A Bilevel Mixed Integer Linear Programming Model for Valves Location in Water Distribution Systems

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Abstract. The positioning of valves on the pipes of a Water Distribution System (WDS) is a core decision in the design of the isolation system of a WDS. When closed, valves permit to isolate a small portion of the network, so called a sector, which can be de-watered for maintenance purposes at the cost of a supply disruption. However, valves have a cost so their number is limited, and their position must be chosen carefully in order to minimize the worst-case supply disruption which may occur during pipe maintenance. Supply disruption is usually measured as the undelivered user demand. When a sector is isolated by closing its boundary valves, other portions of the network may become disconnected from the reservoirs as a secondary effect, and experience supply disruption as well. This induced isolation must be taken into account when computing the undelivered demand induced by a sector isolation. While sector topology can be described in terms of graph partitioning, accounting for induced undelivered demand requires network flow modeling. The aim of the problem is to locate a given number of valves at the extremes of the network pipes so that the maximum supply disruption is minimized. We present a Bilevel Mixed Integer Linear Programming (MILP) model for this problem and show how to reduce it to a single level MILP by exploiting duality. Computational results on a real case study are presented, showing the effectiveness of the approach.

Keywords: Isolation Valves Positioning, Bilevel Programming, Hydroinformatics

1 Introduction

In this Section we introduce a real problem in hydraulic engineering concerning the location of the isolation valves of a Water Distribution System, and reformulate it as a graph based optimization problem. A mathematical model is presented in Section 2, computational results are presented in Section 3 where conclusions are drawn.

1.1 Valves closure and sector isolation

Water Distribution Systems (WDSs) are complex systems whose mission is to supply water to the communities living in their service area. A WDS is made of several components, the main ones being: a set of reservoirs feeding the WDS, a set of pipes delivering water to the system users, a user demand for each pipe, describing the average water consumption by the users served by that pipe (liters per second), a set of junctions each describing the connection of two or more pipes to each other.

Users are connected to their closest pipe by way of smaller pipes through which water is supplied. To this purpose, we can imagine users as being evenly distributed along the pipe. In turn, pipes receive water at an adequate pressure from the reservoirs to which they are connected in the hydraulic network. Usually, the topological layout of hydraulic networks contains a few loops that increase network reliability. Thus, a pipe can be connected to a reservoir by several different paths. Given a pipe, if each connection from the pipe to each reservoir is interrupted, water pressure falls, the pipe no longer supplies its users and it is said to be *isolated*. Failure of ageing pipes frequently occurs. In such a case, the leaking pipe is isolated on purpose, to be dewatered and fixed. Isolation is achieved by closing some of the isolation valves purposely located on the network, in such a way that the failed pipe gets disconnected from the reservoirs. In an ideal situation, each pipe would have one such valve positioned at each of its two extremes, so that only that pipe could be disconnected in case of maintenance by closing just its two valves, but it would require twice as many valves as the network pipes. However, the number of valves is limited due to cost, and their location poses a challenge, as described hereafter.
First, valves must be properly located at pipe extremes in such a way that any pipe can be isolated by closing a proper set of valves. When all valves are closed, the network is subdivided into a set of subnet, called *sectors*; also, the valves that delimit a sector are called the *boundary valves* of that sector. Said in another way, a sector is a set of pipes which stay connected themselves once all valves are closed. It follows that pipes within the same sector share the same status, either isolated or connected to a reservoir, depending on which valves are closed. When a sector is isolated, all its users experience supply disruption.
Second, the WDS engineers who design the network aim to reduce and equally distribute the service disruption among users in case of maintenance operations.

So far, if we suppose that each pipe is equally likely to fail and require maintenance, this target would be achieved by any valves location yielding sectors whose user demand is approximately the same. However, a secondary effect of sector isolation must be accounted for, i.e., *unintended isolation.* A pipe for which all connections to the reservoirs go through the isolated sector will be isolated as well when that sector is closed. Therefore, the supply disruption associated to a sector cannot take into account only the user demand of the sector itself, but must consider the demand of unintentionally isolated pipes as well. We illustrate these concepts on the toy network depicted in Fig. 1. The hydraulic network has a single reservoir, 6 junctions and 7 pipes with positive

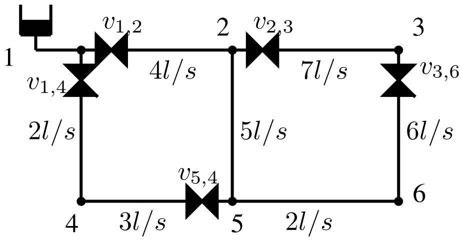

Fig. 1. A simple hydraulic network and its isolation system

demand, plus a 0-demand pipe which connects the reservoir to the rest of the network. 5 valves are positioned at pipe extremes as follows: near junction 1 on pipe $(1, 2)$, near junction 1 on pipe $(1, 4)$, near junction 2 on pipe $(2, 3)$, near junction 3 on pipe $(3, 6)$, and finally near junction 5 on pipe $(4, 5)$. Three sectors are induced by those valves, namely S_1 made of pipes $(1, 4)$, $(4, 5)$ with demand $5l/s$; S_2 made of pipes $(1, 2)$, $(2, 5)$, $(5, 6)$, $(3, 6)$ with demand $17l/s$; S_3 made of pipe $(2, 3)$ with demand $7l/s$. Let $v_{i,j}$ denote a valve located on pipe (i, j) near i, and let $v_{j,i}$ be the one close to j. If pipe $(2, 3)$ needs repairing, valves $v_{1,2}$ and $v_{3,6}$ can be closed, with a supply disruption of $7l/s$. If pipe $(5, 6)$ leaks, the boundary valves of S_2 will be closed, namely $v_{1,2}$, $v_{5,4}$, $v_{3,6}$, and $v_{2,3}$, but the supply disruption will be $24 = 17 + 7$, greater than its sector demand, accounting for the unintended isolation of S_3.

The Isolation System Design (ISD) of WDSs consists of locating a limited number of valves at pipe extremes so that any pipe can be isolated. What an optimal placement is may depend on several criteria that give rise to different objective functions; in particular, the *Bottleneck Isolation Valves Location Problem* (BIVLP) minimizes the maximum undelivered demand. The two main issues related to the ISD problem addressed in the hydraulic engineering literature, are: the identification of the segments and unintended isolation due to the closure of some isolation valves, and the optimal location of isolation valves. Regarding the first topic, among others, [12, 13] exploit a dual representation of the network, with segments treated as nodes and valves as links; [5, 9] exploit the topological incidence matrices to identify the segments. Regarding the second topic, both [9] and [5] tackle the problem by bi-objective genetic algorithms, seeking a compromise between cost and solution quality. In particular, the former minimizes the number of valves and the maximum supply disruption. The latter minimizes the cost of the installed valves related to pipe diameters and the average supply disruption weighted by the probability of pipe failure. Both provide an approximation of the Pareto frontier and no bounds to evaluate the quality of the heuristic solutions.

The only two exact approaches for the BIVLP we are aware of are provided in [2] and [7], which apply two different tools of the Logic Programming field to minimize the maximum undelivered demand given a fixed number of valves. The former is based on Constraint Logic Programming on Finite Domain [11]

(CLP(FD)), and models the problem as a two-players game and three moves: player 1 locates the valves, player 2 chooses a pipe to break, and player 1 closes a set of valves. The latter is based on the Answer Set Programming [8] (ASP) paradigm and uses the concept of "*extended sector*", that is the union of a sector with its (if existing) unintended isolations. Both can be used to compute the Pareto frontier of the problem tackled by [9] by solving a sequence of instances with an increasing number of valves. Different models for the BIVLP have been proposed in each paper, with different pros and cons regarding prototyping and computing times. At present the best computational performance is achieved by the CLP(FD) model [2], which implements a redundant valves elimination technique and bounding procedures, and which is our benchmark.

In the following we reformulate the BIVLP in the framework of graph theory, and set the basis for the mathematical optimization model provided in Section 2.

1.2 Hydraulic sectors and graph partitioning

The Graph Partitioning problem (GPP) is one of the most studied problems in combinatorial optimization, and admits several variants. Recall that a partition of a set is a collection of non-empty disjoint subsets whose union returns the set. Generally speaking, GPP consists of partitioning the vertices of a graph into a set of connected components by removing a minimum (weight) set of edges, according to some criteria. For example, the number of components of the partition is fixed, and the number of nodes in each component is bounded from above. The set of removed edges is usually referred to as a *multicut*. Literature references are many, among which [3] investigates the polytopes associated to the integer programming formulations of the major variants, and [6] studies the convex hull of incidence vectors of feasible multicuts for the capacitated GPP.

The several analogies between the hydraulic network sectorisation and graph partitioning, as recently exploited in the hydraulic engineering literature [14], seem at first to provide an ideal framework for modelling BIVLP. Recall that each pipe must belong to one sector in order to be isolated if required, but only one sector, due to the minimality of sector definition. Therefore, sectors induce a partition on the positive demand pipes, whose associated multicut models the location of the isolation valves at pipe extremes. This simple correspondence, though, does not allow us to address the issue of unintended isolation. To meet this requirement we need to introduce an extended graph that represents the topology of the network, on which flow variables will model water flows, thus capturing unintended supply disruption. We first provide a graph representation of the hydraulic network supporting graph partition, and in the next Section we show how to extend such graph to handle unintended isolation.

First, we suppose to concentrate the user demand of a pipe (i, j) at a single point denoted as ϵ_{ij}, located in the middle of the pipe. Denote by $\delta_{\epsilon_{ij}}$ such demand. Then, we shrink all the reservoirs into a single one, to which we associate node σ. Now we can introduce the undirected graph $G = (V, E)$ defined as follows.
The vertex set V is made of the union of: $\Sigma = \{\sigma\}$, where σ is the super source modelling the set of reservoirs; Ψ, being the union over all pipes of the demand

points ϵ_{ij}; Γ, modelling the junctions of the hydraulic network.
The edge set E is made of: T, the set of the few structural edges corresponding to 0-demand pipes which connect σ to a vertex in Γ; we denote as $\Gamma(\Sigma) \subset \Gamma$ the subset of the junction nodes adjacent to a reservoir. F, made of a pair of edges (i, ϵ_{ij}) and (j, ϵ_{ij}) for each pipe (i, j) in the hydraulic network. Fig. 2 shows how the two adjacent pipes of the hydraulic network are modelled in graph G.

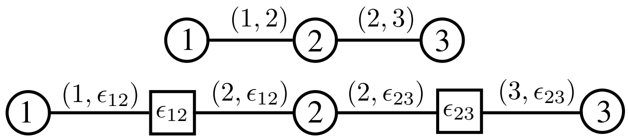

Fig. 2. Two adjacent pipes of the hydraulic network and their representation in G.

Each edge in F may host a valve, while edges in T do not. In real life WDSs, actually, a special valve is always present on each pipe in T in order to isolate the reservoir from the network, if the reservoir needs maintenance. However, such a valve would never be closed for pipe maintenance purposes, so we disregard it here. Therefore, we assume that the edges in T bear no valves, and that these edges, the reservoirs and the nodes in $\Gamma(\Sigma)$ do not belong to any sector. Furthermore, to keep notation simple, we suppose that all pipes other than those incident on the reservoirs have positive demand.

Let h_{max} be the number of available valves, and s_{max} the maximum number of sectors admitted. Denote by Δ_s, for $s \in S = \{1, .., s_{max}\}$, the undelivered demand associated to sector s, and it is given by the sum of $\delta_{\epsilon_{ij}}$ for each pipe (i, j) which gets isolated when the boundary valves of sector s are closed. Note that isolating a sector s on the hydraulic network corresponds to the removal of all edges $(i, \epsilon_{ij}) \in F$ on graph G such that a boundary valve of s is positioned on pipe (i, j) near junction i. We search for the location of at most h_{max} valves on as many edges of F yielding at most s_{max} sectors such that $\max_{s \in S}\{\Delta_s\}$ is minimum. In Section 2 we provide a mathematical model for this problem.

2 Mathematical models for the BIVLP

In 2.1 we present a graph partitioning model on graph G, how to extend G to model unintended undelivered demand, and a bilevel MILP model for BIVLP. In 2.2 it is presented how to reduce the bilevel model to a single level MILP.

2.1 A Bilevel model for BIVLP

Let us introduce the following variables.
$\tau^s_{ij} \in \{0, 1\}\ \forall(u, v) \in F : u = i, v = \epsilon_{ij},\ \forall s \in S$, is a binary variable equal to 1 if a boundary valve for s is located on pipe (i, j) near i in the WDN (edge

(i, ϵ_{ij}) in G), and 0 otherwise. Likewise, $\tau_{ji}^s = 1$ if the valve is near j, that is, on edge (j, ϵ_{ij}) in G.

$z_i^s \in \{0, 1\}$ $\forall i \in \Psi \cup \Gamma \setminus \Gamma(\Sigma)$, $\forall s \in S$, is a binary variable equal to 1 if node i belongs to sector s and 0 otherwise.

Recall that variable z_i^s is not defined for $i \in \Gamma(\Sigma)$ or $i = \sigma$. If $\tau_{ij}^s = 1$ then there must be another sector s' such that $\tau_{ij}^{s'} = 1$, unless i is adjacent to a reservoir. Indeed, one valve separates (at most) two sectors from each other.

The following constraints partition the vertices of G into at most s_{max} sectors with limited user demand, by cutting at most h_{max} edges in F.

$$\sum_{s \in S} z_i^s = 1, \qquad \forall i \in \Psi \cup \Gamma \setminus \Gamma(\Sigma). \tag{1a}$$

$$\sum_{s \in S} \sum_{\epsilon_{ij} \in \Psi} (\tau_{ij}^s + \tau_{ji}^s) \leq h_{max} \tag{1b}$$

$$\sum_{\epsilon_{ij} \in \Psi} z_{\epsilon_{ij}}^s \delta_{\epsilon_{ij}} \leq \delta_{max}, \qquad \forall s \in S, \tag{1c}$$

These constraints impose the following. Each node in $\Psi \cup \Gamma \setminus \Gamma(\Sigma)$ belongs to one and only one sector (1a). At most h_{max} valves are available (1b). The sum of user demands of the edges within the same sector is bounded from above; constraint (1c) excludes very unbalanced partitions with sectors with large demand. However, the threshold δ_{max} as well as the parameters h_{max} and s_{max} must be carefully set so that a feasible solution exists and no optimal solution is cut off. Furthermore, note that it is not required to use all the available valves since the optimal solution value does not necessarily improve as the number of boundary valves increases, and we want to avoid solutions with useless valves, i.e., valves positioned on an edge whose vertices belong to the same sector. Actually, in (2a–2b) we exploit this fact and take for granted that each valve is a boundary valve, that is, if a valve is positioned on an edge then the vertices of that edge belong to different sectors.

The following constraint states the relation between a boundary valve and the sector of the vertices of the edge where the valve is located. Recall that τ_{ij}^s refers to a valve positioned in between vertex i and vertex ϵ_{ij}. We use the symbol $\oplus$ to denote the XOR logical operator and then provide the system of integer linear inequalities that define the operator.

$$\tau_{i,j}^s = z_i^s \oplus z_{\epsilon_{ij}}^s, \qquad \forall (i, \epsilon_{ij}) \in F,\ i, j \notin \Gamma(\Sigma)\ \forall s \in S \tag{2a}$$

$$\tau_{ij}^s = z_j^s, \qquad \forall (i, \epsilon_{ij}) \in F : i \in \Gamma(\Sigma) \tag{2b}$$

Constraints (2a – 2b) state that a boundary valve of sector s is positioned on an edge if and only if exactly one of the two vertices belongs to s, unless the vertex to which the valve is close is adjacent to a reservoir. (2a) can be formulated as the following set of linear inequalities, for each edge $(i, \epsilon_{ij}) \in F$ and for each

$s \in S$.

$$\tau^s_{ij} \leq z^s_i + z^s_{\epsilon_{ij}} \tag{3a}$$

$$\tau^s_{ij} \leq 2 - (z^s_i + z^s_{\epsilon_{ij}}) \tag{3b}$$

$$\tau^s_{ij} \geq z^s_i - z^s_{\epsilon_{ij}} \tag{3c}$$

$$\tau^s_{ij} \geq z^s_{\epsilon_{ij}} - z^s_i \tag{3d}$$

The user demand that is satisfied when sector s is isolated clearly depends on where the sector's boundary valves have been located, according to the current configuration of the isolation system which is expressed in terms of the τ^s variables. Let us denote the satisfied user demand as $DD^s(\tau^s)$ and let $\Upsilon = \sum_{(i,j)} \epsilon_{ij}$ be the total user demand. Therefore, BIVLP can be stated as $min_\tau \Delta$ *s.t.* $\Delta \geq \Upsilon - DD^s(\tau^s)$, $\forall s \in S$ and τ satisfies (1a–1c), (2b), (3a–3d). In the following, we provide a mathematical description of $DD^s(\tau^s)$ as the solution of an optimization problem.

Now we are ready to extend graph G in order to support the introduction of the flow variables required to model water flow on the network corresponding to the satisfied user demand in case a given sector is isolated. Since sectors are isolated one at a time, we must consider separately each such scenario. In order to compute the undelivered demand for a sector, an associated Maximum Flow Problem (MFP) on a graph whose topology depends on τ must be solved.

We start by adding to V a special node P representing a sink collecting all user demand that is satisfied; accordingly, E is extended to include a new set of edges, say D, that carry user demand from each demand-vertex ϵ_{ij} to the sink P. Then, let us introduce s_{max} families of multicommodity flow variables, one for each sector. They are used to represent water flows in the hydraulic network when that given sector is isolated.

First, we define the variables modelling the flow on network pipes. For each $s \in S$ and $(u, v) \in F$, a pair of *non negative* flow variables are introduced, namely, x^s_{uv} and x^s_{vu}. Recall that $v = \epsilon_{ij}$ for some pipe (i, j), and $u \in \{i, j\}$. Therefore, each pipe (i, j) yields four flow variables for each sector, namely, $x^s_{i,\epsilon_{ij}}$, $x^s_{\epsilon_{ij},i}$, $x^s_{j,\epsilon_{ij}}$, and $x^s_{\epsilon_{ij},j}$. All such variables are bounded by the sum of users demand Υ, unless a boundary valve for s is located on the edge, say near i. In such a case, $\tau^s_{ij} = 1$ and $x^s_{i,\epsilon_{ij}} = x^s_{\epsilon_{ij},i} = 0$.
Second, we introduce flow variables on the demand edges in D, connecting each demand vertex ϵ_{ij} to the sink P. For each $s \in S$ and $\epsilon_{ij} \in \Psi$, let $x^s_{\epsilon_{ij},P}$ be such variable. This variable, for any s, is bounded form above by the actual user demand of pipe (i, j), that is $\delta_{\epsilon_{ij}}$ previously introduced.
Finally, for each $s \in S$ and for each edge in T connecting the reservoir $\sigma \in \Sigma$ to a junction $\gamma \in \Gamma$, a non negative flow variable $x^s_{\sigma,\gamma}$ is introduced, with no upper bound. In Fig. 3 the extended graph is depicted. In order to represent the water flow when a given sector s is isolated, we must solve a MFP from the reservoir σ to the sink P with respect to the flow variables indexed by s. For each sector s, the mathematical model of the MFP is defined by balance constraints at the nodes, capacity constraints on the flow variables, and the

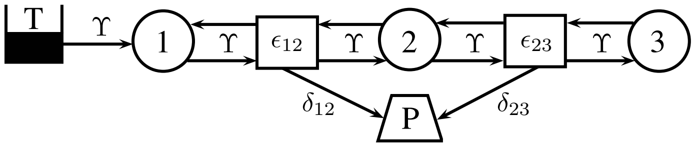

Fig. 3. The extended graph. The arrows show in which direction flow can traverse an edge.

objective function is the maximization of the flow entering P. The optimal value provides the user demand that is satisfied when s is isolated. Capacity goes down to 0 on the arcs where boundary valves of sector s have been located, so the optimal solution value $DD^s(\tau^s)$ is a function of τ^s. Constraints (4–5i) model the MFP for a given sector s. A fake arc going from the sink P to the source σ with non negative flow variable $x^s_{P\sigma}$ is introduced so that the problem can be stated as a circulation problem. Arc (P, σ) is the only arc outgoing from P and entering σ, therefore the objective function can then be stated as maximizing $x^s_{P,\sigma}$. Zero balance constraints at junction nodes are given in (5a), at demand nodes in (5b), at the sink in (5c), and at the reservoir in (5d). Capacity constraints for the flow variables on the edges in F, which depends on valves location, are given in (5e-5f). Similarly, capacity constraints for the flow variables defined on the demand edges in D are given in (5g).

$$DD^s(\tau^s) \;= max \quad x^s_{P,\sigma} \quad s.t. \tag{4}$$

$$x^s_{\sigma,i} + \sum_{j:\epsilon_{ij}\in\Psi} (x^s_{\epsilon_{ij},i} - x^s_{i,\epsilon_{ij}}) = 0 \qquad \forall i \in \Gamma, \tag{5a}$$

$$x^s_{i,\epsilon_{ij}} + x^s_{j,\epsilon_{ij}} - x^s_{\epsilon_{ij},i} - x^s_{\epsilon_{ij},j} - x^s_{\epsilon_{ij},P} = 0 \qquad \forall \epsilon_{ij} \in \Psi, \tag{5b}$$

$$\sum_{\epsilon_{ij}\in\Psi} x^s_{\epsilon_{ij},P} - x^s_{P,\sigma} = 0, \tag{5c}$$

$$x^s_{P,\sigma} - \sum_{i\in\Gamma(\Sigma)} x^s_{\sigma,i} = 0, \tag{5d}$$

$$x^s_{i,\epsilon_{ij}} \leq \Upsilon(1 - \tau^s_{ij}) \qquad \forall (i, \epsilon_{ij}) \in F, \tag{5e}$$

$$x^s_{\epsilon_{ij},i} \leq \Upsilon(1 - \tau^s_{ij}) \qquad \forall (i, \epsilon_{ij}) \in F, \tag{5f}$$

$$x^s_{\epsilon_{ij},P} \leq \delta_{\epsilon_{ij}} \qquad \forall \epsilon_{ij} \in \Psi, \tag{5g}$$

$$x^s_{\sigma,i} \leq \Upsilon \qquad \forall i \in \Gamma(\Sigma), \tag{5h}$$

$$x^s_{i,j} \geq 0, \qquad \forall (i,j) \in E. \tag{5i}$$

Finally, BIVLP can be stated as a bilevel optimization problem.
$min_\tau \Delta$ $s.\ t.$ (1a–1c), (2b), (3a–3d), $\Delta \geq \Upsilon - DD^s(\tau^s)\ \forall s \in S$, $DD^s(\tau^s) =$ (4-5i).
Regarding the ISD problem, the leader select a feasible configuration of the isolation system by setting τ, while the follower determines the quality of a such configuration by solving a maximum flow problem for each sector.

2.2 A one level MILP for BIVLP

Bilevel optimization [4] provides the framework for modelling optimization problems where two decision makers with conflicting objective functions are involved in a hierarchical relationship. The leader takes its decisions aware of the fact that their value depends on how the follower reacts to such decisions. Here, the leader sets the topology of the isolation system, locating the available valves on the pipes. The follower, sector by sector, maximizes the flow from σ to P on a graph whose topology depends on the boundary valves of the sector. Bilevel optimization problems can be tackled by imposing inner problem optimality by adding its optimality conditions, usually expressed as non linear constraints, to the inner problem feasibility constraints, and reformulating the whole problem as a single level optimization problem. When the inner problem is a linear programming problem, duality can be exploited to state optimality by way of linear constraints (see [1]). In our case, given the valves location, each subproblem is a MFP whose dual is the minimum capacity cut provided in (6–7e).

For each $s \in S$ there is a dual variable for each constraint of the MFP model.

- $\omega^s_{i,\epsilon_{ij}}$, $\omega^s_{\epsilon_{ij},i} \geq 0$, $\forall (i, \epsilon_{ij}) \in F$ are the non negative variables associated to capacity constraints (5e) and (5f).
- $\omega^s_{\epsilon_{ij},P} \geq 0$, $\forall \epsilon_{ij} \in \Psi$ are the non negative variables associated to capacity constraints (5g).
- $\omega^s_{\sigma,i} \geq 0$, $\forall i \in \Gamma(\Sigma)$ are the non negative variables associated to capacity constraints (5h).
- π^s_i $\forall i \in \Gamma \cup \Psi$ are the unconstrained node potential variables associated to flow balance constraints (5a) and (5b).
- π^s_P and π^s_σ are the potential variables associated to the sink and the source flow balance constraints (5c) and (5d).

For each sector, an equivalent reformulation of the dual problem of (4-5i) is stated below.

$$min \sum_{(i,\epsilon_{ij})\in F} \Upsilon(1-\tau^s_{ij})(\omega^s_{i,\epsilon_{ij}} + \omega^s_{\epsilon_{ij},i}) + \sum_{(\epsilon_{ij},P)\in D} (\delta_{\epsilon_{ij}} \omega^s_{\epsilon_{ij},P}) \tag{6}$$

$$\pi^s_P - \pi^s_\sigma \geq 1, \tag{7a}$$

$$\pi^s_i - \pi^s_{\epsilon_{ij}} + \omega^s_{i,\epsilon_{ij}} \geq 0 \qquad \forall (i, \epsilon_{ij}) \in F, \tag{7b}$$

$$\pi^s_{\epsilon_{ij}} - \pi^s_i + \omega^s_{\epsilon_{ij},i} \geq 0 \qquad \forall (i, \epsilon_{ij}) \in F, \tag{7c}$$

$$\pi^s_{\epsilon_{ij}} - \pi^s_P + \omega^s_{\epsilon_{ij},P} \geq 0 \qquad \forall (\epsilon_{ij}, P) \in D, \tag{7d}$$

$$\pi^s_\sigma - \pi^s_i + \omega^s_{\sigma,i} \geq 0 \qquad \forall (\sigma, i) \in T. \tag{7e}$$

The dual objective function coefficients of $\omega^s_{i,\epsilon_{ij}}$ and $\omega^s_{\epsilon_{ij},i}$ depend on $\tau^s_{i,\epsilon_{ij}}$. To linearize this non linear expression, for each sector we introduce two non negative variables $\mu^s_{i,\epsilon_{ij}}$ and $\mu^s_{\epsilon_{ij},i}$ for each edge $(i, \epsilon_{ij}) \in F$ and constraints (8a-8b), which realize the equivalence $\mu^s_{i,\epsilon_{ij}} = \omega^s_{i,\epsilon_{ij}} \tau^s_{ij}$.

$$\mu^s_{i,\epsilon_{ij}} \leq \omega^s_{i,\epsilon_{ij}} \qquad \forall (i, \epsilon_{ij}) \in F,\ \forall s \in S, \tag{8a}$$

$$\mu^s_{i,\epsilon_{ij}} \leq \Upsilon \tau^s_{ij} \qquad \forall (i, \epsilon_{ij}) \in F,\ \forall s \in S. \tag{8b}$$

$\mu^s_{i,\epsilon_{ij}}$ is no greater than $\omega^s_{i,\epsilon_{ij}}$ and it is forced to 0 when τ^s_{ij} is 0. Since its coefficient in the objective function to be minimized is $-\Upsilon < 0$, then $\mu^s_{i,\epsilon_{ij}}$ will be equal to $\omega^s_{i,\epsilon_{ij}}$ if $\tau^s_{ij} = 1$.

Now we can replace the objective function (4) of the inner problem for each $s \in S$ by constraint (9) that imposes the well known *max flow – min cut* optimality condition.

$$x^s_{P,\sigma} = \sum_{(i,\epsilon_{ij})\in F} \Big(\Upsilon(\omega^s_{i,\epsilon_{ij}} + \omega^s_{\epsilon_{ij},i}) - \Upsilon(\mu^s_{i,\epsilon_{ij}} + \mu^s_{\epsilon_{ij},i})\Big) + \sum_{(\epsilon_{ij},P)\in D} (\delta_{\epsilon_{ij}} \omega^s_{\epsilon_{ij},P}). \tag{9}$$

Finally, a single level MILP for BIVLP is given by:
$min_\tau \Delta$ *s. t.* (1a–1c), (2b), (3a–3d), and $\Delta \geq \Upsilon - x^s_{P,\sigma}$, (9), (8a-8b), (5a-5i), (7a–7e) $\forall s \in S$
which can be fed into any MILP solver and solved, as described in the next Section.

3 Computational results and conclusions

We developed the MILP model of Section 2 for a real life WDS, the Apulian network serving Puglia, a region in the south of Italy, which was the case study also in [14], [2], and in [7]. A scheme is depicted in Fig. 4: it is made of one reservoir, 23 junctions, and 33 pipes.

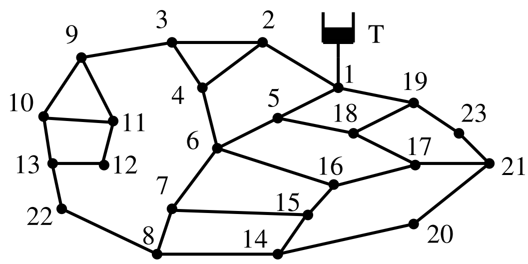

Fig. 4. The Apulian network

Running time is heavily affected by the maximum number of sectors. A rough estimation is the following: $s_{max} = cc + h_{max} - \sum_{\gamma\in\Gamma(\Sigma)}(deg(\gamma) - 1)$, where $\Gamma(\Sigma)$ is the set of junctions γ adjacent to a reservoir $\sigma \in \Sigma$ (in Figure 4, $\Gamma(\Sigma) = \{1\}$); $deg(\gamma)$ is the degree of vertex γ; cc is the number of connected components obtained after edges $(\gamma, \epsilon_{\gamma j})$ have been removed, for each junction $\gamma \in \Gamma(\Sigma)$, except for the component made of σ, γ, and pipe (σ, γ). Any pipe with positive demand and incident on a junction in $\Gamma(\Sigma)$ must have a valve located near the junction to be isolated. Once such valves have been placed, yielding cc sectors, at most one more sector may result from the placement of one additional valve.

This bound can be tight for very sparse networks, such as a tree, but real WDSs have several loops. A tighter estimate for s_{max} and a downsizing of the MILP model can be achieved by more sophisticated procedures exploiting the network topology of the instance, and are currently under investigation.

As in [2], h_{max} varies in $[5, \cdots, 14]$ and the time limit for each instance is $8000''$. In the Apulian network $cc = 1$, $\Gamma(\Sigma) = \{1\}$ and $deg(1) = 4$, then we have that $s_{max} = 1 + h_{max} - (4 - 1) = h_{max} - 2$, hence s_{max} varies in $[3, \cdots, 12]$.

The MILP solver is Gurobi Optimizer 4.6 [10], experiments were run on a *Intel dual core* architecture based on P8400 CPUs, 2.26 GHz, 4GB of RAM.

Figure 5 for each value of h_{max} reports the running time for the optimally solved instances, and the time the best feasible solution was found for the others. For such instances, labels report the gap between the best feasible MILP solution (z^{MILP}) and the optimal one provided in [2] (z^{CLP}). Note that for $h_{max} = 8, 9, 14$ we find the optimal solution within the time limit but we can not certify it.

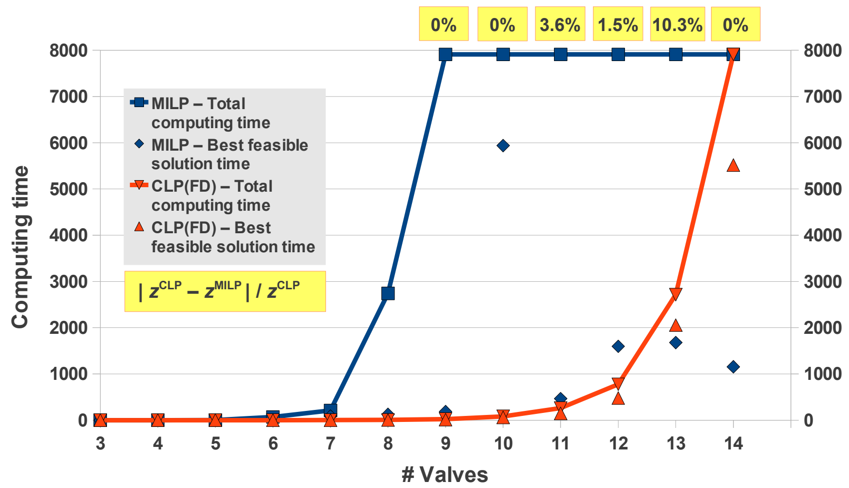

Fig. 5. Computation time and solution gap values with different number of valves

In conclusion, we provided a third exact solution approach to the BIVLP, based on MILP. A first implementation outperforms the ASP based approach and can solve to optimality several instances of those solved by the CLP(FD) based approach in the same time limit, while provides good quality solutions to the others. Regarding the Pareto front generation, some iterations could be saved w.r.t. [2] where the number of used valves is fixed. Current research is devoted to improve the computational performance by: tuning the solver parameters which have been used at their default value; strengthening the GPP constraints taking advantage of the literature studies on its polyhedral structure [3, 6]; reducing symmetries in the model; tightening the estimate on s_{max} exploiting the WDS's graph topology; exploiting the knowledge of the exact solution with one more (one less) available valve in the computation of the Pareto optimal frontier. Furthermore, we aim to tackle different objective functions building on the MILP model here presented to formulate the feasible region of the ISD problem. For example, we plan to handle different probabilities of pipe failures and pipe dependent valve costs. Finally, hybrid approaches integrating MILP and CLP will be explored.

References

1. M. Bruglieri, P. Cappanera, A. Colorni, and M. Nonato. Modeling the gateway location problem for multicommodity flow rerouting. In J. Pahl et al., editor, *Network Optimization*, volume 6701 of *Lecture Notes in Computer Science*. Springer Berlin / Heidelberg, 2011.
2. M. Cattafi, M. Gavanelli, M. Nonato, S. Alvisi, and M. Franchini. Optimal placement of valves in a water distribution network with CLP(FD). *Theory and Practice of Logic Programming*, 11(4-5):731–747, 2011.
3. S. Chopra and M. R. Rao. The partition problem. *Mathematical Programming*, 59(1), 1993.
4. B. Colson, P. Marcotte, and G. Savard. An overview of bilevel optimization. *Annals of Operations Research*, 153:235–256, 2007. 10.1007/s10479-007-0176-2.
5. E. Creaco, M. Franchini, and S. Alvisi. Optimal placement of isolation valves in water distribution systems based on valve cost and weighted average demand shortfall. *Journal of Water Resources Planning and Management*, 24(15), 2010.
6. C. Ferreira, A. Martin, C. de Souza, R. Weismantel, and L. Wolsey. Formulations and valid inequalities for the node capacitated graph partitioning problem. *Mathematical Programming*, 74(3), 1996.
7. M. Gavanelli, M. Nonato, A. Peano, S. Alvisi, and M. Franchini. An ASP approach for the valves positioning optimization in a water distribution system. In F. Lisi, editor, *9th Italian Convention on Computational Logic (CILC 2012), Rome, Italy*, volume 857 of *CEUR Workshop Proceedings*, pages 134–148, 2012.
8. M. Gelfond. Answer sets. In *Handbook of Knowledge Representation*, chapter 7. Elsevier, 2007.
9. O. Giustolisi and D. A. Savić. Identification of segments and optimal isolation valve system design in water distribution networks. *Urban Water Journal*, 2010.
10. Gurobi Optimization, Inc. Gurobi optimizer reference manual. `http://www.gurobi.com`.
11. J. Jaffar and M. J. Maher. Constraint logic programming: A survey. *Journal of Logic Programming*, 19/20:503–581, 1994.
12. H. Jun and G. V. Loganathan. Valve-controlled segments in water distribution systems. *Journal of Water Resources Planning and Management*, 133(2), 2007.
13. J.-J. Kao and P.-H. Li. A segment-based optimization model for water pipeline replacement. *J. Am. Water Works Assoc.*, 99(7):83–95, 2007.
14. A. D. Nardo, M. D. Natale, G. Santonastaso, and S. Venticinque. Graph partitioning for automatic sectorization of a water distribution system. In D. Savic, Z. Kapelan, and D. Butler, editors, *Urban Water Management: Challenges and Opportunities*, volume 3, pages 841–846. Centre for Water Systems, University of Exeter, Exeter (UK), 2011.