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How Hard Is It to Control an Election by Breaking Ties?

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Abstract. We study the computational complexity of controlling the result of an election by breaking ties strategically. This problem is equivalent to the problem of deciding the winner of an election under parallel universes tie-breaking. When the chair of the election is only asked to break ties to choose between one of the co-winners, the problem is trivially easy. However, in multi-round elections, we prove that it can be NP-hard for the chair to compute how to break ties to ensure a given result. Additionally, we show that the form of the tie-breaking function can increase the opportunities for control.

1 INTRODUCTION AND DEFINITIONS

Voting is a general mechanism to combine individual orderings into a group preference. One concern that the individual agents may have is that the chair may manipulate the result by introducing a spoiler candidate or delete some votes. Bartholdi, Tovey and Trick [3] explored an interesting barrier to such manipulation; perhaps it is computationally too difficult for the chair to work out how to perform such control? They proved that many types of control problems are NP-hard for simple voting rules like plurality. Interestingly, one type of control not considered by Bartholdi, Tovey and Trick is control by choosing how ties are broken. In many elections the tie breaking rule is unspecified or ambiguous. The chair therefore has an opportunity to influence the outcome by selecting a beneficial (to him) rule.

We study the computational complexity of control by breaking ties. This problem avoids some of the criticism raised against some other forms of control: since the votes are already cast, it is reasonable for the chair to have complete knowledge. While ties in a real elections may not be that common, they have been observed. US Vice Presidents have had to cast tie-breaking votes in 244 Senate votes. Indeed John Adams, the first Vice President, cast 29 such votes. Often elections that are not closely contested cannot be manipulated [19] and therefore, tied elections being the most closely contested of all, represent an interesting edge case that has not been investigated.

Control by tie-breaking is equivalent to the problem of determining if a chosen alternative can win under *some* tie-breaking rule, an idea known as parallel universes tie-breaking (PUT) [5]. As PUT does not instantiate a particular tie-breaking rule, but rather the set of all tie-breaking rules, there is no longer a dependency on the names of the individual candidates (known as neutrality). Deciding if a candidate is the winner of such a neutral rule with ranked pairs voting has recently been shown to be NP-complete [4]; it follows that control by tie-breaking is NP-complete.

Bartholdi, Tovey and Trick [2] proved that a single agent can manipulate a Copeland election in polynomial time when ties are broken in favour of the manipulators, but manipulation becomes NP-hard for a more complicated tie-breaking rule. With Copeland voting, Faliszewski, Hemaspaandra, and Schnoor [10] proved that the choice of how ties are scored can change the computational complexity of computing a manipulation from polynomial to NP-hard. More recently work by Obraztsova et al. [17], Obraztsova and Elkind [16] and Aziz et al. [1] considered the impact of different randomized tie-breaking schemes on the computational complexity of manipulation.

An election is defined by a set of *candidates* C with $|C| = m$, a *profile* P which is a set of n strict linear orders (votes) over C, and a *voting correspondence* R. Let R be a function $R : P \rightarrow W$ mapping a profile onto a set of *co-winners* where $W \subseteq C$. If $|W| = 1$ then we have a *voting rule*, otherwise we may require a tie-breaking rule T that will return a unique winner (single element) from W.

A *tie-breaking* rule T for an election is a single valued choice function that, for any subset $W \subseteq C$, $W \neq \emptyset$, and profile P, $T(P,W)$ returns a single element $c \in W$ [17]. Commonly, T is a strict linear order over C that is provided aprori (e.g. by age or alphabetically). However, this definition allows us to define non-transitive functions, which are often used in sports competitions (e.g. goal differential).

Name: CONTROL BY TIE-BREAKING
Question: Given profile P and preferred candidate $p \in C$, is there a tie-breaking rule T such that p can be made the unique winner of the election under voting rule R?

In the *manipulation problem* [3], we wish to decide if we can cast one additional vote to make p win. In the *manipulation problem with random tie-breaking* [1, 17], we are also given a probability t want p to win with probability $\geq t$ when ties are broken randomly.

2 SUMMARY OF RESULTS

We start by considering how control by breaking ties is related to other manipulation problems. A little surprisingly, the complexity of control by breaking ties is not related to that of the manipulation problem with random tie-breaking or the standard manipulation problem when ties are broken in a fixed order.

Theorem 1 *There exists a voting correspondence such that the control by tie-breaking problem is polynomial but the manipulation problem with random tie-breaking is NP-complete (and vice versa).*

There also exists a voting correspondence such that the control by tie-breaking problem is polynomial but the manipulation problem is NP-complete (and vice versa).

When tie-breaking only ever takes place once and at the end, then the chair is choosing between the co-winners. In such cases, control by breaking ties is trivially polynomial. The chair can ensure a candidate p wins if and only if p is amongst the co-winners. This statement covers many voting rules including: All scoring rules, Bucklin, Black's Rule, maximin, Copeland$^{\alpha}$ for any α, Plurality with

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runoff, Fallback, Nanson's Rule (where manipulation is NP-hard [15], Schulze's Method, and Kemeny-Young method with $m \leq 3$.

Theorem 2 *The control by tie-breaking problem when we select from among a set of co-winners once is polynomial.*

In multi-round voting rules Conitzer et al. [5] showed that the winner determination under PUT for STV, and therefore control by tie-breaking, is NP-complete. Baldwin and Coombs's voting rules are multi-round rules that successively eliminate candidates based on their Borda or Veto scores, respectively. The manipulation problem for Baldwin's rule is NP-complete and we can modify the proof given by Narodytska et al. [15]. Similarly, for Coombs rule, which successively eliminates the candidate with the largest number of last place votes, we can modify the NP-complete manipulation problem [8] to show control by breaking ties is also NP-complete.

Theorem 3 *The control by tie-breaking problems for Baldwin's rule and Coombs rule are NP-complete.*

Cup and Copeland are used in real life settings involving sports or other competitions where ties must be resolved before the next round. In Copeland, when we select a winner from the elements with highest score Theorem 2 applies. However, in a sports competition, often ties between candidates need to be resolved before the final score can be computed. The frequent use of non-transitive tie-breaking rules in sports also increases opportunities for control by tie-breaking.

Using a result from Conitzer et al. [7] we can determine the best linear tie-breaking order for Cup when each candidate appears only once. However, if a non-transitive order is allowed then, in double elimination style cups such as the Australian Rules Football League Finals Series, the control by tie-breaking problem is hard.

Theorem 4 *When the Cup schedule S can have arbitrary shape and candidates can appear more than once, control by tie-breaking is NP-complete.*

Allowing the tie-breaking rule to be non-transitive increases the potential for control of the tie-breaking rule under Copeland. We note that such tie-breaking is closely to the problem of manipulating a Copeland election with *irrational* voters which is polynomial time computable. In fact, we can use the algorithm presented by Faliszewski et al. [10] to show that the control by tie-breaking problem is polynomial in this case.

When there are only a small, fixed number of tie-breaks, rules can be resistant to control by tie-breaking.

Theorem 5 *There exists a two stage voting rule based on veto and plurality where the control by tie-breaking problem is NP-complete.*

Conitzer and Sandholm [6] give a general construction that builds a two-stage voting rule that often makes it intractable to compute a manipulating vote. This construction runs one round of the Cup rule, eliminating half of the candidates, and then applies the original base rule (X) to the candidates that remain; giving us the notation $Cup_1 + X$. The control by tie-breaking problem is also typically intractable for such two-stage voting rules.

Theorem 6 *The control by tie-breaking problem for $Cup_1 +$ Plurality, $Cup_1 +$ Borda, and $Cup_1 +$ Maximin are NP-complete.*

Elkind and Lipmaa [9] generalize this construction to run a number of rounds, k, of some rule before calling a second rule; making computing a manipulating vote NP-hard in many cases. Manipulation is polynomial for $\textsc{Hyb}(plurality_k, plurality)$ if k is bounded [9]; this result carries to our problem. However, this hybrid is resistant to control by tie-breaking for unbounded k.

Theorem 7 *If k is unbounded, the control by tie-breaking problem for $\textsc{Hyb}(Plurality_k, Plurality)$ is NP-complete. If k or $m-k$ is bounded the control by tie-breaking problem for $\textsc{Hyb}(Plurality_k, Plurality)$ is polynomial time solvable.*

P	NP-complete
scoring rules, Cup, Nanson, Copeland, maximin Bucklin, fallback, Schulze Kemeny-Young ($m \leq 3$)	STV [5], Baldwin ranked pairs [4], Coombs Kemeny-Young ($m \geq 3$)

Table 1. Complexity of control by tie-breaking.

Our results are summarised in Table 1 and full proofs can be found in our fill length technical report [13]. Of course, many of our results are worst-case and may not reflect the difficulty of manipulation in practice. A number of recent theoretical and empirical results suggest that manipulation can often be computationally easy on average (e.g. [11, 12, 18, 19]). We intend to explore the hardness of control by tie-breaking using data from PrefLib [14] and other sources.

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