

Funcons-beta: Integers *

The P_LanCompS Project

`Integers.cbs` | PLAIN | PRETTY

OUTLINE

Integers

- Subtypes of integers
- Natural numbers
- Arithmetic
- Comparison
- Conversion

*Suggestions for improvement: plancomps@gmail.com.
Reports of issues: <https://github.com/plancomps/CBS-beta/issues>.

Integers

| | | |
|--------|-------|-----------------------------|
| [| Type | integers |
| | Alias | ints |
| | Type | integers-from |
| | Alias | from |
| | Type | integers-up-to |
| | Alias | up-to |
| | Type | bounded-integers |
| | Alias | bounded-ints |
| | Type | positive-integers |
| | Alias | pos-ints |
| | Type | negative-integers |
| | Alias | neg-ints |
| | Type | natural-numbers |
| | Alias | nats |
| Funcon | | natural-successor |
| | Alias | nat-succ |
| Funcon | | natural-predecessor |
| | Alias | nat-pred |
| Funcon | | integer-add |
| | Alias | int-add |
| Funcon | | integer-subtract |
| | Alias | int-sub |
| Funcon | | integer-multiply |
| | Alias | int-mul |
| Funcon | | integer-divide |
| | Alias | int-div |
| Funcon | | integer-modulo |
| | Alias | int-mod |
| Funcon | | integer-power |
| | Alias | int-pow |
| Funcon | | integer-absolute-value |
| | Alias | int-abs |
| Funcon | | integer-negate |
| | Alias | int-neg |
| Funcon | | integer-is-less |
| | Alias | is-less |
| Funcon | | integer-is-less-or-equal |
| | Alias | is-less-or-equal |
| Funcon | | integer-is-greater |
| | Alias | is-greater |
| Funcon | | integer-is-greater-or-equal |
| | Alias | is-greater-or-equal |
| Funcon | | binary-natural |
| | Alias | binary |
| Funcon | | octal-natural |
| | Alias | octal |
| Funcon | | decimal-natural |
| | Alias | decimal |
| Funcon | | hexadecimal-natural |
| | Alias | hexadecimal |
| Funcon | | integer-sequence] |

Built-in Type `integers`

Alias `ints = integers`

`integers` is the type of unbounded integers. Decimal notation is used to express particular integer values.

Subtypes of integers

Built-in Type `integers-from(_ : integers) <: integers`

Alias `from = integers-from`

`integers-from(M)` is the subtype of integers greater than or equal to *M*.

Built-in Type `integers-up-to(_ : integers) <: integers`

Alias `up-to = integers-up-to`

`integers-up-to(N)` is the subtype of integers less than or equal to *N*.

Type `bounded-integers(M : integers, N : integers)`
 \rightsquigarrow `integers-from(M) & integers-up-to(N)`

Alias `bounded-ints = bounded-integers`

`bounded-integers(M, N)` is the subtype of integers from *M* to *N*, inclusive.

Type `positive-integers \rightsquigarrow integers-from(1)`

Alias `pos-ints = positive-integers`

Type `negative-integers \rightsquigarrow integers-up-to(-1)`

Alias `neg-ints = negative-integers`

Natural numbers

Type `natural-numbers \rightsquigarrow integers-from(0)`

Alias `nats = natural-numbers`

Built-in Funcon `natural-successor(N : natural-numbers) : \Rightarrow natural-numbers`

Alias `nat-succ = natural-successor`

Built-in Funcon `natural-predecessor(_ : natural-numbers) : \Rightarrow natural-numbers?`

Alias `nat-pred = natural-predecessor`

Assert `natural-predecessor(0) == ()`

Arithmetic

Built-in Funcon `integer-add(_ : integers*) : \Rightarrow integers`

Alias `int-add = integer-add`

Built-in Funcon `integer-subtract(_ : integers, _ : integers) : \Rightarrow integers`

Alias `int-sub = integer-subtract`

Built-in Funcon integer-multiply($_ : \text{integers}^*$) : $\Rightarrow \text{integers}$

Alias int-mul = integer-multiply

Built-in Funcon integer-divide($_ : \text{integers}, _ : \text{integers}$) : $\Rightarrow \text{integers}^?$

Alias int-div = integer-divide

Assert integer-divide($_ : \text{integers}, 0$) == ()

Built-in Funcon integer-modulo($_ : \text{integers}, _ : \text{integers}$) : $\Rightarrow \text{integers}^?$

Alias int-mod = integer-modulo

Assert integer-modulo($_ : \text{integers}, 0$) == ()

Built-in Funcon integer-power($_ : \text{integers}, _ : \text{natural-numbers}$) : $\Rightarrow \text{integers}$

Alias int-pow = integer-power

Built-in Funcon integer-absolute-value($_ : \text{integers}$) : $\Rightarrow \text{natural-numbers}$

Alias int-abs = integer-absolute-value

Funcon integer-negate($N : \text{integers}$) : $\Rightarrow \text{integers}$

\rightsquigarrow integer-subtract(0, N)

Alias int-neg = integer-negate

Comparison

Built-in Funcon integer-is-less($_ : \text{integers}, _ : \text{integers}$) : $\Rightarrow \text{booleans}$

Alias is-less = integer-is-less

Built-in Funcon integer-is-less-or-equal($_ : \text{integers}, _ : \text{integers}$) : $\Rightarrow \text{booleans}$

Alias is-less-or-equal = integer-is-less-or-equal

Built-in Funcon integer-is-greater($_ : \text{integers}, _ : \text{integers}$) : $\Rightarrow \text{booleans}$

Alias is-greater = integer-is-greater

Built-in Funcon integer-is-greater-or-equal($_ : \text{integers}, _ : \text{integers}$) : $\Rightarrow \text{booleans}$

Alias is-greater-or-equal = integer-is-greater-or-equal

Conversion

Built-in Funcon binary-natural($_ : \text{strings}$) : $\Rightarrow \text{natural-numbers}^?$

Alias binary = binary-natural

Built-in Funcon octal-natural($_ : \text{strings}$) : $\Rightarrow \text{natural-numbers}^?$

Alias octal = octal-natural

Built-in Funcon decimal-natural($_ : \text{strings}$) : $\Rightarrow \text{natural-numbers}^?$

Alias decimal = decimal-natural

Literal natural numbers N are equivalent to **decimal-natural** “ N ”.

Built-in Funcon **hexadecimal-natural**($_$: strings) : \Rightarrow natural-numbers?

Alias **hexadecimal** = hexadecimal-natural

Funcon **integer-sequence**($_$: integers, $_$: integers) : \Rightarrow integers*

integer-sequence(M, N) is the sequence of integers from M to N , except that if M is greater than N , it is the empty sequence.

Rule
$$\frac{\text{is-greater}(M, N) == \text{false}}{\text{integer-sequence}(M : \text{integers}, N : \text{integers}) \rightsquigarrow (M, \text{integer-sequence}(\text{integer-add}(M, 1), N))}$$

Rule
$$\frac{\text{is-greater}(M, N) == \text{true}}{\text{integer-sequence}(M : \text{integers}, N : \text{integers}) \rightsquigarrow ()}$$