# COUPLED OSCILLATOR

**Shri Guru Tegh Bahadur Khalsa College, University of Delhi**

**Lab Report For EXPERIMENT NO. 2**

**Team No. 26**

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**B.Sc.(H) Physics Sem II**

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**Submitted to : Dr. Shaan Ameer**

# 1 Objective

1. To excite the given circuit in its possible normal modes and measure the corresponding frequency of oscillations using a virtual electrical timed-circuit simulator
2. Obtain the normal coordinates of the given circuit.
3. Draw schematic of an analogous mechanical coupled system which has one-to-one correspondence with the given circuit.

# 2 Methodology

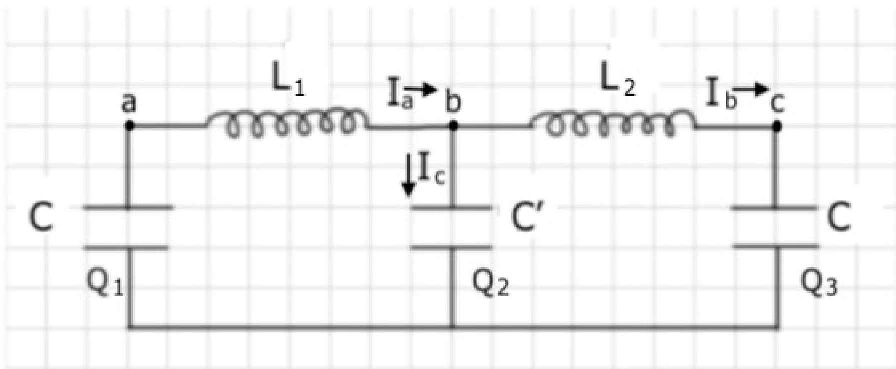
## 2.1 Apparatus

A 2 channel Oscilloscope, Breadboard and connecting wires, two 1300mF Capacitors and one 100mF Capacitor, 1300mH inductors, Battery for charging capacitors.

## 2.2 Procedure

1. The circuit was arranged on the breadboard as shown in the circuit diagram given below.
2. For **mode 1**, charge the capacitors to the same extent by connecting it to a voltage source(like a battery) and connect them so that the charge on the plates connecting the two is of opposite polarity.Positive plate of one should be connected to the negative of other. See how the current flowing through the two inductors varies with time using the using Oscilloscope scopes.
3. For mode 2, charge the capacitors to the same extent by connecting it to a voltage source(like a battery) and connect them so that the charge on the plates connecting the two is of same polarity.Positive plate of one should be connected to the positive of other.

# 3 Theory




Applying KVL in two inner loops,

$$L\frac{dI_a}{dt} = C^{-1}Q_1 - C'^{-1}Q_2 - RI_a \tag{1}$$

$$L\frac{dI_b}{dt} = C'^{-1}Q_1 - C^{-1}Q_3 - RI_b \tag{2}$$

Now, applying KCL at node b,

$$I_a = I_b + I_c \tag{3}$$

In our case setting internal resistance R of inductors zero

$$\frac{dQ_1}{dt} = -I_a, \frac{dQ_b}{dt} = -I_a - I_b, \frac{dQ_3}{dt} = -I_b$$

Differentiating (1) and (2) with respect to time,

$$L\frac{d^2 I_a}{dt^2} = -C^{-1} I_a - C'^{-1}(I_a - I_b) \tag{4}$$

$$L\frac{d^2 I_b}{dt^2} = -C'^{-1}(I_a - I_b) - C^{-1} I_b \tag{5}$$

Adding (4) and (5),

$$L\frac{d^2(I_a + I_b)}{dt^2} = -C^{-1}(I_a + I_b)$$

$$\implies I_a + I_b = A_s \cos(\omega_1 t + \phi_1),\ \text{where } \boxed{\omega_1^2 = \frac{C^{-1}}{L}} \tag{6}$$

Subtracting (5) from (4),

$$L\frac{d^2(I_a - I_b)}{dt^2} = -(2C'^{-1} + C^{-1})(I_a - I_b)$$

$$I_a - I_b = A_f \cos(\omega_2 t + \phi_2),\ \text{where } \boxed{\omega_2^2 = \frac{(2C'^{-1} + C^{-1})}{L}} \tag{7}$$

Solving (6) and (7) for $I_a and I_b$,

$$I_a = \frac{A_s}{2}\cos(\omega_1 t + \phi_1) + \frac{A_f}{2}(\omega_2 t + \phi_2)$$

$$I_b = \frac{A_s}{2}\cos(\omega_1 t + \phi_1) - \frac{A_f}{2}(\omega_2 t + \phi_2)$$

$$\implies \begin{pmatrix} I_a \\ I_b \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 \\ 1 \end{pmatrix} A_s \cos(\omega_1 t + \phi_1) + \frac{1}{2}\begin{pmatrix} 1 \\ -1 \end{pmatrix} A_f \cos(\omega_2 t + \phi_2)$$

**Mode 1 :**

$$\implies \begin{pmatrix} I_a \\ I_b \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 \\ 1 \end{pmatrix} A_s \cos(\omega_1 t + \phi_1) \tag{8}$$

**Mode 2 :**

$$\implies \begin{pmatrix} I_a \\ I_b \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 \\ -1 \end{pmatrix} A_f \cos(\omega_2 t + \phi_2) \tag{9}$$

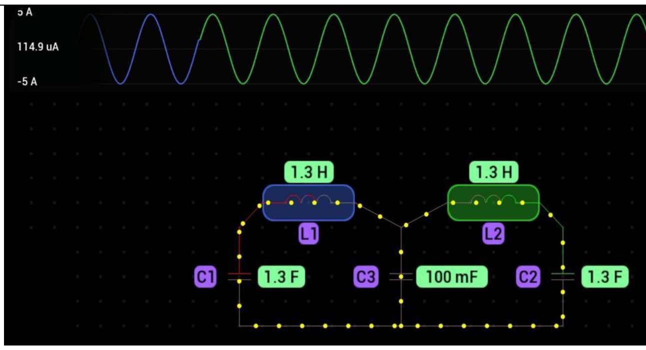
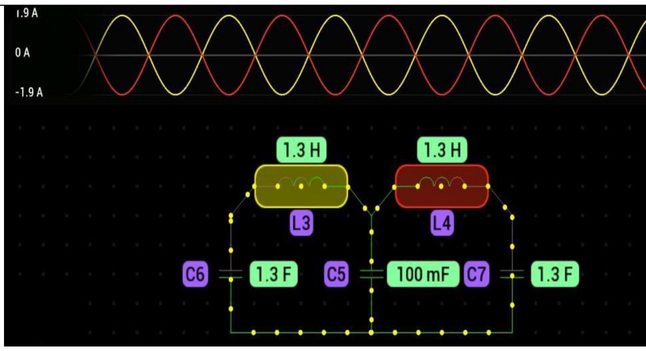
substituting the following values of
C = 1300mF
C' = 100mF
L = 1300mH

Analogous quantities of the circuit and the mechanical system :-

Charge ∼ Displacement of masses
Current ∼ velocity

## 4 Data

For the given circuit, with no internal resistance in circuit elements, using [1]

| Frequency of Normal modes | Normal Coordinates | Current vs Time |
|---|---|---|
| 0.77 Rad. $sec^{-1}$ | $\mathbf{I_a - I_b}$ |   |
| 4.00 Rad. $sec^{-1}$ | $\mathbf{I_a + I_b}$ |   |

## 5 Discussion of Result

When the oppositely charged plates of capacitor are connected we get the normal mode 1 and in the latter case we get mode 2 which agrees with experimental observations and is thus verified.

Normal coordinate associated with any mode is that which has the same frequency of oscillation as that particular mode and the other mode vanishes to zero; there is no direct physical correlation between the Normal co-ordinates and modes, they both are just peculiarities of the coupled system.

World without damping is difficult to imagine and the one with damping is difficult to solve mathematically.

## 6 Sources of Error

1. Experiment should be carried out in a low noise environment for appropriate functioning of the oscilloscope, noise can result in a distorted waveform.
2. **Ensure zero offset** When the two Horizontal and vertical plates of the CRO are not connected to the circuit the dot on the screen should align with the intersection of the vertical and horizontal lines. If it does not, zero errors will be present in measurements.

# 7 Contribution of each Partner

**Partner A**

Name : Shashvat Jain
Roll no. : 2020PHY1114
Contributions :

1. Theory
2. Observations
3. Calculations and Sources of error.

**Partner B**

Name : Prabhujyoti Singh
Roll no. : 2020PHY1210
Contributions :

1. Theory
2. Table formulation

## References

1. *PROTO circuit-simulator* `https://protosimulator.com/`.