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From Default and Autoepistemic Logics to Disjunctive Answer Set Programs via the Logic of GK

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Abstract. We show how the pure logic of GK can be embedded into disjunctive logic programming. The translation we present is polynomial, but not modular, and introduces new variables. The result can then be used to compute the extension/expansion semantics of default and autoepistemic logics using disjunctive ASP solvers.

1 Introduction

Lin and Shoham [6] proposed a logic with two modal operators $\mathbf{K}$ and $\mathbf{A}$, standing for knowledge and assumption, respectively. The idea is that one starts with a set of assumptions (those true under the modal operator $\mathbf{A}$), computes the minimal knowledge under this set of assumptions, and then checks to see if the assumptions were justified in that they agree with the resulting minimal knowledge.

In this paper, for the first time, we consider computing models of GK theories by disjunctive logic programs. We shall propose a polynomial translation from a (pure) GK theory to a disjunctive logic program such that there is a one-to-one correspondence between GK models of the GK theory and answer sets of the resulting disjunctive logic program. The result can then be used to compute the extension/expansion semantics of default logic [9] and autoepistemic logic [8]. To substantiate this claim, we have implemented the translation into a working prototype gk2dlp.[3] A longer version of this paper with more details is available as a workshop paper [5].

2 Main Result: From Pure GK to Disjunctive ASP

Syntactically, the logic of GK is a propositional modal language with modalities $\mathbf{A}$ and $\mathbf{K}$; *pure* GK formulas contain no nested modalities. For space reasons, we cannot present background and refer to [6, 2].

Before presenting the translation, we introduce some notations. Let F be a pure GK formula, we use $tr_p(F)$ to denote the propositional formula obtained from F by replacing each occurrence of a formula $\mathbf{K}\phi$ (called a $\mathbf{K}$-atom) by k_ϕ and each occurrence of a formula $\mathbf{A}\psi$ (an $\mathbf{A}$-atom) by a_ψ, where k_ϕ and a_ψ are new atoms with respect to ϕ and ψ respectively. For a pure GK theory T (a set of pure GK formulas), we define $tr_p(T) = \bigwedge_{F\in T} tr_p(F)$. Intuitively, the new atom k_ϕ will be used to encode $\phi \in \mathbf{K}(M)$ for a GK (Kripke) model M for T, that is, ϕ is known in M. Likewise, a_ϕ encodes $\phi \in \mathbf{A}(M)$, which means that ϕ is assumed in M. Given a propositional formula ϕ and an atom a, we use ϕ^a to denote the propositional formula obtained from ϕ by replacing each occurrence of an atom p with a new atom p^a with respect to a. Intuitively, such new atoms will be used to guarantee the existence of certain interpretations witnessing various technical properties.

We now stepwise work our way towards the main result. We start out with a result that relates a pure GK theory T to a propositional formula. In what follows, $\mathit{Atom}_{\mathbf{K}}(T)$ and $\mathit{Atom}_{\mathbf{A}}(T)$ denote the sets of $\mathbf{K}$-atoms and $\mathbf{A}$-atoms occurring in T, respectively.

Proposition 1 *Let T be a pure GK theory. A Kripke interpretation M is a model of T if and only if there exists a model I^* of the propositional formula $\Phi_T = tr_p(T) \wedge \Phi_{snd} \wedge \Phi_{wit}^{\mathbf{K}} \wedge \Phi_{wit}^{\mathbf{A}}$ with*

$$\Phi_{snd} = \bigwedge_{\phi\in \mathit{Atom}_{\mathbf{K}}(T)} (k_\phi \supset \phi^k) \wedge \bigwedge_{\phi\in \mathit{Atom}_{\mathbf{A}}(T)} (a_\phi \supset \phi^a)$$

$$\Phi_{wit}^{\mathbf{K}} = \bigwedge_{\psi\in \mathit{Atom}_{\mathbf{K}}(T)} \left(\neg k_\psi \supset \left(\neg\psi^{k_\psi} \wedge \bigwedge_{\phi\in \mathit{Atom}_{\mathbf{K}}(T)} (k_\phi \supset \phi^{k_\psi})\right)\right)$$

$$\Phi_{wit}^{\mathbf{A}} = \bigwedge_{\psi\in \mathit{Atom}_{\mathbf{A}}(T)} \left(\neg a_\psi \supset \left(\neg\psi^{a_\psi} \wedge \bigwedge_{\phi\in \mathit{Atom}_{\mathbf{A}}(T)} (a_\phi \supset \phi^{a_\psi})\right)\right)$$

- $\mathbf{K}(M) \cap \mathit{Atom}_{\mathbf{K}}(T) = \{\phi \mid \phi \in \mathit{Atom}_{\mathbf{K}}(T)$ *and* $I^* \models k_\phi\}$*;*
- $\mathbf{A}(M) \cap \mathit{Atom}_{\mathbf{A}}(T) = \{\phi \mid \phi \in \mathit{Atom}_{\mathbf{A}}(T)$ *and* $I^* \models a_\phi\}$*.*

The proposition examines the relationship between models of a pure GK theory and particular models of the propositional formula Φ_T. The first conjunct $tr_p(T)$ of the formula Φ_T indicates that the k-atoms and a-atoms in it can be interpreted in accordance with $\mathbf{K}(M)$ and $\mathbf{A}(M)$ such that $I^* \models tr_p(T)$ iff M is a model of T. The soundness formula Φ_{snd} achieves that the sets $\{\phi \mid \phi \in \mathit{Atom}_{\mathbf{K}}(T)$ and $I^* \models k_\phi\}$ and $\{\phi \mid \phi \in \mathit{Atom}_{\mathbf{A}}(T)$ and $I^* \models a_\phi\}$ are consistent. The witness formulas Φ_{wit} indicate that, if $I^* \models \neg k_\psi$ for some $\psi \in \mathit{Atom}_{\mathbf{K}}(T)$ (resp. $\psi \in \mathit{Atom}_{\mathbf{A}}(T)$) then there exists a model I' of $\mathbf{K}(M)$ (resp. $\mathbf{A}(M)$) such that $I' \models \neg\psi$, where I' is explicitly indicated by newly introduced p^{k_ψ} (resp. p^{a_ψ}) atoms.

While Proposition 1 aligns Krikpe models and propositional models of the translation, there is yet no mention of GK's typical minimization step. This is the task of the next result, which extends the above relationship to GK models.

Proposition 2 *Let T be a pure GK theory. A Kripke interpretation M is a GK model of T if and only if there exists a model I^* of the propositional formula Φ_T such that*

- $\mathbf{K}(M) = \mathbf{A}(M) = \mathit{Th}(\{\phi \mid \phi \in \mathit{Atom}_{\mathbf{K}}(T)$ *and* $I^* \models k_\phi\})$*;*
- *for each* $\psi \in \mathit{Atom}_{\mathbf{A}}(T)$*,*

$$I^* \models a_\psi \text{ iff } \psi \in \mathit{Th}(\{\phi \mid \phi \in \mathit{Atom}_{\mathbf{K}}(T) \text{ and } I^* \models k_\phi\})$$

- *there does not exist another model $I^{*\prime}$ such that*
  - $I^{*\prime} \cap \{a_\phi \mid \phi \in \mathit{Atom}_{\mathbf{A}}(T)\} = I^* \cap \{a_\phi \mid \phi \in \mathit{Atom}_{\mathbf{A}}(T)\}$,
  - $I^{*\prime} \cap \{k_\phi \mid \phi \in \mathit{Atom}_{\mathbf{K}}(T)\} \subsetneq I^* \cap \{k_\phi \mid \phi \in \mathit{Atom}_{\mathbf{K}}(T)\}$.

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In Proposition 2, we only need to consider Kripke interpretations M such that $\mathbf{A}(M) \cup \mathbf{K}(M)$ is consistent. This means that formula Φ_T can be modified to $\Psi_T = tr_p(T) \wedge \Psi_{snd} \wedge \Psi_{wit}$ with

$$\Psi_{snd} = \bigwedge_{\phi \in Atom_{\mathbf{K}}(T)} (k_\phi \supset \phi) \wedge \bigwedge_{\phi \in Atom_{\mathbf{A}}(T)} (a_\phi \supset \phi)$$

$$\Psi_{wit} = \bigwedge_{\psi \in Atom_{\mathbf{K}}(T)} \left(\neg k_\psi \supset \Psi_\psi^{\mathbf{K}}\right) \wedge \bigwedge_{\psi \in Atom_{\mathbf{A}}(T)} \left(\neg a_\psi \supset \Psi_\psi^{\mathbf{A}}\right)$$

$$\Psi_\psi^{\mathbf{K}} = \neg\psi^{k_\psi} \wedge \bigwedge_{\phi \in Atom_{\mathbf{K}}(T)} (k_\phi \supset \phi^{k_\psi}) \wedge \bigwedge_{\phi \in Atom_{\mathbf{A}}(T)} (a_\phi \supset \phi^{k_\psi})$$

$$\Psi_\psi^{\mathbf{A}} = \neg\psi^{a_\psi} \wedge \bigwedge_{\phi \in Atom_{\mathbf{K}}(T)} (k_\phi \supset \phi^{a_\psi}) \wedge \bigwedge_{\phi \in Atom_{\mathbf{A}}(T)} (a_\phi \supset \phi^{a_\psi})$$

Using this new formula, the result of Proposition 2 can be restated.

Proposition 3 *Let T be a pure GK theory. A Kripke interpretation M is a GK model of T if and only if there exists a model I^* of the propositional formula Ψ_T such that*

- $\mathbf{K}(M) = \mathbf{A}(M) = Th(\{\phi \mid \phi \in Atom_{\mathbf{K}}(T) \text{ and } I^* \models k_\phi\})$;
- *for each* $\psi \in Atom_{\mathbf{A}}(T)$,

  if $I^* \models a_\psi$ *then* $\psi \in Th(\{\phi \mid \phi \in Atom_{\mathbf{K}}(T) \text{ and } I^* \models k_\phi\})$
- *there does not exist another model* $I^{*\prime}$ *of* Φ_T *such that*
  - $I^{*\prime} \cap \{a_\phi \mid \phi \in Atom_{\mathbf{A}}(T)\} = I^* \cap \{a_\phi \mid \phi \in Atom_{\mathbf{A}}(T)\}$,
  - $I^{*\prime} \cap \{k_\phi \mid \phi \in Atom_{\mathbf{K}}(T)\} \subsetneq I^* \cap \{k_\phi \mid \phi \in Atom_{\mathbf{K}}(T)\}$.

We are now ready for our main result, translating a pure GK theory to a disjunctive logic program. First, we introduce some notations. Let T be a pure GK theory, we use $tr_{ne}(\Psi_T)$ to denote the nested expression obtained from Ψ_T by first converting it to negation normal form, then replacing "$\wedge$" by "," and "$\vee$" by ";". For a propositional formula ϕ, we use $tr_c(\phi)$ to denote the set of rules obtained from the conjunctive normal form of ϕ (possibly containing new variables) by replacing each clause $(p_1 \vee \cdots \vee p_l \vee \neg p_{l+1} \vee \cdots \vee \neg p_m)$ by a rule $p_1; \ldots; p_l \leftarrow p_{l+1}, \ldots, p_m$. We use $\widehat{\phi}$ to denote the propositional formula obtained from ϕ by replacing each occurrence of an atom p by a new atom $\hat{p}$. We use Φ_T^* to denote the propositional formula obtained from Φ_T by replacing each occurrence of an atom p (except atoms of the form a_ϕ for some $\phi \in Atom_{\mathbf{A}}(T)$) by a new atom p^*.

Intuitively, by Proposition 3, $tr_{ne}(\Psi_T)$ is used to restrict interpretations for introduced k-atoms and a-atoms so that these interpretations serve as candidates M for GK models. By Proposition 1, Φ_T^* constructs possible models M' of the GK theory (with $\mathbf{A}(M') = \mathbf{A}(M)$) that are used to test whether M is a GK model.

Inspired by the linear translation from parallel circumscription into disjunctive logic programs in [3], we have the following theorem.

Theorem 1 *Let T be a pure GK theory. A Kripke interpretation M is a GK model of T if and only if there exists an answer set S of the logic program $tr_{lp}(T)$:*

(1) $\bot \leftarrow not\ tr_{ne}(\Psi_T)$

(2) $p'; \neg p' \leftarrow \top$ (for each atom p' occurring in $tr_{ne}(\Psi_T)$)

(3) $u; A \leftarrow B$ (for each rule $A \leftarrow B$ in $tr_c(\Phi_T^*)$)

(4) $u; c_{\phi_1}; \cdots; c_{\phi_m} \leftarrow \top$ ($\{\phi_1, \ldots, \phi_m\} = Atom_{\mathbf{K}}(T)$)

(5) $u \leftarrow c_\phi, not\ k_\phi$ (for each $\phi \in Atom_{\mathbf{K}}(T)$)

(6) $u \leftarrow k_\phi^*, not\ k_\phi$ (for each $\phi \in Atom_{\mathbf{K}}(T)$)

(7) $u \leftarrow c_\phi, k_\phi^*, not\ \neg k_\phi$ (for each $\phi \in Atom_{\mathbf{K}}(T)$)

(8) $u; c_\phi; k_\phi^* \leftarrow not\ \neg k_\phi$ (for each $\phi \in Atom_{\mathbf{K}}(T)$)

(9) $p^* \leftarrow u$ (for each new atom p^* in $tr_c(\Phi_T^*)$)

(10) $c_\phi \leftarrow u$ (for each $\phi \in Atom_{\mathbf{K}}(T)$)

(11) $\bot \leftarrow not\ u$

(12) $v; A \leftarrow B$

(for each rule $A \leftarrow B$ in the $tr_c(\cdot)$ translation of

$$\bigwedge_{\phi \in Atom_{\mathbf{K}}(T)} (k_\phi \supset \widehat{\phi}) \wedge \neg \bigwedge_{\phi \in Atom_{\mathbf{A}}(T)} (a_\phi \supset \widehat{\phi}))$$

(13) $\hat{p} \leftarrow v$

(for each atom $\hat{p}$ except k-atoms and a-atoms in $tr_c(\cdot)$ of

$$\bigwedge_{\phi \in Atom_{\mathbf{K}}(T)} (k_\phi \supset \widehat{\phi}) \wedge \neg \bigwedge_{\phi \in Atom_{\mathbf{A}}(T)} (a_\phi \supset \widehat{\phi}))$$

(14) $\bot \leftarrow not\ v$

where u, v, and c_ϕ (for each $\phi \in Atom_{\mathbf{K}}(T)$) are new atoms, such that $\mathbf{K}(M) = \mathbf{A}(M) = Th(\{\phi \mid \phi \in Atom_{\mathbf{K}}(T) \text{ and } k_\phi \in S\})$.

Intuitively, rules (1) and (2) in $tr_{lp}(T)$ guarantee that each answer set is a model of the formula Ψ_T. Rules (3) to (8) then create model candidates that violate the minimal knowledge condition; rules (9) to (11) eliminate answer sets for which such models exist. Finally, rules (12) to (14) check whether assumptions and knowledge coincide.

Due to the results of Eiter and Gottlob [2] and Lin and Zhou [7], our Theorem 1 yields a complexity result for the pure logic of GK.

Proposition 4 *Let T be a pure GK theory. The problem of deciding whether T has a GK model is Σ_2^P-complete.*

3 Discussion

We have presented the first translation of pure formulas of the logic of GK to disjunctive answer set programming. Among other things, this directly leads to implementations of default and autoepistemic logics under different semantics. The translation presented in this paper is a generalization of the one presented for Turner's logic of universal causation by Ji and Lin [4]. In recent related work, Chen et al. [1] presented the dl2asp system that implements propositional default logic by translating default theories to (non-disjunctive) ASPs. For their translation, the size of the translated logic program might grow exponentially in the size of the input default theory. In contrast, the size increase of our translation via the logic of GK is polynomial.

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