

# Finite Difference Method For Poisson Equation

Akarsh Shukla , Brahmanand Mishra and Shashvat Jain

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## Plam

- We will start the talk by **introducing** our topic.
- After introdudcing the topic, we would like to exaplain our **question**.
- Then we will proceed to describe the various **methods** we have used in the project.
- After discussion on methods, we would like to discuss our **results**.
- We will conclude the talk with **conclusion** of results and by sharing our **experience** in completing this project.

# Introduction

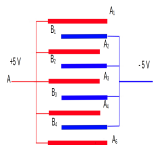
- Poisson Equation is an **elliptic** partial differential Equation.having general form as -:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(r)$$

- Differential Equation can be solved by variety of methods such as spectral methods, finite element methods, FDI.
- Method of finite differences converts differential Equation into a system of linear equation by approxiamting the derivatives using taylor series.
- This system of linear equation obtained can be solved in two ways to obtain the solution of differential equation-:
  - Implicit Methods
  - Explicit Methods
- Implicit methods include many iterative schemes which can be employed to solve this system of linear equation such as **Gauss Seidel, SOR** and **Jacobi Method**.

# Theory

The below figure shows an interleaved capacitor :-



**Figure:** Diagram depicting the arrangement of plates in an interleaved fashion.

After non-dimensionalising we had following poisson equation:-

$$\frac{\partial^2 U'(x', y')}{\partial x'^2} + \frac{\partial^2 U'(x', y')}{\partial y'^2} = -\frac{\rho'(x', y')s^2}{\epsilon_0 \nu}$$

# Problem

$$\rho'(x', y') = \begin{cases} -2\epsilon_0 \times 10^5 & : \text{if } (x', y') \in B_i \text{ where } i = 1, 2, 3, 4 \\ 2\epsilon_0 \times 10^5 & : \text{if } (x', y') \in A_i \text{ where } i = 2, 3, 4 \\ 0 & : \text{elsewhere} \end{cases} \quad (1)$$

With boundary conditions as -:

$$U(0, y) = +5/\nu \quad U(4, y) = +5/\nu \quad (2)$$

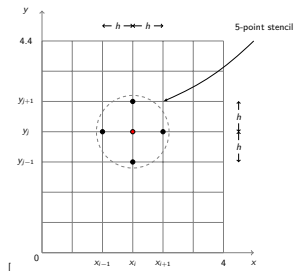
$$U_y(x, 0) = 0 \quad U_y(x, 4.4) = 0 \quad (3)$$

# Methodology

- Finite Difference converts PDE into difference equation.
- Domain is converted into a mesh of equidistant grid points.
- Taylor series is used to approximate the value at these grid points.
- After using Finite Difference operators we get the following stencil for poisson equation

$$U_{i,j} = \frac{1}{4} \left[ U_{i+1,j} + U_{i-1,j} + U_{i,j+1} + U_{i,j-1} + h^2 \frac{\rho'_{i,j} s^2}{\epsilon_0 \nu} \right] \quad (4)$$

This stencil can be represented with the help of following diagram -:



# Iterative Method

- Iterative methods are techniques that exploit the properties of system to solve it implicitly to make computation faster.
- We have used following iteration schemes:-

## Jacobi Method

This method starts with a guess value and with each iteration, it replaces guess values with new obtained values from iteration.

$$x_{i,j}^{(n+1)} = S(X^{(n)}, P, h, i, j)$$

## Gauss Seidel Method

This method uses the obtained value in the same iteration for other unknowns.

$$x_{i,j}^{(n+1)} = S(X^{(n+1)}, P, h, i, j)$$

## SOR

This method involves a relaxation factor (greater than 1) which is multiplied to values obtained from Gauss Seidel before replacing old values.

$$x_{i,j}^{(n+1)} = x_{i,j}^{(n)} + \omega(S(X^{(n+1)}, P, h, i, j) - x_{i,j}^{(n+1)})$$

# Expectation

We expect the following graph-:

- $\alpha_1$  should decrease from positive to negative potential
- $\alpha_2$  should have negatively charged potential
- $\alpha_3$  should increase from negative to positive potential
- $\alpha_4$  should have positively charged potential



# Numerical Result

We obtained following graphs as solution -:

# Conclusion

## Result

- After computing the problem of interleaved capacitor from three different iterative schemes, we have arrived at the conclusion that SOR is the fastest among three.

## Experiences

- We have gained more intuition and it has helped us to grasp the concept of electrostatic potential.
- We have learnt a lot of new techniques and methods related to computational solution of differential equation.
- There are some techniques and methods which we could not implement but surely they have given us some things to ponder upon in future.
- This project has also introduced us to the delight of finding out new things as most of the things we did in this project were completely new to us.

# References

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