Funcons-beta: Trees

The PLanCompS Project

Funcons-beta/Values/Composite/Trees/Trees.cbs*

Trees

```
Datatype trees
         Funcon tree
         Funcon tree-root-value
         Funcon tree-branch-sequence
         Funcon single-branching-sequence
         Funcon forest-root-value-sequence
         Funcon forest-branch-sequence
         Funcon forest-value-sequence
      Meta-variables T <: values
      Datatype trees(T) ::= tree(_{-}: T,_{-}: (trees(T))*)
trees(T) consists of finitely-branching trees with elements of type T. When
V: T, tree(V) is a leaf, and tree(V, B_1, \dots, B_n) is a tree with branches B_1,
\ldots, B_n.
      Funcon tree-root-value(\_: trees(T)): \Rightarrow(T)?
         Rule tree-root-value tree(V: T, _**: (trees(T))^*) \leadsto V
      Funcon tree-branch-sequence(\_: trees(T)): \Rightarrow (trees(T))^*
         Rule tree-branch-sequence tree(_{-}:T,B^*:(trees(T))^*) \rightsquigarrow B^*
```

^{*}Suggestions for improvement: plancomps@gmail.com. Issues: https://github.com/plancomps/CBS-beta/issues.

```
Funcon single-branching-sequence(_{-}: trees(T)): \Rightarrow T^{+}
```

single-branching-sequence B extracts the values in B starting from the root, provided that B is at most single-branching; otherwise it fails.

```
Rule single-branching-sequence \operatorname{tree}(V:T)\leadsto V
Rule single-branching-sequence \operatorname{tree}(V:T,B:\operatorname{trees}(T))\leadsto\operatorname{left-to-right}(V,\operatorname{single-branching-sequence}\ B)
Rule single-branching-sequence \operatorname{tree}(_-:T,_-:\operatorname{trees}(T),_-^+:(\operatorname{trees}(T))^+)\leadsto\operatorname{fail}
```

A sequence of trees corresponds to a forest, and the selector funcons on trees B extend to forests B^* :

```
Funcon forest-root-value-sequence(_{-}: (trees(T))*): \Rightarrow T^*

Rule forest-root-value-sequence(B: trees(T), B^*: (trees(T))*) \rightsquigarrow (tree-root-value B, forest-root-value-sequence() \rightsquigarrow ()

Funcon forest-branch-sequence(_{-}: (trees(T))*): \Rightarrow T^*

Rule forest-branch-sequence(B: trees(T), B^*: (trees(T))*) \rightsquigarrow (tree-branch-sequence B, forest-branch-sequence B forest-br
```

Funcon forest-value-sequence($_{-}$: (trees(T))*): $\Rightarrow T$ *

forest-value-sequence B^* provides the values from a left-to-right pre-order depth-first traversal.

```
Rule forest-value-sequence(tree(V:T,B_1^*:(\operatorname{trees}(T))^*), B_2^*:(\operatorname{trees}(T))^*) \leadsto (V, forest-value-sequence B Rule forest-value-sequence() \leadsto ()
```

Other linearizations of trees can be added: breadth-first, right-to-left, C3, etc.