B.Sc.(Hons.) Physics
PHC401-32221401
Teacher: Mamta

S.G.T.B. Khalsa College
Mathematical Physics III (2021-22)
Lab Assignment # 2b
Fourier Series Representation

Due Date and Time :
27.02.2021, 11:59PM
Max. Marks : 50

Note:

- ***Perform this assignment in groups of two***. Please partner with the same student with whom you wrote the integration module.
- The first line in your code should contain the following information as comment
  1. Your Name and Roll No
  2. Your partner's name and Roll No
- Print your roll no as the first line in your output.

1. **Theory** [20]
   (a) What are **Dirichlet Conditions** for the existence of Fourier Series representation of a function? Are they necessary or sufficient?
   (b) **Fourier Representation of a Periodic Function:**
      i. Write down the Fourier Series Representation for a piecewise continuous periodic function $f(x+2L) = f(x)$.
      ii. Derive the expressions for Fourier coefficients. State what can you say about the coefficients if the function $f(x)$ is an even or odd function.
      iii. Obtain the Fourier Series Representation for the following functions:
      α)

$$\begin{aligned} f(x) &= \begin{cases} 0, & -1 < x < 0 \\ 1, & 0 < x < 1 \end{cases} \\ \text{and} \quad f(x+2) &= f(x) \end{aligned} \tag{1}$$

      β)

$$\begin{aligned} f(x) &= \begin{cases} 0, & -1 < x < -0.5 \\ 1, & -0.5 < x < 0.5 \\ 0. & 0.5 < x < 1 \end{cases} \\ \text{and} \quad f(x+2) &= f(x) \end{aligned} \tag{2}$$

      γ)

$$\begin{aligned} f(x) &= \begin{cases} -0.5, & -1 < x < 0 \\ 0.5, & 0 < x < 1 \end{cases} \\ \text{and} \quad f(x+2) &= f(x) \end{aligned} \tag{3}$$

      What do you mean by even and odd functions? State if the above functions are even, odd or neither even nor odd?
   (c) Explain **Gibbs Phenomenon**?
   (d) **Half Range Expansion:**
      i. How do you write the Fourier series representation for a function $f(x)$ that is defined in a finite range, say, $0 < x < L$?
      ii. What do you mean by half range sine and cosine expansions.
      iii. Write down the half range even and odd periodic extensions for the function defined as

$$f(x) = x, \quad 0 < x < \pi. \tag{4}$$

iv. Derive cosine and sine Fourier representations for the above extensions.

2. **Algorithm or Pseudocode** [5]

   (a) Write the pseudocode for a function "*FourierCoeff*" to evaluate the first N coefficients of the fourier series of a given periodic function $f(x)$ using appropriate integration method in your module MyIntegration.py (assignment A2a) so as to get a result accurate to d number of significand digits. The function should take as an argument the name of the method, d, function f, the half period L, the number of terms N to be evaluated and a variable that is 0,1,-1 if the function is 'even', 'odd' or 'neither even nor odd' respectively. This function *FourierCoeff* should return the fourier coefficients a_0, a_i, b_i, $i = 1, 2, \cdots, N$. Use the 'even' and 'odd' property of function appropriately.

   (b) Which integration method do you think is the most appropriate for the functions in previous question? Give reason for your choice. If you had to approximate the output of a full wave rectifier by Fourier series, which integration method would be the most appropriate. Why?

3. **Programming** [20]

   (a) Write a python program "***A3a-2020PHYxxxx.py***" (*xxxx* being the last four digits of your roll number) that

      i. contains the function "*FourierCoeff*" performing the task as mentioned in previous question.

      ii. uses the function made above and evaluates fourier series coefficients for the functions (1), (2) and (3) using the method mentioned in 2(b).

      iii. evaluates the partial sums S_i using these coefficents of the fourier series approximation of the above function with $i = 1, 2, 5, 10, 20$ terms in the range $[-2.5, 2.5]$ (with 50 evenly spaced values of x in the range).

      iv. plots the Fourier series approximation of the function obtained above with 1, 2, 5, 10, 20 terms (as points of different colors and styles) along with the actual function in the range $[-2.5, 2.5]$ (as continuous curve).

      v. stores the coefficient vectors (in a text file) as well as the graph in pdf format.

      vi. displays the values of the function $f(x)$, the Fourier approximated values and relative error from $f(x)$ for all the above N at $x = -0.5,\ 0,\ 0.5$ in a tabulated form.

   (b) Write another python program "***A3b-2020PHYxxxx.py***" (*xxxx* being the last four digits of your roll number) using the function "*FourierCoeff*" made in previous question to find the half range even and odd fourier series representation for the function (4). Your program should

      i. plot the cosine (sine) Fourier series approximations of the function obtained above with $N = 1$, 2, 5, 10, 20 number of terms along with the even (odd) extended function in the range $[-3\pi,\ 3\pi]$. The two plots should be plotted as subplots.

      ii. display the values of the extended function, the Fourier approximated values and relative error from $f(x)$ for all the above N at $x = 0$, $x = \pi/2$ and $x = \pi$ in tabulated format. Make two tables: one for even and one for odd extension.

4. **Discussion** [5]

   Interpret and discuss your results and graphs. Specifically explain Gibbs Phenomenon from your results.