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Watching and Acting Together: Concurrent Plan Recognition and Adaptation for Human-Robot Teams

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Abstract

There is huge demand for robots to work alongside humans in heterogeneous teams. To achieve a high degree of fluidity, robots must be able to (1) recognize their human co-worker's intent, and (2) adapt to this intent accordingly, providing useful aid as a teammate. The literature to date has made great progress in these two areas – recognition and adaptation – but largely as separate research activities. In this work, we present a unified approach to these two problems, in which recognition and adaptation occur concurrently and holistically within the same framework. We introduce Pike, an executive for human-robot teams, that allows the robot to continuously and concurrently reason about what a human is doing as execution proceeds, as well as adapt appropriately. The result is a mixed-initiative execution where humans and robots interact fluidly to complete task goals.

Key to our approach is our task model: a contingent, temporally-flexible team-plan with explicit choices for both the human and robot. This allows a single set of algorithms to find implicit constraints between sets of choices for the human and robot (as determined via causal link analysis and temporal reasoning), narrowing the possible decisions a rational human would take (hence achieving intent recognition) as well as the possible actions a robot could consistently take (hence achieving adaptation). Pike makes choices based on the preconditions of actions in the plan, temporal constraints, unanticipated disturbances, and choices made previously (by either agent).

Innovations of this work include (1) a framework for concurrent intent recognition and adaptation for contingent, temporally-flexible plans, (2) the generalization of causal links for contingent, temporally-flexible plans along with related extraction algorithms, and (3) extensions to a state-of-the-art dynamic execution system to utilize these causal links for decision making.

1. Introduction

There is a huge demand for humans and robots to work alongside each other to collaboratively achieve tasks. This is apparent in a number of domains, including aerospace manufacturing, household robotics for assisting in daily chores, medical robotics for performing clinical procedures, and countless more. No matter what the domain, it is necessary for the robots to both (1) infer the intent of their human teammates, and (2) adapt to their intent appropriately. Without such fluidity, accomplishing a task in a collaborative manner

with humans would be challenging and humans would likely find working with such robots to be laborious, thus limiting their adoption.

The literature to date has made great progress on these two areas – intent recognition and adaptation – but largely as separate research activities, and rarely with overlap. Numerous approaches to intent and plan recognition have been proposed (Sukthankar, Geib, Bui, Pynadath, & Goldman, 2014; Carberry, 2001). However, these approaches generally focus solely on the recognition task and not on selecting suitable adaptations for the robot once the recognition is achieved. In parallel, numerous approaches to robotic adaptations have been proposed over the years, but few focus explicitly on adapting to inferred intent.

This work takes the viewpoint that intent recognition and robot adaptation can and should be viewed as two sides of the same coin, and that any practicable cognitive robot must integrate both approaches seamlessly. Towards that view, this work presents a single set of algorithms that simultaneously performs intent recognition and selects robot adaptations within one framework and with one model. Both problems are framed in our approach as inference and consistency-based reasoning. By performing intent recognition and adaptation concurrently as opposed to separately, our system is able to achieve a greater degree of robustness and fail less often.

To achieve this duality, a key aspect of our approach is our shared plan representation. We operate on *contingent, temporally-flexible team plans*. Some actions in the plan are targeted at the human, while others are for the robot(s) to perform. Additionally, these plans contain explicit choices which are either controllable by the robot (allowing it to choose to perform or not perform certain actions), or uncontrollable by the robot (up to the human or nature to perform certain actions). The choices in these plans afford flexibility to react to different situations that may arise during execution by encoding different contingencies. An example of such a plan is shown in Figure 1. Our approach exploits the often implicit interconnections between these two sets of choices, controllable and uncontrollable. Namely, only certain combinations of uncontrollable choice outcomes made by the human and controllable choice outcomes made by the autonomous agent would be allowable in successful team plans. Our approach reasons over these sets of choices to determine which outcomes would be possible, and uses that to predict which choices a rational human agent could consistently take (thus inferring intent), and simultaneously infer which choices the robot should take (thus adapting appropriately). By explicitly considering the coupling between human and robot decisions, our reasoning algorithms are thus able to consider intentions and adaptations from a unified viewpoint.

The basis for the interconnections between choices come from several different sources. The first is state: certain actions require other previously-executed actions in order to succeed. The second is temporal requirement: certain choices preclude other choices due to deadlines or other timing constraints imposed in the plan. A third is unanticipated disturbances: certain sets of choices become infeasible given new knowledge that the world has changed in an unexpected way. Our technique deals with all three of these. We handle the first by reasoning about causal links, the second through temporal conflict extraction, and the third through online causal link execution monitoring. At a high level, these three techniques can be summarized as *keeping your eye on the goal*. Our system makes choices and monitors the plan always with respect to the human-robot team's goals.

Our approach is divided into two phases: (1) an offline compilation phase, and (2) online execution. During offline compilation, we first compute implicit temporal relationships in the plan, which allows us to reason about which actions must precede which others. We then use this information to extract *labeled causal links* from the plan, which capture the dependence of certain actions on others in the plan. Labeled causal links generalize the notion of causal links from non-contingent plans to contingent temporally-flexible plans. Their labels capture the choices required for them to hold, which is necessary in our contingent plans where actions may or may not occur (depending on the choices made), and where producers and threats may be partially ordered. After extracting these labeled causal links, they are then translated into propositional state logic constraints where each solution to the constraints represents a successful execution of the team plan. Finally, these propositional constraints are then compiled into a form suitable for fast online use. The output of this compilation is a data structure that represents the space of all successful executions of the plan, and can be quickly queried online. A similar problem has been solved in the past by the Assumption-based Truth Maintenance System (ATMS) (De Kleer, 1986a); our compilation process is inspired by it, and extends the ATMS label propagation algorithms to generate a sound and complete set of prime implicants that are used online. The compilation allows human robot interaction to be performed quickly without the requirement of replanning when new observations or certain unexpected disturbances occur.

Our online execution algorithm is similar in spirit to that introduced by Drake (Conrad & Williams, 2011). We use the compiled constraints to quickly check if the robot is able to make certain choices online, and still respect temporal consistency and causal completeness of the plan. The online algorithm also takes in observations in the form of outcomes to uncontrollable choices decided by the human, as well as a full set of observed state measurements which are the basis for causal-link based execution monitoring. This allows our executive to make incremental changes online to its knowledge base and adapt according to these changes. A grounded example is provided later in this section.

As Pike is a plan executive, it is useful here to briefly say a few remarks about the complexity of executing plans. It is well known that planning is a hard problem – STRIPS planning for example is PSPACE-complete (Bylander, 1994). It is less well known, however, that execution can also be computationally challenging, depending on the features that we desire from our executive. On one end of the spectrum, the easiest form of plan execution – namely dispatching the activities sequentially to agents – is computationally very simple. Each action can be dispatched in near constant time, as no effort is required from the executive to process ordering constraints (let alone metric temporal ones), monitor for disturbances, and more. At the other end of the spectrum, plan execution is much harder for the case where we have more complex plans with metric temporal constraints (some of which have uncontrollable durations), desire rich execution monitoring to discover potential faults before they are problematic, and where the executive is left with discrete choices (i.e., choices for the robot) that must be made in a least-commitment manner online. Such approaches are generally NP-hard mainly because the introduction of discrete choices requires combinatorial reasoning. In the middle of the spectrum, there are a number of approaches that provide a middle ground both in terms of polynomial computational complexity and provided features, such as executing metric temporal plans with execution monitoring but with no choice (Levine, 2012). Our work falls into the more feature-rich, yet computation-

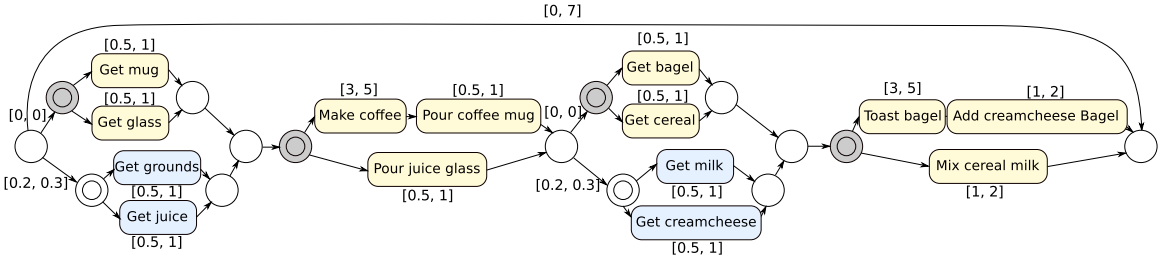


Figure 1: Contingent, temporally-flexible plan (i.e., a TPNU) for making breakfast. Circles denote events, double circles denote controllable choices (made by the robot), and shaded double circles denote uncontrollable choices (made by Alice). Edges denote temporal constraints. Yellow boxes denote activities for Alice, while blue boxes are targeted at the robot. Note that each activity is represented with a start and end event, but illustrated here as a box for compactness. All unlabeled temporal constraints are $[\epsilon, \infty]$ ordering constraints.

ally challenging end of the spectrum. We argue that an executive capable of responding to disturbances, adapting to human intent, monitoring the execution for problems, and addressing metric temporal constraints – all in a flexible, least-commitment way – would provide valuable robustness for autonomous systems and justify its greater computational complexity. Many of the techniques in this paper aim to provide reasonable performance on real-world problems despite the poor worst-case complexity guarantees.

As noted earlier, Pike is designed to take as input a contingent, temporally-flexible team plan for a human-robot team. This input plan can come from a variety of sources, such as a planner capable of outputting contingent, temporally-flexible plans (to the authors knowledge however, no such planner meets this requirements - but the closest are RAO* (Santana, Thiébaux, & Williams, 2016), tBurton (Wang & Williams, 2015), and FOND (Muise, McIlraith, & Beck, 2012)). Additionally, a control program with appropriate choice structure could be created by a human expert in RMPL and compiled into a TPNU (Kim, Williams, & Abramson, 2001; Williams, Ingham, Chung, & Elliott, 2003).

1.1 Approach in a Nutshell

We illustrate a grounded example of concurrent intent recognition and adaptation. Consider the grounded example shown in Figure 1, in which a person named Alice is making breakfast for herself with the help of her trusty robot. The left half of the plan depicts the team either making coffee (for which Alice uses a mug) or getting some juice (for which Alice uses a glass), while the right half depicts either making a bagel with cream cheese or getting some cereal and milk. Alice is running late for work, so she imposes an overall temporal constraint that both her food and drink must be ready within 7 minutes.

Consider just the first half of the plan, in which the team prepares a beverage — either coffee or juice. There are three decisions — at first, Alice chooses to either get a mug or a glass for her beverage. Shortly thereafter and in parallel, the robot chooses to either fetch the coffee grounds or juice from the refrigerator. Finally once all the necessary supplies

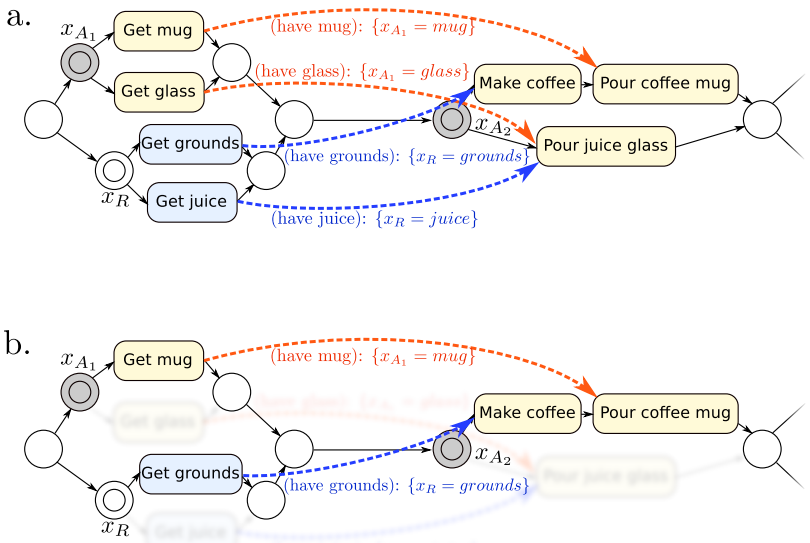


Figure 2: Kitchen example. Part (a) shows the plan annotated with labeled causal links. Part (b) shows the resulting execution if Alice chooses $x_{A_1} = mug$.

have been retrieved, Alice will either make coffee and pour it into her mug, or pour the juice into her glass, depending on her preferred beverage this morning.

Key to our approach is observing that these three choices — two uncontrollable made by Alice and one controllable made by the robot — are not independent. Rather, they are tightly coupled through state constraints in this example, and more generally through temporal constraints as well. For example, if the robot chooses to get the juice out of the refrigerator, Alice will not be able to make coffee as her second choice since she will not have the coffee grounds. Such interrelationships are illustrated in Figure 2a, where we have defined three variables, x_{A_1}, x_R, and x_{A_2}, to represent the different choices. We have also annotated the plan with *labeled causal links*, denoted by dotted arcs and labeled with the environment under which they hold. These labeled causal links capture state requirements in the form of actions preconditions, and their environments imply constraints over feasible choice assignments.

Consider the execution of the example plan depicted in Figure 2b, in which Alice chooses to get the mug instead of the glass for her first choice (i.e., $x_{A_1} = mug$). Alice will therefore have a mug (necessary for pouring coffee into the mug in her second choice), but will not have a glass (necessary for pouring juice into the glass). The top-most labeled causal link, (have mug) contingent upon $x_{A_1} = mug$, will hold, yet the labeled causal link (have glass) contingent upon $x_{A_1} = glass$ will not hold. Thus, Alice cannot choose $x_{A_2} = juice$, since the pour juice action would have an unsatisfied precondition. In this way, the robot infers that Alice – a rational agent – must have the intent of making coffee (i.e., $x_{A_2} = coffee$) since she picked up a mug and not a glass. Our approach therefore makes the assumption that human agents are rational, and will not knowingly make inconsistent choices.

Similar reasoning allows the robot to adapt to Alice's intent. Given that Alice is making coffee, she'll require the coffee grounds — illustrated by the causal link (have grounds) contingent upon $x_R = grounds$. Since this is the only labeled causal link supporting said precondition, the robot infers that it must choose $x_R = grounds$. Thus, the robot adapts by getting the grounds instead of the juice, resulting in the final execution depicted in Figure 2b.

If we step back and consider the larger plan shown in Figure 1, we note that Alice cannot both make coffee and make a bagel because she would run out of time. The minimum time required to make coffee and toast a bagel is more than 7 minutes. Thus, if Alice chooses to grab a mug at the beginning, the robot infers that her intent must be not only to make coffee, but also to choose the less time-consuming cereal option so she'll arrive at work on time. By similar causal link analysis, the robot will adapt by getting milk for her cereal instead of cream cheese for a bagel.

Consider one final case, in which Alice at first chooses $x_{A_1} = glass$. She'll have enough time later for either a bagel or cereal. However, suppose while pouring her juice, an unexpected disturbance occurs: the toaster oven breaks. In this case, a labeled causal link justifying her making a bagel has been dynamically violated at run time. Alice's only option, therefore, is to make cereal. The robot detects this unanticipated change in world state, instantaneously infers Alice's refined intent, and adapts accordingly.

We have just illustrated how a single algorithm, based on constraint satisfaction, concurrently achieves plan recognition and adaptation given state and temporal constraints as well as disturbances. We note that in this specific example, Alice's intent and the robot's adaptations were completely determined after she picked up the mug. This is not generally the case however — often, there may still be multiple consistent options for future choices after constraint propagation (though fewer than before). In these cases, further decisions, either by human or robot, are necessary to hone in on a single intent and adaptation.

1.2 Related Work

There is a rich literature on techniques for both intent recognition and robot adaptation, but largely as separate research activities. To the author's knowledge, there are two other approaches that simultaneously performs explicit intent recognition and robotic adaptation from the a single, core model – a key aspect of our work. The first is described in (Freedman & Zilberstein, 2017). This work extends the foundation laid by (Ramírez & Geffner, 2010) in compiling the probabilistic plan recognition task into a classical planning problem, allowing greater foresight during recognition by using a dynamic prior. A probability distribution over different possible PDDL goal states is computed, and subsequently merged into a single estimated goal state that is used to plan a response for the robot with an off-the-shelf generative classical planner. This process repeats online, resulting in a collaborative execution. Like our approach, the work of (Freedman & Zilberstein, 2017) uses a single model for both recognition and adaptation – in their case a PDDL model, and in ours a TPNU with an action model. In many ways this is a complementary approach to ours, which offers great flexibility online. Our approach focuses on plans and models that contain explicitly defined temporal constraints and choices.

A second approach integrating intent recognition explicitly with robot adaptation is SAM (Pecora, Cirillo, Dell'Osa, Ullberg, & Saffiotti, 2012). Like our approach, SAM reasons about temporally flexible constraints and action preconditions (through a generalization known as synchronizations). SAM has been very successfully applied to a smart home scenario in which the autonomous system watches an elderly individual perform daily routines, and then automatically aids him or her or provides suggestions. While similar in the spirit of performing simultaneous intent recognition and adaptation, our approach is fundamentally different (and in many ways complimentary). To adapt to recognized intentions, SAM employs a form of online, generative temporal planning. We instead focus on having a contingent, shared task available beforehand. Generative planning approaches are well suited to household robotics domains and can enable superb flexibility, whereas having a shared contingent temporal plan is particularly well-suited to other scenarios where considering risk, deadends, low online latency, and predictability are crucial.

To the author's knowledge, most other approaches dealing with either intent recognition or robot adaptation do so as separate processes, without a single core model guiding both simultaneously. We discuss some of these approaches now, beginning with techniques for robot adaptation.

Causal links have been used in a number of AI systems, ranging from partial order planners to execution monitoring systems (Mcallester & Rosenblitt, 1991; Penberthy, Weld, et al., 1992; Veloso, Pollack, & Cox, 1998; Lemai & Ingrand, 2004; Levine, 2012). A causal link from activity A_P to activity A_c with predicate p denotes that activity A_P (the *producer*) achieves p as an effect, which is required as a precondition of a later-occurring activity A_c (the *consumer*) (Russell & Norvig, 1995). A_P must precede A_c in the plan, and there may be no other activity A_T (a *threat*) that negates p between A_P and A_c. Historically, causal links were traditionally used as open conditions during the planning process (Mcallester & Rosenblitt, 1991; Penberthy et al., 1992). When used for execution monitoring, causal links are a form of goal monitoring in which only the relevant conditions with respect to execution are monitored (Veloso et al., 1998; Levine, 2012). This monitoring allows an autonomous system to detect upcoming plan failure before it is imminent, thereby enabling proactive adaptation through replanning. In this work, we extend causal links to apply to contingent temporally-flexible plans, resulting in *labeled causal links*. Because they are crucial to reasoning about plan causal completeness, a key part of Pike's offline compilation stage is to extract these labeled causal links from the plan. They are later used to reason over possible choice outcomes online.

A number of adaptation techniques recover based on violated state constraints. Many such systems have focused on integrated planning and execution, such as IxTeT eXeC (Lemai & Ingrand, 2004), ROGUE (Haigh & Veloso, 1998), IPEM (Ambros-Ingerson & Steel, 1988), and HOTRiDE (Ayan, Kuter, Yaman, & Goldman, 2007). Some focus on planing at reactive time scales or continuously online (Finzi, Ingrand, & Muscettola, 2004; Chien, Knight, Stechert, Sherwood, & Rabideau, 2000). The Human-Aware Task Planner (HATP), combined with SHARY, executes human-robot tasks and replans as needed (Alili, Warnier, Ali, & Alami, 2009; Clodic, Cao, Alili, Montreuil, Alami, & Chatila, 2009). The TPOPExec system generalizes plans and at every time point will try to execute the first action of a subplan consistent with the current state, thus minimizing the need for replanning (Muise, Beck, & McIlraith, 2013). Like our approach, these works all focus on techniques

allowing a robot to adapt to disturbances or situations in its environment. However, these systems generally take a different strategy by focusing on planning, execution with plan repair, and/or responding to disturbances that are not explicitly modeled as human intent. We differ in that we reason explicitly over the possible choices a human is likely to make, and adapt based on this single model.

Beginning with temporally-flexible plans such as Simple Temporal Networks (STNs), efficient dispatchers have been developed that perform fast, online, least-commitment scheduling (Dechter, Meiri, & Pearl, 1991; Muscettola, Morris, & Tsamardinos, 1998; Tsamardinos, Muscettola, & Morris, 1998). Later approaches introduced uncertainty and uncontrollability into these models, resulting in executives that could adapt to many different types of temporal disturbances (Effinger, Williams, Kelly, & Sheehy, 2009; Vidal, 1999). These systems focus on adapting to temporal rather than state constraints, and do not recognize human plans. We do, however, build on the frameworks of some of these approaches. The underlying structure of our temporal plans contains set-bounded temporal constraints like these approaches, and our dispatching algorithms trace their heritage to STN dispatchers (Muscettola et al., 1998).

The Kirk executive (Kim et al., 2001) extends the notion of temporal plans to also incorporate choice, in the temporal planning network (TPN) formalism. By incorporating discrete choices, a plan now represents a set of possible subplans. The TPN is identical to the TPNU formalism used in this work, except that no distinction is made between controllable versus uncontrollable choices. Kirk is designed to assign choices optimally offline, ensuring that the resulting subplan is temporally consistent. A temporal plan dispatcher is then used for online execution. TPNs can be specified in the RMPL language (Williams et al., 2003), allowing a human operator to easily encode flexible plans with choice. The TPNU formalism used in this work derives directly from the TPNs introduced with Kirk. We experimentally compare our work against Kirk in Section 6.

Drake is an executive capable of executing temporally flexible plans with choice (such as TPNUs), scheduling and making choices online with low latency and using minimal space overhead (Conrad, Shah, & Williams, 2009). It responds dynamically to temporal disturbances, both in terms of scheduling and also in terms of making discrete choices that are guaranteed to be feasible given those temporal disturbances. This is accomplished by fusing ideas from temporal plan execution and from the ATMS (Forbus, 1993) in order to compactly maintain the set of feasible subplans online. In this way, Drake can be seen as solving a similar problem to Kirk, but with a different approach. Instead of selecting a single optimal candidate subplan offline, Drake instead compactly maintains the set of all feasible candidate subplans online and makes choices dynamically online. We borrow many techniques and algorithms from Drake in this work, as described throughout this paper.

The Chaski executive performs fast online task reallocation for human-robot teams with temporal constraints (Shah, Conrad, & Williams, 2009). Chaski is capable of inferring if an agent – human or otherwise – is stuck, and will re-allocate tasks for the other agents appropriately to maintain plan consistency. Chaski focuses on the dynamic task assignment problem, while we take a different approach and focus instead on plan recognition and adaptation by reasoning over explicit choices in the plan.

Many approaches to plan, intent, and goal recognition have been proposed, just a few of which are outlined in (Sukthankar et al., 2014; Carberry, 2001; Kautz & Allen, 1986; Bui,

2003; Avrahami-Zilberbrand, Kaminka, & Zarosim, 2005; Goldman, Geib, & Miller, 1999; Ramírez & Geffner, 2010). Most of these approaches, however, perform intent recognition without directly interacting with the human. That is, they do not attempt to interact with the human whose intent is being recognized. Our approach differs in that we perform execution while concurrently monitoring the human. An interesting related (and different) problem is that of goal recognition design (Keren, Gal, & Karpas, 2014). In this problem, the task is to modify the environment or model in which agents operate, so these agents reveal their intentions earlier rather than later.

1.3 Contributions

The main contributions of this work are:

1. Novel framework for concurrent intent recognition and robot adaptation for contingent temporally-flexible plans,
2. Generalization of causal links for contingent temporally-flexible plans, and techniques to extract them from said plans and compile them into propositional constraints,
3. An online, state-of-the art dynamic execution system that employs these causal link constraints to make online decisions.

This work is an extension of the conference paper (Levine & Williams, 2014), and makes the following additions: (1) algorithmic improvements, including the ability to handle potential threats and unordered producers to causal links (thus improving robustness), (2) greatly expanded discussion, theory, and proofs, and (3) much broader experimental validation.

1.4 Organization

This paper is organized as follows. Section 2 introduces some preliminaries and defines Pike's problem statement. We then dive into the online execution algorithm in Section 3. Next, we discuss the offline compilation that makes this online execution possible; Section 4 discusses causal link extraction and a constraint transformation needed for execution. Section 5 briefly describes a knowledge compilation approach that enables fast online querying of the constraints. Empirical evaluations are presented in Section 6. Finally in Section 7, we discuss related work and concluding remarks. An appendix presents additional algorithms and proofs of various theorems in this work.

2. Problem Statement & Solution Architecture

In this section, we define Pike's problem statement — both offline and online. We begin with some preliminaries, present the problem statement, and conclude with an outline of our solution architecture.

2.1 Preliminaries

Pike takes as input a set of possible team plans to be performed, which are represented by a Temporal Plan Network under Uncertainty (TPNU). Importantly, these plans involve

activities performed both by the robot and by the human, activities that are performed concurrently, and constraints on the timing of these activities. In the literature, a TPNU contains two types of uncontrollability: uncontrollable temporal durations (which may be either set-bounded or probabilistic in nature) (Yu, Fang, & Williams, 2014), or uncontrollability in the outcome of discrete choices (Effinger et al., 2009; Santana & Williams, 2014). We focus our attention solely on uncontrollable choices in this paper, though work extending it to handle uncertain temporal durations via strong controllability has been published elsewhere (Karpas, Levine, Yu, & Williams, 2015). The underlying temporal structure of all of these representations have roots in the Simple Temporal Network (Dechter et al., 1991).

An example TPNU is depicted in Figure 1. In this picture, circles denote *events*, each of which represents an instantaneous point in time. Examples of events include "the time at which the robot starts picking up the coffee" or "the time at which the human completed making coffee." Edges in the diagram represent temporal constraints. Colored boxes represent activities.

Definition 2.1 (TPNU)**.** A Temporal Plan Network under Uncertainty (TPNU) $\mathcal{T}$ is a tuple $\langle \mathcal{V}, \mathcal{E}, \mathcal{C}, \mathcal{A} \rangle$ where:

- $\mathcal{V}$ is a set of decision variables, which is partitioned into two groups: $\mathcal{V} = \mathcal{V}_C \cup \mathcal{V}_U$. Each $v \in \mathcal{V}$ is a discrete variable with a finite domain $\textsc{Domain}(v)$. $\mathcal{V}_C$ are *controllable* choices that are decidable by the executive at run time. $\mathcal{V}_U$ are *uncontrollable* choice variables that are not decidable by the executive, but rather by the human or nature.

- $\mathcal{E}$ is a set of events representing notable points in time. Each event $e \in \mathcal{E}$ is associated with a guard (denoted φ_e or $\mathit{guard}(e)$), which is a conjunction of decision variable assignments. An event e is *activated* if φ_e holds. Event e will be *executed* (i.e., a time scheduled for it) by the executive iff it is activated. Additionally, certain events are associated with a decision variable, denoted $v_i = \mathit{variable\text{-}at\text{-}event}(e)$, denoting that v_i must be assigned by the time e is executed.

- $\mathcal{C}$ is a set of temporal constraints. Each $c \in \mathcal{C}$ is a tuple $\langle e_s, e_f, l, u, \varphi_c \rangle$ where e_s is the *from* or *start* event, e_f is the *to* or *finish* event, φ_c is a conjunction of decision variable assignments, and $l, u \in \mathbb{R}$ represent a temporal bound with the meaning that $\varphi_c \Rightarrow (l \leq e_f - e_s \leq u)$. In other words, the temporal constraint must hold if the guard holds (it is *activated*). We require that $\varphi_c \models \varphi_{e_s} \wedge \varphi_{e_f}$, so that whenever a temporal constraint is activated its *from* and *to* events must be executed. We denote a temporal constraint $c \in \mathcal{C}$ sometimes as $e_s \xrightarrow{[l,u]} e_f : \varphi_c$.

- The set $\mathcal{A}$ represent the set of *activities*. An activity $a \in \mathcal{A}$ is a tuple $\langle c, \alpha \rangle$, where $c \in \mathcal{C}$ is a temporal constraint, and α is an action that will be executed online. With $c = \langle e_s, e_f, l, u, \varphi_c \rangle$, action α starts when e_s is scheduled, and terminates when e_f is scheduled. We require that $l > 0$.

The human-robot plans involve actions targeted at both the human and the robot that follow an action model. These actions are denoted by the α field of activities in the TPNU. An *action* α may be anything that an agent can perform. Each action has predicates called *conditions* that are required to all hold true (either at its start or end of execution), and

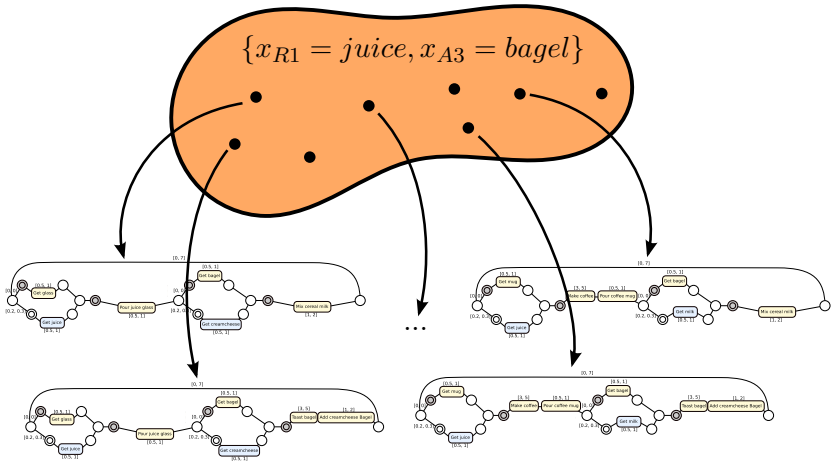


Figure 3: A TPNU is a compact encoding of many different candidate subplans, and an environment should be thought of as representing a subset of those candidate subplans. We use the term candidate subplan and team scenario interchangeably.

other predicates called effects that are asserted by the action and represent the changes to the world as a result of executing the action (again, either at its start or end of execution). In this way, we employ similar semantics to durative actions in PDDL 2.1 (Fox & Long, 2003), though actions could also be represented in other forms such as STRIPS or RMPL operators (Fikes & Nilsson, 1971; Williams et al., 2003). For the remainder of this paper, we treat the start event e_s and end event e_f of an activity as snap actions, each with their own preconditons and effects, denoted Preconditions(e) and Effects(e), respectively.

Central to the notion of Pike are *team scenarios / candidate subplans* and *environments*.

Definition 2.2 (Team Scenario)**.** A *team scenario* φ_S is a full assignment $x_i = v_i \wedge x_j = v_j \wedge \ldots$ to all decision variables in $\mathcal{V}$.

Definition 2.3 (Environment)**.** An *environment* φ is a partial assignment $x_i = v_i \wedge x_j = v_j \wedge \ldots$ to decision variables in $\mathcal{V}$. That is, some decision variables in $\mathcal{V}$ may not be assigned.

Throughout this paper, we sometimes denote team scenarios and environments using set notation for convenience, such as $\{x_i = v_i,\ x_j = v_j,\ \ldots\}$. The empty environment, $\{\}$, is logically equivalent to True.

Intuitively, a team scenario represents a specific *candidate subplan* of the TPNU. Given an assignment to each decision variable, we can evaluate which guard conditions hold, and hence which temporal constraints and events in TPNU are activated. All other inactivated events and temporal constraints can be discarded. A team scenario hence defines a specific *candidate subplan*, which encompasses a set of events, activities, and their associated flexible temporal constraints. For this reason, in this paper we often denote a specific candidate subplan simply by φ_S, its team scenario full assignment to decision variables. This is convenient, for example, so that we may refer to a candidate subplan φ_S satisfying a set of constraints (we mean that φ_S is a solution to those constraints). The underlying network

structure of a candidate subplan is that of an STN (Dechter et al., 1991), though it is distinct from an STN by virtue of having activities each with preconditions and effects.

An environment that is not a team scenario (i.e., a partial assignment to variables), represents a set of candidate subplans. Environment φ represents all candidate subplans φ_S for which $\varphi_S \models \varphi$. Thus, we use environments to compactly reason over large sets of candidate subplans, as illustrated in Figure 3.

Definition 2.4 (Environment Representing Set of Team Scenarios)**.** An environment φ *represents* the set of all team scenarios $\mathcal{S}(\varphi)$, where $\varphi_S \in \mathcal{S}(\varphi)$ iff $\varphi_S \models \varphi$.

For example, suppose we have three discrete variables x, y, and z, each with domain $\{1, 2\}$. The empty environment $\{\}$ is the most general, representing all eight possible scenarios. The environment $\{x = 1\}$ represents the four scenarios that assign $x = 1$. The environment $\{x = 1, y = 2, z = 1\}$, which is itself a scenario, is the most specific (representing just itself).

The conjunction of two environments $\varphi_a \wedge \varphi_b$ represents the intersection of these two sets $\mathcal{S}(\varphi_a) \cap \mathcal{S}(\varphi_b)$, and the disjunction $\varphi_a \vee \varphi_b$ represents their union $\mathcal{S}(\varphi_a) \cup \mathcal{S}(\varphi_b)$. We say that an environment $\varphi_a \wedge \varphi_b$ is *self-consistent* if it does not contain conflicting assignments to the same variable (ex., $\{x = 1, x = 2\}$ is not self-consistent). Generally, as the number of assignments in an environment increases, it gets more specific and represents fewer team scenarios. The most general environment, which represents all team scenarios, is $\{\}$.

Given a candidate subplan φ_S, which contains a set of activated events and temporal constraints, we can consider the scheduling problem of trying to find a time assignment to every event that satisfies those temporal constraints.

Definition 2.5 (Schedule, Temporal Consistency)**.** A *schedule* T_{φ_S} for a candidate subplan φ_S assigns a time value to each event in $\mathcal{E}$ activated by φ_S. We denote the assigned time for a specific event e_i by $T_{\varphi_S}(e_i)$. A schedule is *temporally consistent* iff it satisfies all of the temporal constraints of the candidate subplan (i.e., those $c \in \mathcal{C}$ activated by φ_S). A candidate subplan is *temporally consistent* iff it has at least one temporally consistent schedule.

It is also useful to think in terms of *executions*, which represent not only the schedule but also the choices made. We can then consider not only temporal consistency, but also another very important concept for plan execution known as *causal completeness*. Intuitively, causal completeness says that the preconditions of all activities in the plan are expected to be satisfied by the time those activities are executed.

Definition 2.6 (Execution)**.** An *execution* is a tuple $\langle \varphi_S, T_{\varphi_S} \rangle$ where φ_S is a team scenario (i.e., representing a candidate subplan) and T_{φ_S} is a schedule for that candidate subplan. An execution is *temporally consistent* iff T_{φ_S} satisfies all of the temporal constraints of φ_S. An execution is *causally complete* iff the precondition of every event activated by φ_S is satisfied at the time of its execution in T_{φ_S}, assuming no disturbances. An execution is *correct* iff it is both temporally consistent and causally complete.

The related concept of an *execution trace* represents the state in the midst of execution – some choices have been made so far, and some events have been executed – but not necessarily the entire plan.

Definition 2.7 (Execution Trace)**.** An *execution trace* is a tuple $\langle \varphi_{ex}, \tilde{T}_{\varphi_{ex}} \rangle$ where φ_{ex} is a partial assignment to decision variables in $\mathcal{V}$ (i.e., an environment), and $\tilde{T}_{\varphi_{ex}}$ is a partial schedule that assigns time values to a subset of the events in $\mathcal{E}$. An execution trace $\langle \varphi_{ex}, \tilde{T}_{\varphi_{ex}} \rangle$ can be *extended* to execution $\langle \varphi_S, T_{\varphi_S} \rangle$ iff $\varphi_S \models \varphi_{ex}$ and $T_{\varphi_S}(e_i) = \tilde{T}_{\varphi_{ex}}(e_i)$ for all e_i assigned by $\tilde{T}_{\varphi_{ex}}$.

We can also describe an execution trace as healthy, if additional choices and scheduling decisions can be made that would extend it to a correct execution:

Definition 2.8 (Healthy Execution Trace)**.** An execution trace $\langle \varphi_{ex}, \tilde{T}_{\varphi_{ex}} \rangle$ is *temporally healthy* iff there exists an extending execution $\langle \varphi_S, T_{\varphi_S} \rangle$ that is temporally consistent. An execution trace $\langle \varphi_{ex}, \tilde{T}_{\varphi_{ex}} \rangle$ is *causally healthy* iff there exists an extending execution $\langle \varphi_S, T_{\varphi_S} \rangle$ that is causally complete. Finally, an execution trace $\langle \varphi_{ex}, \tilde{T}_{\varphi_{ex}} \rangle$ is *healthy* iff there exists an extending execution $\langle \varphi_S, T_{\varphi_S} \rangle$ that is correct, i.e. both temporally consistent and causally complete.

Execution begins with the empty execution trace, where $\varphi_{ex} = \{\}$ (i.e., no choices made yet) and $\tilde{T}_{\varphi_{ex}}$ makes no time assignments. As execution proceeds, Pike will make choices that only result in healthy execution traces.

So far, we have not distinguished between choices made by robotic agents versus choices made by human or environmental agents. We therefore further define two related concepts: the *intent* and the *adaptation*. An intent is a set of assignments to the uncontrollable variables made by the human; we think of this as the true intentions of what the human plans to do. An adaptation is similarly an assignment to the controllable variables, and represents a way that the robot may react to the human's intentions.

Definition 2.9 (Intent / Adaptation)**.** An *intent* is an environment consisting solely of assignments to uncontrollable variables in $\mathcal{V}_U$. An *adaptation* is an environment consisting solely of assignments to controllable variables in $\mathcal{V}_C$. Furthermore, an *intent scenario* is an intent that assigns a value to every variable in $\mathcal{V}_U$, and an *adaptation scenario* is an adaptation that assigns a value to every controllable variable $\mathcal{V}_C$.

Based on these definitions, we see that an intent scenario and an adaptation scenario jointly define all choices for the team.

Observation. *Suppose φ_I is an intent scenario, and φ_A is an adaptation scenario. Then $\varphi_I \wedge \varphi_A$ assigns all variables in $\mathcal{V}$, and is hence a team scenario.*

2.1.1 Labeled Value Sets

The *labeled value set* (LVS) is a data structure borrowed from Drake's labeled temporal reasoning algorithms (Conrad & Williams, 2011) for compactly recording the tightest possible constraint over many team scenarios.

Suppose we have a variable t, and we have deduced that $t < 2$ universally holds. Later on, suppose we deduce the additional constraint that when the finite-domain variable $x = 1$, the constraint $t < 1$ holds. This is a tighter constraint, so if we have some information about the world (namely that $x = 1$), we can use the tighter constraint $t < 1$ in our subsequent reasoning. If we write this as a table, we would have two entries; the first would be that

$t < 2$ holds in all circumstances, and the second would be that $t < 1$ holds in the specific circumstance where $x = 1$. Suppose we also deduce that $t < 5$ whenever $x = 2$. This information, while true, is not particularly useful as we already have deduced that the tighter constraint $t < 2$ always holds. It is *dominated* by the other constraints we already have discovered.

Such is the intuition behind the labeled value set, which encodes the tightest value for a constraint as a function of choice. The LVS stays compact using two strategies: (1) not keeping track of unnecessary relations that are dominated by others already deduced, and (2) using environments to compactly represent values over many different scenarios. We use the LVS for several different purposes within Pike.

Labeled value sets operate with respect to a *relation* $<_R$, which provides a total ordering over elements. Following Drake, we use the standard numeric $<$ relation to compare numbers in its temporal reasoning algorithms. However, we additionally use a different relation (defined later) to compare TPNU events chronologically, given temporal flexibility. Therefore we keep the following discussion general with the denoted relation $<_R$. Along with this relation, we also have related quantities ∞_R, $-\infty_R$ (representing the maximal and minimal possible elements of the relation), and operations $\leq_R$, $\max_R$, and $\min_R$.

Definition 2.10 (Labeled Value Pair)**.** A *labeled value pair* is a tuple (a_l, φ_l), where a_l is some value and φ_l is an environment.

A labeled value pair represents that a constraint value of a_l holds whenever φ_l holds. For example, with relation $<$ over a variable t, the labeled value (a_l, φ_l) means that $\varphi_l \Rightarrow t < a_l$.

It is unnecessary to store labeled value pairs corresponding to any constraints implied by other constraints. This is the notion of *dominance*.

Definition 2.11 (Dominance)**.** A labeled value (a_d, φ_d) *dominates* a weaker labeled value (a_w, φ_w), iff 1.) $a_d <_R a_w$, and 2.) $\varphi_w \models \varphi_d$.

Again taking $<$ as our relation, $(1, \{x = 1\})$ dominates $(2, \{x = 1, y = 2\})$. Whenever $\{x = 1, y = 2\}$ holds and the constraint $t < 2$ is active, then $x = 1$ must also hold and the tighter constraint $t < 1$ is also active. Thus, since the dominating pair both applies more generally and is tighter, the weaker, dominated pair may be discarded.

Definition 2.12 (Labeled Value Set)**.** A *labeled value set* $L = \{(a_1, \varphi_1), (a_2, \varphi_2), \ldots\}$ is a set of labeled value pairs, such that no pair in the set dominates any other pair.

Since no labeled value dominates any other, the LVS remains compact by omitting superfluous information. The AddLVS(p, L) method, shown in Algorithm 6 in Appendix B, is responsible for adding a new labeled value pair p minimally to the LVS L and maintaining this non-dominance invariant. We can imagine an analogous algorithm MergeLVS(L, L_{add}) that adds every labeled value of L_{add} minimally to L.

Perhaps the most useful operation on an LVS is *querying* it, which computes the tightest possible constraint value that holds over all scenarios in a given environment. It answers the question: given L, what is the tightest bound that holds over all $\varphi_s \in \mathcal{S}(\varphi)$? In other words, what is the smallest value a that can be guaranteed for the constraint $t <_R a$? We denote the query operation over an LVS L under an environment φ is as $Q_L(\varphi)$. Pseudo code for the query operation is shown in Algorithm 7 in Appendix B.

We can additionally perform certain binary operations on LVS's, such as addition and subtraction. This is quite useful as a "labeled generalization" of normal addition and subtraction, allowing for an elegant formulation of various algorithms such as Floyd Warshall (Conrad & Williams, 2011). To perform a binary operation on two LVS's, every possible combination of their labeled values is considered and the binary operation (ex. addition) is applied to the values. The conjunction of their environments is also taken, forming a new candidate labeled value for the result. Only the dominating labeled values of this result are kept (Conrad & Williams, 2011). For example, suppose we wish to add $L_1 = \{(2, \{x = 1\}), (3, \{y = 2\}), (4, \{\})\}$ to $L_2 = \{(1, \{x = 2\}), (3, \{\})\}$. The resulting LVS contains the sum, taking into account each possible combination of environments but pruning out dominated labeled values: $\{(4, \{x = 2, y = 2\}), (5, \{x = 1\}), (5, \{x = 2\}), (6, \{y = 2\}), (7, \{\})\}$.

2.2 Inputs and Outputs

In order to prepare for execution, Pike takes in the following inputs offline:

- A TPNU $\mathcal{T}$ representation of the human-robot plan.
- An action model specifying conditions and effects of actions in the plan.
- The initial state of the world. Specified as a set of state predicates.
- The desired goal state of the world. Specified as a set of state predicates.

Online during execution, Pike additionally takes in the following inputs:

- A stream of alerts $\mathcal{A}(t)$ for when activities terminate (assumed to be within durations).
- A stream of uncontrollable choice outcomes $\mathcal{U}(t)$ as they occur.
- A stream of predicates $\mathcal{P}(t)$ describing current state estimates.

Pike outputs the following during online execution:

- A stream of dispatched actions, targeted at both the robot and human, at temporally consistent times.
- A stream of controllable choice assignments that maintain a healthy execution trace.
- A flag detecting future plan failure, as soon as it is detected.

We adopt similar temporal controllability semantics as Drake; namely, we treat each activity as begin controllable (Vidal, 2000), meaning that our executive controls the start times of activities but may not arbitrarily and instantaneously choose their end times. Rather, these times are determined by nature or by the human, and the executive will be told online when each activity ends immediately afterward. We do not address the issue of temporal controllability further in this paper (i.e., dynamic controllability), but note that this is an important field and that notable work has been done in characterizing it for conditional plans (Hunsberger, Posenato, & Combi, 2012).

When execution begins, the execution trace is empty – no choices have been made and no events have been scheduled. As execution proceeds, choices and scheduling decisions are made. Given the current execution trace $\langle \varphi_{ex}, \tilde{T}_{\varphi_{ex}} \rangle$, if Pike chooses to execute event e_i at time t, we would have the resulting execution trace with more decision variables assigned and events scheduled: $\langle \varphi_{ex} \wedge \varphi_{e_i}, \tilde{T}_{\varphi_{ex}} \cup \{e_i = t\} \rangle$. Pike is permitted to execute e_i at time t (and hence make associated controllable choices) if and only if this resulting execution trace would be healthy. This execution property is key to Pike's robustness, and will be proved later in this work by Theorem 4.7.

We can also view the above in terms of intents and adaptations. Pike may choose any adaptation (i.e., a controllable choice assignment) such that there remains at least one possible intention for the human consistent with this adaptation and satisfying the plan's preconditions and temporal constraints. We can express this in terms of execution traces as follows. Consider some controllable choice assignments (i.e., an adaptation) represented by environment φ_{A_i}, and an associated event e_i. Pike is permitted to make the choices φ_{A_i} and simultaneously execute event e_i at time t if and only if there exists an adaptation scenario φ_A, an intent scenario φ_I, and a schedule $T_{\varphi_A \wedge \varphi_I}$ where the execution $\langle \varphi_A \wedge \varphi_I, T_{\varphi_A \wedge \varphi_I} \rangle$ is correct and extends the execution trace $\langle \varphi_{ex} \wedge \varphi_{A_i}, \tilde{T}_{\varphi_{ex}} \cup \{e_i = t\} \rangle$. Please note that such semantics allow Pike to restrict the future possibilities for the human, but the end result is a mixed-initiative execution.

We can simplify the above to avoid explicitly differentiating intents and adaptations as separate entities. Pike chooses its controllable choice outcomes φ_{A_i} such that there exists some candidate subplan consistent with φ_{ex} and φ_{A_i} that has a correct execution. By not distinguishing intents from adaptations, we can employ a single framework to check causal completeness and temporal consistency for both kinds of choices – hence concurrently performing intent recognition and adaptation via one mechanism. So long as the execution trace remains healthy, the exists some human and some robot choices that yield a correct execution. Therefore, for the remainder of this paper, we distinguish between intents and adaptations seldomly and only when required, treating both more generally as assignments to decision variables within the context of ensuring a correct execution.

2.3 Solution Architecture

Given this problem statement, we revisit our solution approach. An algorithmic architecture of Pike is shown in Figure 4, and the rest of this paper describes its various components.

First, during offline compilation, a labeled all-pairs shortest path (APSP) algorithm computes a table $D_{i,j}$, which encodes temporal relationships between events in the TPNU (Conrad, 2010). $D_{i,j}$ is used online to schedule activities at appropriate times.

$D_{i,j}$ is also used to extract labeled causal links from the TPNU using an action model. Using these labeled causal links, we generate a set of propositional constraints KB that allow us to reason over sets of choice outcomes that would be causally complete. The set of solutions to these constraints captures the space of all candidate subplans that admit correct executions. We also use the labeled causal links to add additional temporal constraints to the TPNU, effectively creating a new augmented TPNU upon which we must re-compute the labeled APSP a second time to compute an updated $D_{i,j}$ due to the additional constraints.

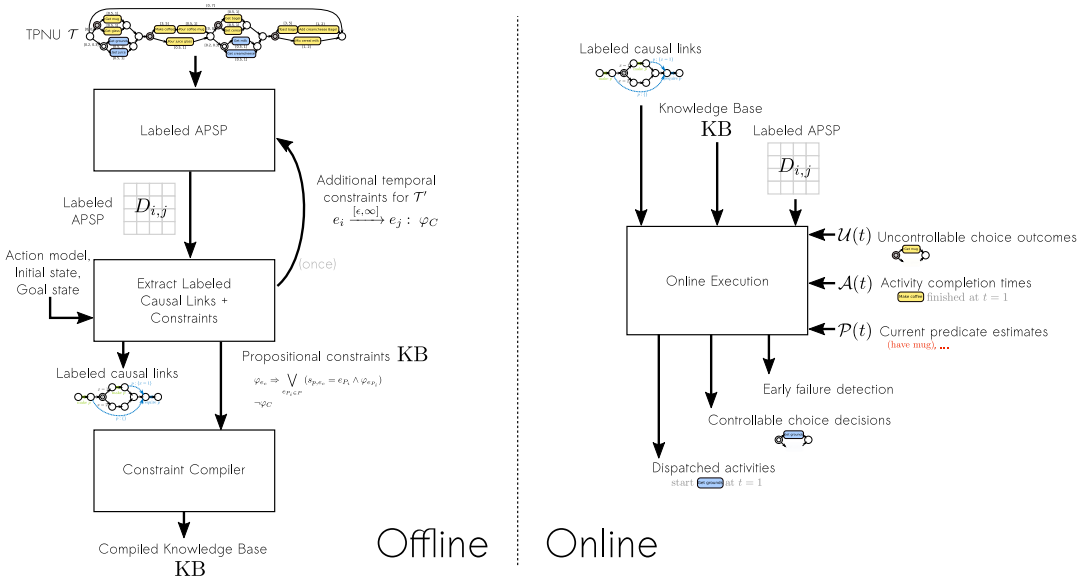


Figure 4: Algorithmic architecture of Pike.

Finally, we generate a compiled representation of our constraints KB via a label propagation algorithm (we introduce the πTMS – which generates prime implicants of KB) for use during online execution. A key requirement is that this KB must be able to answer the following queries efficiently online:

- CorrectTeamPlanExists?(KB). Returns True if and only if KB is satisfiable; i.e., if there exists a candidate subplan that admits at least one correct execution.

- CouldCommitToAdditionalConstraints?(KB, $\{F_1, ..., F_n\}$). Returns True if and only if there would exist a candidate subplan admitting a correct execution after adding the additional set of constraints; that is, if KB $\wedge$ $(F_1 \wedge ... \wedge F_n)$ is satisfiable.

- CommitToConstraints(KB, $\{F_1, ..., F_n\}$). Adds the constraints to KB conjunctively; i.e., KB $\leftarrow$ KB $\wedge$ $(F_1 \wedge ... \wedge F_n)$.

During online execution, we make use of the dispatchable form $D_{i,j}$ to handle dispatching activities at their proper times. We additionally make the above queries to KB, allowing Pike to efficiently reason online about different scenarios and alternatives that it is allowed to choose from and ensuring that its choices are consistent with those of the human. Additionally, online execution makes use of certain data as it becomes observed online: the outcomes to uncontrollable choices made by the human, the durations of dispatched activities, and finally an estimate of the world's state used for execution monitoring. The output of online execution is a live, streaming dispatch of activities targeting the robot, a series of controllable choice decisions for the robot, and early signaling of failure.

3. Online Execution Strategy

The online execution algorithm is responsible for (1) scheduling events and dispatching associated activities at the proper times, (2) assigning values to controllable choices, and (3) monitoring the execution for potential problems. It does so by analyzing choices made by the human, respecting the plan's temporal constraints, and observing the state of the world to detect upcoming failures. Our approach takes a least-commitment approach to execution, leaving as much flexibility as possible to the online executive. This flexibility – in terms of both controllable choices and scheduling decisions – affords a greater degree of robustness than executives operating with grounded plans where such decisions have been already made offline before execution begins. Least-commitment executives such as Pike are able to exploit new information that becomes available online and adapt to it without replanning. While having a greater computational cost and complexity, we argue (and show experimentally later in Section 6.2) that least-commitment execution approaches are well worth the cost due to their significantly improved robustness online.

Our online execution approach takes as input the output of the offline compilation process: (1) a dispatchable form $D_{i,j}$ in the form of a labeled APSP, (2) the set of extracted labeled causal links, and (3) a knowledge base KB representing a compiled version of constraints derived from the causal links. Additionally, the online execution algorithm takes as input certain observations that only become available online, including (1) a stream of uncontrollable choice outcomes $\mathcal{U}(t)$ representing the choices made by the human and/or environment, (2) a stream of activity finish notifications $\mathcal{A}(t)$ denoting when dispatched activities finish, and (3) a stream of estimated state predicates $\mathcal{P}(t)$ used for execution monitoring.

Our executive is heavily inspired by Drake (Conrad, 2010; Conrad & Williams, 2011), which uses an online execution algorithm that is a generalization of the STN dispatching algorithm (Muscettola et al., 1998) for plans with choice. Like Drake, our executive behaves by "greedily" scheduling events, dispatching associated activities, and making choices whenever possible such that doing so would not cause plan failure. Unlike Drake, our approach also considers causal completeness. More specifically, the Pike executive may choose to schedule any event at any time, such that there would remain at least one correct execution of a candidate subplan. Hence, despite the greediness of the approach, choices are never made that lead to execution failure. As execution proceeds and choices are made by both the human and robots, the space of correct candidate subplans becomes successively smaller. The result is a mixed-initiative execution, in which the human and robots make choices together, and possibly constrain each other's future choices (such that at least one candidate team plan remains).

We describe Pike's execution algorithm here, and will illustrate it shortly with an example. Each event e_i is associated with an *execution window*, which captures the earliest and latest allowable absolute times during which e_i can be executed without violating any temporal constraints. Each execution window is divided into two parts: $\mathit{lower\_bound}(e_i)$ and $\mathit{upper\_bound}(e_i)$. In traditional dispatchers for STNs, these lowerbounds and upperbounds for each event are numbers representing absolute times (Muscettola et al., 1998). Whenever an event is executed, its scheduled time is propagated locally to all neighbors, causing a tightening of those events' execution windows. An event may only be executed if

the current time is within its execution window. Following Drake, Pike generalizes this to labeled execution windows, which encode the dependence of each event's execution time on the choice variables. As different choices are made, different temporal constraints become active or inactive, thus affecting each event's execution window. *lower_bound*(e_i) and *upper_bound*(e_i) are each labeled value sets instead of numbers in this formalism. Specifically, a labeled value (l, φ_l) in *lower_bound*(e_i) means that in all scenarios where φ_l holds, event e_i must be executed at l or later; that is, $\varphi_l \Rightarrow e_i \geq l$. Similarly, a labeled value in (u, φ_u) in *upper_bound*(e_i) means that in all scenarios where φ_u holds, e_i must be executed by u or earlier; that is, $\varphi_u \Rightarrow e_i \leq u$. Note that if φ is the empty environment (logically equivalent True), we are equivalent to the unlabeled approach for STNs. The LVS relation $<_R$ for *upper_bound*(e_i) is $<$, and for *lower_bound*(e_i) it is $>$, capturing the notion that earlier upperbounds and later lowerbounds are tightest (Conrad, 2010). Before execution begins, we set up an execution window for each event of the plan, initializing each to the broadest range possible. For each e_i, *lower_bound*(e_i) is initialized to the LVS $\{(-\infty, \{\})\}$, and *upper_bound*(e_i) is similarly initialized to $\{(\infty, \{\})\}$. If an event e_i is executed at some time t, then we propagate the execution window of other events e_j as follows (Conrad, 2010):

$$\begin{aligned} &\textsc{MergeLVS}(\textit{upper\_bound}(e_i), \{(t, \{\})\} + D_{e_i,e_j}) \\ &\textsc{MergeLVS}(\textit{lower\_bound}(e_i), \{(t, \{\})\} - D_{e_j,e_i}) \end{aligned} \tag{1}$$

During execution, Pike must check if event an event e_i can be executed at the current time t. To do so, Pike follows Drake in assembling a set of constraints $F_1, ..., F_n$ that would all need to hold. If there exists a solution (i.e., an assignment to controllable and uncontrollable variables forming a team scenario) satisfying KB, all previously-made choices, and these constraints $\{F_1, ..., F_n\}$, then e_i can safely be executed now (Conrad, 2010). We refer to this procedure as GetConstraintsRequiredToExecute(e_i, t), and it produces the set of constraints $F_1, ..., F_n$ as follows:

1. The guard of e_i, φ_{e_i}.
2. True if e_i is directly executable, otherwise False. Event e_i is not directly executable if (1) it is the end event of an activity (which is scheduled externally and reported to Pike via $\mathcal{A}(t)$), (2) if φ_{e_i} mentions an assignment to a currently unassigned uncontrollable variable (which Pike is not allowed to assign), or (3) if the variable *variable-at-event*(e_i) is uncontrollable and unassigned. Note that if we add False conjunctively, we effectively prevent e_i from being executed now.
3. $\neg\varphi_l$ for any labeled value (l, φ_l) in *lower_bound*(e_i) if $t < l$.
4. $\neg\varphi_u$ for any labeled value (u, φ_u) in *upper_bound*(e_i) if $t > u$.
5. $\neg\varphi$ for any (w, φ) in D_{e_i,e_j} where e_j is a yet un-executed event, and $w < 0$.
6. $\neg\varphi$ for any (w, φ) in D_{e_i,e_j} where e_j is a yet un-executed event and is also not directly executable, and $w \leq 0$.

These constraints are based on those from Drake (Conrad, 2010); we provide brief intuition here. Constraint 1 ensures that the guard of the event holds, a requirement to

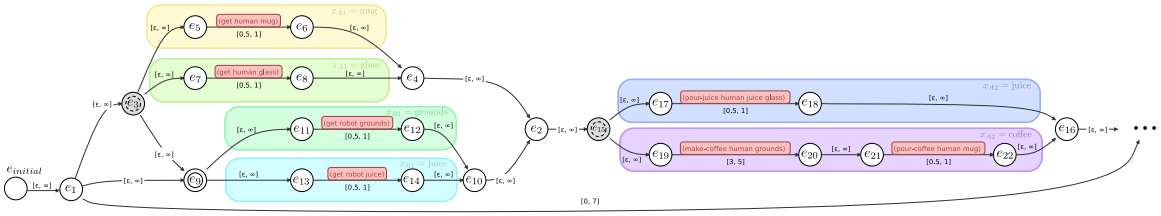


Figure 5: Plan for making breakfast, with events and temporal constraints labeled. ϵ is a small number (0.001 here) to ensure strict ordering constraints. Background colors indicate guard conditions over the indicated events and temporal constraints. The second half of the plan is omitted for brevity.

execute e_i. Constraint 2 prevents Pike from executing non-directly executable events. These are events that are scheduled externally by the environment (for instance, the end event of activities), events whose guards depend on a yet-unassigned uncontrollable choice variable, or events that are "branching points" where uncontrollable choices must be made. This prevents Pike from effectively choosing the outcome of uncontrollable variables. Constraint 3 handles the case of violated lowerbounds: *lower_bound*(e_i) dictates that e_i must be executed *after* time l whenever φ_l holds. If we instead execute it *beforehand* at $t < l$, then φ_l cannot hold. We therefore add $\neg\varphi_l$ to the set of constraints. Similar reasoning holds for the upperbounds in Constraint 4. Constraint 5 handles the case of violated ordering constraints. If the labeled APSP dictates that event e_j must be executed *before* event e_i whenever φ holds, but we're executing e_i now before the un-executed e_j, then φ cannot hold. Finally, Constraint 6 addresses a similar case where e_j is not directly executable (ex., the end event of an activity) and will not be executed by time t.

Theorem 3.1 (GetConstraintsRequiredToExecute is correct)**.** *Let* $\langle \varphi_{ex}, \tilde{T}_{\varphi_{ex}} \rangle$ *be a temporally healthy execution trace. The* GetConstraintsRequiredToExecute(e_i, t) *procedure returns a set of constraints* $F_1, ..., F_n$*, such that* φ_S *satisfies* $F_1, ..., F_n$ *if and only if the execution trace that would result if* e_i *were executed at time* t *– namely* $\langle \varphi_{ex} \wedge \varphi_{e_i}, \tilde{T}_{\varphi_{ex}} \cup \{e_i = t\} \rangle$ *– can be extended to a temporally consistent execution* $\langle \varphi_S, T_{\varphi_S} \rangle$*.*

Proof. Not proven here – see prior work on Drake for details (Conrad & Williams, 2011). □

High-level pseudo code for the online execution algorithm is shown in Algorithm 1, which closely follows the framework introduced by the Drake executive (Conrad, 2010). The process begins by initializing the execution. A queue $Q_{remaining}$ of events remaining to be executed is created. It initially contains every event. Next, the initialization routine sets the initial time to $t = 0$, and associates an execution window with each event. The online algorithm then proceeds to enter a loop, only terminating once there are no more remaining events in $Q_{remaining}$, or failure has been detected. An appropriately tiny pause is introduced at the end of each loop cycle, so that time advances. Within each loop iteration, a number of checks and updates are performed that will be described shortly. Then, an inner loop begins which does the main work of executing the plan by executing individual events. This loop checks all possible events in $Q_{remaining}$, checks if the event can

Algorithm 1: OnlineExecution()

Input: A dispatchable labeled distance graph $D_{i,j}$, a knowledge base KB compiled with causal link constraints, streams of uncontrollable choice assignments $\mathcal{U}(t)$, state estimates $\mathcal{P}(t)$, and activity completions $\mathcal{A}(t)$

Output: Stream of controllable choice assignments, dispatched planned activities, and immediate failure if problem is detected.

```
InitializeExecution()
while |Q_remaining| > 0 do
    CheckActivatedCausalLinks()
    Process any just-finished activities in A(t)
    Call CommitToConstraints(KB, {U}) for any observed uncontrollable choice
     assignments U ∈ U(t)
    Process missed execution windows
    if not CorrectTeamPlanExists?(KB) then
        return Failed
    end
    repeat
        for event e ∈ Q_remaining do
            executable? , {F_1, ..., F_n} ← CanExecuteEventNow?(e, t)
            if executable? then
                ExecuteEvent(e, {F_1, ..., F_n})
                break
            end
        end
    until no event is executed
    Sleep for a small time to allow time t to advance
end
```

Algorithm 2: CanExecuteEventNow?()

Input: An event e, current time t

Output: Returns two values: (1) a boolean for whether e can be executed at time t, and (2) a set of constraints that must hold if so.

```
{F_1, ..., F_n} ← GetConstraintsRequiredToExecuteEvent(e, t)
if CouldCommitToAdditionalConstraints?(KB, {F_1, ..., F_n}) then
    return True, {F_1, ..., F_n}
else
    return False, {False}
end
```

be executed now via CanExecuteEventNow?, and then executes the event if so. The CanExecuteEventNow?(e_i, t) procedure computes a set of constraints $F_1, ..., F_n$ that must hold if e_i is executed at time t, and then checks if these constraints can be satisfied

along with KB. We prove later in this work (Theorem 4.7) that CanExecuteEventNow? is correct.

We illustrate these algorithms by describing an example execution. Figure 5 shows a slightly modified version of the TPNU as discussed earlier, now annotated more precisely with event names (ex., e_1), temporal constraints (ex., ordering constraints such as $[\epsilon, \infty]$ for a small value ϵ), and guard conditions (colored backgrounds). Assume offline compilation is finished, and therefore we have computed (1) the labeled APSP table $D_{i,j}$, (2) a set of labeled causal links (namely those illustrated in Figure 2), and (3) a knowledge base KB can be queried. For this example, a partial list of propositional constraints represented by KB is shown below:

$$
\begin{aligned}
x_{A2} = \text{coffee} &\Rightarrow (s_{(\text{has mug}),e_{21}} = e_6 \wedge x_{A1} = \text{mug}) \\
x_{A2} = \text{coffee} &\Rightarrow (s_{(\text{has grounds}),e_{19}} = e_{12} \wedge x_{R1} = \text{grounds}) \\
x_{A2} = \text{juice} &\Rightarrow (s_{(\text{has glass}),e_{17}} = e_8 \wedge x_{A1} = \text{glass}) \\
x_{A2} = \text{juice} &\Rightarrow (s_{(\text{has juice}),e_{17}} = e_{14} \wedge x_{R1} = \text{juice}) \\
&\ldots \\
&\neg(x_{A4} = \text{bagel} \wedge x_{A2} = \text{coffee})
\end{aligned}
\tag{2}
$$

We discuss the details behind these constraints later, including how they are derived. They capture relationships amongst choice variables that are required to ensure causal completeness as well as avoiding temporal conflicts. The variable assignments of the form $s_{p,e_c} = e_P$ where p is a predicate intuitively mean e_P has been chosen to support e_c: there is a labeled causal link where producer event e_P has an effect p that is a precondition of the consumer event e_c (but more on this later).

Execution now begins. Suppose the current time is $t = 0$. The online execution algorithm will iterate over each event, checking to see if it is executable or not. Let us consider and see if the event $e_{initial}$ is executable. Following the constraints above, Pike derives that the following set $\{F_1, ..., F_n\}$ of additional constraints would need to hold in order to execute $e_{initial}$ now at $t = 0$:

- **Constraint 1:** True ($e_{initial}$ has guard True so it will always be executed)
- **Constraint 2:** True ($e_{initial}$ is directly executable - not the end event of an activity nor does its guard contain an assignment to an unassigned uncontrollable variable)
- **Constraint 3:** None. $t \not< -\infty$ so no constraint is added.
- **Constraint 4:** None. $t \not> \infty$ so no constraint is added.
- **Constraint 5:** None, as $e_{initial}$ has no predecessors.
- **Constraint 6:** None (same reasoning as above).

We have just the single constraint True. As a result, the output of the function CouldCommitToAdditionalConstraints?(KB, F) will be True since adding the constraint True conjunctively will not remove any correct team scenarios. Pike will therefore execute event $e_{initial}$ now at $t = 0$ by calling ExecuteEvent($e_{initial}, F$)

Algorithm 3: ExecuteEvent(e, F)

Input: An event e, a set of constraints $\{F_1, ..., F_n\}$

1 Mark e as executed at time t
2 Remove e from $Q_{remaining}$
3 CommitToConstraints(KB, F)
4 Propagate execution time t time to other events
5 Prune any other events with now-inconsistent guards from $Q_{remaining}$
6 [Optional: prune execution windows with now-inconsistent labels]
7 Dispatch any activities starting at e
8 Activate / Deactivate any labeled causal links starting / ending at e

Algorithm 3 shows the pseudo code for the ExecuteEvent(e, F) procedure, which is called to actually execute event e. The first steps in executing event e are to mark it as executed at time t, remove it from $Q_{remaining}$, and add the required constraints $\{F_1, ..., F_n\}$ to KB. Then, we propagate this time t to all e's neighbors so that their execution windows are appropriately updated. For example, consider event e_6 with execution window $\mathit{lower\_bound}(e_6) = \{(-\infty, \{\})\}$, $\mathit{upper\_bound}(e_6) = \{(\infty, \{\})\}$. Given the previously computed APSP distances $D_{e_6,e_{initial}} = \{(-2.021, \{x_{A1} = \text{mug}, x_{A2} = \text{coffee}, x_{A4} = \text{bagel}\}), (-0.503, \{x_{A1} = \text{mug}\}), (-0.001, \{\})\}$ and $D_{e_{initial},e_6} = \{(\infty, \{\})\}$, we can update via Equation 1 to

$$\begin{aligned}\mathit{lower\_bound}(e_6) = \{&(0.001, \{\}),\\ &(0.503, \{x_{A1} = \text{mug}\}),\\ &(2.021, \{x_{A1} = \text{mug}, x_{A2} = \text{coffee}, x_{A4} = \text{bagel}\})\}\\ \mathit{upper\_bound}(e_6) = \{&(\infty, \{\})\}\end{aligned}$$

Similar updates are computed for the execution windows of all other events remaining to be scheduled.[1]

After propagating execution times to other events, we prune events whose guard conditions are now inconsistent with KB by removing them from $Q_{remaining}$. This is illustrated later in this example when a choice is observed. We may also optionally prune invalid labeled values from execution windows, as they will not affect algorithm correctness but could impact system latency. We found experimentally, however, that this could at times actually increase the worst-case execution latency time (as removal could be expensive). Hence this step is optional.

Next, after any pruning has occurred, Pike will check to see if e is the start event of any activity in the TPNU. If so, then the activity is dispatched at the current time. This is not the case for $e_{initial}$.

The final stage in ExecuteEvent(...) is to *activate* or *deactivate* any labeled causal links starting or ending at event e, respectively. An activated labeled causal link is one that

1. In (Conrad, 2010) and (Muscettola et al., 1998), a minimal dispatchable form is computed that only requires propagating event updates to local neighbors, not to all other events. Such an approach could be adopted here but is omitted for simplicity.

is continually monitored during execution by comparison with the current estimated world state $\mathcal{P}(t)$, which we illustrate later.

After executing $e_{initial}$ and propagating to its neighbors, the execution loop continues trying to find other events that are executable. It turns out that for this example, none are – the plan's temporal constraints force all other events to be executed strictly after $e_{initial}$, but the time has not yet advanced past $t = 0$. For any other event, there must therefore be some labeled value $(l, \{\})$ for $l > 0$ in its lower bound execution window (per the propagation step above). By Constraint 3, $\{F_1, ..., F_n\}$ would contain the constraint $\neg$True = False, which cannot be conjunctively added to KB while still maintaining a solution team scenario. Therefore no other event is executable at $t = 0$, so Pike must wait.

Suppose it is now a short time later, $t = 1$. We may now execute event e_1, as $\{F_1, ..., F_n\}$ would contain the single constraint True. The time $t = 1$ is propagated to other events in the plan. This pattern continues, and guarantees that events will be scheduled at their proper times and that the execution trace remains healthy.

Suppose later that we observe a message in $\mathcal{U}(t)$ that the human is making the choice to pick up the mug, namely x_{A1} = mug. Such inferences are made external to Pike in a separate activity recognition module such as (Lane, 2016), and reported to Pike via $\mathcal{U}(t)$. This triggers a call to CommitToConstraints(KB, $\{x_{A1} = \text{mug}\}$), adding this assignment to the knowledge base. Additionally, since x_{A1} is an uncontrollable variable, other events in the plan may become executable now that it has been observed. For example, event e_3 can now be executed since we have observed the outcome of its requisite choice variable, as can e_9 and e_5. As e_5 is the start event of the `(get human mug)` activity, this action is dispatched (though in practice the human will have already begun doing it as the activity recognition system reported the outcome of x_{A1}).

To illustrate the mechanics by which human choices influence robot adaptations, we point out that the robot may *not* choose to execute event e_{13} (the start of the `(get robot juice)` activity). Doing so would result in $\{F_1, ..., F_n\}$ containing x_{R1} = juice (which is $\varphi_{e_{13}}$). This would cause

$$\text{CouldCommitToAdditionalConstraints?}(\text{KB}, \{x_{R1} = \text{juice}\})$$

to return False, which can be seen by logically analyzing the set of constraints in KB. We show that KB would become inconsistent. KB currently consists of Equation 2, plus the assignment x_{A1} = mug. If the executive were to additionally add the assignment x_{R1} = juice as would be required to execute e_{13}, then the human could not later consistently get coffee x_{A2} = coffee (doing so would require x_{R1} = grounds instead by the second constraint). Thus we infer that the human must later pour juice x_{A2} = juice. By the third constraint, this requires that the human get the glass x_{A1} = glass, which contradicts the earlier added assignment of x_{A1} = mug. We have shown that adding the constraint x_{R1} = juice leads to an unsatisfiable KB; therefore CouldCommitToAdditionalConstraints? returns False and the executive cannot execute e_{13} now.

On the other hand, event e_{11} (the start of the `(get robot grounds)` activity) can be executed now, as its guard condition x_{R1} = grounds is consistent with KB. Once executed, the `(get robot grounds)` activity is dispatched to the robot. This will in turn trigger lower-level motion planning and control algorithms to physically implement this action. We also see that the robot is properly adapting to the human's intent.

Once a choice has been made or observed, certain events are no longer executable because their guards are inconsistent with KB. For example, after observing the choice $x_{A1} = \text{mug}$, events e_7 and e_8, which each have a guard $x_{A1} = \text{glass}$, can never be executed. These events are therefore pruned from execution by removing them from $Q_{remaining}$ and never considered again as candidates to be executed.

Shortly later, the executive receives messages in $\mathcal{A}(t)$ informing that the `(get human mug)` and `(get robot grounds)` activities have finished successfully. Pike hence calls ExecuteEvent(...) for the end events of those two activities. This makes events e_6 and e_{12} now directly executable, so Pike executes them now. This in turn allows other events to become executable, and the execution process continues.

It may be the case that, during execution, an activity takes too long and violates a temporal constraint. This circumstance is detected within the OnlineExecution(...) method by detecting missed execution windows. This routine is borrowed from Drake, and follows logic similar to Constraint 4 in constructing $\{F_1, ..., F_n\}$. If the current time t is greater than the upperbound u for some $(u, \varphi) \in \textit{upper\_bound}(e_i)$, then we add the constraint $\neg\varphi$ to KB. This is because we cannot go back in time, and there is no way that we will meet the given upperbound required under environment φ. For example, if the upper bound part of the execution window for some event e_i is the labeled value set $\{(3, \{x = 1\}), (6, \{\})\}$ and the current time is $t = 4$, then the executive can infer that it has missed the upper bound of 3 required by the choice $x = 1$. Therefore, the conflict $\neg(x = 1)$ is added to KB.

Finally, the last aspect of execution that we wish to illustrate is execution monitoring. Our executive employs causal link based execution monitoring to detect failures early, hopefully before they become critical. Recall that there is a labeled causal link where consumer e_{21} has a precondition of `(have mug)` that is achieved by producer event e_6 (the end of the `(get human mug)` activity). If the predicate `(have mug)` ceases to be true after e_6 is executed – for instance, if the mug somehow gets knocked off of the table by accident – then this labeled causal link is violated. Unlike earlier work in execution monitoring, a violated causal link in our setting does not necessarily imply plan failure, however (Levine, 2012). For example, if the plan contains sufficient flexibility to avoid executing e_{21} (i.e., by choosing $x_{A2} = \text{juice}$), then execution can still succeed. Some TPNUs contain explicit contingency options to address likely causes of failure such as unreliable robotic hardware, where the robot has multiple choices of whether or not to pick something up (this is a form of k-fault tolerance). However, at this point during execution in our breakfast example, the robot cannot "change its mind" and commit to $x_{A2} = \text{juice}$. Hence this causal link violation implies plan failure. We do not need to wait until the consumer activity `(pour-coffee human mug)` is actually executed to report the failure; our executive will detect it as early as possible during execution (such as, for example, immediately after the `(get human mug)` activity finishes). Such early failure detection is extremely beneficial in practice in many autonomous systems, as it provides additional time to replan (Levine, 2012).

A procedure called CheckActivatedCausalLinks() is responsible for detecting causal link violations that occur online. It iterates over all activated labeled causal links from e_P to e_c with predicate p. If $p \notin \mathcal{P}(t)$, then CommitToConstraints(KB, $\{\neg(s_{p,e_c} = e_P)\}$) is called. For example, suppose that after executing e_6 but before e_{12} the executive discovers from the current state estimates $\mathcal{P}(t)$ that `(has mug)` $\notin \mathcal{P}(t)$. Then, the executive commits to the additional constraint $\neg(s_{(\text{have-mug}),e_{21}} = e_6)$. As can be seen from Equation 2, this

Algorithm 4: CompilePlanOffline()

Input: TPNU $\mathcal{T}$ describing human-robot plan, action model, initial and goal states
Output: Dispatchable labeled APSP table, labeled causal links, compiled propositional constraints

```
1 Compute labeled APSP of T
2 Extract labeled causal links
3 Generate augmented TPNU T' and KB using T and labeled causal links
4 if any temporal constraints were added to T' then
5 |   Compute labeled APSP of T'
6 end
7 Add temporal conflict constraints to KB
8 Compile propositional constraints KB
```

invalidates the right-hand side of the first implication constraint, therefore precluding the choice x_{A2} = coffee. Since this choice must be made however, KB becomes inconsistent. If there had been sufficient flexibility in the plan to handle this causal link violation, then KB would still have remained consistent and execution would have succeeded (barring more violations), albeit possibly by forcing some later choice assignments. In this way, Pike is able to react to certain unmodeled disturbances in the world, and predict future plan failure before it occurs.

This concludes our explanation of Pike's online execution algorithm. In summary, we employ a strategy of least commitment (both in terms of timing and controllable choices). The resulting execution obeys the plan's temporal constraints by employing techniques from Drake. Pike additionally makes controllable choices using a compiled knowledge base KB describing the dependencies between controllable and uncontrollable choices, allowing it examine the consequences of hypothetical actions and to ensure that the execution trace remains healthy. The result is a mixed-initiative execution where the robot adapts to the human's intentions, the plan's timing constraints, and unmodeled disturbances.

4. Using Causal Links for Decision Making

In this section, we discuss how we extract labeled causal links from the plan and how we use them for online decision making. These labeled causal links capture important relationships between choices in the plan by making explicit its causal structure, and they will ultimately be used by our executive to make intelligent choices online. From the perspective of human-robot interaction, this causal structure allows Pike to choose robot adaptations that are logically consistent with the human's intentions.

We first introduce some preliminaries, including algorithms and data structures for performing labeled temporal reasoning. These are necessary for causal link extraction, as producers must precede consumers in time in our conditional plans. We then introduce labeled causal links, a generalization of causal links for contingent, temporally-flexible plans. We also introduce a transformation from our input TPNU to an *augmented* TPNU along with a set of constraints based on these causal links, which allow Pike to maintain a healthy execution trace online.

High-level pseudo code for the entire offline compilation procedure is illustrated in Algorithm 4. First, a labeled all-pairs shortest path is computed, which provides temporal reasoning that is necessary both for online execution and for causal link extraction. Next, causal links and threats are extracted. An augmented TPNU is then constructed based on these causal links. This process sometimes results in new temporal constraints being added to ensure causal completeness; if so, the labeled APSP is re-run to reflect these new temporal constraints. Finally, the constraints are compiled into a form suitable for efficient use during online execution.

4.1 Labeled APSP

In this section, we describe the labeled all-pairs shortest path algorithm (APSP), which enables Pike to reason about the temporal constraints in the plan as a function of the choices made. The resulting data structure, a table of shortest path LVS's, is used for extracting labeled causal links and also for online dispatching (as it is a "dispatchable form") (Dechter et al., 1991).

The labeled APSP algorithm was introduced in (Conrad, 2010), and is a strict generalization of the Floyd Warshall all-pairs shortest path algorithm. The Floyd Warshall algorithm takes as input a weighted directed graph (often called a "distance graph" in the scheduling literature) with N vertices, and outputs an $N \times N$ table d_{ij}, where each entry in the table contains the shortest path length (a number) from i to j. These shortest path distances represent required timing constraints (Dechter et al., 1991).

In contrast, the labeled APSP instead generates a table D_{ij} where each entry is a LVS representing the shortest path from i to j as a function of choice. Its input is the *labeled distance graph*, which similarly generalizes of the distance graph from the STN literature. Instead of each edge weight between two vertices being a single number, it is now a labeled value set, representing the edge weight as a function of what choices are made (Conrad & Williams, 2011). Just as an STN can be readily transformed into a distance graph (Dechter et al., 1991), a TPNU (as well as many other temporal representations, such as the DTP) can be readily transformed into a labeled distance graph through a straightforward process. Each vertex in the labeled distance graph is an event from the TPNU, and each labeled simple temporal constraint is converted into two labeled weights going in opposite directions between the associated events, corresponding to the upper and lower bound constraints (Conrad, 2010).

For this work, we use the labeled APSP algorithms as introduced in Drake, with some additional modifications. These modifications compute tighter bounds for each LVS by considering the finite domains of variables and adding additionally, logically implied labeled values when appropriate. Details about the labeled APSP algorithm with supplemental modifications can be found in Appendix C.

Given the resulting output D_{ij} from the labeled all-pairs shortest path, we introduce some necessary terminology. First, we discuss temporal conflicts.

Definition 4.1 (Temporal Conflict)**.** An environment φ is called a *temporal conflict* iff $Q_{D_{e_i,e_i}}(\varphi) < 0$ for any event e_i

Theorem 4.1 (Temporally Inconsistent Candidate Subplan)**.** *A candidate subplan φ_S is temporally inconsistent iff there exists a temporal conflict environment φ where $\varphi_S \models \varphi$.*

Proof. Not proven here – see prior work on Drake for details (Conrad & Williams, 2011). □

Temporal conflicts are important, because they allow us to detect temporally inconsistent candidate subplans so Pike can avoid them online. The definition above is a direct analogy to the negative cycles of distance graphs for STNs (Dechter et al., 1991).

We can also define the very useful concept of precedence, which allows us to determine if some events must be scheduled before other events in all temporally consistent executions.

Definition 4.2 (Precedence)**.** Suppose we have two events, e_i and e_j, with guards φ_i and φ_j respectively. Suppose that we also have a third, *context* environment φ_C. We say that e_i precedes e_j in context φ_C, denoted $e_i \prec e_j\Big|_{\varphi_C}$, iff $Q_{D_{e_j,e_i}}(\varphi_i \wedge \varphi_j \wedge \varphi_C) < 0$.

When the context $\varphi_C = \{\}$, we denote $e_i \prec e_j\Big|_{\{\}}$ as just $e_i \prec e_j$ for brevity. We define a similar *succession* operator $\succ$ analogous to $\prec$: $e_i \succ e_j$ iff $Q_{D_{e_i,e_j}}(\varphi_i \wedge \varphi_j) < 0$.

If $e_i \prec e_j$, then e_i will be executed before e_j in all temporally consistent executions in which both e_i and e_j are activated. If an additional context environment φ_C is provided, it has the effect of conditioning on those sets of choices; e_i is executed before e_j whenever φ_C holds.

A few properties about precedence that hold when $\varphi_i \wedge \varphi_j \wedge \varphi_C$ is not a temporal conflict:

- It is possible for $e_i \prec e_j\Big|_{\varphi_C}$ to hold, or for $e_j \prec e_i\Big|_{\varphi_C}$ to hold, but not both.
- It is possible that neither $e_i \prec e_j\Big|_{\varphi_C}$ nor $e_j \prec e_i\Big|_{\varphi_C}$ hold. In that case, a precise a priori ordering between the two events cannot be determined before the plan is executed. There are temporally consistent executions in which e_i comes before e_j, and other(s) in which e_j comes before e_i.
- There are thus three possibilities: (1) $e_i \prec e_j\Big|_{\varphi_C}$, (2) $e_j \prec e_i\Big|_{\varphi_C}$, or (3) neither.

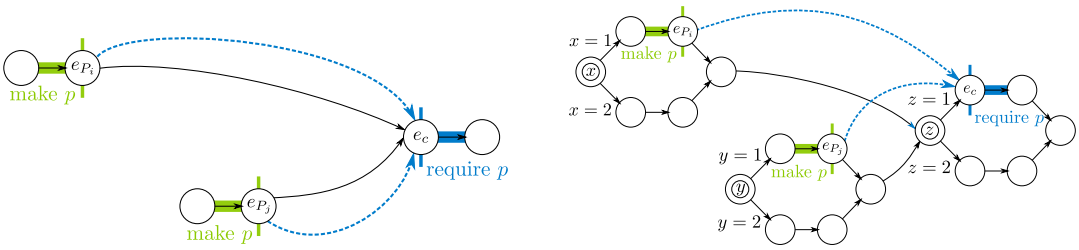
Definition 4.3 (Incomparability)**.** If neither $e_i \prec e_j\Big|_{\varphi_C}$ nor $e_j \prec e_i\Big|_{\varphi_C}$, we say that e_i and e_j are *incomparable* in context φ_C, denoted by $e_i \parallel e_j\Big|_{\varphi_C}$.

4.2 Labeled Causal Links

In this section, we introduce labeled causal links and motivate their use in executing contingent plans.

Given a correct, totally ordered plan and a STRIPS-like action model, the process of extracting causal links from said plan (for use during execution monitoring) is straightforward. For each precondition p of a *consumer* action A_c in the plan, we may find its causal link by regressing backwards from A_c in the plan until we find a *producer* action A_P that produces p as an effect. In this case with totally ordered plans, there is a single, unique causal link associated with every precondition p of every action A_c in the plan.

Things become more complicated when extracting causal links from partially-ordered plans, including those with metric temporal constraints. Aside from inferring precedence



(a) A choice-less plan with two producers and one consumer. Black lines are $[0, \infty]$ temporal constraints, and blue dotted lines represent causal links. Either could be the supporting causal link during execution.

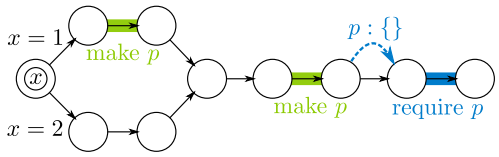
(b) A similar plan to the above, but now with choices surrounding each activity. Labeled causal links are required for this plan, and will allow us to reason about consistent sets of choices amongst x, y, and z.

Figure 6: Labeled causal links enable an executive to ensure that preconditions are satisfied in contingent plans.

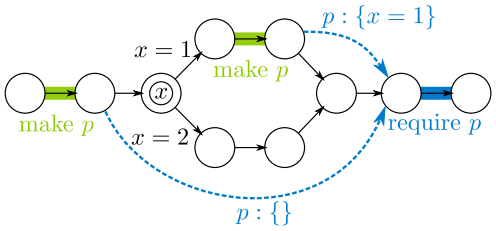
via a transitive closure operation (or in the case of metric temporal constraints, an all-pairs shortest path algorithm), there may in general be multiple *candidate causal links* for each precondition of each action in the plan (Levine, 2012). This is illustrated in Figure 6a, which shows two partially ordered producer actions that produce p as an effect for the later-occurring consumer that requires p as a precondition. The $[0, \infty]$ temporal bounds permit any ordering of the two producers; the top may come first, second, or they may overlap. So, which of the two actions will be the causal link? For the purposes execution monitoring and detecting violated causal links, only the latest producer matters. Hence, there is flexibility during execution since the two producers are unordered. If during execution, the top-most producer finishes first, then we designate the bottom-most action as providing the final support for the consumer – and vice versa. Due to such cases, there may in general be multiple possible candidate causal links for each precondition of each action in the plan. The plan is only at risk of being causally incomplete if the latest-most occurring causal link is violated (Levine, 2012).

Things are complicated further in the case of metric temporal plans with choice, such as the TPNU representation embraced by this work. With the addition of choices, certain producer actions may not even execute depending on what choices are made. As a result, for each precondition p in the plan, we now have a set of *labeled causal links*, at least one of which must hold in order to guarantee that the precondition is met in the plan. The label captures the dependence of the causal link on choices made in the plan.

For example, suppose we generalize the earlier example Figure 6a to Figure 6b, where each of the activities are now conditioned on a choice. In this case, each of the activities may or may not be activated depending on the choice assignments, and may not be executed. The causal links are therefore now contingent upon the choices made. For example, if the executive chooses $x = 2$, then the top-most causal link will vanish since the producer is not executed. Similarly, the bottom-most causal link vanishes if $y = 2$ is chosen. If $z = 1$ is chosen (and hence the consumer will be executed), then either $x = 1$ or $y = 1$ must hold for the consumer's precondition to be supported (and for the plan to be causally complete). But if $z = 2$ is chosen, then there are no constraints on either x or y. This is the core

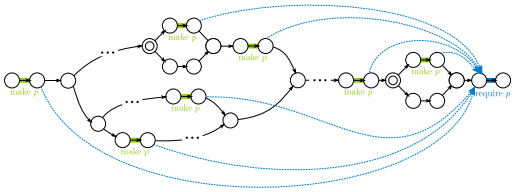


(a) Labeled causal link with later-occurring producer dominates earlier one (not shown).

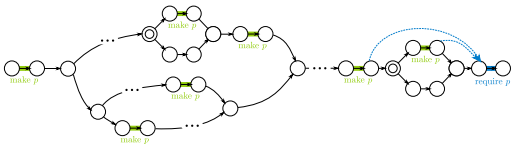


(b) No causal link dominance here.

Figure 7: Three causal link dominance examples. Unlabeled temporal constraints are $[0, \infty]$.



(a) All labeled causal links.



(b) Only non-dominated labeled causal links.

Figure 8: Comparison of labeled causal links before and after dominated ones have been removed. With fewer links, the problem is simplified.

intuition behind labeled causal links: the causal links now depend on the choices made, and for a plan to be causally complete, there must be at least one activated causal link supporting every precondition of every activated event. We therefore *label* each causal links with its requisite choice environments. In the example, the top-most causal link is labeled with $\{x = 1\}$, and the bottom-most one is labeled with $\{y = 1\}$.

The notion of a threat can also be generalized for contingent plans. A threat that asserts $\neg p$ as an effect may or may not be activated depending on the choices made. Note that if a threat is indeed activated, and if the consumer and producer that it threatens are also activated, the executive may still possibly be able to ensure a causally complete plan execution by making certain choices to activate other, later-occurring labeled causal links. This will be described more later.

Given this intuition, we are now prepared to define a labeled causal link.

Definition 4.4 (Labeled Causal Link)**.** A *labeled causal link* is a tuple $\langle e_P, e_c, p, \varphi \rangle$ where e_P is the *producer* event with $p \in \textsc{Effects}(e_P)$, e_c is a consumer event occurring after e_P with $p \in \textsc{Preconditions}(e_c)$, p is a predicate, and the label φ is an environment. The label φ will be the execution environment of e_P, φ_{e_P}.

We define one final useful property related to labeled causal links: causal link *dominance*. While not strictly necessary for correctness, the notion of dominance allows us to prune out many would-be causal links that are superseded by others and need not be considered. A labeled causal link for some consumer event e_c with producer e_{P_i} dominates another labeled causal link with the same consumer but different producer e_{P_j} iff (1) whenever e_{P_j}

is activated then e_{P_i} is also activated, (2) when all three events are activated then e_{P_i} must occur after e_{P_j}, and (3) when all three events are activated then e_{P_i} must precede e_c.

Definition 4.5 (Labeled Causal Link Dominance)**.** A labeled causal link $\langle e_{P_i}, e_c, p, \varphi_{e_{P_i}} \rangle$ *dominates* another labeled causal link $\langle e_{P_j}, e_c, p, \varphi_{e_{P_j}} \rangle$ iff:

1. $e_{P_j} \prec e_{P_i}\Big|_{\varphi_{e_c}}$
2. $\varphi_{e_{P_j}} \models \varphi_{e_{P_i}}$
3. $e_{P_i} \prec e_c\Big|_{\varphi_{e_{P_j}}}$

Some examples of causal link dominance are shown in Figure 7. Consider Figure 7a, in which we have a plan with a single choice of a producer, followed by a second producer. The labeled causal link with the later-occurring producer dominates the other one. This is because whenever the earlier producer is activated, so is the later-occurring one. The situation is however reversed in Figure 7b, where the order of the choice is reversed. In this case, the later-occurring producer is not necessarily activated whenever the earlier one is, so there is no domination. This structure in Figure 7b is useful for modeling contingency or recovery actions in a plan; should there be an unexpected disturbance that negates p sometime before the choice, the executive may recover by choosing $x = 1$ in the later choice, thus ensuring that the precondition will be supported.

It is safe to ignore dominated causal links from consideration during the compilation process, where the augmented TPNU is constructed.

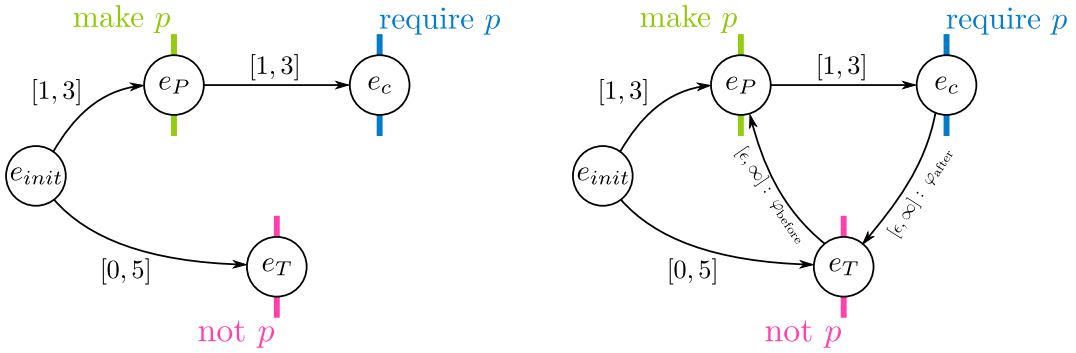
Theorem 4.2 (Dominated Causal Links are Irrelevant)**.** *Dominated labeled causal links do not influence the correctness of an execution.*

Proof. See Appendix A. □

Pruning dominated causal links can have a positive impact on the executive's performance by significantly decreasing the size of the compiled solutions found. This is illustrated in Figure 8b, where a number of labeled causal links can be pruned resulting in much simpler constraint structure.

4.3 Augmented TPNU's

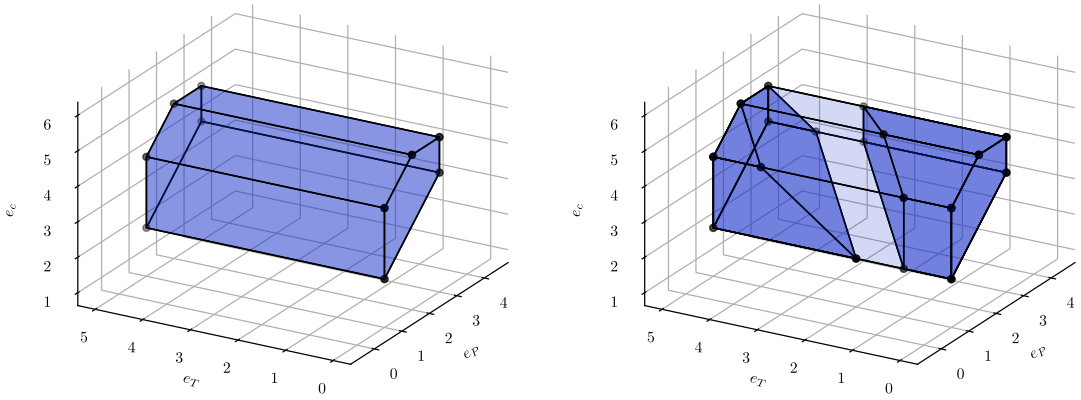
Now that we have introduced labeled causal links, we discuss how they can be used online to guide controllable choices and scheduling decisions. To guarantee correct executions that are causally complete, we transform the original TPNU $\mathcal{T}$ into a new TPNU $\mathcal{T}'$ which we call the *augmented* TPNU, and also generate a knowledge base of propositional constraints we call KB. Together, this augmented TPNU $\mathcal{T}'$ and set of constraints KB describes the space of all possible correct executions of the original TPNU $\mathcal{T}$. As we shall illustrate shortly, $\mathcal{T}'$ along with KB represents every correct execution of $\mathcal{T}$ – but it also prunes out incorrect executions from $\mathcal{T}$ that are temporally consistent yet causally incomplete. Online during execution, Pike uses $\mathcal{T}'$ and KB to make controllable choices and scheduling decisions in such a way as to maintain a healthy execution trace.



(a) Original TPNU $\mathcal{T}$.

(b) The augmented TPNU $\mathcal{T}'$.

Figure 9: A TPNU example, original and augmented.



(a) Space of all temporally consistent executions of $\mathcal{T}$.

(b) Space of all correct (temporally consistent and causally complete) executions of $\mathcal{T}$.

Figure 10: Various spaces of executions.

Consider the example TPNU shown in Figure 9a. This TPNU has four events, each of which must be assigned a time to form a schedule. We assume here that event e_{init} is scheduled at $t = 0$, leaving three remaining events to be scheduled. This TPNU has no choice variables, so there is just one candidate subplan φ_S. What is the space of possible temporally consistent schedules for $\mathcal{T}$? We visualize it in Figure 10a. The x, y, and z axes represent the scheduled times of events e_P, e_T, and e_c, respectively. Each point in this 3D space represents a schedule for $\mathcal{T}$, and the shaded region represents the set of all temporally consistent schedules. This space is convex, as the scheduling problem can be equivalently reformulated as a linear program.

While all schedules visualized in Figure 10a are temporally consistent, they are not all correct. This highlights one of the key differences between the scheduling problem for temporal networks and the plan execution problem: causal completeness must be enforced. Event e_c has precondition p, event e_P asserts p as an effect (and is therefore a producer for a labeled causal link), and event e_T asserts $\neg p$ as an effect and is a potential threat.

Any execution $\langle \varphi_S, T_{\varphi_S} \rangle$ in which e_T is scheduled between e_P and e_c – namely $T_{\varphi_S}(e_P) \leq T_{\varphi_S}(e_T) \leq T_{\varphi_S}(e_c)$ – is causally incomplete. We can also see that $T_{\varphi_S}(e_P) < T_{\varphi_S}(e_c)$ is required in any causally complete execution (any temporally consistent execution satisfies this however, due to the $e_P \xrightarrow{[1,3]} e_c$ temporal constraint). The space of all correct executions is visualized in Figure 10b. We see that a large slice has been removed relative to Figure 10a: these pruned executions were temporally consistent yet causally incomplete, and represent executions where e_T occurred between e_P and e_c.

It is visually apparent From Figure 10b that there are two separate subregions. Our approach in Pike is to represent each of these subregions by a different candidate subplan φ'_S of the augmented TPNU $\mathcal{T}'$. By modifying the original TPNU $\mathcal{T}$ appropriately, the augmented TPNU $\mathcal{T}'$ contains more candidate subplans than the original, and each represents a set of correct executions. Pike covers the entire space of all correct executions in this way. In our visualized example, $\mathcal{T}$ has a single candidate subplan and $\mathcal{T}'$ has two, one for each of the subregions.

Creating the augmented TPNU $\mathcal{T}' = \langle \mathcal{V}', \mathcal{E}, \mathcal{C}', \mathcal{A} \rangle$ and KB is accomplished by copying $\mathcal{T}$, and subsequently adding new controllable decision variables to $\mathcal{V}'$, new temporal constraints to $\mathcal{C}'$, and propositional constraints to KB. In this example, one of the new decision variables added to $\mathcal{V}'$ will choose whether e_T will come before or after the labeled causal link from e_P to e_c. We will introduce the full encoding shortly, but for now we simply call this variable $o \in \{-1, +1\}$ where $o = -1$ indicates e_T must come before and $o = +1$ indicates after. We also add two additional temporal constraints: $e_T \xrightarrow{[\epsilon,\infty]} e_P : \varphi_{\text{before}}$ and $e_c \xrightarrow{[\epsilon,\infty]} e_T : \varphi_{\text{after}}$, resulting in the augmented TPNU shown in Figure 9b. These new temporal constraints will act as ordering constraints that are activated based on o: the guards φ_{before} and φ_{after} are defined such that φ_{before} holds if $o = -1$, and φ_{after} holds if $o = +1$. There are thus two candidate subplans of $\mathcal{T}'$, one corresponding to each possible assignment, differing only in which newly added temporal constraint is activated. Each of these candidate subplans has an associated set of temporally consistent executions, shown as the left and right convex subregions of Figure 10b. By Theorem 4.5 (discussed and proved later), all of these executions are guaranteed to be causally complete and correct. In this way, Pike represents the complex space of correct executions of $\mathcal{T}$ using a factored approach, where $\mathcal{T}$ is transformed into an augmented TPNU $\mathcal{T}'$ with an associated KB. Every candidate subplan φ'_S of $\mathcal{T}'$ that satisfies KB represents a simpler, convex set of correct executions. Taking all of these candidate subplans of $\mathcal{T}'$ together, Pike maintains the full space of possible executions.

Plans may be additionally complicated with the addition of choices, multiple candidate causal links, and multiple threats. A larger example with these features is illustrated in Figure 11. There are five activities, which are conditioned on choice variables w, u, v, x and y. Activity A_1 has a precondition of p. Two activities P_1 and P_2 both produce p as an effect. We also have two potential threats T_1 and T_2 that negate p as an effect. All unlabeled temporal constraints are $[0, \infty]$, so any activity may come before or after (or overlap) other activities. This example involves both *potential producers* and *potential threats*.

Which producers support the precondition p of event e_c? The only events that produce p are e_{P_1} and e_{P_2}. However, neither is guaranteed to precede e_c due to the loose temporal constraints of the problem. These are hence referred to as *potential producers*. We

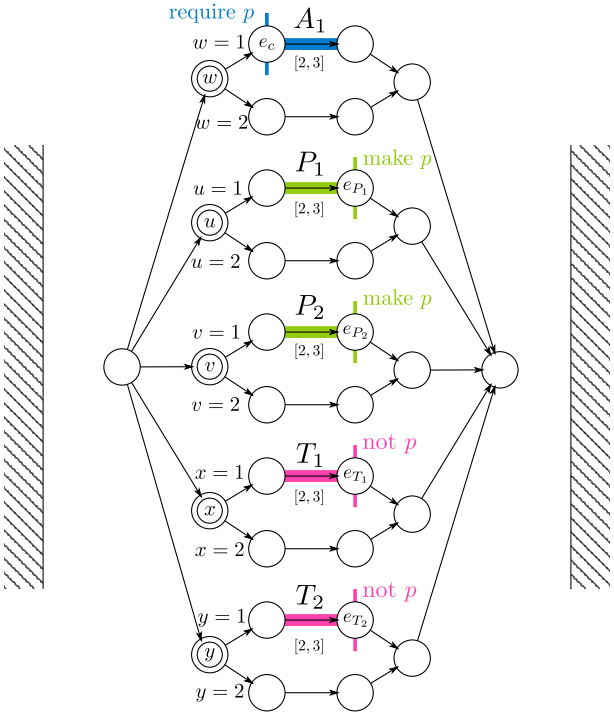


Figure 11: A larger example TPNU. Unlabeled temporal constraints are $[0, \infty]$.

address potential producers by adding additional labeled temporal constraints to $\mathcal{C}'$ of the augmented TPNU, as well as a new controllable decision variable to $\mathcal{V}'$. This new variable chooses which of the different possible producer events — e_{P_1} or e_{P_2} in this case — will be chosen to enforce support for the precondition p of event e_c (i.e., which event will be the producer in the supporting causal link). We define this new decision variable, s_{p,e_c}, as[2]

$$s_{p,e_c} = \begin{cases} e_{P_1} & \text{if } e_{P_1} \text{ will be the producer} \\ e_{P_2} & \text{if } e_{P_2} \text{ will be the producer} \\ \bot & \text{if neither will be the producer} \end{cases}$$

We also add the propositional constraint below to KB, which asserts that if e_c is activated (as is the case when $w = 1$), then one of the producers must also be activated and chosen as the support:

$$(w = 1) \Rightarrow (s_{p,e_c} = e_{P_1} \wedge u = 1) \vee (s_{p,e_c} = e_{P_2} \wedge v = 1)$$

In this way, we translate a labeled causal link into a propositional state logic constraint.

We must also enforce that if $s_{p,e_c} = e_{P_1}$ is chosen, event e_{P_1} must precede e_c. This resolves the potential producer and forms the basis for a labeled causal link. We accomplish this by adding the following temporal constraint to $\mathcal{C}'$, labeled appropriately for the

2. We include $\bot$ in the domain of s_{p,e_c} to represent that s_{p,e_c} may acceptably be left unassigned if e_c is not activated. This is useful for there to be a solution if causal link violations cause $\neg(s_{p,e_c} = e_{P_1})$ and $\neg(s_{p,e_c} = e_{P_1})$ constraints to be added to KB: namely deactivating e_c by choosing $w = 2$.

environment where this choice is made and where the consumer and producer activities are both activated:

$$e_{P_1} \xrightarrow{[\epsilon,\infty]} e_c : \ \{s_{p,e_c} = e_{P_1}, u = 1, w = 1\}$$

Note that we only need to add such additional temporal constraints if the temporal flexibility of the plan could allow for the producer event to happen after the consumer. If $e_{P_1} \prec e_c$, no additional temporal constraints are added to $\mathcal{C}'$.

We add a similar constraint for e_{P_2}:

$$e_{P_2} \xrightarrow{[\epsilon,\infty]} e_c : \ \{s_{p,e_c} = e_{P_2}, v = 1, w = 1\}$$

Next, we address the two potential threats T_1 and T_2. For the following arguments, suppose that P_1 is chosen to be the supporting producer. Hence, the labeled causal link from event e_{P_1} to event e_c over predicate p will be the supporting causal link, and $s_{p,e_c} = e_{P_1}$. To avoid threats to this labeled causal link, no other action may be scheduled during the interval between e_{P_1} and e_c that negates p. Both e_{T_1} and e_{T_2} are *potential threats*, because the loose flexibility in the temporal constraints admits some executions that are causally complete and others that are not. They must each be scheduled either before e_{P_1} or after e_c, as was the case in the earlier example. We resolve these potential threats by adding additional temporal constraints to $\mathcal{C}'$ to enforce this, and by adding a new controllable decision variable to $\mathcal{V}'$ choosing whether it will come before or after:

$$o_{p,e_c,e_{P_1},e_{T_1}} = \begin{cases} -1 & \text{if } e_{T_1} \text{ will precede } e_{P_1} \\ +1 & \text{if } e_{T_1} \text{ will succede } e_c \end{cases}$$

A similar decision variable $o_{e_c,p,e_{P_1},e_{T_2}}$ is also added, representing the ordering of e_{T_2}. We also add the following temporal constraints to $\mathcal{C}'$:

$$e_{T_1} \xrightarrow{[\epsilon,\infty]} e_{P_1} : \ \{s_{p,e_c} = e_{P_1}, \ o_{p,e_c,e_{P_1},e_{T_1}} = -1, \ u = 1, \ x = 1, \ w = 1\}$$
$$e_c \xrightarrow{[\epsilon,\infty]} e_{T_1} : \ \{s_{p,e_c} = e_{P_1}, \ o_{p,e_c,e_{P_1},e_{T_1}} = +1, \ u = 1, \ x = 1, \ w = 1\}$$

These temporal constraints guarantee that if T_1 is activated and would threaten the labeled causal link from e_{P_1} to e_c, then it will be scheduled either before or after the labeled causal link. Two similar labeled temporal constraints are added for e_{T_2}, completing the threat resolutions for the case where e_{P_1} is chosen as the support. We also have similar constraints to the above corresponding to the case where e_{P_2} is chosen as the support. These constraints effectively implement similar threat resolution rules as POCL (Penberthy et al., 1992), though generalized for contingent plans.

4.4 Causal Link Extraction & Constructing the Augmented TPNU

In the previous sections, we have illustrated some examples of labeled causal link extraction, along with examples of constructructing the augmented TPNU $\mathcal{T}'$ and constraints KB. In this section, we codify these examples by introducing appropriate algorithms. Pseudo code is shown in Algorithm 5, and a summary listing of all additional constraints is shown in Table 1.

Algorithm 5: ConstructAugmentedTPNUwithKB()

Input: TPNU $\mathcal{T} = \langle \mathcal{V}, \mathcal{E}, \mathcal{C}, \mathcal{A} \rangle$
Output: Augmented TPNU $\mathcal{T}' = \langle \mathcal{V}', \mathcal{E}, \mathcal{C}', \mathcal{A} \rangle$ with associated constraints KB

$\mathcal{T}' = \langle \mathcal{V}', \mathcal{E}, \mathcal{C}', \mathcal{A} \rangle \leftarrow$ copy of $\mathcal{T}$, KB $\leftarrow$ True
foreach $e_c \in \mathcal{E}$ **do**
  foreach $p \in$ Preconditions(e_c) **do**
    $\xi \leftarrow$ new LVS with relation $<_{e_c}^{dom}$
    foreach $e \in \mathcal{E} \setminus \{e_c\}$ *with* p *or* $\neg p$ *in* Effects(e) **do**
      if *not*$(e_c \prec e)$ *and* $\varphi_e \wedge \varphi_{e_c}$ *is self consistent* **then**
        AddLVS$((e, \varphi_e), \xi)$
      end
    end
    Partition events from ξ into P (producers) and T (threats)
    Add new decision variable s_{p,e_c} to $\mathcal{V}'$ with domain $\{e_{P_1}, e_{P_2}, ..., \perp\}$ for $e_{P_i} \in P$
    KB $\leftarrow$ KB $\wedge \left(\varphi_{e_c} \Rightarrow \bigvee_{e_{P_i} \in P} (s_{p,e_c} = e_{P_i} \wedge \varphi_{e_{P_i}}) \right)$
    foreach $e_{P_i} \in P$ *where not*$(e_{P_i} \prec e_c)$ **do**
      $\mathcal{C}' \leftarrow \mathcal{C}' \cup \left(e_{P_i} \xrightarrow{[\epsilon,\infty]} e_c : \{s_{p,e_c} = e_{P_i}\} \wedge \varphi_{e_{P_i}} \wedge \varphi_{e_c} \right)$
    end
    foreach $e_{P_i} \in P$ **do**
      foreach $e_{T_j} \in T$ **do**
        $\varphi_C \leftarrow \{s_{p,e_c} = e_{P_i}\} \wedge \varphi_{e_{P_i}} \wedge \varphi_{e_{T_j}} \wedge \varphi_{e_c}$
        continue to next if φ_C is self inconsistent or temporally infeasible
        if $e_{P_i} \prec e_{T_j}\big|_{\varphi_C}$ *and* $e_{T_j} \prec e_c\big|_{\varphi_C}$ **then**
          KB $\leftarrow$ KB $\wedge \left(\neg \varphi_C \right)$ ▷ Avoid definite threat
        else if $e_{P_i} \prec e_{T_j}\big|_{\varphi_C}$ *and* $e_{T_j} \parallel e_c\big|_{\varphi_C}$ **then**
          $\mathcal{C}' \leftarrow \mathcal{C}' \cup \left(e_c \xrightarrow{[\epsilon,\infty]} e_{T_j} : \varphi_C \right)$ ▷ Force threat after consumer
        else if $e_{P_i} \parallel e_{T_j}\big|_{\varphi_C}$ *and* $e_{T_j} \prec e_c\big|_{\varphi_C}$ **then**
          $\mathcal{C}' \leftarrow \mathcal{C}' \cup \left(e_{T_j} \xrightarrow{[\epsilon,\infty]} e_{P_i} : \varphi_C \right)$ ▷ Force threat before producer
        else if $e_{P_i} \parallel e_{T_j}\big|_{\varphi_C}$ *and* $e_c \parallel e_{T_j}\big|_{\varphi_C}$ **then**
          ▷ Force threat either before or after
          Add new decision variable $o_{p,e_c,e_{P_i},e_{T_j}}$ to $\mathcal{V}'$ with domain $\{-1, +1\}$
          $\mathcal{C}' \leftarrow \mathcal{C}' \cup \left(e_{T_j} \xrightarrow{[\epsilon,\infty]} e_{P_i} : \{o_{p,e_c,e_{P_i},e_{T_j}} = -1\} \wedge \varphi_C \right)$
          $\mathcal{C}' \leftarrow \mathcal{C}' \cup \left(e_c \xrightarrow{[\epsilon,\infty]} e_{T_j} : \{o_{p,e_c,e_{P_i},e_{T_j}} = +1\} \wedge \varphi_C \right)$
        end
      end
    end
  end
end

Constraint	Reason
Propositional for KB:	
$\varphi_{e_c} \Rightarrow \bigvee_{e_{P_i} \in P} (s_{p,e_c} = e_{P_i} \wedge \varphi_{e_{P_i}})$	At least one activated producer
$\neg\varphi_C$	Avoid definite threat
$\neg\varphi_C$	Avoid temporal conflict
Temporal for $\mathcal{T}'$:	
$e_{P_i} \xrightarrow{[\epsilon,\infty]} e_c : \ \{s_{p,e_c} = e_{P_i}\} \wedge \varphi_{e_{P_i}} \wedge \varphi_{e_c}$	Producers precede consumers
$e_c \xrightarrow{[\epsilon,\infty]} e_{T_j} : \ \varphi_C$	Force threat after consumer
$e_{T_j} \xrightarrow{[\epsilon,\infty]} e_{P_i} : \ \varphi_C$	Force threat before producer
$e_{T_j} \xrightarrow{[\epsilon,\infty]} e_{P_i} : \ \{o_{p,e_c,e_{P_i},e_{T_j}} = -1\} \wedge \varphi_C$	Force threat before producer
$e_c \xrightarrow{[\epsilon,\infty]} e_{T_j} : \ \{o_{p,e_c,e_{P_i},e_{T_j}} = +1\} \wedge \varphi_C$	Force threat after consumer

Table 1: Summary of constraints added for $\mathcal{T}'$ and KB.

Without loss of generality, we assume that there are two auxiliary events in the plan: (1) $e_{initial}$ which precedes all other events and has Effects($e_{initial}$) set to the initial conditions, and (2) e_{goal} which succeeds all other events and has Preconditions(e_{goal}) set to the goal conditions. This encoding technique allows labeled causal links to be extracted from the initial conditions and goals without treating them separately from other events. Similar techniques have been used in other approaches (Muise, Beck, & McIlraith, 2016).

Our algorithm takes the following steps for each precondition p of each event e_c. First, it defines a new partial order relation over pairs of events for determining causal link dominance $<_{e_c}^{dom}$. It is defined as follows:

$$e_i <_{e_c}^{dom} e_j = \begin{cases} \text{True} & \text{if } e_j \prec e_i\Big|_{\varphi_c} \text{ and } e_i \prec e_c\Big|_{\varphi_j} \\ \text{False} & \text{otherwise} \end{cases}$$

This new relation will be helpful for determining labeled causal link dominance, through the use of a LVS, ξ. This LVS ξ will use $<_{e_c}^{dom}$ as its relation $<_R$, and values will be events (as opposed to relation $<$ and values being numbers, as used earlier for temporal reasoning). If $e_i <_{e_c}^{dom} e_j$ and $\varphi_{e_j} \models \varphi_{e_i}$, it can be shown from the definition of labeled causal link dominance that a labeled causal link with producer e_i dominates a labeled causal link with producer e_j (both having consumer e_c). Therefore, a labeled value (e_i, φ_{e_i}) will dominate a different labeled value (e_j, φ_{e_j}) in ξ iff the labeled causal link with producer e_i dominates the labeled causal link with producer e_j. The $<_{e_c}^{dom}$ operator takes care of the first and third criterion required for labeled causal link dominance, and the environment entailment checks of the LVS take care of the second.

	$e_{T_j} \prec e_{P_i}\big\vert_{\varphi_C}$	$e_{P_i} \prec e_{T_j}\big\vert_{\varphi_C}$	$e_{T_j} \parallel e_{P_i}\big\vert_{\varphi_C}$
$e_{T_j} \prec e_c\big\vert_{\varphi_C}$	Case 1: Nothing required.	Case 2: Unresolvable threat — avoid via adding conflict.	Case 3: Force e_{T_j} before e_{P_i} to resolve threat.
$e_c \prec e_{T_j}\big\vert_{\varphi_C}$	Case 4: Impossible.	Case 5: Nothing required.	Case 6: Impossible.
$e_c \parallel e_{T_j}\big\vert_{\varphi_C}$	Case 7: Impossible.	Case 8: Force e_{T_j} after e_c to resolve threat.	Case 9: Force e_{T_j} either before e_{P_i} or after e_c via choice variable.

Figure 12: Labeled causal link threat resolution cases. Moving across horizontally, we have the three possible precedence relations between e_{P_i} and e_{T_j}. Vertically, we have the three possible relations between e_{T_j} and e_c. The environment $\varphi_C = \{s_{p,e_c} = e_{P_i}\} \wedge \varphi_{e_{P_i}} \wedge \varphi_{e_c} \wedge \varphi_{e_{T_j}}$.

During labeled causal link extraction, we add all potential producers, as well as all potential threats, to ξ. We therefore ensure that we find only the relevant, latest-occurring, non-dominated causal link producers and threats. Specifically, for each event e that produces either p or $\neg p$ in the plan, we add (e, φ_e) to ξ as long as $e_c \prec e$ does not hold (hence either $e \prec e_c$ or $e \parallel e_c$). The resulting ξ contains the minimal set of non-dominating possible producers and threats. For the example in Figure 11, $\xi = \{(e_{P_1}, \{u = 1\}), (e_{P_2}, \{v = 1\}), (e_{T_1}, \{x = 1\}), (e_{T_2}, \{y = 1\})\}$.

We then partition each event e in $(e, \varphi_e) \in \xi$ into those that produce p as an effect into set P (producers), and those that produce $\neg p$ into set T (threats).

Next, we make a new controllable decision variable s_{p,e_c} with domain $\{e_{P_1}, ..., \bot\}$ for each e_{P_i} in P. We add to KB the propositional state logic constraint

$$\varphi_{e_c} \Rightarrow \bigvee_i (s_{p,e_c} = e_{P_i} \wedge \varphi_{e_{P_i}})$$

Additionally, for each e_{P_i}, we check whether $e_{P_i} \prec e_c$. If so, then we continue (no additional temporal constraint is needed to order this producer before the consumer). Otherwise, it is possible for e_{P_i} to occur after e_c; we prevent this by adding to $\mathcal{C}'$ the labeled temporal constraint $e_{P_i} \longrightarrow e_c, [\epsilon, \infty] : \{s_{p,e_c} = e_{P_i} \wedge \varphi_{e_c} \wedge \varphi_{e_{P_i}}\}$.

We must also resolve any potential threats of this labeled causal link. Based on the different possible precedence relations amongst the consumer e_c, producer e_{P_i}, and threat e_{T_j}, different constraints may be added to $\mathcal{T}'$ and KB. A case-based analysis is depicted in Figure 12. Depending on the case, either a conflict environment is added, or additional labeled temporal constraints are added to resolve the threat. We iterate over all pairs of potential producer events e_{P_i} and all potential threat events e_{T_j}. The context environment is set such that all of e_c, e_{P_i}, and e_{T_j} are activated, and e_{P_i} is selected as the supporting labeled causal link. Hence, $\varphi_C = \{s_{p,e_c} = e_{P_i}\} \wedge \varphi_{e_{P_i}} \wedge \varphi_{e_c} \wedge \varphi_{e_{T_j}}$. We evaluate all precedence relations with respect to this context environment.

In case 1 above, e_{T_j} is not a threat. Since $e_{T_j} \prec e_{P_i}\big\vert_{\varphi_C}$ and $e_{P_i} \prec e_c\big\vert_{\varphi_C}$, then the threat will not interfere with the causal link. The predicate in question is negated before

the producer event asserts it. Similarly in case 5, the threat in question will occur after the consumer event in the context environment. Therefore, we can be sure that the threat event will not interfere.

Cases 4, 6, and 7 represent impossible situations if φ_C is a temporally consistent environment (which is checked by our algorithm). Since we add appropriately labeled ordering constraints ensuring that producers precede consumers, we will have that $e_{P_i} \prec e_c\big|_{\varphi_C}$. In cases 4 and 7, we also have that $e_{T_j} \prec e_{P_i}\big|_{\varphi_C}$, so we can derive that $e_{T_j} \prec e_c\big|_{\varphi_C}$. This however contradicts the premises in cases 4 and 7. Similar reasoning holds for case 6.

In case 2, the threat is guaranteed to occur temporally between the causal link producer and the consumer. This means that the potential threat is in fact a definite threat, and there is nothing that can be done about it except to avoid it. Therefore, we add a conflict disallowing φ_C to KB.

In case 3, we know that the threat precedes the consumer, but it is incomparable to the producer (which also precedes the consumer). Therefore, we can resolve this threat by adding a labeled temporal ordering constraint forcing the threat to occur *before* the producer event. If this results in a temporally feasible plan after re-running the APSP, we have resolved the threat. If not, we will extract a temporal conflict similar to case 2 above. Similar reasoning also holds for case 8, in which we know that the threat must occur after the producer, but could potentially occur before the consumer e_c. We therefore resolve this threat by forcing the threat to occur *after* the consumer event via an additional labeled temporal constraint.

Case 9 is the most complicated, and involves adding an additional controllable decision variable. In case 9, the threat event can occur anywhere temporally; it could occur before the producer, between the producer and the consumer (which is a problem), and after the consumer. We wish to move the threat so that it occurs either before or after. We hence add a new controllable decision variable $o_{p,e_c,e_{P_i},e_{T_j}}$ with domain $\{-1,+1\}$ to $\mathcal{V}'$. If $o_{p,e_c,e_{P_i},e_{T_j}} = -1$, we choose for the threat to come before e_{P_i}; otherwise, we choose for it to come after e_c. We add two new temporal constraints to enforce this:

- $e_{T_j} \xrightarrow{[\epsilon,\infty]} e_{P_i} : \{o_{p,e_c,e_{P_i},e_{T_j}} = -1\} \wedge \varphi_C$
- $e_c \xrightarrow{[\epsilon,\infty]} e_{T_j} : \{o_{p,e_c,e_{P_i},e_{T_j}} = +1\} \wedge \varphi_C$

If φ_C holds, exactly one of the above two constraints will be activated. This forces the executive to schedule the threat either before or after the labeled causal link interval, thereby resolving it.

Note that we could always do case 9, i.e., for cases 3 and 8. We avoid this however for efficiency, so that we do not need to add unnecessary decision variables and temporal constraints to the problem. This is the key motivation for such case-based reasoning.

This concludes our description of the algorithms that extract dominating labeled causal links and use them generate an augmented TPNU $\mathcal{T}'$ along with the propositional constraints KB. We provide one more grounded example to illustrate. In the TPNU shown in Figure 13, there is one consumer event and two preceding producer events. There is additionally a potential threat that is ordered after the first producer, but may come either before or after the second producer.

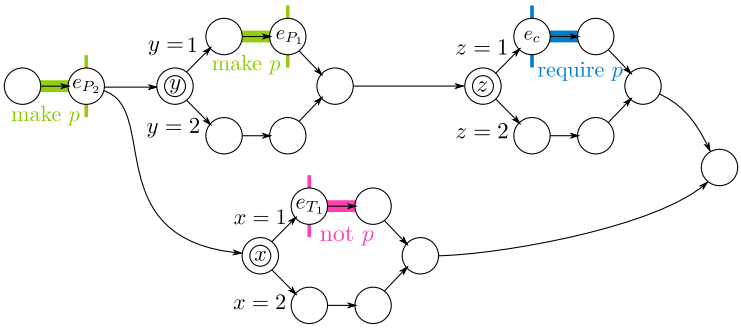


Figure 13: An example TPNU.

The result of running Algorithm 5 on this example will be the following additional constraints for $\mathcal{T}'$ and KB:

- Propositional: $z = 1 \Rightarrow (s_{p,e_c} = e_{P_2}) \vee (s_{p,e_c} = e_{P_1} \wedge y = 1)$
- Temporal: $e_c \xrightarrow{[\epsilon,\infty]} e_{T_1} : \{z = 1, x = 1, s_{p,e_c} = e_{P_2}\}$
- Temporal: $e_{T_1} \xrightarrow{[\epsilon,\infty]} e_{P_1} : \{z = 1, y = 1, x = 1, s_{p,e_c} = e_{P_1}, o_{p,e_c,e_{P_1},e_{T_1}} = -1\}$
- Temporal: $e_c \xrightarrow{[\epsilon,\infty]} e_{T_1} : \{z = 1, y = 1, x = 1, s_{p,e_c} = e_{P_1}, o_{p,e_c,e_{P_1},e_{T_1}} = +1\}$

4.5 Ensuring Healthy Execution Traces

We show in this section that, when Pike uses the augmented TPNU $\mathcal{T}'$ and KB to guide execution, the resulting execution trace will remain healthy. This is a key property of Pike, and a central goal of using the augmented TPNU. In the context of human-robot interaction, this means that Pike will make controllable choices and scheduling decisions such that there is always some way for the plan to succeed (i.e., some way for the human and robot team to make choices that achieve the plan goals). We introduce a series of theorems (proved in Appendix A), culminating in Theorems 4.7 and 4.4, showing that, when Pike executes $\mathcal{T}'$ with KB, it will make online decisions properly in this manner.

We begin by formalizing the relationship between $\mathcal{T}$ and $\mathcal{T}'$ in terms of correct executions and execution traces.

Theorem 4.3 (Executions from $\mathcal{T}$ and $\mathcal{T}'$ correspond)**.** *An execution $\langle \varphi_S, T_{\varphi_S} \rangle$ of the original TPNU $\mathcal{T}$ is correct if and only if there exists a corresponding correct execution $\langle \varphi'_S, T_{\varphi_S} \rangle$ of the augmented TPNU $\mathcal{T}'$ where $\varphi'_S \models \varphi_S$.*

Proof. See Appendix A. □

Theorem 4.4 (Healthy Executions of Original $\mathcal{T}$)**.** *Let $\langle \varphi_{ex}, \tilde{T}_{\varphi_{ex}} \rangle$ be an execution trace at some point during execution of the augmented TPNU $\mathcal{T}'$. This execution trace is healthy with respect to the augmented TPNU $\mathcal{T}'$ if and only if it is also healthy with respect to the original TPNU $\mathcal{T}$.*

Proof. See Appendix A. □

Theorem 4.3 maps correct executions of $\mathcal{T}$ to at least one correct execution in $\mathcal{T}'$ with an identical schedule. The relationship between these two sets is a surjection; every correct execution of $\mathcal{T}$ maps to at least one (possibly multiple) executions of $\mathcal{T}'$, corresponding to different allowable assignments to the s_{p,e_c} or o_{p,e_c,e_P,e_T} variables. Relatedly, in Theorem 4.4, we make a similar connection in terms of execution traces.

The following two theorems highlight key properties of $\mathcal{T}'$, and are visualized by Figure 10b. Theorem 4.5 asserts that every temporally consistent execution of $\mathcal{T}'$ is also correct. Crucially, this property does not hold for $\mathcal{T}$, which could admit temporally consistent yet causally incomplete executions. By constructing $\mathcal{T}'$ and KB, we ensure that Pike can achieve any temporally consistent execution and rest assured that it will also be causally complete.

Theorem 4.5 (Temporally consistent executions of $\mathcal{T}'$ with KB are also causally complete, and correct)**.** *Let $\langle \varphi'_S, T_{\varphi_S} \rangle$ be a temporally consistent execution of $\mathcal{T}'$ where φ'_S satisfies KB. Then this execution is also causally complete (and hence correct).*

Proof. See Appendix A. □

Theorem 4.6 describes the space of candidate subplans whose variable assignments satisfy KB. We can therefore think of the solutions of KB as representing the space of all candidate subplans that admit a correct execution.

Theorem 4.6 (φ'_S satisfies KB iff correct)**.** *φ'_S satisfies KB if and only if there exists a correct execution $\langle \varphi'_S, T_{\varphi_S} \rangle$ of $\mathcal{T}$'.*

Proof. See Appendix A. □

Based on these theorems, we can now prove that the CanExecuteEventNow? procedure, which is crucial for online execution correctness and is shown in Algorithm 2, is correct. Specifically, it will allow an event to be executed and controllable choices to be made by Pike if and only if the resulting execution trace would remain healthy.

Theorem 4.7 (CanExecuteEventNow? is correct)**.** *Let $\langle \varphi_{ex}, \tilde{T}_{\varphi_{ex}} \rangle$ be the current, healthy execution trace of $\mathcal{T}'$. Then* CanExecuteEventNow?(e_i, t) *returns* True *at time t if and only if the execution trace of $\mathcal{T}'$ that would result if e_i is executed at time t – namely $\langle \varphi_{ex} \wedge \varphi_{e_i},\ \tilde{T}_{\varphi_{ex}} \cup \{e_i = t\} \rangle$ – is healthy.*

Proof. See Appendix A. □

By Theorem 4.7, Pike's controllable choices and scheduling decisions keep the execution trace healthy for $\mathcal{T}'$, the TPNU being executed. By Theorem 4.4, healthy execution traces of $\mathcal{T}'$ correspond to healthy execution traces of $\mathcal{T}$. Therefore, we have proven that Pike will maintain a healthy execution trace of $\mathcal{T}$ – a central goal of Pike in ensuring that its choices can match the human's possible intentions.

This concludes our discussion of labeled causal link extraction, and the generation of the augmented TPNU $\mathcal{T}'$ and corresponding constraints. Next, we discuss how to compile the constraints for efficient use during online execution.

5. Constraint Compilation

In this section, we briefly describe a method to compile these constraints into a knowledge base KB for efficient online execution. The goal of this knowledge compilation is to compactly represent all possible candidate subplans φ'_S that admit a correct execution for the human-robot team. Per Theorem 4.6, this means that we must compactly represent the space of all solutions of KB. Towards this goal, we employ a well-known technique from the knowledge compilation community: prime implicants. We briefly introduce the πTMS, a label propagation mechanism building upon the Assumption-based Truth Maintenance System (ATMS) that generates a sound and complete set of prime implicants for our theory incrementally. The resulting prime implicants can be used to efficiently implement the query operations on KB needed for online execution, as discussed shortly.

We begin with background about the ATMS and prime implicants. We then illustrate the key algorithms we use to generate prime implicants. As this is not primary focus of this work, we defer full details to Appendix D.

5.1 Background

5.1.1 ATMS

The Assumption-based Truth Maintenance System, or ATMS, is a knowledge base that allows a problem solver to efficiently reason about facts without prematurely committing to them (De Kleer, 1986a). The ATMS introduces assumptions – facts whose certainty is not known and may be changed by the problem solver with little overhead. This fast context switching is taken advantage of during online execution, when querying which choices can consistently be made online.

We take a propositional logic viewpoint of the ATMS and introduce some terminology. A *node* N is logically equivalent to a proposition. An *assumption* is a special type of node whose truthfulness may be unknown beforehand. A node that represents False is called a *contradiction node*. A *justification* is logically equivalent to the implication $N_{A_1} \wedge ... \wedge N_{A_k} \Rightarrow N_C$ where each node N_{A_i} is an *antecedent* and N_C is the *consequent*. A *nogood* or *conflict* is a conjunction that entails an inconsistency; its resolution is logically equivalent to $\neg(N_{C_1} \wedge ... \wedge N_{C_k})$. An environment is a conjunction of assumption propositions (though it is often denoted as a set for convenience).

An environment φ_e *manifests* a conflict φ_C iff $\varphi_e \models \varphi_C$. An environment φ_e *resolves* a conflict φ_C iff $\varphi_e \models \neg\varphi_C$ (Williams & Ragno, 2007). An environment may manifest a conflict, resolve it, or neither. If φ_e manifests a conflict, then any environment φ whose assignments are a superset of φ_e (i.e., $\varphi \models \varphi_e$) also manifests that conflict. Similar logic holds for resolving a conflict. However, if an environment φ_e neither resolves nor manifests a conflict, then there exists some superset environment that manifest the conflict, and some other superset that resolves it.

Each node in an ATMS is associated with a *label*, which is a set (disjunctive) of environments. A node *holds* in environment φ iff it can be propositionally derived from φ and all justifications in the ATMS, and False cannot be propositionally derived via a conflict (Forbus, 1993). An environment φ_e in node N's label means that N will hold in any superset environment $\varphi \models \varphi_e$ – except those manifesting a conflict. A label is *minimal* if

no environment in it is a subset of any other environment (i.e., no environment entails any other). The ATMS enforces that no environment in an ATMS label may manifest a conflict, but it is possible for supersets of the environment to manifest conflicts (i.e., environments in a label need not resolve all conflicts). The ATMS employs label propagation algorithms to ensure that a sound, complete, and minimal set of environments is incrementally derived for each node's label, given the justifications and conflicts encoded so far (Forbus, 1993).

5.1.2 Prime Implicants

An *implicant* φ of a propositional theory $\mathcal{C}$ is a conjunction of variables (i.e., an environment) that entails the theory, $\varphi \models \mathcal{C}$. A *prime implicant* φ_P is an implicant that is minimal in size; no subset of assignments of φ_P themselves form an implicant. The intuition for this is that if φ is an implicant, then any superset must also be an implicant. We therefore need only maintain the smallest ones, namely prime implicants. There is also the related concept of implicate and prime implicate. An *implicate* θ of a theory $\mathcal{C}$ is a disjunction which logically follows from the theory, namely $\mathcal{C} \models \theta$.

While there are many different knowledge compilation techniques that could be successfully applied to this task (Darwiche & Marquis, 2002), we choose to pursue prime implicants in this work. We are motivated to do so for several reasons, including the potential to be more compact than full model enumeration, and the ability to perform relevant consistency queries online in time polynomial to the prime implicants representation. Similarly, is is possible to update a prime implicant database efficiently (via bounded conjunctions in polynomial time) (Darwiche & Marquis, 2002), which is also relevant for Pike's online execution when we add new constraints to KB. Please note that while it is possible for the number of prime implicants to exceed the number of models for a theory (Quine, 1959), our aim here here is to improve the compactness of our solution representation so as to improve the efficiency of computing our online queries. Experimental results verify this for our problem structure.

5.2 The πTMS

We introduce the πTMS (short for Prime Implicant TMS) – a technique for incrementally and efficiently computing the prime implicants of a theory over finite-domain variables. Like the ATMS, the πTMS uses label propagation to store a set of environments in a label associated with each node. The πTMS, however, uses a form of consensus over finite-domain variables within each label to generate prime implicants and more compactly represent the solution space.

We wish for labels in the πTMS to contain prime implicants of associated theories. To enable this, we modify the semantics of a label. In an ATMS, no environment in a label may manifest any conflicts. The πTMS strengthens this, by requiring that each environment in a label resolve all known conflicts. The difference between these semantics lies in the supersets of an environment: the ATMS permits supersets of environments to manifest conflicts, but the πTMS does not. If some environment φ_e in an ATMS label has a superset φ'_e that manifests a conflict of the theory, then φ_e cannot be an implicant of the theory. In the πTMS, we therefore modify the label semantics to guarantee that that all environments in

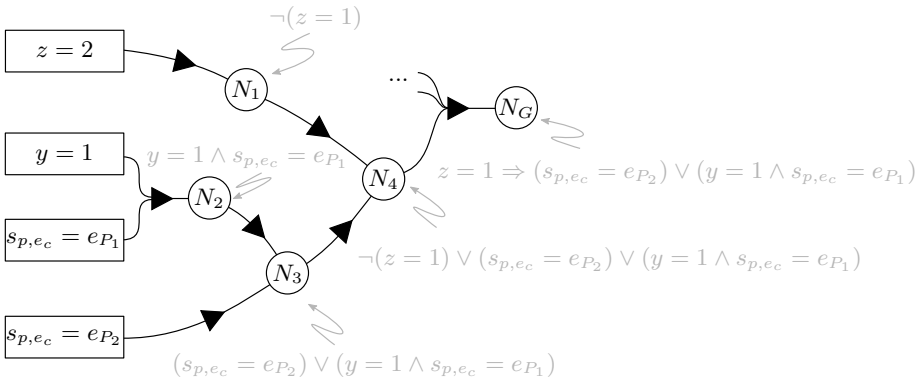


Figure 14: Hierarchical encoding of constraint in πTMS. As in (Forbus, 1993), Boxes represent assumptions, circles represent nodes, and arrows represent justifications with the multiple tails representing a conjunction. Gray annotations represent the constraints represented by the associated nodes.

a node's label must resolve all conflicts, thereby guaranteeing that all supersets also resolve all conflicts.

5.3 Encoding Constraints in the πTMS

In this section, we discuss how we take a theory $\mathcal{C}$ of constraints over finite-domain variables and hierarchically encode it into the πTMS. With our encoding, each constraint in $\mathcal{C}$ is represented by a node in the πTMS, such that the node is propositionally derivable iff the constraint holds. In the earlier example shown in Figure 13, recall that causal link extraction process derived the constraint $z = 1 \Rightarrow (s_{p,e_c} = e_{P_2}) \vee (s_{p,e_c} = e_{P_1} \wedge y = 1)$.

To encode this constraint in the πTMS, it is broken down hierarchically into a series of nodes (each representing a subformulae) where each holds if the corresponding constraint holds. The above implication is first converted to the equivalent disjunction $\Big(\neg(z = 1)\Big) \vee \Big((s_{p,e_c} = e_{P_2}) \vee (s_{p,e_c} = e_{P_1} \wedge y = 1)\Big)$, which can be represented by a node N_4 and the two disjuncts inside by nodes N_1 (representing $\neg(z = 1)$) and N_3 (representing $(s_{p,e_c} = e_{P_2}) \vee (s_{p,e_c} = e_{P_1} \wedge y = 1)$). We can then encode the disjunction via the two implications $N_1 \Rightarrow N_4$ and $N_3 \Rightarrow N_4$ as the only justifications in the πTMS for N_4. This guarantees that N_4 will be propositionally derivable iff either N_1 or N_3 holds, thus capturing the disjunction. We can use a similar technique to encode a conjunction. By justifying a node N_2 with the implication $y = 1 \wedge s_{p,e_c} = e_{P_1} \Rightarrow N_2$ as the only justification for N_2, we guarantee that N_2 will be propositionally derivable iff both $y = 1$ and $s_{p,e_c} = e_{P_1}$ hold – a conjunction. A negation of an assignment can be encoded by creating a disjunction of all other variable domain values. For instance, the N_1 node (which represents $\neg(z = 1)$) can here be justified with the single implication $z = 2 \Rightarrow N_1$, guaranteeing that N_1 will be derivable only if $\neg(z = 1)$ holds.

Following this line of reasoning, any constraint can be decomposed hierarchically into a series of additional nodes and justifications similar in spirit to the well-known Tseitin trans-

formation (Tseitin, 1968), in which additional variables represent the decomposed portions of constraints. πTMS nodes are propositionally derivable iff the associated decomposed constraints hold. We recursively traverse the constraints starting from its top-level conjunction, then following the logical order of operations and applying De Morgan's law as appropriate, to create more nodes hierarchically. The example constraint presented earlier would be encoded into πTMS via the justifications illustrated in Figure 14. The node N_G is the overarching "goal" representing our entire theory, in this case the single constraint we have extracted. In general, more constraints would be conjoined for N_G.

The one exception to this hierarchical decomposition rule applies to conflicts. When encoding a conflict, i.e. a negation of a conjunction of assignments, we instead make use of contradiction nodes (which represent False) originating from the ATMS. Specifically, we create a new contradiction node, and add a single justification with this contradiction as the consequent. The antecedents contain the assignments present in the conflict.

5.4 Incrementally Computing Prime Implicants via Consensuses

Here, we describe our approach to compactly computing prime implicants: consensuses.

Each node's label is expressed in disjunctive normal form (DNF) – i.e., it is a disjunction of environments, which are themselves conjunctions. The following Lemma can be used repeatedly to compute implicants of the DNF. The resulting implicants are smaller than the environments used to generate it, and will eventually result in computing prime implicants when used repeatedly.

Lemma 5.1 (Consensus for Finite Domain Variables)**.** *Suppose theory $\mathcal{C}$ can be expressed in DNF in the form $\left[\bigvee_{i=1\ldots N}(\varphi_i \wedge x = v_i)\right] \vee \ldots$ for some finite-domain variable x with domain $v_1, ..., v_N$. Then $\bigwedge_{i=1\ldots N}\varphi_i$, if self-consistent, is an implicant of $\mathcal{C}$; i.e.:*

$$\bigwedge_{i=1\ldots N}\varphi_i \models \bigvee_{i=1\ldots N}(\varphi_i \wedge x = v_i)$$

Proof. $\bigwedge_{i=1\ldots N}\varphi_i \models \varphi_j$ for all $j = 1 \ldots N$. Hence, each of the φ_i in the disjunction are be entailed. Since x must take some value from its domain, exactly one of the $(\varphi_i \wedge v_i)$ terms must hold, causing the disjunction to be satisfied. □

This result can be seen as the finite-domain generalization of the consensus theorem described in (Kean & Tsiknis, 1990) for boolean variables:

$$\varphi_1 \wedge \varphi_2 \models (\varphi_1 \wedge x) \vee (\varphi_2 \wedge \neg x)$$

Suppose we have a node N with label $L : \{\{x = 1, y = 1, z = 1\}, \{x = 2, y = 1, z = 1\}, \{x = 2, y = 3\}, \{x = 3\}\}$, where each variable x, y, and z has a domain of $\{1, 2, 3\}$. We can express L logically in DNF as

$$\begin{aligned}(x = 1 \wedge y = 1 \wedge z = 1) \vee (x = 2 \wedge y = 1 \wedge z = 1) \vee \\ (x = 2 \wedge y = 3) \vee (x = 3)\end{aligned}$$

Lemma 5.1 applies to this DNF for the variable x and its three domain values $1, 2, 3$, where $\varphi_1 : y = 1 \wedge z = 1$, $\varphi_2 : y = 1 \wedge z = 1$ (note that $\varphi_2 : y = 3$ could equivalently be chosen), and $\varphi_3 :$ True. Therefore, $\varphi_1 \wedge \varphi_2 \wedge \varphi_3$, namely $y = 1 \wedge z = 1$, is an implicant of the theory. This implicant may be added disjunctively to the label L as a new environment, as it will not change its semantics (i.e., the set of represented team scenarios will not change). By "minimally," we refer to the ATMS operation in which an environment φ_i is added to a list L of environments: any $\varphi_j \in L$ that entails φ_i is removed from L, and φ_i is added to L only if it is does not entail any other $\varphi_j \in L$. In our example, the first two terms in the DNF are entailed by the new implicant, so it is added minimally and those terms removed. The resulting label L may be more more compactly with just three environments as $\{\{y = 1, z = 1\}, \{x = 2, y = 3\}, \{x = 3\}\}$.

It is important to note that not all consensuses yield a useful implicant. For example, if we had chosen $\varphi_3 : y = 3$, the resulting consensus would have been $y = 1 \wedge y = 3 \wedge z = 1$. This is not self-consistent due to the conflicting variable assignment for y. While False is indeed an implicant of any theory, it does not provide any useful information in our case, so we do not consider it.

The above-illustrated approach is based on a generalized form of consensus. Since consensus is the dual of resolution, we can also view the above implicant generation in this different light. Hyper-resolution in boolean logic (Robinson, 1965) is a generalization of resolution for performing multiple resolutions in CNF in a single step. We discuss this alternate hyper-resolution view in Appendix D.

5.5 πTMS Label Propagation Algorithms

Now that we have illustrated both the hierarchical node structure of the πTMS as well as the intuition behind computing prime implicants from an existing label, we put the two together to describe the πTMS label propagation algorithms.

We begin by describing the semantics of each node's label. Intuitively, the label of a node contains a set of prime implicants for the theory containing that node's constraint, as described by Invariant 5.2 below:

Invariant 5.2 (Labels of πTMS Contain Prime Implicants)**.** *Let node N represent constraint C in theory $\mathcal{C}$. Then, the label of N contains a sound and complete set of prime implicants for a new theory $\mathcal{C}'$, which consists of the constraint C as well as all conflict constraints in $\mathcal{C}$.*

We illustrate this by continuing with the same example as before. Since there are no conflicts in the theory, the label of node N_4 will contain the complete set of prime implicants for the theory containing just the single constraint: $\{z = 2\}$, $\{s_{p,e_c} = e_{P_2}\}$, and $\{y = 1, s_{p,e_c} = e_{P_1}\}$. Any assignment of variables containing the assignment $z = 2$ will entail the causal link constraint being satisfied. Consider the node N_3, which represents the constraint $(s_{p,e_c} = e_{P_2}) \vee (y = 1 \wedge s_{p,e_c} = e_{P_1})$. By Invariant 5.2, the label for N_3 contains the prime implicants of a theory containing just this constraint. The computed label for N_3 is $\{s_{p,e_c} = e_{P_2}\}$, $\{y = 1, s_{p,e_c} = e_{P_1}\}$.

As another example, suppose that during online execution, Pike adds the conflict $\neg(s_{p,e_c} = e_{P_2})$. This could happen, for example, if the causal link is detected to be violated online.

In that case, the theory we wish to encode is

$$z = 1 \Rightarrow (s_{p,e_c} = e_{P_2}) \vee (s_{p,e_c} = e_{P_1} \wedge y = 1)$$
$$\neg(s_{p,e_c} = e_{P_2})$$

Note in this example that we have a conflict. All prime implicants must resolve this conflict. The computed label for N_G contains prime implicants $\{z = 2, s_{p,e_c} = e_{P_1}\}$, $\{z = 2, s_{p,e_c} = \bot\}$, and $\{y = 1, s_{p,e_c} = e_{P_1}\}$.

In the first example, it is no coincidence that the label of N_3 is a subset of the label of N_4. The ATMS label propagation algorithms – from which the πTMS derives – allow environments to flow through the constraint structure to compute the labels for other nodes hierarchically. The label of any assumption node of a πTMS contains a single environment containing the associated variable assignment. This is the "base case," as that variable assignment is an implicant of the theory containing just that assignment. We may then "build up" hierarchically. For a conjunction (shown visually in Figure 14 as a justification with two tails), we may take all possible combinations of implicants of the subtheories (from each label) and conjoin them to form a new implicant of the new theory – this is accomplished by the ATMS Weave operation (De Kleer, 1986a). For a disjunction, we may simply merge the implicants of the subtheories.

The πTMS label propagation algorithms differ from the ATMS procedures in that they compute prime implicants at every label propagation update – thus aiming to keep propagation to a minimum and find prime implicants early on during propagation. Upon adding a new environment to a label, all possible consensuses (which are implicants of the theory) are computed and added to the label per the above illustration. Note that this does not change the team scenarios represented by that label's prime implicants, but it can make the label more compact. Sometimes, new implicants can be computed based on the implicants that were just added – so this process repeats iteratively until no new implicants can be added to a label. The resulting label is usually more compact than that of the standard ATMS. We also keep track of the new implicants that were ultimately added and only propagate those implicants forward. This can have the effect of reducing unnecessary propagations. For more details, including pseudo code for the πTMS, please see Appendix D.

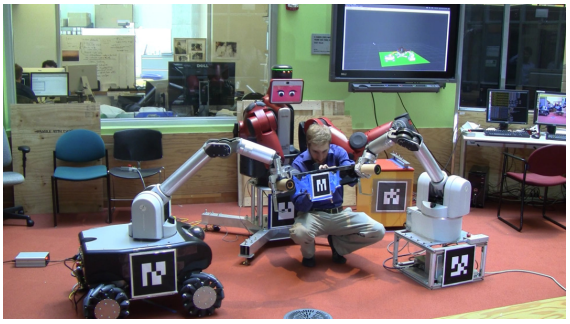
5.6 Using the πTMS for Online Queries

Here, we describe how the prime implicants computed by the πTMS are used to enable Pike's reactive online execution. Specifically, we describe how the queries over KB outlined in Section 2.3 are implemented, given the prime implicants of the goal node N_G.

- CorrectTeamPlanExists?(KB): Returns True iff the label of N_G contains at least one environment; else False if it is empty.

- CouldCommitToAdditionalConstraints?(KB, F): Here, F is a set of constraints. We take advantage of the fact that F contains only assignment and/or conflicts (and not, for example, arbitrary propositional sentences) – this is guaranteed via the constraints noted in Section 3. Without loss of generality, let φ_a denote an environment representing the conjunction of all assignments in F, and C denote the set of conflicts in F. To implement the procedure, we search for some prime implicant φ_i in node

(a) Kitchen example in hardware.



(b) Airplane manufacturing demo.

Figure 15: Various human-robot hardware demos of Pike. At left, the breakfast TPNU as described earlier in this document. At right, an airplane manufacturing scenario.

N_G's label where $\varphi_i \wedge \varphi_a$ does not manifest any of the new conflicts C. If there is such a φ_i, then there exists some scenario extending φ_i, consistent with φ_a, that satisfies all of KB's constraints in addition to F.

- CommitToConstraints(KB, F). Like the above, F will only contain conflict constraints or assignments. For each conflict, we encode the constraint by creating a new contradiction node, and adding a single justification to it corresponding to the conflict. During label propagation, all labels will be split on this conflict, guaranteeing that they resolve the conflict. For assignments $x = v_i$ in F, we encode a conflict $\neg(x = v_j)$ for each domain value $v_j \neq v_i$.

By following the above procedures, we can use the set of prime implicants in the label of N_G to answer the online queries required by Pike's online execution system. This enables a robust user interaction that can adapt to the human's intent as well as other disturbances.

6. Evaluation

In this section, we focus on evaluating two key claims of this work: (1) *concurrent* intent recognition and adaptation is advantageous over other approaches that do so *separately*, and (2) Pike achieves fluid human-robot teamwork through it's goal-directed execution.

We validate (1) above in simulation, comparing Pike against a competing approach based on Kirk (Kim et al., 2001) that recognizes intent and adapts via two separate processes. We validate (2) with two hardware demonstrations of human-robot teamwork. Finally, we provide additional performance measurements on randomly-structured problems to quantify scalability.

6.1 Validation in Robotic Testbed

We begin by discussing (2) above. Specifically, we show two example hardware demonstrations where Pike has successfully been applied to human-robot team collaboration.

Figure 15a shows the familiar household robotics breakfast domain described earlier in this work. A robotic arm is capable of picking up either coffee grounds or orange juice.

The human may pick up a coffee mug or a glass for juice. This demo behaves exactly as described earlier: if the human operator picks up the coffee mug, the robot will infer the human's intent to make coffee and react by fetching the nearby coffee grounds. Similarly, had the human instead picked up the glass, the robot would have adapted by handing the person orange juice.

A computer-vision based sensing system was implemented to act as the activity recognizer in this example. All objects were annotated with AR tags, allowing their 3D location to be measured. To recognize the human's low-level actions, motion was detected on either the mug or the glass. If for example the sensing system detected the mug being raised into the air by the human, then the choice $x_{A1} = \text{mug}$ was published to Pike.

Figure 15b shows a more complex example, in which a human and heterogeneous team of robots collaboratively construct part of an airplane wing in a manufacturing setting. In this demo, two robots (right and left) lift up some internal framing of an airplane wing, for the human and the red Baxter robot to collaboratively assemble together. A second piece, the "rib" must be attached to the frame using a temporary fastener called a cleco. Depending on the action taken next by the human, the Baxter robot adapts accordingly. If the human picks up pliers, Baxter will fetch a cleco fastener. If the human instead picks up a cleco, Baxter will pick up the pliers. If the human picks up both the plier and the cleco, the robot detects this and waits idly. After the first cleco is fastened, a similar process occurs for a second cleco. In this way, the robot is able to adapt to the human's inferred intent in a manufacturing setting.

6.2 Simulations

In addition to the hardware demonstrations above, we also validate Pike in simulation. A key goal of these simulations is to show empirically that Pike's approach, namely concurrently recognizing human intent and adapting, is advantageous over other approaches where intent recognition and adaptation occur as separate processes.

To do this, we compare Pike against an approach based on Kirk (Kim et al., 2001). As discussed earlier, Kirk takes as input a TPN and makes the choices optimally offline based on a cost function, in such a way that the resulting candidate subplan (whose underlying structure is a STN) is temporally consistent. This subplan can then be dispatched online with execution monitoring (Levine, 2012). We use Kirk as a competing approach that performs intent recognition and adaptation separately as follows. Before execution begins, an intent is "inferred" by guessing any intent scenario φ_I (i.e., any full assignment to uncontrollable variables) such there exists a corresponding adaptation scenario φ_A (i.e., a full assignment to controllable variables) where the candidate subplan $\varphi_I \wedge \varphi_A$ admits a correct execution. In other words, some intent "consistent" with at least one adaptation is chosen and assumed to be true, instead of maintaining multiple hypotheses for possible intents as Pike does. After (and separately from) this intent inference, the corresponding adaptation φ_A is chosen. The result is that Kirk has made all of the controllable and uncontrollable choices in the plan, resulting in a single candidate subplan being chosen. This candidate subplan subplan is then executed.

If there are multiple possible feasible intents, this approach may of course guess the intent incorrectly. If this occurs during execution and the Kirk executive determines that it

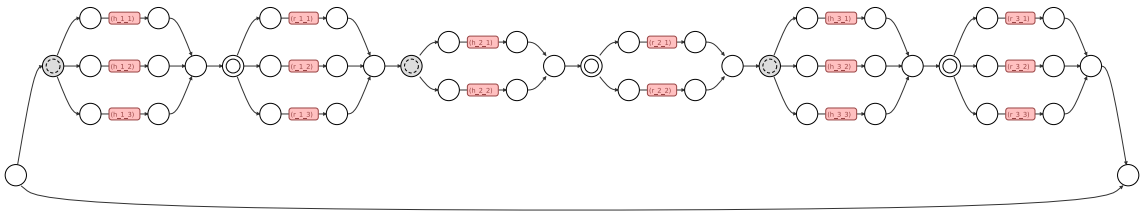


Figure 16: An example k-intents plan for $k = 18$ and structure $[3, 2, 3]$.

made an uncontrollable choice decision incorrectly, replanning is triggered. For the purpose of these tests, we compute the suffix of the existing TPN, backtracking the last incorrect uncontrollable choice and now making it correctly based on the now-observed uncontrollable choice. This approach does not work in general however (as it may not always be possible to backtrack previously made choices), so full generative planning is required in the general case. As generating new contingent plans is a computationally intensive and often time consuming operation, it is crucial for an online executive to minimize execution failures that trigger replanning. Having large delays during execution while the robot is replanning will result in a poor experience for the human working with the robot.

This approach using Kirk represents an incumbent that operates similarly to Pike, with a similar problem statement, model, and inputs / outputs, but performs intent recognition & adaptation separately. This approach also effectively maintains just a single hypothesis intent and adaptation at a time, instead of maintaining a set of candidate hypotheses as in Pike's concurrent approach.

To compare this version of Kirk with Pike in a controlled manner, we introduce an artificial domain called the k-intents domain, generate thousands of example problems from this domain with varying structure, and dispatch each using the incumbent version of Kirk while counting the number of execution failures that trigger replanning during each dispatch. A problem from the k-intents domain consists of a TPNU as well as an action model that imposes causal links. An example is shown in Figure 16. Each TPNU has a structure denoted by a set of positive integers $[N_1, N_2, ..., N_m]$, representing a sequence of choices between different activities. For each N_i, the TPNU contains a human-made uncontrollable choice $y_i \in \{1, 2, ..., N_i\}$ followed sequentially by a robot-made controllable choice $x_i \in \{1, 2, ..., N_i\}$. Each outcome $y_i = j$ activates a single activity for the human denoted h_{ij}, and similarly each outcome $x_i = j$ activates a single activity for the robot denoted r_{ij}. Our action model is such that h_{ij} has a single at-end add effect of predicate p_{ij}, and r_{ij} has a single at-start precondition of p_{ij}, thus forming a causal link from h_{ij} to r_{ij}. We call this a k-intents plan because it has k possible intent scenarios, where $k = \prod N_i$.

A k-intents problem has the property that for each intent scenario φ_I, there exists a single adaptation scenario φ_A that together form a candidate subplan that is correctly executable. This enables our second observation.

Observation (Pike requires no re-plans for k-intents)**.** *Given a k-intents problem, Pike will execute it and never trigger replanning.*

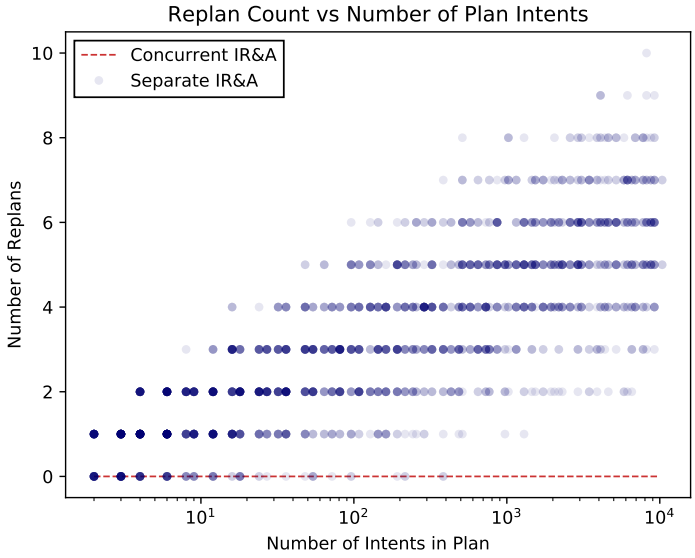


Figure 17: Comparison of concurrent intent recognition and adaptation (red line) versus separate intent recognition and adaptation (blue dots). The x-axis indicates the number of intents (k) in the plan. The y-axis indicates the total number of failures that occurred online due to selecting an incorrect intent, each of which requires replanning.

Proof. Pike compactly encodes the set of all correct team subplans in KB. Whenever an uncontrollable choice $y_i = j$ is observed, Pike is immediately able to infer from KB that the corresponding choice $x_i = j$ must be made. This results in a successful execution where Pike waits for uncontrollable choices to be observed, then makes the corresponding controllable choice correctly – never failing and triggering a replan. □

This nice property of Pike does not hold for Kirk in this context, however.

Observation (Kirk may need to replan for k-intents)**.** *Given a k-intents problem, Kirk may need to replan, possibly multiple times, during execution.*

Proof. In Kirk, intent recognition happens first, in which an intent φ_I is chosen. Then separately, an adaptation φ_A is chosen based on φ_I. However during execution, it very well be the case that the φ_I guessed at start does not match the actual intent chosen by the human, which will be revealed online one assignment at a time through a sequence of uncontrollable choice observations $\mathcal{U}(t)$. If this happens, for example if $\varphi_I \models (y_i = j)$ but Kirk observed that actually the human chose $y_i = j'$ for some $j \neq j'$, then execution will fail and replanning will occur. □

We experimentally validate these observations, the results of which are shown in Figure 17. We compare Kirk's approach of separate intent recognition & adaptation to Pike's concurrent approach that maintains multiple hypotheses. We generate over 2000 instances from the k-intents domain and execute them with Kirk, counting the number of replans.

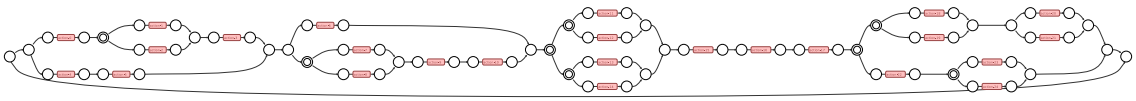


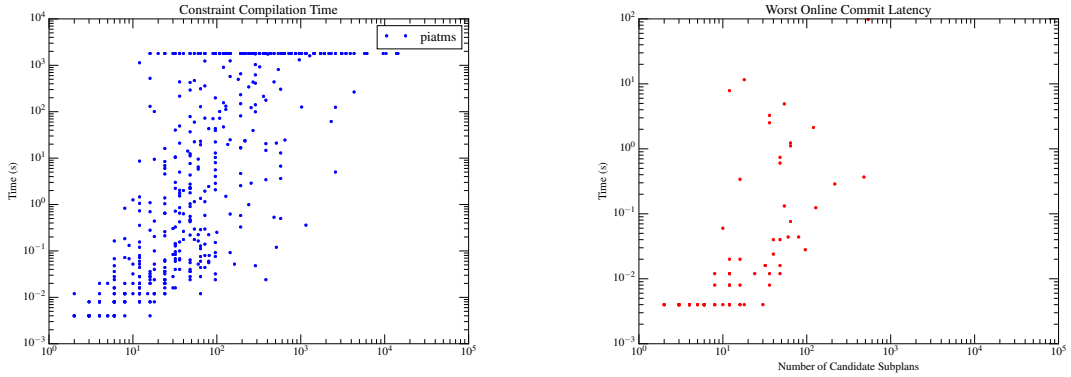
Figure 18: An example randomly-generated, structured TPNU with sequence, parallel, and choice structure. Not shown for brevity are the activity durations, nor the causal link structure imposed by the action model.

$N_i \in \{2, 3\}$ for all i in these tests. To generate a single problem, k is sampled logarithmically (so that the x-axis is uniform), and the closest factorization by 2's and 3's is computed. This factorization is shuffled, yielding a randomized structure for the k-intents problem. Then, Kirk is executed on this problem instance and the number of replans are counted.

Our experiments show that as the number of intents in the plan grows, the average number of replans also grows. As the number of intents is often exponential in the number of uncontrollable choices, this indicates that the number of replans tends to grow roughly linearly with the number of uncontrollable choices in the plan. In the case where generative replanning is costly and time consuming, these results show a notable advantage for approaches such as Pike that concurrently perform intent recognition & adaptation by simultaneously and compactly maintaining many hypotheses.

In addition to the above, we evaluate Pike's performance in terms of compilation speed, execution latency, and KB compactness. For this, we generate different random, structured problem instances. Each problem instance consists of a randomly-generated TPNU, as well as a randomly-generated action model to impose causal link structure. While the problem instances we test on are randomly-generated, we attempt to add structure to the problems to for improved realism. However, as with many artificially, randomized-generated domains, this setting is likely significantly more challenging to Pike than more engineered, real-world problems such as those demonstrated on robotic hardware.

We generate each TPNU as follows. We randomly sample a sequence of elements, where each element is either an activity, a choice, or parallel substructure. For choices and parallel substructure, we further sample more sequences of elements to build up the TPNU recursively. When sampling a sequence, the length is chosen from a binomial distribution whose parameters vary based on the nested depth of the sequence. For example, a top-level sequence of a TPNU may have a maximum of 10 elements, whereas a sequence being generated for a parallel or choice substructure nested one level deep may have a maximum length of 3 elements in our tests. Once the number of elements has been determined, we randomly choose which structure type the element will be: an action, a choice, or a parallel substructure. The probability mass function for this sampling also varies based on nested depth; our tests enforce that no more parallel substructure will be sampled at depths of 3 or more, thus limiting the recursion depth and size of the generated TPNUs. For action substructure, a random activity name is chosen, as are random integer-valued lower and upper bounds. For both parallel and choice substructure, we further sample more sequences of elements at the increased nesting level, corresponding to each "branch." Finally, after all of the substructure has been recursively constructed, the final step in TPNU generation imposes an overall temporal constraint that may "squeeze" the plan. This upper bound



(a) Constraint compilation time

(b) Maximum online commit latency.

Figure 19: Constraint compilation time for the πTMS, and the maximum latency measured during online execution. The x-axis is a rough measure of the complexity of the plan; it is the number of candidate subplans of the TPNU. The y-axis is time in seconds.

of this temporal constraint is currently chosen to be between 0 and 10% shorter than the minimum time required to execute the TPNU, thereby creating temporal conflicts and making certain candidate subplans of the TPNU temporally inconsistent. An example randomly-generated TPNU is shown in Figure 18.

Once the TPNU has been been constructed, we generate random causal link structure via an action model. We use PDDL 2.1 (Fox & Long, 2003) to specify the *at start* conditions and *at end* effects of activities in the TPNU to enforce causal link relations. Our approach works as follows. For each activity A_c in the TPNU, we randomly generate several *at start* conditions p_i for the activity (in our tests, $i \in [1, 4]$ and is chosen via a binomial distribution). For each of these new conditions added, we create (possibly multiple) supporting producer actions $A_{P_{ij}}$ that assert those conditions as effects (thereby creating labeled causal links). To do this, we compute the set of preceding actions by searching the TPNU structure for all actions that are guaranteed to come before our consumer action (we also include a special activity $A_{initial}$ representing the initial conditions). As a result, our tests will never require Pike to add temporal constraints to order a potential producer before its consumer. The producer candidates $A_{P_{ij}}$ will be chosen randomly from this set, and p_i will be added as an *at end* effect to each. If $A_{initial}$ is chosen, then p_i will be added as an initial condition to the PDDL problem. The number of labeled causal links is chosen from a binomial distribution (in our tests, $j \in [1, 4]$). Next, we randomly generate threat activities $A_{T_{ik}}$. Threats are randomly chosen from the set of actions that precede at least one of the producers $A_{P_{ij}}$. The number of threats is chosen from a binomial distribution, whose parameters limit the number to the number of causals links or preceding actions. For each threat $A_{T_{ik}}$, $\neg p_i$ is added as an *at end* effect. Finally, the resulting action model, as initial state, and goal state are written to PDDL domain and problem files.

We generated 500 such random TPNU's with associated action models, and ran Pike on each accordingly. Many of these problems admit no correct execution but still serve as a useful point of reference. Our benchmarking setup ran Pike with an upper time limit

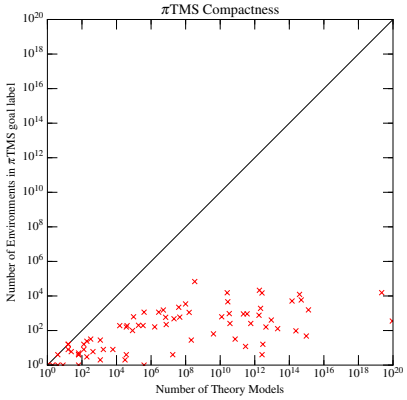


Figure 20: Compactness of the πTMS. The x-axis measures the number of models in the encoded theory as measured by the DSHARP model counter; the y-axis are the number of prime implicants computed by the πTMS. This ratio is generally much less than 1, indicating significant space savings.

of 30 minutes compilation time on each problem. Figures 19a and 19b show the results of running these simulated examples on Pike, using the πTMS to compile the constraints. On the left, we see the time required for Pike to compile the constraints and compute the set of prime implicants (this plot does not include the time to compute the labeled APSP).

As we can see, there is a large variance in compilation times, with a time-out band occurring at the top of the plot (corresponding to 30 minutes). As expected, compilation time increases with increasing number of candidate subplans. We note that for many domains of interest, including manufacturing and high-risk settings, spending a significant amount of time beforehand in a compilation process is an acceptable trade-off to improve online performance.

In Figure 19b, we see the worst case execution latency. Across each simulated execution, this is the worst-case time of the event execution loop in the OnlineExecution procedure shown in Algorithm 1 over the entire course of execution. We note that most calls to this function are actually much faster, due to work in caching solution results and the fact that queries for certain events tends to be much more demanding than for others. The results presented therefore represent a conservative, worst-case estimate for these randomly-generated problems. We see that latency times are generally reactive, with many worst-case decisions happening in a second or less. There are, however, certain cases for which significantly delays have been recorded.

A key goal of the πTMS is to encode the solution space efficiency. Figure 20 quantifies this, by plotting (only for problem instances that admit correct executions) the ratio of the πTMS goal label size to the number of solution models, as measured by the DSHARP model counter (Muise, McIlraith, Beck, & Hsu, 2010). As can be seen, the number of prime implicants never exceeds the number of models (despite this being theoretically possible), but rather in most cases is orders of magnitude less. This indicates that are attemps to compactly represent the entire solution space of KB are fruitful.

7. Conclusion & Future Work

This work introduces Pike, an executive for human-robot teamwork that views intent recognition and robot adaptation as two fundamentally interwoven problems. We argue that any autonomous system must address both of these problems to successfully work alongside humans. Pike uses a single set of algorithms and a single model to concurrently perform intent recognition and find suitable robot adaptations for contingent, temporally-flexible team plans. The result is a mixed-initiative execution in which the human and robot work together and influence one another.

We achieve this through the use of temporal reasoning and causal link analysis, which allows us to find relationships between possible choices in the team plan. This allows Pike to make controllable choices online that will be consistent with choices made by the human, thus adapting to intent. For this purpose, we introduce labeled causal links as well as related extraction algorithms, and show how these labeled causal links can be used to guide correct decisions online by generating additional temporal and propositional constraints. This allows the robot to quickly and effectively make choices online based on the outcomes of the human's decisions, the preconditions and effects of activities in the plan, temporal flexibility, and unanticipated disturbances.

There are a number of avenues for future work. One is to examine the effectiveness of other promising knowledge compilation techniques other than prime implicants, such as d-DNNF for example, to represent the set of candidate subplans admitting a correct execution (Muise et al., 2010). A second avenue that we are currently pursuing takes a probabilistic approach that models the distribution over likely human choices, so that the robot may adapt to likely human intents in a chance-constrained manner and better predict the probability of plan success.

Acknowledgments

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Appendix A. Proofs

Here we provide proofs of key theorems throughout this work. Note that proofs are presented in a different order than in the paper, for logical progression and clarity here.

Theorem 4.3 (Executions from $\mathcal{T}$ and $\mathcal{T}'$ correspond)**.** *An execution $\langle \varphi_S, T_{\varphi_S} \rangle$ of the original TPNU $\mathcal{T}$ is correct if and only if there exists a corresponding correct execution $\langle \varphi'_S, T_{\varphi_S} \rangle$ of the augmented TPNU $\mathcal{T}'$ where $\varphi'_S \models \varphi_S$.*

Proof. $\Rightarrow$. We may assume there exists a correct execution $\langle \varphi_S, T_{\varphi_S} \rangle$ of $\mathcal{T}$. It is possible to choose assignments to the additional s_{p,e_c} and o_{p,e_c,e_P,e_T} variables in $\mathcal{V}'$ not present in $\mathcal{V}$, appending those assignments to φ_S conjunctively to create a φ'_S where $\varphi'_S \models \varphi_S$. We use this to construct an execution with an identical schedule, $\langle \varphi'_S, T_{\varphi_S} \rangle$ of $\mathcal{T}'$. Since $\langle \varphi_S, T_{\varphi_S} \rangle$ is causally complete, every precondition of every event must be satisfied by the time of the event's execution. We also know that there can be no consumers that come before their supporting producers in T_{φ_S}, and no threats may occur between producers or consumers. We can therefore trace this support through properly selected assignments to the s_{p,e_c} and o_{p,e_c,e_P,e_T} variables. The additional guarded temporal constraints present in $\mathcal{T}'$ but not in $\mathcal{T}$ must be satisfied by T_{φ_S}, as the added temporal constraints only serve to preclude such causally incomplete executions. Therefore, the execution $\langle \varphi'_S, T_{\varphi_S} \rangle$ of $\mathcal{T}'$ must be temporally consistent. We now argue that it is also causally complete. Since $\mathcal{T}'$ and $\mathcal{T}$ share the same events $\mathcal{E}$ (each with preconditions and effects from the same activities $\mathcal{A}$), and the events occur at the same time in both executions by virtue of sharing the same schedule T_{φ_S}, the preconditions of all events would be satisfied in $\mathcal{T}'$ at their proper times – just the same as in $\mathcal{T}$. Therefore the execution $\langle \varphi_S, T_{\varphi_S} \rangle$ of $\mathcal{T}'$ is causally complete, and correct.

$\Leftarrow$. We may assume there exists a correct execution $\langle \varphi'_S, T_{\varphi_S} \rangle$ of $\mathcal{T}'$. Let φ_S be the projection of φ'_S (which assigns all the variables $\mathcal{V}'$ of $\mathcal{T}'$) onto the variables $\mathcal{V}$ of $\mathcal{T}$ – i.e., removing the additional s_{p,e_c} and o_{p,e_c,e_P,e_T} variable assignments. Similar to the above case, we know that both executions are causally complete since they share the same schedule, events, and activities with preconditions and effects. Furthermore, since $\mathcal{T}$ contains a subset of the temporal constraints from $\mathcal{T}'$, and since T_{φ_S} satisfies all of those constraints from $\mathcal{T}'$, it must also satisfy all of the temporal constraints of $\mathcal{T}$. The execution $\langle \varphi_S, T_{\varphi_S} \rangle$ is therefore temporally consistent. We have shown it is both temporally consistent and causally complete, so it is correct with respect to $\mathcal{T}$. □

Theorem 4.5 (Temporally consistent executions of $\mathcal{T}'$ with KB are also causally complete, and correct)**.** *Let $\langle \varphi'_S, T_{\varphi_S} \rangle$ be a temporally consistent execution of $\mathcal{T}'$ where φ'_S satisfies KB. Then this execution is also causally complete (and hence correct).*

Proof. We show that the additional propositional and temporal constraints added during Pike's compilation guarantee causal completeness. For φ'_S to be a solution of KB, it must satisfy all of KB's constraints and assign all variables, including the original variables, the s_{p,e_c} variables, and the o_{p,e_c,e_P,e_T} variables.

Let event e_c be any event activated by φ'_S with precondition p; for causal completeness, we show that p is expected to hold at the time e_c is scheduled by T_{φ_S}. Suppose without loss of generality that $s_{p,e_c} = e_P$[3]. By the additional temporal constraints added to the problem guarded in part by this assignment, we know that (1) e_P must also be activated by φ'_S, and (2) e_P must precede e_c in T_{φ_S} (else T_{φ_S} would be temporally inconsistent). Furthermore, any threat e_T cannot occur between e_P and e_c in T_{φ_S}, as required by the other temporal constraints added and possibly the o_{p,e_c,e_P,e_T} variables. Therefore, assuming no unmodeled disturbances, p will hold by the time e_c is scheduled in T_{φ_S}.

3. The reader may wonder why we exclude the possibility of $s_{p,e_c} = \perp$. Any such assignment however is guaranteed to not satisfy KB since e_c is activated, and hence φ_S would not even be represented by KB.

Thus since the execution is temporally consistent and causally complete, it is also correct. □

Theorem 4.6 (φ'_S satisfies KB iff correct)**.** *φ'_S satisfies KB if and only if there exists a correct execution $\langle \varphi'_S, T_{\varphi_S} \rangle$ of $\mathcal{T}$'.*

Proof. $\Rightarrow$. We show that if φ'_S satisfies KB, then there exists a correct execution $\langle \varphi'_S, T_{\varphi_S} \rangle$ of $\mathcal{T}$'. Since all temporal conflicts are encoded in KB, by Theorem 4.1, there exists a temporally consistent execution $\langle \varphi'_S, T_{\varphi_S} \rangle$ of $\mathcal{T}'$. By Theorem 4.5, this temporally consistent execution must also be correct.

$\Leftarrow$. We show that if there exists a correct execution $\langle \varphi'_S, T_{\varphi_S} \rangle$ of $\mathcal{T}'$, then φ'_S satisfies all of the constraints in KB. This execution must be both temporally feasible and causally complete. Since it is temporally feasible, T_{φ_S} satisfies all of the temporal constraints activated by φ'_S. So, φ'_S must satisfy all of the temporal conflict constraints derived by the labeled APSP present in KB. Next we consider the causal completeness constraints in KB. Since the execution is causally complete, we know that there must be some supporting producer e_P for each precondition p of every consumer event e_c, and any threats must be resolved. We may therefore assume that φ_S assigns $s_{p,e_c} = e_P$ and any associated o_{p,e_c,e_P,e_T} variables appropriately to satisfy the additional propositional constraints in KB. Since all constraints in KB are satisfied, then φ'_S satisfies KB. □

Theorem 4.7 (CanExecuteEventNow? is correct)**.** *Let $\langle \varphi_{ex}, \tilde{T}_{\varphi_{ex}} \rangle$ be the current, healthy execution trace of $\mathcal{T}'$. Then* CanExecuteEventNow?(e_i, t) *returns* True *at time t if and only if the execution trace of $\mathcal{T}'$ that would result if e_i is executed at time t – namely $\langle \varphi_{ex} \wedge \varphi_{e_i},\ \tilde{T}_{\varphi_{ex}} \cup \{e_i = t\} \rangle$ – is healthy.*

Proof. The CanExecuteEventNow? procedure begins by obtaining the set of additional constraints $F_1, ..., F_n$ required to execute event e_i at time t (one of these constraints is φ_{e_i}). It then returns whether KB $\wedge \Big(F_1 \wedge ... \wedge F_n\Big)$ is satisfiable. We prove that this conjunction is satisfiable if and only if $\langle \varphi_{ex} \wedge \varphi_{e_i},\ \tilde{T}_{\varphi_{ex}} \cup \{e_i = t\} \rangle$ is healthy.

$\Rightarrow$. We may assume that there exists some φ'_S satisfying KB $\wedge \Big(F_1 \wedge ... \wedge F_n\Big)$. φ'_S must therefore satisfy both KB and $F_1 \wedge ... \wedge F_n$. Since $F_1 \wedge ... \wedge F_n$ is satisfied by φ'_S, then by Theorem 3.1, the execution trace $\langle \varphi_{ex} \wedge \varphi_{e_i},\ \tilde{T}_{\varphi_{ex}} \cup \{e_i = t\} \rangle$ can be extended to a temporally consistent execution $\langle \varphi'_S, T_{\varphi_S} \rangle$ of $\mathcal{T}'$. Considering that KB is satisfied, then by Theorem 4.5, this temporally consistent execution must also be causally complete and hence correct. Therefore, the execution trace is healthy.

$\Leftarrow$. We may assume that $\langle \varphi_{ex} \wedge \varphi_{e_i},\ \tilde{T}_{\varphi_{ex}} \cup \{e_i = t\} \rangle$ is healthy, or namely that there exists an extending execution $\langle \varphi'_S, T_{\varphi_S} \rangle$ of $\mathcal{T}'$ that is correct. By Theorem 4.6, φ'_S satisfies KB. Since this execution is temporally consistent, by Theorem 3.1, φ'_S satisfies $F_1 \wedge ... \wedge F_n$. Therefore, the assignment φ'_S satisfies KB $\wedge \Big(F_1 \wedge ... \wedge F_n\Big)$. □

Theorem 4.4 (Healthy Executions of Original $\mathcal{T}$)**.** *Let $\langle \varphi_{ex}, \tilde{T}_{\varphi_{ex}} \rangle$ be an execution trace at some point during execution of the augmented TPNU $\mathcal{T}'$. This execution trace is healthy with respect to the augmented TPNU $\mathcal{T}'$ if and only if it is also healthy with respect to the original TPNU $\mathcal{T}$.*

Proof. The execution trace $\langle \varphi_{ex}, \tilde{T}_{\varphi_{ex}} \rangle$ is healthy with respect to $\mathcal{T}'$ iff it can be extended to a correct execution $\langle \varphi'_S, T_{\varphi_S} \rangle$ of $\mathcal{T}'$. By Theorem 4.3, this correct execution of $\mathcal{T}'$ exists iff $\langle \varphi_S, T_{\varphi_S} \rangle$ is a correct execution of $\mathcal{T}'$, where $\varphi'_S \models \varphi_S$. This correct execution of $\mathcal{T}$ must be an extension of $\langle \varphi_{ex}, \tilde{T}_{\varphi_{ex}} \rangle$. Since $\langle \varphi_S, T_{\varphi_S} \rangle$ is an extended and correct execution of $\mathcal{T}$ iff $\langle \varphi_{ex}, \tilde{T}_{\varphi_{ex}} \rangle$ is healthy with respect to $\mathcal{T}$, we have proven the claim. □

Theorem 4.2 (Dominated Causal Links are Irrelevant)**.** *Dominated labeled causal links do not influence the correctness of an execution.*

Proof. Suppose we have a TPNU $\mathcal{T}$ containing consumer event e_c with precondition p. Further suppose there is a dominating labeled causal link from producer $e_{P_{dom}}$, and a dominated labeled causal link from producer e_P. As a thought experiment, we introduce a modified version of $\mathcal{T}$, which we call $\mathcal{T}_{rem}$, that is exactly the same as $\mathcal{T}$ except that p has been removed from the effects of e_P (and thus the dominated labeled causal link has disappeared). We will prove that if some execution $\langle \varphi_S, T_{\varphi_S} \rangle$ is correct with respect to $\mathcal{T}$, then it is also correct with respect to $\mathcal{T}_{rem}$ – thus demonstrating the irrelevance of the dominated labeled link with respect to correctness, and highlighting that it can be safely removed from consideration.

Proof by contradiction. We assume that execution $\langle \varphi_S, T_{\varphi_S} \rangle$, even though correct with respect to $\mathcal{T}$, is incorrect with respect to $\mathcal{T}_{rem}$. There can be only one way for this to happen: if the precondition p of e_c is no longer satisfied in the execution of $\mathcal{T}_{rem}$ due to p having been removed as an effect from e_P. If this is the case, then e_P must be the supporting producer of e_c in the execution of $\mathcal{T}$ – i.e., e_c must be activated, e_P must be activated, e_P must be the latest-occurring producer before e_c, and no threat event e_T is executed during the time between e_P and e_c. If e_P were not the supporting producer in the execution of $\mathcal{T}$, then removing p from the effects of e_P would have no bearing on the execution correctness since there would be some other supporting producer for e_c or e_c would not be activated.

As e_P is the supporting producer for e_c in the execution of $\mathcal{T}$, e_P must be activated. By the definition of labeled causal link dominance, $\varphi_{e_P} \models \varphi_{e_{P_{dom}}}$, so $e_{P_{dom}}$ must also be activated. Additionally by the definition of labeled causal link dominance, $e_P \prec e_{P_{dom}}|_{\varphi_{e_c}}$ and $e_{P_{dom}} \prec e_c|_{\varphi_{e_P}}$. Therefore $T_{\varphi_S}(e_P) < T_{\varphi_S}(e_{P_{dom}}) < T_{\varphi_S}(e_c)$. No activated threats may be scheduled in the range between $e_{P_{dom}}$ and e_c, since they are guaranteed to already be scheduled outside the strictly larger range from e_P to e_c. We have therefore shown that the producer $e_{P_{dom}}$ is activated, is a later-occurring producer than e_P, and isn't threatened. This contradicts our earlier conclusion that e_P must have been the supporting producer. Therefore, our initial assumption must be incorrect, and so the execution $\langle \varphi_S, T_{\varphi_S} \rangle$ must be correct with respect to $\mathcal{T}_{rem}$. □

Appendix B. Labeled Value Set Algorithms

The procedure AddLVS(p, L) is shown in Algorithm 6, and $Q_L(\varphi)$ is shown in Algorithm 7.

Appendix C. Extensions to the Labeled APSP

In this appendix, we describe a modified version of the labeled all-pairs shortest path algorithm introduced in Drake (Conrad & Williams, 2011). This modification is designed

Algorithm 6: AddLVS(p, L)

Input: A new labeled value pair $p = (a_N, \varphi_N)$, and an LVS L.
Output: A boolean, True if p was added to L, and False if not.

If any $p_i \in L$ dominates p, **return** False
Remove any p_i from L where p dominates p_i
Add p_i to L
return True

Algorithm 7: QueryLVS(L, φ), denoted $Q_L(\varphi)$

Input: An LVS L with relation $<_R$, and an environment φ
Output: The tightest constraint value guaranteed to hold over all scenarios in $\mathcal{S}(\varphi)$

$P \leftarrow$ all $(a_i, \varphi_i) \in L$ where $\varphi \models \varphi_i$
if $P \neq \{\}$ **then**
  return $\min_R \left\{ a_i \mid (a_i, \varphi_i) \in P \right\}$
else
  return ∞_R
end

to improve the quality of querying the individual table entries, and relies on computing new labeled values logically implied by others. We first introduce those new labeled values below, and then proceed to describe our modified labeled APSP algorithm.

C.1 Resolutions for LVS's

We now present a novel extension to the original LVS that improves compactness and the tightness of the query operator for finite-domain variables through a form of resolution.

Consider an example LVS in which we have a single discrete-domain variable $x \in \{1, 2\}$, and the constraint on t with the following LVS L:

$$t < \{(3, \{x = 1\}), (4, \{x = 2\})\}$$

What is the value $Q_L(\{\})$, the tightest constraint on t over all environments? Since $\{\}$ entails neither $\{x = 1\}$ nor $\{x = 2\}$, neither labeled value in the LVS apply, and hence the LVS cannot guarantee any bound. $Q_L(\{\}) = \infty$, so we are left with the loosest possible constraint $t < \infty$.

We can improve the tightness of the returned constraint, however, if we consider the discrete domain of the variable x. We add *completions* to the LVS, which intuitively are new labeled value sets logically implied by others in the LVS. These completions allow the query operation to return tighter values.

Returning to the above example, the LVS actually can guarantee the bound $t < 4$, which is of course much tighter than $t < \infty$. This is because x will be either 1 or 2 in any scenario, so one of the two labeled values must apply. We can thus be sure that the "loosest" possible value (here, 4) will hold for the constraint. So, given the domain of x, this LVS could equivalently be represented as $\{(3, \{x = 1\}), (4, \{\})\}$.

Here is another example, Suppose we have

$$t < \{(-1, \{x = 1, y = 2\}), (-2, \{x = 2, y = 2\}), (\infty, \{\})\}$$

where $x, y \in \{1, 2\}$. We are interested in the query $Q_L(\{y = 2\})$. The query operation on this LVS would result in $t < \infty$ as the tightest constraint, but again, we can do better. Noting that if $y = 2$ then either one of $\{x = 1, y = 2\}$ or $\{x = 2, y = 2\}$ must hold, we can guarantee the looser of these labeled values, namely $t < -1$.

In general, when computing $Q_L(\varphi)$ and L contains a set of labeled values whose environments partition $\mathcal{S}(\varphi)$, then at least one of these labeled values must hold true. We may therefore take the loosest constraint value among these labeled values.

Theorem C.1. *Suppose we have a discrete variable $x \in \{v_1, v_2, \ldots, v_k\}$, and an LVS over t with relation $<_R$ of the form*

$$t <_R \{(a_1, \varphi_1 \wedge \{x = v_1\}), (a_2, \varphi_2 \wedge \{x = v_2\}), \ldots, (a_k, \varphi_k \wedge \{x = v_k\}), \ldots\}$$

where $\varphi_1, \ldots, \varphi_k$ are arbitrary environments such that

$$\varphi_M = \bigwedge_{i=1}^{k} \varphi_i \neq \bot$$

Then, we can guarantee that $\varphi_M \Rightarrow t \leq_R \max_R \left\{a_1, a_2, \ldots, a_k\right\}$.

Proof. Since $\varphi_M = \varphi_1 \wedge \varphi_2 \wedge \ldots \wedge \varphi_k$, we know that $\varphi_M \models \varphi_i$ for all $i = 1 \ldots k$. Thus, if φ_M holds, then each of the φ_i must hold. We also know that in any scenario, precisely one of the $x = v_j$ constraints must hold. Therefore, exactly one of the $\varphi_i \wedge \{x = v_i\}$ environments must hold, if φ_M holds. In other words, $\varphi_M \models \varphi_i \wedge \{x = v_i\}$ for exactly one i. We do not know which i however, and hence which labeled value applies, but we know that exactly one does. As such, when φ_M holds, we conservatively take the loosest possible constraint among the i labeled values, namely $t \leq_R \max_R \left\{a_1, a_2, \ldots, a_k\right\}$. □

Building upon this theorem, we devise the following inference rule to augment an LVS with additional implied labeled values:

$$\frac{\begin{array}{c}(a_1, \varphi_1 \cup \{x = v_1\}) \\ (a_2, \varphi_2 \cup \{x = v_2\}) \\ \ldots \\ (a_k, \varphi_k \cup \{x = v_k\})\end{array}}{\left(\max_{i\ R} \{a_i\},\ \bigwedge_i \varphi_i\right)}$$

where $x \in \{v_1, ..., v_k\}$ and $\bigwedge_i \varphi_i \neq \bot$.

This inference rule for LVS's is analogous to hyper-resolution in boolean logic. Since the new labeled value is logically implied by the other labeled values (i.e., an implicant), it

Algorithm 8: FindLVSCompletions(L, x)

Input: An LVS L, and a variable x over which to find completions
Output: An LVS C of completions, suggesting pairs to add.

```
1  W ← {(−∞_R, {})}
2  foreach assignment x = v_i in x's domain do
3  |  Y = {}
4  |  foreach (a_L, φ_L) ∈ L do
5  |  |  if φ_L assigns x = v_i then
6  |  |  |  φ_R = φ_L \ {x = v_i}
7  |  |  |  AddLVS((a_L, φ_R), Y)
8  |  |  end
9  |  end
10 |  W ← LVSBinaryOp(max_R(·,·), Y, W)
11 end
12 return W
```

can safely be added to L without changing correctness while improving the tightness of the query operator.

The pseudo code for an algorithm to find all such completions, FindLVSCompletions, is outlined in Algorithm 8. This algorithm finds completions over the variable x for LVS L. It maintains an LVS W, which is incrementally grown to contain all completions that could be added to L. W starts containing only the weakest possible completion, $(\infty_R, \{\})$. W is then updated to contain completions for each possible variable assignment $x = v_i$ in x's domain. For each assignment $x = v_i$, a temporary LVS Y is produced that contains all pairs consistent with that assignment but stripping off $x = v_i$ from the label of the value. This happens in Lines 3 – 9. Line 10 incrementally computes multiple possible φ_M, along with the corresponding maxima by employing a binary operation between W and Y to update W.

C.2 Labeled APSP

We present a modified version of the original labeled APSP algorithm extended to generate LVS completions as described earlier. For the original version, please see Drake (Conrad, 2010). Pseudo code for the labeled APSP is shown in Algorithm 9. It takes in a TPNU with events $\mathcal{E}$ and temporal constraints $\mathcal{C}$, performs the above transformation to a labeled distance graph implicitly, and then runs a generalized version of Floyd Warshall. The algorithm begins in Lines 1 – 10 by initializing a table D with a new LVS for each pair of events. The shortest distances from every event to itself is $\{(0, \{\})\}$. Other off-diagonal entry weights are added corresponding to each episode, and others are all set to $\{(\infty, \{\})\}$.

Lines 11 – 18 provide the "triple for loops" that distinguish the Floyd Warshall algorithm. The key difference between our labeled APSP algorithm and the original presented in Drake is shown in Line 15, which calls a new method MergeWithCompletions(D_{ij}, C_{ij}), instead of the original Merge method. This updated method computes completions for the

Algorithm 9: LabeledTightFloydWarshall

Input: A TPNU $\langle \mathcal{V}, \mathcal{E}, \mathcal{C}, \mathcal{A} \rangle$
Output: D, a table of shortest path distances between pairs of events in $\mathcal{E}$ (each entry an LVS)

```
1  foreach i, j ∈ E do
2  |  D_ij ← {(∞, {})}
3  end
4  foreach i ∈ E do
5  |  D_ii ← {(0, {})}
6  end
7  foreach ⟨i, j, l, u, φ⟩ ∈ C do
8  |  D_ij ← {(u, φ)}
9  |  D_ji ← {(−l, φ)}
10 end
11 foreach k ∈ E do
12 |  foreach i ∈ E do
13 |  |  foreach j ∈ E do
14 |  |  |  C_ij = LVSBinaryOp(+, D_ik, D_kj)
15 |  |  |  D_ij ← MergeWithCompletions(D_ij, C_ij)
16 |  |  end
17 |  end
18 end
19 return D
```

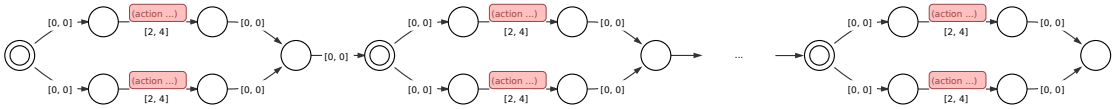

Figure 21: A TPNU with N choices and completely symmetric temporal constraints. Without completions, the LVS from the last event to the first event would in general contain 2^N labeled values. With completions, it would contain just one.

labeled value set, helping to ensure that queries performed upon it will return the tightest possible values.

An example where our modified labeled APSP is beneficial is shown in Figure 21. In this example, a TPNU is shown with N sequential and identical sets of choices of two actions, each with temporal bounds of $[2, 4]$. Without the MergeWithCompletions modification above, the LVS shortest path from the first event to the last event of the plan would contain 2^N entries in it; one for each possible outcome to all of the choices. However, with our addition, this is reduced to just one.

Pseudo code for MergeWithCompletions is shown in Algorithm 10. It takes as input an LVS A, and another LVS L that is to be added to L. The algorithm first adds each labeled value in L to A, maintaining dominance via AddLVS. The algorithm also

Algorithm 10: MergeWithCompletions(A, L)

Input: An LVS A, and another LVS L that should be merged into A
Output: Returns updated A

```
1  Q ← {}
2  foreach pair (a_l, φ_l) in L do
3  |  added? = AddLVS(A, (a_l, φ_l))
4  |  if added? then
5  |  |  Push each variable listed in φ_l to Q if not already present
6  |  end
7  end
8  while Q is not empty do
9  |  x ← pop variable from Q
10 |  C ← FindLVSCompletions(A, x)
11 |  foreach pair (a_c, φ_c) in C do
12 |  |  added? = AddLVS(A, (a_c, φ_c))
13 |  |  if added? then
14 |  |  |  Push each variable listed in φ_c to Q if not already present
15 |  |  end
16 |  end
17 end
18 return A
```

maintains a queue Q over which completions may be found. This queue contains a list of all the variables referenced by any labeled value added to A. The second phase of the algorithm, beginning on Line 8, repeatedly pops variables off of Q and calls the FindLVSCompletions algorithm to find new implied labeled values for L given the current variables that were added. Each newly generated completion is added minimally back to A, and more variables are possibly pushed onto Q if more resolutions could be possible. Finally, the modified A is returned.

While operating with completions can in some cases greatly improve the performance of the labeled APSP algorithm, such as in the example in Figure 21, this is not always the case. We have found that the performance in general is greatly dependent on the numerics of the temporal constraints, and on any encoded "symmetry" across the temporal constraints. For example, if the $[2, 4]$ temporal constraints in Figure 21 were randomized by adding Gaussian noise to them, the performance would drastically decrease, due to the fact that the shortest possible path between different events would now be a much more complex function of what choices are made.

Appendix D. πTMS Label Propagation System

This appendix includes additional information about πTMS label propagation algorithms used by Pike to incrementally generate a sound and complete set of prime implicants for use online.

D.1 Hyper-resolution as Dual of Consensuses

The illustrated approach is based on a generalized form of consensus. Yet since consensus is the dual of resolution, we can also view the above implicant generation in a different light. Hyper-resolution in boolean logic (Robinson, 1965) is a generalization of resolution for performing multiple resolutions in CNF in a single step, re-stated below (in its negative form):

$$\frac{\begin{array}{c} C_1 \vee \neg x_1 \\ C_2 \vee \neg x_2 \\ \ldots \\ C_n \vee \neg x_n \\ x_1 \vee x_2 \vee \ldots \vee x_n \vee D \end{array}}{\bigvee_i^n C_i \vee D}$$

Each x_i is positive literal, and each C_i is a disjunctive clause of literals. The derived conclusion $\bigvee_i^n C_i \vee D$ is the *hyper-resolvent* and is an implicate (as opposed to an implicant in consensus), the disjunction $x_1 \vee x_2 \vee \ldots \vee x_n \vee D$ is the *nucleus*, and each clause $C_i \vee \neg x_i$ is a *satellite*.

To view consensus as the dual of hyper-resolution, we exploit the fact that computing *implicants* (conjunctive) of a theory $\mathcal{C}$ can be accomplished by negating *implicates* (disjunctive) of the theory's negation $\neg\mathcal{C}$ (Elliott, 2004). Continuing with the same example, we can view the implicant $y = 1 \wedge z = 1$ through the lens of hyper-resolution by computing its associated implicate of $\neg\mathcal{C}$. As $\mathcal{C}$ is expressed in DNF, we can easily express $\neg\mathcal{C}$ in CNF by negating each literal and swapping $\wedge$ and $\vee$:

$$\begin{aligned} &(\neg(x = 1) \vee \neg(y = 1) \vee \neg(z = 1)) \wedge \\ &(\neg(x = 2) \vee \neg(y = 1) \vee \neg(z = 1)) \wedge \\ &(\neg(x = 2) \vee \neg(y = 3)) \wedge \\ &(\neg(x = 3)) \end{aligned}$$

Since we operate over finite-domain variables, each of which must take a domain value, we can add in the extra clause on the last line below and perform hyper-resolution:

$$\frac{\begin{array}{r} (\neg(x = 1) \vee \neg(y = 1) \vee \neg(z = 1)) \wedge \\ (\neg(x = 2) \vee \neg(y = 1) \vee \neg(z = 1)) \wedge \\ (\neg(x = 2) \vee \neg(y = 3)) \wedge \\ (\neg(x = 3)) \wedge \\ (x = 1 \vee x = 2 \vee x = 3) \end{array}}{\neg(y = 1) \vee \neg(z = 1)}$$

The first, second, and fourth clauses are the satellites, the nucleus is the final disjunction $x = 1 \vee x = 2 \vee x = 3$, and the hyper-resolvent is $\neg(y = 1) \vee \neg(z = 1)$, which is an implicate of $\neg\mathcal{C}$. Therefore, its negation $y = 1 \wedge z = 1$ is an implicant of $\mathcal{C}$. Effectively, we are taking

advantage of the fact that each variable must take a value from its domain, allowing us to perform hyper-resolution over this disjunction.

Previous work has examined the problem of incorporating hyper-resolution into the ATMS, and argued the complete case intractable (De Kleer, 1986b). However, we argue that our approach in this work, which is incremental and seeks to minimize the number of resolutions performed through efficient propagation, is computationally feasible. Experimental results show that the πTMS is tractable for many of Pike's tested problem instances.

D.2 πTMS Algorithms

Here, we introduce the πTMS label propagation algorithms that modify those of the ATMS.

Algorithm 11: Cross$(L_1, L_2, ...)$, denoted by $L_1 \times L_2 \times ...$

Input: A list of labels $\mathcal{L} = [L_1, L_2, ...L_n]$ (each L_i is a set of environments)
Output: A new label C containing the cross product of all L_i

```
1  Sort labels L
2  C ← {{}}
3  foreach L_i ∈ L do
4  |   C' → {}
5  |   foreach φ_C ∈ C do
6  |   |   foreach φ_L ∈ L_i do
7  |   |   |   Let φ_C' = φ_C ∧ φ_L
8  |   |   |   Add φ_C' to C' minimally if φ_C' is self-consistent
9  |   |   end
10 |   end
11 |   C ← C'
12 end
13 return C
```

The FindCandidateSatellites procedure, outlined in Algorithm 12, finds all environments in a label that contains a single assignment $x = v_i$, and then strips off that assignment. It returns a a list $L_{sat}^{x=v_i}$ of said environments, which are used in constructing the resolutions / consensuses in the next step. Taking the same example as earlier where $L = \{\{x = 1, y = 1, z = 1\}, \{x = 2, y = 1, z = 1\}, \{x = 2, y = 3\}, \{x = 3\}\}$, the result of calling FindCandidateSatellites$(L, x = 1)$ would be $\{\{y = 1, z = 1\}\}$, corresponding to φ_1 described earlier. Calling FindCandidateSatellites$(L, x = 2)$ returns $\{\{y = 1, z = 1\}, \{y = 3\}\}$, which are both of the possible φ_2 values described earlier.

The actual work of computing the implicants occurs in FindConsensuses, outlined in Algorithm 13. Given a variable x, the algorithm calls FindCandidateSatellites$(L, x = v_i)$ for each domain value v_i of x. One environment from each must be chosen, and the result joined conjunctively together if consistent. This is accomplished via the cross product operation $\times$, the pseudo code of which is listed in Algorithm 11. In a nutshell, the $\times$ operation finds the cross product of all possible environments incrementally, prunes inconsistent combinations early before they are fully constructed, and attempts to maintain compactness by only keeping track of minimal, non-entailing environments. The algorithm

Algorithm 12: FindCandidateSatellites$(L, x = v_i)$, denoted by $L_{sat}^{x=v_i}$

Input: Computes $L_{sat}^{x=v_i}$, a set of candidate satellites
Output: $L_{sat}^{x=v_i}$
1 $L_{sat}^{x=v_i} = \{\}$
2 **foreach** $\varphi_i \in L$ *containing* $x = v_i$ **do**
3 | Add $\varphi_i \setminus \{x = v_i\}$ to $L_{sat}^{x=v_i}$ minimally
4 **end**
5 **return** $L_{sat}^{x=v_i}$

Algorithm 13: FindConsensuses(L, x)

Input: A label L, and a variable x over which to find consensuses
Output: A list of consensuses that could be added to L
1 Compute $L_{sat}^{x=v_i}$ for each $v_i \in$ Domain(x)
2 **return** $L_{sat}^{x=v_1} \times L_{sat}^{x=v_2} \times ... \times L_{sat}^{x=v_k}$

is similar to the ATMS Weave operation (Forbus, 1993). Please note that this can, in the worst case, result in a combinatorial explosion of consensuses. Each of the results of the cross product is an implicant of the DNF formula represented by L. Some (but not all) could be prime implicants, and others could entail other terms in L.

Now that we have described how consensuses can be computed for a single variable, we describe how these algorithms are integrated into the overall label propagation algorithm, forming the πTMS. The πTMS borrows the ATMS Propagate and Weave algorithms unchanged, and changes the Update method to the new πTMSUpdate, which incorporates prime implicant generation (Forbus, 1993). Intuitively, Update is called when a new environment is added to a node's label. This in turn triggers propagation to other ATMS nodes via justifications, and the Weave method is called to combine environments conjunctively for this propagation, similar to the $\times$ operator.

Instead of solely adding the new environments to a label minimally as the original Update method does, πTMSUpdate computes prime implicants via the above consensus procedures given the new environments, possibly resulting in new, smaller environments being produced that are entailed by the ones being originally added. Only these entailed environments are added and propagated forward, not the entailing ones – thus reducing unnecessary computation.

Pseudo code for πTMSUpdate(a, L) is shown in Algorithm 14. There are two cases: (1) if node a is a contradiction node, and (2) if it is not. If a is not a contradiction node, then we add new environments to a's label, and also compute new prime implicants for the theory rooted at a by calling AddAndGeneratePrimeImplicants. These new prime implicants can also be added to a, but may entail other environments originally in L. The procedure returns a list L_{added} of environments that were ultimately added to L_a (including new implicants derived) to be propagated forwarded in the similar spirit as the ATMS. Otherwise in case (2) if a is a contradiction node, then all of the environments in L are conflicts. In this case, we split all labels in the πTMS, by ensuring that each one resolves the conflicts. This can be accomplished in a manner similar to (Williams & Ragno, 2007). To

Algorithm 14: πTMSUpdate(a, L)

Input: A node a with associated label L_a, and a list of new environments L to be added to a's label

```
if a is a contradiction node then
    Add each e ∈ L minimally to nogoods
    foreach φ_N actually added to nogoods above do
        SplitAllLabelsOnConflict(φ_N)
    end
else
    L_added = AddAndGeneratePrimeImplicants(L_a, L)
    foreach justification J where a is an antecedent do
        Propagate(J, a, L_added)
        [Optional: Split L_added on any newly-added nogoods]
    end
end
```

split an environment in a label on a conflict, we do nothing if that environment resolves the conflict. If the environment manifests the conflict, we remove it from the label. Otherwise, we create a new series of environments, each containing additional assignments that conflict with conflict. Each of these new environments resolves the conflict. Note that this can at times result in a significant increase in the number of environments in a label.

The AddAndGeneratePrimeImplicants method is shown in Algorithm 15. Its responsibility is to add the environments in L_+ to the label L, generate any new prime implicants of L that now result, add those to L, and return a minimal set of environments L_{added} that have been added to L. Note that L_{added} may contain both environments from L_+ and newly-generated prime implicants.

The AddAndGeneratePrimeImplicants maintains a queue Q of variables, over which to find consensuses. Whenever a new environment φ_i is added to L, all of the variables referenced in φ_i are pushed onto Q (if they're not on there already). This is because it may now be possible to compute new consensuses over those variables. After each $\varphi_i \in L_+$ is added to L minimally and Q is populated, then the routine ConsensusUntilFixedPoint is called with the Q, which continually computes new consensuses until no new ones can be generated and returns the resulting additional environments.

The ConsensusUntilFixedPoint is shown in Algorithm 16. It takes as input a label L as well as a queue Q of variables, and calls the FindConsensuses procedure repeatedly on each variable in the Q to compute new consensuses, each of which is a new implicant that can be added to L. If some newly-computed implicant φ_i is added minimally to L, then its variables are pushed onto Q if not present already. The process is repeated until the queue is empty, meaning that no more consensuses can be computed. This "fixed point" means that L is now a set of prime implicants, as no new, smaller implicants can be derived.

Algorithm 15: AddAndGeneratePrimeImplicants(L, L_+)

Input: A label L, and a label L_+ to be added to L.
Output: Adds the environments from L_+ to L, along with any new consensuses. Returns a label containing all environments actually added to L.

$Q \leftarrow \{\}$
$L_{added} \leftarrow \{\}$
Add each $\varphi_i \in L_+$ minimally to L
foreach φ_i *actually added to* L *above* **do**
    Add φ_i to L_{added} minimally
    Add any variables mentioned in φ_i to Q if not already present
end
$L_{consensuses}$ = ConsensusUntilFixedPoint(L, Q)
Add each $\varphi_i \in L_{consensuses}$ minimally to L_{added}
return L_{added}

Algorithm 16: ConsensusUntilFixedPoint(L, Q)

Input: A label L, and an initial queue of variables Q to find consensuses over.
Output: Keeps finding consensuses / prime implicants for L until a fixed point is reached and no more can be computed. Returns a list of prime implicants added to L in doing so.

$L_{added} \leftarrow \{\}$
while $|Q| > 0$ **do**
    $x \leftarrow$ pop variable from Q
    $L_{consensuses} \leftarrow$ FindConsensuses(L, x)
    Add each $\varphi_i \in L_{consensuses}$ minimally to L
    foreach φ_i *actually added to* L *above* **do**
        Add φ_i to L_{added} minimally
        Add any variables mentioned in φ_i to Q if not already present
    end
end
return L_{added}

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