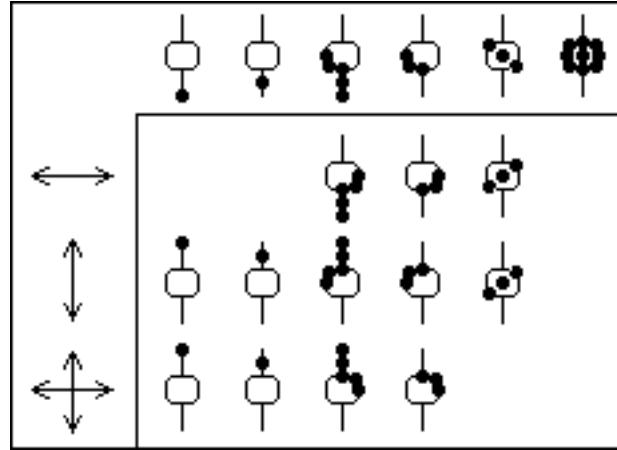


Figure 14: The effect of horizontal and vertical flipping of the 6 distinct entries of the compressed transitivity table. Blank entries indicate that the transformation has no effect and is therefore not required.

5.3 Examples

1) Fine reasoning. Suppose, X is *started by* Y and Y *finishes* Z . What is the relationship between X and Z ? We check the layers of initial conditions for pairs A , B corresponding to the pair si , f . We obtain four matches: a) layer I: bottom right entry; b) layer i: center entry; c) layer ii: top right entry; d) layer III: center entry. a) and c) correspond to non-contributing entries; thus, we only have to consider b) and d). Both sets of initial conditions point to the center entry of the table corresponding to relation yc . The table indicates that entries associated with layer i have to be flipped horizontally; therefore we form the conjunction of yc and its horizontally flipped image sc . We obtain oi . Thus, X is *overlapped by* Z is the final conclusion.

2) Coarse reasoning. Suppose, X is a *younger contemporary* of Y and Y is *head to head with* Z . Layer III contains the matching pair of initial conditions which point to the relation *younger*. Layer III does not indicate that flipping is required; thus X is *younger than* Z is the final conclusion.

3) Combining fine and coarse knowledge. Suppose, X *meets* Y and Y is a *younger contemporary* of Z . How are X and Z temporally related? We check the layers of initial

conditions for pairs A, B of initial conditions corresponding to the pair m, yc. We obtain two matches: top right entry for layer I and center entry for layer II. The top right entry is a non-contributing entry, so we only have to consider the center entry of the table which corresponds to the neighborhood yc. The table indicates that the entries obtained through layer II have to be vertically flipped; this yields the relation bc. Thus, X is a *survived by contemporary* of Z (compare Figure 15).

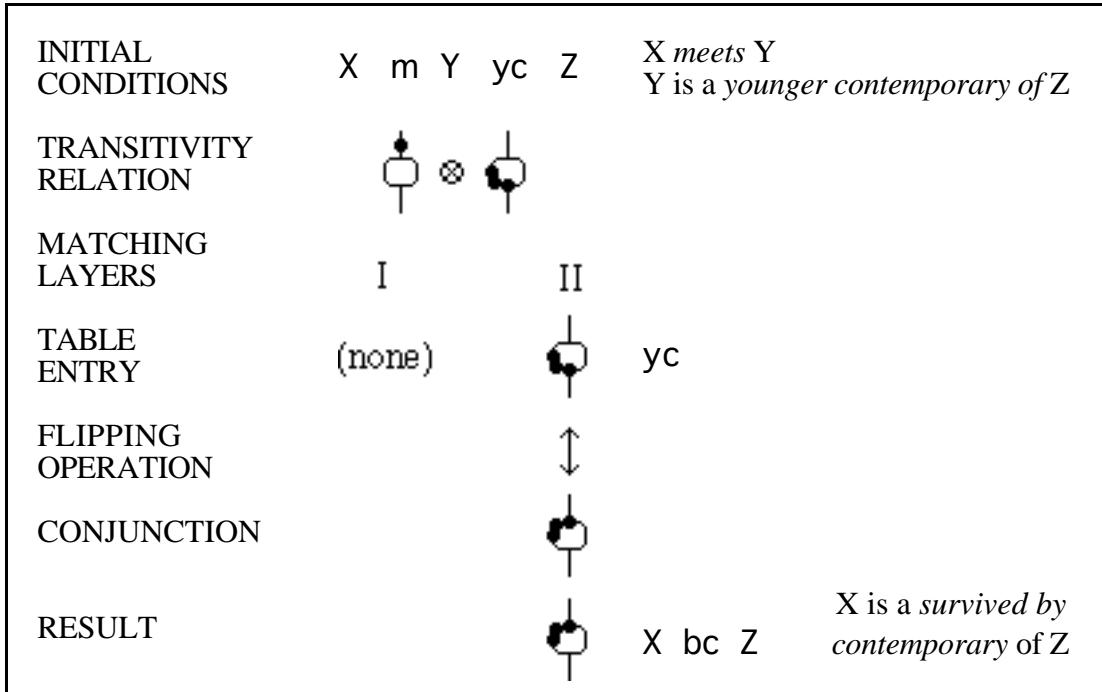


Figure 15: The reasoning steps involved in reasoning with the compressed transitivity table. The example shows how fine and coarse knowledge can be combined.

6 Conclusions

We have modified Allen's approach to interval-based representation of temporal relations in such a way that it can be used rather naturally for reasoning with incomplete knowledge, specifically with coarse knowledge about temporal relationships. Our approach adds flexibility and appears to be cognitively more adequate. It is based on a vision-oriented view of events: events are not treated as isolated entities; rather, they are viewed as conceptual items which are embedded in a network of related events. In this view, the concept of 'conceptual neighborhood' becomes essential.

Conceptual neighborhood plays an important role in cognition. Many cognitive functions rely on the assumption that the world they are dealing with is continuous or quasi-continuous, i.e., changes happen in steps rather than in jumps. For the specific domain of temporal relations we have shown that this assumption is justified.

The concept of neighborhood is a prerequisite for our concept of coarse knowledge. Coarse knowledge allows for short-cuts in reasoning in the following way. Allen's original reasoning strategy conceptually contains four levels of knowledge: 1) problem level in terms of coarse knowledge; 2) initial conditions expressed in terms of fine knowledge; 3) constraints on the transitivity relation corresponding to coarse knowledge; 4) conclusion expressed in terms of fine knowledge. Level 1) is present only if the problem is initially given in coarse terms; level 4) is present only if the result is stated in fine terms. In our approach, we merge levels 1) - 3) by reasoning directly on the coarse level.

I would like to suggest that this short-cut is just one instance of neighborhood-based problem reduction and that the general idea can be applied in various domains of cognition. For example, in natural language representation, concepts are frequently represented in fine terms; as a consequence, semantical ambiguities demand a multiplication of processing effort. If the concepts could be represented on a higher conceptual level, some of the ambiguities would never arise and consequently would not have to be resolved. Another domain is theorem proving⁺ where it is desirable to identify coarser concepts whose conceptual instants share important properties.

The system we have developed in this report can be used as a basic generic structure for the construction of very small sequential reasoners or of parallel reasoners with a very regular structure. For a sequential reasoner, the compressed transitivity table would be sequentially accessed by the layers matching the initial conditions before the entries are flipped and conjoined. For a parallel reasoner, several copies of the table could be accessed simultaneously; the entry-flipping could be wired-in directly. Also, the simple and regular structure of the neighborhood reasoning structure (Figure 8) appears to make implementation by means of an associative memory appropriate.

The presented approach can be extended in various directions. The neighborhood concept can be used for reasoning under uncertainty. Uncertainty or incomplete knowledge correspond to a neighborhood of possibilities compared to a single possibility within this neighborhood in the case of certainty or complete knowledge. This view is in contrast to a view

⁺ this suggestion is due to Steffen Hölldobler.

in which uncertainty or incomplete knowledge correspond to disjunctions of unrelated possibilities. If coarse knowledge becomes coarser due to fuzziness, the same reasoning principles can be applied to coarse knowledge which we have applied to fine knowledge [Freksa, forthcoming]; although full recovery of fine knowledge will no longer be guaranteed.

An obvious extension of the approach is for reasoning with 1-dimensional space which shares many properties with time (compare aquarium metaphor, section 3.3). Extensions for reasoning about 2- or 3-dimensional space are more challenging (compare Freksa & Habel 1990), but a coarse reasoning approach appears to be better tractable than a fine reasoning approach. We expect that the large amount of regularity and the conceptual simplicity of the system will proof helpful for developing representation schemes for more-dimensional spaces.

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