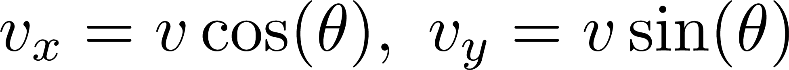
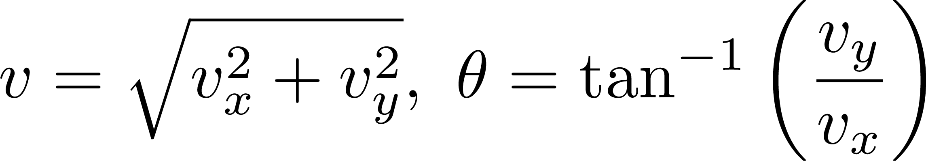
# Material Summary: Linear Algebra

## Vectors

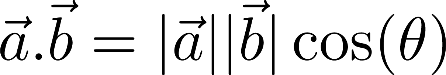
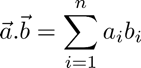
### Vector Definitions

* "Physics definition"
  + A pointed segment in space
* "Computer science definition"
  + A list of objects (usually numbers)
  + Dimensions = length
* Math definition
  + Encompasses both, and allows even more abstraction: *\_*𝑣*\_*
  + Vectors can be added and multiplied
    - By numbers and other vectors
  + Similar to how we defined a field
* Another perspective
  + Transformations
  + Actually, things a just a little more complicated…
    - You can look up "tensors" if you're interested
    - We'll talk a little about tensors later

### Vector Components

* The distances to all coordinate axes: *\_*𝑣*\_*𝑥*\_, \_*𝑣*\_*𝑦*\_*
* Equivalent to
* Polar coordinates:
* Finding components: Pythagoras
* Finding the polar form (magnitude, direction): 
* All these operations generalize to more than 2 dimensions
* We usually denote vectors by *\_*𝑣*\_* or with bold type: 𝒗
  + Another notation: Latin letters for vectors, Greek letters for numbers
  + Reason: The vector 𝒗 and its length 𝑣 can be easily confused

### Vector Operations

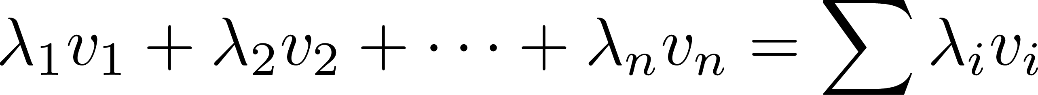
* **Addition**
  + Result:  length = distance from start to end,  direction: start → end
  + In component form: sum all components for every direction
* **Multiplication by a number (scalar)**
  + Result: length = scaled length, direction: same (if scalar *≥0*), opposite otherwise
  + In component form: multiply each component by the number
* **Scalar product of two vectors** 
  + Also called dot product or inner product
  + Result: scalar
  + Definition: 
  + Using the vector components: 
* **Vector product of two vectors**
  + Also called cross product
  + Result: vector, perpendicular to both initial vectors
  + Definition: 
    - – normal vector
    - Magnitude: = volume of parallelogram   
      between and
    - Direction: coincides with the direction of

### Vector Spaces

* A field (usually R or C): 𝐹
* A set of elements (vectors): 𝑉
* Operations
* Addition of two vectors: 𝑤*=*𝑢*+*𝑣
* Multiplication by an element of the field: 𝑤*=*𝜆𝑢
* A "checklist" of eight axioms
* We read this as "vector space (or linear space) 𝑉 over the field 𝐹"
* Examples of vector spaces
  + Coordinate space, e.g., real coordinate space
    - 𝑛-dimensional vectors
  + Infinite coordinate space 
    - Vectors with infinitely many components
  + Polynomial space
    - All polynomials of variable 𝑥 with real coefficients
  + Function space

### Linear Combinations

* Vectors:
* Numbers (scalars):
* **Linear combination:** The sum of each vector multiplied by a scalar coefficient

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* Why linear? No fancy functions, no vector multiplications
* **Span** (linear hull) of vectors: the set of all their linear combinations
* Linear (in)dependence
  + The vectors are **linearly independent** if the only solution to the equation is
  + Conversely, they are **linearly dependent** if there is a non-trivial linear combination which is equal to zero
* Example:

