Project scheduling in the face of uncertainties – The generation of solution and quality robust project schedules

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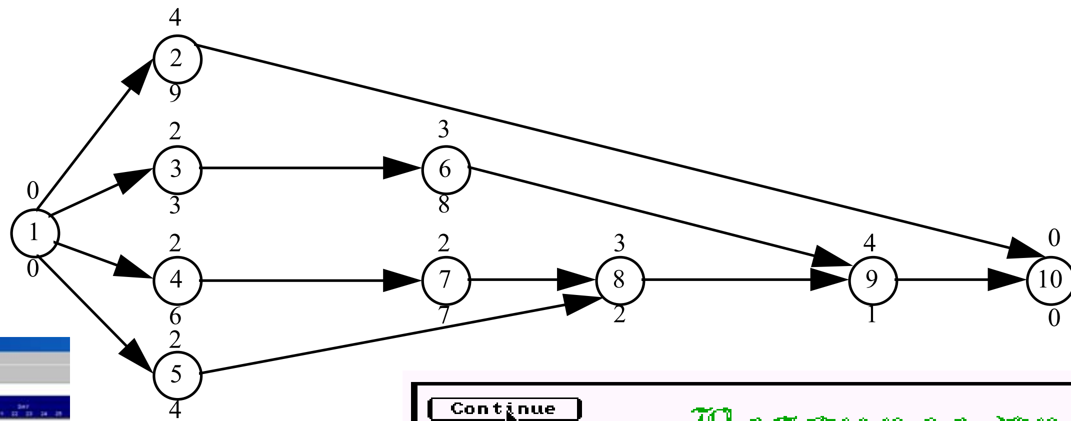
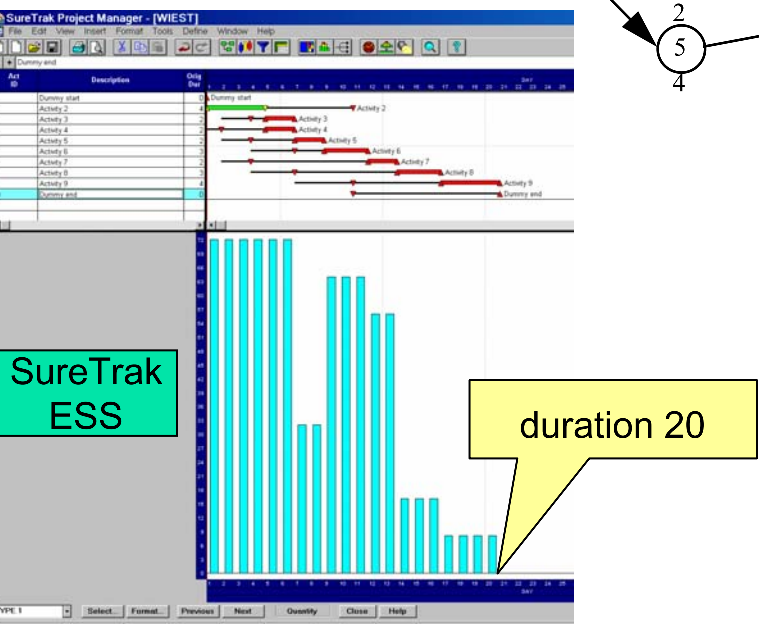
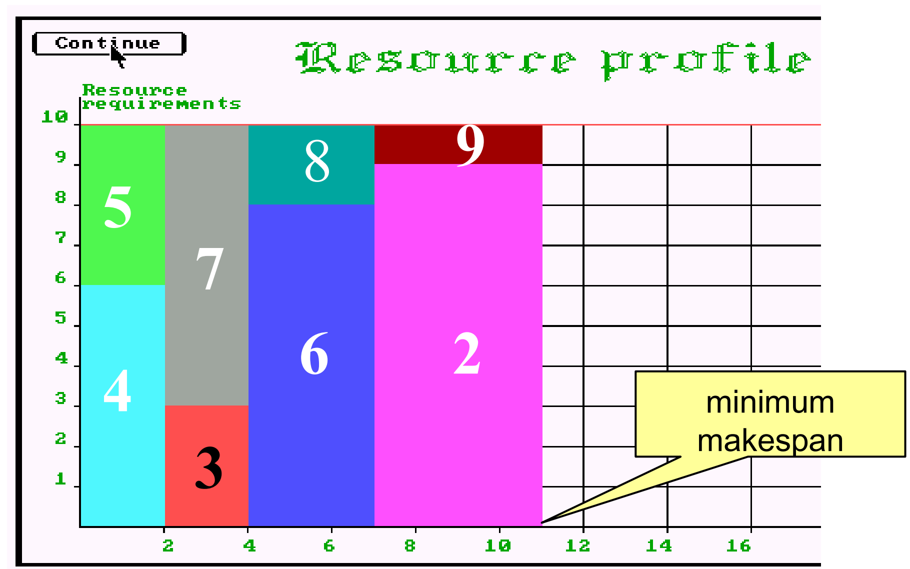
Overview

- **Project scheduling under uncertainty: the problem reloaded**
- **Stochastic project scheduling versus proactive/reactive scheduling: the choice made**
- **Solution robustness versus quality robustness: the objectives defined**
- **The generation of stable schedules in the absence of binding resource constraints: the problem simplified**
- **The critical chain hype: the problems revealed**
- **Resource allocation for stability and the generation of stable baseline schedules : the things to be done**
- **Conclusions**

Facing uncertainty ...







Sources of uncertainty

- **uncertain durations**

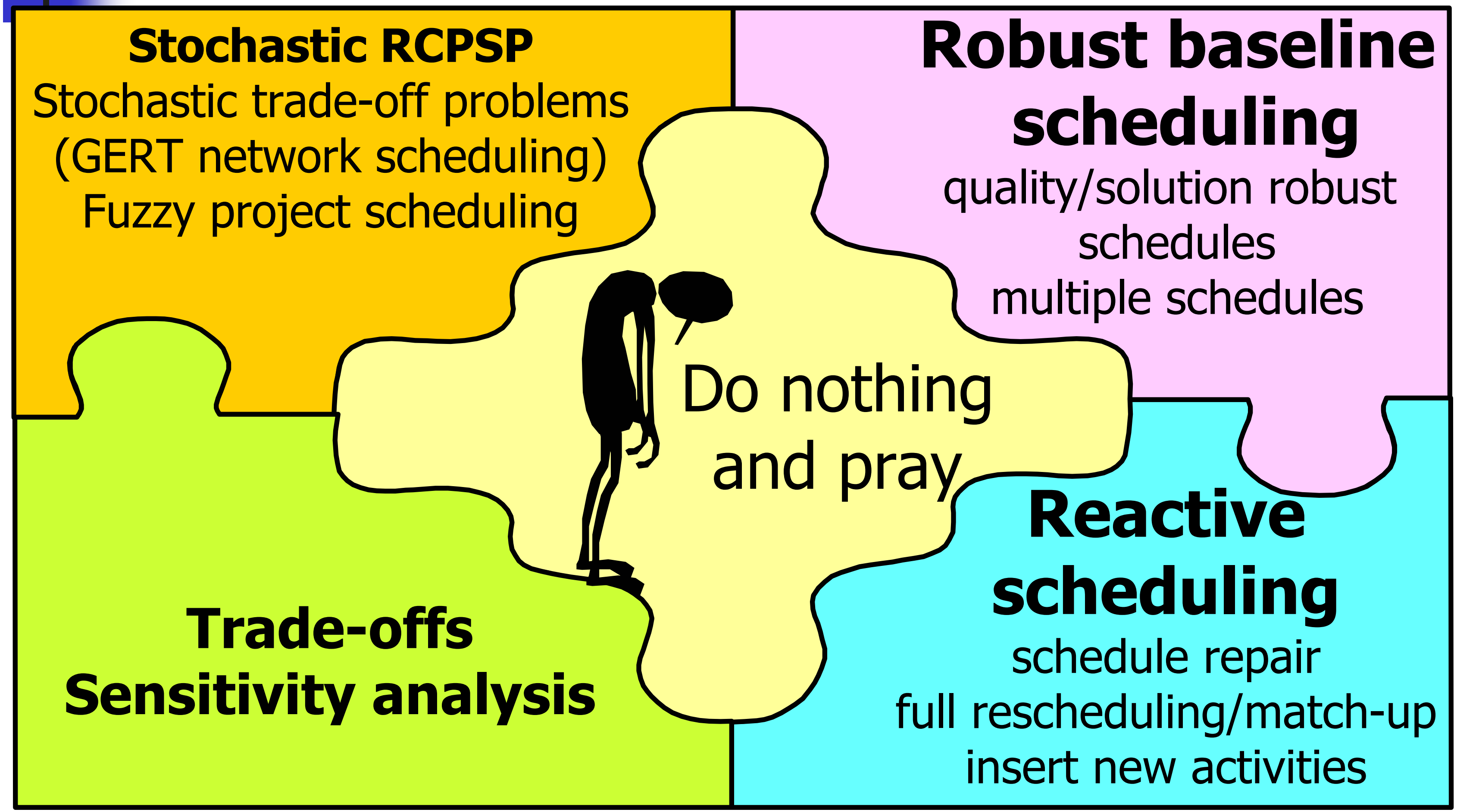
  (processing times, transportation times, set-up times, ...)
- **uncertain time events**

  (ready times, due dates, resource availability, ...)
- **project network and activity characteristics**

  **(network structure, work content,
  precedence relations, execution mode, ...)**
- **cost uncertainty**

  (direct and indirect costs, penalties, priorities, ...)

Coping with uncertainty



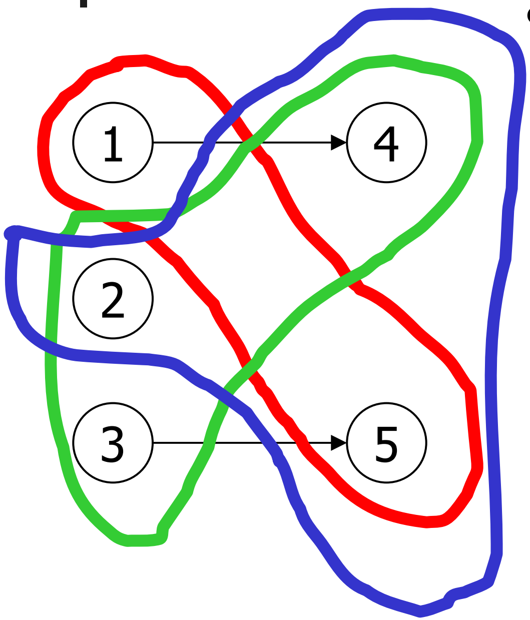
Stochastic project scheduling

Stochastic RCPSP $m,1|cpm,\boldsymbol{d_j}|E[C_{max}]$

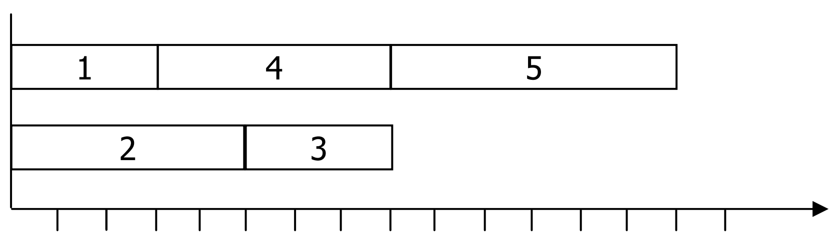
- schedule project activities with uncertain durations in order to minimise the expected project duration under finish-start precedence and renewable resource constraints

- multi-stage stochastic decision process
(Fernandez 1995; Fernandez & Armacost 1996; Fernandez et al. 1998; Pet-Edwards et al. 1998)
  - decisions have to be made at stages g which occur serially through time at random decision points $\boldsymbol{t_g}$
  - *admissibility constraint*: at each stage g, a set of activities has to be selected from the precedence and resource feasible activities at that stage
  - *non-anticipativity constraint*: based on observed past and a priori knowledge about processing time distributions
    - information at $\boldsymbol{t_g}$: precedence constraints, resource requirements and availabilities, distribution for the time remaining for ongoing activities, distributions for the activities not yet scheduled
  - no baseline schedule
  - scheduling policies that define which activities have to be started at $\boldsymbol{t_g}$

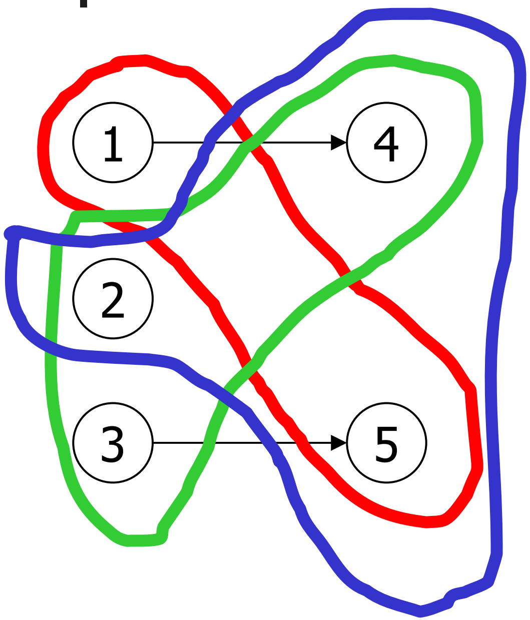
Early start (ES) policies
(Radermacher 1985)



- 3 minimal forbidden sets: $F = \{\{1,5\},\{2,3,4\},\{2,4,5\}\}$
- for each minimal forbidden set, a pair of activities $(i,j), (i,j) \in F, i \neq j$ such that for each sample *d* of activity durations, *j* cannot be started before *i* has finished
- *example* : activity durations are independent and uniformly distributed with variance 2 and expected activity durations ***E*(*d*)**={3,5,3,5,6}
- sample *d*={3,5,3,5,6} add $1 \prec 5, 2 \prec 3, 4 \prec 5$



Preselective policies
(Igelmund & Radermacher 1983)

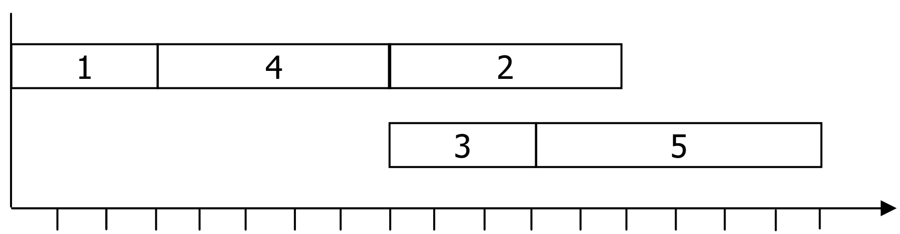


- for each minimal forbidden set, select a waiting activity j that has to wait for *at least one* other forbidden set activit for each sample d of activity durations

~ delaying alternatives

- *selection* = sequence of waiting activities

- the 3 minimal forbidden sets yield 2×3×3=18 selections; e.g. (5,3,2): $1 \prec 5, 4 \prec 3, 4 \prec 2$



- smaller expected makespan than ES-policies
- branch-and-bound procedure (Stork 2000, 2001)

Henry L. Gantt

Do we need a baseline schedule?

- allocate resources to different activities to optimise some measure of performance
- quoting competitive and reliable due dates
- basis for planning external activities (material procurement, preventive maintenance, committing to shipping due dates,…)
- schedule in accord with all parties within the inbound and outbound supply chain (clients, suppliers, workers, other resources,…)
- agree on time windows for work to be done by subcontractors
- share production schedules with suppliers on a continuous basis using Internet technology
- organise production resources to best support smooth schedule execution
- making cash flow projections
- measure performance of both management and shop floor personnel
- project control: measure progress and take corrective actions

Addressing risk

- PMI May 2003 Member Community online survey (*PM Network,* July 2003, p. 12):
  - only 5% of 59 respondents cited "unexpected risk" as the factor that most severely impacts the ability to deliver projects on time and on budget
  - unrealistic estimates/milestones (42%)
    lack of executive support (29%)
    scope changes (25%)

    are recognizible sources of uncertainty so that the risks arising from them are not unexpected, but can be identified, assessed and managed **proactively**
    (D. Hillson, *PM Network*, October 2003, p. 6)

- 'Proactive and reactive planning are not alternatives, they are complementary aspects of planning as a whole'

  Chapman, C. & S. Ward (2003), *Project risk management – Processes, techniques and insights*, Second edition, John Wiley & Sons, p. 15

Proactive/reactive scheduling

- generate a **baseline schedule** that incorporates a degree of anticipation of variability during project execution and/or information about the reactive scheduling approach to be used
- **objectives**:
  - stability or solution robustness
    - insensitivity of activity start times to changes in input data
  - quality robustness
    - sensitivity of schedule performance in terms of objective value
  - flexibility
    - freedom to change schedules during the execution phase (time, sequence, resource allocation, execution modes)

No binding resource constraints

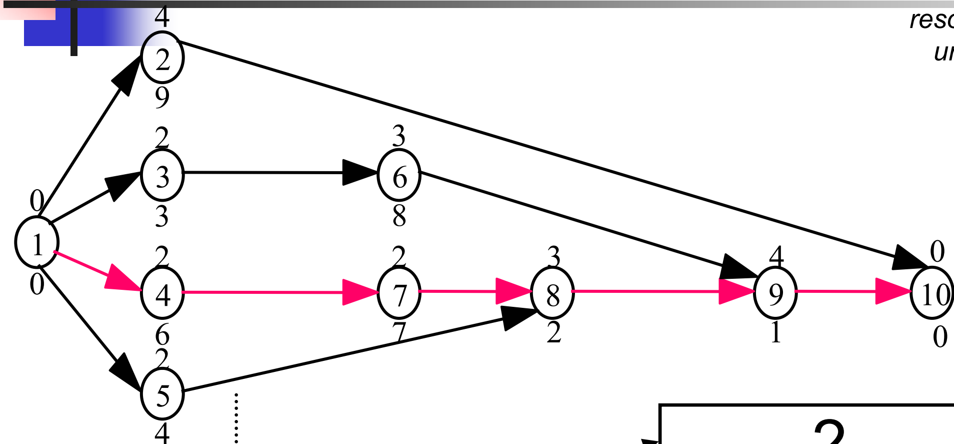
Stable schedule example

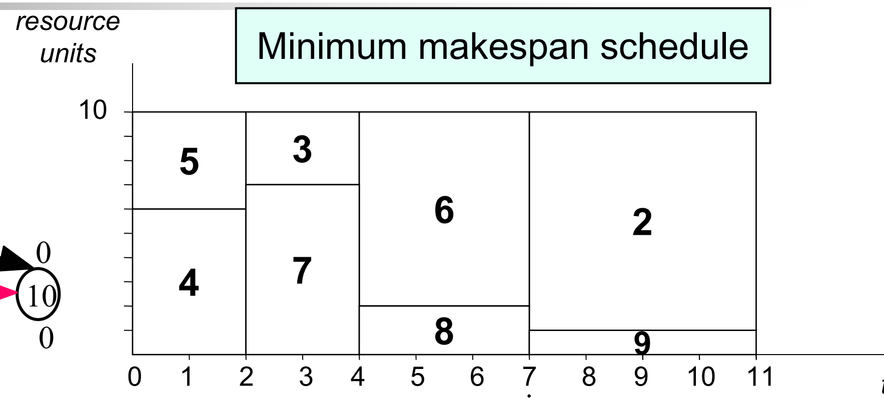
Project deadline=14

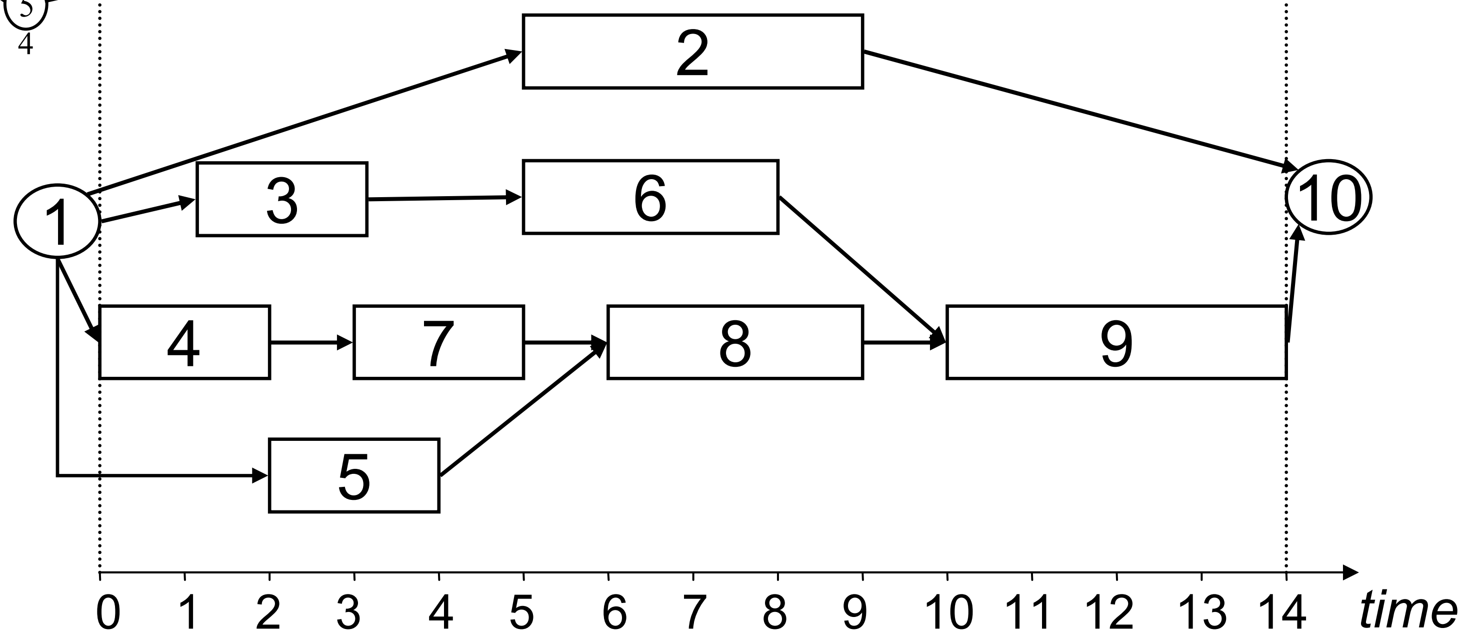
Equal disruption probabilities:

50%: +1 period

50%: +2 periods







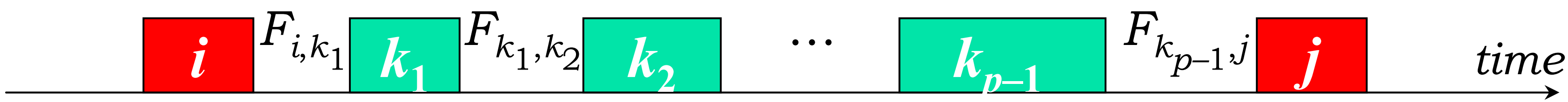
Notation & objective
(Herroelen & Leus, *EJOR*)

- N = set of activities/nodes in the project network
- A = precedence constraints
- TA = precedences in the transitive closure of $G=(N,A)$
  $(i,j)\in TA$ if there is a path from i to j in G
- $\mathcal{S}$ = pre-schedule: specifies start $s_i(\mathcal{S})$ and end $e_i(\mathcal{S})$ times
- d_i = pre-schedule duration of activity i, $s_i(\mathcal{S}) + d_i = e_i(\mathcal{S})$
- $\boldsymbol{D}_i$ = actual (stochastic) duration of activity i
- δ = pre-schedule deadline
- $F_{ij}(\mathcal{S})$= looseness (pairwise schedule float) = $s_j(\mathcal{S})$ - $e_i(\mathcal{S})$
- p_i = probability of disruption of activity i ; $\Sigma p_i=1$
- $\boldsymbol{L}_i$ = increase in d_i if disruption occurs (stochastic variable);
  $\boldsymbol{L}_i$ has scenarios $l_{ik}\in\Psi_i$ and *pmf* $g_i(\cdot)$, $\Sigma_{k\in\Psi_i}\, g_i(l_{ik})=1$
- c_i = non-negative cost per unit time overrun on start of activity i
- $\boldsymbol{S}_i(\mathcal{S})$ = actual starting time of activity i ; $s_i(\mathcal{S}) \leq \boldsymbol{S}_i(\mathcal{S})$

Objective: expected weighted deviation in the start times in the *realised schedule* from those in the *pre-schedule*: $\Sigma_j c_j(\mathrm{E}[\boldsymbol{S}_j(\mathcal{S})]-s_j(\mathcal{S}))$

Objective

- $F_{ij}(S) \quad = s_j(S) - e_i(S) \qquad \forall (i,j) \in TA$



- $MSPF_{ij}(S) \quad = \min_{\text{paths } P(i,j)} \sum_{\text{edges}(k,l) \text{ in } P} F_{kl}(S)$

  = protection of $\boldsymbol{S_j}(S)$ against disruption in i

- $\mathrm{E}[\boldsymbol{S}_j] = s_j + \sum_{i:(i,j)\in TA} p_i \mathrm{E}(\max\{0 ; L_i - MSPF_{ij}\})$

- $\min \sum_j c_j(\mathrm{E}[\boldsymbol{S}_j] - s_j) = \min \sum_{(i,j)\in TA} p_i c_j \mathrm{E}(\max\{0 ; L_i - MSPF_{ij}\})$

  $= \min \sum_{(i,j)\in TA} \sum_{k\in\Psi_i} p_i c_j g_i(l_{ik}) \underbrace{\max\{0 ; l_{ik} - MSPF_{ij}\}}$

Δ_{ijk}=delay in start of j due to disturbance of i according to scenario k

Stability: basic model

$$\min \quad \sum_{(i,j)\in TA} \sum_{k\in \Psi_i} p_i\, c_j\, g_i(l_{ik})\, \Delta_{ijk}$$

subject to

$s_i + d_i + F_{ij} = s_j \qquad \forall (i,j)\in A$

$s_n \le \delta$

$l_{ik} - MSPF_{ij} \le \Delta_{ijk} \qquad \forall (i,j)\in TA,\ \forall k\in \Psi_i$

$MSPF_{ii} = 0 \qquad \forall i\in N$

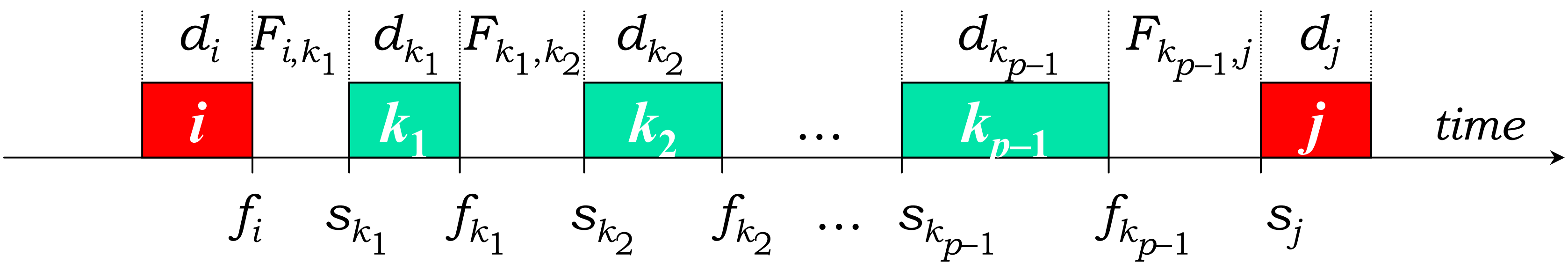
$MSPF_{ij} \le F_{ik} + MSPF_{kj} \qquad \forall (i,j)\in TA,\ \forall k\in isu(i)\cap(tpr(j)\cup\{j\})$

all $\Delta_{ijk},\ s_i,\ F_{ij},\ MSPF_{ij} \ge 0$

Towards an alternative formulation

- we notice that

$$s_i + \sum_P d_k + \sum_P F_{kl} = s_j \qquad \forall (i,j) \in TA, \ \forall \text{ path } P \text{ from } i \text{ to } j$$



$\rightarrow \quad s_i + d_i + \lambda_{ij} + MSPF_{ij} = s_j \qquad \forall (i,j) \in TA$

λ_{ij} = longest path $i \rightarrow j$ in activity durations (excluding i and j)

- rather than replace equations for determination of $MSPF_{ij}$: substitute completely

Alternative formulation

$$\min \quad \sum_{(i,j)\in TA} \sum_{k\in \Psi_i} \alpha_{ijk} \Delta_{ijk} \qquad \begin{cases} \alpha_{ijk} = p_i c_j g_i(l_{ik}) \\ \beta_{ijk} = l_{ik} + d_i + \lambda_{ij} \end{cases}$$

st

$$(x_{ij}) \qquad s_j - s_i \geq d_i \qquad \forall (i,j) \in A$$

$$(v) \qquad s_0 - s_n \geq -\delta$$

$$(y_{ijk}) \qquad s_j - s_i + \Delta_{ijk} \geq \beta_{ijk} \qquad \forall (i,j) \in TA,\ \forall k \in \Psi_i$$

all $\Delta_{ijk} \geq 0$; all s_i unrestricted in sign

- the dual of this formulation is a minimum cost network flow problem with $|M|$ nodes and $1+|A|+\sum_{(i,j)\in TA}(|\Psi_i|)$ arcs
- optimal starting times are optimal node potentials in the dual

Dual formulation

$$\max \quad \sum_{(i,j)\in A} d_i x_{ij} - \delta v + \sum_{\substack{(i,j)\in TA \\ k\in\Psi_i}} \beta_{ijk} y_{ijk}$$

subject to

$$\sum_{(i,j)\in A} x_{ij} - \sum_{(j,i)\in A} x_{ji} + \sum_{\substack{(i,j)\in TA \\ k\in\Psi_i}} y_{ijk} - \sum_{\substack{(j,i)\in TA \\ k\in\Psi_i}} y_{jik} = \begin{cases} 0 & \forall i \in N, i \neq 1, n \\ v & i = 1 \\ -v & i = n \end{cases}$$

$$y_{ijk} \leq \alpha_{ijk} \qquad \forall (i,j) \in TA, \forall k \in \Psi_i$$

Benchmark heuristic (WPF)

- no scenarios
- maximise looseness:
  the weighted sum of pairwise floats
  the weighted sum of buffer sizes (Leus 2003)

$$\max \quad \sum_{(i,j)\in TA} c_j p_i F_{ij}$$

subject to

$$s_i + d_i + F_{ij} \le s_j \qquad \forall (i,j) \in TA$$

$$s_n \le \delta$$

$$\text{all} \quad s_i, F_{ij} \ge 0$$

- dual is a minimum cost network flow problem

Benchmark heuristic 2 (LPH)

- Adapted linear programming based heuristic (LPH) of Mehta & Uzsoy (1998) (Leus 2003)
  - schedule $\Omega(\gamma)$ is the ESS with durations $d_i(\gamma)= d_i+ \gamma n p_i E[\boldsymbol{L}_i]$
    - γ measures the degree to which the expected values of disruptions are propagated throughout the network
    - γn is used to make the parameter independent of the number of activities (average probability = $1/n$)

$$
\begin{aligned}
&\min \quad \sum_{i\in N} c_i \Delta_i \\
&\text{subject to} \\
&s_i(S) + d_i \le s_j(S) && \forall (i,j) \in A \\
&s_n(S) \le \delta \\
&s_i(\Omega(\gamma)) = s_i(S) + \Delta_i && \forall i \in N \\
&\text{all} \quad \Delta_i, s_i(S) \ge 0
\end{aligned}
$$

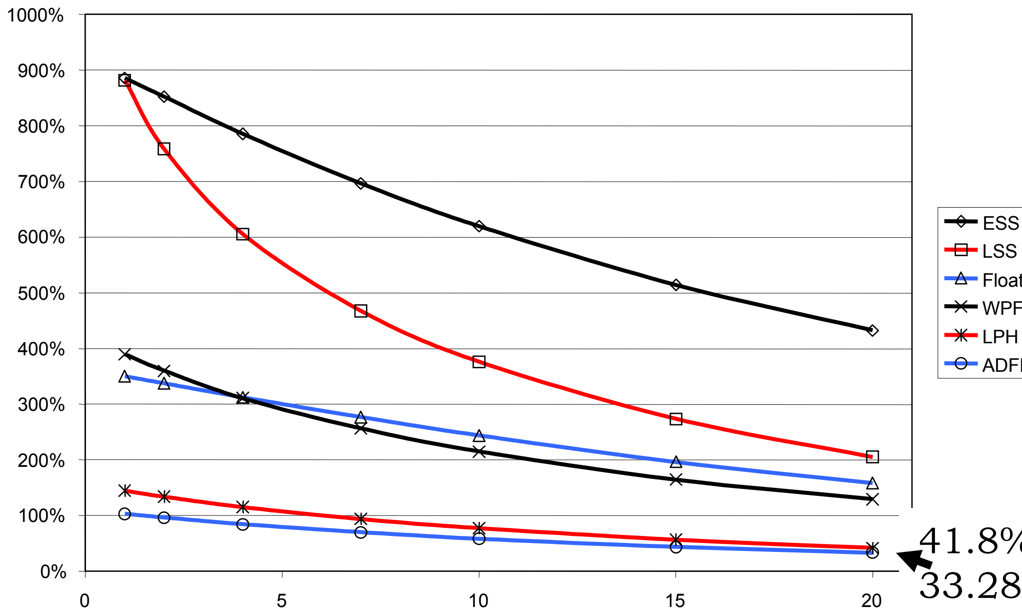
can be written as dual of minimum cost network flow problem

Benchmark heuristic 3 (ADFF)

- Activity-dependent float factor model (Leus 2003)
  - Float factor model (Tavares et al. 1998)
    - ESS(LSS) reduces (increases) the risk of an overall delay but (increases) decreases the project's discounted cost
    - $s_i(\mathcal{S}) = s_i(\text{ESS}) + \alpha(s_i(\text{LSS}) - s_i(\text{ESS}))$; $\alpha \in [0,1]$
  - $$\alpha(i) = \frac{\text{sum of weights } p_k c_l \text{ of arcs before } i \text{ in } G(N,A)}{\text{sum of weights } p_k c_l \text{ of arcs before and after } i \text{ in } G(N,A)}$$

Computational results

- RanGen test instances (Demeulemeester et al. 2003)
  - 300 problems; 100 for each of 3 OS-values 0.25, 0.5 and 0.75 (OS is # of direct and indirect precedence relations divided by the max. # precedence relations)
  - disruption lengths drawn from original continuous distribution function
  - expected weighted deviation as a function of the # of disruptions expressed as a % excess over dual model of Leus

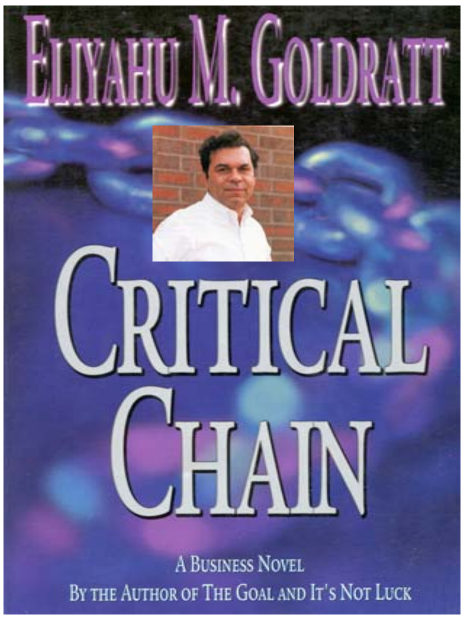


Running times

- Visual C++
- Pentium III, 800 Mhz, 128 Mb RAM
- CS2 (Goldberg 1977) for network flow problems

n	CPU Dual	nr arcs Dual	CPU (WPF)	CPU (LPH)
31	0.00450	903	0.00091	0.00072
61	0.01520	3,273	0.00391	0.00237
91	0.04794	7,393	0.00846	0.00560
121	0.10644	13,313	0.01842	0.01038

The Critical Chain hype ….
(Goldratt 1997)



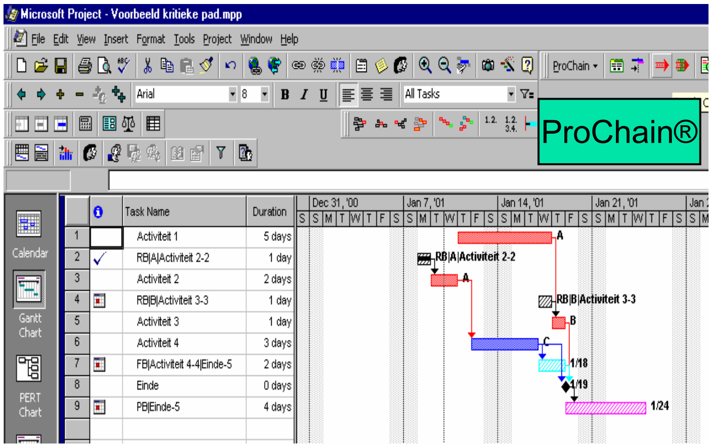
- "The last breakthrough in project management came out in about 1950 and since then there have been immense amounts of articles published on project management, with essentially no new ideas"

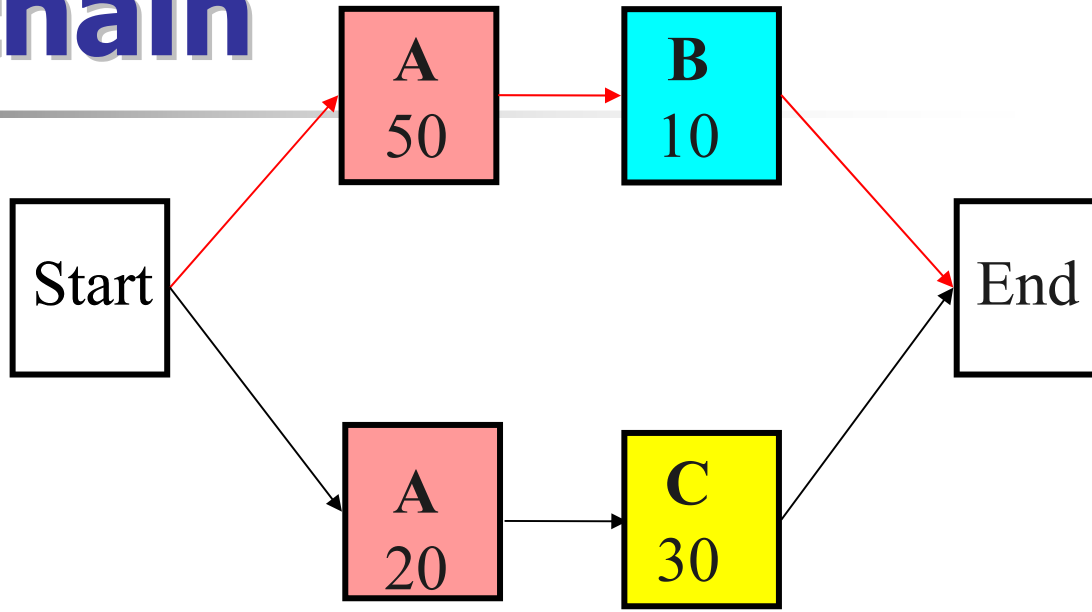
  Cabanis-Brewin, J., 1999, "So…So What??" – Debate Over CCPM Gets a Verbal Shrug from TOC Guru Goldratt, *PM Network*, December, 49-52.

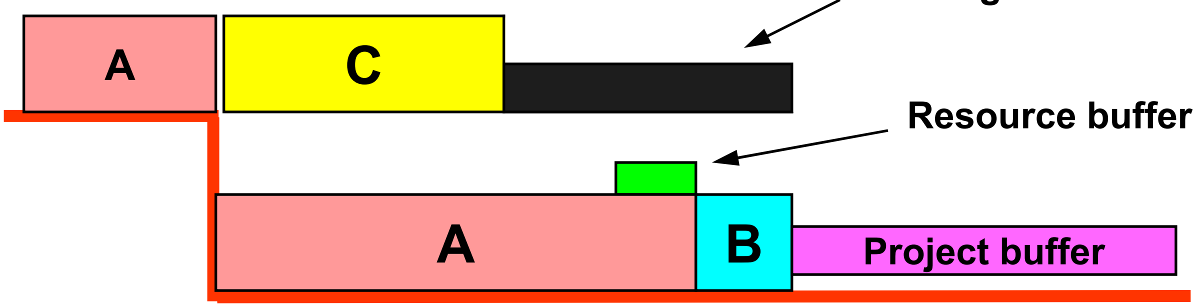
- "Critical Chain is the Best Project Management Philosophy and Tools I have Seen"

  Senior Project Manager of Procter and Gamble, Web Site ProChain Solutions, Inc.

Critical chain





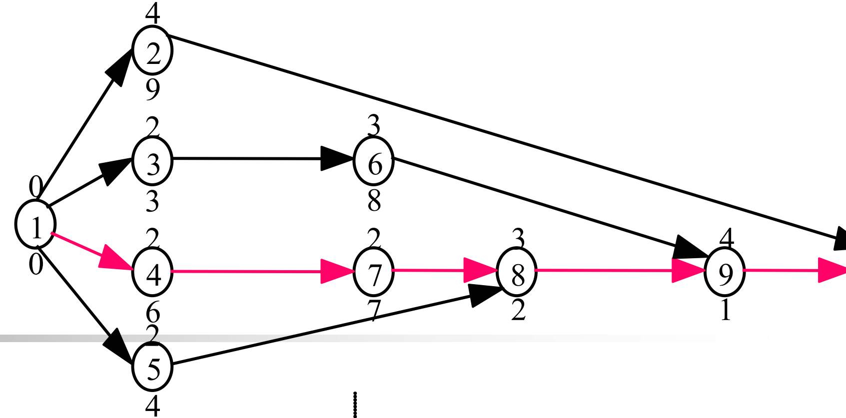


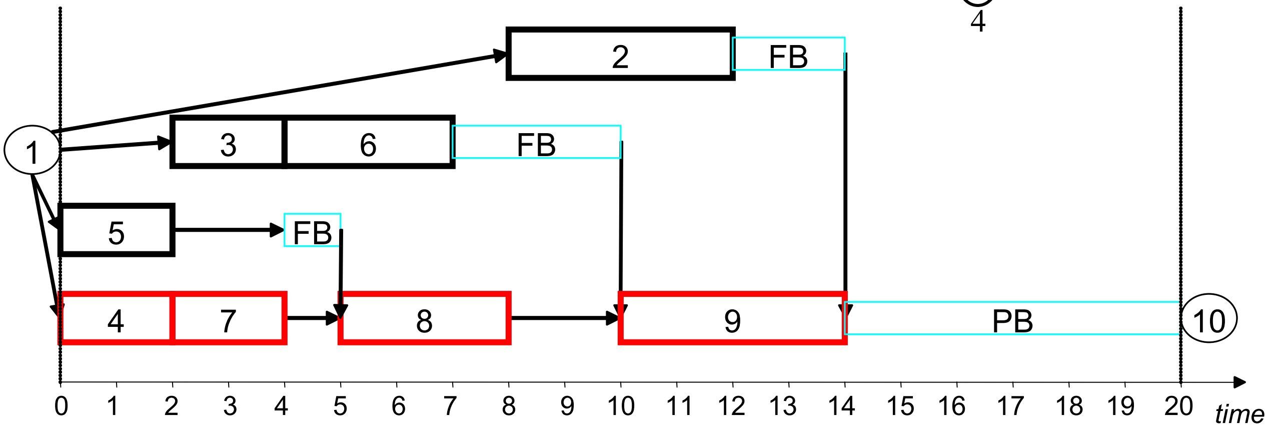
Does it work ?

- eye-opener
- oversimplification
- reveals the need for additional research

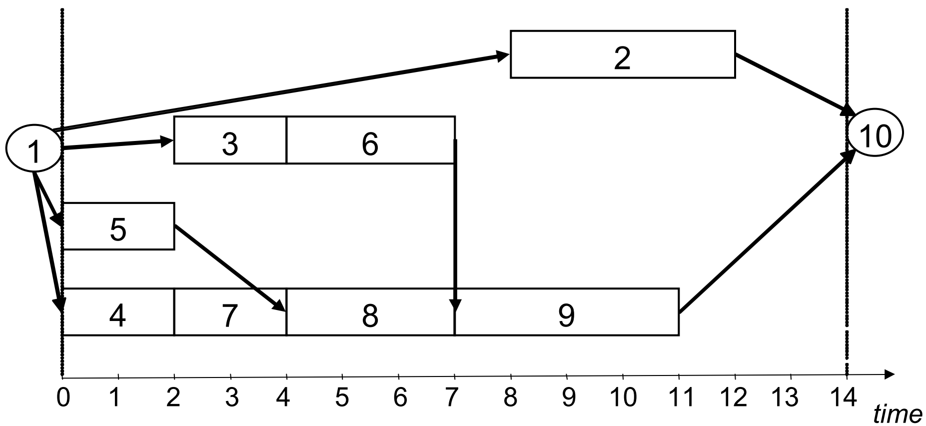
Herroelen, W. and R. Leus, 2001, On the Merits and Pitfalls of Critical Chain Scheduling, *Journal of Operations Management*, 19, 559-577.
Herroelen, W. and R. Leus, 2002, Critical Chain Project Scheduling: Do Not Oversimplify, *Project Management Journal*, 33(4),48-60.
Demeulemeester, E. & W. Herroelen, 2002, *Project Scheduling – A Research Handbook*, Kluwer Academic Publishers.

CC/BM



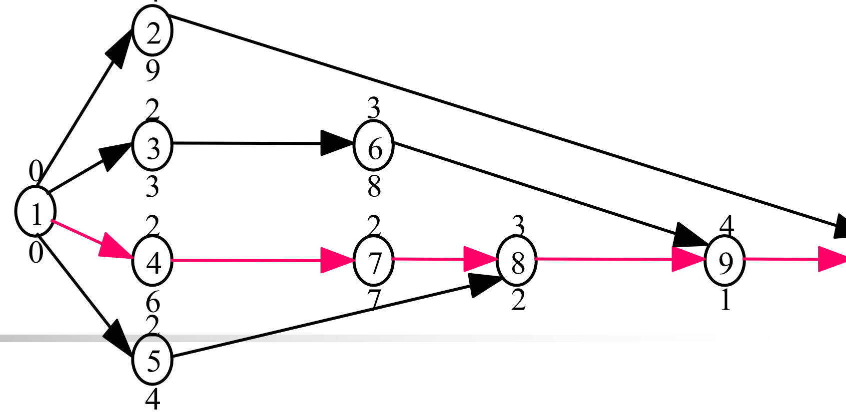


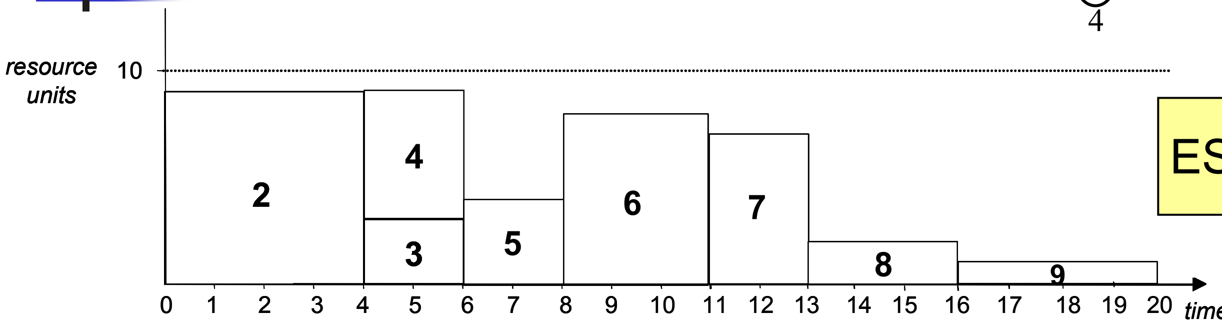
Buffered schedule
ProChain®
50% buffer sizing



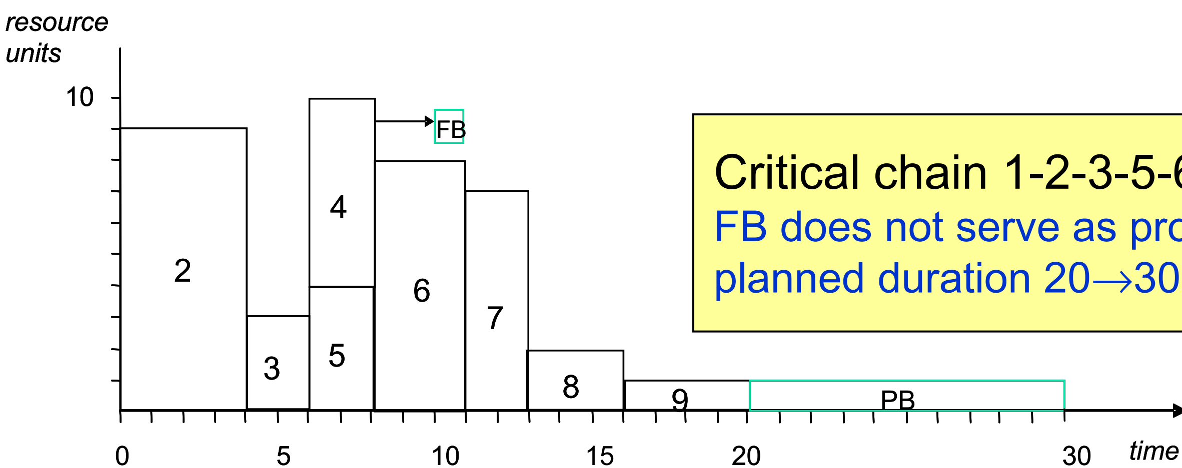
Projected schedule

CC is schedule dependent





ESS schedule

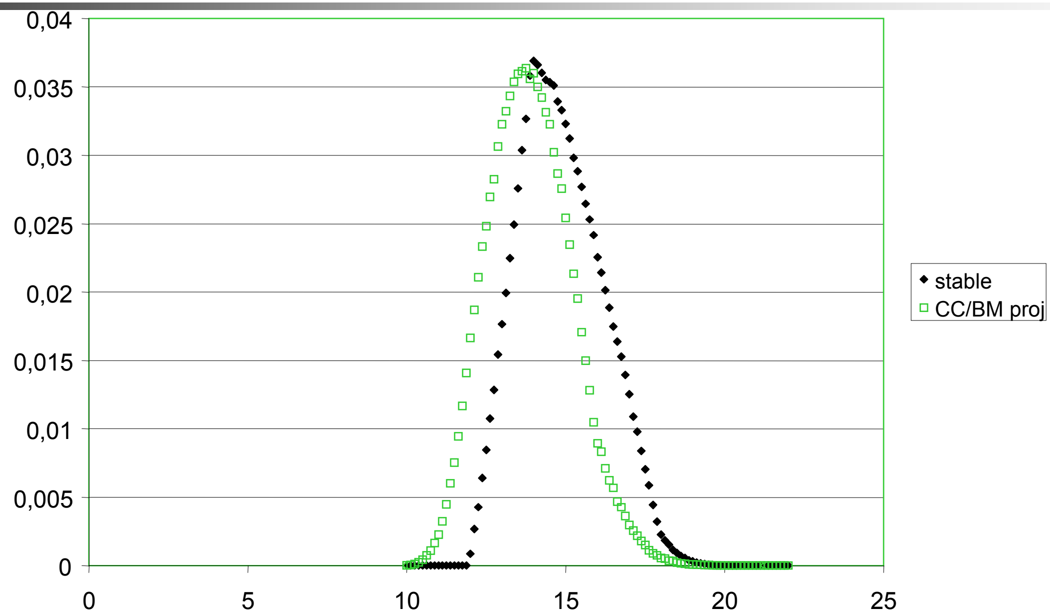


Critical chain 1-2-3-5-6-7-8-9-10

FB does not serve as proactive protective mechanism

planned duration 20→30

Makespan- stability trade-off?

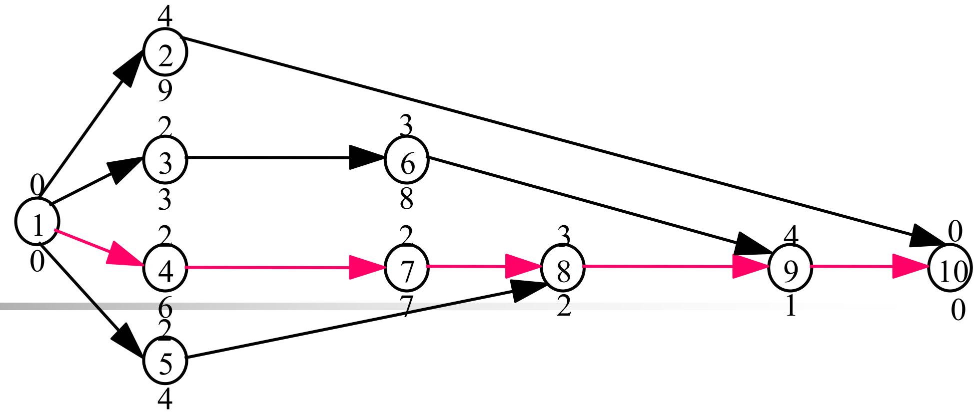


- project simulation: stable input schedule versus CC/BM projected schedule
- activity durations are stochastic variables: triangular distributions
    - mode=original duration
    - minimum=(initial duration)/2
    - maximum=(initial duration)×2

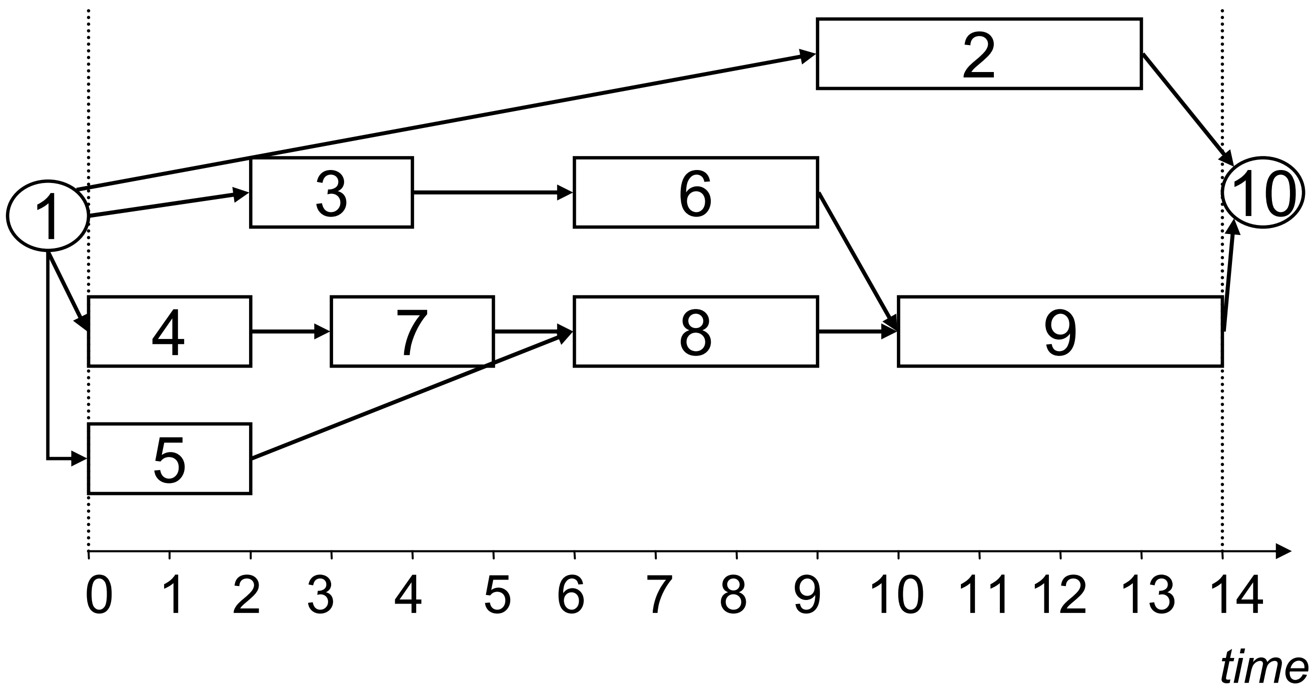
solution robust (stable) schedules 

exhibit acceptable quality robustness 

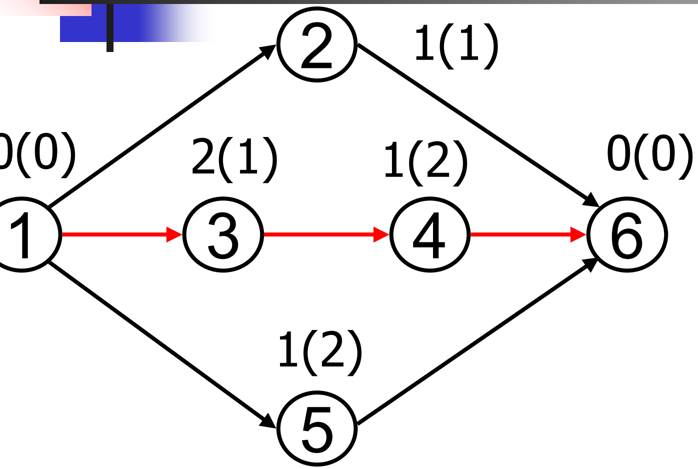
Stable resource feasible schedule

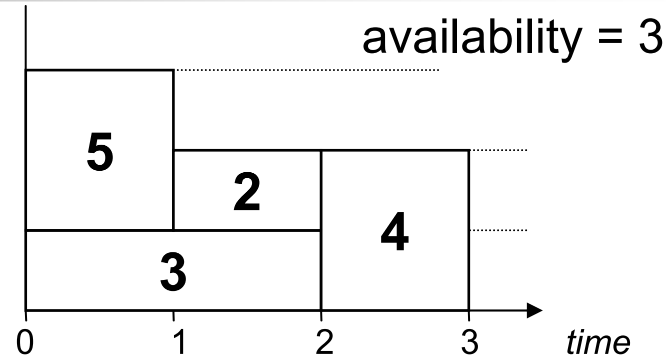


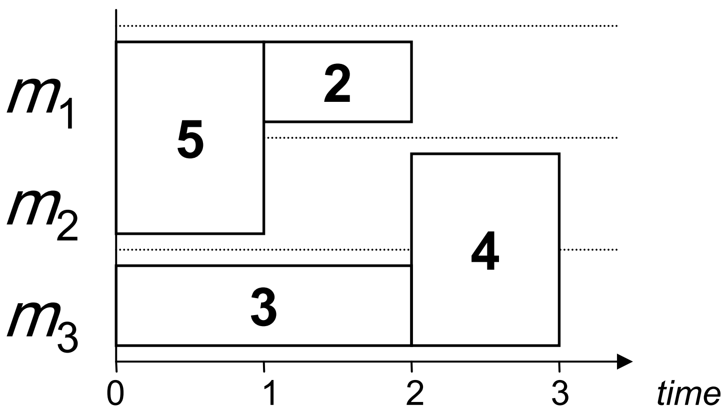
Project deadline=14
Equal disruption probabilities:
50%: +1 period
50%: +2 periods

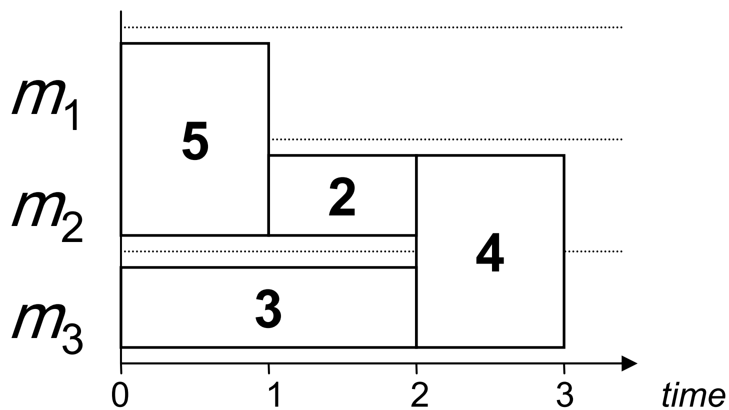


Resource allocation for stability

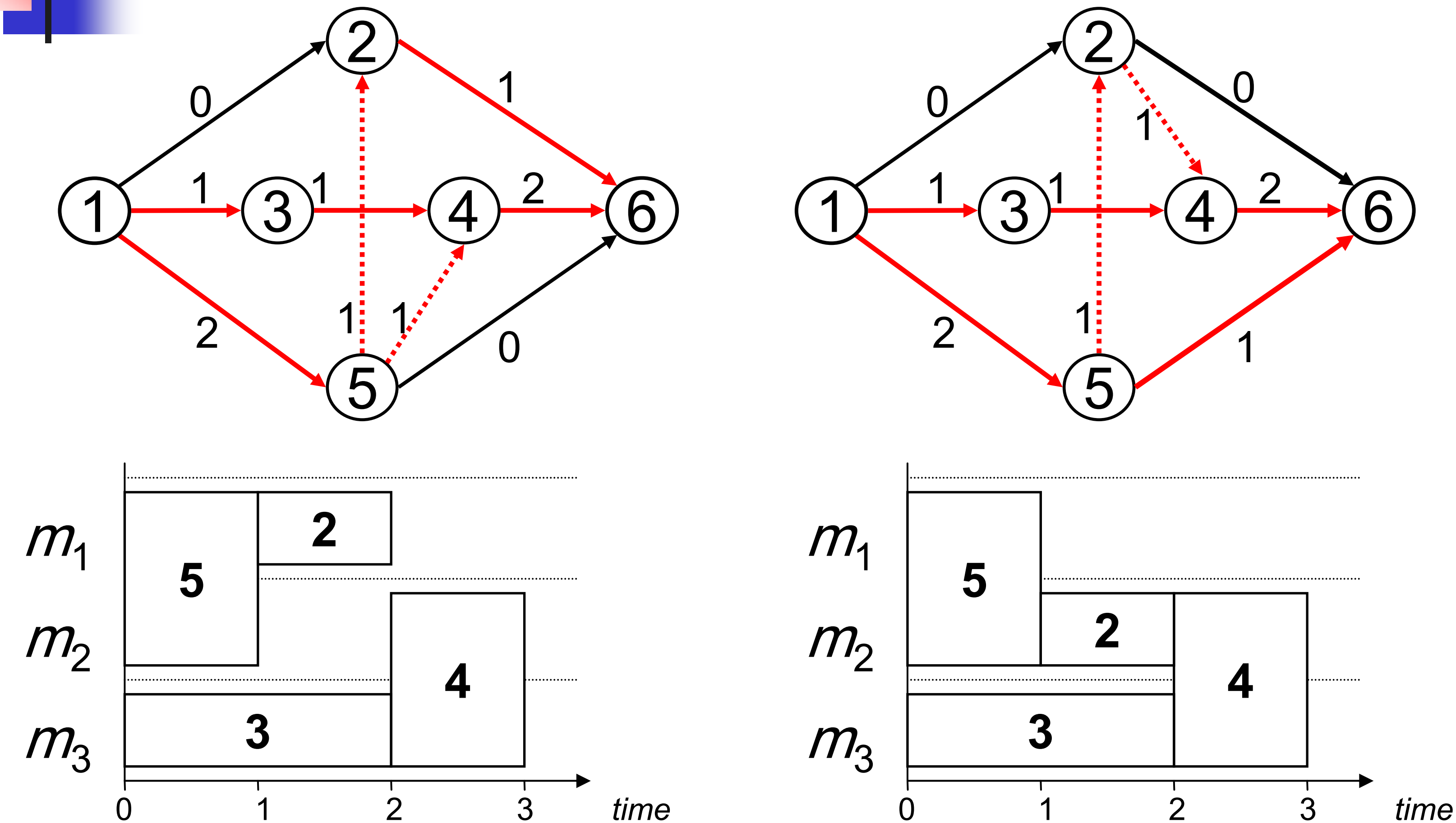








Resource allocation & resource flow networks
(Artiques & Roubellat 2000)



Resource flow network

- defines an ES(early start) policy
- minimal forbidden sets resolved by adding extra flow carrying arcs as additional precedence constraints

- feasible schedule $S(d)$ for a feasible scenario d of activity durations:
  - early start times in the augmented graph are computed by performing a forward CPM (longest path) pass
- assumption that resource allocation remains unchanged during project execution
- scheduling and resource allocation done sequentially
- resource allocation must be done “in line” with pre-schedule

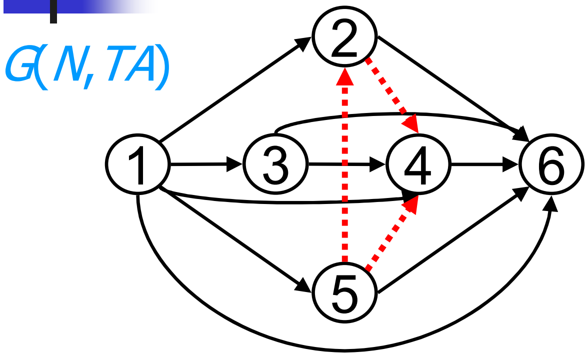
Notation & objective

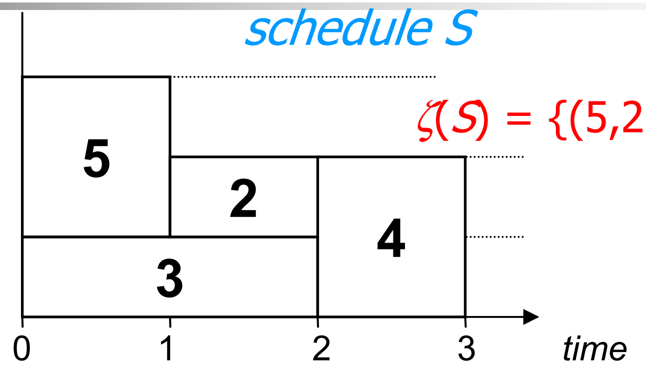
(Leus & Herroelen, IIE Transactions)

- N = set of activities/nodes in the project network
- A = precedence constraints
- $\mathcal{S}$ = pre-schedule: specifies start $s_i(\mathcal{S})$ and end $e_i(\mathcal{S})$ times
- d_i = pre-schedule duration of activity i, $s_i(\mathcal{S}) + d_i = e_i(\mathcal{S})$
- $\boldsymbol{D}_i$ = actual (stochastic) duration of activity i
- r_{ik} = resource requirement for resource type k for activity i
- a_k = per-period availability of resource type k
- f_{ijk} = units of resource type k sent from activity i to j
- TA = transitive closure of A

Problem: construct a resource flow for a feasible schedule S

Problem statement





$\zeta(S) = \{(5,2),(5,4),(2,4)\}$

$\zeta(S) = \{\ (i,j) \in N \times N \mid (i,j) \notin TA \text{ and } e_i(S) \leq s_j(S)\ \}$ for schedule S

$C(f) = \{\ (i,j) \in (N \times N) \setminus TA \mid \exists k \in R\text{: } f_{ijk} > 0\ \}$ for resource flow f

Add set M of arcs from $\zeta(S)$ to TA such that there exists a feasible resource flow f with $C(f) \subseteq M$ and $g(M,S)$ is minimized.
The resulting set of precedence constraints $TA \cup M$ defines an ES-policy.

Stability objective

$s_i(S)$= deterministic starting time of activity i in schedule S
$\boldsymbol{S}_i(M,S)$= actual starting time of activity i (stochastic variable)
(with $M \subseteq \zeta(S)$), based on recursion:

$$\boldsymbol{S}_1(M,S) = s_1(S) = 0$$

and

$$\boldsymbol{S}_j(M,S) = \max\{\ s_j(S)\ ;\ \max_{(i,j)\in A\cup M}\{\ \boldsymbol{S}_i(M,S) + \boldsymbol{D}_i\}\ \}$$

Stability measure:

for a schedule S and extra precedence relations M

$$\boxed{g(M,S) = E\left[\ \Sigma_{i\in N}\, c_i\left[\ \boldsymbol{S}_i(M,S) - s_i(S)\ \right]\ \right]}$$

with c_i = cost per unit time overrun on start of activity i

Branch-and-bound single resource type (Leus 2003)

- decision variables f_{ij}, $(i,j) \in TA \cup \zeta(S)$;
  f_{ij} has domain $[LB_{ij}\,;\ UB_{ij}]$, initialized as $[0;\min\{r_i, r_j\}] \cap \mathrm{IN}$
- at level p of the search tree, $TA \cup \zeta(S)$ can be partitioned into
  - $\alpha_p = \{(i,j) \mid LB_{ij} > 0\}$, the 'included' arcs,
  - $\nu_p = \{(i,j) \mid UB_{ij} = 0\}$, the 'forbidden' arcs, and
  - $\omega_p = \{(i,j) \mid LB_{ij} = 0 \text{ and } UB_{ij} > 0\}$, the 'undecided' arcs

  and we compute $M_p := \alpha_p \backslash TA$
- if a feasible flow exists in constructed network $G(N, TA \cup M_p)$, fathom, otherwise branch
- feasible flow test using maximum flow computations
- binary branching on arc (i,j) in $\zeta(S) \cap \omega_p$:
  - left branch: $LB_{ij} := 1$ (include into α_p and M_p)
  - right branch: $UB_{ij} := 0$ (forbid flow across (i,j) – include into ν_p)

Branch-and-bound

Computational results (Leus 2003)

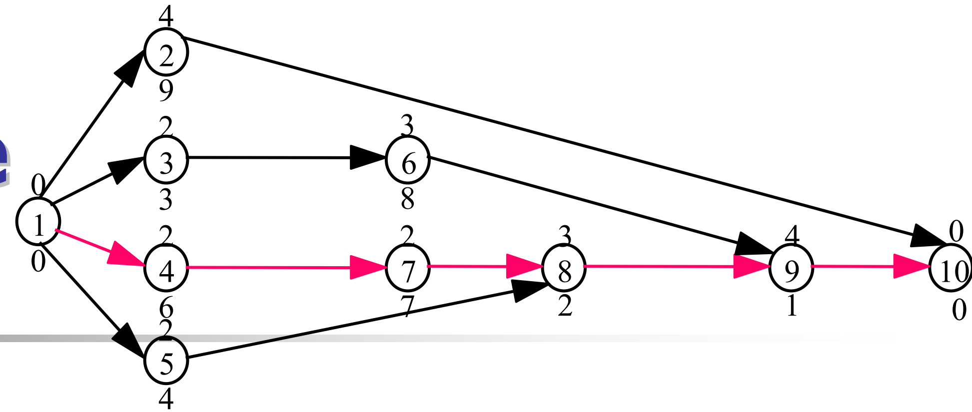
- Visual C++, Pentium III, 800Mhz, 128 Mb RAM
- 200 RanGen instances; OS values 0.2, 0.5; RF 0.7, 0.9; RC 0.2, 0.4; # activities 21, 31, 41 and 61
- schedule generated using B&B of Demeulemeester & Herroelen (1992) (truncated after 1 min)
- CPU-time limit = 150 seconds

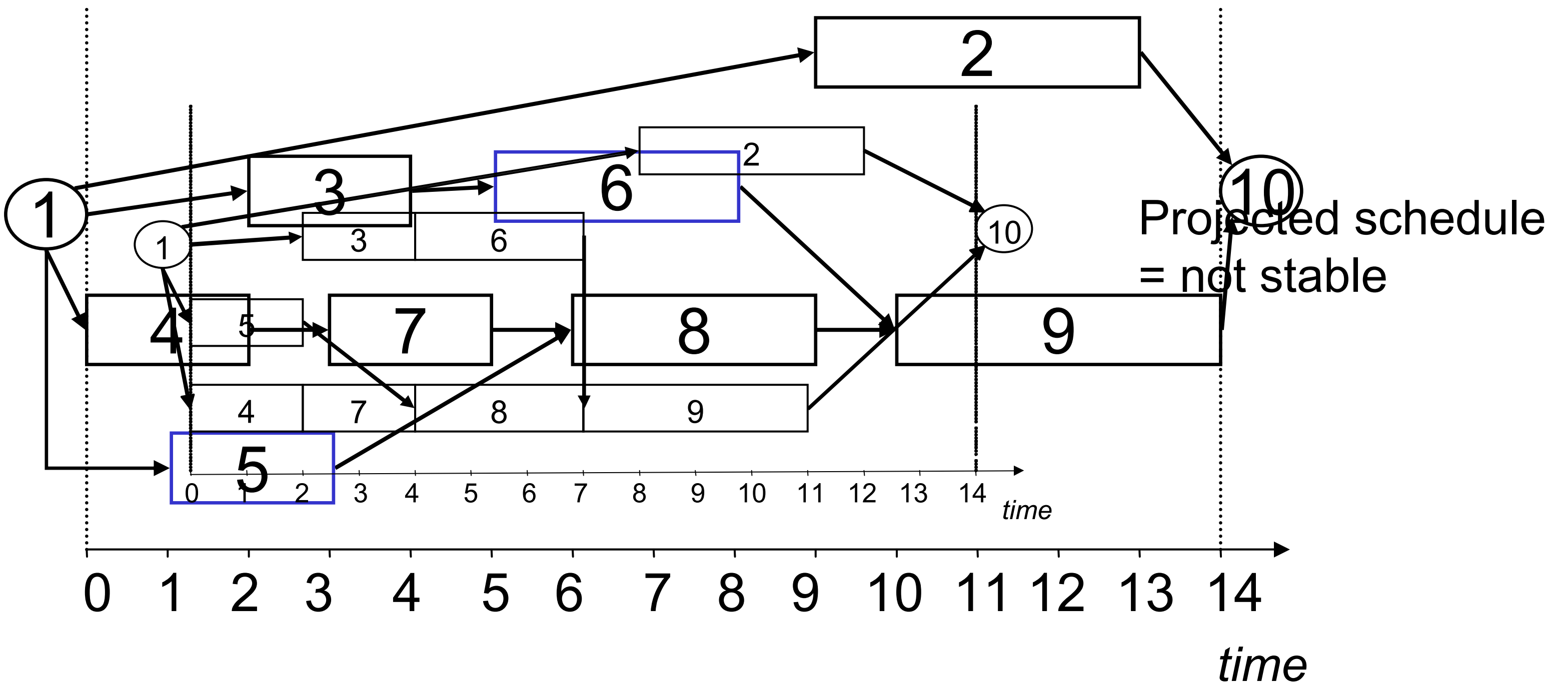
	n = 21	n = 31	n = 41	n = 61
avg. nr. nodes	1503	11018	14680	12971
avg. CPU (sec)	1.82	22.35	36.85	47.07
nr. optimal	100%	89.5%	81%	77.5%
avg %CPU simulation	96.11%	94.9%	95.69%	96.49%

Extensions

- multiple resource types
  - separate resource flow network for each resource type
  - adapt branching decisions and consistency tests
- integrated scheduling+resource allocation
  - trade-off between initial schedule length and stability
- exploit other than ES-policies with constant resource allocation during project execution
  - preselective policies with day-to-day changes in job assignments

Generate stable baseline schedules





- resource-constrained weighted earliness/tardiness procedure (Vanhoucke et al. 2001)
  - due date for each activity set equal to their completion time in unconstrained stable schedule; equal earliness and tardiness penalties
- activities are scheduled as close as possible to their initial time window
- simulation to test quality robustness using different reactive policies

Summary & conclusions

- generating quality and solution robust baseline schedules does make sense
  - co-ordination with outside parties
  - bottleneck resources in multi-project environment

- project planning practice (critical chain) has serious pitfalls and drawbacks
  - maximising stability and minimising due date overrun strongly NP-hard problems

- research opportunities
  - different stability objectives
  - different types of disturbances
  - generalised precedence relations
  - combination of different proactive/reactive policies

Questions?

If so: 

If not:  

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