Supplemental Materials to Submission #1552: Resource-Constrained Scheduling for Maritime Traffic Management

1 Linearization of Constraints (10)-(13)

Using big-M method, constraint (10)-(13) may be linearized as follows:

$$
\begin{aligned}
&\forall i \in \mathcal{N}, e \in \mathcal{E}: t_e \geq (\text{start}_e^{i1}) s_i \\
&\forall i \in \mathcal{N}, e \in \mathcal{E}: S_{im} \geq (\text{start}_e^{im}) t_e \\
&\forall (i,j) \in \mathcal{A}, e, f \in \mathcal{E} | f > e: t_f \geq t_e + (\text{span}_{ef}^{ij}) T_{\min}^{ij} \\
&\forall (i,j) \in \mathcal{A} | j \neq m, e, f \in \mathcal{E} | f > e: t_f \leq t_e + (\text{span}_{ef}^{ij}) T_{\max}^{ij} + (1 - \text{span}_{ef}^{ij}) M_{ef}
\end{aligned}
$$

where M_{ef} is a suitable upper bound. Big-M formulation in MIP may loosen its LP relaxation and incur degenerate behavior if not handled correctly. The following value of U_{ef} gives a tight upper bound:

$$U_{ef} = (f - e) \left[\max_{(i,j) \in \mathcal{A}} \left\{ T_{\max}^{ij}, \max_{i'} s_{i'} - \min_{i'} s_{i'} \right\} \right].$$

2 Classical Benders Decomposition of MTM

[**Note**: The notations used here may overlap with the ones used in the main paper. However, this section is self-contained and should be read independently from the formulation given in the main paper.]

In this section, we present the (classical) Benders decomposition of the problem as described in the main paper. We start with a short introduction of Benders decomposition in Section 2.1 and the notations used in Section 2.2. The master problem, which deals with assignments of activities to events is given in Section 2.3, and the subproblem, which deals with the times of events, in Section 2.4. And finally, Section 2.5 describes the Benders cuts that are added to the master problem.

2.1 Preliminaries

Benders decomposition is a technique for decomposing mixed integer linear programs of the form:

$$
\begin{array}{lll}
[\mathrm{P}] & \min & c_1x + c_2y \\
 & \text{s.t.} & Ax + By \geq b \\
 & & x \geq 0, y \in \mathbb{Y}
\end{array}
$$

where x are continuous real-valued variables and $\mathbb{Y}$ is a discrete (integral) domain, into the master problem [MP]:

$$
\begin{array}{llll}
[\mathrm{MP}] & \min & Q & \\
 & \text{s.t.} & c_2y + \pi(b - By) \leq Q & \forall \pi \in \Pi_p \qquad (1) \\
 & & \pi(b - By) \leq 0 & \forall \pi \in \Pi_r \qquad (2) \\
 & & y \in \mathbb{Y} &
\end{array}
$$

and the subproblem [PSP] (with its dual [DSP]):

$$
\begin{array}{lll}
[\mathrm{PSP}] & \min & c_1x \\
 & \text{s.t.} & Ax \geq b - By \\
 & & x \geq 0
\end{array}
\qquad
\begin{array}{lll}
[\mathrm{DSP}] & \max & (b - By)\pi \\
 & \text{s.t.} & \pi A \leq c_1 \\
 & & \pi \geq 0
\end{array}
$$

where Π_p and Π_r are the set of extreme points and rays, respectively, of the dual subproblem [DSP]. The constraints in (1) are called optimality cuts and the ones in (2) are called feasibility cuts. It has be shown that [P] and [MP] are equivalent. The size of the sets Π_p and Π_r can be extremely large and the idea behind Benders decomposition is to start with initial sets $\Pi_p^0 \subseteq \Pi_p$ and $\Pi_r^0 \subseteq \Pi_r$ (usually empty), and to iteratively add to these sets and resolving [MP], with the hope that optimality can be reached with a much smaller sets $\Pi_p^* \subseteq \Pi_p$ and $\Pi_r^* \subseteq \Pi_r$. Solving the restricted master problem provides a lower bound to the optimal value, while solving the subproblem provides an upper bound. Convergence is guaranteed by the fact that the set of extreme points and rays are finite.

2.2 Notations

Let $\mathcal{R}$ be the set of resources. Examples of a resource are an area of traversable sea space, an anchorage area, a berth, available pilots, etc. Each resource $r \in \mathcal{R}$ has a capacity $K_r \in \mathbb{N}$. Let $\mathcal{N}$ be the set of n vessels. To each vessel $i \in \mathcal{N}$, is associated a list of m activities to be carried out sequentially without time lag between two consecutive activities. For clarity of presentation, we assume without loss of generality, that all vessels have the same number of activities m. In addition, each vessel $i \in \mathcal{N}$ has a release time s_i which is the earliest start time of its first activity. We denote by (i, j) the j-th activity of vessel i, and $\mathcal{A}$ the set of all activities ($|\mathcal{A}| = nm$). Each activity $(i, j) \in \mathcal{A}$ requires a set of resources, and we denote by a_r^{ij}, the amount of resource r required by activity (i,j). Furthermore, each activity (i,j) has a minimum and a maximum processing time, denoted by $T_{\min}^{ij}$ and $T_{\max}^{ij}$ respectively.

Let $\mathcal{E} = \{1,\ldots,nm\}$ be the set of events. The first set of decision variables is $z = \{z_e^{ij} | (i,j) \in \mathcal{A}, e \in \mathcal{E}\}$ where $z_e^{ij} = 1$ iff activity (i,j) is being carried out at t_e, and $z_e^{ij} = 0$ otherwise. When expressing a constraint, the dummy variable z_0^{ij} or z_{nm+1}^{ij} might be used, in which case, its value is always 0 for any activity (i,j). The second set of decision variables is $t = \{t_e | e \in \mathcal{E}\}$, where t_e denotes the time of event e. The times of events are ordered sequentially, that is, $t_1 \leq \cdots \leq t_{nm}$. To simplify notations, we define the following derivative variables $w = \{w_{ef}^{ij} | (i,j) \in \mathcal{A}, e \in \mathcal{E}, f \in \mathcal{E}, e < f\}$, where $w_{ef}^{ij} = (z_e^{ij} - z_{e-1}^{ij}) + (z_{f-1}^{ij} - z_f^{ij})$. In other words,

$$w_{ef}^{ij} = \begin{cases} 2 & \text{if } (i,j) \text{ starts at } e \text{ and ends at } f, \\ 1, 0, -1 & \text{otherwise.} \end{cases}$$

The objective of the problem is to minimize the makespan, i.e., the total time required to complete all activities.

2.3 Master Problem

The master problem [MP] can be formulated as the following integer linear program with a single continuous variable as objective:

$$[\text{MP}] \qquad \min Q \tag{3}$$

subject to:

$$\forall (i,j) \in \mathcal{A}: \sum_{e=1}^{nm} z_e^{ij} \geq 1 \tag{4}$$

$$\forall (i,j) \in \mathcal{A}, e \in \mathcal{E} \setminus \{1\}: \sum_{e'=1}^{e-1} z_{e'}^{ij} \leq (e-1)(1 - (z_e^{ij} - z_{e-1}^{ij})) \tag{5}$$

$$\forall (i,j) \in \mathcal{A}, e \in \mathcal{E} \setminus \{1\}: \sum_{e'=e}^{nm} z_{e'}^{ij} \leq (nm - e + 1)(1 + (z_e^{ij} - z_{e-1}^{ij})) \tag{6}$$

$$\forall (i,j) \in \mathcal{A} | j \neq m, e \in \mathcal{E}: \sum_{e'=1}^{e} z_{e'}^{ij+1} \leq (1 - z_e^{ij}) e \tag{7}$$

$$\forall (i,j) \in \mathcal{A} | j \neq m, e \in \mathcal{E} \setminus \{1\}: z_e^{ij+1} \geq z_{e-1}^{ij} - z_e^{ij} \tag{8}$$

$$\forall e \in \mathcal{E}, r \in \mathcal{R}: \sum_{i=1}^{n} \sum_{j=1}^{m} a_r^{ij} z_e^{ij} \leq K_r \tag{9}$$

$$\forall \pi \in \Pi_p: \phi(\pi, z) \leq Q \tag{10}$$

$$\forall \pi \in \Pi_r: \phi(\pi, z) \leq 0 \tag{11}$$

$$\forall (i,j) \in \mathcal{A}, e \in \mathcal{E}: z_e^{ij} \in \{0,1\}, Q \in \mathbb{R} \tag{12}$$

where $\phi(\pi, z)$ is the expression of the Benders cuts which will be elaborated in the Section 2.5.

2.4 Subproblem

Given an assignment to the binary variables z, the primal subproblem [PSP] is given by the following linear program:

$$\text{[PSP]} \qquad q(z) = \min M \tag{13}$$

subject to:

$$\forall i \in \mathcal{N}, e \in \mathcal{E}\colon\ t_e \geq (z_e^{i1} - z_{e-1}^{i1}) s_i \tag{14}$$

$$\forall i \in \mathcal{N}, e \in \mathcal{E}\colon\ M \geq t_e + (z_e^{im} - z_{e-1}^{im}) T_{\min}^{im} \tag{15}$$

$$\forall e \in \mathcal{E} \setminus \{nm\}\colon\ t_{e+1} \geq t_e \tag{16}$$

$$\forall (i,j) \in \mathcal{A}, e,f \in \mathcal{E} \mid f > e\colon\ t_f \geq t_e + (w_{ef}^{ij} - 1) T_{\min}^{ij} \tag{17}$$

$$\forall (i,j) \in \mathcal{A} \mid j \neq m, e,f \in \mathcal{E} \mid f > e\colon$$

$$t_f \leq t_e + (w_{ef}^{ij} - 1) T_{\max}^{ij} + (2 - w_{ef}^{ij}) U_{ef} \tag{18}$$

$$\forall i \in \mathcal{N},\ e \in \mathcal{E}\colon\ t_e \in \mathbb{R}_{\geq 0}, D_i \in \mathbb{R} \tag{19}$$

where U_{ef} is a sufficiently big number.

2.5 Classical Benders Cuts

Let α, β, γ and δ be the dual variables corresponding to constraints (14), (15), (17) and (18) respectively, in other words, $\pi = (\alpha, \beta, \gamma, \delta)$. We can ignore constraint (16) which has constant term zero and no binary variables, and thus does not contribute any term to the Benders cut. The expression of the Benders cuts $\phi(\pi, z)$ is then given by

$$\phi(\pi, z) =$$

$$\sum_{i=1}^{n} \sum_{e=1}^{nm} \alpha_e^i s_i (z_e^{i1} - z_{e-1}^{i1}) + \sum_{i=1}^{n} \sum_{e=1}^{nm} \beta_e^i T_{\min}^{im} (z_e^{im} - z_{e-1}^{im}) \tag{20}$$

$$+ \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{e=1}^{nm-1} \sum_{f=e+1}^{nm} \gamma_{ef}^{ij} T_{\min}^{ij} w_{ef}^{ij} \tag{21}$$

$$- \sum_{i=1}^{n} \sum_{j=1}^{m-1} \sum_{e=1}^{nm-1} \sum_{f=e+1}^{nm} \delta_{ef}^{ij} (T_{\max}^{ij} - U_{ef}) w_{ef}^{ij} \tag{22}$$

$$- \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{e=1}^{nm-1} \sum_{f=e+1}^{nm} \gamma_{ef}^{ij} T_{\min}^{ij} \tag{23}$$

$$+ \sum_{i=1}^{n} \sum_{j=1}^{m-1} \sum_{e=1}^{nm-1} \sum_{f=e+1}^{nm} \delta_{ef}^{ij} (T_{\max}^{ij} - 2U_{ef}). \tag{24}$$

When π is given, the terms (20)–(22) are a linear expression in z while the terms (23) and (24) are constants. Furthermore, term (24) is a nonpositive constant since, by definition, $U_{ef} \geq T_{\max}^{ij}$ and $\delta_{ef}^{ij} \geq 0$. This is where having a loose U_{ef} values may weaken the cut significantly because of the large negative constant it may have.

3 Example of Feasibility Cuts

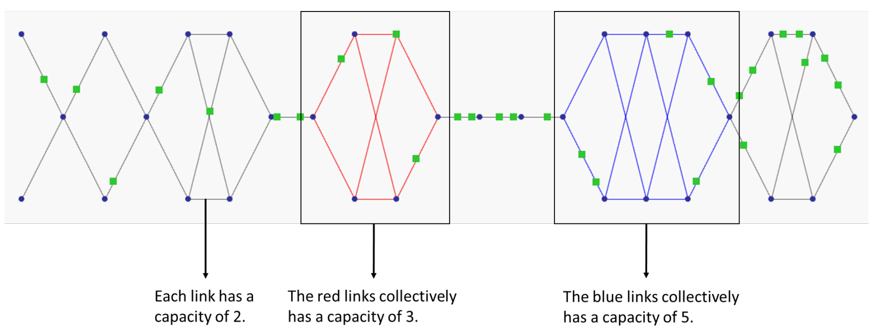
To give an example of feasibility cuts, consider the activity-to-event assignments given in Figure 2. In the assignment 2(a), both activity (5,3) and (2,2) span events 9 to 10. Suppose activity (5,3) a maximum processing time of 5, and activity (2,2) has a minimum processing time of 7. Given this, there is no feasible assignment of times to both events 9 and 10, and the following feasibility cuts can be added to [MP]:

$$\left\{ \text{span}_{ef}^{5,3} + \text{span}_{ef}^{2,2} \leq 1 | e, f \in \mathcal{E}, e < f \right\}.$$

4 Description of the Demo Videos

There are 5 demo videos that are included in the supplementary files. The following is their description:

- synthetic_uncoordinated.mp4 & synthetic_coordinated.mp4:
  These videos contain a sample of the synthetic instances, one for the uncoordinated scenario and one for the coordinated scenario. The edges of the graph represent activities, and the green boxes represent vessels performing those activities. The times to traverse the edges are the processing times of the activities. In this instance, there are three resource types (see the image below).



- ais_uncoordinated.mp4 & ais_coordinated.mp4:
  These videos contain a sample of the instances derived from the AIS data. For the coordinated scenario, we show one with the minimum zone capacity (=1) (for contrast).

- extra_heat_accumulation.mp4:
  This is an extra video showing how heat accumulates within the Singapore port waters. Heat here represents the congestion levels.