Cooperation and Competition When Bidding for Complex Projects: Centralized and Decentralized Perspectives

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A new class of coalition games model cooperation and competition between agents for employment in a complex project.

Consider a complex project—typically, it's involved, intricate, and consists of many varied yet interrelated parts. The successful completion of such a project requires coordinated cooperation of several experts—both people and companies—often organized as teams of subcontractors.[1] For instance, the construction of an apartment building (a rather standard endeavor) usually involves 30 to 40 individual subcontractors in 100 to 150 separate activities.[2]

Assigning subtasks of a complex project to subcontractors is also fairly common. In the UK, the proportions of construction employees employed by subcontractors from 1983 to 1998 grew by 20 percent.[3] Between 2008 and 2011, the number of independent contractors increased by 12 percent in the UK, and in Australia, 17.2 percent of the workforce in 2012 was self-employed (8.5 percent as independent contractors). These are just a few examples of the growing trend to develop projects by employing several specialized subcontractors instead of a single company.

Although this idea is increasingly popular, it isn't immediately clear how to organize the market both for the project's issuer (in this article, called the *client*) and its subcontractors (called *agents*). Interaction between agents applying to work on a project and the client is captured by the *hiring-a-team* problem.[4–6] Agents have private costs for participating in a project and can have different skills; thus, only certain teams are able to complete the project on time. The client organizes an auction in which individual agents place their bids, in this case, their required salaries. After collecting the bids, the client selects the cheapest feasible team: the set of agents able to complete the project on time with the lowest total bid.

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Table 1. Summary of our results.*

	Concept	Existence	Checking	Finding
Decentralized	Rigor. strongly winning team	Not always	$O(n^2 \times \text{FCFT})$	$O(n^5 \log(nv)\text{FCFT})$¶
	Strongly winning team	Not always	Open problem	
	Weakly winning team	Always	$O(n^5 \log(nv)\text{FCFT})$¶	
	Auction winning team	Always	$O(\text{FFT})$	$O(v \times \text{FFT})$
Centralized	Winning team (with asking salaries)	N/A	$O(\text{FCFT})$	
	Strong Nash equilibrium	Always† Not always*	$O(\text{FCFT})$	$O(n^3 \log(nv)\text{FCFT})$¶

* "Existence" denotes whether a team/equilibrium always exists. "Checking" gives the complexity of checking whether a given team satisfies the definition. "Finding" gives the complexity of finding a team/equilibrium. Find feasible team (FFT) and find cheapest feasible team (FCFT) are the complexities of the problems FFT and FCFT, respectively. The symbol † denotes that a result is valid only in the project salary model. The symbol * denotes that a result is valid only in the hourly salary model. The symbol ¶ denotes that a result is valid only if the salaries of the agents are rational numbers.

We can generalize the hiring-a-team problem by exploring two types of markets. The original approach corresponds to the *centralized setting*: agents communicate only with the client by issuing their required salaries (or bids), and it's the client's respon-sibility to select an appropriate team. Our main contribution here lies in considering a different organization of the market, one in which agents first form teams and then bid for the project as consolidated groups rather than as individuals. Because the organization of these agents into teams isn't managed by a central entity, we refer to this setting as *decentralized*. To the best of our knowledge, this formulation of the problem is novel and leads to a new class of games. It even has a natural interpretation: a client might not want to coordinate a project and deal with individual subcontractors but instead expects subcontractors to coordinate among themselves and propose a bid for the whole project.

Additionally, we generalize the hiring-a-team problem by considering two types of agent compensation. In the *project salary model* (corresponding to the original approach), agents are paid for their work irrespectively of their contributed effort. In contrast, we propose a new payment model, the *hourly salary model*, in which agents are paid for their time spent working on the project.

Throughout this article, we assume that we're given an oracle that, for a given team, can determine whether this team is feasible, that is, whether it can successfully complete the project. In particular, given a budget and the requested agents' salaries, the oracle can be asked to find a feasible team or the cheapest feasible team. Elsewhere[7] we show how to create such an oracle for a concrete scheduling model (which moreover generalizes a commodity auction); we also determine the exact complexity of checking whether a given team is feasible in such scheduling model.

Our approach generalizes two models: commodity auctions[8] and path auctions.[9] In a commodity auction, there exists a set of items $I = \{i_1, i_2, \ldots, i_q\}$, and agents own certain subsets of I. A team is feasible if the agents have all the items from I. A commodity auction can be mapped to our problem by considering that I is a set of independent activities; an agent owning a subset corresponds to an agent having skills to complete these activities. A path auction has a graph G with two distinguished vertices: a source s and a target t. The agents correspond to the vertices in the graph, and some vertices are connected by edges. A team is feasible if the participating agents form a path from s to t.

Because we consider teams of agents with sufficient skills, our model resembles cooperative skill games[10] and coalitional resource games.[11] These games, however, consider the stability of the grand coalition and interaction between its members. In contrast, our approach is to expose the competition among multiple teams. Thus, we don't apply the typical cooperative game theory concepts and, instead, model agents' cooperation and competition as a noncooperative game. Our approach is thus closer to endogenous formation of coalitions.[12–14]

In this article, we identify and formalize a new class of coalition games that extend the hiring-a-team games. We propose concepts to characterize the stability of the winning teams and study their computational complexity. Table 1 summarizes our results. All the proofs omitted from the main text are provided elsewhere.[7]

A Complex Project as a Game: A Formal Model

We consider a game in which a client (an issuer) submits a single complex project. The client has a certain valuation v of the project, which is the maximal price that she can pay for completing the project.

There's a set $N = (1, 2, \ldots, n)$ of n agents. For each agent i, we define $\phi_i^m > 0$ to be the agent's *minimal salary* for which i is willing to work. This minimal salary can correspond to the agent's personal cost of participating in the project. The agent prefers to work for ϕ_i^m than not to work (and if they work, then they prefer to work for a higher salary). The value, ϕ_i^m is private to the agent—neither the issuer nor the other agents know ϕ_i^m.

A subset of the agents' population N forms a team to work on the project; our core contribution is on how this process should be organized. A team $\mathcal{C}$ is a triple $\langle N_{\mathcal{C}}, \phi_{\mathcal{C}}, c_{\mathcal{C}} \rangle$ consisting of the set of participating agents $N_{\mathcal{C}} \subseteq N$; a salary function $\phi_{\mathcal{C}} : N_{\mathcal{C}} \to \mathbb{N}$ assigning salaries to member agents; and the total cost of the team $c_{\mathcal{C}} \in \mathbb{N}$, the total amount of money earned by the participants of $\mathcal{C}$. Salaries are discrete (not only money is discrete, but it's also common in real-world auctions to specify a minimal difference between two successive bids). However, to derive some computational results, in some clearly marked places, we assume that the salaries can be rational numbers.

The same team can organize the work of its members on the project in various ways with varying efforts from participants. To capture this property, we introduce a notion of a *schedule*, $\sigma_{\mathcal{C}} : N_{\mathcal{C}} \to \mathbb{N}$, that assigns to each member of a team the amount of time the agent needs to spend on the project. Of course, there could exist many schedules for a single team.[7]

We consider two models of agent compensation. Let $\phi_{\mathcal{C}}^{tot}(i)$ denote the total amount of money agent i gets in team $\mathcal{C}$ (naturally, $c_{\mathcal{C}} = \Sigma\phi_{\mathcal{C}}^{tot}(i)$). In the project salary model, $\phi_{\mathcal{C}}^{tot}(i)$ is equal to the salary of agent $\phi_{\mathcal{C}}(i)$, and thus doesn't depend on the amount of work assigned to that agent. In the hourly salary model, $\phi_{\mathcal{C}}^{tot}(i)$ is equal to the product of the salary $\phi_{\mathcal{C}}(i)$ and the time t_i, during which i processes his or her part of the project (t_i is known from the schedule).

In the project salary model, agents are interested in earning as much money as possible. The hourly salary model represents agents who are interested in having the highest possible hourly wage; thus, an agent would prefer to work $t_i = 1$ time unit with a salary $\phi_i = 3$ to working $t_i = 2$ time units with a salary $\phi_i = 2$.

Different schedules can result in different completion times for the project. If the schedule results in a completion time that's satisfactory for the client, we say that the schedule is *feasible*. For some teams, there might not exist a feasible schedule (say, if the members lack certain skills). We assume that an oracle can answer whether a given schedule is feasible or not. This very general setting can be instantiated by providing a concrete oracle. For instance, elsewhere[7] we show that by appropriately specifying the oracle, our results can be applied to both commodity and path auctions. We also show there how to replace the general oracle with a concrete scheduling model.

Different schedules can result in different completion times for the project.

A team $\mathcal{C}$ is feasible if and only if the asking salaries are no lower than the minimal salaries, $\phi_{\mathcal{C}}(i) \geq \phi_i^m$, there exists a feasible schedule such that the project budget isn't exceeded ($c_{\mathcal{C}} \leq v$), and the cost $c_{\mathcal{C}}$ of the team $\mathcal{C}$ is consistent with the salaries $\phi_{\mathcal{C}}$. Specifically, in the project salary model, $c_{\mathcal{C}} = \Sigma\phi_{\mathcal{C}}(i)$. In the hourly salary model, $c_{\mathcal{C}} = \Sigma t_i \phi_{\mathcal{C}}(i)$.

A team $\mathcal{C}$ is cheaper than $\mathcal{C}'$ if it has a strictly lower cost $c_{\mathcal{C}} \leq v$ or if it has the same cost but it's preferred by a deterministic tie-breaking rule $\prec$, $N_{\mathcal{C}} \prec N_{\mathcal{C}'}$ (for the sake of concreteness, we assume that $\prec$ is the lexicographic order in which a team is represented by a concatenation of the sorted list of the names of its members).

Throughout this article, we use the find feasible team (FFT) and find cheapest feasible team (FCFT) problems.

Problems FFT and FCFT. An instance of FFT consists of a project (with a budget v) and the set N of the agents with (known) minimal required salaries ϕ_i^m. The challenge is to find some feasible team or to claim there's no such team available. In FCFT, we ask for the cheapest feasible team.

Centralized Formation of Teams

In the centralized model, agents submit their asking salaries ϕ_i directly to the client. The client, having the asking salaries, wants to form the cheapest feasible team. We first show that this problem reduces to FFT, the problem of finding a feasible team. Then, we analyze the optimal bidding strategies of agents.

Proposition 1. FCFT can be solved in time $O((\log(v) + n)\text{FFT})$, where FFT is the complexity of FFT. Having the asking salaries of the agents, the problem of finding the winning team can be solved in time O(FCFT), where FCFT is the complexity of FCFT.

The agents can behave strategically and manipulate their asking salaries to maximize their payoffs. We model this problem as a strategic game. An action of agent i is his or her asking salary $\phi_i \geq \phi_i^m$. The payoff of i is ϕ_i if and only if i is a member of the cheapest feasible team; otherwise, the payoff of i is 0.

Interestingly, in the project salary model, there exist sets of vectors of actions that are stable against collaborative strategies of the agents. A vector of the agents' actions is a strong Nash equilibrium (SNE) if no subset of the agents can change its actions so that all deviating agents obtain strictly better payoffs.

For each subset of agents $N' \subseteq N$, by $\mathcal{C}^O(N')$ we denote the cheapest feasible team using only the agents from N'. (If there's no feasible team consisting of the agents from N', the team $\mathcal{C}^O(N')$ doesn't exist.)

Theorem 1. In the project salary model, if there exists a feasible team, then there exists an SNE. In every SNE, the set of the agents who get positive payoffs is the set of agents forming the cheapest feasible team, $N_{\mathcal{C}^O(N)}$.

Proof. Let $N^O = N_{\mathcal{C}^O(N)}$ be the set of the agents participating in the cheapest feasible team. We say that the action ϕ_i of agent i is minimal if and only if $\phi_i = \phi_i^m$. We show how to construct the asking salaries ϕ_i^O of the agents from N^O that, together with the minimal actions of the agents outside N^O, form an SNE. A sketch of the proof is as follows. We show the set of linear inequalities for the variables ϕ_i, $i \in N^O$. Let us denote the maximal values of ϕ_i that satisfy the inequalities as ϕ_i^O (maximal in the sense that if we increase any value ϕ_i^O, then the new values won't satisfy all the inequalities anymore). We show that the actions ϕ_i^O of the agents from N^O, together with the minimal actions of the agents outside of N^O, form an SNE and that the set of the solutions ϕ_i^O that satisfy all the inequalities is nonempty.

The first inequality states that the values ϕ_i must lead to a feasible solution:

$$\sum_{i \in N^O} \phi_i \le v.$$

Next, as $\mathcal{C}^O$ is the cheapest feasible team, for each feasible team $\mathcal{C}'\left(N^O \ne N_{\mathcal{C}'}\right)$ such that $N^O \prec N_{\mathcal{C}'}$, $\mathcal{C}^O$ must have a (weakly) lower cost:

$$\sum_{i \in N^O - N_{\mathcal{C}'}} \phi_i \le \sum_{i \in N_{\mathcal{C}'} - N^O} \phi_i^m.$$

For a $\mathcal{C}'$ preferred over $\mathcal{C}^O$ ($N^O \ne N_{\mathcal{C}'}$ and $N_{\mathcal{C}'} \prec N^O$), $\mathcal{C}^O$ must have a strongly lower cost:

$$\sum_{i \in N^O - N_{\mathcal{C}'}} \phi_i < \sum_{i \in N_{\mathcal{C}'} - N^O} \phi_i^m.$$

First, if the values ϕ_i^O satisfy the above three inequalities and the agents outside of N^O play their minimal actions, then the agents from N^O will get positive payoffs. If they did not get the positive payoffs, it would mean that there exists a feasible cheaper team $\mathcal{C}'$. However, the three inequalities imply that the agents from $N^O - N_{\mathcal{C}'}$ induce the lower total cost than the total cost of the agents from $N_{\mathcal{C}'} - N^O$; this ensures that agents N^O with actions ϕ_i^O form a cheaper team than $\mathcal{C}'$.

Next, we show that no set of agents $N_{\mathcal{C}'}$ can make a collaborative action ϕ after which the payoff for all $N_{\mathcal{C}'}$ agents will be greater than previously. In contrast, assume that there exists such a set of agents $N_{\mathcal{C}'}$ and such an action ϕ. First, we consider the case when the payoff of some agent $i \notin N^O$ would change. This means that after ϕ there would be a new cheapest feasible team $\mathcal{C}'$, where $i \in N_{\mathcal{C}'}$. However, we know that the total cost of the agents from $N^O - N_{\mathcal{C}'}$ is lower than the total cost of the agents from $N_{\mathcal{C}'} - N^O$. This means that $\mathcal{C}'$ can't be cheaper than the team consisting of the agents from N^O. Finally, let's consider the case when only the payoffs of the agents from N^O change (and thus $N_{\mathcal{C}'} \subseteq N^O$). If the strict subset of N^O could form a feasible team, then $\mathcal{C}^O(N)$ would not be the cheapest. Thus, $N_{\mathcal{C}'} = N^O$, which means that every agent from N^O must have played a higher action (and others must have not changed their actions). Because ϕ_i^O were maximal, this means that after the action ϕ, some inequality, for some feasible team $\mathcal{C}''$, wouldn't hold anymore. Thus, we infer that $\mathcal{C}''$ is cheaper than $\mathcal{C}'$.

To check that there always exists a solution, we see that the definition of N^O ensures that the values $\phi_i^O = \phi_i^m$ satisfy all inequalities.

Finally, we prove N^O is formed by the same agents as forming the cheapest team. Assume that the set of the agents that get positive payoffs in some SNE is $N' \ne N^O$. However, if the agents from $(N^O - N')$ play their minimal actions, then the team consisting of the agents from N^O would be cheaper than the team consisting of agents from N'. Thus, agents from $(N^O - N')$ can deviate, getting better payoffs. This completes the proof. ◆

Interestingly, there's no analogous result for the hourly salary model.[7] The proof of Theorem 1 is constructive, but it requires considering all feasible teams and therefore it leads to a potentially high computational complexity. Finding an efficient algorithm for the problem of finding an SNE in the project salary model is open. On the other hand, if the salaries of the agents can be rational numbers, we can find the salary function in an SNE by a polynomial reduction to FCFT. This result is particularly meaningful if the salaries have high granularity; rounding such a rational solution gives an integral solution that's nearly perfect.

Proposition 2. In the project salary model, if the salaries are rational, then finding an SNE can be solved in time $O(n^3 \log(nv)\mathrm{FCFT})$, where FCFT is the complexity of FCFT. Checking whether a given vector of the asking salaries $<\phi_i>$, $i \in N$ is an SNE can be solved in time $O(\mathrm{FCFT})$, where FCFT is the complexity of FCFT.

Decentralized Formation of Teams

If agents can communicate and coordinate their strategies, they form teams and bid for projects as consortiums. We propose the concept of a (rigorously) strongly winning team, in which no subset of agents can successfully deviate. We show how to characterize (rigorously) strongly winning teams and how to reduce the problem of finding them to FCFT. We show that the strongly winning teams might not exist, so we introduce the concept of a weakly winning team. We prove that a weakly winning team always exists (provided there's a feasible team). We demonstrate how to reduce the problem of finding weakly winning teams to FCFT.

We model the behavior of the agents as a strategic game. Agent i's action is a triple $\langle N_{\mathcal{C}}, \phi_{\mathcal{C}}, b_{\mathcal{C}} \rangle$. Intuitively, such an action means that agent i decides to enter team $\mathcal{C} = \langle N_{\mathcal{C}}, \phi_{\mathcal{C}}, b_{\mathcal{C}} \rangle$. The payoff of the agent is equal to $\phi_{\mathcal{C}}(i)$ if $\mathcal{C}$ is feasible, each agent $j \in N_{\mathcal{C}}$ agrees to participate in $\mathcal{C}$ (they all play $\mathcal{C}$, and their payoffs are consistent with the bid of the team, $b_{\mathcal{C}}$), and there's no feasible cheaper team $\mathcal{C}'$ such that all agents from $N_{\mathcal{C}'}$ agree to participate in $\mathcal{C}'$. Otherwise, the payoff of i is 0.

Strongly Winning Teams

As the payoffs depend on whether the others agree to cooperate, rather than the Nash equilibrium, an SNE should be used. In Definition 1, we propose an even more stable equilibrium concept—the rigorously SNE (RSNE), which requires that no subset of agents can deviate such that each agent gets a payoff at least as good as its payoff before deviating (instead of SNE's strictly better). Our approach is motivated by cautious agents. In an SNE, agents have no incentive to deviate if they get the same payoff; however, they also have no incentive not to deviate. Yet, any deviation will result in a serious payoff loss for some agents (changing their payoffs from a positive ϕ to zero). A cautious agent will prefer not to be exposed to the possibility of such a loss.

If agents can communicate and coordinate their strategies, they form teams and bid for projects as consortiums.

Definition 1. The vector of actions π is an RSNE if and only if there's no subset of agents $N_{\mathcal{C}}$. such that the agents from $N_{\mathcal{C}}$ can make a collaborative action $\mathcal{C}$, after which the payoff of each agent i from $N_{\mathcal{C}}$ would be at least equal to his or her payoff under π, and the payoff of at least one agent $i \in \mathrm{N}$ would improve.

An RSNE requires that the payoff of at least one agent $i \in \mathrm{N}$ must change as we treat as equivalent the teams with the same payoffs. For instance, in a game with three agents, a, b, and c, if team a,b gets a positive payoff, it doesn't matter whether c plays $<\{c\}, v + 1>$ or $<\emptyset, v + 1>$: in both cases, all payoffs are the same (recall that v is the client's maximal budget for the project).

Below we introduce additional definitions that help characterize the RSNE in our games.

Definition 2. A feasible team $\mathcal{C}$ is explicitly endangered by a team $\mathcal{C}'$ if $\mathcal{C}'$ is feasible, $N_{\mathcal{C}} \cap N_{\mathcal{C}'} = \emptyset$, and $\mathcal{C}'$ is cheaper than $\mathcal{C}$. A feasible team $\mathcal{C}$ is implicitly endangered by a team $\mathcal{C}'$ if $\mathcal{C}'$ is feasible, $N_{\mathcal{C}} \cap N_{\mathcal{C}'} \neq \emptyset$ and each agent from $N_{\mathcal{C}} \cap N_{\mathcal{C}'}$ gets in $\mathcal{C}'$ at least as good a salary as in $\mathcal{C}$, and either $N_{\mathcal{C}} \neq N_{\mathcal{C}'}$ or $\phi_{\mathcal{C}} \neq \phi_{\mathcal{C}'}$.

If there are agents belonging to both teams ($N_{\mathcal{C}} \cap N_{\mathcal{C}'} = \emptyset$), we don't consider the total cost of the alternative team $\mathcal{C}'$, as the decision whether $\mathcal{C}'$ will be formed depends solely on the agents from $N_{\mathcal{C}} \cap N_{\mathcal{C}'}$: if they decide to form $\mathcal{C}'$, $\mathcal{C}$ won't be formed, thus the client won't be able to choose between $\mathcal{C}$ and $\mathcal{C}'$.

A feasible team $\mathcal{C}$ is (rigorously) strongly winning if and only if there's an RSNE in which the agents from, $N_{\mathcal{C}}$ get positive payoffs $\phi_{\mathcal{C}}$. The following theorem relates endangerment (Definition 2) and a winning team.

Theorem 2. Team $\mathcal{C}$ is rigorously strongly winning if and only if $\mathcal{C}$ isn't explicitly nor implicitly endangered by any team.

The result in Theorem 2 stated for RSNEs transfers to SNEs after a slight modification of the payoffs. It's sufficient to assume that an agent playing an empty team receives a slightly higher payoff than if he or she plays a nonempty losing team. In other words, this modification associates some small costs with the preparation of a bid by the agents. Hereafter, whenever we mention a strictly winning team, we assume that the agents incur such costs. To state the result for SNEs, we also need to use the definition of a team $\mathcal{C}$ being strictly implicitly endangered by $\mathcal{C}'$. This definition differs from being implicitly endangered only by not requiring the agents from $N_{\mathcal{C}} \cap N_{\mathcal{C}'}$ to have at least as good payoffs but strictly better payoffs in $\mathcal{C}'$ than in $\mathcal{C}$.

Theorem 3. If there are small but positive costs of preparing the offer by the agents, then team $\mathcal{C}$ is strongly winning if and only if $\mathcal{C}$ isn't explicitly nor strictly implicitly endangered by any team.

Theorems 2 and 3 lead to a simple brute-force algorithm for checking whether team $\mathcal{C}$ can be a part of some RSNE. It's sufficient to check whether for each set of agents $N' \subseteq N_{\mathcal{C}}$ there exists a payoff function $\phi_{\mathcal{C}'}$ and a cost c such that $\mathcal{C}$ is explicitly or implicitly endangered by $\langle N', \phi, c\rangle$ (such a condition can be checked by enumerating the payoff functions that assign to each agent his or her minimal salary, the salary that he or she obtains in $\mathcal{C}$, or the next higher salary). Below, we characterize RSNEs in the project salary model even more precisely.

Lemma 1. In the project salary model, the set of agents participating in a rigorously strongly winning team is the same as the set of agents participating in the cheapest feasible team.

Lemma 2. In the project salary model, the bid of a strongly winning team is equal to the maximal allowed price v.

Lemmas 1 and 2 show that the problem of finding a strongly winning team reduces to the problem of finding a feasible team. The problem thus becomes an optimization problem; the strategic behavior of agents has no impact.[7] An RSNE (and even an SNE) might not exist in some instances.

Proposition 3. Both in the project and hourly salary models, there might not exist a strongly winning team, even if there exists a feasible team.

It's sufficient to assume that an agent playing an empty team receives a slightly higher payoff than if he or she plays a nonempty losing team.

Proof. Consider a project with budget $v = 5$ and three identical agents a, b, c with minimal salaries $\phi_i^m = 2$ (in the hourly salary model, assume that each agent spends exactly 1 time unit on the project); a team of any two agents is feasible (able to complete the project on time and within budget).

For the sake of contradiction, assume there exists a team $\mathcal{C}$ that gets positive payoffs. Without loss of generality, we assume that $N_{\mathcal{C}} = \{a, b\}$. At least one of the agents, let's say a, has to get salary at most equal to 2.5. However, agents a and c, with salaries equal to 3 and 2, respectively, can form a feasible team in which both a and c get better payoffs. ◆

Weakly Winning Teams

Proposition 3 suggests that a notion of a strongly winning team is too restrictive. Team $\{a, c\}$ can profit by deviating, for example, by playing $\phi(a) = 3$ and $\phi(c) = 2$. But a shouldn't be willing to deviate, as $\{a, c\}$ with payoffs $\phi(a) = 3$ and $\phi(c) = 2$ isn't stable (for instance, team $\{b, c\}$ can play $\phi(b) = 2$ and $\phi(c) = 3$ and successfully deviate from $\{a, c\}$). In the above example, no team strongly wins, even though intuitively there are teams that would agree to work together on the project. Thus, we propose a weaker notion of a winning team.

Definition 3. A feasible team $\mathcal{C}$ is weakly winning if it isn't explicitly endangered by any team and for each feasible team $\mathcal{C}'$ such that $\mathcal{C}$ is implicitly endangered by $\mathcal{C}'$, there exists a feasible team $\mathcal{C}''$ such that $\mathcal{C}'$ is explicitly or implicitly endangered by $\mathcal{C}''$.

Proposition 4. There exists a weakly winning team if and only if there exists a feasible team.

Proposition 5. In the project salary model, if the salaries of the agents can be rational numbers, the problem of finding a weakly winning team and the problem of checking whether a team $\mathcal{C}'$ is weakly winning can be solved in time $O(n^5 \log(nv)\mathrm{FCFT})$.

Finding an efficient algorithm for the same problem with discrete salaries is still an open question.

Mechanism Design

In this section, we analyze two mechanisms that a client can use to find a winning team: the first sets the project's budget v, and the second uses a first-price auction.

Theorem 4. If there exists a feasible team, then there exists a budget v^O for which there exists a strongly winning team. The problem of finding such a v^O. can be solved in time $O(\log(v) \times \text{FFT})$.

In the second approach, we use the first-price auction, in which teams participate. In a standard first-price auction, an item's price starts from some minimal value (the least preferred outcome for the owner of the item). Bidders place bids for the current price. The asking price is gradually increased until there are no further bids; the last bidder wins. Similarly, in our proposed auction, the auction starts from the original budget v (the least preferred outcome for the client); the asking price gradually decreases. Teams place bids for the current asking price (as in the standard first-price auction, multiple bids for the same asking price aren't allowed). The auction stops when no feasible team bids lower than the current asking price. This procedure leads to the concept of an auction-winning team.

Definition 4. A team $\mathcal{C}$ is auction-winning if and only if there's no feasible team $\mathcal{C}'$ such that $b_{\mathcal{C}'} < b_{\mathcal{C}}$, and for each agent $i \in N_{\mathcal{C}} \cap N_{\mathcal{C}'}$ the agent gets better salary in $\mathcal{C}'$, $\phi_{\mathcal{C}'}(i) \geq \phi_{\mathcal{C}}(i)$.

Proposition 6. The problem of checking whether a feasible team $\mathcal{C}$ is auction-winning can be solved in time $O(\text{FFT})$. The problem of finding an auction-winning team can be solved in time $O(v \times \text{FFT})$. ∎

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