

CS 477/677 Analysis of Algorithms

Spring 2024

Homework 1

Due date: February 6, 2024

Note: Students in the 400 section can solve the graduate problem or the extra credit problem for additional points. If both problems are attempted, the highest score will be kept (not both).

1. (U & G-required) [20 points] Arrange the following list of functions in ascending order of growth rate. That is, if function $g(n)$ immediately follows function $f(n)$ in your list, then $f(n)$ should be $O(g(n))$.

$$f_1(n) = \frac{n^4 + 5n \lg n + 3n}{n^3 + n} \log n^2$$

$$f_2(n) = \sqrt{6n^6 + n + 200}$$

$$f_3(n) = n^2 (\log n)^2 + n^2$$

$$f_4(n) = 5^{n+2}$$

$$f_5(n) = 10^n$$

$$f_6(n) = n^5 + 1$$

2. (U & G-required) [20 points] Using the formal definition of the asymptotic notations, prove the following statements:

a) $2n^2 + 6n \in O(n^3)$

b) $3n^3 + 5n \in \Omega(n^2)$

c) $n^n \in \Omega(n!)$

d) $5n^3 + 2n \notin O(n)$

3. (U & G-required) [30 points]

a) If it is known that the running time for algorithm A is $O(\log n)$ and that the running time for algorithm B is $O(n^2)$, what does this statement imply about the relative performance of the algorithms? Explain your answer.

b) If it is known that the running time for algorithm A is $\theta(\log n)$ and that the running time for algorithm B is $\theta(n^2)$, what does this statement imply about the relative performance of the algorithms? Explain your answer.

c) If it is known that the running time for algorithm A is $\theta(\log n)$ and that the running time for algorithm B is $O(n^2)$, what does this statement imply about the relative performance of the algorithms? Explain your answer.

4. (U & G-required) [30 points] Using mathematical induction, show that the following relations are true for every $n \geq 1$:

a) $\sum_{i=1}^n (3i - 2) = \frac{1}{2}n(3n - 1)$

b) $\sum_{i=1}^n 5^{i-1} = \frac{1}{4}(5^n - 1)$

5. (G-Required) [20 points] For each of the following functions, indicate the class $\Theta(g(n))$ the function belongs to. Use the simplest $g(n)$ possible in your answers.

a) $(2n^4 + 5n + 6)^3$

b) $\frac{\sqrt{2n^6 + 2n + 10}}{n^3 + 1}$

c) $n \lg(n + 1)^4 + (n^3 + 2)^3 \lg n$

d) $\frac{9^n}{3^{n+1}}$

6. [Extra credit - 20 points] Indicate whether the first function of each of the following pairs has a smaller, same or larger order of growth (to within a constant multiple) than the second function:

a) $n^2(n + 15)$ and n^3

b) $\log_2(n + 1)$ and $n \ln n$

c) 2^{n-1} and 2^{n+1}

d) $\log_2^2 n$ and $\log_2(n^4)$

e) $(n + 1)!$ and $(n - 1)!$