Randomized Large Neighborhood Search for Cumulative Scheduling

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Abstract

This paper presents a Large Neighborhood Search (LNS) approach based on constraint programming to solve cumulative scheduling problems. It extends earlier work on constraint-based randomized LNS for disjunctive scheduling as reported in (Nuijten & Le Pape 1998). A breakthrough development in generalizing that approach toward cumulative scheduling lies in the presented way of calculating a partial-order schedule from a fixed start time schedule. The approach is applied and tested on the Cumulative Job Shop Scheduling Problem (CJSSP). An empirical performance analysis is performed using a well-known set of benchmark instances. The described approach obtains the best known performance reported to date on the CJSSP. It not only finds better solutions than ever reported before for 33 out of 36 open instances, it also proves to be very robust on the complete set of test instances. Furthermore, among these 36 open instances, one is now closed. As the approach is generic, it can be applied to other types of scheduling problems, for example problems including resource types like reservoirs and state resources, and objectives like earliness/tardiness costs and resource allocation costs.

Introduction

Scheduling can be described as the process of allocating scarce resources to activities over time. Traditionally, the class of *disjunctive* scheduling problems, where each resource can execute at most one activity at a time, has received a lot of attention. In this paper we are concerned with the class of *cumulative* scheduling problems, where resources may execute several activities in parallel, provided the resource capacity is not exceeded.

Many practical scheduling problems are cumulative scheduling problems and in recent years the attention for cumulative scheduling problems in general (Baptiste, Le Pape, & Nuijten 1999) and the resolution of these problems by way of local search in particular has increased (Cesta, Oddi, & Smith 2000; Michel & Van Hentenryck 2003; Palpant, Artigues, & Michelon 2004). Our approach is an implementation of the Large Neighborhood Search framework (Shaw 1998) which is based upon a process of continual relaxation and re-optimization. We apply this approach on a generalization of the Job Shop Scheduling Problem (French 1982) which we call the Cumulative Job Shop Scheduling Problem (CJSSP). Informally, this problem can be stated as follows. Given are a set of *jobs* and a set of *resources*. Each job consists of a set of *activities* that must be processed in a given order. Furthermore, for each activity is given an integer *processing time*, a resource by which it has to be processed, and an integer *demand* that represents the amount of resource required by the activity. Once an activity is started, it is processed without interruption. Resources may process several activities simultaneously. To this end, for each resource is given an integer *capacity*. A resource can simultaneously process only those sets of activities whose total demand do not exceed the capacity of the resource. A *schedule* assigns a start time to each activity. A *feasible* schedule is a schedule that meets the order in which the activities must be processed and in which the capacity of none of the resources is exceeded at any point in time. One is asked to find an *optimal* schedule, i.e, a feasible schedule that minimizes the *makespan* of the schedule, the makespan being defined as the maximum completion time of any of the activities. We remark that the CJSSP is the optimization variant of the Multiple Capacitated Job Shop Scheduling Problem of (Nuijten 1994; Nuijten & Aarts 1996).

The organization of the remainder of the paper is as follows. First we define the CJSSP after which we present the LNS approach we propose. We then present the computational results to finally discuss the conclusions of the paper and potential directions of future research.

The Cumulative Job Shop Scheduling Problem

The Cumulative Job Shop Scheduling Problem (CJSSP) can be formalized as follows. We are given a set $\mathcal{A}$ of activities, a set $\mathcal{R}$ of resources, and a set $\mathcal{J}$ of jobs, each job j consisting of a sequence of activities $(a_1, ..., a_n)$. Each activity $a \in \mathcal{A}$ has a processing time $pt(a)$ and a demand $d(a)$ for resource $r(a)$ to be executed. Each resource $r \in \mathcal{R}$ has a capacity $C(r)$ and a binary relation $\prec$ is given that decomposes $\mathcal{A}$ into chains, such that every chain corresponds to a job. A *schedule* is an assignment $s : \mathcal{A} \rightarrow \mathbb{Z}_0^+$ assigning a positive start time $s(a)$ to each activity $a \in \mathcal{A}$. A schedule s

is *feasible* if it satisfies the *precedence constraints* between each pair of consecutive activities in a job:

$$\forall_{a,a' \in \mathcal{A} | a \prec a'} \; s(a) + pt(a) \leq s(a') \qquad (1)$$

and the *resource capacity constraints* for each resource:

$$\forall_{r \in \mathcal{R}} \; \forall_{t \in \mathbf{Z}_0^+} \sum_{a \in \mathcal{A}_r | s(a) \leq t < s(a)+pt(a)} d(a) \leq C(r) \qquad (2)$$

where $\mathcal{A}_r = \{a \in \mathcal{A} \mid r(a) = r\}$. One is asked to find an *optimal schedule* which is a feasible schedule that minimizes the *makespan* defined as $\max_{a \in \mathcal{A}} s(a) + pt(a)$.

Randomized Large Neighborhood Search

Our approach is an implementation of the Large Neighborhood Search (LNS) framework (Shaw 1998) which is based upon a process of continual relaxation and re-optimization. It is a generalization of the randomized LNS approach for the Job Shop Scheduling Problem of (Nuijten & Le Pape 1998). The LNS framework is illustrated in Figure 1. A first solution is computed and iteratively improved. Each iteration consists of a relaxation step followed by a re-optimization of the relaxed solution. This process continues until some condition is satisfied (for instance a time limit or a number of non-improving iterations).

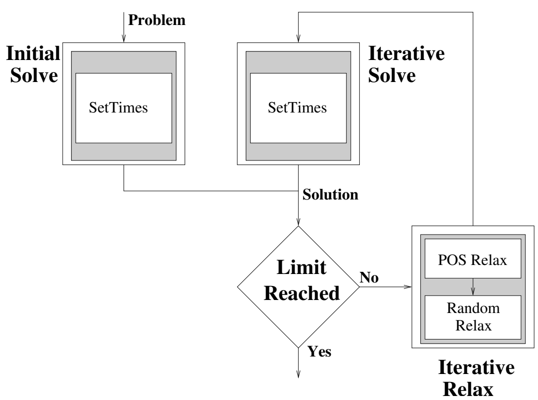

Figure 1: LNS Framework

One of the first approaches that used LNS was on the Job Shop Scheduling Problem (Applegate & Cook 1991). One of the challenges for applying LNS to cumulative scheduling (and to even more complex scheduling problems in general) is that most of the available algorithms to solve cumulative scheduling problems produce solutions with fixed start times. In this context, relaxing a solution by only unfreezing the start time of some activities in the schedule provides limited flexibility to re-optimize the relaxed solution. Previous work on cumulative job shop problems (Cesta, Oddi, & Smith 2000; Michel & Van Hentenryck 2003) avoids this issue by generating precedence constraints as decisions and producing a temporal constraint network which is more flexible thus more adequate for LNS than a completely instantiated schedule. (Palpant, Artigues, & Michelon 2004) also applied an LNS framework on resource-constrained project scheduling problems. They avoid the issue of lack of flexibility by composing a solution method to solve the subproblem which does not necessarily decrease the makespan with a forward/backward heuristic (Li & Willis 1992) to left-shift frozen activities so as to take advantage of the more compact solution with relaxed activities.

In this paper, we investigate a slightly different approach where any algorithm can be used to produce a first solution and to iteratively re-optimize the current solution. The main advantage of this approach is its genericity, i.e., the type of search algorithm used to compute first solutions or for re-optimization is completely orthogonal to the type of LNS relaxation that is used. Consequently one can for instance apply this approach to problems involving minimization of earliness/tardiness costs or resource allocation costs.

To select the combination of search algorithms and LNS relaxations used in our approach, we first focused on a representative subset of the open instances of the CJSSP and made various experiments:

- For building a first solution, we compared various algorithms : some build solutions in chronological order, others are based on minimal critical sets, some are deterministic, others are based on random restart; for each algorithm, we tried different heuristics for selecting an activity or for selecting a pair of activities to order.
- We experimented the same algorithms for the search procedure used after each relaxation to re-optimize the current solution.
- We compared various neighborhoods : some based on selection of activities on the critical path, others based on selection of activities of a job or based on gliding time windows, we tried both deterministic and non-deterministic heuristics for selecting activities; we also compared various combinations of these neighborhoods.

Notice that the approach of (Cesta, Oddi, & Smith 2000) and (Michel & Van Hentenryck 2004) is included among all the combinations we experimented.

In this paper, we focus on the combination that offers the best trade-off between simplicity and quality. We used this combination to conduct an empirical performance analysis on the complete set of benchmarks.

The remaining part of this paper describes this combination and report the computational results. The algorithm we used to generate first solutions for the CJSSP is described in Section *Finding a Solution*. The completely instantiated solution generated by this algorithm is firstly relaxed to obtain a POS (see Section *POS Relaxation*), after which an LNS relaxation is applied on this schedule (see Section *LNS Relaxation*). The overall algorithm is described in Section *Iterative Improvement* and the used constraint propagation in Section *Constraint Propagation*.

Finding a Solution

To find an initial solution to the CJSSP and to re-optimize relaxed solutions, we use the algorithm described in (Le Pape *et al.* 1994). This algorithm is available in ILOG SCHEDULER (ILOG 2005a) and is called *SetTimes*. It fixes the start times of the activities in chronological order and can be summarized as follows.

1. Let S be the set of *selectable* activities. Initialize S to the complete set of activities of the schedule.
2. If all the activities have a fixed start time then exit: a solution has been found. Otherwise remove from S the activities with fixed start time.
3. If the set S is not empty:
   (a) Select an activity from S which has the minimal earliest start time. Use the minimal latest end time to break ties.
   (b) Create a choice point (to allow backtracking) and fix the start time of the selected activity to its earliest start time. Goto to step 2.
4. If the set S is empty:
   (a) Backtrack to the most recent choice point.
   (b) Upon backtracking, mark the activity that was scheduled at the considered choice point as *"not selectable"* as long as its earliest start time has not changed. Goto step 2.

After each decision in step 3b, the earliest start times and latest end times of activities are maintained by constraint propagation. The status *"not selectable"* in step 4b is also maintained by constraint propagation.

This algorithm is clearly sound. It is complete in the case of job shop scheduling (simple temporal constraints with positive delays). On problems with more complex constraints and resource types, this algorithm can still be applied and usually leads to fairly good solutions for minimizing the makespan although it is in general incomplete.

POS Relaxation

As stated above, the solutions produced by the *SetTimes* algorithm have fixed start times. The problem with such a schedule in the context of LNS is its lack of flexibility. If part of the solution is relaxed whereas the rest of the solution remains frozen (fixed start times), there is limited room for re-optimization as there are limited possibilities to reschedule relaxed activities in between frozen activities.

In our approach, the fully instantiated solution is first relaxed into a partial-order schedule (POS). A POS is a graph $G(V, E)$ where the nodes V are the activities of the scheduling problem and the edges are the temporal constraints between pairs of activities, such that any temporal solution to this graph is also a resource-feasible solution. More generally, a POS is a resource temporal network that satisfies the necessary truth criterion as defined in (Laborie 2003). A POS is by nature more flexible and thus more adequate for LNS.

Our POS-generation algorithm was developed independently from the one recently described in (Policella *et al.* 2004) in the context of dynamic and uncertain execution environments.

The main idea behind building a POS for a given resource is to split the resource into two smaller resources of half capacity, to split the activities between these two sub-resources, and to recursively call the POS-generation on each of the two sub-resources with their allocated activities. The leaves of the recursion are sub-resources for which all the allocated activities are in disjunction so that the POS consists of a chain of these activities. More precisely, the recursion to generate a POS $POS(r)$ for a resource r works as follows.

1. Sort all activities $a \in \mathcal{A}$ that are allocated to resource r in chronological order of their start times $s(a)$. Let $l(r)$ be the list of chronological ordered activities. $C(r)$ denotes the capacity of (sub-)resource r. Let $d(a, r)$ denote the demand of activity a for (sub-)resource r and initialize it to $d(a, r) = d(a)$.
2. If less than two activities are allocated to r, or if the sum of the two smallest demands is greater than $C(r)$, then add the chain $l(r)$ to the POS: $POS(r) = POS(r) \cup l(r)$
3. Otherwise:
   (a) Create two sub-resources r_{lower} with associated capacity $C(r_{lower}) = \lceil C(r)/2 \rceil$ and r_{upper} with associated capacity $C(r_{upper}) = C(r) - C(r_{lower})$.
   (b) For each activity $a \in l(r)$ traversed in chronological order of their start times $s(a)$ in the schedule, allocate a to the sub-resource r_{lower} or r_{upper} with the largest capacity slack at time $s(a)$ given the activities already allocated to r_{lower} and r_{upper} and add the activity to $l(r_{lower})$ or $l(r_{upper})$ accordingly. Let $slack_x(a)$ (with $x \in \{lower, upper\}$) denote these slacks. If the largest slack $slack_x(a)$ is smaller than $d(a, r)$, then activity a is allocated to both sub-resources: to sub-resource r_x with demand $d(a, r_x) = slack_x(a)$ and to the other sub-resource with demand $d(a, r) - slack_x(a)$.
   (c) If the list $l(r_{lower})$ is not empty, then recursively goto step 2 with r_{lower}. Similarly, if the list $l(r_{upper})$ is not empty, then recursively goto to 2 with r_{upper}.

In step 2, the recursion reaches a leaf when the sub-resource r is disjunctive, that is, no pair of activities can overlap in time without over-consuming the sub-resource.

In step 3b, activities are considered in chronological order of the start time and, for two activities starting at the same date, the one with largest demand $d(a, r)$ is selected. In case both sub-resources have the same slack for an activity a ($slack_{lower}(a) = slack_{upper}(a)$), ties are broken randomly.

The global POS is then made of the union of temporal constraints of the problem itself and the POS $POS(r)$ of each resource r as generated by the above given algorithm.

Figure 2 illustrates this algorithm on a resource with capacity 5. The resource is first split in two parts, a lower half with capacity 3 and an upper half with capacity 2. Activities are then assigned to each parts as shown in the upper-right part of the figure. As both sub-resources are still not disjunctive, they are split again as shown in the lower-left part of the figure. The POS generated for this resource is depicted in the lower-right part.

Let n denote the number of activities on the resource and C the capacity of the resource. The initial sort of activities by increasing start times can be performed in $O(n \log n)$.

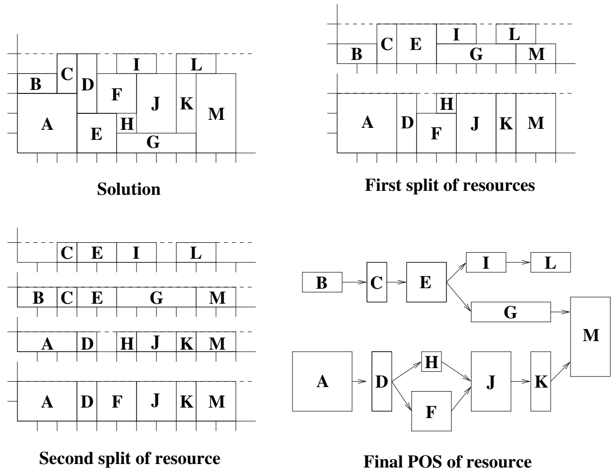

Figure 2: Transforming a fixed-time schedule into a *POS*

There are at most $\log C$ layers of resource splits. At layer i, there are at most 2^i sub-resources, each of which has approximatively $n/2^i$ activities. Allocating an activity a to one of the two sub-resources r_{lower} or r_{upper} can be done in a time proportional to the number of activities allocated to the sub-resource that start or end in the time interval $[s(a), s(a)+pt(a))$. If we assume this quantity to be a constant K, the average complexity for constructing layer i is thus equal to $2^i(Kn/2^i) = Kn$ and the cost for computing all layers is thus in $O(Kn \log C)$. This gives a rough estimate of the average complexity in $O(n \log n + Kn \log C))$. The algorithm described in (Policella *et al.* 2004) is very similar to ours but the authors did not give its complexity : a naive implementation of their algorithm leads to a complexity in $O(n \log n + nC))$ (the discrete resource of capacity C is virtually split into C unary resources), so an $O(C)$ factor compared to the $O(\log C)$ factor in our algorithm. A deeper comparison of the two approaches in terms of efficiency and flexibility is part of our future work.

LNS Relaxation

The basic idea of the LNS relaxation is to select $m \leq |\mathcal{A}|$ activities and to relax those activities in the POS constructed in the previous section so as to leave room for improvement in the next iteration. Let $S = \{a_1, ..., a_m\}$ be the set of selected activities and P be the set of temporal constraints of the problem itself (the ones defined by $\prec$). The relaxed POS is obtained by applying the following algorithm :

1. For each selected activity $a \in S$, remove from the POS all temporal constraints regarding a except the temporal constraints belonging to P.
2. For all removed temporal constraints (a', a), if $a' \notin S$ and $a \in S$ then add new temporal constraints between a' and the first not selected successors of a in the original temporal graph.
3. Optionally remove redundant temporal constraints that may have been introduced in the previous step by applying a topological sort of the temporal graph.

This algorithm is illustrated in Figure 3. The upper-left drawing displays the original POS. We assume this temporal graph contains no temporal constraints belonging to P. Two activities are selected (upper-right drawing): activity D and activity F. At steps 1 and 2, temporal constraints (A,D), (D,H), (D,F), (F,J) are removed and two temporal constraints are added (A, H) and (A,J). The redundant temporal constraint (A,J) is removed in step 3.

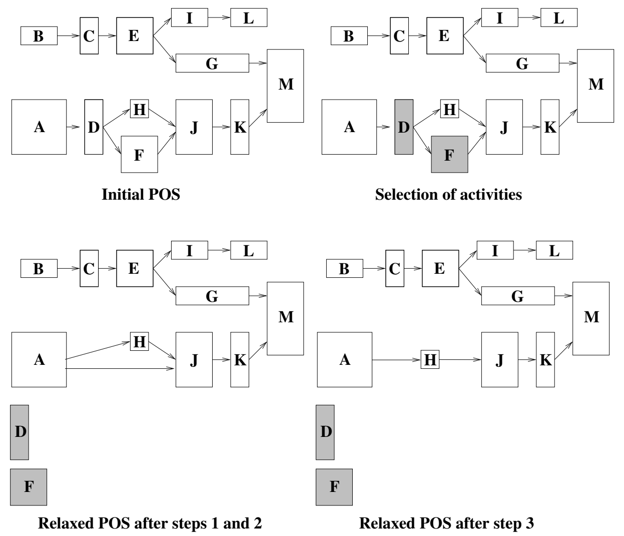

Figure 3: Computing a *Relaxed POS*

The temporal constraints of the obtained relaxed POS are then taken as new start point for the next iteration of the LNS. In our study, to build the set of selected activities S we choose them randomly with a probability α which is a parameter of the algorithm. It means that in average, $m = \alpha\,|\mathcal{A}|$.

This large neighborhood search has been implemented by using the LNS framework available in ILOG SCHEDULER 6.1 (ILOG 2005a).

Iterative Improvement

The iterative improvement procedure we use is described in Algorithm 1. The name of the algorithm - $STRand$ - stands for **S**et**T**imes + **Rand**om relaxation. We used the *SetTimes* procedure with three parameters:

- parameter γ that specifies the maximum allowed number of backtracks (expressed in percentage of the number of activities) for one execution of the procedure,
- a flag $fc \in \{first, cont\}$ that tells whether the search must stop at the first solution found or continue search trying to minimize the makespan until the maximum number of backtracks γ is reached (or an optimal solution found), and
- an upper bound ub on the makespan.

The parameter γ allows to control the *SetTimes* procedure from a simple descent ($\gamma = 0$ no backtracks allowed) to a complete search tree ($\gamma = \infty$).

The input parameters of $STRand$ are: the problem P to solve, the probability α to relax a given activity in the $RandomRelax$ procedure ($0 < \alpha \leq 1$), the improvement step β, the maximum number of backtracks γ, the flag fc, and a global time limit t.

Algorithm 1 Iterative Improvement Algorithm

```
1:  procedure STRand(P,α,β,γ,fc,t)
2:      R := P
3:      m* := ∞                        ▷ m*: Best makespan so far
4:      ub := ∞                        ▷ ub: Upper bound for SetTimes
5:      while time < t do
6:          s := SetTimes(R, γ, fc, ub)
7:          if (makespan(s) < best) then
8:              s* := s                ▷ s*: Best schedule so far
9:              m* := makespan(s)
10:             ub := (1 + β)m*
11:         end if
12:         R := POSRelax(P, s)
13:         R := RandomRelax(R, P, α)
14:     end while
15:     return s*
16: end procedure
```

Constraint Propagation

When re-optimizing a relaxed solution R at line 6, the number of precedence relations may be large as R contains a subset of the temporal constraints generated when converting the fixed-time schedule to a *POS*. To efficiently propagate these temporal constraints, we have implemented two algorithms:

- A topological sort on the direct acyclic graph of temporal constraint is used to compute the initial time bounds of the activities. The complexity of this algorithms is $O(n + p)$ where n is the number of activities and p the number of temporal constraints. This algorithm is run only once at each iteration.
- The algorithm described in (Michel & Van Hentenryck 2003) to incrementally maintain the longest paths in direct acyclic graphs is used to incrementally compute the time bounds of activities. The complexity of this algorithm is in $O(\|\delta\| \log \|\delta\|)$ where $\|\delta\|$ is a measure of the change in the graph since the previous propagation. This algorithm is activated after each decision taken during the search procedure.

This temporal propagation was implemented as a global constraint in the CP framework of ILOG SOLVER (ILOG 2005b) and ILOG SCHEDULER (ILOG 2005a). The capacity of resources is propagated using the *timetable* constraint of ILOG SCHEDULER (Le Pape 1994).

Computational Results

In this section, we report the computational results of the algorithm described in the previous section.

Benchmarks

The benchmarks we used are the standard benchmarks from (Nuijten 1994). These benchmarks are derived from job shop scheduling problems by introducing a certain number of duplicates for each job and increasing the capacity of the resources accordingly. The benchmarks are classified in 5 groups:

- **Set A** : Lawrence LA01-LA10 duplicated and triplicated.
- **Set B** : Lawrence LA11-LA20 duplicated and triplicated.
- **Set C** : Lawrence LA21-LA30 duplicated and triplicated.
- **Set D** : Lawrence LA31-LA40 duplicated and triplicated.
- **Set MT** : MT06, MT10, and MT20 duplicated and triplicated.

Because of the way these instances are constructed, upper bounds on the optimal makespan can be derived from the corresponding job shop scheduling instances. Incidently, the results of (Michel & Van Hentenryck 2004; Cesta, Oddi, & Smith 2000; Nuijten & Aarts 1996) have shown that these upper bounds are not so easy to find as the algorithms used are not aware of the underlying structure of the instances. Notice also that the instances by construction contain equivalent solutions. In our study, as in previous ones, no constraint has been used to break those symmetries.

In terms of size, with these different sets we have quite a large spectrum ranging from 50 to 900 activities and from 5 to 15 resources. Sets A, B, and MT consist of small to medium size instances whereas sets C and D consist of medium to large size instances.

Preparing Time Equivalent Tests

To conduct fair comparisons both between our approach and previously reported approaches as well as between different parameter settings for our approach we aim to do time equivalent tests. To do so we determined maximum running times for each instance in the following way. First, we applied a search procedure that is comparable in complexity to the ones used by (Cesta, Oddi, & Smith 2000; Michel & Van Hentenryck 2004) by allowing no backtracks during search ($\gamma = 0$), i.e., the search procedure *SetTimes* either returns a solution obtained without any backtracking or stops as soon as a backtrack occurs. To allow a better comparison to (Michel & Van Hentenryck 2004) we use the same stop criterion they used, i.e., the algorithm is stopped if for a certain number of iterations the solution was not improved. We set this maximum number of so called *stable iterations* to 1000. The other parameters are as follows : $\alpha = 0.2$, and $\beta = 0$. For each instance we then do 10 runs and take the average CPU time as maximum running time that we will use throughout our experiments for that instance.

The machine used for the computational study is a Pentium 4 at 2.8 Ghz. Average results reported are always computed over 10 runs unless specified otherwise.

Results Summary

We found the best performance for our approach when choosing the following values for the different parameters:

$\alpha = 0.2$, $\beta = 0$, $\gamma = 0.15$, and $fc = cont$. Running time for each instance was limited to the maximum running time computed in the previous section.

Table 1 reports average deviation (in percentage) from the upper bounds (*UB*) reported in (Nuijten & Aarts 1996) and used in the evaluation in (Cesta, Oddi, & Smith 2000) and in (Michel & Van Hentenryck 2004).

As can be observed, our results are on average significantly better both in terms of quality and robustness than the ones published in (Cesta, Oddi, & Smith 2000) and (Michel & Van Hentenryck 2004). Table 1 summarizes prior results of algorithm $IFlat_5$ (iterative flattening procedure with 5 random restarts as in (Cesta, Oddi, & Smith 2000)) and algorithm *IFlatIRelax 4(20)* (relax probability of 20% and a number of relaxations of 4 as in (Michel & Van Hentenryck 2004)) with respectively 1000, 5000 and 10000 iterations. The column *STRand 1000* reports the results obtained when computing average running times.

Set	$IFlat_5$	IFlatIRelax 4(20)			STRand	STRand
		1000	5000	10000	1000	0.2/0/0.15/cont
A	7.76	1.63	1.07	0.90	0.14	**-0.13**
B	7.10	1.04	0.47	0.24	-0.59	**-1.10**
C	13.03	-	4.2	-	1.18	**-0.16**
D	11.92	-	2.4	-	1.11	**0.22**
MT	-	4.76	3.41	3.12	1.30	**0.55**
Total			2.13		0.59	**-0.23**

Table 1: Results summary of $IFlat_5$, $IFlatIRelax$ and $STRand$

On average, $STRand(\alpha = 0.2, \beta = 0, \gamma = 0.15, fc = cont)$ is within -1.10% and 0.55% of *UB*. Furthermore, we have obtained new upper bounds on 33 instances out of 86.

For detailed results, see Section *Detailed Results* below.

Impact of the Relaxation Probability

To study the influence of the different parameters we did a series of tests. We firstly studied the influence of varying the relaxation probability α. Table 2 reports average deviation in percentage from *UB* when α is varying between 0.1 and 0.3. Running time for each instance is limited to the maximum running time computed previously. The other parameters are as follows: $\beta = 0$, $\gamma = 0.15$, and $fc = cont$.

Set	0.1	0.15	0.2	0.25	0.3
A	0.09	-0.09	**-0.13**	-0.12	**-0.13**
B	-0.57	-0.87	-1.10	**-1.26**	-1.25
C	-0.27	**-0.41**	-0.16	0.32	0.58
D	0.52	0.25	**0.22**	0.33	0.58
MT	1.15	0.76	0.55	**0.39**	0.43
Total	0.03	-0.21	**-0.23**	-0.14	0.01

Table 2: Impact of the Relaxation Probability α on $STRand(\alpha, \beta = 0, \gamma = 0.15, fc = cont)$

Overall the best results are obtained with $\alpha = 0.2$ although the quality obtained with $\alpha = 0.15$ is very closed. It gives the best results on 2 sets while $\alpha = 0.15$ works better for set C and $\alpha = 0.25$ works better for set B and set MT.

Impact of the Improvement Step

Table 3 reports the average deviation in percentage from *UB* when the improvement step β is varying from $-\epsilon$ (enforce to find a solution strictly better than the best solution so far) to ∞ (no enforcement at all). Running time for each instance is limited to the maximum running time computed previously. The other parameters are as follows: $\alpha = 0.2$, $\gamma = 0.15$, and $fc = cont$.

Set	$-\epsilon$	0	0.01	∞
A	0.23	**-0.13**	0.16	1.15
B	-0.37	**-1.10**	-0.53	0.77
C	0.76	**-0.16**	3.54	5.50
D	0.92	**0.22**	2.31	3.21
MT	1.58	**0.55**	1.03	3.90
Total	0.47	**-0.23**	1.35	2.74

Table 3: Impact of the Improvement Step β on $STRand(\alpha = 0.2, \beta, \gamma = 0.15, fc = cont)$

For positive improvement steps, increasing the value leads to increasing the average deviation and thus decreasing the quality. The best quality is obtained with a null improvement step, that is when the search procedure is enforced to find a solution better or equal to the previous one. Results obtained when applying $-\epsilon$ as improvement step are not as good. With $-\epsilon$ we observe that the first iterations decrease the cost effectively, but then the search is often trapped in a local optimum.

Impact of the Maximum Number of Backtracks

Table 4 reports the average deviation in percentage from *UB* when the maximum number of allowed backtracks γ is varying from 0% to 20% with the number of activities. Running time for each instance is limited to the maximum running time computed previously. The other parameters are as follows: $\alpha = 0.2$, $\beta = 0$, and $fc = cont$.

Set	0	0.05	0.10	0.15	0.20	0.50
A	0.42	**-0.16**	-0.15	-0.13	**-0.16**	-0.13
B	-0.67	-1.06	-1.07	**-1.10**	-1.09	-1.06
C	0.87	-0.14	**-0.17**	-0.16	-0.06	-0.01
D	0.91	0.24	0.25	0.22	**0.18**	0.28
MT	1.21	**0.38**	0.64	0.55	0.52	0.66
Total	0.44	**-0.23**	-0.22	**-0.23**	-0.22	-0.17

Table 4: Impact of the Number of Backtracks γ on $STRand(\alpha = 0.2, \beta = 0, \gamma, fc = cont)$

Overall, for γ varying between 5% and 20%, there is no much difference in quality. Increasing γ further than 20% leads to increasing the average deviation and thus decreasing the quality.

Impact of the Search Method

Table 5 compares the average deviation in percentage from *UB* when continuing search at each iteration until all allowed backtracks are exhausted with the one obtained when stopping search at the first solution found. Running time for each instance is limited to the maximum running time computed previously. The other parameters are as follows: $\alpha = 0.2$, $\beta = 0$, and $\gamma = 0.15$.

Set	$first$	$cont$
A	**-0.13**	**-0.13**
B	-0.98	**-1.10**
C	0.09	**-0.16**
D	0.38	**0.22**
MT	0.60	**0.55**
Total	-0.11	**-0.23**

Table 5: Impact of the Search Method on $STRand(\alpha = 0.2, \beta = 0, \gamma = 0.15, fc = first/cont)$

When continuing search obviously more time is taken at each iteration: the search returns the best solution found using the limited amount of backtracks instead of returning the first solution found. As the time is limited, this implies less iterations. As can be observed continuing search leads to improved quality on all sets of instances.

Detailed Results

Tables 6, 7, 8, 9 and 10 display in the second column the lower bounds reported in (Nuijten & Aarts 1996) including some recent improvements described in (Laborie 2005) followed by the upper bounds found by (Nuijten & Aarts 1996) or derived from the corresponding job shop scheduling instance. If these two values are equal, only one number is given. The third column reports the upper bounds found by (Michel & Van Hentenryck 2004). The following three columns report the best upper bounds, the average upper bounds and the average running time obtained when performing 1000 stable iterations. The following two columns report the best upper bounds and the average upper bounds obtained with $STRand$ when using the following parameters: $\alpha = 0.2$, $\beta = 0$, $\gamma = 0.15$, and $fc = cont$. Running time for each instance is limited to the maximum running time computed previously. Best and average upper bounds are computed over 10 runs. The last column reports the best overall upper bound obtained with $STRand$ during this study. Results are in bold when a new upper bound is reported. Note that this last column indeed reports the best upper bounds found during all the tests we did during this computational study, not just the best of the two parameter settings reported on in this section.

On set A, new upper bounds for all 4 open instances have been found. All results reported in (Michel & Van Hentenryck 2004) are improved (`la04d` and `la04t`). Instance `la03d` is now closed.

On set B, new upper bounds have been found for all 10 open instances. All improve results reported in (Michel & Van Hentenryck 2004).

Instance	LB/UB	MvH	Avg Running Times			0.2/0/0.15/cont		Best
			best	avg	time	best	avg	
la01d	666	-	666	666	1	666	666	666
la01t	666	-	666	666	1	666	666	666
la02d	655	-	655	659	18	655	657	655
la02t	655	-	655	659	39	655	656	655
la03d	593/597	-	597	604	15	596	603	**593**
la03t	590/597	-	596	600	35	595	599	**592**
la04d	572/590	577	578	607	13	576	578	**576**
la04t	570/590	584	579	608	27	574	577	**573**
la05d	593	-	593	593	1	593	593	593
la05t	593	-	593	593	1	593	593	593
la06d	926	-	926	926	1	926	926	926
la06t	926	-	926	926	1	926	926	926
la07d	890	-	890	890	4	890	890	890
la07t	890	-	890	891	35	890	890	890
la08d	863	-	863	863	2	863	863	863
la08t	863	-	863	863	20	863	863	863
la09d	951	-	951	951	1	951	951	951
la09t	951	-	951	951	1	951	951	951
la10d	958	-	958	958	1	958	958	958
la10t	958	-	958	958	1	958	958	958

Table 6: Detailed Results on Set A

Instance	LB/UB	MvH	Avg Running Times			0.2/0/100/cont		Best
			best	avg	time	best	avg	
la11d	1222	-	1222	1222	1	1222	1222	1222
la11t	1222	-	1222	1222	1	1222	1222	1222
la12d	1039	-	1039	1039	2	1039	1039	1039
la12t	1039	-	1039	1039	34	1039	1039	1039
la13d	1150	-	1150	1150	1	1150	1150	1150
la13t	1150	-	1150	1150	11	1150	1150	1150
la14d	1292	-	1292	1292	1	1292	1292	1292
la14t	1292	-	1292	1292	1	1292	1292	1292
la15d	1207	-	1207	1207	9	1207	1207	1207
la15t	1207	-	1207	1207	47	1207	1207	1207
la16d	892/935	929	931	940	46	925	935	**925**
la16t	887/935	927	922	935	92	921	929	**918**
la17d	754/765	756	756	761	44	755	760	**755**
la17t	753/765	761	764	765	78	755	762	**755**
la18d	803/844	818	815	829	46	811	823	**811**
la18t	783/844	813	816	821	99	808	816	**808**
la19d	756/840	803	816	824	43	804	814	**795**
la19t	740/840	801	808	813	97	792	798	**787**
la20d	849/902	864	872	908	47	859	875	**859**
la20t	842/902	863	872	876	101	862	871	**854**

Table 7: Detailed Results on Set B

Instance	LB/ UB	MvH	Avg Running Times			0.2/0/100/cont		Best
			best	avg	time	best	avg	
la21d	1017/1046	-	1038	1046	203	1037	1042	**1034**
la21t	1012/1046	-	1064	1073	479	1039	1042	**1027**
la22d	913/927	-	939	948	94	926	936	**926**
la22t	913/927	-	936	943	295	929	934	928
la23d	1032	-	1032	1032	34	1032	1032	1032
la23t	1032	-	1032	1033	142	1032	1032	1032
la24d	885/935	932	911	927	168	903	913	**903**
la24t	884/935	929	924	935	414	904	907	**898**
la25d	907/977	-	958	970	140	954	960	**952**
la25t	903/977	965	954	965	502	945	952	**945**
la26d	1218	-	1218	1219	234	1218	1219	1218
la26t	1218	-	1220	1225	615	1218	1219	1218
la27d	1235	-	1244	1255	394	1237	1245	1235
la27t	1235	-	1259	1273	790	1242	1249	1241
la28d	1216	-	1234	1258	265	1222	1235	1216
la28t	1216	-	1256	1270	457	1239	1252	1216
la29d	1117/1152	-	1150	1168	581	1137	1147	**1131**
la29t	1116/1152	-	1199	1212	743	1144	1153	**1130**
la30d	1355	-	1355	1359	129	1355	1355	1355
la30t	1355	-	1355	1358	363	1355	1357	1355

Table 8: Detailed Results on Set C

On set C, 9 new upper bounds upon 10 open instances have been found (`la21d`, `la21t`, `la22d`, `la24d`, `la24t`, `la25d`, `la25t`, `la29d`, and `la29t`). All 3 upper bounds reported in (Michel & Van Hentenryck 2004) are improved.

Instance	LB/UB	MvH	Avg Running Times			0.2/0/100/cont		Best
			best	avg	time	best	avg	
la31d	1784	-	1784	1784	55	1784	1786	1784
la31t	1784	-	1784	1784	211	1784	1784	1784
la32d	1850	-	1850	1850	1	1850	1850	1850
la32t	1850	-	1850	1850	2	1850	1850	1850
la33d	1719	-	1719	1719	65	1719	1719	1719
la33t	1719	-	1719	1719	464	1719	1719	1719
la34d	1721	-	1721	1721	199	1721	1722	1721
la34t	1721	-	1721	1722	695	1721	1721	1721
la35d	1888	-	1888	1892	240	1888	1891	1888
la35t	1888	-	1888	1894	877	1888	1890	1888
la36d	1229/1268	-	1255	1266	251	1251	1260	**1250**
la36t	1227/1268	-	1259	1265	636	1247	1256	**1247**
la37d	1378/1397	-	1444	1458	243	1418	1432	1416
la37t	1370/1397	-	1445	1456	676	1428	1439	1419
la38d	1092/1196	1185	1200	1230	431	1179	1199	**1175**
la38t	1087/1196	1195	1197	1224	1278	1173	1189	**1168**
la39d	1221/1233	-	1242	1255	297	1228	1231	**1226**
la39t	1221/1233	-	1252	1257	731	1227	1237	**1226**
la40d	1180/1222	-	1241	1245	340	1214	1223	**1205**
la40t	1176/1222	-	1245	1253	786	1219	1225	**1206**

Table 9: Detailed Results on Set D

On set D, 8 new upper bounds upon 10 open instances have been found (`la36d`, `la36t`, `la38d`, `la38t`, `la39d`, `la39t`, `la40d`, and `la40t`) and the 2 upper bounds reported in (Michel & Van Hentenryck 2004) are improved.

Instance	LB/UB	MvH	Avg Running Times			0.2/0/100/cont		Best
			best	avg	time	best	avg	
mt06d	55	-	56	56	5	55	56	55
mt06t	55	-	55	56	12	55	55	55
mt10d	837/930	913	899	913	63	901	917	**891**
mt10t	828/930	912	889	897	177	882	890	**879**
mt20d	1165	1186	1209	1223	62	1183	1209	1182
mt20t	1165	1205	1206	1220	157	1189	1211	1179

Table 10: Detailed Results on Set MT

On set MT, new upper bounds have been found for the two open instances (`mt10d` and `mt10t`). On `mt20d` and `mt20t`, $STRand$ also finds a better solution than ever reported before but still does not reach the upper bound derived from the corresponding job shop scheduling instances.

Overall, we improved 33 upper bounds upon 36 open instances. For the 3 remaining open instances (`la22t`, `la37d`, and `la37t`) we found solutions with a makespan that was not yet reported before, although we did not yet find a schedule with a makespan better than or equal to the upper bound derived from the corresponding job shop scheduling instance. This also goes for 3 closed instances (`la27t`, `mt20d`, and `mt20t`). On all other instances, the optimal schedule has been systematically found. Note thus that for many of these instances it is the first time a schedule was found with a makespan equal to the makespan that can be derived from the corresponding job shop scheduling instances.

Conclusion

In this paper we described a generic approach to solve cumulative scheduling problems based on randomized large neighborhood search. We presented a way to calculate a partial-order schedule from a fixed start time schedule, which is a crucial step in the generalization to cumulative scheduling from earlier randomized LNS work for disjunctive scheduling. We applied and tested the approach on the Cumulative Job Shop Scheduling Problem. The empirical performance analysis we performed using a well-known set of benchmark instances showed that our approach obtains the best known performance reported to date on the CJSSP. New upper bounds have been found on 33 out of 36 open instances and among them one instance is now closed. Furthermore, our approach proves to be very robust on the complete set of test instances. As for future work, we plan to study how our approach competes on other types of problems such as the Resource Constrained Project Scheduling Problem. Another direction we plan to study is how this approach can be extended, on one hand to other types of resources such as state resources and discrete reservoirs, and on the other hand to other types of objectives such as earliness/tardiness costs and resource allocation costs. Flexibility of the generated POS is clearly a key factor for the approach. We plan to compare the flexibility of our POS generation procedure with the one of (Policella *et al.* 2004) and work on improving this flexibility by exploiting interactions between resources.

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