Minimizing Total Weighted Completion Time on Batch and Unary Machines with Incompatible Job Families

Journal:	*International Journal of Production Research*
Manuscript ID	TPRS-2017-IJPR-1071.R1
Manuscript Type:	Original Manuscript
Date Submitted by the Author:	29-Nov-2017
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Keywords:	BATCH SCHEDULING, HYBRID PRODUCTION SYSTEMS, FLOW SHOP SCHEDULING, DISCRETE OPTIMISATION
Keywords (user):	batch machine, incompatible job families, total weighted completion time, approximation algorithm, constraint programming

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Response to Reviewers
TPRS-2017-IJPR-1071

The authors appreciate the valuable and constructive comments by the associate editor and reviewers very much. Based on these comments and suggestions, we have made careful modifications on the original manuscript. All changes made to the text are in blue. Our point-by-point responses to the reviewers' comments are given as follows.

<u>Responses to Reviewer 1:</u>

1. *The term "discrete machine" to qualify the second machine is a bit misleading. In many scheduling contexts a discrete machine or a discrete resource is a resource that can perform several activities in parallel provided that its discrete capacity if not execeed. In the present paper it would be better to call it a unary machine or a unit capacity machine.*

   Thanks for this comment. All the terms "discrete machine" have been revised as "unary machine" in this revision.

2. *Even if the proposed problem is clearly of interest in an industrial context (and of course from a theoretical point of view), I think it still has many limitations for tackling a real industrial problem. For instance typically in the semiconductor manufactory scheduling:*
   - the machines are more complex that just the batch machines or the unit capacity machines: there are also some concerns with sequence dependent setup times, with additional constraints like reticles, ...
   - jobs generally consists of more than 2 operations and depending on the job, the process may be quite different
   - of course, there are not single machines in the factory and part of the scheduling problem consists of machine allocation, with candidate machines for a given operation having very different characteristics
   - there are in general some release dates for the jobs and a work-in-process, this will tend to jeopardize the fact that jobs of the same family and with same weight are equivalent which is an implicit assumption of the proposed algorithms for efficient batch formation

   So it could be interesting to consider these limitations and see if and how they can be handled (maybe in a future work section?). In particular, both for the lower bounds and the approximation heuristics, there seem to be some possible generalization of the way the existing results for the individual machines are aggregated (e.g. for LB: GRWC for batch machines and WSPT for unit-capacity ones). Would it be possible to generalize this aggregation and propose LBs or heuristics for problems with more than 2 operations in the job ? This is of course more an open question.

   Thanks for this comment. Considering that no approximation algorithm has been developed the problem $\beta \rightarrow \delta|incompat|\sum w_j C_j$, we mainly study this problem from a theoretical point of view. Thus, the goal of this research is to develop a constant factor approximation algorithm for this basic problem to fill this gap. We hope that this research is capable of providing some helpful insights on solving the related large-scale practical problems. Due to the complexity, some constraints in practice will be addressed in the future work based on this basic problem. In the *Conclusions* section, the future work has been revised as

"*In practice, the machines and operations are more complex. How to extend the lower bounds and approximation algorithm to the general problems with parallel batch machines or more than two operations is an important future work. Some general constraints, such as release dates, set-up time and re-entrant flows, should also be addressed in the future.*"

3. *There has been recent applications of Constraint Programming (CP) techniques to solve some related and in general more complex versions of the problem that I think should be cited. For instance:*

   A. Ham and E. Cakici. Flexible job shop scheduling problem with parallel batch processing machines: MIP and CP approaches. Computers & Industrial Engineering. 102 (2016) 160-165.

   Given that you have been doing your experiments with CPLEX, it would be very interesting to see how a very simple CP Optimizer model compares to your heuristic (CP Optimizer is a CP based optimization engine available together with CPLEX). Also I would suggest using more recent versions of these engines as there has been some performance progress both in CPLEX and in CP Optimizer since version 12.6 (current version is 12.7.1).

   Thanks for this comment and kindly code. Some related references about the constraint programming technique have been added in this revision. We also developed a hybrid constraint programming(CP)-based method to solve this problem (please refer to Section 5). The developed CP-based method integrates the solution of the proposed GBDS algorithm as the initial warm start within the CP technique framework. The pure CP method is also used to make a comparison. The mixed-integer linear programming model and the CP model are all implemented by the version 12.7.1. The table of the optimal solutions in section "Comparison with optimal solutions" has been updated.

   The experiment results show that both GBDS and CP can obtain good solutions. When the batch processing time is enlarged, the GBDS algorithm outperforms the pure CP method. Numerical results also show that when the solutions of GBDS are integrated into CP technique as a warm start, the proposed CP-based method can achieve high-quality solutions and the solutions can be improved a lot.

4. *Beside the description of the problem instance generation (which is very clear), it would be useful to make the actual instances available to the community and to provide the detailed results (for the 3 LBs and GBDS) on each instance in order to allow future comparisons on the same set of instances.*

   I also think that you should give more results on the individual lower bounds LB1-3. How do they compare depending on the different problem characteristics? When does one LB dominate others?

Thanks for this comment. We have uploaded all the instances together with the detailed numerical results in the ScholarOne Manuscript System of IJPR as the supplementary materials, please refer to them. In the near future, we will upload the instances and results on the website.

The discussion about the three lower bounds has been added in the section "Sensitivity analysis" of this revision. Numerically, when the batch processing time is short, LB_3 is tighter than LB_1 and LB_2. When the batch processing time is large, LB_1 and LB_2 are tighter than LB_3. The relationship between LB_1 and LB_2 depends on the data of processing time and weight.

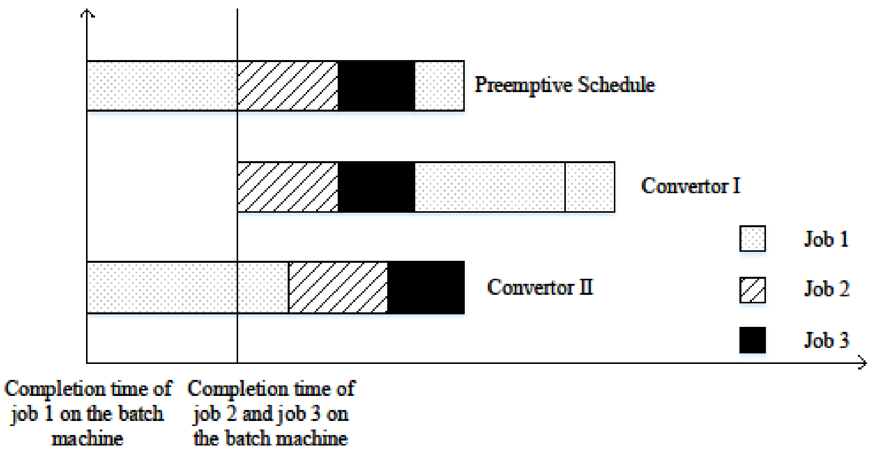
5. *The description of Step 5 of the GBDS algorithm is not very clear:*
- you say that you "Insert p_j-k_j extra units of time in the schedule at time C_j^P". what does that mean exactly?
*- also, I'm wondering if there is not a mistake in the description of convertor II. Given that it considers the *first* scheduled piece of a job i, how can it be that this piece is scheduled from time C_j^P-k_j to C_j^P (given that C_j^P is the preemptive completion time of the job)*

Thanks for this comments. We are sorry for this confusion and the mistake in Convertor II. Given the preemptive schedule, the two convertors work as follows:

In Convertor I, the jobs are processed on the unary machine continuously by non-decreasing order of the completion time of the last piece of jobs with the constraint that no job can start before its completion time on the batch machine.

In Convertor II, the jobs are processed on the unary machine continuously by non-decreasing order of the completion time of the first piece of jobs with the constraint that no job can start before its completion time on the batch machine.

We give an example to illustrate these two convertors:



We have revised the presentation of the GBDS algorithm to make the convertors more clearly in this revison. Thanks again.

6. *On Table 1 and its description, the naming Gap1 and Gap2 is not really explicit, it would be better to for instance use something like "Gap-GBDS" and "Gap-LB".*

Thanks for this comment. We have revised the names of gaps as *"Gap-GBDS"* and *"Gap-LB"* in this revison.

7. *On Table 2, I would switch columns "Time" and "Gap-GBDS" so that the gaps are easier to compare. Also on tables 2-3-4, I would use a bold font for the "best" gaps in each row.*

Thanks for this comment. We have made the adjustments as the suggestions in this revison.

Responses to Reviewer 2:

Major comments:

1. *In the introduction, semiconductor manufacturing is considered as a possible application. The problem tackled in this paper covers some of the properties observed in oxidation area which they are mentioning, however, many of the constraints observed in practice are not addressed (e.g., release dates, re-entrant flows, or parallel machines). Results seem to be very hard to generalize in case release dates are introduced. On page 2, line 60, it is mentioned that the problem* $\beta \rightarrow \delta|incompat|\sum C_j$ *has not been studied in the literature. This is not true in the sense that this problem is a special case of complex job-shop scheduling problems arising in semiconductor manufacturing, which have been studied in, e.g., [1], [2], [3], [4], or [5]. However, no approximation algorithms have been developed for these more general problems, so this, and the provided lower bounds, could be interesting contributions of the paper at hand.*

   Thanks for this comment. As mentioned by the reviewer, the problem $\beta \rightarrow \delta|incompat|\sum w_j C_j$ can be regarded as a special case of the general complex job-shop scheduling problems with batch machines ([5]). However, the problem $\beta \rightarrow \delta|incompat|\sum w_j C_j$ has not been studied specially and no approximation algorithm has been developed for this problem. The goal of this research is to develop a constant factor approximation algorithm for this basic problem to fill this gap. Due to the complexity, some constraints in practice will be addressed in the future work based on this basic problem. In this revision, the sentence "To our knowledge, the problem $\beta \rightarrow \delta|incompat|\sum w_j C_j$ has not been studied in the literature." has been revised as

   "*The problem* $\beta \rightarrow \delta|incompat|\sum w_j C_j$ *can be regarded as a special case of the general complex job-shop scheduling problems with batch machines (Knopp, Dauzère-Pérès, and Yugma 2017). However, the problem* $\beta \rightarrow \delta|incompat|\sum w_j C_j$ *has not been studied specially and no approximation algorithm has been developed for this problem.*"

2. *In the problem formulation (page 3, line 14), it is stated that "We assume that the number of jobs of each family is an integer multiple of the batch capacity". Why has this restriction been introduced? How does this fit to Lemma 1, which allows the last batch of each family to be only partially filled?*
   On page 3, line 21, it is unclear where the parameter a is coming from. This might be related to the observations above, but could be stated more clearly.

   Thanks for this comment. The parameter a is the number of batches and is defined within the definition of the batch set B. To make the definition clear, we have add the definitions in the Notation:

   n : the number of jobs;
   m : the number of families;

a : the number of batches;

The assumption "We assume that the number of jobs of each family is an integer multiple of the batch capacity" is just for convenience and can be removed. Instead, we need to add the following statement about the number of batches in each family.

"*Based on Lemma 1, the number of batches in family k is equal to $\left\lceil \frac{n_k}{b} \right\rceil$, where $\lceil \bullet \rceil$ means the rounding up function. Thus, we can set $a = \sum_{k \in F} \left\lceil \frac{n_k}{b} \right\rceil$ and this problem can be formulated as follows.*"

3. *In the model on page 4, it is not clear why constraint (5) is necessary, since constraints (4) and (6) alone seem to be sufficient.*

   Yes, the original constraint (5) is unnecessary. The purpose of adding this constraint is to get a tighter LP relaxation (can be regarded as the valid inequality). However, the experiments show that the LP relaxation is still not good and we keep this constraint in the formulation. In this revision, we have removed this constraint.

4. *Page 5, Lemma 2: This Lemma, taken from [6], is formulated as "...there exists an optimal schedule, where all batches are sequenced in increasing order of T_l/W_l, where...". However, the Lemma in the original paper is even stronger, stating that all such orderings result in an optimal schedule. Since in Definition 1 the result of the GRWC algorithm is used, Lemma 2 should be clarified in this regard.*

   Thanks for this comment. We have revised the clarification of Lemma 2 as below.

   "*Lemma 2. ...the optimal schedule can be achieved by sequencing all batches in increasing order of T_l/W_l, where...*"

5. *Page 5, line 47: A reference should be given why WSPT yields an optimal solution.*

   Thanks for this comment. We have added the related reference (Smith 1956) in this revision.

6. *Page 6, line 1: I assume that a refers to the number of batches created by the GRWC algorithm, it would be more clear to state this explicitly here.*

   Sorry for this confusion. Yes, the parameter a represents the number of batches. We have added the statement of parameter a in the Notation.

7. *Page 6, line 29: The sentence "The second term of the last expression is same with the problem of scheduling n jobs on a identical discrete machines with additional constraints that each discrete machine must process b jobs." should be rephrased.*

Thanks for this comment. This sentence has been revised as

"*The expression* $\sum_{l=1}^{a}\sum_{i=1}^{h_l^*}\lambda_{li}^*\mu_{li}^*\sum_{j\le i}\nu_{lj}^*$ *can be regarded as the total weighted completion time for the problem of scheduling n jobs on an identical unary machines with additional constraints that each unary machine must process fixed number of jobs.*"

In addition, since the assumption that the number of jobs of each family is an integer multiple of the batch capacity is removed, the number of jobs in one batch may not be equal to *b*. So we add "h_l^*" to denote the number of jobs assigned to batch *l* in the optimal solution.

8. *Page 7, line 56: Step 4 of the algorithm should be presented more clearly. What is meant by the "buffer"?*

   Sorry for this confusion. Here we use "buffer" to represent the queue of the unary (discrete) machine. When jobs are completed on the batch machine, they will join in the queue of the unary machine to wait for processing. We have used "queue of the unary machine" to replace all the related "buffer" in the revision.

9. *Page 8, line 31 and below: In the proof, the intention of the case distinction and its conclusions should be clarified. "idle time" probably refers to discrete machine? It seems unusual to refer to an example in a general proof.*

   Thanks for this comment. Yes, "idle time" refers to the unary (discrete) machine. We have revised it as "idle time on the unary machine". The example used in the proof has been removed in this revision.

10. *Page 9, line 25: Consider providing an informal idea how the algorithm works before diving into the details.*

    Thanks for this comment. We have added the statement on the idea of the algorithm as follows.

    "*The idea is to run the preemptive GBDS algorithm combining the GRWC algorithm and dynamic WSPT rule to get a preemptive schedule. The preemption is allowed only on the unary machine and when one batch is finished on the batch machine while the unary machine is busy. Then the preemptive schedule is converted non-preemptively based on the sequence of the completion time of the last piece or first piece of jobs.*"

11. *Page 9: line 30: Some clarification seems necessary, GBDS is run twice, once for each converter? Or, is the converter changed while the algorithm is run? The converters seem to differ only in the piece they select, so there is a lot of duplication in their description. A combined explanation could be more clear. The formulation "schedule <u>at</u> time" could be clarified. Step 6 seems to be more formal than necessary.*

    Sorry for this confusion. The GBDS algorithm is run once. After the preemptive schedule is achieved, the time period for each piece of each job will be saved. Based on these time periods, the algorithm first uses Convertor I to adjust the schedule non-preemptively and get the solution and objective value. Then the algorithm uses Convertor II to adjust the schedule non-preemptively and get another solution and the objective value. At last, the algorithm returns the better one. We have revised Step 4-6 as follows:

Step 4. Run the preemptive GBDS algorithm to get a preemptive schedule.
Step 5. Adjust the schedule non-preemptively based on Convertor I (II) to get a feasible schedule.

- *Convertor I. The jobs are processed on the unary machine continuously by non-decreasing order of the completion time of the last piece of jobs with the constraint that no job can start before its completion time on the batch machine.*
- *Convertor II. The jobs are processed on the unary machine continuously by non-decreasing order of the completion time of the first piece of jobs with the constraint that no job can start before its completion time on the batch machine.*

Step 6. Return the better schedule with smaller objective value.

12. *Page 10, line 8: "could produce" seems to be too weak.*

Thanks for the suggestion. We have revised "could produce" as "produces" in the new manuscript.

13. *Page 10, line 24 to line 27: The transformation in the third inequality (introducing LB1 and LB3) is not clear enough.*

Sorry for this confusion. We have added one step to make the transformation clear.

$$\leq \frac{2(\Gamma_{GRWC} + \Gamma_{WSPT})}{Z_{opt}^*}$$
$$= 2\left(1 + \frac{\Gamma_{GRWC} + \Gamma_{WSPT} - Z_{opt}^*}{Z_{opt}^*}\right)$$
$$\leq 2(1 + \frac{\Gamma_{GRWC} + \Gamma_{WSPT} - LB_3}{LB_1})$$

14. *Page 11: It is surprising to find meta-heuristic methods being introduced in the section on computational experiments. They should be described before in a dedicated section and it should be motivated why these specific approaches are used, in particular, considering the criticism on Harmony Search given in [7].*

Thanks for this comment. We have added a section "*A constraint programming-based method*" to introduce the methods. In this revison the harmony search method has been removed. Instead, the constraint programming method is used to do the comparison. The Constraint Programming (CP) method has a promising success for solving scheduling problems (Hentenryck and Michel 2009; Öztürk et al. 2013; Ham and Cakici 2016). It has two advantages. On one hand, the natural formulation in CP is closer to the problem description and the scheduling problem can be built as a CP optimizer model easily. On the other hand, CP can find high-quality solutions in a short time for some scheduling and planning problems. The reason to use Differential Evolution (DE) is that DE has been widely applied to various optimisation problems, especially to the batch scheduling problem (Fu, Sivakumar, and Li 2012; Zhang et al. 2017).

Detailed comments:

1. *Regarding related work, the authors could consider citing the survey given in [8].*

   Thanks for the suggestion. The reference [8] has been added in this revision.

2. *Page 2, line 10: What is the "information invariance principle"?*

   The information invariance principle is used as a selection method for GA. In the selection procedure of GA, how many chromosomes in the current population are kept in the new population should be determined. Instead of selecting the chromosomes randomly according to the probability of survival, the ones whose selection probabilities are higher than or equal to the threshold value are picked for the new populations. The information invariance principle is used to calculate the threshold value at each iteration.

3. *Page 2, line 16: "release time" instead of "released time"*

   Thanks for this comment. We have revised “released time” as “release time”.

4. *Page 2, line 51: "constant factor approximation algorithm" seems to be a more common term than "constant approximation algorithm".*

   Thanks for this comment. We have revised “constant approximation algorithm” as "constant factor approximation algorithm".

5. *Page 4, line 1: The model is named "BDMI", though this abbreviation is never explained. Does it mean Batch Discrete Machine Incompatible? This should be explained somewhere.*

   Thanks for this comment. BDMI means “Batch and Discrete Machine with Incompatible job families”. Since the reviewer suggests using “unary” instead of “discrete”, we have used “BUMS (Batch and Unary Machines Scheduling) to replace “BDMI” in this revision.

6. *Page 4, line 19: In constraint (6), the quantifier $\forall k \in F$ seems to be missing.*

   Sorry for this mistake. We have added “$\forall$k ∈ F” in this revision.

7. *Page 4, line 30: Constraint (2) make<u>s</u> sure that ...*

Sorry for this mistake. We have revised this syntax error in this revision.

8. *Page 4, line 34: "Constraint (6) shows ... ", the verb "shows" seems not appropriate here, I suggest, e.g., enforces.*

Thanks for this suggestion. We have revised "shows" as "enforces" in this revision.

9. *Page 5, line 42: Consistency: Why is the term "non-decreasing" used here instead of "increasing" (which is used on the same page 5 in line 6)?*

Thanks for this comment. For consistency, we have used "non-decreasing" and "non-increasing" to represent the sequence rule in this revision.

10. *Page 5, line 48: the authors proposed ~~the~~ a lower bound.*

Thanks for this comment. We have revised "the" as "a" in this revision.

11. *Page 6, line 28: For readability, I recommend using only single letters for the variables CT, PT, and WE.*

Thanks for this suggestion. The variables CT, PT, and WE have been replaced by μ, ν, and λ, respectively.

12. *Page 7, line 20, "time complexities of LB1, ... ", to be precise, the time complexities should not refer to lower bounds, but to the algorithms that calculate them.*

Thanks for this comment. We have revised "time complexities of LB1, ... " as "time complexities to calculate LB1, ..." this revision.

13. *Page 7, line 35: Formulations are unclear. What is the preemptive schedule? The one obtained by the preemptive algorithm? "Each algorithm"? Probably this means "both algorithms"?*

Sorry for this confusion. We have revised this paragraph as follows.

"*In this section, a preemptive algorithm is developed. When one batch is finished on the batch machine while the unary machine is busy, the job that is processed on the unary machine can be disrupted by other jobs. Based on the preemptive schedule achieved by running the preemptive algorithm, a 4-approximation algorithm is proposed to solve this problem. Both algorithms consist of two stages. The first stage is to determine how to form batches and the batch sequence processed on the batch machine. The second stage is to determine the job sequence processed on the unary machine. We denote the approximation algorithm as GBDS (Greedy Batch and Dynamic Selection).*"

14. *Page 7, line 53: It might increase readability to recall the definition of T_l/W_l (from Lemma 2).*

Thanks for this comment. The related definitions of T_l and W_l have been added in the algorithms.

15. *Page 8, line 23: For readability, consider introducing variables right next to their textual description, instead of using identical positions in two separate enumerations.*

Thanks for this suggestion. We have revised this sentence as follows.

"*For job j, the completion time on the unary machine C_j^P, the finished time on the batch machine t_j^P, the waiting time in the queue of the unary machine Δ_j^P, the processing time on the unary machine p_j^P and the weight w_j^P in the solution of the preemptive GBDS algorithm are introduced.*"

16. *Page 10: Section 5 should be improved regarding grammar and language.*

Thanks for this comment. The "Computational experiments" section has been revised, especially in the subsections "Comparison results for instances in G_2" and "Sensitivity analysis".

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Submitted to *International Journal of Production Research*

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To appear in the *International Journal of Production Research*
Vol. 00, No. 00, 00 Month 20XX, 1–17

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(*Received 00 Month 20XX; accepted 00 Month 20XX*)

This paper addresses the problem of scheduling on batch and unary machines with incompatible job families such that the total weighted completion time is minimized. A mixed-integer linear programming model is proposed to solve the problem to optimality for small instances. Tight lower bounds and a 4-approximation algorithm are developed. Numerical results demonstrate that the proposed algorithm can obtain high quality solutions and have a competitive performance. Sensitivity analysis indicates that the performance of the proposed algorithm is also robust on different problem structures.

Keywords: batch machine; incompatible job families; total weighted completion time; approximation algorithm; constraint programming

1. Introduction

This paper focuses on a two-stage scheduling problem comprised of a batch machine in the upstream and a unary machine in the downstream. The batch machine can handle several jobs simultaneously up to its capacity. Once the process begins, the batch machine cannot be interrupted. Only when a batch is finished, the next batch can go into the batch machine. The unary machine processes jobs one by one. Incompatible job families are considered, which means that jobs from different families cannot be assigned to the same batch. The batch processing time depends on the related job family. For jobs in the same family, the batch processing times are identical. The objective is to find a schedule to minimize the total weighted completion time.

Batch machines have important applications in a variety of industrial systems, such as heat-treating ovens in the steel industry, burn-in operation in semiconductor final test and oxidation, diffusion in the wafer fabrication. In such industries, the batch machine usually means a bottleneck process, since it is often time-consuming and expensive. After batch processing, jobs are often needed to be processed one by one on the unary machine. This hybrid process with a batch machine and a unary machine is very common. We present two practical examples as follows. In the steel plant, the steel ingots are heated in the soaking pit before rolling process. The soaking pit is a batch machine which can reheat several steel ingots simultaneously. Then the steel ingots are rolled to a usable form of steel on the rough mill which can be regarded as a unary machine. In the semiconductor manufacturing factory, the oxidation area are batch machines. As to the downstream area of oxidation, such as etch area and photolithography area, machines are almost all unary machines.

We denote this two-stage scheduling problem as $\beta \to \delta|incompat|\sum w_j C_j$, where β, δ and *incompat* mean the batch machine, unary machine and incompatible job families, respectively.

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The third field specifies the objective function. Research on the two-stage scheduling problem that considering batch machines dates back to at least Ahmadi et al. (1992). They investigated the $\beta \to \delta$ problem and proposed a Full Batch-Dealing-Shortest Processing Time heuristic to minimize the total completion time $\sum C_j$. The authors proved that the worst case performance ratio of this heuristic is less than or equal to $1 + K/(2(K+1))$, where K is the total number of batches.

Based on the work of Ahmadi et al. (1992), Hoogeveen and Velde (1998) used the concept of positional completion time to propose a Lagrangian lower bound which can be computed in $O(n \log n)$ time for the problem $\beta \to \delta | \sum C_j$. Potts, and Kovalyov (2000) reviewed the batch machine models considering a single batch machine, parallel batch machines, and shop problems with batch machines. Using the information invariance principle to generate a new selection rule, Kim and Kim (2002) proposed a genetic algorithm for the problem $\beta \to \delta | \sum C_j$. Su (2003) considered the limited waiting time constraints for the second stage and proposed a mixed integer linear programming model for the problem $\beta \to \delta$ to minimize the makespan C_{max}. Chung, Sun, and Liao (2017) and Huang et al. (2017) studied the two-stage scheduling problem with limited waiting time to minimize C_{max}. The release time and size of each job are considered.

All of the above reviewed research did not consider the incompatible job families. Due to the chemical incompatibility or different requirements for temperature, incompatible job families are important features in practical production. The batch machine with incompatible job families was early introduced in Uzsoy (1995) and was studied based on a single batch machine. Some efficient optimal algorithms were proposed to minimize the objective of C_{max}, maximum lateness L_{max} and the total weighed completion time $\sum w_j C_j$ on a single batch machine. Mehta and Uzsoy (1998) studied the problem of minimizing total tardiness on a single batch machine with incompatible job families. They presented a dynamic programming algorithm with polynomial time complexity when the number of jobs and machine capacity were fixed. Koh et al. (2005) considered the scheduling problem on a single batch machine with arbitrary job sizes and incompatible job families. The objective functions they considered are $C_{\max}$, $\sum C_j$ and $\sum w_j C_j$. Yao, Jiang, and Li (2012) considered a single batch machine scheduling problem with incompatible job families and dynamic job arrivals to minimize $\sum C_j$. A decomposed branch and bound algorithm was proposed. Dauzère-Pérès and Mönch (2013) studied a single batch machine with incompatible job families to minimize the weighted number of tardy jobs. They proposed two different mixed-integer linear programming models and a random key genetic algorithm to solve this scheduling problem. Cheng et al. (2014) considered the scheduling problem of a batch machine with incompatible job families. A mixed integer programming model and two polynomial time heuristics based on the longest processing time first rule were developed.

For the $\beta \to \delta$ configuration, the research on the batch machine with incompatible job families is limited. Koh, Kim, and Lee (2009) studied the problem $\beta \to \delta | incompat | C_{\max}$. They presented a mixed integer programming formulation for the problem. Some simple heuristic rules and genetic algorithm were proposed to solve the problem. Yao, Zhao, and Zhang (2012) studied $\beta \to \delta | C_{\max}$ problems with dynamic job arrivals and also considered the extensions with incompatible job families and limited waiting time. They presented TSEDD-based (time-symmetric earliest due date) algorithms for these problems. Fu, Sivakumar, and Li (2012) studied the problem of $\beta \to \delta | incompat | \overline{C}$ with a limited buffer, where $\overline{C}$ means the mean completion time. They presented a lower bound, two constructive algorithms and a differential evolution algorithm. Zhang et al. (2017) studied the problem $\beta \to \delta | incompat | \sum C_j$ with a limited buffer. An approximation algorithm and a hybrid differential evolution algorithm were proposed to solve the problem.

The problem $\beta \to \delta | incompat | \sum w_j C_j$ can be regarded as a special case of the general complex job-shop scheduling problems with batch machines (Knopp, Dauzère-Pérès, and Yugma 2017). However, the problem $\beta \to \delta | incompat | \sum w_j C_j$ has not been studied specially and no approximation algorithm has been developed for this problem. The goal of this research is to develop a constant factor approximation algorithm for this problem to fill this gap. In this paper, we first propose a mixed-integer linear programming model for problem $\beta \to \delta | incompat | \sum w_j C_j$. Based on the structure of this problem, three different lower bounds are developed. An approximation algorithm is proposed, and we prove that its worst-case performance ratio is bounded by the constant 4. Numerical results show that the proposed algorithm has competitive and robust performances compared with the meta-heuristics.

The remainder of this paper is organized as follows. Section 2 presents the problem definition and mathematical formulation. In Section 3, we develop three lower bounds for this problem. An approximation algorithm and the worst-case analysis are proposed in Section 4. In Section 5, a constraint programming-based method is developed. Computational experiments are conducted in Section 6, and we conclude this paper in Section 7.

2. Problem formulation

There are n jobs which are grouped into m incompatible families. In the first stage, these jobs are assigned to some batches to be processed on the batch machine. In the second stage, the jobs are processed one by one in an arbitrary sequence on the unary machine. All the jobs are available at time zero. Preemption is not allowed on both the batch machine and unary machine. For a better presentation, the notations and decision variables are illustrated as follows.

Notations

n : the number of jobs;
m : the number of families;
a : the number of batches;
N : the job set, where $N = \{1, 2, \cdots, n\}$;
F : the job family set, where $F = \{1, 2, \cdots, m\}$;
n_k : the number of jobs in family k;
B : the batch set, where $B = \{1, 2, \cdots, a\}$;
F_k : the set of jobs which belong to family k;
p_i : the processing time of job i on the unary machine;
w_i : the weight of job i.
q_k : the processing time of batches consisting jobs of family k;
b : the batch capacity;
M_1 : a large positive number;
M_2 : a large positive number.

Variables

x_{il} : $x_{il} = 1$ if job i is assigned to batch l, and otherwise, $x_{il} = 0$;
y_{kl} : $y_{kl} = 1$ if batch l consists of the jobs of family k, and otherwise, $y_{kl} = 0$;
u_{ij} : $u_{ij} = 1$ if job i is processed before job j on the unary machine, and otherwise, $u_{ij} = 0$;
C_i : the completion time of job i on the unary machine.

Uzsoy (1995) proved the full batch property for a single batch machine under a regular measure of performance which is monotonically non-decreasing in the completion time of the individual job. Here, we prove that this full batch property can be extended to this two-stage problem in the following lemma.

Lemma 1. *There exists an optimal schedule of the problem $\beta \to \delta|incompat|\sum w_j C_j$, which consists of no partially full batches except possibly the last batch of each family.*

Proof. Let B_k^* denote the set of batches in which jobs belong to family k in an optimal schedule. From the first batch to the penultimate batch in B_k^*, if one batch is not full, the jobs in the later batches can be inserted to this batch until it is full. After this exchange, the completion time of each job in family k on the batch machine is smaller than or equal to that in the optimal schedule. Thus we can keep the starting time of each job in family k on the unary machine unchange. Repeating this inserting procedure, we can get a new schedule which consists of no partially full batches except possibly the last batch of each family. Since the starting time of each job on the unary machine in the new schedule can be the same as that in the optimal schedule, the new schedule is also an optimal schedule. □

Based on Lemma 1, the number of batches in family k is equal to $\lceil \frac{n_k}{b} \rceil$, where $\lceil \bullet \rceil$ means the rounding up function. Thus, we can set $a = \sum_{k \in F} \lceil \frac{n_k}{b} \rceil$ and this problem can be formulated as the following mixed-integer linear programming model which is denoted as BUMS (Batch and Unary Machine Scheduling) model.

$$
\begin{aligned}
(BUMS) \min \sum_{j \in N} w_j C_j & & (1)\\
\text{s.t.} \sum_{l \in B} x_{il} = 1, & \quad \forall i \in N, & (2)\\
\sum_{i \in N} x_{il} = b, & \quad \forall l \in B, & (3)\\
\sum_{k \in F} y_{kl} = 1, & \quad \forall l \in B, & (4)\\
x_{il} \le y_{kl}, & \quad \forall i \in F_k, k \in F, l \in B, & (5)\\
u_{ij} + u_{ji} = 1, & \quad \forall i, j \in N, i < j, & (6)\\
C_i - p_i \ge C_j - M_1 u_{ij}, & \quad \forall i, j \in N, i \ne j, & (7)\\
C_i - p_i \ge \sum_{h=1}^{l} \sum_{k \in F} y_{kh} q_k - M_2(1 - x_{il}), & \quad \forall i \in N, l \in B & (8)\\
u_{ij} \in \{0, 1\}, & \quad \forall i, j \in N, i \ne j, & (9)\\
y_{kl} \in \{0, 1\}, & \quad \forall k \in F, l \in B, & (10)\\
x_{il} \in \{0, 1\}, & \quad \forall i \in N, l \in B. & (11)
\end{aligned}
$$

The objective function (1) is to minimize the total weighted completion time. Constraint (2) makes sure that one job can be assigned to only one batch. Constraint (3) guarantees all batches are full. By constraint (4), one batch only consists of jobs which belong to one family. Constraint (5) enforces that only when batch l consists of the jobs in family k, the jobs in family k can be assigned to batch l. Constraint (6) makes sure that jobs have processing precedence on the unary machine. Constraint (7) makes sure that only when a job finishes processing on the unary machine, the latter ones can start processing on the unary machine. Constraint (8) makes sure that job i can be processed on the unary machine only when the batch that job i belongs to finishes processing on the batch machine. Constraints (9), (10) and (11) are the range of the variables. Herein, we can set $M_1 = \sum_{k \in F} \lceil \frac{n_k}{b} \rceil q_k + \sum_{i \in N} p_i$ and $M_2 = \sum_{k \in F} \lceil \frac{n_k}{b} \rceil q_k$.

If we set the number of incompatible job families and all the weights to be 1, this problem reduces to problem $\beta \to \delta | \sum C_j$, which has been shown to be strongly NP-hard by Ahmadi et al. (1992). Thus the problem $\beta \to \delta | incompat | \sum w_j C_j$ is also NP-hard in strong sense.

3. Lower bounds

In this section, we develop three lower bounds for problem $\beta \to \delta | incompat | \sum w_j C_j$, which can be used to estimate the solution qualities of the proposed algorithms. Let t_j and r_l to denote the completion time of job j and batch l on the batch machine, respectively. Next, we first introduce the optimal Greedy Weighted Completion Time (GRWC) algorithm for the single batch scheduling problem with the objective to minimize total weighted completion time in the Lemma 2, and readers may refer to Uzsoy (1995) for details.

Lemma 2. *(Uzsoy (1995), GRWC) Index all jobs of the same family in non-increasing order of their weights w_i. For each family, starting from the first job, form full batches greedily. For the single batch scheduling problem with the objective to minimize total weighted completion time, the optimal schedule can be achieved by sequencing all batches in non-decreasing order of T_l/W_l, where T_l denotes the processing time of the batch l and W_l is the total weight of the jobs in the batch l.*

Definition 1. *Γ_{GRWC} is defined as the total weighted completion time achieved by the GRWC algorithm for the single batch scheduling problem.*

Considering the situation that all the jobs after the batch machine can be processed on the unary machine immediately, i.e., the waiting time on the unary machine is omitted, we derive the first lower bound denoted by LB_1 in the Theorem 1.

Theorem 1. *Form and sort the batches based on the GRWC algorithm. Then*

$$LB_1 = \Gamma_{\text{GRWC}} + \sum_{j=1}^{n} w_j p_j$$

is a lower bound of the problem $\beta \to \delta|incompat|\sum w_j C_j$.

Proof. Let (x^*, y^*, u^*, C^*) be an optimal solution of this problem. For the completion time C_j^* of job j, let w_j^* and p_j^* denote its related weight and processing time. Obviously, each C_j^* consists of three parts: the completion time t_j^* on the batch machine, the waiting time $\triangle_j^* \geq 0$ in the queue of the unary machine and the processing time p_j^* on the unary machine. Thus, in terms of the optimal objective value $\sum_{j=1}^{n} w_j^* C_j^*$, we have the following inequalities

$$\begin{aligned}\sum_{j=1}^{n} w_j^* C_j^* &= \sum_{j=1}^{n} w_j^* (t_j^* + \triangle_j^* + p_j^*) \\ &\geq \sum_{j=1}^{n} w_j^* t_j^* + \sum_{j=1}^{n} w_j^* p_j^* \\ &\geq \Gamma_{GRWC} + \sum_{j=1}^{n} w_j p_j = LB_1.\end{aligned}$$

Note that the last inequality holds according to Lemma 2, and this completes the proof. □

Definition 2. *Sort all the jobs in non-decreasing order of the ratios p_j/w_j for all $j \in N$. Let $\hat{p}_j$ and $\hat{w}_j$ denote the related processing time and weight of job j after sorting and define Γ_{WSPT} as the total weighted completion time obtained by the weighted shortest processing time (WSPT) rule for the single unary machine, i.e., $\Gamma_{\text{WSPT}} = \sum_{i \in N} \sum_{j \leq i} \hat{w}_i \hat{p}_j$.*

The WSPT rule gives the optimal solution for the single unary machine to minimize the total weighted completion time (Smith 1956). In Eastman, Even, and Isaacs (1964), the authors proposed a lower bound for the problem of scheduling n jobs on some identical unary machines with the objective to minimize the total weighted completion time. We introduce it in Lemma 3 and this helps us to develop another lower bound LB_2 in Theorem 2.

Lemma 3. *(Eastman, Even, and Isaacs 1964) $\frac{1}{M}\Gamma_{\text{WSPT}} + \frac{M-1}{2M} \sum_{i \in N} w_i p_i$ is a lower bound for* *the problem of scheduling n jobs on M identical unary machines with the objective to minimize the total weighted completion time.*

Theorem 2. *Form and sort the batches based on the GRWC algorithm. Then*

$$LB_2 = \Gamma_{\text{GRWC}} + \frac{1}{a}\Gamma_{\text{WSPT}} + \frac{a-1}{2a}\sum_{i\in N} w_i p_i$$

is a lower bound of the problem $\beta \to \delta|incompat|\sum w_j C_j$.

Proof. Let (x^*, y^*, u^*, C^*) be an optimal solution of this problem. When one batch is finished on the batch machine, we let the jobs of this batch be processed immediately and consecutively on the unary machine without changing the relative positions in the optimal solution. Let C_j^0 denote the new completion time of job j. For each job j, it is easy to check that C_j^0 would be smaller than or equal to C_j^*. Thus we have the following inequalities

$$\begin{aligned}\sum_{j=1}^{n} w_j^* C_j^* &\geq \sum_{j=1}^{n} w_j^* C_j^0 \\ &= \sum_{l=1}^{a}\sum_{i=1}^{h_l^*} \lambda_{li}^* \mu_{li}^* + \sum_{l=1}^{a}\sum_{i=1}^{h_l^*} \lambda_{li}^* \sum_{j\leq i} \nu_{lj}^* \\ &\geq \Gamma_{GRWC} + \sum_{l=1}^{a}\sum_{i=1}^{h_l^*} \lambda_{li}^* \sum_{j\leq i} \nu_{lj}^*,\end{aligned}$$

where μ_{lj}^*, ν_{lj}^* and λ_{lj}^* mean the completion time on the batch machine, the processing time and the weight of job j assigned to batch l in the optimal solution, respectively. h_l^* denotes the number of jobs assigned to batch l in the optimal solution. The expression $\sum_{l=1}^{a}\sum_{i=1}^{h_l^*} \lambda_{li}^* \sum_{j\leq i} \nu_{lj}^*$ can be regarded as the total weighted completion time for the problem of scheduling n jobs on a identical unary machines with additional constraints that each unary machine must process fixed number of jobs. According to Lemma 3, we have

$$\begin{aligned}\sum_{j=1}^{n} w_j^* C_j^* &\geq \Gamma_{GRWC} + \sum_{l=1}^{a}\sum_{i=1}^{h_l^*} \lambda_{li}^* \sum_{j\leq i} \nu_{lj}^* \\ &\geq \Gamma_{GRWC} + \frac{1}{a}\Gamma_{WSPT} + \frac{a-1}{2a}\sum_{i\in N} w_i p_i \\ &= LB_2.\end{aligned}$$

This completes the proof. □

Next, we take a contrary position to develop another lower bound, denoted by LB_3. Considering the situation that all the jobs are available on the unary machine after the first batch is finished at time r_1, this problem is relaxed to the single unary machine scheduling problem with identical release time r_1 to minimize the total weighted completion time. The relaxed problem can be solved to optimality by the WSPT rule.

Theorem 3. *Let q_0 denote the minimum value of q_k for $k \in F$. Then*

$$LB_3 = \sum_{j=1}^{n} w_j q_0 + \Gamma_{\text{WSPT}}$$

is a lower bound of the problem $\beta \to \delta|incompat|\sum w_j C_j$.

Proof. Let (x^*, y^*, u^*, C^*) be an optimal solution of this problem, and for the completion time C_j^* of job j, let w_j^* and p_j^* denote its related weight and processing time. Thus, in terms of the optimal objective value $\sum_{j=1}^{n} w_j^* C_j^*$, we have the following inequalities

$$\begin{aligned}\sum_{j=1}^{n} w_j^* C_j^* &\geq \sum_{j=1}^{n} w_j^*(r_1 + \sum_{i=1}^{j} p_i^*)\\ &= \sum_{j=1}^{n} w_j r_1 + \sum_{j=1}^{n} w_j^* \sum_{i=1}^{j} p_i^*\\ &\geq \sum_{j=1}^{n} w_j q_0 + \Gamma_{WSPT} = LB_3.\end{aligned}$$

□

The time complexities to calculate LB_1, LB_2 and LB_3 are all $O(n \log n)$ and can be calculated very fast. Based on the derivation process of these three lower bounds, their qualities depend heavily on the batch processing time and the sum of the processing time of jobs in one batch on the unary machine. When the batch processing time is small enough, LB_3 is tighter than LB_1 and LB_2. Under this situation, the batch processing time has little influence on the objective, and jobs tend to enter the queue of the unary machine earlier. On the other hand, if the batch processing time is large enough, the jobs within one batch should be processed on the unary machine consecutively. This situation matches the structures of LB_1 and LB_2, and thus LB_1 and LB_2 are tighter than LB_3. The relationship between LB_1 and LB_2 depends on the data of processing time and weight. In this paper, we set the lower bound, LB, as the maximum value of LB_1, LB_2 and LB_3.

4. A 4-approximation algorithm

In this section, a preemptive algorithm is developed. When one batch is finished on the batch machine while the unary machine is busy, the job that is processed on the unary machine can be disrupted by other jobs. Based on the preemptive schedule achieved by running the preemptive algorithm, a 4-approximation algorithm is proposed to solve this problem. Both algorithms consist of two stages. The first stage is to determine how to form batches and the batch sequence processed on the batch machine. The second stage is to determine the job sequence processed on the unary machine. We denote the approximation algorithm as GBDS (Greedy Batch and Dynamic Selection).

4.1 *Preemptive GBDS algorithm*

We first introduce the preemptive GBDS algorithm. The preemption occurs only when one batch is finished on the batch machine while the unary machine is busy. It is described as follows.

Preemptive GBDS

Step 1. For each k, sort jobs of F_k in the non-increasing order of $w_i,\ i \in F_k$;
Step 2. Form full batches greedily from the first job in each family;
Step 3. Calculate the value of T_l/W_l for all batches, where T_l denotes the processing time of the batch l and W_l is the total weight of the jobs in the batch l. The batches are processed on the batch machine in non-decreasing order of T_l/W_l;
Step 4. When the unary machine is available, select the job i^* which has the smallest value of p/w in the current queue of the unary machine,

- If no batches are finished during the processing of job i^*, process job i^*;
- Else, next batch l^* is finished and enters the queue of the unary machine;
  - If the ratio p_i^*/w_i^* is smaller than or equal to that of all jobs in the batch l^*, process job i^*;
  - Else, process job i^* partially until the finished time of batch l^*. Then stop processing job i^* and return the remaining part of job i^* with the criterion p_i^*/w_i^* to the queue of the unary machine;

Step 5. Repeat the process in Step 4 until all jobs are completed. Return the schedule π.

Figure 1 is an illustration of the preemptive GBDS algorithm. The horizontal axis represents the time. The ratios p/w for all the jobs satisfy the following inequalities: $p_{l1}/w_{l1} \le p_{l2}/w_{l2} \le p_{l3}/w_{l3}$ for batch $l = 1, 2, 3, 4$; $p_{23}/w_{23} \le p_{31}/w_{31}$; $p_{42}/w_{42} \le p_{32}/w_{32}$ and $p_{33}/w_{33} \le p_{43}/w_{43}$. Based on the above preemptive GBDS algorithm, it is easy to check that the job 23 should not be preempted, but the job 32 should be preempted. The gray boxes represent the preempted jobs. Next, we give a upper bound of the solution of the preemptive GBDS algorithm in Theorem 4.

Figure 1. An illustration of the preemptive GBDS algorithm.

Theorem 4. *Let π_P denote the schedule generated by the preemptive GBDS algorithm, and $Z(\pi_P)$ denote its corresponding objective value. The following equality holds*

$$Z(\pi_P) \le \Gamma_{\text{GRWC}} + \Gamma_{\text{WSPT}}.$$

Proof. For job j, the completion time on the unary machine C_j^P, the finished time on the batch machine t_j^P, the waiting time in the queue of the unary machine Δ_j^P, the processing time on the unary machine p_j^P and the weight w_j^P in the solution of the preemptive GBDS algorithm are introduced. For each job j, we have

$$w_j^P C_j^P = w_j^P (t_j^P + \Delta_j^P + p_j^P).$$

For a preempted job, the completion time of this job is equal to the completion time of the last part of this job. Note that there is no idle time on the unary machine between t_j^P and C_j^P for any job j. When one batch is finished and the unary machine is busy, there are two cases:

- Case 1. The preemption occurs. For job j in this batch, the waiting time Δ_j^P is the sum of the processing time of jobs of which ratios p/w are smaller than $\frac{p_j^P}{w_j^P}$ in the queue of the unary machine. It is smaller than or equal to the sum of the processing time of jobs of which ratios

p/w are smaller than $\frac{p_j^P}{w_j^P}$ in N, i.e., $\sum_{i<\hat{j}} \hat{p}_i$, where $\hat{j}$ is the position of job j in the WSPT order;

- Case 2. There is no preemption. For job j in this batch, the waiting time Δ_j^P is the sum of the processing time of jobs of which ratios p/w are smaller than $\frac{p_j^P}{w_j^P}$ in the queue of the unary machine and partial processing time of the job i_0 which occupies the unary machine when the batch is finished. But because the ratio $\frac{p_{i_0}}{w_{i_0}}$ is smaller than $\frac{p_j^P}{w_j^P}$, Δ_j^P is also smaller than $\sum_{i<\hat{j}} \hat{p}_i$ which contains p_{i_0}.

When one batch is finished and the unary machine is available, it is easy to check that the waiting time Δ_j^P is the sum of the processing time of which $\frac{p_i^P}{w_i^P}$ is smaller than $\frac{p_j^P}{w_j^P}$ in the queue of the unary machine. It is smaller than or equal to $\sum_{i<\hat{j}} \hat{p}_i$, where $\hat{j}$ is the position of job j in the WSPT order. Therefore we have the following inequalities

$$\begin{aligned} Z(\pi_P) &= \sum_{j\in N} w_j^P(r_j^P + \Delta_j^P + p_j^P) \\ &\leq \sum_{j\in N} w_j^P(r_j^P + \sum_{i<\hat{j}} \hat{p}_i + p_j^P) \\ &= \sum_{j\in N} w_j^P r_j^P + \sum_{j\in N} w_j^P \sum_{i\leq\hat{j}} \hat{p}_i \\ &= \Gamma_{GRWC} + \Gamma_{WSPT} \end{aligned}$$

This completes the proof. □

4.2 *GBDS algorithm*

The solution of the preemptive GBDS algorithm is not feasible for the problem $\beta \to \delta|incompat|\sum w_j C_j$, since preemption exists. Based on the preemptive solution, we derive the GBDS algorithm to obtain a feasible solution. The idea is to run the preemptive GBDS algorithm combining the GRWC algorithm and dynamic WSPT rule to get a preemptive schedule. The preemption is allowed only on the unary machine and when one batch is finished on the batch machine while the unary machine is busy. Then the preemptive schedule is converted non-preemptively based on the sequence of the completion time of the last piece or first piece of jobs.

GBDS

Step 1. For each k, sort jobs of F_k in the non-increasing order of w_i, $i \in F_k$;
Step 2. Form full batches greedily from the first job in each family;
Step 3. Calculate the value of T_l/W_l for all batches, where T_l denotes the processing time of the batch l and W_l is the total weight of the jobs in the batch l. The batches are processed on the batch machine in non-decreasing order of T_l/W_l;
Step 4. Run the preemptive GBDS algorithm to get a preemptive schedule.
Step 5. Adjust the schedule non-preemptively based on Convertor I (II) to get a feasible schedule.

- *Convertor I.* The jobs are processed on the unary machine continuously by non-decreasing order of the completion time of the last piece of jobs with the constraint that no job can start before its completion time on the batch machine.
- *Convertor II.* The jobs are processed on the unary machine continuously by non-decreasing order of the completion time of the first piece of jobs with the constraint

that no job can start before its completion time on the batch machine.
Step 6. Return the better schedule with smaller objective value.

In Figure 2, we give an example in which there are three jobs to illustrate the convertors in the GBDS algorithm.

Figure 2. An example of the convertors.

In Theorem 5, we prove that the relationship between the preemptive GBDS and GBDS has the following result.

Theorem 5. *Let π_P and π_N denote the schedules generated by the preemptive GBDS and GBDS, respectively. Z is defined as the related objective function value. We have*

$$Z(\pi_N) \leq 2Z(\pi_P).$$

Proof. In the preemptive GBDS algorithm and the GBDS algorithm, the *Step 1* to *Step 3* are same and determine how to form batches and the sequence of batches on the batch machine. After *Step 3*, each job j has a finished time t_j on the batch machine, which can be regarded as the release time to the unary machine. So the sequence decisions on the unary machine are same with the problem $1|r_j|\sum w_jC_j$. Phillips, Stein, and Wein (1998) proved that given a preemptive schedule P for $1|r_j, pmtn|\sum w_jC_j$, the procedure of *Convertor I* in the GBDS algorithm produces a non-preemptive schedule N in which $C_j^N \leq 2C_j^P$ for each job j. Hence we have $Z(\pi_N) \leq 2Z(\pi_P)$, and this completes the proof. □

The time complexity of the GBDS algorithm is $O(n^2)$. Next, we prove that the worst-case performance ratio of the GBDS algorithm is bounded by 4 in Theorem 6.

Theorem 6. *The GBDS algorithm is a 4-approximation algorithm for problem $\beta \rightarrow \delta|incompat|\sum w_jC_j$.*

Proof. Let Z_{opt}^* denote the optimal objective value. Combining the results in Theorem 4 and Theorem 5, we have the following inequalities

$$\begin{aligned}
\frac{Z(\pi_N)}{Z_{opt}^*} &\leq \frac{2Z(\pi_P)}{Z_{opt}^*} \\
&\leq \frac{2(\Gamma_{GRWC}+\Gamma_{WSPT})}{Z_{opt}^*} \\
&= 2(1+\frac{\Gamma_{GRWC}+\Gamma_{WSPT}-Z_{opt}^*}{Z_{opt}^*}) \\
&\leq 2(1+\frac{\Gamma_{GRWC}+\Gamma_{WSPT}-LB_3}{LB_1}) \\
&= 2(1+\frac{\Gamma_{GRWC}-\sum_{j=1}^n w_jq_0}{\Gamma_{GRWC}+\sum_{j=1}^n w_jp_j}) \\
&< 2(1+1) \\
&= 4.
\end{aligned}$$

This completes the proof. □

5. A constraint programming-based method

Constraint programming (CP), a method for solving the constraint satisfaction problem, now becomes an complementary solution technique for the combinatorial optimization problems and has a promising success for solving scheduling problems (Hentenryck and Michel 2009; Öztürk et al. 2013; Ham and Cakici 2016). In this section, a constraint programming-based method, CPWS (Constraint Programming with Warm Start), is developed to search the solution space further. In the CPWS method, the solution of the GBDS algorithm is attached as the initial solution of the CP technique. The CP technique has two advantages. On one hand, the natural formulation in CP is closer to the problem description and the scheduling problem can be built as a CP optimizer model easily. On the other hand, CP can find high-quality solutions in a short time for some scheduling and planning problems (Hentenryck and Michel 2009).

5.1 *CPWS method*

In CP, the interval variables are used as the decision variables to denote the tasks in scheduling or planning problems. Each interval variable has a start time, an end time and the given duration. The interval variables and some functions used for the problem $\beta \to \delta | incompat | \sum w_j C_j$ are defined as below:

- $batch_i$, an interval variable representing the processing task of job $i \in N$ in the batch machine;
- $unary_i$, an interval variable representing the processing task of job $i \in N$ on the unary machine;
- $state^{batching}$, state of batch machine representing a batch. The intervals of the state function represent the different batches;
- $cumul^{capacity} = \sum_{i \in N} pulse(batch_i, 1)$. Each assignment of job to the batch machine increases the cumulative function at the start of the usage of batch machine, and decreases the function when the batch machine is released at its end time.

With the variables and functions defined above, we can reformulate the problem $\beta \to \delta | incompat | \sum w_j C_j$ via CP as follows:

$$\min \sum_{i \in N} w_i [endOf(unary_i)] \tag{12}$$

$$\text{s.t.} \quad endBeforeStart(batch_i, unary_i), \forall i \in N, \tag{13}$$

$$alwaysEqual(state^{batching}, batch_i), \forall i \in N, \tag{14}$$

$$noOverlap(unary), \tag{15}$$

$$capacity \le b. \tag{16}$$

The objective (12) is to minimize the total weighted completion time. Constraint (13) ensures the precedence relation between the batch machine and the unary machine for each job. Constraint (14) specifies that the jobs in the same batch have the same start and end time in the batch machine. Constraint (15) imposes that the jobs on the unary machine should be processed one by one. Constraint (16) ensures that the total number of jobs in one batch cannot exceed the batch capacity.

The CPWS method integrates the solution of the GBDS algorithm as a wram start, then the CP solver with default configuration is used to solve the CP model. The pure CP method without warm start is also used to solve the problem. The time limits for the CPWS method and the pure CP method are set as 5 minutes.

5.2 *Differential evolution method for comparison*

In order to test the competitive performance of the proposed GBDS algorithm and CPWS method, the Differential Evolution (DE) are applied to make a comparison. DE is first proposed by Storn and Price (1997), which has been widely applied to various optimisation problems and batch scheduling recently (Fu, Sivakumar, and Li 2012; Zhang et al. 2017). The mutation operator is as below,

$$V_i(g+1) = X_\alpha(g) + \lambda(best(g) - X_\alpha(g)) + F(X_\beta(g) - X_\gamma(g)).$$

where $X_i(g)$ is the ith individual at generation g and $V_i(g+1)$ is a mutant individual for target individual $X_i(g)$. This mutation operator is proved to have good convergent performance in Fu, Sivakumar, and Li (2012). The crossover operator, selection operator and parameter settings of DE are referred to Fu, Sivakumar, and Li (2012). In detail, the values of three control variables are that $\lambda = 0.4$, $F = 0.7$, $CR = 0.5$. The population size is set as $2a$.

The time limit for DE is set as 20 minutes. To apply DE to solve this problem, we need to code the feasible solutions. The code consists of $m+1$ parts. The former m parts are used to decide the batch forms in m job families. The last part is used to get the batch sequences on the batch machine. The job sequences on the unary machine is same as that in the batch machine, i.e., follows first in and first out rule. The value of each position in the code is randomly generated from uniform distribution $[-1, 1]$. The related sequence is based on the non-increasing order of the code value. We give an example in which $n = 8, m = 2, b = 2$ to illustrate the code process in Figure 3.

Figure 3. Illustration on the code process of DE.

6. Computational experiments

This section conducts the computational experiments to test the performances of the proposed formulation and algorithm. All runs are made on a 64-bit Window 10 platform with Intel Core 3.2GHz CPUs and 8.0GB RAM. The mathematical formulation BUMS is solved by Cplex 12.7.1 with a time limit 3600 seconds under default configuration. The CP method is coded with OPL language in Cplex 12.7.1.

6.1 *Problem instances generation*

We use the combination "n-m-b" to present the problem case, where n is the number of jobs, m is the number of job families and b is the batch capacity. As used by Ahmadi et al. (1992) and Koh et al. (2005), we present the instance generation rule below.

For each combination "n-m-b", set $n_k = b\lfloor \frac{n}{mb} \rfloor, k = 1, 2, \cdots, m-1$, and $n_m = n - \sum_{k=1}^{m-1} n_k$. For each job j, the weight w_j is randomly generated from the uniform distribution $[1, 2]$, and the processing time p_j on the unary machine is randomly generated from the uniform distribution $[1, 10]$. The batch processing time in family k, q_k, is randomly generated from the uniform distribution $[b(\sum_{j=1}^{n} p_j)/n - m, b(\sum_{j=1}^{n} p_j)/n + m]$, which can avoid the situation that the objective value is dominated by the batch machine or the unary machine. Let G_1 and G_2 denote two different sizes of instances, respectively. The configuration of each group is given as follows

G_1: $n = \{8, 12, 16, 20\}$, $m = \{2, 4\}$ and $b = \{2, 4\}$;
G_2: $n = \{50, 100, 200, 400\}$, $m = \{2, 5, 10\}$ and $b = \{10, 20\}$.

We use an additional requirement $\frac{n}{mb} \geq 1$ to get rid of some simple combinations. For G_1, we generate 10 instances for each combination to test the problem size that can be solved to optimality by the formulation BUMS. For G_2, we generate 50 instances for each combination to

Table 1. Comparison results of GBDS and LB with the optimal solutions of instances in G_1.

n-m-b	*#Opt*	*Time (s)*	*Gap-GBDS (%)*	*Gap-CPWS (%)*	*Gap-LB (%)*
8-2-2	10	0.28	2.16	0.00	4.55
8-2-4	10	0.19	0.59	0.00	1.82
8-4-2	10	0.13	2.15	0.00	4.81
12-2-2	10	70.56	3.49	0.00	4.06
12-2-4	10	6.40	0.46	0.00	2.56
12-4-2	10	2.60	2.99	0.00	3.66
16-2-2	1	501.08	0.60	0.00	1.42
16-2-4	9	517.09	1.68	0.00	2.24
16-4-2	4	1869.06	1.60	0.00	2.73
16-4-4	10	13.73	1.73	0.00	2.24
20-2-2	0	-	-	-	-
20-2-4	0	-	-	-	-
20-4-2	0	-	-	-	-
20-4-4	4	1495.50	1.54	0.00	1.60

test the performance of the proposed algorithm. Thus, a total number of 1090 instances are used to test the performances of the proposed algorithm.

6.2 *Comparison with optimal solutions*

We present the numerical results based on the instances in G_1. For a convenient table size, we use the following notations to present the numerical results:

n-m-b: the combination of numbers of jobs and families, and batch capacity;
#Opt: the number of instances solved to optimality within one hour by the BUMS formulation;
Time: the average time in seconds to solve the instances to optimality by the BUMS formulation;
Gap-GBDS: the average gap between GBDS and optimal solutions;
Gap-CPWS: the average gap between CPWS and optimal solutions;
Gap-LB: the average gap between LB and optimal solutions.

In Table 1, the columns of *Time*, *Gap-GBDS* and *Gap-LB* report the average results for the instances that are solved to optimality in each combination. When no instance in one combination is solved to optimality within one hour by the BUMS formulation, the *#Opt* is zero and the columns of *Time*, *Gap-GBDS* and *Gap-LB* are displayed as "-". As shown in Table 1, the problem size that can be solved to optimality by BUMS is reported as 12 jobs within one hour on a single PC. When $n \geq 16$, only some instances are solved to optimality within one hour. The values of *Gap-GBDS* are all very small. The maximum average value of *Gap-GBDS* is 3.49% for the combination 12-2-2, which indicates that the proposed GBDS algorithm is capable of obtaining the near-optimal solutions for small-scale instances. Moreover, the proposed lower bound is also very tight to the optimal objective value. The maximum average value of *Gap-LB* appearing in the combination 8-4-2 is 4.81%. The *Gap-CPWS* column shows that the CPWS method can obtain the optimal solutions in a short time, which indicates the advantage of the CP technique.

6.3 *Comparison results for instances in G_2*

Table 2 presents the detail comparison results for instances in G_2. Here we use CP to denote the pure CP method. *Gap-GBDS*, *Gap-CPWS*, *Gap-CP* and *Gap-DE* are the average values of gaps between the results achieved by the related algorithms and the lower bound. We calculate the gap between the *Algorithm* and *LB* as 100(*Obj(Algorithm)*-*LB*)/*LB* for each instance, then get the average value of gap among the 50 instances for each combination, where *Obj(Algorithm)* is the objective value achieved by the related *Algorithm* (GBDS, CPWS, CP or DE). *Time* is the computing time in seconds for the GBDS algorithm. From the values of *Gap-GBDS* in Table 2, we

Table 2. Comparison results for instances in G_2.

n-m-b	*Time (s)*	*Gap-GBDS (%)*	*Gap-CPWS (%)*	*Gap-CP (%)*	*Gap-DE (%)*	*GBDS-CP*	*GBDS-DE*	*CPWS-CP*
50-2-10	0.27	2.44	**1.29**	1.38	6.73	1.00E-00	1.39E-25	2.71E-02
50-5-10	0.26	2.12	**1.48**	1.57	2.68	1.00E-00	1.28E-05	3.18E-02
100-2-10	0.52	2.44	**1.11**	1.17	9.64	1.00E-00	2.78E-43	2.98E-02
100-2-20	0.50	1.46	**1.05**	1.10	8.22	1.00E-00	3.04E-43	5.35E-02
100-5-10	0.52	2.26	**1.50**	1.56	7.51	1.00E-00	1.85E-41	1.48E-01
100-5-20	0.50	1.25	**1.01**	1.25	2.76	4.82E-01	2.36E-26	2.16E-05
100-10-10	0.52	3.10	**2.24**	2.34	4.02	1.00E-00	2.23E-14	4.41E-02
200-2-10	1.18	2.18	**0.84**	0.98	11.37	1.00E-00	6.03E-53	1.35E-07
200-2-20	1.12	1.60	**0.85**	1.07	10.43	1.00E-00	6.21E-54	4.00E-07
200-5-10	1.17	2.07	**1.29**	1.45	10.14	1.00E-00	7.71E-50	4.25E-05
200-5-20	1.12	1.59	**1.34**	1.46	8.42	9.91E-01	1.67E-50	5.08E-03
200-10-10	1.18	2.52	**1.96**	2.05	8.60	1.00E-00	4.56E-41	7.43E-02
200-10-20	1.12	1.37	**1.11**	1.46	2.89	7.60E-02	4.77E-33	4.20E-08
400-2-10	2.77	1.80	**0.60**	0.78	11.98	1.00E-00	8.23E-61	8.89E-09
400-2-20	2.65	1.39	**0.64**	0.81	11.53	1.00E-00	4.19E-58	1.13E-08
400-5-10	2.83	1.83	**0.96**	1.28	11.52	1.00E-00	3.39E-58	3.79E-12
400-5-20	2.56	1.36	**1.07**	1.23	10.71	1.00E-00	1.08E-59	1.51E-06
400-10-10	2.82	2.73	**2.12**	2.58	11.64	9.58E-01	1.02E-51	9.65E-11
400-10-20	2.76	1.47	**1.35**	1.82	9.03	3.87E-06	5.00E-58	3.35E-09

Note: The time limit for CPWS and CP is 5 minutes.
The time limit for DE is 20 minutes.
The bold font represents the best gap.

can see that all the values are less than 4%. The maximum value of *Gap-GBDS* is 3.10% in the combination 100-10-10. The time seconds for GBDS are also very small. These results indicate that the GBDS algorithm can obtain high quality solutions in a short time. All values of *Gap-GBDS* are less than those of *Gap-DE*. When compared to CP, the CP method outperforms the GBDS algorithm in most combinations from the values of gap. When the solutions of the GBDS algorithm are integrated into the CP technique, the *Gap-CPWS* column shows that the CPWS method seems to yield the best results among these methods.

To further justify this observation, we establish the hypothesises as follows:

- GBDS-CP (H_0: *Gap-GBDS*$-$*Gap-CP*≥ 0 and H_1: *Gap-GBDS*$-$*Gap-CP*< 0)
- GBDS-DE (H_0: *Gap-GBDS*$-$*Gap-DE*≥ 0 and H_1: *Gap-GBDS*$-$*Gap-DE*< 0)
- CPWS-CP (H_0: *Gap-CPWS*$-$*Gap-CP*≥ 0 and H_1: *Gap-CPWS*$-$*Gap-CP*< 0)

In Table 2, the columns of *GBDS-CP*, *GBDS-DE* and *CPWS-CP* represent the p-values obtained by the T-test when comparing the related two algorithms. The null hypothesises of all the combinations are rejected when comparing GBDS and DE , since the p-values are less than the 5% significance level. When comparing GBDS and CP, the *GBDS-CP* column shows that the CP method is better than the GBDS algorithm in most combinations. But when the solutions of the GBDS algorithm are attached as the initial solutions of the CP technique, the *CPWS-CP* column shows that the results of the CP technique can be improved. These results indicate that the GBDS algorithm can provide high-quality solutions and has a competitive performance.

6.4 *Sensitivity analysis*

The relationship between the batch processing time and the sum of the processing time of jobs in one batch on the unary machine has an effect on the problem structure. To test the robustness of the GBDS algorithm, we reduce and enlarge the batch processing time as $0.5q_k$ and $1.5q_k$ for each family k, respectively. The results are reported in Table 3 and Table 4. The columns are same with that in Table 2.

When the batch processing time is reduced, the batches are finished earlier and more batches enter the queue of the unary machine to wait to be processed on the unary machine. In Table 3, the values of *Gap-GBDS* for all the combinations are small, where the maximum value of *Gap-GBDS* is 9.71% in the combination 200-10-20. The hypothesis test results show that the proposed GBDS algorithm outperforms the DE algorithms. When the numbers of jobs and families are large, the

Table 3. Comparison results for instances in G_2 with reduced batch processing time.

n-m-b	*Time (s)*	*Gap-GBDS (%)*	*Gap-CPWS (%)*	*Gap-CP (%)*	*Gap-DE (%)*	*GBDS-CP*	*GBDS-DE*	*CPWS-CP*
50-2-10	0.25	5.59	**1.48**	1.53	11.82	1.00E-00	1.39E-21	1.80E-01
50-5-10	0.23	7.98	**6.69**	6.95	20.21	1.00E-00	1.58E-34	3.40E-02
100-2-10	0.47	6.12	**1.18**	1.51	20.06	1.00E-00	5.06E-41	9.67E-08
100-2-20	0.44	4.75	**1.14**	1.31	15.95	1.00E-00	7.13E-34	8.27E-03
100-5-10	0.47	7.19	**2.65**	2.87	18.97	1.00E-00	5.39E-36	1.47E-04
100-5-20	0.45	7.66	**7.19**	7.86	23.35	4.95E-02	3.48E-42	1.44E-07
100-10-10	0.48	9.66	**8.12**	8.48	25.92	1.00E-00	1.57E-40	3.00E-04
200-2-10	1.05	6.97	**1.29**	2.25	27.45	1.00E-00	1.14E-50	1.86E-22
200-2-20	1.03	6.10	**1.25**	2.24	24.77	1.00E-00	2.09E-47	1.38E-19
200-5-10	1.11	7.07	**1.86**	2.90	26.54	1.00E-00	7.06E-48	3.95E-17
200-5-20	1.05	6.89	**3.11**	3.84	23.26	1.00E-00	5.91E-48	6.37E-07
200-10-10	1.17	7.86	**3.59**	4.11	26.33	1.00E-00	1.17E-47	1.45E-05
200-10-20	1.02	9.71	**9.11**	10.29	29.30	7.21E-09	8.93E-50	2.50E-19
400-2-10	2.74	7.42	**1.94**	6.89	32.75	9.79E-01	9.30E-59	3.41E-25
400-2-20	2.64	6.88	**1.86**	6.52	31.11	9.17E-01	8.57E-60	3.48E-24
400-5-10	2.79	7.31	**2.58**	7.86	31.55	3.16E-02	1.68E-58	4.88E-24
400-5-20	2.54	6.90	**2.85**	8.23	29.86	2.97E-06	9.99E-55	1.87E-24
400-10-10	2.64	7.49	**4.08**	9.00	31.35	4.23E-10	6.33E-62	1.43E-29
400-10-20	2.45	7.80	**4.89**	11.12	30.09	2.28E-24	2.49E-61	4.64E-33

Note: The time limit for CPWS and CP is 5 minutes.
The time limit for DE is 20 minutes.
The bold font represents the best gap.

Table 4. Comparison results for instances in G_2 with enlarged batch processing time.

n-m-b	*Time (s)*	*Gap-GBDS (%)*	*Gap-CPWS (%)*	*Gap-CP (%)*	*Gap-DE (%)*	*GBDS-CP*	*GBDS-DE*	*CPWS-CP*
50-2-10	0.23	0.44	**0.41**	0.42	4.65	1.00E-00	1.23E-38	9.00E-02
50-5-10	0.23	**0.25**	**0.25**	0.27	0.91	5.14E-04	1.93E-27	5.17E-04
100-2-10	0.49	0.30	**0.28**	0.30	7.22	3.78E-01	6.28E-52	1.44E-04
100-2-20	0.50	**0.33**	**0.33**	0.41	6.59	9.05E-04	3.70E-49	6.12E-04
100-5-10	0.48	**0.26**	**0.26**	0.28	5.23	1.33E-04	7.64E-46	1.16E-05
100-5-20	0.49	**0.12**	**0.12**	0.30	1.26	2.52E-08	4.96E-41	2.52E-08
100-10-10	0.48	0.17	**0.16**	0.19	1.20	5.20E-06	2.24E-37	7.12E-07
200-2-10	1.09	0.17	**0.16**	0.21	8.89	5.26E-14	1.66E-60	9.10E-17
200-2-20	1.06	**0.22**	**0.22**	0.29	8.45	3.94E-14	4.66E-63	3.35E-14
200-5-10	1.06	**0.16**	**0.16**	0.20	7.83	4.97E-11	1.75E-60	4.87E-11
200-5-20	1.00	**0.19**	**0.19**	0.26	6.50	4.05E-08	9.41E-56	3.84E-08
200-10-10	1.02	**0.15**	**0.15**	0.23	5.70	5.50E-10	1.60E-52	3.38E-10
200-10-20	1.02	**0.08**	**0.08**	0.30	1.26	1.13E-14	1.08E-49	1.13E-14
400-2-10	2.42	0.09	**0.08**	0.15	10.03	1.63E-15	1.24E-70	6.18E-17
400-2-20	2.39	**0.12**	**0.12**	0.24	9.93	2.68E-18	5.69E-72	1.59E-18
400-5-10	2.41	**0.09**	**0.09**	0.20	9.49	8.90E-20	1.06E-63	8.50E-20
400-5-20	2.38	**0.12**	**0.12**	0.21	9.11	1.04E-15	3.96E-72	1.03E-15
400-10-10	2.30	**0.09**	**0.09**	0.30	8.01	6.00E-22	5.98E-61	7.19E-22
400-10-20	2.32	**0.10**	**0.10**	0.31	7.17	2.11E-12	2.44E-67	2.11E-12

Note: The time limit for CPWS and CP is 5 minutes.
The time limit for DE is 20 minutes.
The bold font represents the best gap.

proposed GBDS algorithm can obtain better solutions than those of the CP method. As shown in the column *Gap-CPWS*, when the solutions of GBDS algorithm are integrated as a warm start of the CP technique, the performance of the CP technique can be improved a lot.

When the batch processing time is enlarged, the jobs in one batch tend to be processed on the unary machine consecutively. As shown in Table 4, all the values of gap between the GBDS and lower bound are very small. The maximum value of *Gap-GBDS* is 0.44% in the combination 50-2-10. These results show that the GBDS algorithm can obtain near-optimal solutions when the batch processing time is large. When comparing the GBDS algorithm and the CP (DE) method, most of the p-values are less than 5%, which indicates that the proposed GBDS algorithm has a competitive performance when enlarging the batch processing time. Since the solutions of the GBDS algorithm is good enough, most of the solutions of the CPWS algorithm is same with those of the GBDS algorithm.

The performances of LB_1, LB_2 and LB_3 also depend on the relationship between the batch processing time and the sum of the processing time of jobs in one batch on the unary machine. In Figure 4, LB_3 is tighter than LB_1 and LB_2 for all instances with reduced batch processing time. This is reasonable since the batch processing time is small. When processing one batch of jobs on the unary machine, many other batches have already been completed. This is very close to the structure of LB_3. However, when we enlarge the batch processing time, Figure 4 shows that LB_1 and LB_2 are tighter than LB_3. When the batch processing time are all larger than the sum of the processing time of jobs in one batch on the unary machine, the unary machine will process jobs in one batch consecutively and the waiting time become small, which leads that LB_1 and LB_2 are tighter than LB_3.

Figure 4. Performances of different lower bounds.

7. Conclusions

In this paper, we study the two-stage scheduling problem with a batch machine followed by a unary machine such that the total weighted completion time is minimized. The incompatible job families are considered. The problem is formulated as a mixed-integer linear programming model. Based on the structure of this problem, three lower bounds are developed. An approximation algorithm is proposed and we prove that the worst-case performance ratio of this algorithm is bounded by a constant 4. Numerical experiments are conducted to test the efficiency of the proposed algorithm. When compared to the optimal solutions and lower bounds, the proposed algorithm can obtain high quality solutions. When compared to the CP method and DE algorithm, the proposed algorithm can provide good solutions and has a competitive performance. Sensitivity analysis indicates that the proposed algorithm also has a robust computational performance for different problem structures.

In practice, the machines and operations are more complex. How to extend the lower bounds and approximation algorithm to the general problems with parallel batch machines or more than two operations is an important future work. Some general constraints, such as release dates, set-up time and re-entrant flows, should also be addressed in the future.

Acknowledgements

This research was supported in part by National Science Foundation of China [grant numbers 71690232 and 71371015] and by National Science Foundation [grant numbers CMMI-1435800].

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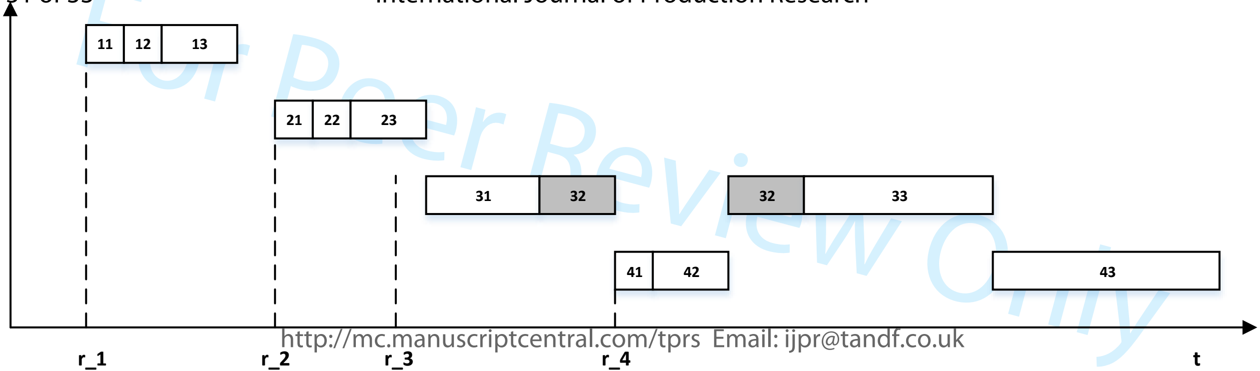
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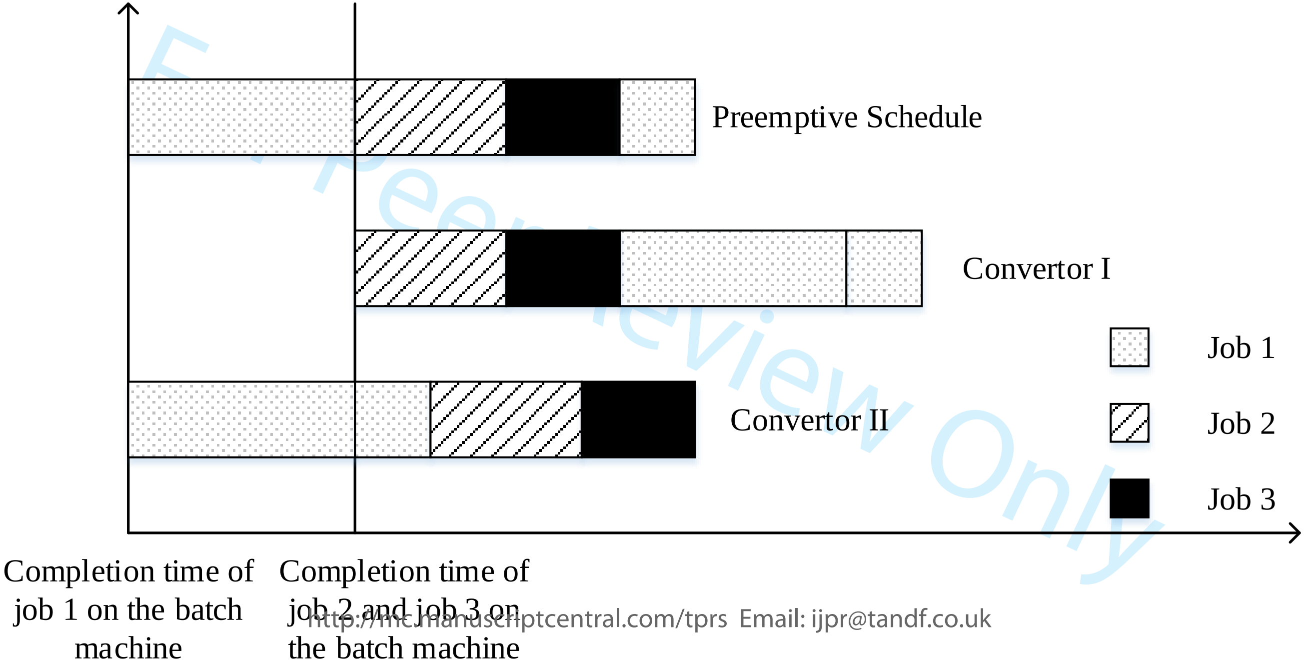
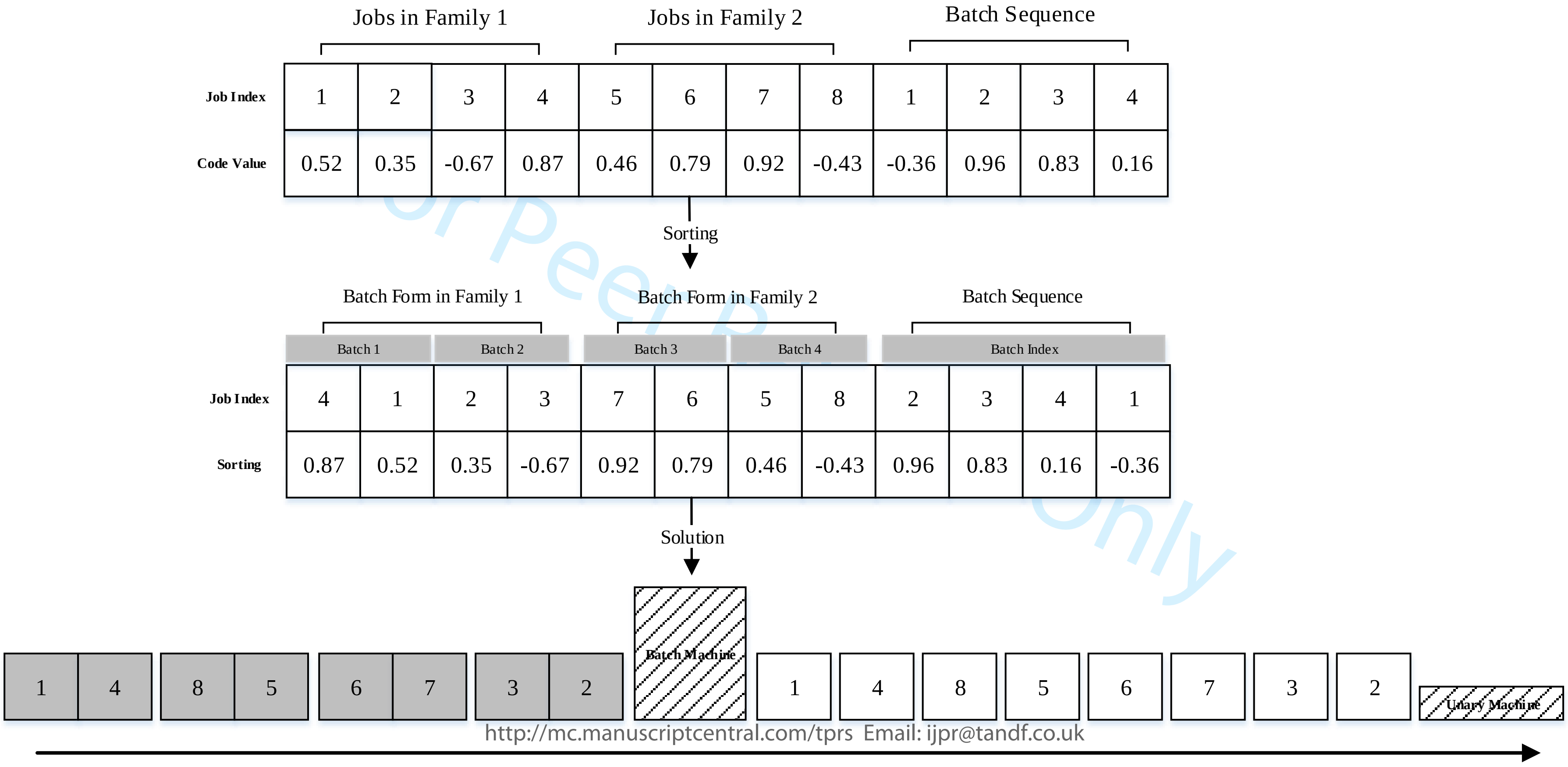
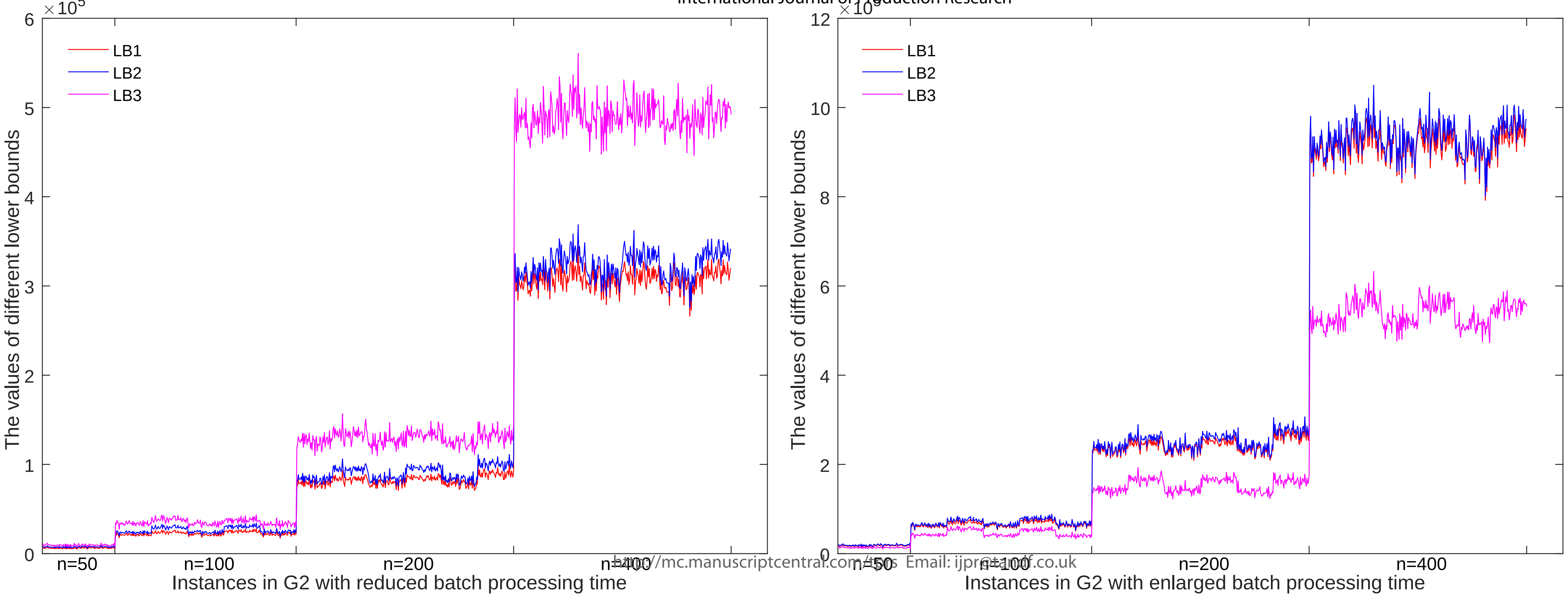

Preemptive Schedule
Convertor I
Convertor II
Job 1
Job 2
Job 3
Completion time of job 1 on the batch machine
Completion time of job 2 and job 3 on the batch machine


Jobs in Family 1
Jobs in Family 2
Batch Sequence
Job Index
1 2 3 4 5 6 7 8 1 2 3 4
Code Value
0.52 0.35 -0.67 0.87 0.46 0.79 0.92 -0.43 -0.36 0.96 0.83 0.16
Sorting
Batch Form in Family 1
Batch Form in Family 2
Batch Sequence
Batch 1
Batch 2
Batch 3
Batch 4
Batch Index
Job Index
4 1 2 3 7 6 5 8 2 3 4 1
Sorting
0.87 0.52 0.35 -0.67 0.92 0.79 0.46 -0.43 0.96 0.83 0.16 -0.36
Solution
1 4 8 5 6 7 3 2
Batch Machine
1 4 8 5 6 7 3 2
Unary Machine


×10^5
LB1
LB2
LB3
The values of different lower bounds
n=50
n=100
n=200
n=400
Instances in G2 with reduced batch processing time
×10^6
LB1
LB2
LB3
The values of different lower bounds
n=50
n=100
n=200
n=400
Instances in G2 with enlarged batch processing time