

# A Detailed Mathematical Model for the Aircraft Landing Problem on a Multiple Runway System

## Abstract

The real time *Aircraft Landing Problem (ALP)* within an *Air Traffic Control (ATC)* system is typically a large scale problem. Since the modeling is a crucial step in solving the **ALP**, we are concerned in the following paper with modeling the landing procedure of aircrafts for a multiple runway system. We proposed a detailed *Mixed Integer Programming (MIP)* model where we treat respectively the three basic decisions, the scheduling of aircraft landings, their assignment to an airport with multiple runways and the control practice.

**Keywords:** Aircraft Landing Problem, Detailed Model, Multiple Runway System, Terminal Maneuvering Area.

## Introduction

Over the last few decades, the air traffic has increased significantly, and on the basis of forecasts, this trend is predicted to continue in the future. Indeed, the scenario, known by EUROCONTROL's forecast experts as regulated growth, declared that the flight demand in Europe is predicted to be 14.4 million movements in 2035, which is the 1.5 times the 2012 volume (**European Organization for the safety of Air Navigation 2013**). This continued growth in civil aviation project to have wide ranging consequences for the air transportation system. On the one hand, the air navigation system in areas of high traffic density reached a degree of utilization close to the limit of capacity (i.e., the operational capacity exceed the physical one), which will lead on the other hand to delays and air traffic congestion. The first casualties' areas of congestion and delays have been historically terminal areas surrounding the airport known as *Terminal Maneuvering Area (TMA)*. These parts constitute the critical interface between airspace and airport. Therefore, this latter dynamic and uncertain region has been highlighted as the outstanding areas key and remains the main bottleneck for the nation's *Air Traffic Control (ATC)* network.

Economic loss owing to delays respectively the airborne one coupled with the expected increase in the demand for air traffic, has become serious. Thus, reducing them has become an important focus and the most countries of the world have called for urgent solutions for the near problem. On one side, the considerable air transportation system complexity has motivated a vibrant

and innovative body of research. On the other side, the need for increasing efficiency and capacity while reducing delays persuade many researchers. Thus, the issue of airport capacity modeling has been extensively studied by several researchers since the 1970s in different countries and has been the subject of many papers. Hence, the purpose of this paper is to propose a detailed model for the *Aircraft Landing Problem (ALP)* on a multiple runway system. It is a significant problem of the more general scheduling, sequencing and assignment of aircraft in the **TMA** including if necessary the control action. In the rest of the paper, *Section 2* provides a description of the aircraft landing operation from a practical stand point. Next, the existing constrains, objectives and modeling technique related to the **ALP** are discussed and categorized in *Section 3*. We, then, introduce in *Section 4* our detailed model formulation before ending the paper in *Section 5*, with some conclusions.

## The ALP from practical standpoint

The arrival phase of flight is characterized by the passage of the en-route environment to the terminal area and airport environment, where the aircraft monitor an arrival routing, and then performs the approach, the landing and finally taxing to the parking position. The aircraft landing operation may be seen as a series of decisions where the air traffic controller must organize the inbound flow from the *Extended Terminal Maneuvering Area (ETMA)* of the airport towards the prescribed runway threshold. Typically, as stated by (**Diallo et al. 2012**), he is in charge of determining approach paths, runway allocation, and landing times of several aircrafts in the radar horizon. The decisions encountered several times by the air traffic controller can be broadly divided into: *routing decisions*, *timing decisions* and *runway assignment*. *Routing Decisions* means an origin-destination route for each aircraft has to be chosen regarding air segments and runways under traffic regulation constraints (it is a spatial decision as appointed by (**Kjenstad et al. 2013**)). Once a route has been assigned to each aircraft, the ATC problem in the **TMA** (with pre-defined routes) is reduced to the *Aircraft Scheduling Problem (ASP)* (**Samà et al. 2015**), where the aircraft passing timing has to be determined in each air segment, runway and (eventually) holding circle (*timing decisions*) (**D'Ariano et al. 2012 - Zhang et al.**

2007). This means giving a *Scheduling Landing Time (SLT)* for each aircraft. It is defined in terms of the time that the aircraft could land flying straight to the runway at its cruise speed.

Once the **SLT** is determined for the aircraft, the air traffic controller appeal to the sequencing process where the sequence of aircraft landings on the *Instrument Landing System (ILS)* path is established. At present, all major airports all over the world such as *Doha International airport* (Ghoniem et al. 2014), at least those were **ATC** advisory tools are not implemented; employ a policy based on *First Come First Served (FCFS)* to make flight scheduling in the **TMA**. This means as shown in Fig.1 that aircrafts are scheduled according to their *Estimated Time Arrivals (ETAs)* by simply applying a required minimum separation between them.

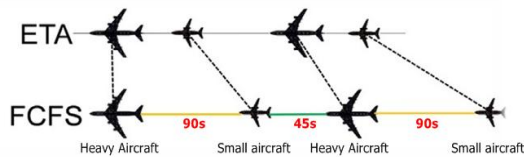


Fig.1. Aircrafts sequencing according to **FCFS**

The *separation requirements* are among the more sensitive tasks for which the responsibility of the air traffic controller. The separation is described in three dimensions: vertical, lateral and longitudinal distances. Often, the lateral and longitudinal separation standards are combined into only one called *horizontal separation* standard. Therefore, as said by (Furini et al. 2012), the air traffic controller is supposed to keep each aircraft in its jurisdiction separated by at least 1000 feet vertically and 3 nautical miles (nm) horizontally, at least around the airport and 5 nm horizontally elsewhere.

The air traffic controllers are also responsible for ensuring that a minimum separation between succeeding aircraft throughout the same *Final Approach Fix (FAF)* is maintained. Therefore, add to the “*radar separation*”, which is a longitudinal spacing of 5 nm (3 nm in congested area) and a vertical spacing of 1000 feet (Furini et al. 2012), we could mention another most applied notion, the “*wake turbulence separation*”. It constitutes a *minimum separation distances* (in nm) required between successive aircrafts based on the preceding aircraft categories. A heavy aircraft usually generates more severe wake turbulence than a small one (Girish 2016). Consequently, a small aircraft flying behind a large aircraft needs a less *minimum separation distance* than the reverse order (and vice versa). Finally, once the aircrafts are sequencing, an

“*allocated runway*” is assigned to the aircraft. The aircraft can be eligible for allocation to more than one runway as some airports have more than one active landing runway.

The management philosophy presented previously can be yield for the normal traffic period. However, in cases of emergency, like air space saturation, small imbalances between demand and capacity during peak times (International Civil Aviation Organization 2007); the aircraft cannot initiate its landing procedure and it is forced by the air traffic controller to “wait” in the sky. Therefore, some undesired maneuvers in flight have to be performed by the air traffic controller to make an aircraft wait before landing. The possible aircraft control actions in the **TMA** are speed control, sequencing, holding and routing (D’Ariano et al. 2010).

## Previous Work

### Operating Constraints of ALP

The *landing time window* and the *required separation* constraints were handled as an indispensable constraint for the **ALP**. In connection with this clarification, the *Constrained Position Shifting (CPS)* concept (Dear 1976) was presented also as a crucial rule that each aircraft must obey in the sequencing process.

The *landing time window constraint* means that the **SLT** for each aircraft to arrive at a certain reference point (e.g., *Meter Fix (MF)* or **FAF**), has to be bounded by an *Earliest Landing Time (ELT)* and a *Latest Landing Time (LLT)*. The **ELT** represents the earliest time when the aircraft could land flying directly to the runway at its fastest speed with no maneuver (vectoring or speed adjustments), and the **LLT** represents the latest time when the aircraft could land flying at its most fuel efficient airspeed while also holding for the maximum allowable time (Diallo et al. 2012).

For the *minimum separation distance constraint*, since the variability of aircraft speed in the **TMA** has been limited, this distance can be translated into a *minimum separation time* (Samà et al. 2015). Therefore, these separation distances are dealt with by converting them into separation time using a fixed landing speed depending on the aircraft category (Beasley et al. 2001). A representative one is shown in Fig. 2. These separation times were derived from the official separation distances issued by *Federal Aviation Administration (FAA)* (Balakrishnan and Chandran 2010).

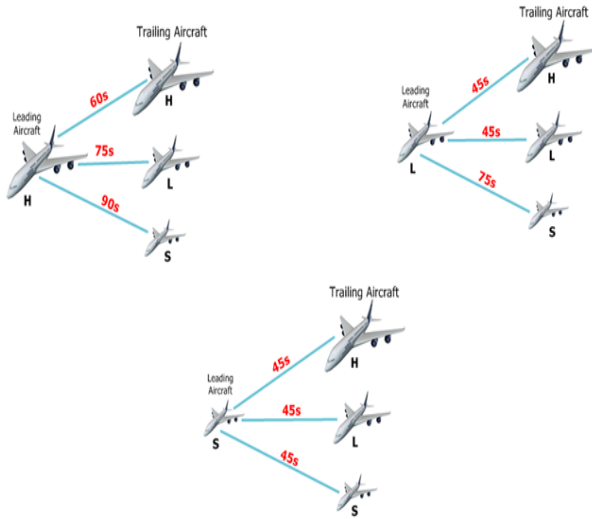


Fig.2. Separation Time at the Runway Threshold (RT) (in sec) between a leading aircraft (Heavy, Large or Small) and a trailing aircraft (Heavy, Large or Small)

Specific constraints can exist for the ALP, having a look at the real world. We note what it is called *precedence/ priority constraints*. Indeed, with respect to landings, certain precedence relationships related to the landing sequence may be required. (Andreussi et al. 1981) have introduced precedence constraints when they have given the priority to the aircraft which arrives in a critical situation of kerosene and to the aircraft which made a maximum delay. Aircraft priorities are taken into account, in the approaches proposed by (Ghoniem et al. 2015) and (Ghoniem and Farhadi 2015), where they have given predominately the priority to the 'Heavy' and 'Large' aircraft as opposed to 'Small' aircraft. In addition, *overtake constraint* is introduced by several researchers since the extra workload by overtakes could be too intensive to be accepted by air traffic controllers in the TMA. Indeed, the detailed model presented in (D'Ariano et al. 2012) precise that the overtaking is not allowed precisely within an air segment in the TMA.

## Objective Functions

The air transportation system is a multi-stakeholder system comprising technical, human and natural factors. Every one of them aims to achieve a set of goals which meet their needs. If, for instance, we place ourselves at the level of airlines which are the major stakeholder in the air transportation system (Balakrishnan and Chandran 2010); we notice that their goals would be to lower the cost of the aircraft (e.g. minimizing fuel consumption). Elsewhere, governments and the public are paying more attention to the environmental impact of airline operations

due to noise and emission issues (Chen and Solak 2015). As noted, the goals of various stakeholders are contradictory, as well as their achievements should be in a reasonable given time. In face of that acute system, as (Sabar and Kendall 2015) emphasizes, the overall system performance is naturally sensitive to the chosen objectives. In this respect, the ALP can be classified in the following objectives functions.

It is clear that among the important aspects to be taken into account when solving the ALP, is the use of existing critical airport resources (runway). In turn, a goal commonly pursued by the service providers is to maximize the processing capacity of the runway system, i.e., the maximization of the number of operations that can be accommodated in the runway per unit of time. This consideration translates into the maximization of the use of airport capacity (i.e., throughput), which can be viewed as minimizing the completion time or the makespan (Bianco et al. 1999).

On the other hand, the delay objective attracted many researchers. Knowing that delay of an aircraft is partly due to the entrance delay in the TMA if it is delayed at the Entry Fix (EF) and partly due to the additional delay caused by the resolution of potential aircraft conflict. A stream of researchers choose to minimize the first part (minimizing the total of deviation from the *Target Landing Times* (TLTs); average delay minimization; minimizing the total airborne delay (Zhan et al. 2010) and the second part (maximum delay minimization, see. e.g. (D'Ariano et al. 2010) - (Lee and Balakrishnan 2008) - (Bianco et al. 2006) - (Samà et al. 2015). We note also the contributions to minimize the costs incurred by delays were numerous (see e.g. BenCheikh et al. 2009). We have also observed another objective functions in which the researchers tackled for a special aspect of the ALP. We note the works of (Beasley et al. 2000) and (Zhou and Jiang 2015) which opted for balancing the controllers' workload. We note that, notwithstanding its importance, there is a very limited literature on operational models in aviation that directly consider *environmental objective* which is a source of interest for the government. These few studies include (Mori et al. 2013) when the authors tried to minimize the total fuel burn and (Moktharimousavi et al. 2015) where they minimize fuel consumption cost and, therefore, the carbon dioxide pollutions of the air is minimized. Finally, (Artiouchine et al. 2008) presented a *control objective* when they minimize the number of times which an aircraft enters in the holding pattern.

## Modeling Techniques

It was normal to envisage the ALP as a particular system of queues. In this context, (Bauerle et al. 2007) examined a model for the landing procedure of aircrafts at an airport

with up to two runways based on queuing theory concepts. Moreover, based on the hypothesis that the *Mixed Integer Linear Program* (MILP) presents the advantage of a great flexibility modeling and allows introducing easily the desired extensions (such as the permissible *landing time window* and the *minimum separation time*) (Ji et al. 2015); most researchers such as (Beasley et al. 2000) – (Samà et al. 2016) – (Murça and Muller 2015) deal with *Mixed Integer Programming* (MIP) formulations for the ALP modeling. On the other hand, it was being seen that the ALP bears similarities with other optimization problems. In this respect, it was being seen that the *Arrival Scheduling and Sequencing* (ASS) problem is known to be an application of the *Traveling Salesman Problem* (TSP) (Bertsimas and Frankvish 2015). (Bianco et al. 1993) make the first attempt to model the ASS as a permutation problem similar to the TSP. From their point of view, (Briskorn and Stolletz 2014) mention that the ALP and the *Vehicle Routing Problem* (VRP) have resemblances. Finally, ASS problem towards an airport platform including a single or multiple runways bears similarities with several well known job sequencing problems in manufacturing. Therefore, many academic researchers such as (Samà et al. 2015) – (D'Ariano et al. 2010) – (Briskorn and Stolletz 2014) – (D'Ariano et al. 2015) viewed the ASP as an extension of the Job Shop Scheduling (JSS).

## Proposed Mathematical Formulation

### Problem Description

Upon analyzing the large number of researches about ALP, it is possible to mention that an ALP universal description does not exist in the literature. In most cases, the description depends on the researcher's choice. For example, (Samà et al. 2015) has linked the ATC and the *Air Traffic Flow Management* (ATFM) problems in such a way that the potential conflicts between single aircrafts are efficiently solved in each TMA resource (holding circles, air segments, runways).

In this paper, the problem is figuring out how a set of approach aircraft class is scheduling and sequencing to a set of runways since the proposed models are intended for multiple runway airport. For each aircraft class descending through the surrounding airspace to enter the ETMA is associated with a minimum entrance time at the EF and an earliest, target and latest times at the FAF. Since the latest landing time depends on the remaining fuel, a maximal latest time is given for the aircraft class. On the basis of these data, some variable decisions are defined. Among these decisions, it is the SLT that permit the aircraft to land efficiently to the assigned runway. Once the SLT is attributed to the aircraft, the ATC should sequence the

aircrafts by applying the separation requirement. Knowing that, the successive separation method is established.

In our modeling step, for the purpose of model reasonably reflects reality, we will tackle two cases. The first one is when the arrival aircraft entering the TMA could have touched down without control action. The second is when the air traffic controller encounters unforeseeable events such as gate unavailability, bad visibility, weather conditions or the occupation of the air segment or runway by the landing of another aircraft. Thus, in such a situation, the aircraft cannot initiate its landing procedure and is forced by the air traffic controller to perform a maneuver.

Then, to respond to the two cases, we decompose the ALP into three sub problems that can be solved independently. For the first case, we propose two mathematical models: the *arrival aircraft scheduling sequencing model* and the *arrival aircraft runway assignment*. Then, in the latter case, in addition to the two latter models (scheduling/ sequencing and assignment), the air traffic controller could appeal to the *arrival aircraft control action*.

### Type of Model Choice

Recently, the ALP models can be classified as basic or detailed. The former includes only the runways in the TMA while the latter schedules aircraft on other relevant TMA resources like the air segments. Since it was deducted that the complex runway system is not always the system bottleneck, but that the latter may occur, also, in other airspaces of the TMA, for example the air segments (Sabar and Kendall 2015), the detailed model to solve the ASP has been the subject of several studies (Samà et al. 2015) - (D'Ariano et al. 2015) - (Samà et al. 2012) - (Samà et al. 2013) and it is in fact our choice in this paper.

### Modeling Technique Choice

At present, and according to (Sabar and Kendall 2015), MIP formulations approaches have been able to find good quality solutions for the ALP, even obtaining optimal solutions for small instances. In fact, our choice was based on this mature field to formulate the ALP in it's entirely.

### MIP Formulation

- Arrival Aircraft / Control Action

- *Set*

- A: Set of approach aircraft waiting for scheduling on multiple runway airport
- W: Set of aircraft classes

▪ **Parameters**

- $n_{w(a)}^V$ : Maximum number of holding procedures performed by aircraft class  $w(a)$
- $TM_{w(a)}$ : **Time of Maneuver** for a single holding procedure for the aircraft class  $w(a)$
- $AT_{w(a)}$ : **Actual Time** of the aircraft class  $w(a)$

▪ **Decision variables**

- $X_{w(a)}^H = \begin{cases} 1 & \text{if aircraft class } w(a) \text{ performed} \\ & \text{a holding procedure} \\ 0, & \text{otherwise} \end{cases}$
- $V_{w(a)}$ : Number of holding procedure performed by aircraft class  $w(a)$

▪ **The objective function**

The objective is to minimize for the aircraft class the time of maneuver which will lead indirectly to minimize the air traffic controller workload.

$$\text{Min} \sum_a^A X_{w(a)}^H \times (AT_{w(a)} + (TM_{w(a)} \times V_{w(a)}))$$

▪ **Constraint**

$$V_{w(a)} \leq n_{w(a)}^V, \text{ for } \forall a \in A, w(a) \in W \quad (1)$$

The most important types of **ATFM** control actions: airborne speed control and airborne holding. The providers of air traffic services usually make these control decisions. In this respect, the constraint given by (1) imposes a maximum number of holding procedures that can be performed by each aircraft class.

• Arrival Aircraft Scheduling Sequencing Model

▪ **Sets**

- $A$ : Set of approach aircraft waiting for scheduling on multiple runway airport
- $W$ : Set of aircraft classes
- $R$ : Set of runways

▪ **Parameters**

- $D_{w(a)}$ : Delay imposed to arrival aircraft class  $w(a)$  if it performs a holding procedure
- $MET_{w(a)}^{EF}$ : **Minimum Entrance Time** for the aircraft class  $w(a)$  at the **EF**
- $EAT_{w(a)}^{FAF}$ : **Earliest Approach Time** for the aircraft class  $w(a)$  at the **FAF**
- $TAT_{w(a)}^{FAF}$ : **Target Approach Time** for the aircraft class  $w(a)$  at the **FAF**

- $LAT_{w(a)}^{FAF}$ : **Latest Approach Time** for the aircraft class  $w(a)$  at the **FAF**
- $LTA_{w(a)}^{MAX}$ : **Maximal Latest Time of Arrival** for the aircraft class  $w(a)$
- $ADT_{w(a)}^{MAX}$ : **Maximum Approach Delay Time** for the aircraft class  $w(a)$
- $MPT_{w(a)}^{FAF}$ : **Minimum Processing Time** for the aircraft class  $w(a)$  at the **FAF**
- $ST_{w(a),w(a')}^{FAFr}$ : **Minimum Separation Time** between successive approaching aircrafts  $w(a)$  and  $w(a')$  landing on the same runway at the **FAF**
- $ST_{w(a),w(a')}^{FAFr,r'}$ : **Minimum Separation Time** between aircraft class  $w(a)$  landing on runway  $r$  and aircraft class  $w(a')$  landing on runway  $r'$  at the **FAF**
- $AET_{w(a)}^{EF}$ : **Actual Entrance Time** for the aircraft class  $w(a)$  arriving at the **EF**
- $AAT_{w(a)}^{FAF}$ : **Actual Arrival Time** for the aircraft class  $w(a)$  at the **FAF**

▪ **Decision variable**

- $SLT_{w(a)}^{FAF}$ : **Scheduling Landing Time** for the aircraft class  $w(a)$  at the **FAF**
  - $\alpha_{w(a),w(a')}^{FAF} = \begin{cases} 1, & \text{the aircraft class } w(a) \text{ kept landing} \\ & \text{after aircraft class } w(a') \\ 0, & \text{otherwise} \end{cases}$
  - $\gamma_{w(a),w(a')}^{FAF,r} = \begin{cases} 1, & \text{the aircraft class } w(a) \text{ and aircraft class } w(a') \\ & \text{are landing on the same runway } r \\ 0, & \text{otherwise} \end{cases}$
  - $\theta_{w(a),w(a')}^{FAF,rr'} = \begin{cases} 1, & \text{the aircraft class } w(a) \text{ is landing} \\ & \text{on the runway } r, \\ & \text{and the aircraft class } w(a') \\ & \text{is landing on the runway } r' \\ 0, & \text{otherwise} \end{cases}$
  - $P_{w(a)}$ : **Priority weight** assigned to aircraft class  $w(a)$  based on its weight class (*Heavy* and *Large* aircraft than *Small* aircraft) and on the maximum airborne delay

▪ **The objective function**

We are working on a real time scheduling which consists on the *delay minimization*; specifically, the *minimization of the maximum consecutive delay* over all aircrafts at the two meter fixes, the **EF** in the **TMA** and at the **FAF** of the air segment. This implies that the objective formulation takes account the delay minimization of landing aircraft class subject to *time window* constraint for each aircraft's landing time, *minimum longitudinal separation time* between consecutive landings, and *overtake* by introducing precedence constraints.

$$\text{Min Max} \sum_a^A \sum_w^W P_{w(a)} \times (AET_{w(a)}^{EF} - MET_{w(a)}^{EF}) + (SLT_{w(a)}^{FAF} - TAT_{w(a)}^{FAF})$$

The item  $(AET_{w(a)}^{EF} - MET_{w(a)}^{EF})$  is introduced to denote the *Entrance Delay (ED)* of each aircraft class at the **EF** which can due for example to other aircraft scheduled on its entrance air segment.

The item  $(SLT_{w(a)}^{FAF} - TAT_{w(a)}^{FAF})$  is introduced to denote the *Consecutive Delay (CD)* of each aircraft class caused for example by waiting in the air or performing a maneuver.

#### ▪ Constraint

$$SLT_{w(a)}^{FAF} \geq TAT_{w(a)}^{FAF} \text{ for } \forall a \in A, w(a) \in W \quad (1)$$

$$EAT_{w(a)}^{FAF} < SLT_{w(a)}^{FAF} < LAT_{w(a)}^{FAF} + D_{w(a)} \text{ for } \forall a \in A, w(a) \in W \quad (2)$$

$$AAT_{w(a)}^{FAF} - TAT_{w(a)}^{FAF} > 0 \text{ for } \forall a \in A, w(a) \in W \quad (3)$$

$$SLT_{w(a)}^{FAF} - TAT_{w(a)}^{FAF} < ADT_{w(a)}^{MAX} \text{ for } \forall a \in A, w(a) \in W \quad (4)$$

$$SLT_{w(a)}^{FAF} < SLT_{w(a')}^{FAF} \text{ for } \forall a, a' \in A, w(a), w(a') \in W \quad (5)$$

$$SLT_{w(a')}^{FAF} > (SLT_{w(a)}^{FAF} + ST_{w(a)w(a')}^{FAFr}) \times \gamma_{w(a)(a')}^{FAF,r} \text{ for } \forall a, a' \in A, w(a), w(a') \in W, r \in R \text{ such that } w(a) \neq w(a') \quad (6)$$

$$SLT_{w(a')}^{FAF} > (SLT_{w(a)}^{FAF} + ST_{w(a),w(a')}^{FAFr,r'}) \times \theta_{w(a)w(a')}^{FAF,rr'} \text{ for } \forall a, a' \in A, w(a), w(a') \in W, r, r' \in R \text{ such that } w(a) \neq w(a'), r \neq r' \quad (7)$$

$$\alpha_{w(a)w(a')}^{FAF} + \alpha_{w(a')w(a)}^{FAF} \leq 1 \text{ for } \forall a, a' \in A, w(a), w(a') \in W \quad (8)$$

$$\sum_A \sum_W \alpha_{w(a)w(a')}^{FAF} = 1 \quad (9)$$

$$SLT_{w(a)}^{FAF} - SLT_{w(a')}^{FAF} \geq ST_{w(a')w(a)}^{FAFr} \times \alpha_{w(a')w(a)}^{FAF} + M \times (\alpha_{w(a)w(a')}^{FAF} + \gamma_{w(a)(a')}^{FAF,r}) \text{ for } \forall a, a' \in A, w(a), w(a') \in W, r \in R \text{ such that } w(a) \neq w(a') \quad (10)$$

$$SLT_{w(a')}^{FAF} - SLT_{w(a)}^{FAF} \geq ST_{w(a)w(a')}^{FAFr} \times \alpha_{w(a)w(a')}^{FAF} + M \times (\alpha_{w(a')w(a)}^{FAF} + \gamma_{w(a')(a)}^{FAF,r}) \text{ for } \forall a, a' \in A, w(a), w(a') \in W \text{ such that } w(a) \neq w(a') \quad (11)$$

$$SLT_{w(a)}^{FAF} - SLT_{w(a')}^{FAF} \geq ST_{w(a')w(a)}^{FAFr,r'} \times \alpha_{w(a')w(a)}^{FAF} + M \times (\alpha_{w(a)w(a')}^{FAF} + \theta_{w(a)w(a')}^{FAF,rr'}) \text{ for } \forall a, a' \in A, w(a), w(a') \in W, r \in R \text{ such that } w(a) \neq w(a') \quad (12)$$

$$SLT_{w(a')}^{FAF} - SLT_{w(a)}^{FAF} \geq ST_{w(a)w(a')}^{FAFr,r'} \times \alpha_{w(a)w(a')}^{FAF} + M \times (\alpha_{w(a')w(a)}^{FAF} + \theta_{w(a')w(a)}^{FAF,rr'}) \text{ for } \forall a, a' \in A, w(a), w(a') \in W, r \in R \text{ such that } w(a) \neq w(a') \quad (13)$$

$$SLT_{w(a)}^{FAF} \leq MPT_{w(a)}^{FAF} \leq LTA_{w(a)}^{MAX} \text{ for } \forall a, a' \in A, w(a), w(a') \in W \quad (14)$$

*Constraint (1)* explains the fact that the **SLT** for the aircraft class  $w(a)$  has to be as close as possible to the **TAT**. The *Constraint (2)* accounts that the **SLT** must belong to a *time window*, bounded by an **EAT** and a **LAT** with the delay imposed to arrival aircraft class  $w(a)$  if it performs for example a holding pattern. In most cases of approach aircrafts, their **AAT** is almost impossible to be the same as their **TAT**; it is demonstrated in (3). *Equation (4)* is to restrain the delay time of the approach aircraft class  $w(a)$ ; it should not be larger than the maximum approach delay time. The constraint given by (5) indicates that an aircraft class lands before other aircraft in the case that its **SLT** at the **FAF** is earlier than the others, in other words, overtakes is not allowed. *Equation (6)* indicates the safety interval that the aircrafts need to meet if there are two successive aircrafts landing on the same runway (suppose that aircraft class  $w(a)$  is the former and the aircraft class  $w(a')$  is the latter). Indeed, *equation (7)* indicates the safety interval that the aircrafts need to meet if there are two successive aircrafts landing on different runway ( $r$  and  $r'$ ). *Equations (8)* indicates that, for two approach flights  $w(a)$  and  $w(a')$ , either  $w(a)$  is the leading and  $w(a')$  is the trailing or  $w(a)$  is the trailing and  $w(a')$  is the leading. *Equation (9)* indicates that any approach aircraft class has only one leading/trailing aircraft. The constraints given by (10), (11), (12) and (13) establish the *longitudinal separation* between each pair of aircraft landing on the same or different runways and for any landing sequence. If aircraft class  $w(a')$  lands before aircraft  $w(a)$  on the same runway, *expression (10)* is naturally satisfied since  $M$  is a sufficiently large constant used to ensure that the equation is redundant in the case when  $w(a)$  lands before  $w(a')$  ( $\alpha_{w(a)w(a')}^{FAF} = 1$ ) and *expression (11)* ensures the separation between  $w(a)$  and  $w(a')$ . On the other hand, if aircraft  $w(a')$  lands before aircraft  $w(a)$  on the same runway, *expression (10)* is naturally satisfied and *expression (11)* ensures the separation between  $w(a)$  and  $w(a')$ . The expressions (12) and (13) have the same goal than the two latter equations when they ensure the separation between a leading and trailing aircraft and in this case landing on different runways. Finally, (14) explains that at the **FAF**, the processing time varies between a pre-defined *time windows*, due to a limited possibility of aircraft speed changes.

- Arrival Aircraft Runway Assignment

- **Set**

- $A$ : Set of approach aircraft waiting for assigning on a runway
- $W$ : Set of aircraft classes
- $R$ : Set of runways

- **Parameters**

- $EAT_{w(a)}^{RT}$ : **Earliest Arrival Time** of the aircraft class  $w(a)$  at the **RT**
- $PLT_{w(a)}^{RT}$ : **Predicted Landing Time** of the aircraft class  $w(a)$  at the **RT**
- $LAT_{w(a)}^{RT}$ : **Latest Arrival Time** of the aircraft class  $w(a)$  at the **RT**
- $RO_{w(a)}^r$ : **Runway Occupancy Time** by arriving aircraft class  $w(a)$
- $\Delta_{w(a),w(a')}^r$ : Minimum separation time between aircraft class  $w(a)$  and aircraft class  $w(a')$  landing on the same runway  $r$
- $\Delta_{w(a),w(a')}^{r,r'}$ : Minimum Separation Time between aircraft class  $w(a)$  and aircraft class  $w(a')$  landing on different runway ( $r$  and  $r'$ )

- **Decision variables**

- $ALT_{w(a)}^{RT}$ : **Assigned Landing Time** of the aircraft class  $w(a)$  at the **RT**

$$H_{w(a)}^r = \begin{cases} 1, & \text{the aircraft class } w(a) \text{ is assigned to the runway } r \\ 0, & \text{otherwise} \end{cases}$$

$$\delta_{w(a),w(a')}^r = \begin{cases} 1 & \text{if aircraft class } w(a) \text{ and aircraft class } w(a') \\ & \text{are scheduled to land} \\ & \text{on the same runway } r \\ 0, & \text{otherwise} \end{cases}$$

$$Z_{w(a),w(a')}^{r,r'} = \begin{cases} 1 & \text{if aircraft class } w(a) \text{ is assigned to runway } r \\ & \text{and aircraft class } w(a') \text{ is assigned to} \\ & \text{runway } r', \\ & \forall w(a) \neq w(a') \in W, \forall r, r' \in R \\ 0 & \text{otherwise} \end{cases}$$

- **The objective function**

Even in the context of delay, for the objective formulation we want to *minimize the maximum consecutive delay* at the **RT** subject to the constraint type: *time window* and *separation requirement* and some constraints related to the runway resource aspect.

$$\text{Min Max} \sum_a \sum_w \sum_r H_{w(a)}^r \times (ALT_{w(a)}^{RT} - PLT_{w(a)}^{RT})$$

- **Constraint**

$$\sum_r^R H_{w(a)}^r = 1, \text{ for } \forall a \in A, w(a) \in W \quad (1)$$

$$ALT_{w(a')}^{RT} - ALT_{w(a)}^{RT} \geq \Delta_{w(a),w(a')}^r \times \delta_{w(a),w(a')}^r \\ \in A, w(a)w(a') \text{ for } \forall a, a' \in W, r \in R \quad (2)$$

$$ALT_{w(a')}^{RT} - ALT_{w(a)}^{RT} \geq \Delta_{w(a),w(a')}^{r,r'} \times Z_{w(a),w(a')}^{r,r'} \\ \text{for } \forall a, a' \in A, w(a)w(a') \in W, r, r' \in R, \text{ such that } w(a) \neq w(a'), r \neq r' \quad (3)$$

$$EAT_{w(a)}^{RT} \leq ALT_{w(a)}^{RT} \leq LAT_{w(a)}^{RT} \text{ for } \forall a \in A, w(a) \in W \quad (4)$$

$$ALT_{w(a)}^{RT} \geq ALT_{w(a)}^{RT} + RO_{w(a)}^r \times (\delta_{w(a),w(a')}^r + Z_{w(a),w(a')}^{r,r'}) \\ \text{for } \forall a, a' \in A, w(a)w(a') \in W, r \in R \quad (5)$$

$$\delta_{w(a),w(a')}^r \geq H_{w(a)}^r + H_{w(a')}^r \text{ for } \forall a, a' \in A, w(a)w(a') \in W, r \in R \quad (6)$$

*Constraint (1)* explains the pigeonhole principle where the runway is a blocking resource as the presence of an aircraft on the runway imposes that no other aircraft can use it. Therefore, since the runway can be occupied by only one aircraft at a time, a safety check must be ensured between any pair of aircraft. In this respect, we have two cases, that one if the pair of aircrafts class land on the same runway and on different runways. The first case is presented by (2); and (3) presents the case of different runways. *Constraint (4)* explains that the **ALT** must belong to a *time window*; bounded by an **EAT** and a **LAT**. The *equation (5)* explains the fact that the trailing aircraft class of any pair cannot touchdown on the runway before the leading aircraft class is clear of the runway after passing a certain occupation time knowing that the latter time is different from an aircraft class to another. Finally, the *equation (6)* ensures that when both aircraft class  $w(a)$  and aircraft class  $w(a')$  are allocated to the same runway; the runways must be identical.

## Conclusions

To present and in view of the above literature investigation features, the modeling of the aircraft landing operation still an important step in the resolution of the problem. Moreover its importance, it is not an easy task for several reasons. On the one hand, it requires depth knowledge of several components which are affected by high uncertainty. On the other hand, it remains to the air transportation system complexity and the differences between airports and different stakeholder aims. In addition, it is proved that the **ALP** is divided into a decision problem and a control one. In many operation research models, it has been claimed that separating the two aspects of the problem is beneficial. Thus, it will be better to study each problem separately, but, it is noticeable that for the majority of

models associated with the sequencing and scheduling landings, the control problem leaves aside and it is the responsibility of the air traffic controller. Notice that there should be a lack of works which assist air traffic controllers in establishing control actions during the approach to the airport. Taking account of all these distinctions, this work provides not only an overview of the various constraints and objectives related to the multiple runways ALP. Moreover, we were developed mathematical models that dissect the scheduling, sequencing of aircraft and their assignment to a runway taking account the control aspect.

## Acknowledgements

This work was supported by the Research Center of College of Computer and Information Sciences, King Saud University. The authors are grateful for this support.

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