1 CP Optimizer model

$$
\begin{aligned}
\min \quad & w_1 \cdot \sum_{j \in J} \max(0, \mathrm{endOf}(a_j) - \overline{\omega}_j) && (1)\\
& + w_2 \cdot \sum_{j \in J} \sum_{e \in (E \setminus E_j^{Pr})} \mathrm{presenceOf}(a_{ej}^{Em}) && (2)\\
& + w_3 \cdot \sum_{p \in P} \sum_{e \in E} \Big(0 < \sum_{j \in J_p} \mathrm{presenceOf}(a_{ej}^{Em})\Big) && (3)\\
& + w_4 \cdot \sum_{p \in P} \mathrm{lengthOf}(b_p) && (4)
\end{aligned}
$$

$$
\begin{aligned}
& \mathrm{span}(b_p, [a_j]_{j \in J_p}) && \forall p \in P && (5)\\
& \mathrm{endBeforeStart}(a_k, a_j) && \forall k \in P_j && (6)\\
& \mathrm{alternative}(a_j, [a_{ij}]_{i \in M_j}, 1) && \forall j \in J && (7)\\
& \mathrm{alternative}(a_{ij}, [a_{eij}^{EmM}]_{e \in E_j}, r_{ij}^{Em}) && \forall j \in J, i \in M_j && (8)\\
& \mathrm{alternative}(a_{ej}^{Em}, [a_{eij}^{EmM}]_{i \in M_j}, 1) && \forall j \in J, e \in E_j && (9)\\
& \mathrm{noOverlap}([a_{eij}^{EmM}]_{j \in J, i \in M_j : e \in E_j}) && \forall e \in E && (10)\\
& \mathrm{noOverlap}([a_{ej}^{Em}]_{j \in J : e \in E_j}) && \forall e \in E && (11)\\
& \mathrm{presenceOf}(a_{ej}^{Em}) == \mathrm{presenceOf}(a_{ek}^{Em}) && \forall j \in J, k \in L_i, e \in E && (12)\\
& \mathrm{alternative}(a_j, [a_{bj}^{Wb}]_{b \in B_j}, r_j^{Wb}) && \forall j \in J && (13)\\
& \mathrm{noOverlap}([a_{bj}^{Wb}]_{j \in J : b \in B_j}) && \forall b \in B && (14)\\
& \mathrm{alternative}(a_j, [a_{gdj}^{Eq}]_{d \in G_{gj}}, r_{gj}^{Eq}) && \forall j \in J, g \in G^* && (15)\\
& \mathrm{noOverlap}([a_{gdj}^{Eq}]_{j \in J : d \in G_{gj}}) && \forall g \in G^*, d \in G_g && (16)\\
& \text{interval } b_p && \forall p \in P && (17)\\
& \text{interval } a_j \subset [\alpha_j, \omega_j) && \forall j \in J && (18)\\
& \text{interval } a_{ij} \text{ optional size } d_{ij} && \forall j \in J, i \in M_j && (19)\\
& \text{interval } a_{eij}^{EmM} \text{ optional} && \forall j \in J, i \in M_j, e \in E_j && (20)\\
& \text{interval } a_{ej}^{Em} \text{ optional} && \forall j \in J, e \in E_j && (21)\\
& \text{interval } a_{bj}^{Wb} \text{ optional} && \forall j \in J, b \in B_j && (22)\\
& \text{interval } a_{gdj}^{Eq} \text{ optional} && \forall j \in J, g \in G^*, d \in G_{gj} && (23)
\end{aligned}
$$

Decision variables are defined in equations 17-23:

- In Eq. 17, b_p represent a sub-project p, it starts at the start time of the project and ends at its end time.
- In Eq. 18, a_j represents a job j. A project b_p spans all its jobs (constraint in Eq. 5).

- Eq. 19 defines a set of optional interval variables a_{ij} for each job j and each possible model i. Interval variable a_{ij} will be present if and only if mode i is selected for job j. The duration of jobs is mode-dependent that is why each optional interval variable a_{ij} is defined its own size d_{ij}. Constraint 7 specifies that among the different modes for job j one and only one will be selected.

- For a given job j executed in a given mode i, several employee allocations are possible. Eq. 20 defines one optional interval a_{eij}^{EmM} for each possible allocated employee perform job j in mode i. Constraint 8 specifies that exactly r_{ij}^{Em} employees will have to be selected (intervals will be present) for each interval a_{ij}. The r_{ij}^{Em} selected intervals will start and end at the same value as the master interval variable a_{ij}.

- Eq. 21 defines a set of optional interval variables a_{ej}^{Em} for each job j and each possible employee e. Constraint 9 states that if an employee is selected for a given job j then it must be selected by one of the possible modes. This type of multi-level alternative constraints are common in CP Optimizer models.

- Intervals defined in Eq. 22 represent the possible allocation of a workbench to a job j. The selection of r_j^{Wb} for a job j is formulated by constraint 13.

- Similarly, for each group of equipments, intervals defined in Eq. 23 represent the possible allocation of an equipment from a given group to a job j. The selection of r_{gj}^{Eq} for a job j is formulated by constraint 15.

The unary resource constraints (employees, workbenches, equipments) are formulated as no-overlap constraints in Eq. 11,14,16. Equation 10 is a redundant constraint.

Precedence constraints between jobs are modeled as endBeforeStart constraints in Eq. 6.

Linked jobs are formulated in Eq. 12. In practice, this constraint can be posted only on the subset of common employees between job j and job k. A constraint $!\,\mathrm{presenceOf}(a_{ej}^{Em})$ or $!\,\mathrm{presenceOf}(a_{ek}^{Em})$ should be posted on the employees not in $E_j \cap E_k$.

The objective function is similar to the one in the paper, except that the project span can directly be formulated as the length of the project interval variables b_p.

Note that the same redundant constraints as in the paper can be formulated using cumulative functions in the CP Optimizer model. For instance a constraint similar to (20) would be:

$$\sum_{j \in J, i \in M_j} \mathrm{pulse}(a_{ij}, r_{ij}^{Em}) \leq |E| \tag{1}$$