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# A Theoretic and Practical Framework for Scheduling in a Stochastic Environment\*

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## Abstract

There are many systems and techniques that address stochastic planning and scheduling problems, based on distinct and sometimes opposite approaches, especially in terms of how generation and execution of the plan, or the schedule, are combined, and if and when knowledge about the uncertainties is taken into account. In many real-life problems, it appears that many of these approaches are needed and should be combined, which to our knowledge has never been done. In this paper, we propose a typology that distinguishes between proactive, progressive, and revision approaches. Then, focusing on scheduling and schedule execution, a theoretic model integrating those three approaches is defined. This model serves as a general template to implement a system that will fit specific application needs: we introduce and discuss our experimental prototypes which validate our model in part, and suggest how this framework could be extended to more general planning systems.

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# 1 Introduction

Scheduling usually starts from a given set of activities, which must satisfy a set of temporal and resource constraints, and search for a schedule in which precise start times are set and precise resources are allocated to each activity. Very often an optimal schedule is actually looked for, taking into account some optimization criteria (minimal makespan, minimal resource consumption, etc.). In Job-Shop Scheduling, usually addressed by Operations Research techniques, the set of activities to schedule comes from so-called jobs, consisting of sequences of predetermined activities, which must be included in a shop, accounting for resource demands. In Artificial Intelligence planning, the set of activities is produced by the planning search engine, which reasons about goals to achieve, available generic activities, and causality relations among them, to provide a plan of activities (i.e. instantiated activities ordered through their preconditions and effects), which usually disregards resource usage conflicts and may not commit to a precise complete ordering.

But the frontier between Artificial Intelligence planning and scheduling is not that clear, especially when time, resources and uncertainty must be considered [44]. This is why people are more and more interested in a more general planning and scheduling framework in which all kinds of planning and scheduling decisions to make to reach the eventually executed schedule are considered all at once: we propose in this paper to use the general term of *schedule generation* to refer to such an extended framework.

It is particularly true when uncertainties come into the picture: in that case, in both Job-Shop Scheduling and Artificial Intelligence planning, even the usual strict distinction between the generation and the execution of the schedule must be relaxed. Classical approaches for solving planning and scheduling problems are both predictive and deliberative, in that a schedule is designed offline and then sent to the execution controller, which can execute it online in a straightforward manner, but with no ability to reconsider anything. However, in practical applications, we have to plan or schedule with incomplete, imprecise, and/or uncertain data: simply executing a strictly and completely determined predictive schedule is not sufficient, as there is high chance that such a schedule will not fit the real situation that will arise. As we are going to see it, in stochastic environments, both planning (i.e. which activities to choose) and scheduling (i.e. when to start them and on which resources) decisions might either be anticipated at generation time, or postponed until execution time, when some uncertainties are resolved.

That is exactly the focus of this paper: we are interested in the way the generation and the execution of the effective schedule should be jointly considered to best fit uncertainty management needs. Which means we are not deeply concerned with the actual planning search or job-shop scheduling techniques that are used

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5 to generate that schedule. The questions we want to address are: How a global  
6 generation/execution loop should be designed? If available, should knowledge  
7 about possible deviations be used, and if yes, how could one integrate them in the  
8 solution process? How much one should commit to the predictive decisions made  
9 offline? Should online revision of such decisions be allowed? To what extent?  
10 Alternatively, how much of the decision process may be postponed and only be  
11 taken online? How do our choices influence the online efficiency, memory con-  
12 sumption, or the quality of the eventually executed schedule? In other words, our  
13 goal is to study planning and scheduling under uncertainty in terms of how and  
14 when decisions are made.  
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16 In this paper, we have chosen to focus on a basic optimal scheduling problem:  
17 we want to allocate resources and assign start times to a set of activities, so that  
18 temporal and resource constraints are satisfied and so as to optimize a given qual-  
19 ity metric. We will also use constraint-based techniques to validate our model.  
20 However, the work presented here can be extended to a more general schedule  
21 generation framework in which planning decisions are integrated, since we are  
22 interested in when and how various decisions get made, and not what those deci-  
23 sions are. Somehow, as we will see, some limited planning concerns already fit in  
24 the picture, since we allow alternative sequences of activities to be generated, the  
25 selection being done at execution time, in a conditional planning like manner.  
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27 After having proposed some basic definitions in Section 3, we provide a brief  
28 review of some relevant work in planning and scheduling to exhibit a thorough  
29 classification of the techniques for planning and scheduling under uncertainty,  
30 discussing their strengths and weaknesses. We show that in real-life applications,  
31 mixing those techniques within a single system appears to be highly desirable.  
32 For that purpose we propose (in Section 4) a new conceptual model encompass-  
33 ing both the generation and the execution of schedules and in which a variety of  
34 techniques for dealing with uncertainty can be concurrently implemented. Sec-  
35 tion 5 presents experimental prototypes that validate our model in part. Finally  
36 we discuss how our framework can be extended to planning in Section 6.  
37

## 38 2 Some Basic Definitions 39

40 Before presenting our formal model in Section 4, we informally describe the prob-  
41 lem we are interested in, only to introduce basic concepts and properties that are  
42 relevant when addressing schedule generation and schedules in an uncertain envi-  
43 ronment.  
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45 A standard scheduling problem comprises a set of activities and a set of re-  
46 sources. Each activity has a duration, there are temporal relations between ac-  
47 tivities, and each resource has a limited capacity. The objective is to assign re-  
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sources and times to activities given temporal and resource constraints. To achieve this, we have to make scheduling decisions which are typically: choosing allocations, choosing sequences, and setting activity start times. We endow the traditional scheduling system with the capability of representing alternative subsets of partially-ordered activities, among which only one must be chosen and executed.

In general, scheduling problems are also optimization problems: typical optimization criteria are makespan, number of tardy activities, tardiness or allocation cost, etc.

If we assume an execution environment without uncertainty, one usually generates a schedule offline that is then executed online without any problem. There are, however, many possible sources of uncertainty in scheduling; e.g., some activity durations or some resource capacities are imprecise (as resources may break down).

We now give some definitions to avoid ambiguity of terms commonly used by different communities.

**Definition 2.1 (Complete Schedule)** *A complete schedule is the solution of a scheduling problem; i.e., all decisions are made: activity start times are set, resource allocations are done, sequencing decisions are made, and no alternative subsets of partially ordered activities remain.*

**Definition 2.2 (Flexible Schedule)** *A flexible schedule is an incomplete schedule: decisions have still to be made. Constraints are associated with a flexible schedule to restrict the set of complete schedules that can be derived from it.*

**Definition 2.3 (Conditional Schedule)** *A conditional schedule is a schedule in which distinct alternative subsets of partially ordered activities can be modeled. The schedule contains a condition to test for each choice between such alternatives at execution time.*

**Definition 2.4 (Predictive Schedule)** *A predictive schedule is a schedule that is generated before the end of its execution. A predictive schedule is generated before or during execution.*

We may have to modify predictive schedules to adapt them to online situations we do not know beforehand.

**Definition 2.5 (Executable Schedule)** *A schedule is executable at time  $t$ , if and only if it does not violate any constraint known at time  $t$ .*

**Definition 2.6 (Adaptive Scheduling System)** *An adaptive scheduling system is a system that is able to generate a new executable schedule whenever the currently executing schedule is no longer executable.*

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5     **Definition 2.7 (Robust Predictive Schedule)** A predictive schedule is said to be  
6     robust, if the quality of the eventually executed schedule is close to the quality  
7     of the predictive schedule. More formally, we can state that a predictive schedule  
8     with quality  $q_{\text{pred}}$  is  $\epsilon$ -robust for a given  $\epsilon$  and optimization criterion, if the quality,  
9      $q_{\text{exec}}$ , of the eventually executed schedule is such that  $(1 - \epsilon) \times q_{\text{pred}} \leq q_{\text{exec}} \leq$   
10     $(1 + \epsilon) \times q_{\text{pred}}$  given an execution controller and known online perturbations.  
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13     The possible values of  $\epsilon$  are in  $[0, 1]$ . We use  $\epsilon$  values to compare the robustness  
14     of predictive schedules: the smaller the  $\epsilon$  value, the more robust the predictive  
15     schedule.  
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18     **Definition 2.8 (Stable Predictive Schedule)** A predictive schedule is said to be  
19     stable, if the decisions made in the eventually executed schedule are close to the  
20     decisions made in the predictive schedule. More formally, we can state that a pre-  
21     dictive schedule containing the set of decisions  $DES_{\text{pred}}$  is  $\Delta$ -stable for a given  
22      $\Delta$ , if the set of decisions,  $DES_{\text{exec}}$ , in the eventually executed schedule is such  
23     that  $(1 - \Delta) \times |DES_{\text{pred}}| \leq |DES_{\text{pred}} \cap DES_{\text{exec}}|$  given an execution controller  
24     and known online perturbations.  
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27     The possible values of  $\Delta$  are in  $[0, 1]$ . We use  $\Delta$  values to compare the stability  
28     of predictive schedules: the smaller the  $\Delta$  value, the more stable the predictive  
29     schedule.  
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32     Stability and robustness may be independent as long as the quality of the pre-  
33     dictive schedule is low when compared to the optimal one: it will not be very  
34     difficult to maintain such a low quality online, still changing no decision. But as  
35     soon as the schedule quality is high enough the two may quickly become antago-  
36     nistic. A flexible schedule can be stable but not robust, which means that we do  
37     not change decisions but its actual quality (obtained after execution) deviates from  
38     its predictive quality (i.e. the quality we estimate before execution). On the con-  
39     trary, a flexible schedule can be robust but not stable in the case where we change  
40     decisions online to keep its effective quality after execution close enough to its  
41     predictive quality (i.e. the quality that was estimated before changing decisions).  
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### 45     3 Classification 46

47     In this section, we concisely describe a taxonomy of techniques for scheduling  
48     and planning under uncertainty that is independent of any specific representation  
49     or reasoning technique. Such classifications have already been done, especially  
50     in the Operations Research community (as in Billaut *et al.* [11], in Herroelen  
51     and Leus [23]), but none is totally satisfactory to our needs since they only dis-  
52     tinguish between offline and online techniques: proactive techniques take into  
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5 account knowledge about uncertainty to make decisions offline, while reactive  
6 techniques are used online to adapt the current schedule when an unpredicted  
7 event occurs such as a machine that breaks down. We go beyond this distinction  
8 and consider issues such as how, when, and what decisions are made, optimality  
9 requirements, etc.  
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### 13 3.1 Our Taxonomy in Brief

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15 We will in the following distinguish between three main families of techniques,  
16 which refer to three main ways of balancing schedule generation and execution:  
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18 **Proactive** techniques: such techniques stick to the classical idea of building a  
19 global solution at generation time, which will never be reconsidered at ex-  
20 ecution time. It is proactive (and not only predictive) in that knowledge  
21 about the uncertainties is taken into account in order to generate more reli-  
22 able schedules. To reach such a goal, one may  
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- 25 • generate one complete *generic* schedule which is proved to cover most  
26 cases, i.e. to execute correctly in most of the possible situations that  
27 will arise at execution time,
- 28 • generate a *flexible* solution, i.e. in which some decisions have not been  
29 made and are postponed until execution time,
- 30 • or generate a *conditional* solution, i.e. a solution in which various mu-  
31 tually exclusive decisions are developed, the one being effectively cho-  
32 sen being dependent on some condition which will only be observed  
33 at execution time.
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38 **Revision** techniques: such techniques generate a complete schedule in the classi-  
39 cal predictive way, and at execution time, whenever that solution does not fit  
40 the observed situation, it is revised, i.e. some of the decisions already made  
41 are modified through online schedule re-generation.  
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44 **Progressive** techniques: also known as continuous, such techniques generate and  
45 execute in the classical way, but only locally, in the short term: a new part  
46 of the global schedule is generated online, either at predefined timestamps,  
47 or whenever a condition expressing that some uncertainties are resolved is  
48 satisfied.  
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### 52 3.2 Proactive Techniques

53 A proactive technique takes into account the knowledge about uncertainty to pro-  
54 duce schedules that are more robust, more stable, or both more stable and more  
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5 robust than they would be without using this piece of information.  
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A first naive method for making a schedule insensitive to online perturbations is to produce offline a complete, predictive, robust schedule by taking into account the worst-case scenario. A more balanced view is to account for the knowledge on the possible deviations, and hence on all possible scenarios that may emerge online, to compute a complete but somehow *generic* solution that will be able to fit most of such situations. In other words, one generates a cautious complete schedule because one wishes to maximize the executability of such a solution online. That can be done for instance when one knows distributions (e.g. of probability) of the possible deviations, and can compute the solution that has the highest probability of success.

Different uncertainty models (probability distributions, possibility theory, etc.) can be used in proactive techniques for the representation of the problem, and for solving it (e.g., find the schedule that will have the highest probability that the makespan will not exceed a given value).

The research work of Dubois, Fargier, and Prade [17] is a good example of such *generic schedule* generation: they use fuzzy logic, tackling scheduling problems with fuzzy activity durations, and try to find a schedule that minimizes the possibility of performance less than a threshold. Standard search techniques are used but the constraint-satisfaction requirement is replaced by “reasonably sure that no constraint will be violated.” This is a realistic approach that lies between accepting only schedules that are sure to work and accepting a schedule without taking into account possible deviations.

Another approach consists in introducing some flexibility in the schedule: only a subset of decisions are made offline with a search, and the rest are made online with no or only very limited search; this is a kind of least-commitment approach with respect to decision-making since we only make decisions when information is more precise, and/or more certain. Morris, Muscettola, and Vidal [30] for instance maintain a simple temporal network representing a schedule with uncertain activity durations in which start times are not set: they provide algorithms to guarantee the executability of such schedules whatever the actual durations will be. Here we have a *flexible schedule*.

In the same vein, redundancy-based scheduling can also be considered as a proactive technique for scheduling. For example, Davenport, Gefflot, and Beck proposed an approach in which they add slack times to critical activities, i.e. the activities that are allocated on possibly breakable resources [15]. This work extended the Master’s thesis of Gao [20]. The new temporally protected problem can then be tackled with techniques usually used to solve deterministic scheduling problems; e.g., constraint propagation algorithms can be used to make tree search more efficient. More precisely, activity durations are set to be longer than the original ones to generate a predictive schedule that can face possible machine

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5 breakdowns. The activity durations used are based on breakdown statistics; e.g.,  
6 the mean time between failures or mean time to repair may be used.  
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8 Experiments on job-shop problems show that this technique significantly reduces  
9 the gap between the predictive quality and the effective quality, but it results in an  
10 increase of tardiness. However, the temporal protection given to activity  $a_1$ , which  
11 should allow the following activity  $a_2$  allocated to the same resource to start earlier  
12 if no breakdown occurs, can be “lost,” if a constraint prevents  $a_2$  from starting  
13 earlier. This observation is at the outset of two methods, time-window slack and  
14 focused time-windows slack, presented by Davenport, Gefflot, and Beck who pro-  
15 posed to post additional constraints such that each activity has a minimum tem-  
16 poral slack. Simulation results show that these two methods are able to generate  
17 schedules whose tardiness is smaller than the tardiness of the schedules deter-  
18 mined by the temporal protection technique. It is also shown that effective quality  
19 is predicted more accurately; i.e., predictive quality *a priori* (before execution) is  
20 closer to the actual quality, observed *a posteriori* (after execution). Although this  
21 approach is not based on real theoretical foundations, it is simple and pragmatic  
22 such that it can be readily applied to a real scheduling problem.  
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24 Generating partial-order schedules (POSs) is another example of such flexible  
25 approaches: a subset of sequencing decisions are made offline, and the remainder  
26 being made online with using a dispatching rule. For example, Policella *et al.* con-  
27 sidered the problem of generating POSs for problems with discrete resources [36].  
28 They tackle resource-constrained project scheduling problems using filtering algo-  
29 rithms and propose two orthogonal procedures for constructing a POS. The first,  
30 which is called the resource-envelope-based approach, uses computed bounds on  
31 cumulative resource usage to identify potential resource conflicts, and progres-  
32 sively reduces the total set of temporally feasible solutions into a smaller set of  
33 resource-feasible solutions by resolving detected conflicts.<sup>1</sup> The second, referred  
34 to as the earliest-start-time approach, instead uses conflict analysis of an earliest-  
35 start-time solution to generate an initial fixed-time schedule, and then expands this  
36 solution to a set of resource-feasible solutions in a post-processing step. As might  
37 be expected, the second approach, by virtue of its more focused analysis, is found  
38 to be a more efficient POS generator based on experimental results.  
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40 Wu, Byeon, and Storer proposed another way of producing partial-order sched-  
41 ules [49]. They identify a critical subset of decisions that, to a large extent, dictate  
42 global schedule performance. These critical decisions are made offline, and the  
43 rest of the decisions are done online. They address job-shop problems in which  
44 a weight and a due date are associated to each job. Before execution, they solve  
45 an Order Assignment Problem (OAP) optimally; i.e., they partition activities into  
46 groups such that original precedence constraints are respected, and introduce a  
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48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 <sup>1</sup>Earlier, Muscettola proposed such an approach [33].

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5 set of precedence constraints to the problem between groups. Each possible as-  
6 signment of activities to these groups is associated with a cost representing the  
7 minimum possible weighted tardiness given the precedence constraints imposed  
8 between the groups; the objective is to choose the assignment that minimizes this  
9 cost. After solving the OAP, we get a job-shop scheduling problem that can be  
10 solved to evaluate the partition (the solution of the OAP).

11 Another way to address flexibility is to build a *conditional schedule*: every-  
12 thing is set but with alternative branches. Just-In-Case scheduling illustrates this  
13 method perfectly. It is plainly a proactive technique for scheduling. Drummond,  
14 Bresina, and Swanson proposed a contingent scheduling method, called Just-In-  
15 Case (JIC) scheduling, applied to a real-world telescope observation scheduling  
16 problem where observation durations are uncertain [16]. The solution consists in  
17 building a contingent schedule that takes into account likely failures of the auto-  
18 matic telescope and that does no waste valuable observing time. The objective is  
19 to increase the percentage of the schedule that can be executed without breakage  
20 by assuming there is a scheduling algorithm for solving the deterministic problem,  
21 and probability distributions of the observation durations are known; i.e., the mean  
22 values and the standard deviations are known. The JIC scheduling proceeds as fol-  
23 lows. Before execution, a schedule is generated assuming deterministic durations,  
24 then the algorithm identifies the activity with the highest break probability, the  
25 break point in the schedule is split into two hypothetical cases accordingly: one  
26 in which the schedule breaks and one in which it does not, and then the procedure  
27 finds an alternative schedule assuming the breakage. There are of course several  
28 likely breakages, so the procedure is applied several times. During execution, if  
29 there is no breakage, we keep executing the same schedule, otherwise, if the situ-  
30 ation is covered by the contingent schedule, we switch to a new schedule. This is  
31 a successful, real-world solution. The online portion is trivial since we only have  
32 to switch to an alternative schedule. However, this method is applied to a one-  
33 machine scheduling problem, and the combinatorial complexity seems to hinder  
34 its generalization to multiple machine problems.

35 A last example of conditional models can be found in the work in planning  
36 made by Tsamardinos, Vidal, and Pollack [46]. They propose a conditional tem-  
37 poral model in which a plan contains alternative branches depending on the state  
38 of the world. They analyze the conditions under which observations of such states  
39 must be made prior to branching, taking into account quantitative temporal con-  
40 straints (durations).

41 In addition, all the literature on Markov Decision Processes [37, 21] (which  
42 we will not develop here) can be viewed as examples of such conditional proactive  
43 models for planning.

### 3.3 Revision Techniques

Revision techniques consist in changing decisions during execution when it is necessary or desirable; e.g., we change decisions when the current predictive schedule becomes inconsistent, when estimated quality deviates too much from the predictive one, or in a more opportunistic way when a positive event occurs (for example, an activity finishes earlier than expected). In other words, we need an execution-monitoring system able to react and indicate when it is relevant to change decisions of the current predictive schedule. Note that such a procedure is called rescheduling, and a revision method is usually called predictive-reactive in the literature [38, 27, 11].

Sadeh, Osuka, and Schelbach, with the system called Micro-Boss, worked on a job-shop scheduling problem where machines may break down [40]. Large Neighborhood Search is used when partially rescheduling; i.e., a set of activities to unschedule (“conflict propagation”) is identified by using recovery rules: all the activities whose start times are changed by these rules are unscheduled. The original scheduling algorithm is then used to reschedule the unscheduled activities. Simple rules are followed for identifying the neighborhood and for conflict propagation:

- Right Shift Rule: activities are moved later in time while preserving sequence;
- Right Shift and Jump Rule: activities are moved later in time, jumping over ones that do not need to be moved.

The idea is to use a simple rule to quickly repair each schedule. When fully rescheduling Micro-Boss is used. This system focuses on resource conflicts on a limited time period. Micro-Boss has the ability to detect the emergence of new bottlenecks during the construction of the schedule and change the current scheduling strategy. Micro-Boss is also able to schedule an entire bottleneck (or at least a large chunk of it); i.e., it considers resource conflicts over the complete time horizon. Resource contention is monitored during the construction of a schedule, and the problem-solving effort is constantly redirected towards the most serious bottleneck resource.

During execution, dispatching rules are used to adapt the current schedule. Activities are scheduled one at a time; i.e., every time a machine becomes free, a ranking index is computed for each remaining activity, and the activity with the highest ranking index is then selected to be processed next as follows:

- Weighted Shortest Processing Time (WSPT) tries to reduce overall tardiness by giving priority to short jobs and taking into account the priority of each job;

- Slack per Remaining Processing Time (SRPT) combines the Minimum Slack (MSLACK) rule and the durations of the remaining activities; MSLACK computes the slack time of each activity with respect to the due date of its job and selects the activity with the smallest slack time;
- Weighted Cost OVER Time (WCOVERT) combines WSPT, the expected waiting time of each remaining activity, and MSLACK;
- Apparent Tardiness Cost (ATC) combines WSPT and MSLACK.

According to Sadeh these heuristics are well suited to solving problems in which some activities have to be performed within non-relaxable time windows as well as repairing schedules in the face of contingencies [39]. Using Micro-Boss is a reasonable, pragmatic approach where there is no explicit reasoning about time to find a solution. The system does not generate stable schedules: the number of activities to reschedule may be large.

Another work of interest about partial rescheduling was done by Smith [45]. OPIS is a full scheduling system based on repeatedly reacting to events. It is a more sophisticated reasoning mechanism for analysis of conflicts than Micro-Boss on the basis of constraint propagation. This approach to incremental management of schedules is based on a view of scheduling as an iterative, constraint-based process.

El Sakkout and Wallace used a different approach to fully reschedule [41]. Their approach consists in a minimal-perturbation rescheduling. Given a schedule and a reduction in resource capacity, their system has to find a new schedule which minimizes the sum of absolute deviations from activity start times in the original schedule. The main idea is to combine Linear Programming (LP) and Constraint Programming (CP), and using potentially good assignments (probes) to solve the problem. The cost function for measuring change involves a kind of earliness/tardiness which is usually well solved by Simplex, if resource constraints are relaxed. One represents temporal constraints and the cost function in a linear program and temporal constraints and resource capacity constraints in CP. One uses the relaxed optimal start times from the linear program to drive the CP branching heuristic; i.e., one adds new linear constraints in the linear program when one branches in the decision tree (CP model) and propagates these decisions. This technique is named “Probe Backtrack Search.” The experiment consists of one (non-unary) resource with a given schedule; when an event reduces resource capacity over some time interval the system reschedules. This method is only practical in situations where the time-to-solve is irrelevant. The optimization criterion of the original schedule is ignored. Their experiments show that this hybrid branch-and-bound technique is better than pure CP, other combinations of CP and LP, and pure Mixed Integer Programming (MIP). These hybrid algorithms are

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5 not intended to be used online; to be considered revision solution techniques,  
6 the solution approach has to be adapted to take into account the execution of sched-  
7 ules; i.e., unary temporal constraints are added to the problem during execution  
8 corresponding to committed activity start and end times.  
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10 In AI planning, Wang and Chien [48] focused on replanning which preserves  
11 elements of the original (predictive) plan in order to use more reliable domain  
12 knowledge and to facilitate user understanding of produced plans. They presented  
13 empirical results documenting the effectiveness of these techniques in a NASA  
14 antenna operations application. The authors assume there is a default value for  
15 each state goal. They suppose there are well-known methods (activities) for es-  
16 tablishing the default value for each state goal. They assume the original plan is  
17 applicable from a state where each relevant state goal is at its default value. When  
18 an unexpected state change occurs, the replanning algorithm then re-uses as much  
19 of the original plan as possible while minimizing the amount of re-execution. The  
20 replanner returns a plan consisting of the activities that need to be re-executed and  
21 those not executed, as well as additional ordering constraints.  
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### 27 **3.4 Progressive Techniques**

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29 The idea behind progressive techniques is to interleave scheduling and execution,  
30 by solving the problem piece by piece, where each piece can correspond to the  
31 activities in a time window for example. Reasoning is done as a background  
32 task online; i.e., we can afford more time to search, we incrementally commit to  
33 scheduling decisions periodically or when new information arrives, and no deci-  
34 sions are changed.  
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36 One way of proceeding when using a progressive approach is to select and  
37 schedule new subsets of activities to extend the current executing schedule on a  
38 gliding time horizon. A decision is made when the uncertainty level of the infor-  
39 mation has become low enough, and/or when the *anticipation time horizon*, the  
40 interval between the current time and the expected end time of the last scheduled  
41 activity, has become too small. We can distinguish between the *commitment time*  
42 *horizon*, which is the temporal window in which decisions that are taken should  
43 not be reconsidered during execution, and the *reasoning time horizon*, which is the  
44 temporal window in which possible actions and expected events are considered for  
45 the search process to make relevant decisions. These two time horizons overlap  
46 but are not necessarily equal. One thus needs an execution-monitoring system able  
47 to react and indicate when we have to make new decisions, what information to  
48 reason about to make new decisions, and what decisions to make.<sup>2</sup> A short-term  
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54 <sup>2</sup>Alternatively, new subsets of activities can simply be integrated periodically, and so no com-  
55 plex conditions are monitored.  
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5 schedule on which decisions are made is sometimes called an “overlapping plan”  
6 in manufacturing.  
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8 Vidal, Ghallab, and Alami [47] presented an approach in which they allocate  
9 container transfer activities in a harbor to robots only as long as temporal uncer-  
10 tainty remains low enough to be reasonably sure that the chosen robot will actually  
11 be the best to choose. The objective is to minimize makespan. There is an antici-  
12 pation time horizon, but the commitment time horizon merges with the reasoning  
13 time horizon.  
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### 17 **3.5 Mixed approaches**

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19 Interestingly, there are not that many approaches that combine different techniques  
20 along the classification that we just proposed. Most of them combine a revision  
21 approach with a progressive approach.  
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23 In the CASPER planning system [14], planning is done on board an autono-  
24 mous spacecraft, mixing a replanning process when a failure occurs or a science  
25 opportunity is observed (i.e., new goal specifications are added to the planning  
26 problem), and continuous planning. At each time, the system updates the current  
27 plan, goals, state, and next predictive states by using a telescoping-time horizon  
28 approach. The current plan is generated given what is expected to happen. When  
29 a plan failure happens, an iterative-repair procedure, which is a greedy search  
30 algorithm, is applied in the short-term horizon to change allocations, move, renew,  
31 or remove actions.  
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33 Branke and Mattfeld tackled dynamic job-shop scheduling problems making  
34 decisions on a gliding time horizon and changing some decisions [12]. Note,  
35 however, that the progressive approach is not chosen by the authors but required  
36 to solve such a dynamic problem in which jobs arrive stochastically over time. No  
37 monitoring of execution is used to decide when to select and schedule activities.  
38 Activity durations are deterministic. Each job is associated with a due date, and  
39 the goal is to minimize the summed tardiness. An evolutionary algorithm is im-  
40 plemented to find schedules with short makespans and low early idle times. The  
41 generation-execution approach proceeds as follows. An initial schedule is created  
42 with the set of known jobs and the execution of this schedule begins. At some  
43 point a new job arrives. All the already executed and executing activities are re-  
44 moved from the scheduling problem, the new job is added and a new solution  
45 is found by changing some decisions. When the final activity in a job executes,  
46 the contribution to the summed tardiness is calculated. There is no anticipation  
47 time horizon, and the commitment time horizon merges with the reasoning time  
48 horizon.  
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50 Shafaei and Brunn published an empirical study of a number of dispatching  
51 rules on the basis of a gliding-time-horizon approach for a dynamic job-shop en-  
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5 vironment [42]. The first purpose of the study was to find the best dispatching  
6 rule, and the second was to investigate the effects of the rescheduling interval on  
7 performance and examine whether there is a policy that can always improve per-  
8 formance. Activity durations are imprecise and randomly picked with equiproba-  
9 bility in ranges. The job arrival rate follows a Poisson distribution, and shop load  
10 is either heavy or moderate. A release date and a due date are associated with each  
11 job. The performance measure considered is an economic objective, which is to  
12 minimize the sum of the cost of starting jobs too early and the cost of work-in-  
13 progress inventory and tardiness. In general, under tight due-date conditions, the  
14 rescheduling interval has a much more significant effect on performance than un-  
15 der loose due-date conditions: the smaller the interval, the lower the costs. There  
16 is no anticipation time horizon, and the commitment time horizon merges with the  
17 reasoning time horizon.  
18

19 In another report, Shafaei and Brunn investigated how efficient these dispatching  
20 rules are in a dynamic and stochastic shop environment [43]. The number of  
21 activities per job is uniformly sampled, job routes are randomly selected, activity  
22 durations are imprecise, and machines may break down: machine breakdown  
23 intervals and repair times follow exponential probability distributions. The sim-  
24 ulation results, under various conditions in a balanced and unbalanced shop, are  
25 presented and the effects of the rescheduling interval and operational factors in-  
26 cluding shop load conditions and a bottleneck on the schedule quality are studied.  
27 They conclude that more frequent rescheduling generally improves performance  
28 in an uncertain situation. There is no anticipation time horizon, and the commit-  
29 ment time horizon merges with the reasoning time horizon.  
30

31 There is one approach which, as opposed to the previous ones, does mix proac-  
32 tive (instead of progressive) and revision techniques: the one presented by Bresina  
33 *et al.* [13], which deals with planning for space rover applications. The authors  
34 claim that it is not possible to do timely replanning on board the rover when a  
35 failure occurs. They have to deal with uncertainty about the power required, the  
36 necessary data storage, the position and orientation of the rover, and environmen-  
37 tal factors that influence actions, such as soil characteristics, etc. Therefore, it  
38 is necessary to plan in advance for some of the potential contingencies. But the  
39 planner can only afford to account for the “important” contingencies, and must  
40 leave the rest to run-time replanning. They are also concerned with looking for  
41 what we call a *generic* (though flexible here) schedule, through decision-theoretic  
42 planning: their objective is to find the plan with the maximum expected utility:  
43 each goal has a value or award associated with it, and an action’s uncertain effects  
44 on continuous variables are characterized by probability distributions.  
45

46 In his PhD dissertation, Hildum [24] presented the Dynamic Scheduling Sys-  
47 tem (DSS), an agenda-based blackboard system, which is capable of dealing with  
48 a wide range of dynamic Resource-Constrained Scheduling Problems, and pro-  
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ducing quality schedules under a variety of real-world conditions. It handles a number of additional domain complexities, such as inter-order tasks (common tasks that are shared by two or more jobs) and mobile resources with significant travel requirements. The solution approach of DSS is a proactive-revision-progression approach, since flexible and stable schedules (with slack time) are generated when a resource brakes down or a new order arrives. Slack time is inserted into the schedules because some resource allocations are still to be done and there are alternative and mobile resources. This solution approach is applied to industrial applications. Furthermore, the basic DSS problem-solving method is essentially a multiple-attribute, dynamic heuristic approach that focuses on the most urgent unsolved subproblem at any point in time. The system can deal with both hard and soft constraints and uses constraint programming techniques.

### 3.6 Discussion

We can compare the three families of techniques with respect to the following criteria: online memory need, online CPU need, schedule robustness and stability, commitment time horizon, and use of the knowledge about uncertainty.

Revision techniques do not consume a lot of memory online, since we only have to store one schedule. They may require a lot of CPU online, depending on the time spent on rescheduling, which may be done through backtrack search. And, since one will usually not have enough time to process a complete search, the quality of revised solutions may quickly degrade, which means the robustness of schedules is not guaranteed. Stability may be very low, if we change a lot of decisions. Commitment time horizon could be expected to be high, as one computes a global schedule, but it will effectively be very short, since we do not know in advance what decisions will be changed online. The knowledge about uncertainty is not taken into account to make decisions.

Online memory may vary for proactive techniques, depending on whether we have to store one or more schedules: conditional schedules may require a lot of memory. In general, online computational need is low, since we do not have to search online for solutions with backtracking. We can expect to generate highly robust or stable schedules, since we take into account what may occur online to make decisions, but with generic approaches and sometimes with flexible schedules the solution found amounts to a compromise which does not assure to get the highest quality solution in the observed situation. That is not the case with conditional approaches where each branch is expected to be the optimal sub-schedule in the matching situation. The commitment time horizon appears to be long with flexible schedules, but that is at the price of not taking all decisions. In conditional approaches, the commitment time horizon is short as alternatives prevent from being committed to one specific branch.

Progressive techniques permit us to limit our online memory need to the minimum, since we only store a piece of schedule. The requirement in CPU online is balanced, since we need to search but only to solve sub-problems. Decisions are made with a more or less short commitment time horizon and a time granularity. This limited commitment time horizon may prevent from getting very robust schedules, since one loses a global view of the problem to solve, but that actually depends a lot on the type of decisions which are done progressively. If precise scheduling (as in job-shop scheduling) might often be done only in the short term with no loss in the global optimality, that is of course not the case with classical planning decisions, where one looks for a causal sequence from the initial state to the final goal state, and hence can hardly do without a global view of the solution. Continuous planning is actually only feasible in applications where new goals arrive periodically and can be treated separately and successively, as in [14]. However, this family of techniques always generates stable schedules.

	Online memory need	Online CPU need	Quality and Robustness	Stability	Commitment time horizon	Knowledge about uncertainty
Revision	Average	High	Low	Low	Very short	No
Proactive generic	Low	Very low	Average	Very high	Long	Yes
Proactive incomplete	Low	Low	High	Very high	Long	Yes
Proactive conditional	Very high	Low	Very High	Very high	Short	Yes
Progressive	Very low	Average	Average	Very high	Short	No

Table 1: The properties of each family of solution-generation techniques.

Table 1 synthesizes the different properties of each family of solution-generation techniques used to handle uncertainty, change, or both for scheduling. These features can help a user to choose a technique in a specific application domain: if memory usage is limited, then conditional schedules are probably not the right answer; or if optimality is a key concern, then proactive techniques must be favored, provided that knowledge about uncertainty is available.

Of course such properties only apply to “pure” proactive, revision, or progressive techniques, and will be qualified somehow when one mixes them: for instance, a progressive conditional approach (i.e. compute several alternatives but with limited commitment time horizon) will need average online memory, with both reasonable online CPU need and rather high robustness. Therefore, one can see how such mixed techniques are interesting for finding better compromises

with respect to those features a user wishes to meet in her application. But there are also more pragmatic reasons, making “pure” techniques usually unrealistic: in a highly stochastic world, a pure revision approach would amount to almost permanent rescheduling, which shall be limited, by mixing the revision technique with a proactive or progressive approach (see Section 3.5). On the contrary, a pure proactive technique is not always realistic, since there will often be unpredicted or unmodeled deviations that can only be dealt with by a revision technique. And as suggested above, purely conditional global schedules suffer from combinatorial explosion and are often simply infeasible: developing only some of the branches progressively is a way to overcome it.

As a matter of conclusion, a user should be given a global system encompassing the three kinds of approaches, allowing her to tune the levels of proactivity, progression, and revision that will best fit her needs. As we have seen, few mixed techniques have been proposed for making scheduling decisions in a stochastic environment, but as far as we know, no one has proposed a system or an approach that combines the three ways of scheduling under uncertainty.

## 4 Representation

This section describes a model for scheduling in a stochastic execution environment. This model integrates the three families of approaches presented in the previous section. Our model is not intended to be formal on each aspect of the scheduling problem, but only on the key and new features related to when and how decisions are made: in order to also remain as generic as possible with respect to different problem formulations and search techniques existing in related literature, the model will be less formal on less relevant aspects (as for instance the representation of resource constraints).

### 4.1 Schedule

We are interested in extended scheduling problems with mutually exclusive subsets of activities, in a way similar to what was done by Tsamardinos, Vidal, and Pollack [46] and by Beck and Fox [6]. At the roots of our model, we need variables and constraints inspired by the constraint paradigm.

**Definition 4.1 (Variable)** *A variable is associated with a domain of values or symbols, and it is instantiated with one and only one of the values or symbols in this domain.*

**Definition 4.2 (Constraint)** *A constraint is a Boolean function relating one (unary constraint), two (binary constraint) or more ( $k$ -ary constraint) variables that*

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5      restrict the values that these variables can take.  
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10     The domain of a variable is reduced by removing values that can be proved to  
11 not take part of any solution (given the decisions already made).

12     We distinguish between two types of variables in the problem: the *controllable*  
13 variables and the *contingent variables*.<sup>3</sup>  
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**Definition 4.3 (Controllable Variable)** A *controllable variable* is a variable instantiated by a decision agent.

16     One of the issues we are interested in in this paper, with respect to controllable  
17 variables, is to decide when to instantiate them. For example, it may be difficult to  
18 set activity start times in advance when activity durations are imprecise because  
19 of temporal uncertainty.

20     **Definition 4.4 (Contingent Variable)** A *contingent variable* is a variable instantiated by Nature.

21     Moreover, a (probabilistic/possibilistic/etc.) distribution of possible values may  
22 be attached to each contingent variable, when knowledge about uncertainty is  
23 available and one wishes to use it. Such distributions are updated during execution:  
24 they can be revised when new information arises, or they can be truncated  
25 when one has observed, for example, that an activity with an imprecise duration  
26 has not yet finished.

27     We can now define the basic objects of a scheduling problem, namely resources and activities.

28     **Definition 4.5 (Resource)** A resource  $r$  is associated with one or more variables,  
29 that represent its capacity, efficiency, and/or state. Its capacity is the maximal  
30 amount that it can contain or accommodate at one time point. Its efficiency de-  
31 scribes how fast or how much it can do with respect to its available capacity. Its  
32 state describes its physical condition. A resource capacity, efficiency, and state  
33 can all vary over time. These variables are either controllable or contingent.  $r$  is  
34 related to a global resource constraint  $ct_r$  on all its variables and the variables  
35 of the activities that require it. The scheduling problem comprises a finite set of  
36 resources noted  $\mathcal{R}$ .

37     We can model the state of the execution environment as a set of state resources;  
38 e.g., the outside temperature is modeled by a resource that can be in only one of  
39 three states depending on time: hot, mild, and cool.

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55     <sup>3</sup>Controllable variables correspond to decision variables, and contingent variables to state vari-  
56 ables in the Mixed Constraint-Satisfaction Problem framework [19].  
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 5 **Definition 4.6 (Activity)** An activity  $a = \langle start_a, d_a, end_a, [\mathcal{CT}_a] \rangle$  is defined by  
 6 three variables: a start time variable  $start_a$ , a duration variable  $d_a$ , and an end  
 7 time variable  $end_a$ . These variables are either controllable or contingent.  $a$  may  
 8 be associated with an optional set of resource constraints  $\mathcal{CT}_a$  that involve the  
 9 variables of the resources it requires.  
 10

11 In a constraint-based model, we usually post the following constraint for each  
 12 activity:  $end_a - start_a \geq d_a$ . Of course, constraints of any type between variables  
 13 can be posted on our scheduling problem; e.g., we can post temporal constraints.  
 14

15 We assume that the distributions of possible values for contingent variables  
 16 are independent from the values chosen for controllable variables, except for the  
 17 end-time variables of the activities that have contingent duration variables. The  
 18 activity end-time variables are contingent, and their distributions depend on the  
 19 start times chosen for these activities. For example, when we choose the resource  
 20 to allocate to an activity with a probabilistic duration, this does not effect the  
 21 probability distribution associated with the activity duration.  
 22

23 Our scheduling problem is composed of resources, activities, and constraints  
 24 relating them, with possibly additional variables describing the state of execution  
 25 environment.  
 26

27 To fit the classification described in Section 3, additional constraints may have  
 28 to be posted by the schedule-generation algorithm to set resource allocations,  
 29 make sequencing decisions, and/or set precise activity start times. For example,  
 30 when we want to set the start time of an activity, we post a new unary temporal  
 31 constraint that relates to the activity start time variable.  
 32

33 Central to our model is the notion of *conditions* that are subsets of variables  
 34 related by logical and/or arithmetic relations: such conditions guide the branch-  
 35 ing within conditional schedules, the selection of new subsets of activities in a  
 36 progressive technique, etc.  
 37

38 **Definition 4.7 (Condition)** A condition  $cond = \langle func, [atw] \rangle$  is a logical and/or  
 39 arithmetic relation  $func$  in which at least one variable is involved. It may be  
 40 associated with an optional active temporal window that is an interval  $atw =$   
 41  $[st, et]$  between two time-points  $st$  and  $et$  in the current schedule. If  $st = et$ , then  
 42 it means the condition must be checked at a precise time-point in the schedule.  
 43

44 A condition can involve characteristics of the distributions of contingent vari-  
 45 ables. A condition can be expressed with conjunctions and disjunctions of condi-  
 46 tions.  
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48 A typical example of a condition is what we will call a *branching condition*.  
 49 A branching condition will be attached to one of mutually exclusive subsets of  
 50 activities (see below) and will be checked at a specific time point that we will call a  
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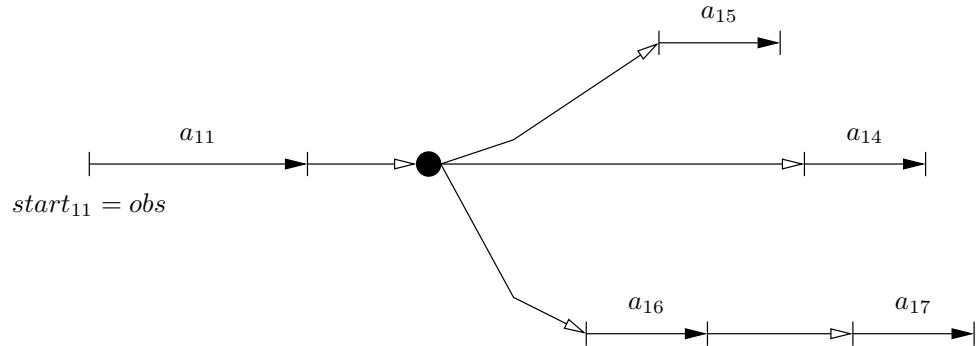


Figure 1: Example of a branching condition with three mutually exclusive subsets of activities.

*branching node.* For example, given three mutually exclusive subsets of activities and a contingent variable representing the outside temperature, we observe the value of this contingent variable at the start time of activity  $a_{11}$  (see Figure 1). The three branching conditions are the following: temperature is below 20 degrees, temperature is between 20 and 30 degrees, and temperature is above 30 degrees. When the first condition is met, then we choose  $a_{15}$ ; when the second condition is met, then we choose  $a_{14}$ ; when the third condition is met, then we choose  $a_{16}$  and  $a_{17}$ .

We propose the following recursive definition of a schedule to describe our model with respect to these particular mutually exclusive subsets of activities.<sup>4</sup>

**Definition 4.8 (Schedule)** *A schedule  $\mathcal{S}$  is either*

- *void  $\mathcal{S} = \emptyset$ , or*
- *$\mathcal{S} = \langle a_{\mathcal{S}}, \{\mathcal{CT}_{\mathcal{S}}\}^*, \mathcal{S}' \rangle$  is an activity  $a_{\mathcal{S}}$  partially ordered via constraints in  $\{\mathcal{CT}_{\mathcal{S}}\}^*$  with respect to the activities of a schedule  $\mathcal{S}'$ , or*
- *$\mathcal{S} = \langle bnd_{\mathcal{S}}, nbs_{\mathcal{S}}, \{rcps_{\mathcal{S}}\}^*, cnd_{\mathcal{S}} \rangle$  is a set of  $nbs_{\mathcal{S}}$  mutually exclusive recipes  $rcps_{\mathcal{S}}$ ; i.e., mutually exclusive recipes represent different ways of attaining the same goal, as defined below; such recipes follow a branching node  $bnd_{\mathcal{S}}$  and lead to a converging node  $cnd_{\mathcal{S}}$ ; i.e., there are precedence constraints that enforce these ordering relations. A node is a dummy activity  $a^{\text{dum}}$  of null duration that does not require any resource:  $a^{\text{dum}} = \langle start_a^{\text{dum}}, 0, end_a^{\text{dum}} \rangle$ , with  $start_a^{\text{dum}} = end_a^{\text{dum}}$ .*

**Definition 4.9 (Recipe)** *A recipe  $rcp = \langle \mathcal{S}, [Py_{rcp}], bc_{rcp} \rangle$  consists of a schedule  $\mathcal{S}$  associated with an optional probability, possibility, or plausibility of being ex-*

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<sup>4</sup>One should notice that what we call a schedule would be better referred to as a *solution* to a scheduling problem, which is possibly not fully set but only defines a partial order. In the Operations Research community, a schedule implies that all activity start times are set.

ecuted  $Py_{rcp}$  and a branching condition  $bc_{rcp}$ : it will be executed if and only if  $bc_{rcp}$  is met.

A recipe can describe one of several possibilities for performing an action; e.g., a device can be built in different ways that are mutually exclusive. At execution, for each set of mutually exclusive recipes, only one will be executed.

The first two ways of defining a schedule are just an alternative way to recursively define a classical partially ordered schedule without alternatives. The third introduces parts of a schedule that divide, at some given branching node, into mutually exclusive recipes: each recipe  $rcp_i$  will be executed, if a branching condition is met at that point.

It should be noted that conditions must be designed such that they are actually mutually exclusive and cover all cases. Moreover, as usual with probability distributions, if ever they are used, the sum of probabilities associated to the  $n_{bs}$  branches must equal 1.

The previous recursive definitions permit us to build a schedule piece by piece, building subsets of partially ordered activities that are then composed into a set of mutually exclusive recipes, this set being in turn integrated into a subset of partially ordered activities that is in turn one of several mutually exclusive recipes, and so on: alternatives may be nested within alternatives.

For tractability reasons, we assume there are no temporal constraints between two activities that do not belong to the same recipe. However, some precedence constraints must be added to constrain branching conditions to be checked before their related recipes would be executed.

## 4.2 Generation and Execution

We defined a model that a proactive method could use to generate a schedule that would then be entirely sent to the execution controller. To make it possible to use revision and progressive techniques, we need to consider a situation in which a schedule is executed in a highly stochastic environment, thus requiring scheduling decisions to be made and/or changed while executing. Hence, we need to design a model interleaving schedule generation and execution: the resulting system must be able to react, to know what to do (e.g., reschedule, schedule next subset, make new scheduling decisions, etc.), and to know how to do it.

Two types of algorithms will be needed: *execution algorithms* will be in charge of dealing with the current schedule and both making the scheduling decisions that remain and actually executing activities; *generation algorithms* will be in charge of changing the current schedule, either because some part is not valid anymore and must be modified (revision approach), or because new activities must be added (progressive approach).

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 5     The dynamic evolution of our model will be monitored via condition meeting:  
 6     if such a condition is met, then we know we have to make or change decisions.  
 7     A branching condition is actually used by the execution algorithms, guiding them  
 8     into the proper alternative. We need to introduce here two new types of condition:  
 9     a *generation condition*, when met, activates a new generation step through the  
 10    generation algorithm; then a *fire condition* will actually enforce the global moni-  
 11    toring system to turn to this newly generated schedule. Such generation and fire  
 12    conditions are needed both in revision and progressive approaches.  
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14  
 15     Typical examples of generation and fire conditions are violations of some con-  
 16     straints in the current schedule, arrivals of new activities to execute, critical re-  
 17     sources no longer available (implying a revision mechanism), or the anticipation  
 18     time horizon has become too small and so we need to schedule a new subset of  
 19     activities to anticipate execution (implying a progressive mechanism).  
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21  
 22     The generation and execution model can be represented by a dynamic directed  
 23     acyclic graph (DAG), similar to a finite-state machine, whose nodes are called  
 24     execution contexts.  
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 27     **Definition 4.10 (Execution Context)** *An execution context  $\text{ect} = \langle \mathcal{S}_{\text{ect}}, \alpha_{\text{ect}} \rangle$  is  
 28       composed of a schedule  $\mathcal{S}_{\text{ect}}$  and an execution algorithm  $\alpha_{\text{ect}}$ .*  
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30  
 31      $\mathcal{S}_{\text{ect}}$  is a flexible or complete schedule. In addition, it is possible for an execu-  
 32     tion context to not contain all recipes starting from a branching node, but only  
 33     those with the highest values  $P_y$ ; another example of generation condition is  
 34     hence that when the value  $P_y$  of a remaining recipe becomes high enough, we  
 35     generate in a progressive way a new schedule composed of the current schedule  
 36     and a new recipe.  
 37

38      $\alpha_{\text{ect}}$  makes decisions (start time setting, resource allocations, branching on one  
 39     recipe among several candidates, etc.) greedily: it cannot change decisions that  
 40     are already made. In case of pure execution approach, such as dispatching,  $\alpha_{\text{ect}}$   
 41     makes all decisions.  
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43     The arcs of our dynamic DAG are transitions between execution contexts.  
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 46     **Definition 4.11 (Transition)** *A transition  
 47        $tr = \langle ect_{tr}^{src}, ect_{tr}^{tat}, cond_{tr}^{gen}, cond_{tr}^{fir}, \beta_{tr} \rangle$  is composed of a source execution con-  
 48       text  $ect_{tr}^{src}$ , a target execution context  $ect_{tr}^{tat}$ , a generation condition  $cond_{tr}^{gen}$ , a fire  
 49       condition  $cond_{tr}^{fir}$ , and a generation algorithm  $\beta_{tr}$ .*  
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51  
 52     The default situation for the temporal windows of the generation and fire con-  
 53     ditions of transition  $tr$  is the whole source execution context  $ect_{tr}^{src}$ ; i.e., their  
 54     temporal windows equal the maximal interval between the start point and the end  
 55     points of  $ect_{tr}^{src}$ .  
 56

When generation condition  $cond_{tr}^{gen}$  is met, generation algorithm  $\beta_{tr}$  generates target execution context  $ect_{tr}^{tat}$  from source execution context  $ect_{tr}^{src}$  and the data of the problem model. Execution algorithm  $\alpha_{ect_{tr}^{tat}}$  is set by  $\beta_{tr}$  from a library of template execution algorithms, that depend upon the degree of flexibility (incompleteness or branching) of the generated schedule.  $\beta_{tr}$  can decide or change a part of or all decisions; in particular, it can select a subset of activities to include into  $ect_{tr}^{tat}$  (progressive approach). Transition  $tr$  is fired when its fire condition is met. When  $tr$  is fired, we change contexts: we go from source execution context  $ect_{tr}^{src}$  to target execution context  $ect_{tr}^{tat}$ . Generation condition  $cond_{tr}^{gen}$  must be more general than or equal to fire condition  $cond_{tr}^{fir}$  since  $cond_{tr}^{gen}$  must be met before or when  $cond_{tr}^{fir}$  is met.

Template transitions are defined offline, and each of them is an implicit description of many transitions that may appear in a model. A template transition is active when it can be instantiated; i.e., it is active when one of its implicit transitions can generate a new context. For example, a template revision transition associated with a resource constraint  $rct_1$  is active each time one of the activities involved in  $rct_1$  is executing and allocated to the resource involved in  $rct_1$ : when the associated resource breaks down, then we have to change a number of allocation decisions.

The generation algorithm generating  $ect_{tr_l}$  and the execution algorithm associated with  $ect_{tr_l}$  are complementary: the former makes some decisions for a subset of activities, and the latter makes the remaining decisions for these activities; e.g., the former makes allocation and sequencing decisions, and the latter sets activity start times.

It should also be noted that all conditions are checked by execution algorithms. When a branching condition is met, we do not change execution contexts. When a generation condition is met, a new execution context is generated. When a fire condition is met, we change execution contexts.

Our first assumption is that uncertainty level decreases when executing a context. Ergo, we leave some decisions to the execution algorithm to limit the computational effort that would be used to revise decisions, and the perturbations and instability due to such revision. Decisions that can be made in advance because they concern variables with low uncertainty are taken by generation algorithms, while remaining decisions will be taken later either by generation or execution algorithms when their uncertainty will be lower.

Our second assumption is that dynamics of the underlying controlled physical system are low enough with respect to the time allotted to the reasoning system to search for schedules online. Therefore, one has enough time to generate new contexts before they are executed. Generation algorithms should be anytime; i.e., generation algorithms should be able to produce a schedule whose quality, robustness, or stability increases with search time. In principle, the decisions made by

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5 generation algorithms are better with respect to an optimization criterion than the  
6 decisions made by execution algorithms. The former have more time to reason  
7 and choose the best schedules among a set of executable schedules, whereas the  
8 latter are greedy and return the first executable they find. Generating a new exe-  
9 cution context and switching into it are not necessarily synchronous, this permits  
10 us to generate new execution contexts during the time period in which the current  
11 execution context is under control and executing.  
12

13 One can see that any ‘pure’ technique can be easily instantiated with our  
14 model. A pure proactive technique will only need a single context, generation  
15 being made once offline, the remaining decisions being taken by the sole execu-  
16 tion algorithm. In a pure revision approach, contexts contain complete predictive  
17 schedules with basic execution algorithms, and generation/fire conditions asso-  
18 ciated with failures or quality deviations in the current context, and generation  
19 algorithms change the current schedule to fit the new situation. In a pure progres-  
20 sive approach, contexts contain complete schedule pieces with standard execu-  
21 tion algorithms, and generation/fire conditions related to the anticipation time horizon  
22 getting too small or the uncertainty level decreasing, and generation algorithms  
23 change the current schedule by inserting a new subset of activities into it. The  
24 strength of the model is that now all three kinds of approaches can be integrated  
25 and parameters can be tuned to put more or less flexibility, more or less revision  
26 capabilities, etc., upon needs that are driven by the application.  
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### 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 **4.3 An Illustrative Example**

This section presents an illustrative example in the context of the construction of a dam that shows how our generation-execution model is applied in practice.

We assume we have to schedule a set of activities:

- construction of three roads  $road_1$ ,  $road_2$ , and  $road_3$ ;
- construction of two houses  $house_1$  and  $house_2$ ;
- digging of a tunnel in one of two alternative ways  $tunnel_1$  and  $tunnel_2$  depending on geological conditions observed at the end of  $road_1$ ;
- preparation of dam foundations  $foundation$ .

In addition, we assume we can use two trucks, and each activity requires one of them except  $house_1$  that requires both. There are precedence relations between activities, due dates, and a tardiness-cost function. Activity durations are imprecise and modeled by probability distributions. Our objective is to generate a schedule that minimizes the expected tardiness cost.

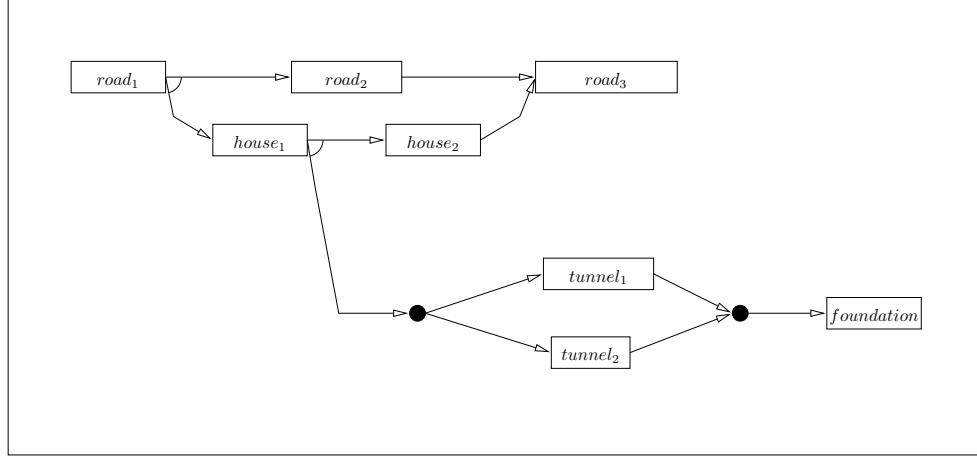


Figure 2: Temporal representation of a small dam construction project.

Figure 2 represents the temporal constraints between activities of this small dam construction project. This is a directed AND/OR graph. Each box symbolizes an activity, and the box lengths depend on activity durations. Arcs represent precedence constraints between activities; e.g., *road<sub>2</sub>* must start after the end of *road<sub>1</sub>*. Conjunctions are represented by arcs connected with small arcs of a circle; e.g., activities *house<sub>2</sub>* and *road<sub>2</sub>* might be executed in any order or in parallel. Disjunctions are represented by arcs that are not connected with arcs of a circle; e.g., activities *tunnel<sub>1</sub>* and *tunnel<sub>2</sub>* are mutually exclusive. Each recipe is associated with a probability of being executed that may vary during execution.

When creating a generation-execution model for this illustrative problem, the first phase consists in deciding whether we want to monitor schedule execution. Since our knowledge about uncertainty is limited to probability distributions, we want to be able to change decisions when expected tardiness cost deviates too much from its predictive value. In addition, we want to be able to make decisions on a gliding time horizon to limit our commitment. For these reasons, we need to monitor schedule execution. We decide to design two template transitions. The revision transition is active during execution, whereas the progression transition is active as long as there is at least one activity that is still not scheduled. We decide to use the same execution algorithm for any execution context. This execution algorithm fixes activity start times as early as possible.

The first execution context *ect<sub>1</sub>* is generated offline since we need to schedule at least one activity before starting execution. Figure 3 represents *ect<sub>1</sub>*. Notice that *ect<sub>1</sub>* only includes a subset of activities since our progression generation algorithm makes scheduling decisions on a short-term horizon. *tunnel<sub>1</sub>* is scheduled because its probability of being executed is much higher than the one associated

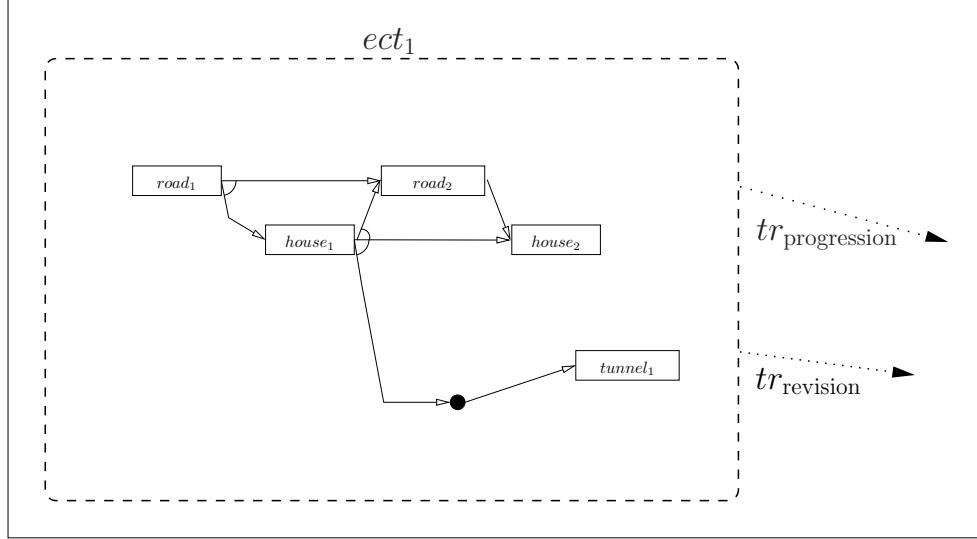


Figure 3: Execution context generated before starting execution.

with  $tunnel_2$  at this time. Two additional precedence constraints are included in  $ect_1$  as we cannot use more than two trucks at the same time:  $road_2$  starts after the end of  $house_1$ ;  $house_2$  starts after the end of  $road_2$ . These sequencing decisions are made to minimize the expected tardiness cost given probability distributions.

During execution we monitor the probabilities associated with recipes. We start executing  $ect_1$ . During the execution of  $road_1$ , we assume the probability associated with  $tunnel_1$  decreases, so the probability associated with  $tunnel_2$  increases. These changes fire the progression transition, and a new context  $ect_2$  is generated. Figure 4 represents  $ect_2$  that includes both  $tunnel_1$  and  $tunnel_2$ .

When  $road_1$  finishes, we observe geological conditions and choose  $tunnel_2$  accordingly. Figure 5 represents  $ect_2$  at this time. Note that the branching node disappears in  $ect_2$ .

The execution of  $ect_2$  continues until we observe that the mean end time of  $house_1$  is later than predicted in such a way that  $house_2$  is delayed and the expected tardiness cost increases too much with respect to its predicted value. This fires the revision transition that creates a new context  $ect_3$ . Figure 6 represents  $ect_3$ , where  $road_2$  is now sequenced after  $house_2$ .

We are now executing  $ect_3$ . When  $house_2$  is executing, the progression transition is activated since we need to select and schedule new activities to maintain the lead of generation over execution. A new context  $ect_4$ , which includes all activities, is generated, see Figure 7. Notice that the template progression transition is no longer active as all activities have now been scheduled.

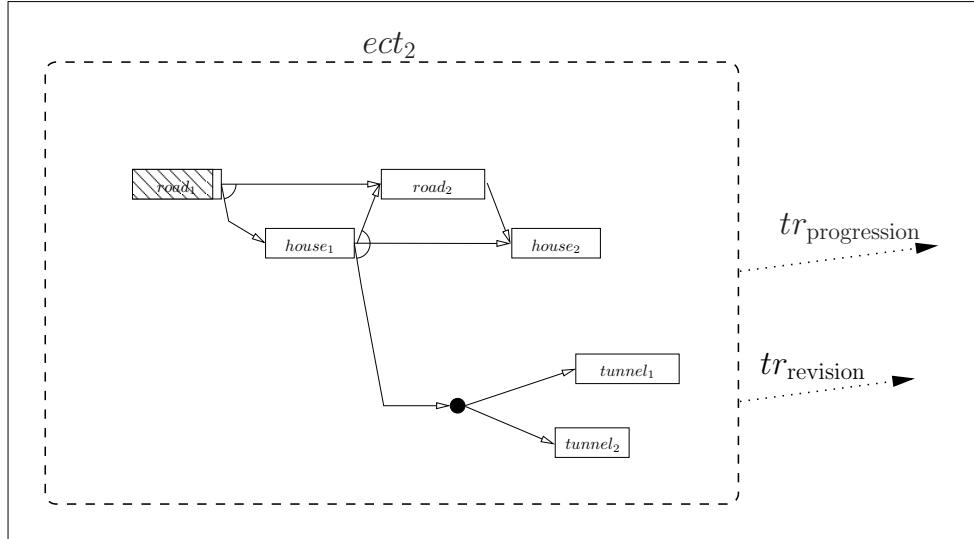


Figure 4: Execution context generated during the execution of  $road_1$ .

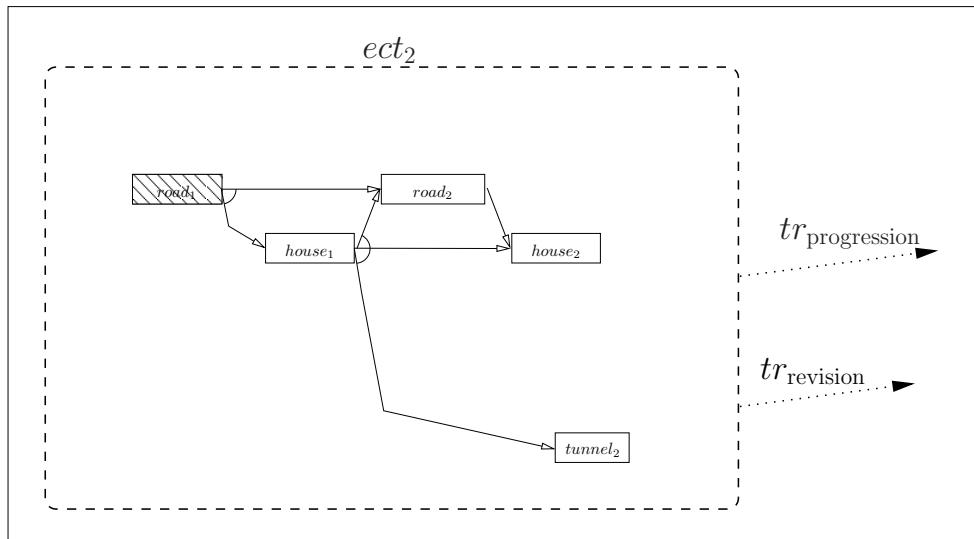


Figure 5: Execution context generated when  $road_1$  ends.

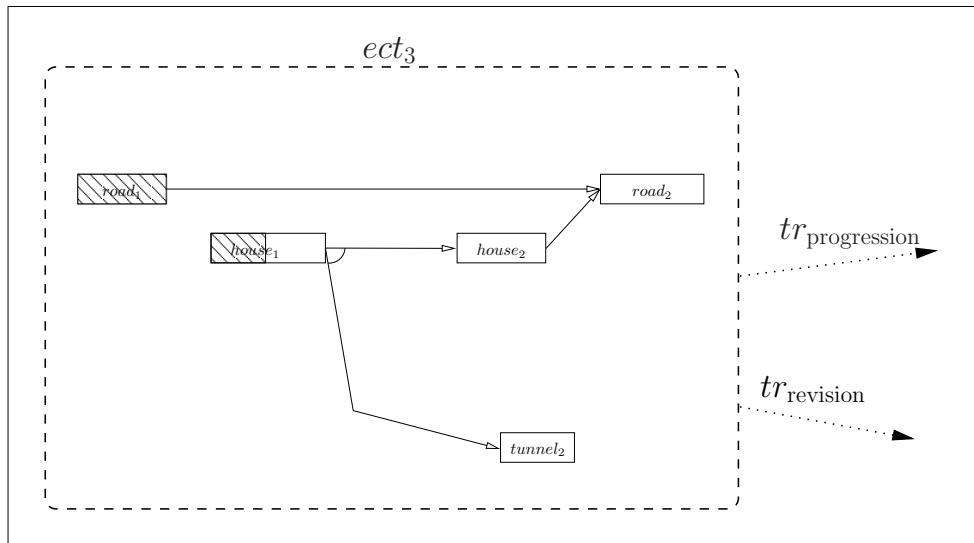


Figure 6: Execution context generated during the execution of  $house_1$ .

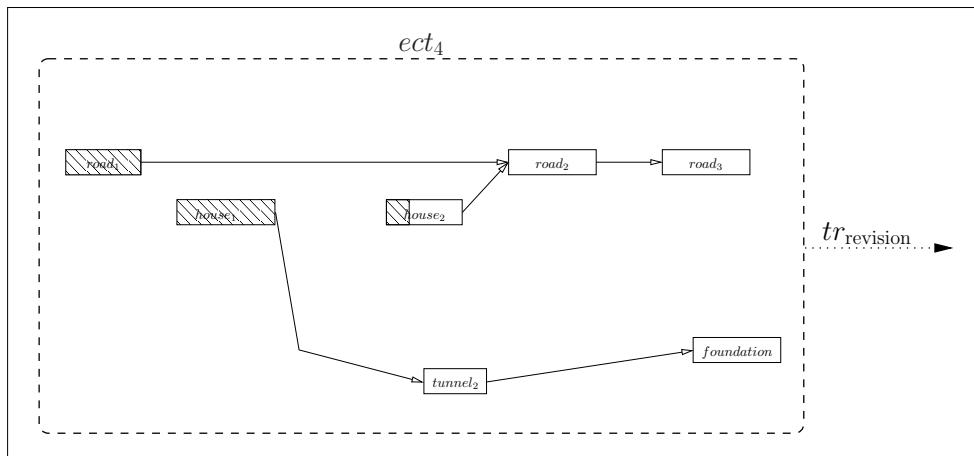


Figure 7: Execution context generated during the execution of  $house_2$ .

## 5 Experimental System

In this section, we present a few software prototypes that we implemented, and we show how they are special cases of our global model and hence validate it. Detailed experimental results using these prototypes appear in the cited papers.

### 5.1 Scheduling Problem

The *flexible job-shop scheduling problem* (flexible JSSP) is a scheduling problem where the set of activities  $A$  is partitioned into *jobs*, and with each job is associated a total ordering over a subset of the activities of  $A$ . Each activity specifies a set of alternative resources. An activity must be assigned to one of its alternative resources and execute without interruption on the assigned resource. No activities that are assigned to the same resource can overlap their executions. We represent these non-overlapping activities formally by a partition of  $A$  into *resource sets*. The partition of  $A$  into resource sets is based on which resource each activity is assigned to; i.e., resource sets are made during the search.

A *solution* corresponds to allocating one resource to each activity and a total ordering on each resource set such that the union of the resource and job orderings is an acyclic relation on  $A$ .

For our experimental investigations, we focused on probabilistic flexible job-shop problems. Activity end times are observed during execution. Moreover, resources may break down: for each resource, the duration between two consecutive breakdowns and breakdown durations are observed during execution as well. Random variables are fully independent and associated with probability distributions. Random variables represent activity durations, durations between breakdowns, and breakdown durations. We focused our experimental study on two criteria to minimize: makespan, and sum of tardiness and allocation costs.

#### 5.1.1 Tardiness Cost

Each job  $job_i \in JOB$  is associated with a due date  $due_i$ . If the last executed activity of  $job_i$  finishes later than  $due_i$ , then a *tardiness cost* is incurred:

$$tardiCost_i^{\text{eff}} = \phi_i \times \max(endJob_i^{\text{eff}} - due_i, 0),$$

where  $\phi_i$  is a positive weight, and  $endJob_i^{\text{eff}}$  is the effective end time of  $job_i$ , observed during execution. A schedule is associated with a tardiness cost  $K_{\text{tardiness}}$  defined as follows.

$$K_{\text{tardiness}} = \sum_{\forall job_i \in JOB} tardiCost_i^{\text{eff}}$$

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5   **5.1.2 Allocation Cost**  
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Another cost, called *allocation cost*, is associated with each resource allocation: when a given activity  $a_{ij} \in A$  is executed with a given resource  $r_l \in R_{ij}$ , it incurs a cost  $allocCost_{ijl}$ . A schedule is defined by a set of allocations and associated with an allocation cost  $K_{\text{allocation}}$ : each activity  $a_{ij}$  is effectively allocated to a resource  $r^{\text{eff}} \in R_{ij}$  and this allocation costs  $allocCost_{ij}^{\text{eff}}$ .

$$K_{\text{allocation}} = \sum_{\forall a_{ij} \in A} allocCost_{ij}^{\text{eff}}$$

18   **5.1.3 Global Cost**  
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The global cost  $K$  represents the whole cost of a solution; this cost takes both allocation and tardiness costs into account. It is formally defined as:

$$K = K_{\text{allocation}} + K_{\text{tardiness}}.$$

Each scheduling problem is characterized by a maximal global cost,  $K^{\max}$ , that defines an upper bound with respect to the schedule costs. We want to maximize the probability that the global cost is less than the maximal global cost (robustness); i.e., we want to maximize the level of service of each schedule; this can be formally expressed as follows:

$$\max \Pr(K \leq K^{\max}).$$

Note that tardiness and allocation costs are antagonistic. The tardiness cost is likely to be high, if we only want to reduce allocation cost by choosing systematically the cheapest resources when allocating activities so that a small number of activities can be scheduled in parallel. Conversely, the allocation cost is likely to be high, if we only want to reduce tardiness cost by choosing systematically the most expensive resources when allocating activities so that a large number of activities can be scheduled in parallel.

46   **5.2 Architecture**  
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Our experimental system is composed of the following modules: a solver, a controller, and a world.<sup>5</sup> Figure 8 represents our software architecture. Plain arcs represent data flows, while dotted arcs represent data controls. Boxes represent architecture modules.

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54   <sup>5</sup>The world module is not a real execution environment but a simulator of it, it instantiates  
55   random variables.  
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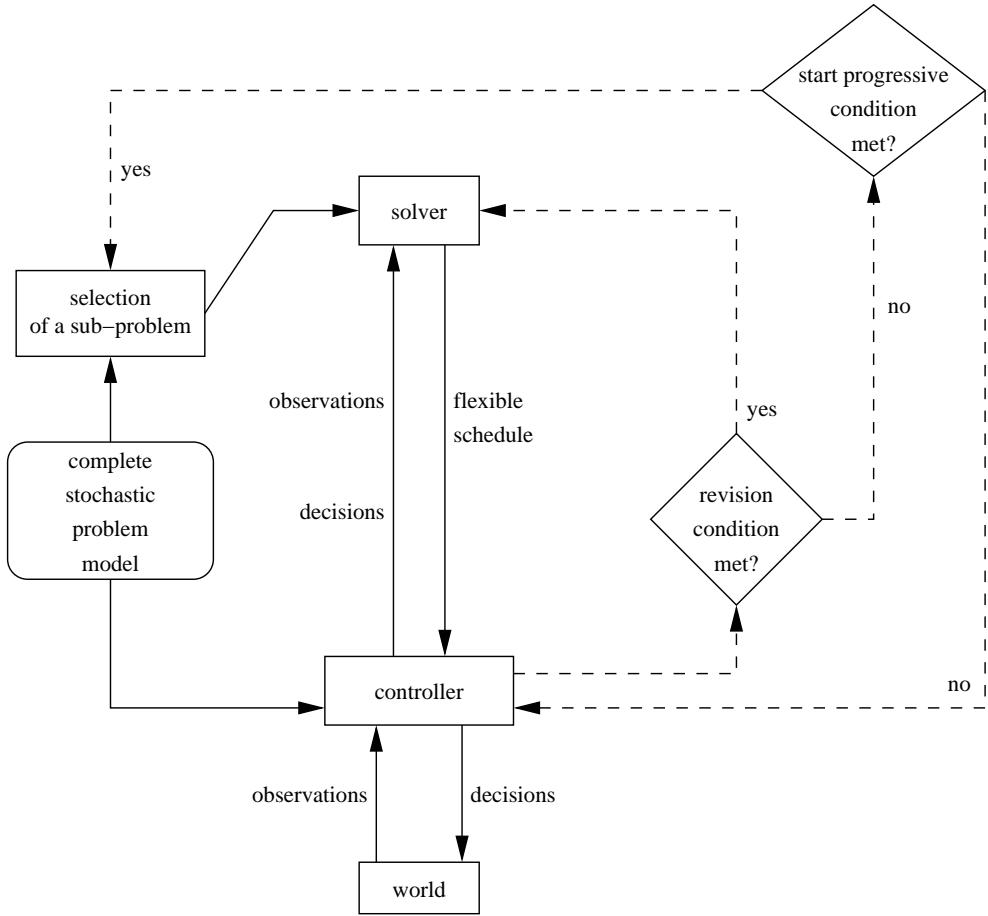


Figure 8: Architecture of our experimental system.

A solver module is in charge of making decisions with a backtrack search and constraint propagation. The input of the solver module is a stochastic model with variables and conditions. The resolution procedure first computes a deterministic approximation of this model; i.e., we pick a value for each random variable given its probability distribution. The computation can be parametrized, see Section 5.5.2. Then, we partially explore the search tree corresponding to the optimization of this deterministic model in depth-first search. The search procedure sequences the activities on each resource and thus, produces flexible solutions because activity start times are not set. We search for flexible solutions until the allotted time for searching is elapsed; i.e., we use an anytime algorithm. When we stop the resolution procedure the best flexible schedule is sent to the controller module.

The controller module is responsible for choosing activity start times given de-

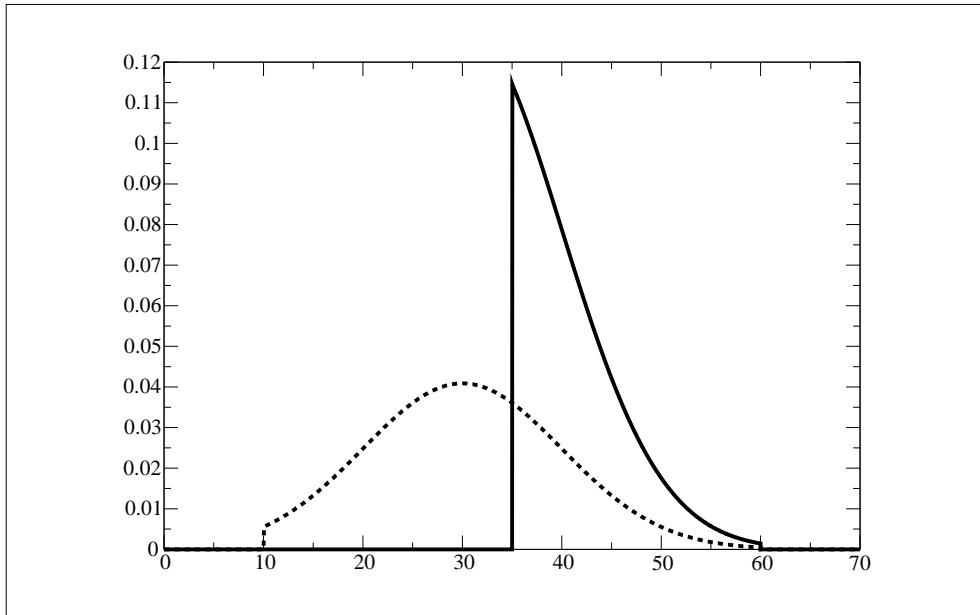


Figure 9: Two truncated and normalized normal laws.

cisions made by the solver module and what happens during execution (observations sent by the world module). The controller module monitors progression and revision conditions to start either a selection of new activities or a re-optimization, when it is relevant. The controller module is also in charge of maintaining known and unknown probability distributions online. For updating unknown probability distributions, associated with contingent variables, such as activity end times, we use Monte Carlo simulation. For computing expected activity end times, we run a series of simulations on the flexible schedule at hand by assuming activities are executed as soon as possible. For each activity  $a_{ij}$  that is allocated and sequenced but not yet executed, we randomly pick a set of possible durations. The probability distributions we know, such as the probability distributions associated with activity durations, are truncated and normalized. Suppose we have an activity whose execution started at date  $t = 0$  and whose duration, which is between 10 and 60, follows a normal law of mean 30 and standard deviation 10. This activity duration is depicted by the dotted curve in Figure 9. Now suppose that at date  $t = 35$  the activity is still executing, we will assume that the probability distribution of the activity end time is the initial law truncated on the interval  $[35, 60]$  and renormalized as shown by the solid curve in Figure 9. More formally, if the initial probability law of an activity duration is described by a distribution function  $p_0(dur)$  defined in  $[dur^{\min}, dur^{\max}]$ , and if the activity has been executed  $dur_0$  units of

time, the current distribution is defined by the probability distribution  $p_{dur0}(dur)$  as follows:

$$\forall dur \in [dur0, dur^{\max}], \quad p_{dur0}(dur) = \frac{p_0(dur)}{\int_{dur0}^{dur^{\max}} p_0(t) dt}.$$

A Monte Carlo simulation run consists in randomly picking a value for each random variable such that the two sets of values that are randomly picked for two consecutive simulation runs may be completely different.

For computing expected activity end times, we run a series of simulations on the flexible schedule at hand by assuming activities are executed as soon as possible. For each activity  $a_{ij}$  that is allocated and sequenced but not yet executed, we randomly pick a set of possible durations, and for each resource that is allocated to  $a_{ij}$ , we randomly pick a series of breakdowns. For generating resource breakdowns, we proceed as follows: for each resource allocated to  $a_{ij}$ , the last breakdown generated occurs after the end time of  $a_{ij}$ ; if a breakdown overlaps with the activity, then the activity end time is delayed. For running simulation we use a precedence graph that is generated for the temporal constraint network: each node represents an activity, and each arc represents a precedence constraint between two activities. We then topologically sort the precedence graph and use this ordering in the simulations. The simulation horizon equals the sum of all activity durations. The complexity of a single simulation run is in  $O(nbBk + nbAct + nbPCt)$ , where  $nbBk$  is the number of breakdowns,  $nbAct$  is the number of activities, and  $nbPCt$  is the number of precedence constraints; it is in  $O(nbBk + nbAct)$  for our problem, since there are fewer unary resources than activities and our problem has the same number of precedence constraints as a job-shop scheduling problem.

When progression conditions are met, we have to select and schedule a new subset of activities to anticipate execution; e.g., when the time period between the current time and the end time of the last scheduled activity is too short, we need to select a new set of activities. This permits progressive decision-making since the whole scheduling problem is split into sub-problems and solved piece by piece during execution with uncertainty level decreasing. We have to change scheduling decisions; i.e., we have to revise the current schedule when revision conditions are met: the current schedule is no longer executable or its expected quality deviates too much with respect to its baseline quality computed just after its generation by the solver module. For example, we reschedule when a resource breaks down. The generation-execution loop is controlled by the controller module on the basis of these conditions.

## 5.3 Revision Approach

Our experimental revision approach is parametrized by choosing a *revision criterion* and a *sensitivity factor*. A revision criterion is a condition that is monitored during execution. For example, we monitor the absolute difference between the expected quality, computed before execution, and the current expected quality, computed during execution based on what we observe. We compare this absolute difference with a reference value, and if the revision criterion is met, then we reschedule. A sensitivity factor sets the sensitivity of the revision criterion with respect to perturbations that occur during execution. The sensitivity factor is set to indirectly choose the search effort that depends on the number of reschedulings that occur online [8].

### 5.3.1 Experimental Results

In this section, we report the results of two experimental studies aiming at determining the impacts of the revision criterion and the uncertainty level on makespan with a limited computational effort. We address JSSPs with probabilistic durations, and we assume resources cannot break down. All the programs run to perform these tests have been written in C++, use ILOG Scheduler 5.3 [25] for scheduling, and have been compiled with Microsoft Visual C++ 6. The results presented here have been obtained with using a dual processor Pentium II Xeon 450MHz with 256MB.

**Impact of Revision Criterion on Makespan** We investigated three different revision criteria for deciding when to reschedule.<sup>6</sup>

First, we describe formally the three criteria. All these criteria depend on a strictly positive parameter called the sensitivity factor  $\omega$  that can tune the sensitivity of the revision criteria. When  $\omega$  is very small, there will not be any rescheduling. The larger the sensitivity factor, the higher the number of reschedulings (and the smaller the stability of the schedule).

The first criterion revCrit<sub>1</sub> consists in monitoring the makespan and is defined as follows: we reschedule when

$$M_{\text{exp}} > \frac{M_{\text{pre}}}{\omega},$$

where  $M_{\text{exp}}$  is the current expected makespan, and  $M_{\text{pre}}$  is the makespan of the current predictive schedule. This criterion will not result in rescheduling, if the effective activity durations are shorter than expected. In other words, it does

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<sup>6</sup>The work reported here was partially published [8].

not allow “opportunistic” rescheduling that would take advantage of unexpectedly short execution times.

A second variation revCrit<sub>2</sub>, based on the first, is opportunistic because it reschedules based on the absolute difference between the expected makespan and the predictive makespan. We reschedule when

$$|M_{\text{exp}} - M_{\text{pre}}| > \frac{D}{\omega},$$

where  $D$  is the mean of all activity durations of the initial problem.

The two versions based on makespan monitoring are *a priori* rather crude: we mainly take into account the observed durations of the activities of the critical paths. If these activities do not slip too much, then, as a consequence, the makespan will not slip too much either. However, it is possible that the expected makespan remains approximately the same while the executions of the activities that do not belong to the critical paths are such that it is possible to find a much better solution by rescheduling. We thus propose a third variation of the approach revCrit<sub>3</sub> that takes into account each activity duration. We reschedule when

$$\frac{\sum_{A_{\text{new}}} |end_{\text{exp}} - end_{\text{pre}}|}{nbNewAct} > \frac{D}{\omega},$$

where  $A_{\text{new}}$  is the set of  $nbNewAct$  activities that were executing the last time we rescheduled, and  $end_{\text{exp}}$  and  $end_{\text{pre}}$  are the expected and predictive activity end times, respectively.

The problem instances with imprecise durations are generated from classical JSSP instances [28, 3, 1]. Each activity is associated with a normal probability distribution with mean,  $p$ , corresponding to the duration in the classical instance, and with standard deviation  $\alpha \times p$  with  $\alpha > 0$ .  $\alpha$  characterizes the degree of uncertainty of the problem. The higher  $\alpha$ , the larger the standard deviation of each activity duration so the more uncertainty in the system.  $\alpha$  is constant and equals 0.3 for each experiment done in this first study.

During schedule execution, whenever we are informed by the environment that an activity ends, we update all our data structures, and simulate the continued execution of the schedule. The updating frequency is sufficiently low to permit 1,000 simulation runs each time. When the end of an activity is observed, there might be other concurrent activities still executing for which distributions are updated. We use simulation to calculate the revision criterion, and then we reschedule only if the revision criterion is met.

Scheduling and rescheduling are done using the constraint-programming approach with a standard chronological backtracking algorithm and a time limit of one second. The new schedule is the best solution found within the time limit

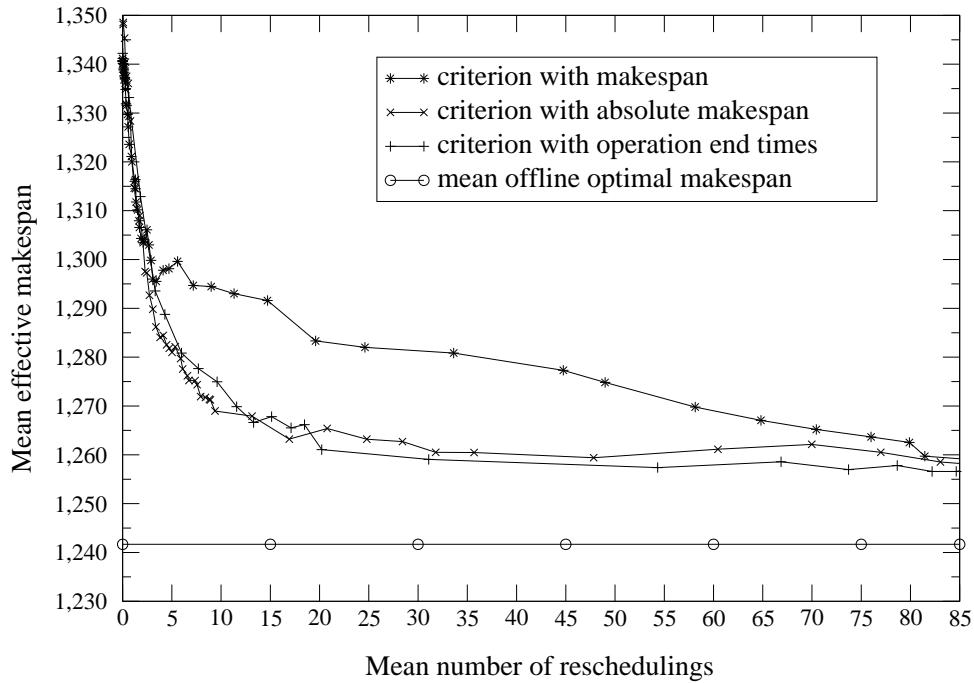


Figure 10: Mean effective makespan for la11 with a low uncertainty.

using a depth-first search. Two heuristic rules are used to make sequencing decisions: first, we have to select the resource and second, we have to select the activity to rank first on this resource. The resource  $r$  with the minimal global slack is selected with priority, and then the activity requiring  $r$ , with the minimal latest start time is ranked first on  $r$ . The global slack of a resource is the slack time of the activities that are allocated to it. The global slack is computed on the basis of the temporal window of each activity; i.e., this temporal window is the time period between its earliest start time and its latest end time. During search, constraint propagation is used to prune the search tree, in particular Edge-Finder [35, 4]. Note that we do not use simulation to filter solutions found by the tree search because we assume that a solution with a low deterministic makespan is also a solution with a low expected makespan [7].

The results shown on Figure 10 come from running 100 different execution scenarios per value of the sensitivity factor  $\omega$ . These results are obtained from experimenting with the problem la11 that consists of 100 activities [28]. We observe that monitoring schedule execution with the use of each rescheduling criterion improves schedule quality; i.e., mean effective makespan is smaller than the mean effective makespan obtained without rescheduling that equals 1,350: for example, after 2 reschedulings the mean effective makespan approximately equals 1,290

(4.4% of improvement). The most significant improvement is obtained with less than about ten reschedulings. The mean offline optimal makespan equals the average over the 100 optimal makespans, each of them is computed offline assuming we know the execution scenario in advance; i.e., we have to solve 100 deterministic JSSPs to optimality. Each of these 100 optimal makespans is a lower bound on the best schedule quality that can be achieved. We actually experimented with other literature JSSPs of the same size: la12, la13, la14, abz5, abz6, orb1, orb2, orb3, and orb4. The results obtained with these instances corroborate these observations [28, 3, 1]. These curves confirm the criterion based on the activity end times provides the smallest makespan for a given computational effort since it is more opportunistic than the first two revision criteria. We also think this is due to the fact that the activities that do not belong to the critical path have a smaller impact on makespan than the activities of the critical path because of activity slack times.

**Impact of Uncertainty Level on Makespan** All experiments have been run with the same revision criterion. We chose the revision criterion revCrit<sub>3</sub> presented above. This revision criterion performs best when about ten reschedulings are done.

Our study consists in varying the uncertainty level  $\alpha$  and observing how the effective makespan changes.  $\alpha$  directly changes the standard deviations of activity durations: the higher  $\alpha$ , the larger the standard deviations. We considered the same ten JSSP instances as those used in the study of the impact of the revision criterion on makespan. We conducted a series of three experiments, each of them corresponds to a fixed value of  $\alpha$ . The curves represented on the next four figures have been obtained by testing 30 problem instances, 100 different realizations per problem instance. The search procedure is the same as the one presented above. The tree search is limited to one second when rescheduling and we run 1,000 simulation runs each time an activity ends.

The curves on the next three figures represent the relationship between the mean number of reschedulings and the mean relative error of effective makespan for thirty problem instances. We tested ten problem instances per uncertainty level. The mean relative error of makespan is computed over the  $nbExecSce = 100$  execution scenarios of each problem instance as follows.

$$\text{mean relative error} = \frac{1}{nbExecSce} \times \sum_{sce=1}^{nbExecSce} \frac{M_{sce}^{\text{eff}} - M_{sce}^{\text{opt}}}{M_{sce}^{\text{opt}}},$$

where  $M_{sce}^{\text{eff}}$  is the effective makespan obtained after scheduling and rescheduling execution scenario  $sce$ ,  $M_{sce}^{\text{opt}}$  is the optimal makespan of  $sce$  computed offline knowing  $sce$  in advance. Each  $M_{sce}^{\text{opt}}$  has been computed with a texture-based

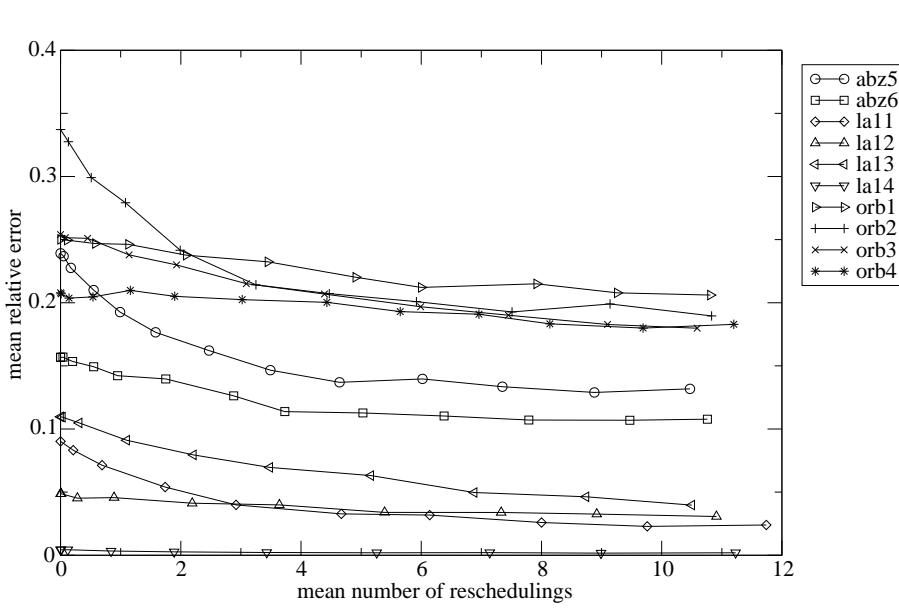


Figure 11: Mean relative error with a low uncertainty.

search [5]. Note that we have actually experimented with 3,000 execution scenarios since there are three uncertainty levels, ten problem instances per uncertainty level, and one hundred execution scenarios per problem instance.

Figure 11 represents the results obtained when  $\alpha = 0.3$ .

Figure 12 represents the results obtained when  $\alpha = 0.5$ .

Figure 13 represents the results obtained when  $\alpha = 0.8$ .

Note that all these uncertainty levels are actually rather high since even when  $\alpha = 0.3$  the standard deviation equals about the third of the mean value of the probability distribution.

Figure 14 represents the aggregated results obtained when  $\alpha$  varies. These curves have been obtained by using linear approximation of the preceding results shown on Figures 11, 12, and 13, and by computing the average curve over the ten problem instances for a given uncertainty level. We have to approximate linearly the curves to perform mathematical operations on them, such as computing the average curve, because we do not control directly the number of reschedulings that are performed but we control it via sensitivity factor  $\omega$ .

### 5.3.2 Conclusions

From the experimental results presented in Section 5.3.1, we conclude that it is worth using a revision approach when dealing with a JSSP with probabilistic ac-

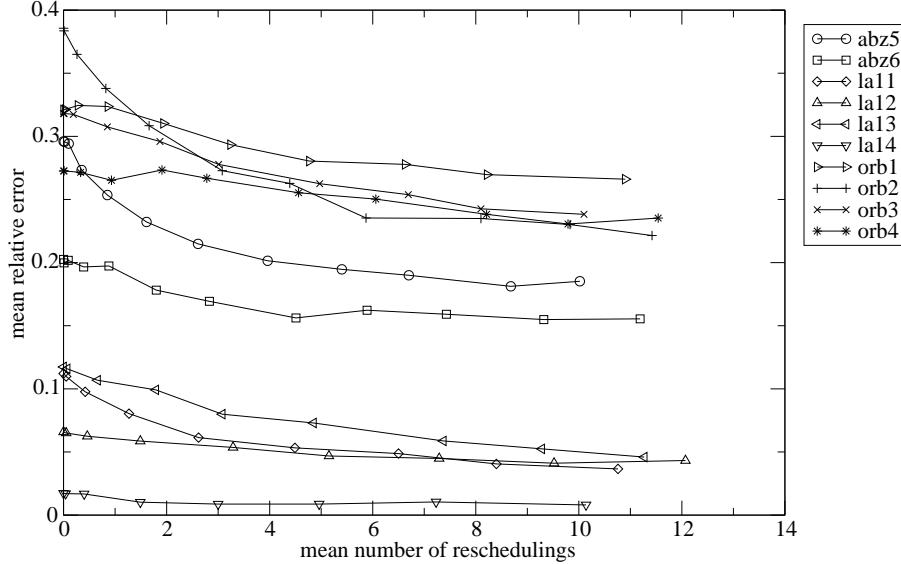


Figure 12: Mean relative error with a medium uncertainty.

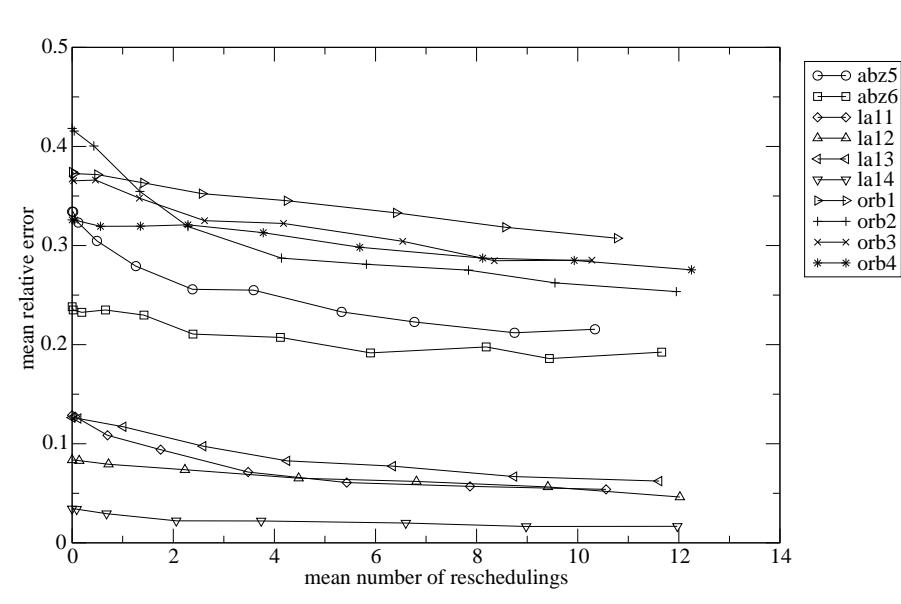
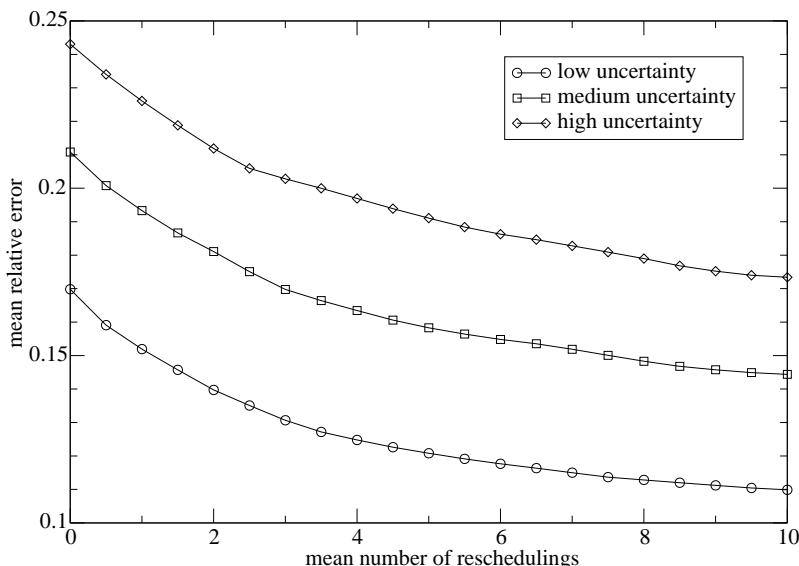


Figure 13: Mean relative error with a high uncertainty.



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Figure 14: Mean relative error for different uncertainty levels.

tivity durations even with a high imprecision. The higher the number of reschedulings, the lower the effective makespan. We observe that the higher the imprecision, the higher the effective makespan, and the improvement of the mean effective makespan is the same for the three uncertainty levels; moreover, it is about 6% smaller than the mean effective makespan obtained without rescheduling. Note also that the effective makespans of the schedules obtained without rescheduling are not longer than 25% of their corresponding optimal makespans on average. After about 10 reschedulings effective makespans are not longer than 18% of their corresponding optimal makespans on average. These optimal makespans are strict lower bounds since they are obtained when we know the execution scenario in advance. These lower bounds are idealistic and are, therefore, relatively far from the qualities of the best ever possible solutions to actually compute and execute.

#### 5.4 Progressive Approach

Our progressive approach is characterized by four parameters that can be set to choose indirectly the anticipation time horizon and the size of each sub-problem (commitment time horizon):  $\delta t^{\min}$  controls the anticipation time horizon with respect to time,  $\sigma t^{\min}$  controls the anticipation time horizon with respect to the uncertainty level,  $\delta t^{\max}$  controls the size of each sub-problem with respect to time, and  $\sigma t^{\max}$  controls the size of each sub-problem with respect to the uncertainty

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The problem with such an approach is that we must be very careful when taking an allocation decision or an ordering decision. (i) We have to wait until the uncertainty level around the decision is low enough so that the decision is well informed, and (ii) we cannot wait too long because we do not just want to have a reactive and myopic decision process. Determining when to select, allocate, and order a new subset of activities will be done based on monitoring a progression criterion during execution. Determining what activities to select will be done by using heuristics and Monte Carlo simulation. Determining how to allocate and order the activities of the selected subset will be done using constraint-based search, combined possibly with Monte Carlo simulation.

More precisely, suppose that we are at a given time  $t$  and we are executing a flexible schedule. We assume that a subset of activities  $A_{\text{allocOrder}}$  of the problem have already been allocated and ordered given all constraints at  $t$  and the rest of the activities  $A_{\text{pending}}$  of the problem are not yet allocated and only ordered given the precedence constraints of the original problem. A subset of activities  $A_{\text{executed}} \subseteq A_{\text{allocOrder}}$  have already been executed, a second one  $A_{\text{executing}} \subseteq A_{\text{allocOrder}}$  are executing, and a third one  $A_{\text{toBeExe}} \subseteq A_{\text{allocOrder}}$  have to be scheduled and executed. An activity is scheduled when its start time is fixed at a date before, at, or after the current time. (i) Of course, we do not want to wait until the last activity of  $A_{\text{toBeExe}}$  finishes execution before allocating and ordering subsequent activities of  $A_{\text{pending}}$  because we would then have very little time to react and could not easily come up with a good decision, and (ii) we do not want to make decisions too far in the future in regions where there is still a lot of uncertainty. Furthermore, we do not want to take the allocation and ordering decisions one by one, which would be very myopic and certainly lead to a poor schedule quality. We want to select a subset of activities and perform a combinatorial search on this sub-problem in order to satisfy temporal, resource, and cost constraints. So there are three questions here: (1) how to design conditions that will be monitored during execution and say “now, we can try to extend the current flexible schedule by allocating and ordering a new part of the problem,” (2) when these conditions are met, how to select the subset of activities to be allocated and ordered, and (3) when the subset of activities is selected, how to allocate and order the activities of this subset in such a way that we maximize the probability that the constraints will be satisfied and the global cost will be minimal at the end of execution.

51     **5.4.1 When to Try Extending the Current Flexible Schedule?**  
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To extend the current flexible schedule, we need to assess what time we still have before being forced to select, allocate, and order a new subset of activities of  $A_{\text{pending}}$  and how high the uncertainty level of the end times of the activities of

$A_{\text{toBeExe}}$  is. We need to monitor two conditions during schedule execution and when at least one of them is met, we can try to extend the current flexible schedule.

We propose to evaluate the trade-off between the fact that  $\delta t^{\min}$ , controlling the anticipation time horizon, should be large enough to have time to perform a combinatorial search leading to a good solution and to ensure the stability of the schedule, and the fact that execution has advanced far enough to get reduced uncertainty on the expected end times of the activities of  $A_{\text{toBeExe}}$ .

#### 5.4.2 Temporal Condition for Starting Selection

Given  $A_{\text{toBeExe}}$ , there exists a time point  $t_D$  that is the last time point before which we have to make at least an allocation decision if we want to anticipate execution.  $t_D$  is equal to the earliest of the expected end times of the activities of  $A_{\text{allocOrderLast}} \subseteq A_{\text{toBeExe}}$  that are ordered at last positions in jobs:  $t_D = \min_{\forall job_i \in JOB} (\max_{\forall a_{ij} \in A_{\text{toBeExe}}} (end_{ij}^{\text{exp}}))$ , where the min is over all jobs  $job_i$ , the max is over all activities  $a_{ij}$  of fixed job  $job_i$  that still have to be scheduled and executed,  $end_{ij}^{\text{exp}}$  is the expected end time of activity  $a_{ij}$ , and  $JOB$  is the set of jobs.  $t_D$  is maintained using Monte Carlo simulation. We can try to extend the current flexible schedule from the date at which  $t_D - t \leq \delta t^{\min}$ .  $\delta t^{\min}$  is a parameter of the software prototype, depending on the application domain of interest.

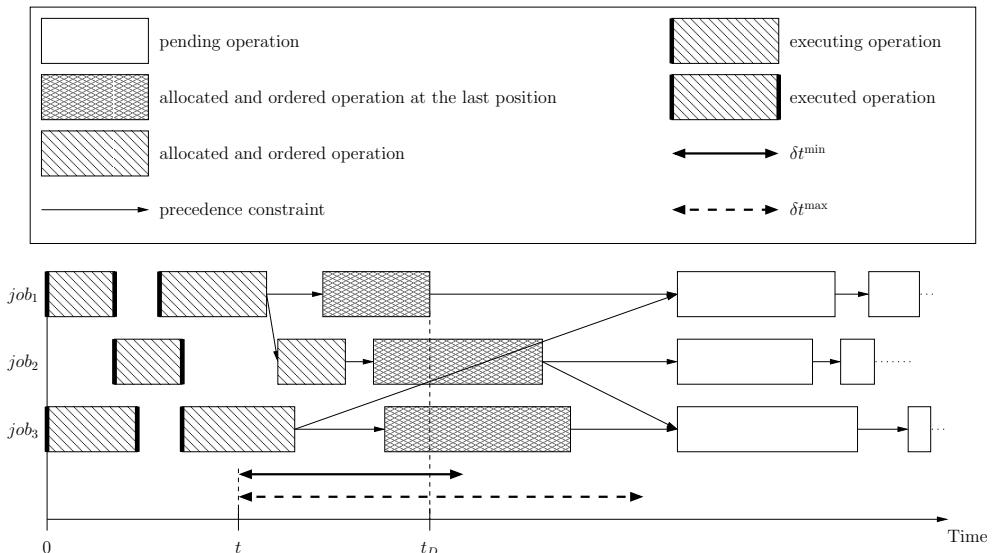


Figure 15: An example of a progressive scheduling approach.

Figure 15 represents the execution of a flexible schedule; only three jobs are partially represented;  $A_{\text{allocOrder}}$  is the subset of activities represented by nine

shaded rectangles;  $t_D$  is the earliest expected end time of the activity of  $job_1$  ordered at the third position;  $A_{\text{allocOrderLast}}$  is composed of the three activities represented by the most shaded rectangles and ordered at the last positions of jobs.  $A_{\text{pending}}$  is composed of the activities represented by white rectangles. Note that all pending activities are not represented. Precedence constraints are represented by arrows. The start times and end times of the executed activities and the start times of the executing activities are highlighted.

**Uncertainty Condition for Starting Selection** When the highest standard deviation of the end times of the activities to be executed is less than a given threshold  $\sigma t^{\min}$ , we can select a subset of pending activities. These standard deviations are maintained using Monte Carlo simulation. In a more formal way, we extend the current flexible schedule from the date at which

$$\min_{\forall job_i \in JOB} (\max_{\forall a_{ij} \in A_{\text{toBeExe}}} (\sigma(end_{ij}^{\text{exp}}))) \leq \sigma t^{\min},$$

where  $\sigma(end_{ij}^{\text{exp}})$  is the standard deviation of the end time of activity  $a_{ij}$ .  $\sigma t^{\min}$  is a parameter of our experimental system and depends on the application domain.

### 5.4.3 How to Select the Subset of Activities to Be Allocated and Ordered?

As soon as one of the two conditions defined above is satisfied, we still need to select a subset of activities to allocate and order. We do not want to select a too large problem because: (i) we do not have an infinite time to allocate and order it (in any case less than  $t_D - t$ ) and (ii) we do not want to take allocation and ordering decisions too far in the future as they concern data with too much uncertainty. We thus need to monitor two conditions during the selection process. To select the subset of pending activities to be allocated and ordered, we proceed in two steps as follows: (i) we compute and associate priorities to a subset of pending activities called the eligible activities to determine the order in which we assess each of them and (ii) we assess the eligible activities by using a temporal condition and an uncertainty condition to determine which of them are selected.

**Assessment Order** It is important to assess the eligible activities in a relevant order because we need to consider the eligible activities that have priority in terms of resource contention and tardiness costs. An eligible activity is the pending activity that is ordered at the first position of a job. There is thus one and only one eligible activity per job at the beginning of the selection process. We proceed in several steps. (1) We use a heuristic that gives the order in which we iterate through the current eligible activities and assess the current flexible schedule to

determine which of them we select. (2) Once all eligible activities have been labeled by a priority value we consider each eligible activity in the decreasing order of priority and assess the probability distribution of the end time of its preceding activity with respect to the job it belongs to:

- if this distribution does not meet at least one of the two conditions defined in the next paragraph, then the eligible activity is selected and no longer eligible (and no longer pending); this selected activity is ordered and allocated in such a way that its mean end time is minimized; the next activity of the same job, which is a pending activity, becomes eligible and the priorities of the current eligible activities are then (re)computed;
- if this distribution meets the temporal condition and the uncertainty condition defined in the next paragraph, then the eligible activity remains pending and is no longer eligible concerning the current selection process. Both conditions must be met to stop the selection because we want to be sure to avoid selecting too small subsets of pending activities.

(3) If there are no more eligible activities, then the selection process is stopped, otherwise the selection process goes on by executing alternately (2) and (3).

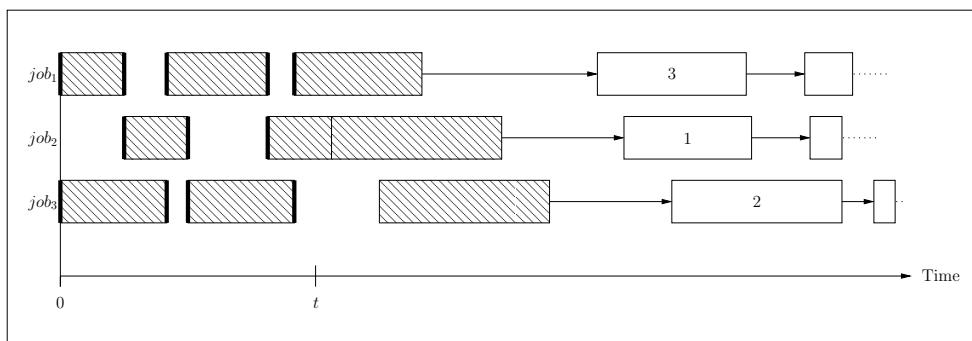


Figure 16: Example of the assessment order of eligible activities.

Figure 16 gives an example of such an order; the three eligible activities are labeled in the assessment order. We compute the priority of each eligible activity by using a heuristic that is based on both an energy-based heuristic and the Apparent Tardiness Cost rule described below.

We do not need to consider the online continued execution, and we can behave as if schedule execution was stopped during the selection process since the dynamics of the underlying physical system are much slower than the dynamics of the decision-making system.

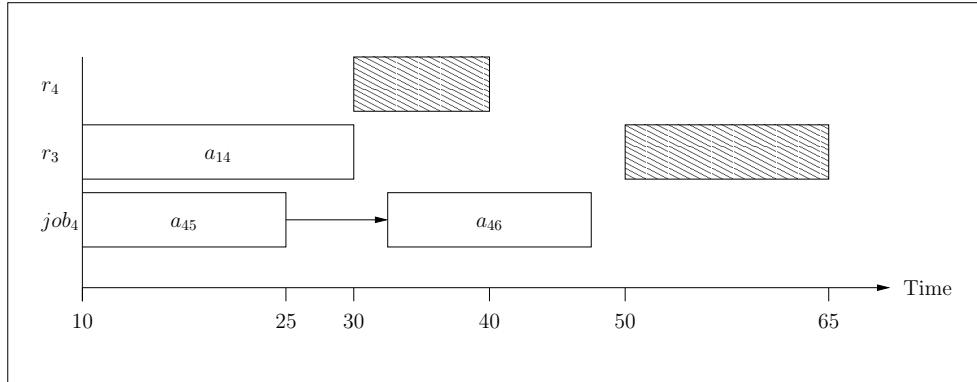


Figure 17: Example of simulation of a selected activity.

In fact, we are only interested in the probability distributions of the end times of the activities of  $A_{allocOrderLast}$  and in the probability distributions of resource breakdown end times at time  $t$  when selecting and making decisions on a new subset of activities. When an eligible activity is selected, we run a set of simulations concerning this activity, its preceding activities, and the alternative resources it requires to decide what resource to allocate during the selection process. These allocation decisions are not definitive since a combinatorial search is done when the selection is finished. Figure 17 shows one simulation run of an eligible activity  $a_{46}$  associated with two alternative resources  $r_3$  and  $r_4$ . This simulation is run at time  $t = 10$ . The duration of  $a_{46}$  that is randomly picked given its probability distribution equals 15 time units. The end time of its direct predecessor  $a_{45}$  is randomly picked using the current distribution and equals 25.  $a_{46}$  cannot start execution before 25 as there is a precedence constraint between  $a_{45}$  and  $a_{46}$ , and  $a_{46}$  cannot finish before 40 (this value equals the sum of its start time and its duration). Each of its alternative resources may be already allocated, not available and/or broken down. After picking activity end times, resource breakdown start times, and resource breakdown end times randomly, we can compute the expected end time of  $a_{46}$  for this simulation run.  $r_3$  is allocated to  $a_{14}$  that finishes at 30, and  $r_3$  breaks down at 50 and is fixed at 65.  $r_4$  breaks down at 30 and is repaired at 40. From this simulation run, we can update the means and standard deviations of the two end times of  $a_{46}$  (a mean and a standard deviation for each resource) as follows: if  $a_{46}$  is executed with  $r_3$ , then its execution starts at 30 and finishes at 45 without interruption; if  $a_{46}$  is executed with  $r_4$ , then its execution starts at 25, it is suspended at 30, it resumes execution at 40 and finishes thus at 50. After updating the two expected end times of  $a_{46}$  we allocate the resource that minimizes the expected end time of  $a_{46}$  ( $a_{46}$  would be allocated to  $r_3$ , if only considering this

unique run).

**Temporal and Uncertainty Conditions for Stopping Selection** Given the assessment order, we have to make sure both that we will select enough pending activities to schedule these activities long before executing them and that the uncertainty level of the end time of each selected activity is higher than a given threshold. An eligible activity  $a_{ij}$  is selected, if the mean end time of its preceding activity of the same job  $a_{ij-1}$  is less than  $t + \delta t^{\max}$  and/or the standard deviation of  $a_{ij-1}$  is less than  $\sigma t^{\max}$ , otherwise it is not selected and is no longer eligible during the current selection process.  $\delta t^{\max}$  and  $\sigma t^{\max}$  are parameters. We assume  $\delta t^{\max} \geq \delta t^{\min}$  and  $\sigma t^{\max} \geq \sigma t^{\min}$ , otherwise we could not select any pending activity.

**Energy-Based Heuristic** The energy-based heuristic is combined with the Apparent Tardiness Cost rule based heuristic [2] to compute the priorities of eligible activities, and the estimations of the job queues and the allocation costs of pending activities. We take resource constraints into account by extending the mean durations of the pending activities using an energy-based heuristic [18]. We compute the criticality of each activity that depends on the average loads and costs of the resources it requires, we then modify its mean duration accordingly. We first compute the artificial duration  $durRes_{ijl}$  of activity  $a_{ij}$  executed with resource  $r_l$  as follows:

$$durRes_{ijl} = \frac{mDur_{ij}}{allocCost_{ijl} \times \sum_{\forall r_k \in R_{ij}} \frac{1}{allocCost_{ijk}}},$$

where  $mDur_{ij}$  is the mean duration of  $a_{ij}$ ,  $allocCost_{ijl}$  is the allocation cost when  $r_l$  is allocated to  $a_{ij}$ , and  $R_{ij}$  is the set of all alternative resources required by  $a_{ij}$ . Note that  $durRes_{ijl}$  can be computed once offline.

It is also useful to compute the artificial duration of the pending activities requiring  $r_l$ :  $durRes_l = \sum_{\forall a_{ij} \in A_{\text{pending}} \cap A_l} durRes_{ijl}$ , where  $A_l$  is the set of activities that require  $r_l$ . We can then compute the criticality degree  $crit_{ij}$  of each pending activity  $a_{ij}$ :

$$\forall a_{ij} \in A_{\text{pending}} : crit_{ij} = 1 + \sum_{\forall r_l \in R_{ij}} 1 - \frac{durRes_{ijl}}{durRes_l}.$$

$crit_{ij}$  represents how high the probability is that the effective duration of  $a_{ij}$  will be greater than the original mean duration of  $a_{ij}$  given resource constraints and allocation costs. We then extend the mean duration  $mDur_{ij}$  of each pending activity  $a_{ij} \in A_{\text{pending}}$  with respect to how critical  $a_{ij}$  is:

$$extendedMDur_{ij} = mDur_{ij} \times crit_{ij}.$$

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 5 **Heuristic Based on Apparent Tardiness Cost Rule** We compute the priority of  
 6 each eligible activity, the estimated job queues and the allocation costs of pending  
 7 activities by using a modified version of the Apparent Tardiness Cost rule in which  
 8 its weight is redefined [2]. A weight  $weight_{ij}$  is associated with each pending  
 9 activity. It equals the sum of the expected tardiness cost of  $job_i$  and the expected  
 10 allocation cost of the eligible activity  $a_{ij}$ :  
 11

$$13 \quad \forall a_{ij} \in A_{\text{eligible}} : weight_{ij} = tardiCost_i^{\text{exp}} + allocCost_{ij}^{\text{exp}}.$$

14 The expected tardiness cost of  $job_i$  is computed as follows:  
 15

$$16 \quad tardiCost_i^{\text{exp}} = \begin{cases} \phi_i \times endJob_i^{\text{exp}} - due_i & \text{if } endJob_i^{\text{exp}} - due_i \geq 0 \\ (1 - \phi_i) \times (endJob_i^{\text{exp}} - due_i) & \text{otherwise,} \end{cases}$$

17 where  $endJob_i^{\text{exp}} = end_i^{\text{allocOrderLast}} + queue_i$ , and  $end_i^{\text{allocOrderLast}}$  is the expected  
 18 end time of the selected activity of  $job_i$  ordered at the last position.  $end_i^{\text{allocOrderLast}}$   
 19 is computed by running Monte Carlo simulations.  
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21  $allocCost_{ij}^{\text{exp}}$  is the mean allocation cost associated to  $a_{ij}$ :  
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$$23 \quad allocCost_{ij}^{\text{exp}} = \frac{\sum_{r_l \in R_{ij}} allocCost_{ijl}}{k},$$

24 where  $k$  is the number of alternative resources required by  $a_{ij}$ . In case no new  
 25 activity is yet selected for  $job_i$ ,  $end_i^{\text{selectedLast}}$  is the expected end time of the al-  
 26 located activity of  $job_i$  ordered at the last position and is computed by running  
 27 Monte Carlo simulations.  $\phi_i \in [0, 1[$  is used to balance the expected tardiness cost  
 28 among the jobs. Note that  $allocCost_{ij}^{\text{exp}}$  can be computed once offline. We set  
 29 the look-ahead parameter  $\chi = 2$ , as suggested by Morton and Pentico [31], in the  
 30 formula to compute each priority  $\pi_{ij}$ :  
 31

$$32 \quad \forall a_{ij} \in A_{\text{eligible}} : \pi_{ij} = \frac{weight_{ij}}{queue_i} \times e^{\frac{-\max(0, due_i - t - queue_i)}{\chi \times queue_i}},$$

33 where  $t$  is the current time,  $queue_i$  is the mean duration of the pending part of  
 34  $job_i$ , and  $\overline{queue}_i$  is the average over the mean durations of the pending activities  
 35 of all the jobs but  $job_i$ .  $queue_i$  and  $\overline{queue}_i$  are computed by using the extended  
 36 mean durations. In a more formal way:  
 37

$$38 \quad queue_i = \sum_{\forall a_{ij} \in A_{\text{pending}} \cap A_i} extendedMDur_{ij}$$

39 and  
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$$41 \quad \overline{queue}_i = \frac{\sum_{\forall job_q \in JOB \setminus \{job_i\}} queue_q}{n - 1},$$

42 where  $n$  is the number of jobs.  
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5     **5.4.4 How to Allocate and Order the Subset of Activities?**  
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#### 5.4.4 How to Allocate and Order the Subset of Activities?

10     The set of all the selected activities on all the jobs constitutes the sub-problem to  
11     solve. After the selection process is finished, we need to approximate the con-  
12     tribution of each job in terms of cost; i.e., the allocation cost and the length of  
13     the pending activities of each job. This approximation is done by using the same  
14     heuristic as the one dedicated to the selection of activities. To make decisions,  
15     we generate a deterministic problem; i.e., we use the mean durations of the se-  
16     lected activities extended depending on resource breakdown distributions, and job  
17     queues estimated heuristically. We use standard constraint programming tech-  
18     niques to explore and reduce the search space.

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20     **5.5 Discussion**  
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22     **5.5.1 Revision and Progressive Scheduling**  
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24     With respect to the experimental study of our revision approach, we set the value  
25     of the sensitivity factor that indirectly determines the number of reschedulings.  
26     When rescheduling, we are looking for solutions that optimize a criterion, there  
27     always exists a solution in our case. It could be interesting to tackle more con-  
28     strained problems with the revision approach.  
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31     For the progressive approach, an experimental study has still to be conducted  
32     to understand the relationships between the different and numerous parameters,  
33     indicators, and problem characteristics.  
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35     If  $\delta t^{\min}$  equals zero, then it amounts to only deciding to extend the current  
36     flexible schedule when the uncertainty condition is met. The temporal condition is  
37     met when at least one of the activities of  $A_{\text{allocOrderLast}}$  finishes execution, and the  
38     uncertainty condition should be met before this date in principle, if  $\sigma t^{\min}$  is not too  
39     small, see below. If  $\delta t^{\max}$  and  $\sigma t^{\max}$  are small, then it means we frequently extend  
40     the current flexible schedule with small subsets of activities: this is a reactive  
41     approach. If  $\delta t^{\max}$  and/or  $\sigma t^{\max}$  are very large, then it amounts to selecting and  
42     scheduling all activities: this is a predictive approach.  
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45     We can change both  $\delta t^{\min}$  and  $\sigma t^{\min}$  to choose when we want to consider a  
46     subset of pending activities during execution of the current flexible schedule. If  
47      $\delta t^{\min}$  is small and  $\sigma t^{\min}$  is large, then it amounts to extending the current flexible  
48     schedule when the uncertainty condition is met. If  $\delta t^{\min}$  is large and  $\sigma t^{\min}$  is  
49     small, then it means we extend the current flexible schedule when the temporal  
50     condition is met. If both  $\delta t^{\min}$  and  $\sigma t^{\min}$  are small, then it means we extend the  
51     current flexible schedule at the last time: the temporal anticipation is short. If  
52     both  $\delta t^{\min}$  and  $\sigma t^{\min}$  are large, then it amounts to extending the current flexible  
53     schedule very early: the temporal anticipation is long.  
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5 **5.5.2 Integrating Proactive Scheduling**  
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We focused on revision and progressive scheduling approaches in our experimental prototypes because they are more complex than pure proactive approaches. Because revision and progressive approaches combine reasoning with execution, it is necessary to build a framework such as the one suggested here that combines generation and execution algorithms as well as the ability to sense the environment. In contrast, proactive techniques can be purely generative, building schedules to minimize some measure of expected performance and not making any assumptions about how the schedule will be executed.

In this section, we discuss a particular example of proactive scheduling from the literature, show how it can be conceptualized as an instance of our framework, and furthermore demonstrate how this conceptualization opens up a number of areas for future work which blur the boundaries between proactive, progressive, and revision approaches. As our thesis in this paper is that a system for scheduling under uncertainty needs to combine these different approaches, we see this blurring as a positive step toward an integrated scheduling system.

**Proactive Scheduling with Uncertain Durations** Beck and Wilson [7] address the job-shop scheduling problem where all activities and resource capacities are fixed but where the duration of each activity is represented as a normally distributed random variable with a known mean and standard deviation. The objective is to find a sequence of activities on each resource, consistent with the temporal constraints, that minimizes the makespan that is achievable with a given probability threshold. For example, the activity sequence that results in makespan,  $D$ , that is achievable with 0.95 probability means that in 95% of the execution scenarios for this schedule, the real makespan will be less than or equal to  $D$ . The authors seek the activity sequence that leads to the minimal such  $D$ . Unfortunately, just evaluating the probability of achieving a given makespan for a potential solution is in #P [22].

A number of algorithms are investigated, all using Monte Carlo simulation to evaluate full and partial activities sequences. A branch-and-bound method evaluates partial sequences at each node in the search tree while a set of filtering algorithms generate candidate solutions based on a deterministic approximation and evaluate them with Monte Carlo simulation.

It should be noted that the use of Monte Carlo simulation in this context is different from that discussed in Section 5.2. Here the simulation is used as a low complexity way of evaluating a solution to a hard problem. In Section 5.2, the simulation was used as part of the experimental apparatus to take the place of the “real” execution of the schedule.

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5     **Using the Framework to Model and Extend Beck and Wilson** The algorithms  
6     studied by Beck and Wilson are generation algorithms that create flexible sched-  
7     ules: the order of the activities on each resource is defined but no activity start  
8     times are assigned.  
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10     While, in general, a proactive technique does not require an execution context  
11     or execution algorithm, our framework does provide an interesting perspective on  
12     the algorithms proposed by Beck and Wilson. In order to evaluate a solution using  
13     Monte Carlo simulation, multiple scenarios are generated which assign a duration  
14     to each activity. Beck and Wilson evaluate the makespan of an activity sequence  
15     in a single scenario by assuming that each activity starts as soon as possible after  
16     its predecessors have ended: the start time of an activity is the maximum over the  
17     end times of all its direct predecessors.<sup>7</sup> Another way of viewing this assump-  
18     tion is as the right shift execution algorithm: each activity is shifted to the right  
19     just far enough to incorporate the realized durations of its preceding activities.  
20     In other words, a particular execution algorithm has been incorporated into the  
21     Monte Carlo simulation which evaluates the proactive schedules.  
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36     This conceptualization immediately suggests that other execution algorithms  
37     could be incorporated. For example, rather than completely sequencing the ac-  
38     tivities on each resource, the generation algorithm might leave some activities  
39     unsequenced and rely on a more sophisticated simulation of an execution algo-  
40     rithm. The execution algorithm may follow the shortest expected processing time  
41     rule to decide on the complete sequence of activities and the right shift rule to set  
42     activity start times.  
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### 35     5.5.3 Comparison of the Three Families of Techniques

38     Given information about uncertainty, an instantiation of our framework can be  
39     used, offline, with simulation, to evaluate a proactive algorithm, as in Beck and  
40     Wilson, a progressive approach, such as the one described above, or a purely  
41     revision approach, by simulating an execution algorithm. In terms of offline rea-  
42     soning, *when there is uncertainty information available*, the differences among  
43     these approaches become simply a matter of degree of offline decision-making.  
44     A sophisticated system may be able to decide that in some situations (e.g., high  
45     uncertainty) a revision approach will lead to better results, while in others (e.g.,  
46     low uncertainty) a more proactive approach will allow better coordination with  
47     outside suppliers.  
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50     A proactive algorithm can be used online as well; e.g., when we need to change  
51     decisions (i.e., we combine revision and proactive approaches), or when we have  
52     to make new decisions (i.e., we combine progressive and proactive approaches).  
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55     <sup>7</sup>Recall that the direct predecessors of an activity consist of the activity immediately preceding  
56     it in the same job and the activity immediately preceding it on the same resource.  
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## 6 Adapting our Framework to Planning

First of all, our study which is focused on a scheduling problem is of course relevant in a more general planning framework since uncertainties on resources and time, which have been the focus in this paper, usually also play the main part in stochastic planning, and precise scheduling is an important sub-problem of planning, especially when time and resources are explicitly taken into account [44], as in plan-space and/or constraint-based planning approaches.

In some sense, the capability of dealing with conditional subsets of partially-ordered activities helps bridging the gap between scheduling and planning as such choices introduce some level of (limited) reasoning about actions, suggesting for instance to represent the problem with AND/OR graphs, as it is often done in the planning community [34].

But of course more can be done to introduce some more planning “flavor” in the global picture. Let us look at what can be done revisiting our taxonomy:

- *Revision* planning techniques can be easily incorporated, tracking causal inconsistencies (e.g. a precondition which does not hold as expected): one can first try to adapt the plan locally, resolving the conflict through adding new constraints (e.g. strictly ordering some unconstrained activities) or inserting new activities that will restore the broken causal link; in case that fails, then complete replanning can be run.
- As it has been suggested for instance in [14], *continuous planning* can be considered to address the arrival of new goals, simply extending the current plan to satisfy the new goals: partial-order plan-space planning is usually the best choice to address such issues.
- *Flexible* and *conditional* plans are now well-known techniques for building plans, especially in temporal constraint-based planning [30, 46], which is perfectly compatible with our constraint-based scheduling framework: as we have already said, our conditional schedules are actually inspired by the literature in conditional planning.
- Last, *probabilistic* planning [26] is another field of intensive research which proposes techniques to look for a *generic* solution in terms of the most probable plan to execute securely, considering distributions over known uncertainties, for instance the probability of alternative effects of some activities. Adding such probabilities (or possibilities, as in [17]) and computing the overall expected probability of success of the resulting plan is no more difficult than adding probabilities on resource breakdowns or activity durations.

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5 Our framework can hence easily be extended, both theoretically, incorporating  
6 in our model new sources of uncertainty (e.g. imprecise effects of actions)  
7 and specific algorithms for switching contexts (replanning, plan adaptation), and  
8 experimentally: techniques in the field of plan-space search, and more specifically  
9 in partial-order and constraint-based temporal planners, such as IxTeT [29]  
10 or HSTS [32], are highly compatible with our constraint-based scheduling proto-  
11 types.  
12

13 To conclude with this section, and focusing on the IxTeT-Exec system [29],  
14 one should notice that it does actually cover our three main types of approaches:  
15 revision (plan repair and replanning are incorporated into the execution module),  
16 proactiveness (plans are flexible) and progressiveness (new goals can be integrated  
17 at any time). Therefore, IxTeT-Exec, which lies in the area of plan-space temporal  
18 planning and uses widely constraint-based techniques, is already an example of  
19 the relevance of our work in planning. But it cannot be considered as an extension  
20 of our work, for (at least) four reasons: conditional branches are not possible in  
21 IxTeT, uncertainty on resources is not tackled, the models of uncertainty are very  
22 basic (e.g. no probabilistic nor possibilistic models), and optimization is not for-  
23 mally addressed in that work. We strongly believe that such work and ours could  
24 definitely benefit from each other, towards the goal of a complete and expressive  
25 formal architecture for planning and scheduling under uncertainty.  
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## 32 7 Conclusion and Future Work

33 In this paper, we presented a general framework for planning and scheduling when  
34 the execution environment is stochastic. Our model acts as a generic conceptual  
35 representation that can integrate three general complementary families of tech-  
36 niques to cope with uncertainty: proactive techniques use information about un-  
37 certainty to make decisions; revision techniques change decisions when it is rele-  
38 vant during execution; progressive techniques solve the problem piece by piece on  
39 a gliding time horizon. We showed this model can address diverse and complex  
40 problems, from classical scheduling settings to concerns inspired by conditional  
41 planning: in particular, it is possible to handle mutually exclusive subsets of activ-  
42 ities. In addition, we described software prototypes directly instantiated from our  
43 model and controlled by several parameters. The prototypes can address a large  
44 range of types of uncertainties modeled with probabilities.  
45

46 To summarize, the main contributions of this paper are the following:  
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- 51 • a three-dimension classification of the systems and techniques for planning  
52 and scheduling in a stochastic environment;
- 53 • a representation model of an extended scheduling process interleaved with  
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5 execution, integrating the three families of techniques from our classifica-  
6 tion;  
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- 8 • software prototypes and experimental results that support our representation  
9 when problems are probabilistic.  
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11 Apart from the extension of our model to more general planning concerns,  
12 which has been discussed in last section, interesting future work consists in en-  
13 hancing our experimental system, mixing the proactive, progressive, and revision  
14 techniques and in studying the relationships between the different parameters, in-  
15 dicators, and problem characteristics. For example, we could set a proactive ap-  
16 proach that takes into account 60% of execution scenarios, a middle progressive  
17 time horizon, and a small number of reschedulings. Another study could consist  
18 in setting a proactive approach that takes into account 80% of execution scenarios,  
19 a large progressive time horizon, and a small number of reschedulings. This work  
20 paves the way to the development of a software toolbox gathering a large set of  
21 algorithms to manage scheduling and schedule execution in a stochastic environ-  
22 ment. Users of that toolbox would use the general architecture designed through  
23 our model but they would be able to design their own application by selecting  
24 relevant modules and correctly tuning parameters (e.g., anticipation time horizon,  
25 sensitivity factor, etc.). Our future work is to implement such a complete toolbox  
26 and perform additional experiments to check how parameter tuning will influence  
27 stability and robustness of the solutions that the system will generate.  
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## List of responses to the reviewers' comments

> Both reviewers observe that the paper is an extension of your IJCAI'07 paper, in particular, yet this is not referenced. It is clearly important to indicate for readers what the relationship is and to include the citation.  
>

We have cited our IJCAI paper at the beginning of our JOSH paper.

> Apart from specific comments raised by the reviewers, which you should check and address as appropriate, the key concern I have is that the paper is not strongly related to the subject of the call: the interface between planning and scheduling. I think that it is fair to claim that you have, between you, some knowledge and experience of planning material and could make some concession to the planning audience, drawing people into the paper by with some motivation in terms of planning research. One of the reviewers observes that it is possible to see a relationship to planning and I am equally confident that this is the case. I think the chief requirement here is to make the material more obviously relevant to a planning audience and to demonstrate a deeper concern with the issues that lie at the interface between planning and scheduling in order to show relevance to the special issue.  
>

We have rewritten Section 1 (Introduction), we have inserted some planning examples in Section 3 (Classification) and have changed some text of Section 3, we have inserted Section 6 (Adapting our Framework to Planning) to discuss some extension of our work to AI Planning, and we rewrote Section 7 (Conclusion and Future Work).

> Reviewer #1: The paper focuses on scheduling in a stochastic environment, where machines can break down, and the duration of activities is uncertain. It classifies previous work on this topic, including proactive, revision and progressive techniques. The paper goes on to present a notation and formalism for combining all these types of approaches, enabling a broad range of new solutions/policies for scheduling problems to be generated. This is an interesting and potentially valuable idea. However the formalism is not described very clearly. One also doesn't get a good sense of what is possible in the framework, and how the generalised schedules could be generated in practice. There are some substantial experimental results described, but only concerning the use of a revision approach.  
>

> In general, the paper is often hard to follow, and doesn't seem to have been very carefully written. In my view, there's a potentially good journal paper here, but the current version needs a good deal of rewriting before it is publishable.  
>  
>

> In the following detailed comments, I'm using the left-hand column numbers (from 1 to 65 on each page). So that e.g. 2: 22 means page 2, line around 22.  
>

> It would natural to state that this paper extends an IJCAI'07 paper by the authors.  
>

We have referenced this paper at the beginning of our paper.

> 2:22 contoller -> controller  
>

We have changed our text accordingly.

> Before the definitions on page 4, it would be good to say something about what is meant by a "schedule". (Footnote 4 on page 17 indicates that this is not a priori clear.) Also, "predictive schedule", as used e.g., in 5:30.  
>

We have rewritten Section 2 dedicated to basic definitions.

> 4:34, Definition 2.6: The second sentence is confusing and doesn't seem to fit in with the first. Should "expected quality" be something like "actual quality" (similarly line 56)?  
>

We have corrected this issue by rephrasing text.

> Section 3, Classification, page 5 etc. A couple of bodies of work that it sounds like the authors are not aware of: by Erik Demeulemeester and Willy Herroelen and co-authors, from K.Univ.Leuven, and by Rolf Moehring and others at Technische Universitaet Berlin.  
>

We have added a reference to a paper of Herroelen and Leus.

> 5:34 - "In other words..." this sentence doesn't seem to fit with the previous one. If one were to maximize the executability, then this would often presumably lead to an extremely cautious schedule, with a very poor makespan.  
>

We don't think this remark makes sense, as we think there is no inconsistency in the sentence. Anyway, we have rephrased a little bit the sentence and have replaced "rigid" by "complete" to better fit with our definitions.

> 6:13 - "incomplete flexible schedule" - this is tautologous since a flexible schedule was defined to be incomplete.  
>

We have corrected the text accordingly. We have found another place where tautologies were written (in the discussion after mixed approaches, Section 3.5). I

> 7: 9 - It would be good to have more detail regarding the second approach.  
>

We have inserted Section 3.1 (Our Taxonomy in Brief) to give the big picture of our classification. In addition, we have inserted another example at the end of Section 3.3 (Revision Techniques).

> 8:15 - some references for MDP approaches would be appropriate here.  
>

We have inserted two citations (Geffner and Puterman) there.

> 8:55 "Opportunistic scheduling has the ability...": Is "Opportunistic scheduling" a specific technique? If so, a description or reference would be nice.  
>

We have rephrased to remove this expression from our text.

> 9:12 etc: the four bullet points could do with more explanation.  
>

We have inserted some comments.

> 9:47 "is better than pure CP...": In what way?  
>

We rephrased this paragraph.

> Page 10: I think that the different kinds of horizon need clearer explanation. Also, the the telescoping-time horizon and the gliding-time horizon on page 11.  
>

We think the text is now clear enough concerning these terminologies, since we have added some examples to illustrate our classification.

> 11:12 "planning is done on-board". On-board what?. This paragraph needs some more explanation.  
>

We have corrected the sentence and rewritten this paragraph.

> 11:31 "Activity durations are deterministic": it would be good to say explicitly where the uncertainty is in the problem.  
>

We don't understand this misunderstanding, since we say that jobs arrive stochastically in this paragraph.

> 12:8 "The objective is then to minimize the cost of starting jobs too early and the cost of work-in-progress inventory and tardiness." Is the sum of these costs being minimised? This paragraph is also quite hard to follow.  
>

We have corrected the text to explain that the sum of these costs is being minimized. We have rephrased this paragraph to make it more understandable.

> 13: 13 "The commitment time horizon equals the complete  
problem time horizon: even though decisions will be changed online," This seems strange: if the decision are changed, then they presumably weren't really committed to!  
>

We have rephrased to make this paragraph consistent with our definition of "commitment time horizon." I modified Table 1 accordingly.

> 13: 55 "a pure proactive technique is not realistic since there will always  
be unpredicted or unmodeled deviations that can only be dealt with by a revision  
technique." I don't think that this is necessarily the case, if the proactive technique is sufficiently complete in the cases that it considers.  
>

We have changed our text to give more "flexibility" in the sentence.

> Page 14, table 1: why is there a single column for quality and robustness, when these would seem to be opposing criteria? One may need to sacrifice quality to achieve a more robust solution.  
>

That's right, but we present our properties so because we make the hypothesis we optimize only one criterion at a time (quality, robustness, or stability).

> 14: 52 "extended scheduling problems with mutually exclusive" : I guess "mutually exclusive" should be "disjoint". Similarly on page 17.  
>

We don't think that the mutually exclusive subsets of activities are necessary disjoint.

> Page 15, definition 4.2: why is a constraint being defined as a function rather than a relation?

>

A constraint is a Boolean function, since it permits to check whether the domains of variables are consistent with respect to some relation; e.g.,  $f = (\text{startA} > \text{startB}) \text{ AND } (\text{endC} < \text{endD})$ .

> 15:35 "a set of (probabilistic/possibilistic/etc.) distributions": Why a set rather than just a single distribution?

>

We have changed our text and written that only a single distribution can be associated with a contingent variable, since contingent variables are independent of controllable variables. This simplifies the reading of our text.

> Page 17, definition 4.8: this is not easy to understand. The second case seems to be defining a schedule as an activity, but I doubt that this is what was intended. What does it mean to say that a "recipe follows a branching node"? Is this enforced by a precedence constraint? The definition seems to need a semantics, or a better explanation, perhaps with small examples.

>

We have changed our text accordingly. Some precedence constraints are enforced when dealing with recipes.

> 23:47 "During the execution of road1, we assume the probability associated with tunnel1 decreases,": by what mechanism? What kind of inputs allows one to deduce this? This confused me further when I noticed that Tunnel seems to be a decision variable (line 51).

>

We have included a sentence saying that there is a continued monitoring of probabilities associated with recipes, information is given by Nature.

> 25: 42 "We simply present a few software prototypes that we implemented,  
> and we show how they are actually special cases of our global model and hence  
> largely validate it. Detailed experimental results appear in the cited papers."  
> It's not completely clear what has been implemented, what has been tested experimentally, and what is just a theoretical idea. For example, 5.4 is part of Section 5, Experimental System, but it's not clear whether it's been implemented or not.  
>

The revision approach has been completely implemented and experimented. The progressive approach has been designed but not completely implemented. The proactive approach has been implemented and tested by Beck and Wilson.

> 28: 51 What's the significance of the following? "Note that the computation of a deterministic approximation can be parametrized, see Section 5.5.2".

>

We have rephrased the paragraph.

> 30: 18 The method for generating resource breakdowns is not completely clearly explained.

>

We don't think we need to add explanations for this. We have replaced "operation" by "activity" twice in this paragraph, it was perhaps the reason of misunderstanding, we guess. In addition, we have split the paragraph into two parts.

> 31: 45etc: varsigma is a rather unusual character to use, and many readers (myself included, before I looked it up) may not know how to pronounce it.

>

We have replaced it by omega.

> 37: 39 "true lower bounds" -> "strict lower bounds"

>

We have changed our text accordingly.

> Page 37 etc: I found Section 5.4 often to be very hard to follow, especially pages 43 and 44.

>

> 39:25, the formula for  $t_D$ : it perhaps should be clarified that  $i$  is intended to be fixed for the max (whereas  $j$  varies).

>

We have changed our text accordingly.

> 46: 42 "evaluating the probability of achieving a given makespan for a

> potential solution is in #P [20]." Saying that a problem is in #P expresses an upper bound, rather than a lower bound on the complexity. I guess what was meant was "is #P-hard", or "is #P-complete".

>

We have not changed this sentence. The person interested by this aspect can read the cited reference.

>

>

> Reviewer #2: This paper addresses the problem of stochastic scheduling. The paper seeks to provide three main contributions:

>

> 1) A survey of current stochastic scheduling algorithms, providing a taxonomy of current techniques.

>

> 2) A framework that can capture any of (and even a mixture of) these techniques.

>

> 3) Experimental analysis to validate the model

>

>

>

> The work in this paper is both novel and interesting to the planning and scheduling communities. The authors correctly identify that when dealing with stochastic problems, scheduling and execution have to be considered simultaneously. The contribution of a framework in which joint scheduling/execution systems can be developed is a powerful idea. The paper has several flaws that mean I cannot recommend the work for publication at this time. However, it should be clear that the work is worthy of publication provided sufficient amendments are made.

>

>

> General criticism of the paper relates to the flow and consistency between the different sections. Several parts of the work are difficult to follow, and can be improved. The empirical analysis is not extensive enough and does not validate the model, in my view.

>

> The paper is strictly a paper about scheduling and no real attempt has been made to relate it to AI Planning. I say this, as there are a couple of references to AI Planning that are entirely unqualified. One which refers to POMDP planning, without citations.

We have inserted citations, rewritten Section 1 (Introduction), inserted some examples in Section 3 (Classification), inserted Section 6 (Adapting our Framework to Planning), and reworked Section 7 (Conclusion).

> Another which refers to AND/OR graphs in planning: again with no citations.

We have added a citation for AND/OR graphs.

> To me, it is clear that the framework could be used in a planning context. But in order for the references to planning to be useful, the authors should comment on this.  
>

We have added Section 6 that deals with extending our framework to AI planning.

> Much of Sections 1 to 4 are repeated verbatim from the IJCAI '07 paper:  
>  
> A General Framework for Scheduling in a Stochastic Environment  
> Bidot, Vidal, Laborie and Beck  
>  
> And yet, no reference is made to it. Also, a reasonably large amount of text is reused from your ref [9]. It would be clearer to the reader to highlight that the paper does reuse previous published work. You do add a footnote that "The work reported in this paragraph was partially published [8]". [8] appears to be a workshop publication, but [9] and the unreferenced IJCAI publication are not. And so, it is more important that these are mentioned.  
>

We have cited our IJCAI publication at the beginning of our paper.

> Specific Comments:  
>  
> It is perhaps worrying to read in the abstract of a journal article that it contains "first experimental prototypes". Perhaps just modesty, but not in this case. More on that later.  
>

We have changed our abstract in this sense to reflect what we have really done from an experimental perspective.

> Section 2 seems almost redundant to me. If you are providing definitions, they should be precise, even if they are not defined in formal language. Precision is the exact reason to use formality in definitions: it avoids the ambiguity inherent in linguistic definitions. There are several places in which they are imprecise:  
>  
> \* Surely an "Executable Schedule" should not violate any constraints for all t  
>

We have kept this definition in our text, as the executability of a schedule is monitored (revision approach): it is an important aspect of our paper.

> \* How can an "Adaptive Scheduling System" generate a new executable schedule WHENEVER the current schedule is no longer executable? In some circumstances, surely there will be no solution.  
>

We have addressed this issue to make the definition more realistic.

> \* You define a "Stable Schedule" as one in which NO changes are made. However, in the following paragraph, you say that "A schedule can be stable but not robust, which means that we do not change many decisions...". So in fact, a stable solution is one in which "not many decisions" are changed.  
>

We have changed the text accordingly (the definition of a stable predictive schedule).

> Section 3 is generally good. The text in Section 3.1 could benefit from being broken into smaller subsections, it seems to be quite verbose presently. One problem with Section 3 is that the definitions of Proactive, Revision and Progressive techniques (crucial to the work) are mixed in with the literature survey. It would perhaps be better to have definitions of the three techniques up-front before discussing any literature. As it is, a less than careful reader may miss crucial information.  
>

We have added Section 3.1 to give an overview of our classification.

- > Section 4.1 is generally okay. When you mention mathematical relations, you should say arithmetic relations, as the logical relations you mention are also mathematical relations.
- >

We have rephrased this section accordingly.

- > Page 17 includes a forward reference: this should be avoided.
- >

We have not found this forward reference in our text.

- > Sections 4.2 and 4.3 are good. The example is useful.
- >

That's fine.

- > Section 5.1 is a good description of the studied problem, Section 5.2 is good, and the figures are illustrative.
- >

That's fine

- > Section 5 does, however, need to be extended and be more coherent. It introduces an interesting set of results for the Revision approach, and then describes an unimplemented model for the Progressive approach.
- >
- > I believe that for this work to be of real value, it needs to be seen that there are circumstances in which each of the three approaches is better than the others. Otherwise, one of the techniques may dominate, and there actually is no need for the general architecture.
- >
- > Since only one of the approaches is implemented, how can comparisons really be made?
- >

We cannot conduct any additional experiments. However, we have rewritten Section 3 (Classification) to make clearer the differences between the three approaches.

- > Another real problem in Section 5 is the flow between subsections and even within subsections. Incorrect use of the word "paragraph" makes it difficult to understand pages 34 and 41. Page 41 contains forward references to page 43.
- >

We have rephrased our text accordingly.

- > Section 5.3 on the Revision approach is quite strong. The paper needs two more sections of that strength, on the Progressive and Proactive approaches, with empirical results.

We could not work again on the software prototype, so we cannot produce any new experimental results.

- > In 5.5.2, Beck & Wilson [7] is discussed. Surely this should be discussed in the literature survey in Section 3, and not in the discussion of your results.
- >

We do not agree with this proposal, since we have taken part in the original design of this work.

- > Typos and English:
- >
- > In the abstract: "Hence it is first desirable" should be "it is"
- >

We have corrected our text accordingly.

> Section 1  
>  
> Sentence starting "In order for that plan..." should probably be split into two sentences to improve readability.  
>

We have corrected our text accordingly.

> Use of "in in" page 2 (sentence starting "What we are"), maybe consider using "In this paper we are interested in" instead.  
>

We have corrected our text accordingly.

> "use here" -> "use"  
>

We have corrected our text accordingly.

> "jointly reconsidered" -> "jointly considered"  
>

We have corrected our text accordingly.

> "executed schedule? etc. In other words" -> Remove the "etc."  
>

We have corrected our text accordingly.

> "reconsidered" -> "considered"  
>

We have corrected our text accordingly.

> "Therefore, after having..." -> "After having..."  
>

We have corrected our text accordingly.

>  
> Section 2  
>  
> "we only present here" -> "we present"  
>

We have corrected our text accordingly.

> Use of "in in" again. Reword to something like "Before presenting the formal model in Section 4, we informally present the problem..."  
>

We have corrected our text accordingly.

> "as is often done in the planning community" (citation required)  
>

We have corrected our text accordingly.

>  
> Section 3  
>  
> "that are more robust, more stable, or both, than they would be..." -> "that are more robust, more stable, or both more stable and more robust than they would be..."  
>

We have corrected our text accordingly.

> "worst scenario" -> "worst-case scenario"  
>

We have corrected our text accordingly.

> Several paragraphs not indented.  
>

We have corrected our text accordingly.

> I don't understand the sentence starting "This observation is at the outset of two methods". What do you mean by outset in this context?  
>

We have corrected our text accordingly.

> "tackle problems with using" -> "tackle problems using"  
>

We have corrected our text accordingly.

> "par excellence" -> suggests personal opinion  
>

We have changed this expression.

> It is often stated that "we" do things, when really it is an algorithm / system doing it.  
>  
> "Last but not least" is quite idiomatic and should be avoided  
>

We have replaced it by "In addition."

> You introduce the abbreviation LP for Linear Programming, and then use it to mean Linear Program ("constraints and the cost function in an LP...")  
>

We have changed this paragraph.

> "We can also tell about an approach" -> "An approach"  
>

We have corrected our text accordingly.

>  
> Section 4  
>  
> "litterature" -> "literature"  
>

We have corrected our text accordingly.

```
> "synthesize" -> "synthesizes"  
>
```

We have corrected our text accordingly.

```
> ellipsis appears at the end of the paragraph ending "about unceratinty is available . . . ". Why?  
>
```

We have changed this paragraph accordingly.

```
> "We only defined a model" -> "We defined a model"  
>
```

We have corrected our text accordingly.

```
>  
> Section 5  
>  
> "most shadowed" -> "most shaded"  
>
```

We have corrected our text accordingly.

```
> page 43 begins with a 10 line sentence. Break into smaller sentences  
>
```

We have corrected our text accordingly.

```
> page 44  
>  
> "\gamma \in ]0,1[" -> "\gamma \in [0,1]"  
>  
> "\gamma \in [0,1[" -> "\gamma \in [0,1]"  
>  
>
```

The formula for tardicCost\_i^exp is faulty (we don't need \gamma actually). We have corrected it and rephrased the text accordingly.

```
> Section 6  
>  
> "An interesting future work" -> "Interesting future work"  
>
```

We have corrected our text accordingly.

```
>  
> References  
>  
> Page numbers missing for references 4,
```

We have not found the pages for the reference of Philippe Baptiste and Claude Le Pape.

```
> 8,
```

We have not found the pages for this paper.

> 10,

This a book, so we don't need to give the number of pages.

> 11

We have modified this reference accordingly.

> and 37  
>

We have corrected this reference.

> Does Technical Report ref. 4 have a number?  
>

We have changed this reference.

> Why give editors for ref 13 if not for every other conference?  
>

We have removed editors for conferences.

> Is month of publication important? If so, then several are missing.  
>

We have removed this field for every paper.

We have found few "operation" that we have replaced by "activity" (Sections 5.1.1, 5.1.3, and 5.2). We have found few "reactive" that we have replaced by "revision" (Sections 5.5.2 and 5.5.3).

We have corrected a few typos as well

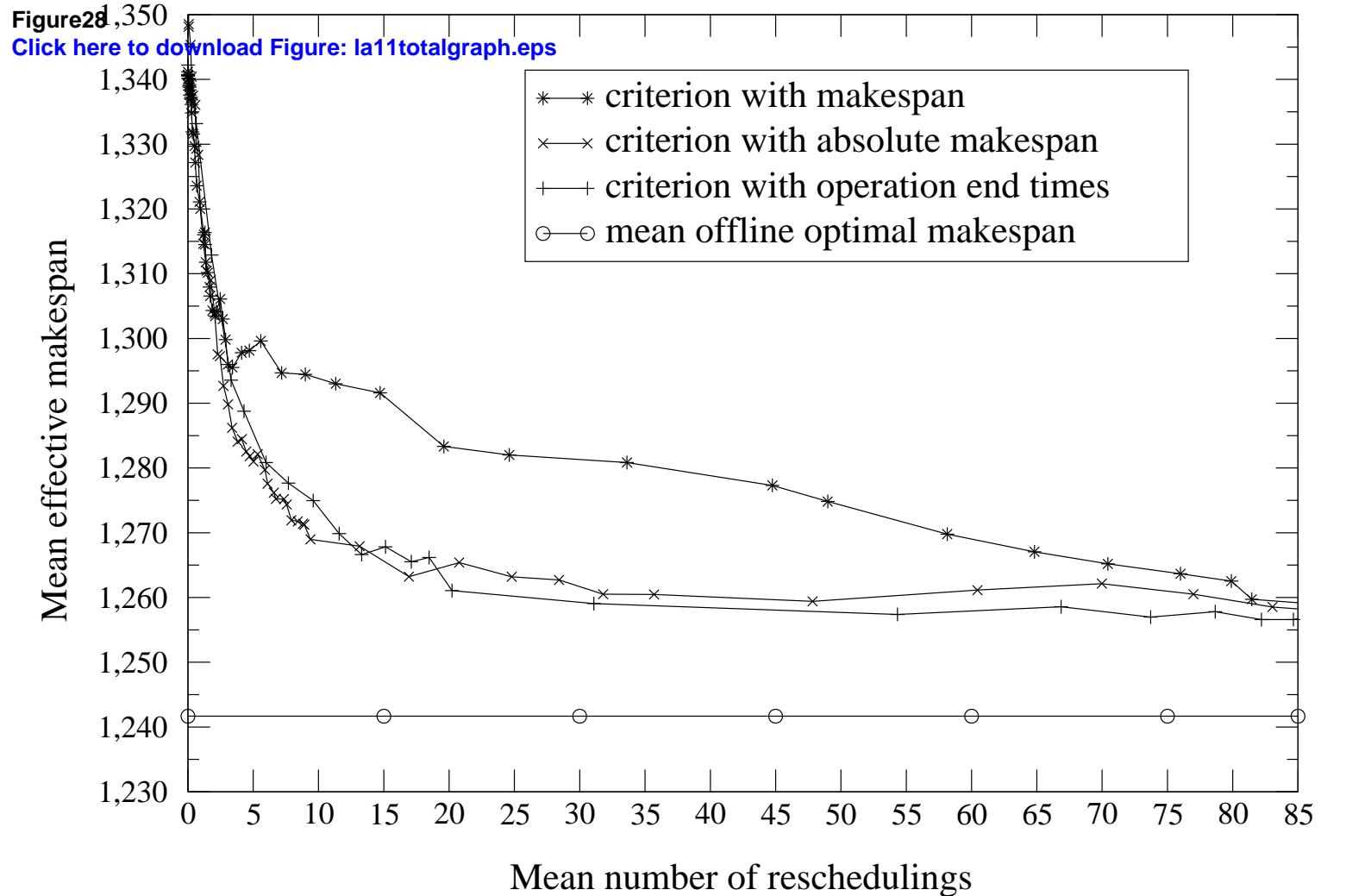


Figure 29  
Click here to download Figure: la11totalgraph.pdf

