

# Faustian Dynamics in Sarkar's Social Cycle

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**Abstract.** Recently Bai and Lagunoff [1] have studied the question of Faustian Dynamics (FD) of policy and political power using a formal game theoretic framework. Specifically, they studied the conflict between implementing a (personally) optimal policy and maintaining political power. However, these works assumed that the policy makers come from the same population that empowers them.

In contrast, in this paper we study a society that has a political class, hence policy makers are detached from the general population. Specifically, we study a society where members of the political class compete via pre-election propaganda campaigns – a competition form characteristic to modern democracies. We assume that the society is characterised by an inherent cyclical Faustian dynamics, such as the Sarkar Cycle, and concentrate on the strategic behaviour of the political class members. We show that their propaganda over time tends to become extreme (single issue oriented). In addition, the equilibrium behaviour of the political class members precludes them from adopting a persistent agenda. Rather, to optimise their political gain over time, they must lack any permanent agenda or views.

## 1 Sarkar Elections Model

Consider the following scenario, that we term *Sarkar Elections (SE)*. Two political parties repeatedly compete for votes in elections. Each party is given a unit of resources for propaganda and has to distribute it between three key issues,  $\mathcal{A}$  (e.g., Defence),  $\mathcal{B}$  (Science) and  $\mathcal{C}$  (Economics). In other words, the space of *propaganda resource allocation (PRA)* strategies available to the party  $i$  during  $t$ 'th election is described by distributions  $q_t^i(\cdot) \in \Delta(\mathcal{I})$ , where  $\mathcal{I} = \{\mathcal{A}, \mathcal{B}, \mathcal{C}\}$ .

During any given election the public's interest places the issues in one of the three orders of importance described by a function of the form  $\sigma : \mathcal{I} \rightarrow \{1, 2, 3\}$ , where 1 is the highest and 3 is the lowest level: (i)  $\sigma^A = (1, 2, 3)$ , so that the order of preference between issues is  $\mathcal{A} > \mathcal{B} > \mathcal{C}$ ; (ii)  $\sigma^B = (2, 3, 1)$  for the ordering  $\mathcal{B} > \mathcal{C} > \mathcal{A}$ ; and (iii)  $\sigma^C = (3, 1, 2)$  for the ordering  $\mathcal{C} > \mathcal{A} > \mathcal{B}$ . Between elections the public's preference shifts with probability  $0 < \alpha \in [0, 1]$  along the cycle  $\sigma^A \rightarrow \sigma^B \rightarrow \sigma^C \rightarrow \sigma^A \rightarrow \dots$

In our model the general public remembers who has won the previous elections,  $x_t^{win} \in \{1, 2\}$ , and keeps a trust vector for each party  $i$ ,  $r_t^i \in [0, 1]^3$ , that expresses how likely it is for a voter to yield to the  $i$ 'th party propaganda on each issue. This memory is short term and changes as follows. Let  $\sigma_t$  be the current issue preference order of the public, and let  $q_t^i$  be resource distributions used by the party  $i$ . Also let  $I_t^*(i) = \arg \min_{I \in \mathcal{J}(i)} \sigma_t(I)$ , where  $\mathcal{J}(i) = \arg \max_{I \in \mathcal{I}} q_t^i(I)$ . That is,  $\mathcal{J}(i)$  is the set of issues to which the party  $i$  devoted most resources, and we assume that the public assigns the highest trust to just one issue in  $\mathcal{J}(i)$ , and will use  $\sigma_t$ 's ordering to break ties. This sets the trust

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vector during the next election to  $r_{t+1}^i(I) = (1 - \epsilon)\delta(I, I_t^*(i)) + \epsilon$ , where  $\epsilon > 0$  is a small constant, and  $\delta$  is Kronecker delta. Since the trust vector is computed based on the PRA  $q_t^i$ , we say that  $r_{t+1}^i$  is the *trust signature* of  $q_t^i$ .

The winner of the election,  $x_{t+1}^{win}$ , is the party that received the most votes at  $t$ 'th elections, and ties are broken by preferring the party that won the previous elections. We assume that the number of votes received by a party is directly proportional to the probability of a voter to be exposed to a particular combination of propaganda, and the trust that the voter has towards the party. The utility of party  $i$  during  $t$ 'th election with issue importance ordering given by  $\sigma_t$  is:

$$\begin{aligned} u^i(q_t^i, q_t^{-i} | x_t^{win}, r_t^i) = \\ \rho \left[ \epsilon r_t^i(\sigma_t^{-1}(2)) q_t^i(\sigma_t^{-1}(2)) \left( q_t^{-i}(\sigma_t^{-1}(1)) + q_t^{-i}(\sigma_t^{-1}(3)) \right) \right. \\ \left. + r_t^i(\sigma_t^{-1}(1)) q_t^i(\sigma_t^{-1}(1)) \left( q_t^{-i}(\sigma_t^{-1}(2)) + q_t^{-i}(\sigma_t^{-1}(3)) \right) \right], \end{aligned} \quad (1)$$

where  $\rho < 1$  is a normalisation factor chosen so that for all  $q_t, x_t^{win}, r_t^i, \sigma_t$ , it holds that  $0 < u^i(q_t^i, q_t^{-i} | x_t^{win}, r_t^i) + u^{-i}(q_t^i, q_t^{-i} | x_t^{win}, r_t^i) < 1$ . Combined  $\rho, \epsilon$  and  $r^i$  have an additional effect of simulating general public's interest in elections. E.g., it is possible that the sum of player's utilities will be less than 1, which means that some part of the public abstained. In the case when both parties fully concentrate their PRA on the same singular issue, nobody comes to vote at all, leading to zero utility  $u^i = u^{-i} = 0$ .

We assume that parties cannot reason about mixed strategies over the space  $\Delta(\mathcal{I})$ . However, parties can *simulate* a higher order strategy by a dynamic process that modifies  $q_t^i$  between elections. Finally, we assume that each party wishes to maximise its utility, and that such a utility is greedily accumulated, i.e. that winning the future election is less important than winning the current one.

In spite of its complexity, the SE model is easily packed into a two-players Markovian stochastic game  $\langle S, s_0, A, T, u^1, u^2 \rangle$ , where:

- $S$  is the state space of the problem and is the Cartesian product of:
  - (i) the set of issue importance orderings  $\{\sigma^A, \sigma^B, \sigma^C\}$ , (ii) binary variable  $x^{win}$ , (iii) two trust vectors  $r^1, r^2 \in [0, 1]^3$ .
- $s_0$  is the initial state.  $\sigma_0$  and  $x_0^{win}$  are arbitrarily created by some historical forces, and  $r_0^1 \neq r_0^2$  are, in a sense, coherent with them. E.g.  $r^i(I) = \delta(I, \sigma_0^{-1}(2 - \delta(x_0^{win}, i)))$ .
- $A = A^1 \times A^2$  is the joints actions space with  $A^1 = A^2 = \Delta(\mathcal{I})$ .
- $T : S \times A \rightarrow \Delta(S)$  is the probabilistic transition function that describes the game's state dynamics. For a joint PRA  $q_t = (q_t^1, q_t^2)$ , new state  $s_{t+1} = (\sigma_{t+1}, x_{t+1}^{win}, r_{t+1}^1, r_{t+1}^2)$  and source state  $s_t = (\sigma_t, x_t^{win}, r_t^1, r_t^2)$  it holds that  $T(s_{t+1}|s_t, q_t) = 0$ , unless the following conditions hold, in which case  $T(s_{t+1}|s_t, q_t) = \alpha$ , where  $\alpha$  is the tendency of the socium to shift interests:
  - $\sigma_{t+1}$  is left shift of  $\sigma_t$
  - $r_{t+1}^i$  is calculated from  $q_t^i$  and  $\sigma_t$  (for tie-breaking).

$$x_{t+1}^{win} = \begin{cases} 1 & u^1(q_t|x_t^{win}, r_t^1) > u^2(q_t|x_t^{win}, r_t^2) \\ 2 & u^1(q_t|x_t^{win}, r_t^1) < u^2(q_t|x_t^{win}, r_t^2) \\ x_t^{win} & \text{otherwise} \end{cases}$$

- The utility functions are defined by the probability of a member of the public to give his/her vote to the party,  $u^1$  and  $u^2$  respectively.

We assume that the political greed drives parties to maximise  $\sum_{t=0}^{\infty} \gamma^t u_t^i$ , where  $u_t^i$  is the expected number of votes (equivalently the probability of a single member of the public giving his/her vote to the party) that the party  $i$  receives during  $t$ 'th elections, and  $\gamma \in [0, 1]$ .

It is a priori possible that no stationary propaganda policy  $\pi^i$  exists so that  $q_t^i = \pi^i(s_t)$ . However, the SE structure, combined with a mild rationality-like assumption on PRA sequences, allows us to characterise and compute such a stationary behaviour limit.

## 2 Sarkar Elections Equilibrium

The construction rationality of a *propaganda resource allocation* (PRA) sequence will be qualified by *Greedy Modelling Action* (GMA) concepts. Though specific to Sarkar Elections (SE), their nature is similar to the better and best response in normal form games.

**Definition 1** Let  $s = (x^{win}, \sigma, r^1, r^2) \in S$  be a SE model state. An action,  $q = (q^1, q^2)$ , is a GMA at state  $s$  if the following condition holds. For each  $i \in \{1, 2\}$ , let  $\tilde{q}^i(I) = \delta(1, r^i(I))$ . Also, let  $Q = \{a^i \in A^i | u^i(\tilde{q}^i, \tilde{q}^{-i}) < u^i(a^i, \tilde{q}^{-i})\}$ . Condition: If  $|Q| > 0$  then  $q^i \in Q$ , otherwise  $q^i = \arg \max_{a^i \in A^i} u^i(a^i, \tilde{q}^{-i})$ , where ties are broken by preferring PRAs with greater affinity for  $\sigma^{-1}(1)$ .

**Definition 2** Let  $s = (x^{win}, \sigma, r^1, r^2) \in S$  be a SE model state. Extreme GMA,  $q = (q^1, q^2)$ , at state  $s$  is defined as follows. For each  $i \in \{1, 2\}$ , let  $\tilde{q}^i(I) = \delta(1, r^i(I))$ . Then  $q^i = \arg \max_{a^i \in A^i} u^i(a^i, \tilde{q}^{-i})$ , where ties are broken by preferring PRAs with greater affinity for  $\sigma^{-1}(1)$ .

**Definition 3** Let  $s = (x^{win}, \sigma, r^1, r^2) \in S$  be a SE model state. A GMA,  $q = (q^1, q^2)$ , is a Significantly GMA, if it has the same trust signature as an Extreme GMA.

Now, the concept of (*Extreme*) GMA allows us to break down the state space of the SE model into transient and *sink* states. In particular, the *sink set* of states will be composed by the Nash Equilibrium (NE) points of stage games, as is shown in Lemmata 1 and 2.

**Lemma 1** Given a state  $s = (x^{win}, \sigma, r^1, r^2)$ , all equilibrium strategies of its stage game are Significantly GMAs. Furthermore, exists a pure NE in extreme strategies (PNEE). I.e., exists  $q = \{q^1, q^2\}$ , so that  $q \in A$  is a NE and  $q^i(I) \in \{0, 1\}$  for all players  $i$  and issues  $I \in \mathcal{I}$ . For any PNEE exists  $i$  so that  $q^i(I) = 1 \iff I = \sigma^{-1}(1)$  and  $q^{-i}(I) = 1 \iff I = \sigma^{-1}(2)$ . PNEEs dominate any non-extreme strategy equilibrium with respect to utility.

**Lemma 2** Let  $\widehat{S} \subset S$  be a subset of SE model states, so that  $s = (x^{win}, \sigma, r^1, r^2) \in \widehat{S}$  if and only if exists  $i \in \{1, 2\}$  so that  $r^i(I) = \epsilon + (1-\epsilon)\delta(I, \sigma^{-1}(1))$  and either  $r^{-i}(I) = \epsilon + (1-\epsilon)\delta(I, \sigma^{-1}(2))$  or  $r^{-i}(I) = \epsilon + (1-\epsilon)\delta(I, \sigma^{-1}(3))$ .

Then the set of states  $\widehat{S}$  in the SE model is closed under the probabilistic transition of the model limited to Extreme GMAs.

**Lemma 3** The set  $\widehat{S}$  is closed under the probabilistic transition of the SE model limited to significantly greedy modelling actions.

We conclude that the Extreme and the Significantly GMAs support a sink-like equilibrium  $\widehat{S}$ . We now complete this by showing that if parties are capable of beneficially stabilising their behaviour over time, then their PRAs necessarily force the SE into  $\widehat{S}$ .

**Lemma 4** Let  $\{q_t = (q_t^i, q_t^{-i})\}_{t=0}^{\infty}$  be a PRA sequence used by the two parties in the SE game, and let  $\{s_t\}_{t=0}^{\infty}$  be a sequence of SE model states that resulted from applying the sequence  $\{q_t\}$ . In addition, denote  $\widehat{u}^i(q_t, s_t) = \max_{a^i \in A^i} u^i(a^i, q_t^{-i}|s_t)$ , and

$$\widehat{u}_{[T,*]}^i = \sum_{t=T}^{\infty} \gamma^{t-T} (\widehat{u}^i(q_t, s_t) - u^i(q_t|s_t)),$$

where  $\gamma$  is the future utility discount factor of the game.

If for all  $i \in \{1, 2\}$  holds that  $\lim_{T \rightarrow \infty} \widehat{u}_{[T,*]}^i = 0$ , then for all  $\eta$  there is  $T$  so that for all  $t > T$  the PRA  $q_t$  is an (approximate)  $\eta$ -equilibrium of the stage game determined by  $s_t$ .

For sufficiently small  $\eta$ , the trust profile of an  $\eta$ -equilibrium strategy is equivalent to that of a true equilibrium strategy.

**Corollary 1** Let  $\eta$  be sufficiently small so that all  $\eta$ -equilibrium strategies are Significantly GMAs. Under the conditions of Lemma 4, let  $T$  be so that for all  $t > T$  strategy  $q_t$  is  $\eta$ -equilibrium. If for some  $t_0 > T$  holds that  $s_{t_0} \in \widehat{S}$ , then for all  $t > t_0$   $s_t \in \widehat{S}$ .

In fact, if we allow the players to choose which equilibrium to play, they will play the extreme strategies equilibria, as they dominate any other (see Lemma 1). This also holds for PRA sequences designed to maximise the discounted accumulated utility.

**Corollary 2** Let  $\widehat{Q}$  be the family of PRA sequences,  $\mathbf{q} = \{q_t\}$ , that satisfy the conditions of Lemma 4. Then for any sequence  $\mathbf{q} \in \widehat{Q}$  either the first or the second condition holds: **a)** Exists  $T$  so that for all  $t > T$  holds that  $q_t$  has trust signature of an extreme strategy equilibrium; **b)** There's an alternative  $\tilde{\mathbf{q}} \in \widehat{Q}$  that has higher discounted accumulated reward than  $\mathbf{q}$ .

Now, extreme equilibrium strategies are *Extreme GMAs* and have trust signatures coherent with  $\widehat{S}$  of Lemma 2, hence the Corollary.

**Corollary 3** Let  $\{q_t\}$  be a sequence that satisfies conditions of Corollary 2. Then there exists  $T$  so that for all  $t > T$ ,  $s_t \in \widehat{S}$ .

Finally, the above discussion allows us to formulate and prove the properties of a Markov equilibrium policy for the SE.

**Theorem 1** Consider the following complete PRA strategy  $\pi^i : S \rightarrow A^i$  defined by its behaviour in states within set  $\widehat{S}$  and its complementary:

- For all  $s \in \widehat{S}$  compute the corresponding extreme greedy modelling action,  $q = (q^1, q^2)$  with respect to  $s$ . Set  $\pi^i(s) = q^i$ .
- For all  $s = (x^{win}, \sigma, r^1, r^2) \in S \setminus \widehat{S}$ , compute both extreme actions equilibria and their associated rewards. Let  $q = (q^1, q^2)$  be so that the winner  $x^{win}$  receives higher reward. Set  $\pi^i(s) = q^i$ .

Then the joint PRA strategy  $\pi = (\pi^1, \pi^2)$ , where  $\pi^1 = \pi^2 = \pi^i$ , is an equilibrium of SE.

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## REFERENCES

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