Appendix A

Non-dimensional coefficients and reduced-order phugoid model

A.1 Non-dimensional coefficients

The notation for the derivatives used in this report has the same relationship to fully non-dimensional stability derivatives as in reference (2). The definitions given there are reproduced here:

$$x_u = \frac{V}{c} \cdot \frac{C_{X_u}}{2\mu_c} \qquad z_u = \frac{V}{c} \cdot \frac{C_{Z_u}}{2\mu_c - C_{Z_{\dot\alpha}}} \qquad m_u = \frac{C_{m_u} + C_{Z_u} \cdot \dfrac{C_{m_{\dot\alpha}}}{2\mu_c - C_{Z_{\dot\alpha}}}}{2\mu_c K^2_Y}$$

$$x_\alpha = \frac{V}{c} \cdot \frac{C_{X_\alpha}}{2\mu_c} \qquad z_\alpha = \frac{V}{c} \cdot \frac{C_{Z_\alpha}}{2\mu_c - C_{Z_{\dot\alpha}}} \qquad m_\alpha = \frac{C_{m_\alpha} + C_{Z_\alpha} \cdot \dfrac{C_{m_{\dot\alpha}}}{2\mu_c - C_{Z_{\dot\alpha}}}}{2\mu_c K^2_Y}$$

$$x_\theta = \frac{V}{c} \cdot \frac{C_{Z_o}}{2\mu_c} \qquad z_\theta = \frac{V}{c} \cdot \frac{-C_{X_o}}{2\mu_c - C_{Z_{\dot\alpha}}} \qquad m_\theta = \frac{-C_{X_o} \cdot \dfrac{C_{m_{\dot\alpha}}}{2\mu_c - C_{Z_{\dot\alpha}}}}{2\mu_c K^2_Y}$$

$$z_q = \frac{V}{c} \cdot \frac{2\mu_c + C_{Z_q}}{2\mu_c - C_{Z_{\dot\alpha}}} \qquad m_q = \frac{C_{m_q} + C_{m_{\dot\alpha}} \cdot \dfrac{2\mu_c + C_{Z_q}}{2\mu_c - C_{Z_{\dot\alpha}}}}{2\mu_c K^2_Y}$$

$$z_\delta = \frac{V}{c} \cdot \frac{C_{Z_\delta}}{2\mu_c - C_{Z_{\dot\alpha}}} \qquad m_\delta = \frac{C_{m_\delta} + C_{Z_\delta} \cdot \dfrac{C_{m_{\dot\alpha}}}{2\mu_c - C_{Z_{\dot\alpha}}}}{2\mu_c K^2_Y}$$

A.2 <u>Refined phugoid approximation in non-dimensional form</u>

In reference (3), a reduced order approximation is derived in fully non-dimensional form, by setting $D_c q = 0$ and $D_c \alpha = 0$ in a θ-based system of the form (for horizontal flight and $C_{X_q} = 0$, $C_{Z_q} = 0$):

$$\begin{pmatrix} C_{X_u} - 2\mu_c D_c & C_{X_\alpha} & C_{Z_o} & 0 \\ C_{Z_u} & C_{Z_\alpha} + (C_{Z_{\dot\alpha}} - 2\mu_c) D_c & 0 & 2\mu_c \\ 0 & 0 & -D_c & 1 \\ C_{m_u} & C_{m_\alpha} + C_{m_{\dot\alpha}} D_c & 0 & C_{m_q} - 2\mu_c K_Y^2 D_c \end{pmatrix} \begin{pmatrix} \hat{u} \\ \alpha \\ \theta \\ q\bar{c}/V \end{pmatrix} = \underline{0} \qquad \text{(A.1)}$$

The resulting characteristic equation is:

$$\lambda^2 \, . \, (2\mu_c (C_{Z_\alpha} . C_{m_q} - 2\mu_c . C_{m_\alpha}))$$
$$+ \lambda \, . \, (2\mu_c (C_{X_u} . C_{m_\alpha} - C_{m_u} . C_{X_\alpha}) + C_{m_q} (C_{Z_u} . C_{X_\alpha} - C_{X_u} . C_{Z_\alpha}))$$
$$+ \; C_{Z_o} (C_{m_u} . C_{Z_\alpha} - C_{Z_u} . C_{m_\alpha}) \qquad \text{(A.2)}$$

The equivalent of the derivation given in paragraph 2.4 (b) of this report requires transforming the system to state vector $(\hat{u} \;\; \alpha \;\; \gamma \;\; q\bar{c}/V)^T$. This can be accomplished by simply substituting $\theta = \gamma + \alpha$ into the four equations (A.1), to give:

$$\begin{pmatrix} C_{X_u} - 2\mu_c D_c & C_{X_\alpha} + C_{Z_o} & C_{Z_o} & 0 \\ C_{Z_u} & C_{Z_\alpha} + (C_{Z_{\dot\alpha}} - 2\mu_c) D_c & 0 & 2\mu_c \\ 0 & -D_c & -D_c & 1 \\ C_{m_u} & C_{m_\alpha} + C_{m_{\dot\alpha}} D_c & 0 & C_{m_q} - 2\mu_c K_Y^2 D_c \end{pmatrix} \begin{pmatrix} \hat{u} \\ \alpha \\ \gamma \\ q\bar{c}/V \end{pmatrix} = \underline{0} \qquad \text{(A.3)}$$

If $D_c q$ and $D_c \alpha$ are set to zero in (A.3), we have:

$$\lambda^2 \, . \, (2\mu_c (C_{Z_\alpha} . C_{m_q} - 2\mu_c . C_{m_\alpha}))$$

$$+ \lambda \, . \, (2\mu_c (C_{X_u} C_{m_\alpha} - C_{m_u} C_{X_\alpha}) + C_{m_q} (C_{Z_u} C_{X_\alpha} - C_{X_u} C_{Z_\alpha}) + C_{Z_o} (C_{Z_u} C_{m_q} - 2\mu_c C_{m_u}))$$

$$+ \; C_{Z_o} (C_{m_u} . C_{Z_\alpha} - C_{Z_u} . C_{m_\alpha}) \tag{A.4}$$

That is, the C_{X_α} term in the damping has been replaced by $C_{X_\alpha} + C_{Z_o}$. Since:

$$C_{X_\alpha} \approx C_L \left(1 - \frac{C_{L_\alpha}}{\pi Ae}\right)$$

according to ref.3, and $C_{Z_o} = -C_L$, these two terms partly cancel to leave only the change in induced drag due to perturbations in α, where

$$C_{L_\alpha} \approx 2\pi/\text{rad}.$$

The difference between (A.2) and (A.4) can be explained, by the different "amounts" of $D_c\alpha$ that are cancelled in (A.1) and (A.3). In a physical interpretation, the derivation in the θ-based system allows the short period approximation to have a quasistatic solution $\alpha \neq 0$, $q \neq 0$ proportional to $\hat{u}$, but it does not account for the fact that this quasistatic solution also requires that $\theta \neq 0$; while this effect of $\theta = \alpha$ for horizontal flight is implicitly present if γ and α are state variables instead of θ and α.

There is no doubt that the γ-based formulation is more correct. The difference between the two approximations accounts for the fact that the refined phugoid model does not perform very well in ref.(3), due to overestimating the damping.

APPENDIX B

Examples of CASPAR input data sets

B.1 Examples of CASPAR input data sets

In this appendix five examples of CASPAR root locus program (ROLO) input data sets are given, one for each flight condition.

The models used for the computations have been left fully dimensional and in body axes, the way they were produced by the CASPAR LINMAIN linearization program. The state variables in stability axes have been defined as outputs when they were required for feedback. Integrations have been added as needed, by expanding the order of the system matrix. The different root loci in the report have been obtained by direct editing of the feedback gains in the data sets reproduced here. The results, which were plotted point by point by ROLO on rather large and varying scales, have been replotted to a common scale on a microcomputer with a small plotter, and converted to line root loci by hand.

table B.1: Model for 50 m/s, normal centre of gravity

```
'P.LAMMERTSE',

8,3,8,

-.035711   0.26871   -9.7375   -5.9269   0      0   0   0
-.14789    -1.3128   -1.1144   46.879    0      0   0   0
 0          0         0         1        0      0   0   0
 .04120    -.19451    0        -3.5434   0      0   0   0
 .11370    -.99351    50        0        0      0   0   0
 .99351    .11370     0         0        .1961  0   0   0
 0          0         0         0        1      0   0   0
 -5.0683 -0.5815      0         0        1      0   0   0
 0          .074575   0
-7.4618 -.14023       0
 0          0         0
-13.525 -.028336      0
 0          0         0
 0          0         0
 0          0         0
 0          0         0
.99351 .11370 0 0 0.1961 0.2 0 0
.002274 -.0198702 1 0 0 0 0 0
-.14286 -1.3349 0 -2.7509 0 0 0 0
0 0 0 1 0 0 0 0
-5.0683 -0.5815 0 0 1 0 0 0
0 0 0 0 1 0 0 0
0 0 0 0 0.1 0 0.04 0
0 0 1 0 0 0 0 0
0 0 0
0 0 0
0 0 0
0 0 0
0 0 0
0 0 0
0 0 0
0 0 0
0 0 0 0 0 2 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 -4 0.06 -0.5 0 -0.04 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0.008 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
-11,

-2.25,0.25,-0.25,2.25
'(TAS+G/VH) (1+1/5S)'
'GAMMA'
'AZ'
'Q',
'H-V/G.U'
'H'
'PI ACTION'
'THETA'
'ELEVATOR',
'PZ ("HG)'
'INPUT 3'
```

table B.2: <u>Model for 80 m/s, normal centre of gravity</u>

```
'P.LAMMERTSE',

 8,3,8,

 -.062973   0.22551   -9.8004   -2.7413    0       0      0  0
 -.073865   -2.3783    .11133   75.492     0       0      0  0
    0          0         0         1       0       0      0  0
  .028146   -.26414      0      -5.6695    0       0      0  0
 -.011359   -.99993   79.932       0       0       0      0  0
  .99993    .011359      0         0     .1226     0      0  0
    0          0         0         0       1       0      0  0
    0          0         1         0       0       0      0  0
   0        .051429   0
 -19.102 -.096709     0
  0           0       0
 -34.624 -.019541     0
   0          0       0
   0          0       0
   0          0       0
   0          0       0
 .99993 .011359 0 0 0.1226 0.2 0 0
 -.000142 -0.0125 1 0 0 0 0 0
 -.07196 -2.3839 0 -4.469 0 0 0 0
 0 0 0 1 0 0 0 0
 0 0 0 0 1 0 0 0
 0 0 0 0 0 1 0 0
 0 0 1 0 0 0 0 0
 -.000142 -0.0125 1 0 0.01 0 0 0
 0 0 0
 0 0 0
 0 0 0
 0 0 0
 0 0 0
 0 0 0
 0 0 0
 0 0 0
 0 0 0 0 2 0 0 0
 0 0 0 0 0 0 0 0
 0 0 0 0 0 0 0 0
 0 -2.5 0.0225 -0.3125 -0.015625 0 0 0
 0 0 0 0 0 0 0 0
 0 0 0 0 0 0 0 0
 0 0 0 0 0.003125 0 0 0
 0 0 0 0 0 0 0 0
 0 0 0 0 0 0 0 0
 -11,

-2.25,0.25,-0.25,2.25
  '(1+.2/S)(DLT TAS + G/V.H)'
  'GAMMA'
  'AZ S',
  'Q',
  'H'
  'INT.U+G/V.H'
  'THETA'
  'GAMMA+.01H'
  'ELEVATOR',
  'PZ ("HG)'
  'INPUT 3'
```

table B.3: <u>Model for 35 m/s, normal centre of gravity</u>

```
'P.LAMMERTSE',
 8,3,8,
 -.025526   0.35902   -9.4515   -9.3972    0       0       0  0
 -.20731   -0.73242   -2.5941    31.802    0       0       0  0
    0         0          0         1       0       0       0  0
 .075989    -.16540      0       -2.4804   0       0       0  0
 .26468     -.96433    35.0        0       0       0       0  0
 .96433      .26468      0         0     .28019    0       0  0
    0         0          0         0       1       0       0  0
    0         0          1         0       0       0       0  0
   0        .093489   0
 -3.6563 -.1758       0
  0          0        0
 -6.6272 -.035523     0
   0         0        0
   0         0        0
   0         0        0
   0         0        0
 0.96433 0.26468 0 0 0.28019 0.2 0 0
 .0075623 -.027552 1 0 0 0 0 0
 -0.1932 -0.7991 0 -1.8452 0 0 0 0
 0 0 0 1 0 0 0 0
 0 0 0 0 1 0 0 0
 0 0 0 0 0 1 0 0
 0 0 1 0 0 0 0 0
 .96433 .26468 0 0 0 0 0 0
 0 0 0
 0 0 0
 0 0 0
 0 0 0
 0 0 0
 0 0 0
 0 0 0
 0 0 0
 0 0 0 0 2 0 0 0
 0 0 0 0 0 0 0 0
 0 0 0 0 0 0 0 0
 0 -5.71 0.1275 -0.7 -0.08125 0 0 0
 0 0 0 0 0 0 0 0
 0 0 0 0 0 0 0 0
 0 0 0 0 0.01625 0 0 0
 0 0 0 0 0 0 0 0
 0 0 0 0 0 0 0 0
 -11,
-2.25,0.25,-0.25,2.25
 'PI ENERGY'
 'GAMMA'
 'AZ S',
 'Q',
 'H'
 'INT ENERGY'
 'THETA'
 'U S'
 'ELEVATOR',
 'PZ "HG'
 'INPUT 3'
```

table B.4: <u>Model for 50 m/s, forward centre of gravity</u>

```
'P.LAMMERTSE',

 8,3,8,

 -.03468    0.27991   -9.7283   -6.2855   0      0      0  0
 -.14240   -1.2973   -1.1919    46.836   0      0      0  0
    0         0         0          1     0      0      0  0
 .060369   -.28877      0       -3.7485  0      0      0  0
  .1212    -.99257     50          0     0      0      0  0
 0.99257   0.1216       0          0    .1960   0      0  0
    0         0         0          0     1      0      0  0
    0         0         0          0     0      0      0  0
   0       .074733   0
 -7.4618  -.14053    0
  0          0       0
 -14.072  -.038709   0
   0         0       0
   0         0       0
   0         0       0
   0         0       0
 .99257 .1216 0 0 0.1960 0.2 0 0
 .002432 -.0198514 1 0 0 0 0 0
 -0.13715 -1.3216 0 -2.7469 0 0 0 0
 0 0 0 1 0 0 0 0
 0 0 0 0 1 0 0 0
 0 0 0 0 0 1 0 0
 0 0 0 0 0 0 1 0
 0 0 1 0 0 0 0 0
 0 0 0
 0 0 0
 0 0 0
 0 0 0
 0 0 0
 0 0 0
 0 0 0
 0 0 0
 0 2 0 0 0 0 0 0
 0 0 0 0 0 0 0 0
 0 0 0 0 0 0 0 0
 0 -2.5 0.06 0 0 0 0 0
 0 0 0 0 0 0 0 0
 0 0 0 0 0 0 0 0
 0 0.5 0 0 0 0 0 0
 0 0 0 0 0 0 0 0
 0 0 0 0 0 0 0 0
 -11,

-2.25,0.25,-0.25,2.25
 '(TAS+G/VH)(1+1/5S)'
 'GAMMA'
 'AZ S'
 'THETA',
 'H'
 'INT U'
 'INT H'
 'THETA'
 'ELEVATOR',
 'PZ "HG'
 'INPUT 3'
```

table B.5: <u>Model for 50 m/s, aft centre of gravity</u>

```
'P.LAMMERTSE',

8,3,8,

-.036395   0.26134   -9.7429   -5.6900    0      0      0  0
-.15163   -1.323     -1.0656   46.905     0      0      0  0
    0         0         0         1       0      0      0  0
.028835   -.13014       0      -3.4065    0      0      0  0
.10872    -.99407      50         0       0      0      0  0
    1         0         0         0       0      0      0  0
.99407     .10872     0         0       0.1960 0      0  0
    0         0         0         0       0      0      0  0
   0        .07447      0
-7.4618  -.14003        0
 0            0         0
-13.159  -.021437       0
   0          0         0
   0          0         0
   0          0         0
   0          0         0
.99407 .10872 0 0 0.1960 0 0.2 0
.0021744 -.0198814 1 0 0 0 0 0
-0.14675 -1.3436 0 -2.7486 0 0 0 0
0 0 1 0 0 0 0 0
0 0 0 0 1 0 0 0
0 0 0 0 0 1 0 0
0 0 0 0 0 0 1 0
0 0 0 0 5 1 0 0
0 0 0
0 0 0
0 0 0
0 0 0
0 0 0
0 0 0
0 0 0
0 0 0
0 2 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 -2.5 0.06 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0.5 0 0 0 0 0 0
0 0 0 0 0 0 0 0
-11,
-2.25,0.25,-0.25,2.25
'(TAS+G/VH)(1+1/5S)'
'GAMMA'
'AZ S',
'THETA',
'H'
'INT U'
'INT H'
'DUMMY'
'ELEVATOR',
'PZ "HG'
'INPUT 3'
```