

# Economically-Augmented Job Shop Scheduling

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## Abstract

We present economically augmented job shop scheduling (EJSP) as an example of a coordination problem among self-interested agents with private information. We discuss its significance in modern organizational supply-web structures, analyze its complexity and present a specific type of combinatorial auctions as a solution mechanism. We relate EJSP to results from the area of economic mechanism design and especially emphasize the need for solution mechanisms, which give the agents no incentive to lie. This requirement is a significant extension to the side constraints that are usually considered in the scheduling literature.

“Cooperation in the economic tradition is mutual assistance between egoists.”

*Hervé Moulin in (Moulin 1995), p. 5*

## Introduction

In recent years, two related trends influenced the development of organizational structures in and between production enterprises. Within enterprises, an emphasis on local competencies and decentralized decision making promoted the creation of (semi-)autonomous substructures. Between enterprises, this was complemented by the formation of innovatively designed cooperations of otherwise economically independent enterprises. Both tendencies were driven by the conviction that a network of “units”, each of which concentrates on its core competency and is equipped with enough authority for locally autonomous and economically sensible decision making, is an answer to the challenges caused by an increasingly complex environment with increasingly agile competitors. This network of units is called an *economic cooperation* below.

This was accompanied by a shift from the typical centralized decision making in monolithic, hierarchically organized structures to a decentralized decision mode with its need for the explicit *coordination of objectives and activities*. While actors and groups of actors have often been (ignorantly) treated as well-functioning, self-motivating entities in the former paradigm, the (partial) economic autonomy of the actors in a collaboration required to put an emphasis on their

motivation (or, to speak in economic terms, their *incentive*) to collaborate properly.

As usual, the cooperation is facing a number of problems (producing goods, designing new products, acquiring financial and physical resources etc.). The cooperation is driven by the *individual objectives* of the participants. We assume that the *individual objective is to maximize utility* (which is a common assumption made for economic environments with rational agents). We also assume that the participants share some objectives (if this would not be the case, there would not be a point in speaking of a cooperation). We will concentrate on types of cooperations whose objective is *to maximize economic efficiency of the cooperation as a whole*. Note that this makes sense from the perspective of the participants that will negotiate their overall goal before the precise details of day-to-day business will be known: if the *cooperation as a whole* strives for economic efficiency, the amount of money that will be earned (or saved) will be maximized – and, in consequence, more money (or less costs) will be distributed to the participants than it would be possible with other objectives. Of course, if judged from the perspective of an individual actor, other objectives may be more beneficial (e.g., “let me do every interesting task and let me keep all the benefits”), but, as we discuss cooperations, it seems sensible to use efficiency as a least common denominator – the actual mode of distributing earnings or costs can still be discussed, but if more money than necessary has been spent or less money than possible has been earned, the amount to distribute will inevitably be smaller than necessary.

In total, the cooperation will search for a *mechanism* that implements a solution to the problem at hand which satisfies the individual objectives “utility maximization” and the common objective “efficiency”. The solution will coordinate the activities of the actors and their individual objectives. We will therefore call the mechanism an (economic) coordination mechanism. To be able to guarantee efficiency, it is necessary that the actors *report the required information* about their objectives, their utilities etc. *truthfully*. Therefore, the mechanism should give the right incentives to do so. Note that paying respect to the assumption of utility maximizing behavior, we cannot assume that the actors would report information truthfully without an economic incentive to do so. Although, from the assumption of (partial) autonomy, we cannot assume that we will be able generally

to enforce or verify truthfulness – neglecting this is one of the most significant weaknesses of old-fashioned decision-making mechanisms.

A significant and important subset of the problems that economic cooperations are facing are resource allocation problems.

While numerous mechanisms for the general case (Parkes & Ungar 2000; Wurman & Wellman 2000; Conen & Sandholm 2002c; 2002b) and for certain limited subcases (Ausubel & Milgrom 2002; Lehmann, Lehmann, & Nisan 2001; Tennenholz 2000) have been investigated recently, only very few studies have been carried out to understand and solve specifically structured problems in relevant application domains.

An important subclass of resource allocation problems, which frequently occurs in manufacturing and logistics, are of the shop scheduling type. Its occurrence is not confined to single production shops. It can also be used to model a large set of structurally equivalent or similar problems in fine- and medium-grain manufacturing planning, in supply-web logistics, resource-bounded project planning, task assignment and so forth.

After previewing the contribution of the paper and after discussing related work, I introduce *economically augmented job shop scheduling problems* formally. I discuss some complexity issues, cast the problem as a specific type of combinatorial auction problem, discuss complexity again and relate the resulting economic coordination problem to results from economic mechanism design. In the final section, I discuss the potential benefits of the approach, show related pitfalls and outline future research topics.

A key objective of the paper is to demonstrate how standard scheduling problems, if considered in a truly economic setting with self-interested agents, give rise to specific economic coordination problems that can be tackled with instruments known from the mechanism design literature. The basic concept is straightforward: economically-augmented (job shop) scheduling problems are problems, where the participating agents have *monetary utilities for schedules*. We are looking for a mechanism that implements a schedule that maximizes the sum of the individual utilities (an *efficient schedule*). To cast this as an combinatorial auction problem, we model jobs as agents and units of resource time as goods. Money is used to *transfer utility* among agents. Prices give an instrument to make lying unattractive.

By reading the paper, it should become clear that

- the underlying allocation problem is, of course, NP-hard, and communicating the information efficiently that is required to solve the allocation problem is in itself a significant tractability problem.
- with self-interested agents, information is inherently distributed. Agents have to be motivated to communicate interests truthfully – otherwise, optimality/efficiency of the allocation cannot be guaranteed.
- while being able to give guarantees for efficiency is an important objective for the cooperation as a whole, tractability considerations (from the perspectives of computation and communication) may require heuristic solutions for

certain instances. If this is the case, (unrestricted) agents will generally behave strategically and it becomes very difficult to guarantee properties of the chosen mechanism.

In total, considering a setting with self-interested, autonomous agents requires to understand that global properties of the solution will be the emergent result of individual behavior. The design of the coordination mechanism has to make sure that incentives for the agents exist to behave such that the wanted global properties emerge,

## Related Work

Related work is numerous, basically all of the microeconomic literature is motivated by and related to resource allocation problems. The study of scheduling problems in an economic context has also a significant tradition in AI literature. For an excellent overview with an emphasis on economics, as well as for interesting and significant recent results related to both areas, see (Parkes 2001) or (Wurman 1999). For an instructive overview of methods and problems in the wider context of self-interested agents, consider (Sandholm 1996), whose further work, for example (Sandholm 2002), also contributes significantly to advances in the area. Also the nice results of Nisan and others, e.g. (Nisan & Ronen 1999; Nisan 2000), have to be mentioned as an introductory source.

A very significant example for work related to the economics of production scheduling is the work of Wellman et al (Wellman et al. 2001; Walsh, Wellman, & Ygge 2000). Especially (Wellman et al. 2001) tackles “factory” scheduling problems restricted to the single machine case. It is, however, straightforward to extend a number of their results, especially those on the (non-)existence of equilibria, to various shop scheduling settings. The paper gives also an excellent introduction to the microeconomic notion of equilibria.

Closely related to the application of economic principles in manufacturing environments are, for example, the work of Van Dyke Parunak (Parunak 1996) or A.D. Baker (Baker 1996).<sup>1</sup> More recently, this has also been discussed in the context of new modes of manufacturing as a promising paradigm for controlling scheduling and planning processes in a distributed environment with local goals, private information and general efficiency objectives (see, for example, (Adelsberger & Conen 2000)).

Most of the economic literature does not consider the specifics of resource allocation problems that exhibit the structure of shop scheduling problems. On the other hand, most of multi-agent system literature on production and logistics does not consider the basic abstract setting with the required rigidity – the key question is: how can we design a mechanism that pays respect to self-interest (individual utility maximization) and autonomy (enforcement impractical) while still guarantee a certain quality level of the outcome (optimality/efficiency/tractability).

The following sections have been written with some formality. It is not necessary to consider every detail, the indi-

<sup>1</sup>Though the concept of (Pareto) efficiency has been introduced much earlier into the scheduling literature, see (Wassenhove & Gelders 1980).

vidual results are straightforward, and they will hopefully provide some solid grounds for further studies. Some of these further studies are outlined in the final part of the paper. Depending on the background of the reader, the part on scheduling or the part on combinatorial auctions may be more relevant to prepare the reader for the discussion and for further, own work. If open questions remain or if you want to share or develop new insights together, don't hesitate to contact the author (ie., me).

## From Shop Scheduling to Economic Coordination

Scheduling allocates resources over time to enable the execution of tasks. We study an subclass of scheduling problems, namely job shop problems. The tasks are given by a set of jobs. Each job consists of a sequence of operations. The operations have to be performed on specific machines in a given order. We base the following on the constraint optimization version of discrete, deterministic Job Shop Scheduling (JSS) as defined in (Ausiello *et al.* 1999).

### Definition 1 (Job Shop Scheduling – Basic Setting).

An instance of the class of job shop scheduling problems (JSS) consists of a set  $M = \{1, \dots, m\}$  of  $m$  machines, and a set  $J = \{1, \dots, n\}$  of  $n$  jobs, each consisting of a set  $O_j = \{o_j^1, \dots, o_j^{n_j}\}$  of  $n_j$  operations. For each such operation,  $o_j^i$ , a machine  $m_j^i \in M$  and a processing time  $p_j^i \in \mathbb{N}$  is given. A ready time,  $r_j \in \mathbb{N}_0$ , denotes the earliest possible start time for the first operation of each job  $j \in J$ .

The following example with two agents and three machines is used throughout the paper:

**Example 1.** Job 1: Ready time 0, Operation 1 on machine 2 requires 2 time units. Operation 2 on machine 1 requires 3 time units, operation 3 on machine 3 requires 2 time units.

Job 2: Ready time 0, Operation 1 on machine 3 requires 1 time unit. Operation 2 on machine 2 requires 2 time units, operation 3 on machine 1 requires 2 time units.

In the following, we refer to an arbitrary, but fixed JSS problem  $P$ . A potential schedule for a set of operations is a mapping from the operations to start times. A potential schedule which assigns start times to all operations in  $P$  is called *complete*. A potential schedule is called *feasible* if (1) no first operation starts too early, (2) no sequence constraint is violated, and (3) no overlap in the processing times of assigned operations occurs on any machine. A schedule that is complete and feasible with respect to  $P$  is called a *valid* schedule or a *solution* of  $P$  (s. Fig. 1 for an example). Formally:

**Definition 2 (Valid Schedule).** Given an instance  $P$  of JSP. A mapping  $s : \cup_{j \in J} O_j \rightarrow \mathbb{N}_0$  is a valid schedule for  $P$  iff (1)  $s(o_j^1) \geq r_j$  for all  $j \in J$ , (2)  $s(o_j^i) + p_j^i \leq s(o_j^{i+1})$  for all  $j \in J$  and all  $i \in \{1, \dots, n_j - 1\}$ , and (3), for every pair of distinct<sup>2</sup> operations  $o_j^h, o_k^i$ , either  $m_j^h \neq m_k^i$  or  $s(o_j^h) + p_j^h \leq s(o_k^i)$  or  $s(o_k^i) + p_k^i \leq s(o_j^h)$ .

<sup>2</sup>That is,  $j, k \in J$ ,  $h \in \{1, \dots, n_j\}$ ,  $i \in \{1, \dots, n_k\}$  and, if  $j = k$ ,  $h \neq i$ .

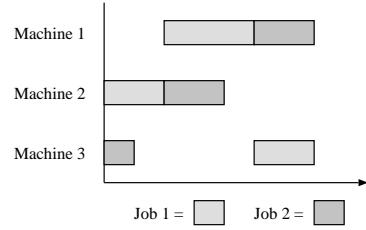


Figure 1: The valid schedule  $(0, 2, 5)_1, (0, 2, 5)_2$  (schedules are given as  $n_j$ -ary sequences of start times for each job  $j$ ) for example 1.

We frequently study a problem  $P$  relative to a time horizon  $[TS, TE]$ ,  $TS, TE \in \mathbb{N}_0$  and write  $P^{[TS, TE]}$ . The set of all valid schedules restricted to a specific time horizon is denoted with  $S^{[TS, TE]}$ . If the time horizon is irrelevant, no index is shown. For a given set  $S$ , the subset  $S_j$  contains all schedules that are restricted to the operation of a given job  $j$ . For a given schedule  $s$ , the sub-schedule  $s_j$  denotes the elements of  $s$  that schedule operations of job  $j$ .

We now turn our attention to comparisons of the *quality* of schedules. Often measures that depend on the completion times of the jobs are used, for example, minimize  $C^{max} = \max_{j \in J} C_j$ , where  $C_j = s(o_j^{n_j}) + p_j^{n_j}$  is the time of completion of the last operation  $o_j^{n_j}$  of job  $j$  in a given (valid) schedule  $s$ . Regularly it is also assumed that each job has a due date  $d_j$  which allows us to use derivations from this due date as a measure.

In a value-oriented business environment, this way of measuring the quality of schedules is only convincing if the measure has a close correspondence to the cash-flow pattern that is induced by the imposed solutions (usually, it is tried to find a solution that optimizes the chosen measure or that approximates an optimal solution). With respect to the core measure of the success of management and operations, ie. value, such time (or capacity etc.) related measures can only be considered as being surrogates that may render decisions intransparent because they introduce incomparabilities between manufacturing shops competing for resources and give no handle to tie local decisions to the decisions of upstream and downstream manufacturing/logistics units. This, however, would be an important prerequisite for an unified analysis of the plans, schedules, decisions and operations of all interrelated business units.

But even within one shop, it is usually not trivial to map the goals that are related to jobs to *one* measure only. Some jobs may be produced directly to customer orders (due date is important), some jobs are produced to fill up the stock (minimizing production cost is important) etc. The resulting multi-criteria problem does not seem to be convincingly solvable without introducing a unifying measure. As the *value* related to strategic, tactic, and operational decisions determines the success of business operations, mapping goals to value and measuring quality with money is a natural choice.

We will now augment job shop problems with value information and view them as economic coordination problems.

## JSS as a Economic Coordination Problem

Job shop scheduling problems can be extended to economic coordination problems—the jobs are identified with agents competing for resources. The goods to be traded are slots of machine time. To augment a JSS problem  $P$  economically, we assume that each agent has utility for schedules, that is, a value function  $v_j' : S \rightarrow \mathbb{Q}_0$  is available for every job agent  $j$  which assigns a non-negative value to each possible valid schedule (note that it can also be useful to consider incomplete feasible schedules).

**Definition 3 (EJSP).** An instance  $P$  (possibly restricted to a time horizon) of the class JSS of job shop scheduling problems and a set  $V$  of  $j$  value functions  $v_j' : S \rightarrow \mathbb{Q}_0$ , one for each job  $j$ , defines an instance  $EP$  of the class of economically augmented job shop scheduling problems (EJSP).<sup>3</sup>

The general objective of solving EJSPs is to select a schedule from the set of possible valid schedules that achieves *economic efficiency*:

$$\arg \max_{s \in S} \sum_{j \in J} v_j'(s) \quad (1)$$

We show below that this class of scheduling problems encompasses a number of traditional job shop scheduling problems. To do so, we first consider a restricted variant of the EJSP, where, for each agent  $j$ , the value of a schedule does only depend on the operations in  $O_j$ . In the general variant, an agent  $j$  may value two schedules differently though both assign the same start times to the operations in  $O_j$ . For example, this allows an agent  $k$  to express that she prefers if a machine, say 2, is assigned to agent  $l$  instead of agent  $m$  in the interval [3, 5]. This expressiveness is not always required – though, to map a very common objective, namely *max completion time*, it is required, see below.

**Definition 4 (EJSP without allocative externalities).** Let  $EP$  be given as above.  $EP$  is an EJSP without allocative externalities iff, for any agent  $j$  and any pair of schedules  $a, b \in S$ , such that the restriction  $a_j$  of  $a$  to operations of  $j$  coincides with the restriction  $b_j$  of  $b$ ,  $v_j'(a) = v_j'(b)$ .

In the three-field categorization of scheduling problems, I suggest to denote the general problem without allocative externalities as  $n|m; v_j(S_j)|\sum v_j$  and the general problem with allocative externalities as  $n|m; v_j(S)|\sum v_j$ . In the later case,  $v_j(S)$  denotes that the individual agents have preferences for complete schedules, while  $v_j(S_j)$  denotes the case where the value of a schedule for an agent depends only on the sub-schedule of her operations.

Now, we will study the complexity of EJSP without allocative externalities by exploiting its relationship to minsum criteria in scheduling. This is a little bit more awkward than necessary (weighted set packing or weighted matching for easier subclasses in the one-machine case could be

<sup>3</sup>Remember that  $S$  is the set of all valid schedules that solve  $P$ . If a time horizon is given, this set may be empty. If no time horizon is given, the set is countably infinite. We will therefore usually assume that a time horizon is given (without displaying it).

used instead), but may give some hints on how classical job shop scheduling criteria relate to economically augmented job shop scheduling.

Now consider a JSS problem  $P'$  and a typical minsum criterion (Hoogeveen, Schuurman, & Woeginger 1998), *total job completion time*. We map this  $J||\sum C_j$  problem to a restricted EJSP  $EP'$  as follows (the mapping will be denoted with  $g(\cdot)$  below):

First, determine a valid schedule by timetabling the operations of job 1 as early as possible and without slack and proceed with the operations of job 2, starting with  $o_2^1$  at time  $C_1$ . Continue this until all operations are scheduled. This produces (with effort linear in the number of operations) a valid schedule  $s'$  of length  $l = \sum_{j \in J} \sum_{1 \leq i \leq n_j} p_j^i$ . This determines the time horizon  $[0, l]$  to be considered. Now, a value function  $v_j'(\cdot)$  for each agent  $j$  can be given that determines, with effort linear in the number of operations, the value of any given input schedule:

**Function**  $v_j'(\text{In: Valid Schedule } s, \text{Out: Value } v)$

$$C_j \leftarrow s(o_j^{n_j}) + p_j^{n_j}; \\ v \leftarrow l - C_j; \text{return } v;$$

Note that this transformation is polynomial because it makes use of the fact that there is significant structure in the problem. Would we have to enumerate all (or almost all) schedule/value pairs explicitly, the transformation would not be polynomial.

**Proposition 5.** Let  $P'$  be an arbitrary, but fixed instance of  $J||\sum C_j$  and set  $EP' = g(P')$ . A schedule  $s$  minimized the criterion  $\sum C_j$  in  $P'$  if and only if it maximizes the economic efficiency in  $EP'$ .

*Proof.* Let  $s$  be a schedule that maximizes efficiency in  $EP'$ . Assume that  $s$  does not minimize the total completion time in  $P'$ , that is, there is a schedule  $r \in S$  such that  $\sum_{j \in J} s(o_j^{n_j}) + p_j^{n_j} > \sum_{j \in J} r(o_j^{n_j}) + p_j^{n_j}$ , or, shorter,  $\sum_{j \in J} s(o_j^{n_j}) > \sum_{j \in J} r(o_j^{n_j})$ . Because  $s$  maximizes efficiency in  $EP'$ ,  $\sum_{j \in J} (l - s(o_j^{n_j}) + p_j^{n_j}) \geq \sum_{j \in J} (l - r(o_j^{n_j}) + p_j^{n_j})$ , or, written differently,  $\sum_{j \in J} l - \sum_{j \in J} s(o_j^{n_j}) - \sum_{j \in J} p_j^{n_j} \geq \sum_{j \in J} l - \sum_{j \in J} r(o_j^{n_j}) - \sum_{j \in J} p_j^{n_j}$  respectively  $\sum_{j \in J} r(o_j^{n_j}) \geq \sum_{j \in J} s(o_j^{n_j})$ , contradicting the assumption. The other direction follows immediately as well.  $\square$

Similar results can be obtained for other minsum criteria (e.g., total tardiness, weighted total completion/tardiness, holding costs, early/tardy penalties). Note also that EJSP without allocative externalities cannot model criteria like  $C^{\max}$ , because a global optimum for the desired criterion is not obtainable from local considerations. The jobs *cannot* be modeled as being independent in this case. If the value function should reflect the benefit of achieving a minimal  $C^{\max}$ , they *must* reflect this dependency in their valuation of schedules, or otherwise, self-interest prevents the optimization of the desired criterion. For  $C^{\max}$ , the jobs should value schedules with an earlier completion time for all jobs

higher than, for example, schedules that give them an earlier individual completion time (the time the last *operation* of  $j$  finishes) but a later overall completion time (the time the last *job* finishes)—in other words, the value of a schedule does not only depend on the operations in  $O_j$  but on all (final) operations (that is, allocative externalities are present).

**Proposition 6.** *EJSP is (strongly) NP-hard.*

*Proof.* First note that if we formulate EJSP as a decision problem that asks, if there exists a schedule with utility above a certain bound, we can check if a certain schedule is a “Yes” instance in time polynomial to the size of the input easily. Thus, the decision problem variant of EJSP is in NP. As the above proposition 5 shows,  $J||\sum C_j$  can be reduced to EJSP. Furthermore, the transformation  $g$  is polynomial (see above, which also applies to the decision problem variants). From the (strong) NP-hardness of  $J||\sum C_j$  (s. (Lawler *et al.* 1992) or (Hoogeveen, Schuurman, & Woeginger 1998)) the proposition follows.  $\square$

To adapt the problem to our economic setting with self-interested, (bounded) rational agents, the following assumption are necessary.

In line with our basic motivation, we assume that the value function is *information private to the agent*, that is, no (*central*) institution has access to this information without prior consent of the agent. In addition, *utility is transferable* between agents (ie, a meaningful currency has to be available to express valuations and to transfer payments). Technically, the value functions are quasilinear in money, compare, for example, (Mas-Colell, Whinston, & Green 1995). Quasilinearity allows us to interpret the utility for a good (time slots in our case) as the willingness to pay for it.

We also assume that no agent can be forced to act against his will. With these assumptions, we can translate economically augmented job shop scheduling problems to a specific subclass of combinatorial auction problems (CAPs), which are currently studied extensively in AI and microeconomic literature, s. (de Vries & Vohra 2001) for a survey.

## Transforming EJSP to CJSAP

Let  $EP$  be in EJSP and let  $[TS, TE]$  be a time horizon. An economic coordination problem can now be formulated as follows:

*Agents:* A set of  $n$  job agents,  $N = J = \{1, \dots, n\}$ , and an arbitrator, 0. The arbitrator is modeled as a *supplier*. The job agents are modeled as *consumers*.

*Goods:* The set of goods,  $\Omega$ , is the set of all machine-specific unit intervals within the time horizon (remember that  $M$  is the set of machines), that is:

$$\Omega = \{[z, z+1]_i \mid i \in M, z \in \mathbb{N}_0, z = TS, \dots, TE - 1\}$$

Any subset  $B \subseteq \Omega$  (alternatively writable as  $B \in 2^\Omega$ , the power set of  $\Omega$ ) is called *bundle*.

*Value functions:* Before we can define value functions, a function  $A(\cdot)$  that partitions a schedule  $s$  into the covered machine-specific unit intervals is required (note that  $s$  is a function, and thus a relation, so that it is appropriate to write

$(o, t) \in s$  instead of  $t = s(o)$ ). This function  $A : S \rightarrow 2^\Omega$  is defined as

$$A(s) = \{[z, z+1]_m \mid (o_j^i, t) \in s, m = m_j^i, \\ z = t, \dots, t + p_j^i - 1\}.$$

Furthermore, let  $B$  be a bundle and  $s$  a schedule.  $B$  covers  $s$  iff  $A(s) \subseteq B$ .  $B$  precisely covers  $s$  iff  $A(s) = B$ . We will also say that  $s$  covers  $B$  precisely.

In an allocation, every consumer will obtain a possibly empty subset of the unit intervals, that is a bundle. The value of a bundle for the consumer is equal to the value of the best schedule that is covered by the bundle. The value function of consumer  $j$ ,  $v_j : 2^\Omega \rightarrow \mathbb{Q}_0$  can now be defined as follows:

$$u_j(B) = \begin{cases} 0 & \text{if no } s \in S_{r[j]} \text{ exists, such that } A(s) \subseteq B \\ \max_{\{s: s \in S_{r[j]}, A(s) \subseteq B\}} u'_j(s) & \text{otherwise.} \end{cases}$$

Here,  $S_{r[j]}$  is the set of all schedules such that their restriction to the operations of agent  $j$  is feasible.

**Example 2.** Reconsult example 1. The time horizon to be considered is  $[0, 9]$ . The following consideration leads to the preference relations, which underly the value functions: let  $j$  be a job agent and  $s_1$  and  $s_2$  be schedules which are complete with respect to  $j$ . If the last operation of agent  $j$  in  $s_1$  is completed earlier than  $s_2$ , agent  $j$  prefers schedule  $s_1$  (written as  $s_1 \succ s_2$ ). If the completion times are equal, the agent is indifferent between  $s_1$  and  $s_2$  (written as  $s_1 \sim s_2$ ).

For agent 1 the preference relation given below follows. for convenience, a rank is assigned to the equivalence classes of equally preferred schedules.

- 1:  $(0, 2, 5) \succ$
- 2:  $(0, 2, 6), (0, 3, 6), (1, 3, 6) \succ$
- 3:  $(0, 2, 7), (0, 3, 7), (0, 4, 7), (1, 3, 7), (2, 4, 7) \succ$
- 4: All invalid schedules.

For agent 2, the preference relation is only partially displayed:

- 1:  $(0, 1, 3) \succ$
- 2:  $(0, 1, 4), (0, 2, 4), (1, 2, 4) \succ \dots$
- 6: All invalid schedules.

Agent 1 values the earliest possible completion (at 7) with 20 currency units (CU) and agent 2 (at 5) with 16 CU. A delay per time unit reduces the value of the schedule for agent 1 by 3 CU and for agent 2 by 2 CU, resulting in the following functions  $v_j^R(\cdot)$ , which assign values to ranks:

$$\text{Agent 1: } v_1^R(1) = 20, v_1^R(2) = 17, v_1^R(3) = 14, v_1^R(4) = 0.$$

$$\text{Agent 2: } v_2^R(1) = 16, v_2^R(2) = 14, v_2^R(3) = 12, \\ v_2^R(4) = 10, v_2^R(5) = 8, v_2^R(6) = 0.$$

For a given bundle  $B$ , its valuation can be determined by mapping  $B$  with a function  $r$  to the preference rank which corresponds to the best schedule which is covered by  $B$ . For example for the largest bundle,  $\Omega$ , which covers all schedules that lie within the time horizon, rank 1 is the result of the mapping for both agents, that is  $v_1(\Omega) = v_1^r(r(\Omega)) = v_1^r(1) = 20$  respectively  $v_2(\Omega) = 16$ .

**Proposition 7 (Monotony).**

$v_i(A) \leq v_i(B)$  for all  $A \subseteq B \subseteq \Omega$ ,  $i \in N$ .

*Proof.* This follows immediately from the construction of the value functions: a bundle  $B$  that contains at least as many unit intervals as a bundle  $A$  covers at least the schedules that  $A$  covers, thus its value cannot be smaller than the value of  $A$  because the maximum value over all covered schedules in the EJSP instance is at least as large for  $B$  as for  $A$ .  $\square$

This formalizes the *Free Disposal* condition, because adding further unit intervals will not reduce value, or, in other words, superfluous goods can be disposed off at no cost.

We also assume that *no budget restrictions* exist: every consumer  $j$  is in possession of enough money to be able to pay up to the amount of the valuation for his most preferred bundle. The consumers have no endowment beyond money. All goods belong to the arbitrator. If the consumer does not receive any good, his utility shall be 0 (w.l.o.g.).

The objective of the allocation of the goods (from the perspective of the arbitrator) is to maximize the aggregated utility of the consumers. Here, this directly corresponds to maximizing the sum of individual utilities. An allocation is an  $(n+1)$ -ary sequenced partition of the set of goods,  $\Omega$ , which assigns to agent  $j$  the goods in the bundle  $X_j$  (some  $X_j$  may be empty, so it is not a partition with non-empty subsets, as it is usually assumed). An allocation  $X^* = (X_0^*, \dots, X_n^*)$  conforms to the above mentioned objective if and only if the allocation is *efficient*, that is,  $X^*$  has to be a maximizer for the following problem

$$\max_X \sum_{j=1}^n v_j(X_j) \quad (X \text{ iterates over all allocations}) \quad (2)$$

**Example 3.** For the above example, the following assignments lead to efficient allocations (consecutive unit intervals are consolidated into bundles):  $X_1 \supseteq \{[0, 2]_2, [2, 5]_1, [5, 7]_3\}$ ,  $X_2 \supseteq \{[0, 1]_3, [2, 4]_2, [5, 7]_1\}$ ,  $X_0 = \Omega \setminus (X_1 \cup X_2)$ .

**Definition 8 (CJSAP).** Let EP be an instance of EJSP. The sets  $\Omega$  and  $N$ , the value functions  $v_j(\cdot)$  of the consumers that are obtained by the above transformation of EP, and the objective (2) of the arbitrator define an instance C of the class of Combinatorial Job Shop Auction Problems, CJSAP.

We call an allocation that conforms to the objective of the arbitrator a *solution*. A solution exists for every possible instance of a CJSAP. This follows with a straightforward combinatorial argument immediately from the finiteness of the problem (which we assume throughout the paper).

**Proposition 9.** For any  $EP \in EJSP$ , if a valid schedule exists, a solution of the corresponding problem  $C \in CJSAP$  can be transformed into an optimal schedule for the original, economically augmented job shop scheduling problem EP and vice versa.

*Proof.* Let  $X = (X_0, X_1, \dots, X_n)$  be a tight solution of  $C$ . A tight solution is a solution that does not contain time slots that are not used. Note that a tight solution always exists

because the value of a bundle is the value of the best covered schedule, and, in consequence, the bundle that covers just the best schedule, is tight and optimal (that is: unused slots cannot add to the value of a bundle due to the construction of the value functions). Further note that it is always possible to tighten a solution with costs less than exponential. Now, construct a schedule  $s^X$  from  $X$  as follows: for each agent  $j \in J$ , find the best schedule  $s_j^X$  that is covered by  $X_j$  by simply timetabling the operations as unit intervals are available (due to the tightness of  $X$ , this is possible in cost linear to the number of time slots in  $X$ ). Combine the schedules  $s_j^X$  to the schedule  $s^X$ . Note that the construction of  $C$  ensures that this construction is always possible if a valid schedule exists. Furthermore, as the allocation  $X$  is a partition of  $\Omega$ , no overlap on a machine can occur. The other direction is even simpler: each reservation for an operation in the optimal schedule can be split into the machine-specific unit intervals. The information in the schedule can be used to assign the unit intervals immediately to the correct part of the allocation (cost linear to the processing time).  $\square$

We will now turn our attention to the *problems related to finding an efficient allocation*. They can be outlined as follows: (1) a certain amount of value information is necessary to determine an efficient allocation; (2) once the required information is available, the actual computation has to be performed; (3) an incentive has to be given to the agents to report their part of the required information truthfully, or otherwise, efficiency of the solution cannot be guaranteed; and (4) the agents have to be satisfied with the determined outcome. We neglect issue (1), briefly discuss issue (2) and concentrate on (3) and (4). We will, however, return to the issue of complexity later.

## Determination of Efficient Allocations

If we assume for now that the (complete and true) value functions of all agents are known, an efficient allocation of the unit intervals to the job agents can be computed with one of the well-studied methods of winner determination (compare (Sandholm 2002; Sandholm *et al.* 2001; Fujishima, Leyton-Brown, & Shoham 1999)). To save communication, approaches have been suggested that only partially reveal the value functions of the consumers (compare (Parkes 2001) for indirect, iterative auctions and (Conen & Sandholm 2002c; 2002b) for progressive direct mechanisms based on (Conen & Sandholm 2001)). Note that the winner determination problem is essentially a set-packing problem (Rothkopf, Pekeč, & Harstad 1998; de Vries & Vohra 2001) and that it is NP-hard. We show below that this also holds for CJSAP.

## Prices

In the above example, two core problems remain: first, because only agent 1 can realize his best alternative, agent 2 will envy him. Second, we have tacitly assumed that the agents report their utility truthfully—but why should they do so under our assumption of preferences being private information? Certainly, agent 2 could expect to benefit from over-exaggerating his valuations. If agent 1 would expect

agent 2 to over-exaggerate, he would do the same, and so forth. Without an additional way to ensure satisfaction and to make lying unattractive, we cannot expect to compute allocations that are efficient with respect to the true preferences. The way to go is to introduce prices. Each consumer has a net utility function which reflects the impact of prices (negative transfers in the following definition) on the realizable utility.

**Definition 10 (Net utility).** The net utility function  $u_j(\cdot) : 2^\Omega \times \mathbb{Q} \rightarrow \mathbb{Q}$  for each consumer  $j$  is defined as  $u_j(x, t) = v_j(x) + t$ .

To keep every consumer satisfied with the outcome (consisting of allocation and payment),  $u_j(X_j, p) \geq u_i(A, p)$  must hold for every consumer  $j$  and every bundle  $A \subseteq \Omega$ , that is, the net utility of the bundle he receives must be at least as good as it would be for any other bundle at the given prices (or, otherwise, the consumer would prefer to receive the bundle that gives him the best net utility).

This leads immediately to the notion of equilibria. An outcome, that is, an allocation and a payment vector (determined by the prices), is an equilibrium if both sides of the market are satisfied with it (the market is *cleared*). In our case this is true if the above condition holds for all consumers and if the allocation is efficient (to satisfy the supplier). There are different restrictions that one can impose on prices—prices for bundles have to be the sum of prices for individual goods (see (Gul & Stacchetti 1999; Kelso & Crawford 1982)), prices are independent for every bundle (see (Wurman & Wellman 2000)), prices are determined for the bundles in the efficient allocation and prices for bundles of these bundles are additive (see (Connen & Sandholm 2002a)). Only the independent pricing of all bundles can guarantee the existence of equilibrium price vectors. However, implementing the determined outcome may require enforcement: each agent is only allowed to buy one bundle—thus, if he is interested in  $AB$  and we have  $p(AB) = 6$ ,  $p(A) = 2$ ,  $p(B) = 2$ , he would want to enter the auction with two identities to buy  $A$  and  $B$  separately. For pricing modes without enforcement, equilibrium price vectors need not exist. This negative result holds also for instances of CJSAP. We will therefore not discuss (anonymous) equilibrium pricing in detail and turn our attention to a solution concept for which solutions always exists: Vickrey payments. We will demonstrate in the example below that implementing Vickrey payment-based coordination mechanisms give the participating agents no incentive to lie (a very relevant design objective in a private information setting). Please, consider the continued example below.

**Example 4.** Bundles on offer in the efficient allocation:

$$\begin{aligned} A &= \{[0, 2]_2, [2, 5]_1, [5, 7]_3\} \\ B &= \{[0, 1]_3, [2, 4]_2, [5, 7]_1\} \end{aligned}$$

Demand for these bundles:

	$\emptyset$	$A$	$B$	$AB$
Agent 1	0	<b>20</b>	0	20
Agent 2	0	0	<b>12</b>	16

*Vickrey Payments (the vector of payments also fulfill the equilibrium condition if interpreted as prices for the bundles instead of personalized payments):*

	$A$	$B$	$AB$
Agent 1's payment	4		
Agent 2's payment		0	
Equilibrium prices	4	0	4

The payments ensure that there is no (individual) incentive for the agents to lie, as can be seen as follows. First note that the price each agent has to pay does only depend on the reported utilities of the competing agents—it represents the loss of utility that the other agents experiences due to the participation of the former agent. Now, assume that agent 1 would under-bid his valuation with, say, 17. Then he risks that agent 2 (or any other agent) would bid just above 17 but below 20, say 18, and would thus receive the good. As the price he has to pay is independent of his own bid and will equal the bid of the other agent, he could have done better by bidding truthfully (exactly  $2 = 20 - 18$  instead of 0). If he would have known beforehand what the other agent will bid, say  $x$ , he would not have a reason to under-bid either, because there would be no difference in net value for agent 1 in bidding 20 or  $x + \epsilon$  (as the price will be the bid of the other agent anyway). He would also not over-bid, because he risks that he receives the over-bid bundle for a price between his true valuation and his bid and would, thus, realize a loss. If he would over-bid in the fully-informed situation, he cannot gain any net value from it either. An analogous reasoning applies to agent 2, thus both agents will not have an incentive to misrepresent their valuation if they act rational. This corresponds to an equilibrium in dominant strategies (weakly dominant in the case of informed agents).

The general principle invoked here has been mentioned in the example: the payments that an agent has to transfer do not depend on her own bid but captures instead the effect of her participation on the other participating agents. The principle has been discovered and applied independently by Vickrey (in 1961) (Vickrey 1961), Clarke (in 1971) (Clarke 1971) and Groves (in 1973) (Groves 1973). It is intuitively clear that, as soon as there is a dependency for a bundle  $X_j$  between the reported utility of agent  $j$  and the price  $j$  has to pay,  $j$  will have an incentive to minimize this price by misrepresenting his utility whenever possible. To do so, he might start to collect information about the other participating agents (which is not necessary above) to become able to behave *strategically*. This, in turn, may make it impossible for the arbitrator to pick the efficient allocation—all that he could do would be to pick the allocation that is efficient with respect to the *reported* utilities. This brief digression into the issue of *incentive compatibility* may suffice to demonstrate one of the most prevalent problems in environments where the agents have private information: the problem of eliciting their preference truthfully to enable a truly optimal decisions (for pointers to recent work, see (Bikhchandani *et al.* 2001; Parkes 2001)).

**Proposition 11 (Existence of Vickrey Outcome).** An outcome consisting of an efficient allocation and a vector of related Vickrey payments exists for every instance of CJSAP.

*Proof.* The proof follows from a straightforward combinatorial argument for finite sets of bundles and agents. A computable solution of the maximization problem (2) (to determine the efficient allocation) and the  $n$  (or less) maximization problems following from the initial problem by leaving out, for each non-empty bundle in the allocation, the agent that receives it (to determine the effect of his participation, that is the Vickrey payments), is immediately available from enumerating all possible complete and reduced allocations and picking the optima.  $\square$

## Complexity Issues

Two immediate problems of auction mechanisms that try to solve a CJSAP are (1) that the mechanisms may require communication that is exponential in the number of unit intervals (compare the general result of (Nisan & Segal 2002), which extends to CJSAP) and (2) that solving the actual allocation problem once all required value information is received is NP-hard, as the following corollary demonstrates:

**Corollary 12.** *CJSAP is (strongly) NP-hard.*

*Proof.* The decision problem variant of CJSAP is in NP. The proposition follows immediately from Proposition 6 and the polynomiality of the transformation given in Proposition 9.  $\square$

Of course, CJSAP is a subclass of the general combinatorial auction problem CAP which is equivalent to maximum weighted set-packing (compare (de Vries & Vohra 2001)). If we allow partially ordered sequences of operations and alternative routings, the corresponding variant of EJSP and the resulting CJSAP could be mapped to the general combinatorial auction problem bijectively.

Note that we cannot expect to obtain optimal solutions for every problem with reasonable computational efficiency. Furthermore, the approximability results obtained for maximum weighted set packing (Ausiello *et al.* 1999) are not very encouraging. We also do not expect that CJSAP is an especially well-behaved subclass in this respect due to its proximity to EJSP and, thus, typical job shop scheduling problems.

## Discussion

This paper has discussed economically augmented job shop problems and their transformation into a specific subclass of combinatorial auction problems. We have briefly discussed the problem of finding efficient solutions and of doing this in a way to ensure that the agents participate and report their valuations truthfully. The presented approach is applicable in the context of self-interested agents with private information. We consider the study of such application scenarios in the context of scheduling for manufacturing and logistics as important due to the increasing tendencies to (1) restructure companies towards collaborations of (semi-)autonomous units on all levels of granularity and to (2) form (potentially) volatile collaborations between autonomous enterprises. In both cases, monetarian value may turn out to be a key issue for the integration of diverging interests—economically efficient mechanisms that keep the

participating agents individually satisfied can contribute to this integration without violating autonomy and information privacy more than necessary.

Some questions and related answers may further focus the discussion:

- *Why is the approach interesting?* Because real-world scenarios involving human actors tend to exhibit self-interested behavior and distributed information. In such a setting, we cannot assume that all (better: all *required*) information will be truthfully communicated without an incentive to do so. We can now either design a mechanism that guarantees truthful communication (within the applicability constraints set by the underlying assumptions) or, at least, try to understand and to control the consequences of being unable to do so. An important immediate consequence is that agents will behave strategically (collecting information about others, sending out false information).
- *How does it relate to multi-criteria decision making?* The modeling assumes that all jobs can (or are required to) value the available choices. Each job, of course, is still facing a multi-criteria decision problem (how to order the choices rationally?), so this is still an interesting issue. Once you have a rational preference order (which is a common assumption in decision theory and economics), this order can *always* be expressed as a utility function (which gives an “individual” valuation). Now, the crucial requirement is that these valuations need to be comparable relatively across agents (this is expressed in the assumption of transferable utility and a common currency). *Only* this gives a handle to *solve* conflicts in economic terms, which is precisely what has to be done in a business environment anyway: in the end, you must focus on the economic impact of your decisions! In other words: in an economic setting, there really is no point in identifying a solution on the Pareto frontier while neglecting economic value—you want to shrink the frontier by introducing economic value to enable an economically *best* choice. In total, there still is a need for multi-criteria decision making, but on the side of the individual participants, not on the side of the “institution” (for example, an auctioneer) that implements the economic coordination mechanism. As a consequence, local competency can act to the best of its knowledge, while being required to communicate its preferences to the other participants in an economic language.
- *What are important topics for future research?* Technically, the “best” coordination mechanism for combinatorial auctions has not been found yet. If an instance is hard to solve, non-optimizing mechanisms are required. But even if an instance would be computationally tractable, information generation (on the side of the agents) and communication can be intractable. It is therefore interesting to look for mechanism that minimize communication respectively information generation (Parkes 2002; Conen & Sandholm 2002c; 2002b). This is also still an issue of interest, if non-optimizing allocations are allowed, ie. if we turn our attention to exact mechanism that do not discourage dishonest behavior generally or to heuris-

tic mechanisms. If the design of a mechanism makes dishonest behavior attractive, a detailed analysis of the strategic behavior of agents becomes relevant. For a head start into this issue in relation to a simplified factory scheduling model, see (Wellman *et al.* 2003) (with very nice but not very encouraging results – the practicability limits of the analysis of the interaction among strategically behaving agents becomes pretty clear). Also very interesting is the design of heuristic mechanisms that take into account the specifics of shop scheduling scenarios (see the section on decision point bidding in (Conen 2002)) and are, thus, easy to understand and to apply for the participating agents. Some more issues are found in the following subsection.

## Extensions and future work

The mapping of economic job shop problems to combinatorial auction problems can easily be extended to accommodate other classes of job shop problems, for example problems with alternative routings, reservation costs for resources, or sequence-dependent set-up costs, or of other open and flow shop scheduling problems.

Further effort is necessary to develop a useful set of benchmark problems and to study the specific structure of real-world problem scenarios. The impact of the utilization of heuristic modifications of standard combinatorial auction procedures on the desired properties efficiency and incentive-compatibility has to be studied in these specific scenarios.

On the other hand, the detailed knowledge about various subclasses of shop scheduling problems (with specific one or multi-dimensional optimization criteria) can have an impact on the analysis of structural phenomena in the corresponding combinatorial auction problems that may help to develop improved mechanisms and algorithms for specific problem scenarios.

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