## Függvények határértéke

1. 
$$\lim_{x \to \pm \infty} \frac{2x^3 + x + 1}{x^2 - x + 10}$$

1. 
$$\lim_{x \to \pm \infty} \frac{2x^3 + x + 1}{x^2 - x + 10}$$
 2.  $\lim_{x \to \infty} \frac{x\sqrt{x} + \sqrt{2x^3 + 3}}{x^2 + x + 1}$ 

$$3. \lim_{x \to -\infty} \frac{x}{\sqrt{x^2 + 1}}$$

4. 
$$\lim_{x \to 1} \frac{\sin(x-1)}{x-1}$$
 5.  $\lim_{x \to 1} \frac{\sin(x-1)}{x^2-1}$ 

5. 
$$\lim_{x \to 1} \frac{\sin(x-1)}{x^2 - 1}$$

6. 
$$\lim_{x\to 0} \frac{1-\cos x}{1-\cos 2x}$$

$$7. \lim_{x \to 0} x^2 \sin \frac{1}{x}$$

$$8. \lim_{x \to \infty} x \sin \frac{1}{x}$$

9. 
$$\lim_{x \to \infty} arctgx \cdot \frac{\sin x}{x}$$

10. 
$$\lim_{x \to \infty} \left( x - \sqrt{x^2 - 8x} \right)$$
 11.  $\lim_{x \to \infty} \left( \sqrt{3x^2 + 1} - x \right)$ 

11. 
$$\lim_{x \to \infty} (\sqrt{3x^2 + 1} - x)$$

9. 
$$\lim_{x \to \infty} arctgx \cdot \frac{\sin x}{x}$$
12. 
$$\lim_{x \to \infty} \frac{\sqrt{x^2 + 1} - x}{\sqrt{x + 1} - \sqrt{x}}$$

13. 
$$\lim_{x \to 0^+} \frac{2\sqrt{x} + 3\sqrt[3]{x}}{6\sqrt[3]{x} - 8\sqrt{x}}$$

14. 
$$\lim_{x \to \infty} \frac{2\sqrt{x} + 3\sqrt[3]{x}}{6\sqrt[3]{x} - 8\sqrt{x}}$$

$$15. \lim_{x\to 0\pm} e^{1/x}$$

16. 
$$\lim_{x\to +\infty} e^{1/x}$$

17. 
$$\lim_{x\to 0\pm} x\cdot e^{1/x}$$

18. 
$$\lim_{x \to \pm \infty} x \cdot e^{1/x}$$

19. 
$$\lim_{x\to\infty} \frac{3-a^x}{2+3a^x}$$
, a pozitív

13. 
$$\lim_{x \to 0+} \frac{2\sqrt{x} + 3\sqrt[3]{x}}{6\sqrt[3]{x} - 8\sqrt{x}}$$
14.  $\lim_{x \to \infty} \frac{2\sqrt{x} + 3\sqrt[3]{x}}{6\sqrt[3]{x} - 8\sqrt{x}}$ 
15.  $\lim_{x \to 0} \frac{1}{6\sqrt[3]{x} - 8\sqrt{x}}$ 
16.  $\lim_{x \to \pm \infty} e^{1/x}$ 
17.  $\lim_{x \to 0\pm} x \cdot e^{1/x}$ 
18.  $\lim_{x \to 0} \frac{3 - a^x}{2 + 3a^x}$ ,  $a \text{ pozitiv}$ 
20.  $f(x) = \frac{e^{1/x}}{1 + e^{1/x}}$ ,  $\lim_{x \to 0\pm} f = ?$   $\lim_{x \to \pm \infty} f = ?$ 

21. 
$$f(x) = \frac{e^{1/x}}{1 - e^{1-x}}$$
,  $\lim_{x \to 0\pm} f = ?$   $\lim_{x \to \pm \infty} f = ?$   $\lim_{x \to 1\pm} f = ?$ 

22. 
$$f(x) = \frac{e^{1/x}}{e^x - e^{1/x}}$$
,  $\lim_{x \to 0\pm} f = ?$   $\lim_{x \to \pm \infty} f = ?$   $\lim_{x \to 1\pm} f = ?$   $\lim_{x \to -1\pm} f = ?$ 

23. 
$$\lim_{x \to \infty} x^2 \left( e^{2/x^2} - 1 \right)$$

23. 
$$\lim_{x \to \infty} x^2 \left( e^{2/x^2} - 1 \right)$$
 24.  $\lim_{x \to \infty} e^x \cdot \ln(1 + 2e^{-x})$ 

25. 
$$f(x) = e^x$$
,  $g(x) = f(f(f(1/x)))$ ,  $\lim_{x \to 0+} g(x) = ?$ 

26. 
$$f(x) = x \sin \frac{1}{x}$$
 ha  $x \neq 0$  és  $f(0) = 0$ ,  $g(x) = 0$  ha  $x \neq 0$  és  $g(0) = 1$ ,  $h(x) = \cos x$  
$$\lim_{x \to \infty} h(f(x)) = ? \quad \lim_{x \to 0} h(f(x)) = ? \quad \lim_{x \to 0} g(f(x)) = ?$$

27. Ábrázolja vázlatosan az alábbi függvényeket

$$a) \quad f(x) = \frac{x-1}{x^2}$$

a) 
$$f(x) = \frac{x-1}{x^2}$$
 b)  $f(x) = \frac{(x-1)^2}{x^2}$ 

c) 
$$f(x) = \frac{(x-1)^3}{x^2}$$

d) 
$$f(x) = \frac{e^x}{1 - e^{x-1}}$$
 e)  $f(x) = \frac{e^{1/x}}{1 - e^{x-1}}$ 

e) 
$$f(x) = \frac{e^{1/x}}{1 - e^{x-1}}$$