Sequential Core-Set Monte Carlo

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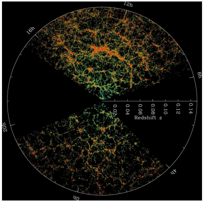
Motivation—Scalable incremental Bayesian inference

(*Simultaneous localization & mapping*)



(*Sloan Digital Sky Survey*)

Necessity of streaming memory

- **online** inference, **continual** learning
- **large**-**scale** data (limited memory)
- analysis by **synthesis** (dictionary)
- model interpretation by **examples**

Problems of previous approaches

- repeated access to **all data**
- **heuristic** approximations
- **model-specific** methods
- **opaque** representations

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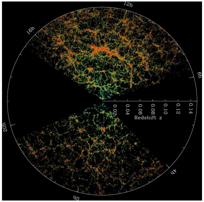

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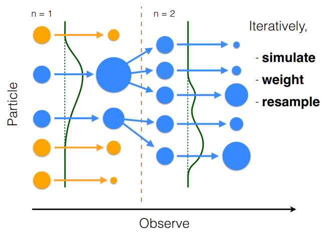
Our Contribution

__Online__ variant of sequential Monte Carlo for __generic__ exchangeable Bayesian models with __optimization__-based __streaming__ memory compression

Sequential Monte Carlo (SMC) with rejuvenation (MCMC)



(*SMC*)

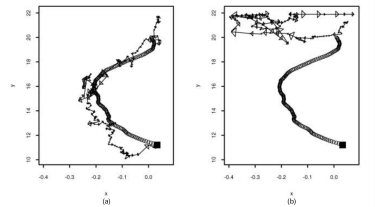


Fig. 2. Comparing the performances of (a) the resample–move method and (b) a standard SIR filter: ○, the ship's true trajectory; ■, final position; ———, the filter's output

(Gilks and Berzuini, 2001)

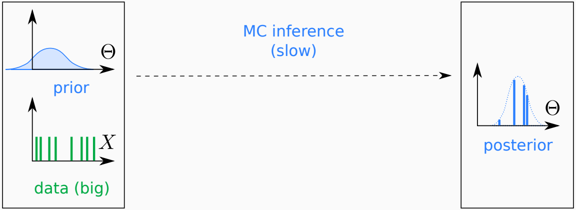
Sequential importance resampling

- particle population
- recursive updates via *reweighting* & *resampling*

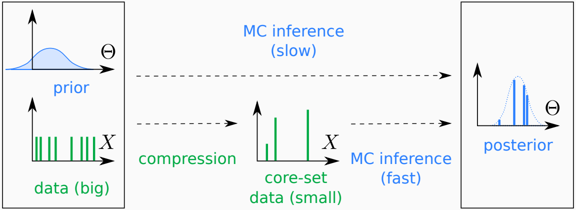
Variance reduction

- interleaved MCMC moves (*rejuvenation*/*resample-move*/*mutation*)
- crucial for approximation quality
- **computation** & **memory** $\propto$ **observations** !

Bayesian core-sets—Faster approximate inference



Bayesian core-sets—Faster approximate inference



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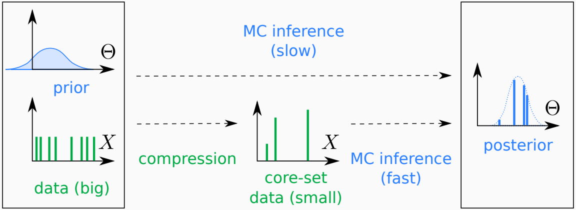
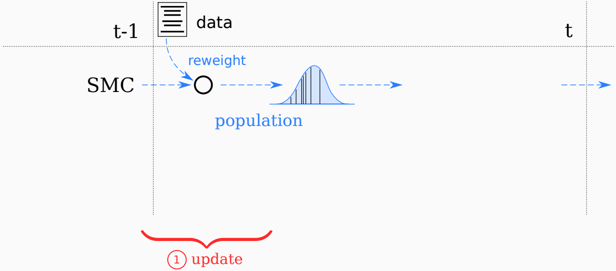


Complementary belief representations

- Θ: MC particles
- X: Bayesian core-sets (Campbell and Broderick, 2017)

Dataflow in sequential core-set Monte Carlo (SCMC)



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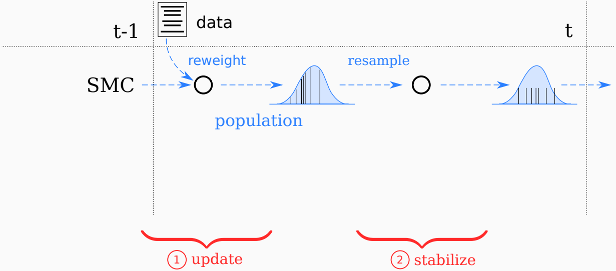

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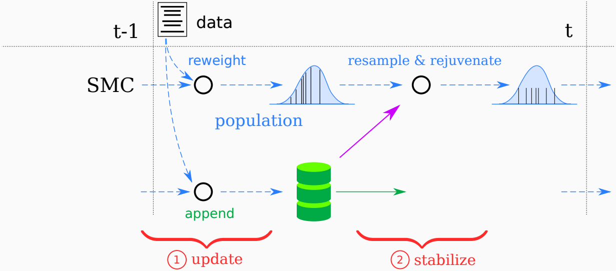

Filtering step in SCMC

- *rejuvenation* in sequential Monte Carlo (SMC) leverages core-set memory

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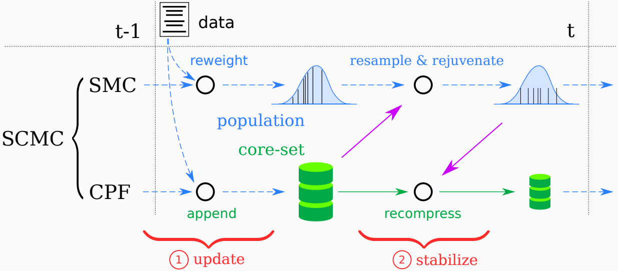

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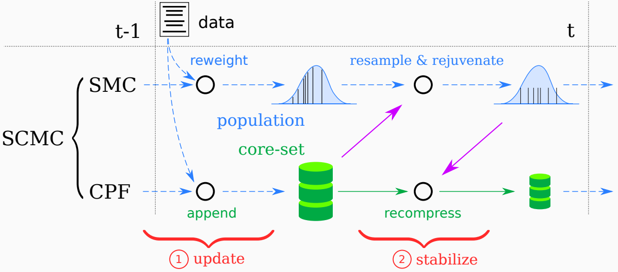

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Intuition

- **SMC** $\approx$ '*incrementalized* **MCMC**'
- **Core-set** compression $\approx$ sparse **VI** (Campbell and Beronov, 2019)
- **SCMC** $\approx$ 'sparse **SMC**' $\approx$ '*incrementalized* & sparse **MCMC** & **VI**'

Approximate rejuvenation in SCMC

Rejuvenation target

prior	$p_0(\theta)$		
likelihood	$p(x \mid \theta)$		
exact MCMC target	$p(\theta \mid x_1, \ldots, x_N)$	$\propto$	$p_0(\theta) \cdot \prod_{j=1}^{N} p(x_j \mid \theta)$
approximate MCMC target		$\approx$	$p_0(\theta) \cdot \prod_{j=1}^{C} p(\bar{x}_j \mid \theta)^{\bar{w}_j}$

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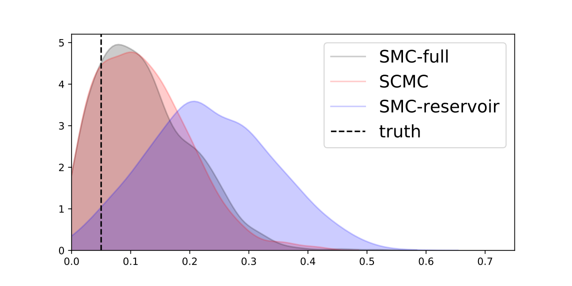
Example MCMC method—Metropolis-Hastings
acceptance ratio $\alpha(\cdot, \theta)$ for proposal $g(\cdot \mid \theta)$

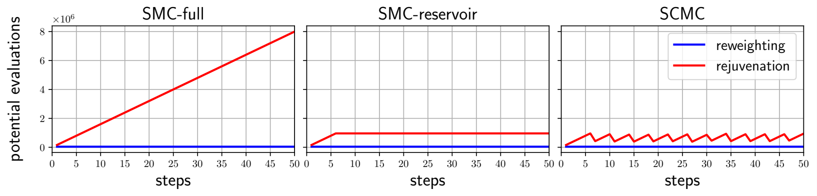
$$\alpha(\hat{\theta}, \theta) := \min\left(1, \frac{g(\theta \mid \hat{\theta})}{g(\hat{\theta} \mid \theta)} \cdot \frac{p(\hat{\theta} \mid x_1, \ldots, x_N)}{p(\theta \mid x_1, \ldots, x_N)}\right) \approx \min\left(1, \frac{g(\theta \mid \hat{\theta})}{g(\hat{\theta} \mid \theta)} \cdot \frac{p_0(\hat{\theta})}{p_0(\theta)} \cdot \prod_{j=1}^{C} \frac{p(\bar{x}_j \mid \hat{\theta})^{\bar{w}_j}}{p(\bar{x}_j \mid \theta)^{\bar{w}_j}}\right)$$

Posterior approximation quality

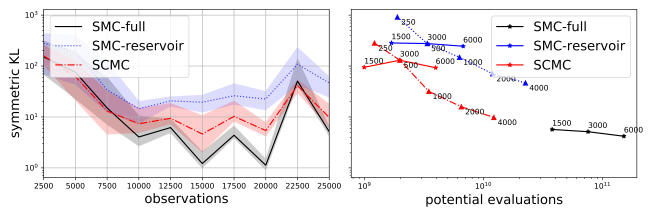
Qualitative improvement: AR(1) system identification (1 parameter)



Computation cost reduction: Normal-inverse-Wishart (27 parameters)



Error & variance reduction: Bayesian logistic regression (11 parameters)



Related methods—Online Bayesian inference

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(Extended/Unscented/Ensemble) Kalman filters

(locally) Gaussian models

Sparse online Gaussian processes

(Csató and Opper, 2002)

Particle filters / Sequential Monte Carlo

non-adaptive/offline approximation of rejuvenation, or growing cost

(Kitagawa, 1998; Chopin, 2002; Del Moral, Doucet, and Jasra, 2006; Doucet and Johansen, 2009; Naesseth, Lindsten, and Schön, 2019; Gunawan et al., 2020)

Variational filters

non-adaptive, asymptotically inconsistent variational family

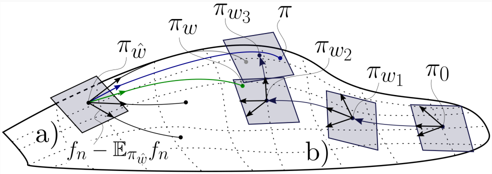
(Friston, 2008; Broderick et al., 2013; Nguyen et al., 2017; Marino, Cvitkovic, and Yue, 2018)

Information geometric projection filters

non-adaptive variational family

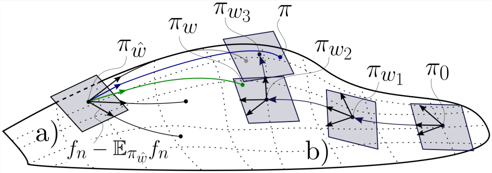
(Kulhavý, 1990; Kulhavy, 1992; Kulhavý, 1996; Brigo, Hanzon, and Le Gland, 1995; Brigo, Hanzon, and Le Gland, 1999; Armstrong and Brigo, 2016)

Bayesian statistics ↔ Information geometry



Bayesian statistics	↔	Information geometry
Hilbert space	↔	*exponential* tangent space
of centered *log*-likelihood functions		of tangent vectors
at weighting distribution		at point of statistical manifold
log density ratio $\log(p/q)$	↔	*logarithmic* map $\mathrm{Log}_q(p)$
tuple of observations	↔	coordinate frame
Fisher information metric	↔	Riemannian metric

Bayesian statistics $\leftrightarrow$ Information geometry



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SparseVI (Campbell and Beronov, 2019): approximate 'future' **Log**$_{\text{prior}}$ (posterior)

SCMC: approximate 'past' **Log**$_{\text{belief}}$ (prior)

Conclusion—Sequential Core-Set Monte Carlo

Contributions

- **bounded-resource** *data tempering* SMC via core-set rejuvenation
- generic & statistically sound **streaming** construction of Bayesian core-sets
- **generalization** of *projection filtering* to core-set posterior exponential family

Future work

- convergence theory, adaptive choice of core-set & population size
- distributed streaming inference via core-set messages
- generalization to non-exchangeable/non-stationary models
- applications: 'Bayes optimal subsampling' for variance reduction & active learning in SL, episodic memory in RL, evolutionary optimization ...

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***SMC*.** https://www.robots.ox.ac.uk/~fwood/talks/2015/agi-keynote-2015-probabilistic-programming.pdf.