CS 477/677 Analysis of Algorithms

Spring 2024

Homework 5

Due date: March 19, 2024

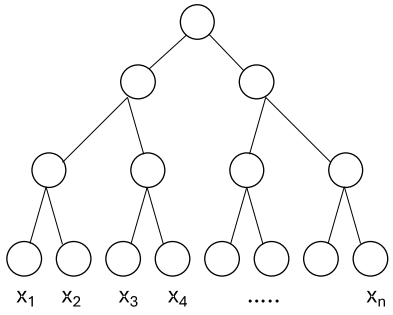
1. (U&G-required) [20 points] Answer the following questions:

(a) [10 points] Illustrate the operation of RADIX_SORT on the following array `A = [29134, 20134, 9134, 134, 34, 4]`. Show the order of the elements after sorting for each digit. **Note:** missing digits should be considered as 0s.

(b) [10 points] Illustrate the operation of COUNTING_SORT on the following array `A = [15, 3, 17, 2, 9, 10, 8]`. Show the counting and output arrays after each iteration.

2. (U&G-required) [20 points]

Write pseudocode for a procedure BUILD_NEW_MAX_HEAP that takes as input an array $A = [x_1, \ldots, x_n]$ of *n* integer numbers and builds a **max-heap** in which the values at the bottom level are, from left to right, the integers $x_1, \ldots x_n$ (as in the figure on right). Indicate the size of the heap that is created. Assume that *n* is a power of 2.



3. (U&G-required) [20 points]

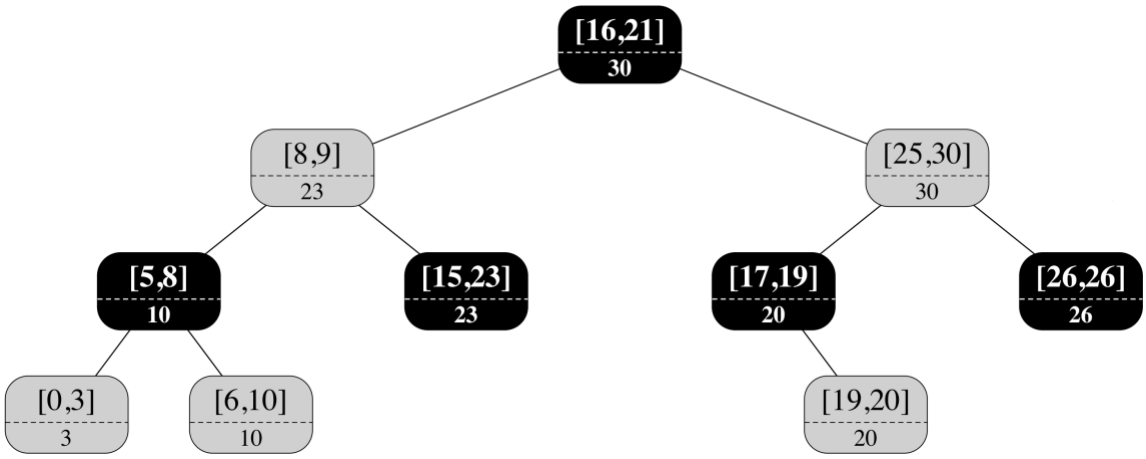
Give a justification to show that the longest simple path from a node x in a red-black tree to a descendant leaf has length at most twice that of the shortest simple path from node x to a descendant leaf.

4. (U & G-required) [20 points]

(a) [10 points] Show how INTERVAL-SEARCH(T, i) operates on the tree T shown in the figure below, with `i = [22, 24]`.

(b) [10 points] Show the tree that results after inserting interval `i = [11, 40]` into the

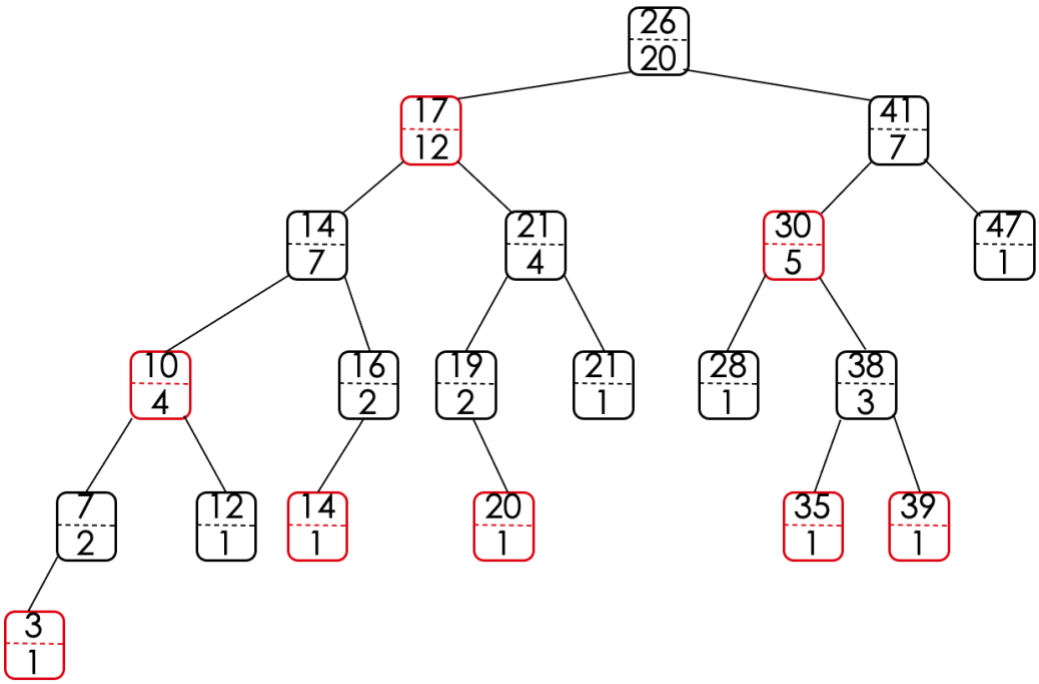
tree T shown in the figure below (black nodes have dark background). Make sure to restore any red-black tree properties that may be affected during the insert.



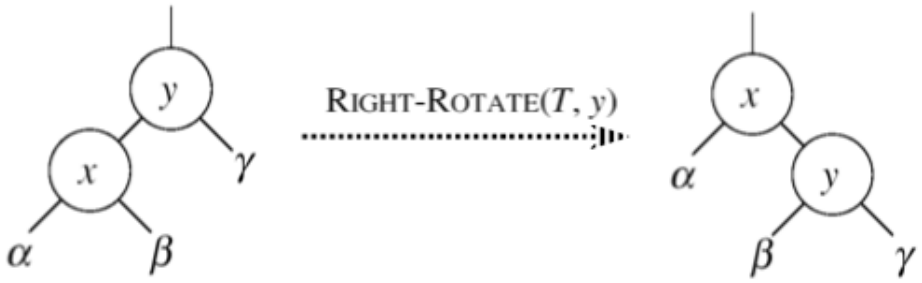
5. (U & G-required) [20 points]

(a) [10 points] Show how OS-SELECT (*T.root*, 16) operates on the red-black tree shown in the figure below.

(b) [10 points] Show how OS-RANK(*T*, *x*) operates on the red-black tree shown in the figure below and the node x with *x.key* = 20.



6. (G-required) [20 points] Let *a*, *b* and *c*, be arbitrary nodes in subtrees α, β, and γ in the left figure below. Indicate how the **depths** of *a*, *b* and *c* change after this transformation.



Extra credit:

7. [20 points]

a) [10 points] Illustrate the operation of `MAX-HEAPIFY(A, 3)` on the array `A = [27, 17, 3, 16, 13, 10, 1, 5, 7, 12, 4, 8, 9, 0]`.

b) [10 points] Illustrate the operation of `BUCKET-SORT` on the array `A = [.73, .15, .11, .67, .32, .24, .83, .53, .70, .45]`.