B.Sc.(Hons.) Physics
PHC401-32221401
Teacher: Mamta

S.G.T.B. Khalsa College
Mathematical Physics III (2021-22)
Lab Assignment # 7
Dirac Delta Funtion

Due Date and Time :
19.03.2022, 11:59PM
Max. Marks : 30

Note:
Do this assignment in groups of two using your module for integration

1. **Theory** [10]
   (a) Explain the Dirac Delta Function. Explain that it is not the function in the sense we define a function but rather a distribution.
   (b) Give five representations of Dirac Delta function $\delta(x)$ as a limit of sequence of functions:
   $$\delta(x) = \lim_{\epsilon \to 0+} f_\epsilon(x)$$
   where $f_\epsilon(x)$ is an absolutely integrable function on $\mathbb{R}$ s.t.
   $$\begin{aligned} \int_{-\infty}^{+\infty} f_\epsilon(x)\, \mathrm{d}x &= 1 \\ \text{and} \qquad f_\epsilon(x) &= \frac{1}{\epsilon} f\left(\frac{x}{\epsilon}\right) \end{aligned}$$
   (c) State the properties of a Dirac Delta Function $\delta(x-a)$ and also its 3-dimensional version $\delta^3(\vec{r}-\vec{a})$.
   (d) Evaluate
      i. $\displaystyle\int_{-\infty}^{\infty} \delta(x-2)\,(x+1)^2\, \mathrm{d}x$
      ii. $\displaystyle\int_{-\infty}^{\infty} 9x^2 \delta(3x+1)\, \mathrm{d}x$
      iii. $\displaystyle\int_{-\infty}^{\infty} 5\mathrm{e}^{t^2} \cos(t)\delta(t-3)\, \mathrm{d}t$

2. **Programming** [15]
   (a) Choose any two representations from your answer to question 1(b). Write the python program to verify that these two sequences of functions behave like a Dirac Delta function $\delta(x-a)$ in the limit $\epsilon \to 0$ for two different values of a. For this
      i. Plot the above functions for $\epsilon = \epsilon_0/2^n$ with $\epsilon_0 = 0.4$ and $n = 1, 2, \ldots, 5$ and discuss the behaviour.
      ii. Evaluate
      $$\begin{aligned} \text{(a)} \quad I_{1\,\epsilon} &= \int_{-\infty}^{\infty} f_\epsilon(x)\, \mathrm{d}x \\ \text{(b)} \quad I_{2\,\epsilon} &= \int_{-\infty}^{\infty} f_\epsilon(x)\,(x+1)^2\, \mathrm{d}x \\ \text{(b)} \quad I_{3\,\epsilon} &= \int_{-\infty}^{\infty} 9x^2 f_\epsilon(3x+1)\, \mathrm{d}x \end{aligned}$$
      for above values of epsilon using
      - Simpson$_{1/3}$ or Gauss Legendre quadrature
      - Gauss Hermite quadrature

      methods using the functions in your module.
   (b) Your program should also print the values of integrals for different ϵ in a tabulated format.

3. **Discussion** [5]
   Interpret and discuss your results and graphs.