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Context-free and Context-sensitive Kernels: Update and Deletion Equivalence in Abstract Argumentation[1]

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Abstract. Notions of equivalence which guarantee inter-substitutability w.r.t. further modifications have received considerable interest in nonmonotonic reasoning. This paper is within the context of abstract argumentation and we focus on the most general form of a dynamic scenarios, so-called *updates* as well as certain sub-classes, namely *local*, *normal* and *arbitrary deletions*. We provide characterization theorems for the corresponding equivalence notions and draw the relations to the recently proposed kinds of expansion equivalence [15, 3]. Many of the results rely on abstract concepts like *context-free* kernels or semantics satisfying *isolate-inclusion*. Therefore, the results may apply to future semantics as well as further equivalence notions.

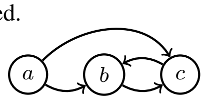
1 Introduction

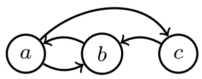
In the last two decades *argumentation theory* has received growing interest within the AI-community (cf. [17, 16] for excellent overviews). Dung's *abstract argumentation frameworks* (AFs) [11] play a dominant role in this area. In AFs arguments and attacks between them are treated as undefined primitives, i.e. the internal structure of arguments is not considered. In order to determine acceptable sets of arguments a huge variety of *semantics* were introduced. In 2007 Baroni and Giacomin presented a first systematic classification of argumentation semantics [2]. This paper was an important step since until its publication comparisons between semantics were almost exclusively example driven.

More recently several problems regarding dynamic aspects of abstract argumentation have been addressed in the literature [7, 5, 6, 4]. One exceptional work in this context is the characterization of *strong equivalence* via so-called *kernels* which are purely syntactical concepts [15]. Strong equivalence guarantees inter-substitutability in any dynamic scenario, which is relevant, for instance, for simplification issues. In case of instantiation-based argumentation [10], i.e. if arguments and attacks stem from an underlying knowledge base we observe that more restrictive kinds of expansions naturally appear. For instance, *normal* and *local expansions* correspond to re-instantiations if a new piece of information is added [3] or if we change to a less restrictive notion of attack (see [13] for different attack relations).

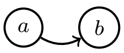
In this paper we study the most general form of a dynamic scenario, so-called *update* where adding as well as deleting of arguments and attacks is allowed. Furthermore, we consider certain sub-classes, namely *local*, *normal* as well as *arbitrary deletions* which naturally complement the already existing expansion types.

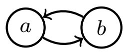
Consider the following motivating example under preferred semantics, which selects maximal conflict-free and self-defending sets of arguments. The AF $\mathcal{F}_U$ is an update of $\mathcal{F}$ which might stem from a re-instantiation process where the underlying notion of attack is changed.

$\mathcal{F}$: 

$\mathcal{F}_U$: 

At first glance it seems that there is no semantical impact of this update since $\{a\}$ is the unique preferred extension in both frameworks. Retracting the argument c reveals the inherent difference between $\mathcal{F}$ and $\mathcal{F}_U$.

$\mathcal{F} \setminus \{c\}$: 

$\mathcal{F}_U \setminus \{c\}$: 

Now $\{b\}$ becomes preferred in $\mathcal{F}_U \setminus \{c\}$ but still not in $\mathcal{F} \setminus \{c\}$. This leads to the open problem of characterizing update and deletion equivalence. The study is motivated by the following observations.

- *very nature of argumentation*: Argumentation is inherently dynamic. During a discussion or dispute it is natural to come up with new arguments interacting with the former and/or question old arguments as well as attacks between them, respectively. Quite surprisingly, the input of retracting arguments and attacks has not yet received much attention, apart from [7, 6]. We provide novel and complementing results for this line of research.
- *instantiation-based argumentation*: In general, changing the underlying knowledge base cause an update on the abstract level. Furthermore, deleting facts or changing to a more restrictive notion of attack correspond to normal and local deletions. Consequently, characterizations w.r.t. the mentioned dynamic scenarios may help to identify redundant information on the underlying knowledge base.
- *implicit vs. explicit information*: As demonstrated by the introductory example, an update does not necessarily influence the semantics of an AF immediately but it may change the implicit stored information. This means, further dynamic scenario like retracting arguments may make the implicit difference explicit. The equivalences studied in this paper characterize whether two AFs share the same implicit information w.r.t. updates and deletions completing the study in [15, 3].

Our main contributions can be summarized as follows:

- We formalize and characterize update, deletion and local deletion equivalence for nine prominent semantics. In particular, we show that all mentioned notions collapse to identity. Instead of proving this one by one for any considered semantics we follow the line in [2] and provide abstract criteria guaranteeing the coincidence with syntactical identity.
- In this paper we start with a first systematic analysis of kernels. In particular, we build a bridge between abstract properties of kernels and features of equivalence notions characterizable through kernels.

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In this respect, the notion of *context-free* kernels plays a decisive role to show commutativity of kernels and normal deletions. As a by-product we obtain that normal expansion equivalence implies normal deletion equivalence for any considered semantics.

- The second main part of the paper provides characterization theorems for normal deletion equivalence w.r.t. stable, admissible, complete and grounded semantics. Quite surprisingly, the *context-sensitive* σ-*-kernels originally introduced to characterize strong expansion equivalence [3] are crucial to characterize normal deletion equivalence.

The paper is organized as follows. The Section 2 reviews the necessary definitions at work in abstract argumentation frameworks and introduces the new dynamic concepts. In Section 3 we present characterizations for update, deletion as well as local deletion equivalence and introduce the notion of context-free and context-sensitive kernels. Section 4, the second main part, contains the characterization theorems for normal deletion equivalence w.r.t. stable, admissible, complete and grounded semantics. Finally, in Section 5 we discuss related results and present our conclusions. Due to the limited space we omit all proofs.

2 Preliminaries

Argumentation frameworks (AFs) $\mathcal{F} = (A, R)$ are set-theoretically just directed graphs whose nodes are interpreted as *arguments* and whose edges represent *conflicts* between them. We assume a fixed infinite set $\mathcal{U}$ of arguments, called *universe*. All AFs possess a finite set of arguments A being a subset of the universe. The set of all AFs is denoted by $\mathscr{A}$. If $(a, b) \in R$ holds we say that a *attacks* b, or b is *defeated* by a in $\mathcal{F}$. An argument $a \in A$ is *defended* by a set $A' \subseteq A$ in $\mathcal{F}$ if for each $b \in A$ with $(b, a) \in R$, b is defeated by some $a' \in A'$ in $\mathcal{F}$. For a set $E \subseteq A$ we use $R^+_{\mathcal{F}}(E)$ for $E \cup \{b \mid (a, b) \in R, a \in E\}$. This set is called the *range* of E in $\mathcal{F}$. Furthermore, we say that a set $A' \subseteq A$ is *conflict-free* in $\mathcal{F}$ if there are no arguments $a, b \in A'$ such that a attacks b. The set of all conflict-free sets of an AF $\mathcal{F}$ is denoted by $cf(\mathcal{F})$. We call an argument *isolated* if it neither attacks an argument in $\mathcal{F}$ nor is defeated by an argument in $\mathcal{F}$. For an AF $\mathcal{F} = (B, S)$ we use $A(\mathcal{F})$ to refer to B and $R(\mathcal{F})$ to refer to S. We use $L(\mathcal{F}) = \{a \mid (a, a) \in R(\mathcal{F})\}$ for the set of all self-defeating arguments and $NL(\mathcal{F}) = A(\mathcal{F}) \setminus L(\mathcal{F})$ for the set of non-looping arguments in $\mathcal{F}$. Finally, we introduce the union of $\mathcal{F}$ and $\mathcal{G}$ as well as the restriction of $\mathcal{F}$ to a A as expected, namely $\mathcal{F} \cup \mathcal{G} = (A(\mathcal{F}) \cup A(\mathcal{G}), R(\mathcal{F}) \cup R(\mathcal{G})$ and $\mathcal{F}|_A = (A, R(\mathcal{F}) \cap (A \times A))$.

2.1 Semantics

A *semantics* is a function σ which assigns to each $\mathcal{F} = (A, R)$ a set $\sigma(\mathcal{F}) \subseteq \mathfrak{P}(A)$ of σ-*extensions*. Numerous semantics are available. Each of them captures different intuitions about how to reason about conflicting knowledge (see [1] for an excellent overview). We consider here $\sigma \in \{stb, ad, pr, co, gr, ss, stg, id, eg\}$ for stable, admissible, preferred, complete, grounded, semi-stable, stage, ideal and eager semantics [11, 8, 18, 12, 9].

Definition 1. Let $\mathcal{F} = (A, R)$ be an AF and $E \subseteq A$.

1. $E \in stb(\mathcal{F})$ iff $E \in cf(\mathcal{F})$ and for each $a \in A \setminus E$, $(E, a) \in R$,
2. $E \in ad(\mathcal{F})$ iff $E \in cf(\mathcal{F})$ and each $a \in E$ is defended by E in $\mathcal{F}$,
3. $E \in pr(\mathcal{F})$ iff $E \in ad(\mathcal{F})$ and for each $E' \in ad(\mathcal{F})$, $E \notin E'$,
4. $E \in co(\mathcal{F})$ iff $E \in ad(\mathcal{F})$ and if $a \in A$ defended by E in $\mathcal{F}$, $a \in E$,
5. $E \in gr(\mathcal{F})$ iff $E \in co(\mathcal{F})$ and for each $E' \in co(\mathcal{F})$, $E' \notin E$,
6. $E \in ss(\mathcal{F})$ iff $E \in ad(\mathcal{F})$ and if $E' \in ad(\mathcal{F})$, $R^+_{\mathcal{F}}(E) \notin R^+_{\mathcal{F}}(E')$,
7. $E \in stg(\mathcal{F})$ iff $E \in cf(\mathcal{F})$ and if $E' \in cf(\mathcal{F})$, $R^+_{\mathcal{F}}(E) \notin R^+_{\mathcal{F}}(E')$,
8. $E \in id(\mathcal{F})$ iff $E \in ad(\mathcal{F})$, $E \subseteq \bigcap_{P \in pr(\mathcal{F})} P$ and for each $A \in ad(\mathcal{F})$ satisfying $A \subseteq \bigcap_{P \in pr(\mathcal{F})} P$ we have $E \notin A$,
9. $E \in eg(\mathcal{F})$ iff $E \in ad(\mathcal{F})$, $E \subseteq \bigcap_{P \in ss(\mathcal{F})} P$ and for each $A \in ad(\mathcal{F})$ satisfying $A \subseteq \bigcap_{P \in ss(\mathcal{F})} P$ we have $E \notin A$.

In the following we use the term *any considered semantics* (σ) as a shorthand for *any considered semantics* (σ) *introduced in Definition 1*. In [2] several general criteria for comparing and evaluating semantics were introduced. One of those principles is the basic prerequisite of *conflict-freeness* ($\mathcal{CC}$). This principle requires $\sigma(\mathcal{F}) \subseteq cf(\mathcal{F})$ for each AF $\mathcal{F}$ and is satisfied by any considered semantics σ.

2.2 Expansions, Equivalences and Kernels

In the following we introduce typical dynamic scenarios of argumentation firstly introduced in [5, 15]. *Normal expansions* add new arguments and possibly new attacks which concern at least one of the fresh arguments. *Strong expansions* are normal and only add strong arguments, i.e. the added arguments never are attacked by former arguments. Finally, *local expansions* do not introduce any new arguments but possibly new attacks among the old arguments.

Definition 2. An AF $\mathcal{G}$ is an *expansion* of AF $\mathcal{F} = (A, R)$ (for short, $\mathcal{F} \preceq_E \mathcal{G}$) iff $\mathcal{G} = (A \cup B, R \cup S)$ where $A \cap B = R \cap S = \emptyset$. An expansion is called

1. *normal* ($\mathcal{F} \preceq_N \mathcal{G}$) iff $\forall ab\ ((a, b) \in S \rightarrow a \in B \vee b \in B)$,
2. *strong* ($\mathcal{F} \preceq_S \mathcal{G}$) iff $\mathcal{F} \preceq_N \mathcal{G}$, $\forall ab\ ((a, b) \in S \rightarrow (a \notin A \vee b \notin B))$,
3. *local* ($\mathcal{F} \preceq_L \mathcal{G}$) iff $B = \emptyset$.

Equivalence tells us whether two syntactically different object represent the same implicit information w.r.t. a certain property. The decisive properties for the following notions of equivalence are possessing the same σ-extensions w.r.t. the dynamic scenarios introduced above. For the sake of clarity and comprehensibility we use *expansion equivalence* instead of *strong equivalence* (the term originally coined in [15]) to indicate that arbitrary expansions are allowed.

Definition 3. Given a semantics σ. Two AFs $\mathcal{F}$ and $\mathcal{G}$ are

1. *standard equivalent* w.r.t. σ ($\mathcal{F} \equiv^\sigma \mathcal{G}$) iff $\sigma(\mathcal{F}) = \sigma(\mathcal{G})$,
2. *expansion equivalent* w.r.t. σ ($\mathcal{F} \equiv^\sigma_E \mathcal{G}$) iff for each AF $\mathcal{H}$, $\mathcal{F} \cup \mathcal{H} \equiv^\sigma \mathcal{G} \cup \mathcal{H}$ holds,
3. *normal expansion equivalent* w.r.t. σ ($\mathcal{F} \equiv^\sigma_N \mathcal{G}$) iff for each AF $\mathcal{H}$, such that $\mathcal{F} \preceq_N \mathcal{F} \cup \mathcal{H}$ and $\mathcal{G} \preceq_N \mathcal{G} \cup \mathcal{H}$, $\mathcal{F} \cup \mathcal{H} \equiv^\sigma \mathcal{G} \cup \mathcal{H}$ holds,
4. *strong expansion equivalent* w.r.t. σ ($\mathcal{F} \equiv^\sigma_S \mathcal{G}$) iff for each AF $\mathcal{H}$, such that $\mathcal{F} \preceq_S \mathcal{F} \cup \mathcal{H}$ and $\mathcal{G} \preceq_S \mathcal{G} \cup \mathcal{H}$, $\mathcal{F} \cup \mathcal{H} \equiv^\sigma \mathcal{G} \cup \mathcal{H}$ holds,
5. *local expansion equivalent* w.r.t. σ ($\mathcal{F} \equiv^\sigma_L \mathcal{G}$) iff for each AF $\mathcal{H}$, such that $A(\mathcal{H}) \subseteq A(\mathcal{F} \cup \mathcal{G})$, $\mathcal{F} \cup \mathcal{H} \equiv^\sigma \mathcal{G} \cup \mathcal{H}$ holds.

One main result in [15] is that expansion equivalence can be decided via so-called *kernels*. A kernel is a function $k : \mathscr{A} \mapsto \mathscr{A}$ where each $k(\mathcal{F}) = \mathcal{F}^k$ is obtained from $\mathcal{F}$ by deleting certain *redundant* attacks. We say that a relation $\equiv$ is *characterizable through kernels* if there is a kernel k, s.t. $\mathcal{F} \equiv \mathcal{G}$ iff $\mathcal{F}^k = \mathcal{G}^k$. In [3] it was shown that even weaker notions like strong expansion equivalence are characterizable through kernels. Therefore, more sophisticated definitions were introduced, so-called σ-*-*kernels*. Here are the relevant kernel definitions. Observe that kernels are efficiently computable concepts.

Definition 4. Given an AF $\mathcal{F} = (A, R)$ and a semantics σ. We define σ-kernels $\mathcal{F}^{k(\sigma)} = \left(A, R^{k(\sigma)}\right)$ respectively σ-*-kernels $\mathcal{F}^{k^*(\sigma)} = \left(A, R^{k^*(\sigma)}\right)$ whereby

1. $R^{k(stb)} = R \setminus \{(a, b) \mid a \neq b, (a, a) \in R\}$,
2. $R^{k(ad)} = R \setminus \{(a, b) \mid a \neq b, (a, a) \in R, \{(b, a), (b, b)\} \cap R \neq \emptyset\}$,

3. $R^{k(gr)} = R \setminus \{(a,b) \,|\, a \neq b, (b,b) \in R, \{(a,a),(b,a)\} \cap R \neq \emptyset\}$,
4. $R^{k(co)} = R \setminus \{(a,b) \,|\, a \neq b, (a,a),(b,b) \in R\}$,
5. $R^{k^*(ad)} = R \setminus \{(a,b) \,|\, a \neq b, ((a,a) \in R \wedge \{(b,a),(b,b)\} \cap R \neq \emptyset)$, $\vee ((b,b) \in R \wedge \forall c\, ((b,c) \in R \rightarrow \{(a,c),(c,a),(c,c),(c,b)\} \cap R \neq \emptyset))\}$,
6. $R^{k^*(gr)} = R \setminus \{(a,b) \,|\; a \neq b, ((b,b) \in R \wedge \{(a,a),(b,a)\} \cap R \neq \emptyset)$ $\vee ((b,b) \in R \wedge \forall c\, ((b,c) \in R \rightarrow \{(a,c),(c,a),(c,c)\} \cap R \neq \emptyset))\}$,
7. $R^{k^*(co)} = R \setminus \{(a,b) \,|\, a \neq b, ((a,a),(b,b) \in R) \vee ((b,b) \in R$ $\wedge (b,a) \notin R \wedge \forall c\ ((b,c) \in R \rightarrow \{(a,c),(c,a),(c,c)\} \cap R \neq \emptyset))\}$.

A kernel k is said to be *node-* or *loop-preserving* iff for each AF $\mathcal{F}$, $A(\mathcal{F}) = A(\mathcal{F}^k)$ or $L(\mathcal{F}) = L(\mathcal{F}^k)$, respectively. Observe that both properties are satisfied by any kernel introduced above. It will not escape the attentive reader that no corresponding *stb*-*-kernel has been introduced. The following theorem shows that there is no need for this since even strong expansion equivalence is characterizable through the traditional *stb*-kernel.

Theorem 1. *[3, cf. Figure 2] Let* $\sigma \in \{stb, ad, co, gr\}$ *and define* $k^*(stb) = k(stb)$. *For any AFs* $\mathcal{F}$ *and* $\mathcal{G}$ *we have:*

$$\mathcal{F} \equiv^\sigma_N \mathcal{G} \Leftrightarrow \mathcal{F}^{k(\sigma)} = \mathcal{G}^{k(\sigma)} \text{ and } \mathcal{F} \equiv^\sigma_S \mathcal{G} \Leftrightarrow \mathcal{F}^{k^*(\sigma)} = \mathcal{G}^{k^*(\sigma)}.$$

2.3 New Concepts: Update and Deletion

We now introduce the most general form of dynamic scenarios, so-called *updates* where arguments and attacks can be deleted and added. Furthermore, we consider certain sub-classes of *deletions*[3] representing the natural counter-parts (more precisely, inverse operations) to arbitrary, normal and local expansions.

Definition 5. Given an AF $\mathcal{F} = (A, R)$, a set of arguments B and a set of attacks S as well as a further AF $\mathcal{H}$. The AF

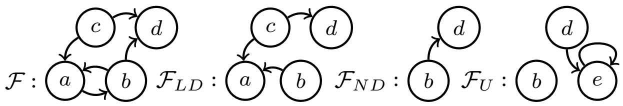
$$\mathcal{G} = (\mathcal{F} \setminus [B,S]) \cup \mathcal{H} := ((A, R \setminus S)|_{A \setminus B}) \cup \mathcal{H}$$

is called an *update* of $\mathcal{F}$ (for short, $\mathcal{F} \asymp_U \mathcal{G}$). An update is called a

1. *deletion* ($\mathcal{F} \succeq_D \mathcal{G}$) iff $\mathcal{H} = \emptyset$,
2. *normal deletion* ($\mathcal{F} \succeq_{ND} \mathcal{G}$) iff ($\mathcal{F} \succeq_D \mathcal{G}$) and $S = \emptyset$,
3. *local deletion* ($\mathcal{F} \succeq_{LD} \mathcal{G}$) iff $\mathcal{F} \succeq_D \mathcal{G}$ and $B = \emptyset$.

Normal deletions retract arguments and their corresponding attacks. *Local deletions* in contrast delete attacks only. Note that $[B,S]$ does not necessarily have to be an AF. Therefore we use $[B,S]$ instead of (B,S). If clear from context we use B and S instead of $[B,\emptyset]$ or $[\emptyset,S]$.

Example 1. Let $B = \{a,c\}$, $S = \{(a,b),(b,d)\}$ and $\mathcal{H} = (\{d,e\},\{(d,e),(e,e)\})$. The AFs $\mathcal{F}_{LD} = \mathcal{F} \setminus S$, $\mathcal{F}_{ND} = \mathcal{F} \setminus B$ and $\mathcal{F}_U = (\mathcal{F} \setminus [B,S]) \cup \mathcal{H}$ represent a local deletion, a normal deletion and an update of $\mathcal{F}$. Conversely, $\mathcal{F}$ is a local resp. normal expansion of $\mathcal{F}_{LD}$ or $\mathcal{F}_{ND}$.



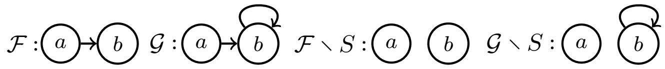
We now introduce the corresponding equivalence notions.

Definition 6. Given a semantics σ. Two AFs $\mathcal{F}$ and $\mathcal{G}$ are

1. *update equivalent* w.r.t. σ ($\mathcal{F} \equiv^\sigma_U \mathcal{G}$) iff for any pair $[B,S]$ and any AF $\mathcal{H}$ we have: $(\mathcal{F} \setminus [B,S]) \cup \mathcal{H} \equiv^\sigma (\mathcal{G} \setminus [B,S]) \cup \mathcal{H}$,
2. *deletion equivalent* w.r.t. σ ($\mathcal{F} \equiv^\sigma_D \mathcal{G}$) iff for any pair $[B,S]$ we have: $\mathcal{F} \setminus [B,S] \equiv^\sigma \mathcal{G} \setminus [B,S]$,
3. *normal deletion equivalent* w.r.t. σ ($\mathcal{F} \equiv^\sigma_{ND} \mathcal{G}$) iff for any set of arguments B we have: $\mathcal{F} \setminus B \equiv^\sigma \mathcal{G} \setminus B$,
4. *local deletion equivalent* w.r.t. σ ($\mathcal{F} \equiv^\sigma_{LD} \mathcal{G}$) iff for any set of attacks S we have: $\mathcal{F} \setminus S \equiv^\sigma \mathcal{G} \setminus S$,

To familiarize the reader with the new notions we give the following simple example.

Example 2. Consider the AFs $\mathcal{F}, \mathcal{G}$. Let $S = \{(a,b)\}$ and $B = \{a\}$. Thus, $\{a,b\} \in \sigma(\mathcal{F} \setminus S) \setminus \sigma(\mathcal{G} \setminus S)$ and $\{b\} \in \sigma(\mathcal{F} \setminus B) \setminus \sigma(\mathcal{G} \setminus B)$ for any considered semantics, i.e. $\mathcal{F} \not\equiv^\sigma_{LD} \mathcal{G}$ as well as $\mathcal{F} \not\equiv^\sigma_{ND} \mathcal{G}$.



The following figure provides a first overview about the interrelations that arise from the definitions immediately. It strikes the eye that the newly introduced notions complement the existing ones naturally. For equivalences Φ and Ψ, $\Phi \subseteq \Psi$ iff there is a link between Φ and Ψ.

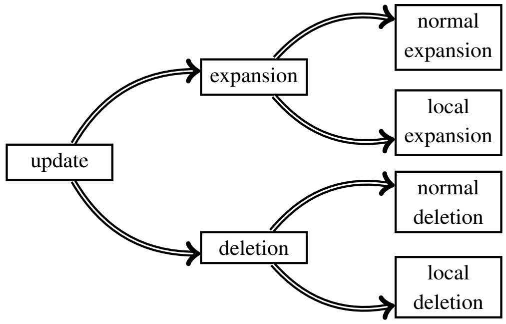


Figure 1. Preliminary Relations

We close this section with some further reflections on the relations considered in this section. First of all, it is easy to see that any relation introduced in Definition 3 and 6 is indeed an equivalence. As an aside, expansion equivalence is even a congruence for the operator $\cup$. This means, $\mathcal{F} \equiv^\sigma_E \mathcal{G}$ and $\mathcal{F}' \equiv^\sigma_E \mathcal{G}'$ implies $\mathcal{F} \cup \mathcal{F}' \equiv^\sigma_E \mathcal{G} \cup \mathcal{G}'$.[4] Additionally, certain properties are implied by definition, e.g. if $\mathcal{F} \equiv^\sigma_{ND} \mathcal{G}$, then for any set of arguments B, $\mathcal{F} \setminus B \equiv^\sigma_{ND} \mathcal{G} \setminus B$. While this property (so-called *subset-inheritance*) is not surprising for normal deletion equivalence it is not expected for normal expansion equivalence. It is one main result of this paper to build a bridge between abstract properties of kernels and the fulfillment of subset-inheritance of the corresponding equivalence notions.

3 Comparing Update, Deletion and Expansion Equivalence

The main goal of this section is to refine Figure 1. In particular we will compare expansion and deletion equivalence as well as their normal and local versions. We will see that the equivalences w.r.t. update, deletion as well as local deletion coincide with identity for any considered semantics. Moreover, we identified two weak criteria for argumentation semantics guaranteeing such a behavior. In the second part we show that any equivalence characterizable through a context-free and node-preserving kernel satisfies subset-inheritance. Context-free kernels allow to consider local parts of a given AF when deciding redundancy of attacks. As a by-product we will show that normal expansion equivalence implies normal deletion equivalence for any considered semantics.

[3] Boella et. al [7] called this kind of dynamic scenarios *abstractions*.

[4] The proof is astonishing simple, namely $\mathcal{F} \cup \mathcal{F}' \equiv^\sigma_E \mathcal{G} \cup \mathcal{F}' \equiv^\sigma_E \mathcal{G} \cup \mathcal{G}'$.

3.1 (Weak) Requirements for Identity

It is the surprising result of this section that local deletion (and therefore deletion as well as update) equivalence collapse to identity for any semantics satisfying conflict-freeness and the newly introduced principle of *isolate-inclusion* ($\mathcal{II}$). The latter is fulfilled by a semantics σ iff for any AF $\mathcal{F}$, the set of all isolated arguments is contained in at least one σ-extension. Observe that any considered semantics apart from stable semantics satisfy $\mathcal{II}$.[5]

Theorem 2. *Given a semantics σ satisfying $\mathcal{CF}$ and $\mathcal{II}$. For any two AFs $\mathcal{F}$ and $\mathcal{G}$ we have:*

$$\mathcal{F} \equiv_U^\sigma \mathcal{G} \Leftrightarrow \mathcal{F} \equiv_D^\sigma \mathcal{G} \Leftrightarrow \mathcal{F} \equiv_{LD}^\sigma \mathcal{G} \Leftrightarrow \mathcal{F} = \mathcal{G}.$$

In order to refine Figure 1 we obtain the following relations.

Corollary 3. *Given AFs $\mathcal{F}$, $\mathcal{G}$. For any considered semantics σ,*

1. $\mathcal{F} \equiv_U^\sigma \mathcal{G} \Leftrightarrow \mathcal{F} \equiv_D^\sigma \mathcal{G} \Leftrightarrow \mathcal{F} \equiv_{LD}^\sigma \mathcal{G} \Leftrightarrow \mathcal{F} = \mathcal{G}$ *($k = id$)*
2. $\mathcal{F} \equiv_D^\sigma \mathcal{G} \Rightarrow \mathcal{F} \equiv_E^\sigma \mathcal{G}$, *(deletion vs. expansion)*
3. $\mathcal{F} \equiv_{LD}^\sigma \mathcal{G} \Rightarrow \mathcal{F} \equiv_L^\sigma \mathcal{G}$. *(local versions)*

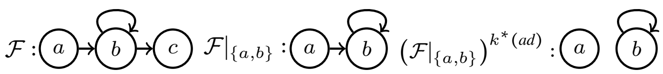
3.2 Context-free Kernels

In this section we start with a first systematic analysis of equivalence relations characterizable through *context-free* kernels. We start with the formal definition.

Definition 7. A kernel $k : \mathscr{A} \times \mathscr{A}$, $\mathcal{F} \mapsto \mathcal{F}^k$ is said to be *context-free* iff $(a,b) \in R(\mathcal{F}^k) \Leftrightarrow (a,b) \in R((\mathcal{F}|_{\{a,b\}})^k)$. Otherwise, k is called *context-sensitive*.

This means, in case of context-free kernels the decision whether (a,b) has to be deleted does not depend on further arguments than a and b. Put differently, the reason of being redundant stems from the arguments themselves. The following Example 3 shows that the *ad*-*-kernel is context-sensitive.

Example 3. Consider the AF $\mathcal{F}$. Applying Definition 4 we obtain $\mathcal{F} = \mathcal{F}^{k^*(ad)}$. We observe $(a,b) \in \mathcal{F}^{k^*(ad)}$. On the other hand, $(a,b) \notin (\mathcal{F}|_{\{a,b\}})^{k^*(ad)}$ since $(b,b) \in R(\mathcal{F}|_{\{a,b\}})$ and there are no further c's, s.t. $(b,c) \in R(\mathcal{F}|_{\{a,b\}})$.



Context-sensitivity carries over to all σ-*-kernels. Furthermore, all traditional σ-kernels are context-free as stated below.

Proposition 4. *The σ-kernels $k(stb), k(ad), k(gr)$ and $k(co)$ are context-free and the σ-*-kernels $k^*(ad), k^*(gr)$ as well as $k^*(co)$ are context-sensitive.*

We show now the decisive property of context-free and node-preserving kernels paving the way for the main theorem. Loosely speaking, equality of kernels of normal deletions guarantees equality of normal deletions of kernels and vice versa. We point out that any σ-*-kernel introduced in Definition 4 does not satisfy the mentioned commutativity of applying kernels and normal deletions.[6]

Lemma 5. *Given two AFs $\mathcal{F}$,$\mathcal{G}$ and a context-free as well as node-preserving kernel k. For any set of arguments B we have,*

$$(\mathcal{F} \smallsetminus B)^k = (\mathcal{G} \smallsetminus B)^k \Leftrightarrow \mathcal{F}^k \smallsetminus B = \mathcal{G}^k \smallsetminus B.$$

Now we are prepared to prove the main theorem showing subset-inheritance of any equivalence relations characterizable through context-free and node-preserving kernels.

Theorem 6. *Given a context-free, node-preserving kernel k as well as an equivalence $\equiv^k \subseteq \mathscr{A} \times \mathscr{A}$ characterizable through k, then*

$$\mathcal{F} \equiv^k \mathcal{G} \Leftrightarrow \text{ for any } B : \mathcal{F} \smallsetminus B \equiv^k \mathcal{G} \smallsetminus B.$$

Since normal expansion equivalence w.r.t. any considered semantics is characterizable through traditional σ-kernels we obtain subset-inheritance. Remember that normal expansion equivalence is concerned with dynamic scenarios where new arguments and attacks come into play but former arguments and corresponding attacks remain unchanged. From this perspective satisfying subset-inheritance constitutes a remarkable result. Furthermore we derive the missing relation between normal expansion and normal deletion equivalence completing statements 2 and 3 in Corollary 3.

Corollary 7. *Given two AFs $\mathcal{F}$ and $\mathcal{G}$ and a set of arguments B, then for any considered semantics σ, we have:*

1. $\mathcal{F} \equiv_N^\sigma \mathcal{G} \Rightarrow \mathcal{F} \smallsetminus B \equiv_N^\sigma \mathcal{G} \smallsetminus B$ *(subset-inheritance)*
2. $\mathcal{F} \equiv_N^\sigma \mathcal{G} \Rightarrow \mathcal{F} \equiv_{ND}^\sigma \mathcal{G}$. *(normal versions)*

3.3 Summary

The following Figure 2 summarizes the results presented in this section. In contrast to Figure 1 we obtain a more compact picture of the interrelations between all considered equivalence notions. We mention an interesting kind of inversion w.r.t. expansions and deletions. It was one main result in [3] that expansion and normal expansion equivalence coincide. Here, we have shown that deletion and local deletion equivalence coincide. In [15] it was shown that local expansion equivalence needs a more involved characterization and in the following we will see that the same applies to normal deletion equivalence.

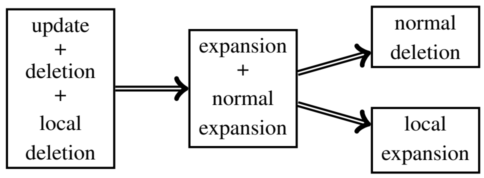


Figure 2. Non-trivial Relations

4 Characterizing Normal Deletion Equivalence

In this section we present characterization theorems for normal deletion equivalence w.r.t. stable, admissible, complete and grounded semantics. We point out two special features of normal deletion equivalence. Firstly, in contrast to all other dynamic equivalence notions mentioned in this paper we have: for any AF $\mathcal{F}$ there exist infinitely many normal deletion equivalent frameworks $\mathcal{G}$. Secondly, it is the unexpected result of this section that σ-*-kernels, originally introduced to characterize strong expansion equivalence (compare Theorem 1), play a decisive role w.r.t. characterizing normal deletion equivalence.

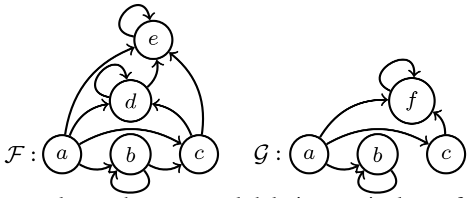
4.1 Stable Semantics

What are the characterizing properties for normal deletion equivalence w.r.t. stable semantics? We already know that normal expansion equivalence is sufficient for normal deletion equivalence for any considered semantics (statement 2, Corollary 7). The converse does not hold as shown by the example below.

[5] A counter-example is given by $\mathcal{G} \smallsetminus S$ depicted in Example 2. Obviously, a is isolated but $stb(\mathcal{G} \smallsetminus S) = \emptyset$. Nevertheless, Theorem 2 applies to stable semantics too which can be shown with reasonable effort.

[6] A counter-example is already given in Example 3. Set $\mathcal{F} = \mathcal{F}$, $\mathcal{G} = \mathcal{F}|_{\{a,b\}}$, $B = \{c\}$ and $k = k^*(ad)$, then $(\mathcal{F} \smallsetminus B)^k = (\mathcal{G} \smallsetminus B)^k$ but $\mathcal{F}^k \smallsetminus B \neq \mathcal{G}^k \smallsetminus B$.

Example 4. Consider the following AF $\mathcal{F}$ and $\mathcal{G}$. Obviously, $A(\mathcal{F}) \neq A(\mathcal{G})$. Thus, $\mathcal{F} \not\equiv_N^{stb} \mathcal{G}$ since the characterizing *stb*-kernel is node-preserving (Theorem 1). Furthermore, we observe $stb(\mathcal{F}) = stb(\mathcal{G}) = \{\{a\}\}$. Even more, for any set of arguments B we have[7]: $\mathcal{F} \setminus B \equiv^{stb} \mathcal{G} \setminus B$. Consequently, $\mathcal{F} \equiv_{ND}^{stb} \mathcal{G}$.



What can we learn about normal deletion equivalence from these two frameworks? One necessary condition is obviously standard equivalence. A closer inspection of Example 4 reveals two promising properties fulfilled by both AFs. Firstly, all non-shared arguments are self-defeating (arguments e, d and f) and secondly, all non-looping arguments being simultaneously in $\mathcal{F}$ and $\mathcal{G}$ (arguments a and c) attack all arguments in $A(\mathcal{F}) \setminus A(\mathcal{G})$ or $A(\mathcal{G}) \setminus A(\mathcal{F})$, respectively. The following definition describes these two properties formally. We use Δ for symmetric difference.

Definition 8. For any two AFs $\mathcal{F} = (A, R)$ and $\mathcal{G} = (A', R')$,

1. $Loop(\mathcal{F},\mathcal{G}) \Leftrightarrow_{def} L(\mathcal{F} \cup \mathcal{G}|_{A\Delta A'}) = A\Delta A'$,
2. $Att^{stb}(\mathcal{F},\mathcal{G}) \Leftrightarrow_{def} \forall b \in A \setminus A'\ \forall a \in NL(\mathcal{F}|_{A\cap A'}) : (a,b) \in R \wedge \forall b \in A' \setminus A\ \forall a \in NL(\mathcal{G}|_{A\cap A'}) : (a,b) \in R'$.

The following proposition states that the mentioned properties are indeed necessary for being normal deletion equivalent.

Proposition 8. *For any two AFs $\mathcal{F}$ and $\mathcal{G}$ we have:*

$$\mathcal{F} \equiv_{ND}^{stb} \mathcal{G} \Rightarrow Loop(\mathcal{F},\mathcal{G}), Att^{stb}(\mathcal{F},\mathcal{G}) \text{ and } \mathcal{F} \equiv^{stb} \mathcal{G}.$$

The following simple AFs show that we still not reached a characterization.

Example 5 (Example 4 continued)**.** Let $\mathcal{G}' = \mathcal{G} \cup (\{b,c\},\{(c,b)\})$. It easy to see that $Loop(\mathcal{F},\mathcal{G}')$ and $Att^{stb}(\mathcal{F},\mathcal{G}')$. Furthermore, we observe $stb(\mathcal{F}) = stb(\mathcal{G}') = \{\{a\}\}$. Define $B = \{a,e,d,f\}$. Hence, $stb(\mathcal{F} \setminus B) = \emptyset \neq \{\{c\}\} = stb(\mathcal{G} \setminus B)$. This means, $\mathcal{F} \not\equiv_{ND}^{stb} \mathcal{G}'$.

What is the decisive difference between Examples 4 and 5? The crucial difference is that although the entire AFs $\mathcal{F}$ and $\mathcal{G}$ do not possess the same stable-kernels their restrictions to the shared arguments do $\left(\text{i.e., } (\mathcal{F}|_{\{a,b,c\}})^{k(stb)} = (\mathcal{G}|_{\{a,b,c\}})^{k(stb)}\right)$ in contrast to $\mathcal{F}$ and $\mathcal{G}'$. And indeed, this can be shown in general.

Proposition 9. *For two AFs $\mathcal{F} = (A,R)$ and $\mathcal{G} = (A',R')$ we have:*

$$\mathcal{F} \equiv_{ND}^{stb} \mathcal{G} \Rightarrow (\mathcal{F}|_{A\cap A'})^{k(stb)} = (\mathcal{G}|_{A\cap A'})^{k(stb)}$$

Finally, we are prepared to present the first characterization theorem w.r.t. normal deletion equivalence. The following theorem states that the stable-kernel-condition stated above $\left((\mathcal{F}|_{A\cap A'})^{k(stb)} = (\mathcal{G}|_{A\cap A'})^{k(stb)}\right)$, the loop-condition, i.e. any non-shared argument has to be self-attacking ($Loop(\mathcal{F},\mathcal{G})$) as well as the attack-condition, i.e. all non-looping arguments being in $\mathcal{F}$ and $\mathcal{G}$ has to attack arguments belonging exclusively to $\mathcal{F}$ or $\mathcal{G}$ $\left(Att^{stb}(\mathcal{F},\mathcal{G})\right)$ are not only necessary but even sufficient if considered collectively. We want to mention that all three relations satisfy subset-inheritance which play a major role in the proof.

[7] We encourage the reader to verify this assertion for some sets of arguments.

Theorem 10. *Given two AFs $\mathcal{F}$, $\mathcal{G}$ and define $I = A(\mathcal{F}) \cap A(\mathcal{G})$:*

$$\mathcal{F} \equiv_{ND}^{stb} \mathcal{G} \Leftrightarrow Loop(\mathcal{F},\mathcal{G}),\ Att^{stb}(\mathcal{F},\mathcal{G}), (\mathcal{F}|_I)^{k(stb)} = (\mathcal{G}|_I)^{k(stb)}.$$

Consequently, in the special case where both AFs share the same arguments we obtain the coincidence of normal deletion and normal expansion equivalence as well as strong expansion equivalence.

Corollary 11. *For any two AFs $\mathcal{F}$ and $\mathcal{G}$, s.t. $A(\mathcal{F}) = A(\mathcal{G})$*

$$\mathcal{F} \equiv_{ND}^{stb} \mathcal{G} \Leftrightarrow \mathcal{F}^{k(stb)} = \mathcal{G}^{k(stb)} \Leftrightarrow \mathcal{F} \equiv_N^{stb} \mathcal{G} \Leftrightarrow \mathcal{F} \equiv_S^{stb} \mathcal{G}.$$

4.2 The Role of Context-sensitive Kernels

Let us consider now admissible, complete and grounded semantics. The following example gives the first hint that these semantics behave differently compared with the characterization of stable semantics. More precisely, traditional σ-kernels do not serve as parts of the characterizations.

Example 6. Let $\sigma \in \{ad, co, gr\}$ and consider the following AFs $\mathcal{F}$ and $\mathcal{G}$. We observe $\mathcal{F}^{k(\sigma)} = \mathcal{F} \neq \mathcal{G} = \mathcal{G}^{k(\sigma)}$. On the other hand, it is easy to see that $\sigma(\mathcal{F} \setminus B) = \sigma(\mathcal{G} \setminus B)$ for any set of arguments B. This means, $\mathcal{F} \equiv_{ND}^{\sigma} \mathcal{G} \not\Rightarrow \mathcal{F}^{k(\sigma)} = \mathcal{G}^{k(\sigma)}$ even if the considered AFs share the same arguments (compare Corollary 11).

A detailed inspection of AFs $\mathcal{F}$ and $\mathcal{G}$ reveals that their corresponding σ-*-kernels coincide $\left(\mathcal{F}^{k^*(\sigma)} = \mathcal{G}^{k^*(\sigma)} = G\right)$ and surprisingly, this observation holds in general as stated in the following proposition. The proof contains very lengthy case-distinctions which has to be done for any semantics separately.

Proposition 12. *Let $\sigma \in \{ad, co, gr\}$. For any two AFs $\mathcal{F}$ and $\mathcal{G}$:*

$$\mathcal{F} \equiv_{ND}^{\sigma} \mathcal{G} \Rightarrow (\mathcal{F}|_{A\cap A'})^{k^*(\sigma)} = (\mathcal{G}|_{A\cap A'})^{k^*(\sigma)}.$$

We have already shown that σ-*-kernels do not satisfy the exchangeability property stated in Lemma 5. Nevertheless, we can prove the following characteristic needed to prove the main theorem.

Lemma 13. *Let $\sigma \in \{ad, co, gr\}$. For any two AFs $\mathcal{F}$ and $\mathcal{G}$ and any set of arguments B we have,*

$$(\mathcal{F} \setminus B)^{k^*(\sigma)} = (\mathcal{G} \setminus B)^{k^*(\sigma)} \Leftarrow \mathcal{F}^{k^*(\sigma)} = \mathcal{G}^{k^*(\sigma)}.$$

Analogously to Theorem 6 we show subset-inheritance of any equivalence characterizable through a certain σ-*-kernel. The proof of Theorem 6 relies on abstract properties of kernels in contrast to this theorem which has to be shown one by one for any kernel.

Theorem 14. *Let $\sigma \in \{ad, co, gr\}$. Given an equivalence relation $\equiv^{k^*(\sigma)} \subseteq \mathscr{A} \times \mathscr{A}$ characterizable through $k^*(\sigma)$, then*

$$\mathcal{F} \equiv^{k^*(\sigma)} \mathcal{G} \Leftrightarrow \text{ for any } B : \mathcal{F} \setminus B \equiv^{k^*(\sigma)} \mathcal{G} \setminus B.$$

It was one main result in [3] that strong expansion equivalence is characterized through σ-*-kernels for $\sigma \in \{ad, co, gr\}$ (compare Theorem 1). Consequently, we obtain subset-inheritance for strong expansion equivalence. Furthermore, identical σ-*-kernels are sufficient for normal deletion equivalence and even characterizing in case AFs sharing the same arguments.

Corollary 15. *Let $\sigma \in \{ad, co, gr\}$. Given two AFs $\mathcal{F} = (A,R)$ and $\mathcal{G} = (A',R')$ and a set of arguments B, then:*

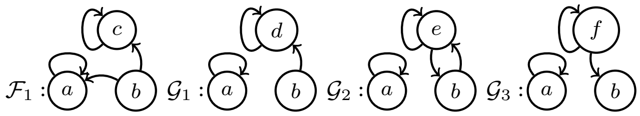
1. $\mathcal{F} \equiv_S^{\sigma} \mathcal{G} \Rightarrow \mathcal{F} \setminus B \equiv_S^{\sigma} \mathcal{G} \setminus B$, *(subset-inheritance)*

2. $\mathcal{F}^{k^*(\sigma)} = \mathcal{G}^{k^*(\sigma)} \Rightarrow \mathcal{F} \equiv^{\sigma}_{ND} \mathcal{G}$, *(sufficient criterion)*
3. *If* $A = A'$, *then* $\mathcal{F} \equiv^{\sigma}_{ND} \mathcal{G} \Leftrightarrow \mathcal{F}^{k^*(\sigma)} = \mathcal{G}^{k^*(\sigma)} \Leftrightarrow \mathcal{F} \equiv^{\sigma}_{S} \mathcal{G}$. *(conditional coincidence)*

In order to detect the missing ingredients for characterization theorems consider the following example.

Example 7 (Example 6 continued)**.** It can be checked that

1. $\mathcal{F}_1 \equiv^{\sigma}_{ND} \mathcal{G}_1$ for any $\sigma \in \{ad, co, gr\}$,
2. $\mathcal{F}_1 \equiv^{ad}_{ND} \mathcal{G}_2$ but $\mathcal{F}_1 \not\equiv^{\tau}_{ND} \mathcal{G}_1$ for any $\tau \in \{co, gr\}$ (set $B = \{a\}$),
3. $\mathcal{F}_1 \not\equiv^{\sigma}_{ND} \mathcal{G}_3$ for any $\sigma \in \{ad, co, gr\}$ (again, consider $B = \{a\}$).



The AFs presented in Example 7 give rise to the assumption that the former loop-condition, i.e. all non-shared arguments are self-defeating even applies to admissible, complete as well as grounded semantics. Furthermore, the stable-attack-condition has to be weakened in accordance with the less demanding semantics, namely instead of "attack all non-shared arguments" we have "counter-attack if attacked" for admissible semantics or "it is forbidden to be attacked" for complete and grounded semantics. The following definition takes this observation into account.

Definition 9. Let $\sigma \in \{co, gr\}$, $\mathcal{F} = (A, R)$ and $\mathcal{G} = (A', R')$, then

1. $Att^{ad}(\mathcal{F},\mathcal{G}) \Leftrightarrow_{def} \forall b \in A \smallsetminus A' \; \forall a \in NL(\mathcal{F}|_{A\cap A'}) : ((b,a) \in R \rightarrow (a,b) \in R) \wedge \forall b \in A' \smallsetminus A \; \forall a \in NL(\mathcal{G}|_{A\cap A'}) : ((b,a) \in R' \rightarrow (a,b) \in R')$
2. $Att^{\sigma}(\mathcal{F},\mathcal{G}) \Leftrightarrow_{def} \forall b \in A \smallsetminus A' \; \forall a \in NL(\mathcal{F}|_{A\cap A'}) : (b,a) \notin R \wedge \forall b \in A' \smallsetminus A \; \forall a \in NL(\mathcal{G}|_{A\cap A'}) : (b,a) \notin R'$

Finally, we present the characterization theorems for admissible, complete and grounded semantics.

Theorem 16. *Let* $\sigma \in \{ad, co, gr\}$. *Given two AFs* $\mathcal{F} = (A, R)$ *and* $\mathcal{G} = (A', R')$ *and let* $I = A \cap A'$,

$$\mathcal{F} \equiv^{\sigma}_{ND} \mathcal{G} \Leftrightarrow Loop(\mathcal{F},\mathcal{G}),\; Att^{\sigma}(\mathcal{F},\mathcal{G}),\; (\mathcal{F}|_I)^{k^*(\sigma)} = (\mathcal{G}|_I)^{k^*(\sigma)}.$$

5 Conclusions and Related Work

We studied and characterized new dynamic notions of equivalence in the context of Dung's abstract AFs. In particular, we considered the evolution of AFs in its most general form, so-called updates where arguments and attacks can be deleted and added. In contrast to logic programming where update equivalent programs are almost identical [14] we prove their exact identity in case of AFs. The result was shown for any semantics satisfying the basic requirements of conflict-freeness and isolate-inclusion. Moreover, and quite surprisingly, these weak properties guarantee that deletion equivalence and even local deletion equivalence, where the retraction of attacks is considered, collapse to identity. Put differently, any argument/attack may play a crucial role with respect to further evaluations if updates, deletions or local deletions are considered.

In contrast, normal deletion equivalence, where the retraction of arguments is considered, is exceptional in several regards. Firstly, the characterizations for stable, admissible, complete and grounded semantics rely on kernel definitions originally introduced to deal with certain kinds of expansions [15, 3]. Secondly, normal deletion equivalent AFs do not even have to share the same arguments and thus give space for simplifications.

Dynamic scenarios including retractions are addressed in few works only. Boella et al. studied how the semantics of an AF remains unchanged after the removal of a single argument or attack [7]. Their results allow to identify redundant attacks in order to preserve existing extension. The approach does not handle multiple extension semantics. In [6] the authors studied the impact of the removal of a single argument together with its attacks. They introduced the notion of narrowing as well as expansive change and provide some first conditions for obtaining a certain kind of change. We believe that our results complement this line of research.

There are at least two natural directions in which this research can be further pursued. Due to the connection between abstract properties of kernels and features of equivalence notions characterizable through kernels it seems worthwhile to study and analyze further properties of kernels. This line of research can be seen as an analogon to the principle-based evaluation of semantics initiated in [2]. A further future goal is to use our results in context of instantiation-based argumentation [10]. Since normal deletions naturally occur if we retract information from the underlying knowledge base we may use normal deletion equivalence to identify redundant parts of the knowledge base.

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