Extracting individual spectra of spectroscopic binaries using the Fourier transform

Jean-Baptiste Delisle (jean-baptiste.delisle@unige.ch)

November 23, 2021

1 Context

A spectroscopic binary is a binary star whose global spectrum shows significant contributions from both components (primary and secondary stars). The two components are in most cases too close to each other to be resolved with our telescopes, so the individual spectrum of each star cannot be obtained directly. However, due to the orbital motion of the binary, the two spectra are shifted (Doppler effect) periodically with opposite phases. When the primary is moving toward us (blue-shifted spectrum) the secondary is moving away from us (red-shifted spectrum).

In this exercise we will use this effect to extract the spectrum of each component from a series of ”global” spectra taken at different orbital phases.

We will concentrate on a small region of the spectra around the sodium D-lines doublet (around 589.5 nm).

2 Data loading and visualization

We have 34 HARPS spectra taken at different epochs. The orbital motion of the system has already been solved, so the velocities of both stars at the epoch of each spectrum are known. All the data are available in the sb2.npz file (which you can read using np.load):

- *vA_m_s* is the velocity time series of the primary star,
- *vB_m_s* is the velocity time series of the secondary star,
- *la_nm* is the wavelength at the center of each pixel in the spectra,
- *spectra* is the 2D array of spectra, indexed by the time (first axis) and wavelength (second axis). For the sake of simplicity, we assume equal errors for all the data (each pixel in each spectrum).

To do:

(a) Load the data in sb2.npz.

(b) Plot the first spectrum.

(c) Plot this spectrum restricted to the wavelength range $[587, 592]$ nm.

3 Doppler shift of the spectra

Let us recall the formula for the relativistic Doppler effect. Considering a source with radial velocity v with respect to the observer, the light emitted by the source at a wavelength λ_s is received with a wavelength

$$\lambda_r = \lambda_s \sqrt{\frac{1+\beta}{1-\beta}}. \tag{1}$$

with $\beta = v/c$. The shift in wavelength is thus:

$$\Delta\lambda = \lambda_r - \lambda_s = \lambda_s \left(\sqrt{\frac{1+\beta}{1-\beta}} - 1 \right). \tag{2}$$

We note that this shift $\Delta\lambda$ depends on the considered wavelength λ_s. In order to simplify the computations, we will assume in the following that the considered range of wavelength is sufficiently small such that the shift is approximately constant. Since we are interested in the region around the sodium D-lines doublet, we approximate it with:

$$\Delta\lambda = \lambda_{\text{ref.}} \left(\sqrt{\frac{1+\beta}{1+\beta}} - 1 \right) \tag{3}$$

with $\lambda_{\text{ref.}} = 589.5$ nm.

Let $F_A(\lambda)$ and $F_B(\lambda)$ be the spectra of the stars at rest, and $F(\lambda)$ one of our 34 measured spectra.

(a) Give the expression of F as a function of F_A, F_B, $\Delta\lambda_A$, and $\Delta\lambda_B$.

(b) Compute the values of $\Delta\lambda_A$ and $\Delta\lambda_B$ for each spectrum.

4 Doppler shifts in Fourier space

Extracting the contributions of F_A and F_B is not easily done directly in the wavelength space. However, the problem simplifies much by switching to Fourier space.

(a) For a given spectrum $F(\lambda)$, give the expression of its Fourier transform $\hat{F}(\nu)$ as a function of the Fourier transforms of both components ($\hat{F}_A$ and $\hat{F}_B$).

(b) Since we have 34 spectra taken at different epochs, we have 34 different Fourier transforms $\hat{F}_k(\nu)$ corresponding to different shifts $\Delta\lambda_{A,k}$, $\Delta\lambda_{B,k}$. We now want to solve for the values of $\hat{F}_A(\nu)$ and $\hat{F}_B(\nu)$. Show that $\hat{F}_A(\nu)$ and $\hat{F}_B(\nu)$ can be solved for, at each frequency ν independently.

(c) Show that for a given frequency ν, solving for $\hat{F}_A(\nu)$ and $\hat{F}_B(\nu)$ requires to solve a linear problem of the form $y = A\theta + \epsilon$. What does play the role of y, A, θ, and ϵ here?

5 Side effects and apodization

To avoid side effects which can be strong if the values at both edges of the window are very different (this introduces a step when folding), it is a good idea to apodize the spectra before applying the Fourier transform.

We thus subtract a constant value to the spectra such that the level is close to 0 at both edges, and then we multiply by a window function. The window function w is typically equal to 1 at the center of the window and 0 on the edges. Here, we use the Hann window:

$$w(k) = \sin^2(\pi k/n), \tag{4}$$

where k is the index of the wavelength λ, and n the total number of pixels. The apodized spectrum is thus

$$F_{\text{apo}}(\lambda_k) = w(k)(F(\lambda_k) - C). \tag{5}$$

We use $C = 54440$ in the following.

(a) Compute the 2D array of apodized spectra.

(b) Compute the 2D array of their Fourier transforms (using np.fft).

(c) Loop over the frequency $\nu \neq 0$ and solve for $\hat{F}_A(\nu)$ and $\hat{F}_B(\nu)$.

(d) What happens when $\nu = 0$? Comment on the reasons and consequences.

(e) Compute the individual apodized spectra of the two stars from their Fourier transforms.

(f) Finally correct the individual spectra from the apodization factor. Comment your results.