B.Sc.(Hons.) Physics
PHC401-32221401
Teacher: Mamta

S.G.T.B. Khalsa College
Mathematical Physics III (2021-22)
Lab Assignment # 6
Weighted Least Square Fitting

Due Date and Time :
14.03.2023, 11:59PM
Max. Marks : 30

Note:

- ***Perform this assignment in groups of two***. Please partner with a student in the same practical group (A1 or A2.
- Use the template for the cover page as first page in your assignment.
- Submit one pdf file that contains theory and discussion (may be either typed in Latex and converted to pdf or handwritten neatly on A4 size sheets and scanned), the source code and the output (containing the input you have given, any table or graphs). Besides this pdf you are also required to submit any data file and the source code(.py file). Name all files as 2020PHYXXXX_A6.pdf, 2020PHYXXXX_A6.py, 2020PHYXXXX_A6_graphs.jpeg, etc. Here 'XXXX' are the last four digits of your class roll no and 'AA' is the assignment number(like A1, A2 etc.).
- The first line in your code should contain the following information as comment
  1. Your Name and Roll No
  2. Your partner's name and Roll No
- Print your roll no as the first line in your output.

1. **Theory** [10]
   (a) What is the principle of maximum likelihood?
   (b) Describe the method of weighted least squares for fitting a linear function to a given set of N data points $(x_i, [y_{ij}])$ ($i = 1, \cdots, N$ and $j = 1, \cdots, N1$), where $N1$ is the number of $y-$values observed for each x. Take weights to be $w_i = 1/\sigma_i^2$, σ_i being the value of error in $\bar{y}_i$ and $\bar{y}_i$ is the mean of all $[y_{ij}]$ for each i.
      i. How is Least square fitting related to the principle of maximum likelihood?
      ii. Derive the formulae for mean, error in mean, slope, intercept, errors in slope and intercept.
      iii. Under what condition does weighted least square fitting (WLSF) reduces to ordinary least square fitting(OLSF)? Show that under this condition above formulae reduce to the formulae for OLSF.
   (c) Define correlation coefficient and adjusted correlation coefficient.

2. **Programming** [15]
   (a) Write the python functions 'Mylsf' and 'Mywlsf' to fit $y = mx + c$ using the methods of least square fitting and weighted least square fitting respectively to the data given as $N \times 3$ matrix $(x_i, \bar{y}_i, w_i)$. The data is to be read from a file provided to you (rename the file as 'xxxx.dat', 'xxxx' being the last four digits of your roll number). Your functions should take the data points as input and return slope, intercept, error in slope, error in intercept, sum of residuals $\sum(y_i - mx_i - c)$, sum of squares of residuals $\sum(y_i - mx_i - c)^2$ and correlation coefficient.
   (b) In a mechanics laboratory, a spring is hung from a rigid support, and a mass M is hung on the spring. If the mass is pulled down and released, the system will oscillate with a period T given by the expression $T = 2\pi[(M + m)/k]^{1/2}$, k, m being the spring constant and effective spring mass respectively. The observations for various masses (M_i) and corresponding time periods $(T_{ij}, j = 1, 2, \cdots, 10)$ are given in the file '*data-lsf.xlsx*'.
   (c) Show that T^2 vs. M follows a linear law. What are the slope and intercept of this line? How is σ_y^2 related to σ_T^2.
   (d) Write a python program "*2020PHYxxxx_A6.py*" that performs the following tasks:

i. modifies the given data (M_i, $[T_{ij}]$) to obtain data ($x_i = M_i, [y_{ij}] = [T_{ij}^2]$), and hence from it evaluates $\bar{y}_i$ and $w_i = \sigma_{\bar{y}_i}^2$ to obtain the matrix (x_i, $\bar{y}_i$, w_i) and store it as a text file named 'xxxx.csv' where xxxx are the last four digits of your roll number.

ii. fits the above data (x_i, $\bar{y}_i$, w_i) using the functions made above.

iii. prints the values obtained from output of function in the above file. Plot the data points (along with error bars) as well as the fitted line and store the graph as 'xxxx-graph' in png or jpeg or pdf format.

iv. compares the sum of squares of deviations and the correlation coefficients of lsf and wlsf.

(e) Extend your code to evaluate the value of k and m along with their errors and store in the same output file ('xxxx.out').

(f) Use the inbuilt function "*linegress*" from scipy stats and compare your results of 'lsf' with this function's output.

3. **Discussion** [5]
Interpret and discuss your results and graphs.