Analysis II HS21

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2 ODE (ordinary differential equation)

Theorem 2.1.6

Let $F : \mathbf{R}^2 \to \mathbf{R}$ be differentiable. Let $x_0 \in \mathbf{R}$ and $y_0 \in \mathbf{R}^2$. Then the ODE $y' = F(x, y)$ has a unique solution f defined on a "largest" open interval I containing x_0 such that $f(x_0) = y_0$.

Definition 2.2.1

Let $I \subset \mathbf{R}$ be an open interval and $k \in \mathbb{N}_0$. An homogeneous linear ODE of order k on I is of the form $y^{(k)} + a_{k-1}y^{(k-1)} + \cdots + a_1y' + a_0y = 0$ where the coefficients $a_0, \ldots, a_{k-1}$ are complex-valued functions on I, and the unknown is a function $I \to \mathbf{C}$ that is k-times differentiable on I. An equation of the form $y^{(k)} + a_{k-1}y^{(k-1)} + \cdots + a_1y' + a_0y = b$, where $b : I \to \mathbf{C}$ is another function, is called an inhomogeneous linear ODE.

Recognize an ODE

1. no coefficients before the highest derivative
2. all coefficients are continuous
3. no products of y or their derivatives
4. no powers of y or their derivatives
5. no functions depending on y or their derivatives

Proposition 2.3.1

Any solution of $y' + ay = 0$ is of the form $f(x) = z\exp(-A(x))$ where A is a primitive of a. The unique solution with $f(x_0) = y_0$ is $f(x) = y_0 \exp(A(x_0) - A(x))$.

Solving inhomogeneous equations

Case 1: Make a guess. For example $y' = y + x^2$ guess $f(x) = ax^2 + bx + c$, and solve the equation.
Case 2: Use the variation of the constant. Assume $f_p = z(x)\exp(-A(x))$ for $z : I \to \mathbb{C}$. Then $z'(x) = b(x)\exp(A(x)) \implies k(x) = \int b(x)\exp(A(x))dx$.