TRUCK ROUTE PLANNING IN NON-STATIONARY STOCHASTIC NETWORKS WITH TIME-WINDOWS AT CUSTOMER LOCATIONS

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Abstract: Most existing methods for truck route planning assume known static data in an environment that is time varying and uncertain by nature which limits their widespread applicability. The development of intelligent transportation systems such as the use of information technologies reduces the level of uncertainties and makes the use of more appropriate dynamic formulations and solutions feasible. In this paper, a truck route planning problem called Stochastic Traveling Salesman Problem with Time Windows (STSPTW) in which traveling times along the roads and service times at the customer locations are stochastic processes is investigated. A methodology is developed to estimate the truck arrival time at each customer location. Using estimated arrival times, an approximate solution method based on dynamic programming is proposed. The algorithm finds the best route with minimum expected cost while it guarantees certain levels of service are met. Simulation results are used to demonstrate the efficiency of the proposed algorithm.

Keywords: Truck routing, Stochastic traveling salesman problem, Time windows, Container movement, Uncertain environment.

I. INTRODUCTION

In the trucking industry, time is money. The ability of a trucking company to succeed economically rests on its ability to move goods reliably and efficiently, with minimal delay. In many traffic networks especially in major cities, traffic congestion has already reduced mobility and system reliability, and has increased transportation costs. In addition to contributing to truck-drivers' inefficiency, traffic congestion is a major source of air pollution (especially diesel toxins), wasted energy, increased maintenance cost caused by the volume of trucks on roadways, etc. (Barton, 2001).

With the expected substantial increase in the volume of international and national cargos entering and moving through the U.S. highway system, together with the anticipated growth in the number of personal vehicles in use, it is expected that the condition of traffic congestion will only get worse, unless careful planning is initiated. For instance, according to the Federal Highway Administration, currently nearly half of the California's urban highways are congested. It is expected that, between 2000 and 2025 in California, personal vehicle trips will be increased by 38 percent (CalTrans 2004), and the volume of containers moving in and out of the major California ports will be tripled (Mallon and Magaddino, 2001).

There are numerous ways to improve traffic congestion, and therefore reduce transport times associated with goods movements. Options include developing new and expanding current facilities, deploying advanced technologies, and improving operational characteristics and system management practices. It should be noted that the scarcity of land in major cities has made the option of developing new facilities, if not infeasible, significantly costly.

Goods movements, in nature, contain uncertainties. For instance, the customer demands, travel costs, and travel times are uncertain, time-dependent variables. In the presence of a high degree of uncertainties, it is widely expected that optimal solutions for goods movements will be outperformed, over time, by algorithms that are more local in nature (Powell et al., 2000). However, the deployment of advanced technologies such as the use of information technologies can reduce the level of uncertainties to a manageable level and makes the use of dynamic formulations and solutions feasible. The focus of this paper is to investigate methods to improve the operational characteristics of goods movements by developing techniques that can be easily implemented using new but currently available computer and information technologies.

The objective of this paper is to develop methods for routing and scheduling of trucks in uncertain environments where they are used to transfer containers between service stations (e.g., marine terminals, intermodal facilities, and warehouses) and end customers. We assume that each of these customers/facilities may have imposed time-window constraints on pick-up/drop-off containers. Jula et al. (2005) has shown that the container movement problem by trucks with time-window constraints at both origins and destinations could be modeled as an asymmetric Traveling Salesmen Problem with Time Windows (TSPTW). In the TSPTW problem, a vehicle, initially located at the depot, must serve a number of geographically dispersed customers such that each customer is served within a specified time window. The objective is to find the optimum route with minimum total cost of travel.

The TSPTW problem is a special case of the Vehicle Routing Problem with Time Windows (VRPTW) in which the capacity constraints are relaxed. During the last two decades, the Vehicle Routing Problem (VRP) and a variety of its practical applications have been the subject of a wide body of research (Golden and Assad 1988, Laporte 1992, Fisher 1995, Toth and Vigo 2002). The Stochastic Vehicle Routing Problem (SVRP) arises whenever some or all elements of the VRP problem are random. The most studied area in SVRP has been the VRP problem with Stochastic Demands (VRPSD), and with Stochastic Customers (VRPSC) (Bertsimas 1992, Gendreau et al. 1995, Gendreau et al. 1996, Bertsimas and Simchi-Levi 1996, Secomandi 2001, Branke et al., 2005). Despite its importance and practical application, especially in major cities with traffic congestion, research efforts on the stochastic VRP with Stochastic Travel Times (VRPST) have been limited (Laporte et al. 1992, Lambert et al. 1993, Kim et al. 2005).

The objective in the VRPST problem is to find the optimal vehicle routes in the presence of random travel and service times. Most VRPST solution methods require the knowledge of the distribution of the sum of the travel and service times along the routes (Laporte et al. 1992, Gendreau et al. 1996).

In this paper, the Stochastic TSPTW problem with non-stationary stochastic travel and service times is investigated. This problem hereafter will be called Stochastic TSPTW (STSPTW) for convenience. In the deterministic traveling salesman problem with hard time windows, one can clearly group routes into feasible or infeasible routes. A route is feasible if it visits all customer locations before their demanded latest times. In the STSPTW, however, such a black and white definition cannot be easily put forward. To avoid any confusion, in this paper, we will define and use the terms *acceptable routes* and *service level* in the stochastic scenario. A route is considered acceptable if the probability of arriving to each customer location in the route within their time window is greater than the service level. To determine the acceptable routes, we develop a methodology to estimate the first and second moments of arrival time at each customer location. One of the major difficulties in estimating the arrival time at each node is the existence of the non-linearity formed by time windows. In this paper, this non-linearity is thoroughly investigated and an approximate methodology is developed to address the existing time windows in the STSPTW problem. Furthermore, we propose an approximate solution algorithm based on a modified dynamic programming method to find the least-cost route for the STSPTW problem, which meets the required service level at the customer locations.

The paper is organized as follows. In Section II, the problem of stochastic TSPTW with random travel and service times is described. In Section III, we explain why it is difficult to obtain the probability density function (PDF) of arrival times at customer locations even for a simplified STSPTW problem. Therefore, we develop a method to estimate the mean and variance of the arrival times at customer locations. The concept of confidence coefficient at a node, and the approximate solution method for STSPTW are proposed in Section IV. The experimental results are given in Section V, and Section VI presents the conclusion.

II. Problem Description

Let $G=(ND,A)$ be a graph with node set $ND=\{o\}\cup\{d\}\cup N$ and arc set $A=\{(i,j)\,|\,i,j\in ND\}$. The nodes o and d represent the single depot (the origin-depot and destination-depot), and $N=\{1,2,\ldots,n\}$ is the set of customers. Associated with each arc $(i,j)\in A$, is a stochastic process $X_{ij}(t)$ with argument t representing the travel time on that arc. The argument t indicates the time when a vehicle enters arc (i,j). We assume that the travel times on the individual links at any particular time t are statistically independent.

A cost coefficient denoted by c_{ij} is associated to each arc $(i,j)\in A$. The cost c_{ij} can be either a deterministic number (e.g., the distance between nodes i and j), or a random variable (e.g., the travel time on the arc (i,j)). Associated with each node $i\in ND$ is a service time s_i representing the duration of time for a vehicle to be served at that node. We assume that the service time s_i is a random variable, which is independent of the time service starts at node i. Furthermore, to each node $i\in ND$ a time window $[a_i,b_i]$ is associated where a_i and b_i are the earliest and the latest time to visit node i, respectively. We assume that if a vehicle arrives at the customer location at any time earlier than a_i, it has to wait till a_i to start servicing. If it arrives at any time later than b_i, it cannot be served.

We define a route r in graph G as an ordered set of nodes $r=\{o,w_1,w_2,\ldots,w_k,d\}$, where $w_i\in N$, $i=1,\ldots,k$, with associated arc set $A^r=\left\{(w_i,w_j)\,\middle|\,w_i,w_j\in r,(w_i,w_j)\in A,\ w_j \text{ is visited immediately after } w_i\right\}$.

Let Y_o be the departure time from node *o*. Given the departure time, i.e. $Y_o = y_o$, the random variable Y_i^r denotes the arrival time at node *i* taking the path starting at node *o*, passing through the nodes in route *r,* in the specified order, and ending at node *i*. The route *r* is '*acceptable*' if the probability of visiting every node *i* on route *r* before its latest time, b_i, is greater than an arbitrary constant Υ_i, which is called the *service level* at node *i*. Precisely speaking, the confidence coefficient at node *i* on route *r*, denoted by γ_i^r, is defined as the probability of arriving at node *i* no later than b_i, i.e.

$$\gamma_i^r = P\left\{Y_i^r \le b_i\right\}. \tag{1}$$

We say that the arrival time at node *i* taking route *r* meets the required *service level* at this node, say Υ_i, if the confidence coefficient at node *i,* γ_i^r, is greater than or equal to Υ_i.

The cost of route *r,* denoted by C^r, is the cost of traveling between nodes *o* and *d* in the specified order of nodes on route *r*. As discussed, the cost C^r can be either a deterministic number (e.g., the distance) or a random variable (e.g., the travel time). The objective of this paper is to find the least-cost (i.e., minimum C^r, for deterministic numbers, and minimum $E[C^r]$ for random variables) acceptable route starting from origin *o*, visiting all nodes in *N* and ending at destination *d*, such that each node is visited exactly once.

III. ESTIMATING THE FIRST TWO MOMENTS OF THE ARRIVAL TIME

Let $r=\{o,1,\ldots,i,j,\ldots,m,d\}$ be a route in graph *G* with associated arc set A^r. Figure 1 graphically illustrates a typical route *r*.

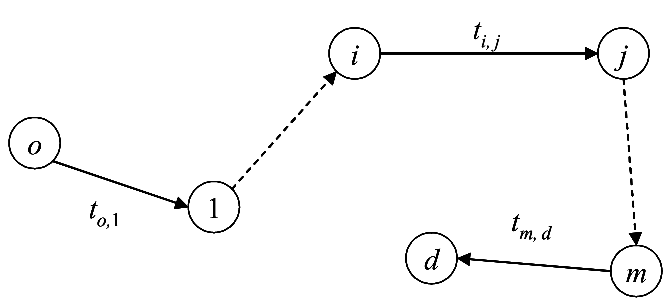

Figure 1: A graphical representation of a typical route.

Let $f_{X_{ij}}(x_{ij},t)$ be the non-negative first-order probability density function (PDF) of the stochastic process $X_{ij}(t)$ (the travel time on arc $(i,j)\in A^r$), and let $f_{S_i}(s_i)$ be the PDF of the service time at node $i\in r$. Given the departure time from the origin, the time window at node i, $f_{S_i}(s_i)$, and $f_{X_{ij}}(x_{ij},t)$, we are interested in finding the PDF of the arrival time at node i on route r, i.e., $f_{Y_i^r}(y_i^r)$.

Without service time and time windows

In this subsection, we assume that there are no time windows and service times associated with the nodes of route r, i.e., $[a_i,b_i]=[0,\infty)$ and $s_i=0\ \forall i\in r$. In other words, as soon as a vehicle arrives at node i it immediately continues its travel toward node j.

Suppose the arrival time at node i on route r, Y_i^r, is given. Since there is no waiting time at node i, Y_i^r also indicates the departure time from this node. Hence, the arrival time at successor node j on route r can be computed by

$$Y_j^r = Y_i^r + Z_{ij}, \tag{2}$$

where the random variable Z_{ij} is the travel time on link (i,j) given the departure time from node i, i.e., $Z_{ij} = X_{ij}(y_i^r)$.

Equation (2) indicates that given the arrival time at node i (the present), the arrival time at node j (the future) is not influenced by the past but only by the present state. In other words, equation (2) implies a Markovian process (Papoulis, 1991).

Let $Y_o = y_o$ be the departure time from the origin o. Using the Markov chain rule and the *Chapman-Kolmogoroff equation,* the conditional PDF of travel time between the origin and node j on route r can be computed recursively by

$$f\left(y_m^r \middle| y_0\right) = \int_0^\infty f\left(y_m^r \middle| y_{m-1}^r\right) f\left(y_{m-1}^r \middle| y_o\right) dy_{m-1}^r, \tag{3}$$

where $m = 2,\ldots, j \in r$, and $(m, m-1) \in A^r$.

Generally speaking, it is very hard and tedious to compute (3) for every node on route *r* and for an arbitrary PDF $f(\cdot|\cdot)$. Instead, it is more practical to find the first two moments of the travel time between the origin and node *j*. In the rest of this section, we shall describe how the first and second moments of the arrival time at each node of route *r* can be estimated.

Let $\eta_{ij}(t)$ and $\sigma_{ij}^2(t)$ denote the mean and variance of the stochastic process $X_{ij}(t)$ corresponding to its first-order probability density function, respectively. These parameters are defined by

$$\eta_{ij}(t) = E\left[X_{ij}(t)\right] = \int_0^\infty x_{ij} \cdot f_{X_{ij}}\left(x_{ij}, t\right) dx_{ij} , \tag{4}$$

$$\sigma_{ij}^2(t) = E\left[\left(X_{ij}(t) - \eta_{ij}(t)\right)^2\right] = \int_0^\infty \left(x_{ij}(t) - \eta_{ij}(t)\right)^2 \cdot f_{X_{ij}}\left(x_{ij}, t\right) dx_{ij} . \tag{5}$$

According to (2), given the mean arrival time at node *i*, the mean arrival time at node *j* can be computed by (Papoulis, 1991)

$$\begin{aligned} E\left[Y_j^r\right] &= E\left[Y_i^r\right] + E\left[E\left[Z_{ij} \middle| Y_i^r = y_i^r\right]\right] \\ &= E\left[Y_i^r\right] + E\left[E\left[X_{ij}\left(Y_i^r\right)\right]\right] \end{aligned} . \tag{6}$$

Using (4), equation (6) becomes

$$E\left[Y_j^r\right] = E\left[Y_i^r\right] + E\left[\eta_{ij}\left(Y_i^r\right)\right], \tag{7}$$

where

$$E\left[\eta_{ij}\left(Y_i^r\right)\right] = \int_0^\infty \eta_{ij}\left(y_i^r\right) \cdot f_{Y_i}\left(y_i^r\right) dy_i^r . \tag{8}$$

Using Taylor's series expansion and assuming that the function $\eta_{ij}(t)$ is differentiable at and around $E\left[Y_i^r\right]$, the first order approximation of (8) can be obtained by (Fu and Rilett, 1998)

$$E\left[\eta_{ij}\left(Y_i^r\right)\right] \cong \eta_{ij}\left(E\left[Y_i^r\right]\right). \qquad (9)$$

Substituting (9) in (7), we get

$$E\left[Y_j^r\right] \cong E\left[Y_i^r\right] + \eta_{ij}\left(E\left[Y_i^r\right]\right). \qquad (10)$$

Similarly, by using Taylor's series expansion, the first order approximation of the second moment of Y_j^r in (2) can be obtained by (see also, Fu and Rilett 1998, and Papoulis 1991)

$$\operatorname{var}\left(Y_j^r\right) \cong \left(1+\eta_{ij}'\left(E\left[Y_i^r\right]\right)\right)^2 \cdot \operatorname{var}\left(Y_i^r\right) + \sigma_{ij}^2\left(E\left[Y_i^r\right]\right). \qquad (11)$$

where $\eta_{ij}'(.)$ is the first derivative of $\eta_{ij}(.)$. Therefore, given the departure time $Y_o = y_o$ and the mean and variance of the travel time on each arc $(i, j) \in A^r$, the mean and variance of the arrival time at each node $i \in r$ can be calculated using equations (10) and (11), recursively.

The first order approximation model given by (10) and (11) are obtained by assuming that the second and higher derivatives of $\eta_{ij}(t)$ and $\sigma_{ij}^2(t)$ are equal to zero. Obviously, the first order approximation can be improved if higher order terms of the Taylor's series are also included. Finding the third and fourth central moments of these stochastic variables are neither practically nor computationally feasible in many situations. In this paper, we will use only the first order approximation.

With service time

In this subsection, we assume that there is no time window associated with the nodes of route *r*. However, as soon as a vehicle visits a node $i \in r$, it will be served for a period of time before departing for the next node. We assume that the service times at some or all the nodes on route *r* are random variables, which are independent of the time the service starts at those nodes.

Let random variable S_i denote the service time at node *i* with non-negative PDF $f_{S_i}(s_i)$. Given the arrival time at node *i* taking route *r*, Y_i^r, the departure time from node *i*, W_i^r, is obtained by

$$W_i^r = Y_i^r + S_i . \tag{12}$$

The arrival time at node *j*, Y_j^r , is given by

$$Y_j^r = W_i^r + Z_{ij} . \tag{13}$$

where the random variable Z_{ij} is the travel time on link (*i*,*j*) given the departure time from node *i*, i.e. $Z_{ij} = X_{ij}(w_i^r)$. According to (12), the first moment of W_i^r can be calculated as

$$E\left[W_i^r\right] = E\left[Y_i^r\right] + E\left[S_i\right]. \tag{14}$$

Since random variable S_i is independent of the arrival time Y_i^r, we have $\text{cov}(Y_i^r, S_i) = 0$. Hence, the second moment of W_i^r can be computed by

$$\text{var}\left(W_i^r\right) = \text{var}\left(Y_i^r\right) + \text{var}\left(S_i\right). \tag{15}$$

Given the mean and variance of the departure time W_i^r in (14) and (15), the first order approximation model of the mean and variance of Y_j^r can be obtained according to equations (10) and (11) by

$$E\left[Y_j^r\right] \cong E\left[W_i^r\right] + \eta_{ij}\left(E\left[W_i^r\right]\right), \text{ and} \tag{16}$$

$$\text{var}\left(Y_j^r\right) \cong \left(1 + \eta'_{ij}\left(E\left[W_i^r\right]\right)\right)^2 \cdot \text{var}\left(W_i^r\right) + \sigma_{ij}^2\left(E\left[W_i^r\right]\right), \tag{17}$$

where now W_i^r depends on the service time as described by equations (14) and (15). Hence, given the departure time $Y_o = y_o$, the mean and variance of the travel time on each arc $(i, j) \in A^r$, and the mean and variance of the service time at each node $i \in r$, the first two moments of the arrival time at each node *i* can be calculated using equations (14) to (17), recursively.

With time windows

Here, we consider the case in which some or all the nodes on route r have time windows. At each node, we are interested in investigating the effect of the existing time window on the characteristics of the arrival time. In other words, given the first and second moments of the arrival time at each node with time window, we would like to approximate the first and second moments of the departure time.

We start by assuming that there is no service time associated with nodes of route r. We also assume that the non-linear function $g(y_i)$, illustrated graphically in Figure 2, denotes the departure time as a function of the arrival time y_i at each node i.

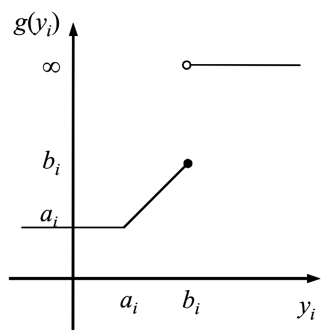

Figure 2: The graphical representation of the time window at node *i* considered in this paper.

As shown in Figure 2, if a vehicle arrives at node i earlier than a_i, the vehicle waits till time a_i before departing. However, if the arrival time at node i is greater than b_i the departure time will be infinite, which indicates that the vehicle will not be served if it arrives later than b_i.

Given the arrival time Y_i^r at node i taking route r, the departure time W_i^r is given by

$$W_i^r = g\left(Y_i^r\right). \qquad (18)$$

Note that for W_i^r in (18) to be a random variable, the function $g(.)$ must have the following properties (Papoulis, 1991):

1. Its domain must include the range of the random variable Y_i^r.

2. It must be a *Baire* function, i.e. for every w_i^r the value of y_i^r such that $g(y_i^r) \leq w_i^r$ must consist of the union of a countable number of intervals.

3. The events $\{g(Y_i^r) = \pm\infty\}$ must have zero probability.

The departure time window $g(.)$, shown in Figure 2, violates property 3. That is, whenever $y_i^r > b_i$ the departure time becomes infinity, which implies that the probability of the events $\{g(Y_i^r) = +\infty\}$ is not zero. To overcome this problem, we assume that if a vehicle arrives at node ***i*** at any time $y_i^r > b_i$, the departure time will be $M \cdot y_i^r$, where *M* is a very large number. Figure 3 illustrates the modified function $g(.)$.

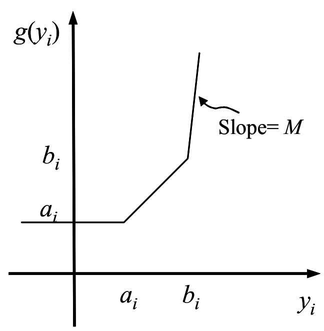

Figure 3: The modified time window g(.).

Note that the value of *M* is chosen to be very large such that it would be almost impossible for a vehicle arriving at node ***i*** at time $y_i^r > b_i$ to continue its path toward the next node. In other words, the value of *M* does not penalize (as in the case of soft time windows) the late vehicles, but prohibits them in reaching the other nodes on the route within a reasonable amount of time. Hence, one may think of selecting the value of *M* asymptotically close to infinity.

Let $f_{Y_i^r}(y_i^r)$ be the PDF of the random variable Y_i^r. The first moment of the departure time $W_i^r = g(Y_i^r)$ can be obtained by

$$E\left[W_i^r\right] = \int_0^{\infty} g\left(y_i^r\right).f_{Y_i^r}\left(y_i^r\right)dy_i^r \;. \qquad (19)$$

As discussed earlier, it is very difficult to obtain the PDF of the random variable Y_i^r at each node *i*. Hence, it is more practical to find means to estimate the first and second moments of the random variable W_i^r in terms of the function $g(.)$ and moments of Y_i^r. These estimates can be obtained by approximating the function $g(Y_i^r)$ at or around $E\left[Y_i^r\right]$ using Taylor's series expansion as follows

$$g\left(y_i^r\right)=g\left(E\left[Y_i^r\right]\right)+g'\left(E\left[Y_i^r\right]\right)\cdot\left(y_i^r-E\left[Y_i^r\right]\right)+\frac{1}{2}g''\left(E\left[Y_i^r\right]\right)\cdot\left(y_i^r-E\left[Y_i^r\right]\right)^2+\cdots. \quad (20)$$

Note that for the Taylor's series expansion in (20) to be valid, the mean arrival time at node *i*, $E\left[Y_i^r\right]$, should be far enough from points $y_j=a_j$ and $y_j=b_j$, where the function $g(.)$ is not differentiable. In the following, we first start by assuming that the above condition holds. Later, we propose an approximation method to deal with cases in which $E\left[Y_i^r\right]$s are close to the kink points.

From Figure 3, one can easily observe that the second and higher derivatives of function $g(.)$ are zero, i.e. $g^{(n)}=0$ for $n\geq 2$. Thus, according to (20) the first central moment of the random variable W_i^r can be obtained by

$$E\left[W_i^r\right]=g\left(E\left[Y_i^r\right]\right), \quad (21)$$

and the variance of W_i^r can be calculated by

$$\operatorname{var}\left(W_i^r\right)=\operatorname{var}\left(g\left(Y_i^r\right)\right)=E\left[g^2\left(Y_i^r\right)\right]-\left(E\left[g\left(Y_i^r\right)\right]\right)^2. \quad (22)$$

Likewise, the function $g^2\left(Y_i^r\right)$ in (22) can be approximated at or around $E\left[Y_i^r\right]$ using Taylor's series expansion as follows

$$g^2\left(y_i^r\right)=g^2\left(E\left[Y_i^r\right]\right)+\left(g^2\right)'\left(E\left[Y_i^r\right]\right)\cdot\left(y_i^r-E\left[Y_i^r\right]\right)+\frac{1}{2}\left(g^2\right)''\left(E\left[Y_i^r\right]\right)\cdot\left(y_i^r-E\left[Y_i^r\right]\right)^2+\cdots. \quad (23)$$

Knowing that $g^{(n)}=0$ for $n\geq 2$, we have

$$\left(g^2\right)' = 2gg'$$
$$\left(g^2\right)'' = \left(2gg'\right)' = 2\left(g'\right)^2 . \tag{24}$$
$$\left(g^2\right)^{(n)} = 0 \quad , n \geq 3$$

Therefore, according to (23) and (24), $E\left[g^2\left(Y_i^r\right)\right]$ in (22) can be obtained by

$$E\left[g^2\left(Y_i^r\right)\right] = g^2\left(E\left[Y_i^r\right]\right) + \left(g'\right)^2\left(E\left[Y_i^r\right]\right) \cdot \operatorname{var}\left(Y_i^r\right). \tag{25}$$

Substituting (25) and (21) in (22), we obtain

$$\operatorname{var}\left(W_i^r\right) = \left(g'\right)^2\left(E\left[Y_i^r\right]\right) \cdot \operatorname{var}\left(Y_i^r\right). \tag{26}$$

Equations (21) and (26) lead to estimating the mean and variance of the departure time W_i^r at node *i* in terms of the function $g(.)$ and the mean and variance of the arrival time Y_i^r . However, it should be noted that by using equations (21) and (26) we may face the following problem: a slight difference in the mean arrival time may lead to a large difference in the variance of the departure time.

By means of an example, we address this problem. Consider the graph G shown in Figure 4 in which two stationary yet stochastic routes, labeled 1 and 2, connect node *i* to node *j*. We assume that the mean and standard deviation of the travel time between nodes *i* and *j* on route 1 are η_1=1:55 hour and σ_1=45 minutes and those of route 2 are η_2=2:05 hour and σ_2=10 minutes, respectively. We also assume that the time window at node *j* is [11:00, 12:30], and the departure time at node *i* is 9:00 a.m.

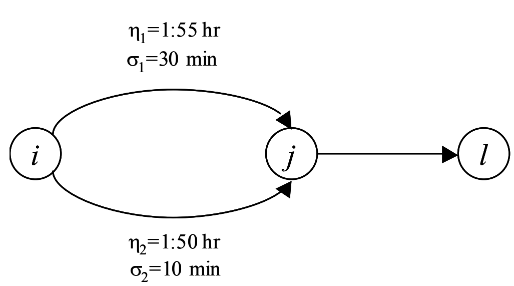

Figure 4: Two routes with different stochastic characteristics connecting node *i* to node *j*.

According to (21) and (26), the mean and variance of the departure time having taken route 1 will be $E\left[W_j^1\right]$=11:00 and $\operatorname{var}\left(W_j^1\right)$=0, and for the route 2 are $E\left[W_j^2\right]$=11:05 and $\operatorname{var}\left(W_j^2\right)$=100. That is, a slight difference in the mean arrival times may lead to a large difference in the variance of the departure time.

It should be noted that at points y_j=a_j and y_j=b_j in Figure 3, the right and left derivatives of $g(.)$ are not equal. More precisely, Equation (26) is valid as long as

$$\begin{aligned} &\left|E\left[Y_j^r\right]-a_j\right| >> \sigma\left(Y_j^r\right), \quad \text{and} \\ &\left|E\left[Y_j^r\right]-b_j\right| >> \sigma\left(Y_j^r\right) \end{aligned} \tag{27}$$

hold. For the cases in which the inequalities in (27) do not hold, the differences in the slope of $g(.)$ around points y_j=a_j and y_j=b_j should also be taken into account. Here, we approximate the value of the standard deviation $\sigma\left(W_j^r\right)$ by

$$\sigma\left(W_j^r\right) \approx \frac{1}{2} \cdot \int_{E\left[Y_j^r\right]-\sigma\left(Y_j^r\right)}^{E\left[Y_j^r\right]+\sigma\left(Y_j^r\right)} g'(x)\,dx, \tag{28}$$

where $g(.)$ is given in Figure 3 and $\operatorname{var}\left(W_j^r\right) = \sigma^2\left(W_j^r\right)$.

For the above discussed example, we notice that traveling on any route 1 or 2 results in $\left|E\left[Y_j^r\right]-a_j\right| < \sigma\left(Y_j^r\right)$ for r=1, 2, which violates the inequalities in (27). Approximating the standard deviation of the departure time using (28), we get $\sigma\left(W_j^1\right)$=20 minutes, and $\sigma\left(W_j^2\right)$=7.5 minutes which shows the advantage of taking route 2 over route 1 as far as the minimum $\operatorname{var}\left(W_j^r\right)$ is concerned.

Recall that we started this subsection by assuming that there is no service time associated with the nodes of route r. Now if the random variable S_i, which denotes the service time at node $i \in ND$, is given, the time when service starts at node i can be approximated by equations (21) and (28), respectively. Hence, given the mean and variance of the service time at each node i, the mean and variance of the departure time from node i, W_i^r, can be calculated according to equations (14) and (15) by

$$E\left[W_i^r\right] = E\left[g\left(Y_i^r\right)\right] + E\left[S_i\right], \text{ and} \tag{29}$$

$$\operatorname{var}\left(W_i^r\right) = \operatorname{var}\left(g\left(Y_i^r\right)\right) + \operatorname{var}\left(S_i\right), \tag{30}$$

where $E\left[g\left(Y_i^r\right)\right]$ and $\operatorname{var}\left(g\left(Y_i^r\right)\right)$ are the mean and variance of the time when service starts at node i and are given by (21) and (28), respectively. Consequently, the first two moments of the arrival time at node j on route r can be computed using equations (16) and (17).

In summary, given the departure time $Y_o = y_o$, the mean and variance of the travel time on each arc $(i, j) \in A^r$, the mean and variance of the service time, and the time window at each node $i \in r$, the mean and variance of the arrival time at each node i taking route r can be recursively calculated using equations (16) - (17), (21), (28) - (30).

IV. Proposed Solution Method for the STSPTW

Consider the graph G as defined in Section III. Recall from Section II that the arrival time at node i taking route r is said to meet the required *service level* at this node, say Υ_i, if the confidence coefficient at node i, γ_i^r, is greater than or equal to Υ_i. For the sake of simplicity, hereafter, we assume that the service level at all the nodes are equal, i.e., $\Upsilon_i = \Upsilon$, $\forall i \in r$. Thus, route r is said to be *acceptable* if the arrival times at all nodes $i \in ND$ meet the service level Υ.

Recall also that, in the STSPTW problem, the objective is to find the least-cost acceptable route starting from origin o, visiting all nodes in N and ending at destination d, such that each node is visited exactly once.

In this section, we propose an approximate solution method based on a modified dynamic programming method to find the least-cost acceptable route in the STSPTW. We assume that the mean and variance of the traveling time $X_{ij}(t)$ on arc $(i,j) \in A$, denoted by $\eta_{ij}(t)$ and $\sigma_{ij}^2(t)$ and defined in (4) and (5), are given. We also assume that the mean and variance of the service time s_i at each node $i \in ND$ denoted by $E[s_i]$ and $var(s_i)$, respectively, are known a priori. We assume that the cost of traveling on a route equals to the mean travel time on that route.

In order to apply dynamic programming, we define a state by a 2-tuple (S,i), where $S \subseteq N$ is an unordered set of visited nodes, and $i \in S$ is the last visited node. Associated to each state are:

- The mean, $E\left[Y_i^S\right]$, and variance, $\mathrm{var}\left(Y_i^S\right)$, of the arrival time at node i taking the path starting from the origin passing through every node of S exactly once and ending at node i,
- A cost denoted by C_i^S, defined as the cost of traveling on the path described above, and
- A confidence coefficient γ_i^S.

Definition (acceptable arc): Let the arrival time at node i meet the required service level, i.e., Y. An arc $(i,j) \in A$ is said to be 'acceptable' if the arrival time at node j traveling on arc (i,j) also meets that service level.

Definition (state expandability): Let node $i \in S$ be the last visited node of the state (S,i). The state S is said to be expandable to node j, $j \in ND$ and $j \notin \{S,o\}$, if the arc $(i,j) \in A$ is 'acceptable'.

The modified dynamic programming method is briefly described in the following. We note that the worst-case computational complexity of the algorithm is exponential in the number of nodes in graph G. At each state (S,i), the approximate solution algorithm for the STSPTW looks for uncovered nodes (i.e., $j \in ND$ and $j \notin \{S,o\}$), which could be added to the set of nodes in S. In order to reduce the computational time, two types of elimination tests are performed: arc elimination and state elimination.

1) *Arc elimination test*: The arc elimination test looks one step ahead to reduce the number of states. Assume that the state (S,i) is expandable to node j. The state (S,i) will be expanded to node j if all arcs (j,k), $k \in ND/\{S, j, o\}$, are acceptable. Otherwise the state will be eliminated.

2) *State elimination test*. This test implements the dynamic programming algorithm to reduce the number of states. Given states (S_1,i) and (S_2,i), where S_1 and S_2 cover the same set of nodes in

ND, the second state is eliminated if $C_i^{S_1} \le C_i^{S_2}$ (or for random variables $E\left[C_i^{S_1}\right] \le E\left[C_i^{S_2}\right]$) and the random variable $Y_i^{S_1}$ dominants $Y_i^{S_2}$ (e.g., $E\left[Y_i^{S_1}\right] \le E\left[Y_i^{S_2}\right]$ and $\text{var}\left(Y_i^{S_1}\right) \le \text{var}\left(Y_i^{S_2}\right)$).

The following is the summary of the approximate solution algorithm for the STSPTW.

<u>Algorithm:</u>

Step 1 (Initialization):

Level l=1, $E\left[W_o\right] = Y_o$, $\text{var}(W_o) = 0$;

For all expandable nodes $i \in N/\{o\}$ from origin o:

1) Generate (S,i): $S=\{o,i\}$,

2) Compute $E\left[Y_i^S\right]$, $\text{var}(Y_i^S)$, C_i^S according to (16) and (17).

Step 2: (During):

Level $l=l+1$,

For all states (S,i) at level $l-1$, and

For all expandable nodes $j \in N / S$ from state (S,i):

Perform Arc and State elimination tests,

For the remaining nodes:

1) Generate $(S \cup \{j\},j)$:

2) Compute $E\left[Y_j^S\right]$, $\text{var}(Y_j^S)$, C_j^S according to (16)-(17), (21), (28) -(30).

Step 3 (Termination):

If $l<|ND|$ go to step 2.

If $l=|ND|$, go to step 2 and set $j=\{d\}$.

If $l=|ND|+1$, find the minimum cost C_d^S among all states (S,d), that is the best route.

V. Experimental Results

To evaluate the efficiency of the proposed STSPTW algorithm, experiments are performed in this section on graphs with stationary as well as non-stationary travel and service times. The algorithm is coded in Matlab 6.5 developed by MathWorks, Inc.

Experiment 1 (Stationary Stochastic Network): In this experiment, we consider graph G in Figure 5 consisting of 5 nodes in which node 1 represents the single depot (origin-depot and destination-depot).

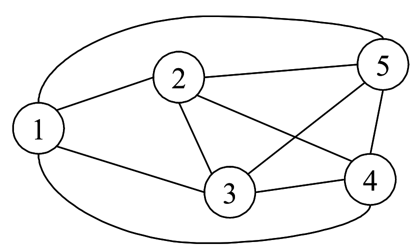

Figure 5: The graph *G* consists of 5 nodes in Experiment 1.

We assume that the travel time between every pair of nodes i and j in graph G is stationary, i.e., the travel time X_{ij} is independent of time t, the time when the vehicle enters the arc $(i,j) \in A$. The mean and standard deviation of the travel time in graph G is given by

$$\eta = \begin{bmatrix} 0 & 60 & 90 & 120 & 150 \\ 60 & 0 & 60 & 90 & 105 \\ 90 & 60 & 0 & 60 & 105 \\ 120 & 90 & 60 & 0 & 60 \\ 150 & 105 & 105 & 60 & 0 \end{bmatrix}, \tag{31}$$

$$\sigma = \begin{bmatrix} 0 & 6 & 9 & 12 & 15 \\ 6 & 0 & 6 & 9 & 36.7 \\ 9 & 6 & 0 & 6 & 10.5 \\ 12 & 9 & 6 & 0 & 6 \\ 15 & 36.7 & 10.5 & 6 & 0 \end{bmatrix} \tag{32}$$

in matrix form, where η_{ij} and σ_{ij} are the mean and standard deviation of the travel time between nodes i and j in minutes, respectively. For the sake of simplicity, we assume that graph G is symmetric. It should be noted that the proposed algorithm is general and its application is not limited to symmetric networks.

The mean and standard deviation of the service time of the nodes of graph G are given by

$$\mu = \begin{bmatrix} 0 & 15 & 15 & 15 & 15 \end{bmatrix}, \tag{33}$$

$$\Sigma = \begin{bmatrix} 0 & 1.5 & 1.5 & 1.5 & 1.5 \end{bmatrix} \tag{34}$$

in vector forms, where μ_i and Σ_i are the mean and standard deviation of the service time at node i in minutes, respectively. Likewise, the earliest and latest time to visit nodes of graph G are given by

$$\alpha = \begin{bmatrix} 8 & 10 & 9 & 12 & 11 \end{bmatrix}, \tag{35}$$

$$\beta = \begin{bmatrix} 18 & 15 & 13 & 16 & 15 \end{bmatrix} \tag{36}$$

where α_i and β_i are the earliest and latest hours of the day to visit node i, respectively.

The developed algorithm is applied on graph G with different values of service level Υ. The lower bound for the confidence coefficient in (1) is found by applying the Tchebycheff and Chernoff bounds (see also Papoulis 1991) assuming a normal distribution for all travel and service times in graph G. Table 1 summarizes the computational results for each service level. The cost is equal to the mean travel time.

Table 1: Computational results for stationary, STSPTW in Experiment 1.

Service level	Tchebycheff Bound		Chernoff Bound	
	Best Route	Cost of the Route [min.]	Best Route	Cost of the Route [min.]
Υ=80%	r={1 3 5 4 2 1}[a]	465	r={1 3 2 5 4 1}	495
Υ=90%	r={1 3 2 5 4 1}	495	r={1 3 2 5 4 1}	495
Υ=95%	r={1 2 3 4 5 1}	510	r={1 3 2 5 4 1}	495
Υ=97.5%	r={1 2 3 4 5 1}	510	r={1 2 3 4 5 1}	510
Υ=99%	NA[b]	NA	r={1 2 3 4 5 1}	510

a) The optimum route for deterministic graph, i.e. δ=0 and Σ=0, see also (Dumas et al., 1995).

b) NA: Not Available, i.e. no route could be found.

Table 1 indicates that as the value of the service level increases, the cost of the best route increases too. That is, with a high value of Υ, more routes become unacceptable, thus, reducing the number of acceptable low-cost routes in the algorithm.

The estimated mean and standard deviation of the arrival times at the nodes of the best routes in Table 1 are shown in Table 2 rows 2 and 3. These estimated values are found using the developed method discussed in Section III. To evaluate the estimated arrival times at the nodes and to validate the results of Table 1, the best routes obtained in Table 1 are simulated 10,000 times using the same parameters given in (31) to (36). In each trial, the travel time on the links and the service time at the nodes of the best routes are generated using a normal random number generator with the mean and standard deviation given in (31) to (34).

Table 2 rows 4 and 5 present the mean and standard deviation of the arrival times at nodes of the simulated best routes. Table 2 row 6 shows the percentage of times that the simulated routes met the time window constraints at the nodes given in (35) and (36). This percentage is called the route success rate in Table 2. Note that the mean and standard deviation of the arrival times at the nodes given in rows 4 and 5 are calculated for simulated routes which managed to meet all time window constraints.

Table 2: Estimated vs. simulated first two moments of arrival times at the nodes of the best routes in Experiment 1.

Route		**r={1 3 5 4 2 1}**	**r={1 3 2 5 4 1}**	**r={1 2 3 4 5 1}**
Estimated arrival times at nodes	**Mean [hr]**	{8 9:30 11:30 12:45 14:30 15:57}	{8 9:30 10:45 12:45 14:0 16:15}	{8 9:0 11:15 12:30 13:45 16:30}
	Std[a] [min]	{0 9 13.91 15.22 17.78 18.79}	{0 9 10.92 38.37 38.86 40.7}	{0 6 6.18 8.74 10.71 18.49}
Simulated arrival times at nodes	**Mean [hr]**	{8 9:29.5 11:28.8 12:43.6 14:28.0 15:43.1}	{8 9:29.9 10:44.8 12:44.4 13:59.4 16:14.4}	{8 9:0 11:15.1 12:30 13:45 16:30.1}
	Std [min]	{0 8.64 13.07 14.01 15.83 16.93}	{0 8.97 10.89 38.07 38.46 40.22}	{0 6.06 6.19 8.66 10.7 18.54}
Route success rate		95.61%	99.39%	100.0%

a) Std: Standard Deviation.

Comparison between Table 2 rows 2 and 3 with rows 4 and 5 shows how good the developed estimation method is. In all cases, our estimation method is close to the simulated values, especially for r={1 2 3 4 5 1}.

Table 2 row 6 also indicates that the proposed approximate solution method was able to find a good solution to the STSPTW problem. Moreover, it shows that the Tchebycheff bound generates tighter lower bounds for the lower values of the service level, i.e. Υ=80%; the Chernoff bound is much tighter for the higher values of the service level, i.e. $\Upsilon \in \{95\%, 97.5\%, 99\%\}$.

Experiment 2 (Stationary STSPTW): In this experiment, we are interested in evaluating the computational aspects of the proposed algorithm. Here, the experimental test consists of a Euclidean plane in which the nodes' coordinates are uniformly distributed between 0 and 50, and the coordinates of the depot is at 0 and 0. The mean travel time equals distance, and the standard deviation of the travel time is set to be one tenth of the mean travel time. We assume that there is no service time associated with the nodes.

The time window at each node is generated around the time to begin service at that node according to the first nearest neighbor TSP tour (Reinelt, 1994). That is, assuming travel time equals distance, we found the best deterministic TSP tour based on the first nearest neighbor heuristic algorithm. Accordingly, the time to reach node i taking the generated tour, say T_i, is calculated and the time window $[a_i, b_i]$ is generated around this time by

$$a_i = \max\left(0\,,\, T_i - \frac{w}{2}\right), \tag{37}$$

$$b_i = T_i + \frac{w}{2}, \tag{38}$$

where w=20, 30, 40, 60, and 80 minutes.

The proposed solution method is applied to the generated graph for a service level Υ=90% using the Tchebycheff bound. Table 3 presents the experimental results with a different number of nodes (customers), N, and different time window widths, w. The cost is equal to the mean travel time. The second column in Table 3 shows the deterministic cost of taking the first nearest neighbor TSP solution. This solution, however, may not be acceptable for the STSPTW.

The experiments are tested on an Intel Pentium M, 1.6 GHZ. In Table 3, each set of nodes (row) is built upon the previous row. For instance, for the number of nodes equal to 30, we kept the same randomly generated nodes for N=20 and added 10 newly generated ones. A new first-nearest-neighbor TSP tour is then found and a time window is assigned to each node according to (37) and (38).

Table 3: Computational results for stationary, STSPTW in Experiment 2 using Tchebycheff bound with Υ=90%.

No of nodes	Init[a]	*w*=20 Min		*w*=30 Min		*w*=40 Min		*w*=60 Min		*w*=80 Min	
		Best[b]	CPU[c]	Best	CPU	Best	CPU	Best	CPU	Best	CPU
20	180.6	178.7	1.3	178.7	2.6	178.7	3.1	178.0	6.6	178.0	14.1
30	267.6	265.2	4.1	262.8	10.5	262.8	14.8	255.9	43.1	254.6	168.0
40	289.9	288.8	15.3	281.1	25.8	280.6	49.0	274.6	184.1	270.7	725.8
50	309.1	306.3	25.0	304.1	49.1	304.1	110.7	301.0	435.0	NA[d]	NA
60	382.2	376.6	53.2	374.3	119.9	374.3	229.0	NA	NA	NA	NA
80	455.3	445.7	510	440.3	1381	NA	NA	NA	NA	NA	NA

a) The cost of the initial route in minutes generated by the deterministic first nearest neighbor TSP heuristic.

b) The cost of the best solution in minutes using the proposed approximate STSPTW method.

c) The CPU time in seconds on a Pentium M (1.6 GHz) personal computer.

d) NA: Not Available, i.e., no route could be found.

The results in Table 3 indicate that the approximate algorithm is successful in solving problems up to 80 nodes with fairly wide time windows. The results also show that the CPU time increases with the width of the time windows. It should be noted that as the width of the time windows and the number of customers increase, the number of acceptable routes increases sharply. Thus, the algorithm needs more time to find the best route.

Experiment 3 (Non-stationary Stochastic Network): Consider graph G in Figure 5 again. In this experiment, we assume that the traveling time on some arcs of graph G are non-stationary stochastic processes. Let $\eta_{ij}(t)$ and $\sigma_{ij}(t)$ in (40) and (41) be the mean and standard deviation of the stochastic process $X_{ij}(t)$ (the travel time between nodes i and j in minutes),

$$\eta(t) = \begin{bmatrix} 0 & x(t) & y(t) & 120 & 150 \\ x(t) & 0 & x(t) & 90 & 105 \\ y(t) & x(t) & 0 & 60 & 105 \\ 120 & 90 & 60 & 0 & x(t) \\ 150 & 105 & 105 & x(t) & 0 \end{bmatrix}, \tag{39}$$

$$\sigma(t) = \begin{bmatrix} 0 & 0.1x(t) & 0.1y(t) & 12 & 15 \\ 0.1x(t) & 0 & 0.1x(t) & 9 & 36.7 \\ 0.1y(t) & 0.1x(t) & 0 & 6 & 10.5 \\ 12 & 9 & 6 & 0 & 0.1x(t) \\ 15 & 36.7 & 10.5 & 0.1x(t) & 0 \end{bmatrix}, \tag{40}$$

where the functions $x(t)$ and $y(t)$ are given graphically by Figure 6 and Figure 7, respectively. In other words, we assume that the travel times on arcs (1,2), (1,3), (2,3), and (4,5) are non-stationary random processes. Since graph G is assumed to be symmetric, the travel times on arcs (2,1), (3,1), (3,2), and (5,4) are also non-stationary with the same characteristics as their counterparts (1,2), (1,3), (2,3), and (4,5), respectively.

Three different scenarios are considered in Figure 6 and Figure 7 for $x(t)$ and $y(t)$: a) stationary travel time, b) non-stationary travel time with relatively low dynamics, and c) non-stationary travel time with relatively high dynamics. In Experiment 1 scenario ‘a’ was investigated.

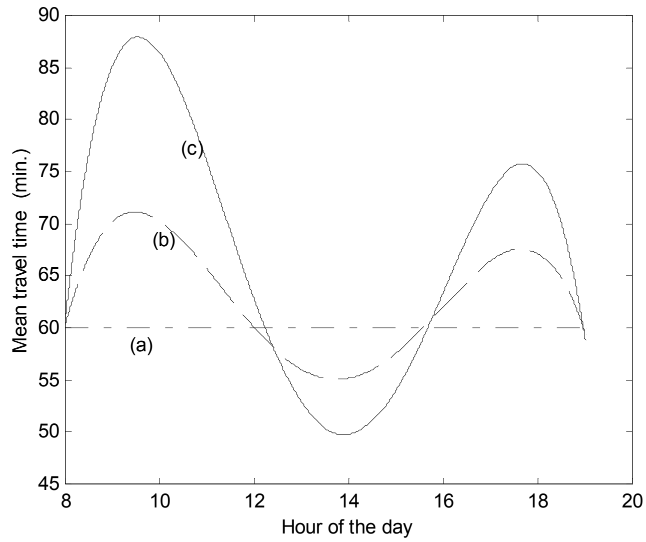

Figure 6: The mean travel time *x*(*t*) in minutes versus the time of the day. (a) stationary, (b) non-stationary with low dynamics, and (c) non-stationary with high dynamics.

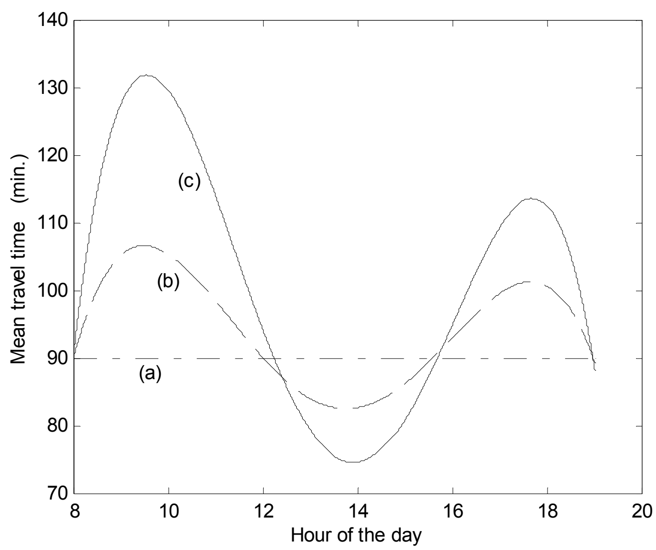

Figure 7: The mean travel time *y*(*t*) in minutes versus the time of the day. (a) stationary, (b) non-stationary with low dynamics, and (c) non-stationary with high dynamics.

In this experiment, the proposed solution method is applied to graph G with non-stationary travel times on some arcs as explained above. Different values of service level Υ and different lower confidence bounds are considered. Table 4 summarizes the computational results in which the cost is equal to the mean travel time.

Table 4: Computational results for non-stationary, STSPTW in Experiment 3.

Service level	Lower bound	Non-stationary with low dynamic		Non-stationary with high dynamic	
		Best Route	Cost of the Route	Best Route	Cost of the Route
ϒ=80%	Tchebycheff	r={1 3 5 4 2 1}	463.5	r={1 3 2 5 4 1}	514.2
	Chernoff	r={1 3 2 5 4 1}	502	r={1 3 2 5 4 1}	514.2
ϒ=90%	Tchebycheff	r={1 3 2 5 4 1}	502.0	r={1 2 3 4 5 1}	526.3
	Chernoff	r={1 3 2 5 4 1}	502	r={1 3 2 5 4 1}	514.2
ϒ=95%	Tchebycheff	r={1 2 3 4 5 1}	515.6	r={1 2 3 4 5 1}	526.3
	Chernoff	r={1 3 2 5 4 1}	502	r={1 2 3 4 5 1}	526.3
ϒ=97.5%	Tchebycheff	r={1 2 3 4 5 1}	515.6	NA[a]	NA
	Chernoff	r={1 2 3 4 5 1}	515.6	r={1 2 3 4 5 1}	526.3
ϒ=99%	Tchebycheff	NA	NA	NA	NA
	Chernoff	r={1 2 3 4 5 1}	515.6	r={1 2 3 4 5 1}	526.3

a) NA: Not Available, i.e. no route could be found.

Using the developed estimation method in Section III, the estimated mean and standard deviation of the arrival times at the nodes of the best routes of Table 4 are calculated and shown in rows 2 and 3 of Table 5 and Table 6. Table 5 shows the results for the non-stationary travel times with relatively low dynamics, while Table 6 is for the non-stationary travel times with high dynamics.

The results in Table 4 and the estimated arrival times in Table 5 and Table 6 are validated and simulated through running a simulation program for 10,000 times. In each trial, the travel times on the links and the service time at the nodes of graph G are generated using a normal random number generator with the mean and standard deviation given above. The mean travel times on arcs (1,2), (1,3), (2,3) and (4,5) are generated according to Figure 6 and Figure 7 at each instant of time. Table 5 and Table 6 also present the percentage of times (route success rate) that the simulated routes met the time window constraints in the non-stationary, stochastic graph G described above.

Table 5: Estimated vs. simulated first two moments of arrival times at the nodes of the best routes in Experiment 3, non-stationary network with low dynamics.

Route		r={1 3 5 4 2 1}	r={1 3 2 5 4 1}	r={1 2 3 4 5 1}
Estimated arrival times at nodes	Mean [hr]	{8 9:30.5 11:30.5 12:46.8 14:31.8 15:43.5}	{8 9:30.5 10:56.4 12:56.4 14:07.0 16:22.0}	{8 9:00.3 11:24.4 12:39.4 13:50.6 16:35.6}
	Std[a] [min]	{0 9.05 13.94 14.14 16.83 18.67}	{0 9.05 11.37 38.50 37.71 39.6}	{0 6.03 7.08 9.40 10.67 18.47}
Simulated arrival times at nodes	Mean [hr]	{8 9:30.0 11:29.5 12:45.8 14:30.2 15:42.0}	{8 9:30.5 10:56.2 12:55.1 13:06.4 16:21.2}	{8 9:00.3 11:24.4 12:39.4 13:50.7 16:35.6}
	Std [min]	{0 8.87 13.28 13.1 15.11 17.0}	{0 9.12 11.36 37.43 36.61 38.25}	{0 6.01 7.11 9.35 10.7 18.48}
Route success rate		95.49%	98.98%	100.0%

Table 6: Estimated vs. simulated first two moments of arrival times at the nodes of the best routes in Experiment 3, non-stationary network with high dynamics.

Route		r={1 3 5 4 2 1}	r={1 3 2 5 4 1}	r={1 2 3 4 5 1}
Estimated arrival times at nodes	Mean [hr]	{8 9:31.2 11:31.2 12:51.8 14:36.8 15:44.9}	{8 9:31.2 11:13.8 13:13.8 14:19.2 16:34.2}	{8 9:00.8 11:39.4 12:54.4 14:01.3 16:46.3}
	Std[a] [min]	{0 9.05 13.94 14.14 16.83 18.67}	{0 9.12 12.34 38.80 37.08 39.00}	{0 6.1 8.5 10.5 11.0 18.6}
Simulated arrival times at nodes	Mean [hr]	{8 9:30.7 11:29.8 12:50.4 14:34.6 15:42.7}	{8 9:31.2 11:13.5 13:12.2 14:19.1 16:33.8}	{8 9:01.0 11:39.5 12:54.4 14:01.3 16:46.1}
	Std [min]	{0 8.86 12.92 11.52 13.64 16.09}	{0 9.04 12.28 36.93 34.86 36.57}	{0 6.1 8.5 10.5 11.0 19.0}
Route success rate		92.81%	97.89%	99.99%

a) Std: Standard Deviation.

Comparison between Table 5 rows 2 and 3 with rows 4 and 5 shows that our developed method was able to estimate the arrival times at the nodes of the non-stationery stochastic network with very small errors. These errors are slightly higher in Table 6 due to the higher dynamics of the network.

Table 5 and Table 6 rows 6 indicate that the proposed algorithm is successful in finding a good approximate solution for the non-stationary STSPTW problem. As expected, the method tends toward finding more conservative solutions for highly dynamic problems. The results also demonstrate the advantage of using the Chernoff bound over the Tchebycheff bound for highly dynamic problems.

Experiment 4 (Non-stationary STSPTW): Here, we evaluate the computational aspects of the proposed algorithm for the non-stationary, STSPTW. In this experiment, we adopt the same generated networks as described in Experiment 2. We kept half of the links in that network stationary and altered the other half to non-stationary links. More precisely, the travel time of the link (i,j) is kept unchanged (stationary) if $i+j$ is an even number. If the sum is an odd number, the mean traveling time on the link (i,j) is assumed to follow the scheme shown in Figure 8. Shown in Figure 8 is the mean travel time $\eta_{ij}(t)$ normalized by its static counterpart, η_{ij}, from Experiment 2.

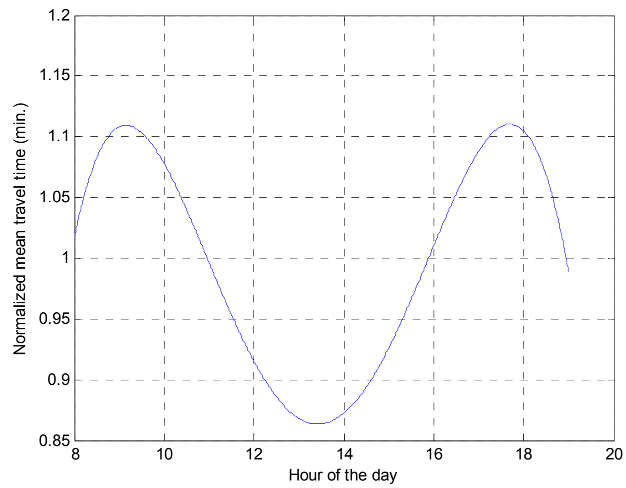

Figure 8: The normalized mean travel time on link (*i,j*) in minutes versus the time of the day. The mean travel time in normalized by its static counterpart η_{ij} from simulation Experiment 2.

In this experiment, we assume that the standard deviation of the travel time at each time is one tenth of the mean travel time at that time, and that no service time is associated with the nodes of the network. We also assume that the cost of the traveling is equal to the mean travel time at that time, and that the service level is Υ=90%. Table 7 presents the experimental results for different number of nodes (customers), N, where the Tchebycheff bound is used. The time window widths are assumed to be w=30, 40, 60, and 80 minutes

Table 7: Computational results for non-stationary, STSPTW in Experiment 4 using Tchebycheff bound with Υ=90%.

No of nodes	*w*=30 Min		*w*=40 Min		*w*=60 Min		*w*=80 Min	
	Best[a]	CPU[b]	Best	CPU	Best	CPU	Best	CPU
20	184.8	1.9	184.8	2.56	184.8	5.9	184.4	11.0
30	267.4	5.9	267.4	9.9	260.4	35	260.4	154
40	291.8	10.4	291.8	40.4	283.1	144.7	274.1	600
50	305.4	41.8	304.9	85.5	304.6	328.7	NA[c]	NA
60	374.8	114.7	374.9	233.8	NA	NA	NA	NA
80	435.0	478	NA	NA	NA	NA	NA	NA

a) The cost of the best solution in minutes using the proposed approximate STSPTW method.

b) The CPU time in seconds on a Pentium M (1.6 GHz) personal computer.

c) NA: Not Available.

The results in Table 7 indicate that the approximate STSPTW algorithm was able to solve non-stationary stochastic problems with up to 80 nodes with fairly wide time windows. As expected, the CPU time increases with the time window width and problem size.

VI. Summary and Conclusion

In this paper, a truck route planning problem called Stochastic Traveling Salesman Problem with Time Windows (STSPTW) in which traveling times along the roads and service times at the customer locations are stochastic processes is investigated. The objective is to find the least-cost route among all routes which meet the required service level at the customer locations. To do so, we developed a methodology to estimate the first and second moments of arrival time at each customer location. In addition, we developed a methodology to address the existing time windows in the STSPTW problem. We further proposed an approximate solution methodology based on a modified dynamic programming method to find the least-cost route in the STSPTW.

The results from various experiments indicate that our developed method was able to estimate the mean and standard deviation of the arrival times at the nodes of both stationary and non-stationary networks with very small errors. The results also show that the approximate solution method is efficient for both stationary and non-stationary STSPTW problems and that the approximate algorithm is successful in solving STSPTW problems with up to 80 nodes with fairly wide time windows.

In this paper, we compared the use of the Tchebycheff and Chernoff bounds in finding the confidence levels for the approximate solutions. We observed, through experimental results, that the Tchebycheff bound generates tighter lower bounds for low values of the service level, whereas the Chernoff bound is much tighter for the higher ones. The experimental results also demonstrated the advantages of using the Chernoff bound over the Tchebycheff bound for the highly dynamic non-stationary STSPTW problem. However, it should be noted that, in general, the Chernoff bound is more difficult to compute and knowledge of the characteristic function is needed a priori.

The findings of this research have significant practical relevance. One of the primary criticisms of applying stochastic optimization methods to solving routing problems is the lack of techniques that are robust in handling the inherent uncertainties of the system. The approach developed in this research is a simple and robust method to estimate the arrival times in the presence of time windows. Therefore, planners can use these estimators instead of the deterministic counterparts in developing less costly routes.

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