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Assertional-based Prioritized Removed Sets Revision of $DL\text{-}Lite_R$ Knowledge Bases[1]

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Abstract. The paper proposes an extension of "Prioritized Removed Sets Revision" (PRSR) to $DL\text{-}Lite_R$ stratified knowledge bases. The revision strategy is based on inconsistency minimization and consists in determining smallest subsets of assertions to be dropped from the current $DL\text{-}Lite_R$ knowledge base, taking the stratification into account, in order to restore consistency and accept the input. We consider different forms of input: membership assertion, positive inclusion axiom or negative inclusion axiom. We show that according to the form of input and under some conditions PRSR can be achieved in polynomial time.

1 INTRODUCTION

DL-Lite [3], a family of lightweight DLs provides a powerful framework that allows for a flexible representation of knowledge with a low computational complexity for the reasoning process [1]. In particular, *DL-Lite* is specifically tailored for applications that use huge volume of data, like Web applications, in which query answering is the most important reasoning task. In the last years, DLs knowledge base evolution gave rise to increasing interest and often concerns the situation where new information should be incorporated while ensuring the consistency of the result. Such problem is well-known as a belief revision problem. Recently, an assertional-based "Removed Sets Revision" (RSR) approach has been proposed in [2] to revise $DL\text{-}Lite$ knowledge bases. This approach, consists in determining minimal subsets of assertions that should be dropped from the current *DL-Lite* knowledge base in order to restore consistency and accept the input. In this approach, minimality is defined in terms of cardinality and not in not in terms of set inclusion. Moreover, computing the set of minimal information responsible of inconsistency can be done in polynomial time. However, in real word applications, assertional facts are often provided by several and potentially conflicting sources. Concatenating them gives a prioritized or a stratified assertional base, and to our best knowledge, revising prioritized *DL-Lite* knowledge bases has not been addressed so far.

In this paper, we only consider $DL\text{-}Lite^{H}_{core}$ (known as $DL\text{-}Lite_R$) that underlies *OWL2-QL* language and generalizes $DL\text{-}Lite_F$. However results of this work can be easily generalized for others DL-Lite logics [1]. In *DL-Lite* a TBox $\mathcal{T}$={PIs,NIs} can be viewed as composed of positive inclusion axioms, denoted by (PIs), and negative inclusion axioms, denoted by (NIs). Two kinds of inconsistency can be distinguished in DL-based knowledge bases: incoherence and inconsistency. Namely, a knowledge base is said to be inconsistent iff it does not admit any model and it is said to be incoherent if there exists at least a non-satisfiable concept, namely for each interpretation I which is a model of $\mathcal{T}$, we have C^I=$\emptyset$. The negative closure of $\mathcal{T}$, denoted by $cln(\mathcal{T})$, performs interaction between PIs and NIs and is obtained using the rules detailed in [3]. A new property was given for consistency checking: $\mathcal{K}$ is said to be consistent iff $\langle cln(\mathcal{T}), \mathcal{A}\rangle$ is consistent [3]. In this paper, the revision leads to ignoring some assertional facts, namely we give a priority to TBox over ABox. Let $\mathcal{K}$ be an inconsistent knowledge base, we define the notion of conflict which is a minimal inconsistent subset of $\mathcal{A}$, more formally:

Definition 1. *Let $\mathcal{K} = \langle \mathcal{T}, \mathcal{A}\rangle$ be an inconsistent DL-Lite knowledge base. A conflict set C is a set of membership assertions such that: $i)$ $C \subseteq \mathcal{A}$, $ii)$ $\langle \mathcal{T}, C\rangle$ is inconsistent, $iii)$ $\forall C'$, $C' \subset C$, $\mathcal{T} \cup C'$ is consistent.*

We denote by $\mathcal{C}(\mathcal{K})$ the collection of conflicts in $\mathcal{K}$. Since $\mathcal{K}$ is assumed to be finite, if $\mathcal{K}$ is inconsistent then $\mathcal{C}(\mathcal{K}) \neq \emptyset$ is also finite. Within the *DL-Lite* framework, in order to restore consistency while keeping new information, the Prioritized Removed Sets Revision strategy removes exactly one assertion in each conflict minimizing the minimum number of assertions from $\mathcal{A}_1$, then the minimum number of assertions in $\mathcal{A}_2$, and so on. Using lexicographic criterion instead of set inclusion one, will reduce the set of potential conflicts. Taking the stratification of the $ABox$ into account has not been considered before for revising or repairing *DL-Lite* knowledge base.

2 PRSR FOR DL-LITE KNOWLEDGE BASES

In the following, we consider stratified knowledge bases such that $\mathcal{T}$ is coherent and not stratified but, on contrast, the ABox is stratified. The ABox is partitioned into n strata, $\mathcal{A}$=$\mathcal{A}_1 \cup \cdots \cup \mathcal{A}_n$ such that the assertions in $\mathcal{A}_i$ have the same level of priority and have higher priority than the ones in $\mathcal{A}_j$ where $j > i$. We assume that $\mathcal{K}$ is consistent and let us denote by N a new consistent information to be accepted. The presence of this new information may leads to inconsistency.

We first define a lexicographic preference relation between subsets of the ABox. Let X and X' be two subsets of $\mathcal{A}$. X is strictly preferred to X', denoted by $X <_{lex} X'$ if and only if $i)$ $\exists i$, $1 \leq i \leq n$, $|X \cap \mathcal{A}_i| < |X' \cap \mathcal{A}_i|$, $ii)$ $\forall j$, $1 \leq j < i$, $|X \cap \mathcal{A}_j| = |X' \cap \mathcal{A}_j|$. We now more formally present PRSR according to the nature of the input information.

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2.1 PRSR by a membership assertion

We first consider the case where N is a membership assertion. $\mathcal{K} \cup \{N\}$ denotes $\langle \mathcal{T}, \mathcal{A} \cup \{N\}\rangle$ where $\mathcal{A}$ is a prioritized ABox.

Definition 2. *A* prioritized removed set*, denoted by X, is a set of membership assertions such that $i)$ $X \subseteq \mathcal{A}$, $ii)$ $\langle \mathcal{T}, (\mathcal{A}\backslash X) \cup \{N\}\rangle$ is consistent, $iii)$ $\forall X' \subseteq \mathcal{A}$, if $\langle \mathcal{T}, (\mathcal{A}\backslash X') \cup \{N\}\rangle$ is consistent then $X \leq_{lex} X'$.*

We denote by $\mathcal{PR}(\mathcal{K} \cup \{N\})$ the set of the prioritized removed sets of $\mathcal{K} \cup \{N\}$. If $\mathcal{K} \cup \{N\}$ is consistent then $\mathcal{PR}(\mathcal{K} \cup \{N\}) = \emptyset$.

Proposition 1. *If $\mathcal{K}\cup\{N\}$ is inconsistent then $|\mathcal{PR}(\mathcal{K}\cup\{N\})| = 1$.*

Definition 3. *The revised knowledge base $\mathcal{K} \circ_{PRSR} N$ is such that $\mathcal{K} \circ_{PRSR} N = \langle \mathcal{T}, \mathcal{A} \circ_{PRSR} N\rangle$ where $\mathcal{A} \circ_{PRSR} N = (\mathcal{A}\backslash X) \cup \{N\}$ with $X \in \mathcal{PR}(\mathcal{K} \cup \{N\})$.*

When N is a membership assertion and the ABox is prioritized the PRSR give the same result as RSR [2] in the flat case (where all the assertions in the ABox have the same priorities).

2.2 Revision by a positive or a negative axiom

We now consider the case where new information N is a PI axiom or a NI axiom. In this case, $\mathcal{K} \cup \{N\}$ denotes $\langle \mathcal{T} \cup \{N\}, \mathcal{A}\rangle$.

Definition 4. *A prioritized removed set, denoted by X, is a set of assertions such that $i)$ $X \subseteq \mathcal{A}$ $ii)$ $\langle \mathcal{T} \cup \{N\}, (\mathcal{A}\backslash X)\rangle$ is consistent and $iii)$ $\forall X' \subseteq \mathcal{A}$, if $\langle \mathcal{T} \cup \{N\}, (\mathcal{A}\backslash X')\rangle$ is consistent then $X \leq_{lex} X'$.*

The following proposition provides the condition of the existence of exactly one prioritized removed set.

Proposition 2. *If for each $C \in \mathcal{C}(\mathcal{K}\cup\{N\})$ there exists i and j, $i \neq j$, such that $C \cap \mathcal{A}_i \neq \emptyset$ and $C \cap \mathcal{A}_j \neq \emptyset$ then $|\mathcal{PR}(\mathcal{K}\cup\{N\})| = 1$.*

This situation holds when each stratum is consistent with $\mathcal{T}\cup\{N\}$ for example, when the stratification comes from several experts with different degrees of reliability. In this case, computing the prioritized removed set is polynomial. The following proposition gives the condition of existence of several prioritized removed sets.

Proposition 3. *If there exists $C \in \mathcal{C}(\mathcal{K} \cup \{N\})$ such that, there exists i, $C \cap \mathcal{A}_i \neq \emptyset$ and for all j, $j \neq i$, $C \cap \mathcal{A}_j = \emptyset$ then $|\mathcal{PR}(\mathcal{K} \cup \{N\})| \geq 2$.*

There are several prioritized removed sets as soon as there are conflict sets included in a stratum where each conflict leads to two prioritized removed sets. Namely, let NC be the number of conflicts such that each one is included in a stratum, the number of prioritized removed sets is bounded by 2^{NC}. In such case, each prioritized removed set leads to a possible revised knowledge base: $\mathcal{K}_i$=$\langle \mathcal{T} \cup \{N\}, (\mathcal{A}\backslash X_i)\rangle$ with $X_i \in \mathcal{PR}(\mathcal{K} \cup \{N\})$. Within the *DL-Lite* language it is not possible to find a knowledge base that represents the disjunction of such possible revised knowledge base. In order to keep the result of revision within the *DL-Lite* language define a selection function, where the revised knowledge base is defined as follows.

Definition 5. *Let f be a selection function, the revised knowledge base $\mathcal{K}\circ_{PRSR}N$ is such that $\mathcal{K}\circ_{PRSR}N$=$\langle \mathcal{T} \cup \{N\}, \mathcal{A} \circ_{PRSR} N\rangle$ where $\mathcal{A} \circ_{PRSR} N$=$(\mathcal{A}\backslash f(\mathcal{PR}(\mathcal{K} \cup \{N\})))$.*

When N is a NI or a PI axiom PRSR generalizes RSR [2] and if $\mathcal{A}$ is not stratified then $\mathcal{K} \circ_{PRSR} N = \mathcal{K} \circ_{RSR} N$.

3 COMPUTING PRIORITIZED REMOVED SETS

From a computational point of view, several cases arise depending on the nature of the input N, the content of the knowledge base and the form of the conflicts. First of all, if the TBox $\mathcal{T}$ only contains PI axioms, and if the input N is a PI axiom or a membership assertion, no inconsistency can occur, so the revision operation trivially becomes a simple union. Among the remaining cases, we distinguish two different situations: (i) N is a membership assertion. (ii) N is a PI axiom or a NI axiom.

When N is an assertion, thanks to Proposition 1, there exists only one prioritized removed set. The computation of this set amounts in picking in each conflict the assertion which is different from N. This operation follows from a simple and non costly adaptation of the algorithm given in [3] for checking consistency of a *DL-Lite* knowledge base: It enumerates all assertional facts that conflict with the input. Then, since every conflict set that contradicts a NI axiom is of the form $\{\alpha, N\}$ where $\alpha \in \mathcal{A}$, there exists exactly one prioritized removed set. Hence, in this case the removed set computation can be performed in polynomial time.

When N is a PI or a NI axiom, according to Definition 4, a prioritized removed set of $\mathcal{K}$ is formed first by computing the prioritized removed sets of the set $(\mathcal{T} \cup \{N\}) \cup A_1$, then extending the process to $(\mathcal{T} \cup \{N\}) \cup \{A_1 \cup A_2\}$, and so on.

However, according to the form of conflicts, two situations hold: (i) Each conflict involves two elements having different levels of priority. (ii) There exists at least one conflict involving two elements having the same priority level. The first situation leads to a single removed set, and thus we have a specialized algorithm for this case, which runs in polynomial time. For the second situation, we also have an algorithm, which relies on the use of the hitting sets of the collection of conflicts, similarly to [4]. A hitting set is a set which intersects each set in a collection. A minimal hitting set, with respect to set inclusion, is called a kernel. Among the kernels, prioritized removed sets are the minimal ones according to the lexicographic pre-ordering $\leq_{lex}$. Our algorithm for the compuation of prioritized removed sets is based on an incremental construction of kernels which are minimal w.r.t. lexicographic pre-ordering, which proceeds stratum by stratum.

4 CONCLUSION

In this paper, we investigated the extension of Prioritized Removed Sets Revision to *DL-Lite* knowledge bases. A future work will focus on the extension of Removed Sets Fusion defined in a propositional setting, to the merging of *DL-Lite* knowledge bases.

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REFERENCES

[1] A. Artale, D.Calvanese, R.Kontchakov, and M. Zakharyaschev, 'The dl-lite family and relations', *J. Artif. Intell. Res. (JAIR)*, **36**, 1–69, (2009).

[2] S. Benferhat, Z. Bouraoui, O. Papini, and E. Würbel, 'Assertional-based removed sets revision of DL-Lite$_R$ knowledge bases', in *ISAIM*, (2014).

[3] D.Calvanese, G.De Giacomo, D.Lembo, M.Lenzerini, and R.Rosati, 'Tractable reasoning and efficient query answering in description logics: The *dl-lite* family', *J. Autom. Reasoning*, **39**(3), 385–429, (2007).

[4] E. Würbel, R. Jeansoulin, and O. Papini, 'Revision: an application in the framework of GIS', in *KR*, eds., A. G. Cohn, F. Giunchiglia, and B. Selman, pp. 505–515. Morgan Kaufmann, (2000).