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**Mathematics: analysis and approaches**  
**Higher level**  
**Paper 1**

Thursday 6 May 2021 (afternoon)

Candidate session number

2 hours

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**Instructions to candidates**

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.



5. [Maximum mark: 8]

(a) Show that  $\sin 2x + \cos 2x - 1 = 2 \sin x (\cos x - \sin x)$ . [2]

(b) Hence or otherwise, solve  $\sin 2x + \cos 2x - 1 + \cos x - \sin x = 0$  for  $0 < x < 2\pi$ . [6]

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Do **not** write solutions on this page.

## Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 16]

A biased four-sided die, A, is rolled. Let  $X$  be the score obtained when die A is rolled. The probability distribution for  $X$  is given in the following table.

$x$	1	2	3	4
$P(X=x)$	$p$	$p$	$p$	$\frac{1}{2}p$

(a) Find the value of  $p$ . [2]

(b) Hence, find the value of  $E(X)$ . [2]

A second biased four-sided die, B, is rolled. Let  $Y$  be the score obtained when die B is rolled. The probability distribution for  $Y$  is given in the following table.

$y$	1	2	3	4
$P(Y=y)$	$q$	$q$	$q$	$r$

(c) (i) State the range of possible values of  $r$ .

(ii) Hence, find the range of possible values of  $q$ . [3]

(d) Hence, find the range of possible values for  $E(Y)$ . [3]

Agnes and Barbara play a game using these dice. Agnes rolls die A once and Barbara rolls die B once. The probability that Agnes' score is less than Barbara's score is  $\frac{1}{2}$ .

(e) Find the value of  $E(Y)$ . [6]

