

# Funcons-beta: Integers

The P<sub>L</sub>anCompS Project

`Funcons-beta/Values/Primitive/Integers/Integers.cbs*`

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\*Suggestions for improvement: [plancomps@gmail.com](mailto:plancomps@gmail.com).  
Issues: <https://github.com/plancomps/CBS-beta/issues>.

## Integers

```
[ Type integers
  Alias ints
  Type integers-from
  Alias from
  Type integers-up-to
  Alias up-to
  Type bounded-integers
  Alias bounded-ints
  Type positive-integers
  Alias pos-ints
  Type negative-integers
  Alias neg-ints
  Type natural-numbers
  Alias nats
Funcon natural-successor
  Alias nat-succ
Funcon natural-predecessor
  Alias nat-pred
Funcon integer-add
  Alias int-add
Funcon integer-subtract
  Alias int-sub
Funcon integer-multiply
  Alias int-mul
Funcon integer-divide
  Alias int-div
Funcon integer-modulo
  Alias int-mod
Funcon integer-power
  Alias int-pow
Funcon integer-absolute-value
  Alias int-abs
Funcon integer-negate
  Alias int-neg
Funcon integer-is-less
  Alias is-less
Funcon integer-is-less-or-equal 2
  Alias is-less-or-equal
Funcon integer-is-greater
  Alias is-greater
Funcon integer-is-greater-or-equal
  Alias is-greater-or-equal
Funcon binary-natural
```

*Built-in Type* `integers`

*Alias* `ints = integers`

`integers` is the type of unbounded integers. Decimal notation is used to express particular integer values.

### Subtypes of integers

*Built-in Type* `integers-from`(`_ : integers`) `<: integers`

*Alias* `from = integers-from`

`integers-from`(`M`) is the subtype of integers greater than or equal to `M`.

*Built-in Type* `integers-up-to`(`_ : integers`) `<: integers`

*Alias* `up-to = integers-up-to`

`integers-up-to`(`N`) is the subtype of integers less than or equal to `N`.

*Type* `bounded-integers`(`M : integers`, `N : integers`) `↔ integers-from`(`M`) `integers-up-to`(`N`)

*Alias* `bounded-ints = bounded-integers`

`bounded-integers`(`M`, `N`) is the subtype of integers from `M` to `N`, inclusive.

*Type* `positive-integers` `↔ integers-from`(`1`)

*Alias* `pos-ints = positive-integers`

*Type* `negative-integers` `↔ integers-up-to`(`-1`)

*Alias* `neg-ints = negative-integers`

### Natural numbers

*Type* `natural-numbers` `↔ integers-from`(`0`)

*Alias* `nats = natural-numbers`

*Built-in Funcon* `natural-successor`(`N : natural-numbers`) `: ⇒ natural-numbers`

*Alias* `nat-succ = natural-successor`

*Built-in Funcon* `natural-predecessor`(`_ : natural-numbers`) `: ⇒ natural-numbers ?`

*Alias* `nat-pred = natural-predecessor`

*Assert* `natural-predecessor`(`0`) `== ( )`

## Arithmetic

*Built-in Funcon* `integer-add(_ : integers *) :  $\Rightarrow$  integers`

*Alias* `int-add = integer-add`

*Built-in Funcon* `integer-subtract(_ : integers, _ : integers) :  $\Rightarrow$  integers`

*Alias* `int-sub = integer-subtract`

*Built-in Funcon* `integer-multiply(_ : integers *) :  $\Rightarrow$  integers`

*Alias* `int-mul = integer-multiply`

*Built-in Funcon* `integer-divide(_ : integers, _ : integers) :  $\Rightarrow$  integers?`

*Alias* `int-div = integer-divide`

*Assert* `integer-divide(_ : integers, 0) == ( )`

*Built-in Funcon* `integer-modulo(_ : integers, _ : integers) :  $\Rightarrow$  integers?`

*Alias* `int-mod = integer-modulo`

*Assert* `integer-modulo(_ : integers, 0) == ( )`

*Built-in Funcon* `integer-power(_ : integers, _ : natural-numbers) :  $\Rightarrow$  integers`

*Alias* `int-pow = integer-power`

*Built-in Funcon* `integer-absolute-value(_ : integers) :  $\Rightarrow$  natural-numbers`

*Alias* `int-abs = integer-absolute-value`

*Funcon* `integer-negate(N : integers) :  $\Rightarrow$  integers`

$\rightsquigarrow$  `integer-subtract(0,  
                          N)`

*Alias* `int-neg = integer-negate`

## Comparison

*Built-in Funcon* `integer-is-less`(`_ : integers`, `_ : integers`) :  $\Rightarrow$  `booleans`

*Alias* `is-less` = `integer-is-less`

*Built-in Funcon* `integer-is-less-or-equal`(`_ : integers`, `_ : integers`) :  $\Rightarrow$  `booleans`

*Alias* `is-less-or-equal` = `integer-is-less-or-equal`

*Built-in Funcon* `integer-is-greater`(`_ : integers`, `_ : integers`) :  $\Rightarrow$  `booleans`

*Alias* `is-greater` = `integer-is-greater`

*Built-in Funcon* `integer-is-greater-or-equal`(`_ : integers`, `_ : integers`) :  $\Rightarrow$  `booleans`

*Alias* `is-greater-or-equal` = `integer-is-greater-or-equal`

## Conversion

*Built-in Funcon* `binary-natural`(`_ : strings`) :  $\Rightarrow$  `natural-numbers?`

*Alias* `binary` = `binary-natural`

*Built-in Funcon* `octal-natural`(`_ : strings`) :  $\Rightarrow$  `natural-numbers?`

*Alias* `octal` = `octal-natural`

*Built-in Funcon* `decimal-natural`(`_ : strings`) :  $\Rightarrow$  `natural-numbers?`

*Alias* `decimal` = `decimal-natural`

Literal natural numbers `N` are equivalent to `decimal-natural "N"`.

*Built-in Funcon* `hexadecimal-natural`(`_ : strings`) :  $\Rightarrow$  `natural-numbers?`

*Alias* `hexadecimal` = `hexadecimal-natural`

*Funcon* `integer-sequence`(`_ : integers`, `_ : integers`) :  $\Rightarrow$  `integers*`

`integer-sequence`(`M`, `N`) is the sequence of integers from `M` to `N`, except that if `M` is greater than `N`, it is the empty sequence.

Rule 
$$\frac{\text{is-greater}(M, N) == \text{false}}{\text{integer-sequence}(M : \text{integers}, N : \text{integers}) \rightsquigarrow (M, \text{integer-sequence}(\text{integer-add}(M, 1), N))}$$

Rule 
$$\frac{\text{is-greater}(M, N) == \text{true}}{\text{integer-sequence}(M : \text{integers}, N : \text{integers}) \rightsquigarrow ()}$$