BENCHMARKS FOR BASIC SCHEDULING PROBLEMS

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Abstract

In this paper, we propose 260 scheduling problems whose size is greater than that of the rare examples published. Such sizes correspond to real dimensions of industrial problems.

The types of problems that we propose are : the permutation flow shop, the job shop and the open shop scheduling problems.

We restrict us to basic problems : the processing times are fixed, there are neither set-up times nor due dates nor release dates, etc. Then, the objective is the minimization of the makespan.

Keywords : Combinatorial optimization, Scheduling, Benchmarks.

Introduction

The types of problems discussed in this paper (permutation flow shop, job shop and open shop scheduling problems) have been widely studied in the literature using exact or heuristic methods, but a common comparison base is missing. We hope that this paper will fill a gap in this domain.

The three-field nomenclature described in Lawler et al. [5] names these problems F| |C_{max}, J| |C_{max} and O| |C_{max} respectively. They certainly belong to the most studied ones among the scheduling problems. Let us describe them briefly.

There are n jobs that have to be performed on m unrelated machines ; in our case, every job consists of m non preemptible operations. Every operation of a job uses a different machine during a given time and may wait before being processed.

For the permutation flow shop problem, the operations of every job must be processed on machines 1 … m in this order. Moreover, the processing order of the jobs on the machines is the same for every machine. The problem consists in finding a permutation of the n jobs that minimizes the makespan.

In the case of the job shop problem, any processing order of the jobs on the machines is allowed. For every job, the operations must be processed in a given order on the machines, but this order may differ according to the jobs.

For the open shop problem, every operation is assigned to a given machine but the order of the operations of every job is totally free.

The aim of this paper is to present unsolved problems whose size corresponds to the one of industrial problems. These problems must be easy to generate.

Generating interesting problems

As we do not know any exact method to solve exactly the problems we want to propose, we have used heuristic methods to get hopefully good solutions of these problems. These heuristic methods are based on taboo search techniques. Taboo search is described very generally in Glover [4] and one can find some of its practical applications to the flow shop sequencing problem in Taillard [8] and Widmer et al. [10], and to the job shop scheduling problem in Taillard [9]. Taboo search is very easy to implement and generally provides excellent results, but it requires a great amount of CPU time.

In order to propose problems that are as difficult as possible (the most interesting ones), we have generated many instances of problems that we have “solved” in a summary way with taboo search. Then, we have chosen the 10 problems that seemed to be the hardest ones and we have solved them once more, allowing our heuristic method to perform a higher number of iteration.

Obviously, the choice of the hardest problems is very subjective. We decided that a problem was interesting if the best makespan we found was far from a lower bound of the makespans and if many attempts to solve the problem (starting from various initial solutions) did not provide the same solution. Such a method enabled us to detect the simplest problems but we may not propose problems that have a local optimum with a large attraction basin.

The problems

The problems we propose are randomly generated with a good random number generator proposed in Bratley [1]. We recall its implementation so that this paper is self contained.

A problem will be entirely defined by the initial value of the seed of the random generator and by the way of generating it.

For every type of problem, we give a simple manner of computing a lower bound of the makespan ; in particular, this permits to verify the generation of the problem.

For every size of problem, we give the total number of instances we have generated (summary resolution), the maximum number of iteration of taboo search that were done (long resolution) and the proportion of problems that were solved up to the lower bound, that is to say optimally. For every type and every size of problem, we give 10 instances.

For each instance, we give the initial value of the random generator seed, the best value of the makespan we have found (i. e. an upper bound of the optimal makespan) and a lower bound of all the makespans.

The random number generator

Let us recall the implementation of the linear congruential generator we have used which is based on the recursive formula $X_{i+1} = (16\ 807\ X_i) \bmod (2^{31} - 1)$. This implementation uses only 32-bit integers and provides a uniformly distributed sequence of numbers between 0 and 1 (not contained) :

0) Initial seed and constants : X_0 $(0 < X_0 < 2^{31} - 1)$
$a = 16\,807$, $b = 127\,773$, $c = 2\,836$, $m = 2^{31} - 1$

1) Modification of the seed : $k := \lfloor X_i/b \rfloor$
$X_{i+1} := a(X_i \bmod b) - kc$
If $X_{i+1} < 0$ then let $X_{i+1} := X_{i+1} + m$

2) New value of the seed : X_{i+1}
Current value of the generator : X_{i+1}/m

Below, we shall denote by U(0,1) the pseudorandom number that this generator provides. We have $0 < U(0,1) < 1$ for every generated number.

We shall denote by U[a,b] (with $a < b$, a and b integers) the integer number $\lfloor a + U(0,1)\cdot(b-a+1) \rfloor$. For every random integer generated, we have $a \le U[a,b] \le b$ and every integer between a and b has the "same" probability of being chosen. In order to implement the integer random procedure only with 32-bit integers, the problems have been chosen in such a way that one never has to deal with a seed X such that :

$$\left\lfloor a + \frac{X \cdot (b-a+1)}{m} \right\rfloor \neq \left\lfloor a + \frac{X}{\lfloor m/(b-a+1) \rfloor} \right\rfloor$$

Flow shop problems

There are in the literature some problems of this type ; let us quote for example eight small and simple problems proposed in Carlier [2] and solved exactly in this reference.

The flow shop problems are characterized by the processing times d_{ij} of job *j* on machine *i* $(1 \le i \le m,\ 1 \le j \le n)$. We have generated the values of d_{ij} by the following way :

For i = 1 to m
    For j = 1 to n
        $d_{ij} := U[1,99]$

We propose problems with 5, 10 and 20 machines and from 20 to 500 jobs. We compute the lower bound of the makespan as presented below.

Let b_i be the minimum amount of time before machine *i* starts to work and a_i be the minimum amount of time that it remains inactive after its work up to the end of the operations, and let T_i be its total processing time. We have :

$$b_i = \min_j \left(\sum_{k=1}^{i-1} d_{kj} \right)$$

$$a_i = \min_j \left(\sum_{k=i+1}^{m} d_{kj} \right)$$

$$T_i = \sum_{j=1}^{n} d_{ij}$$

Clearly, the optimal makespan C^*_{max} is greater than or equal to

$LB = \max_i (b_i + T_i + a_i) \leq C^*_{max}$

This lower bound is easy to compute and we conjecture that :

$\lim_{n/m \to \infty} Prob(C^*_{max} = LB) = 1$

For every size of problem we give the following information (Table 1) :

Nb jobs :	The number of jobs.
Nb machines :	The number of machines.
Nb instances :	The total number of problems generated.
LB reached :	The proportion of problems for which we found a solution for which the makespan was equal to the lower bound (or equal to the lower bound augmented by 2% for the 500-job 20-machine problems).
Nb iterations :	The maximum number of iterations performed by taboo search (long resolution).
Nb resolutions :	The number of attempts to solve the problem from various initial solutions (long resolution).

Nb	Nb	Nb	LB	Nb	Nb
20	5	100	35%	10^4	3
20	10	100	1%	10^4	3
20	20	100	0%	$2 \cdot 10^4$	3
50	5	70	41%	$5 \cdot 10^3$	3
50	10	70	3%	10^4	3
50	20	70	0%	$5 \cdot 10^4$	3
100	5	10 000	54%	$2 \cdot 10^3$	4
100	10	50	6%	$2 \cdot 10^4$	3
100	20	50	0%	10^4	3
200	10	300	28%	$2 \cdot 10^3$	3
200	20	25	0%	$2 \cdot 10^3$	3
500	20	100	14% *	10^3	3

* The value reached for this size was less than or equal to 1.02 times the lower bound.

Table 1. Flow shop problems.

Then we give ten instances for every size of problem with the following information (Table 2) :

Time seed :	The initial value of the random generator's seed.
UB :	An upper bound of the optimal makespan (the best value we got).
LB :	A lower bound of the makespans.

As the aim is to give an upper bound as good as possible but not a fast solving method, the computation time does not have much importance. However, let us mention that an iteration of taboo search needs about $4 \cdot 10^{-6} \cdot n^2 \cdot m$ seconds on a "Silicon Graphics" personal workstation (10 Mips).

20 jobs, 5 machines	Flow shop	
Time seed	UB	LB
873654221	1278	1232
379008056	1359	1290
1866992158	1081	1073
216771124	1293	1268
495070989	1236	1198
402959317	1195	1180
1369363414	1239	1226
2021925980	1206	1170
573109518	1230	1206
88325120	1108	1082

20 jobs, 10 machines	Flow shop	
Time seed	UB	LB
587595453	1582	1448
1401007982	1659	1479
873136276	1496	1407
268827376	1378	1308
1634173168	1419	1325
691823909	1397	1290
73807235	1484	1388
1273398721	1538	1363
2065119309	1593	1472
1672900551	1591	1356

20 jobs, 20 machines	Flow shop	
Time seed	UB	LB
479340445	2297	1911
268827376	2100	1711
1958948863	2326	1844
918272953	2223	1810
555010963	2291	1899
2010851491	2226	1875
1519833303	2273	1875
1748670931	2200	1880
1923497586	2237	1840
1829909967	2178	1900

50 jobs, 5 machines	Flow shop	
Time seed	UB	LB
1328042058	2724	2712
200382020	2836	2808
496319842	2621	2596
1203030903	2751	2740
1730708564	2863	2837
450926852	2829	2793
1303135678	2725	2689
1273398721	2683	2667
587288402	2554	2527
248421594	2782	2776

50 jobs, 10 machines	Flow shop	
Time seed	UB	LB
1958948863	3037	2907
575633267	2911	2821
655816003	2873	2801
1977864101	3067	2968
93805469	3025	2908
1803345551	3021	2941
49612559	3124	3062
1899802599	3048	2959
2013025619	2913	2795
578962478	3114	3046

50 jobs, 20 machines	Flow shop	
Time seed	UB	LB
1539989115	3886	3480
691823909	3733	3424
655816003	3689	3351
1315102446	3755	3336
1949668355	3655	3313
1923497586	3719	3460
1805594913	3730	3427
1861070898	3744	3383
715643788	3790	3457
464843328	3791	3438

100 jobs, 5 machines	Flow shop	
Time seed	UB	LB
896678084	5493	5437
1179439976	5274	5208
1122278347	5175	5130
416756875	5018	4963
267829958	5250	5195
1835213917	5135	5063
1328833962	5247	5198
1418570761	5106	5038
161033112	5454	5385
304212574	5328	5272

100 jobs, 10 machines	Flow shop	
Time seed	UB	LB
1539989115	5776	5759
655816003	5362	5345
960914243	5679	5623
1915696806	5820	5732
2013025619	5491	5431
1168140026	5308	5246
1923497586	5602	5523
167698528	5640	5556
1528387973	5891	5779
993794175	5860	5830

100 jobs, 20 machines	Flow shop	
Time seed	UB	LB
450926852	6345	5851
1462772409	6323	6099
1021685265	6385	6099
83696007	6331	6072
508154254	6405	6009
1861070898	6487	6144
26482542	6393	5991
444956424	6514	6084
2115448041	6386	5979
118254244	6544	6298

200 jobs, 10 machines	Flow shop	
Time seed	UB	LB
471503978	10927	10816
1215892992	10570	10422
135346136	11004	10886
1602504050	10936	10794
160037322	10550	10437
551454346	10378	10255
519485142	10885	10761
383947510	10808	10663
1968171878	10473	10348
540872513	10727	10616

200 jobs, 20 machines	Flow shop	
Time seed	UB	LB
2013025619	11441	10979
475051709	11549	10947
914834335	11537	11150
810642687	11580	11127
1019331795	11484	11132
2056065863	11416	11085
1342855162	11659	11194
1325809384	11587	11126
1988803007	11498	10965
765656702	11569	11122

500 jobs, 20 machines	Flow shop	
Time seed	UB	LB
1368624604	26699	25922
450181436	27303	26353
1927888393	26928	26320
1759567256	27009	26424
606425239	26771	26181
19268348	26959	26401
1298201670	26870	26300
2041736264	27104	26429
379756761	26586	25891
28837162	26910	26315

Table 2. Instances of flow shop problems.

Job shop problems

In the literature, we may find instances of small problems in Lawrence [6] and Muth et al. [7] ; most of the optimal values of these problems are given in Carlier et al. [3]. We can consider that problems up to ten machines may be solved satisfactorily with existing methods. This is why we propose problems with 15 and 20 machines and from 15 to 100 jobs.

The processing time d_{ij} of the jth operation of job *i* , ($1 \le i \le n$, $1 \le j \le m$) is obtained as follows :

```
For i = 1 to n
    For j = 1 to m
        dij := U[1,99]
```

The machine M_{ij} on which the jth operation of job *i* has to be performed is given by the following procedure :

```
0)  Mij := j (1 ≤ i ≤ n, 1 ≤ j ≤ m)
1)  For i = 1 to n
        For j = 1 to m
            Swap Mij and MiU[j,m]
```

Let us note the use of another initial seed for the choice of the machines : *Machine seed.*

An instance of a small open shop problem, obtained with the same procedures, is given extensively in Table 7.

The lower bound of the makespans corresponds to the maximum amount of time that a job or a machine requires, i.e. :

$$LB = \max\left\{\max_j\left(\sum_i d_{ij}\right), \max_i\left(\sum_{j,k|M_{kj}=i} d_{kj}\right)\right\}$$

We conjecture again that this bound is tight if $n/m \to \infty$, because we have always found an optimal schedule if $n/m \ge 6$, considering more than 2000 problems whose size was varying from 20 jobs, 2 machines to 150 jobs, 15 machines, passing by 500 jobs, 4 machines.

The time needed to perform one iteration of taboo search is about $20 \cdot 10^{-6} \cdot n \cdot m$ seconds on the same computer as for the flow shop problems.

Tables 3 and 4 are analogous to Tables 1 and 2, but, for job shop problems, we give in addition *Machine seed* in Table 4.

Nb jobs	Nb machines	Nb instances	LB reached	Nb iterations	Nb resolutions
15	15	50	0%	$5{\cdot}10^5$	4
20	15	50	4%	10^6	3
20	20	50	0%	10^7	4
30	15	50	18%	$5{\cdot}10^5$	4
30	20	50	0%	$2{\cdot}10^6$	4
50	15	100	65%	$2{\cdot}10^5$	4
50	20	26	15%	$5{\cdot}10^5$	4
100	20	100	89%	$3{\cdot}10^5$	3

Table 3. Job shop problems.

15 jobs, 15 machines		Job shop	
Time seed	Machine seed	UB	LB
840612802	398197754	1247	977
1314640371	386720536	1263	942
1227221349	316176388	1233	921
342269428	1806358582	1181	911
1603221416	1501949241	1236	940
1357584978	1734077082	1247	889
44531661	1374316395	1235	935
302545136	2092186050	1221	963
1153780144	1393392374	1289	982
73896786	1544979948	1270	911

20 jobs, 15 machines		Job shop	
Time seed	Machine seed	UB	LB
533484900	317419073	1376	1139
1894307698	1474268163	1381	1251
874340513	509669280	1368	1178
1124986343	1209573668	1356	1130
1463788335	529048107	1375	1148
1056908795	25321885	1385	1181
195672285	1717580117	1495	1257
961965583	1353003786	1432	1153
1610169733	1734469503	1378	1202
532794656	998486810	1383	1186

20 jobs, 20 machines		Job shop	
Time seed	Machine seed	UB	LB
1035939303	773961798	1663	1217
5997802	1872541150	1626	1240
1357503601	722225039	1574	1185
806159563	1166962073	1665	1271
1902815253	1879990068	1598	1256
1503184031	1850351876	1679	1207
1032645967	99711329	1704	1331
229894219	1158117804	1633	1269
823349822	108033225	1635	1267
1297900341	489486403	1616	1212

10 jobs, 10 machines		Open shop	
Time seed	Machine seed	UB	LB
1344106948	1868311537	652	637
425990073	1111853152	596	588
666128954	1750328066	617	598
442723456	1369177184	581	577
2033800800	1344077538	657	640
964467313	1735817385	545	538
1004528509	967002400	623	616
1667495107	818777384	606	595
1806968543	1561913259	606	595
938376228	344628625	604	596

30 jobs, 15 machines		Job shop	
Time seed	Machine seed	UB	LB
98640593	1981283465	1770	1764
1839268120	248890888	1853	1774
573875290	2081512253	1864	1729
1670898570	788294565	1852	1828
1118914567	1074349202	2015	1729
178750207	294279708	1844	1777
1549372605	596993084	1823	1771
798174738	151685779	1714	1673
553410952	1329272528	1824	1641
1661531649	1173386294	1723	1602

30 jobs, 20 machines		Job shop	
Time seed	Machine seed	UB	LB
1841414609	1357882888	2064	1830
2116959593	1546338557	1983	1761
796392706	1230864158	1905	1694
532496463	254174057	2031	1787
2020525633	978943053	2038	1731
524444252	185526083	2057	1856
1569394691	487269855	1950	1690
1460267840	1631446539	2014	1744
198324822	1937476577	2013	1758
38071822	1541985579	1973	1674

50 jobs, 15 machines		Job shop	
Time seed	Machine seed	UB	LB
17271	718939	2791	2760
660481279	449650254	2800	2756
352229765	949737911	2768	2717
1197518780	166840558	2845	2797
1376020303	483922052	2757	2679
2106639239	955932362	2833	2781
1765352082	1209982549	2977	2943
1105092880	1349003108	2928	2885
907248070	919544535	2722	2655
2011630757	1845447001	2777	2723

50 jobs, 20 machines		Job shop	
Time seed	Machine seed	UB	LB
8493988	2738939	2961	2868
1991925010	709517751	3013	2848
342093237	786960785	2859	2755
1634043183	973178279	2790	2691
341706507	286513148	2813	2725
320167954	1411193018	2921	2845
1089696753	298068750	2907	2812
433032965	1589656152	2840	2764
615974477	331205412	3129	3063
236150141	592292984	3173	2995

100 jobs, 20 machines		Job shop	
Time seed	Machine seed	UB	LB
302034063	1203569070	5582	5464
1437643198	1692025209	5221	5181
1792475497	1039908559	5671	5552
1647273132	1012841433	5345	5339
696480901	1689682358	5573	5392
1785569423	1092647459	5403	5342
117806902	739059626	5450	5436
1639154709	1319962509	5459	5394
2007423389	749368241	5360	5358
682761130	262763021	5278	5183

Table 4. Instances of Job shop problems.

Open shop problems

We do not know instances of such problems in the literature. This is why we give problems of small size. These problems are obtained using exactly the same procedures as those used for the job shop problems, and the lower bound remains the same too.

Because one has to choose the order of the operations of a job, one can find very often an optimal schedule, except for the problems in which the number of jobs is about the number of machines. In this case, either an optimal solution is easily reached, or the problem is harder than a job shop problems of the same size.

For problems with $n \gg m$, we have observed empirically that the mean complexity of taboo search applied to open shop problems — $O(n^{2.37} \cdot m^{3.69})$ — is lower than the complexity of taboo search applied to job shop problems — $O(n^{2.50} \cdot m^{3.81})$.

In table 7, we describe extensively the first 4-job 4-machine problem we propose, i.e. the processing times d_{ij} of the operation *j* of job *i* and its associated machine M_{ij}. We give a good (optimal ?) schedule of this problem in the Gantt chart of Figure 1.

The time needed by taboo search to perform one iteration is about $23 \cdot 10^{-6} \cdot n \cdot m$ seconds.

Tables 5 and 6 are analogous to Tables 3 and 4.

Nb jobs	Nb machines	Nb instances	LB reached	Nb iterations	Nb resolutions
4	4	50'000	98.5%	10^5	5
5	5	45'000	99.7%	$5 \cdot 10^5$	4
7	7	1'000	94%	10^6	5
10	10	300	89%	$2 \cdot 10^6$	5
15	15	40	52%	$3 \cdot 10^5$	3
20	20	25	24%	$3 \cdot 10^6$	3

Table 5. Open shop problems.

4 jobs, 4 machines		Open shop	
Time seed	Machine seed	UB	LB
1166510396	164000672	193	186
1624514147	1076870026	236	229
1116611914	1729673136	271	262
410579806	1453014524	250	245
1036100146	375655500	295	287
597897640	322140729	189	185
1268670769	556009645	201	197
307928077	421384574	217	212
667545295	485515899	261	258
35780816	492238933	217	213

5 jobs, 5 machines		Open shop	
Time seed	Machine seed	UB	LB
527556884	1343124817	300	295
1046824493	1973406531	262	255
1165033492	86711717	328	321
476292817	24463110	310	306
1181363416	606981348	329	321
897739730	513119113	312	307
577107303	2046387124	305	298
1714191910	1928475945	300	292
1813128617	2091141708	353	349
808919936	183753764	326	321

7 jobs, 7 machines		Open shop	
Time seed	Machine seed	UB	LB
1840686215	1827454623	438	435
1026771938	1312166461	449	443
609471574	670843185	479	468
1022295947	398226875	467	463
1513073047	1250759651	419	416
1612211197	95606345	460	451
435024109	1118234860	435	422
1760865440	1099909092	426	424
122574075	10979313	460	458
248031774	1685251301	400	398

15 jobs, 15 machines		Open shop	
Time seed	Machine seed	UB	LB
1561423441	1787167667	956	937
204120997	213027331	957	918
801158374	1812110433	899	871
1502847623	1527847153	946	934
282791231	1855451778	992	946
1130361878	849417380	959	933
379464508	944419714	931	891
1760142791	1955448160	916	893
1993140927	179408412	951	899
1678386613	1567160817	935	902

20 jobs, 20 machines		Open shop	
Time seed	Machine seed	UB	LB
957638	9237185	1215	1155
162587311	1489531109	1332	1241
965299017	1054695706	1294	1257
1158457671	1499999517	1310	1248
1191143707	1530757746	1301	1256
1826671743	901609771	1252	1204
1591533998	1146547719	1352	1294
937297777	92726463	1269	1169
687896268	1731298717	1322	1289
687034842	684013066	1284	1241

Table 6. Instances of Open shop problems.

	Operation *j*			
Job *i*	1	2	3	4
1	54	34	61	2
2	9	15	89	70
3	38	19	28	87
4	95	34	7	29

a) Processing times (d_{ij})

	Operation *j*			
Job *i*	1	2	3	4
1	3	1	4	2
2	4	1	2	3
3	1	2	3	4
4	1	3	2	4

b) Machines (M_{ij})

Table 7. The first instance of the 4-job 4-machine open shop problem.

Concluding remarks

We hope that the problems that we propose will constitute a comparison base for the future resolution methods.

Everyone may send us his own results about these problems, specifying whether his solutions are proved optimal or not, in order to update the best solutions known.

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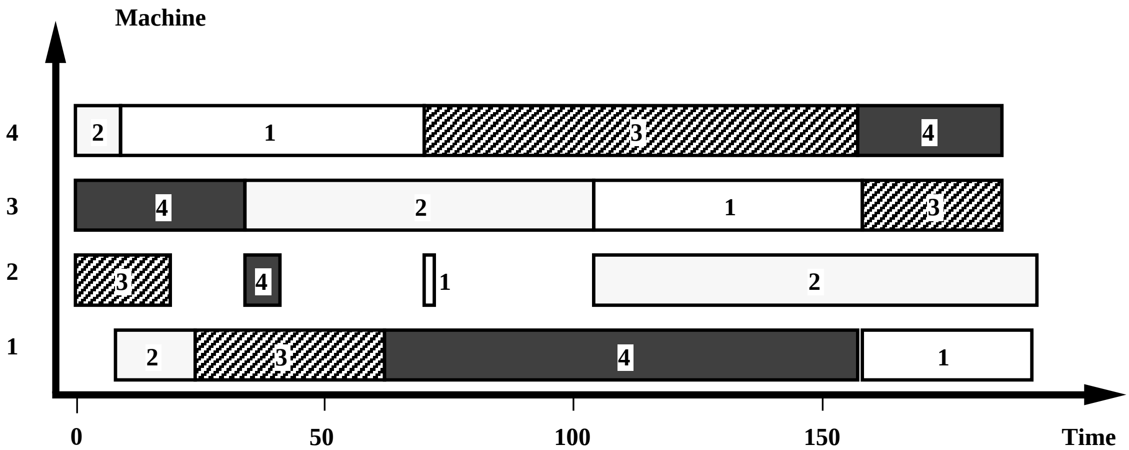

Figure 1. A good schedule for the problem of Table 7.